MINING ENGINEERS' HANDBOOK

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SECTION 15

COMPRESSED AIR PRACTICE

FIRST AND SECOND EDITIONS

BY

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

COMPRESSED AIR PRACTICE

1. ADVANTAGES AND USES OF COMPRESSED AIR POWER

Advantages for mining service: (*a*) air drills are of rugged construction, and (*b*) less susceptible to damage by moisture than electric drills; (*c*) air lines are readily maintained and extended; (*d*) leaks easily detected and repaired; (*e*) air power is safe in gaseous mines; (*f*) exhaust air aids ventilation.

Uses of compressed air in mining, quarrying and smelting (2, 21). Compressed air is widely applied in mining and allied fields, for operating drills and hoists; pneumatic tools, as grinders, drills, riveters, chippers, pneumatic diggers and spades; air-driven sump pumps; direct-acting and air-lift pumps; pile drivers for shaft sheathing; drill-steel sharpeners: air pistons for unloading cars; air motors; compressed-air locomotives; oil furnaces for heating steel; shank- and detachable-bit grinders; cement and paint sprays; stone channellers; mine ventilation; supplying air for blowing converters; agitating cyanide tanks; transferring metallurgical solutions; starting Diesel engines; coal preparation.

2. TERMINOLOGY AND DEFINITIONS (1, 2, 21)

Compressor capacity is usually expressed in vol of FREE AIR, *i e*, air under the press and temp conditions at the compressor intake, the normal atmos conditions of the locality.

Reciprocating compressors. In these, the compressing element is a piston moving in a cylinder.

Positive displacement blowers have two inter-meshing gear-type elements, rotating in opposite directions within a stationary casing. Rotary blowers are positive displacement units which compress by eccentrically-mounted vanes, with a rotary seal separating intake and discharge openings. Liquid-seal blowers are a variation of rotary blower, in which a vaned impeller operates in an elliptical casing, the seal between intake and discharge being water, which is thrown outward during operation, following the form of the casing and acting as a liquid piston. All the above are of the positive-displacement type.

Turbo or centrifugal compressors. Compression is effected by centrifugal action of bladed impellers rotating at high velocities.

Hydraulic compressors are those in which water descending in a vert pipe compresses the entrained air

Single-stage compressors. Compression from initial to final pressure is completed in one step or stage. In reciprocating machines compression is completed in a single stroke of the piston.

Multi-stage compressors. Compression from initial to final press is completed in two or more distinct steps or stages. In a 2-stage reciprocating compressor compression is carried to an intermediate pressure in one cylinder, and the air then passes through an intercooler to another cylinder in which compression reaches the final pressure.

Compressor drives are classified as steam- or power-driven. Also as DIRECT-CONNECTED, in which the driver is an integral part of the machine, as gas or Diesel-engine-driven units, where the power cylinders and air pistons act on the same crankshaft; DIRECT-STEAM-DRIVEN, in which the steam and air pistons are on the same rod and act on the same crankshaft; DIRECT-CONNECTED ELECTRIC, in which the motor is mounted on compressor crankshaft; WATER-WHEEL-DRIVEN, in which the water wheel is mounted on the compressor crankshaft; DIRECT COUPLED, in which the driver is connected to the compressor crankshaft through a flexible coupling, clutch, or gears; BELT DRIVE, in which the driver is connected to the compressor by a flat belt, multiple V-type belt, or rope; CHAIN DRIVE, in which the driver is connected to the compressor by a chain and sprocketed pulleys.

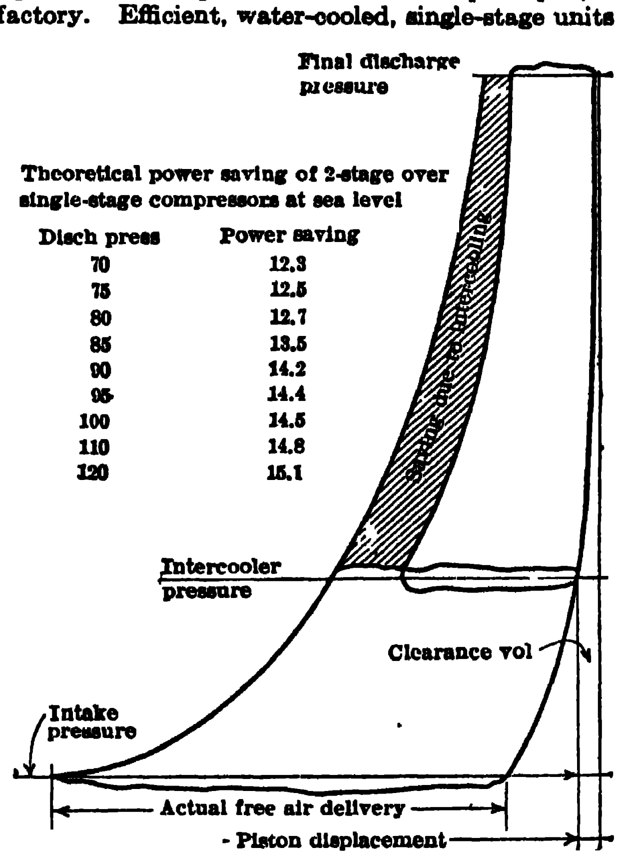
Classification of reciprocating compressors (2, 21). VERTICAL COMPRESSORS have the compressing element in a vert plane. A variation is the V-type, with two or more compressing cylinders oppositely inclined. HORIZONTAL COMPRESSORS have the compressing elements in a horiz plane. ANGLE COMPRESSORS are of the multi-cylinder type, with the cylinder axes at an angle to each other; there are usually one or more horiz cylinders and a corresponding number of vert cylinders. RADIAL COMPRESSORS are those having a series of compressing elements arranged in the same plane around the common crank

from which they are driven. STRAIGHT-LINE COMPRESSORS are usually horiz, of the center-crank type, with a common axis for cylinders and frame. DUPLEX COMPRESSORS have two parallel sets of compressing elements, driven by individual cranks on a common crank-shaft. SINGLE-ACTING COMPRESSORS. Compression takes place on only one stroke per revolution in each compressing element. DOUBLE-ACTING COMPRESSORS. Compression takes place on both strokes per revolution in each compressing element. In WATER-COOLED COMPRESSORS some of the heat of compression is removed by water circulated in jacketed cylinders. The spraying of water directly into the air is now obsolete. In AIR-COOLED COMPRESSORS some of the heat of compression is removed by air circulated over fins cast on the cylinders.

3. OPERATING DATA

Application of single- and multi-stage units. For pressures below 60 lb per sq in, a single-stage compressor is usually satisfactory. Efficient, water-cooled, single-stage units are built for pressures to 125 lb and in sizes to 100 hp. For larger sizes, 2-stage compressors are preferred for the usual 100-lb rating. Except in the smaller sizes, two-stage units are lower in power costs than single-stage, for pressures of 80 to 100 lb, which are normal at most mines and quarries. This is especially true at high altitudes. For starting Diesel engines, air at 250 lb is required; since this is usually the only application for air at this pressure, small two-stage, air-cooled units are used.

Advantages of stage compression: (*a*) less hp required to compress a unit of air (Fig 1); (*b*) lower final temp; (*c*) lower aver press on bearings and greater mech effic; (*d*) increased safety and ease of lubrication. With high final temp, part of lubricant is destroyed, increasing wear on piston and cylinder; (*e*) greater actual air delivery from same piston displacement. The final press in the low-press cyl is much lower than in single-stage work, and clearance air, when expanded to atmos press, occupies less space. Hence, the inflow through suction valves begins earlier in the stroke; (*f*) air delivered by a stage compressor is dryer than from a single cylinder. At constant press, the capacity of air for water vapor decreases with its temp, and in passing through the intercooler much of the original moisture in the air is precipitated; hence, less trouble from condensation in delivery pipe.



Theoretical power saving of 2-stage over single-stage compressors at sea level

Disch press	Power saving
70	12.3
75	12.5
80	12.7
85	13.5
90	14.2
95	14.4
100	14.5
110	14.8
120	15.1

Fig 1

Air compression at altitudes above sea-level (1, 2). The primary effect of altitude is to reduce the hp per unit of vol delivered (Fig 2, 3), permitting larger air cylinders in 2-stage machines. Volumetric effic of 2-stage units is only slightly affected by altitude operation; for single-stage, there is actually a loss in volumetric effic, because of the greater vol of free air trapped in clearance spaces due to the higher compression ratios required for the same discharge pressure. As the reduction in hp per unit of volume at altitudes is smaller for single-stage than for 2-stage units, cylinder sizes in the former are seldom changed. Although compressors deliver more air for the same hp at altitudes, air-operated equipment as rock drills, requires a greater volume of air (Table 2).

Power required for compressors (2). The theoretical hp for single- and 2-stage compressors at sea-level is given in Table 1; for altitudes, see Fig 2, 3. Actual brake hp for both single- and 2-stage compressors is greater than the theoretical, by an amount varying with the press, altitude, and size and type of compressor; for 100 lb press, it is normally 1.2 to 1.3 times the theoretical (Table 2).

Theoretical adiabatic hp required by a single-stage compressor is calculated by:

$$\text{Hp} = 0.0043636\, VP_1 \frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

in which: V = vol of air delivered, in cu ft free air per min; P_1 = abs intake press, lb per sq in; P_2 = abs discharge press, lb per sq in; n = ratio of specific heats. Absolute press equals gage press plus atmos press, both in lb per sq in.

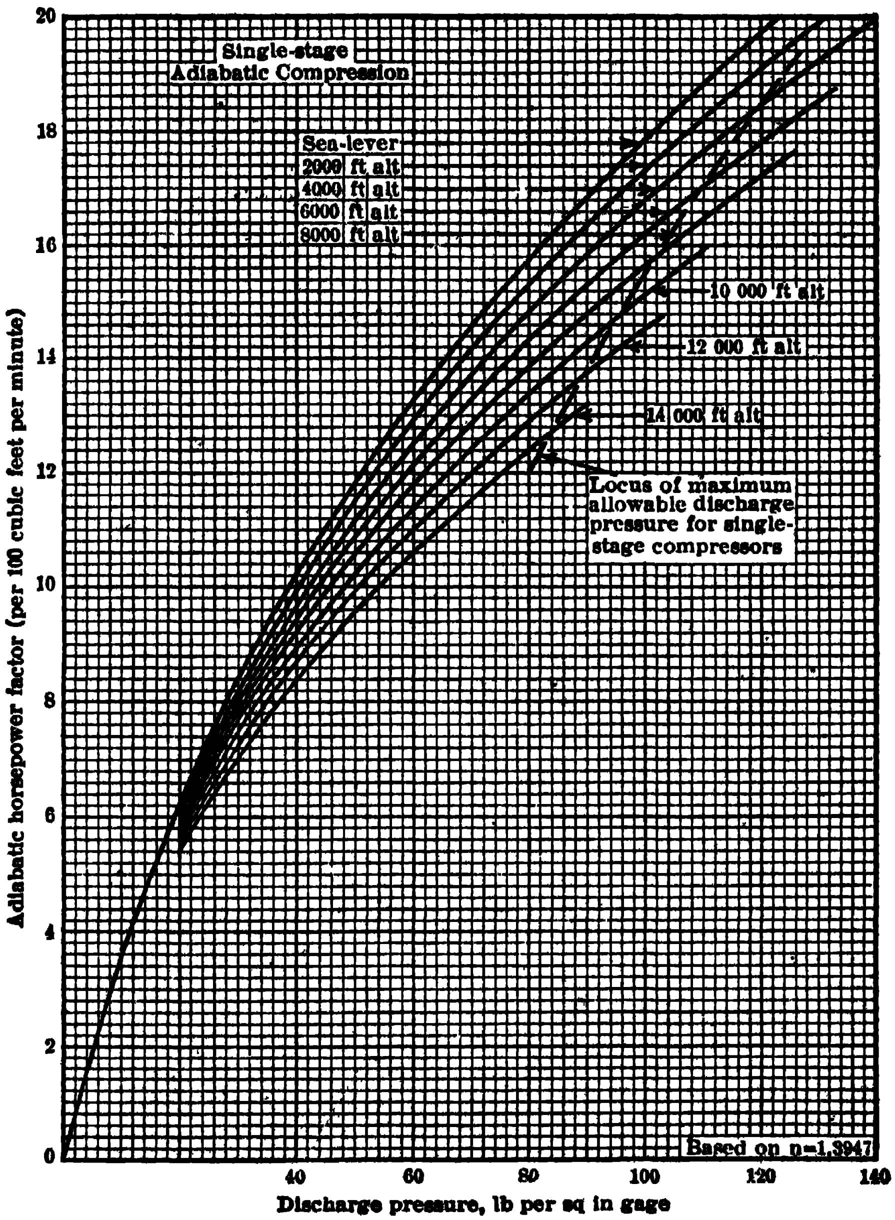


Fig 2. Theoretical Horsepower required by Single-stage Air Compressors (2)

Assuming air containing 0.57% moisture by weight (an aver for a temperate climate), n = 1.3947 and the formula becomes:

$$\text{Hp} = 0.01542\, VP_1\left[\left(\frac{P_2}{P_1}\right)^{0.283} - 1\right]$$

This formula applies to any number of compression stages, by using it for each stage individually with the correct volumes and pressures; and taking the sum of the horse

powers. The formula for the theoretical adiabatic power required by a 2-stage compressor, assuming perfect intercooling and equal division of work between the cylinders, is:

$$\mathrm{Hp} = 0.03084\, VP_1 \left[\left(\frac{P_2}{P_1} \right)^{0.1415} - 1 \right]$$

Capacity of reciprocating compressors (1, 2) is usually stated in terms of piston displacement, which is the vol swept through by the piston, expressed in cu ft per min.

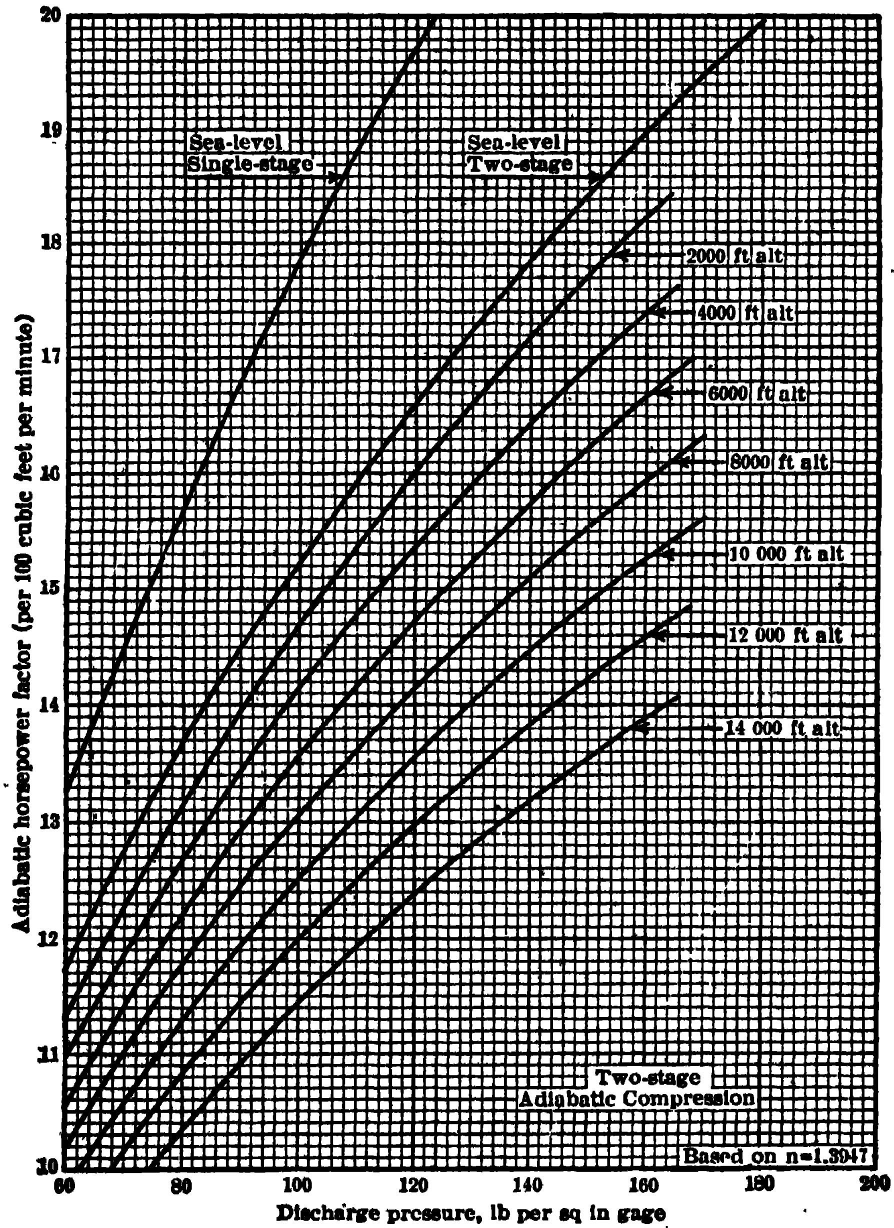


Fig 3. Theoretical Horsepower required by Two-stage Air Compressors (2)

Actual delivered air capac is less than piston displacement, due to clearance, slippage and valve leakage. Ratio of the vol of air delivered, divided by piston displacement is the volumetric effic. For single-stage compressors the actual delivery is usually 65 to 75% of piston displacement; for 2-stage compressors the delivery is 75 to 90% of piston displacement. Capac of portable compressors is usually given in terms of actual free air delivery. Formula for calculating piston displacement of compressors with single-acting cylinders is:

$$PD = \frac{A \times S \times n \times \mathrm{rpm}}{1\,728}$$

Table 1. Theoretical Hp Required at Sea-level to Compress 100 cu ft of Air from Atmos Press (14.7 lb) to Various Gage Pressures (based on n = 1.3947)

Discharge press, lb gage	Discharge press, lb absolute	Discharge press, atmos absolute	Single- or multi-stage compression, isothermal	Single-stage compression, adiabatic	Two-stage compression, adiabatic	Theoretical inter-cooler gage press	% of power saved by two-stage over single-stage adiabatic compression
			Theoretical hp per 100 cu ft per min				
5	19.7	1.34	1.8	2.0	....	....	....
10	24.7	1.68	3.3	3.6	....	....	....
15	29.7	2.02	4.5	5.0	....	....	....
20	34.7	2.36	5.5	6.2	....	....	....
25	39.7	2.70	6.4	7.4	....	....	....
30	44.7	3.04	7.1	8.4	....	....	....
35	49.7	3.38	7.8	9.3	....	....	....
40	54.7	3.72	8.4	10.2	....	....	....
45	59.7	4.06	9.0	11.0	....	....	....
50	64.7	4.40	9.5	11.8	....	....	....
55	69.7	4.74	10.0	12.6	....	....	....
60	74.7	5.08	10.4	13.3	....	....	....
65	79.7	5.42	10.8	13.9	....	....	....
70	84.7	5.76	11.2	14.6	12.8	20.6	12.3
75	89.7	6.10	11.6	15.2	13.3	21.6	12.5
80	94.7	6.44	12.0	15.7	13.7	22.7	12.7
85	99.7	6.78	12.3	16.3	14.1	23.6	13.5
90	104.7	7.12	12.6	16.9	14.5	24.5	14.2
95	109.7	7.46	12.9	17.4	14.9	25.5	14.4
100	114.7	7.80	13.2	17.9	15.3	26.3	14.5
110	124.7	8.48	13.7	18.9	16.1	28.1	14.8
120	134.7	9.16	14.2	19.8	16.8	29.8	15.1
130	144.7	9.84	14.7	20.7	17.3	31.5	16.4
140	154.7	10.52	15.1	21.5	17.9	32.9	16.7
150	164.7	11.20	15.5	22.3	18.5	34.5	17.1
160	174.7	11.88	15.8	....	19.0	36.1	....
170	184.7	12.56	16.2	....	19.5	37.3	....
180	194.7	13.24	16.6	....	20.0	38.8	....
190	204.7	13.92	16.9	....	20.4	40.1	....
200	214.7	14.60	17.2	....	20.9	41.4	....
250	264.7	18.00	18.6	....	22.7	47.6	....
300	314.7	21.40	19.7	....	24.5	53.4	....
350	364.7	24.81	20.6	....	26.1	58.5	....
400	414.7	28.21	21.4	....	27.4	63.3	....
450	464.7	31.61	22.3	....	28.6	67.8	....
500	514.7	35.01	22.9	....	29.6	72.1	....
550	564.7	38.41	23.4	....	30.6	76.3	....
600	614.7	41.81	23.9	....	31.3	80.5	....

Table 2. Approx Brake Hp Required at Altitudes to Compress 100 cu ft Free Air Delivered per min*

Altitude, ft	Single-stage			Two-stage			
	Lb per sq in gage			Lb per sq in gage			
	60	80	100	60	80	100	125
0	16.3	19.5	22.1	14.7	17.1	19.1	21.3
1 000	16.1	19.2	21.7	14.5	16.8	18.7	20.9
2 000	15.9	18.9	21.3	14.3	16.5	18.4	20.5
3 000	15.7	18.6	20.9	14.0	16.1	18.0	20.0
4 000	15.4	18.2	20.6	13.8	15.8	17.7	19.6
5 000	15.2	17.9	20.3	13.5	15.5	17.3	19.2
6 000	15.0	17.6	20.0	13.3	15.2	17.0	18.8
7 000	14.7	17.3	19.6	13.0	14.9	16.6	18.4
8 000	14.5	17.1	19.3	12.7	14.6	16.2	18.0
9 000	14.3	16.8	18.9	12.5	14.3	15.9	17.6
10 000	14.1	16.5	18.6	12.3	14.1	15.6	17.2
12 000	13.6	15.9	17.9	11.8	13.5	15.0	16.5
14 000	13.1	15.2	17.2	11.3	12.9	14.3	15.7

* Note: Brake hp varies considerably with size and type of compressor.

in which PD = piston displacement, cu ft per min; A = piston area, sq in; S = stroke, in; n = number of first-stage pistons; rpm = rev per min. With double-acting cylinders,

$$PD = \frac{A \times S \times n \times \text{rpm}}{874}$$

in which a reasonable deduction is made for vol occupied by piston rod. Without this deduction, the divisor would be 864.

In calculating piston displacement, the first-stage cylinders only are considered, as all the air passing through the first-stage passes also through the higher stages.

Discharge temperatures (1). Table 3 gives the theoretical discharge temp of single and 2-stage compressors. Variations occur in practice, due to water jacketing and radiation, tending to lower the temp at the higher pressures. Formulas for computing these temperatures (adiab compression) under conditions not shown in the tables are:

For single-stage compressors, $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{0.283}$, where T_1, T_2 = intake and disch temp, F deg abs; P_1, P_2 = abs intake and disch press.

For 2-stage compressors (assuming perfect intercooling and equal division of work between the cylinders), $T_2 = T_1 \sqrt{\left(\frac{P_2}{P_1}\right)^{0.283}}$

Table 3. Theoretical Cylinder Temperatures at End of Stroke (based on adiabatic compression, n being 1.3947 with perfect intercooling, intake temp, 60° F and intake press, 14.7 lb abs)

Final press		Final temp, deg F		Final press		Final temp, deg F	
Gage press, lb	Abs press, lb	Single-stage	Two-stage	Gage press, lb	Abs press, lb	Single-stage	Two-stage
10	24.7	142	...	100	114.7	470	235
20	34.7	203	...	110	124.7	492	244
30	44.7	252	...	120	134.7	513	251
40	54.7	290	...	130	144.7	531	259
50	64.7	331	181	140	154.7	552	266
60	74.7	364	194	150	164.7	570	272
70	84.7	394	206	200	214.7	651	300
80	94.7	421	217	250	264.7	718	323
90	104.7	446	227				

Transmission of compressed air is in wrought iron or steel pipe; "extra-heavy" pipe being used for high pressures such as for Diesel engine starting. For sizes and weights of pipe and fittings, see Sec 41. PIPE JOINTS are made by sleeve couplings, or by flanges into which the pipe ends are expanded or threaded. Sleeve couplings, suitable for all except very large sizes, should be put on with white or red lead, especially where leaks may develop due to shifting ground. Gaskets are used for flange couplings: asbestos near receiver, fiber type elsewhere. EXPANSION JOINTS are necessary on long lines. Welded pipe is widely used, for its convenience and elimination of joints.

Transmission losses in pipes (see also Sec 39, Art 2). The heat of compression is quickly lost in the first few hundred ft of air main, and can not economically be retained by non-conducting covering.

Transmission line hints. Losses from leaky joints or unsound pipe often exceed all other transmission losses. Pipes should be inspected regularly to eliminate waste of power. Pipe of too small diam reduces effective press by causing high velocity and undue friction (Table 4, 5). Veloc in mains should not exceed 20 to 25 ft per sec; in short branch pipes, it may be 40 or 50 ft. Pipe with rough interior causes excessive friction loss. Each length should be cleaned of foreign substances before coupling. Lead forced into the pipe at couplings makes obstructive ridges. Surface mains should be protected, to avoid freezing of the moisture and consequent obstruction. Tees, elbows, and other fittings cause friction and should be avoided wherever possible (Table 7).

Friction losses of pressure in pipe lines are important, and can be determined by the formula (E. C. Harris, *Bull* Univ of Mo, Vol 1, 1912):

$$f = \frac{0.1025LQ^2}{d^{5.31}}$$

where: f = press drop, lb per sq in, L = length of pipe, ft, r = ratio of compression at pipe entrance, Q = vol of air flowing, cu ft per min, and d = internal diam of pipe, in. Table 5, 6 are based on this formula. Table 7 gives loss of press through hose, and friction loss through pipe fittings. Another formula for press loss is that of D'Arcy:

$$V = c\sqrt{\frac{d^5(p_1 - p_2)}{wL}},$$

where: V = cu ft compressed air delivered at final press; c = experimental constant; d = diam pipe, in; L = length pipe, ft; w = wt in lb per cu ft of air at press p_1; p_1, p_2 = initial and final gage press.

Table 4. Values of c in D'Arcy's Formula

Pipe, in	c	Pipe, in	c
1	43.3	7	60.3
2	52.6	8	60.7
3	56.5	9	61.0
4	58.0	10	61.2
5	59.0	11	61.6
6	59.8	12	62.0

Graphic solution of D'Arcy's formula (Fig 4). Find in left-hand margin the vol of compressed air. Pass horizontally to diagonal line for length of pipe; project vertically to intersect the diagonal for initial gage press, then horiz to right to vert line of initial pressures. Using this point as a pivot, swing a straight-edge to cut lines for size of pipe and press drop. To find cu ft of compressed air delivered, divide vol of free air by the ratio of final abs press in the pipe to atmos press. Use of chart can be reversed to find max vol of air transmitted through a given pipe line. To have several branch lines equal in carrying capac to a main line, the pipes should be to each other as the sq root of the fifth powers of their diam.

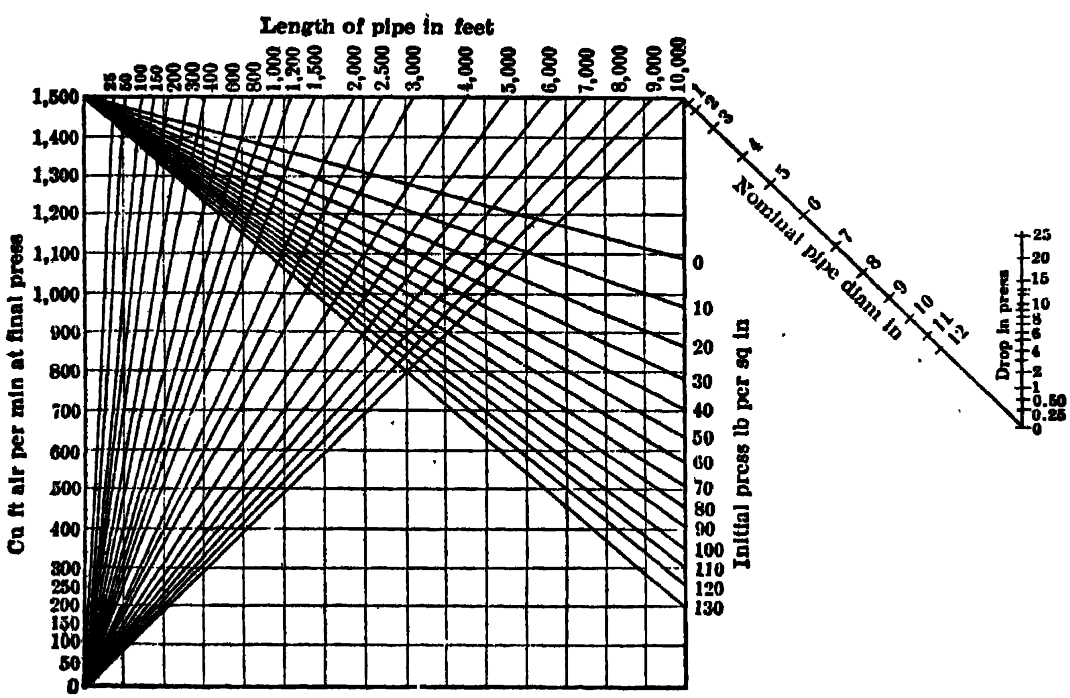


Fig 4. Graphic Solution of D'Arcy's Formula (N. Herz)

Table 5. Friction of Air in Pipes

Divide the number corresponding to diam and vol by the ratio of compression; result is loss in lb per sq in, in 1 000 ft of pipe

Cu ft free air per min	Nominal diam, in								
	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/2	3
5	12.7	1.2	....	....	....	....	....	....	...
10	50.7	7.8	2.2	....	....	....	....	....	...
15	114.1	17.6	4.9	....	....	....	....	....	...
20	....	30.4	8.7	2.0	....	....	....	....	...
25	....	50.0	13.6	3.2	....	....	....	....	...
30	....	70.4	19.6	4.5	....	....	....	....	...
35	....	95.9	26.6	6.2	2.7	....	....	....	...
40	....	125.3	34.8	8.1	3.6	1.9	....	....	...
45	....	....	44.0	10.2	4.5	2.4	....	....	...
50	....	....	54.4	12.6	5.6	2.9	....	....	...
60	....	....	78.3	18.2	8.0	4.2	2.2	....	...
70	....	....	106.6	24.7	10.9	5.7	2.9	....	...
80	....	....	139.2	32.3	14.3	7.5	3.8	....	...
90	....	....	....	40.9	18.1	9.5	4.8	....	...
100	....	....	....	50.5	22.3	11 7	6.0	....	...
110	....	....	....	61.1	27.0	14.1	7.2	2.8	...
120	....	....	....	72.7	32.2	16.8	8.6	3.3	...
130	....	....	....	85.3	37.8	19.7	10.1	3.9	...
140	....	....	....	98.9	43.8	22.9	11.7	4.6	...
150	....	....	....	113.6	50.3	26.3	13.4	5.2	...
160	....	....	....	129.3	57.2	29.9	15.3	5.9	...
170	....	....	....	....	64.6	33.7	17.6	6.7	...
180	....	....	....	....	72 6	37.9	19.4	7.5	...
190	....	....	....	....	80.7	42.2	21.5	8.4	2.1
200	....	....	....	....	89.4	46.7	23.9	9.3	2.9
220	....	....	....	....	108.2	56.5	28.9	11.3	3.5
240	....	....	....	....	128.7	67.3	34.4	13.4	4.2
260	....	....	....	....	....	79.0	40.3	15.7	4.9
280	....	....	....	....	....	91.6	46.8	18.2	5.7
300	....	....	....	....	....	105.1	53.7	20.9	6.6

Cu ft free air per min	Nominal diam, in								
	2	2 1/2	3	3 1/2	4	4 1/2	5	6	8
320	61.1	23.8	7.5	3.5	....	....	....	....	...
340	69.0	26.8	8.4	3.9	....	....	....	....	...
360	77.3	30.1	9.5	4.4	....	....	....	....	...
380	86.1	33.5	10.5	4.9	....	....	....	....	...
400	94.7	37.1	11.7	5.4	2.7	....	....	....	...
420	105.2	40.9	12.9	6.0	3.1	....	....	....	...
440	115.5	44.9	14.1	6.6	3.4	....	....	....	...
460	125.6	48.8	15.4	7.1	3.7	....	....	....	...
480	137.6	53.4	16.8	7.8	4.0	....	....	....	...
500	150.0	58.0	18.3	8.5	4.3	....	....	....	...
525	165.0	64.2	20.2	9.4	4.8	2.6	....	....	...
550	181.5	70.2	22.1	10.2	5.2	2.9	....	....	...
575	197.0	76.7	24.2	11.2	5.7	3.1	....	....	...
600	215.0	83.5	26.3	12.2	6.2	3.4	....	....	...
625	233.0	92.7	28.5	13.2	6.8	3.7	....	....	...
650	253.0	98.0	30.9	14.3	7.3	4.0	....	....	...
675	272.0	105.7	33.3	15.4	7.9	4.3	....	....	...
700	294.0	113.7	35.8	16.6	8.5	4.6	....	....	...
750	337.0	130.5	41.1	19.0	9.7	5.3	2.9	....	...
800	382.0	148.4	46.7	21.7	11.1	6.1	3.3	....	...
850	433.0	165.0	52.8	24.4	12.5	6.8	3.8	....	...
900	486.0	185.0	59.1	27.4	14.0	7.7	4.2	....	...
950	541.0	209.4	65 9	30.5	15.7	8.6	4.7	....	...
1 000	600.0	232.0	73.0	33 8	17.3	9.5	5.2	....	...
1 050	....	256.0	80.5	37.3	19.1	10 4	5.8	....	...
1 100	....	280.6	88 4	40.9	21.0	11.5	6.3	2.4	...
1 150	....	306.8	96.6	44.7	22.9	12.5	6.9	2.6	...
1 200	....	344.0	105.2	48.8	25.0	13.7	7.5	2.8	...
1 300	....	392.0	123.4	57.2	29 3	16.0	8 8	3.3	...

Cu ft free air per min	Nominal diam, in							
	3 1/2	4	4 1/2	5	6	8	10	12
1 400	66.3	33.9	18.6	10.2	3 8	....	....	....
1 500	76.1	39 0	21.3	11.8	4.4	....	....	....
1 600	86.6	44.3	24.2	13.4	5.1	....	....	....
1 700	97.8	50.1	27.4	15.1	5.7	....	....	....
1 800	110.0	56.1	30.7	16.9	6.4	....	....	....
1 900	....	62.7	34.2	18.9	7.1	....	....	....
2 000	....	69.3	37.9	21.3	7.8	....	....	....
2 100	....	76.4	40.8	23.0	8.7	2.0	....	....
2 200	....	83.6	45.8	25.3	9.5	2.2	....	....
2 300	....	91.6	50.1	27.6	10.4	2.4	....	....
2 400	....	99.8	54.6	30 1	11.3	2.6	....	....
2 500	....	108.3	59.2	32.6	12 3	2.9	....	....
2 600	....	117.2	64.0	35.3	13.3	3.1	....	....
2 700	....	....	69.1	38.1	14.3	3.3	....	....
2 800	....	....	74.3	41.0	15.4	3.6	....	....
2 900	....	....	79.8	43.9	16.5	3.9	....	....
3 000	....	....	85.2	47.0	17.7	4.1	....	....
3 200	....	....	97.1	53.5	20.1	4.7	....	....
3 400	....	....	109.5	60.4	22.7	5.3	....	....
3 600	....	....	122.8	67.7	25.4	5.6	....	....
3 800	....	....	....	75 5	28.4	6.6	....	....
4 000	....	....	....	83.6	31.4	7.3	....	....
4 200	....	....	....	92.1	34.6	8.1	....	....
4 400	....	....	....	101.2	38.1	8.9	....	....
4 600	....	....	....	110.5	41.5	9.7	2.9	....
4 800	....	....	....	120.4	45.2	10.5	3.2	....
5 000	....	....	....	....	49 1	11.5	3.4	....
5 250	....	....	....	....	54.1	12.6	3.8	....
5 500	....	....	....	....	59.4	13.9	4.2	....
5 750	....	....	....	....	64.9	15.2	4.6	....
6 000	....	....	....	....	70 7	16.5	5.0	....
6 500	....	....	....	....	82.9	19.8	5.9	2.3
7 000	....	....	....	....	96.2	22.5	6.8	2.6
7 500	....	....	....	....	110.5	25.8	7.8	3.0
8 000	....	....	....	....	125.7	29.4	8.8	3.6
9 000	....	....	....	....	....	37.2	11.2	4.4
10 000	....	....	....	....	....	45.9	13.8	5.4
11 000	....	....	....	....	....	55.5	16.7	6.5
12 000	....	....	....	....	....	66.1	19 8	7.7
13 000	....	....	....	....	....	77.5	23.3	9.0
14 000	....	....	....	....	....	89.9	27.0	10.5
15 000	....	....	....	....	....	103.2	31.0	12 0
16 000	....	....	....	....	....	117.7	35.3	13.7
18 000	....	....	....	....	....	148.7	44.6	17.4
20 000	....	....	....	....	....	....	55 0	21.4
22 000	....	....	....	....	....	....	66.9	26.0
24 000	....	....	....	....	....	....	79 3	30.1
26 000	....	....	....	....	....	....	93.3	36.3
28 000	....	....	....	....	....	....	108.0	42 1
30 000	....	....	....	....	....	....	123.9	48.2

Table 6. Friction of Air in Pipes, at 80-lb Gage

Loss of press, lb per sq in per 1 000 ft of pipe

Cu ft free air per min	Equivalent cu ft compressed air per min	Nominal diam, in										
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2
10	1.55	7.90	1.21	.34	.....	.....	.....	.....	.....	.....	.....	.....
20	3.10	31.4	4.72	1.35	.31	.....	.....	.....	.....	.....	.....	.....
30	4.65	70.8	10.9	3.05	.69	.31	.....	.....	.....	.....	.....	.....
40	6.20	.....	19.5	5.40	1.25	.56	.....	.....	.....	.....	.....	.....
50	7.74	.....	30.5	8.45	1.96	.87	.....	.....	.....	.....	.....	.....
60	9.29	.....	43.8	12.16	2.82	1.24	.34	.....	.....	.....	.....	.....
70	10.82	.....	59.8	16.6	3.84	1.70	.45	.....	.....	.....	.....	.....
80	12.40	.....	78.2	21.6	5.03	2.22	.59	.....	.....	.....	.....	.....
90	13.95	.....	.....	27.4	6.35	2.82	.75	.....	.....	.....	.....	.....
100	15.5	.....	.....	33.8	7.85	3.47	.93	.36	.....	.....	.....	.....
125	19.4	.....	.....	46.2	12.4	5.45	1.44	.56	.....	.....	.....	.....
150	23.2	.....	.....	76.2	17.7	7.82	2.08	.81	.....	.....	.....	.....
175	27.2	.....	.....	.....	24.8	10.6	2.87	1.10	.....	.....	.....	.....
200	31.0	.....	.....	.....	31.4	13.9	3.72	1.44	.45	.....	.....	.....
250	38.7	.....	.....	.....	49.0	21.7	5.82	2.25	.70	.33	.....	.....
300	46.5	.....	.....	.....	70.6	31.2	8.35	3.24	1.03	.47	.....	.....
350	54.2	.....	.....	.....	96.0	42.5	11.4	4.42	1.39	.65	.33	.....
400	62.0	.....	.....	.....	.....	55.5	14.7	5.76	1.82	.84	.42	.....
450	69.7	.....	.....	.....	.....	70.2	18.7	7.25	2.29	1.06	.55	.....
500	77.4	.....	.....	.....	.....	86.7	23.3	9.0	2.84	1.32	.67	.30
600	92.9	.....	.....	.....	.....	.....	33.4	12.9	4.08	1.89	.96	.53
700	108.2	.....	.....	.....	.....	.....	45.7	17.6	5.52	2.58	1.32	.72
800	124.0	.....	.....	.....	.....	.....	59.3	23.1	7.15	3.36	1.72	.95
900	139.5	.....	.....	.....	.....	.....	75.5	29.2	9.17	4.26	2.18	1.19
1 000	155.	.....	.....	.....	.....	.....	93.2	36.1	11.3	5.27	2.68	1.48
		2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
1 500	232	.....	81.7	25.5	11.8	6.0	3.32	1.83	.69	.....	.....	.....
2 000	310	.....	.....	45.3	21.0	10.7	5.9	3.30	1.21	.29	.....	.....
2 500	387	.....	.....	70.9	32.9	16.8	9.2	5.1	1.91	.45	.....	.....
3 000	465	.....	.....	.....	47.4	24.2	13.2	7.3	2.74	.64	.19	.....
3 500	542	.....	.....	.....	64.5	32.8	17.8	10.1	3.70	.85	.26	.....
4 000	620	.....	.....	.....	84.1	43.0	23.4	13.0	4.87	1.14	.34	.....
4 500	697	.....	.....	.....	.....	54.8	29.8	16.4	6.15	1.44	.43	.....
5 000	774	.....	.....	.....	.....	67.4	36.7	20.3	7.65	1.78	.53	.21
6 000	929	.....	.....	.....	.....	96.5	53.0	29.2	11.0	2.57	.77	.29
7 000	1 082	.....	.....	.....	.....	.....	72.1	39.8	14.8	3.50	1.06	.40
8 000	1 240	.....	.....	.....	.....	.....	94.2	52.1	19.5	4.57	1.36	.56
9 000	1 395	.....	.....	.....	.....	.....	.....	65.8	24.7	5.78	1.74	.69
10 000	1 550	.....	.....	.....	.....	.....	.....	81.3	30.5	7.15	2.14	.84
11 000	1 710	.....	.....	.....	.....	.....	.....	.....	36.8	8.61	2.60	1.01
12 000	1 860	.....	.....	.....	.....	.....	.....	.....	43.8	10.3	3.08	1.19
13 000	2 020	.....	.....	.....	.....	.....	.....	.....	51.7	12.0	3.62	1.40
14 000	2 170	.....	.....	.....	.....	.....	.....	.....	60.2	14.0	4.20	1.63
15 000	2 320	.....	.....	.....	.....	.....	.....	.....	68.5	16.0	4.82	1.86
16 000	2 480	.....	.....	.....	.....	.....	.....	.....	78.2	18.2	5.48	2.13
18 000	2 790	.....	.....	.....	.....	.....	.....	.....	98.8	23.0	6.95	2.70
20 000	3 100	.....	.....	.....	.....	.....	.....	.....	.....	28.6	8.55	3.32
22 000	3 410	.....	.....	.....	.....	.....	.....	.....	.....	34.5	10.4	4.04
24 000	3 720	.....	.....	.....	.....	.....	.....	.....	.....	41.0	12.3	4.69
26 000	4 030	.....	.....	.....	.....	.....	.....	.....	.....	48.2	14.4	5.65
28 000	4 350	.....	.....	.....	.....	.....	.....	.....	.....	55.9	16.8	6.5
30 000	4 650	.....	.....	.....	.....	.....	.....	.....	.....	64.2	19.3	7.5

Table 6. Friction of Air in Pipes—*Continued*

At 100-lb Gage

Cu ft free air per min	Equivalent cu ft compressed air per min	Nominal diam, in										
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2
10	1.28	6.50	.99	.28	...	...	...	...	...	...	...	...
20	2.56	25.9	3.90	1.11	.25	.11	...	...	...	...	...	...
30	3.84	58.5	9.01	2.51	.57	.26	...	...	...	...	...	...
40	5.12	...	16.0	4.45	1.03	.46	...	...	...	...	...	...
50	6.41	...	25.1	6.96	1.61	.71	.19	...	...	...	...	...
60	7.68	...	36.2	10.0	2.32	1.02	.28	...	...	...	...	...
70	8.96	...	49.3	13.7	3.16	1.40	.37	...	...	...	...	...
80	10.24	...	64.5	18.7	4.14	1.83	.49	.19	...	...	...	...
90	11.52	...	82.8	22.6	5.23	2.32	.62	.24	...	...	...	...
100	12.81	...	...	27.9	6.47	2.86	.77	.30	...	...	...	...
125	15.82	...	...	48.6	10.2	4.49	1.19	.46	...	...	...	...
150	19.23	...	...	62.8	14.6	6.43	1.72	.66	.21	...	...	...
175	22.40	...	...	...	19.8	8.72	2.36	.91	.28	...	...	...
200	25.62	...	...	...	25.9	11.4	3.06	1.19	.37	.17	...	...
250	31.64	...	...	...	40.4	17.9	4.78	1.85	.58	.27	...	...
300	38.44	...	...	...	58.2	25.8	6.85	2.67	.84	.39	.20	...
350	44.80	...	...	...	...	35.1	9.36	3.64	1.14	.53	.27	...
400	51.24	...	...	...	...	45.8	12.1	4.75	1.50	.69	.35	.19
450	57.65	...	...	...	...	58.0	15.4	5.98	1.89	.88	.46	.25
500	63.28	...	...	...	...	71.6	19.2	7.42	2.34	1.09	.55	.30
600	76.88	...	...	...	...	...	27.6	10.7	3.36	1.56	.79	.44
700	89.60	...	...	...	...	...	37.7	14.5	4.55	2.13	1.09	.59
800	102.5	...	...	...	...	...	49.0	19.0	5.89	2.77	1.42	.78
900	115.3	...	...	...	...	...	62.3	24.1	7.6	3.51	1.80	.99
1 000	126.6	...	...	...	...	...	76.9	29.8	9.3	4.35	2.21	1.22
		2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
1 500	192.3	...	67.0	21.0	9.8	4.9	2.73	1.51	.57	...	...	...
2 000	256.2	...	...	37.4	17.3	8.8	4.9	2.72	.99	.24	...	...
2 500	316.4	...	...	58.4	27.2	13.8	8.3	4.2	1.57	.37	...	...
3 000	384.6	...	...	84.1	39.1	20.0	10.9	6.0	2.26	.53	...	...
3 500	447.8	...	...	...	58.2	27.2	14.7	8.2	3.04	.70	.22	...
4 000	512.4	...	...	...	69.4	35.5	19.4	10.7	4.01	.94	.28	...
4 500	576.5	...	...	...	...	45.0	24.5	13.5	5.10	1.19	.36	...
5 000	632.8	...	...	...	...	55.6	30.2	16.8	6.3	1.47	.44	.17
6 000	768.8	...	...	...	...	80.0	43.7	24.1	9.1	2.11	.64	.24
7 000	896.0	...	...	...	...	...	59.5	32.8	12.2	2.88	.87	.33
8 000	1 025	...	...	...	...	...	77.5	42.9	16.1	3.77	1.12	.46
9 000	1 153	...	...	...	...	...	...	54.3	20.4	4.77	1.43	.57
10 000	1 280	...	...	...	...	...	...	67.1	25.1	5.88	1.77	.69
11 000	1 410	...	...	...	...	...	...	...	30.4	7.10	2.14	.83
12 000	1 540	...	...	...	...	...	...	...	36.2	8.5	2.54	.98
13 000	1 668	...	...	...	...	...	...	...	42.6	9.8	2.98	1.15
14 000	1 795	...	...	...	...	...	...	...	49.2	11.5	3.46	1.35
15 000	1 923	...	...	...	...	...	...	...	56.6	13.2	3.97	1.53
16 000	2 050	...	...	...	...	...	...	...	64.5	15.0	4.52	1.75
18 000	2 310	...	...	...	...	...	...	...	81.5	19.0	5.72	2.22
20 000	2 560	...	...	...	...	...	...	...	...	23.6	7.0	2.74
22 000	2 820	...	...	...	...	...	...	...	...	28.5	8.5	3.33
24 000	3 080	...	...	...	...	...	...	...	...	33.8	10.0	3.85
26 000	3 338	...	...	...	...	...	...	...	...	39.7	11.9	4.65
28 000	3 590	...	...	...	...	...	...	...	...	46.2	13.8	5.40
30 000	3 850	...	...	...	...	...	...	...	...	53.0	15.9	6.17

Table 6. Friction of Air in Pipes—*Concluded*

At 125-lb Gage

Cu ft free air per min	Equivalent cu ft compressed air per min	Nominal diam, in															
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	6	8	10	12
10	1.05	5.35	.82	.23	....	....	....	....	....	....	....	....	....	....	....	....	....
20	2.11	....	3.21	.92	.21	....	....	....	....	....	....	....	....	....	....	....	....
30	3.16	....	7.42	2.07	.47	....	....	....	....	....	....	....	....	....	....	....	....
40	4.21	....	13.2	3.67	.85	.38	....	....	....	....	....	....	....	....	....	....	....
50	5.26	....	....	5.72	1.33	.59	....	....	....	....	....	....	....	....	....	....	....
60	6.32	....	....	8.25	1.86	.84	.23	....	....	....	....	....	....	....	....	....	....
70	7.38	....	....	11.2	2.61	1.15	.31	....	....	....	....	....	....	....	....	....	....
80	8.42	....	....	14.7	3.41	1.51	.40	....	....	....	....	....	....	....	....	....	....
90	9.47	....	....	....	4.30	1.91	.51	....	....	....	....	....	....	....	....	....	....
100	10.50	....	....	....	5.32	2.36	.63	....	....	....	....	....	....	....	....	....	....
125	13.15	....	....	....	8.4	3.70	.98	.38	....	....	....	....	....	....	....	....	....
150	15.79	....	....	....	12.0	5.30	1.41	.55	....	....	....	....	....	....	....	....	....
175	18.41	....	....	....	....	7.2	1.95	.75	....	....	....	....	....	....	....	....	....
200	21.05	....	....	....	....	9.4	2.52	.98	.31	....	....	....	....	....	....	....	....
250	26.30	....	....	....	....	....	3.94	1.53	.48	....	....	....	....	....	....	....	....
300	31.60	....	....	....	....	....	5.62	2.20	.70	....	....	....	....	....	....	....	....
350	36.80	....	....	....	....	....	7.7	3.00	.94	.44	....	....	....	....	....	....	....
400	42.10	....	....	....	....	....	10.0	3.91	1.23	.57	.28	....	....	....	....	....	....
450	47.30	....	....	....	....	....	12.7	4.92	1.55	.72	.37	....	....	....	....	....	....
500	52.60	....	....	....	....	....	....	6.10	1.93	.89	.46	....	....	....	....	....	....
600	63.20	....	....	....	....	....	....	8.8	2.76	1.28	.65	.36	....	....	....	....	....
700	73.80	....	....	....	....	....	....	11.9	3.74	1.75	.89	.49	....	....	....	....	....
800	84.20	....	....	....	....	....	....	....	4.85	2.28	1.17	.64	.35	....	....	....	....
900	94.70	....	....	....	....	....	....	....	6.2	2.89	1.48	.81	.44	....	....	....	....
1 000	105.1	....	....	....	....	....	....	....	7.7	3.29	1.82	1.00	.55	....	....	....	....
1 500	157.9	....	....	....	....	....	....	....	....	8.0	4.1	2.25	1.24	.47	....	....	....
2 000	210.5	....	....	....	....	....	....	....	....	....	7.3	4.0	2.24	.82	....	....	....
2 500	263.0	....	....	....	....	....	....	....	....	....	11.4	6.2	3.4	1.30	.31	....	....
3 000	316.0	....	....	....	....	....	....	....	....	....	....	9.0	4.9	1.86	.43	....	....
3 500	368.0	....	....	....	....	....	....	....	....	....	....	12.1	6.9	2.51	.57	....	....
4 000	421.0	....	....	....	....	....	....	....	....	....	....	....	8.9	3.30	.77	....	....
4 500	473.0	....	....	....	....	....	....	....	....	....	....	....	11.1	4.2	.98	....	....
5 000	526.0	....	....	....	....	....	....	....	....	....	....	....	....	5.2	1.21	.36	....
6 000	632.0	....	....	....	....	....	....	....	....	....	....	....	....	7.5	1.74	.52	....
7 000	738.0	....	....	....	....	....	....	....	....	....	....	....	....	10.0	2.37	.72	.27
8 000	842.0	....	....	....	....	....	....	....	....	....	....	....	....	13.2	3.10	.93	.38
9 000	947.0	....	....	....	....	....	....	....	....	....	....	....	....	....	3.93	1.18	.47
10 000	1 051	....	....	....	....	....	....	....	....	....	....	....	....	....	4.85	1.46	.57

Table 7. Pressure Loss of Air in Hose

(Correct for hose with smooth inside lining. Rough inside lining may cause 50% greater loss than the figures given)

Size of hose	Gage press at pipe line	Cu ft free air per min through 50-ft lengths													
		20	30	40	50	60	70	80	90	100	110	120	130	140	150
		Loss of press, lb per sq in, 50-ft hose length													
1/2"	50	1.8	5.0	10.1	18.1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
with	60	1.3	4.0	8.4	14.8	23.4	.....	.....	.....	.....	.....	.....	.....	.....	.....
coup-	70	1.0	3.4	7.0	12.4	20.0	28.4	.....	.....	.....	.....	.....	.....	.....	.....
lings	80	.9	2.8	6.0	10.8	17.4	25.2	34.6	.....	.....	.....	.....	.....	.....	.....
at	90	.8	2.4	5.4	9.5	14.8	22.0	30.5	41.0	.....	.....	.....	.....	.....	.....
each	100	.7	2.3	4.8	8.4	13.3	19.3	27.2	36.6	.....	.....	.....	.....	.....	.....
end	110	.6	2.0	4.3	7.6	12.0	17.6	24.6	33.3	44.5	.....	.....	.....	.....	.....
3/4	50	.4	.8	1.5	2.4	3.5	4.4	6.5	8.5	11.4	14.2	.....	.....	.....	.....
with	60	.3	.6	1.2	1.9	2.8	3.8	5.2	6.8	8.6	11.2	.....	.....	.....	.....
coup-	70	.2	.5	.9	1.5	2.3	3.2	4.2	5.5	7.0	8.8	11.0	.....	.....	.....
lings	80	.2	.5	.8	1.3	1.9	2.8	3.6	4.7	5.8	7.2	8.8	10.6	.....	.....
at	90	.2	.4	.7	1.1	1.6	2.3	3.1	4.0	5.0	6.2	7.5	9.0	.....	.....
each	100	.2	.4	.6	1.0	1.4	2.0	2.7	3.5	4.4	5.4	6.6	7.9	9.4	11.1
end	110	.1	.3	.5	.9	1.3	1.8	2.4	3.1	3.9	4.9	5.9	7.1	8.4	9.9
1"	50	.1	.2	.3	.5	.8	1.1	1.5	2.0	2.6	3.5	4.8	7.0	.....	.....
with	60	.1	.2	.3	.4	.6	.8	1.2	1.5	2.0	2.6	3.3	4.2	5.5	7.2
coup-	70	.....	.1	.2	.4	.5	.7	1.0	1.3	1.6	2.0	2.5	3.1	3.8	4.7
lings	80	.....	.1	.2	.3	.5	.7	.8	1.1	1.4	1.7	2.0	2.4	2.7	3.5
at	90	.....	.1	.2	.3	.4	.6	.7	.9	1.2	1.4	1.7	2.0	2.4	2.8
each	100	.....	.1	.2	.2	.4	.5	.6	.8	1.0	1.2	1.5	1.8	2.1	2.4
end	110	.....	.1	.2	.2	.3	.4	.6	.7	.9	1.1	1.3	1.5	1.8	2.1
1 1/4"	50	.....	.....	.1	.2	.2	.3	.4	.5	.7	1.1	.....	.....	.....	.....
with	60	.....	.....	.....	.1	.2	.3	.3	.5	.6	.8	1.0	1.2	1.5	.....
coup-	70	.....	.....	.....	.1	.2	.2	.3	.4	.4	.5	.7	.8	1.0	1.3
lings	80	.....	.....	.....	.....	.1	.2	.2	.3	.4	.5	.6	.7	.8	1.0
at	90	.....	.....	.....	.....	.1	.2	.2	.3	.3	.4	.5	.6	.7	.8
each	100	.....	.....	.....	.....	.....	.1	.2	.2	.3	.4	.4	.5	.6	.7
end	110	.....	.....	.....	.....	.....	.1	.2	.2	.3	.3	.4	.5	.5	.6
1 1/2"	50	.....	.....	.....	.....	.....	.1	.2	.2	.2	.3	.3	.4	.5	.6
with	60	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2	.3	.3	.4	.5
coup-	70	.....	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2	.3	.3	.4
lings	80	.....	.....	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2	.3	.4
at	90	.....	.....	.....	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2	.3
each	100	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2
end	110	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.1	.2	.2	.2

For longer or shorter lengths, friction loss is proportional to the length for 25 ft, 1/2 of the above; for 150 ft, 3 times the above, etc.

Table 8. Friction Losses in Screw Pipe Fittings (Loss given in equiv length of straight pipe)

Nominal pipe diam, in	Actual inside diam, in	Gate valve	Long-radius ell, or on standard tee	Medium-radius ell, or on tee reduced in size 25%	Standard ell, or on tee reduced in size 50%	Angle valve	Close return bend	Tee through side outlet	Globe valve
Resistance factor for type of fitting		0.25	0.33	0.42	0.67	0.90	1.00	1.33	2.00
1/2	0.622	0.31	0.41	0.52	0.84	1.1	1.3	1.7	2.5
3/4	0.824	0.44	0.57	0.73	1.2	1.6	1.8	2.3	3.5
1	1.049	0.57	0.77	0.98	1.6	2.1	2.3	3.1	4.7
1 1/4	1.380	0.82	1.1	1.4	2.2	2.9	3.3	4.4	6.5
1 1/2	1.610	0.98	1.3	1.6	2.6	3.5	3.9	5.2	7.8
2	2.067	1.3	1.7	2.2	3.6	4.8	5.3	7.1	10.6
2 1/2	2.469	1.6	2.2	2.8	4.4	5.9	6.6	8.7	13.1
3	3.068	2.1	2.8	3.6	5.7	7.7	8.5	11.4	17.1
4	4.026	3.0	3.9	5.0	7.9	10.7	11.8	15.8	23.7
5	5.047	3.9	5.1	6.5	10.4	13.9	15.5	20.7	31.0
6	6.065	4.8	6.4	8.1	12.9	17.4	19.3	25.6	38.5
8	7.981	6.7	8.9	11.2	17.9	24.1	26.8	35.6	53.8
10	10.020	8.8	11.5	14.7	23.4	31.5	35.0	46.6	70.0
12	12.000	10.9	14.4	18.4	29.3	39.3	43.7	58.1	87.4

Based on Foster's formula: $L = 43.7 \times r \times d^{1.2}$, in which L = equivalent length straight pipe, r = resistance factor, d = diam of fitting, ft (*Trans* A S M E, Vol 42, p 648 (1920).

Table 9. Relative Carrying Capacities of Pipes

Size	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	6
1	1.0	0.48	0.27	0.14	0.07	0.05	.....	.....	.....	.....	.....
1 1/4	2.1	1.0	0.55	0.29	0.16	0.10	0.06	.....	.....	.....	.....
1 1/2	3.7	1.8	1.0	0.53	0.29	0.18	0.11	0.08	.....	.....	.....
2	7.14	3.4	1.9	1.0	0.53	0.33	0.21	0.14	0.10	.....	.....
2 1/2	13.4	6.3	3.5	1.9	1.0	0.63	0.40	0.27	0.20	0.15	.....
3	21.4	10.3	5.7	3.0	1.6	1.0	0.67	0.43	0.32	0.23	0.15
3 1/2	.....	16.1	9.0	4.7	2.5	1.5	1.0	0.68	0.50	0.38	0.23
4	.....	.....	13.1	6.9	3.7	2.3	1.4	1.0	0.71	0.56	0.34
4 1/2	.....	.....	.....	9.6	5.0	3.1	2.0	1.4	1.0	0.77	0.47
5	.....	.....	.....	.....	6.7	4.4	2.6	1.8	1.3	1.0	0.63
6	.....	.....	.....	.....	.....	6.5	4.3	2.9	2.1	1.6	1.0
7	.....	.....	.....	.....	.....	.....	6.5	4.4	3.2	2.4	1.5
8	.....	.....	.....	.....	.....	.....	.....	6.5	4.7	3.5	2.2
10	.....	.....	.....	.....	.....	.....	.....	.....	8.3	6.3	3.9
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	10.3	6.3

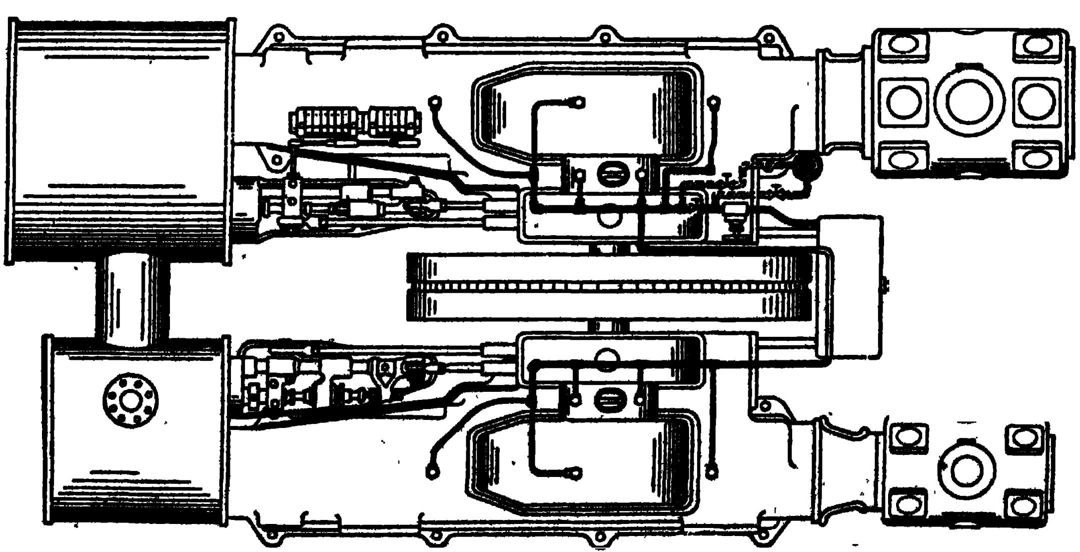
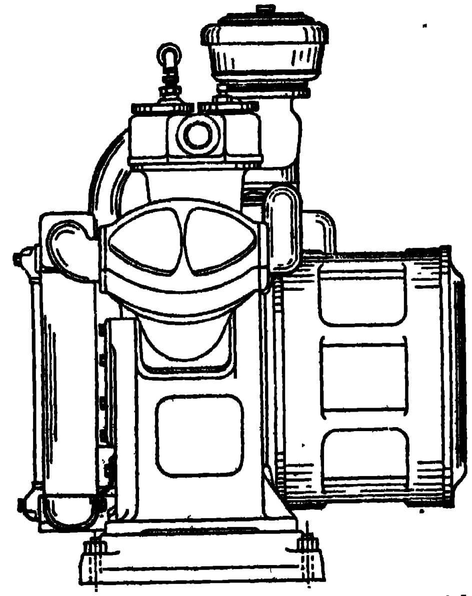
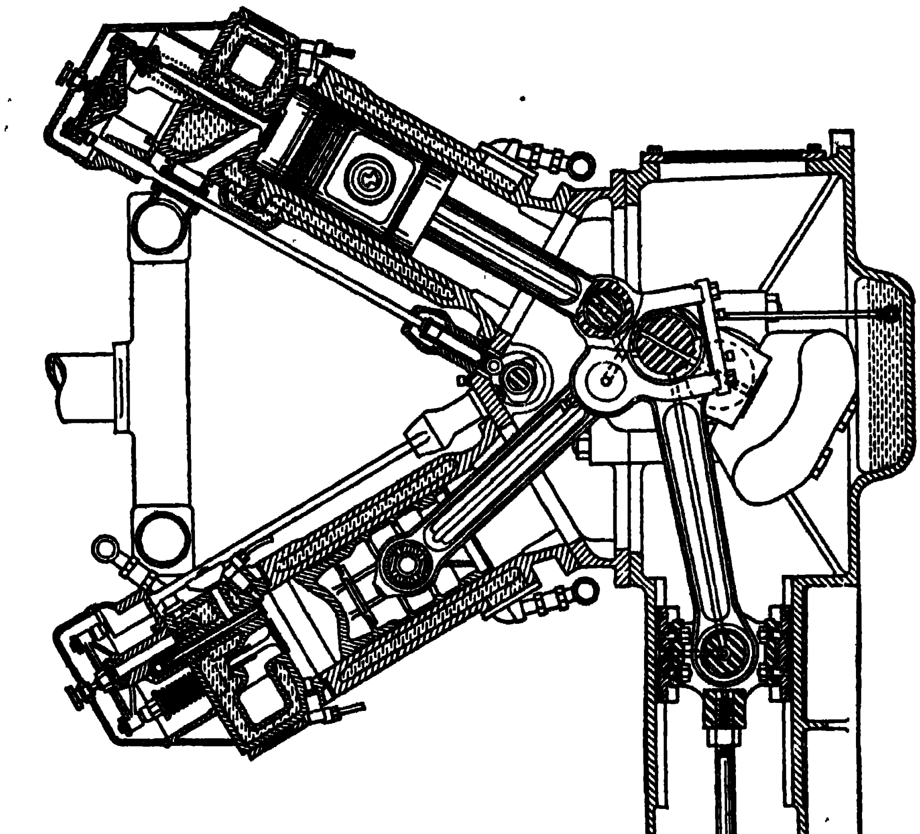


Fig 5. Ingersoll-Rand Duplex, Steam-driven Compressor (intercooler not shown)

4. RECIPROCATING COMPRESSORS

Steam-driven compressors are used where steam power is most economical; as at coal mines, or where fuel oil or gas is available at low cost, or where steam must be provided for other uses. They are built in capacities from 100 to 8 000 cu ft per min (Fig 5).





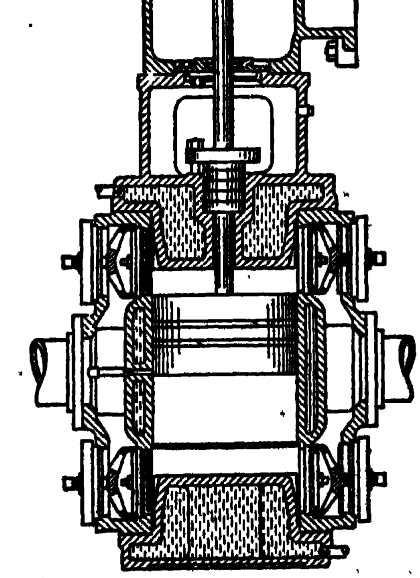


Fig 6. Small Air-cooled Compressor with Direct-connected Motor (Ingersoll-Rand)

Fig 7. Ingersoll-Rand Compressor, Direct-connected with Diesel Engine

Direct-connected, electric-driven compressors are used where electric power is most economical, and where the value of compactness, high efficiency, and the power-factor correction capacity of synchronous motors offsets the slightly higher first cost of this type

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Capacities, from 80 to 15 000 cu ft per min. Compact air-cooled units (Fig 6) are available in capacities to 450 cu ft per min.

Diesel engine- and gas-engine-driven compressors are also built as direct-connected machines (Fig 7). They are most used in petroleum production, but are suitable for metal mines where fuel oil or gas is cheap.

Direct-connected compressors are of two forms: (*a*) horizontal, with the power cylinder on one end of the frame and the air cylinder on the other, the connecting rods acting on a

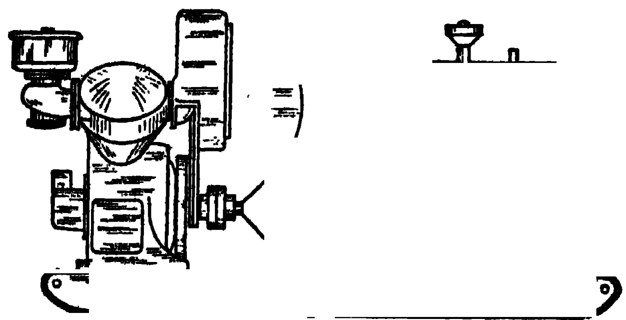


Fig 8. Sullivan Compressor, Direct-connected to Engine

common crankshaft; (*b*) vertical, with power cylinders in a vertical or V-arrangement, and the compressor cylinders in a horiz plane, the connecting rods acting on a common crankshaft. Type (*b*), lighter in weight and more compact, is built in capacities to 4 000 cu ft per min, at 100 lb discharge pressure. Direct-connected compressors may also be driven by an impulse water wheel, mounted on the crankshaft (now obsolete).

Direct-coupled compressors (Fig 8) are largely used where the driving unit can be economically operated at normal compressor speeds, as for portable or mine-car types, and small motor-driven or medium size Diesel-driven compressors. Capacities vary with the kind of driver.

Portable compressors (Fig 9, 10) are well suited to contract work, and exploration or operation of small mines; driven by gasolene engine, fuel oil (Diesel) engine, or elec motor.

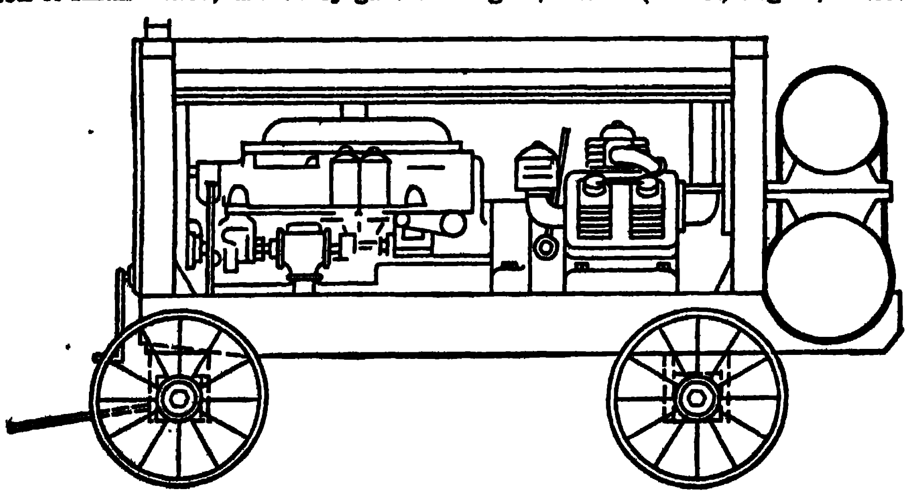


Fig 9. Portable Compressor

They are mounted on skids, steel wheels, solid rubber or pneumatic-tired wheels, or on trucks. Units with flanged wheels, or crawler-type treads, are used for tie tamping on railroads. Most makers offer 2-stage, air-cooled machines; some, water-cooled machines. Portable compressors are rated in actual air deliveries and the sizes are quite well standardized at 60, 85, 160, 210 and 315 cu ft per min. Water-cooled units of about 420 cu ft capac are also built.

Mine-car compressors are designed for electrically-operated mines for local supply of compressed air underground, thus eliminating long pipe lines. They are equipped with standard motors, or, for gaseous mines, with explosion-proof motors and control approved by the U S Bur of Mines. These compressors are air-cooled, usually coupled directly to the motor, and mounted on a frame having flanged wheels to run on mine track. Overall height above the rails is 32 in or less, depending on size and make; capacities, to about 170 cu ft per min (Fig 10).

Belt-driven compressors, widely used for stationary plants, are built in many forms and sizes. Most of them are driven by multiple V-type belts in grooved pulley sheaves, or by short, flat belt with idler, or short belt with driver on pivoted base, or long-belt drive (Fig 11). Long-belt drive, because of space required, is now little used. Any of

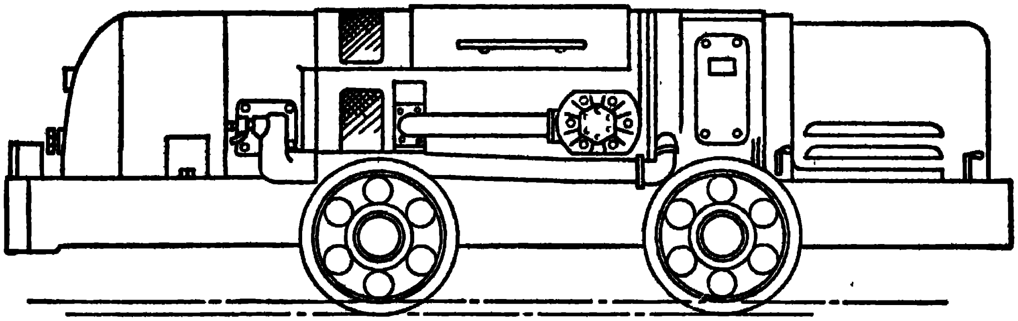


Fig 10. Sullivan Mine-car Compressor

the 3 types of short-belt drives give good results if properly installed. Capacities: 4 to 1 450 cu ft per min; sizes from 4 to 70 cu ft are almost always air-cooled; those from 70 to 450 cu ft are either air- or water-cooled; larger sizes, water-cooled.

Chain-driven compressors are rarely used, because the reciprocating motion introduces severe service and design problems for the chain manufacturer, and usually requires spring-loaded sprockets. Where supplied, they are generally the same as for belt drive, but have sprockets instead of pulleys.

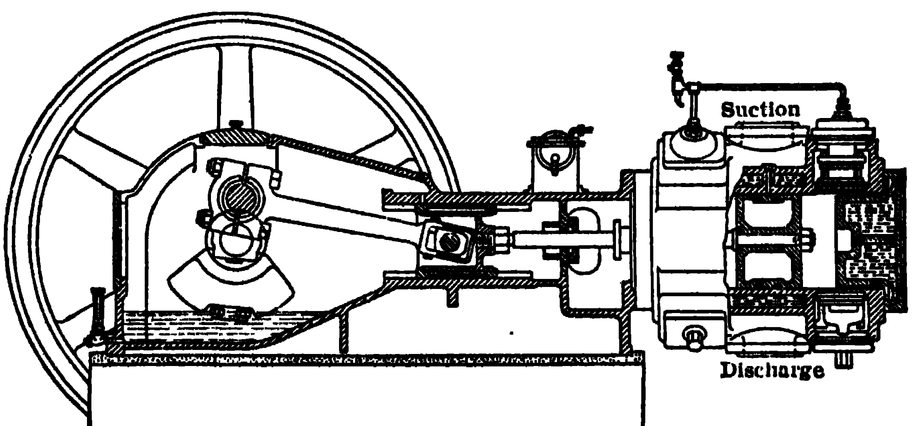


Fig 11. Worthington Belt-driven Single-stage Compressor

Air valves for reciprocating compressors (1, 2, 5). Practically all discharge and inlet valves on present-day reciprocating compressors are of the plate type; other forms, as poppet, Corliss and piston-inlet, are found only on older machines. Poppet valves are controlled by a coil spring. Their weight is a disadvantage, and they are now little used, except in third- or fourth-stage cylinders of some multi-stage compressors.

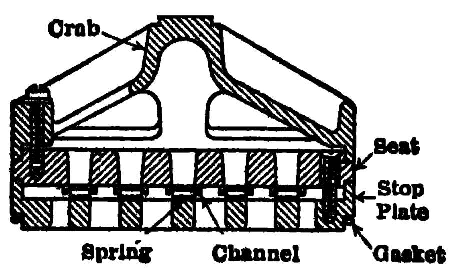


Fig 12. Ingersoll-Rand "Channel Inlet" Plate Valve

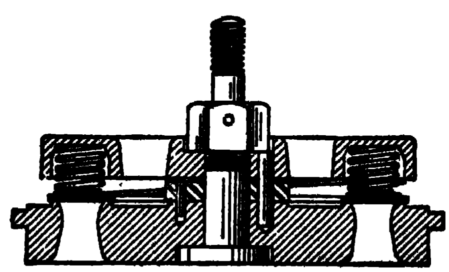


Fig 13. Plate Valve

Plate valves (Fig 12, 13) consist of a seat or grid, having rectangular or annular ports, upon which the valve seats; also a closing spring and a stop plate. The valve, of thin, light-weight alloy steel, is composed of annular rings, rectangular strips, or disks. The

springs may be separate (spiral-coiled, flat-coiled, flat-volute, flat-annular, flat-strip), or form a part of the valve itself. The stop- or cushion-plate limits the valve's lift. A cushioning device, often provided, usually consists of an air space in the stop-plate, a spring stop-plate, or means for trapping air between spring and valve. Fig 14, 15 show other forms. The valve lift is from .03 to .28 in, depending on size and type; valve area, for both inlet and discharge, from 12 to 20% of piston area.

Advantages of plate valves: simplicity, minimizing of inertia by their light weight, large valve area, prompt opening and closing, and more effective water jacketing is permitted.

Governors, regulators and unloaders for reciprocating compressors (1). There are three methods of controlling the output: (*a*) by varying the speed; (*b*) by varying the output at constant speed; (*c*) by automatically starting and stopping the compressor at predetermined pressure limits; (*d*) a combination of (*a*) and (*b*) is sometimes used. APPLICATIONS: method (*a*) to steam and internal-combustion engines, the speed of which is easily varied; (*b*) to belt- and electric motor-driven units running at constant speed; (*c*) is common for moderate capacity compressors, where the demand for air is intermittent; (*d*) is used where method (*a*) does not give the required range of control. Nearly all governors and regulators are controlled by variations of air pressure.

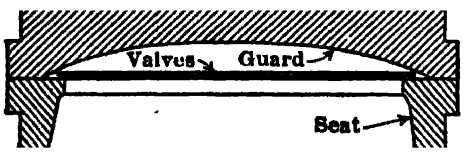


Fig 14. Worthington "Feather" Valve

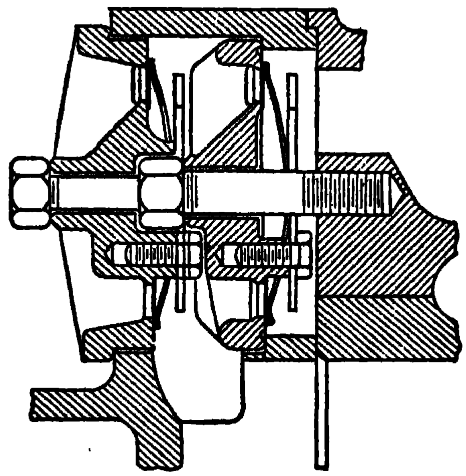


Fig 15. Sullivan "Wafer" Valve

Regulators for steam-driven compressors. AUTOMATIC CUT-OFF GOVERNORS, used on several makes, vary the cut-off point of the steam, thus changing the speed. They are actuated by both speed and air pressure, thus securing proper speed for a given capac and maintaining the most economical steam cut-off for each speed. DIRECT THROTTLING GOVERNORS are also controlled by air pressure and speed, throttling the steam at point of inlet. Cut-off is adjusted by hand. This method gives poorer steam economy than the automatic cut-off governors.

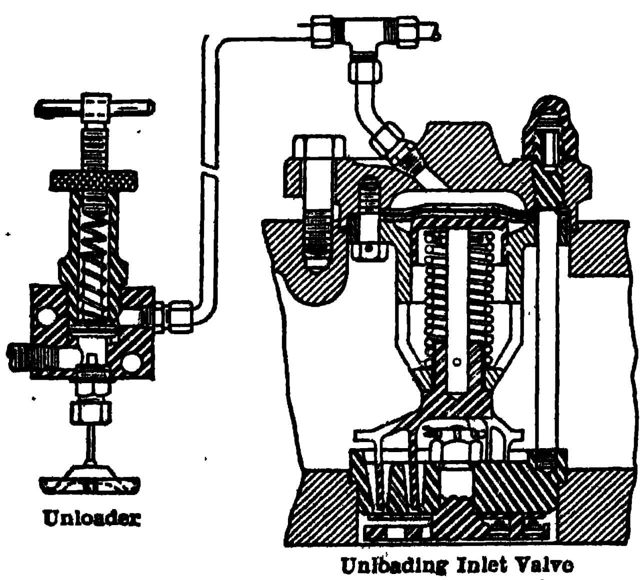


Fig 16. Inlet-valve Unloader and Pilot Valve (Chicago Pneumatic Tool Co)

Cylinder unloaders, for constant-speed compressors (see method *b* above).

Inlet-valve unloader holds the valve open by a fingered yoke, thereby allowing the air to move in and out of the cylinder without compression. This unloader is usually operated by air pressure, controlled by a pilot valve. On duplex compressors, partial unloading is obtained by opening the inlet valves in only one end of the cylinders (Fig 16).

Clearance unloaders consist of clearance pockets at each end of a double-acting cylinder. The pockets are connected with the cylinder by valves. As the pocket volume is added to the normal cylinder clearance, the amount of new air taken into the cylinder is reduced by that amount. By providing enough clearance pockets, an unlimited range of capacity is obtainable. In practice, there are 4 pockets in each cylinder, which are opened in pairs to insure balanced operation. On a duplex compressor this provides five-step control: 0, $^1/_4$, $^1/_2$, $^3/_4$ and full (Fig 17). The

clearance valves are operated by air pressure, controlled by air-operated pilots or by pressure-operated solenoid switches.

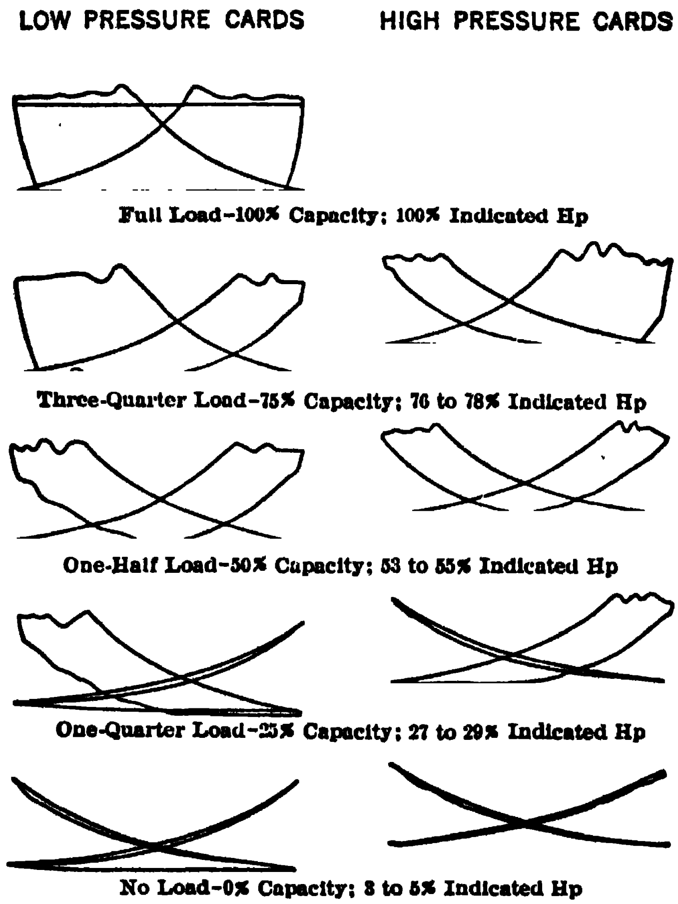


Fig 17. Indicator Cards, showing Operation of Ingersoll–Rand 5-step Clearance Control

Inlet plug unloader, used on some compressors of moderate capacity, consists of a valve which closes the compressor intake passage. It is pressure-operated by means of a small pilot valve (Fig 18).

Centrifugal unloader is an auxiliary used on some compressors having automatic start- and stop-control, so that the motor can come up to speed before compression begins. A small flyball governor opens a line to the high-press cylinder and exhausts it to atmosphere when the compressor stops; when the compressor is started, it closes this line only after operating speed is reached.

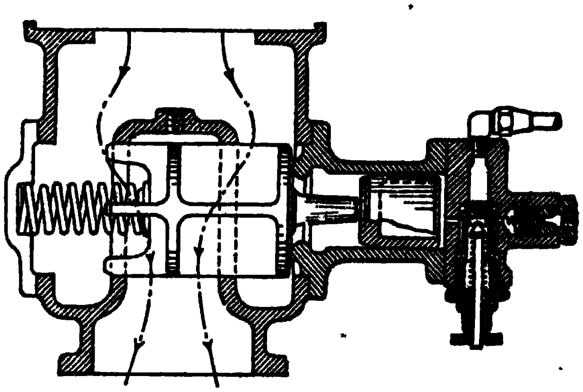


Fig 18. Inlet Plug Unloader

Speed governors are of the regulation flyball type, the compressor cylinders being unloaded by one of the methods already described. Throttling regulators can be employed to slow down the compressor when unloaded. This method is often applied to portable compressors driven by gasolene or oil engines.

Intercoolers cool the air in its passage between stages of a multi-stage compressor. Since cooling of compressed air reduces its volume, less air is handled by the high-press cyl, and proper cooling at this point largely determines the effic of stage compression. In good practice the air is cooled in the intercooler to within 20° F of the intake temp. Fig 19 shows a common type of intercooler; a shell containing horis " finned " tubes, through which water circulates, baffle plates distributing the air around and between the tubes. In AIR-COOLED COMPRESSORS, the compressed air passes through the intercooler

tubes, the fins conducting the heat to the atmospheric air blown over them by a fan. Intercooler press is a good gage for detecting trouble in the compressor cylinders. The correct abs intercooler press is roughly equal to abs intake press × cylinder ratio. Cylinder ratio is the displacement of low-press cyl or cylinders, divided by displacement of the

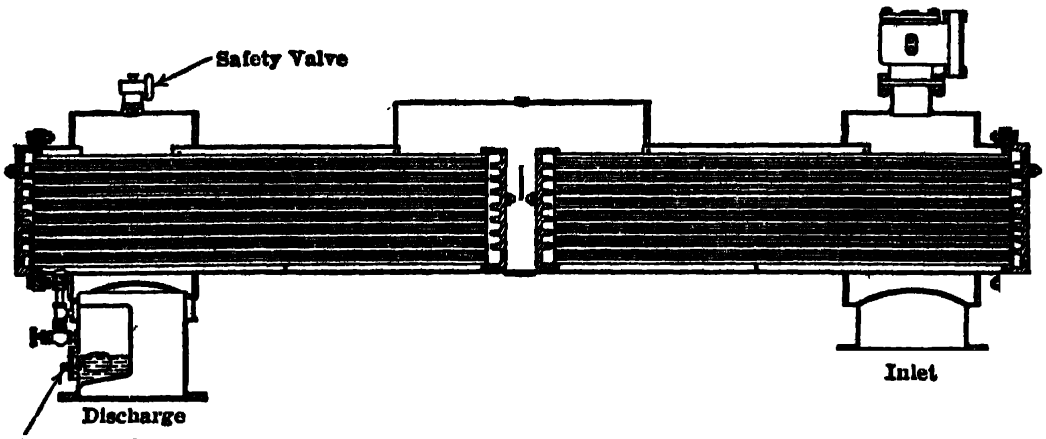


Fig 19. Intercooler (Ingersoll–Rand)

high-press cyl. On 100 lb, 2-stage, sea-level compressors, the intercooler press is usually 27–29 lb gage.

5. POSITIVE PRESSURE AND ROTARY BLOWERS

Positive pressure blowers work on the principle of rotary pump; lobed impellers mesh with each other like gear teeth, and, revolving within a casing, sweep the air before them.

Between casing and impellers, and between the impellers themselves, are clearance spaces through which some compressed air leaks back. This leakage (slip) is constant with speed of the blower, but varies with square foot of the press. Slip is measured by the number of revolutions required to offset it, and is about 20%. Capac is stated in terms of displacement per rpm, and delivery equals displacement times rotative speed, minus slip. Omitting slippage, 5–10% should cover all losses. Intake ports being large, the entering air is not heated; hence, a good vol effic, which, based upon power intake, is 60–85%. Rotative speeds range from 600 in the smaller, to 100 rpm in the larger blowers. They are not well suited to variable delivery, nor to pressures above 10 to 12 lb gage.

Rotary blowers and liquid-seal blowers are of the positive-press type, operating at comparatively high speeds. At present, they are used chiefly for very small vacuum pumps and compressors having little use in the mining field.

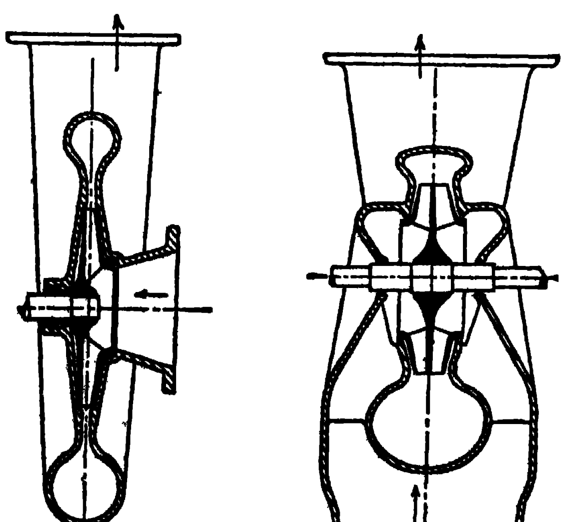


Fig 20. Single-stage Centrifugal Blowers (uncooled)

6. TURBO BLOWERS AND COMPRESSORS (1, 2, 10)

These operate by the centrifugal action of impellers rotating within a casing like a centrifugal pump. They have 1 to 15 stages, depending on the press, each consisting of a separate impeller followed by a diffuser and suitable return passage (Fig 20, 21). Advantages: Simplicity, economy of space and foundation, freedom from vibration, non-pulsating discharge, uniform press with wide variations in capac, light weight, few mechanical parts and oil-free discharge.

Types and applications. Motor-driven single-stage turbo blowers may supply air for mine ventilation, flotation, converters, or scavenging; also for cylinders of 2-cycle Diesel engines and supercharging them. They have capacities of 300–40 000 cu ft per min for

pressures of $^3/_4$ to 3 lb. In the smaller sizes, impellers may be mounted directly on the motor shaft. Steam turbine-driven units are also built. Small, multi-stage blowers for

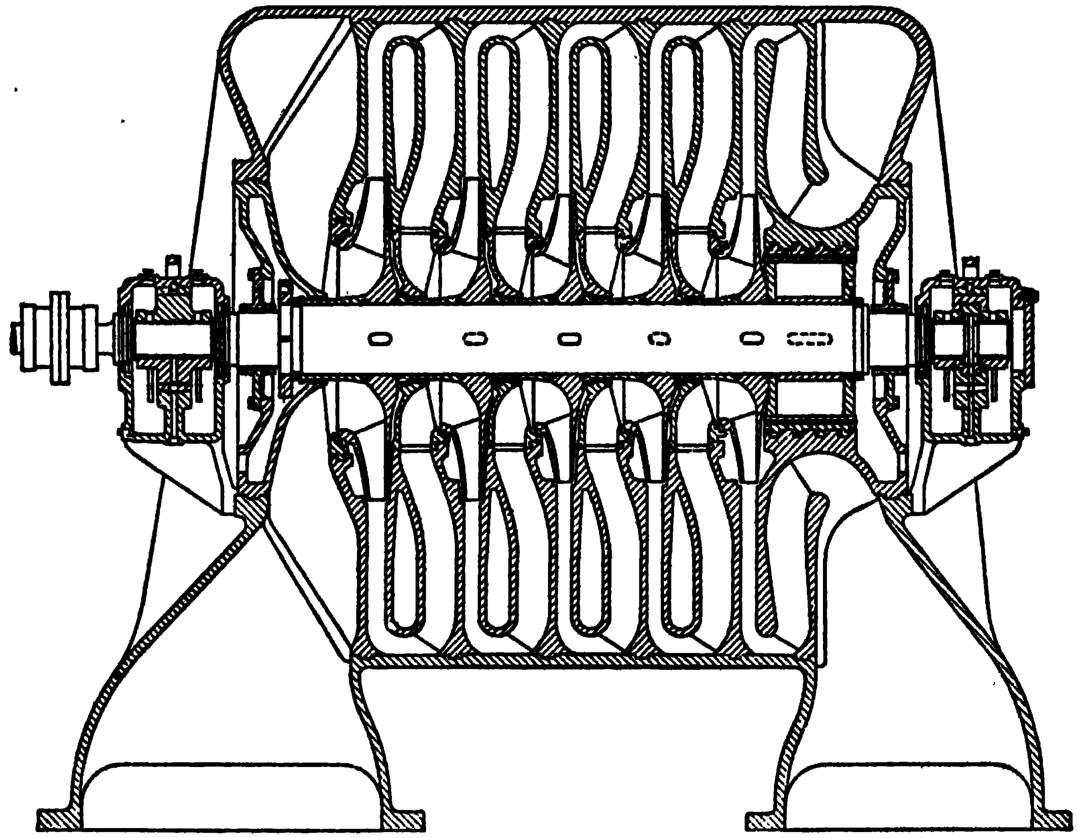


Fig 21. Ingersoll-Rand 5-stage Blower

pressures of 1–6 lb are made in sizes from 300 to 3 000 cu ft per min. Larger units, having 3–5 stages, as used for copper converter work, are available for pressures of 10–25 lb, in sizes to 30 000 cu ft per min, or higher.

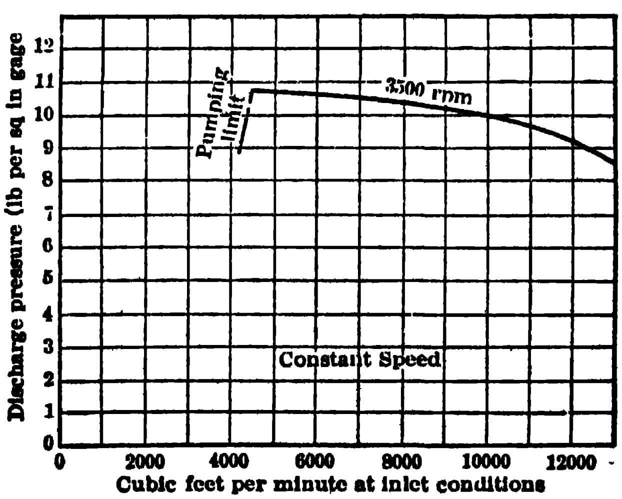


Fig 22. Characteristic Curve of Constant-speed Blower

Multi-stage blowers or turbo compressors for higher pressures can be obtained in capacities of 3 000 to 70 000 cu ft per min. In general, turbo-compressors for 100 lb press, as required in mine service, are practical as to first cost only in capacities above 5 000 cu ft per min.

Turbo units are regulated by constant pressure, constant volume, constant intake-pressure, or constant air-weight methods, as required by local conditions (Fig 22, 23).

The Flin-Flon mine (12), Manitoba, of Hudson Bay Mining and Smelting Co, uses a 20 000 cu ft per min turbo blower, which delivers air at 16 lb press for the copper converter. Fresnillo Mines have in service a 12 000 cu ft per min, 9-stage turbo compressor, operating at 95 lb gage press, and capable of 20% overload. It weighs 6 lb per cu ft per min of capacity (17).

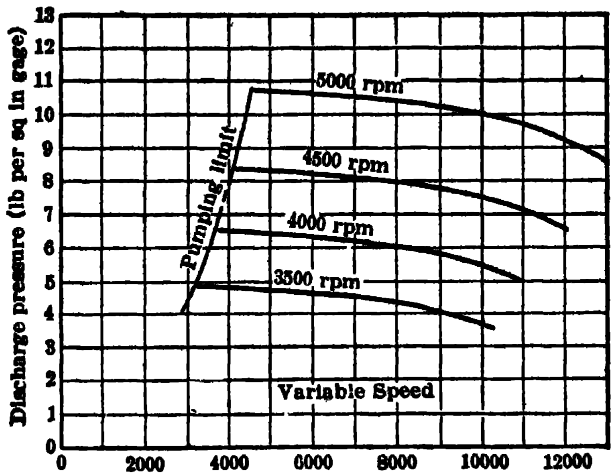


Fig 23. Characteristic Curves of Variable-speed Blower

7. HYDRAULIC AIR COMPRESSORS (1, 5)

These are of two types: (*a*) driven by water wheels (see Art 4); (*b*) operating by direct action of falling water.

Compression by direct action of falling water. Principle: If a stream of water be allowed to fall in a vertical pipe, any air entrained and mixed with the water on beginning its downward motion is compressed by weight of the water. Then, if the direction of flow be suddenly changed to the horiz, and the velocity lessened, the air will be liberated, and may be collected in a suitable container or separating chamber.

This method is most suitable for compressing large volumes of air, and is naturally limited to localities where the necessary quantity and fall of water are available. Air thus compressed has about 3% less oxygen than atmospheric air. A number of installations have been made, the first one at Magog, Quebec, in 1896 (1). Local circumstances will determine the method's feasibility. The alternative of using the water to generate electric power for driving reciprocating compressors should be considered.

8. COMPRESSOR ACCESSORIES

Aftercoolers (Fig 24) cool the air after compression, and hence deposit moisture between compressor and delivery pipe.

This is important, as moisture carried through the pipe lines to machine drills or other tools washes out lubricant and causes wear, freezing and water hammer; also, without aftercooling, pipe lines alternately contract and expand, developing leaks. A 2 000-cu ft per min compressor, running at capacity for 8 hr on intake air at 82° F and 75% humidity, will take in 1 200 lb of water vapor with the air (Fig 25). Condensed, this amounts to 3 bbl of water (43). Some moisture can also be removed by pipe-line separators, placed near the point of air usage (Fig 26).

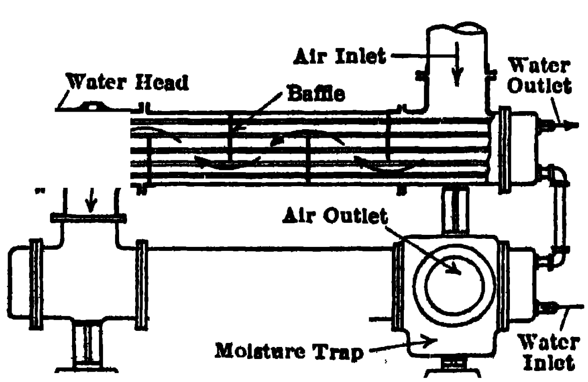


Fig 24. Ingersoll-Rand Horiz Aftercooler

Air receivers act as storage capacity during intervals when the demand exceeds the compressor output, and also help to eliminate pulsations in the discharge line. Some moisture is condensed from the air in passing through the receiver (Fig 27). Receiver size may be

roughly found by the formula $V_2 = \dfrac{14.7\,V_1}{P_2 + 14.7}$, where V_2 = vol of receiver, cu ft; V_1 = displacement of compressor, cu ft per min; and P_2 = discharge press, lb per sq in. This gives the approx minimum size; a larger receiver is preferable (2).

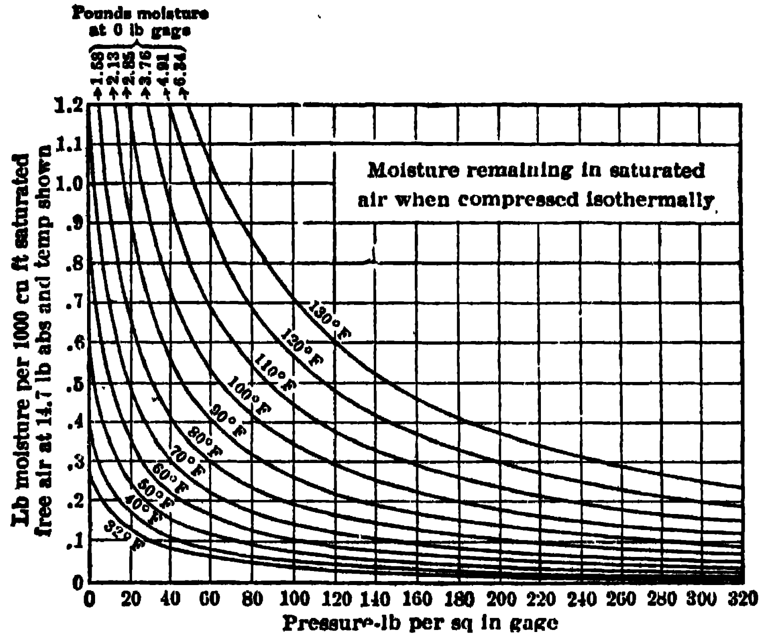


Fig 25. Moisture Carried in Air at Different Pressures

The Anglo American Mining Corp, So Africa, uses a 10 000-cu ft per min compressor to supply dehumidified air underground. Compression is in 3 stages to 130 lb, and air is then expanded in a motor cylinder to freezing point at about 90 lb per sq in. This devaporizes the air, which lowers the mine temp, and reduces the volume of ventilating air required (23, 24, 25).

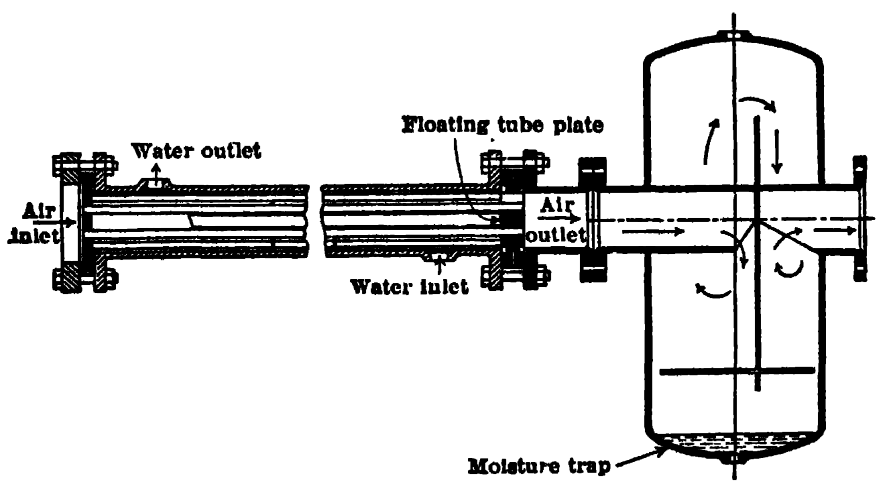


Fig 26. Pipe-line Type of Aftercooler

9. CARE AND OPERATION OF COMPRESSOR PLANTS

Installation layout (Fig 27). The compressor should be in a clean, light room, with ample space for making repairs, removing pistons, rods and intercooler tube nests; also for cleaning and inspection. The compressor intake must not be near any source of heat, as pipes discharging exhaust steam, nor exposed to water, dust, or other waste that can be blown by the wind into the range of intake suction. Concrete foundations are always preferable, though brick or stone can be used if more convenient.

Lubrication is vitally important for proper operation and life of the compressor, but must not be excessive. Oil specifications and instructions of compressor makers should always be followed (2, 21). CRANKCASE. Good quality lubricating oil does not wear out,

and may be re-used in the circulating system when treated by efficient filtering. CYLINDERS. Inlet and discharge valves should be removed occasionally and cleaned; by examining them it can be determined whether the cylinder is receiving proper lubrication. All wearing surfaces should show a greasy appearance; not dry. Oil should not accumulate

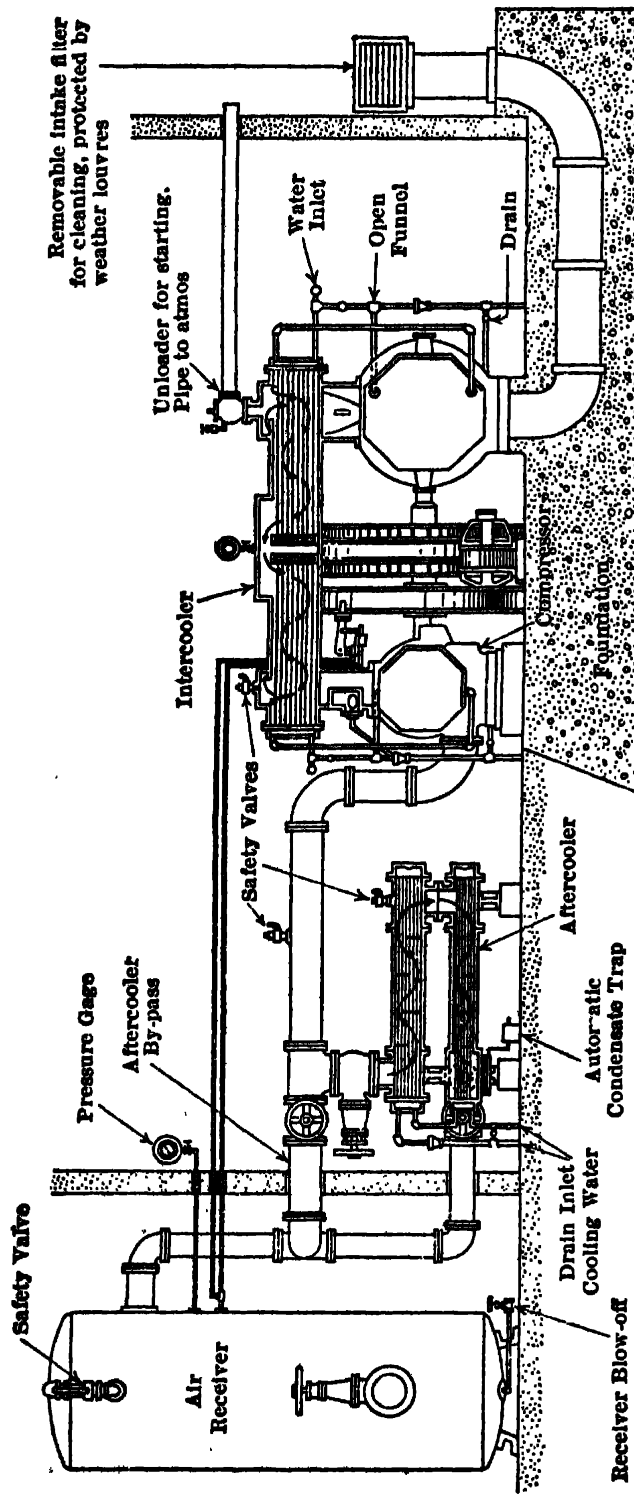


Fig 27. Typical Layout for Compressor Installation

Table 10. Compressor Air-cylinder Lubricant (2)

	Naphthene or asphalt base	Paraffin base
Flash point (open cup)	375° F, min	400° F, min
Viscosity at 100° F (S U V)	260–450 sec	250–400 sec
Pour point	5° F, max	30° F, max
Mineral acid neutralization No	Nil	Nil
Carbon residue (Conradson)	0.10%, max	0.50%, max

Table 11. Compressor Crankcase Lubricant

	Naphthene or asphalt base	Paraffin base
Flash point (open cup)	335° F, min	385° F, min
Viscosity at 100° F (S U V)	225–350 sec, min	200–300 sec, min
Pour point	5° F, max	30° F, max
Acid neutralization No.	0.10, max	0.10, max
Carbon residue (Conradson)	0.10%, max	0.50%, max
Steam emulsion value	75, max	75, max

in valve pockets or bottom of air cylinder. Table 10, 11 give specifications for suitable lubricants and Table 12, normal quantity of oil.

Intake air and piping. Wherever possible, the intake duct should be run to outside of the building, preferably on the north or coolest side, as the aver outdoor temp is con-

siderably below that of the engine room. Irrespective of the outside temp, the compressed air in a shop pipe-line will be near the temp of the shop when it reaches the tools. If the indoor temp is 70° F, and the compressor takes in 1 000 cu ft free air directly from the shop, it will deliver the same vol at the tools, because initial and final temperatures are the same. If the compressor intake air comes from outdoors, at 40° F, only 943 cu ft of free air will be required to deliver 1 000 cu ft at the indoor temp, a saving of 5.7%. The cooler the climate, the more pronounced the effect (Table 13).

If the compressor intake air comes from an engine room at 100° F, 1 057 cu ft free air will be required to deliver 1 000 cu ft at the indoor temp of 70° F. A case is recorded where, without a cold-air intake duct, 1 500 000 cu ft of free air delivered was costing $43.96 per day. After installing a duct from outside, the cost dropped to $40.42, an annual saving of $1 062 (43). Such savings are obtainable at the small expense of the intake duct.

Table 12. Air-cylinder Oil Required per 10 hr (2)

Cyl diam, in	Piston displacement cu ft	Rubbing surface, sq ft	Oil feed: Drops per min	Oil feed: Pints per 10 hr
Up to 6	Up to 65	Up to 500	1	0.05
6–8	65–125	500–750	1	0.08
8–10	125–225	750–1 100	1	0.11
10–12	225–350	1 100–1 500	1–2	0.14
12–15	350–600	1 500–2 000	2–3	0.20
15–18	600–1 000	2 000–2 600	3–4	0.27
18–24	1 000–1 800	2 600–3 600	4–5	0.36
24–30	1 800–3 000	3 600–4 800	5–6	0.48
30–36	3 000–4 500	4 800–6 000	6–8	0.60
36–42	4 500–6 500	6 000–7 500	8–10	0.74
42–48	6 500–9 000	7 500–9 000	10–12	0.90

Above figures are for 1 cyl only; for duplex or compound compressors, both cyls must be considered; based on approx 8000 drops per pint, at 75° F.

Table 13. Effect of Intake Temperature on Compressor Capacity (intake vol required to produce 1 000 cu ft free air at 70° F)

Temp of intake, °F	Relative intake vol required, cu ft	% hp saved	Temp of intake, °F	Relative intake vol required, cu ft	% hp saved
30	925	7.5	80	1 019	−1.9
40	943	5.7	90	1 038	−3.8
50	962	3.8	100	1 057	−5.7
60	981	1.9	110	1 076	−7.6
70	1 000	0.0	120	1 095	−9.5

The intake for a single compressor should have an area of at least 25% of area of air piston. For a multiple installation, care should be taken to provide a duct of sufficient size. Area of intake duct at the compressor nearest the air source should equal the sum of the areas of all the individual intake pipes; but the area of the main duct can be decreased after each branch to a compressor.

Air filters (2) on each intake duct are highly advisable. Tests show that they return 30% or more on the investment by increasing life and reducing maintenance. There are several kinds. In the ADHESIVE IMPINGEMENT TYPE (Fig 29), air is drawn in against baffles of metal, crimped wire, or glass-wool coated with a viscous substance. Metal filters are cleaned at intervals; the glass-wool, replaced. REPLACEMENT FILTERS are pre-coated with a suitable solution. After cleaning they are dipped in an oil adhesive. For large installations, AUTOMATIC FILTERS are used, in which the filter elements are constantly moving and passing through oil carried in the base (Fig 30). OIL-BATH FILTERS (Fig 31) are of the adhesive impingement type, but the filter element is partially immersed in oil. Capillary attraction and action of the intake air keep the elements properly coated. In FABRIC FILTERS, the air passes through filter cloths, cleaned by a reverse air flow, or a vacuum cleaner, or may be dry cleaned.

From 1 to 4 lb of dirt are taken into a 2 000-cu ft per min compressor, operating 10 hr a day for 10 days (26). The aver dust intake of a 1 000-cu ft compressor is 77 1/2 lb per year (44).

Without a filter, valves on a 5 000-cu ft per min compressor required cleaning every 2 weeks; after installing a filter, the valves were cleaned every 6 months (26).

On a large compressor, filters effected an annual saving of $2 228, or a return of 69.9% on filter cost. In a plant of 2 small compressors, filters effected a yearly saving in maintenance equal to 62.8% on cost of the filters (26).

Explosions in compressors and receivers (1, 32). CAUSES: excessively high internal press, due less to the air press carried than to ignition, in compressor, piping, or receiver, of

an explosive mixture of air and gas from lubricant. The underlying causes are: (*a*) excessive and improper lubrication, resulting in coating receiver, intercooler and air passages with gummy deposits of partly dried oil and dust; (*b*) ignition and volatilization of these deposits by unusual temp, due to broken or leaky valves. LUBRICATION should be in accordance with compressor builders' or oil companies' specifications. All oils give off combustible

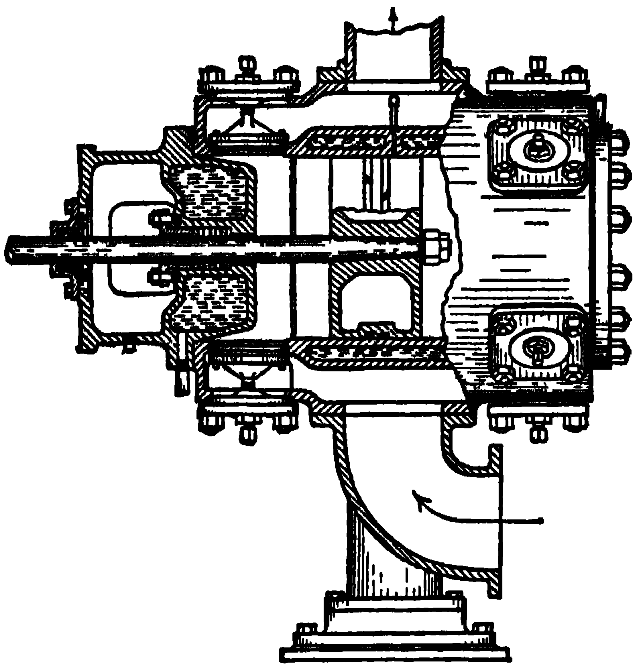


Fig 28. Typical Compressor Air Cylinder

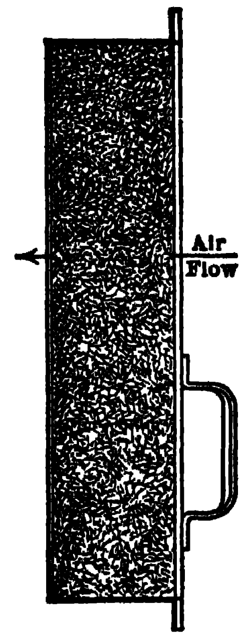


Fig 29. Metal Adhesive Impingement Air Filter

gases when heated. The lowest temp at which this begins is the FLASH POINT, the ignition temp being the BURNING POINT. As ordinary lubricating oils flash at about 250° F (a temp below the usual working temp of compressors), special HIGH-FLASH cylinder oils should be used (see Table 10, 11).

Temp due to compression depends upon initial temp, working press, and effic of the cooling devices (Art 8). A single-stage compressor, working at 100 lb, has a normal discharge temp of less than 470° F; a 100 lb 2-stage compressor at 100 lb, about 235° F (Table 3, Art 3). With leaky exhaust valves, the temp may be materially higher (1).

Precautions for avoiding high temp: (*a*) the compressor should be adapted to the conditions; (*b*) intake pipe covered by insulating material, and air taken from as cool a place as possible, outside of the engine room; a lowering of 5° F may increase effic by 1%;

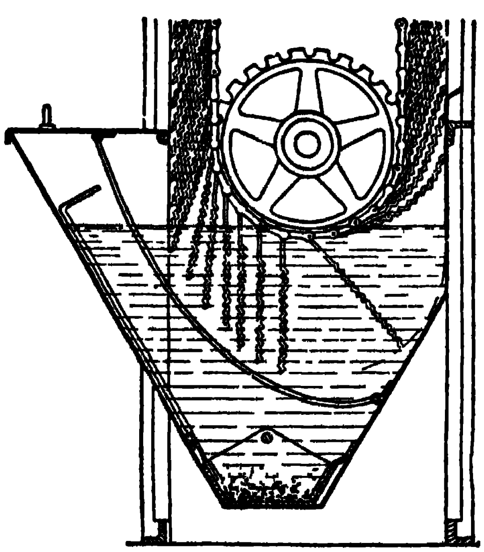


Fig 30. "Automatic" Filter Air

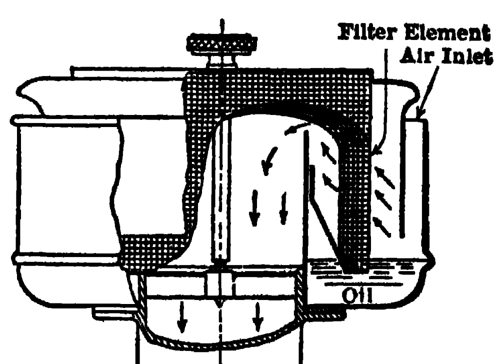


Fig 31. Oil-bath Impingement Air Filter

(*c*) unloader should be such as not to cause excessive heating when in operation; (*d*) largest possible area of cyl surface should be jacketed, and plenty of water used; (*e*) a stage compressor must have effic intercoolers; (*f*) aftercoolers increase effic; they eliminate oil vapor as well as moisture, thus minimizing danger of explosions; (*g*) if circulating water be reused, provide ample water coolers; (*h*) place receiver inlets near the top, and outlets about 1 ft above the bottom, to insure clean air; (*i*) receiver should have blow-off cocks at the bottom, and a man-hole for inspecting the interior; (*j*) place a recording thermometer between high-press cyl and receiver; (*k*) install an automatic safety valve with no shut-off valves between it and the compressor; (*l*) inlet air should be free from dust; (*m*) while running, never inject kerosene into the compressor to cut carbon deposit; (*n*) keep intercooler, aftercooler and receiver clean.

Freezing of moisture in compressed air. Moisture in air, under atmos conditions of 75% relative humidity, amounts to about 1 lb water per 1 000 cu ft air. MOISTURE-CARRYING CAPACITY of air depends upon its temp, but is independent of its press or density. Hence, if saturated air be compressed isothermally to 0.1 of its original volume, its water carrying capac is reduced to 0.1, and 0.9 of the water present is deposited. Hot air from the compressor cyl, on entering the receiver, deposits some of its moisture; the remainder is carried into the mains where it may freeze and so reduce the effective area of the pipe. Lastly, in air tools, the intense cold of expansion at exhaust freezes the moisture, which may obstruct ports and valves (1).

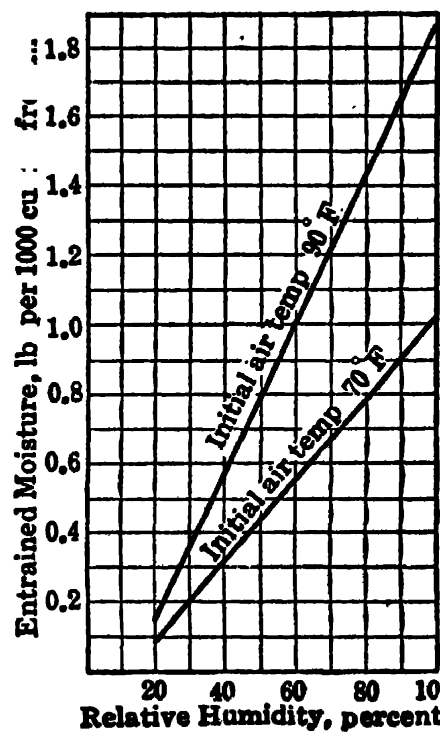


Fig 32. Variation of Entrained Moisture with Relative Humidity and Initial Air Temp (59)

Prevention of freezing. To remove moisture before the air reaches the points at which freezing may take place, several devices may be used, singly or in combination: (*a*) in aftercooler (Art 8), set as close to compressor as possible; (*b*) automatic water traps, at low points in air line; (*c*) a second receiver, near the compressed-air tools, the air being transmitted under higher press, and press reduced as it enters second receiver; (*d*) reheaters.

Quantity of water in compressed air can be approx estimated from Fig 32. To use the chart, start at bottom, with the per cent of relative humidity of the outside air at compressor intake; thence go vertically to the marked or an interpolated temp line; thence horiz to the left-hand scale (see also Fig 25).

Reheating compressed air (1, 3, 5, 35). Large savings are theoretically possible by reheating, due to increase of air volume. But, as it is difficult to apply reheaters to air lines serving rock drills underground, their use is limited to air hoists, pumps and motors. Cost of reheating is about $^1/_8$ of cost of producing the same increase in vol by compressing additional air (5). METHODS IN GENERAL USE: (*a*) passing the air through a cast-iron chamber or coil of pipe, heated by a fire or current of hot gases or steam; (*b*) adding heat within the body of air itself, by combustion of fuel, injection of steam or hot water, or placing in the air line an electric-resistance coil (1). Reheaters in general consist of a hollow shell, having inlet and outlet connections with the air main, and containing tubes, coils or deflecting plates for bringing the compressed air in contact with the heating elements.

10. COST OF COMPRESSED AIR EQUIPMENT AND ITS OPERATION

Installed cost of compressors varies with their size and type, and transport costs involved. Table 14 gives approx costs of compressors without accessories. In general, small compressors cost more per cu ft of capac than large ones.

Cost of compressed-air power includes a number of factors: (*a*) operating cost of compressor, for power attendance, lubricating oil and repairs; (*b*) depreciation; (*c*) interest on investment, plus taxes and insurance. Power cost is the largest item, especially for

continuous operation. The other charges amount to so small a proportion of the total that comparisons are often made on basis of power costs alone (2).

Table 14. Approx Cost of Compressors, Without Accessories (various sources, 1940). Based on compressors working at 100 lb, except as noted; freight not included

Type of compressor for 100-lb delivery press	Piston displacement, cu ft per min	Cost per cu ft per min output	Type of compressor for 100-lb delivery press	Piston displacement, cu ft per min	Cost per cu ft per min output
Small air-cooled, including motor	1–50	$46–12	Two-stage, duplex, power-driven	325–1 350	$ 7–4
Two-stage, air-cooled, including motor	70–450	16–5	Two-stage, duplex, steam-driven	275–7 200	13–5
Single-stage, power-driven, straight-line	70–630	7–3	Two-stage, duplex, including direct connected motor	340–6 200	13–5
Single-stage, steam-driven, straight-line	100–600	16–5	Portable, including engine	50–420	24–13
Single-stage, straight-line, including direct-connected motor	200–630	13–6	Mine-car, including motor	60–180	31–18
			Turbo-compressors	5 000–10 000	10–7

Mark W. Booth (22) gives cost of steam- and motor-driven compressors for inbye power as $7.50 to $10 per cu ft per min.

In general, compressed air costs between 2 and 20 cts per 1 000 cu ft, based on the factors listed above. "Compressed-air Data" gives a cost of 6.1 cts per 1 000 cu ft, including labor, interest, deprec, power and repairs, for an elec-driven compressor, based on a sliding current cost of 1 to 20 cts per kw-hr (2).

Cost of compressed-air tools varies considerably. Table 15 gives approx costs of machines commonly used in mines.

Table 15. Approx Cost of Air-operated Equipment (various sources, 1940)

Type of tool	Weight	Cost	Type of tool	Weight	Cost
Hand-held drill	27–80	$165–300	Drill-steel sharpeners	350–4 500	$400–2 400
Sinkers (heavy, hand-held drills)	75–120	300–400	Oil furnaces for heating drill steel	550–900	200–350
Drifters	120–300	350–650	Air-line lubricators	7–12	8–15
Stopers, hand-rotated	60–105	250–300	Wagon-mounted drills	900–2 400	900–1 600
Stopers, automatic rotation	80–125	350–400	Columns and tripods	100–250	80–125
Air-operated sump pumps	50–120	150–350	Clay diggers	20–30	90–115

Tables 16, 17 and 18 give costs of compressed-air service, including drill steel, at different mines.

Table 16. Costs of Air, Drills and Steel per Ton (2 000 lb) of Ore Hoisted (39)

Lead and zinc mines		Lead and zinc mines	
No 1, Tri-State District, 1928	$0.127	Bunker Hill & Sullivan, Idaho, 1928	$0.229
No 2, Tri-State District, 1928	0.114	Bunker Hill & Sullivan, Idaho, 1931	0.204
No 3, Tri-State District, 1927	0.075	Page, Idaho, 1928	0.280
Waco, Tri-State District, 1927–29	0.068	Silver King Coalition, Idaho, 1929	0.854
Barr, Tri-State District, 1929	0.066	Park-Utah, 1928	0.139
Hartley-Grantham, Tri-State District, 1929	0.052	Tintic Standard Mine, Idaho, 1929	0.770
Hartley, Tri-State District, 1930	0.078	Ground Hog, N M, 1930	0.169
No 8, Southeastern Mo, 1928	0.081	Pecos, N M, 1929	0.141
Mascot No 2, Tenn, 1929	0.047	Morning Mine, Idaho, 1928	0.122
		Hecla & Star Mines, Idaho, 1928	0.070

Table 17. Direct Stoping Costs for Air, Drills and Steel per Ton (2 000 lb) of Ore Mined (41)

Mine	Cost	Mine	Cost
Open-stope mines		**Cut-and-fill stopes**	
Aver of 7 lead mines in Tri-State district, 1927–1930	$0.102	Cold Springs, sorted ore hoisted, 1931	$1.116
D C & E Mine, Orango, Mo, 1937 (45)	0.094	Cold Springs, crude ore broken, 1931	0.172
No 8, Southeast Mo, 1928	0.080	Lucky Tiger, Sonora, N M, 1924	1.052
Mascot, Tenn, 1929	0.042	Eighty-five, aver 2 stopes, 1929–30	0.49
Marquette Range, No 1 hard-ore mine, 1928	0.093	Tesuitlan, Mex, 1930–31	0.252
Vanadium Rifle, Colo, 1931	0.090	Champion, 1930	0.128
Mineville, N Y, 1927	0.101	Pilares, Mex, 1929	0.175
Burra-Burra, Ducktown, Tenn, 1928	0.051	Morning, 1928	0.189
Burra-Burra, Ducktown, Tenn, 1932	0.081	Hecla, 1928	0.215
No 1, Menominee Range, Mich, 1928	0.071	LaColorado, Mex, 1929	0.100
No 2, Marquette Range, 1928	0.027	Campbell, Ariz, aver 3 stopes, 1926–30	0.255
Spring Hill, Mont, 1929–30	0.124	Magma, 1928	0.414
Sherritt Gordon, 1931–32	0.150	**Square-set stopes**	
Hanover Bessemer, Fierro, N M, 1930	0.075	Argonaut, 1929	$0.101
Shrinkage-stopes		United Verde Extension, 1928	0.244
Nevada-Massachusetts, 1928	$0.309	Bunker Hill & Sullivan, 1928	0.194
Harmony, 1929	0.430	Page, Idaho, 1928	0.16
Hillside, 1929	0.202	Park Utah, 1928	0.192
Daisy, 1929	0.190	Tintic Standard, 1929	0.521
Cortey, 1929	0.250	Silver King Coalition, 1929	0.472
Eighty-five, aver 4 stopes, 1925–29	0.470	**Block-caving stopes**	
Verde Central, 1929–30	0.485	Ray, 1928	$0.0027
Teck Hughes, 1928	0.309	Inspiration, 1928	0.005
Kirkland Lake, 1930	0.230	Humboldt, 1928	0.045
Vipond, 1929–30	0.250	Braden, Chile, 1928	0.0128
Elkoro, 1930	0.162	**Top-slicing stopes**	
Mogollon, N M, 1922	0.340	Mesabi, No 1, 1929	$0.0231
Sylvanite, 1930	0.468	Marquette, No 9, 1929	0.085
Engels, 1928	0.135	Marquette, No 10, 1928	0.040
Mount Hope, 1930	0.179	Miami, 1916	0.030
Sub-level caving stopes		Calumet & Arizona, 1916	0.100
Eureka-Asteroid, 1929	$0.070		
Montreal, 1928	0.18		

Table 18. Cost of Air, Drills and Steel, Michigan Copper Mines (40)

Kind of work	Aver cost per foot of development, 1927				
	Large, open stopes with stulls	Large, open stopes with pillars	Long, narrow open stopes with narrow pillars	Shrinkage stopes	Cut-and-fill stopes
Shaft sinking	......	$1.40	$1.21	$1.01	$3.36
Drifting	$0.562	0.518	0.447	0.529	0.926
Raising	......	0.437	......	0.66	......
Crosscutting	......	......	0.447	0.530	0.667
Winzes	......	......	......	0.66	......

At United Verde copper mine, Ariz, cost of compressed air per ft of drift was $1.30, and drill steel, $1.10 (38).

During experiments at the DC&E mine, Orango, Mo, in 1937, cost of compressed air only was 30 cts per ton mined; compressed-air cost for sharpening bits, 1.62 cts per bit (45).

11. ROCK DRILLS (1, 2, 34)

Reciprocating or piston drill, in which the steel is clamped to the piston, was the original type; not now used, except in special cases, as underwater drilling. In HAMMER

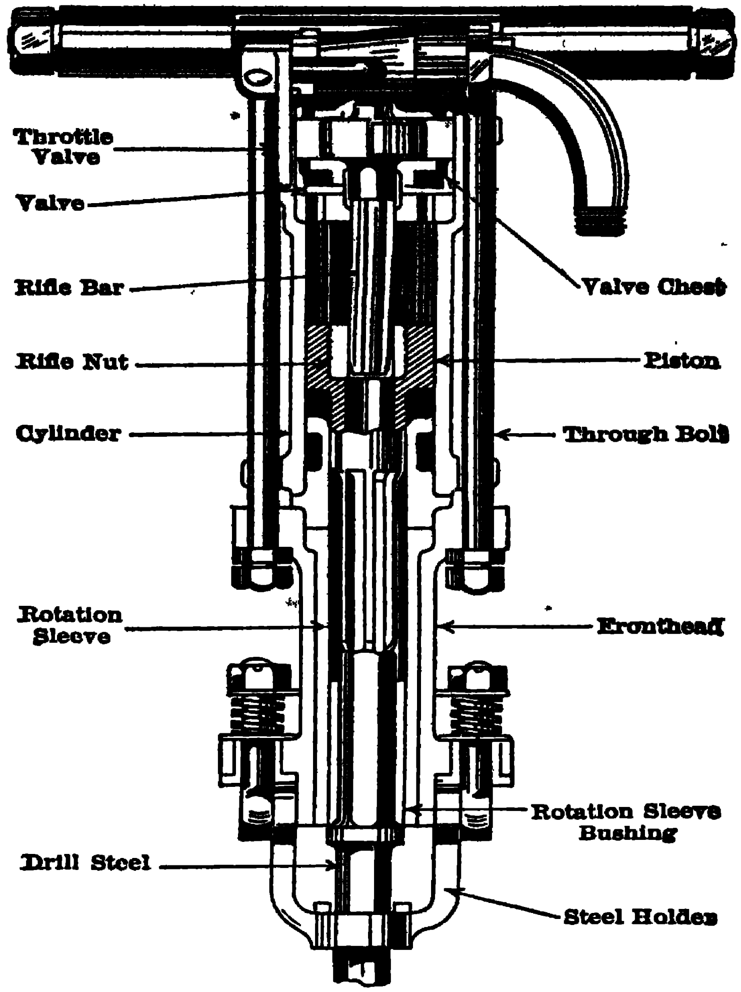


Fig 33. Typical Hand-held Hammer Drill

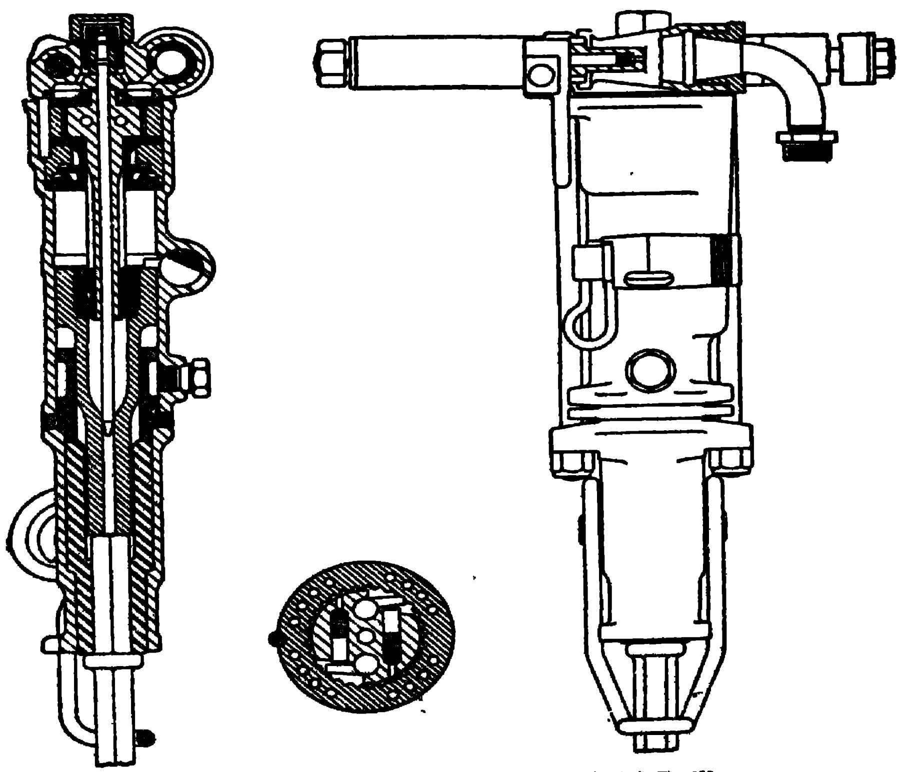


Fig 34. Ingersoll-Rand Hand-held Drill

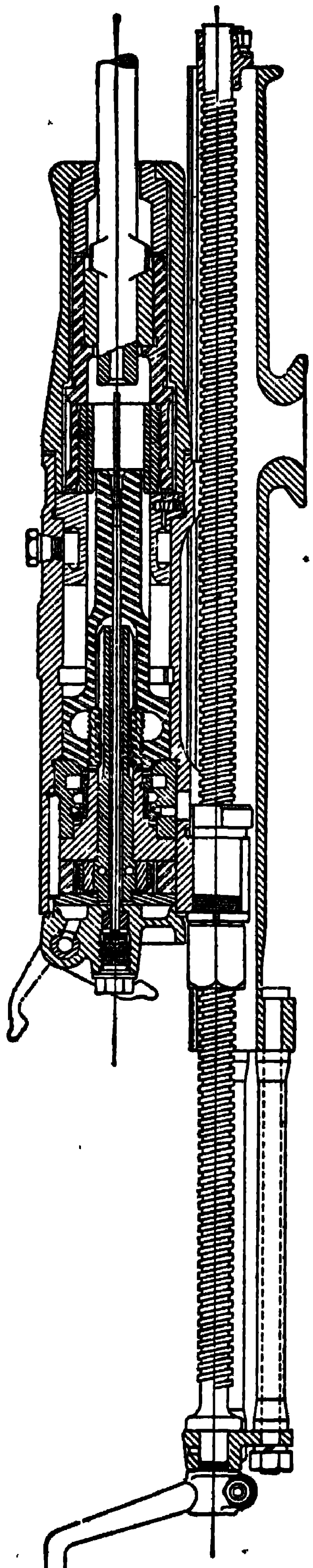


Fig 35. Sullivan Drifter Drill

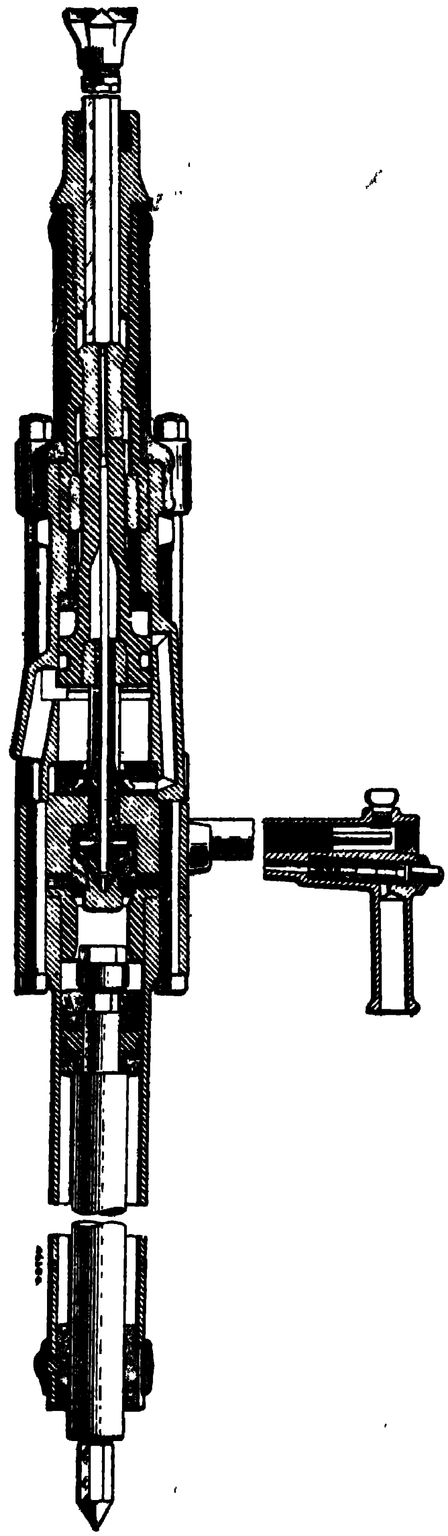


Fig 36. Ingersoll-Rand Automatically-rotated Stoper Drill

DRILLS, the piston strikes the drill steel, which is loosely held in a chuck, the bit remaining in contact with the bottom of the hole except during a slight rebound. In certain types, an " anvil block " is placed between the piston and end of drill steel.

Drill steel ordinarily used has a small hole running through it longitudinally; it is called " hollow steel." " Solid steel," without a hole, is sometimes used for auger drills and hand-rotated stopers. Either water or air, or a combination of both, is forced through hollow steel to remove cuttings from the hole. According to the mode of cleaning the

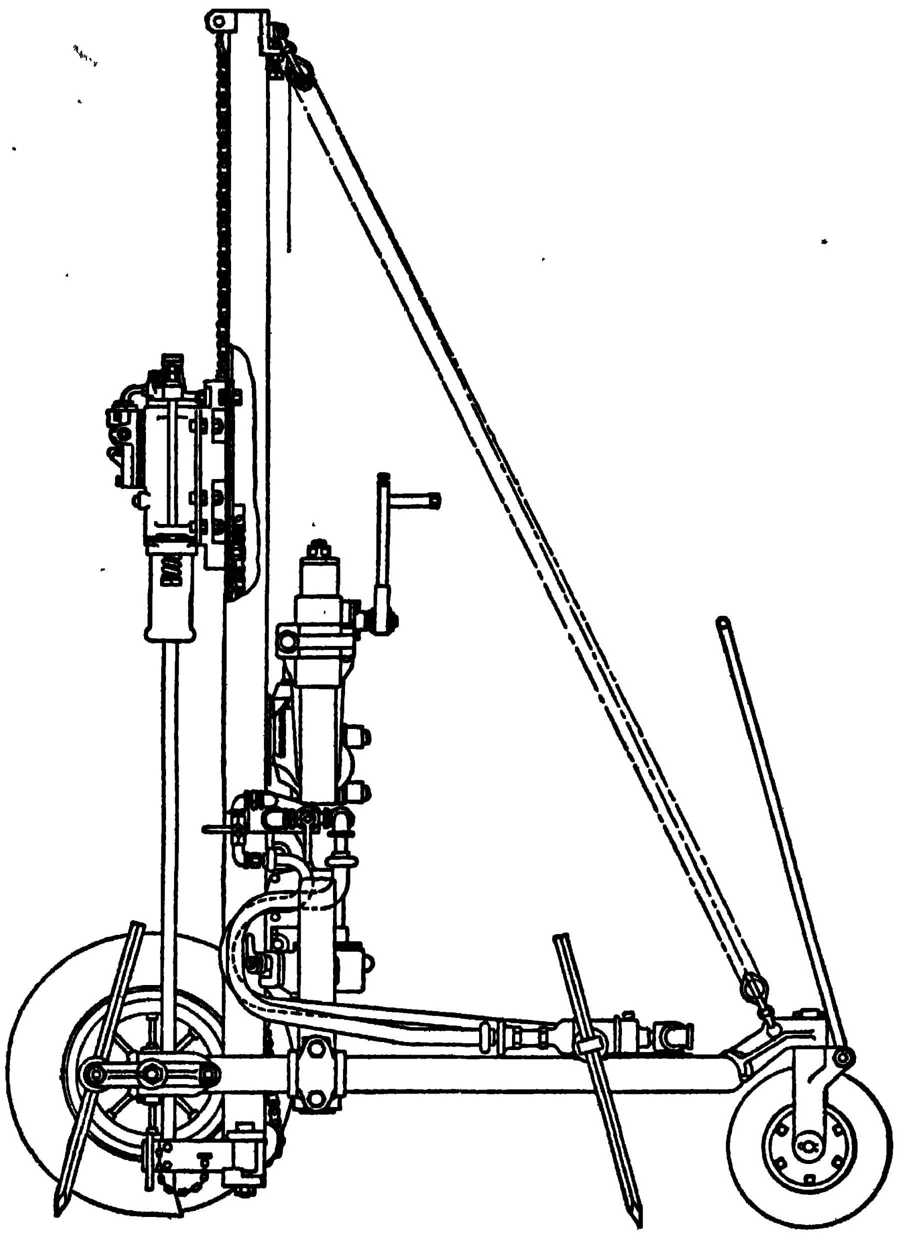


Fig 37. Ingersoll-Rand Wagon Drill

hole, drills are classed as " dry " and " wet." Modern drills strike from 1 600 to 2 200 blows per min.

Hammer drills. Principal classes, and the field of work for each are: (*a*) HAND-HELD DRILLS, mounted, and self-rotating, used chiefly for down-hole work (Fig 33, 34); (*b*) DRIFTERS, mounted drills designed for holes at or near the horiz (Fig 35); (*c*) STOPERS, designed for drilling upward (Fig 36); (*d*) WAGON-DRILLS, drifter or hand-held type, mounted in guides on portable rigs (Fig 37). Guides usually adjustable, for drilling at any angle. Other classes, variations of the above, might be cited; as " SINKERS," large, powerful hand-held drills, for shaft sinking and other heavy work. Hand-held drills are sometimes mounted for horiz drilling, and drifters set on tripods or quarry bars for down

holes. Drills are of many sizes, varying in weight and rotative and blowing power. The most effective drill for given service depends on character of the rock, depth of holes, air press available and other factors; experience is the best guide.

Table 19. Classification of Rock Drills (21)

Hand-held machines	Hand-held machines proper	Non-rotating	Drills	Plug drills	Light drills for plug-and-feather work
			Breakers	Known as paving breakers and classified by wt	Light, 30–40 lb
					Medium, 50–60 lb
					Heavy, 80 lb
			Drivers	Pile-drivers, classified by size of sheeting they will drive	Light, 120–130 lb
					Heavy, 150 lb
		Self-rotating	Usually known as hand-held sinkers; classified by wt		Very light, 25–40 lb
					Light, 40–50 lb
					Medium, 50–65 lb
					Heavy, 65 lb and up
	Converted drifters	Heavy sinkers; classified by cyl bore		3 in	Some intermediate sizes are made, but these are the usual sizes
				3 1/2 in	
Drills on fixed mountings	Hand feed	Drifters, classified by cyl bore	3 in	Some intermediate sizes are made, but these are usual sizes	
			3 1/2 in		
			4 in	Drifters may be mounted on portable carriages or jumbos; differ from wagon-drills in that the mounting is not an integral part of the unit, and not sold with it	
	Mechanical feed		3 in		
			3 1/2 in	Some drifters have air feeds; differ from stopers in that the drill is not supported by feed cyl	
			4 in		
Drills on air legs	Hand-rotated	Stopers, classified partly by wt, partly by bore, into 2 classes: light and heavy			Light, 60–75 lb
					Heavy, 80–100 lb
	Self-rotating	Distinction based on kind of work they are suited for			Light, 80–100 lb
					Heavy, 110–130 lb
Wagon-mounted drills	Classified according to length of feed and bore of drill			Light wagon-drill, feed 6 ft or less	With 3-in drill, or smaller
					With 3 1/2-in drill
					With 4-in drill
				Heavy wagon-drill, feed 10 ft or more	With 3 1/2-in drill
					With 4-in drill

Hand-held drills (Figs 33, 34) are made in dry, wet or blower types. The blowers have greater hole-cleaning capacity than the standard dry type, due to the greater vol of air blown through the steel. They weigh from 30 to 85 lb. Sinker drills weigh from 75 to 135 lb. A special drill, weighing about 40 lb, and having strong rotation and light blow, uses solid twisted auger-steel for drilling coal and soft ground. Hand-held drills are ordinarily employed for down holes; sometimes for drifting by using a conversion mounting or a plank or sling. They are also adapted for stoping by adding an air-feed leg (38).

Drifter drills (Fig 35) are generally self-rotating, and used wet. A few have independent air-motor-operated rotation. Drifters weigh from 115 to 225 lb. They are usually

clamped to a cross arm on a column. In quarries, the mounting is often a horiz bar, supported on legs or a tripod. In some regions, as used for stoping, they are fed by a screw which pushes the drill forward on its carriage, or withdraws it for changing the steel. Recently, several mechanical feeds have come into use; some are air feeds, using air pressure or air motors to move the drill forward; a successful form has a ratchet device operated by the vibration of the drill. Drifters usually have 3-, 3 $^1/_2$-, or 4-in cylinders. The 3-in drill weighs about 125 lb, the 3 $^1/_2$-in 145 lb, and the 4-in 180 to 220 lb. Other sizes, as 2 $^5/_8$ in, 3 $^1/_8$-in and 3 $^3/_4$-in, are furnished by some makers.

Stoper drills are primarily for overhead work, as raising and back stoping. Standard-feed stopers have the air-feed cylinder attached to the backhead of the drill, the air-feed-

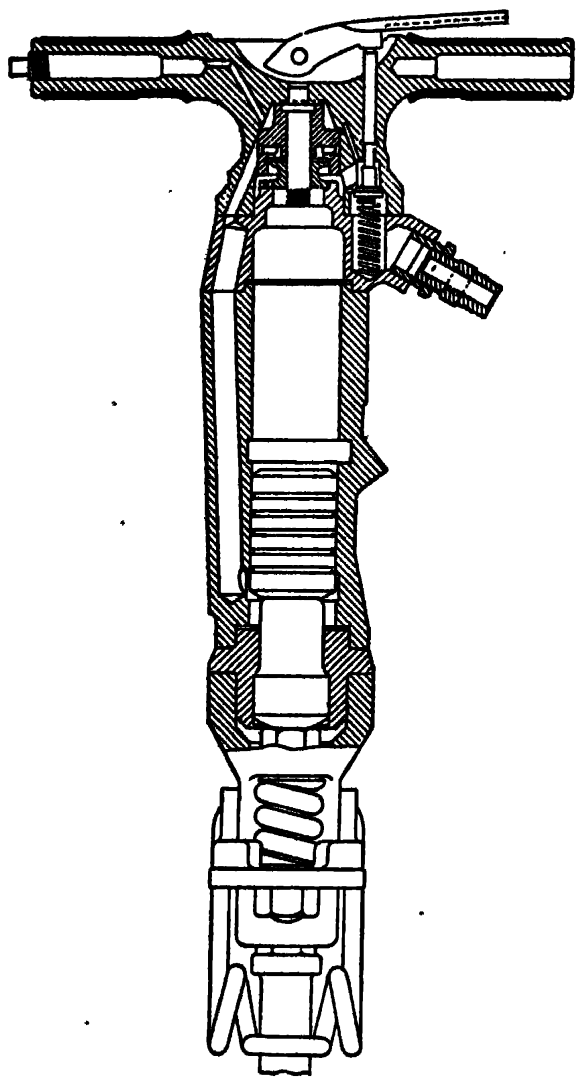


Fig 38. Chicago Pneumatic Demolition Tool

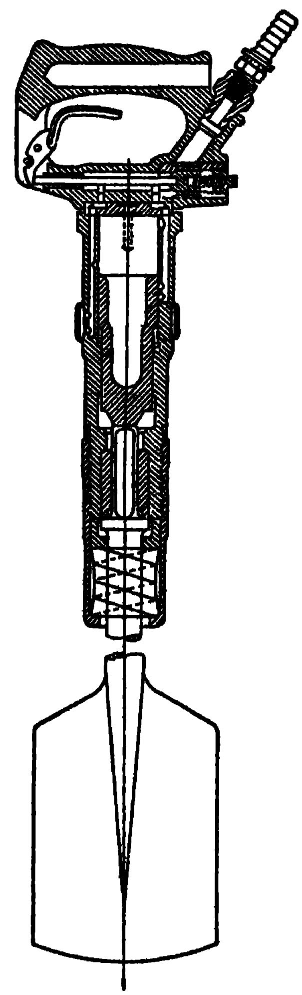


Fig 39. Thor Clay Digger

piston extending out of the lower end. In reverse-feed stopers the feed piston is attached to the backhead, the air-feed cyl moving away from it. This permits clamping the drill to a bar, for either vert or horiz holes. Hand-rotated stopers, both wet and dry, weigh 80 to 100 lb. They usually have chucks for hexagon or quarter-octagon steel. Automatically-rotated stopers (Fig 36), weighing 90 to 125 lb, are made for wet or dry drilling, the wet being ordinarily used. Length of feed, 20 to 25 in; total length of drill extended, 70 to 90 in. In excavating a 15 × 8 ft mine shaft, 866 ft deep, with stopers, the cost in moderately hard schist was $52.00 per ft (29). Other air-driven tools are shown in Fig 38, 39.

Valves for hammer drills comprise: SPOOL VALVES, contained in a short valve-chest; TUBULAR VALVES, of several forms, sometimes called sliding-sleeve or double-seated spools. They reciprocate on a central core, which is often the upper portion of the rifle-bar of the drill. In some, the air passes through the center of the valve. DISK VALVES are circular

disks or annular rings seating on the flat surface of the ring. They are usually placed on the upper portion of the rifle bar. FLAPPER and BUTTERFLY VALVES rock back and forth, have a fulcrum or pivot, seating on flat surfaces at either side. BALL VALVES are spherical steel balls, seating on ground seats. In VALVELESS DRILLS the piston acts as the valve. This construction is used mainly on the lighter tools, as clay spades.

Rotation devices comprise: RATCHET AND RIFLE-BAR ROTATION, in which the rear end of the helical fluted rifle-bar has pawls working in a ratchet ring in the backhead. Rotation is transmitted to the chuck or rotation sleeve by straight flutes in the front part of the piston. PNEUMATIC ROTATION utilizes the fluted piston and rifle-bar as in the preceding method, but the pawls are forced into position by compressed air instead of springs. INDEPENDENT ROTATION is separate from the piston action. In some types an air motor

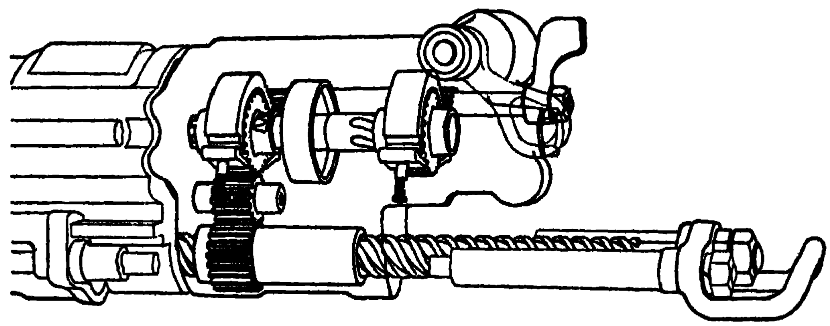


Fig 40. Gardner-Denver Motor Feed for Drifter Drill

drives the chuck sleeve through gears; or the motor drives eccentrics, transmitting power to gyrating yokes, which in turn drive the chuck sleeve.

Drill feeds. HAND-FEED for drifters consists of a long screw turned by a crank engaging a nut attached to the drill head. One type of AUTOMATIC FEED uses a ratchet device, which turns the feed screw by the drill's vibration; another form uses an air motor (Fig 40). PNEUMATIC FEEDS use air pistons similar to the stoper feed. Air-motor feeds are also used on wagon-drills and heavy drills for down hole work in quarries.

Drill mountings. TRIPOD MOUNTING is used for drifter drills in open-cut work and quarries. COLUMN MOUNTINGS are employed in tunnelling and general underground work. PNEUMATIC-FEED COLUMNS are telescopic; extended and held in place by compressed air. HYDRAULIC COLUMNS are similar to single-screw columns, but a hydraulic jack operated by a short lever replaces the jack screw. HAND-HELD DRILL MOUNTINGS are separate feed-screw mountings, to which a hand-held drill can be clamped for converting it temporarily to a light drifter. QUARRY-BARS are 10- or 12-ft horiz bars, supported on 4 legs. Chief application is for dimension-stone quarrying, with drifter drills and channelling tools. They usually have rack and pinion to control travel of the drill on the bar. BROACHING MOUNTINGS are heavy mountings, usually quarry bars, with air-motor feed, for heavy drills on broaching and down-hole work. DRILL CARRIAGES (jumbos) are portable mountings for 1 to 12 or more drills, standing on a track; used in driving headings of large tunnels. WAGON MOUNTINGS are portable mountings for drifter drills (see *ante*).

Note. Use of tunnel columns and carriages, and shaft bars, is treated in Sec 6; tripods, quarry bars, gadders, channelers, and towers for submarine drilling, in Sec 5.

Tripod parts (Fig 41): *a*, saddle clamp; *b*, clamp jaw; *c*, saddle throughbolt; *d*, front leg and ear bolts; *e*, side and back-leg bolts; *f*, right ear; *f'*, left ear; *i*, weight hanger; *j*, *k*, leg collar and set-screw; *l*, leg pointer; *m*, weight. VARIATIONS of the tripod: Lewis-hole tripod, for drilling parallel holes close together; quadrant tripod, for submarine drilling from a platform. With the latter, the drill is swung aside when changing bits.

Parts of double- and single-screw columns (Fig 42): 1, column top; 2, column pipe; 3, jack screw for double-screw column; 4, base-block bolt (large); 5, base-block bolt (small); 6, jack-screw cup; 7, column base block (2 pcs); 8, jack-screw nut, double-screw column; 10, safety clamp; 11, column arm; 12, column-arm cap and bolts; 15, cap set-screw; 16, jack-screw cap; 17, jack-screw; 18, lock nut; 19, jack-screw nut. COLUMN CLAMP: *b*, clamp-jaw; *c*, clamp through-bolt; *p*, *q*, column clamp, top and bottom; *r*, *s*, front and back clamp bolts.

Air pressure for rock drills. Most drills are designed for press of 70–100 lb, and are most efficient at these pressures. Higher press cause abnormal wear and breakage of drill parts, and increased steel breakage. Low press decreases drilling speed; at press as low as 40 lb, stopers are damaged by pounding of the steel on anvil block and front head.

Relation of air consumption and pressure to drilling speed. Elaborate tests at United Verde mine, Ariz (70), to determine the most ECONOMICAL AIR PRESS for hammer drills, lead to following

conclusions: 1. Pressure exceeding 90 lb gives little increase in mech effic. 2. Depth drilled per air ihp is greatest for sinker type of drill at 90 lb press, and increases slowly at higher press for other hammer drills. 3. Factor of desirability increases only slowly at press above 100 lb. 4. Press higher than about 85 lb at the drill increases upkeep. 5. High press increases breakage and loss of drill runner's time. 6. Increased breakage of drill steel tends to limit the press. 7. Under conditions at this mine, the most economical press appears to be 90–95 lb.

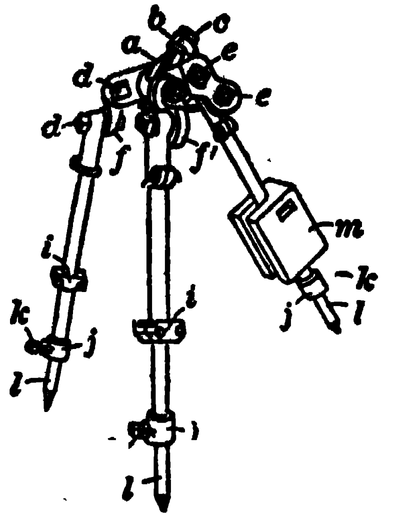


Fig 41. Tripod Drill Mounting

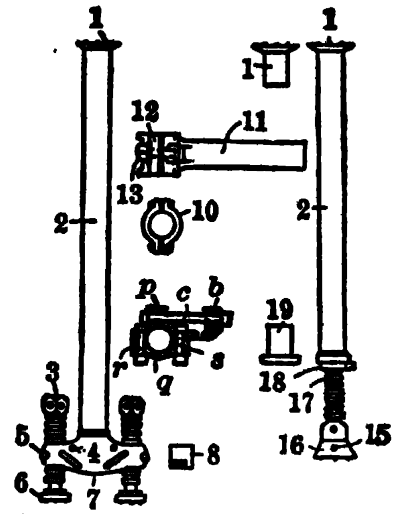


Fig 42. Double- and Single-screw Columns

For tunneling, where overall economy may depend upon speed of advance, drills are run at max rather than at most economical speed. In Table 20, based upon above data, drilling speed at 80 lb press is taken as 100%.

Factory tests with blocks of selected stone (71): drills were mounted on column, and run at press of 60, 70, 80, 90 and 100 lb (Fig 43). Each machine drilled 1 min with each of 3 bits, 1 7/8, 1 3/4 and 1 5/8-in gage. Air consumed was measured by a displacement meter (Art 19).

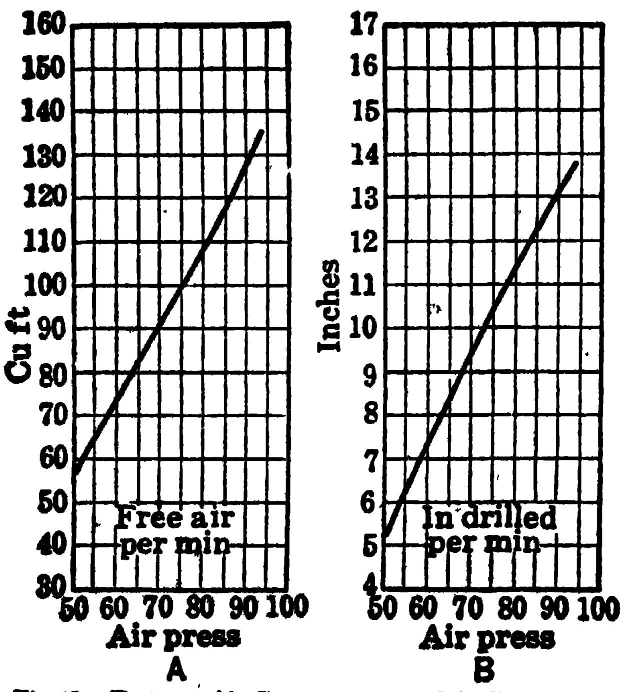


Fig 43. Tests on Air Consumption and Drilling Speed

Standardization of drills and their equipment, though desirable, can not be expected in machines undergoing frequent improvement; but, where different makes or types of drills are run in the same mine, they should be so selected as to use the same sizes of steel, and interchangeable hose connections. Columns, column-arms, and fittings should be reduced to fewest number of types that will do the required work. Nuts and bolts on arm, collar, and clamp, that are frequently used, should take the same wrench; or, at least, their number should be minimised.

Table 20. Relation of Drilling Speed to Air Press

Gage press	Speed, %	Gage press	Speed, %
40	31	90	119
50	47	100	140
60	64	110	160
70	82	120	180
80	100	130	198

Suggestions to rock drillers (33). Keep side rods drawn up snug, but not so tight as to close the springs. If piston repeatedly strikes the cyl head, when feeding, send machine to the shop. Don't run with wide-open throttle when cranking a bit out of the hole, or when cranking back in soft ground; crowding the crank reduces drilling speed. Don't remove feed-screw support. Keep oil plugs tight. Don't twist off lubricator or valve

bolts by careless use of wrench. Don't use red rubber gaskets in water spud. Don't drill with machine out of line. Don't drill with loose swing-clamp. Don't use too much water, especially in down holes. Don't take a chance on using a chipped or slightly upset shank. Examine anvil block frequently, to see if it is causing shank trouble. Don't run bits too far when drilling. Call the nipper's attention to the trouble with the machine, when sending it to shop for repairs.

Care and treatment of hammer drills (34). The bit must be held down to its work; otherwise the shank slips and hammer strikes the cyl head, causing breakage; bit shank should receive full force of blow. Drills should be washed with kerosene about once a week, and should be oiled before reassembling. The hardened steel bushing, taking the wear incident to screwing the air hose on and off the drill, must not be removed from the handle by drill runner, otherwise the threads are sooner worn or stripped. Examine and gage bit shanks at intervals, to see that they are not too short nor too badly worn to fit the socket properly.

Air consumption. In hammer drills the type of valve and piston, and the speed of blows per min, vary so greatly that it is impossible to prepare a general table of air consumption. For any drill it will vary widely with hardness of rock, air press, and skill of the driller. Manufacturers can furnish the approx air consumption of a particular drill, or it can be measured by an air meter (Art 19). Table 21 gives approx figures.

Table 21. Approx Air Consumption of Rock Drills (various sources, 1940)

Type of drill	Air consumed, cu ft free air per min, compressed to 90 lb gage	Type of drill	Air consumed, cu ft free air per min, compressed to 90 lb gage
25–35-lb hand-held drills..	60–70	200–300-lb drifters........	190–250
35–45-lb hand-held drills..	75–90	Hand-rotated stopers.....	65–120
45–65-lb hand-held drills..	85–120	Automatically-rotated stopers....................	120–180
65–90-lb hand-held drills..	100–135	Drill-steel sharpeners......	30–145
115–140-lb drifters.........	145–175	Oil furnaces.............	7– 25
140–200-lb drifters....	165–250		

Table 22. Multipliers to Find Compressor Capacity for Operating 1 to 70 Drills at Altitudes above Sea-level (1, 3)

Altitude, ft	Number of drills: 1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	50	60	70
	Multipliers																		
0	1.	1.8	2.7	3.4	4.1	4.8	5.4	6.0	6.5	7.1	8.1	9.5	11.7	13.7	15.8	21.4	25.5	29.4	33.2
1 000	1.03	1.85	2.78	3.5	4.22	4.94	5.56	6.18	6.69	7.3	8.34	9.78	12.05	14.1	16.3	22.0	26.26	30.3	34.2
2 000	1.07	1.92	2.89	3.64	4.39	5.14	5.78	6.42	6.95	7.60	8.67	10.17	12.52	14.66	16.9	22.9	27.28	31.46	35.52
3 000	1.10	1.98	2.97	3.74	4.51	5.28	5.94	6.6	7.15	7.81	8.91	10.45	12.87	15.07	17.38	23.54	28.05	32.34	36.52
4 000	1.14	2.05	3.08	3.88	4.67	5.47	6.15	6.84	7.41	8.09	9.23	10.83	13.34	15.62	18.01	24.4	29.07	33.52	37.8
5 000	1.17	2.10	3.16	3.98	4.8	5.62	6.32	7.02	7.61	8.31	9.48	11.12	13.69	16.03	18.49	25.04	29.84	34.4	38.84
6 000	1.20	2.16	3.24	4.08	4.9	5.76	6.48	7.2	7.8	8.52	9.72	11.4	14.04	16.44	18.96	25.68	30.6	35.4	39.84
7 000	1.23	2.21	3.32	4.18	5.04	5.9	6.64	7.38	7.99	8.73	9.96	11.68	14.39	16.85	19.43	26.32	31.36	36.16	40.84
8 000	1.26	2.27	3.40	4.28	5.17	6.05	6.8	7.56	8.19	8.95	10.21	11.97	14.74	17.26	19.9	29.96	32.13	37.04	41.83
9 000	1.29	2.32	3.48	4.39	5.29	6.19	6.96	7.74	8.38	9.16	10.45	12.26	15.09	17.67	20.38	27.6	32.9	37.92	42.83
10 000	1.32	2.38	3.56	4.49	5.41	6.34	7.13	7.92	8.58	9.37	10.69	12.54	15.44	18.08	20.86	28.25	33.66	38.8	43.82
12 000	1.37	2.47	3.70	4.66	5.62	6.57	7.4	8.22	8.9	9.73	11.1	13.02	16.03	18.77	21.64	29.32	34.94	40.28	45.48
15 000	1.43	2.57	3.86	4.86	5.86	6.86	7.72	8.58	9.3	10.15	12.58	13.58	16.73	19.59	22.59	30.6	36.46	42.04	47.47

Table 22 is used as shown by the following example: Required the vol of free air to operate 30 drills at 9 000 ft altitude, at a gage press of 80 lb. Sea-level consumption per

drill assumed as 190 cu ft per min. From Table, the factor for 30 drills at 9 000 ft is 20.38; multiplying 190 cu ft by 20.38 gives 3 872 cu ft free air per min, which is the actual free air delivery of a compressor for the above outfit under aver conditions, to which must be added pipe-line losses of friction and leakage. The multipliers in the table allow for time consumed in changing bits, cleaning hole and moving drill; all drilling operations being intermittent. In soft rock, where the actual time of drilling is short, more drills can be run with a compressor of given size than when working in hard rock, where the drills work continuously for longer periods. Soft material, therefore, increases the air consumption for a given number of drills, and hard material decreases it. Compressor capacities are those required at the drills, no allowance being made in the table for losses due to transmission pipe friction or leaky pipes. In selecting a compressor, these factors must be taken into account.

Following are formulas for dealing with rock-drill work:

Time to change bits and clean hole, e min; to drill 1 ft, d min per ft; to move drill and lost time, l min; to set up drill, g min.

Length of feed, ft, f; depth of hole, ft, D; time to drill length of feed, $fd = r$. Number of bits per hole, $D \div f$.

Total time per hole $= (e + fd)(D \div f) + (l + g)$, including moving and setting up drill. Working minutes per day $= M$ (say, 600).

$$\text{Number of holes per day} = \frac{M}{(e + fd)(D \div f) + (l + g)}$$

$$\text{Ft hole drilled per day} = \frac{DM}{(e + fd)(D \div f) + (l + g)}$$

Cost per day $= C$ on standard basis: hence, cost per ft of hole

$$= \frac{C(e + fd)(D \div f) + (l + g)}{DM} = R = \frac{C}{M}\left[\frac{e}{f} + d + \frac{l + g}{D}\right]$$

If $l + g = S$, or aver time to move drill and set up; and $e + r = T$, or aver time for changing bits, cleaning hole, and drilling the length of feed, the above formula reduces to:

$$R = \frac{CT}{Mf} + \frac{CS}{MD}$$

Note that length of feed (f) is the difference in length between successive bits, which is ordinarily 2 ft. In the formula the possible length of feed is immaterial; the value is affected only by the lengths of bit. If it be difficult to get a follower bit into the hole, the feed should be several inches longer than the difference between lengths of bits, since this affects time of changing bits and cleaning the hole.

Water pressure for wet drills is from 20 to 90 lb; if greater than the air press, water will be forced into the drill cyl and carry away lubricant. Normally, 25 to 35 gal of water

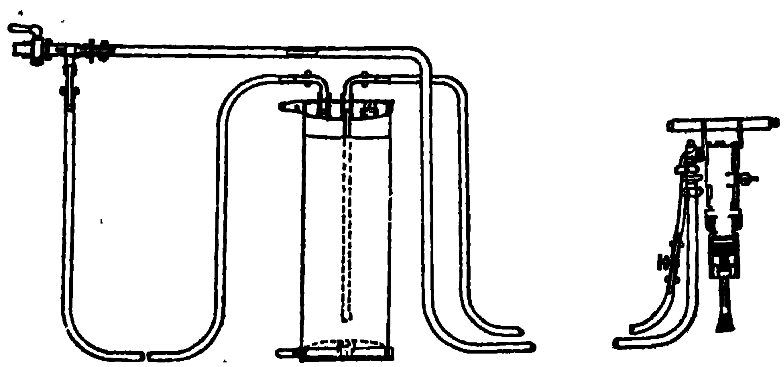


Fig 44. Water-feed Tank for Hand-held Drills

are required per drill per shift. A mode of regulating water press is to set up in the mine workings, at about 200-ft intervals, several tanks overflowing into one another. Drill-makers furnish 15 to 18-gal tanks, in which compressed air gives the desired water press (Fig 44).

Lubrication of drills (3) is of great importance. There is little possibility of their getting too much oil; with enough, oil will appear on the drill-steel shanks. Oil for rock drills must resist washing away, withstand low temp (often below 32° F) caused by sudden expansion of air in the tool, and flow to all parts requiring lubrication. Poor oil becomes sticky. All major oil companies supply lubricants that have been tested and approved.

Most hammer drills have automatic lubricators. Air-line lubricators (Fig 45) are equally satisfactory; an oil container is placed in the pipe leading to each drill, the oil being picked up by the air and carried under press to all parts of the drill. They are dependable, but not vital to adequate lubrication, as the regular oiling system suffices, if the oil is suitable. Drills must be kept clean. They should be dismantled at regular intervals, the parts cleaned with kerosene, and oiled as they are reassembled. If any part has been scored because of inadequate lubrication, the scored surfaces should be smoothed.

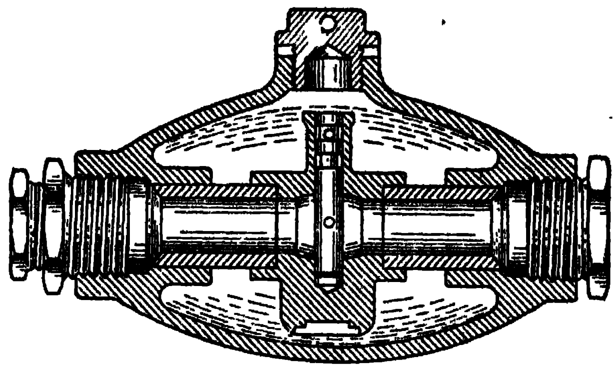


Fig 45. Gardner-Denver Air-line Lubricator

Ingersoll-Rand Co (Drill Doctor's Book) gives the following specifications for a lubricant for drills, drill-steel sharpeners, and bit and shank punches. 1. QUALITY: (*a*) lubricant shall be well-refined petroleum oil, free from suspended matter and water, or a good petroleum oil compounded with enough animal oil to form a satisfactory lubricant for drills where water or wet air is encountered, or a good grade of free-flowing liquid grease, which will not separate upon standing; (*b*) Castor machine oil (aluminum soap), and oils containing graphite, are not approved. 2. PHYSICAL PROPERTIES of oils found good in actual field service are: flash point (open cup), 350° F min; viscosity 100° F (Saybolt Universal), 450 sec min, 700 sec max; pour point, 15° F max; mineral-acid neutralization number, 0.10 max; free fatty acid (% Oleic), 0.40 max; steam emulsion number, 300 max. Viscosity of liquid greases can not always be determined; in any event, the lubricant should be free-flowing, conforming to above specifications except for viscosity.

12. OIL FURNACES AND SHARPENERS FOR DRILL STEEL

Note. Data on Drill Steel, Bits for Hand and Machine Drilling, Hand Sharpening and Tempering of Bits, are given in Sec 5.

Power sharpeners, operated by compressed air, are now in general use at large mines and wherever rock drilling is done on a large scale. Chief advantages over hand-sharpen-

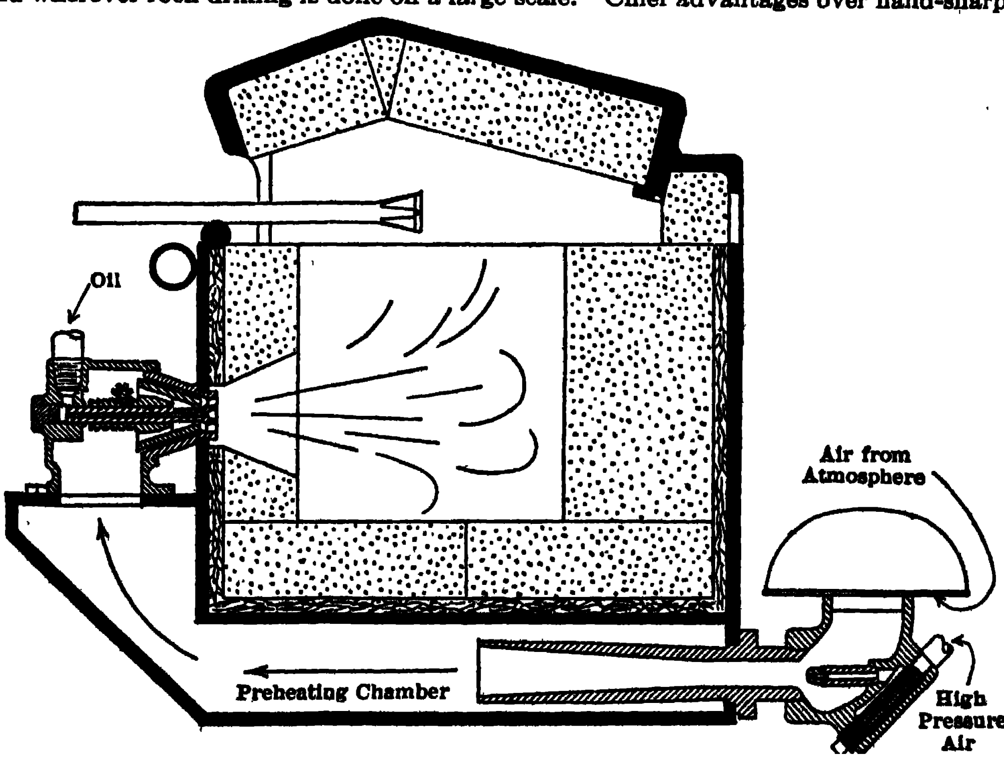


Fig 46. Ingersoll-Rand Oil-fired Furnace for Heating Drill Steel

ing: (*a*) greater rapidity and economy, since 1 smith can sharpen as many bits as several hand blacksmiths with their helpers; (*b*) more accurate shaping and gaging of bits compos-

ing a set. Dies being used, all bits of same shape and gage are identical, with symmetrical wings and sharp, square corners. Hence, mechanically forged bits always " follow " well, decreasing trouble from stuck (fitchered) bits. Accurate shanks formed by power sharpeners decrease drill piston breakage. Shanking done by hand is facilitated by small devices, giving quicker and more accurate results than hand work alone.

Detachable bits are widely used. They can be resharpened several times before discarding. Air, elec- and gasolene-driven grinders, with special bit holders and grinding guides, are furnished for this purpose. Some makes of detachable bits are attached to drill rod by threads, which may be forged on rod by a power sharpener and special dies.

Oil-fired furnaces (Fig 46) with compressed-air blowers are widely used to heat steel for forging. See also Fig 47.

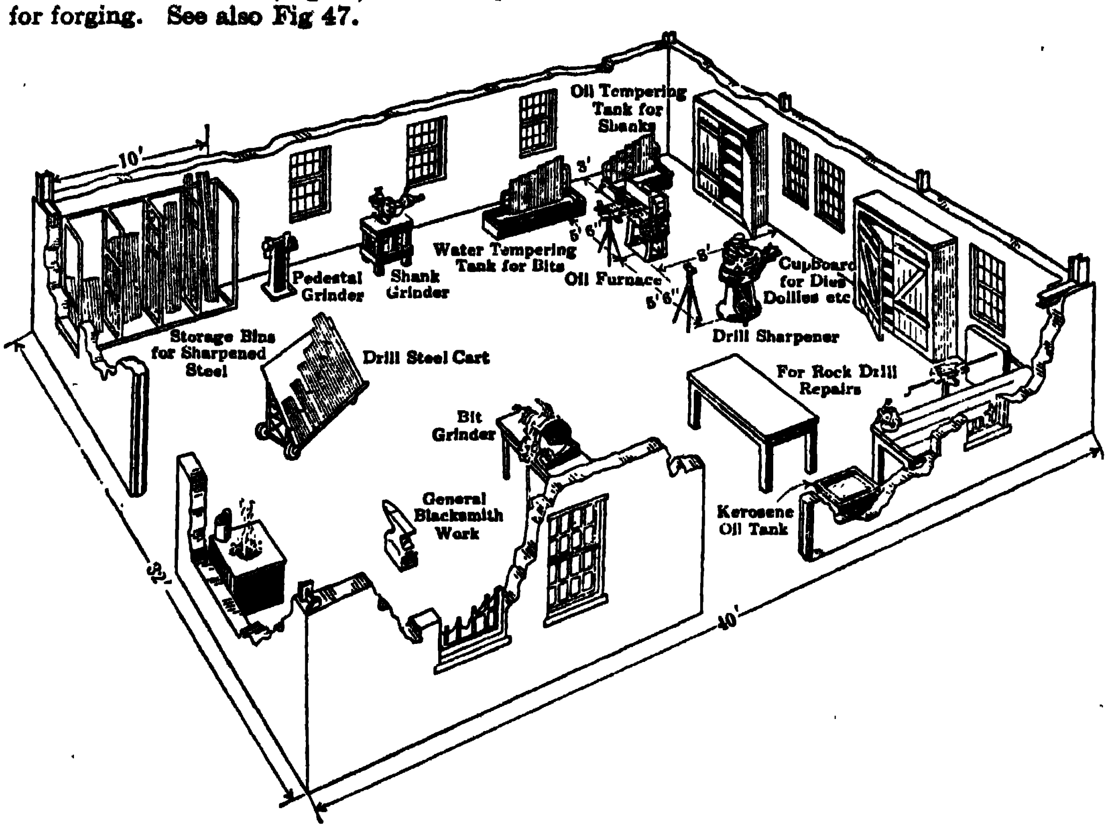


Fig 47. Diagram of Well Laid-out Mine Blacksmith Shop

13. PNEUMATIC QUARRY TOOLS

Channelers cut out blocks of stone without blasting. They can cut a continuous slot, at any angle from vert to horiz, by a gang of chisels, the entire machine moving back and forth along the slot at a speed that will keep the bottom smooth and even.

Power required. For operating by compressed air, the makers state that the air consumption when actually cutting is as follows for single-cyl channelers: 4 1/2-in cyl, 190 cu ft free air per min; 6 1/2-in, 230; 7-in, 300; 8-in, 400 cu ft per min. Duplex channeler requires 5–8% less than twice as much air as a simplex. A reheater reduces air consumption about 20%. Using steam, a 25 to 30-hp boiler serves a channeler having a 6 1/2–7-in cyl. Few air-operated machines are now used.

Rate of cutting depends on size of channeler and hardness of rock: 4 1/2-in cyl machines cut 60–75 sq ft per day in slate, 200 sq ft in soapstone; 6 1/2-in cyl machines cut 80–150 sq ft in marble, 260 sq ft in soapstone; 7-in cyl machines, 60–75 sq ft in tough sandstone, 130–210 sq ft in limestone; 8-in cyl machines, 60–76 sq ft per day in gneiss.

Channeling with drills (line drilling). A piston or hammer drill is mounted on a quarry bar, and drills a row of holes close together. The rock between holes is broken by a drill with broaching bit.

Stone surfacing (dressing) can be done with small hammer drills, so mounted as to move in a horiz plane over the block of stone.

14. COMPRESSED-AIR COAL CUTTERS (1)

These are widely used to replace hand work, for undercutting the face of coal, preparatory to breaking it by blasting or wedging. Objects: to economize cost of mining; to

decrease proportion of fines; to increase rate of production from a given extent of workings (1). Air-driven machines only are considered here; for elec cutters, see Sec 16.

Endless-chain cutters are furnished in longwall types for longwall mining (Fig 49) and in short-wall types for room-and-pillar mining. The bits are carried on an endless chain, driven by an air motor; the machine is fed forward by a chain or rope anchored at one end, and feeding into sprockets or winding on a drum in the machine itself.

Rotary-bar cutters are almost obsolete. They are used like shortwall machines, but have a toothed bar instead of an endless chain.

Pick machines are either unmounted or mounted on low wheels. The bit makes a cut 40–80 in deep. Unmounted hand picks are light-weight hammer drills, 15–18 in long and weighing 20–30 lb.

"Radialaxe" coal cutter is a rock drill, with bit rotation device, mounted on a column, and provided with worm gear, so that the machine may be swung radially in undercutting or shearing.

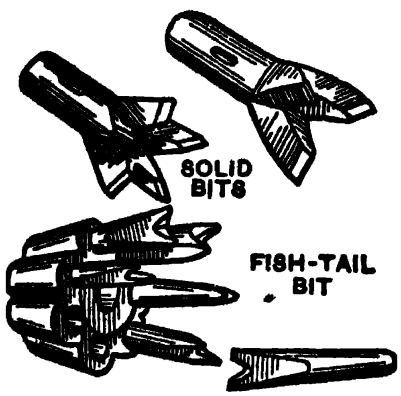


Fig 48. Bits for "Radialaxe" Coal Cutter

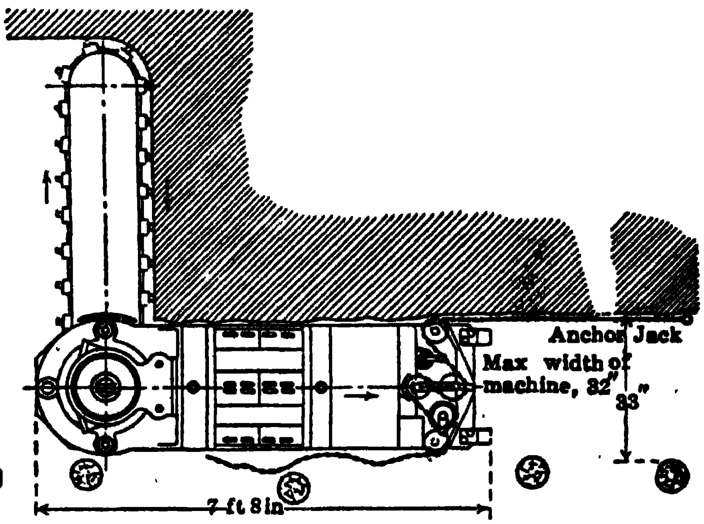


Fig 49. Sullivan Longwall Ironclad Coal Cutter

"Radialaxe" bits are of 2 forms (Fig 48). Solid bits, forged with 3 or 5 prongs, are sometimes used, but most bits are of the combination "fish-tail" type. All are easily forged and sharpened.

15. COMPRESSED-AIR HOISTS

Large air hoists for hoisting in mine shafts are dealt with in Sec 12.

Air "lifts" for raising heavy weights through small distances at slow speeds, as for cranes in iron works and machine shops, are made in several forms. SINGLE-CYLINDER TYPE consists of a long-stroke cyl, suspended vertically from a crane or cable, with the load hooked to lower end of piston rod. TELESCOPE TYPE, for low headroom, has 2 cyl, telescoping into each other. Oil or water, acted on by air press, may be used in both of the above, to prevent accidental lowering of the load through leakage of air, the oil being retained in the cyl by a stopcock, and passing into a reservoir for lowering.

Drum hoists are widely used for scraping, slushing, loading slides, handling timber in shafts, and pulling cars. They are driven by air, elec, or steam (Sec 16, 27). Engines designed for steam may be run by compressed air, but not economically. On replacing its boilers by a Diesel engine, an Ariz gold mine successfully converted its steam hoists to air operation. A reheater, heated by exhaust from the Diesel, was used between the compressor and hoist (27).

Air hoists are driven by a piston motor, turbine, or geared motor, with rated capacities of 750–18 000 lb. Cable speeds for slushing and scraping are 120–400 ft per min; for pulling cars, 25–100 ft per min. They are made with 1, 2 or 3 drums. Table 23 includes power required to hoist skip and rope, and to overcome engine friction which is taken at 20%. It is assumed that the skip weighs 0.5 as much as its contents, that the traction on a horiz mine track is 30 lb per ton and that there are 500 ft of wire rope. Engine is plain slide-valve, in good condition, and loaded to about full capacity. If not, and the air is throttled, more air is needed for same work.

Table 23. Air Required to Hoist 1 Ton (2 240 lb) on Slopes (Ingersoll-Rand Co)

Angle of slope to horiz, deg	Cu ft free air per min of actual hoisting, @ 350 ft per min rope speed	Cu ft free air per 100 ft of hoist	Angle of slope to horiz, deg	Cu ft free air per min of actual hoisting, @ 350 ft per min rope speed	Cu ft free air per 100 ft of hoist
0	11	3.1	35	520	148
5	90	25.7	40	590	168
10	100	45.6	45	650	186
15	230	65.7	50	700	200
20	320	91.2	60	790	225
25	400	114	75	880	250
30	470	134	90	900	257

16. COMPRESSED-AIR LOCOMOTIVES (1)

These were formerly widely used for mine haulage; now largely displaced by electric locomotives. **Note.** Sec 11 deals with all means of Underground Transport, including electric and gasolene locomotives (see also Sec 16). Sec 15 treats of compressed-air locomotives only.

General construction. There are 1 or 2 storage tanks, which, with piping, etc, are carried on a frame mounted on 4 or 6 driving wheels. The cyls are usually simple, sometimes compound. The high-press air of the storage tanks passes into a small auxiliary tank, where it is reduced to proper working press, before going to the cyls.

Air pressure in storage tanks, 700 to 1 200 lb. Cyl press: simple engine, 125–150 lb; compound engine, 200–250 lb. Intermediate press for compound cyls, 50–60 lb. TANK CAPACITY, for hauls of 1 to 1.5 mile, 50–150 cu ft. For long hauls, it is better to increase press, rather than size of tanks.

Tractive force of compressed-air locomotives may be found by: $T = (D^2 \times L \times 0.98\ P) \div d$: where T = tractive force, lb; D = diam of piston, in; L = stroke, in; P = initial press, lb; $0.98\ P$ = effective cyl press; d = diam of driving wheels.

17. PUMPING BY COMPRESSED AIR (1, 2)

General. Pumps designed for steam are often operated by compressed air, though rarely efficiently, due to: (*a*) difference in expansion curves of steam and air; (*b*) the frequently improper selection of cyl diam for the available air press.

Volume of air for pumps working without expansion. If V = vol of free air, cu ft per min; H = head under which pump works, ft; G = gal of water raised per min; P = receiver air press, lb; V_2 = vol of free air corresponding to 1 cu ft at press P; then $V = 0.093\ V_2\ (H \times G) \div P$. This formula is based on 100 ft piston speed, with 15% added volume of air to cover losses.

Table 24. Values of V_2 and $0.093\ V_2$, Mean Press per Stroke, and Hp per Cu Ft Free Air

Air press P, lb	V_2	$0.093\ V_2$	Mean air press per stroke	Hp per cu ft free air	Air press P, lb	V_2	$0.093\ V_2$	Mean air press per stroke	Hp per cu ft free air
25	2.70	0.2511	17.01	0.0743	60	5.08	0.4724	30.75	0.1340
30	3.04	0.2827	19.40	0.0847	65	5.42	0.5040	32.32	0.1406
35	3.38	0.3143	21.60	0.0943	70	5.76	0.5357	33.83	0.1468
40	3.72	0.3459	23.66	0.1033	75	6.10	0.5673	35.27	0.1527
45	4.06	0.3776	25.59	0.1117	80	6.44	0.5989	36.64	0.1583
50	4.40	0.4092	27.39	0.1196	85	6.78	0.6305	....	....
55	4.74	0.4408	29.11	0.1270	90	7.12	0.6621	....	....

Example. Required the vol of free air per min to raise 200 gal of water 150 ft, gage press being 80 lb. From Table 24, $0.093\ V_2$ corresponding to 80 lb = 0.5989; hence, $V = 0.5989\ (200 \times 150) \div 80 = 224.4$ cu ft free air. Hp developed in using 224.4 cu ft free air at 80 lb is $224.4 \times 0.1583 = 35.6$ hp. At 40 lb gage press, $V = 0.3459\ (200 \times 150) \div 40 = 259.0$ cu ft free air, and $259.0 \times 0.1033 = 26.8$ hp. These examples show the advantage of using low-press air for simple pumps.

Since machine drills require air at high press, they generally control the design of the compressed-air plant. To get the best results, the press for the pumps must be reduced below that in the mains. A receiver should be placed between the reducing valve and the pumps, to drain the air, lessen probability of freezing, and to equalize the press (1).

Foot-gallons pumped per cu ft of free air range from say 135 to 155; efficiency is increased by stage compression, and by reheating (Art 9).

Table 25. Air Consumption of Reciprocating Pumps (2)

The table gives the press and vol of air for any size pump. Reasonable allowances have been made for losses due to clearance in pump and friction in the pipe

Height to which water is pumped, ft	Ratio of diam of air cylinder (steam cyl) to diam of water cyl											
	1 to 1		1 1/2 to 1		1 3/4 to 1		2 to 1		2 1/4 to 1		1 1/2 to 1	
	Air press at pump, lb	Cu ft free air per gal water	Air press at pump, lb	Cu ft free air per gal water	Air press at pump, lb	Cu ft free air per gal water	Air press at pump, lb	Cu ft free air per gal water	Air press at pump, lb	Cu ft free air per gal water	Air press at pump, lb	Cu ft free air per gal water
25	15.3	.38	......	......	......	......	......	......	......	......	......	......
50	30.3	.57	13.5	.81	......	......	......	......	......	......	......	......
75	45.5	.77	20.3	1.00	15.0	1.16	......	......	......	......	......	......
100	60.6	.96	27.8	1.20	19.8	1.35	15.3	1.53	......	......	......	......
125	75.7	1.15	33.8	1.39	24.8	1.54	19.0	1.71	15.0	1.91	......	......
150	91.8	1.34	40.5	1.59	29.8	1.74	22.8	1.91	18.0	2.11	14.5	2.33
175	106.0	1.54	47.5	1.79	34.8	1.93	26.5	2.10	21.0	2.30	17.0	2.52
200	121.5	1.73	54.0	1.97	39.8	2.13	30.3	2.29	24.0	2.50	19.5	2.72
225	136.8	1.93	61.0	2.18	44.5	2.31	34.0	2.48	27.0	2.69	21.8	2.91
250	......	......	67.5	2.36	49.5	2.51	37.8	2.66	30.0	2.89	24.3	3.10
300	......	......	81.0	2.75	59.0	2.87	45.5	3.06	36.0	3.28	29.0	3.48
350	......	......	94.5	3.13	69.3	3.27	53.0	3.44	42.0	3.66	34.0	3.65
400	......	......	108.0	3.51	79.0	3.66	61.0	3.86	48.0	4.04	39.0	4.28
450	......	......	......	......	89.3	4.05	68.3	4.23	54.0	4.43	43.8	4.65
500	......	......	......	......	......	......	76.0	4.63	60.0	4.82	48.5	5.03

To find the vol of air required to pump a given quantity of water a given height, find the ratio of diam between water and air cylinders and multiply the gal of water by the figure found in the column for the required lift. The result is the number of cu ft free air. Press required on the pump is in the column directly opposite. For example: the ratio between cylinders being 2 to 1, required to pump 100 gal to a height of 250 ft. Opposite 250, at ratio 2 to 1, is the figure 2.66; 2.66 × 100 = 266 cu ft free air. Press required is 37.8 lb on the pump piston.

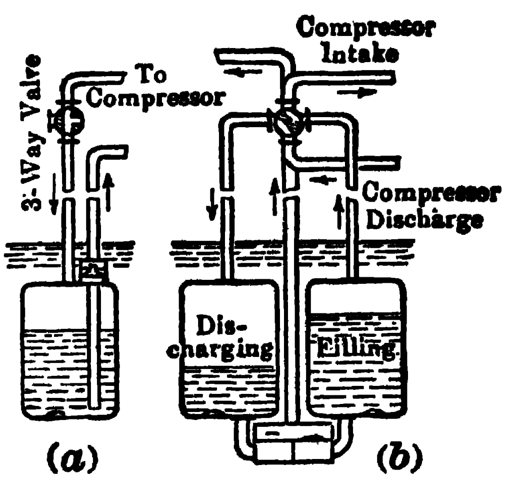


Fig 50. Direct Displacement Pump

Direct displacement pumps operate by displacement of a vol of water by an equal vol of air, the air press corresponding to height of lift. Air is used without expansion. SINGLE-CHAMBER PUMPS (Fig 50 *a*) are suited to low lifts. Compressed air entering the top of the chamber drives out the water, and when all is expelled, the air escapes through the 3-way valve. Then the water discharge valve closes automatically, and water enters the chamber through the bottom valve. The 3-way valve is operated by a float. DOUBLE-CHAMBER PUMPS (Fig 50 *b*). The chambers operate alternately, the action of the pump being practically constant. AIR REQUIRED. The press depends upon height of lift. Press per sq in per foot of head = 0.434 lb; hence, the height to which a given air press will raise water = gage press ÷ 0.434. 1 cu ft of water will be displaced by 1 cu ft of air at the required press, plus about 20% to cover losses. Effic of these pumps is about 25%, which compares favorably with effic of single-cyl direct-acting steam pumps.

Small, air-driven centrifugal pumps are available for pumping water from sumps, shafts and low places in the workings; capacities to 200 gal per min, against heads to 50 ft, single-stage, and 200 ft, two-stage.

Air-lift pump (See also Sec 13) (1, 2). The delivery pipe is partly submerged in the water to be raised. Compressed air is admitted to foot of the pipe. In a small diam pipe, the air forms piston-like layers, between masses of water; in a large one, air enters a number of ports or nozzles, mixing with water in form of bubbles. The water is raised partly by vis viva of compressed air, but chiefly by aëration and consequent reduction of sp gr of rising column. This is not an efficient method of raising water, if considered from a power basis only, but in its proper field it does unique service.

Submergence is depth below surface of water at which compressed air is admitted; PER CENT OF SUBMERGENCE is ratio of submergence to total length of delivery pipe; it must be greater for low than for high lifts. For best effic, range is about 66% for 20-ft lift (vert distance from surface of water to point of disch), to 41% for 500-ft lift; aver, say 55–60%.

Construction. Generally, when the cross-section of the well is large enough, the outside air pipe system is used, in which the water discharge pipe and the air pipe are placed side by side (Fig 51, No 1 and 2). Where well space is limited, the air pipe is inside the water discharge pipe (Fig 52, No 4 and 5). Fig 52, No 3 shows a typical footpiece.

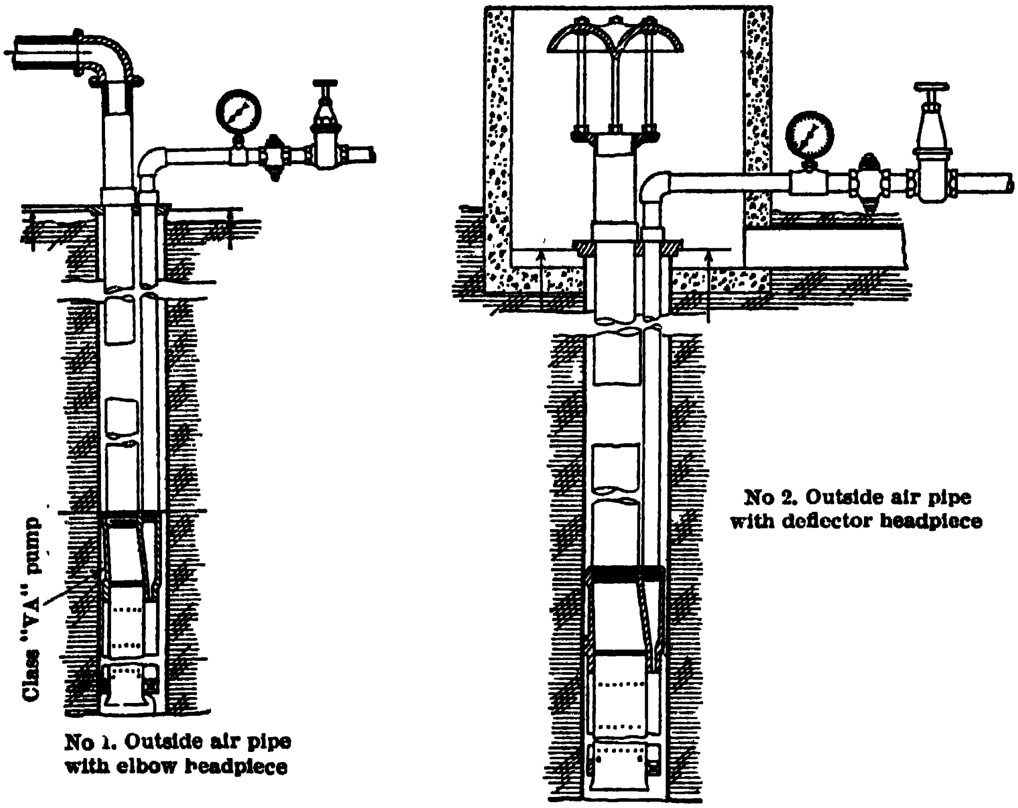


Fig 51. Air-lift Wells with Outside Air Pipes

Efficiency of the air lift (60, 61). Causes of loss: (*a*) slippage due to relative motion between air bubbles and water; (*b*) friction of the mixed air and water in discharge pipe; (*c*) friction in flow of water before air is admitted; (*d*) kinetic energy absorbed, due to veloc of discharge. High discharge veloc tends to low slippage loss. Friction in the discharge pipe varies directly with square of veloc, slippage varying inversely with veloc. Losses (*a*) and (*b*) are difficult to measure; (*c*) and (*d*) are more or less under control. For economy, the veloc of discharge should be such as to minimize (*a*) and (*b*). Important variables affecting performance of the air-lift are: percentage of submergence, lift, vol water discharged, vol and press of air.

Air-lift calculations (1, 62, 63). Essential factors: Va = cu ft free air per 1 gal water; h = total vert lift, ft; H = submergence, ft; C = constant (effic of system) (Table 26). Submergence governs the press required, which is greater for starting than for running. The air-lift differs from other pumps in that the press required to raise a column of water depends on submergence (H), not on lift (h); H being actual head against which air press acts. To find starting air press, multiply starting submergence by 0.434. Following formula closely approximates results of practice:

$$Va = \frac{h}{C \log \dfrac{H + 34}{34}}$$

For liquids other than water, the general formula becomes:

$$Va = \frac{h \times \text{sp gr}}{C \log \dfrac{(H + 34) \times \text{sp gr}}{34 \times \text{sp gr}}}$$

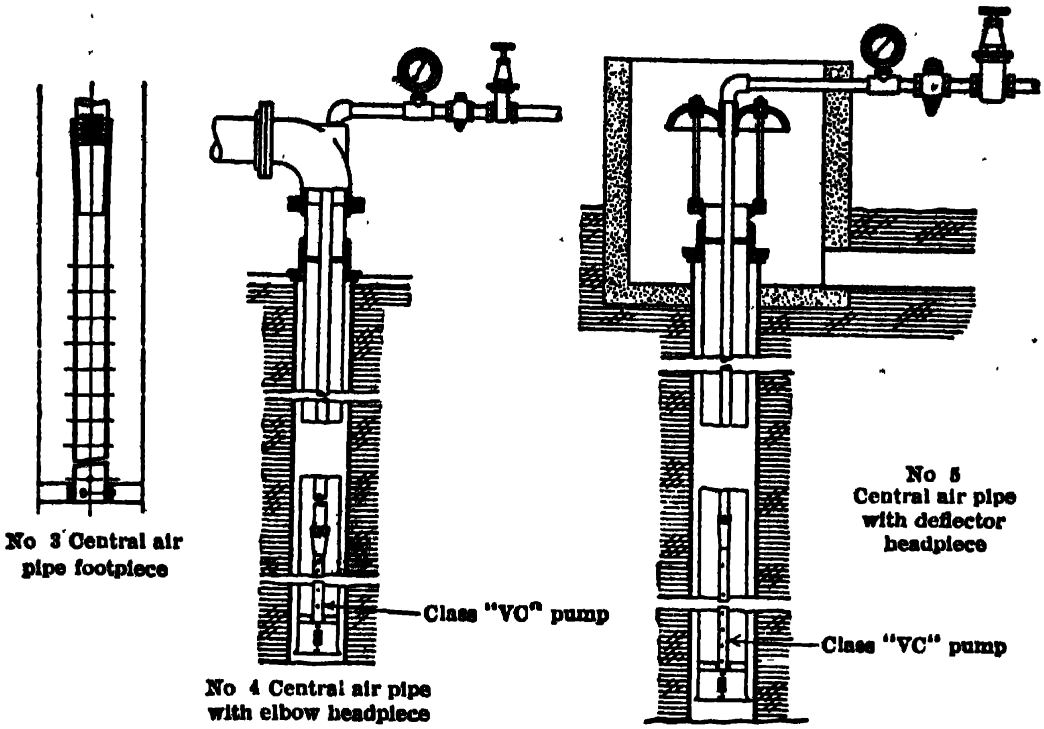


Fig 52. Air-lift Wells with Central Air Pipes

Values of C (Table 26) are based on best ratio of submergence to lift. As this can not always be secured, constants based on customary and best submergences are given in Table 27, for use in above general formulas. Actual pumping- or water-level in a well is seldom known in advance. It is customary to assume lift and submergence, based on experience, and pipe the well accordingly. When working conditions are arrived at, the submergence is adjusted by raising or lowering the pipes until best results appear.

Table 26. Values for constant C

Submergence, %	Value of C	
	Outside air line	Inside air line
75	366	330
70	358	322
65	348	306
60	335	285
55	318	262
50	296	238
45	272	214
40	246	185
35	216	162

Size of discharge pipe. $d = 13.54\sqrt{\frac{Q}{v}}$, in which d = inside diam of pipe, in; Q = vol of mixture of air and water, cu ft per min; v = veloc of mixture in pipe, ft per min.

With pipe of uniform diam, best discharge veloc of the mixed air and water, for lifts from 40 to 200 ft, varies from 2 000 ft per min at 35% submergence, to 700 ft per min at 70% submergence. With tapered pipe, best discharge veloc is 1 400 ft per min at 35% submergence, and 550 ft per min at 70% submergence. Best veloc for the mixed water and air at the bottom of the discharge pipe is 800 ft per min at 35% submergence, and 450 ft per min at 70% submergence.

Table 27. Submergences

Lift, ft	Customary allowable submergence, %	Best submergence, %
20 –125	50–70	65–70
125–175	40–65	60–65
175–250	40–60	55–60
250–350	37–55	50–55
350–650	37–50	45–50
650–750	35–45	40–45

Starting pressure, $P_s = 0.434\ H_m$, in which P_s = starting press, lb per sq in; H_m = starting submergence, ft.

Working pressure, $P_w = 0.434\ H_s + P_f$, in which P_w = working press, lb per sq in; H_s = working submergence, ft; P_f = friction drop in air line from compressor to footpiece, lb per sq in.

Compound air-lift is mainly used in shallow shaft sumps. When the depth allows about 25% submergence, water may be lifted halfway in one pipe and allowed to run back to the bottom of another at same depth as the first. This permits 50% submergence for a third pipe, in which

water is raised to point of discharge. In a deep shaft, Bendigo, Australia, water was raised in a series of lifts, a total of 1 385 ft, by air at 60 to 80 lb press (1).

Table 28. Cost of Pumping with Air-lift (from different sources, 1915)

Output, gal per min	Lift, ft	Cost of coal	Cost per 1 000 gal, ¢	Output, gal per min	Lift, ft	Cost of coal	Cost per 1 000 gal, ¢
2 775	50	....	0.33	175	75	....	1.33
2 775+	35	$2	0.15	695	50	$1.50	0.10
925	75	2	1.00	10 400	30	2	0.25
2 000	75	....	4.50	100	60	2	4.50
2 000	50	....	3.50	....	......	....	....

Air-lift for water-works. The strainer openings should be of such size as to admit fine material into the well, whence it can be pumped, and exclude coarser particles, which form a natural gravel filter outside (Fig 53). Back-blowing, applicable to all wells, requires sealing the top of well casing. By closing the discharge pipe, air is forced through the footpiece, and, driving the water

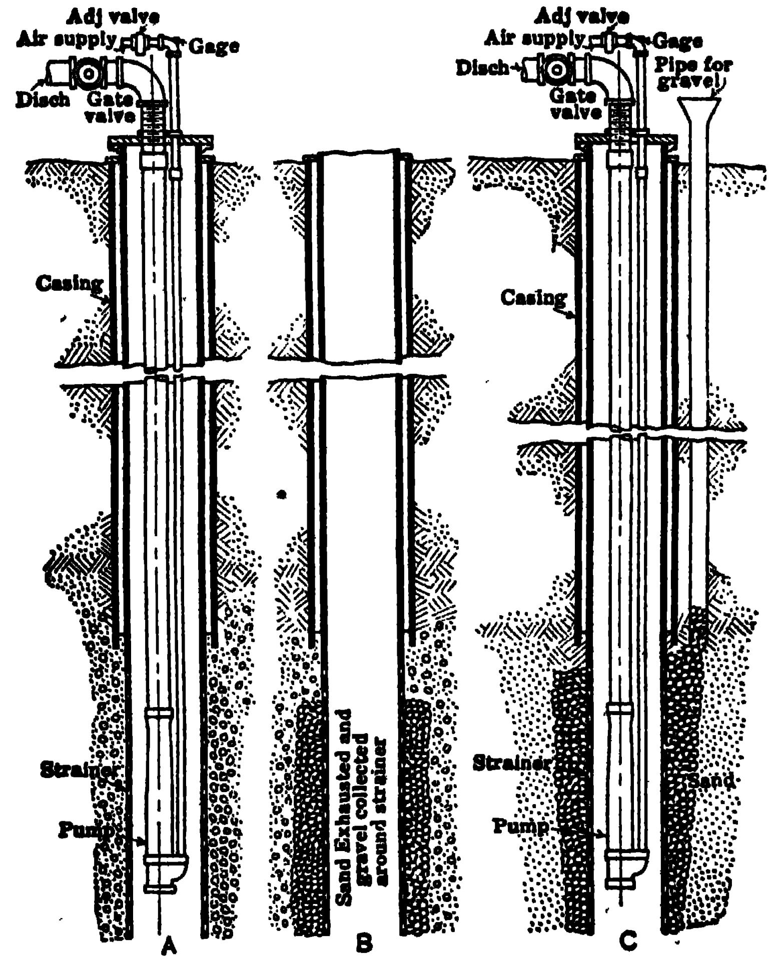


Fig 53. Well Construction under Different Conditions of Water-bearing Sand

through the strainer floats the finer sand. Then, by opening the discharge, the flow resumes its course and carries some of the sand with it. The operation is repeated as necessary, discharging the sand at the surface.

Fig 53 shows 3 modes of equipping wells: *A* has a proper strainer for a stratum of sand and gravel; *B* shows gravel collected around the strainer after the finer sand has been pumped out. The gravel strainer, which forms outside, increases the inflow area, keeps the strainer openings clear, and increases output. *C* shows a water-bearing stratum of fine sand, into which gravel is introduced through auxiliary holes, drilled nearby and terminating in the sand, so that when the sand is removed by back-blowing, the gravel enters around the strainer as before.

Air-lift in conjunction with mine pumps (65) is convenient as an auxiliary for drainage. By a suitable connection, live air is turned into the pump column, which thus becomes an air-lift. Exhaust air from a pump has been used for the same purpose, enabling the pump to work under a greater

head. The air-lift will unwater shafts to within a few ft of the bottom, after which its discharge pipe serves as the permanent pump column.

Improvised air-lifts (66) can readily be made with materials on hand at any plant, by inserting a small air pipe in any cased well. Footpiece is made by drilling holes, or cutting slits with hack-saw, in the capped end of the pipe. A small drill hole in the bottom of the cap will let out sand that may work its way in.

Unwatering mines with air-lift. For this the air-lift is especially well adapted, because: (*a*) its capac can be made very large; (*b*) as the water level in the mine is lowered, the air and disch pipes can be extended more readily than the usual forms of pumps can be moved to new positions.

I. Glen Alden Coal Co, Scranton, Pa (67). The Lackawanna River broke into the mine, flooding a pumproom containing five 5 000-gal electric pumps to a depth of 14–16 ft. Three air-lifts were installed, the first 2 discharging through 16-in pipes, the third through a 12-in pipe. When the water in the pumproom was lowered, some of the mine pumps were started before the air-lifts were completely installed. Both pumps and air-lifts operated 3.5 months. It was estimated that the air-lifts raised 660 000 000 gal, discharging 2 360 gal per min. Initial submergence, 77%; final submergence, about 20%; air press about 45 lb.

II. An inclined shaft, Idaho-Maryland Mine, Cal (68), unused since 1914, was unwatered in 1920. Discharge pipe comprised 900 ft 10-in oil casing, 200 ft 12-in machine-wound wooden pipe, and 1 200 ft 4-in air pipe. Compressor capac, 2 200 cu ft free air per min. Clamps for supporting the pipe were placed every 100 ft, and 2 1-in wire ropes were used for lowering. Dip of shaft to about 850 ft, 70°. At 700 ft, the pipe stuck, and water was lowered to 400-ft level, where a plunger pump was installed. By air-lifting to this level, the trouble point was reached, after which the pipe was extended to 1 013 ft, the 800-level was unwatered, and a pump there installed. From 800 ft down, unwatering proceeded to 953 ft, at which point the air-lift could just handle the inflow. Three compressed-air plunger pumps took 2 weeks to lower the water 47 ft to the 1 000-ft level.

The main air-lift unwatered the mine to a point 560 ft on the slope (519 ft vert), in about 1 month. Max capac attained, 1 450 gal per min, with 49% submergence and 97 lb air press. Capac dropped to 250 gal per min with 60-lb air and 18.3% submergence. Near bottom of the 10-in pipe, shaft flattened to 62.5° and then to 49.5°. For the portion from 560 to 886 ft (282 ft vert), capac varied from 1 500 gal per min with 77 lb of air to 200 gal with 50 lb air; submergence, 19%. From 886 to 1 068 ft, water was raised in a 10-in pipe. Having heavy submergence and small head, this lift had enormous capac; in one 24-hr period, the water was lowered 50 ft on the incline.

Elevating sands and slimes in concentrating and cyanide mills, and pumping solution in cyanide treatment, are special applications of the air-lift.

Table 29. Cases of Compressed-air Illness, East River Tunnels, N Y (52)

	Press, lb	Cases	Number of decompressions
8-hr shifts	15	1	73 956
	16	1	17 625
	17	1	93 333
	18	2	45 256
	19	4	66 997
	20	4	70 605
	21	3	65 074
Two 2-hr shifts; rest interval, 2 hr	22	0	19 187
	23	3	24 018
	24	10	17 009
	25	5	23 239
	26	19	42 458
	27	29	35 827
	28	96	102 851
	29	139	56 092
Two 2-hr shifts; interval, 2 hr	30	14	28 538
	31	36	56 291
	32	37	41 356
	33	50	63 846
	34	113	75 111
Two 1.5-hr shifts; interval, 3 hr	35	16	20 816
	36	8	8 629
	37	3	5 048
	38	3	9 972
	39	11	13 253

18. WORKING IN COMPRESSED AIR

Caisson disease, compressed-air illness, or air embolism, is caused by a too rapid decompression after exposure to high air pressure for a time. It is characterized by presence of free nitrogen in the tissues and body fluids, and by one or more of following symptoms: localized pain, vertigo, prostration, or symptoms referable to central nervous system (52). In selecting men for work in compressed air, the essentials are normal lungs, kidneys and heart; in the older men, blood pressure must not be high.

In the East River tunnels, N Y, where constant medical attendance was maintained, among 87 men over 50 years old, there were only 3 cases of compressed-air illness; indicating that, for sound men, the question of age may not be especially important. Men with moderate adipose tissue are not [illegible]ceptible than the aver individual, but corpulent men should be rejected.

Temperature and humidity apparently do not conduce to compressed-air illness, although humidity increases fatigue. During a period of very hot weather, a lock at one shaft of East River tunnel was exposed to the sun, causing intense heat inside the lock while men were decompressing. This did not tend to illness, but the men suffered from exhaustion after leaving the lock.

Hours of labor and working pressure. In the East River tunnels, N Y, no cases of illness occurred until 15 lb press was reached, although by that time there had been 188 496 decompressions. Table 29 shows that, for working press of 15–21 lb, there were only 16 cases in 621 342 decompressions, and as all were trivial, it appears safe to subject normal men to press up to 22 lb per sq in, for 8-hr shifts.

Lengths of shift in Table 29 are actual hr of labor at tunnel face; total time spent in compressed air, including decompression periods, was considerably longer.

Note that cases of illness reached their peak at 29 lb press, with 2 3-hr shifts. With the change, at 30 lb, to 2-hr shifts, the number of cases greatly decreased, but rose sharply at 34 lb. Had this change of shift been made at 29 lb, instead of 30, many of the 139 cases recorded at 29 lb would probably have been avoided. The benefit of the further change to 1.5-hr shifts, at 35–39 lb press, is clear. The relatively small number of decompressions above 34 lb did not furnish adequate data, though the percentage of cases was not large.

Table 30. Relative Cases of Illness in Subaqueous Tunnels of Penn R R and Pub Service Commission, N Y (52)

Tunnels	Decompressions	Press, lb	Cases No	Cases %
Penn R R.........	8 510	40	139	1.630
Pub Serv........	5 325	41	5	0.094
	8 456	42	12	0.142
	5 730	43	6	0.105
	4 702	44	1	0.021
	33 085	45	24	0.073

Tables 30, 31 cover a period of 1 month (Compressed Air Illness, Keays, 1909). In the Pennsylvania tunnel work, less insistent medical supervision is indicated.

Effect of gases in compressed-air illness (Sec 23, Art 5). Carbon monoxide (CO) is so dangerous that it should be kept at not more than about 0.01 of 1%. CO_2 seems to have no bearing on number of cases, except that it accelerates respiration, thereby causing more rapid saturation of the blood and tissues with nitrogen. During compression, the N absorbed by the blood increases approx 1% per each added atmosphere (14.7 lb). Haldane found that half saturation requires 15 min; nearly complete, 1 hr. Effort of labor hastens heart action and hence saturation.

Table 31. Hours of Labor, in Tunnels in Table 30

Pub Serv Comm tunnels					Penn tunnel				
Press, lb	Hr	Shifts			Press, lb	Hr	Shifts		
1–22	8	0.5 hr for lunch			1–31	8	0.5 hr for lunch		
		Hr on	Hr off	Hr on					
22–30	6	3	1	3					
30–35	4	2	2	2			Hr on	Hr off	Hr on
35–40	3	1.5	3	1.5	32–42	6	3	3	3
40–45	2	1	4	1					
45–50	1.5	0.75	5	0 75					

In decompression, the above process is reversed. Supersaturated body tissues give off N to the blood, which, after desaturation by the lungs, takes up more N from the tissues. This goes on until equilibrium is reached at normal atmos press. If decompression be too rapid to permit the blood to carry to the lungs all the N freed from tissues, or if the lungs can not rid the blood of excess gas, nitrogen bubbles (emboli) form in the tissues and circulatory system, causing severe pain. This is the present theory of the cause of caisson disease.

Symptoms. (*a*) Disturbances of central nervous system, due to pressure of the bubbles upon brain or spinal cord; (*b*) Unconsciousness or collapse, due to much gas distributed throughout the circulation; (*c*) Localization of pain depends on the particular tissues invaded by the bubbles; (*d*) Vertigo, due to bubbles in the middle ear, or to disturbances of nervous system; (*e*) Difficult breathing, due to bubbles forced through pulmonary arteries into the lungs (" chokes ").

Treatment. Cases of illness are usually relieved by recompression, but, if this be delayed in symptom (*a*), the nerve elements may be permanently injured. Symptom (*b*), with abdominal pain, may be serious, though prompt treatment should produce recovery. It is believed that illness could be limited to (*c*), if periods of labor are properly adjusted, and with sufficient time for decompression.

Analysis of cases of illness (52). In 1 361 461 decompressions in N Y Pub Serv Comm tunnel work, there were 680 cases, of which: 91.8% were of localized pain (see above); 6.5%, vertigo; 1.6%, nervous system. Only 1 case of localized pain (in abdomen, followed by collapse) was fatal. Of nervous cases, all but 2 recovered completely.

Care in entering compressed air. To prevent blocking of ear passages, men should " swallow " repeatedly; or, inflate the middle ear by firmly closing nostrils and mouth, and making a strong expiratory effort (Valsalva method). This equalizes internal press and opens the Eustachian tubes, unless the individual has cold in head. If equalization fails to take place, severe pain is caused by pressure on ear-drum, which may even produce rupture.

Precautions in " locking out." As a quick drop in press causes rapid fall in temp, men must be well clothed in the lock, to avoid chilling. While in the lock, they should exercise, to stimulate circulation, thus causing more rapid desaturation of N.

Medical air-lock is a steel cyl, 6 ft diam by 18.5 ft long; one end solidly closed, the other having a door opening inward. It is divided into 2 compartments with a door in the partition opening to inner compt, which contains a cot for use if necessary. The lock has elec lights and heater, telephone, clock, press gage, thermometer, and medical equipment. Press in either compt can be regulated from both inside and outside. Doors have bull's eye windows, for inspection from outside.

When a patient enters the lock, air press is rapidly raised to that in which he had been working. This compresses the bubbles of N in the tissues, which are again taken up by the circulation. Careful decompression follows in stages (details given below). In affections of spinal cord, ample time must be allowed for recompression; a second or even third recompression may be necessary.

Regulations of N Y State Labor Dept. Rule 1151 limits the number of working hours in any 24 hours, according to air press in working place, as follows:

(*a*) Not exceeding 22 lb press, 8 hr, with 30-min interval in open air; (*b*) 22–30 lb, 2 3-hr shifts, 1-hr interval; (*c*) 30–35 lb, 2 2-hr shifts, 2-hr interval; (*d*) 35–40 lb, 2 1.5-hr shifts, 3-hr interval; (*e*) 40–45 lb, 2 1-hr shifts, 4-hr interval; (*f*) 45–50 lb, 2 45-min shifts, 5-hr interval.

On passing out of the working place, Rule 1152 requires stage decompression, in which a drop of half the max press is at rate of 5 lb per min; the remainder, at uniform rates, varying according to working press, as follows: (*a*) for press below 15 lb, minimum decompression rate is 3 lb per min; (*b*) for 15–20 lb, 2 lb; (*c*) for 20–30 lb, 3 lb every 2 min; (*d*) over 30 lb, 1 lb per min.

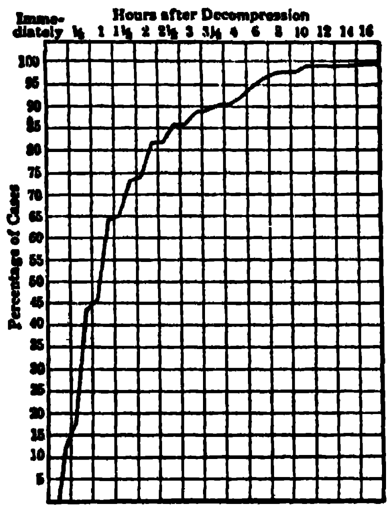


Fig 54
Relative to Time after Decompression Before Illness Began (52)

Rules 1153, 1171 and 1172 specify equipment of the decompression lock, and requirements as to sanitation, ventilation and medical attendance, which can not be detailed here.

19. MEASUREMENT OF COMPRESSED AIR (1, 2, 8, 9, 31)

This is desirable for determining the air consumption of a drill, quantity of air flowing through a pipe, and the compressor output. The appliances, time and experience necessary for accurate measurement are seldom available and their cost is rarely warranted. In most cases, an approx result only is obtained.

Direct-reading meters are simple, compact, and well suited to field use. Fig 55 shows a meter with the scale calibrated to read directly in cu ft free air per min, when measuring air at 80 lb per sq in. A table of multipliers is furnished with the meter, for other pressures. As the accuracy of this type of meter is affected by pulsating flow, small receivers should be placed in the line both before and after the meter, to minimize pulsations.

Water displacement meters (Fig 56) are accurate, but, due to their size, are suitable only for permanent installation.

Fig 56 shows a meter with 2 tanks, 2 ft diam by 4 to 5 ft high, connected at the bottom by a 3-in pipe, and about half filled with water. Each has a gage glass for observing the water level, and in top of each is a pipe and 4-way valve. When the valve is set as shown by the full line, water in right-hand tank is forced into the other by air press, and the air in the latter is driven out through the valve to the machine being tested. When the water level reaches a definite point at top of gage glass on left-hand tank, the valve is thrown to position shown by dotted lines. Then the process is reversed, the air in right-hand tank passing to the machine under test. The number of times each tank is emptied in a given period shows volume of compressed air used, expressed in cu ft free air by the formula, $P_1V_1 \div P = V$, in which P_1 = abs press of compressed air, V_1 = its vol, P and V = barom press and vol of free air.

Venturi meters are inserted in pipe lines for measuring the air. They cause a comparatively small press loss. They are usually fitted with a recording gage, the accuracy of

which is affected by pulsating flow. The following formula is by G. A. Goodenough:

$$Q = 1292\frac{D^2P_1}{\sqrt{T_1}}\sqrt{\left(\frac{P_2}{P_1}\right)^{1.434} - \left(\frac{P_2}{P_1}\right)^{1.717}}$$

in which Q = flow of air, cu ft per min, 68° F, 14.7 lb per sq in, and 40% relative humidity; P_1 = entrance press, lb per sq in abs; P_2 = throat press, lb per sq in abs; T_1 = entrance temp, abs; A = area of throat, sq in; D = throat diam, in; W = wt of air flowing per sec.

Orifice meters (Fig 57), with recording gage, are compact, easily installed and suitable for all usual pipe sizes. Their accuracy is effected by pulsating flow. The following formula is by S. A. Moss (*Trans* A S M E, Vol 50):

$$Q = \frac{19.16\,D^2\,C\,T_3}{P_3}\sqrt{\frac{(P_1 - P_2)P_2}{T_1}}$$

in which Q = cu ft free air flowing per min, at press P_3 and temp T_3; D = smallest diam of orifice, in; T_1 = abs temp, ° F, at upstream side of nozzle; T_3 = abs temp, ° F, at which vol of air delivered is expressed, usually atmospheric; P_1 = total abs press, lb per sq in, at upstream side of nozzle; P_2 = abs static press, lb per sq in, at downstream side of nozzle; P_3 = abs press, lb per sq in, at which vol of air delivered is expressed, usually atmospheric; C = coef of discharge, which must be known for each orifice. For well-rounded orifices, C varies between 0.95 and 0.99; for sharp-edged orifices, approx 0.6. Constants C are based on air containing 0.57% moisture by wt. The formula is accurate to within 2% when $P_2 - P_1$ is less than 10% of P_1.

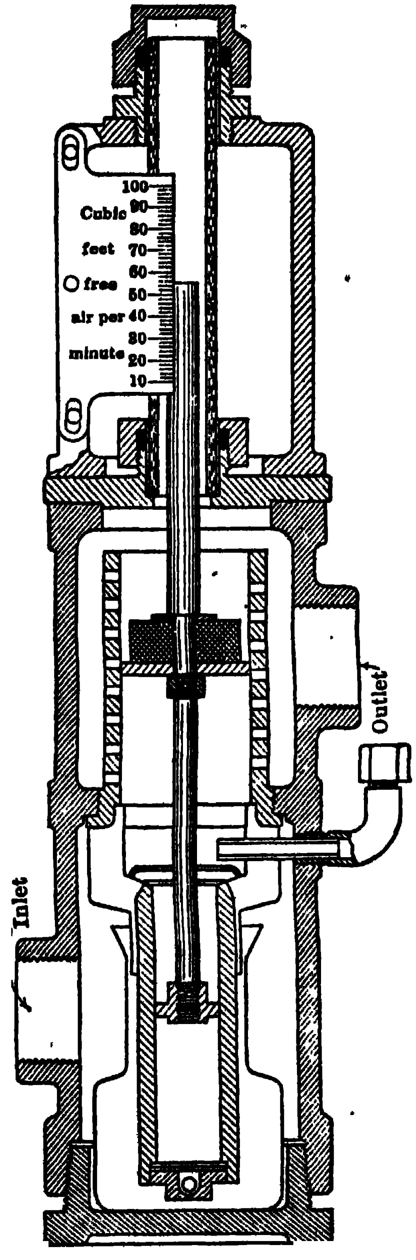


Fig 55. Direct-reading Air Meter (New Jersey Meter Co)

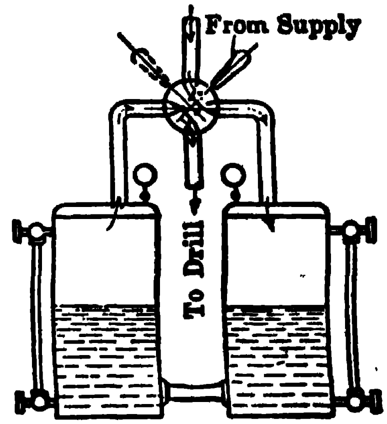


Fig 56. Displacement Tank Apparatus

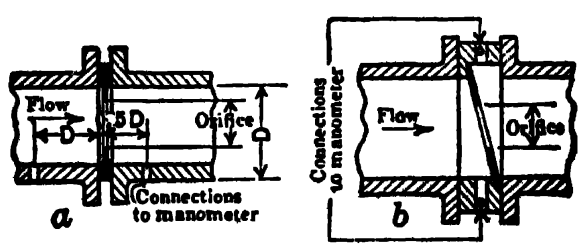


Fig 57. Orifice Meters for Pipe Lines

Non-pulsating flow, as produced by centrifugal compressors, can be more accurately measured. Properly calibrated Venturi and orifice meters give good results. On the Rand, where many centrifugal compressors are installed, a large number of meters of the "weighted-gate" type are used. In these a movable gate changes the size of orifice for different volumes of flow (1).

Measuring air-compressor output accurately requires considerable time and careful attention to the apparatus used. The procedure and precautions are outlined in "The

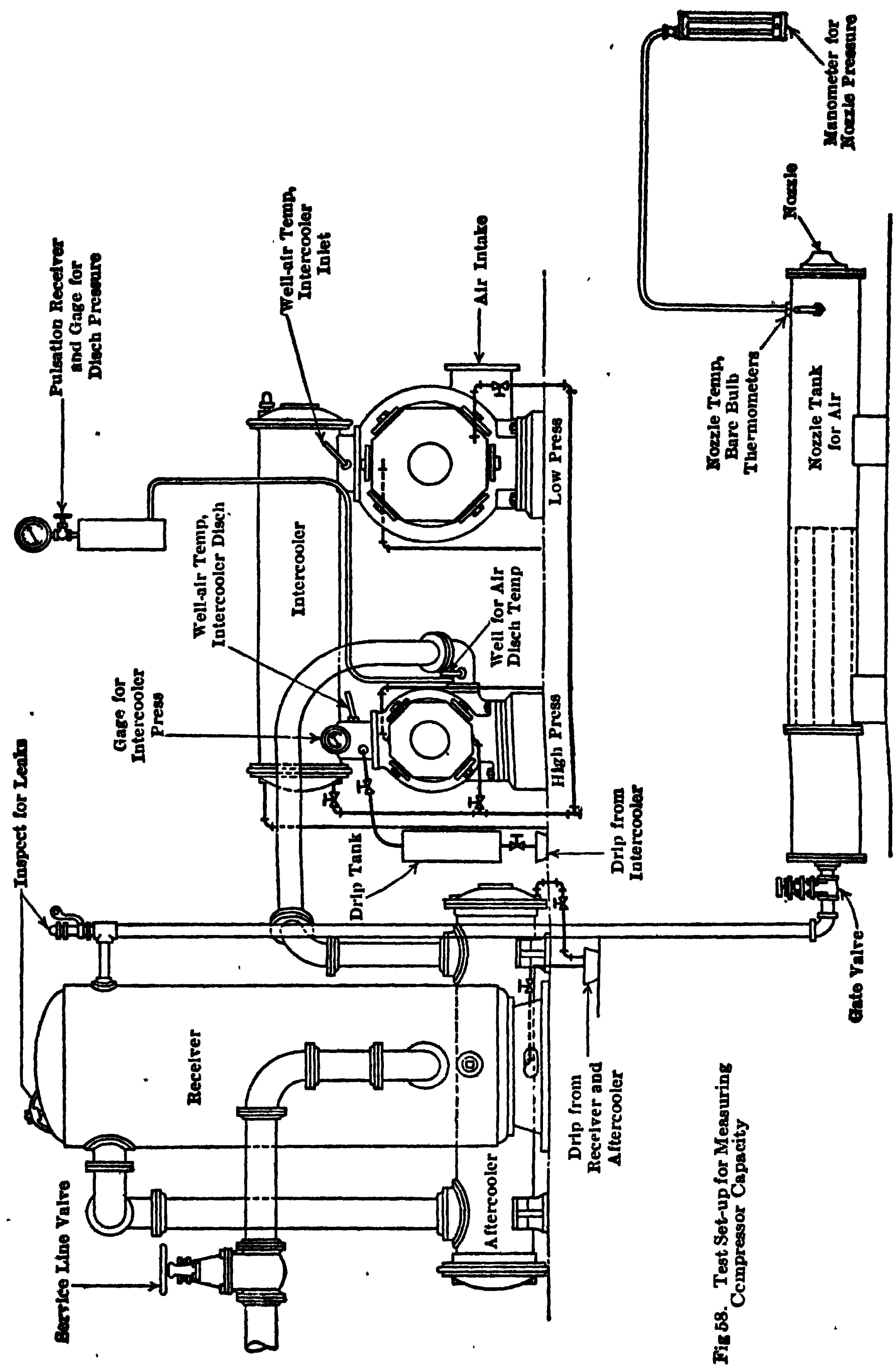
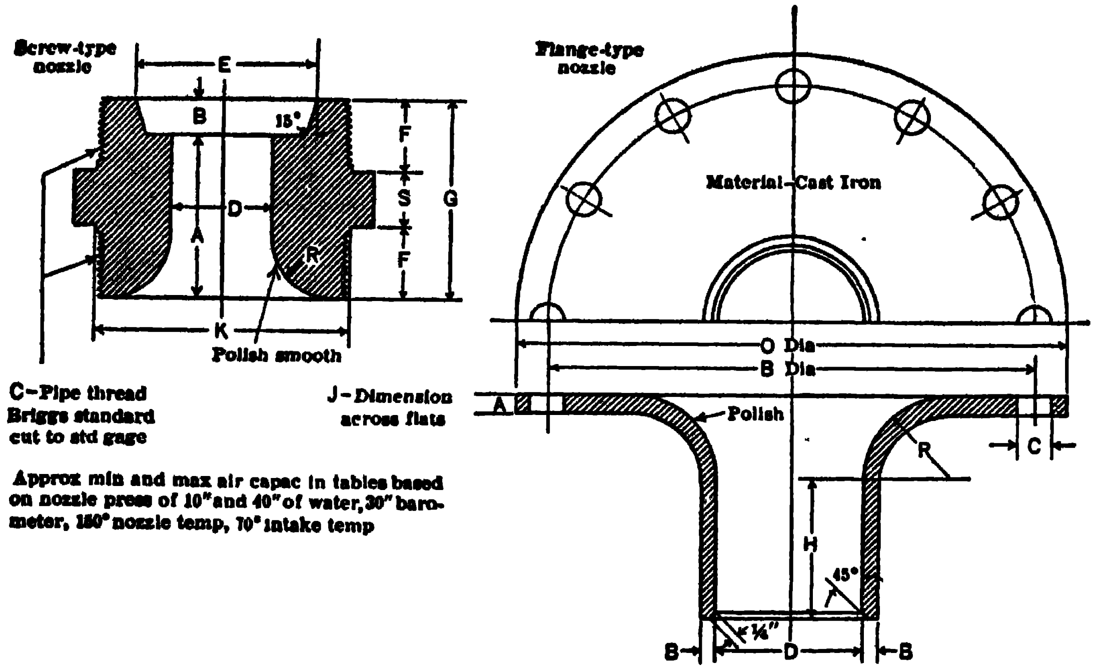


Fig 58. Test Set-up for Measuring Compressor Capacity



(*a*) (*b*)

Fig 59. Standard Nozzles for Compressor Tests

Sizes in inches (Type *a*, Fig 59,)											Min cap free air cu ft/ min	Max cap free air cu ft/ min
D	*A*	*B*	*C*	*E*	*F*	*G*	*J*	*K*	*R*	*S*		
1/8	7/32	1 25/32	1 1/2	1 9/16	3/4	2	2 1/4	1.9	3/32	1/2	.988	1.98
3/16	5/16	1 11/16	1 1/2	1 9/16	3/4	2	2 1/4	1.9	1/8	1/2	2.235	4.45
1/4	13/32	1 19/32	1 1/2	1 9/16	3/4	2	2 1/4	1.9	5/32	1/2	3.95	7.91
3/8	5/8	1 3/8	1 1/2	1 9/16	3/4	2	2 1/4	1.9	1/4	1/2	8.89	17.8
1/2	13/16	1 3/16	1 1/2	1 9/16	3/4	2	2 1/4	1.9	5/16	1/2	15.8	31.6
3/4	1 1/4	1 1/8	2	1 5/8	15/16	2 3/8	2 1/2	2 3/8	1/2	1/2	35.6	71.2
1	1 5/8	3/4	2	1 5/8	15/16	2 3/8	2 1/2	2 3/8	5/8	1/2	63.2	127
1 3/8	2 1/4	1/2	3	2 1/2	1	2 3/4	3 3/4	3 1/2	7/8	3/4	119	239
2	3 1/4	1/2	4	3 1/2	1 1/2	3 3/4	4 3/4	4 1/2	1 1/4	3/4	253	506

Sizes in inches (Type *b* Fig 59)								Number of bolts	Standard flange size	Max cap free air cu ft/ min	Min cap free air cu ft/ min
D	*A*	*B*	*C*	*H*	*R*	*OD*	*BC*				
2 1/2	5/8	1/2	7/8	2 1/2	1.5	13.5	11.75	8	8	780	390
3	5/8	1/2	7/8	3	1.8	13.5	11.75	8	8	1 140	570
4	5/8	1/2	1	4	2.4	16.0	14.25	12	10	2 020	1 010
5	5/8	1/2	1	5	3.0	19.0	17.0	12	12	3 160	1 590
6	3/4	1/2	1 1/8	6	3.6	21.0	18.75	12	14	4 510	2 260
8	7/8	5/8	1 1/4	6	4.8	27.5	25.0	20	20	8 100	4 050
10	1	3/4	1 3/8	8	6.0	32.0	29.5	20	24	12 600	6 350
12	1 1/8	3/4	1 3/8	8	7.2	38.75	36.0	28	30	18 200	9 100

Test Code for Reciprocating Compressors," by the Amer Soc of Mech Engrs, and in the "Standards of the Compressed Air Institute," of the U S. The procedure outlined in these publications should be followed. The flowing air is measured through a low-press orifice, after the pulsations have been dampened in a receiver and nozzle tank. Fig 58 shows a typical set-up. The formula given above for orifice meters may be used. Fig 59 shows dimensions of standard nozzles for such tests.

Measuring leakage in air lines. A rough method is to find the percentage of compressor capac required to maintain press, when no air is being used. A meter in the main line can also be used to determine leakage. A more accurate method of testing leakage, with a low-press nozzle like that used in a compressor capac test, is shown in Fig 60 (49). Air flows from the receiver and through valve *A* to tank *B*, and then out through orifice *F*. At *D* is a water manometer *E*. Temp in the tank is taken by a thermometer at *C*. The orifice has a smooth, rounded inner surface, as in Fig 59.

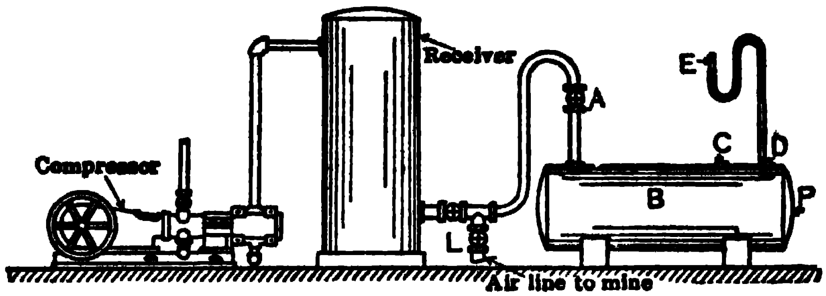


Fig 60. Apparatus for Testing Air Lines

To determine the leakage: Close valve *L* on air line. Regulate compressor so it will not unload at press at which test is to be made, and discharge all the air through orifice *F*. Adjust valve *A* until the compressor exactly holds the desired press. When press is constant, the meter shows the vol delivered at that press. Then, on opening valve *L*, the air line and meter are in parallel. Readjust valve *A* until receiver press is again constant at desired value. Compressor is now delivering same vol as when all air was discharged through *F*, with some leaks from air line, the rest going through *F*, where it is measured. Difference between first and second readings is the vol of leakage.

Table 32. Discharge of Air to Atmos Through an Orifice ("Comp Air Data")

In cu ft per min at 14.7 lb per sq in atmos press and 70° F

Gage press before orifice, lb per sq in	Diam of orifice, in										
	1/64	1/32	1/16	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1
1	.028	.112	.450	1.80	7.18	16.2	28.7	45.0	64.7	88.1	115
2	.040	.158	.633	2.53	10.1	22.8	40.5	63.3	91.2	124	162
3	.048	.194	.775	3.10	12.4	27.8	49.5	77.5	111	152	198
4	.056	.223	.892	3.56	14.3	32.1	57.0	89.2	128	175	228
5	.062	.248	.993	3.97	15.9	35.7	63.5	99.3	143	195	254
6	.068	.272	1.09	4.34	17.4	39.1	69.5	109	156	213	278
7	.073	.293	1.17	4.68	18.7	42.2	75.0	117	168	230	300
9	.083	.331	1.32	5.30	21.2	47.7	84.7	132	191	260	339
12	.095	.379	1.52	6.07	24.3	54.6	97.0	152	218	297	388
15	.105	.420	1.68	6.72	26.9	60.5	108	168	242	329	430
20	.123	.491	1.96	7.86	31.4	70.7	126	196	283	385	503
25	.140	.562	2.25	8.98	35.9	80.9	144	225	323	440	575
30	.158	.633	2.53	10.1	40.5	91.1	162	253	365	496	648
35	.176	.703	2.81	11.3	45.0	101	180	280	405	551	720
40	.194	.774	3.10	12.4	49.6	112	198	310	446	607	793
45	.211	.845	3.38	13.5	54.1	122	216	338	487	662	865
50	.229	.916	3.66	14.7	58.6	132	235	366	528	718	938
60	.264	1.06	4.23	16.9	67.6	152	271	423	609	828	1 082
70	.300	1.20	4.79	19.2	76.7	173	307	479	690	939	1 227
80	.335	1.34	5.36	21.4	85.7	193	343	536	771	1 050	1 371
90	.370	1.48	5.92	23.7	94.8	213	379	592	853	1 161	1 516
100	.406	1.62	6.49	26.0	104	234	415	649	934	1 272	1 661
110	.441	1.76	7.05	28.2	113	254	452	705	1 016	1 383	1 806
120	.476	1.91	7.62	30.5	122	274	488	762	1 097	1 494	1 951
125	.494	1.98	7.90	31.6	126	284	506	790	1 138	1 549	2 023

Values are based on 100% coeff of flow. For well-rounded entrance orifice, multiply by 0.97; for sharp-edged orifice, multiply by 0.65; results are approx.

Loss of power due to leakage (50). At a mining plant tested, consisting of two 1 700-cu ft cross-comp non-condensing steam-driven compressors, supplying hoists, pumps and drills, one pump was operated by air continuously, for which purpose pressure was always maintained in the entire system.

Results of test: Aver compressor delivery during working periods, 1 950 cu ft per min; aver hp input, 340; flow of compressed air to supply leaks, 950 cu ft per min; hp to supply leaks, 157; total power cost per year, $18 200; power cost per year to supply leaks, $12 400; power cost to supply machines using the air, $5 800.

In another case, a pump of 200 gal capac ran 24 hr per day against a 30-ft head. For the pump only, the compressor input was 105 hp; whereas, with no leakage, the pump could be run by 13 hp. At a press of 70 lb sq in, 20 cu ft of air per min will escape through a $^1/_8$-in hole.

20. MISCELLANEOUS APPLICATIONS OF COMPRESSED AIR

Ore sticking in bins has been successfully freed by a compressed-air poke rod, consisting of a nozzle in the end of a pipe which is thrust into the bin. Wet slimy ore has also been freed by a series of 1 $^1/_4$- to 2-in pipes on the sides and bottoms of the bins, with $^1/_8$- to $^1/_4$-in orifices (13). A nozzle spraying a mixture of compressed air and castor oil from a line oiler has been used to remove dust from a mine heading after a blast. Castor oil does not create a fire hazard (14, 42).

Mine lamps are cleaned by compressed air at Alliance Colliery, Kasha, Pa, of Lehigh Nav Coal Co more quickly and thoroughly than by hand (15).

Mines at Kalgoorlie and Broken Hill, Australia, use compressed air to ventilate dead-end headings. A $^3/_{16}$-in jet in a 6-in Venturi induces 600 cu ft per min of air, which moves to the working face 500 ft away. With a 10-in Venturi, the air will travel 1 000 ft (18).

In the Hollinger Consol Gold Mines, Timmins, Ont, ethyl mercaptan is distributed through compressed-air mains to warn miners of fire in the workings or near a shaft (19).

In driving a sewage disposal tunnel, 11 ft 9 $^1/_2$ in diam, for the Minneapolis-St Paul Sanitary district, the heading-and-bench method was used. The face was undercut in sandstone by high-press air issuing from a 5-jet manifold, at end of a $^3/_4$-in pipe, and then advanced 4 or 5 ft with pick machines before another undercutting. No blasting was necessary (30).

21. MAKERS OF COMPRESSED-AIR MINE EQUIPMENT

Compressors for Mine Service

United States and Canada

Allis-Chalmers Co, Milwaukee, Wis; Bury Compressor Co, Erie, Pa; Chicago Pneumatic Tool Co, New York; Fuller Company, Catasauqua, Pa; Gardner Denver Co, Quincy, Ill; Hardie Tynes Mfg Co, Birmingham, Ala; Ingersoll-Rand Co, New York; The Norwalk Company, South Norwalk, Conn; Pennsylvania Pump and Compressor Co, Easton, Pa; Boots-Connersville Blower Corp, Connersville, Ind; Sullivan Machinery Co, Michigan City, Ind; Worthington Pump and Machinery Corp, Harrison, N J; Canadian Ingersoll-Rand Co, Sherbrooke, Canada.

Great Britain

Alley & MacLellan Ltd, London; Bellis & Morcom, Birmingham; Broom & Wade Ltd, London; Climax Rock Drill & Eng Works, Carn Brea, Cornwall; Consol Pneumatic Tool Co, Fraserburgh, Scotland; Daniel Adamson, Manchester; Hick Hargreaves & Co, Bolton, England; Holman Bros, Broad St House, London; Reavell & Co, London; Armstrong (Sir W. G.) Whitworth & Co, Newcastle-on-Tyne; Peter Brotherhood Limited, Peterborough, England; Browett & Lindley, Letchworth, England; Ingersoll-Rand Co, Manchester; Tilgham's Air Compressor Co, Altringham, England; Wilson, Alexander, Aberdeen, Scotland; Worthington Simpson, Newark-on-Trent, England.

Continent of Europe

Demag, Duisburg, Germany; Flottmann A G, Herne, Germany; Fma/Pokorny, Frankfurter Maschinenbau A G, Frankfurt on Main; Klein, Schanzlin & Becker Aktiengesellschaft, Frankenthal; Germany; Maschinenbau A G, Frankenthal; Neumann & Esser, Aachen; Rheinmetall-Borsig A G, Berlin; Zwickauer Maschinenfabrik, Zwickau in Sachsen; Alling, C K, Copenhagen, Denmark; Atlas, Copenhagen; J. Kruger, Copenhagen; Dansk Trykluft Kompagnie, Copenhagen; Espholine Maskinfabrik A/S, Copenhagen; Henning Klee, Copenhagen; V. Lowener, Copenhagen; A B Atlas Diesel, Stockholm, Sweden; Ateliers de Meudon, Meudon, France; Crepelle, Lille, France; Dujardin, Lille; Rateau, Paris; Sebia, Paris; Spiros, Paris; Konigsfelder Maschinenfabrik, Brunn, Germany; Skoda-Werke A G, Pilsen; Witkowitser Machinenfabrik, Witkowitz.

Rock Drills for Mine Service

United States and Canada

Chicago Pneumatic Tool Co, Chicago and New York; Cleveland Pneumatic Tool Co, Cleveland: Gardner-Denver Co, Quincy, Ill; Ingersoll-Rand Co, New York; Sullivan Machinery Co, Claremont, N H; Worthington Pump & Machinery Co, Harrison, N J.

Great Britain, Europe and So Africa

Atlas Diesel Co, London; British Flottmann Drill Co, Cardiff, Wales; Broom & Wade Ltd, High Wycombe, England; Climax Rock Drill & Eng Works, Johannesburg, Transvaal; Climax Rock Drill & Eng Works, London; Consolidated Pneumatic Tool Co, Fraserburgh, Scotland; Consol Pneumatic Tool Co, Johannesburg; Delfos Ltd, Benoni, Transvaal; Holman Bros, London; Ingersoll-Rand Co, London; Ruston, W & Co, Sheffield; Scott, F & Son, Sheffield; Sullivan Machinery Co, Transvaal; Ateliers de Meudon, Meudon, France; Demag A G, Duisburg, Germany; Flottmann A G, Herne; Frolich & Klupfel, Barmen; Krupp Kraftwerkzeug-Vertrieb, Essen, Germany; A B Atlas Diesel, Stockholm.

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SECTION 16

ELECTRIC POWER FOR MINE SERVICE

BY

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

ELECTRIC POWER FOR MINE SERVICE

1. PURCHASED POWER

Electric power may be purchased from Public Service companies in most coal and metal mining regions, usually at a lower cost than by the operation of an isolated plant (1) and can be supplied to stationary motors at any required voltage. Direct current is required for traction haulage, coal cutters and storage battery charging. Rate schedules for alternating current usually include a demand charge, either expressed, or made a part of the energy charge, based upon integrated peaks of 5–15 minutes' duration, or on momentary peaks as created by hoists. Demand is usually expressed in k v a; energy, in kw-hr. Total cost varies from 0.9 to 3¢ per kw-hr. Power factor, load factor, delivered voltage and quantity used affect total cost.

2. CHOICE OF CURRENT AND VOLTAGE

Extent of territory to be served and density of load should determine voltage: 250 volt d c is preferable for underground use, because of safety; 500 volt is more effic in large mines or where loads are heavy (but certain states prohibit its use in new operations); 220, 440 and 2 200 volt a c can be applied to stationary motors and transformers; for lighting at shaft bottoms, and in pump and hoist rooms, a c at 110 volts. For underground a-c transmission, 2 200, 4 000 and 6 600 volts usually meet requirements, though higher voltages can be used if necessary.

3. POWER PLANT (see also Sec 40, 42)

Steam power is now rarely used at large metal mines, but has definite though limited application at coal mines where there is an abundance of waste fuel or unmarketable coal. Parts of certain coal seams contain impure bands that must be rejected, or that are not amenable to cleaning, but that can be burned under boilers with special firing equipment. In some districts there is a very limited market for the fines, and it may be more economical to burn at point of production. The boiler water problem can usually be solved in any

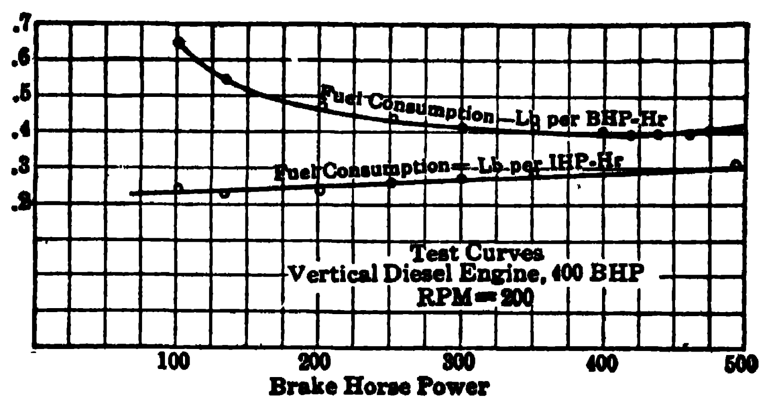

Fig 1. Test of Diesel Engine

district. Such plants can be economically connected electrically to existing utility circuits; cost, $60–$100 per kw of capacity.

Oil-burning engines are being widely installed where purchased power is not available. Engines of the Diesel type, now built from a few hp to 10 000 hp in single units, have been so perfected that they are as reliable as the steam engine with its boiler and auxiliaries. The modern oil engine will burn practically any liquid fuel, from kerosene to heavy tar oils, provided the latter are heated sufficiently to enable the fuel pumps to handle them. Fuel is burned directly in the cylinders, avoiding the well-known losses in the steam plant.

Total thermal effic of the oil engine is approx 34–36%, as against about 10% effic of boiler and steam engine. The builders usually guarantee an oil consumption of 0.41–0.5 lb per hp hr.

Ignition of the fuel oil is directly by heat of compression of the charge of air in the cyl, as the oil is sprayed into it. The atomised spray, coming into contact with the hot air, which is at about 500 lb press, has a temp of about 1 000° F. There is no explosion in the cyl; after ignition during the first part of the stroke, the oil-vapor expands and acts on the piston at nearly constant press until the exhaust port is opened (Sec 39, 40).

Commercial oil engines are: 4- and 2-cycle, single-acting; 4- and 2-cycle, double-acting. They are also designated, with reference to design of the piston, as: crosshead type, which is the more reliable for heavy duty and constant work; trunk type, which is longer, but requires lower headroom, and costs less. Oil engines are also classified as: AIR-INJECTION, which require an air compressor to atomise the fuel oil (the compressor power being usually from 8 to 10% of the engine power); SOLID-INJECTION, which have no compressor, the oil being injected into the cyl by a pump. The latter type is probably best for the smaller size engines (Sec 39).

Cost of a 500-hp oil engine, directly connected to an a-c generator, with exciter, switchboard, piping, auxiliaries, foundation, freight and erecting, delivered in central portion of U S, is \$80–\$90 per hp, depending upon type of engine, station equipment, and the cooling-water supply, which costs more if cooling towers are required, instead of allowing the water to go to waste.

Attendance required is less than for steam plant, since there are no boiler and yard crews, and no handling of ashes. Fuel can be stored in tanks by pumping or gravity flow from tank cars, with no loss or deterioration.

Conclusions. Oil engines are ideal for mining plants. They require small space, there are no standby losses, as all costs except fixed charges stop when engine is shut down, and should produce power for about 1.5–3¢ per kw-hr. (For more details as to sizes, costs, effic and thermodynamic operation of oil engines, see Sec 39.

4. SUB-STATIONS

Sub-stations: OUTDOOR SUB-STATIONS (not inclosed), for transforming a c to a higher or lower voltage and for controlling the various circuits. In general, they comprise transformers, oil switches and lightning arresters, mounted on a concrete foundation with a steel structure to support disconnecting switches, bus-bars and transmission circuits. For a small sub-station, the entire equipment can be mounted on wood poles. INDOOR SUB-STATIONS are usually small fireproof buildings, housing motor-generator sets (Sec 42, Art 6), rectifiers, or rotary converters (Sec 42, Art 11), with the necessary control for converting a c to d c for use underground. The d c is carried underground from these stations by suspending cables in shafts or boreholes. UNDERGROUND SUB-STATIONS are used to house step-down transformers, rectifiers, motor-generator sets and rotaries. Housing should be fireproof and adequately ventilated. When oil is used to cool transformers, or in switches and similar equipment, provision is made to entrain the oil in case of leakage. MERCURY ARC RECTIFIERS have advantages of high effic, high momentary overload capac and portability, and make them attractive conversion units for mine service. Two types, multiple anode and single anode (ignitron), are available. For installations inside the mine, the rectifier and auxiliaries are usually in 3 units, each mounted on its own 4-wheel truck. One truck carries rectifier, heat exchanger and vacuum pumps; another carries transformers (filled with non-combustible liquid dielectric to avoid need for fireproof vault) and an oil circuit-breaker; and the third, a direct-current automatic reclosing circuit-breaker, also relays and meters. As each truck is permanently wired as a unit, the entire sub-station equipment may readily be moved on the mine track after breaking a few electrical connections. Fig 2 shows COMPARATIVE EFFICIENCIES of the types of conversion equipment, any of which can be arranged for automatic operation.

Automatic control is successfully applied to all sub-station equipment, and its cost is fully warranted by saving in labor and increased reliability of operation.

Motor-generator sets should be used where there is variation in the a-c supply, or where it is desired to create over-compounding of d-c, to compensate for voltage drop due to line resistance. By increasing the excitation of the synchronous motor field, a leading power factor can be created that in many cases has considerable value (Sec 42).

Rotary converters have higher effic than motor-generator sets, and will stand severe momentary overloads without injury, but the d-c voltage varies with the a-c voltage. They operate at about unity power factor and give no correction to the system.

Commutating-pole type of rotary converter (Sec 42, Art 6) is especially adapted to mine service, because of its enormous overload capacity. For load factors of 30% and less, which are usual at mines, it is common to use sub-station apparatus which will

average half load and occasionally carry peaks of 50% overload. With commutating-pole rotaries, the machine may be operated near its rated capacity, carrying the peaks at overloads of 200%. This gives much higher all-day effic and satisfactory service, with a machine of half the usual size. These machines may be obtained as standard in sizes of

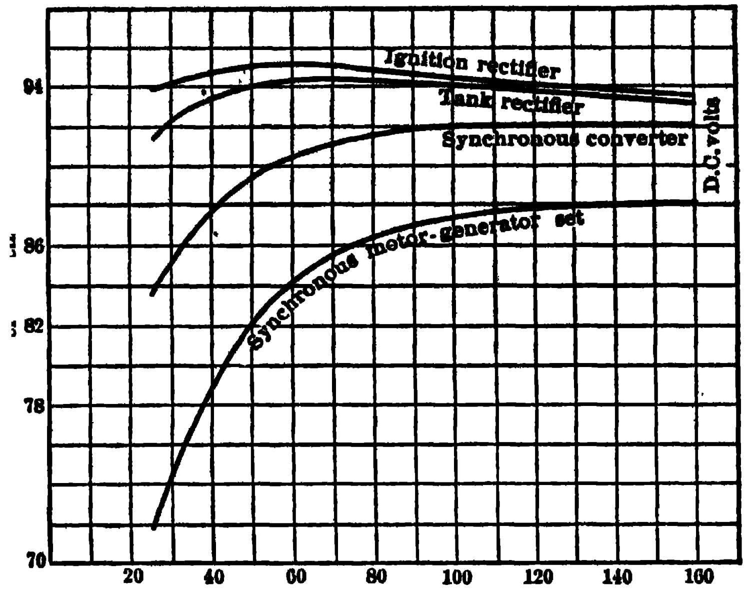

Fig 2. Efficiencies of Conversion Equipment

100 kw and up for 60 cycles and as small as 300 kw for 25 cycles. They are usually connected 6-phase and have remarkably good effic; the 150-kw, 60-cycle, 275-volt machine attaining 94.7% at 1.5 load, and 93.5% at 0.75 load.

5. TRANSMISSION LINES (see also Sec 42)

Calculation of d-c 2-wire circuits: if I = current in amperes, or watts ÷ volts; L = length of transmission, ft; V = volts lost in transmission; then circular mils in conductor = $(22 \times I \times L) \div V$.

For haulage work, where the track forms the return circuit, the rail capacity should be at least equal to that in trolley and feeders. Steel rails in general have 12 times the resistance of copper, and for 2 rails will be equal to 20 000 cir mils of copper per lb, per yd. Thus, a track of 40-lb rail will be equivalent to 800 000 cir mils. When capacities of wire and rail are unequal, the drop in each should be figured separately, using the formula $V = 11 \times I \times L \div$ cir mils, in which case V is the drop in half the circuit. If the load is distributed, L represents aver distance.

Table 1. Distances to which 100-kw 3-phase power can be transmitted at different potentials, assuming energy loss of 10% and a power factor of 85%. Distances are proportional to squares of voltages and inversely proportional to the kw

Amer wire gage	Area in circular mils	Voltages					
		2 000	3 000	4 000	5 000	6 000	8 000
		Distance of transmission in miles for various potentials at receiving end					
6	26 250	1.38	3.11	5.53	8.65	12.5	22.2
5	33 100	1.75	3.93	7.00	10.9	15.7	28.0
4	41 740	2.20	4.96	8.80	13.8	19.8	35.3
3	52 630	2.77	6.25	11.1	17.3	25.0	44.4
2	66 370	3.50	7.88	14.0	21.9	31.5	....
1	83 690	4.41	9.93	17.6	27.6	39.7	....
0	105 500	5.58	12.5	22.3	34.9	50.1	....
00	133 100	7.02	15.8	28.0	43.9	....	....
000	167 800	8.85	19.9	35.4	55.3	....	....

Calculation of short 3-phase transmission circuits, capacity neglected:

e_g = Volts, line to neutral at generator end; e_r = Volts, line to neutral at receiver end; $E = \sqrt{3}\,e$ = volts phase to phase; R = Resistance of one conductor, ohms; X = Reactance of one conductor, ohms; I = Current per phase; $= \frac{\text{Three-phase watts}}{\sqrt{3}\,E\,(p\text{-}f)}$; $p\text{-}f$ = power factor = $\cos\theta$; power loss = $3I^2R$; $e_g = \sqrt{(e_r\cos\theta + IR)^2 + (e_r\sin\theta + IX)^2}$. Cos θ and sin θ correspond to the power-factor angle at the receiver end. For leading power-factor, sin θ will be negative.

Permissible losses may be determined from standpoint of economy in transmission or from operating conditions. It is common practice to figure all d-c lines with a loss of say 10%; but large saving in transmission cost may be effected by considering that, as a general rule, for best economy the value of power lost should approximately equal the INTEREST charges on the conductor.

Example. Assume 500-kw aver load at 500 volts, delivered 1 000 ft with 5% loss. With power costing 2¢ per kw-hr, this loss amounts to $4 320 per year. The required copper, from above data, is 5 200 lb, which, at 15¢, will cost $780, and interest on this cost at 6% is $46.80. Total yearly cost of delivering this power is $4 367 per year, of which the interest charge is about 1%. It is evident that a much larger investment in copper with a reduced power loss would be more economical. For trial, assume 5 times as much copper, with 0.2 as much power loss. Interest charges will then be $234, power loss $864, and total transmission cost is $1 098, a saving of $3 269 per year. Evidently a saving of about $200 might be effected by still further reducing the loss to about 0.5%, but this would involve an investment of about $4 000 for added copper. On BASIS OF COST the investment in above cases would be large, as the copper required for least annual cost would be nearly 9 000 000 cir mils. An even larger installation would theoretically be justified were the power cost higher or copper cost lower; on the other hand, with power cost halved, the size of conductors would be halved.

From OPERATING POINT OF VIEW, it is advisable to allow not more than 33% drop at max load, which permits satisfactory operation of mining machinery under the worst conditions, and at lower loads conditions will be greatly improved. This appears an excessive drop, but voltmeter records at a large number of mines show max drop of 50% and over. With 10-hr load factors of 20 to 30%, this max loss represents an aver loss of 6.6 to 10%. With potentials of 300 to 600 volts at the mine openings, the minimum voltages underground will be 200 to 400, which will give satisfactory operation of machinery designed for 220 and 500 volts. The low cost of power, usual in coal mining, makes a large copper investment inadvisable.

With a-c transmissions, full-rated voltage should be maintained, since output and torque of induction motors vary as the square of the applied voltage. With step-down transformers and where reduced voltage taps may be used, line drops of 10 to 15% may be advisable; but, in general, and particularly with synchronous converters, it is best to design transmission lines for not over 5% drop at rated capacity, which should be taken as the max 15-min integrated peak. Where this is not known the continuous rating of primary apparatus connected may be used.

Construction. Location of transmission lines for either d c or a c should be considered relative to accessibility, economy, and safety.

Accessibility for inspection and repairs is of first importance, and if conditions permit location near a road, it pays to make a considerable detour, as it will facilitate finding sources of trouble. In case of ground or short circuit, a repairman can cover the ground in 0.2 to 0.5 the time if on horseback or with automobile. In many cases, particularly in coal-mining districts, a mile or more added length of line will give advantages worth many times the first cost. If such location is not available, lines should be straight as possible; should have clearance of at least twice the line spacing over or under any other lines; and through towns or villages should be kept high (30 to 40 ft), because if low, short circuits and groundings may be caused by objects thrown over the wire.

Poles and cross-arms (Sec 42, Art 14). Steel cross-arms and pins are best, as the life of wood arms is short, unless treated, in which case the cost will approach or exceed that of steel. The Bo-arrow and similar constructions are light, strong, cheap, and do not require gaining and weakening a wooden pole. Steel poles or towers are common for long high-voltage lines, with spans of 200 to 500 ft. For ordinary mining work, wood poles are satisfactory for spans of 100 to 125 ft, where wires are not larger than No 0.

To a point 1 ft above ground, the butts of wood poles should be treated with creosote or other preservative. All joints and wherever poles are drilled or cut should be similarly treated. Where brush or grass fires are expected, place concrete around pole to height of 2 ft, or protect base with rock, or, if conditions justify, use steel poles.

Lightning arresters (Sec 42, Art 13), for protection of sub-stations, should be made up of capacitors and suitable station or line-type arresters.

Ground wires are an efficient protection for transmission lines, and if several are used, partially surrounding the lines, they afford complete defense against lightning, though arresters will still be necessary to prevent trouble from arcing grounds. Pole lines usually have a single ground wire, supported from the pole top on a bayonet or ridge iron.

In moist climates or where exposed to fumes and gases, as from coke ovens or smelters, ground wires should be of copper; elsewhere they may be of galvanized steel strand, double dipped, 3/8 or 7/16-in diam, and should be tied with wire of the same material, NEVER COPPER WITH STEEL. Good groundings may be made at power plants, as to water pipes, etc; but at sub-stations and on transmission lines it is often necessary to drive iron pipes into the ground, or to bury copper ground-plates. Galvanized iron pipes, 0.75 to 1-in diam, driven 6 to 8 ft in earth, will have 15 to 20 ohms resistance. Where good grounds can not be obtained for a considerable distance, larger and better ones should be provided where possible at each end of such section (2).

6. UNDERGROUND WIRING

Underground circuits not exceeding 550 volts use insulated or bare wire, supported on insulated fittings. Wires may be supported directly on hangers or insulators attached to roof or sides of passageway, or may be attached to timbers supporting the roof, but in such manner as will allow timbers to be changed without disturbing the electrical circuit. In all cases wires must be kept free from contact with roof, sides or timbers, whether insulated or not.

Circuits exceeding 550 volts are insulated for proper voltage, and protected by armor (metallic or nonmetallic), or carried in iron conduit; metallic armor or conduit to be grounded. For circuits of 2 300 volts a c and higher, cables are available without metal armor. Individual conductors are shielded by 5-mil tape, with bare grounding conductors in contact with the tape. In case of insulation failure, grounding conductors carry fault to ground. Circuits are mechanically protected by burying in trenches in floor or sides of passageway.

Table 2. Current-carrying Capacity of Bare Copper Conductors Used in Mines

Conductor, B & S gage	Current capac, amperes	Conductor, circ mils	Current capac, amperes	Conductor, circ mils	Current capac, amperes
10	80	250 000	690	750 000	1 520
8	105	300 000	790	800 000	1 590
6	145	350 000	880	850 000	1 660
4	210	400 000	965	900 000	1 730
2	280	450 000	1 050	950 000	1 800
1	320	500 000	1 140	1 000 000	1 870
0	375	550 000	1 215		
00	435	600 000	1 285		
000	525	650 000	1 370		
0000	615	700 000	1 450		

Trolley wire. Hard-drawn GROOVED WIRE is used exclusively for underground trolley, and of sizes shown in Table 3.

Table 3. Average Properties of Hard-drawn Copper Trolley Wire

(American Standard Grooved Section)

Nominal area in circular mils	Area, sq in	Dimensions, in				Pounds per		Breaking strength, lb	Ohms per 1 000 ft, 68° F, 97.2% conductance
		Section depth	Upper lobe width	Lower lobe width	Web thickness	1 000 ft	Mile		
417 000	0.3142	0.750	0.357	0.563	0.250	1 262	6 663	14 500	0.02667
350 000	.2758	.620	.376	.620	.268	1 063	5 612	12 600	0.03014
300 000	.2355	.574	.376	.574	.267	908	4 792	10 515	0.03529
211 600	.1665	.482	.376	.482	.267	642	3 387	7 751	0.04992
168 100	.1314	.430	.340	.429	.237	506	2 674	6 366	0.06326
133 225	.1083	.392	.318	.388	.217	417	2 204	5 432	0.07675

Trolley wires may be supported from roof, walls, timbers, or pipe, as conditions require, and are strung 4 to 6 in outside the gage line, on the side opposite to safety or manholes, room necks, or chutes (Sec 23). MINIMUM HEIGHT is about 6 in clearance above the locomotive frame; general range, 32 to 56 in; max height, usually 7 ft. Where the wire is less than 6 ft above rail, it must be protected at all cross-headings, on partings and switches, or wherever men or mules are required or permitted to cross underneath or work near it. This protection consists in placing the wire in an inverted trough of wood or other material, so that sides project at least 2 in below wire, or by cutting a groove in the roof, so that the wire is supported 2 in above roof level. Wooden troughs may be supported on roof bolts, or held by trolley wire suspensions, spaced not over 10 ft apart. Special fittings have been devised for clamping, insulating, supporting, and stretching trolley wire underground, as shown in catalogs of makers. As much of the underground trolley wire is not permanently located, soldered ears, frogs, etc, are not used.

Trolley lines should be sectionalized, so that any grounded section may be isolated and repaired without interference with other sections. It is usual to install SECTION INSULATORS for each cross or panel heading of the mine, their number and location being determined by local conditions. Where roof is very bad, it may be desirable to sectionalize every room heading or branch.

Bonding rails is done by welded, compressed and pin-terminal bonds. WELDED BONDS have a direct weld between the copper wire and steel terminal which is welded to the rail;

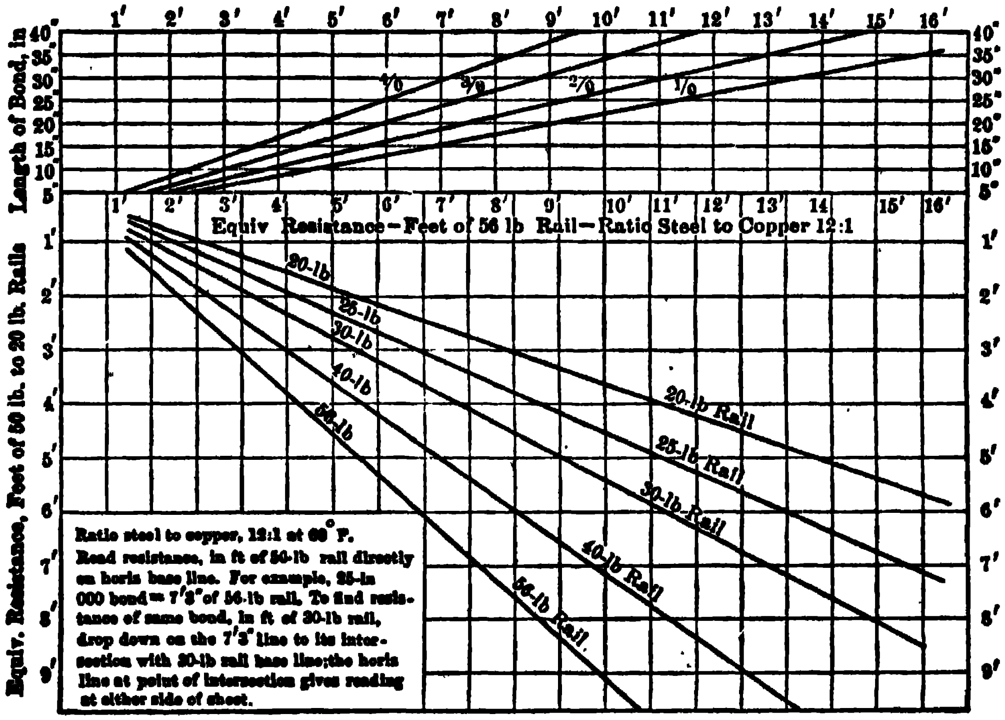

Fig 3. Rail Bond Resistance Chart

they are easy to apply and give permanency of contact; any form of arc-welding apparatus can be used. Welded bonds have a contact area equal approx to 6 times nominal capacity of the bond. COMPRESSED-TERMINAL or PIN-TERMINAL BONDS are either concealed under the splice bar or are outside of it. CONCEALED BONDS can be used satisfactorily only where the rail is 60 lb and over, as with lighter rail there is insufficient room between splice bar and rail to avoid cramping and cutting the bond. The copper stud of the pin-terminal bond should be same size as the hole in rail. A small hole is drilled in stud, and after stud is placed in the rail, a drift pin is driven through, expanding the stud to a tight fit. Following the drift pin a short, tapered steel pin is driven and remains in the stud, maintaining contact with the rail. With the compressed-terminal bond, the stud should be expanded in the rail by a screw or hydraulic compressor, and enough pressure applied to cause the copper stud to flow into and make intimate contact with the rail metal. A head is formed on each side of the hole, sealing it against moisture and maintaining contact of the bond. The compressed-terminal bond is the most efficient, with lower resistance throughout a long life, but the pin-terminal type is more easily applied and with intelligent

application gives excellent results in mining work. EARTH RESISTANCE in coal and metal mines is usually very low, because of acid water, and in many mines a large part of return current travels through the earth. This causes ELECTROLYSIS of rails, pipes, etc, which can be reduced or eliminated by good bonding of track, with return feeders if required, and by bonding track rails to air or water piping at intervals of say 500 ft (Fig 3).

Capacity of bonds should be 0.5 that of trolley and feeders, but not less than 500 circular mils per ampere should be allowed, based on aver load. Area of steel rails, sq in, = 0.1 the wt per yd, and conductivity is about $^1/_{12}$ that of copper. 1 sq in = 1 250 000 circular mils (nearly), and a short cut from lb per yd to equivalent circular mils of copper is: 1 lb per yd equals 10 000 mils. THERMIT RAIL WELDING is applicable to heavy, much used underground tracks, eliminating use of bonds and bonding troubles; reduces track maintenance and gives a smooth track. This follows the practice of street railway tracks embedded in paving.

Mine telephone wiring must have special attention, because, on account of the small sizes required, the conductors are mechanically weak and subject to breakage and trouble.

Where phones are near shaft bottoms or pit mouths, it is true economy to install wires or cables in an acid-waterproof, strong and flexible, non-metallic conduit. Along entries and gangways, where a conduit would be too expensive, rubber-insulated wire should be used. This must be supported on insulators and kept clear of coal, rock or timbers, as carefully as though it were bare, since insulation can not be depended upon underground. METALLIC CIRCUIT should be used for telephone service, to avoid noise and interference from power circuits, and lines must either be transposed frequently or TWISTED-PAIR used.

Signal systems. For these the same precautions as above should be observed and, for bare signal wires along haulage roads, potentials should not exceed 30 volts.

7. ELECTRIC HOISTING (see Sec 12 for general subject of hoisting)

Electric hoisting is more economical than steam, except where fuel is cheap and cost of power high (3). Where a c is available, either purchased or produced at the mine's central plant, hoisting by electricity eliminates boiler plant and the stand-by losses attendant upon the use of steam. Choice of TYPE OF HOIST is largely influenced by power requirements and conditions under which power is furnished. Slight changes in duty cycle will greatly modify power requirements. A cycle consisting entirely of acceleration and retardation is inherently wasteful with any type of hoist. Fig 4 shows hoist diagrams for various duty cycles, ranging from 2 to 10 sec acceleration to full speed, for a hoisting period of 17 sec, with conical drum. The fig shows effect of too rapid or too slow acceleration. The quantity $\sqrt{\text{Mean}^2}$ hp ("root mean square hp") is obtained by multiplying the square of the current input in each cycle period by the number of sec duration of the period, adding these and dividing by total elapsed time (sec), and extracting the square root (Bib 3, p 323).

Types of hoist drive (4) commonly used are: (*a*) INDUCTION MOTOR, geared to drum; (*b*) D-C MOTOR, direct-coupled to drum, with variable voltage supply from an independent motor-generator set; (*c*) D-C DRIVE, same as above, with flywheel type motor-generator set, supplying variable d-c voltage. Type (*a*) is lowest in first cost, with effic equal to or greater than type (*b*), and is usually recommended in sizes up to 600 hp, under power conditions where heavy starting peaks are not prohibitive or penalized. Type (*b*) has the advantage of ease and certainty of control (5), adjustable hoisting speed and elimination of gears. Reduced speed will reduce peaks while developing a mine, when hoisting men, and in similar service. It is more expensive than type (*a*), and has the disadvantage of requiring the operation of 2 separate machines, *viz*, the motor-generator set and the hoist. Motor-generator set may include an additional generator for mine use, to which part of the stand-by loss may be charged. Type (*c*) is suitable only for large hoists, where the power contract heavily penalizes peak loads, as the effic is 10 to 12% less than that of type (*b*). The effect of the flywheel is to cut down peaks of the load curve, and theoretically permits uniform input to the induction motor driving the hoist generator. Stand-by losses are still larger than those of type (*b*), approximating 2 kw per ton of flywheel wt, and it is not possible to reduce this charge by driving another generator (6).

Fig 5 shows typical diagram, line *AB* representing aver power input to the hoist motor. For complete elimination of peak loads the flywheel must, within each complete cycle, deliver power in hp-sec, corresponding to area of diagram above the aver line.

Fig 6 is a diagram for obtaining effective wt of flywheels. The example shown by the heavy lines assumes 20% drop in speed to deliver 25 000 hp-sec, with radius of gyration assumed as 4 ft, with set running initially at 600 rev per min, and shows that an effective wt of 39 000 lb will be required. Use of conical drums often improves the load curve by reducing the acceleration peak

and compensating for wt of rope, which is a large factor in deep shafts (Sec 12). Elec shaft hoisting is chiefly an elec problem, and design of drums will depend on the most economical duty cycle.

Control of induction-motor hoists may be by liquid rheostat or magnetic contactors. **Liquid rheostat** consists of a tank, filled with electrolyte (as soda or salt water) arranged to carry electrodes, usually triangular or trapezoidal. Electrodes are preferably fixed in

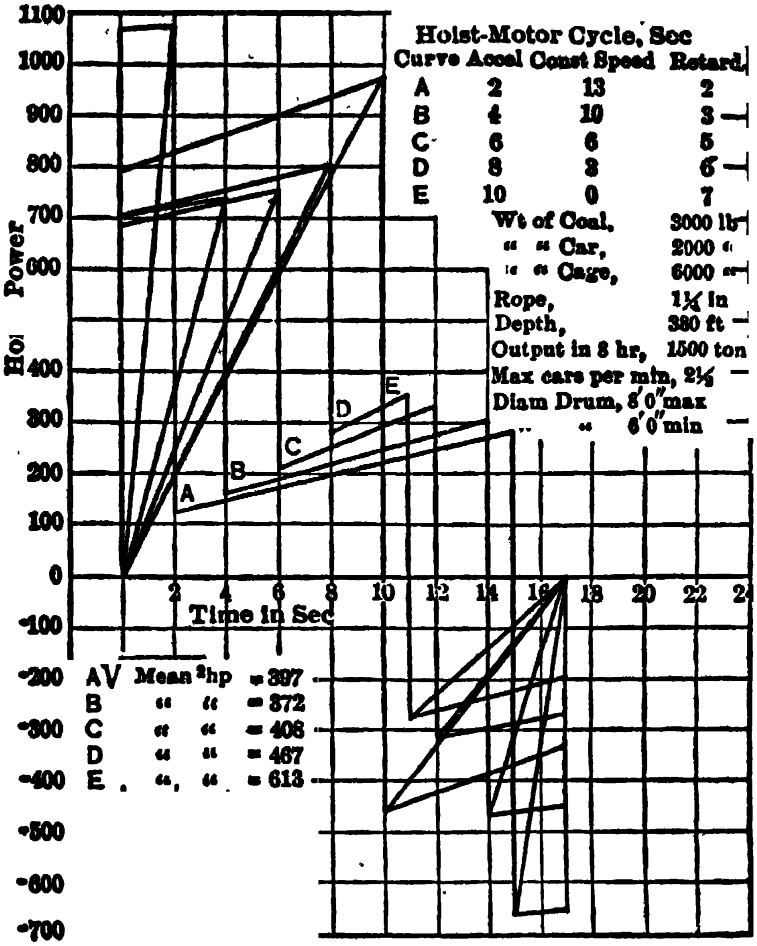

Fig 4. Electric Hoist Diagrams

position, and the electrolyte, circulated by a small pump, rises and falls in the tank. Electrodes are connected in the wound rotor secondary circuits, and motor torque and speed vary with the submerged area of electrodes. As the secondary circuit must not be opened, the electrode tips remain submerged. The primary switch of the motor is inter-locked with a lever operating a weir; the first movement of this lever closes the primary circuit, and further movement raises the weir controlling overflow of the electrolyte, allowing the liquid to rise on the electrodes. Assuming the weir to be raised at once to its full height, the rise of electrolyte on the plates, and consequently the rate of ACCELERATION of the motor, depend on the capacity of the circulating pump, thus providing a smooth and certain control of acceleration (7). This type of rheostat is preferable, because of low first cost and upkeep, and smooth acceleration.

Resistance of ordinary hydrant water, per sq ft cross-sec and 1 ft in length, is approx 100 ohms. Table 4, A, gives resistance when containing commercial table salt; Table 4, B, gives resistance when containing commercial H_2SO_4.

Table 4. Data for Liquid Rheostats

Table A		Table B	
% Salt by wt	Resist, ohms	% Acid by wt	Resist, ohms
0.23	7.84	0.174	4.12
0.46	4.65	0.435	1.75
0.70	3.12	0.724	1.10
0.93	2.38	0.985	0.85
1.16	1.90	.....	....
1.39	1.48	.....	....

For liquid rheostats without forced circulation, use from 600 to 1 000 cu in of solution per kw absorbed. Plates or electrodes should have an area of 1 sq in per ampere.

Contactor control consists of a series of magnetic contactors, arranged with current-limit control, so that the contactors close automatically in succession as the current falls

to a predetermined point. Time control may also be used where the cycle of operation is such as will permit it, and may possess advantages not present with current-limiting contactors. Position of a MASTER CONTROLLER determines the number of contactors which may close, and operator may thus start more slowly, but can not exceed the fixed limit of acceleration.

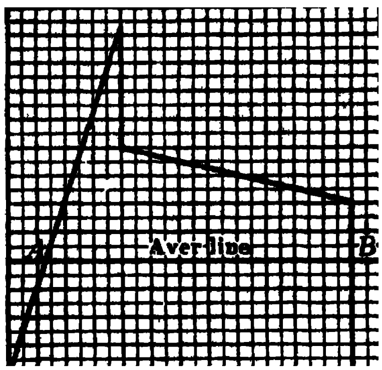

Fig 5. Hoist Diagram, Showing Aver for the Cycle

Control of d-c hoist motors by field control of variable-voltage generator is by rheostat in generator field, which is separately excited. Generator is of interpole type, and readily commutates current from 0 to max voltage, positive or negative, while running at a constant speed. Hoist-motor field is separately excited, so that its speed varies directly as impressed voltage. Same mode of control is used for ILGNER SYSTEM, with addition of a slip regulator in the induction motor circuit, allowing the set to slow down when heavily loaded, thus drawing energy from flywheel for all loads above a fixed capacity (Sec 12).

Slip regulator in a simple form consists of a liquid rheostat, in series with main induction-motor secondary, with electrodes raised by a small induction motor in series with main motor primary current, usually through series transformers. These electrodes are counterbalanced, so that, with normal current in main motor, they are completely submerged with minimum resistance in main-motor secondary. When line current exceeds predetermined amount, the torque of the small regulator motor is correspondingly increased, lifting the electrodes and thereby

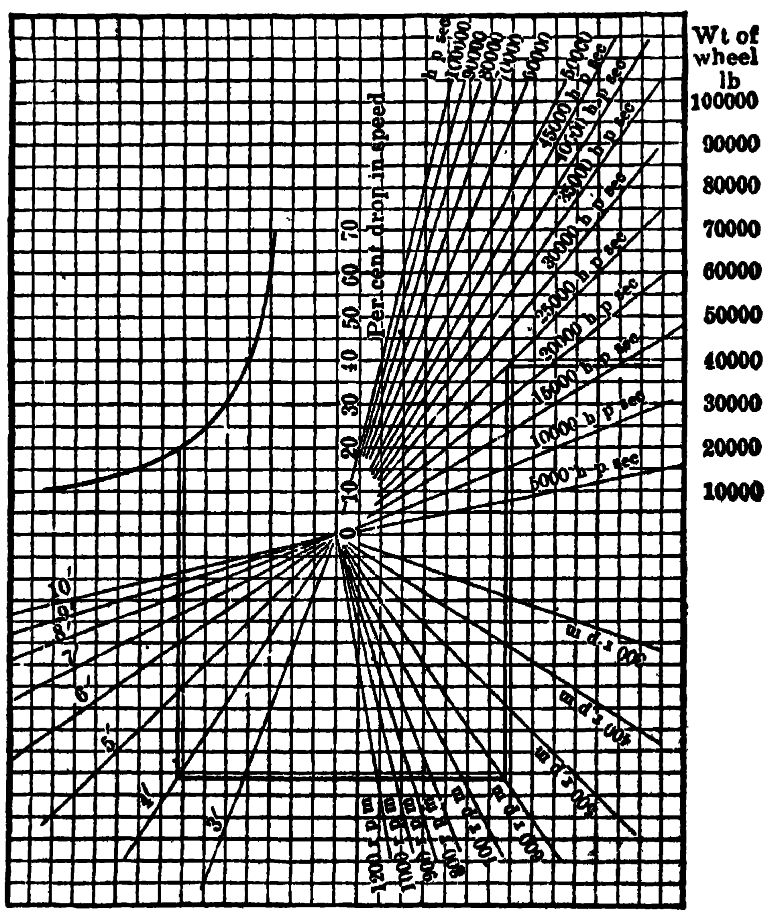

Fig 6. Diagram for Obtaining Effective Wt of a Flywheel

increasing the secondary resistance of main motor, slowing it down and allowing flywheel to deliver energy to the d-c generator.

Safety requirements demand that electric shaft hoists be provided with an approved device to prevent overwinding (Sec 12). Where men are hoisted, additional provision

must be made to prevent overwinding at their special landings, if any. In such case, there is also a visual signal to indicate at all man landings that the overwind device is set for hoisting men; and it must not be possible to operate the hoisting signal lights without setting the man-landing overwind device. Besides the main brakes, shaft hoists must have a hand brake, capable of keeping the drum under control of the operator. If the power supply fails in case of overwinding or overspeeding, an automatic trip or release applies this brake, which must be able to hold the max unbalanced load. If any contingency causes an emergency application of the brake, power is shut off from the motor.

The hoist must not be maneuvered unless all equipment, including protective devices, is in normal working condition. Should the legal rate of speed be exceeded when men are on the cage, the hoist is automatically brought to a stop. For a hoist designed to operate in balance, the driving motor must have sufficient power to raise full loads of men in a max unbalanced condition, in case of emergency. Electrical safety devices must be tested at the beginning of each shift; a record of such tests being made and signed by an authorized person, and kept on file at the mine.

Automatic control is most readily applied to D-C HOISTS with field control, since at any position of the controller speed is practically constant, even with varying load. It follows that an interlock may be arranged, which will maintain the speed in a definite relation to the position of the cage in the shaft. Hence, both acceleration and retardation may be automatic, and an overwind is impossible; and in case of overbalance or rapid retardation, current will be returned to the line. With INDUCTION-MOTOR HOIST, the speed at all resistance points varies with the load. Against overwinding, some centrifugal device should be used, in connection with a limit switch to open the circuit and apply the brakes, if the speed at certain points is not suitably reduced. If the landing is approached at proper speed, the device will not operate, but if the proper speed is exceeded at any one of a number of points, the current will be cut off and brakes applied.

Slope hoists are particularly adapted to electric drive, which eliminates losses of the steam hoist when idle, while lowering unbalanced, waiting for trips or on account of accidents; it gives uniform torque for starting, and maintains uniform speed with varying loads (as on variable grades). Motor-driven hoists may be located at any desirable point underground, where it might not be economical or possible to operate otherwise. Nearly all slope hoists have either single or double reduction gearing; the former is preferred, particularly since the perfection of the double-helical gear.

Small geared hoists are largely employed in local dips in collieries. With rope pull of 500 lb and equipped with 3-hp motor, these have been standardized by several makers, at prices approximating $400, or somewhat less. Using track sheaves, a single hoist may serve 2 or more dip headings or rooms. Some extensive bituminous mines use large numbers of these convenient hoists.

8. ROPE HAULAGE AND CONVEYERS

Endless and tail-rope haulages (Sec 11) are readily adapted to electric drive, although endless rope, with rope speeds of about 3 to 4 miles per hr, requires large gear reduction. The greater flexibility of locomotive haulage has eliminated rope haulages except where grades are very heavy. In such cases the use of motor drive permits location of haulage engines at any point, outside or underground.

Car hauls, conveyers, and elevators are almost always motor driven (see Sec 27). The proper type of motor must be selected to suit the character of work; whether induction a-c or shunt, compound or series-wound d-c motors are chosen, depends on the speed and torque characteristics of load. Enclosed and ventilated motors should be selected for dusty or wet service. Since the motors are run continuously without attendance, full overload and no-voltage protection and preferably current-limit starters should be provided.

Sometimes, as with retarding conveyers, power is required for starting, after which mechanical or regenerative braking is necessary. Frequently it is desirable to install emergency control, so that in case of trouble the conveyer or car-haul may be instantly stopped from any one of a number of points. A small switch short-circuits the trip coil on a no-voltage release, opening circuit breaker. The motor thus stopped can not be started until this switch is opened.

9. ELECTRIC LOCOMOTIVES (for "Underground Haulage," see Sec 11)

Standard mine locomotive (Fig 7) has a heavy steel frame, on journal springs over axles, gear-driven by series-wound motors, which are supported partly from axles and partly from frame. A rheostatic control, usually with series-parallel connections, and C-I grid resistances, serves for starting and running in either direction. Contactor con-

trol with master controller is used on the larger locomotives. Hand or compressed air brakes, sand boxes, valves, trolley and headlights are all of special mine types.

Details. Frames may be of C I, riveted steel plates, solid steel slabs, or cast-steel bar type. Riveted-plate and slab frames are unbreakable, as in collision. Cast bar-steel frame has the advantage of affording ready access to brakes, sand rigging, and other apparatus, and gives best ventilation. Bearings, for journals, axle linings through motors, and armature shafts, must be carefully designed to exclude dust and sand, and to give good service with minimum attention. Journal bearings are of bronze, easily removable when weight is taken off, and lubricated by waste-packed oil cellars. Axle-bearing linings may be of bronze or babbit, lubricated in the same way. Armature bearings are preferably ball bearings, though bronze-lined waste-packed bearings are also successful. It is highly important that the wear on these bearings be minimized, because wear shifts the air gap, affects commutation and gear mesh, and may destroy the armature by rubbing the pole faces. Well-designed and constructed ball bearings have eliminated such troubles almost completely on small locomotives, and are becoming standard practice on large ones of 75 hp and over. The limited space on narrow-gage locomotives makes ball bearings desirable. Motors for mine locomotives vary but little in general design from standard railway motors, though because of the lower voltages used, the coils, commutators, and brushes are often heavier for the same hp. Coils should be form-wound, interchangeable, impregnated, and baked, making them practically waterproof and increasing the conduction of heat from the copper conductor. The usual narrow track gages require very compact motors, making it increasingly difficult to obtain sufficient heat-radiating surface. Good commutation is difficult to obtain on account of

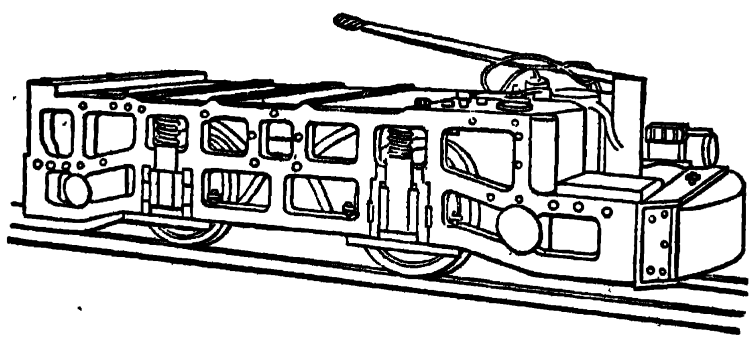

Fig 7. Electric Mine Locomotive

rough track, many frogs and switches, and severe peak loads, both on starting trips and on heavy grades. Commutating-pole motors are very advantageous, eliminating sparking due to overloads and minimizing effect of jar and vibration on commutator and brushes. They also have a higher continuous rating, due to lower magnetic densities in armature core and pole face. Where gear faces are reduced on very narrow gages, special heat-treated alloy steels may be used, and would be an advantage in standard practice. Controllers for most locomotives below 10 ton are of the magnetic blowout type, with separate interlocked cylinders for speed control and reversing; 10-ton locos or larger are of the contactor type. Most controllers have both series and parallel connections on reverse cylinders, but series connection is of no special value except for very light work, as motors will not operate satisfactorily in series unless mechanically connected, as with side-rods. Some large controllers use double contacts, with separate blowout magnets for each resistance and running point.

Resistances for speed control are practically all of the C-I grid type. Grids are strong, elastic, and easily replaced. Resistances should be placed at the ends of locomotives, where ventilation is good, and not above or near motors or cables.

Successful operation of electric locomotives depends primarily on motor rating per ton of wt; also on type of wheel, wt of rail, amount of sand used, etc.

Drawbar pull (Table 5). Maximum available for starting depends on the wt of locomotive, and in extreme cases, with sharp sand, is as high as 50% of the wt; ordinarily, say, 40%. Without sand a coeffic of adhesion of 25% may be estimated for steel wheels, or 20% for chilled C-I wheels.

Usual rating for adhesion is 25%, and motors are available which will exert this drawbar pull at rated speed for 1 hr with a temp rise of 75° C. This so-called railway rating is satisfactory where length of haul is short and grades not excessive, so that sand is unnecessary except on starting, and approximates 10 hp per ton, at 8 miles per hr. On long hauls (over 1 mile) with adverse grades requiring much sand and where motors can not

Table 5. Locomotive Drawbar Pull and Haulage Capacities

Figures are for locomotives with steel wheels; for C-I wheels, take 80% of values in table

Locomotive wt, tons (steel wheels)	Rated drawbar pull and gross train load (tons). Coeffic of friction, 30 lb per ton on level track, and 20 lb per ton for each per cent of grade													
	Level track		1% grade		2% grade		3% grade		4% grade		5% grade		6% grade	
	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons	Draw-bar pull, lb	Haul-age capac, tons
3	1 500	50	1 440	29	1 380	20	1 320	15	1 260	12	1 200	9	1 140	8
4	2 000	70	1 920	39	1 840	26	1 760	20	1 680	15	1 600	12	1 520	10
5	2 500	84	2 400	48	2 300	33	2 200	24	2 100	19	2 000	15	1 900	13
6	3 000	100	2 880	58	2 760	39	2 640	29	2 520	23	2 400	18	2 280	15
7	3 500	117	3 360	67	3 220	46	3 080	34	2 940	27	2 800	22	2 660	18
8	4 000	133	3 840	77	3 680	53	3 520	39	3 360	30	3 200	25	3 040	20
10	5 000	167	4 800	96	4 600	66	4 400	49	4 200	38	4 000	31	3 800	26
13	6 500	216	6 240	126	5 980	85	5 720	63	5 460	50	5 200	40	4 940	33
15	7 500	250	7 200	144	6 900	99	6 600	73	6 300	57	6 000	46	5 700	38
17	8 500	283	8 160	163	7 820	112	7 480	83	7 140	65	6 800	52	6 460	43
20	10 000	333	9 600	192	9 200	132	8 800	98	8 400	76	8 000	62	7 600	51
25	12 500	416	12 000	240	11 500	164	11 000	122	10 500	96	10 000	77	9 500	61

cool off between trips, the standard 1-hr capacity will not suffice. In such cases, the motor rating for continuous service is specified, and should not be less than 6 hp per ton. CONTINUOUS RATING of enclosed, series motors, as used on locomotives, ranges between 35 and 48% of the 1-hr rating. FORCED VENTILATION, by separate fan outside the casing, increases this by 60 to 85%. Chief objection is that sand, dust, or moisture may be drawn or forced into the motor windings, but, by taking in air at top and discharging it at bottom of motor casing, this difficulty is largely avoided. In mines of large output, with narrow-gage tracks and limited height of gangways, ventilated motors are necessary, and experiments made on the above lines show reduction in temp rise, as in Table 6.

Table 6. Forced Ventilation for Haulage Motors

Wt of motor, tons	Trips	Aver no cars	Length haul, ft	Temp, deg C, without fan		Temp, deg C, with fan		Mine temp	Remarks
				Com	Wind'gs	Com	Wind'gs		
24	14	72	12 000	111	104	76	72	14°	Air inlet at gear end
30	10	100	15 000	124	106	64	70	14°	Air inlet over commutator

Use of sand increases the tractive resistance of mine cars from 20 to 100%; the smallest possible quantity of the best quality obtainable should be used.

Cost of locomotives of same wt and rating is fairly uniform among the different builders. Following are examples of common sizes:

4 ton...........	$3 160	8 ton...........	$4 230	13 ton..........	$6 300
6 ton...........	3 850	10 ton.	565	15 ton...........	7 290

These prices are for chilled-iron wheels; add $152–$182 for steel-tired, and $90–$152 for rolled-steel wheels. Add $490 for cable-reel gathering locomotive, and $400 for crab-reel type (see below).

Gathering locomotives are of the same general type as main-line haulage locomotives, with the addition of a trailing-cable or rope, which permits the locomotive to enter rooms or headings where no trolley wire is strung. These locomotives require much less headroom than mules, and a locomotive with 2 men will gather from 2 to 5 times as many cars as 1 mule with driver, the advantage being greatest where grades are worst. Where the vein is only 3 to 4.5 ft thick the saving is great, because for mule haulage the top must be taken down or bottom lifted to make headroom. Gathering locomotives should handle 85 to 150 cars per day, or even more where several cars may be placed at once in each room or stope. Haulage should be planned so that cars are delivered and removed regularly; it is usually necessary to arrange headings in pairs so that on one trip empties may be placed in one and loaded cars taken from the other, rather than to handle empties and loaded cars in the same heading.

Cable-reel type has a 2-conductor twin or concentric trailing cable, attached to the trolley and to rail at the room neck, and paid out as the locomotive travels into the room, the cable as reeled out lying on the floor. A single-conductor cable is sometimes used, with the room track forming the return circuit. On returning, the cable is again wound on the reel, a tension of 10 to 20 lb being maintained to prevent over-running and cutting the cable. In some types the reel is driven by a friction clutch from the locomotive axle, but a later form drives the reel by a small series-wound motor, with resistance for continuous connection across the line, maintaining uniform tension like a spring of infinite length. Cables may be overrun and injured by the friction-clutch type, should braking cause wheels to skid, or by the motor type should the current fail. Cable-reel type may be used for pushing empty cars or pulling loaded cars up grade.

Traction-cable or crab type carries a small and very compact motor-driven hoist. In operation, the "spragger" or trip-rider pulls the wire rope into the room, couples it to car, and motorman winds up rope, pulling car out to locomotive on heading. One or more loaded cars may thus be hauled up grades too steep for locomotives. This type may be used where grades are against loaded cars only; in special cases, locomotives carry both forms of reel.

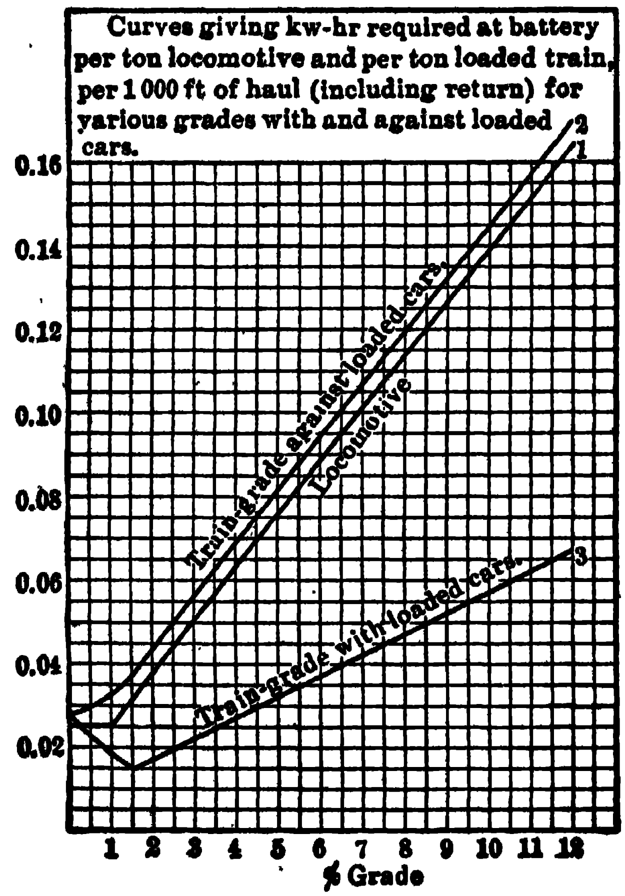

Fig 8. Battery Capac for Storage-battery Locomotive

Storage-battery locomotives, for light main-line haulage and for gathering service, are made in a number of sizes and types, and are guaranteed for specific installations. Battery equipment forms a large part of the locomotive wt, and the frame, motors, and other parts are as light as is consistent with strength. High-speed automobile-type motors are commonly used, with double-reduction or worm gearing, the voltage used ranging from 80 to 100, with a few at 150 volts and higher (16). BATTERIES are of 2 types, the acid or lead type, or the alkaline nickel-iron. Each has advantages for specific installations, but the alkaline battery is 30% lighter, and is more rugged, while the lead type is about 15% more efficient and is 30% cheaper in first cost.

Calculation of battery capacity for a given installation. Fig 8 contains 3 curves: 1, for the locomotive; 2, for the train, with grade against the load; 3, for the train on a grade with load. The curves are based on frictional resistance of 30 lb per ton on level track, plus 20 lb for each per cent of grade. Effic of locomotive and motor equipment is assumed to be 60%. These curves can be used for calculating battery capacity for any kind of duty; given the wt of loaded train, number of cars per train, grade, haulage distance, and wt of locomotive. The proper wt of locomotive, which must be sufficient to haul the max load on specified grades, is selected from Table 7, which gives max drawbar pull (lb) and haulage capac (tons) for a given wt of locomotive equipped with steel wheels. Track friction, 30 lb per ton and resistance of 20 lb per ton for each per cent of grade.

Calculation of battery capacity for a gathering locomotive. It is convenient to divide the length of haul into 3 parts: distance from side track to first room on entry; distance on entry in front of rooms; max depth of room (total depth is used in calculation to allow for switching, etc.).

Example. Assume the locomotive must gather 120 cars in 8 hr, delivering each empty car to the face. Trains are of 10 cars, making total number of trips 12. Wt of loaded car, 5 200 lb; length of haul from side track to first room, 600 ft; depth of room, 250 ft; length of haul on entry

Table 7. Haulage Capacity of Storage-battery Locomotives on Grades

Loco wt, tons	Level		1%		2%		3%		4%		5%	
	Draw-bar pull, lb	Haulage capac, tons	Draw-bar pull, lb	Haulage capac, tons	Draw-bar pull, lb	Haulage capac, tons	Draw-bar pull, lb	Haulage capac, tons	Draw-bar pull, lb	Haulage capac, tons	Draw-bar pull, lb	Haulage capac, tons
3	1 500	50	1 440	29	1 380	20	1 320	15	1 260	12	1 200	9
4	2 000	70	1 920	39	1 840	26	1 760	20	1 680	15	1 600	12
5	2 500	84	2 400	48	2 300	33	2 200	24	2 100	19	2 000	15
6	3 000	100	2 880	58	2 760	39	2 640	29	2 520	23	2 400	18
7	3 500	117	3 360	67	3 220	46	3 080	34	2 940	27	2 800	22
8	4 000	133	3 840	77	3 680	53	3 520	39	3 360	30	3 200	25

in front of rooms, 400 ft. Rooms are level and grade on entry is 1.5% against the load. Distance the locomotive travels, and distance the train is hauled on different parts of a trip, are given in tabular form:

	Equiv length of haul, ft	
	Loco	Loaded train
Side track to first room..	600	600
On entry in front of rooms	400	400 + 2 = 200
In rooms................		10 × 250 = 2 500

Loaded train weighs.... $\frac{5\,200 \times 10}{2000} = 26$ tons

Assume locomotive weighs 5 tons. Kw-hr required: to haul 1 ton of locomotive 1 000 ft on level, 0.025; to haul 1 ton of locomotive 1 000 ft on 1.5% grade, 0.031; to haul 1 ton of loaded train 1 000 ft on level track, 0.026; to haul 1 ton of loaded train 1 000 ft on 1.5% grade, 0.037.

Kw-hr required for each part of haul over which LOCOMOTIVE travels = (distance (ft) ÷ 1 000) × loco wt × kw-hr (read from curve). Per trip for level track, (2 500 ÷ 1 000) × 5 × 0.025 = 0.312. Per trip for 1.5% grade, (1 000 ÷ 1 000) × 5 × 0.031 = 0.155.

Kw-hr required for each part of haul over which TRAIN travels = (distance (ft) ÷ 1 000) × tons of loaded train × kw-hr (read from curve). Per trip on level track, (250 ÷ 1 000) × 26 × 0.026 = 0.17. Per trip on 1.5% grade, (800 ÷ 1 000) × 26 × 0.037 = 0.77. Total kw-hr per trip: locomotive, 0.467; train, 0.952; or 1.419. Hence, total for 12 trips = 17 kw-hr.

Table 8. Kw-hr Capacity at Normal Discharge Rates

(Normal battery charge for standard locomotives)

Alkaline		Lead	
Type	Kw-hr	Type	Kw-hr
63 A–4	11.3	48 MV 9	11.9
63 A–6	16.0	48 MV 11	14.8
63 A–8	22.6	48 MV 13	17.9
126–A–4	22.6	48 MV 15	20.8
....	.....	96 MV 9	23.8

To allow for decreased capacity of battery when discharging at a high rate, as required in gathering service, switching extra cars, pulling empties and loads on track, hauling locomotives from charging station to working place, and running with brake partly set, the kw-hr obtained by calculation is multiplied by 1.35, to give the capacity at normal discharge rate, which in this case is 23 kw-hr. Consult Table 8 for battery capacities.

Note that for above example the nearest battery capacity with least number of cells is 63 A-8 alkaline, or 48 MV 15 lead. Either will need at least 1.5-hr boosting charge, whenever the battery has been in use for 4 hr, in which case alkaline battery will have 15% greater capacity, and lead 25% greater capacity than given in table. Example shows how Fig 8 is used when calculating battery capacity required for gathering, and it is evident that this method can be used for any kind of installation on any kind of haul.

10. ELECTRIC-DRIVEN PUMPS (for "Mine Drainage," see Sec 13)

Piston and plunger pumps are being superseded by centrifugal pumps for every use underground, because of lower first cost, less room required and less maintenance. Electric drive is now almost exclusively used for centrifugal mine pumps. Details are given in Sec 13, Art 10.

Main underground centrifugals usually have cast-iron casings and bronze runners; or bronze casings with chrome-iron runners, or entirely of chrome, depending on character of the mine water. These pumps are designed for specific heads, stages being added as head increases. For PRIMING centrifugal pumps no foot valve is required in the suction, but a check valve in the discharge is essential. With automatic control, a float switch or electrode is customary for starting and stopping. A time clock or remote control can be used. A relay will stop pump when three attempts have been made to start it; a suitable signal indicates when this occurs. In case steam, compressed air or water under pressure is available for priming, the vacuum pump can be dispensed with. It is essential that all air in the pump or suction line be exhausted before the pump will start.

Submerged pumps, requiring no priming device, are sometimes useful; but, due to their cost, they are generally limited to relatively small units, say not over 20 ft in length. For UNWATERING MINES, the cost of BORE-HOLE PUMPS may be fully warranted. They consist of a vertical motor mounted on the surface on top of the discharge, with a shaft extending to a pump at the bottom. Where the water is acid, protection of exposed parts is expensive. Such pumps, having capacities to 5 000 gal per min against heads of 600 ft, are built by the Sterling Pump Co, Hamilton, Ohio, and other makers. Fig 9 shows a submersible pump, made by the Byron Jackson Co, Los Angeles, Calif, the motor of which is mounted at the bottom inside the casing, power being carried to it by a power cable and copper oil tube carried outside the discharge pipe to the surface. It can be installed in a crooked slope, shaft or bore hole, and turned at any angle at the bottom. Sizes are from 10 to 10 000 gal per min.

Gathering pumps, discharging into main sumps, usually require 3-10 hp motors, arranged for "across-the-line" starting, and are connected to the trolley system or feeder lines. They can be provided with a float switch for starting and stopping.

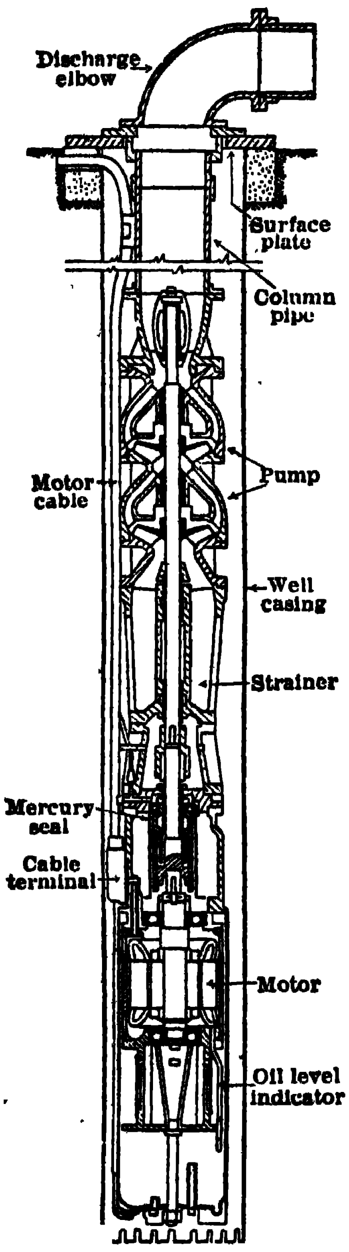

Fig 9. Submersible Electric-driven Centrifugal Pump

11. ELECTRIC COAL CUTTERS (for compressed-air cutters, see Sec 15, Art 14 and Bib 9)

Principal types: shortwall, longwall, turret or top cutter, and universal cutter, the earlier chain-breast type having given way to the shortwall machine. In 1939, the tendency is toward larger capacity motors, to meet the demands of double or triple shifting. For some special applications, motors of 100 hp (intermittent rating) are employed. Contactor control, with thermal and magnetic overload protective features, is in general use (20–25).

Shortwall machine (Fig 10) has no stationary guide frame, and for moving is mounted on a truck, propelled by gearing connected to the cutter motor. It carries a reel holding 200–500 ft of trailing cable. The cutter head is a narrow rectangle, with driving sprocket at one end and an idler at the other. By rope or chain feed, the machine pulls itself into the coal face to the length of cutter bar and then travels sidewise, cutting close to the floor and sliding on it. The machine can also cut at a determined height above the floor. In mines worked with conveyer systems, the truck is not generally used, as the machine remains in place until the mining is completed there. A recent departure from heavy designs in shortwall machines is the Sullivan "Buddy," with 15-hp motor. Being designed for conveyer mining, it has no cable reel or truck.

Longwall machine (Fig 11) resembles the shortwall, but has a longer and narrower body, with the cutter bar at right angles to the frame. Little used in this country at present, but its use is likely to increase in thin seams.

Turret or top cutter (Fig 12) is generally track-mounted, with two motors, one propelling the machine, the other driving the cutter chain. It is now less used than formerly. The cutter bar operates in a horiz plane, with limited vert adjustment for variations in height of seam, or dirt bands to be cut out. This machine is also available with cutter bar arranged to cut at or near bottom of the seam. One design of turret machine is not track-mounted, but moves across the coal face in essentially the same manner as the shortwall machine.

Universal or track-mounted cutter (Fig 13). The cutter bar is adjustable for working at any desired angle and height, thus permitting shearing, or top, center, or bottom cutting to be made as conditions require.

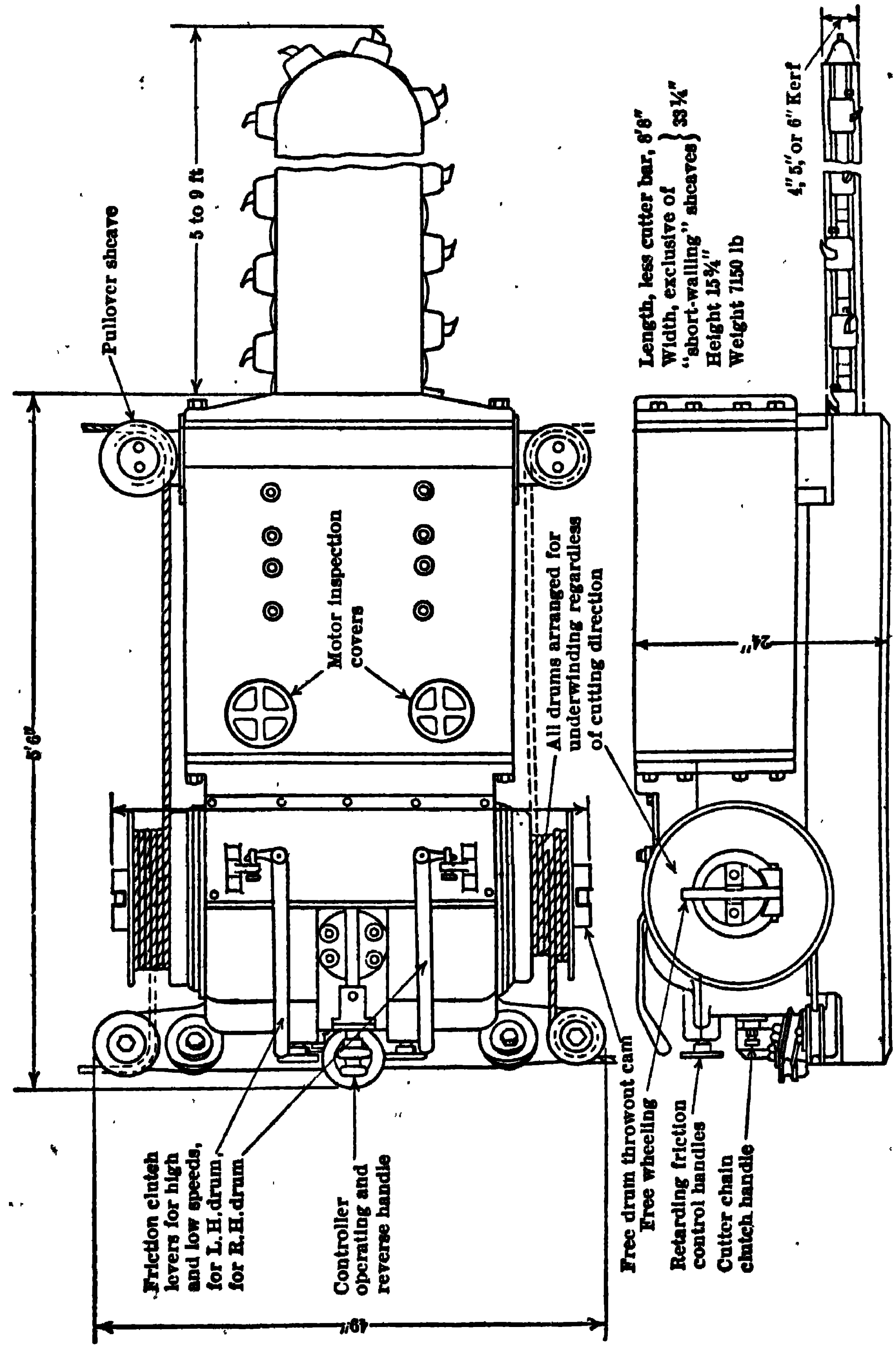

Fig 10. Sullivan Shortwall Coal Cutter (representative of all standard makes)

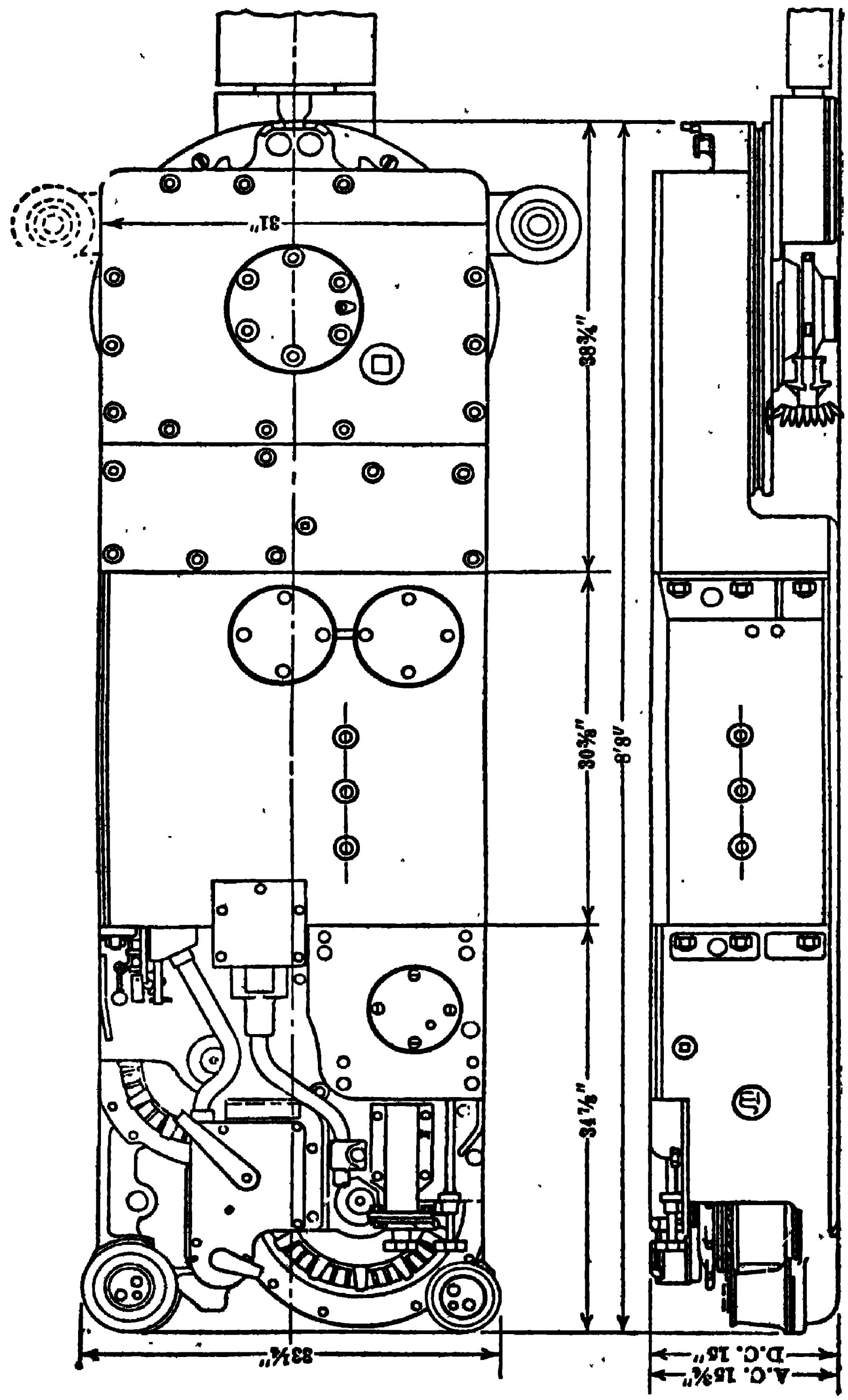

Fig 11. Jeffrey Longwall Coal Cutter

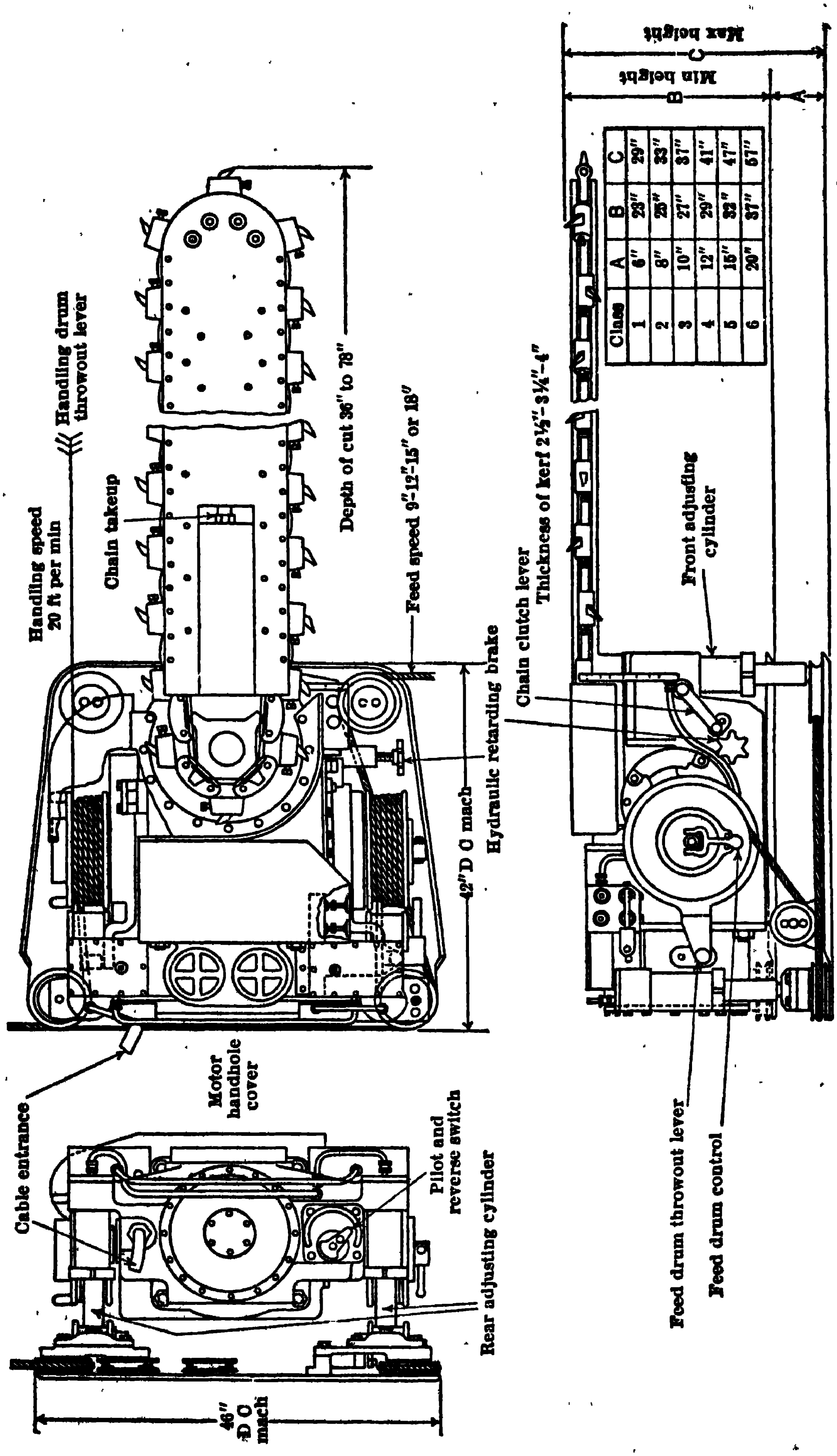

Fig 12. Sullivan Floor-type, Turret, or Top Cutter (9B)

Coal cutters in general have one or more motors, to a max of about 100 hp. Present tendency is to increase the power, because of continual service in machine mines. The current is usually a c at 250–500 volts, but d c can be used.

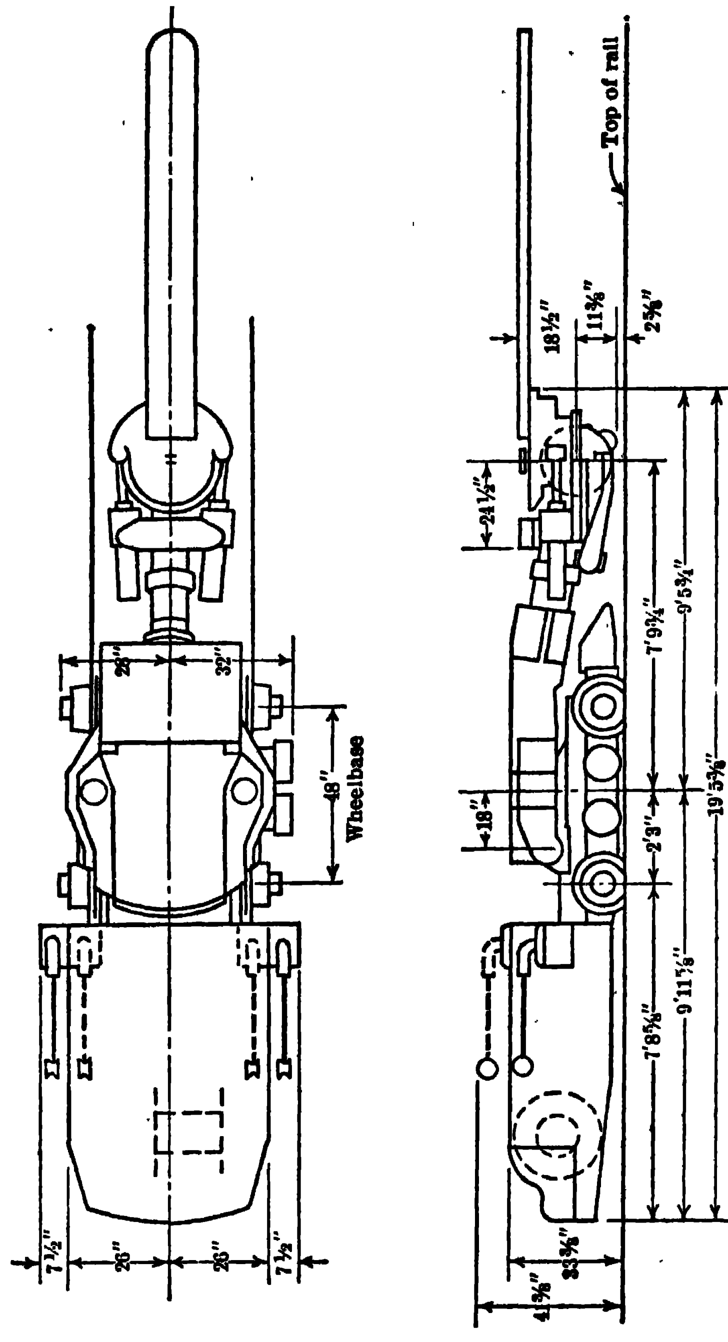

Fig 13. Jeffrey Track Coal Cutter, Universal Type, Bottom, Top, or Center Cuts, or for Shearing

12. MINE LIGHTING BY ELECTRICITY (see also Sec 23)

Lighting of mine buildings, offices, stores, shops and dwellings, should be at low current potentials, preferably not over 125 volts, to reduce fire risk and shock hazard. This involves the use of a c current, with distribution at standard voltage of 2 200 and step-down transformers of suitable capacity at centers of distribution. Large buildings, as storehouses, should have 3-wire systems at 110–220 volts, to reduce wiring costs. Dwellings may be grouped in blocks of 10 to 20 or more, supplied from a single transformer.

Underground lighting of haulageways is done by connecting 275-volt lamps singly or in groups in parallel to the mine d c trolley and feeder circuits, except for 500-volt trolley systems, in which case 110, 125 or 275-volt lamps may be connected in series as required.

At SHAFT BOTTOMS and similar areas of important activity, where a c circuits serve pumps and hoists, a small step-down transformer may be used to reduce 2 200 volts to 110, or 110-volt lamps used in series for 220 and 440-volt a c circuits.

Miners' storage-battery cap lamps (see Sec 23) are widely used in nearly all mines. They are generally installed on a rental basis, under which the miner is assessed 6 to 8¢ per shift, the mine operator paying 65 to 85¢ per month to cover parts and servicing. There are two types: (*a*) Wheat lamp, having a 2-cell lead-acid storage battery, and (*b*) Edison lamp, with 2 or 3-cell alkaline batteries. Both lamps are highly efficient and have the approval of the U S Bur of Mines for use in gaseous mines. The Wheat lamp complete weighs 3.6–4.5 lb, depending on the model; the Edison 4–5.4 lb. Makers also supply lamps fitted with handles, for officials and others in inspection service. Special lamps, with red globes for trip or tail light duty, can be had with either storage or dry cell batteries. Flash lamps, having dry cells, are now made in permissible types.

13. UNDERGROUND MECHANICAL LOADERS

These comprise a large variety of equipment, nearly all of which is operated electrically. Full details of construction and applications are given in Sec 27.

14. ELECTRIC-DRIVEN FANS AND BLOWERS

Electric drive is now used almost exclusively for all kinds of mine ventilators. Constant speed motors are best adapted to this service, though variable speed is sometimes used. Details of fans and blowers are given in Sec 14.

15. MISCELLANEOUS ELECTRICAL DEVICES

Block-signal systems (Fig 14), with red, green or yellow lights, for control of main-line haulage are available with manual or automatic operation, for both of which current is taken from trolley circuits. Only one motor at a time can enter a single-track line. Signals reduce waiting time at sidings, making faster and more effic haulage. Manual operation involves opening and closing the circuits by hand; automatic systems are usually operated by relays, receiving current by the passage of the trolley wheel or glider (see also Sec 11, 23).

Rock-dust distributers of various sizes and capacities are available. Fig 15 is an example. One type has a 20-hp motor, with push-button control, which drives feed screw, agitator, and a high pressure blower capable of distributing 60–100 lb of dust per minute through 500 ft of 3-in hose, or through a fan-shaped nozzle. A recent design has 2 motors, one driving the blower, the other the agitator and feed screws. Rock dust distributers are not self propelling, but are usually moved by a cable-reel or storage-battery locomotive; or the distributer can be connected direct to the trolley wire with a trolley tap and rail clamp. For the general subject of rock-dusting see Sec 23.

Photo-electric cells (electric eyes) and relays are applied in and about mines to promote safety and efficiency by automatic control or operation of equipment for: (*a*) opening and closing of doors for passage of locomotives and cars; (*b*) throwing of track switches; (*c*) operating block-signal systems that protect main-line haulage; (*d*) giving warning of approach of locomotives and cars; (*e*) counting cars in trips; (*f*) checking number of men entering or leaving mines; (*g*) insuring proper spotting of mine cars on cages; (*h*) weighing cleaning plant rejected material.

Methane detectors use the Wheatstone bridge principle of measuring elec resistance, to indicate the amount of methane present in mine atmos by measuring the change in resistance of a heated platinum filament. A miner's cap-lamp battery or dry cells supply the current for operating these devices (for details, see Sec 23).

Mine fan signal systems. One device consists of a contact attachment to the fan shaft, which interrupts a circuit to one or more incandescent lamps. The rate of flickering gives a visual indication of the fan speed. Another device employs an air vane in the fan housing. When the fan stops, or the speed drops too low, the decrease in air pressure causes the vane to drop and close contacts to complete the circuit to an audible signal. A propeller-driven generator is connected to a voltmeter, with scale calibrated in rpm.

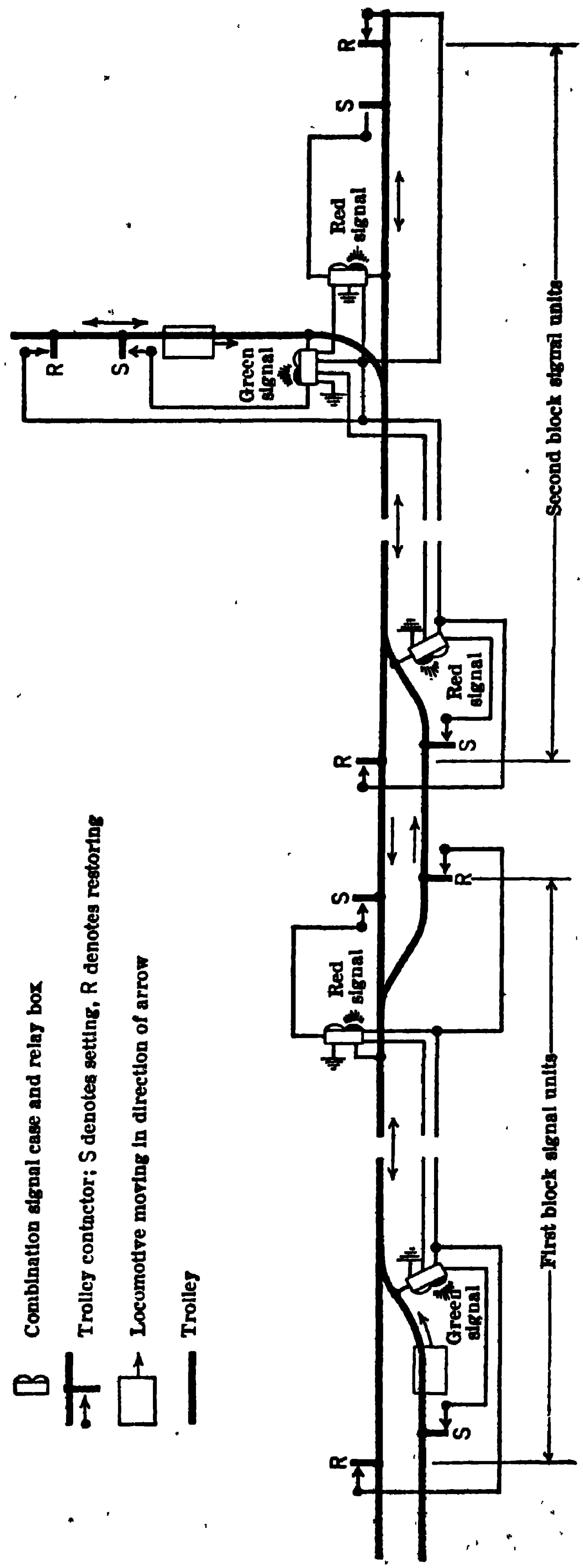

Fig 14. Typical Signal Layout for Main Haulway (Nachod & U S Signal Co, Louisville, Ky)

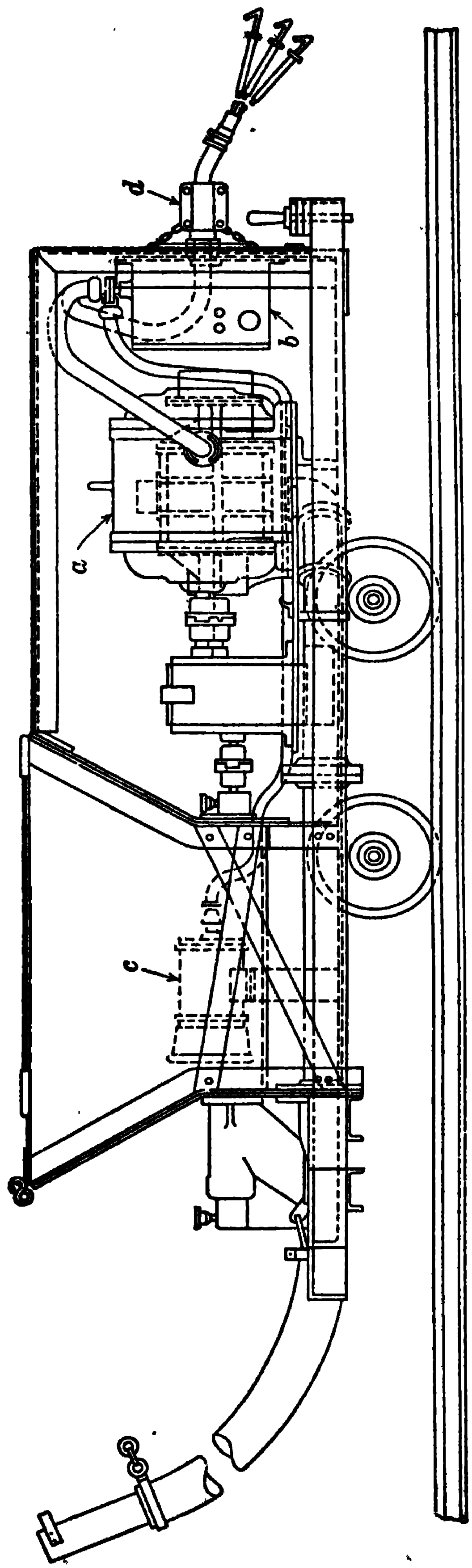

Fig 15. Rock-dust Distributer (Mine Safety Appliance Co, Type 65AC) *a*, Motor. *b*, Magnetic starter. *c*, Headlight. *d*, Cable clamp.

16. TYPES OF ELECTRIC MOTORS FOR MINE SERVICE

The following tables of sizes and prices of motors are condensed from catalogues of the General Electric Co, dated Jan 1, 1939. Prices of other makers correspond closely with these tabulations.

Table 9. Squirrel-cage Induction Motors, Normal Torque, 25-Cycle

Type K, normal-starting current; KF, low-starting current. Constant speed, 3-phase; each unit at 220, 440 and 550 volts

Hp	Synchronous, rpm	Price, $, motor only							
		Type K, open		Type KF, open		Standard		Explosion-proof	
		Sleeve-bearing	Ball-bearing	Sleeve-bearing	Ball-bearing	Type K, ball-bearing	Type KF, ball-bearing	Type K, ball-bearing	Type KF, ball-bearing
10	1 500	....	....	208	218	....	295	....	340
	750	208	218	208	218	295	295	340	340
15	1 500	....	....	286	300	....	373	....	418
	750	286	300	286	300	390	390	456	456
20	1 500	....	....	338	355	....	442	....	508
	750	338	355	338	355	544	544	635	635
25	1 500	....	....	420	441	....	826	....	917
	750	420	441	420	441	626	626	717	717
30	1 500	....	....	486	510	....	974	....	1 065
	750	486	510	486	510	758	758	849	849
40	1 500	....	....	573	602	....	1 226	....	1 317
	750	573	602	573	602	1 165	1 165	1 256	1 256
50*	1 500	....	....	685	719	....	1 426	....	1 517
	750	685	719	685	719	1 345	1 345	1 480	1 480
60*	1 500	....	....	791	831	....	1 730	....	1 903
	750	791	831	791	831	1 795	1 795	1 975	1 975
70*	1 500	....	....	920	966	....	2 020	....	2 222
	750	920	966	920	966	2 082	2 082	2 290	2 290

* These motors are recommended only for direct connection

The above 25-cycle motors are now little used in mines; due to the development of 60-cycle motors (see Table 10).

Table 10. Squirrel-cage Induction Motors, Constant-speed, 3- or 2-phase, 60 Cycles

Type K, normal-torque, normal-starting current; KF, normal-torque, low-starting current; KG, high-torque, low-starting current

Hp	Synchronous, rpm	Volts	Price,* $, Type K: Sleeve-bearing	Price,* $, Type K: Ball-bearing
1/2	3 600	220	23.25	27.00
	"	110	25.60	29.35
	"	440, 550	25.60	29.35
	1 200	220/440	36.00	40.00
	"	110, 550	39.60	43.60

Hp	Synchronous, rpm	Volts	Price,* $, Type K: Sleeve-bearing	Price,* $, Type K: Ball-bearing
3/4	3 600	220/440	29.25	33.25
	"	110, 550	32.20	36.20
1	3 600	220/440	36.00	40.00
	"	110, 550	39.60	43.60

Hp	Synchronous, rpm	Price,* $ Sleeve-bearing K, KF	Sleeve-bearing KG	Ball-bearing K, KF	Ball-bearing KG
For 110, 220, 440 and 550 volts					
1/2	900	44a	....	49a	.....
	600	66a	....	70a	.....
3/4	1 200	39a	....	43a	.....
	900	51a	....	55a	.....
	600	80a	....	84a	.....
	514	93a	....	98a	.....
1	1 800	35a	....	39a	.....
	900	58a	....	62a	.....
	720	80a	....	84a	.....
	514	101a	....	106a	.....
1 1/2	3 600	44a	....	48a	.....
	1 800	42a	....	46a	.....
	900	70a	....	74a	.....
	720	89a	....	93a	.....
	514	129a	....	135a	.....
2	3 600	51a	....	55a	.....
	1 800	49a	....	53a	.....
	900	81a	....	85a	.....
	720	97a	....	102a	.....
	514	154a	....	162a	.....
3	3 600	58a	....	62a	.....
	1 800	55a	58	59a	62
	900	92a	97	97a	102
	720	123a	....	129a	.....
5	3 600	70a	....	74a	.....
	1 800	67a	70	71a	74
	900	118a	124	124a	130
	720	147a	....	154a	.....
7 1/2	3 600	92b	....	97b	.....
	1 800	88	92	92	97
	900	141	148	148	155
10	3 600	118b	....	124b	.....
	1 800	112	118	118	124
15	3 600	141b	....	148b	.....
	1 800	134	141	141	148
20	3 600	168b	....	176b	.....
For 220, 440 and 550 volts					
3	514	202a	....	212a	.....
5	514	248a	....	260a	.....
7 1/2	720	194	....	204	.....
	514	292a	....	307a	.....
10	900	176	189	185	198
	720	238	....	250	.....
	514	336a	....	353a	.....
15	900	216	232	227	244
	720	279	....	293	.....
	514	419a	....	440a	.....

Hp	Synchronous, rpm	Price,* $ Sleeve-bearing K, KF	Sleeve-bearing KG	Ball-bearing K, KF	Ball-bearing KG
For 220, 440 and 550 volts					
20	1 800	160	172	168	181
	900	254	273	267	287
	720	321	....	337	.....
	514	484a	....	508a	.....
25	3 600	194b	....	204b	.....
	1 800	185	199	194	209
	900	292	336	307	353
	720	400	....	420	.....
	600	463	....	486	.....
	514	570a	....	599a	.....
30	3 600	267b	....	280b	.....
	1 800	254	273	267	287
	900	364	419	382	440
	720	463	....	486	.....
	600	546	....	573	.....
	514	640a	....	672a	.....
40	3 600	322b	....	338b	.....
	1 800	307	353	322	371
	1 200	364	419	382	440
	900	421	484	442	508
	720	546	....	573	.....
	600	615	....	646	.....
	514	748a	....	785a	.....
50	3 600	401b	....	421b	.....
	1 800	382	439	401	461
	1 200	421	484	442	508
	900	496	570	521	599
	720	632	....	664	.....
	600	696	....	731	.....
	514	846a	....	888a	.....
60	3 600	464b	....	487b	.....
	1 800	442	508	464	533
	1 200	496	570	521	599
	900	572	658	601	691
	720	699	....	734	.....
	600	791	....	831	.....
	514	946a	....	993a	.....
75	3 600	599b	....	629b	.....
	1 800	521	599	547	629
	1 200	595	684	625	718
	900	672	773	706	812
	720	803	....	843	.....
	600	909	....	954	.....
	514	1 077a	....	1 131a	.....
100	3 600	833b	....	875b	.....
	1 800	666	766	699	804
	1 200	779	896	818	941
	900	824	948	865	995

Table 10. Squirrel-cage Induction Motors, Constant-speed, 3- or 2-phase, 60 Cycles (*Cont'd*)

Hp	Synchronous, rpm	Price,* $ Sleeve-bearing K, KF	Sleeve-bearing KG	Ball-bearing K, KF	Ball-bearing KG	Hp	Synchronous, rpm	Price,* $ Sleeve-bearing K, KF	Sleeve-bearing KG	Ball-bearing K, KF	Ball-bearing KG
	For 220, 440 and 550 volts						For 220, 440 and 550 volts				
	720	986	....	1 035	.....		1 200	1 078	....	1 132	......
	600	1 116	....	1 172	.....		900	1 205	....	1 265	......
	514	1 274*a*	....	1 338*a*	.....		720	1 290	....	1 355	......
125	3 600	1 032*b*	....	1 084*b*	.....		600	1 436	....	1 508	......
	1 800	794	....	834	.....		514	1 574*a*	....	1 653*a*	......
	1 200	928	....	974	.....	200	3 600	1 528*b*	....	1 604*b*	......
	900	1 043	....	1 095	.....		1 800	1 175	....	1 234	......
	720	1 152	....	1 210	.....		1 200	1 410	....	1 481	......
	600	1 280	....	1 344	.....		900	1 520	....	1 596	......
	514	1 425*a*	....	1 496*a*	.....		720	1 550	....	1 628	......
150	3 600	1 196*b*	....	1 256*b*	.....		600	1 729	....	1 815	......
	1 800	920	.....	966	. .		514	1 877*a*	....	1 971*a*	......

* Motor only. *a*, Type K only. *b*, Type KF only. Motors of 1/6, 1/4 and 1/3 hp are also made; also motors of other speeds than those listed. All are open-type, 60-cycle, polyphase motors, rated 40° C; when operated on 50 cycles at listed voltages will run without injurious heating, not exceeding 50° C rise; 60-cycle ratings and prices apply. Sync speeds are 5/6 of those at 60 cycles.

Table 11. Totally-enclosed, Fan-cooled, Squirrel-cage Induction Motors, Standard and Explosion-proof; Constant-speed, 3- or 2-phase, 60 Cycles

Type K, normal-torque, normal-starting current; KF, normal torque, low-starting current; KG, high-torque, low-starting current; all units ball bearing

Hp	Synchronous, rpm	Price,* $ Standard K, KF	Standard KG	Explosion-proof K, KF	Explosion-proof KG	Hp	Synchronous, rpm	Price,* $ Standard K, KF	Standard KG	Explosion-proof K, KF	Explosion-proof KG
	For 110, 220, 440 and 550 volts						For 2 200 volts				
3/4	720	93*a*	....	111*a*	.....	30	1 200	571	....	662	......
1	720	112*a*	....	134*a*	.....		900	679	....	770	......
	600	121*a*	....	143*a*	.....	40	3 600	623*b*	....	689*b*	......
1 1/2	720	121*a*	....	143*a*	.....		1 800	594	....	685	......
	600	140*a*	....	164*a*	.....		1 200	679	....	770	......
2	3 600	78*a*	....	96*a*	.....		900	816	....	907	......
	1 800	76*a*	....	94*a*	.....		720	1 022	....	1 113	......
	1 200	82*a*	....	100*a*	.....		600	1 191	....	1 293	......
	900	113*a*	....	135*a*	.....	50	3 600	796*b*	....	887*b*	......
	720	140*a*	....	164*a*	.....		1 800	763	....	854	......
	600	183*a*	....	214*a*	.....		1 200	816	....	907	......
3	3 600	85*a*	....	103*a*	.....		900	939	....	1 030	......
	1 800	82*a*	85	100*a*	103		720	1 173	....	1 277	......
	1 200	99*a*	102	121*a*	124		600	1 314	....	1 429	......
	900	135*a*	140	159*a*	164	60	3 600	885*b*	....	976*b*	......
	720	183*a*	....	214*a*	.....		1 800	850	....	941	......
	600	207*a*	....	238*a*	.....		1 200	939	....	1 030	......
5	3 600	97*a*	....	115*a*	.....		900	1 098	....	1 192	......
	1 800	99*a*	102	121*a*	124		720	1 266	....	1 381	......
	1 200	131*a*	135	155*a*	159		600	1 442	....	1 573	......
	900	178*a*	184	209*a*	215	75	3 600	1 179*b*	....	1 270*b*	......
	720	207*a*	....	238*a*	.....		1 800	1 085	....	1 176	......
7 1/2	3 600	124*b*	....	146*b*	.....		1 200	1 234	....	1 344	......
	1 800	131	135	155	.....		900	1 374	....	1 498	......
	1 200	172	178	203	.....		720	1 575	....	1 724	......
	900	201	208	232	.....		600	1 803	....	1 971	......
10	3 600	161*b*	....	185*b*	.....	100	3 600	1 562*b*	....	1 706*b*	......
	1 800	172	178	203	.....		1 800	1 356	....	1 483	......
	1 200	194	201	225	.....		1 200	1 601	....	1 749	......
15	3 600	201*b*	....	232*b*	.....		900	1 693	....	1 850	......
	1 800	194	201	225	.....		720	1 919	....	2 106	......
20	3 600	228*b*	.....	259*b*	.....		600	2 191	....	2 403	......

Table 11. Totally-enclosed, Fan-cooled, Squirrel-cage Induction Motors, Standard and Explosion-proof; Constant-speed, 3- or 2-phase, 60 Cycles (*Continued*)

Type K, normal-torque, normal-starting current; KF, normal-torque, low-starting current; KG, high-torque, low-starting current; all units ball-bearing

Hp	Synchronous, rpm	Price,* $ Standard K, KF	Price,* $ Standard KG	Price,* $ Explosion-proof K, KF	Price,* $ Explosion-proof KG
		For 2 200 volts			
125	3 600	1 880*b*	....	2 056*b*	......
	1 800	1 595	....	1 746	......
	1 200	1 874	....	2 050	......
	900	2 067	....	2 265	......
	720	2 254	....	2 473	......
150	3 600	2 145*b*	....	2 349*b*	......
	1 800	1 830	....	2 005	......
	1 200	2 151	....	2 356	......
	900	2 360	....	2 589	......
	720	2 509	....	....	......
200	3 600	2 678*b*	....	2 938*b*	......
	1 800	2 291	....	2 514	......
	1 200	2 737	....	3 005	......
		For 220, 440 and 550 volts			
5	600	281*a*	....	326*a**	......
7 1/2	720	281	....	326	......
	600	342	....	408	......
10	900	263	276	308	......
	720	342	....	408	......
	600	383	....	449	......
15	1 200	263	276	308	......
	900	320	336	386	......
	720	383	....	449	......
	600	482	....	573	......
20	1 800	247	259	292	......
	1 200	320	336	386	......
	900	358*c*	377	424	......
	720	482	....	573	......
	600	600	....	691	......
25	3 600	281*b*	....	326*b*	......
	1 800	272	286	317	331
	1 200	358	377	424	443
	900	438	482	529	573
	720	600	....	691	......
	600	764	....	855	......
30	3 600	354*b*	....	420*b*	......
	1 800	358	377	424	443
	1 200	438	482	529	573
	900	546	601	637	692
	720	764	....	855	......
	600	901	....	992	......
		For 220, 440 and 550 volts			
40	3 600	477*b*	....	543*b*	......
	1 800	461	507	552	598
	1 200	546	601	637	692
	900	695	758	786	849
	720	901	....	992	......
	600	1 015	....	1 117	......
50	3 600	650*b*	....	741*b*	......
	1 800	630	687	721	778
	1 200	695	758	786	849
	900	818	....	909	......
	720	1 043	....	1 147	......
	600	1 148	....	1 263	......
60	3 600	752*b*	....	843*b*	......
	1 800	729	795	820	886
	1 200	818	....	909	......
	900	944	....	1 038	......
	720	1 153	....	1 268	......
	600	1 305	....	1 436	......
75	3 600	1 046*b*	....	1 137*b*	......
	1 800	964	1 042	1 055	1 133
	1 200	1 101	....	1 211	......
	900	1 243	....	1 367	......
	720	1 486	....	1 635	......
	600	1 682	....	1 850	......
100	3 600	1 441*b*	....	1 585*b*	......
	1 800	1 265	....	1 392	......
	1 200	1 480	....	1 628	......
	900	1 566	....	1 723	......
	720	1 873	....	2 060	......
	600	2 120	....	2 332	......
125	3 600	1 759*b*	....	1 935*b*	......
	1 800	1 509	....	1 660	......
	1 200	1 763	....	1 939	......
	900	1 982	....	2 180	......
	720	2 189	....	2 408	......
150	3 600	2 038*b*	....	2 242*b*	......
	1 800	1 748	....	1 923	......
	1 200	2 048	....	2 253	......
	900	2 290	....	2 519	......
	720	2 451	....	....	......
200	3 600	2 603*b*	....	2 853*b*	......
	1 800	2 233	....	2 456	......
	1 200	2 679	....	2 947	......

a, Type K only. *b*, Type KF only. *c*, for Type K; $338 for Type KF. *Motor only.

Explosion-proof motors for hazardous gas conditions are tested and listed by the Underwriters' Laboratories, and must be specified for this service, that they may bear the Underwriters' Label which indicates factory inspection by Underwriters' representatives.

The motors listed in Tables 10 and 11 are in quite general use. In ordering, consult the General Electric Co's catalogue of Jan 1, 1939, which also contains details of the starting devices, both manual and magnetic.

Table 12. General-service, Wound-rotor Induction Motors; Constant and Adjustable-varying-speed, 3- and 2-phase, 60 Cycles

Hp. continuous, 40° rise	Synchronous, rpm	Price,* $ 110, 220, 440, 550 Volts: Sleeve-bearing	Price,* $ 110, 220, 440, 550 Volts: Ball-bearing
1/2	1 200	100	105
3/4	1 200	109	114
	900	133	140
1	1 800	113	119
	1 200	122	128
	900	144	151
	600	224	235
1 1/2	3 600	173	182
	1 800	115	121
	1 200	133	140
	900	156	164
	600	231	243
2	3 600	180	189
	1 800	120	126
	1 200	144	151
	900	171	180
	600	242	254
		220, 440, 550 volts	
3	3 600	204	214
	1 800	136	143
	1 200	165	173
	900	194	204
	600	277	291
5	3 600	249	261
	1 800	166	174
	1 200	201	211
	900	240	252
	600	402	422
7 1/2	3 600	296	311
	1 800	197	207
	1 200	241	253
	900	288	302
	600	455	478
10	3 600	345	362
	1 800	230	242
	1 200	275	289
	900	341	358
	600	495	520
15	3 600	482	506
	1 800	321	337
	1 200	347	364
	900	422	443
	600	592	622
20	1 800	384	403
	1 200	419	440
	900	496	521
	600	685	719

Hp continuous, 40° rise	Synchronous, rpm	Price,* $ 220, 440, 550 Volts: Sleeve-bearing	Price,* $ 220, 440, 550 Volts: Ball-bearing	Price,* $ 2 200 Volts: Sleeve-bearing	Price,* $ 2 200 Volts: Ball-bearing
25	1 800	432	454	....	....
	1 200	476	500	....	....
	900	548	575	693	728
	600	759	797	....	....
30	1 800	478	502	....	....
	1 200	525	551	679	713
	900	606	636	743	780
	720	759	797	883	927
	600	825	866	939	986
40	1 800	566	594	676	710
	1 200	617	648	767	805
	900	699	734	836	878
	720	864	907	978	1 027
	600	944	991	1 044	1 096
50	1 800	643	675	754	792
	1 200	703	738	843	885
	900	789	828	917	963
	720	961	1 009	1 064	1 117
	600	1 050	1 103	1 143	1 200
	514	1 153	1 211	....	....
	450	1 256	1 319	....	....
60	1 800	719	755	826	867
	1 200	791	831	916	962
	900	865	908	998	1 048
	720	1 053	1 106	1 148	1 205
	600	1 152	1 210	1 239	1 301
	514	1 260	1 323	1 349	1 416
	450	1 361	1 429	1 454	1 527
75	1 800	825	866	932	979
	1 200	909	954	1 016	1 067
	900	973	1 022	1 113	1 169
	720	1 180	1 239	1 269	1 332
	600	1 293	1 358	1 387	1 456
	514	1 407	1 477	1 488	1 562
	450	1 523	1 599	1 594	1 674
100	1 800	993	1 043	1 085	1 139
	1 200	1 100	1 155	1 182	1 241
	900	1 130	1 187	1 295	1 360
	720	1 375	1 444	1 448	1 520
	600	1 509	1 584	1 577	1 656
	514	1 630	1 712	1 702	1 787
	450	1 744	1 831	1 828	1 919
125	1 800	1 151	1 209	1 226	1 287
	1 200	1 284	1 348	1 353	1 421
	900	1 413	1 484	1 476	1 550
	720	1 550	1 628	1 620	1 701
	600	1 704	1 789	1 767	1 855
	514	1 845	1 937	1 914	2 010
	450	1 972	2 071	2 058	2 161
150	1 800	1 302	1 367	1 371	1 440
	1 200	1 476	1 550	1 523	1 599
	900	1 594	1 674	1 653	1 736
	720	1 717	1 803	1 780	1 869
	600	1 897	1 992	1 941	2 038
	514	2 033	2 135	2 087	2 191
	450	2 168	2 276	2 230	2 342
200	1 800	1 592	1 672	1 627	1 708
	1 200	1 718	1 804	1 755	1 843
	900	1 888	1 982	1 936	2 033
	720	2 098	2 203	2 136	2 243
	600	2 254	2 367	2 274	2 388
	514	2 409	2 529	2 458	2 581
	450	2 564	2 692	2 624	2 755

For mine service, the use of these motors is usually limited to sizes larger than 25 hp.

All are open-type, 60-cycle, polyphase motors, rated 40 C, when operated on 50 cycles at listed voltages, will operate without injurious heating, not exceeding 50 C rise; sixty-cycle horsepower ratings and prices apply; synchronous speeds will be 5/6 of those at 60 cycles. * Motor only.

Table 13 lists synchronous motors, which have limited application where power factor correction is required. Usually in mines, synchronous motor-generator sets are employed for this purpose. To increase their use, rotaries or rectifiers may be advantageous.

Table 13. General-service, High-speed Synchronous Motors

	Volts	1.0 Power-factor				0.8 Power-factor			
		220, 440, 550		2 200		220, 440, 550		2 200	
Hp	Speed, rpm	Motor only	Exciter	Motor only	Exciter	Motor only	Exciter	Motor only	Exciter
20	1 200	....	....	....	....	$570	$149	$601	$149
25	1 200	$570	$149	$601	$149	583	149	614	149
	900	....	....	....	....	670	252	711	252
30	1 200	583	149	614	149	607	149	639	149
	900	670	252	711	252	712	252	749	252
	720	....	....	....	....	837	288	881	288
40	1 800	....	....	....	....	1 030	128	1 085	128
	1 200	607	149	639	149	638	173	672	173
	900	712	252	749	252	748	252	788	252
	720	837	288	881	288	878	323	923	323
	600	....	....	....	....	1 043	363	1 098	363
50	1 800	1 030	128	1 085	128	1 067	128	1 123	128
	1 200	638	173	672	173	656	173	691	173
	900	748	252	788	252	796	289	837	289
	720	878	323	923	323	936	323	986	323
	600	1 043	363	1 098	363	1 101	363	1 170	363
60	1 800	1 067	128	1 123	128	1 195	128	1 195	128
	1 200	656	173	691	173	755	173	755	173
	900	796	289	837	289	862	289	862	289
	720	936	323	986	323	1 000	384	1 000	384
	600	1 101	363	1 170	363	1 179	427	1 179	427

220, 440, 550 and 2 200 Volts

Hp	Speed, rpm	1.0 power-factor		0.8 power factor		Hp	Speed, rpm	1.0 power-factor		0.8 power-factor	
		Motor only	Exciter	Motor only	Exciter			Motor only	Exciter	Motor only	Exciter
75	1 800	1 195	128	1 317	128	125	600	1 398	427	1 487	543
	1 200	755	173	876	252		514	1 597	485	1 680	613
	900	862	289	974	289	150	1 800	1 549	128	1 742	146
	720	1 000	323	1 106	384		1 200	1 085	252	1 289	292
	600	1 179	427	1 295	427		900	1 210	289	1 416	329
100	1 800	1 317	128	1 432	128		720	1 307	323	1 537	480
	1 200	876	252	976	252		600	1 487	427	1 700	543
	900	974	289	1 095	329		514	1 680	485	1 863	613
	720	1 106	323	1 210	384	200	1 800	1 742	146	....	....
	600	1 295	427	1 398	427		1 200	1 289	252	....	....
	514	1 506	485	1 597	613		900	1 416	329	....	....
125	1 800	1 432	128	1 549	146		720	1 537	384	....	....
	1 200	976	252	1 085	292		600	1 700	427	....	....
	900	1 095	289	1 210	329		514	1 863	485	....	....
	720	1 210	323	1 307	384						

Direct-current, constant speed motors are widely employed for operating coal-cutters, underground mechanical conveyers and loaders. Sizes smaller than those listed in Table 14 (from $^1/_{20}$ hp to $7^1/_2$ hp) are also made and are useful for general service.

Table 14. General-service, Constant-speed, Direct-current Motors, Shunt-, Series-, and Compound-wound, Commutating-pole

Hp	Rated full-load basic speed, rpm	Max speed*	Price, $, motor only, shunt-wound: Sleeve-bearing	Price, $, motor only, shunt-wound: Ball-bearing	Additions †
			H5 and 230 volts		
10	3 500	......	345	362	*ajs*
	1 750	*A*	282	296	*aks*
	1 150	*C*	345	362	*akt*
	850	*D*	411	432	*akt*
	690	*F*	472	496	*blt*
	575	*I*	546	573	*blt*
15	3 500	......	421	442	*aks*
	1 750	*A*	344	361	*akt*
	1 150	*C*	422	443	*akt*
	850	*D*	506	531	*blt*
	690	*F*	553	581	*blt*
	575	*I*	684	718	*cmu*
20	3 500	......	490	515	*blt*
	1 750	*A*	398	418	*akt*
	1 150	*C*	492	517	*blt*
	850	*D*	598	628	*cmt*
	690	*F*	674	708	*cmu*
	575	*I*	805	845	*cmu*
25	1 750	*A*	448	470	*blt*
	1 150	*C*	559	587	*blt*
	850	*D*	685	719	*cmu*
	690	*F*	762	800	*cmu*
	575	*I*	924	970	*cmv*
30	1 750	*B*	494	519	*blt*
	1 150	*C*	621	652	*cmu*
	850	*D*	764	802	*cmu*
	690	*F*	857	900	*cmv*
	575	*I*	1 022	1 073	*cmw*
			115 volts		
25	3 500	......	836	878	*b*
30	3 500	......	935	982	*c*
40	3 500	......	1 089	1 143	*c*
	1 750	*B*	607	637	*cmu*
	1 150	*E*	780	819	*cmu*
	850	*H*	958	1 006	*cmv*
	690	*F*	1 061	1 114	*cmw*
	575	*I*	1 276	1 340	*dny*
50	3 500	......	1 271	1 335	*c*
	1 750	*B*	737	774	*c*
	1 150	*E*	903	948	*cmv*
	850	*H*	1 104	1 159	*dnw*
	690	*K*	1 271	1 335	*dny*
	575	*I*	1 466	1 539	*dny*

Hp	Rated full-load basic speed, rpm	Max speed*	Price, $, motor only, shunt-wound: Sleeve-bearing	Price, $, motor only, shunt-wound: Ball-bearing	Additions †
			230 volts		
25	3 500	......	557	585	*b*
30	3 500	......	623	654	*c*
40	3 500	......	726	762	*c*
	1 750	*B*	578	607	*cmu*
	1 150	*E*	743	780	*cmu*
	850	*H*	912	958	*cmv*
	690	*F*	1 010	1 061	*cmw*
	575	*I*	1 215	1 276	*dny*
50	3 500	......	847	889	*c*
	1 750	*B*	702	737	*c*
	1 150	*E*	860	903	*cmv*
	850	*H*	1 051	1 104	*dnw*
	690	*K*	1 210	1 271	*dny*
	575	*I*	1 396	1 466	*dny*
60	1 750	*B*	756	794	*c*
	1 150	*E*	978	1 027	*cmu*
	850	*J*	1 185	1 244	*dny*
	690	*K*	1 361	1 429	*dny*
	575	*L*	1 565	1 643	*dnz*
75	1 750	*B*	894	939	*c*
	1 150	*E*	1 142	1 199	*dnx*
	850	*J*	1 366	1 434	*dny*
	690	*K*	1 591	1 671	*dnz*
	575	*L*	1 815	1 906	*dnz*
100	1 750	*B*	1 117	1 173	*d*
	1 150	*E*	1 405	1 475	*d*
	850	*J*	1 655	1 738	*dnz*
	690	*K*	1 912	2 008	*eoz*
	575	*L*	2 160	2 268	*eoY*
125	1 750	*B*	1 331	1 398	*d*
	1 150	*G*	1 655	1 738	*d*
	850	*J*	1 932	2 029	*eoz*
	690	*L*	2 160	2 268	*eoY*
	575	*M*	2 529	2 655	*eoY*
150	1 750	*B*	1 538	1 615	*d*
	1 150	*G*	1 896	1 991	*e*
	850	*J*	2 193	2 303	*e*
	690	*L*	2 529	2 655	*eoY*
	575	*M*	2 892	3 037	*gpZ*
200	1 750	*B*	1 949	2 046	*e*
	1 150	*G*	2 361	2 479	*e*
	850	*J*	2 691	2 826	*f*
	690	*L*	3 110	3 266	*hqZ*
	575	*M*	3 533	3 710	*irZ*

* By field control (shunt-wound constant-speed motors), rpm:

A, 2 190	*D*, 1 700	*F*, 1 380	*H*, 1 275	*J*, 1 065	*L*, 865
B, 1 925	*E*, 1 440	*G*, 1 325	*I*, 1 150	*K*, 1 035	*M*, 720
C, 1 725					

† Additions (not to be used as separate prices): First symbol refers to additional cost of compound- or series-winding; second symbol, additional cost of base; third symbol, additional cost of pulley; all as listed below.

Compound- or series-winding		Base		Pulley	
a, $10	*f*, $ 81	*j*, $10	*o*, $ 59	*s*, $ 4	*x*, $18
b, $16	*g*, $ 87	*k*, $15	*p*, $116	*t*, $ 5	*y*, $25
c, $30	*h*, $ 93	*l*, $18	*q*, $124	*u*, $ 7	*z*, $34
d, $39	*i*, $106	*m*, $24	*r*, $141	*v*, $ 8	*Y*, $46
e, $67		*n*, $36		*w*, $16	*Z*, $67

The foregoing tables of different types and sizes of elec motors, with their list prices, are intended to be illustrative only, in aiding engineers to make preliminary estimates of mine plant. As already stated, sizes having other horsepowers, speeds and voltage are also manufactured.

Besides the types listed in the tables, there are variable-speed motors, geared motors, induction motor-generator sets, and direct-current generators and exciters, most of which have rather limited fields of use for mine service. For information regarding them, apply to the makers.

17. MAKERS OF ELECTRICAL MINE EQUIPMENT
(Approved by the U S Bureau of Mines)

Air compressors

General Electric Co. Erie, Pa
Ingersoll-Rand Co. Painted Post, N Y
Sullivan Machinery Co. Michigan City, Ind

Loading machines and conveyers

Jeffrey Mfg Co. Columbus, Ohio
Myers-Whaley Co. Knoxville, Tenn
Joy Mfg Co. Franklin, Pa
Sullivan Machinery Co. Claremont, N H
Goodman Mfg Co. Chicago, Ill
Duncan Foundry & Machine Works. Alton, Ill
Brown Faro Co. Johnstown, Pa
Northern Conveyor & Mfg Co. Milwaukee, Wis
Fairfield Engineering Co. Marion, Ohio
Bertrand P. Tracey Co. Pittsburgh, Pa
Mancha Storage Battery Locomotive Co. Chicago, Ill
Fairmont Mining Machinery Co. Fairmont, W Va
Crawford Machinery Co. Pittsburgh, Pa
La-Del Conveyor & Mfg Co. New Philadelphia, Ohio
Vulcan Iron Works, Wilkes-Barre, Pa

Coal drills

Chicago Pneumatic Tool Co. Cleveland, Ohio
Martin-Hardsocg Co. Pittsburgh, Pa
Jeffrey Mfg Co. Columbus, Ohio
Ohio Brass Co. Mansfield, Ohio
Sullivan Machinery Co. Claremont, N H
Colonial Supply Co. Pittsburgh, Pa

Mining machines (coal cutters)

Sullivan Machinery Co. Claremont, N H
Goodman Mfg Co. Chicago, Ill
Jeffrey Mfg Co. Columbus, Ohio
Joy Mfg Co. Franklin, Pa

Room hoists

Brown Faro Co. Johnstown, Pa
Sullivan Machinery Co. Claremont, N H
Flood City Brass & Electric Co. Johnstown, Pa
Ingersoll-Rand Co. Athens, Pa

Mine pumps

Fairmont Machinery Co. Fairmont, W Va
Demine Co. Salem, Ohio
Scranton Pump Co. Scranton, Pa
Weinman Pump Mfg Co. Columbus, Ohio
Boyts-Porter Co. Connellsville, Pa
Harris Pump & Supply Co. Pittsburgh, Pa
Brown Faro Co. Johnstown, Pa
Ingersoll-Rand Co. Phillipsburg, N J

Rock-dust distributers

Mine Safety Appliance Co. Pittsburgh, Pa
Diamond Machine Co. Monongahela, Pa
American Mine Door Co. Canton, Ohio

Storage-battery locomotives

Jeffrey Mfg Co. Columbus, Ohio
Westinghouse Elec & Mfg Co. East Pittsburgh, Pa
General Electric Co. Erie, Pa
Atlas Car & Mfg Co. Cleveland, Ohio
Goodman Mfg Co. Chicago, Ill
Ironton Engine Co. Ironton, Ohio

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SECTION 17

SURVEYING

BY

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SURVEYING AND DRAFTING INSTRUMENTS

LAND SURVEYING

LEVELING AND CONTOURS

TOPOGRAPHIC, AERIAL, MINERAL AND RAILROAD SURVEYING

SURVEYING AND DRAFTING INSTRUMENTS

1. TAPES AND RODS

Steel tapes are commonly 25, 50, 75, 100, 200, 300, or 500 ft long. To 100 ft, they are usually thin steel ribbons; the longer tapes are thicker and narrower, to lessen liability of kinking and to stand rougher usage. The lighter tapes are graduated throughout in feet, tenths, and hundredths; the heavier are usually marked by notched sleeves, etchings, or brass rivets at every 1-, 5-, 10-, or 20-ft point, with last foot on each end divided into tenths or hundredths. When 5-, 10-, or 20-ft points alone are marked, the last 5, 10, or 20 ft is usually graduated at every foot.

Some 200 or 300-ft tapes are made 201 ft or 301 ft long, the added foot graduated into tenths or hundredths. In most lighter tapes the zero-point is at end of ring, in some at the end of the steel ribbon, while in others it is 0.2 or 0.3 ft from end. The heavier tapes have a handle for each end, the zero and other end of the graduations then being marked on the ribbon. Bronze tapes are made for use in wet places, such as mines. Metric tapes are in lengths of 10, 15, 20, 25, 30, and 50 m; many being graduated in feet and inches on reverse side. Common lengths of light metric tapes are 15 and 30 m, the heavier, 25, 50 and 100 m long. Most long tapes are on reels, which should be of ample size, for a wet and dirty tape fills a larger reel than a clean one, and long tapes for surface surveying at mines are also often used underground, where they become wet and dirty. INVAR TAPES are for very accurate work. Due to temperature changes, their length changes only about 1/25 as much as ordinary steel tape. U S Gov't uses 15 kg tension for 50-meter Invar tapes. They are not made at present in the U S, but 100-ft Invar tapes are obtainable.

Steel tapes fitted with thermometer for temperature corrections, and with spring balance handle for measuring amount of pull, are used chiefly in city surveying. Broken tapes can be mended by riveting on back of tape a piece of old tape of same width; complete tape-mending kits are sold for this purpose. Tapes graduated in feet and inches are used particularly in building construction.

Cloth and metallic tapes are common; the former stretch readily and are useless for surveying. A metallic tape is of cloth with fine brass wires woven into it to prevent stretching. These are usually graduated into feet, tenths, and half-tenths, and made in lengths of 25 to 100 ft; sometimes with feet divided into inches. A 50-ft metallic tape may be in error as much as 0.5 ft, owing to rough usage and alternate wetting and drying; hence it should be used only for rough work.

The cost of a 100-ft tape and box is from \$5 to \$20. The heavier tapes, exclusive of reels, cost \$5 to \$12 per 100 ft, and can be obtained up to 1 000 ft; the ordinary reels for long tapes cost \$3 to \$20; certain special reels for mine surveying cost as high as \$25. Metallic tapes 50 ft long cost \$3 to \$4. Bronze tapes are a little more expensive than ordinary steel tapes.

Pocket steel tapes from 3 ft up are for accurate measuring when long tapes, marked only at the 5-, 10-, or 20-ft points, are used. Lengths: 3, 5, 6, 8, 10 ft; 1, 1.5, 2, 2.5 meters. To CONVERT MEASUREMENTS recorded in tenths and hundredths of a foot into inches and fractions, the following equivalents may be used:

Decimal of foot	= 0.01	0.08	0.17	0.25	0.50	0.75
Inches	= 1/8 −	1 −	2 +	3	6	9

If these values are memorized, decimals of a foot may be quickly transposed into inches, for example: 0.72 ft = 0.75 ft − 0.03 ft = 9 in − 3/8 in = 8 5/8 in.

Odometer is an instrument attached to a wheel of a vehicle, to record number of revolutions made by wheel in traversing between two points; rev × circumf = distance.

Pedometer is an instrument to record the distance traversed by a person carrying it; it is adjustable for different lengths of pace.

Stadia method of measuring distance (see Art 22).

Leveling rods are of two types, TARGET and SELF-READING; former is read by rodman, latter by levelman. Commonest forms of target rods are the Boston, the New York, and the Philadelphia; the Philadelphia can be used also as a self-reading rod (Fig 1). Most rods are made of 2 or more sections, so that they may be extended. Target of Boston rod is permanently fastened to one strip of the rod; this requires that for readings higher

than the unextended rod the rod must be inverted, which introduces an error if foot of rod is worn so that the target is not in correct place with reference to the zero of vernier scale. The other target rods are always used right-side up and have movable targets. The New York rod has a vernier for reading to a thousandth of a foot (Art 2); most Philadelphia rods have a scale resembling a vernier, which gives the hundredths and makes it easy to estimate thousandths, thus serving every purpose of a vernier. All these rods are usually 6 or 7.5 ft long and are extensible for readings up to 11 or 13 ft. They are also made 3 ft long, with extension to 5 ft, and 5 ft long with extension to 9 ft, for use underground.

A self-reading rod has graduations (usually every hundredth of a foot) painted on its face, so that it can be read through telescope by the levelman. The figures and graduations on face of rod are usually of a definite pattern, thus aiding levelman to read hundredths and estimate half-hundredths of a foot. Self-reading rod for ordinary work has many advantages over target rod. A rod equipped with clamps having screw-heads, which lie close to rod and are protected by a metal guard, is preferable to a rod with outstanding screws, especially if rod is to be shipped from place to place. Ordinary leveling rods cost $10 to $20.

One-piece 12-ft rods are accurate and cheap, but not readily portable. They have hardwood edges, extending slightly beyond face of rod, to protect edges and face.

Stadia (or telemeter) **rods** are usually of soft pine, with a face 3 to 4 in wide, on which distinctive diagrams are painted, to be clearly distinguished at distances of 1 000 ft or more. For stadia sights not greater than 300 ft, the ordinary Philadelphia rod is suitable; for sights up to about 600 ft, either the first or second rod shown in Fig 2 is suitable; for longer sights the third rod in Fig 2 is best. Many surveyors make their own stadia rods, because those sold have too narrow a face, with too many figures and graduations, and are not marked with clear, distinctively shaped diagrams. For ease in transportation, stadia rods are frequently in two parts, hinged at about the 7-ft mark; if hinged, it is well to provide a hard-wood cleat for back of rod, to strengthen hinged joint. In Fig 2 two of the rods have cross-hairs marked on them, to show how they are read (Art 22 and 23).

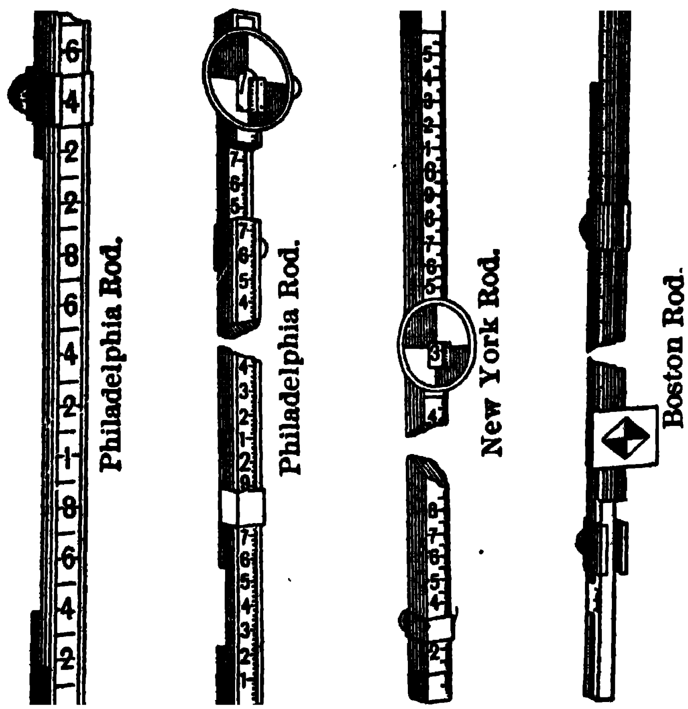

Fig 1. Leveling Rods

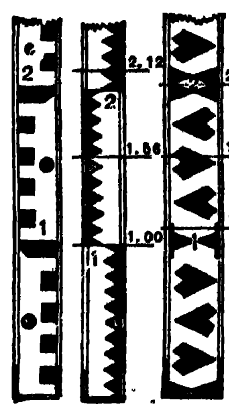

Fig 2. Stadia Rods

2. VERNIERS

Vernier is used for determining fractional part of smallest division of a scale more accurately than it can be estimated by eye. The simplest form (used on some leveling rods) is shown in Fig 3, where one unit on rod is divided into 10 equal parts. The entire length of vernier equals space occupied by 9 divisions of scale; if then the vernier length be subdivided into 10 equal parts, one division on vernier will be $^9/_{10}$ as large as one division on scale. If the index (zero-point of vernier) in Fig 3 is at 3 on the scale, then distance

ab must be $^1/_{10}$ of one of scale divisions, and *cd* is $^2/_{10}$ of a scale division. If vernier be raised until *a* comes opposite *b*, then reading will be 301; if *c* comes opposite *d*, the reading is 302. Hence, in the case of a decimal vernier, the number of that line on vernier which coincides with any line on scale gives the number of tenths of the smallest division on scale by which the index point lies beyond the last preceding scale division. For example, reading at right of Fig 3 is 326.

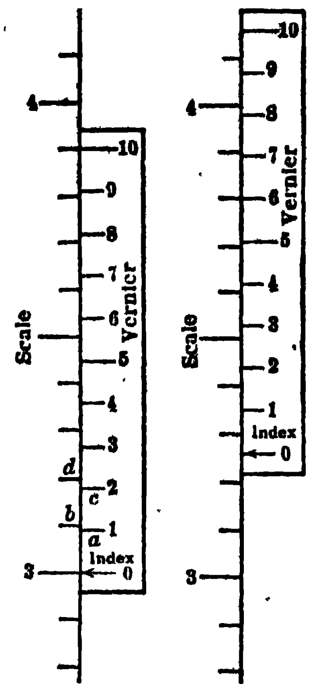

Fig 3. Leveling Rod Vernier

Transit verniers are usually double, one on each side of the index, so that angles can be measured either clockwise or counter-clockwise. Against the horiz circle of ordinary transit there are usually four verniers, in pairs (Art 4). A 1-min instrument has its horiz circle divided in half-degrees and there are 30 divisions on the vernier; a 30-sec instrument has circle divided to 20 min, with 40 divisions on vernier; a 20-sec instrument has circle divided to 20 min, with 60 divisions on vernier; and a 10-sec instrument has circle divided to 10 min, with 60 divisions on vernier.

The vernier of a 1-min instrument is constructed by dividing into 30 equal parts the space occupied by 29 half-degrees on circle; the difference between one division on circle and one of vernier is therefore 1 min of arc. A pair of verniers is thus constructed on both sides of a zero-point (or index). If the pair of verniers is then placed so that their common zero-point coincides with one of the divisions, say 0° point on circle, then the 30-min line on each vernier will also coincide with the 14° 30′ line on circle, and the first lines on either side of zero of vernier will fail to coincide with the corresponding lines on circle by 1 min, the second line on either side of the zero of the vernier will fail to coincide with its nearest mark on circle by 2 min, and so on. If now the vernier be moved so that the next line to its zero-point coincides with its nearest line on circle, one vernier will read 1 min and the other 29 min; and the reading will be either 0° 01′ or 359° 59′, depending upon whether angle is read clockwise or counter-clockwise. Therefore, to read an angle on a transit, note the last division on circle beyond which vernier index has passed; then follow along vernier in same direction to the first line which coincides with any line on circle, and add the number of this line to circle reading. Thus, Fig 4 represents a 1-min horiz circle with double vernier, which reads (remembering that it is the vernier that moves) 350° 44′ clockwise, or 9° 16′ counter-clockwise.

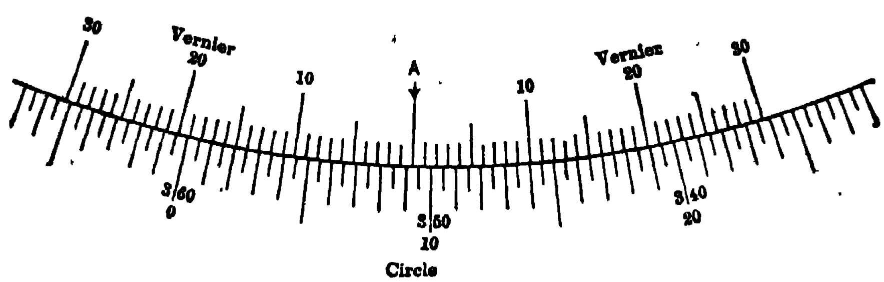

Fig 4. Horizontal Circle, with Double Vernier

Single verniers are used with the horiz circle of some instruments, on which a double vernier would be in the way of the telescope standards; for example, on a 20-sec transit a single vernier corresponds in length to about 20° of arc. A single vernier has two zero-points, one at each end, and is numbered to read from either end. To measure an angle, one or other vernier index-point is set at 0° of the circle, depending upon the direction in which angle is to be measured.

Folded vernier is a short, single vernier, sometimes used with vert circles of transits and plane tables. It is designed to shorten the single vernier to half its ordinary length, so that it can be placed between legs of the standards. The index point is the middle division; the numbering goes from middle to left end, and then begins again at right end and runs to middle. Similar numbers run in reverse direction, starting at middle and running to right. In reading such a vernier, if a coincidence is not reached between middle and farther end, return to the other end of vernier and continue in same direction, toward the center, until coincidence is found.

Retrograde verniers are those in which vernier divisions are longer than circle divisions. They are found only on old surveying instruments. In the ordinary verniers described above, a vernier division is always smaller than a circle division; these are called DIRECT VERNIERS.

3. THE COMPASS

Surveyor's compass is used to determine the MAGNETIC BEARING of a line, that is, the horizontal angle between the line and a magnetic needle. The needle is balanced on a pivot in center of a circular box, the edge of which is usually graduated in half-degrees and is numbered from 0° to 90° in both directions from N and S ends of compass-box; the needle swings freely in a horiz plane. Attached to the compass-box, in exact line with its N and S points, are two vert sights (sometimes a telescope) used for pointing the instrument. The compass is connected to a tripod by ball-and-socket joint, which is clamped after instrument has been made horiz by two spirit levels attached to frame. The frame has a spindle which fits into the ball-and-socket joint, so that after instrument has been leveled it can be swung around in a horiz plane. A small counterweight is attached to S end of needle (in the northern hemisphere) to prevent needle from dipping and thus rubbing against glass top of compass-box. For compasses used in mines, and their limitations, see Sec 18, Art 4.

To use the compass, first set it up at the point, level it, let the needle down on its pivot, then point the sights along the line of which the bearing is required. Since the needle points north, and the box turns under it, the letters E and W on the box are commonly reversed from their natural position so that the reading of needle will give the proper quadrant directly. The BEARING of a line, sometimes called the FORWARD BEARING, is the angle it makes with needle when looking along line AB from A toward B. The bearing of same line, looking from B toward A, is called the REVERSE BEARING. To obtain the bearing of a line, always read N end of the needle if N end of the box is toward the station of which the bearing is desired; when S end of box is toward that station, read S end of needle (the one to which counterbalance is attached in the northern hemisphere). If wrong end of needle be read, the bearing noted will be the reverse bearing. For difference between MAGNETIC and TRUE BEARING see Art 10.

Precautions. When not in use, needle should be kept raised from pivot by the screw and lever provided for that purpose, to avoid dulling pivot-point. Before reading bearing, be sure the needle has been let down on its pivot, and tap the glass lightly over end of needle, to be sure it is not touching under side of glass. If needle appears to cling to the cover, the glass has probably become electrified; that can be corrected by placing a moistened finger on the glass. The clinging of the needle to the glass may also be due to carelessness in leveling the compass, to loss of magnetism, or to shifting of counterweight. In using compass, take care that no iron or steel is near instrument, to attract needle from true position. (Steel tape, pins, ax, pocket-knife, or steel in eyeglass frames are sources of error.) Electric currents affect needle so seriously that a compass is of little use in cities, or in underground elec haulage ways, for obtaining even an approx magnetic meridian. Some compasses have a movable inner circle, on which the declination can be laid off; care should be taken that this circle is set at 0° if declination is not to be laid off (Art 10).

It is good practice to take both forward and reverse bearings of all important lines, because of liability of needle to be deflected from magnetic meridian by presence of ferruginous rocks or other iron material; this deflection is called LOCAL ATTRACTION. If the bearing of AB, taken from Sta A, and the bearing of BA, from Sta B, do not agree, local attraction exists at either A or B. To determine at which station it is present, take bearing of BC with compass at B, and then with compass at C determine bearing of CB. If these agree, local attraction is indicated at A but not at B.

Adjustments of compass. (*a*) The bubbles are adjusted in same manner as those on horiz plate of a transit (Art 4). (*b*) Straightening needle and centering pivot-point, the need for which is detected by noting whether S end reads 180° from N end; failure to do so may be due to a bent needle or a bent pivot-point, or both. A bend in needle may be detected by reading S end of needle when N end points at some selected graduation, and then reading N end when S end points at same graduation. Bend the needle at the middle, where metal is soft, until ends read the same in either position (though not necessarily reading 180° apart at either trial). The pivot may be centered by finding (by trial) the position of max difference between end readings in different parts of circle and bending pivot until any two end readings agree. (*c*) Re-magnetizing the needle; by rubbing it with a bar magnet from middle outward to ends, applying N end of magnet for the S end of needle, and *vice versa.*

The pocket compass is a hand instrument for obtaining roughly the bearing of a line. There are two kinds, PLAIN and PRISMATIC. Former is much like surveyor's compass, except that it usually has no sights. In prismatic compass the graduations, instead of being on compass-box, may be on a card fastened to needle (like a mariner's compass) and moving with it. This compass is generally provided with sights; bearing is read by

means of a prism, or folding mirror, at same instant that compass is sighted along line. Pocket compasses for surveying range in price from $10 to $60, the latter being small, delicately constructed compasses, to carry in pocket for use with a tripod. The ordinary surveyor's compass costs from $25 to $75, including tripod.

In using the **Brunton pocket transit**, the observer sees simultaneously the compass and the object sighted. This is accomplished by a hinged mirror, which reflects the line of sight upward, toward observer. A CLINOMETER (Art 5) is attached, for measuring the inclination of line of sight.

4. TRANSIT AND ITS ADJUSTMENTS

Engineer's transit (sectional view, Fig 5) has two spindles, one fitting inside the other, to each of which is fastened a horiz circular plate; the outer spindle is attached to lower plate, on which is a graduated circle, and inner spindle carries upper plate, to which the verniers and standards holding telescope are attached. The motion of these horiz plates is controlled by clamps and tangent screws. On upper plate are two spirit-levels; the plates are leveled by 4 (sometimes 3) leveling screws, working between base and lower plate. Transits having telescope bubble and vert arc are called ENGINEER'S (OR SURVEYOR'S) TRANSITS; a plain transit lacks these attachments.

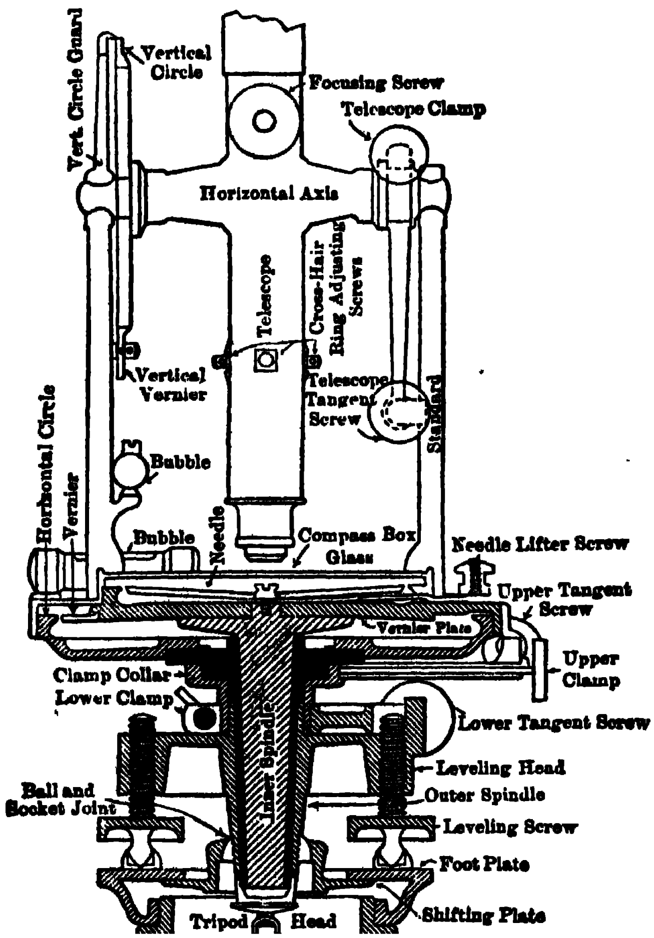

Fig 5. Engineer's Transit (Sectional View)

Telescope has a compound objective, and an eye-piece composed of 4 lenses, if instrument is an erecting transit, two lenses if inverting; an inverting instrument gives much better illuminated field. Between objective and eye-piece is the cross-hair ring, held in its concentric position by four opposing screws tapped into the ring, the heads being outside the telescope tube.

Special attachments are: (*a*) Dark eye-piece for observing sun. (*b*) Diagonal or prismatic eye-piece for sighting high altitudes. (*c*) Reflector attached to telescope for illuminating cross-hairs when working in dark. (*d*) Gradienter screw for measuring rates of grade directly. (*e*) Stadia hairs, which are two extra horiz cross-hairs for measuring distances by stadia. (*f*) Solar attachment for determining meridian by observation on the sun.

Precautions in care of transit. Neither tripod legs nor their shoes should be allowed to become loose. In taking instrument out of its box, always lift it by placing the hands beneath leveling base. When about to move transit from one point to another, be sure that the 4 leveling screws are properly bearing, that needle is lifted, and that clamps are set just firmly enough so that any slight shock will allow motion. Carry a waterproof bag at all times to protect instrument from rain or dust. When transit must stand out in rain, the cap should be put over object glass and eye-piece closed; if water gets into telescope, take out eye-piece, cover open end of the telescope tube with a cloth, and allow it to dry out. Any parts that have been wet should be wiped dry, especially vert arc, but be careful not to touch edges of arcs; always avoid placing the hands on exposed graduations, as they will tarnish the metal. In cleaning lenses, use fine camel-hair brush; for screw threads, use stiff toothbrush and apply a very little watch oil after cleaning, wiping

off exposed screws after oiling. Carry transit in a box when traveling on a vehicle. When shipping, always enclose transit in its case, with all clamps tightened, pack paper about it, and enclose case in another box packed with excelsior. Cover tripod threads with wooden or metal cap when shipping.

Adjustments of the transit are: (*a*) to make plane of plate bubbles perpendicular to vert axis of instrument; (*b*) to make line of sight perpendicular to horiz axis; (*c*) to make horiz axis perpendicular to vert axis of instrument.

(*a*) **Bubble tubes.** Bring plate bubbles to centers of respective tubes by turning leveling screws. Turn 180° and correct half of apparent error by turning capstan screws of bubble tubes. Adjust each bubble independently.

(*b*) **Line of sight.** First, adjust vert cross-hair to lie in plane perpendicular to horiz axis. Set up and level the instrument. Sight the vert hair on a well-defined point, clamp both plates, and move telescope up and down. If the point does not appear to travel along vert cross-hair, loosen screws holding cross-hair ring, and by tapping lightly on one screw, rotate ring until above condition is fulfilled. Then tighten screws and proceed with second part of adjustment as follows: Sight telescope at point A (200 to 300 ft away), and clamp vert axis; invert telescope on its horiz axis and set a point B at about same elev as A, in line of sight, and about same distance from instrument as A. Loosen clamp, turn horizontally and sight A again; clamp horiz motion, invert telescope on horiz axis and if line of sight falls on B it is in adjustment. If not, set point C in line of sight beside point B. Mark or note a point D one-fourth the distance between C and B, measured from C. To adjust, move cross-hair ring until D is intersected, by loosening screw on one side of telescope and tightening opposite one. If transit has an erecting eye-piece, move ring in direction D to C; if an inverting eye-piece, move ring in direction C to D.

(*c*) **Telescope standards.** Set up and level the transit. Sight on some high point A and clamp vert axis. Lower the telescope and set point B in line of sight about level with telescope. Reverse telescope, turn instrument 180° horizontally, sight again on A and clamp. Lower telescope until point B is visible, and mark point C in line of sight and at same height as B. If C does not fall on B, raise or lower adjustable end of horiz axis by the capstan-headed screw under it. Bring adjusting screw into position by a right-hand turn; otherwise the block on which horiz axis rests may stick and not follow the screw. Then tighten capstan screws just enough to avoid looseness of the bearing.

If an instrument is badly out of adjustment, it is better to bring it gradually into adjustment as a whole, rather than to attempt to adjust completely one part at a time. Then the adjustment of one part will not disturb preceding adjustments, parts are not subjected to strains, and instrument will remain longer in adjustment.

Telescope bubble (see "peg" adjustment, explained in Art 5).

Vernier of the vert circle should read 0° when telescope bubble is in center of its tube. If not, loosen screws holding vernier and tap lightly until the zeros coincide. This adjustment eliminates what is called the INDEX ERROR in measuring vert angles, provided line of sight is in adjustment parallel to bubble tube.

Objective slide should move parallel to line of sight. Adjust line of sight as described under (*b*) above, but use very distant objects; then repeat same adjustment using points close by; if an error is observed in this last test, it indicates that the objective slide does not move parallel to line of sight. Adjustment is made by moving adjusting screws of objective slide apparently to increase the error by one-quarter of observed error. Then test adjustment of line of sight again on distant points. The adjustments of objective slide, centering eye-piece tube, and centering the circles, are usually done by instrument makers.

To eliminate effects of errors in adjustment, so as to obtain accurate results, the instrument must be used as follows: To avoid errors in plate bubbles, turn 180° horizontally and bring bubbles half-way back to the middle by the leveling screws. Errors in line of sight and horiz axis are avoided by sighting telescope first in its direct and then in its reversed position, and taking the mean of the results, when running lines or measuring angles. Errors of eccentricity are eliminated by taking mean of readings of two opposite verniers, and errors of graduation of the circle are nearly eliminated by reading the angle in different parts of circle or by measuring the angle by repetition. When only one vernier is read in determining an angle, take care to read the one that was set at 0°.

Transits cost from $125 to $500. They weigh 5 to 15 lb, exclusive of tripods, which weigh 7 to 12 lb. Accurate results can be obtained with the lighter transits having 3 1/2- to 4 1/2-in horiz circle. It is advisable not to use too light a tripod with light-weight instruments, on account of wind. Stiff-leg tripods cost $15 to $25 and extension-leg tripods, $20 to $35.

5. THE LEVEL AND ITS ADJUSTMENTS

Engineer's level is a telescope to which a delicate spirit-level is attached parallel to line of sight, so that, when bubble is in center, the line of sight is horiz. The two common types are the Y- and the dumpy-levels. In both the telescope is mounted on a vert axis, about which telescope can swing horizontally, and is leveled by 4 leveling screws.

Y-level has its spirit-level attached to telescope, which rests in two Y-shaped supports, fastened to a horiz bar rigidly connected to vert axis. The telescope can be taken out of the Ys, turned end for end, and replaced, when testing bubble for adjustment.

Dumpy-level has its vert axis, horiz bar, and supports for telescope, all cast in one piece, to which spirit-level is attached. The dumpy-level will stand much rougher usage than the Y-level, has fewer wearing parts, and permits fully as precise work. Practically the only advantage the Y-level has over dumpy-level is that the adjustment of line of sight can be a little more conveniently tested.

Locke hand level is a metal tube with plain glass covers at ends and a small spirit-level on top. When looking through the tube an image of the bubble is seen in one-half of tube, being reflected by a prism. In other half of tube, the landscape is viewed. When bubble is in center, the observer can note where the horiz line, which appears in center of bubble tube, cuts a leveling rod (Art 19).

Clinometer is similar to a hand level, except that the bubble is attached to an arm, free to move in a vert plane; used for measuring slopes by sighting telescope along inclined line, moving the arc until bubble is centered, and then reading angle of inclination.

Adjustments of the Y-level are: (*a*) To make horiz cross-hair truly horiz and to make line of sight coincide with axis of the pivots, or be parallel to it; (*b*) to make line of sight and axis of bubble tube parallel; (*c*) to make axis of bubble tube perpendicular to vert axis.

(*a*) **Line of sight.** First make horiz cross-hair horiz, when instrument is level. This is tested after leveling, by sighting on a point and noting whether the cross-hair appears to remain on the point, as telescope is slowly rotated on its vert axis. Adjustment is made, if necessary, by rotating the cross-hair ring, as described under second adjustment of transit. When this is done, loosen clips which hold telescope in the Ys. Set a fine mark on a wall in the intersection of the cross-hairs, and clamp the vert axis. Rotate telescope 180° about its own axis. Correct half apparent error by the screws of cross-hair ring, adjusting first one hair and then the other.

(*b*) **Bubble adjustment by indirect method.** Bring bubble to center of its tube and clamp vert axis. Rotate telescope in the Ys a few degrees on its horiz axis; if bubble moves, correct entire error by the capstan screws at one end of bubble tube. Then clamp telescope over a pair of leveling screws and bring bubble into center of tube; lift telescope from Ys, turn it end for end, and replace in Ys without jarring the instrument. Correct half apparent error by the vert adjusting screw of bubble tube.

(*c*) **Adjustment of Ys.** Level the instrument, then bring bubble exactly to middle, over a pair of leveling screws. Turn telescope 180° on its vert axis and correct half apparent error by adjusting screw of the Y support. Since the bubble is brought to center of tube at each reading of the rod, this last adjustment in no way affects accuracy of leveling, but is a convenience.

Adjustments of dumpy-level are the same in purpose as for Y-level, but are, owing to construction of instrument, done in a different order and in some cases by different procedure. They are: (*a*) to make horiz cross-hair truly horiz when instrument is level; (*b*) to make axis of bubble tube perpendicular to vert axis; (*c*) to make line of sight parallel to axis of bubble.

(*a*) **Cross-hairs** are adjusted as described in paragraph *a* above.

(*b*) **Bubble tube.** Level the instrument and carefully center the bubble over a pair of leveling screws. Turn telescope 180° horizontally and correct half apparent error in the bubble by adjusting screws of level tube.

(*c*) **Line of sight by direct or "peg" method.** Select points *A* and *B*, 200 ft or more apart. Set up level beside *A*, so that when a rod is held on *A* the eye-piece will just clear the rod. Look through the large end of telescope at the rod and take reading opposite center of field of sight. Point telescope toward *B* and read the rod in usual manner, being sure that bubble is in the middle of tube. Then set up level at *B* and repeat above operation. These observations give two independent determinations of difference in elev between the two points; the true difference in elev is the mean of the two differences. (Same result can be obtained from a single setting of level, at a point exactly midway between *A* and *B*.) Leaving instrument at *B*, set rod at *A*, with target set to read height of instrument above *B*, plus or minus the true difference in elev between *A* and *B*. If line of sight is then brought to intersect the rod target, it will define a level line. The bubble being in center

of its tube, the line of sight is now brought to intersect target by moving cross-hair ring by its adjusting screws. Example:

Instrument at Sta A		Instrument at Sta B	
Rod reading on Sta A	= 4.243	Rod reading on Sta B	= 4.618
Rod reading on Sta B	= 5.724	Rod reading on Sta A	= 3.177
Diff in elev of A and B	= 1.481	Diff in elev of B and A	= 1.441

Mean of two differences = $^1/_2$ (1.481 + 1.441) = 1.461, the true difference in elev.

Instrument at B is now 4.618 above the station. Rod reading on Sta A should be 4.618 − 1.461 = 3.157, to give a level sight.

This peg adjustment may be applied to line of sight of the Y-level, except that in this case, after target at A has been set at correct elev to define a level line, the line of sight is made to bisect target by the leveling screws and then bubble is brought to mid-position by its adjusting screws. This method is also used for adjusting the hand level. In applying it for transits, the adjustment may be made by moving either cross-hair ring or one end of level tube. If cross-hair is moved, the adjustment of line of collimation must be tested; if bubble tube is moved, the vert arc vernier must be adjusted.

To eliminate effect of errors, in adjustment of line of sight of bubble tube, or of the Ys, the observer must be sure that bubble is in center of tube at instant rod is read, and that backsights and foresights are about equal in length.

Precise levels, designed for running long lines of bench levels, are usually equipped with three leveling screws, an inverting telescope of high power, stadia hairs, very sensitive bubble to be read by a mirror simultaneously with the rod, and a slow-motion screw for leveling (Art 18).

Leveling instruments cost from $100 (architect's levels, with horiz circle) to $250; precise levels cost from $400 upward; hand levels, about $8.

6. DRAFTING INSTRUMENTS

Only a few of the more uncommon drafting instruments are here described.

Planimeter is for determining area of an irregular plane figure. The method is to follow perimeter of figure with the tracing point of planimeter, reading recording wheel at beginning and end of process; the difference in readings gives area expressed in a certain unit. They are also called integrators.

Amsler polar planimeter, the most common form, has two arms, one of fixed length and the other adjustable, the length of latter being set to give any desired unit of measurement, as indicated by scale marked on arm by maker. At end of fixed arm is a weighted needle point, called the anchor point, which is pressed into the paper to hold instrument in a fixed position. The tracing point is at end of adjustable arm. The planimeter thus rests on three points, anchor point, tracer, and recording wheel. To measure an area, press anchor point into the paper outside area to be measured, and in such position that the tracer can follow around entire perimeter of area. Before proceeding further, run tracer around perimeter and note whether recording wheel, during this process, runs over any creased or irregular part of drawing paper; if so, change position of anchor to avoid it. Start tracer from a definite point in perimeter, preferably such a point that the two arms will be about at right angles to each other. Read the scales on disk, wheel, and vernier, obtaining four digits in the reading, the first from the disk, the next two from wheel, and the fourth from vernier. Move tracer carefully around perimeter until starting point is reached, when the disk, wheel, and vernier scales are again read. The difference of the two readings gives area in the unit at which length of adjustable arm is set. Check the area by another determination, with anchor point in some other position, so that recording wheel will travel over a different path on the paper; for in its first trip there might have been some irregularity in paper, or dirt on it, which caused wheel to slip and not accurately record all the path over which it traveled. If unit of measurement recorded by planimeter is unknown, find it by running planimeter around any known area. If area is so large that anchor point can not be set outside its limits, divide it into segments, determining area of each separately; or anchor may be placed inside the area, provided the area of a CORRECTION CIRCLE is added to result, which value is given by maker. Results correct to within 1% may easily be obtained with this instrument.

Some planimeters are made with both arms of fixed length, thereby giving all results in one unit. These cost $15 to $30, whereas those with adjustable arm cost from $30 to $60.

Rolling planimeter has a tracing point at end of an adjustable pivoted arm, which is fastened to a frame supported on two rollers. The whole instrument is rolled forward and backward in a straight line, while the tracing point traverses outline of area. A precision within 0.1% is easily reached with this instrument, provided corrections are

made to allow for shrinkage or swelling of drawing paper which may have occurred after drawing was made. These instruments cost $125 to $175.

Pantograph is an instrument for copying a plan to the same or a different scale; used principally for enlarging or reducing a plan having many irregular lines, such as a topographic map with contours. It is a jointed framework of several pieces of wood or metal, assembled to form a parallelogram. It rests upon three points, one of which, *A*, is fixed, while the other two, *B* and *C*, are movable. There are other bearing points, but they simply support the instrument and are not essential to its principle. The two movable points *B* and *C* are in such positions that they will trace exactly similar figures and by varying position of these points on their respective arms, the relative scales of the figures traced by them can be adjusted. The essential condition which the instrument must fulfil before it is ready for use is that *A*, *B*, and *C* shall lie in a straight line, and that each point shall be attached to one of three different sides (or sides produced) of the parallelogram. Any one of the three points can be the fixed point. These instruments are usually provided with scales on the arms, indicating proper settings for various reductions or enlargements. Because of lost motion in joints, very accurate results can not, as a rule, be reached, but the best metal pantographs are sufficiently accurate for most topographical maps; more care must be taken in the use of the instrument for enlarging than for reducing. Cost ranges from $1 to $300; reliable work requires use of a pantograph costing $100 or more. An instrument called the EIDOGRAPH, costing about $125, has been made for the same purpose as a pantograph.

7. DRAWING AND BLUEPRINT PAPERS

Drawing papers for working plans are of all grades, from manila detail paper, at about 13¢ per yd, to well-seasoned, muslin-mounted paper, which is affected but slightly by changes in humidity, costing $2.00 to $5.00 per yd, depending on width and quality. Mounted paper comes in 10-, 20-, and 30-yd rolls, in widths of 36, 42, 58, 62, and 72 in. The best grades of drawing papers can also be obtained in sheets, either plain or mounted; common sizes: Demy, 15 by 20 in; Medium, 17 by 22; Royal, 19 by 24; Double Royal, 24 by 36; Imperial, 22 by 30; Double Elephant, 27 by 40; Antiquarian, 31 by 53 in.

Transparent paper is used largely for studies and for temporary copies of plans. For more permanent copies tracing cloth is used; there is a growing use of bond papers for permanent tracings. Tracing paper comes in sheets of same standard sizes as mounted paper; also in 10-, 20-, and 50-yd rolls and in widths 24, 27, 30, 36, 40, 42, 48, and 54 in, costing 5 to 25¢ per yd.

Tracing cloth is a uniform quality of finely woven linen, coated with a preparation to make it transparent. It is made in 24-yd rolls, in widths 30, 36, 42, 48, and 54 in, and costs $1 to $2.50 per yd. Most tracing cloth has to be rubbed with powdered chalk or specially prepared pumice before it will take ink. It has a smooth and a rough side; most draftsmen prefer the rough side, because it takes ink and pencil lines more readily, and will not show erasures so plainly, when process prints are made from it.

Cross-section and profile papers can be procured with colored lines, both on heavy paper and on transparent paper or cloth; also mounted on muslin or linen. Cross-section paper is printed in orange, green, red, and blue, in sheets 16 by 20 in, 17 by 22 in, 15 by 42 in, and 40 by 50 cm; also on smaller sheets called "coordinate paper," for rectangular or polar coordinates, and for logarithmic scale and isometric drawings. Cross-section papers are also made in 20- and 50-yd rolls with the engraving, 20, 24, and 30 in wide, and in metric units 50 cm and 75 cm wide. Profile papers are printed in green, orange, and red, in three scales, called Plates *A*, *B*, and *C*. *A* has 4 spaces horizontally to the inch and 20 vertically; *B*, 4 horizontally and 30 vertically; *C*, 5 horizontally and 25 vertically. These are mostly manufactured in 20- and 50-yd rolls, with the engraving 5, 9, 10, and 20 in wide, and cost 15 to 25¢ per yd on paper, 90¢ to $1.25 per yd mounted on cloth, and $1 to $1.35 per yd on tracing cloth.

Blueprint paper is the most common of process papers. It is a white paper, coated on one side with a solution which is sensitive to light. When fresh it has a yellowish green color; if it has a green, dark green, or blue color, it is old and will not give clear white lines. The more permanent blueprints are made on coated cloth. Good rag stock paper gives whiter lines than wood pulp paper.

Prepared blueprint paper and cloth is made in 10- and 50-yd rolls, in widths 24, 30, 36, 42, and 54 in; paper costs 15 to 25¢ and cloth 60¢ to $1.50 per yd, according to width.

In making a print, expose the tracing (with blueprint paper under it) in a printing frame a length of time depending on sensitiveness of paper and brilliancy of light. Then wash paper thoroughly in water, to remove all chemicals. Don't let the tracing become wet, for wet spots can not be eradicated from tracing cloth. In most large cities are concerns engaged in making blueprints,

supplying prints at from 3 to 5¢ per sq ft. Blueprints always shrink to a smaller size than the tracing; linear shrinkage usually varies from 0.5 to 2%, depending upon quality of paper. To write white lines on a blueprint, use solution of bicarb soda or Chinese white water color, applied with ordinary clean pen.

Brown process paper is a thin sensitized paper for making a NEGATIVE print from a tracing. The negative is then used to make prints having either blue or brown lines on white ground. This double process yields prints which are shrunk 2% or more from the original drawing. The process requires a bath in hyposulphite of soda, as well as a washing in water. Prints from brown process negatives can be made to produce lines almost as black and perfect as the original drawings. Negatives cost 5¢ per sq ft.

Direct black and white process prints give black lines on white background. Prints from this process do not shrink. Cost is about double the cost of blueprints. There is also a direct process blackprint paper and cloth, of German manufacture, used the same as blueprint paper. It requires washing in water only.

Photostat prints. Drawings are photographed on a sensitive paper, which is washed in fixing bath and dried. First print from an ink drawing on white paper has white lines on dark brown background. If this is again photographed, there will be brown lines on white background. The latter effect is obtainable directly from a blueprint by a first photograph. Most photostat machines reduce or enlarge only about 2 diameters; but the process can be repeated until the required scale is obtained. Since photostats shrink considerably and more or less unevenly in drying, a graphical scale should be put on the drawings. Cost of photostats, 7–10¢ per sq ft.

8. PLOTTING TRAVERSES

By protractor and scale. The courses of a traverse should be plotted by a more exact method than may be used for plotting details; but the latter can usually be plotted safely by protractor and scale. A closed traverse must necessarily close on the plan; any error of closure on plot therefore indicates error in scaling a distance or in laying off an angle. The bearing of any line of traverse can be computed from bearing of some other line, assumed to be correct, by means of the intermediate deflection angles; thus any line can be extended until it meets the plotted meridian, and its direction can be checked independently. Such checks are absolutely necessary if traverse is not a closed circuit.

Rectangular coordinate method is useful in plotting, especially when the area of a closed traverse has to be determined, because same computations required for finding the area can be used for plotting. First, compute latitude and departure of each course in traverse; then the total latitude and total departure (or longitude) of each point. N latitude and E departure are called plus, S latitude and W departure being minus (Art 12). If a meridian through the most westerly point and an east and west line through most southerly point be chosen for axes, there will be no negative coordinates. The total latitude of any point is the algebraic sum of all the latitudes of preceding courses, beginning with the most southerly point, and similarly for the total departure (or longitude), beginning with the most westerly point. In the case of a closed polygon, construct a rectangle of which the height equals difference in latitude of most northerly and southerly points, and the width equals difference in longitude of most westerly and easterly points. This rectangle should be plotted very accurately by straight-edge and reliable triangles, or constructed by beam compass, and checked by scaling the diagonals. To plot any point, lay off its total latitude on both the easterly and westerly sides of rectangle, measuring from southerly side; the point will lie on a line connecting these two points. Along this line scale off from westerly side of rectangle the total departure of the station, thus determining position of point. The other points are similarly plotted. To check a plot of this character, scale lengths of the traverse lines. Lines running nearly parallel to the axes should be checked by some other method, preferably the tangent method, explained below.

Fig 6 shows computations and coordinate method of plotting the same traverse of which the area is computed in Art 12; total latitudes and departures used in this plot are computed from balanced latitudes and departures in example in Art 12. Advantages of this method are: each point is plotted independently of others; all plotting is done by scale only; and plotting can be readily checked.

To apply the coordinate method of plotting to a traverse which does not close, draw two series of coordinate lines accurately spaced and intersecting at right angles; number these lines northward and eastward from origin of coordinates, the latter being chosen preferably at or beyond extreme southwest corner of traverse. Coordinates of the stations are computed from the unbalanced latitude differences and departures, and plotted as described above. In relatively unimportant work an unclosed traverse may sometimes be plotted by the chord or the tangent method, avoiding computation of coordinates.

Tangent method consists in laying off the deflections of a traverse by plotting the natural tangents of the angles. First compute bearings of all the courses on the basis of deflection angles. Then obtain algebraic sum of all the deflection angles, and add it to the bearing of first course; if result checks bearing of final course, the bearing computations are correct. Next plot the traverse roughly with scale and protractor, to determine its shape and extent; this is for purpose of starting first course on final drawing, at such point and direction that the drawing will be centered properly on the sheet. Then proceed to plot as follows (Fig 7). Draw through starting point A a line representing the meridian, in such manner that first course AB will run in desired direction. Construct a right triangle with A as the vertex, Ab the base, and cb the natural tangent of bearing of AB, measured in same unit as Ab; Ab should be of ample length, say 10 in, because from this base line the entire traverse is projected. Draw Ad through c and scale off AB. From Bd erect another perpendicular ef equal to tangent of the deflection angle at B, and then scale BC. Similarly plot courses CD, DE, and so on. Check every third or fourth course, as DE, by transferring the meridian to pass through D; then construct a right triangle Ddh, scale gh, and compare it with tangent of calculated bearing of DE. If error in bearing is 10 min or less, scale off on gh the tangent of calculated bearing, and draw DE through this new point, so that error in direction of DE will not be carried further along the traverse; if error found is greater than 10 min, repeat previous plotting, beginning at last checked course, and find error. A scale reading 10 ft to the inch is convenient for constructing legs of right triangles. When deflection angle much exceeds 45°, it is more accurate to plot the complement of deflection angle rather than angle itself, in which case a perpendicular is erected at station point and used as base of right triangle.

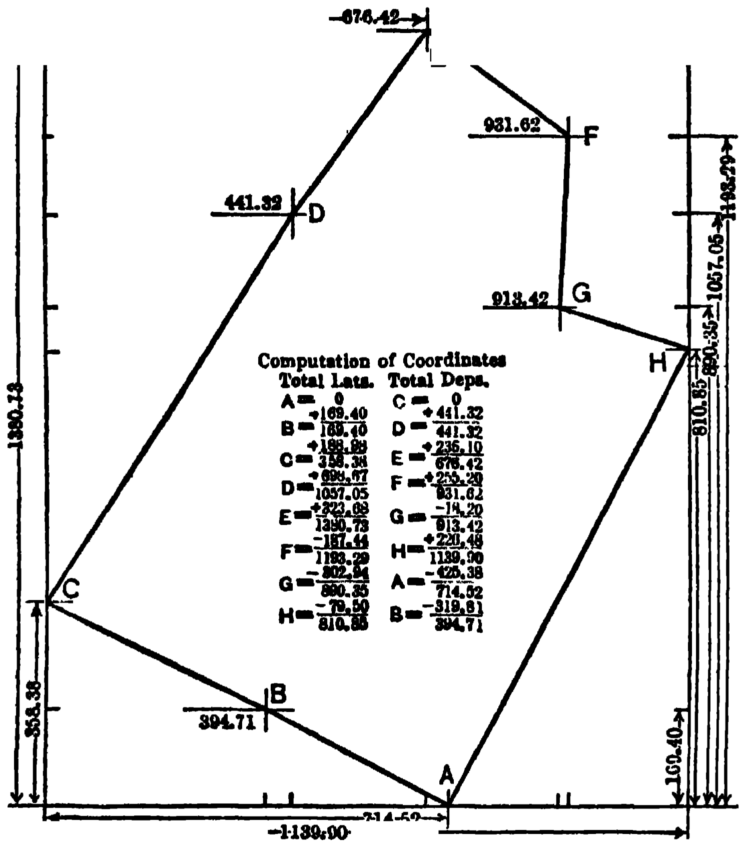

Fig 6. Coordinate Method of Plotting Closed Traverse

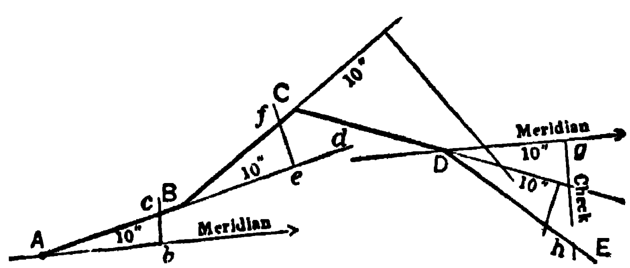

Fig 7. Tangent Method of Plotting Traverse

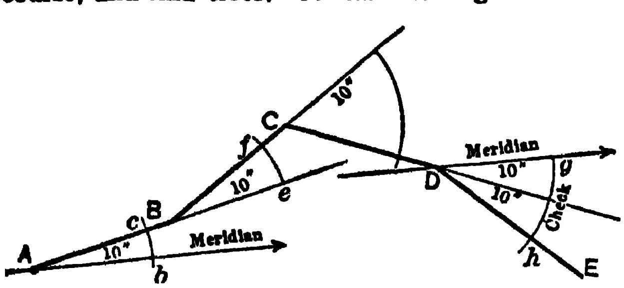

Fig 8. Chord Method of Plotting Traverse

Chord method consists in laying off deflection angles of a traverse by plotting chords of arcs which will subtend the angles. Fig 8 represents same traverse as Fig 7, but plotted here by chord method. Draw Ab representing meridian; draw bc with 10-in radius, and

scale from b the chord distance bc, which equals 10 times twice the sine of half of the deflection angle. Ac is drawn through c, and AB scaled. Then draw arc ef with 10-in radius, and plot deflection angle at B by scaling chord ef equal to 20 times the sine of half the angle, draw BC through f and scale BC. Similarly all of the angles are plotted. Check every third or fourth course, as DE, by transferring the meridian through D; draw arc gh, scale chord gh, divide it by 20 to obtain sine of angle; find the corresponding angle, double it, and obtain calculated bearing. If it is in error 10 min or less, correct it by plotting chord corresponding to calculated bearing, and draw DE in corrected position; if the error is greater than 10 min, re-plot and correct error. If deflection angle is greater than 90°, lay off 90° of it by a triangle and the rest by method of chords. In looking up twice the sine of half the angle, for plotting chords, the necessity for multiplying by 20 can be avoided by plotting natural sine of half the angle directly as a chord, with a scale of 10 ft to the inch, while the radius is measured with a 20-ft scale.

For maps of large areas, such as a state or a portion of a country, it is not sufficiently accurate to draw meridians and parallels of latitude as forming rectangles. The most common form of projection used is the POLYCONIC, in which the surface of a sphere representing the earth is developed on a series of cones tangent to parallels of latitude. In the U S Coast Survey, Special Publication No 5, is a table giving the x and y coordinates of points of intersection of meridians and parallels, for plotting parallels of latitude and meridian lines for this kind of projection.

9. FINISHING THE PLAN

Every plan should have a complete title written or lettered in pencil when drawing is begun, so that it can readily be identified. If plan is a land map, a meridian should be shown, and designated whether true or magnetic. State names of abutting owners, and limits of their property. Insert all essential dimensions in their proper places. In case of a land plan, the area is usually expressed in square feet or acres and is lettered in middle of the parcel; lengths of sides are printed at middle of each line, and bearings of each side, or angles between them, should be given; also section corners, stone bounds, iron pipes, or other physical boundaries which may exist are represented by abbreviations, such as S B for stone bound, I P for iron pipe, sp for spike, and stk for stake. It is the practice of many surveyors to omit from plan the calculated bearings of lines, or angles; this custom can not be too severely condemned. It is good practice to letter in some inconspicuous place, preferably near border, a reference to the notebook containing survey from which the plan was plotted, and initials of draftsman. If any leveling has been done in connection with the plan, the location and elevation of bench-marks should be described in a note, or plotted on map, preferably both.

On working drawings, and sometimes on finished plans, the traverse lines of the survey are drawn, usually as fine, colored, full lines; the stations being indicated either by very small circles, the centers of which mark the exact points, or by short lines drawn through the points in such manner as to bisect angles. Triangulation stations are represented by equilateral triangles, stadia location stations by small squares, and other auxiliary stations by circles. The elevation of bench-marks is frequently represented by a small cross and figures, thus, B M × 72.63.

The boundaries of a property and the physical features, such as streets or buildings, are usually drawn as full black lines. Shore lines are drawn in black or Prussian blue, and should, as a rule, be the heaviest lines on the plan, unless they are a very unimportant feature. Whenever colors are to be used, better results can always be obtained with water colors than with bottled inks. Red drawing ink gives very unsatisfactory results; in a few years it spreads and gradually fades away, leaving the tracing discolored. The following colors are commonly used by surveyors: burnt sienna, raw sienna, yellow ochre, scarlet vermilion, carmine, sepia, cadmium yellow, chrome yellow, gamboge, Hooker's green, Prussian blue, and indigo. All of these, except gamboge, may be used on tracing cloth without danger of running, but if process prints are to be made from the tracing, Prussian blue and indigo will not print well. Some colors which do not give good prints may be made to do so by adding a little Chinese white or cadmium yellow to give them body, but not enough to change the color. A most satisfactory way to color a plan is to shade the line or areas by colored pencil on back of tracing. Before using the color, rub places where it is to be applied with a pencil eraser and after color has been applied rub it down with a cloth to produce a uniform shading. On blueprints, for a red line, use scarlet vermilion; for yellow, Winsor & Newton's chrome yellow "Winchester."

Lettering on a drawing should, as a rule, read from bottom of plan, that is, all words and figures should read from left to right when the plan lies with its title horizontal; lettering running perpendicular to bottom should read from bottom upward. Roman or

Gothic letters are appropriate on maps for published reports, or where a neatly finished appearance is desirable, but, since most plans are prepared chiefly for utility, the Reinhardt style is applicable for all lettering except possibly main title of map.

Reinhardt lettering is rapidly executed, presents a workman-like appearance, and is used more than any other (Fig 9). For fine lettering, such as Roman, No 170 Gillott pens are suitable. Some draftsmen prefer a little coarser pen, such as Gillott's No 303, and a still coarser pen, Gillott's No 604 EF, will be found suitable for the finer letters of single-stroke work, as for numbering contours or for the smaller dimensions on a map. For still heavier lines, King's No 9 Nonpareil (similar to stub pen) is serviceable. For heavier lines than these, and particularly for executing the Reinhardt style, the Paysant patent pens are good; they come in sets of graded size, costing say $1.50 per pen, including holder. No 6 and 7 Paysant gives about right weight of line for notes on tracing cloth, while No 4 pen will give a line heavy enough for the main line of a title. These pens make lines which have somewhat rounded ends, so it is necessary for neat results, when making the heavier letters, to square up the corners with an ordinary fine pen, as Gillott's No 170. The slot of a Paysant pen should not be cleaned with a cloth because lint attaches to slot; it should be cleaned

Fig 9. Style Sheet for Lettering

with a small piece of tough paper, and then wiped with a cloth. The so-called ball-point pens are also used to some extent for single-stroke work.

Title should comprise: brief description of drawing, owner's name, location of property, scale, date of survey, surveyor's name and address. It should be so laid out that the most important information is on the most prominent line. The lines of title should be centered, arranged in compact form, but not crowded. It is advisable to letter first the most important or longest line. In other lines, when two words occur on same line separated by a considerable space, first letter right-hand word and then start left-hand word at same distance from center line of title as is the end right-hand word. The size of title should conform to size of plan. Never waste time on a fancy title; it is hard to read and as most draftsmen are not familiar with such alphabets, the results are apt to be poor. Put title in lower right-hand corner of a plan, so that it can be read by turning up the corner of sheets that lie above it in a drawer. In some offices, titles are set up in type and printed on plan. A rolled plan should have a brief title on back, at both ends, so it will be visible when sheet has been rolled in either direction. A graphical scale should be drawn on plans that are to be reproduced photographically.

Notes are an essential part of most plans; they are usually lettered in Reinhardt style. Border lines are not necessary on construction drawings, but on finished drawings and maps a single firm line, drawn 1/2 to 1 in from edge of sheet, gives a good appearance. A fine line just inside of a heavier one makes a good border line; many draftsmen make border lines too heavy.

Meridian should be represented on all maps as a full arrow, if it be a true meridian, or as a half-arrow, if it represent the magnetic meridian, the half-arrow head being on same side as the declination. Make arrows of simple design, and not too conspicuous in size or breadth of lines. It is good practice to show both the true and the magnetic meridian, and to indicate on latter the declination existing at place and time of survey. Do not let a meridian line cut any other lines of a drawing.

To clean a drawing use sponge eraser, art gum, or dry bread crumbs. For tracings, gasolene or benzene will remove pencil marks without affecting ink lines, but it takes the life out of the tracing and makes it turn yellow with age; it also causes a tracing to collect dirt more readily. A tracing upon which lettering has been stamped must not be cleaned with gasolene.

On working plans requiring accurate scaling, it is advisable to plot, at the outset, two long scales

at right angles to each other, by which to measure amount of shrinkage or swelling of the paper, and to make allowance for it in subsequent plotting or scaling from the plan.

Tracings from blueprints or from penciled working drawings can be made by laying them on a drawing table having a glass top, and placing table where plenty of light will fall under it, or else placing electric lights under the table.

Conventional signs are used to represent topography and other common details. A few of the most common are shown in Fig 10. They are used mostly on topographic maps and are produced

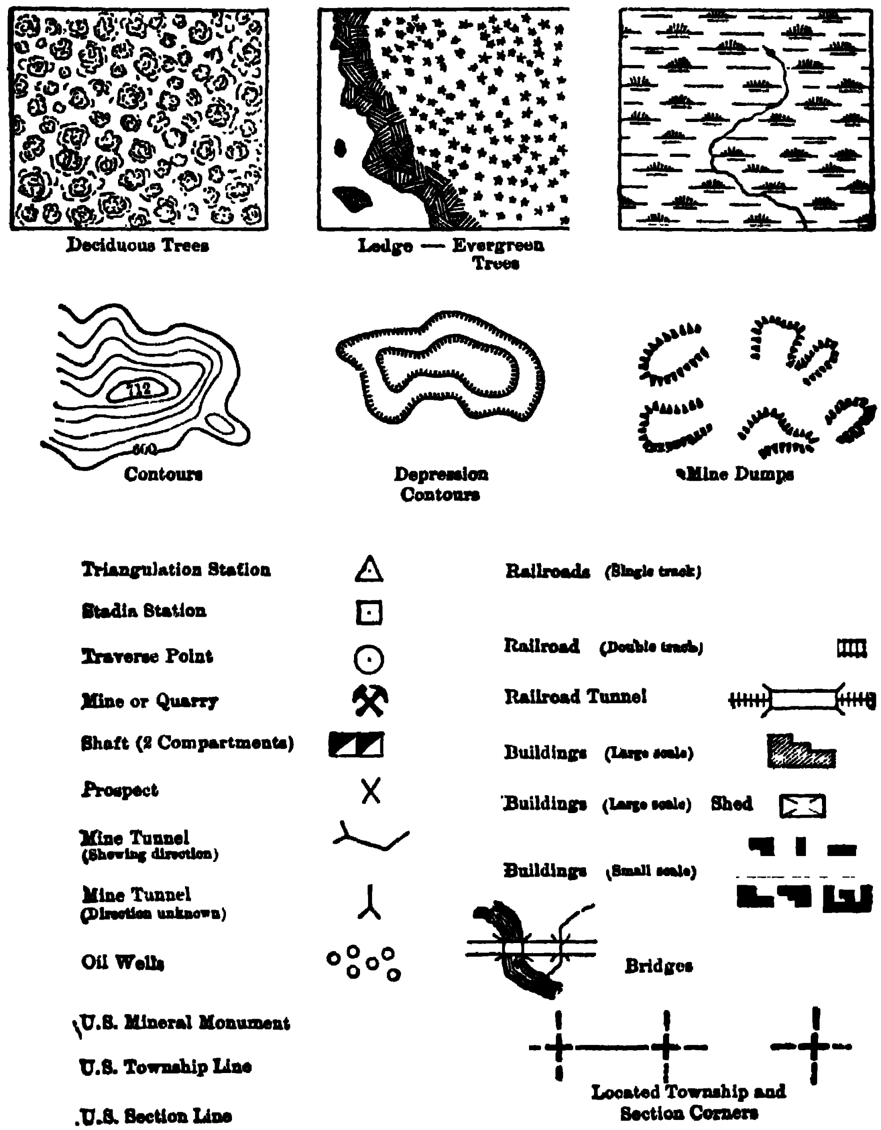

Fig 10. Conventional Topographic Signs

either in black, as is custom with U S Coast and Geodetic Survey, or in color, as on maps of U S Geol
for
part... to bottom of map. ...
full line about half as heavy as shore line, drawn as close to the shore line as possible, and should follow very carefully every irregularity. The next line is parallel to first, but a little lighter and with a little more space, and so on; 5 to 10 lines are sufficient. Water-lining and fresh marsh symbols are frequently drawn in Prussian blue. In executing the symbol for evergreen trees, draw each individual symbol in lines of uniform weight, although the various symbols are made of lines of different weights (Fig 10). To avoid tendency to make them in rows, it is well to draw irregular groups of symbols in different parts of area to be covered, and to fill in between these groups with smaller or lighter symbols. In the symbol for deciduous trees it is intended to represent the plan of a tree in foliage, with a slight shading toward lower right-hand side. On the back of the U S Geol Surv maps are printed the conventional signs used by that office.

Contour lines are almost always drawn in burnt sienna water-color, using either a Gillott No 303 or a contour pen. Every fifth or tenth contour is represented by a line slightly wider and a little darker in color. In numbering contours, give no more figures than are necessary to find the eleva-

tion of any contour without difficulty; they should be small figures in burnt sienna. A common mistake is to make contours so heavy that they subordinate the more important features on map.

Sub-aqueous contours are usually represented on hydrographic maps by dot-and-dash black lines, the shallowest contour having one dot between dashes, the next contour in depth having two dots between dashes, and so on. In some cases, contours representing fathoms of water are shown as single dots for first fathom, two dots and a space for second fathom, dots in groups of three for the third, and so on.

LAND SURVEYING

10. TRAVERSE WITH COMPASS AND TAPE

Surveys with compass and tape are adapted only to surveys of inexpensive land, in localities where there is little danger of local magnetic attraction. Even serious local

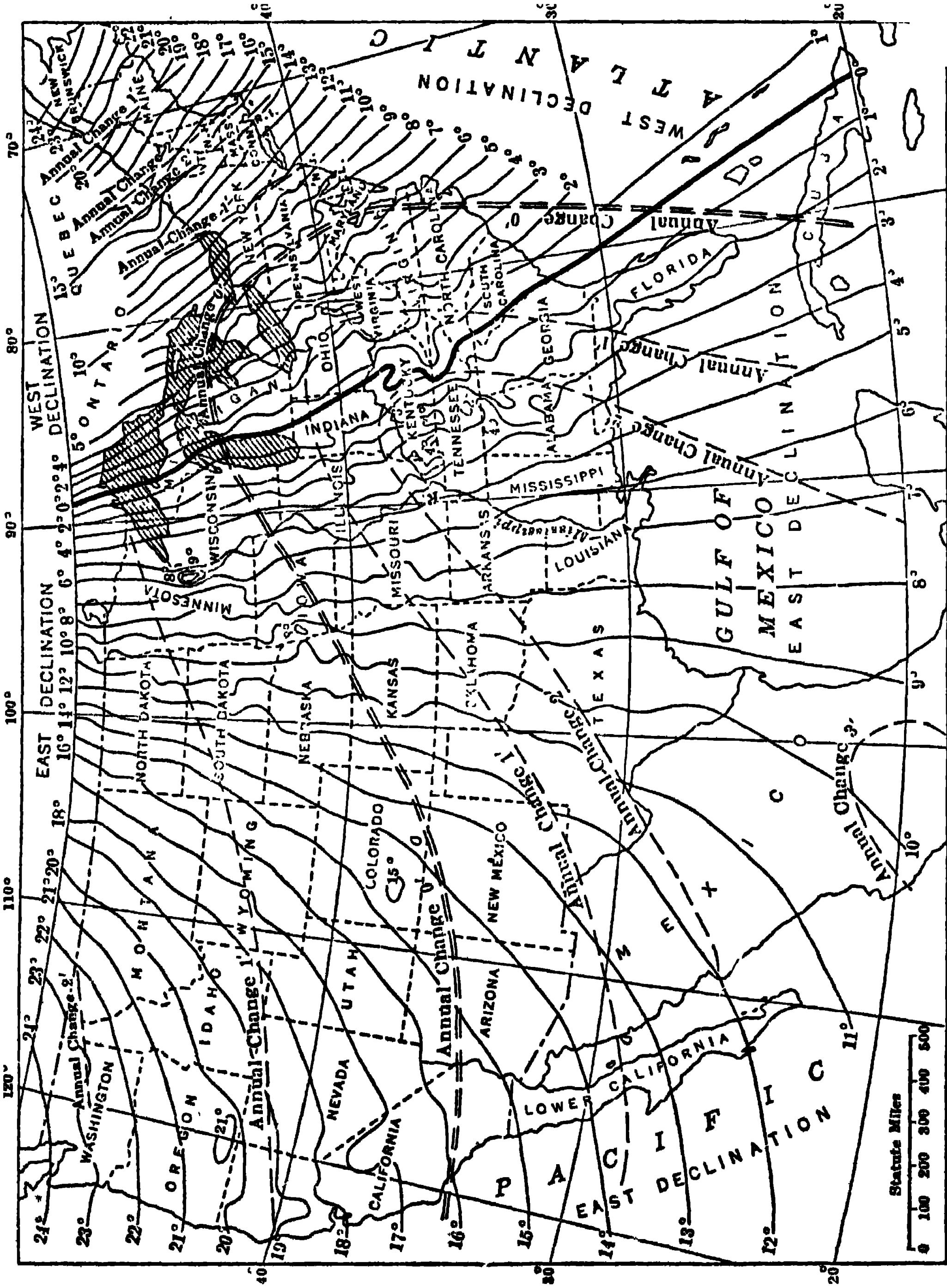

Fig 11. Isogonic Chart of the United States for 1935 (Permission of U S Coast and Geodetic Survey)

attraction may be circumvented if the surveyor has warning of its occurrence, but when rocks containing magnetic iron are distributed generally throughout a district, it is difficult, if not impossible, to obtain consistent compass readings. At every station the reverse as well as forward bearing should be read, to detect local attraction, and, if found, its amount should be determined and the erroneous bearings corrected (Art 3). Bearings are not usually read closer than $^1/_4°$.

If a compass survey were made of a square lot 1 000 ft on a side, an error of $^1/_8°$ (which corresponds to reading bearings to nearest $^1/_4°$) in an angle would correspond to an error of 2.2 ft on a side. In a compass survey, therefore, it is obviously not consistent to measure the sides closer than to nearest half-foot or foot. Under favorable conditions compass surveys can not be relied upon for an accuracy exceeding 1 part in 500. Formerly surveys were often made by compass and chain. In re-running old lines, the compass on transit can be used to aid in finding the physical features which mark the corners, and a reliable transit and tape survey is then substituted for the old approx compass survey (Art 11 and 15).

Magnetic declination is the angle which the needle makes with the true meridian. When necessary to determine this angle accurately, the true meridian is found by observation on the sun or on Polaris (Art 13), and then noting its compass bearing. If north end of needle points east of true meridian, it is called an east declination. Even at a given place the declination changes continually by small amounts; these changes are called VARIATIONS.

To set off the declination so that the compass will give true bearings, turn N point of compass box toward north. Then, if the declination is east, turn graduated circle to the right by the amount of the declination; if the declination is west, move the zero to the left. Needle readings will then be true bearings.

Isogonic lines are curves connecting points at which the magnetic declination is the same. Isogonic charts are prepared by the U S Coast and Geodetic Survey, by plotting on a map the observed declination at numerous magnetic stations, and interpolating declinations at intermediate places (see Fig 11). These charts do not give precise results, but are useful for learning approx declinations. Observed declinations at interpolated places are frequently quite different from those stated on isogonic charts; hence, whenever the magnetic declination is desired with precision, it is necessary to make observations on the true meridian (Art 13).

Changes in declination make it necessary, in re-running old lines, to modify given bearings by an amount equal to the change in declination which has taken place since lines were first run. To determine declination for some past date, records of U S Coast Survey are useful, provided a magnetic station was situated near location of survey; but it is not safe to rely on records of a magnetic station, even only 5 miles distant, except in a general way. It is always safer to determine the present declination, and compare it with the declination at the time lines were first run, which may have been recorded with old survey. If it was not then recorded, the best way to find the correction in bearing is to determine the present magnetic bearing of any well-defined line of the old survey, such as between two identified stone monuments, and apply the difference between present and original bearing to all original bearings (Art 15).

11. TRAVERSE WITH TRANSIT AND TAPE

Precision with which measurements should be made depends upon object of survey. If it is a city survey of valuable property, angles will be measured to the nearest 10 sec, and distances to a hundredth of a foot; while in a farm survey, angles to nearest minute and distances to a tenth or even to within a foot (by stadia method, Art 22) are exact enough. Transits for general surveying read to minutes or half-minutes. It is therefore necessary in precise surveys to measure angles by repetition (see end of this article), to obtain results consistent with accuracy of tape measurements. For better appreciation of relation between distances and angles bear in mind that 0.03 ft, at a distance of 100 ft, subtends 1 min of angle. The degree of precision of transit and tape surveys varies from 1 in 1 500 to 1 in 40 000; to attain an error of less than 1 in 10 000 requires special care, particularly in taping.

Errors in tape measurements are due to: (*a*) erroneous length of tape; (*b*) incorrect amount of pull on tape; (*c*) inaccurate plumbing; (*d*) incorrect alinement of tape; (*e*) sag of tape; (*f*) changes in temperature; (*g*) effect of wind.

(*a*) **Erroneous length** can be corrected by comparing the tape with a standard; most cities maintain a 100-ft standard at some convenient place. To test a tape stretch its full length beside the standard, and leave it until it acquires the same temperature, before comparison is made. Test at intermediate points as well as for total length. Its tempera-

ture and pull should be noted; also whether it is suspended or supported. For a nominal charge the U S Bureau of Standards, Washington, will test a tape and forward results. If tape is too long, measurements made with it are recorded too short, and proper correction should be added. It is good practice to have a new tape standardized by the Bureau of Standards, and then keep it for the sole purpose of standardizing other tapes in daily use. A tape subjected to rough usage will soon have several repaired places, which often cause errors in its length.

(*b*) **Amount of pull** has a very appreciable effect upon the length of a tape; ordinary, light 100-ft tapes will stretch 0.01 to 0.02 ft with an increase of 10 lb above the ordinary pull of 10 lb. The amount of stretch in any given tape can be readily determined by fastening its ring end to a stout nail in the floor, and applying different tensions as determined by a spring balance, the tape lying throughout on floor. Spring-balance handles are manufactured to be attached to tape in the field for measuring amount of pull.

(*c*) **Inaccurate plumbing** is the most fruitful source of errors in taping; if plumbing is unavoidable, more accurate results can be obtained when measuring downhill than uphill. In spite of greatest care, so much error is introduced by plumbing that for accurate results it is better to measure the inclined distance from horiz axis of instrument to a nail in the stake at next station; read, on the vert arc, the inclination of tape, and compute horiz distance from cosine of this angle; *hor dist* = *incl dist* × *cosine vert angle*. If angle is small, the computation may be simplified by using versed sine of angle, thus: *hor dist* = *incl dist* − (*incl dist* × *versin vert angle*). The versed sine (1 − cos), which will be a small number, may be taken from a table of natural functions and the multiplication can be made with sufficient accuracy on a slide rule. This method requires a transit set-up at every other tape-length, but it is more expeditious and gives much more accurate results than plumbing. Another way is to tape directly from stake to stake in one tape-length; set up instrument at every alternate stake and measure vert angles to both adjoining stations, by sighting at a rod target set at same height at which the horiz axis stands above the stake under transit; this gives inclination of tape; then compute horiz distance by either method given above. Still another method is to measure inclined distance from stake to stake, and obtain difference in elevation by leveling; then compute horiz distance by formula in following paragraph. When much inclined work is to be done, it is advisable to use a 200-ft or 300-ft tape.

(*d*) **Alinement errors** are not likely to be large; lining in the tape by eye is exact enough for most measurements. If, in measuring 100-ft tape-lengths, a point is 1 ft out of line, it will introduce an error of only 0.005 ft in that tape-length. Error due to poor alinement may be computed by the formula: $c - a = h^2 \div 2c$, where c is the taped distance, h the offset from the line, and a the correct length. In this formula c is the hypotenuse, a the long leg, and h the short leg of a right triangle. Thus, if one end of a 100-ft tape is on line and the other 0.8 ft off line, the error in that one tape-length is $0.8^2 \div 200 = 0.0032$ ft. Of course there will be a similar error in next tape-length, making a total error of 0.0064 ft in the 200 ft. The shorter the taped distance the greater proportionately is the error due to faulty alinement. This formula is correct to the nearest 0.01 ft, if taped distance is less than 300 ft and offset does not exceed 10% of taped distance.

(*e*) **Sag.** Measurements frequently have to be taken with tape unsupported between its ends. It is customary, in this case, to exert a stronger pull than normally, to counteract effect of sag. With a 12-lb pull, on an ordinary 100-ft ribbon-steel tape supported at its ends, the increased tape reading due to sag is about 0.01 ft. This may be found for any particular tape by the formula: lengthening due to sag = $(L \div 24) \times (wl \div t)^2$, where w = wt of tape in lb per ft of length, t = pull in lb, l = length of tape between supports in ft, and L = total length of tape in ft. The result will be in ft. If the ordinary pull of 10 to 12 lb is given, the correction for sag may be applied by use of this formula, though this is seldom done. The most practical and accurate way of eliminating the error due to sag is to determine by actual test the amount of pull necessary to apply to a suspended tape to bring its ends exactly 100 ft apart. Support plumb lines from points previously set 100 ft apart by a standardized tape when supported horizontally throughout, then note pull required to stretch the suspended tape to be tested so that its end marks will coincide with the plumb lines. U S Bur Standards will do this and report results.

(*f*) **Temperature correction.** A new tape is of standard length at 68° F, with a pull of 10 lb when supported throughout its length. As the average coeff of expansion is desired, the temp of the tape must be noted and temp correction applied. Small tape-thermometers are made for this purpose; the thermometer should be in contact with tape, to obtain as closely as possible the temp of tape itself. Even then, in sunlight, it is difficult to obtain correct temp.

Cumulative and compensating errors. In all measurements it is of utmost importance to distinguish between errors which tend to balance and those which accumulate, the latter

being far more important. For those which tend to balance, the number of errors which will probably remain uncompensated, according to method of least squares, will be the square root of total number of opportunities for error. For example, if a 100-ft tape is 0.02 ft too long a cumulative error of about 1 ft will be made in measuring a one-mile line. If, on other hand, tape-lengths are not marked closer than 0.05 ft, the total error made by this compensating error of 0.05 ft will be only $\sqrt{52} \times 0.05 = 0.36$ ft.

Angle measured may be the deflection angle, the actual angle, or the azimuth angle. The DEFLECTION angle is that between the last course produced and the forward course. The ACTUAL ANGLE measured is usually the smaller of the two angles formed by two diverging lines, although, in case of a closed polygon, many surveyors prefer to read consistently either interior or exterior angles. The AZIMUTH angle is the angle a course makes with the meridian (true or assumed) direction measured clockwise, usually from the south. In measuring a deflection angle the telescope has to be inverted, whence any error in line of collimation will be introduced into deflection angle; such error may be compensated by turning the angle with the telescope, first direct and then inverted, and taking half final reading as correct angle. Deflection angles are recorded R (right) or L (left), according to direction turned.

To measure azimuth, set up the instrument at A with the vernier reading 0°, and lower clamp loose. Point telescope at magnetic south, true south, or other arbitrarily selected meridian, and clamp the lower motion. Unclamp the upper plate, sight B, and read the angle in a clockwise direction. This is azimuth of line AB. Take the instrument to B, being careful not to let the upper plate be moved, invert telescope and backsight on A; clamp the lower motion, reverse telescope to its direct position, loosen upper clamp, and sight C. The reading is the aximuth of BC, referred to same meridian as AB. By this method an error in line of collimation becomes cumulative; the survey should therefore be closed, if possible, by noting final azimuth of starting line AB. The difference between the first and the final determination of azimuth of AB indicates at once the total error in the angular work.

As a check against large errors in azimuth readings, the magnetic bearing of each line should be read, when practicable, and compared with calculated bearing. The calculated bearing of first line is assumed to be its observed bearing; the calculated bearing of any line is obtained from calculated bearing of the line next preceding, combined with the measured angle between the two lines. If a right deflection, add it to NE and SW bearings and subtract from NW and SE bearings; if a left deflection, reverse the process. If resulting angle is between 90° and 180°, take its supplement; if over 180°, subtract 180° from it. Care should be exercised in noting quadrant in which the new line lies.

Since azimuths are read from the S direction, clockwise through 360°, azimuths between 0° and 90° have a SW bearing of the same number of degrees; azimuths between 90° and 180° have a NW bearing equal to supplement of the azimuth; azimuths from 180° to 270° have a NE bearing equal to the azimuth minus 180°; azimuths between 270° and 360° have a SE bearing equal to 360° minus the azimuth. (It is the practice of astronomers to assume the south point as zero azimuth; whence this method has been officially adopted by the U S Coast and Geodetic Survey, and the Public Lands surveyors. North is assumed as zero by the U S Army, in the Infantry, but not in the Artillery.—ED.)

Checking closed traverses. The algebraic sum of the deflection angles should equal 360°, right deflection being considered positive and left deflection negative. If the interior angles are read, or computed, the sum of all interior angles of a closed traverse should be $(n - 2) \times 180°$, where n is number of lines in traverse. If angles are read to nearest min, each angle is probably correct within 30 sec. If a is the number of angles in a traverse, the allowable error in their sum is $30'' \times \sqrt{a}$. Thus, if $a = 9$, allowable error is 1 min 30 sec. This check should be investigated before leaving the field; if angular error is much larger than that allowable, a mistake is indicated, which should be found and corrected. For method of checking distances see Art 12.

Checking unclosed traverses. The only check on distances is re-measurement of the lines, preferably in the direction opposite that first traversed. (A portion of a traverse can sometimes be closed and checked by the method of Art 12.) Angles can be checked by determination of the true bearing (to nearest minute) by solar or stellar observation (Art 13) of the first and any subsequent course of the traverse.

To measure an angle by repetition, set up transit at A, with circle reading 0°, sight B, and clamp lower motion. Loosen upper clamp and sight C; read and record angle. Leaving the two plates clamped together, invert telescope, and sight B, with the lower motion, unclamp upper circle and sight C; read and record the angle. One-half this angle should check closely the first reading, but will be more exact than the first. By repeating an angle six times, and observing final reading to nearest minute, an accuracy within 10 sec can be obtained. The method might be carried even further, starting with the vernier

set at other points on the circle, but ordinary transits are not reliable for results closer than 10 sec. The purpose of making half the repetitions with telescope direct, and half with it inverted is to compensate errors of adjustment in line of collimation and horizontal axis.

To lay off an angle this method can be applied as follows: The angle is first laid off as usual and a point is set; this angle is then measured by repetition. If found to be in error, say 15 sec, the point is moved in proper direction a distance computed by multiplying *tan* 15 sec by distance from the point to the instrument.

For determination of an area three general methods of traversing are: (*a*) Set transit at corners of the property and measure angles directly; also measure distances along property lines. (*b*) If boundary lines are occupied by high fences or other obstacles which prevent setting up instrument at corners, but still permit distances to be measured along property lines, the angles are obtained by measurement between lines staked out parallel to property lines, by equal offsets. (*c*) If boundaries of property are such that it is practicable neither to set up transit at corners, nor to measure distances directly along property lines, an independent traverse is run, regardless of boundaries. To convenient stations of this traverse the property corners are connected by the reading of angles and distances. In this case, a transit station should be set far enough from a property corner to permit telescope to be focused on it (not closer than 6 to 8 ft). The property lines should also be measured, where possible, as a check.

12. COMPUTATION OF AREAS

Adjustment of notes. First correct all slight errors in angles, provided they are not large enough to indicate mistakes, by altering value of those which were taken from short sights, or those in which error is most likely to lie, until the sum of all interior angles is $(n - 2) \times 180°$. Next compute bearing of each course, starting from one of which the bearing is known or assumed. Next correct tape measurements for erroneous length of tape, for sag, or for temp, if necessary. Field data are now in form for computing area.

Double meridian distance method. The data are conveniently tabulated as shown below. The LATITUDE of a course is its length times the cosine of its bearing; its DEPARTURE is its length times the sine of its bearing. Latitudes to the northward and departures to the eastward are called positive, and southward latitudes and westward departures are called negative. After these have been computed for all courses, and entered in their appropriate note-book columns, find total of each column.

(**Note.**—The parallelism between other synonymous terms for latitude and departure, which have been and are used by many surveyors, may be indicated thus: Easting or westing (used in old practice) = departure. Northing or southing = latitude difference = latitude. In this article, the word latitude is used, not in its geodetic sense, but as an abbreviation of latitude difference.—ED.)

If the work were exact, the sums of the positive and of the negative latitudes would be equal; similarly with positive and negative departures. If the discrepancies between positive and negative totals are not large enough to indicate mistakes in tape measurements or computations, errors may be distributed among the latitudes and departures according to following rule, provided angles have been read by TRANSIT: The correction to be applied to the latitude (departure) of any course is to the total error in latitudes (departures) as the latitude (departure) of that course is to the sum of all the latitudes (departures), without regard to algebraic signs. This rule assumes that errors are more likely to occur in tape measurements than in angles. Any knowledge of difficulties met in the field, which would lead surveyor to suspect that the error lay in certain lines, should take precedence over this rule. Furthermore, it is probable that, on account of sag of tape and small obstacles on the line, recorded distances are too long rather than too short; as a general rule, therefore, it is not good practice to apply the above rule when it would lengthen any of the distances. The algebraic sum of balanced latitudes and departures must be zero. In the tabulated example given below latitudes and departures have been balanced so as not to increase length of any lines, and so as to decrease particularly the line HA, which is known to have been measured under difficulties.

In the case of a COMPASS survey, the errors are as likely to be in bearings as in distances; hence, if nothing is suspected as to the probable locus of errors, they should be assumed to be proportional to lengths of courses, and the survey is balanced by the following rule, which alters not only the lengths of courses but also their directions: The correction to be applied to the latitude (departure) of any course is to total error in latitudes (departures) as length of that course is to perimeter of traverse

Having balanced the latitudes and departures, next compute the double meridian distance (D M D) of each course. The D M D of first course equals departure of first

course. The D M D of any other course equals the D M D of preceding course, plus departure of preceding course, plus departure of course itself. The D M D of last course should be numerically equal to departure of last course, but should have the opposite algebraic sign. Algebraic signs must be carefully observed, not only in computing D M D's, but in the whole computation of area. If D M D's are computed beginning at the most westerly point in the traverse, all D M D's will be positive. The last column of positive and negative double areas is found by multiplying each D M D by its corresponding latitude. Half the algebraic sum of the double areas gives area of the traverse.

Computation of Area by Double Meridian Distance Method

Course	Bearing	Dist, feet	Lat	Dep	Balanced Lat	Balanced Dep	D M D	Double area
AB	N 62° 05′1/2 W	361.92	+ 169.40	− 319.83	+169.40	−319.81	− 319.81	− 54 176
BC	N 64° 25′ W	437.64	+ 188.98	− 394.73	+188.98	−394.71	−1 034.33	−195 471
CD	N 32° 16′3/4 E	826.38	+ 698.67	+ 441.32	+698.67	+441.32	− 987.72	−690 088
DE	N 35° 59′1/2 E	400.05	+ 323.68	+ 235.10	+323.68	+235.10	− 311.30	−100 762
EF	S 53° 42′ E	316.65	− 187.46	+ 255.20	−187.44	+255.20	+ 179.00	− 33 552
FG	S 3° 26′1/4 W	303.52	− 302.97	− 18.20	−302.94	− 18.20	+ 416.00	−126 023
GH	S 70° 39′1/2 E	240.03	− 79.50	+ 226.48	− 79 50	+226.48	+ 624.28	− 49 629
HA	S 27° 41′ W	915 72	− 810.90	− 425.43	−810.85	−425.38	+ 425.38	−344 919
....	....	3 801.91	+1 380 73 −1 380.83 error −0.10	+1 158.10 −1 158.19 error −0 09	0.00	0.00	....	2) 1 594 620 797 310 sq ft

Note.—In extensive work it is better to enter positive and negative latitudes and departures in separate columns, for convenience in adding.

Checks on accuracy. Close agreement between the sums of plus and minus latitudes and between plus and minus departures gives a check on the accuracy of multiplication by cosine and sine of the bearings. The D M D's are checked as shown above, but there is no check on the double areas. The final area can be roughly checked by planimeter (Art 6), or by dividing the plotted traverse into rectangles or triangles, scaling their dimensions and computing their areas. A more exact check on the area is to compute from the latitudes the double parallel distances (D P D), just as the D M D's were computed from the departures, and then obtain double areas by multiplying each D P D by its departure.

If l be error in latitudes and d error in departures, the ERROR OF CLOSURE of survey equals $\sqrt{l^2 + d^2}$. Thus, the error of closure of survey shown above is

$$\sqrt{0.09^2 + 0.10^2} = 0.135 \text{ ft},$$

and the relative error of closure is 0.135 ÷ 3 802, or 1 part per 28 300.

Large mistakes in taping distances will be detected in the process of computation. If latitudes and departures do not balance within reasonable limits, the error in departures divided by error in latitudes equals tangent of bearing of that line which would represent error of closure on the plan, and if only one mistake has been made in measuring distances it probably occurred on a line having approximately this bearing.

If error between plus and minus latitudes, or departures, should be seen to be about double the latitude, or departure, of any one course, it is probable that the bearing of that course was read or recorded in wrong quadrant.

Traverse tables shorten the work of computations. One of the best is that by R. L. Gurden, computed to four decimal places, for every minute of angle and up to 100 units of distance.

The same tables can be used for computing horiz and vert distances corresponding to inclined tapings. Nearly every textbook of surveying and civil engineers' pocketbooks contain a traverse table computed to 15 min of angle. In the absence of a traverse table computed to single minutes, a table of log sines and cosines can be used (Sec 45).

When a traverse does not follow property lines (Fig 12), areas may be computed as simple geometric figures, as in following example:

In Fig 12, ABC is property line and XYZ, traverse line.

$$aA = 25.2 \times \sin 76° 24' = 24.5$$
$$aX = 25.2 \times \cos 76° 24' = 5.9$$

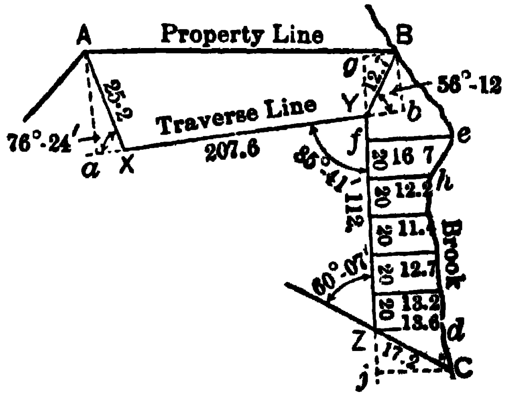

Fig 12. Traverse not on Property Lines

$$ab = 5.9 \quad + 207.6 + 7.1 \quad = 220.6$$
$$bB = \quad 12.7 \times \sin 56° \, 12' \quad = \quad 10.6$$
$$bY = \quad 12.7 \times \cos 56° \, 12' \quad = \quad 7.1$$
$$AB = 220.6 + \frac{(24.5 - 10.6)^2}{2 \times 220.6} = 221.0$$
$$\text{Area } AabB = 220.6 \times \frac{24.5 + 10.6}{2} = 3\,872$$
$$\text{Area } \quad AXa = \frac{24.5 \times 5.9}{2} \qquad = -72$$
$$\text{Area } \quad BYb = \frac{10.6 \times 7.1}{2} \qquad = \underline{-38}$$
$$\text{Area } AXYB \qquad\qquad = 3\,762 \text{ sq ft}$$

Another method, of special application to mine surveys, is to compute area from the COORDINATES of the corners of a tract (see Art 8, this Sec, and Art 9, Sec 18). Coordinates of the corners are computed in ordinary manner by noting bearing and distance of each from nearest, or most accessible, traverse station, of which the coordinates are ascertained by method previously described. Number the corners consecutively around the tract. Multiply each latitude (ordinate) by the difference between following and preceding longitude (abscissa), always subtracting the preceding from the following. One-half the sum of the products is the area.

Meandering boundaries, such as brooks, roads, etc, are usually located by measuring perpendicular offsets from a traverse line which has been run conveniently near the boundary. If curvature of boundary be fairly uniform, offsets are taken at regular intervals, but if it be erratic, offsets must be taken at each point where direction changes. In either case, areas of the several trapezoids are computed, and their sum added to or subtracted from area within traverse lines. If offsets be taken at equal intervals, the computation may be made by one of the following methods.

Trapezoidal rule. Area $= \frac{1}{2} d \, (h_e + 2 \, \Sigma h + h_e')$, where d = common interval between offsets, h_e and h_e' = end offsets of the series of trapezoids, and Σh = sum of intermediate offsets. This rule assumes that the boundary is a straight line between adjacent offsets.

Simpson's one-third rule. Area $= \frac{1}{3} d \, (h_e + 2 \, \Sigma h_{odd} + 4 \, \Sigma h_{even} + h_e')$, where d = common interval between offsets; h_e and h_e' = first and last offset; $2 \, \Sigma h_{odd}$ = twice the sum of the odd offsets (3d, 5th, 7th, etc); $4 \, \Sigma h_{even}$ = four times the sum of the even offsets (2d, 4th, 6th, etc). To apply this rule there must be an odd number of offsets; if the number is even, compute the area of one end trapezoid separately. This rule assumes that between adjacent offsets the boundary is a parabola, through 3 consecutive points.

In Fig 12 the area of *efZd* by the trapezoidal rule $= \frac{1}{2} \times 20 \times [13.6 + 2(13.2 + 12.7 + 11.4 + 12.2) + 16.7] = 1\,293$ sq ft. By Simpson's rule the same area $= \frac{1}{3} \times 20 \, [13.6 + 2 \times 12.7 + 4 \, (13.2 + 11.4) + 12.2] + \frac{1}{2} \times 20 \, (12.2 + 16.7) = 1\,286$ sq ft. Area *BYfe* is computed by finding area of trapezoid *Bgfe* and subtracting triangle *BgY*, giving 235 sq ft. Similarly *dZC* is computed by finding area of trapezoid *dZjC* and subtracting triangle *ZjC*, giving 58 sq ft. The length of side *BC* can be found by computing the several parts *Be*, *eh*, etc, by method of Art 10. For example

$$eh = 20 + \frac{(16.7 - 12.2)^2}{2 \times 20} = 20.5 \text{ ft.}$$

13. DETERMINING THE TRUE MERIDIAN

The three methods best adapted to the needs of an engineer are: (*a*) by observing with ordinary transit the bearing of sun, found from its altitude; (*b*) by use of the solar attachment for transit; (*c*) by observing the bearing of pole-star at its greatest elongation. The observations consist in measuring directly the horizontal angle between sun or star and the survey line of which the bearing is to be determined; preliminary observations on other factors required for solution of problem, such as the altitude of sun or star, the latitude of place, or the time, are usually also necessary.

In solar observations certain times of day are more favorable than others, while at certain times the observations should not be made at all. During summer (in northern hemisphere) observations may be made at almost any time between 7 and 10 a m, and between 2 and 5 p m. Observations before 7 a m or after 5 p m are usually unsatisfactory, because the sun is so near horizon that atmospheric refraction introduces a large error into the measured altitude of the sun; for best results, sun should be at least 10° above horizon. In midwinter it will therefore be necessary to wait until nearly 9 a m before making an observation. Between 10 a m and 2 p m astronomical conditions are not favorable for

an accurate solution of the spherical triangle that must be employed to compute the sun's azimuth. In general, when sun is well to the east or west, conditions are favorable; when it is near meridian they are unfavorable.

Observation of sun's bearing. The only additional equipment necessary for an ordinary engineer's transit is a colored glass over the eye-piece. Even this may be dispensed with in the following manner: Draw out the eye-piece tube a little way and hold a card behind it; if telescope be then pointed at sun, an image of sun and of the cross-hairs can be seen on card. The image may be focused by moving card to or from eye-piece; the cross-hairs can not usually be seen except when projected on to the sun's disk. In computing sun's azimuth it is necessary to know the time within 5 min or less. If possible the watch used should be regulated to standard (railroad) time.

The observation consists in measuring simultaneously the horiz angle from some reference line, and the altitude of sun's center. Set the vernier at 0°, and sight telescope along the reference line. Then unclamp the upper motion, and sight at the sun. The horiz and vert angles, and time of pointing are recorded. For a comparatively rough result, say, when an error of 1 to 2 min is not serious, it will be sufficient to bisect the sun's disk, by estimation, with both vert and horiz cross-hair; for more precise results, it is necessary to observe upper and then lower edge of sun for altitude, and right and left edges for horiz angle. The means of these pairs may be taken as altitude and horiz angle to the center, as though they were both measured directly at same instant of time. To make these double observations on the sun with least effort, the edges to be sighted should be so selected that only one tangent screw will have to be turned at a time. In the morning (in northern hemisphere) the sun is rising and moving to right. If telescope be set so that sun appears in upper left-hand quadrant of field, the sun will approach the vert and recede from the horiz cross-hair. If transit has an inverting eye-piece the motion will appear reversed. By setting the horiz hair a little above lower edge of sun's disk, the sun's motion itself will bring cross-hair into contact with lower edge of the disk; it is therefore necessary only to keep vert cross-hair in contact with right edge, by means of the upper tangent screw, stopping this motion the instant both cross-hairs are simultaneously in contact with the edge of disk. When this position is reached the watch time should be recorded, the vert and horiz circles read, and the index error of vert circle determined. The second half of the observation consists in bringing the disk into the lower right-hand quadrant, setting the vert hair a little to right of left edge, and keeping the horiz hair steadily in contact with upper edge of disk, by the vert tangent screw, until both hairs are again in contact. The watch time is recorded, both circles are read again, and index error of vert arc determined. The mean of the two vert angles is assumed to correspond to the mean of the two horiz angles, and to the aver of the two times. These two pointings should give sun's bearing to nearest min of angle; for greater precision, observations may be repeated, 3 readings being made in each position of cross-hairs. If instrument has a complete vert circle, so that altitudes may be read with telescope inverted, it is advisable to take the second half-set of readings with telescope in inverted position, thus compensating errors of adjustment (Sec 4). It is good practice, after observation is made, to sight back at the reference mark: if vernier reads 0°, it indicates the horiz circle did not move during observation. It is undesirable to let the observations extend over more than 10 or 12 min of time, since the curvature of sun's path will introduce error into result. If observation be made in afternoon, the disk should be placed in upper right-hand and in lower left-hand quadrants.

To calculate the sun's bearing, take the mean of measured altitudes, subtract from it the refraction correction in following table, and call this *h* in formulas below.

Refraction Correction to the Altitude (Minutes to be Subtracted)

Altitude	Refraction	Altitude	Refraction	Altitude	Refraction
5°	9′.9	13°	4′.1	25°	2′.1
6	8.5	14	3.8	30	1.7
5	7.4	15	3.6	35	1.4
8	6.6	16	3.3	40	1.2
9	5.9	17	3.1	45	1.0
10	5.3	18	3.0	50	0.8
11	4.9	19	2.8	55	0.7
12	4.5	20	2.6	60	0.6

Next find sun's " declination " for the instant of observation, from (*a*) American Ephemeris and Nautical Almanac, (*b*) Nautical Almanac, or (*c*) handbook of any instrument maker who publishes a " Solar Ephemeris." (*a*) and (*b*) are published by Navy Dept, Wash, D C. In (*a*) the " apparent declination " is given for each day of month for the instant of " 0 hr Greenwich Civil Time," together with the tabular difference for 24 hr. In (*b*) it is given for every 2 hr, together with the

"hourly difference." In (c) it is given as in (a), together with the "variation per hr." To correct the declination for the number of hours since 0 hr (midnight), multiply, if (a) is used, the tabular difference for 24 hr by the number of hr of Greenwich Civil Time, divide by 24 and add this correction algebraically to the declination for 0 hr. To find number of hr of Greenwich Time, add 12 hr if time is P M; then add 5 hr if given time is Eastern Standard, 6 hr if Central Time, 7 hr if Mountain Time, 8 hr if Pacific Time. If given time is local, add west longitude in hr, min, and sec. If given time is "apparent" (true sun time), first subtract from it algebraically the "equation of time"; which is difference between Civil and Apparent time at any instant. Values of the "Eq Time" are given in the above publications in same manner as the declination. When converting "apparent time" to mean time, it is necessary, since the Eq Time is tabulated for civil time, to first find the civil time with sufficient accuracy to find the correct Eq Time. Corrected declination is called D in the formulas.

Example. Compute sun's declination at 2 P M Eastern Standard Time, Jan 8, 1938.

Sun's decl Jan 8, 1938, at Greenwich Civ Time 0 = $-22° 21' 25.6''$. Tab diff for 24 hr = 476.2″, Greenwich Civ Time, corresponding to 2 P M, E S T = 19^h. Hence, declination = $-22° 21' 25.6'' + \frac{476.2 \times 19}{24} = -22° 15' 07.8''$.

Example. Compute the sun's decl, Jan 8, 1938, when the "apparent time" is 12^h (noon) in longitude 71° 04′ W (= $4^h\ 44^m\ 18^s$ = 4.74 hr W). Sun's decl at G C T 0^h = $-22° 21' 25.6''$; tab diff for 24 hr = 476.2″. Eq Time at G C T, 0^h = $-6^m\ 25^s.60$; tab diff. for 24 hr = $25^s.59$. The Greenwich Apparent Time = $12^h\ 00^m\ 00^s + 4^h\ 44^m\ 18^s = 16^h\ 44^m\ 18^s$ = 16.74 hr. The approx Gr Civil Time = $16^h\ 44^m\ 18^s + (-6^m\ 25^s.6) = 16^h\ 50^m\ 43^s.6$ = 16.85 hr, a value sufficiently accurate for ordinary purposes for computing the Eq Time. Hence, the corrected equation of time = $-6^m\ 25^s.60 + \frac{16^h.85 \times (-25^s.59)}{24} = -6^m\ 43^s.56$; then the Gr C T = $16^h\ 44^m\ 18^s + (-6^m\ 43^s.6) = 16^h\ 51^m\ 01^s.6$ = 16.85 hr. Required declination is $-22° 21' 25.6'' + \frac{16.85 \text{ hr} \times 476.2''}{24} = -22° 15' 51.3''$.

The latitude of the place must be obtained either from a map or by direct observation. If taken from a map it must be obtained within about 1/2 min. If observed directly, it is found as follows: The max altitude of the sun should be found at noon, with the transit, by setting horiz cross-hair on lower edge of sun and following it as long as it continues to rise. When sun drops below cross-hair, the altitude is read from the vert arc. This reading must be corrected for index error, refraction, and semi-diameter. The semi-diameter may be taken from the Nautical Almanac at the same opening as the declination. Call the corrected altitude h. The latitude is then found from the equation, $L = 90° - (h - D)$. South declination must be considered negative.

Example. On Jan 8, 1938, in longitude 71° 04′ W (= $4^h\ 44^m\ 18^s$ W = $4^h.74$ W), observed meridian altitude of the sun's lower limit was 25° 06′; index-error, +01′; refraction correction, −02′; sun's semi-diam, 16′.3; hence true altitude of sun's center = 25° 21′.3. Sun's declination (from Ephemeris) at 0 hr Greenwich Civil Time −22° 21′ 25.6″; tabular difference for 24 hr = 476.2″. Apparent time at place of observation is 12 hr (noon); hence Gr Apparent Time is $16^h\ 44^m\ 18^s$. Equation of time at Gr Civ Time 0^h = $-6^m\ 25^s.6$. Tabular diff for 24 hr = $25^s.59$. The required Equation of Time is $-6^m\ 43^s.56$ (see preceding page); adding this to the Gr App Time = $16^h\ 51^m\ 01^s$ = 16.85 hr is found for the Gr Civ Time. Declination D, at instant of observation, is −22° 15′ 51.3″. The formula then gives the latitude, L = 42° 23.1′ N.

The azimuth of the sun's center at instant of observation is now found by the formula:

$$\cos Z = \frac{\sin D - \sin h \sin L}{\cos h \cos L}$$

in which Z is azimuth from north point (either east or west).

Example. Azimuth observation on sun, April 12, 1925.

Instrument at sta 19
Reading at sta 20 = 0° 00′

Upper and left edges	Horiz angle (to right) = 84° 28′	Vert angle = 41° 30′
Lower and right edges	Horiz angle = 86° 11′	Vert angle = 41° 16′
Watch, Eastern Time, $9^h\ 07^m$ AM	Mean = 85° 19′.5	Mean = 41° 23′
Greenwich Civil Time, $14^h\ 07^m$		Refraction = − 1′.2
		h = 41° 21′.8

Declination at "0^h Greenwich Civil Time" = + 8° 24′ 44″.3
Correction = + 54″.09 × 44^h.12 = + 12 56. 6
D = + 8° 37′ 40″.9

Angles	Nat Sin	Log Sin	Log Cos
D +8° 37′.7	0.15002		
h 41° 21′.8		9.82009	9.87537
L 42° 23′.8		9.82882	9.86834
product	−0.44557	9.64891	9.74371
numerator	−0.29555		9.47063n
		57° 46′.5	9.72692n

Z = 122° 13′.5(N E)
Angle = 85 19 .5
Bearing = N 36° 44′.0° E
As = 216° 44′.0

In case it is not convenient to use the natural functions, Z may be found by

$$\cos^2\frac{Z}{2} = \left(\frac{\cos s \cos (s - p)}{\cos L \cos h}\right)$$

in which $p = 90 - D$, and $s = 1/2\,(L + h' + p)$.

In case there is doubt about error of the watch on standard time, local time may be obtained by formula:

$$\sin^2 1/2P = \frac{\cos s \sin (s - h)}{\cos L \sin p}$$

Angle P (expressed in hr, min, and sec), added to west longitude of place gives Greenwich apparent time. To obtain Greenwich Civil Time subtract "equation of time" given in the Almanac.

Solar attachment is a small appliance, which may be attached to an engineer's transit and used for finding the meridian by an observation on the sun. There are several different forms, but the general principle is that the attachment itself can be rotated about a polar axis (parallel to earth's axis), so as to follow the sun's daily motion, while the whole instrument can be rotated about vertical axis of transit, to swing transit telescope into plane of meridian. By a combination of these two motions the telescope of the attachment (or its equivalent) may be pointed at sun. If proper settings be made for observer's latitude and sun's declination, this pointing at sun can be accomplished only by bringing transit telescope into plane of meridian.

The three steps required in the process are as follows: First lay off declination of sun in such manner that the solar telescope may be pointed at sun, while the polar axis is

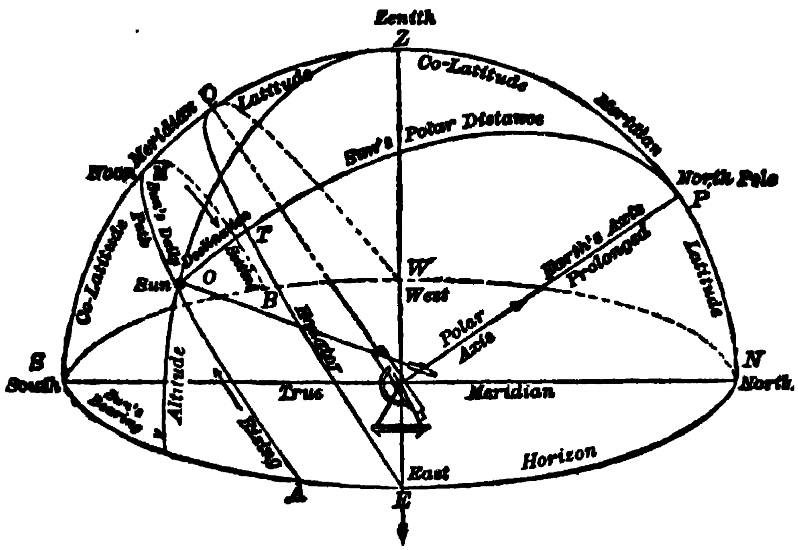

Fig 13. Half Celestial Sphere

parallel to earth's axis. The method of doing this depends upon construction of instrument. In the Burt solar attachment the declination is set off directly on a special arc provided for this purpose. In the Saegmuller pattern it is set off by means of vertical arc of transit and an attached level, the small telescope being in same plane as transit telescope when declination is laid off. In other forms of attachment, such as Shattuck's and Smith's, which contain mirrors, this setting is made by special devices depending upon construction of instrument. After the declination setting is made, the polar axis of the attachment is given an inclination to horizon equal to latitude of place of observation. In the Burt and Saegmuller attachments this is done by making vert arc read the co-latitude of the place. In the Shattuck and Smith attachments the transit telescope is placed in position by setting vert circle to read latitude of the place. When these two settings have been made the solar telescope is sighted at sun, or, in case of those attachments having mirrors, the mirrors are turned so that sun can be seen in telescope. When sun's disk is bisected exactly by the cross-hairs, the plane of transit telescope must be coincident with plane of meridian.

The declination referred to in preceding paragraph may be taken from the Nautical Almanac for given date and for the instant of 0^h Greenwich Civil Time; the tabular difference for 24 hr is given in the next column. To obtain correct declination take number of hours in the Greenwich Civil Time and multiply this by the tab diff for 24 hr ÷ 24. The number of hours in the Greenwich Civil Time is obtained as follows: If the time is P M add 12 hr; then, if it is Eastern Standard Time, add 5 hr, if Central Time add 6 hr, if Mountain Time add 7 hr, and if Pacific Time, add 8 hr.

If local time is given, add the west longitude expressed in hr, min, and sec. The corrected declination has still to be corrected for effect of atmospheric refraction, as found in the handbooks of instrument makers, or determined by Comstock's method, as follows: Set vert hair on one edge of sun and note time; set vernier of plate 10 min ahead and note time when same edge of sun is again on cross-hair. If n be seconds of time between observations and h the altitude, degrees, the refraction in minutes equals $2\,000 \div h \times n$. Add this correction to north declinations, and subtract from south declinations. The latitude used in solar observations may be taken from a reliable map, or directly observed by method already described.

Chief objection to solar attachments is that, for even fair results, the adjustments must be frequently inspected. Direct solar observations with an engineer's transit are more accurate, and take very little more time than with the attachment.

For details of construction and use of different solar attachments, see catalogs of makers. W. & L. E. Gurley, Troy, N Y, make the Burt solar attachment; Ainsworth & Sons, Denver, Colo, the Shattuck; Young & Sons, Phila, Pa, the Smith. Nearly all instrument-makers manufacture attachments similar to the Saegmuller Solar.

Burt solar compass is an instrument for determining direction of the true meridian by observations on the sun. It consists of a horiz plate upon which are mounted two vert sights, like those of the ordinary needle compass and also a polar axis which is fixed in same vert plane as sights; the whole instrument revolves about a vert axis. An arm, carrying a lens and a screw is so attached to the polar axis that it may be set at any inclination to it, and may be revolved about it.

To determine the meridian, set the arm at proper inclination to polar axis by setting off sun's declination on the declination arc. Next give polar axis an inclination equal to latitude of the place, by setting off this latitude on the latitude arc. If the arm is now turned about the polar axis and the whole instrument about the vert axis, so that the lens throws an image of sun on center (square) of screen, the sights are in plane of the meridian.

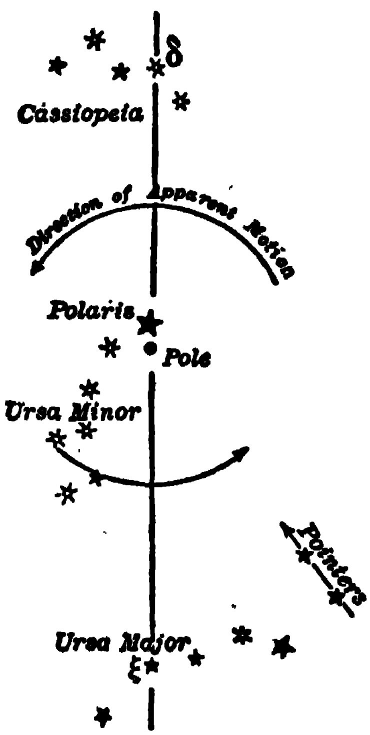

Fig 14. Circumpolar Stars

Observations on the pole-star are less conveniently made than those on the sun, but the calculations are simpler and accuracy is somewhat greater. To identify Polaris, first find Great Dipper, comprising 7 fairly bright stars forming a dipper with bent handle (Fig 14). The 2 stars on side of bowl farthest from handle are called the "Pointers," because a line through them, prolonged about 5 times the distance between them, passes through a point close to Polaris. Its position may also be verified by noting that Cassiopeia (the W) is on opposite side of Polaris from the dipper. Once each sidereal day, Polaris describes a small circle about the invisible pole, with radius (in 1938) of about 1° 01′. Twice each day the star is on the meridian; once when directly above the pole, called "upper culmination," and again when below the pole, at "lower culmination." Its extreme eastern position is the "eastern elongation"; its extreme western position, the "western elongation." Observations are best made at either elongation. Fig 14 shows the constellations as they appear when Polaris is at its upper culmination; by looking at the figure inverted they are seen as they appear at lower culmination; by looking at it from left and right margins of page, the positions are seen for western and eastern elongations of Polaris.

Equipment necessary, besides the transit, comprises a flash light or lantern, and a reflector to throw light into the telescope field and make the cross-hairs visible. It is not necessary to know accurately the instant at which elongation will occur, but it is convenient to know the time within a few min, so as to have ample time to prepare for the observation. Position of the constellations gives a fair idea of the time before Polaris will reach the position of elongation. A closer time estimate can be made from following table for latitude 40° N.

Date	Eastern elongation	Western elongation	Date	Eastern elongation	Western elongation
Jan 1	$13^h\ 04^m$	$0^h\ 59^m$	July 1	$1^h\ 11^m$	$13^h\ 03^m$
Feb 1	11 01	22 52	Aug 1	23 06	11 01
Mar 1	9 10	21 02	Sept 1	21 05	9 00
Apr 1	7 08	19 00	Oct 1	19 07	7 03
May 1	5 10	17 02	Nov 1	17 06	5 01
June 1	3 09	15 00	Dec 1	15 08	3 03

Note. The figures given are local civil time (those greater than 12 hr being P M): local time of east or west elongation does not vary more than a min in different parts of the U S. To change to standard time, take difference in longitude between standard meridian and observer's meridian, and convert this into time; 1 hr corresponds to 15° of longitude. Add this to local time if place is west of standard meridian, and *vice versa*. Standard meridians are those divisible by 15. To find the time for a date other than first day of month, interpolate between values given.

Example. Find Central Standard Time of eastern elongation of Polaris on Aug 3, at longitude 91° W of Greenwich. The time for Aug 1 is $23^h\ 06^m$, or $11^h\ 06^m$ P M; the difference is about 2 hr for the month, or about 4 min per day. Hence, for Aug 3, the time is about $10^h\ 58^m$ P M; the place is 1° or 4 min west of Central (90°) meridian; Central standard time is therefore $11^h\ 02^m$ P M.

About 0.5 hr before time of elongation, transit should be set up and star bisected by vertical cross-hair. As it slowly moves right or left toward its greatest elongation it is followed by using lower tangent screw. At eastern elongation the star will be rising; at western elongation, setting. When extreme position is reached, the line may be transferred to a stake on the ground, or an angle measured to a reference line. For greater accuracy, a second observation is made immediately afterward, with telescope in reversed position, the plate bubble being leveled again before sighting. Mean of the results will be free from errors of adjustment.

It now remains to find the true bearing of the line marked out, or the bearing of the star at elongation, which is computed by:

Sin azimuth = *sin* polar distance × *sec* latitude. To establish the meridian line, this angle is laid off to right for western elongation, to left for eastern elongation. This may be done by transit, or by a perpendicular offset measured with tape.

Latitude used in formula need not be known with great accuracy; it may be taken from a map if it can be found within 1 or 2 min of true value. For a direct observation, use following method: If the altitude of Polaris be observed when above the pole (upper culmination), its max altitude found by trial is the meridian altitude. Latitude is then computed by: $L = h - p$. To obtain true altitude h, the measured altitude must be corrected for refraction and for index error. POLAR DISTANCE p is found by finding declination of Polaris in Nautical Almanac, in Table headed "Circumpolar Stars," and subtracting it from 90°. If the star be observed at lower culmination, the polar distance must be added to altitude.

Example, observation for latitude. Observed altitude of Polaris at upper culmination = 43° 37′ 00″; index correction = + 30″; refraction correction = 1′ 01″; hence, true altitude = 43° 36′ 29″. Declination of Polaris is +88° 58′ 24″; polar distance 1° 01′ 36″ and latitude is therefore 42° 34′ 53″ N.

Example, observation for meridian. Direction of Polaris is observed at its western elongation Mch 11, 1938, in latitude 42° 30′ N. Declination of Polaris is +88° 58′ 27″; polar distance, 1° 01′ 33″. Azimuth is computed as follows:

$$\begin{aligned} \log \sin p &= 8.25293 \\ \log \sec L &= \underline{0.13237} \\ \log \sin Z &= 8.38530; \text{ hence, } Z = 1^\circ\ 23'\ 29''\ \text{W}. \end{aligned}$$

This angle is to be laid off to the right, to fix a point on meridian 310 ft N of transit. The offset is found as follows:

$$\begin{aligned} \log 310.0 &= 2.49136 \\ \log \tan 1^\circ\ 23'\ 29'' &= \underline{8.38541} \\ & \ 10.87677; \text{ hence, offset } = 7.530 \text{ ft}. \end{aligned}$$

Azimuth is obtained with greater accuracy by this method than by observation on the sun, provided the instrument is used in both direct and reversed positions, and plate is releveled before each pointing on star.

14. OBSTACLES AND INACCESSIBLE DISTANCES

Random line is often necessary when obstacles occur on a traverse line. If it be required to run line AB (Fig 15), neither end being visible from the other, and there is no intermediate point from which both ends are visible, a random line AX can be run. Measure perpendicular distance CB and also AC. Compute AB by trigonometry (or by method of Art 10, if BC is short and AC long), and set any desired point D on AB as follows: $AE = AD \times (AC \div AB)$, and $ED = BC \times (AD \div AB)$. By this method points on AB can be set without running out and actually measuring line AB. $Tan\ CAB = CB \div AC$.

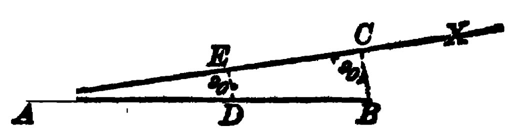

Fig 15. Running Random Line

When a straight line AX can not be run, a traverse from A to B can be run in which line AB would form closing side; its direction and length can then be computed by methods of Art 12 and 17. Any point D can be set on AB by computing its latitude and departure and comparing these with latitude and departure of nearest transit point H on the random traverse, as follows: the difference in their departures divided by difference in their latitudes gives tangent of bearing of line HD, and length is

$$HD = \sqrt{\text{diff lat}^2 + \text{diff dep}^2}.$$

By setting transit at H, pointing in direction of calculated bearing, and laying off distance HD, point D can be accurately set on AB. Since this traverse method involves measurement of angles, it will not lend itself so readily to accurate results as the case first described, where random line is straight. In running any straight line, when necessary to produce the line by reversing telescope, the mean result of a double reversal should always be taken, the telescope being first erect and then inverted, to compensate errors in adjustment.

Running parallel line past obstacle. One of the most exact methods of extending a straight line past an obstacle of limited size is to run a parallel offset line past the obstacle. The instrument is set at B (Fig 16) and BB' is laid off at right angles; BB' is made any convenient distance which will bring the auxiliary line beyond obstacle. Similarly, point A' is set opposite point A; although if AA' be short the transit need not be used for laying off right angle at A. Points A and B must have been accurately set on line. The instrument is then set at B', backsighted on A', the telescope inverted, and points C' and D' set on line. Leaving telescope inverted, another backsight is taken on A', and process repeated to compensate instrumental errors; points C' and D' will be set on mean of these two lines if any errors are evident. Transit is then moved to C', a right angle turned off, and point C set, the distance $C'C$ being made equal to $B'B$. By then setting at C and sighting ahead on $D(DD' = CC')$, the original transit line is then again run forward. Distance $B'C'$ is measured carefully, to give distance BC; this is why it is necessary that lines BB' and $C'C$ shall be laid off at right angles by the transit.

Fig 16. Running Parallel Line Past an Obstacle

The other offsets AA' and DD' are not in any way connected with measurement along the line; they simply define its direction; hence, if convenient, it is often necessary only to show these distances as swing offsets for transitman to sight on. Offsets AA' and DD' should be taken 200 ft or more from obstacle, if practicable.

Should obstacle be in a hollow, so that it is possible to see over it with instrument at A, the point D, or a foresight of some sort, should be set on line beyond obstacle, to be used as foresight when transit returns to original line. The distance may be obtained, as above, by an offset line around obstacle. Sometimes it is possible to place a nail exactly on line on ridgepole of a building (if that is the obstacle), which gives an excellent backsight for extending line on other side.

Equilateral triangle traverse. In Fig 17, set instrument at C, lay off 120°, set stake D a sufficient distance away, measure CD. Then set up at D, lay off 60°, and set point E, making $DE = CD$. Set up at E, lay off 120°, which defines line EB. Evidently $CE = CD = DE$.

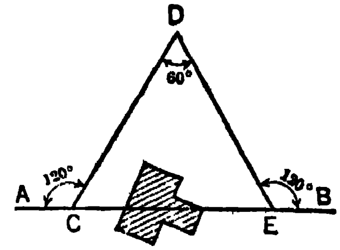

Fig 17. Triangular Traverse Past an Obstacle

To pass obstructed portions of main traverse an auxiliary traverse with small deflection angles may be run which will lie near the former. Since the deflection angles are small the distances measured on auxiliary traverse can be applied as though they were measured on main traverse, from point of departure to point where main traverse is again reached. The simplest case is where an isosceles triangle is formed with main traverse as the base. Suppose at A on main traverse a deflection of 0′ 24′ R is taken to B on auxiliary traverse (AB = 421.7 ft): at B deflection 0° 48′ L (2 × 0° 24′) is turned and BC made 421.7 ft. Then C lies on main traverse and by laying off at C deflection 0° 24′ R, the main traverse line is resumed. This method can be extended to comprise a series of isosceles triangles, the bases of which are either main traverse line or lines parallel to it.

To measure an inaccessible distance, where line is visible, such as across a pond, any of following methods may be employed:

(a) Lay off from transit line AC (Fig 18) a line AB, which clears end of pond and can be taped, and set stake B; then lay off ABC = 90° and set point C by intersecting main transit line. Measure angle CAB and side AB, from which $AC = AB + \cos CAB$; or

better, $AC = AB + AB\ exsec\ CAB$. Angle C and distance CB can be computed, and checked against their field measurements.

(*b*) With instrument at A (Fig 19) and a swing offset of 100 ft from C (some point on

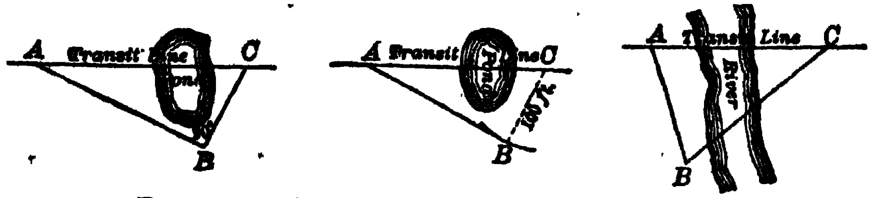

Fig 18, 19, and 20. Measuring Inaccessible Distances

main traverse line on other side of pond) measure angle between CA and a tangent AB to the swing offset. $AC = 100 \div sin\ CAB$.

(*c*) Any line AB (Fig 20) may be laid off along shore of a river, and a point C set on main traverse line across the river. Measure angles A and B, and also C as a check, and from AB as a base compute AC by trigonometry. If AB be made perpendicular to CA and some multiple of 100 ft long, computation is simplified.

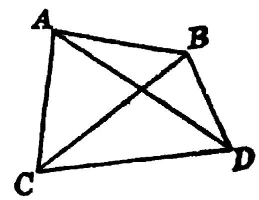

Fig 21

To obtain distance between two inaccessible points A and B (Fig 21), by observations from two accessible points C and D, measure DC and angles ADC, ADB, ACB, and BCD. Compute CB in triangle CBD, AC in triangle ACD, and then in triangle ACB compute AB.

To obtain the inaccessible distance AB (Fig 21) between two accessible points, by observation on two inaccessible points C and D when distance CD is known, measure angles CAD, DAB, CBD, and ABC. Assume $AB = 1$. Then compute CD by process described in above paragraph. This gives a ratio between AB and CD, and since CD is known, the actual length of AB may be computed.

15. RE-RUNNING OLD LINES

In re-running old property lines, surveyor must first determine where original boundaries of property lie, and then survey those boundary lines. He should not attempt to correct original lines, even though sure that errors exist in them. He must first of all look for physical evidence of monuments or other marks, known to have existed on the boundary lines; failing in this, he should base his judgment as to their location on such evidence as occupancy, dimensions stated in deeds, or the testimony of competent witnesses. It must not be assumed that a boundary is missing because not at once visible. Stone bounds are often buried two or three feet deep; the top of a stake soon rots off, but evidences of the stake are often found many years after the top has disappeared, and the supposed location should be carefully dug over to find traces of the old stake.

In interpreting a deed it is assumed that the document was intended to convey property the boundaries of which form a closed traverse. Therefore, if it be found that the omission of a whole chain-length, or the reversing of direction of a bearing, will cause a deed description to close, this change may properly be made. Where record of original survey does not close, deeds of adjoining property are often of assistance. Where artificial features are mentioned as boundaries, these always take precedence over the recorded distances and angles, but such marks must be mentioned in deed to acquire force or authority of monuments. When area does not agree with boundaries, as described in deed, the boundaries control. All distances, unless otherwise specified, are to be taken as straight lines; but distances stated as so many feet along a wall, or highway, are supposed to follow these lines even if not straight. When a deed refers to a plan, the dimensions on this plan become a part of description of property.

Legal boundaries. Where property is bounded on or by a highway, the abutters own the fee to middle line, but where it is an accepted street each abutter yields his portion of street for public use. If, however, the street be abandoned, the land reverts to owner if his deed reads that the property is bounded *on* the street; but if it reads that it is bounded *by* the street he owns no fee in street. If a street has been opened and used for a long series of years, bounded by walls or fences, and there has been no protest regarding them, these lines usually hold as legal boundaries; but if street lines have been defined by proper officials, and evidence of these lines is such that they can be re-run, then the law usually requires a much longer period of occupancy before a private owner can acquire rights from the public in the street. In the case of a line between private owners, acquiescence in location of the boundary will, in general, make it the legal line; but if there be a

mistake in its location, and it has not been brought to attention of the interested parties, nor any question raised as to position, then occupancy for many years does not make it a legal line.

Property bounded by a non-navigable stream extends to the thread of stream, the legal meaning of which is a line midway between shore lines, not necessarily the thread in time of drought. If property be described as running to a river, it is interpreted to mean to low-water mark, unless otherwise stated; if described as running to bank of a river, it means to edge of the uplands; if to shore line, it means to water's edge when stream has its average regimen of flow. Where original ownership ran to shore of a navigable stream, and the water has subsequently receded, the proper subdivision is one which gives to each owner along shore his proportional share of river channel; in general, these lines will therefore run perpendicular to channel of stream from the original intersection of division and shore lines. A more complete statement of the principles mentioned above, particularly with reference to U S Public Land Surveys, will be found in an address on "The Judicial Functions of Surveyors," by Chief-Justice Cooley of Michigan Supreme Court. (*Proc* Mich Assoc of Eng and Surveyors, 1882, pp 112–122.)

Should all evidence of artificial boundaries of a property be missing, surveyor will have to accept deed description as best evidence. If directions of lines be given as magnetic bearings, it is necessary first to determine declination of needle at date of survey. The declination should be stated in deed or on original plan, but frequently it appears in neither. If the date can be established, the declination for that year and place may be obtained from records of local surveyors or from past U S Coast Survey records. If one line can be identified as a boundary, the difference between its present magnetic bearing and its original bearing gives difference in declination directly, and the remaining deed lines can be run out by correcting all bearings by this amount. The chain used in original survey may have been of different length from the one now used; this can be readily determined by measuring length of any well-defined line of the property, and the difference thus found is applied proportionally in measurements locating lost corners.

16. UNITED STATES PUBLIC LANDS

The system. The United States system of surveying public lands, inaugurated in 1784 and since modified by various acts of Congress, requires that public lands "shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles so as to form townships six miles square," and that corners of townships thus surveyed "must be marked with progressive numbers from the beginning." Also, that townships shall be subdivided into 36 sections, each of which shall contain 640 acres, as nearly as may be, by a system of two sets of parallel lines, one governed by true meridians and the other by parallels of latitude, the latter intersecting the former at right angles, at intervals of a mile. Since meridians converge, it is evident that the requirement that the lines shall conform to true meridians, and also that townships shall be six miles square, is mathematically impossible.

A surveyor whose work lies in a district formerly a part of the U S public lands should procure from General Land Office, at Washington, a copy of "Manual of Surveying Instructions for Survey of the Public Lands of the United States." In this will be found all general and many specific instructions regarding the system, only the general scheme of which is described below.

Subdivision work is carried on as follows: *First*, the establishment of an INITIAL POINT, a PRINCIPAL MERIDIAN, by astronomical observations, which is a true meridian through initial point; and a BASE-LINE, which is a true parallel of latitude through initial point (Fig 22).

These operations are performed in different localities as a basis for surveys in those regions. The principal meridian is a straight line and the base-line a curve, being at every point at right angles to the meridian through that point. The base-line is laid out by first running a straight line and measuring offsets from it to locate points on the parallel of latitude. Two methods are used for this, called the secant method and the tangent method, complete explanations of which, with tables, will be found in the manual issued by the General Land Office.

Second, the division of the area into tracts approx 24 miles square, by establishment of STANDARD PARALLELS, sometimes called CORRECTION LINES, which are true parallels of latitude extending east and west through 24-mile points on the principal meridian; and also establishment of GUIDE MERIDIANS, which are true meridians through 24-mile points on base-line and on standard parallels, and extending north to intersection of next standard parallel or base-line.

Since these guide meridians converge, the tracts will be 24 miles on their southern and less on their northern boundaries (Fig 22). The southerly end of these guide meridians, where they leave the standard parallel or base-line, are called STANDARD CORNERS, and points where they meet the next standard parallel to the north are called CLOSING CORNERS.

Third, the division of each 24-mile tract into TOWNSHIPS (Fig 22), each approximately 6 miles square, by establishment of MERIDIONAL LINES, or RANGE LINES, which are true

meridians through standard township corners, established at intervals of 6 miles on the base-line and on the standard parallels, and extending north to an intersection with the next standard parallel or base-line; also establishment of LATITUDINAL LINES, or TOWNSHIP LINES, joining township corners previously established at intervals of 6 miles on principal meridian, guide meridians, and range lines.

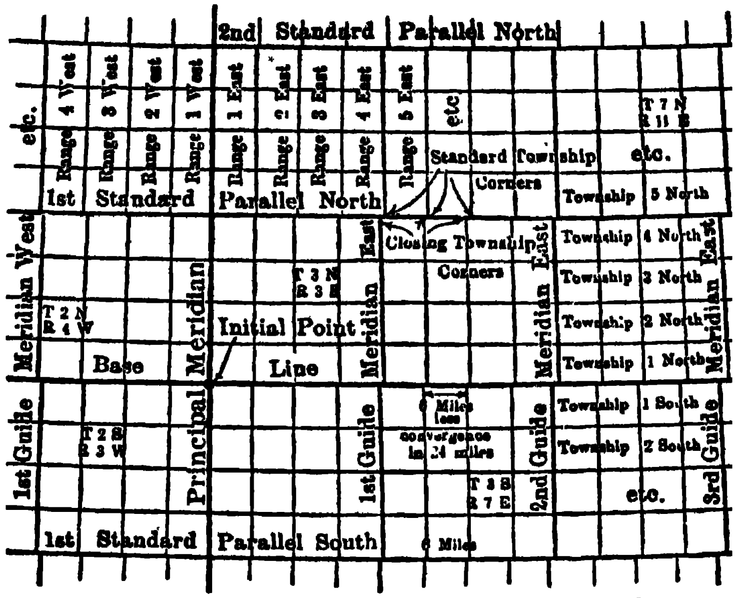

Fig 22. Subdivision of 24-mile Tract into Townships

Neglecting discrepancies in fieldwork, the E and W boundaries of all townships will be 6 miles in length, but the north and south will vary in length, being a max of 6 miles at standard parallel or base-line forming southern limit of a 24-mile tract, and a minimum at that forming its northern limit. Townships are designated by numbering them in order north and south from base-line and east and west from principal meridian. Any series of contiguous townships or sections situated north and south of each other constitute a RANGE, while such a series in an east and west direction constitutes a TIER. Thus "Township 2 North, range 4 west of the sixth principal meridian" locates its position; usually abbreviated to "T 2 N, R 4 W, 6th P M" (Fig 22). The half-mile intervals on range lines are made full 40 chains for the entire 24 miles, except the most northerly half-mile, into which all excess or deficiency due to irregularities of measurement is thrown. Similarly, corners on E and W lines are so placed that excess or deficiency of measurements is thrown into the most westerly half-mile of each latitudinal township boundary.

Fourth, the division of each township into SECTIONS (Fig 23), each approx 1 mile square (640 acres), by establishing SECTION LINES, both meridional and latitudinal, parallel to and at intervals of 1 mile from eastern and southern boundaries of township. Sections in all townships are numbered.

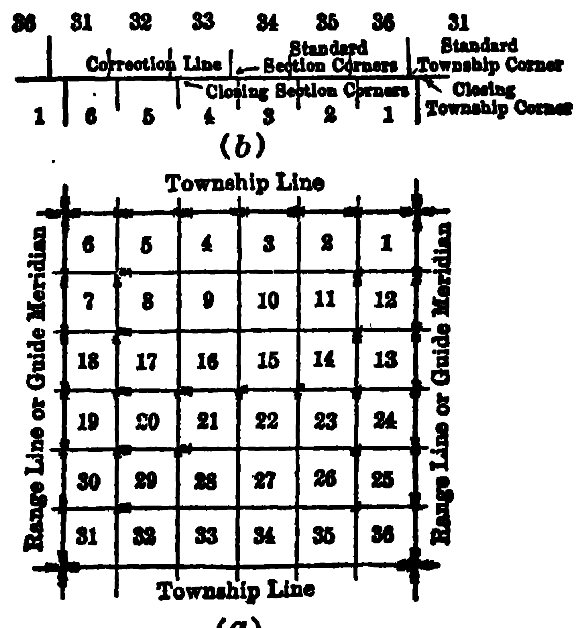

Fig 23. Subdivision of Township into Sections and Method of Marking Corners

In staking out section corners, surveyor sets up instrument at southeast corner of township, observes the meridian, and retraces range line northward 1 mile, and township line westward the same distance. This is for a comparison of his retracement of those lines with the record field notes, to ascertain possible discrepancies to be dealt with. The retracements are extended where necessary. Then from southwest corner of section 36 he runs north on a line parallel with east boundary of township, setting a quarter-section corner at 40 chains and a section corner at 80 chains. Then from section corner just set he runs east on a random line, parallel to S boundary of section, setting a

temporary quarter-section corner at 40 chains. On intersecting the range line he notes the falling of his random line and also distance it overruns or falls short of length of S boundary of section. If the falling is not more than 50 links (33 ft, representing angular deviation of 21 min), and if distance overruns or falls short of length of southern boundary of section 36 by not more than same amount, a return course which will join the two section corners is calculated; this new line is then run toward the W, the permanent quarter-section corner being set at its middle point.

From the section corner just regained, the survey is now continued north between sections 25 and 26, parallel with E boundary of township, the direction being changed slightly to E or W according to whether latitudinal section line just completed exceeded or fell short of desired length. At 40 and 80 chains on this line quarter-section and section corners, respectively, are set: from the section corner a random line is run across to the range line, a return course being calculated and run as before. This process is continued until 5 of the 6 sections in the series are enclosed. Then, if N boundary of township is not a correction line, from the section corner last established a random is run N to township boundary, and from data thus secured a true line is calculated and run from section corner on township line back to initial corner. If N boundary of township is a correction line, however, the point at which the random intersects this boundary is established as a CLOSING CORNER, and its distance from nearest STANDARD CORNER is measured and recorded. In either case the permanent quarter-section corner is established at 40 chains north of the initial corner, the excess or deficiency being thrown into the most northerly half-mile.

In a similar manner the succeeding ranges of sections are enclosed, randoms being run eastward to section corners previously established, and true lines corrected back. But, from the fifth series of section corners thus established, random lines are projected to westward also, and are closed on corresponding section corners in range line forming western boundary of township. In correcting these lines back, however, the permanent quarter-section corners are established at points 40 chains from initial corners of the randoms, thereby throwing all fractional measurements into the most westerly half-miles. This method of subdivision is shown in Fig 23.

Subdivision of sections. When public lands were parceled out to settlers, the quarter-section was usually the unit area granted as a "homestead"; this required establishment of the quarter-section corner at center of a section. Also, the subsequent division of original "quarters" into "eighties," "forties," or other minor subdivisions, has necessitated location of numerous corners in addition to those originally established by government.

The interior quarter-section corner of a section is always located at the intersection of straight lines joining quarter corners on opposite sides of section. This holds wherever the section may be located within township; that is, it applies to those in N and W sides as well as to other sections of township. A modification of this method is required in W and N sides of a township; in case of sections lying in W range the meridional line from the quarter corner on N or S boundary of section is run parallel to E line of section; similarly, in case of sections lying in N tier, the latitudinal interior line initiated at the quarter corner on E or W boundary is run parallel to S line of section. The reasons for this procedure in these special cases are apparent from consideration of methods previously described for establishment of original quarter-section corners in such sections. For subdivisions smaller than quarter-sections the same general methods are employed. Normally, no modification of this method is required, except that in a fractional section, where no opposite corresponding quarter-section corner has been or can be established, the center line must be run from the proper quarter-section corner as nearly in a cardinal direction as due parallelism with the section boundaries will permit.

In current practice, beginning in 1908, all regular corners are marked with an iron post having a bronze cap, and filled with concrete; older corners are marked by various kinds of monument, depending upon character and importance of the corner to be perpetuated, the soil, and materials available. Stone or wooden posts are common. Where these materials are not available, a mound of earth may be raised over the corner, a small marked stone, a charred stake, or a quart of charcoal, being deposited beneath it. Occasionally, in timber lands, the corner falls on a spot occupied by a tree, which may stand as the monument.

Stones or posts are marked with horiz notches, to indicate their respective positions in the township. Section corners on range lines, including under this term principal and guide meridians, are marked with notches on their north and south faces, the number of notches being equal to number of miles to next adjacent township corner north or south. Similarly, section corners on township lines, including base-lines and standard parallels, are notched on their east and west faces. Township corners, being located on both range and township lines, are marked with six notches on each of the four sides. Besides being notched, corners on correction lines are marked SC on their northern or CC on their southern faces, depending upon whether they are standard or closing corners. Section corners in interior of a township are given notches on their east and south faces, corresponding to number of miles to east and south boundaries of township. Thus, the corner common to sections 20, 21, 28, and 29 would have two notches on south and four on east face, as indicated in Fig 21. Quarter-section corners are marked with the fraction "1/4," those on meridional lines on their west and those on latitudinal lines on their north faces. Wherever possible, a monument set at a corner is witnessed by several nearby objects which may easily be found, are not readily moved or obliterated, and are comparatively permanent. In timbered country the stone or post is usually witnessed by "bearing trees," situated near the corner.

Field notes. Duplicate copies of all field notes and plats of public land surveys are on file in the General Land Office. The notes contain complete descriptions of all corner monuments, and give in narrative form complete data of alinement and topographic features crossed or near the lines. Original notes and plats of these surveys in the following states have been transferred to the state authorities, from whom may be obtained copies on request.

Alabama, Secretary of State, Montgomery; Arkansas, Commissioner of State Lands, Little

Rock; Florida, Commissioner of Agriculture, Tallahassee; Illinois, Auditor of State, Springfield; Indiana, Auditor of State, Indianapolis; Iowa, Secretary of State, Des Moines; Kansas, Auditor of State and Register of State Lands, Topeka; Louisiana, Register of State Lands, Baton Rouge; Michigan, Director, Dept of Conservation, Lansing; Minnesota, Secretary of State, St. Paul; Mississippi, Commissioner of State Lands, Jackson; Missouri, Secretary of State, Jefferson City; Nebraska, Commissioner of Public Lands and Buildings, Lincoln; North Dakota, State Engineer, Bismarck; Ohio, Auditor of State, Columbus; Oklahoma, Commissioner of General Land Office, Wash, D C; South Dakota, Commissioner of School and Public Lands, Pierre; Wisconsin, Commissioners of Public Lands, Madison.

In many of the states, copies of the field notes and plats have been secured for the various counties, and are kept for reference and inspection in office of County Register of Deeds, County Surveyor, or other official.

Photo-lithographic copies of township plats and field notes of surveys of area covered by public land surveys in above states may also be obtained from General Land Office, Washington, at nominal prices. In other public-land states, copies of records can be procured on application to Surveyor General at State Capitol, except in California and Oregon, whose Surveyors General are at San Francisco and Portland respectively.

Surveys are continuing in the other public-land states, where the records may be examined in the U S public survey offices, as follows: Arizona, Phoenix; California, Glendale; Colorado, Supervisor of Surveys, Denver; Idaho, Boise; Montana, Helena; Nevada, Reno; New Mexico, Santa Fé; Oregon, Portland; Utah, Salt Lake City; Washington, Olympia; Wyoming, Cheyenne; Territory of Alaska, Juneau.

Relocating lost corners. An act of Congress specifically provides that corners actually located in the field shall be established as proper corners of sections or quarter-sections which they were intended to designate, irrespective of whether or not they were properly located in the first place. A further provision is that " the boundary lines actually run and marked " (in the field) " shall be established as the proper boundary lines of the sections, or subdivisions for which they were intended, and the length of such lines as returned by . . . the surveyors aforesaid shall be held and considered as the true length thereof." The principles upon which present practice in relocating corners of original surveys are based are given in " Circular on the Restoration of Lost or Obliterated Corners and Subdivisions of Sections," published by General Land Office, Washington.

17. SPECIAL PROBLEMS

To straighten a crooked boundary. The new division line AB (Fig 24) is to be a straight line, so laid out as to make no change in areas of lots P and Q. First run trial line AC. Measure necessary offsets to the crooked line and compute areas X, Y, and Z lying between AC and the crooked boundary (Art 12). The condition desired is that $X + Z = Y$. As difference between $X + Z$ and Y is the amount which the trial line has taken from one parcel and given to the other, the whole difference must be returned to proper lot.

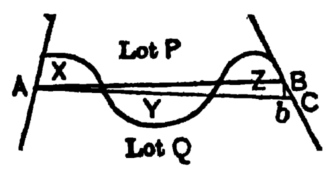

Fig 24. To Straighten a Crooked Boundary

For example, suppose Lot P originally contained 20 000 and Q, 30 000, and it is found that area $X = 500$, $Y = 800$, and $Z = 600$. The trial line has, therefore, made lot $P = 20\,000 + 500 - 800 + 600 = 20\,300$, and lot $Q = 30\,000 - 500 + 800 - 600 = 29\,700$. Lot P has been made 300 too large and lot Q 300 too small, which is equal to $X + Y - Z$. This 300 must be taken from lot P and returned to lot Q by running line AB in such manner that the area of triangle $ACB = 300$. Multiply area of triangle ACB by 2 and divide product by AC. This gives perpendicular bB, from which, together with angle at C, compute BC and set point B. Compute AB and its bearing.

Supplying closing side of a traverse. If the latitudes and departures of the several courses of a traverse which does not close be computed, the algebraic sum of latitudes gives latitude of closing side, and the algebraic sum of departures gives departure of closing side (Art 12). The tangent of its bearing is its departure divided by its latitude; length of closing side is the latitude divided by *cos* of bearing, or departure divided by *sin* of bearing.

To cut off a given area by a straight line MX from a given point M on one of the sides of a field. Plot the field and known point; draw a trial line ML from this point to a corner on other side of field, so as to lay off approximately the required area. The purpose of the plot is simply to determine to which corner to draw this trial line. As lengths and bearings of all lines except ML are known, its length and bearing can be computed as above, and the area of portion cut off by ML computed. The difference between this area and required area is a triangle MLX, of which trial line ML is the base; the altitude Xa can be computed by dividing twice the area by ML. The distance along side LX can then be computed, since it is the hypotenuse of right triangle LaX. The length and bearing of required closing side MX, which is hypotenuse of right triangle MaX, can now be computed, and the total intercepted area calculated, as a check.

To find area cut off by a line in given direction from given point. Let the cut-off line be AM. This problem may be solved by drawing a line from A in the traverse to corner E, which lies nearest other extremity M of cut-off line. The area of portion of traverse cut off by AE is then computed (Art 12) and to this area is added, or from it is subtracted, the area of triangle AEM.

Setting batter-boards for a foundation. For a brick or stone foundation the lines to be defined are the outside lines of building, and the elevation desired is usually the top of first floor. For a wooden building, the line usually given is outside line of brick or stone underpinning, and the elevation given is top of this underpinning, on which sill of structure is to rest. Sometimes the outside line of sill is desired, instead of outside line of underpinning; there should be a definite understanding in regard to these points before work of staking out is begun.

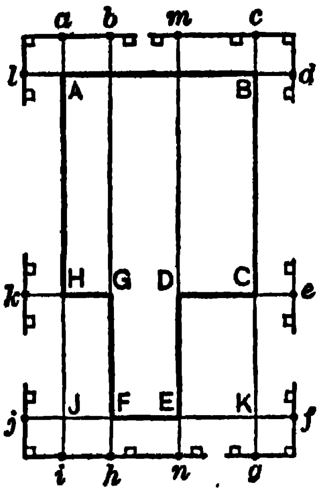

Fig 25. Setting Batter-boards for a Foundation

First stake out accurately the location of building by temporary stakes at all corners, as in Fig 25, at A, B, C, D, E, F, G, and H. Set stakes at J and K also, so that entire work can be checked by measuring diagonals AK and BJ, AC and BH. These checks should always be applied where possible. The posts for batter-boards are next driven into the ground 3 or 4 ft outside line of cellar, so that they will not be disturbed when cellar walls are being constructed. On the posts, which are usually 2 by 4 in, 1-in boards are nailed. The boards are so set by surveyor that their upper edges are level with top of underpinning, or with whatever other part of building of which the grades are required. After batter-boards are all in place, they should be checked roughly by sighting across them; all should appear at same level. Sometimes, however, on account of slope of ground, some of them have to be set a definite number of feet above or below grade.

The lines are then marked by nails driven in upper edges of batter-boards. The transit is set up on one of corner stakes of house, at B for example, and a sight is taken on K. This line is then marked on the farther batter-board (at g) and on the one near transit (at c). A sight is next taken along BA, this line is produced both ways, and nails are set on batter-boards at l and d. All lines are similarly marked on the batters.

Staking out lines and grades for streets, sewers, and drains. Street lines are usually marked by monuments, but best practice requires that accurate offsets to the underpinning of buildings along the line shall be measured and recorded, so that if monuments be disturbed, by building operations or road building, they can be exactly replaced. These monuments are usually stone bounds 3 or 4 ft long and 4 to 8 in square on top. The stone should be long enough to reach below frost. A drill-hole near center of top marks the point; this drill-hole may be filled with lead and a small copper tack driven into it, or a copper plug may be inserted and exact point marked with a center-punch.

To set a stone bound to take place of a stake marking a corner, first drive four temporary stakes in such manner that cords stretched between tacks in opposite stakes will intersect over the tack in corner stake. Then remove corner stake and dig the hole for the monument, going no deeper than is necessary to set it on firm earth. Then set monument plumb in hole, and fill around it with care, so that mark in top coincides with intersection of strings. Where a very substantial bound is required, or where ground is too soft to furnish a secure foundation, a concrete footing may be put in and concrete filled around monument to within 1 ft of surface.

Grade stakes for sewers or drains are usually set parallel to line stakes and 3 to 6 ft to one side of them, their tops being a full number of feet above flow line of pipe. Some engineers prefer to drive a grade stake to a good bearing and then mark on side of it a line which shall be a full number of feet above flow line. When ground is hard, it is good practice to drive heavy spikes flush with ground and to take the elevation of their tops. Record is made of amount by which the several spikes are above or below grade.

A target rod is especially useful, although a self-reading rod is satisfactory, for "shooting in a grade," which is done as follows: If grade from sta 0 to 5 be straight and if stakes have been set at 0 and 5, the instrument is set up just to one side of sta 0, the rod is held on that grade stake, and target clamped at height of telescope; the rod is then held on stake at sta 5, the telescope is inclined to sight target and clamped. The line of sight is now parallel to grade line, and any intermediate stake may be set by driving it until target is bisected by horizontal hair of telescope when rod is held on stake.

After grade stakes are set, batter-boards are placed to span the trench, the upper edge of every board being a full number of feet above flow line of pipe; they are so set by

leveling from grade stakes by a carpenter's level. The center line is marked on batter-boards by measuring over from line stakes, which have been set on, say, a 4-ft offset line. In absence of engineer, all this work can be done by foreman. It is well, therefore, to set grade stakes and offset-line stakes even though engineer be nearby, for the foreman will then be able to check lines and grades at any time.

To lay off any angle with a tape only, first lay off a perpendicular, say 100 ft from vertex A of angle, by the 3–4–5 method, using any of following combinations of dimensions for the two legs and hypotenuse of right triangle CAB: (3–4–5), (6–8–10), (9–12–15), (12–16–20), (15–20–25), (18–24–30), (21–28–35), etc. On the perpendicular, measure distance BC equal to 100 times natural tangent of angle; angle CAB is the required angle.

LEVELING AND CONTOURS

18. PROFILE LEVELING

Profile leveling consists in determining elev of ground at enough points on a line so that its profile can be plotted and computations made from it. The line is first marked off in 100-ft stations, or at shorter intervals if desired. The level is set up and a reading, called a backsight (B S or + S), is taken on a leveling rod held on a BENCH-MARK (B M), which is a point of which the elev above mean sea level or any other assumed datum plane has been accurately determined. The B S is added to elev of B M, giving height of instrument (H I), or elev of line of sight above datum plane. Rod-readings, called foresights (F S or − S), are then read on as many station points on line as can conveniently be seen from instrument, and elevations of these points are found by subtracting F S readings from H I. Rod-readings are taken at all distinct changes in slope which occur on the line, whether at full-station points or not; such intermediate points are located by tape (sometimes by pacing) and recorded as plus stations, as shown below.

Example of Profile Field Notes

Profile of proposed drain line from Shaft A to Pine Brook. Aug. 18, 1913 { Haddock, level. Kelley, rod.

Sta.	$+S$	H I	$-S$	Surface elev	B M & T P elev	Description
B M_1	1.76	963.28	.....	.....	961.52	Top of N E cor stone foundation hoisting engine ho. See Book 1, p 2
0	....	......	4 6	958.7	......	....
1	....	......	5.2	958.1	......	....
2	....	......	5.5	957.8	......	....
3	....	......	5.8	957.5	......	....
3+36	....	......	4 8	958.5	......	....
4	....	......	1.1	962.2	......	....
T P	3.21	965.51	0.98	.....	962.30	Top of boulder 8 ft E 4+30
4+77	....	......	0.5	965.0	......	....
5	....	......	1.7	963.8	......	....
6	....	......	4.9	960.0	......	....
6+75	....	......	6.2	959.3	......	W bank Pine Brook
6+92	....	......	13.7	951.8	......	Channel of Pine Brook
.....	....	......	8.9	956.6	......	Approx high water mark
B M_2	....	......	7.66	.....	957.85	Top N. W. cor concrete step, storehouse No 4. See Book 1, p 2. Elev 957 84

When necessary to move level to a new position along line, a TURNING POINT (T P) is selected and its elevation determined, to be used at the next set-up of level for computing its new H I. When new H I is found, by a backsight, F S readings are taken at points ahead on the line, after which the elevation of a new T P is established, and so on. The turning point may be a regular station, or any other solid point.

Usually B M's are established before profile levels are run, in which case readings should be taken on those B M's which are passed during profile leveling, and all such B M's should if possible be used as T P's. If B M's have been previously established, the closing check is made on the B M nearest end of line of profile levels, as indicated in above notes. Sometimes the line of levels is made to form a closed circuit by returning to original

B M. By comparing its final elevation with that used at beginning of the work, the error of closure is ascertained as a check on the T P readings, although a mistake may still occur in an individual reading at a station point.

Bench-marks should be chosen with regard to permanence, and should be points readily described and found. The rod readings on B M's and T P's should usually be taken to one more decimal place than those read for profile. To eliminate errors of adjustment, the distances between level and rod, when held on a T P or B M, should be about equal for backsight and foresight. The proper length of sight will depend upon distance at which rod can be read distinctly, and upon the precision required. Generally, sights should not exceed 300 ft where elevations are required to nearest 0.01 ft, and even at a much shorter distance "boiling" of the air may prevent such precision.

Calculation of level notes may be checked by computing the sum of the backsights and of foresights on the first and last bench-mark and on intermediate turning points; the difference of these sums should check the difference in elevation between first and final bench-marks. It is well to apply this check at bottom of every page of level notes. If station points have been marked in advance of the level party, two experienced men can run a line of profile levels over country smooth enough to permit a set-up every 600 ft, at rate of about 1 mile per hr, or say 10 set-ups per hr.

Double rodded lines are frequently adopted when it is impracticable to complete a circuit, or for a profile through new country, where no B M's have been established. Instead of foresighting on a single T P, readings are taken on two different T P's, near together and varying in elevation by at least 1 ft. From next set-up, a backsight is taken on each T P and two H I's are computed, which should agree within whatever limit of error is allowable for class of work in hand. This is an excellent method; it requires little more time than to run a line of levels by the ordinary method described above, and gives a valuable check at every new set-up, by detecting mistakes as soon as made.

Precise levels for establishing B M's are sometimes run by a precise level (Art 5), or by taking the following special precautions with ordinary level. The observations are made in such manner as to eliminate the recognized common errors in leveling, which are: (*a*) settling of level on soft ground; (*b*) unequal expansion and contraction of parts of instrument, due to changes of temperature; (*c*) variable refraction of air near ground; (*d*) unequal length of backsight and foresight; (*e*) selecting unstable turning points; (*f*) rod not held plumb; (*g*) bubble not in center of tube at instant of reading.

Errors due to settling of tripod are eliminated by making two readings on backsight and two on foresight at each set-up, in following manner; first, B S is read, then F S; F S is then read again, and lastly B S once more; two rodmen are employed. Correct elevation of T P is obtained from average B S and F S readings. Errors due to changes of temp may be partly prevented by shortening time between reading of B S and F S; in all cases, the instrument should be shielded from sun's rays, and from wind, by an umbrella or special shield. Rapidity is of great advantage in eliminating errors due to settling of tripod. Errors due to refraction of air may be partly avoided by using long-legged tripods, setting the level high above ground, and by making observations in the middle of the day, rather than in early morning or late afternoon. Backsights and foresights are kept nearly equal in length by reading these distances with the stadia hairs; notes should be so kept that they will show at a glance whether the sums of F S and B S distances are equal or not. A foot-plate or wooden pin may be driven into the ground to use for a T P, when a satisfactory point can not be found at proper distance from instrument. To aid in holding rod plumb various types of spirit-level are used.

In work of precise character, allowance must be made for curvature of the earth and refraction of atmosphere. These corrections are usually combined, and amount to the following: for sights of 300 ft, combined correction is 0.002 ft; for 500 ft, 0.005 ft; for 1 000 ft, 0.020 ft. The correction is applied by subtracting it from any single rod-reading, but if rod be held equally distant from instrument on F S and B S, the effect of curvature and refraction is compensated.

Allowable errors in precise leveling of various surveys are as follows: U S Coast Survey, $4^{mm} \times \sqrt{\text{kilometers}}$; U S Lake Survey, $10^{mm} \times \sqrt{\text{kilometers}}$; Mississippi River Survey, $5^{mm} \times \sqrt{\text{kilometers}}$; U S Geological Survey, $0.017^{ft} \times \sqrt{\text{miles}}$. Results actually reached fall well within these limits. A high grade of leveling has been done on the Barge Canal Survey of New York with an ordinary Y-level of good construction; allowable error of closure was only $0.02^{ft} \times \sqrt{\text{miles}}$. An accuracy almost equal to that attained by a precise leveling instrument can be reached with ordinary Y-level or the dumpy level, but not so economically.

19. CROSS-SECTION LEVELING

Leveling for cross-sections is required for determining volume of material, whether excavated from a cutting or a borrow-pit, or removed by hydraulic mining, or occurring in a mine dump. The entire upper surface of volume is divided by transit and tape into 10- to 100-ft squares (or rectangles); elevations to nearest tenth are taken at all corners of squares, and at as many intermediate points as necessary, in case of irregular ground. Lines running in one direction are usually lettered, numbers being used for the other system of lines. Thus, point F, 8 lies at intersection of line F and line 8; and an intermediate point (not at a corner) is designated thus $(H + 6,\ 7 + 2)$, which means that it lies 6 ft from line H toward line I and 2 ft from line 7 toward line 8. By laying out and leveling over the same system of cross-section lines, before and after excavation, the differences between original and final elevations at corners give lengths of vertical edges of a series of vertical truncated rectangular prisms. Toward edges of a borrow-pit, for example, will probably occur several triangular and trapezoidal truncated prisms. If A = area of horiz section of truncated prism, h_1, h_2, h_3, h_4 = lengths of vert edges of prism, and V = vol, then:

For truncated triangular prism, $V = A \times (h_1 + h_2 + h_3) \div 3$.

For truncated rectangular prism, $V = A \times (h_1 + h_2 + h_3 + h_4) \div 4$.

When additional elevations are taken at intermediate points, a rectangular prism should be divided into triangular prisms; the additional measured height forms the vertical edge at one corner of each triangular prism. Where several rectangular prisms occur, having same area of cross-section, A, the computation of volume may be simplified thus: Volume of assembly of truncated rectangular prisms $= A(p_1 + 2\,p_2 + 3\,p_3 + 4\,p_4) \div 4$, in which p_1 = sum of heights common to one prism, p_2 = sum of heights common to two prisms, p_3 = sum of those common to three, p_4 = sum of those common to four.

If several intermediate elevations have been ascertained, it is well to compute volume from original surface down to some plane below final surface. Then make similar computations for volume between final surface and the same plane. The difference between these two volumes will give required volume.

Road cross-sections, as a basis for estimating quantity of earthwork in railroad or highway construction, are taken by a method entirely different from the above. From plan of proposed road its alinement is staked out, a profile is taken along center line (Art 18), and is subsequently plotted. On this profile the grade line is drawn, which corresponds either to sub-grade or to finished grade of road. Roads are usually first finished to sub-grade, which is below the completed surface by an amount equal to thickness of road covering, as pavement of a highway, or ballast of a railroad. The width of base of road and inclination of side slopes are known. For ordinary earth the slope is usually 1.5 ft horiz to 1 ft vert, called "a slope of 1.5 to 1."

For construction work the engineer sets grade stakes at every full station, or oftener, on center line, and slope stakes at both sides, where finished slope will intersect surface of ground. All three stakes are marked, stating amount of "cut" or "fill" to be made at center point, or, in case of slope stakes, stating the vertical distance from base of the road to surface of ground at these points. Cross-sections are taken not only at every full station, but at every distinct change in slope along center line, and also where surface on either side of center line demands an intermediate section properly to represent the volume included between cross-sections. Cross-sections are taken perpendicular to center line of road, and radially on curves.

To find the cut or fill at the center, set up the level and find the H I to nearest 0.01 ft, by sighting on nearest station or bench-mark of which the elev is known. From the profile obtain grade elev at given station. The H I minus grade elev gives ROD-READING FOR GRADE, which is computed to nearest 0.1 ft. Take a rod-reading on ground at center stake; the difference between rod-reading for grade and surface rod-reading gives cut or fill at that point. It is customary to record cuts as + and fills −. The surface elev is obtained by adding the cut to grade elev, or by subtracting, in case of fill. A grade stake on center line is marked with cut or fill, thus: "C 3.2" or "F 6.7."

Slope stakes are set at the points where side slopes meet surface. These stakes are also marked with the distance the ground lies above or below base of section; it is called the cut or fill at the side slope, but, strictly speaking, there is no cut or fill at slope stakes. The position of a slope stake is found by trial as follows: In the case of a cut, estimate from the center cut just determined, and from slope of surface, what the probable side cut will be. The distance from center stake to a point on side slope having this cut equals ($1/2$ base + cut × slope). Make this computation roughly, measure out this distance from center stake, and take a rod-reading at that point. The rod-reading for grade (distance from H I to base of section) minus this surface rod-reading gives cut at trial point. Compute the exact distance out from center stake to a point on side slope having this cut. If this computed distance equals measured distance from center to rod, the trial point was correct·

if not, a second trial must be made by holding rod on another point and repeating operation; the difference between measured and calculated distance aids in judging where to hold the rod for second trial. When correct point is found, at which the measured distance equals calculated distance, the stake is marked " C," followed by the vert distance this point lies above the base. The depth of cut, and distance from center to slope stakes, are entered in notes. The same process is repeated for slope stake on other side of center. Rod-readings are taken at intermediate points on the cross-section, if needed to define shape of surface; their positions are located by measuring distances from center stake, while the cuts at these points are the differences between rod-reading for grade and surface rod-readings. The field notes of a section of embankment having 30-ft base and slopes 1.5 to 1, which required 4 readings to represent its slope properly, would be entered thus: $\frac{21.3}{-4.2}$ $\frac{14.7}{-6.1}$ -3.2 $\frac{17.1}{-1.4}$.

The area of each cross-section may be computed by dividing it into triangles, or cross-sections may be plotted and their areas determined by planimeter (Art 6). By the end area method the volume between sections A_1 and $A_2 = \frac{1}{2}(A_1 + A_2) \times$ distance between sections.

Cross-sections for dams, canals, and other engineering structures are begun by making short profiles at right angles to line of structure, at intervals of 10 to 100 ft, depending upon conditions, and extending far enough on either side to include the possible location of structure. These cross-section lines, representing the surface, are plotted for each station, and on them are also plotted the cross-sections of proposed completed structure at the respective stations. This gives a complete record of conditions before work is started, and as it progresses a continuous graphic record of work may be kept from month to month by use of different colored lines on the various cross-sections.

Masonry volumes are computed either by methods of solid geometry or by the Prismoidal Formula: Vol $= L \div 6\,(A_1 + 4\,M + A_2)$, in which L is the length of the solid, A_1 is cross sec area at one end normal to length, A_2 is cross sec area of other end, and M is cross sec area midway between the two ends, which is seldom the aver of the end areas.

20. BAROMETRIC LEVELING

The barometer is based upon the principle that atmos pressure is a function of elev above sea level, a change of 1 in in mercury column corresponding to a change of about 900 ft in elev. Atmos pressure varies also with changes in temp and humidity, whence it is necessary, while measuring differences of altitude with a barometer, to determine these variations and make proper allowance for them. Both mercurial and aneroid barometers are used in surveying, but the latter, due to its compactness, is generally employed.

Barometric leveling is used for reconnoissance, or for determining contours for a small-scale map. With an aneroid, results as close as 10 ft may be obtained, but not without repeating observations and using instrument with great care. The error in elevations determined by barometer is approx constant, whence the percentage error is smaller for large differences in elevation than for small differences. The barometer does not give actual elevations, but the difference between two readings will be a function of difference in elev of the two points, provided atmospheric conditions have not so changed as to affect readings.

The aneroid barometer has two scales, the inner one corresponding to inches of mercury and the outer one to feet of altitude, the zero of the altitude scale being, in most instruments, at 31 in on mercury scale. The outer scale *should not be movable* with respect to inner scale, for the number of feet of altitude corresponding to 1 in of mercury varies in different parts of scale. Aneroids marked "compensated" are supposed to be so adjusted that changes in temp of the instrument will not affect readings. The instrument should be handled carefully, to avoid disturbing its delicate mechanism. When it is to be read, tap the case lightly to make sure that the instrument has adjusted itself to the changed pressure; it should then stand a few minutes to allow it to come to the true reading. It should not be heated by the sun nor by the body. It may be held in either vertical or horizontal position when being read, but as the readings in these two positions are different it should be held in same position at all stations. As accurate results may be obtained from small as from large aneroids.

The best method of determining elevations by barometer is to use two instruments, one being kept at a fixed station of known elev and read at regular intervals to indicate changes in atmos pressure, while the other is carried to the various points of which elevations are to be determined. The differences in elevation shown by the traveling barometer, corrected for differences indicated by fixed barometer, will be the true differences in elev. The time should be noted whenever the field barometer is read, in order to apply proper correction as indicated by the fixed barometer. Air temp should always be noted whenever barometer is read. If only one barometer be available, it should be read at initial station

and again at same station after completing the other observations, atmos changes during the interval being interpolated from difference between first and last readings.

For Determining Difference in Elevation by Barometer

Barom Inches	Feet	Barom Inches	Feet	Barom Inches	Feet	Barom Inches	Feet	Barom Inches	Feet
16.00	12 280	20.00	18 110	22.75	21 466	25.50	24 457	28.25	27 133
16.10	12 442	20.05	18 175	22.80	21 533	25.55	24 578	28.30	27 179
16.20	12 604	20.10	18 240	22.85	21 590	25.60	24 559	28.35	27 225
16.30	12 765	20.15	18 305	22.90	21 647	25.65	24 610	28.40	27 271
16.40	12 925	20.20	18 370	22.95	21 704	25.70	24 661	28.45	27 317
16.50	13 084	20.25	18 434	23.00	21 761	25.75	24 712	28.50	27 362
16.60	13 242	20.30	18 499	23.05	21 818	25.80	24 762	28.55	27 409
16.70	13 398	20.35	18 563	23.10	21 874	25.85	24 813	28.60	27 454
16.80	13 554	20.40	18 627	23.15	21 931	25.90	24 864	28.65	27 500
16.90	13 709	20.45	18 691	23.20	21 987	25.95	24 914	28.70	27 545
17.00	13 864	20.50	18 755	23.25	22 044	26.00	24 964	28.75	27 591
17.10	14 017	20.55	18 818	23.30	22 100	26.05	25 014	28.80	27 637
17.20	14 169	20.60	18 882	23.35	22 156	26.10	25 065	28.85	27 682
17.30	14 321	20.65	18 945	23.40	22 212	26.15	25 115	28.90	27 727
17.40	14 471	20.70	19 008	23.45	22 267	26.20	25 164	28.95	27 772
17.50	14 621	20.75	19 071	23.50	22 323	26.25	25 214	29.00	27 817
17.60	14 770	20.80	19 134	23.55	22 378	26.30	25 264	29.05	27 862
17.70	14 918	20.85	19 197	23.60	22 434	26.35	25 314	29.10	27 907
17.80	15 065	20.90	19 260	23.65	22 489	26.40	25 363	29.15	27 952
17.90	15 211	20.95	19 322	23.70	22 544	26.45	25 412	29.20	27 997
18.00	15 357	21.00	19 384	23.75	22 599	26.50	25 462	29.25	28 041
18.10	15 502	21.05	19 446	23.80	22 654	26.55	25 511	29.30	28 086
18.20	15 646	21.10	19 508	23.85	22 709	26.60	25 560	29.35	28 131
18.30	15 789	21.15	19 570	23.90	22 764	26.65	25 609	29.40	28 175
18.40	15 931	21.20	19 632	23.95	22 818	26.70	25 658	29.45	28 220
18.50	16 073	21.25	19 694	24.00	22 873	26.75	25 707	29.50	28 264
18.55	16 143	21.30	19 755	24.05	22 927	26.80	25 755	29.55	28 308
18.60	16 214	21.35	19 816	24.10	22 982	26.85	25 805	29.60	28 352
18.65	16 284	21.40	19 877	24.15	23 036	26.90	25 853	29.65	28 396
18.70	16 354	21.45	19 938	24.20	23 090	26.95	25 902	29.70	28 440
18.75	16 423	21.50	19 999	24.25	23 144	27.00	25 950	29.75	28 484
18.80	16 493	21.55	20 060	24.30	23 198	27.05	25 999	29.80	28 528
18.85	16 562	21.60	20 120	24.35	23 251	27.10	26 047	29.85	28 572
18.90	16 632	21.65	20 181	24.40	23 305	27.15	26 095	29.90	28 616
18.95	16 701	21.70	20 241	24.45	23 358	27.20	26 143	29.95	28 659
19.00	16 769	21.75	20 301	24.50	23 412	27.25	26 191	30.00	28 703
19.05	16 838	21.80	20 361	24.55	23 465	27.30	26 239	30.05	28 746
19.10	16 907	21.85	20 421	24.60	23 518	27.35	26 287	30.10	28 790
19.15	16 975	21.90	20 481	24.65	23 571	27.40	26 334	30.15	28 833
19.20	17 043	21.95	20 540	24.70	23 624	27.45	26 382	30.20	28 877
19.25	17 111	22.00	20 600	24.75	23 677	27.50	26 430	30.25	28 920
19.30	17 179	22.05	20 659	24.80	23 730	27.55	26 477	30.30	28 963
19.35	17 246	22.10	20 718	24.85	23 782	27.60	26 524	30.35	29 006
19.40	17 314	22.15	20 777	24.90	23 835	27.65	26 572	30.40	29 049
19.45	17 381	22.20	20 836	24.95	23 887	27.70	26 619	30.45	29 092
19.50	17 448	22.25	20 894	25.00	23 940	27.75	26 666	30.50	29 135
19.55	17 516	22.30	20 954	25.05	23 992	27.80	26 713	30.55	29 178
19.60	17 582	22.35	21 012	25.10	24 044	27.85	26 760	30.60	29 220
19.65	17 648	22.40	21 071	25.15	24 096	27.90	26 807	30.65	29 263
19.70	17 715	22.45	21 129	25.20	24 148	27.95	26 854	30.70	29 306
19.75	17 781	22.50	21 187	25.25	24 199	28.00	26 900	30.75	29 348
19.80	17 847	22.55	21 245	25.30	24 251	28.05	26 947	30.80	29 391
19.85	17 913	22.60	21 303	25.35	24 303	28.10	26 994	30.85	29 433
19.90	17 979	22.65	21 360	25.40	24 354	28.15	27 040	30.90	29 475
19.95	18 044	22.70	21 418	25.45	24 406	28.20	27 086	30.95	29 518

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Calculating the altitude. The Laplace formula for calculating difference in elevation (neglecting small corrections) is:

$$D = 60\,158.6\,(\log h - \log H)\,\{1 + (t_a' + t_a - 64°) \div 900\},$$

where D = difference in elev in feet; h = height of mercury at lower station in inches; H = height of mercury at upper station in inches; t_a' and t_a, the observed air temp, Fah. The temp correction is the only one necessary to apply to the aneroid; it is often large. In case a mercurial barometer

is used, H is height of mercury at upper station reduced to temp of mercury at lower station by the equation:

$$H = h' \{1 + 0.00008967 (t_m - t_m')\},$$

where t_m = temp of mercury at lower station and t_m' = temp of mercury at upper station. Table on preceding page, condensed from that of Guyot, is based on the Laplace formula; it gives the values of log height × 60 158.6.

Example, using one aneroid to find difference in elev between Neponset River and Blue Hill:

Station	Time	Inches, barom	Air temp
Neponset River	1.00 p m	30.395	53° F
Blue Hill	1.45 "	29.730	47
Neponset River	2.25 "	30.390	42

Time going up = 45 min; total time = 85 min. Difference in readings at lower station = 0.005. Probable reading at Neponset River at 1.45 p m = 30.395 − (45 ÷ 85 × 0.005) = 30.392 in.

From barometric table 30.395 in = 29 042 ft
29.730 " = 28 466
Difference = 576 ft
Temp corr = 576 [{1/2 (53 + 42) + 47} − 64] ÷ 900 = + 20
Difference in elev = 596 ft

Boiling-point thermometer. Great differences in elev may be determined roughly by means of the boiling-point thermometer. Such measurements depend upon the fact that the boiling point of water diminishes as atmos pressure diminishes and therefore the temp of boiling point is a function of the height of the instrument. The observation consists in noting temp of boiling water by a thermometer placed in a boiler made especially for the purpose. The approx height above sea level may then be found by referring to the following table.

Boiling point (Fah)	Alt in feet above sea level	Boiling point (Fah)	Alt in feet above sea level
190	11 720	208	2 050
195	8 950	209	1 545
200	6 250	210	1 020
202	5 185	211	510
204	4 130	212	0
206	3 085	213	−505

This table is taken from Wilson's *Topographic Surveying.*

21. CONTOURS

A contour line is the intersection of a level surface with surface of the ground. The shore line of a lake is a contour, and if lake level were raised 1 ft the new shore line would be of different shape and would form a contour line of 1 ft greater elevation. Contours, therefore, connecting points of same elevation, can be used to represent the slope of ground, and provide a means for reading directly from a map the approximate elevation of any point. Contours are usually located a full number of feet above the datum and at regular intervals, say, every 5, 10, or 50 ft; the number marked on contour is its elevation above datum.

Characteristics of contours are: (*a*) All points on any contour have same elevation. (*b*) Every contour closes on itself, either within or beyond limits of map. (*c*) A contour which closes within limits of map encloses either a summit or a depression. In depressions will usually be found a pond or a lake; but where there is no water the contours are usually marked in some way to indicate a depression. (*d*) Contours can never cross one another, except where there is an overhanging cliff, in which case there must be two intersections. (*e*) On a uniform slope contours are spaced equally. (*f*) On a plane surface they are straight and parallel. (*g*) In crossing a valley, contours run up the valley on one side and, turning at the stream, run back on other side. Since contours are always at right angles to the lines of steepest slope they will generally be at right angles to the thread of stream at point of crossing. (*h*) Contours cross ridge lines (divides) at right angles.

If a line be drawn across a contour map, the profile of that line may be constructed, since the elevations of points at which contours are cut by the line are known, and horizontal distances between these points can be scaled or projected from the map. Conversely, if profiles of a sufficient number of lines are given, it is possible to plot these lines on a map, mark the elevations, and sketch the contours.

In locating contours it should be borne in mind that they can be interpolated on uniform slopes and that the data necessary to obtain in the field are therefore elevations of characteristic points on profiles, such as tops of knolls, low places in valleys, profiles of ridges and of streams. Contours are usually farther apart at top and bottom of a natural eroded slope (where no wave action is present) than in middle. Where contour interval is small, and much detail is desired, the ground may be cross-sectioned (Art 19) and from elevations thus determined contours may be sketched; or contours may be actually traced out and located in the field by stadia; in this case the rodman moves up or down slope until a level rod-reading shows that the foot of his rod is on a contour. The position of rod is then located by distance and azimuth (Art 23).

A hand level (Art 5) is often used to determine elevation of additional points, taking elevations of points on transit line as B M's, as follows: First measure distance from ground to levelman's eye, which may be, say, 5.2 ft. Then, from a reading of the rod held on a point of known elevation the height of levelman's eye is determined. The leveler then directs rodman uphill or down until rod-reading is such as will correspond to that computed for a contour; the point thus found is located by tape or by pacing. If contours are being located on a side-hill, the levelman then moves uphill past the located point, on which rodman stands, until he reads the contour interval plus 5.2 on the rod; he is now standing on a contour, which is then located by taping. The levelman remains on this point, while rodman travels uphill until levelman reads on rod 5.2 minus the contour interval; rod is now resting on the next higher contour. In going downhill, the rod is held on a contour while levelman backs down the hill until he reads 5.2 minus the contour interval, when he will be standing on contour next below rodman; the rodman then passes levelman and backs down the hill until the rod-reading is 5.2 plus the interval, which determines next lower contour. Hand-leveling is not accurate enough to be used for more than about 400 ft away from a B M. If several profiles be run along characteristic lines, to be plotted on map, and then a few necessary side shots be located by hand level, contours can be sketched. The best method of locating contours is by stadia or plane-table methods (Art 22 to 25).

TOPOGRAPHIC, AERIAL, MINERAL AND RAILROAD SURVEYING

22. STADIA METHOD

The stadia method measures distances by observing, through telescope of a surveying instrument, the space on a graduated rod intercepted between two horizontal hairs, called STADIA HAIRS, which are spaced equal distances above and below the middle horizontal cross-hair. When rod is held at different distances from instrument, proportional spaces on the rod will be included between the stadia hairs. This is a rapid method of measuring distances while filling in details of topographic and hydrographic surveys. It has the great advantages that the intervening country does not have to be traveled, that inaccessible distances, as across water surfaces, can be measured, that errors of measurement are compensating rather than cumulative, while its accuracy is sufficient for many kinds of work, even for determination of area in some cases since an accuracy of one part in 500 may be attained with the stadia. STADIA RODS are described in Art 1.

Fundamental principle of the stadia is the geometric theorem that in similar triangles homologous sides are proportional. The common vertex of the triangles, in this case, is not the center of the transit, but a point in front of telescope objective, at a distance equal to F, the focal length of objective. Hence the distance from center of instrument to rod, when the sight is horiz and the rod vert $= (F \div i)s + (F + c)$, where i is distance between upper and lower stadia hairs, s is the intercepted space on the rod, and c the distance from center of instrument to objective. Evidently $(F + c)$ is practically a constant for any given instrument (varied only by focussing the telescope); for ordinary purposes it is taken as 1 ft for transits and 2 ft for plane-table alidades. The other function $(F \div i)\ s$ is a variable, but as it is customary to space stadia hairs so that $F \div i = 100$, the equation for transit instruments reduces to: *distance* $= 100\ s + 1$. Every 0.01 ft on rod therefore corresponds to a distance of 1 ft.

Inclined sights. It is necessary always to hold rod vertically when readings are being taken. When line of sight is inclined, the distance intercepted on a vert rod represents a distance greater even than the inclined distance to rod, while inclined distance is greater also than horiz distance. All inclined readings must therefore be reduced to the

horiz, and if difference in elev between instrument station and rod station is desired the vert distance also must be computed from the inclined sight.

By trigonometry, making one slight assumption, it may be shown that in Fig 25a:

$$\textit{Vertical Distance} = (F + i)\, s \times {}^1/_2 \sin 2\alpha + (F + c) \sin \alpha.$$

$$\textit{Horizontal Distance} = (F + i)\, s \times \cos^2 \alpha + (F + c) \cos \alpha.$$

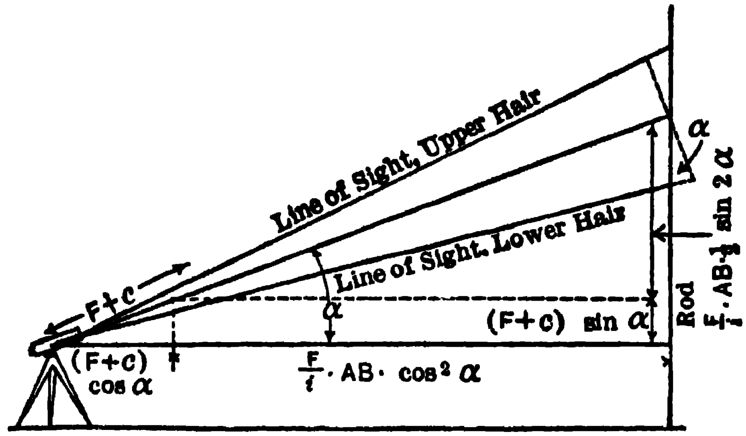


Fig 25a. Diagram of Stadia Reading

For the ordinary conditions (when $F \div i = 100$, and $F + c = 1$), and making the approximations that $\sin \alpha = {}^1/_2 \sin 2\alpha$, and that $\cos \alpha = \cos^2 \alpha$, in last terms of above formulas, which assumptions are nearly true for small angles,

$$\textit{Vertical Distance} = (100 \times \textit{rod interval} + 1)\ {}^1/_2 \sin 2\alpha.$$

$$\textit{Horizontal Distance} = (100 \times \textit{rod interval} + 1) \cos^2 \alpha.$$

These are the formulas used for ordinary stadia work. The reduction of field-notes can be best accomplished, however, by use of tables, diagrams, or stadia slide-rules, all of which are based upon these formulas.

23. STADIA FIELDWORK

Points are located by (*a*) azimuth, (*b*) distance, and (*c*) angle of elev or depression. If elevations are not required, the vert angle is read only to nearest 10 min, since that is close enough for determining horiz distance; but if elevations are required, vert angles should be read to nearest minute, sometimes to half-minute, and vernier index correction noted for each important vert angle. Distance and azimuth angle are read with whatever refinement is necessary for work in hand; as a rule, distances are recorded to nearest foot, and azimuths to nearest minute, although for side-shots, which are to be plotted by protractor, the azimuth to nearest 5 min is close enough. Traverse lines which control the survey are measured by tape, or by stadia, as accuracy of survey may demand. Some surveyors prefer to run traverse lines by transit and tape; then to take a level over traverse to determine elev of certain transit stations, boulders, or other bench-marks, thus establishing an accurate skeleton for a stadia survey, which follows over same ground for filling in details. For large-scale maps it may be advisable to tape the courses of control survey, but for small-scale maps the stadia method is sufficiently accurate. In latter case, distances should be read on both foresight and backsight.

Azimuths. In starting a survey, if the true azimuth of any course be known, all azimuths of survey may refer to the true meridian; but if no meridian has been established, as is frequently the case, the azimuths may be referred to magnetic meridian, or to any other arbitrary direction. Azimuth angles are most commonly read from the south clockwise for 360°. The horiz vernier is set on 0°, the telescope turned to point toward magnetic S, and lower clamp tightened. The upper clamp is next loosened and telescope sighted on next station; the horiz arc is then read clockwise to determine azimuth of first course, referred to the magnetic meridian through first station. The azimuth to any other point is obtained by sighting telescope on it and reading the vernier, the lower motion remaining clamped. On moving to the next instrument station, the telescope is oriented by inverting and backsighting, as explained in Art 11. Magnetic bearings of courses may be noted as a rough check on azimuths.

Distances are read by setting one of stadia hairs on a whole foot-mark, by vertical tangent screw, and noting the intercept between the stadia hairs to nearest 0.01 ft. Care

must be taken not to mistake middle horizontal cross-hair for a stadia hair; this can be avoided by always making a mental estimate of the distance, or by adding oblique hairs to the instrument. Occasionally a half-interval may be read when an object obstructs view of the whole interval; it should be recorded as twice the interval read (or, better, the sum of the two "halves" taken separately).

Vertical angles are read by sighting middle cross-hair on the point on rod corresponding to distance from horiz axis of telescope to station beneath transit. This distance is HEIGHT OF INSTRUMENT (H I); it is not the same as H I in leveling, which is the elevation of telescope above datum. The line of sight is thus made parallel to a line from station to station, and vert angle read is the inclination of this line, whence difference in elev of the two stations can be directly computed. Same vert angle is used in computing horiz distance. If it be impossible to sight on H I point of rod, owing to obstacles in line of sight, the angle is noted when sighting at any convenient foot-mark on rod, this reading of middle hair being recorded in notes, with the vert angle.

Fieldwork is carried on as follows: When transit has been set up and properly oriented for reading azimuth angles, the transitman loosens the upper motion and sights on a rod held on point to be located. He first reads the stadia intercept from that part of rod nearest the H I point, and records it; then sets middle cross-hair on H I point of rod, and the vert hair bisecting rod. He then signals rodman to go to next point, and meanwhile reads the azimuth, vert angle, and index error, if necessary. If elevations are not required, he does not set on the H I, nor read vertical arc unless angle is greater than 3°; if it exceeds 3°, it is recorded only to the nearest 10 min, since it is used only for horiz corrections.

Stadia notes are kept on left-hand page in columns headed, from left to right, as follows: Station, Distance, Vertical Angle, Difference in Elevation, Elevation. The last two columns, as a rule, are computed in the office (Art 24). On the right-hand page should be drawn such sketches and details as are required to convey sufficient information to enable a draftsman unfamiliar with locality to plot the notes. Transit stations are usually designated by letters, and side shots by numbers. These numbers should be recorded on sketch within small circles, or in some other manner, so that they will not be mistaken for measured distances, which will also appear in sketch. Where purpose of survey is to produce a contour map, the accuracy of map depends in large measure upon the completeness of description given in notes regarding slope of the ground between located points. The elev of points on ridges, valleys, knolls, and depressions will be determined in the field; but, without a full description of intervening slopes, a correct interpolation of contours is often almost impossible. Notes like the following convey the required information: "Straight slopes from pt 16 to 17, 17 to 12, 13 to 17." Supplementary sketches illustrating shape of the ground are often necessary. The interpretation of field-notes, and their expression on map, are the weakest features of topographical surveys made by stadia; whence the importance of complete descriptive notes can not be overestimated. In open rolling country, where a 5-ft contour interval is required, a party of three men should cover about 5 acres per day.

24. STADIA COMPUTATIONS

Reduction of stadia notes includes computation of DIFFERENCE IN ELEVATION (vert heights) and HORIZ DISTANCES (or horiz corrections). Computations are almost always made by tables, diagrams, or stadia slide-rule, the latter being the most convenient means for reductions both in field and in office. Horiz distances can be quickly computed also by use of HORIZ CORRECTIONS, which are the distances to be subtracted from inclined readings to obtain horiz distances; to the latter must be added the instrument constant $(F + c)$, provided character of work in hand requires taking account of this 1 or 2 ft. Since the rod intercepts are usually read to nearest 0.01 ft, distances are recorded only to nearest foot; hence, in most topographic work the constant $(F + c)$ may be neglected, and there is no need of applying horiz correction for vert angles under 3°.

The stadia diagram, Fig 26 (by J. K. Finch, C. E., of Columbia University), gives 100 × rod interval, on vert lines; vert angles, on inclined lines, differences in elev, on horiz lines; and horiz corrections, on curved lines. The diagram is used as follows: Rod interval × 100, as recorded in field-notes under Distance, was 427; vert angle 3° 51′; instrument constant $(F + c) = 1$ ft. Follow up vert line 428 to inclined line 3° 51′; horiz line reads 28.7, which is entered in the field-notes in the column headed Diff in Elev. It is then applied to the elev of transit station to give elev of the point desired; if vert angle is upward (+) it is added to the elev of transit point and if downward it is subtracted. The horiz correction is read from the curved line which lies nearest the intersection of vert line 428 and inclined line 3° 51′; in this case it is a correction of 2 ft, to be subtracted after the value of $(F + c)$ has been added, as follows: $427 + 1 - 2 = 426$ ft.

This diagram will be found of great assistance in applying rough checks on computations, especially in the use of plane table, if it be remembered that on a vertical angle of 34′, the vert height is 1% of inclined distance; on a vert angle of 2° 52′, it is 5%; and on an angle of 5° 46′, it is 10% of inclined distance.

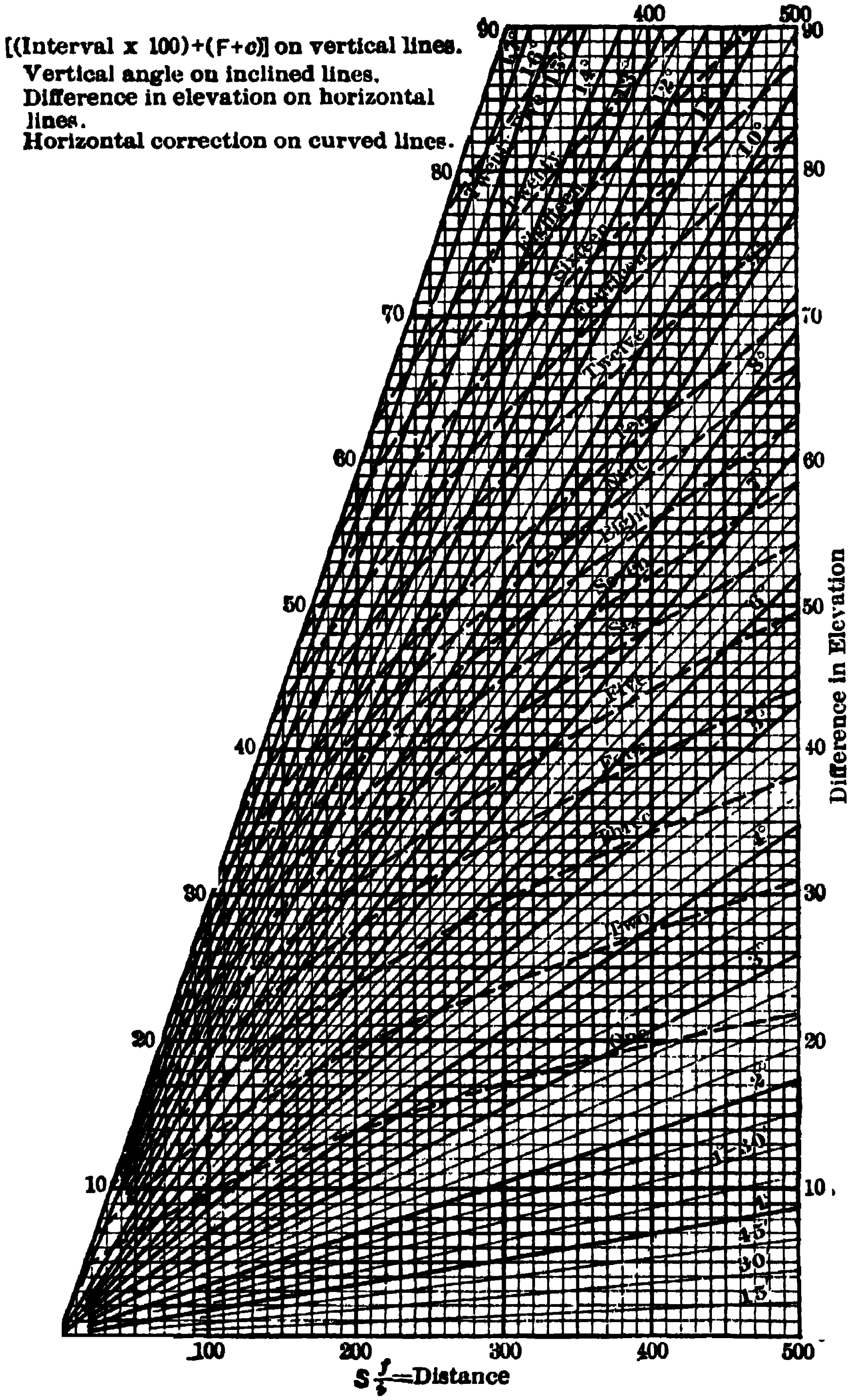
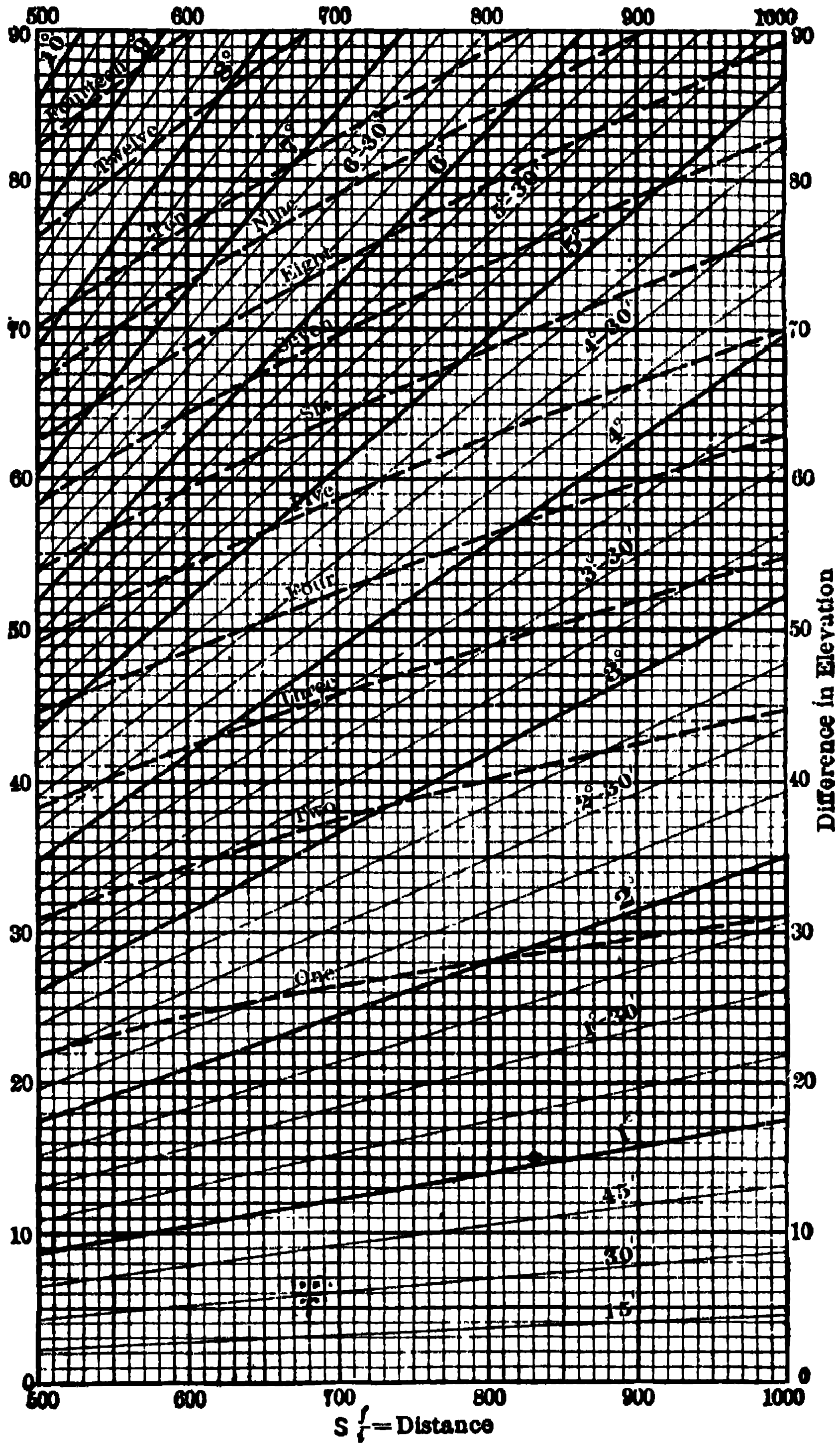


Fig 26. Stadia Diagram for Obtaining

Even without this diagram, differences in elev may be computed if the short table of horiz corrections is available. After horiz distance has been found, multiply it by tangent of vert angle, to get vert distance; this can be conveniently performed on the tangent scale of an ordinary slide-rule.

When vertical angle is not read to the H I mark on rod, this fact must be taken into account in computing differences in elevation, as follows: Suppose the distance read was 135; vert angle,

−8° 50′ on 2.5; H I, 4.5. From diagram, diff elev = − 20.6, to which add +2.0, giving −18.6 The 2 ft is positive because the vert angle was measured to a point on rod 2 ft lower then H I point, which made observed vert angle greater than it would have been if sight had been taken at H I point on rod.



Vertical and Horizontal Distances (J. K. Finch)

Beaman's stadia arc is an attachment to the vertical arc, giving readings which obviate the use of stadia tables, diagrams or slide-rule. It requires more manipulation of the instrument than the usual method, but is of advantage particularly when elevations must be computed in the field as work progresses.

25. THE PLANE TABLE

The plane table is used for constructing a map directly in the field. The paper is fastened to a board, like a drafting board, supported on a tripod, with a device for leveling and clamping so that it can be oriented in any desired azimuth. The usual size of table top is 24 by 30 in, and the tripod most commonly used is the Johnson, which has an ingenious ball-and-socket leveling device and clamp. On the map an ALIDADE is used to define directions and measure distances. It has a telescope equipped with stadia hairs and is mounted on a horizontal axis resting in Y supports; these are connected to a metal column which rises from center of a flat piece of metal about 18 in long, of which both edges are straight and parallel to plane of telescope. On this base are two spirit-levels for leveling the table. The telescope has only a vert motion and a vert arc. The entire alidade is moved about on top of map after table has been leveled.

The use of the plane table has generally been limited to topographical maps drawn on a small scale, but its legitimate field is much broader than that. A system of triangulation (Art 26) or a transit and tape traverse may first be carefully surveyed and plotted on map; levels also may be run to determine elevation of triangulation or traverse points, which will then be used as a control for plane-table survey. In this case the plane table is used only to fill in details, and accurate maps on a scale as small as 40 ft to the inch can be rapidly made. The most important advantage of the plane-table over other topographic methods is that all the sketching is done in the field, where topographer can see form of ground that he is mapping. He can sketch details at once in their proper position, without burdening his memory and without making elaborate notes. For this reason details may be accurately sketched from a much smaller number of located points than would be required, for instance, by transit and stadia method.

If not practicable to run out a system of triangulation, or a transit and tape traverse, the plane table can be used for making such a survey graphically, as explained below. In all cases, however, the accuracy of those points located by plane table is limited by precision of the plotting rather than of the measurements. In the field the map must be protected from distortion by moisture, so as to preserve accuracy of plot. Celluloid sheets are obtainable, which can be used even in rainy weather.

The plane-table method usually, though not always, requires more time for fieldwork than transit and stadia methods; it is also more dependent upon favorable weather. But taking into account both the field and the office work, the plane table has proved more economical than the transit and stadia for work in open country, and the results, as a whole, are more reliable. In open rolling country, a contour interval of 5 ft being required, a party of 3 men should survey about 8 acres per day.

Locating points by intersection. To begin a plane-table survey it will generally be necessary to have on the map two plotted points corresponding to two points on the ground, distance between which is known, and of which one, at least, can be occupied with the table. A simple method of locating points, without measuring any distance, is as follows: The base-line ab is plotted on plane-table sheet, representing the measured base AB to the adopted scale. The table is first leveled and so set that a on the map is vertically above A on the ground (a special plumbing appliance is used for this adjustment). One edge of alidade is placed along base-line ab drawn on map, the table is turned in azimuth until telescope sights signal B, and horiz motion is clamped. Line ab is now parallel to AB and the table is said to be oriented The alidade is now placed with its straight-edge passing through a, the telescope is sighted to some signal C, and an indefinite line is drawn toward C; the point c on map is somewhere on this line. Now moving table to B (setting b vertically over B) and repeating the process of orienting the table and sighting toward C, point c is located at intersection of lines ac and bc. In similar manner any number of points may be located.

Locating points by direction and distance. The commonest way to locate points by plane table is to obtain direction with alidade and measure distance by stadia, which is then laid off to scale along the straight-edge. If a control traverse has been run, the table can be set up over any of the plotted points and oriented as explained above.

Locating points by resection. It is sometimes desired to locate a plane-table station from a base of which only one end can be occupied by the table. A and B are the points on ground at ends of base-line; C is the point to be located; and ab is base-line plotted on plane-table sheet. Set up at A, the accessible end of the base, and orient table by sighting B with alidade pointing along ab. Then, centering alidade on a, draw an indefinite line toward C, for full length of the alidade. The table is then taken to C and oriented by the indefinite line just drawn. Since position of c on the indefinite line is not known, its position on map must be estimated for setting table over point C. The alidade is now sighted toward B, with the edge of its base on b and a resection line is drawn, which cuts the first indefinite line, thus locating point c desired. If point c thus found is not over point C on ground, reset over C and repeat process. The position of c found by this method should be checked, if possible, by resection lines from other points of which the positions are known to be correct.

26. TRIANGULATION AND TRIGONOMETRIC LEVELING

Triangulation consists in locating certain points by observation of angles, starting from a base-line which has been accurately measured; the method is often used as a control for a topographic or hydrographic survey. A triangulation system comprises a series of triangles, the corners of which are the observing stations or points which it is desired to locate. In this system of triangles, the length of one side of some triangle must be measured; then if all the angles in each triangle be observed, lengths of all other lines in the system may be calculated by trigonometry. Three types of triangulation system may be recognized: First, a series of approximately equilateral triangles; second, a series of central polygons, for example, a row of hexagons each having an interior station; third, a series of quadrilaterals with both diagonals drawn. The first system is the cheapest when it is desired to extend survey along a narrow belt, as in surveying a river. It has the disadvantage of having but few "checks," that is, few geometric conditions which must be satisfied by the measurements. The second system is adapted to surveying an area of greater width than the first. The third is the most accurate, because it has the greatest number of checks. No attempt will here be made to describe the precise work carried on by the U S Coast Survey in mapping large areas, the following notes relating only to such work as the mapping of a few square miles.

Fieldwork consists in first making a reconnaissance of the country, selecting suitable triangulation points with due regard to shape of the triangles, which should be as nearly equilateral as possible, choosing a favorable position for the base-line to be measured by tape, and ascertaining that the chosen points are visible each from the others.

Base-line should be located on fairly even ground, for convenience in measurement, and its position should be such as to admit of accurate connection with the triangulation system. After clearing line of obstructions, stakes are lined in by transit one tape length apart. Grade of each tape length is determined by direct leveling. In measuring each length the intermediate supports are lined in vert and horiz, proper tension is applied with a spring balance, and temperature is read on two or more thermometers attached to the tape. Marks indicating ends of tape length are scratched on metal strips tacked to tops of posts. At least 2 measurements of entire base should be made. INVAR tape (Art 1) is used by U S Geodetic Survey for base-line measurements. For description of methods of base measurement, see U S Coast and Geod Surv, Pubs 120 and 145.

Angle measurements. The transit is set up at each end of base-line and angles are carefully measured by repetition (Art 11), to the signals erected at those triangulation points which are intended to be connected directly with the base. The transit is then taken to the various points of triangulation system, and the necessary angles are measured so that length of each line in the system can be computed, and as many as desired can be checked by computation through different triangles.

Signals over triangulation points are frequently made of a 4 by 4-in pole supported by a tripod; on the pole are tacked black and white cloth bands of definite width, or flags. The foot of pole is supported 7 or 8 ft above the ground, so that theodolite may be set up beneath signal when measuring angles. Tripod signals may also be made of young saplings, and in the smaller ones the pole usually rests on the station point; this necessitates removal of the entire signal when angles are measured at that station. Where necessary to build a high signal, on account of forests, a tall mast may be made of 2 or 3 poles spliced together, and braced by wire guys. In all cases it is advisable to tack a black and white band to that part of the pole which is *vertically over the station*, so that observations may always be made on that particular point.

Trigonometric leveling consists in computing difference in elevation of two points on the basis of horiz distance and vertical angle between them; it is usually combined with triangulation work, the vert angles being measured at same time as horiz angles. A vert angle is measured to some definite point on signal, the height of which above station was determined when signal was erected; the height of instrument above its own station should also be measured and recorded. In very precise work, angles are measured with a special vertical-circle instrument. In less precise work, an ordinary theodolite, the vert arc of which reads to 30 sec or to 20 sec, may be used, but with such instruments only single readings can be made; in which case the best results are obtained by averaging several independent readings, half of which are taken with telescope direct and the other half with telescope inverted. In every case the index correction, or reading of vert arc when telescope is level, must be recorded.

The chief difficulty in obtaining accurate results by trigonometric leveling is the uncertainty in the coefficient of refraction of the air. This varies with locality, temperature, and atmospheric pressure, and the only way its effect can be eliminated is by taking simultaneous observations between two stations.

Observations at one station. Multiply horiz distance by tangent of vert angle, and apply a single correction for curvature and refraction by the formula: $h = K^2 \div 1.7426$, where K is distance between stations in miles, and h is correction in feet. If K be in units of 1 000 ft, then $h = 0.02\,K^2$ (nearly). This correction is applied to increase difference in elevation if vert angle is positive, or to decrease it if angle is negative.

When two simultaneous observations can be made, $h_1 - h_2 = K\,tan\,{}^1/_2\,(\alpha_2 - \alpha_1)\,\{1 + (h_1 + h_2) \div 2\,R + K^2 \div 12\,R^2\}$, where $h_1 - h_2$ = difference in elevation of stations in feet; K = arc of the earth subtended between plumb lines through the two stations (approx equal to horiz distance between the two stations) in feet; α_1 and α_2 are simultaneously observed vert angles; R = radius of earth at latitude of stations; for most work it is close enough to take $\log R$ (in feet) = 7.32068.

27. TERRESTRIAL PHOTOGRAPHIC SURVEYING

A rapid method of locating topographic details for construction of small-scale maps (say 2 or 3 miles to 1 in) is afforded by photographic surveying. Best results are obtained where country has characteristic shapes, and is not too thickly wooded to afford good positions for taking the views. To locate a point, it is photographed from two stations, the positions of which are known. It is necessary to know the direction in which camera was pointed when each photograph was taken and the focal length f of lens. For further details, see "Higher Surveying," by Breed and Hosmer, 5th Edn, 1938, Chap VII, VIII.

Instrument used for this work sometimes consists of a transit, so constructed that standards and telescope can be removed and a camera mounted in their place, so that azimuth angles can be measured, or laid off, by the horizontal circle. The instrument is leveled and horiz angles between certain well-defined points are measured while telescope is on transit; when the camera is substituted, the same angles are laid off on the circle and the exposures are made. Such instruments cost about $500.

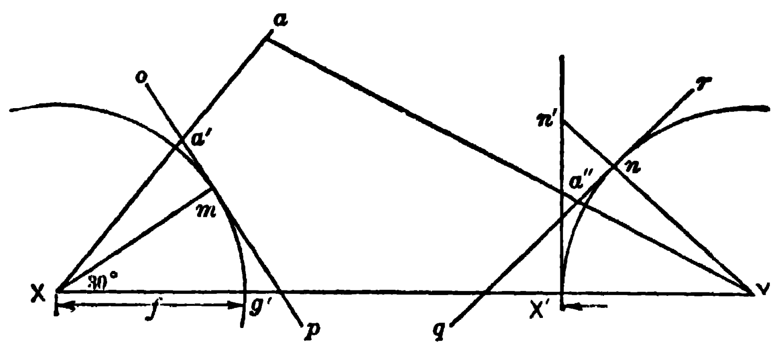


Fig 27. Plotting Camera Stations

Notches on the 4 sides of the camera opening yield corresponding points on edges of every picture; lines are then drawn across photograph connecting these 4 points. The vertical line is known as the PRINCIPAL and the other as the HORIZON line; their intersection is called the PRINCIPAL POINT.

If photograph be held at distance f in front of the eye, and normal to line of sight, points a, b, and c on the picture will appear to cover corresponding points A, B, and C in the landscape. The photograph, when held in this position, thus gives a measure of the angles between various points in the view; a fact utilized in plotting.

To plot map, first plot the base line XY to the desired scale (Fig 27). With each base station as a center draw a circle of radius f (true focal distance), intersecting the base at x' and y'. Next plot the traces of the photographs. If the angle between the base and direction the camera was pointed has been measured, say 30°, lay off radial line Xm. A line op at m, perpendicular to Xm is the trace of photograph taken at X. If the angle is not known and the image of X appears in photograph taken at Y, as x', erect a perpendicular to base at x'. Lay off distance $x'n'$ = to distance measured on print from PRINCIPAL POINT along horizon line to foot of perpendicular dropped from image of the base to horizon line. The intersection n, of $n'Y$ and the circle, is the location of the principal point. Line rq, perpendicular to $n'Y$ at n, is the trace of the photograph. To plot point a, first measure on photograph op, the distance from a to the principal line, and plot that distance on line po as ma'. Similarly, the distance na'' is measured on the other photograph and plotted along trace rq. The plotted position of A thus lies at intersection of Xa' and Ya'', or at a. In same manner are plotted as many points as desired.

Differences in elevation are computed as follows: Scale the distance of point a above horiz line on photograph. This distance divided by distance from camera station to point a' (scaled from map) gives the natural tangent of angle of elevation or depression. The actual horiz distance Xa is then scaled from map, after the point has been plotted, and is multiplied by tangent of vert angle, to give difference in elevation between camera station and point a; or it can be found graphically. Details of map are sketched in from a study of the photograph, after characteristic points have been plotted.

28. AERIAL SURVEYING

For further details, see "Higher Surveying" by Breed and Hosmer, 1938 edn, Chap VIII and IX.

Aerial photographic surveying is used for making maps for highway, railway, waterway, pipe-line and transmission line location; for irrigation, water supply, flood control, river and harbor improvements, soil conservation, geological research, timber estimates, topographic and exploration surveys.

Aerial photographs are classed as vertical, oblique or composite. In taking vert photographs, the optical axis of camera is held as nearly vert as possible at instant of exposure. In oblique photographs, the camera axis is purposely inclined to the vert and may or may not include the horizon. In composite photographs, taken with a multi-lens camera, one photograph is usually vert and the others oblique. Some multi-lens cameras have no vert chamber. The oblique views of composite photographs are rectified (transformed into vert prints) before being used for plotting. Then all prints will appear as verticals, although the oblique prints will no longer be rectangular in shape.

Aerial cameras are precision instruments, comparable to the engineer's transit, and are of single or multi-lens type. The latter consists of two or more cameras mounted *en bloc*, all exposures being made simultaneously. For taking vert or composite photographs, the camera is mounted in gimbals attached to a frame rigidly fastened to the airplane. Exposures are made through a hole in the floor of the plane. By inspection of a level attached to the camera, the operator holds the optical axis as nearly vert as possible at instant of exposure. In taking single oblique photographs the camera is sometimes held in a standard attached to edge of the cockpit, and may be moved in any direction. Present practice in Canada is to mount 3 cameras on a cross beam in the plane; they are inclined at angle of 21° with the horiz when the plane is on even keel.

The cameras are of the fixed focus type. At instant of exposure, the film is held flat in focal plane by a vacuum back or a glass pressure plate. To locate the principal point, notches or collimating marks in the focal plane are photographed on the negative at instant of exposure.

View finders are commonly used. For taking oblique photographs the view finder has a sighting arm, which may be set at any desired angle. The camera is held at the required depression angle by directing the sight at the horizon. For vertical photographs, a vert view finder is mounted, between camera and photographer, over a hole in the floor of the plane. This instrument is essentially a camera with a ground glass plate in its focal plane, by which the terrain is viewed. It is leveled by an attached bubble and the instrument may be rotated about its optical axis. The vert view finder is to determine: (1) when points are vertically under the camera; (2) the "angle of crab," so that the camera may be adjusted along the true line of flight; (3) the interval of time between exposures. The ground glass plate is ruled with lines 0.5 in apart in the direction of flight, and normal to these lines are 2 others. To determine the "angle of crab" (angle between longitudinal axis of the plane and direction of flight) the view finder is turned about its optical axis until objects on the ground appear to move parallel to the lines ruled on the ground glass along the direction of flight. In order that successive pictures shall be directly in front of one another, the camera is turned about its optical axis to an amount equal to "angle of crab." The interval between exposures is determined by the time it apparently takes a ground point to travel between the transverse lines ruled on the ground glass plate.

Taking the pictures. When taking photographs of the area to be mapped, the airplane is flown back and forth over parallel straight lines (" flight lines "), and usually at a predetermined altitude. The proper distances between the flight lines are computed and before the flight the lines are plotted on a map to aid the pilot in keeping his course. Altitude is given by an "altimeter." Where vert photographs are to be used for compiling maps, it is best to show the principal points of preceding and following prints in each print. Hence, exposures are timed so that the overlap between successive pictures in direction of flight will be about 60%. To avoid hiatus between adjacent strips, the

flight lines are so spaced that the photographs will also overlap. This " side lap " is usually 30%–50%.

Relation between scale of print and altitude of camera. Strictly, where the ground varies in elevation, there is no such thing as the scale of a photograph. It is only when the ground is level, and the photograph has been taken truly vert, that the print has a definite, uniform scale. However, any elevation may be selected as a datum, and, if the elev of the camera is known at instant of exposure, the scale of the print may be computed for the given datum.

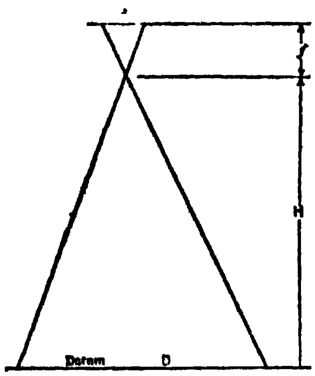


Fig 28

There are two ways of expressing the scale of an aerial photograph: (1) by the representative fraction (R F); (2) by the scale factor. The R F is the usual scale, expressed by a fraction in which the numerator is unity; it is the ratio of a distance on the map to the corresponding distance on the ground, denoted as s. The scale factor, expressed as S, denotes the number of feet on the ground corresponding to 1 in on the map. In Fig 28, if distance D on the ground (datum) is represented by d on the print, then the R F scale $s = d \div D$. Also, $d \div D = f \div H$, where f is focal length of lens and H the altitude of the plane above the datum, f and H being measured in the same units. If the scale factor is desired, $S = H \div f$, where H is in ft and f in inches. Both formulas show the relation between scale of the print and altitude of the plane above the datum.

Example. Compute s and S, when $f = 10$ in and $H = 10\,000$ ft:

$$s = \frac{f}{H} = \frac{10}{10\,000 \times 12} = \frac{1}{12\,000} \text{ and } S = \frac{H}{f} = \frac{10\,000 \text{ ft}}{10 \text{ in}} = \frac{1\,000}{1}, \text{ that is, 1 in} = 1\,000 \text{ ft.}$$

Another form of the scale equation is $s = f \div (H - h)$, where H is the altitude of camera above datum and h the elevation of the datum. From these formulas the altitude at which the plane must fly to give the print a desired scale above a definite datum may be computed.

Time interval between exposures may be determined by the "view finder" (see *ante*). But, if speed of plane, focal length of lens, altitude of exposure station, size of print and end lap are known, then the interval between exposures can be computed.

Example. Speed of plane = 120 mile per hr; $f = 10$ in; altitude of exposure station, 10 000 ft; size of print, 7 by 9 in (the 7-in dimension being in direction of flight); end lap 60%, then

$$s = \frac{10}{10\,000 \times 12} = \frac{1}{12\,000}$$

Total distance covered by the 7 in on print = 7 × 12 000 ÷ 12 = 7 000 ft. Net distance covered by one print = 7 000 × 40% = 2 800 ft. Speed of plane, ft per sec = (120 × 5 280) ÷ (60 × 60) = 176 ft. Interval between exposures = 2 800 ÷ 176 = 15.9 sec.

Number of negatives to cover a given area. When the dimensions of the area, scale and size of print, and the end and side overlaps are known, then the number of prints required may be computed.

Example. Size of area, 15.5 by 20 miles; prints, 7 by 9 in; scale 1 to 1 000; end lap 60%; side lap 30%. Total area covered by 1 print = (7 × 12 000)/12 × 5 280) × (9 × 12 000) ÷ (12 × 5 280) = 2.26 sq miles. Net area covered by one print = 2.26 (1.00 − .60) (1.00 − .30) = 0 633 sq mile. Required number of prints to cover area = (15.5 × 20) ÷ 0.633 = 490 prints. To this value add a sufficient number of prints to cover the margins. It is customary to make 1 or 2 additional exposures at beginning and end of each flight line, to insure proper coverage. In the example, if 1 additional print was taken at both beginning and end of each flight line, 26 prints should be added to the 494; total, 520.

Scale errors. For mapping, the plane is often flown at a precomputed altitude. As altimeters are not precise instruments, and because of varying meteorological conditions, the altitude will vary, thus causing a change in scale of photographs. If $f = 10$ in and $H = 5\,000$ ft, the scale S is 1 in = 500 ft. If, due to above mentioned changes, H is 4 800, scale S would be 1 in = 480 ft. Under good flying conditions the altitude should not vary more than 100 ft.

Definitions. Nadir or ground plumb point is a point on the ground vertically beneath the lens at instant of exposure. Its image on the print is the plate nadir or plate plumb point. " Principal point " is the point where the optical axis of the camera intersects the film. If it is not shown on the print, it is taken at the intersection of the lines between the reference marks on margins of the prints. When the optical axis is truly vert, the

nadir and principal point coincide. When the plate has some tilt (usually the case), these points do not coincide. A point approx midway between nadir and principal point in a tilted photograph is the "isocenter," or center of distortion. Positions of the plumb point and isocenter could be located on the print, provided the amount and direction of tilt are known, which may be determined if the position and elevation of 3 or more ground control points on the print are known.

Displacement caused by relief. As all photographs are perspectives, all objects in the photograph, except the plumb point and those in the datum, are displaced. In Fig 29 the top of chimney A will be shown on the datum at A'. Displacement on the datum is $A''A'$ and on the print $a''a$. Displacement of point B is $B''B'$. Plumb point P' has no displacement. The displacement of points below the datum is inward, and, as it is in a vert plane, the error on the print is radial from the plate plumb point, which in a truly vert photograph coincides with the principal point. Displacement on the print is calculated as follows. The hill A (Fig 30) whose height above datum is h, appears on the

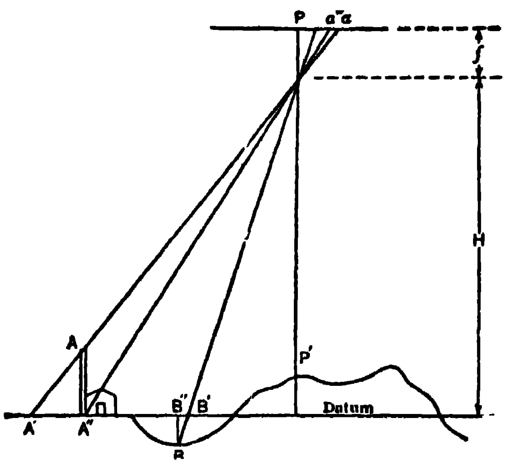


Fig 29

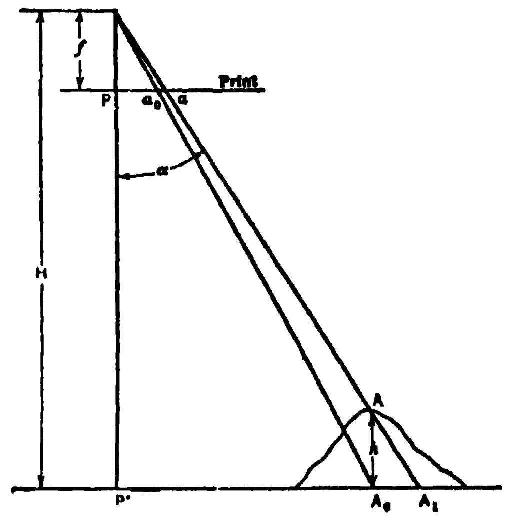


Fig 30

print at a; ground displacement is A_1A_0 and displacement on the print is aa_0. By measuring distance Pa on the print and dividing it by f the angle α = angle A_0AA_1 may be found. Now $a_0a : A_0A_1 = f : H$; whence $a_0a = (f \div H) \times A_0A_1 = s \cdot h \tan \alpha$. Another formula for computing the displacement is derived as follows. In the last equation, $s = f \div H$ and $\tan \alpha = Pa \div f$. Substituting, $a_0a = f \div h \cdot h \cdot Pa \div f = Pa \cdot h \div H$.

Example. H = 10 000 ft, f = 10 in, h = 500 ft, Pa measured on the print = 3.075 in. By the first formula, displacement is a_0a = 10 ÷ (10 000 × 12)·500·12·tan 3.075 ÷ 10 = 0.154 in; by the second formula, a_0a = 3.075 × 500 ÷ 10 000 = 0.154 in. Thus, by computing the displacement of a point, its correct position on the print may be plotted.

Tilt. In taking vert photographs, the ideal position for the optical axis at the instant of exposure is truly vert; that is, the principal point and plumb point coincide. This condition seldom exists. The angle of inclination of the optical axis and the vert is called the "tilt," which experience shows may be kept within 1° and seldom exceeds 3°. When tilt exists the film, which is at right angles to the optical axis, is not truly horiz, and the images of objects on the film are displaced. If the tilt angle is known, the displacement may be computed. But, there is no simple way of determining the tilt angle. To do so, the image points of at least 3 ground-control points, whose positions have been found by ground surveying, must appear on the photograph. Then the tilt may be determined analytically, graphically, or by rectifying instruments.

Effect of displacement on plotting caused by relief and tilt. Although displacement caused by relief is radial from the plumb point, and that caused by tilt is radial from the isocenter, the positions of these points are indeterminate unless the tilt is known. In plotting, however, relief and tilt are assumed to be radial from the principal point. When both relief and tilt are small, the error in plotting is negligible.

Ground control is the term applied to points identifiable in the photographs, whose horiz positions and elevations referred to a selected datum are known. Their locations and elevations are determined by ordinary ground surveying. The amount of ground control may comprise many or few points, according to accuracy required and the mapping method employed. In deciding the location of control points, it should be observed that

at least 3 points must be known, to fix independently the position of a vert photograph, from which a planimetric map is to be compiled. Where stereoscopic processes are used, at least 2 points known in position and elev, and a third point known in elev, must be available in the area of common overlap. In either instance, the control should be so positioned as to form as strong a geometrical figure as can be drawn with either the single photo or the overlap. Thus, in deciding how control should be located, consideration should be given to the proposed use of the photographs. Note that triangulation stations are usually elevated points, subject to large relief displacements in the pictures. Traverses, on the other hand, may be run along highways, in valleys, near the level of the selected datum, and are therefore nearly free from this objection. In taking levels, the points selected should be easily identified in the pictures, and so placed as to give the best determination of tilt. If the photographs are taken before the ground control has been established, points with little or no displacement and which can be identified on the photographs (as intersections of roads with other roads, railways and streams) are chosen for control and then located in the field. Where there are no easily identifiable objects on the photographs, it may be best to lay on the ground white cheesecloth about a yard wide in form of crosses or circles, so located in open spaces that they will show clearly on the photographs. These cheesecloth stations can be joined to the ground control either before or after photographs are taken. This should be done fairly promptly, unless the cloths are weighted down to prevent displacement.

Photographic maps. This term is a misnomer, because, in general, a photograph can not be both a true photograph and a true map, or projection. As the photograph is a perspective, all points in it above or below the datum are displaced. When pictures are pieced together to form a continuous representation of the ground, the result is a MOSAIC. Maps made on a true horiz projection from data derived from photographs are true maps. In spite of geometric defects of mosaics, they are very useful for certain studies, and are more readily understood by the non-technical man than engineers' plans. In estimating timber, and in other applications, the mosaic is often preferable to the map itself. For

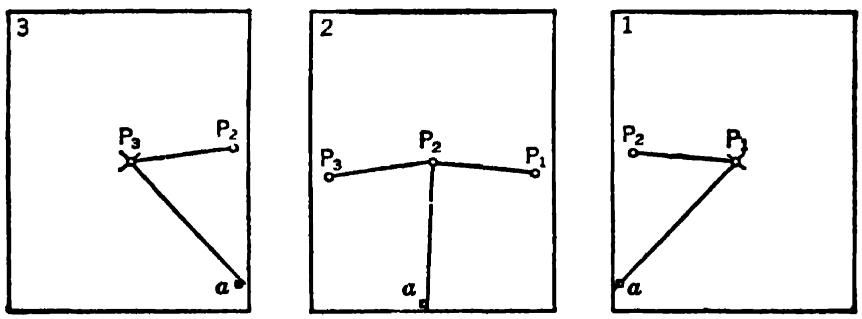


Fig 31

preliminary engineering investigations, engineers often like to have, besides the mosaic, a complete set of photos for stereoscopic inspection.

Photographic maps comprise: UNCONTROLLED MOSAIC, a representation of the ground made by matching aerial photographs without reference to ground control points; CONTROLLED MOSAIC, a representation of the ground made from aerial photos by bringing them to a uniform scale and fitting them to ground control stations; PLANIMETRIC MAP, showing the natural and cultural features in plan only, often called "line map"; TOPOGRAPHIC MAP, made from aerial photographic data, including contour lines derived by ground surveying; STEREOMETRIC MAP, a relief map made by applying the stereoscopic principle to aerial or terrestrial photos.

Orienting prints and compiling maps graphically. All photographs are true perspectives, and for practical purposes maps may be considered as orthographic projections. The perspective or photograph is a view in which the light rays from the object points converge to the lens of the camera, and diverge from lens to the negative. Excepting in the optical axis of the lens, the light rays are not perpendicular to the negative; but, in the map all objects are represented as they would appear if the eye were on a line everywhere perpendicular to the plane of the map. The map compiler must take the perspectives (photographs) and make an orthographic projection from them. A common method is RADIAL TRIANGULATION, RADIAL INTERSECTION, or RADIAL LINE CONTROL, based on the assumption that the photo gives the true horiz angles at the principal point.

In plotting, it is assumed that relief and tilt displacements are radial from the principal point. Suppose that on 3 consecutive overlapping pictures the principal points are P_1, P_2, P_3 (Fig 31), and that from each a radial line is drawn on the corresponding print

to the corner of a building a. By superimposing the pictures (Fig 32) all 3 lines intersect at a common point, but this point of intersection does not coincide exactly with any of the positions of the building on the pictures, as would be supposed. The angles P_2P_1a, P_1P_2a, etc, are true angles because their sides are radial from the principal or the plumb point. No appreciable error is caused in plotting by using the principal point for the plumb point. The true position of the corner of the house is at the intersection of the 3 radial lines b. The error in position on the print may be due to the camera being higher in one case than in another, or to the elevation of the ground on which the building stands. If the 3 lines so drawn have no common intersection, an error should be looked for; if not discovered, one of the pictures is probably distorted by tilt and should be rejected when identified. If the point appears on 4 or 5 prints, it is easier to identify the one causing the error.

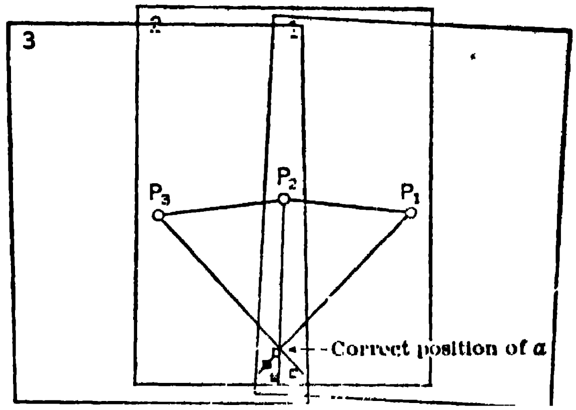


Fig 32

In applying radial intersection to a strip of aerial photos, the approx scale of the prints is first determined, by comparing known distances on the ground with corresponding distances on the prints. When this is done, a map projection of the area is made to this scale. All ground control points from ground surveys are located on the projection. The photos are next examined, the ground control points identified and marked on all prints on which they occur. In Fig 33 these points are shown on photos 1 and 2 at T_1, T_2 and T_3. The principal point of each photo is next identified and marked on all preceding and succeeding photographs in which it appears. These points are shown at P_1, P_2 and P_3. While the principal point is easily located on some one print, it may not be accurately identifiable on adjacent prints. In this case, an identifiable point near the principal point is chosen, and used as if it were the principal point. When such points are near each other, the error caused in plotting by the substitution is negligible. In Fig 33, print III, PS_3 is chosen in place of P_3 (it is also shown on print II). The picture control points are next identified and marked on all prints where they occur: they should be well defined, and at least nine chosen on each print. However, the number of points depends on whether the terrain is flat, medium or heavily accidented. To insure correct plotting, the greater the accidentation the greater the number of picture control points; 3 should be chosen across and near bottom of print, 3 across the center, and 3 across the top. Print II (Fig 33) shows how these points should be distributed normal to direction of flight. A stereoscope greatly aids in properly locating the points.

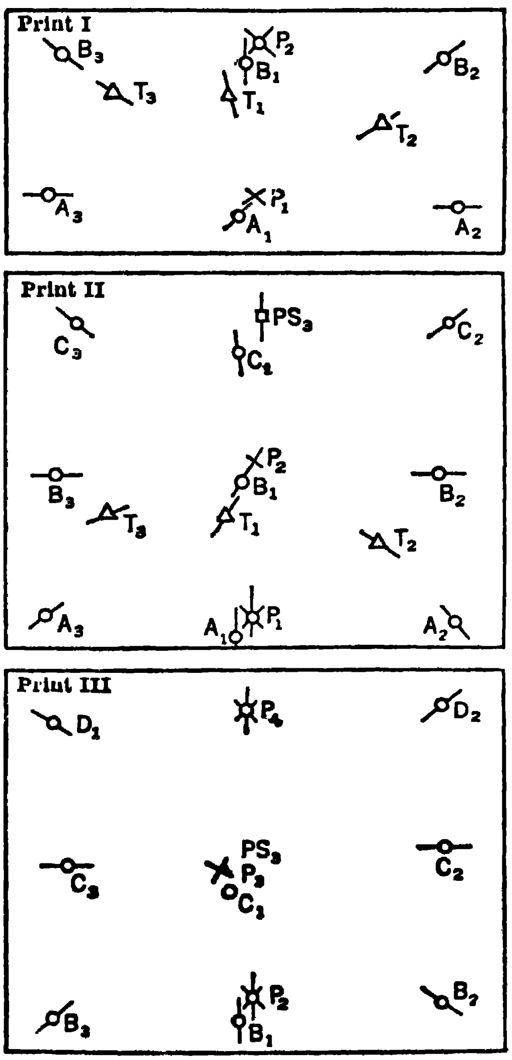


Fig 33

After locating the ground control points, principal points or their substitutes, and picture control points, radial lines are drawn on each print (Fig 33) from the principal point or its

substitute to all control points. The orientation of the successive pictures is based on the principle of resection. Intersection of the radials indicate their orthographic projection, for which the tracing-cloth solution of the 3-point problem is used. The first photo is oriented under the tracing cloth, so that the radial lines from center to ground control points fall through the plotted positions of these points on the tracing. The center of the print is then oriented, and so marked on the tracing. Without moving the photo, radial lines are drawn on the tracing to the selected picture control points, and to the photograph centers. The first print is now removed, the second placed under the tracing and similarly oriented. Radials are drawn from the center to the control points and to the centers, as in the first print. Some of the radials on the tracing from the first print will be intersected by some of those traced from the second print. These intersections may now be used for orienting the third print. Fig 34 is the plot of radial control of the 3 prints in Fig 33, which is used for orienting the prints for making the finished map. The process is continued until a new set of ground control points have been reached, when a check is obtained. If the points on the tracing do not check with the ground control points, they are adjusted to correct the errors. In locating on each print the radial line to its center, radials through the previously established ground control or picture control points are used. If these do not all fit, and no gross error is evident, each radial is shifted slightly, points on opposite edges of the print being on opposite sides of the radial line. Because of errors due to the assumptions that displacements caused by relief and small tilt-angles are radial from the principal point, perfect checks can not be expected, but the tracing may be adjusted so that these errors are equally divided and a compromise position adopted.

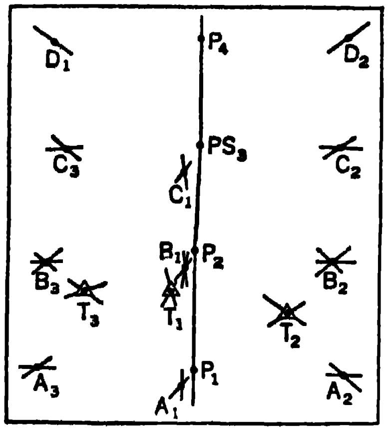


Fig 34

Controlled mosaics. From control established by ground survey methods, or as a result of a radial plot, it is possible to rectify the individual photographs and bring them to a common scale between control points. A mosaic may now be laid with these scale-rectified prints, which will contain very small displacement errors.

Transferring details to the tracing. After the skeleton of the map has been prepared, details are filled in from the original photos. In each case the photo is placed under the tracing cloth and oriented with respect to the points whose positions are now fixed. Detail is taken off along the line joining any two points. The print is oriented so that one point on the photo lies under its map position, and the second point lies on the line joining the two points on the map. Differences in scale between print and plot are then corrected by estimation in the compilation process, due regard being taken for any known relief differences along this line. In the case of controlled mosaics, there will be no apparent difference in scale between positions of control points on the print and on the plot. Detail can be taken from the mosaic, or from rectified prints, and a planimetric map drawn. The operator must use his best judgment based on knowledge of the terrain in selecting the position of detail where mismatches occur as a result of relief.

There is now used a projection camera, like an enlarging camera, into which a photograph may be inserted and its image cast on a drawing board. The drawing board may be tilted by 3 leveling screws. The map projection is placed on the board, and small sections of photographs made to fit the control points by varying the enlargement of the photo and tilting the board. Details may thus be transferred to the map.

Contours. There are two modes of obtaining the contours on a map made from aerial photos: (1) by the ordinary ground survey methods, after completing the horiz locations; (2) by the stereoscopic principle applied to overlapping photos. In the first method there must be enough lines of direct levels to furnish all bench marks necessary, as well as points located horizontally for the plane-table work. Planimetric locations can be checked, and any residual errors removed. For details of stereoscopic work, see Bib references to books listed under *Photographic Surveying.*

29. HYDROGRAPHIC SURVEYING

Hydrographic surveying is usually applied to the determination of shape of bottom of a body of water, sometimes including a determination of character of material composing the bottom. It is customary first to establish points on shore, by triangulation or traverse,

to which the hydrographic survey may be referred, then to measure (usually from a boat) the depth of water at various points, and determine position of these points. Besides ordinary transit and tape outfit, a sounding-pole (or lead-line in deep water) and a boat with necessary equipment will be required. In tidal waters, and in lakes where water level changes rapidly, a tide gage should be installed. Sounding-poles are similar to self-reading rods, graduated to 0.1 ft; a shoe is sometimes attached to lower end, provided with a cup-shaped cavity which, when smeared with tallow or soap, brings up samples of the bottom. The lead-line is a chain, or hemp line, to which is attached a lead weight. A brass sash-chain is satisfactory, having cloth tags of various colors for footmarks. Where there is little current, a 6- to 10-lb weight suffices for depths to 40 ft.

Locating soundings. There are 6 general methods: (*a*) The boat is rowed on a range line, at uniform speed, and soundings are located by time intervals. (*b*) Boat is rowed on a range line, while positions of soundings are "cut in" by a transit reading from shore, or by an angle taken with sextant from boat. (*c*) Boat may be rowed on a range line, or not, while its position is located by angles taken simultaneously by two transits on shore, or by angles taken simultaneously to shore stations from boat by two sextants. (*d*) Soundings are located by stadia method. (*e*) Soundings are defined by the intersection of fixed ranges. (*f*) In case of a stream, a wire or cord is stretched from shore to shore, and soundings are taken at measured points along this line.

Locating by range and angle. In still water, where the boat is easily held on any desired course, soundings are conveniently located by proceeding along a range line of known position, marked by two objects on shore, such as range poles, and then "cutting in" the position of sounding pole by a transit angle observed from a previously determined shore point at instant the sounding is made. The ranges may be fan-shaped if pivot-point of system be a steeple or some other object of known position situated far enough back from shore so that the lines will not diverge too rapidly. The recorder writes in his notebook the depths as called off by leadsman, and also the times of sounding. He also notifies leadsman by calling out "Sound" about 5 sec before each sounding is desired. The leadsman takes soundings as quickly as possible, usually within 2 or 3 sec of the desired time. In hydrographic work of Corps of Engineers, U S Army, in which nearly all soundings are located by two angles taken with transits on shore, soundings are usually taken at 15, 20, or 30-sec intervals, and an observation is made each minute by the instrument men at the instant signal is given from boat. The chief of party usually acts as signalman, directing work in the boat, and seeing that boat is kept on the ranges (if any are used). The signal is given by holding up a flag for about 10 sec and dropping it suddenly at instant the sounding is taken, at which moment the transitmen on shore read angles to the leadsman, or to the pole, if visible. In work of U S Engineers, white, red, and sometimes black flags are used for signaling. Both the recorder's and the instrument men's notes should report color of signal, as well as the time for each located sounding, thus giving two means of identifying angles with their corresponding soundings. This double check is of particular value in taking long lines of soundings. In waters with changing level, the time record is required also, to reduce soundings to Mean Low Water, or other datum. The tide-gage readings are recorded at regular intervals throughout the day.

When purpose of survey is to locate every slight change in shape of bottom, it is desirable to take soundings as frequently as leadsman can handle the pole, instead of at timed intervals. In this case, the boat is usually rowed on a range and the instrument men on shore "cut in" only those soundings that are designated by signalman in boat, the time being recorded to nearest second. As a rule every sixth or eighth sounding is located. In plotting such notes, the soundings being quite close together, the points that were "cut in" are plotted on map and intermediate readings are interpolated, the soundings of which are assumed to be equally spaced between the located ones, unless time record shows a different condition.

30. SURVEYS OF MINERAL LANDS

By Prof G. L. Hosmer, Mass Inst of Tech. Field notes furnished by Prof James Underhill, Colo Sch of Mines, author of "Mineral Land Surveying"

Requirements of the U S Mining Laws respecting location of a claim comprise: discovery of a mineral deposit, marking claim boundaries, posting location notice and filing certificate (Sec 24).

Legal dimensions of claim: Length, not more than 1 500 ft on course (strike) of vein; width, not more than 300 ft on each side of middle of vein; normal claim is thus 600 by 1 500 ft. State legislation may make the width less than the maximum, as in N and S Dak, Wyo, and in 4 counties of Colo (Sec 24). Two surveys are necessary: (*a*) location survey; (*b*) survey for patent.

Location survey, which is comparatively simple, may legally be made by the claimant himself, even without use of instruments; but it is advisable to employ a competent surveyor, to avoid possible loss due to inaccuracies. In making this survey, the strike of the vein is determined as accurately as possible, and used as the center line of claim. This line must pass through the point of discovery, its direction being referred to either

magnetic or true meridian; but, as the subsequent patent survey uses the true meridian, this is advisable also for the location survey (Art 13).

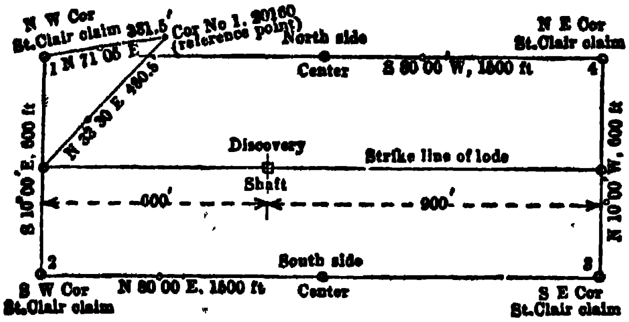


Fig 35. Typical Claim. NOTE —Corner angles may be greater or less than right angles (see Fig 36)

Distances may be measured on slope when necessary, and then reduced to horiz. Corner stakes are set, and marked with corner numbers and name of lode. A marked stake is also set at middle point of each side line. End lines must be parallel, but are not necessarily at right angles to center line. Claims may be of various forms (Fig 35–37), but always within legal dimensions. At least one point on the survey (usually a corner) must be tied to a corner of a neighboring patented claim, to a government survey corner or U S locating monument, or, in absence of these, to any permanent object. If other reference points are not obtainable, bearings of mountain peaks may be taken.

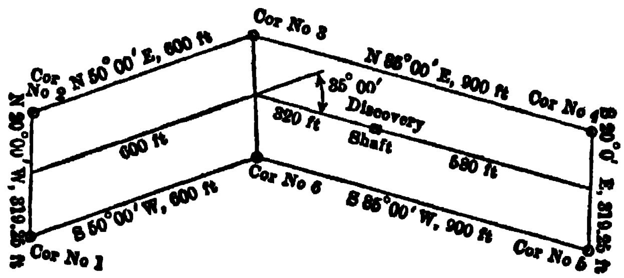


Fig 36. Claim with Broken Side Lines

Location certificate may be written by the claimant, but is usually drawn up for him by the surveyor. It must state: name of lode, name of locator, date of location, number of feet claimed on each side of point of discovery, description of boundaries by bearings and distances, and tie lines to established points. Following is a form that may be purchased in blank:

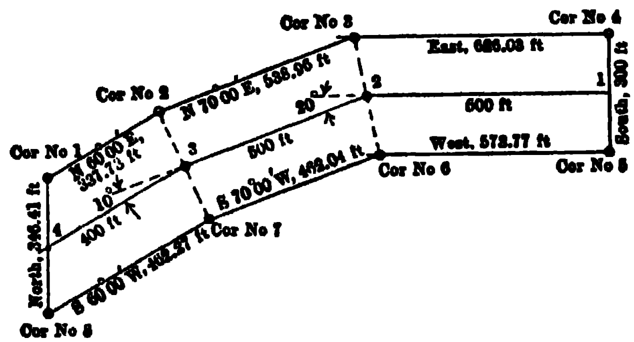


Fig 37. Claim with Broken Side Lines; End Lines Parallel

State of ______________ } ss.
County of ______________ }

Know All Men by These Presents: That ______________, the undersigned, has this ____ day of ______________, ______, located and claimed and by these presents does locate and claim by right of discovery and location, in compliance with the Mining Act of Congress, approved May 10, 1872, and all subsequent acts, and with local customs, laws and regulations, ____________linear feet and horizontal measurement on the ____________ lode, vein, ledge or deposit, along the vein thereof, with all its dips, angles and variations as allowed by law, together with 300 feet on ____________ side of the middle of said vein at the surface, so far as can be determined from present developments; and all veins, lodes, ledges, or deposits and surface ground within the lines of said claim, ________ feet running ____________ from center of discovery ____________ and ____________ feet running ____________ from center of discovery ____________, said discovery ____________ being situate upon said lode, vein, ledge or deposit, and within the lines of said claim in ______________ Mining District, County of ______________, and State of ______________, described by metes and bounds as follows, to-wit:

Beginning at Corner No 1 (here follows detailed description).

Said lode was discovered on the ____________ day of ____________, A. D. ________

Date of location, ____________ A. D. ________.

Date of certificate, ____________ A. D. ________.

______________________ (Seal)

Placer claims (see Sec 24 and Art 13). If situated on lands already surveyed by gov't, and the claim boundaries conform to gov't sub-divisions, no further survey may be necessary; elsewhere the boundaries must be acceptable to the Com'r of the General Land Office. In general, as the boundaries are meridians and east-west lines, an area much larger than the claim itself may sometimes have to be surveyed, to obtain a location that will be approved.

Mill sites are located and surveyed like mining claims, but may be on non-mineral lands only. Their area is limited to 5 acres, and boundaries must be straight lines (Sec 24).

Tunnel site gives a right of way for a tunnel, not more than 3 000 ft long. Rights covering veins cut by the tunnel, and not previously located, extend 1 500 ft on each side of center line. A dump area not over 250 ft square may also be taken. Center line of tunnel is staked out so that each stake can be sighted from the preceding one. The boundaries are also staked, the bearings being obtained by true meridian. Claims on veins cut by the tunnel may be patented, but tunnel itself can not be patented.

Survey for patent. Title to a mining claim is by deed or patent from the U S. A survey must be made by a commissioned U S Deputy Mineral Surveyor, permanent monuments set at the claim corners, and a preliminary plat prepared. Notes, plat. copy of location certificate, and affidavits of surveyor and assistants are sent to Surveyor General's Office, where they are examined and checked; if found correct, they are approved and copies returned to the Deputy, who delivers them to the claimant or his attorney.

Before the surveyor is officially in charge of a survey, the claimant must obtain an order from the Surveyor General; application for which states name and address of claimant, name of claim, name and address of Deputy Mineral Surveyor, and is signed by the claimant or his attorney. This must be accompanied by a certified copy of location certificate, and a certificate showing that the required fees have been paid. The surveyor can not act as attorney for the claimant. When the order is issued, the designated surveyor is directed by the Surveyor General to make the final survey. Although the surveyor is not authorized to do the work before receiving the order, a preliminary verification of the location survey is advisable, to discover possible errors or inconsistencies. If modifications are required, an amended location certificate or a relocation certificate may be filed, saving time and trouble later on. The order for survey contains the survey number to be used, and is accompanied by a certified copy of location certificate, which shows name of claimant, name of location, date of location, date and place of record and certificate of County Clerk.

In a patent survey the final claim must coincide with or lie wholly within the limits of the location survey. All bearings must refer to the true meridian (Art 13). Errors in closure not exceeding one part in 2 000 are permitted.

If the claim overlaps or conflicts with claims already patented or located, the surveyor makes all ties and measurements necessary for a closed traverse around the portion in conflict and for showing its exact relation to surrounding claims. The area of conflict is deducted from that of the claim being surveyed. Survey notes must give a complete statement of all such areas, and the net area claimed.

Field notes are written on printed forms furnished by Surveyor General, and must follow the form given in the "Manual of Instructions for Survey of Mineral Lands of the United States" (Circular No 430, General Land Office). They include a description of the claim boundaries by true bearings and distances, distances to intersections, computed areas, ties to patent corners, and distance and bearing along the lode line on each side of point of discovery. They must also certify that at least $500 worth of work or improvements has been done on the claim. In making this estimate the surveyor sees that it conforms to decisions of General Land Office (Sec 24).

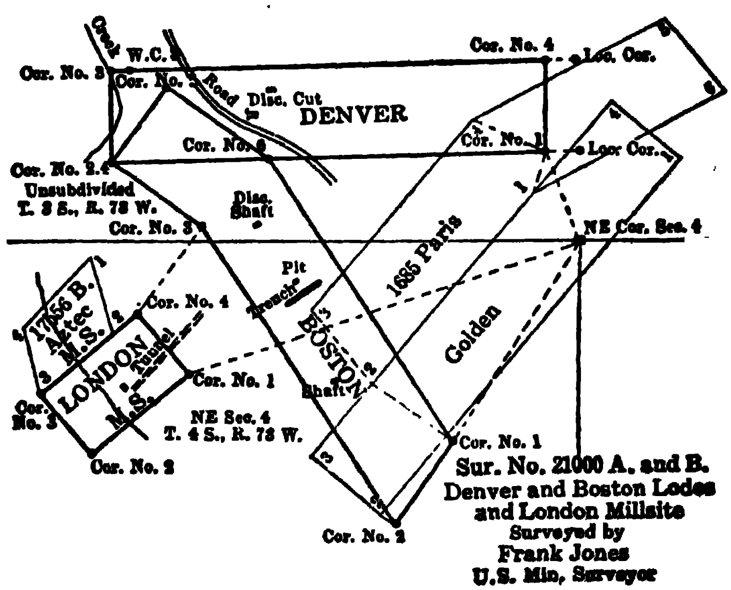


Fig 38. U S Mineral Surveyor's Plat of Patent Survey, showing Conflicts (from James Underhill's "Mineral Land Surveying," by permission)

Surveyor's plat (usually on scale of 1 in = 200 ft) shows the claim boundaries, with bearings, distances, all adjoining or conflicting claims, and all ties to corners or monuments. It must show lengths and bearings of all lines used in the notes. Fig 38 is the plat of the claims described in the following field notes (by Prof James Underhill). They contain additional data not on the map.

Survey notes, preliminary plat, affidavits of surveyor and assistants, copy of location certificate and the order from Surveyor General are sent to Surveyor General's office. When returned with approval, the Deputy surveyor delivers them to the claimant or his attorney. For his own protection and convenience, the surveyor preserves copies of all these documents, as he may have to make further surveys, or act as witness in court proceedings.

Example. Mineral Survey No 21000 A and B, Denver Land District.

FIELD NOTES of the Survey of the Mining Claim of TIMOTHY BROWN, known as the Denver and Boston Lodes and the London Millside, Coral Mining District, Clear Creek Co, Colo.

Unsubdivided Township 3 S. and Section 4, Township 4 S., Range 73 W. 6th P. M. Surveyed under instructions dated Sep 1, 1919.

by Frank Jones
U. S. Mineral Surveyor.

Claim located.........................., 19...; survey begun Sep 5, 1919; completed Oct 31, 1919. Address of claimant, Box 567, Denver, Colorado.

Denver Lode, Survey No. 21000A.

Beginning at Cor. No. 1. A spruce post 5 ft. long, 5 ins square, set 2 ft. in the ground with mound of stone, scribed cross (x) at corner point and D. 1–21000A, whence

The N.E. Cor. Sec. 4 T. 4 S. R. 73 W. 6th P. M. bears S 19° 30′ E, 309 ft.

Cor. No. 1, Sur. No. 1685, Paris lode, claimant unknown, bears S 19° W, 142 ft.

Position of Cor. No. 4, Sur. No. 1685 Paris lode, bears N 66° 36′ W 259.93 ft.

A corner of the location bears N. 88° 25′ E. 100 ft.

A cross (x) and B. R. D. 1–21000A chiseled 4 ft. above the ground on a granite cliff 20 ft. high bears N. 45° E. 25.3 ft. Thence S. 88° 25′ W.

Feet	
333.02	Intersect line 3–4, Sur. No. 1685 Paris lode, at S. 40° W. 146.77 ft. from Cor. No. 4.
770	Road, 10 ft. wide, course Southeast and Northwest.
896.67	Cor. No. 6 Boston lode of this survey.
1400	To Cor. No. 2; identical with a corner of the location. A rock in place 6 × 4 × 2 ft. above the general surface, chiseled cross (x) at corner point and D. 2–21000A. Thence N. 1° 35′ W.
85	Creek, 4 ft. wide, flows Southwest.
300	To Cor. No. 3, identical with position for a corner of the location. Not set, as it falls in the center of the creek, 4 ft. wide, flowing Southeasterly, where corner could not be established. Thence N. 88° 25′ E.
65	Witness Corner to Cor. No. 3. A granite stone, 27 × 12 × 8 ins., set 14 ins. in the ground, with mound of stone, chiseled cross (x) at corner point and W. C. D. 3–21000A, whence, a pine tree, 8 ins. diam. blazed and scribed B. T. W. C. × D. 3–21000A, bears N. 28° E, 15 ft.
250	Road, 10 ft. wide, course Southeast and Northwest.
1400	To Cor. No. 4. A cross (x) at corner point and D. 4–21000A chiseled on a ledge of rock, whence a corner of the location bears N. 88° 25′ E. 100 ft. Thence S. 1° 35′ E.
73.17	Intersect line 4–5, Sur. No. 1685 Paris lode, at N. 62° E. 263.08 ft. from Cor. No. 4.
300	To Cor. No. 1, the place of beginning.

Boston Lode.

Lode Lines. As near as can be determined from present developments, the vein of the Denver location extends 935 ft. N. 88° 25′ E. and 465 ft. S. 88° 25′ W. from face of discovery cut. The vein of the Boston location extends 1000 ft. S. 33° E. and 125 ft. N. 33° W, thence 375 ft. N. 55° W. from the discovery shaft.

Survey No. 21000B. London Millsite.

Beginning at Cor. No. 1. A granite stone 24 × 14 × 7 ins. set 13 ins. in the ground with mound of stone chiseled cross (x) at corner point and L. M. S. 1–21000B, whence, the N. E. Cor. Sec. 4 T. 4S. R. 73 W. 6th P. M. bears N. 71° 32′ E. 1331.22 ft. Thence S. 50° W.

Feet	
250	Creek, 4 ft. wide flows Southeast.
400	To Cor. No. 2. A pine post 4 ft. long, 4 ins. square, set 2 ft. in the ground with mound of stone, scribed cross (x) at corner point and L.M.S. 2–21000B. Thence N. 40° W.
250	To Cor. No. 3. Identical with Cor. No. 3, Sur. No. 17556B, Aztec mill site, claimant unknown. A rock in place showing 4 × 5 × 7 ft. above the general surface, chiseled 3–17556B, cross (x) at corner point, and L.M.S. 3–21000B. Thence N. 50° E.
160	Creek, 4 ft. wide, flows Southeast.
290	Cor. No. 2, Sur. No 17556B, Aztec millsite.
400	To Cor. No. 4. A granite stone 24 × 15 × 6 ins. set 14 ins. in the ground, with mound of stone, chiseled cross (x) at corner point and L.M.S. 4–21000B: whence Cor. No. 3, Sur. No. 21000A Boston lode, bears N 35° 40′ E. 350 ft. Thence S. 40° E.
250	To Cor. No. 1, the place of beginning. Variation at all corners 14° 30′ E. No bearing objects available at the several corners of this survey.

Area	*Acres*	
Total area, Denver lode		9.642
Area in conflict with Sur. No. 1685 Paris lode	1.032	
Total area, Boston lode		10.330
Area in conflict with Sur. No. 1685 Paris lode	0.919	
Golden lode, unsurveyed	2.160	
Denver lode of this survey	1.392	
Total area, London millsite		2.296

Surveys of Boston lode and London millsite are identical with their respective locations as staked upon the ground.

Location. This claim is located in the unsubdivided T. 3 S., R. 73 W., and the N.E. 1/4 of Sec. 4, T. 4 S., R. 73 W., 6th P M.

Expenditure of five hundred dollars. I certify that the value of the labor and improvements made upon, or for the benefit of, each of the lode locations embraced in said mining claim by the claimant or his grantors, is not less than five hundred dollars, and that said improvements consist of: The discovery cut of the Denver lode, the face of which, being the discovery point, is on the center line 465 ft. from the center of line 2–3, 5 ft. wide, 10 ft. face, running S. 88° 25′ W. 20 ft. to mouth. Value, $200.

No. 1. The discovery shaft of the Boston lode which is on the center line, 1 000 ft. from the center of line 1–2, 4 × 6 ft., 25 ft. deep. Value, $250.

No. 2. A pit on the center line 775 ft. from the center of line 1–2, 4 × 4 ft., 10 ft. deep. Value, $50.

No. 3. A trench, the east end of which bears from Cor. No. 6, S. 22° 20′ E. 425 ft., 4 ft. wide, 8 ft. deep, running S. 57° W. 130 ft. Value, $600.

A common improvement, being a crosscut tunnel 5 × 6 ft., the mouth of which, situated on London millsite, bears N. 77° 08′ W. 950 ft. from Cor. No. 2 Boston lode. Tunnel runs thence N. 43° E. 350 ft. to breast. Value, $3 500.

This improvement is in course of construction for development of Denver and Boston lodes, being all the contiguous lode locations owned in common within the range of benefit, and as described has been wholly constructed subsequent to the time since contiguity and common ownership have prevailed as between said lodes. The surface embraced in this claim ascends rapidly from mouth of tunnel towards Cor. No. 4, Denver lode. Therefore, the tunnel, continued with laterals run therefrom, will cut the veins of said locations at great depth, and thus afford an advantageous and economical means of development. An undivided one-half interest in the value of the improvement is therefore hereby credited as patent expenditure to each lode. No portion thereof or interest therein has heretofore been credited as patent expenditure.

Other improvements. A shaft on Boston lode, 4 × 6 ft., 30 ft. deep, which bears from Cor. No. 2, N. 23° 45′ W. 510 ft. Claimant unknown.

A cabin on Denver lode, the N. W. Cor. of which bears from Witness Corner to Cor. No. 3, S. 83° 25′ E. 445 ft., 25 × 15 ft., course of long sides East. Claimant unknown.

A compressor house on London millsite, the S.E. Cor. of which bears from Cor. No. 1, S. 72° 25′ W. 205 ft., 20 × 15 ft., course of long sides S. 75° E. Claimant herein.

Instrument. These surveys were made with a Buff and Berger transit, No. 2 345. The courses were deflected from the true meridian as determined by direct solar observation. The distances were measured with steel tapes.

Report. The boundary lines and connections of these surveys, as well as the lode lines of the two lode locations, were run directly upon the ground. The N.E. Cor. Sec 4, T. 4 S., R. 73 W. 6th P. M. is a stone properly marked. The N. 1/4 Cor. of said section is missing. The N.W. Cor. Sec 4, T. 4 S., R. 73 W. 6th P. M. is a stone properly marked. The N.W. Cor. Sec 4, T. 4 S., R. 73 W. 6th P. M. bears West 5 280 ft. from the N.E. Cor. said section.

Sur. No. 1685 Paris lode: Cors. Nos. 1 and 2 are stones properly set and marked. Cor. No. 6 is a post, standing, properly set and scribed. Cors. Nos. 3, 4 and 5 are missing. Lines 1–2 and 6–1 were found correct as approved. Lines 2–3, 3–4 and 4–5 are shown as approved. From Cor. No. 1, the N.E. Cor. Sec. 4, T. 4 S. R. 73 W. 6th P. M. bears S. 43° 34′ E. 216.71 ft., instead of S. 42° E. 220.5 ft. as approved.

Sur. No. 17556B. Aztec millsite: Cor. No. 3, identical with Cor. No. 3, London millsite of this claim, and Cor. No. 2 are rocks in place properly marked. Line 2–3 was found to be S. 50° W. 290 ft. instead of S. 47° W. 300 ft. as approved.

Location certificates. (NOTE by Editor of this Handbook: Certificates of above LODE CLAIMS are drawn up in the form shown near beginning of this Art.) Certificate for the London millsite:

STATE OF COLORADO }
County of Clear Creek } ss.

TO ALL WHOM THESE PRESENTS MAY CONCERN:

Know ye that I, JAMES FRANKLIN, of Denver, Colorado, do hereby declare and publish as a legal notice to all the world that I have a valid right to the occupation, possession and enjoyment of all and singular that tract or parcel of land not exceeding five acres, situate, lying and being in Coral Mining District, in the County of Clear Creek, in the State of Colorado, bounded and described as follows, to wit: The London millsite, beginning at Cor. No. 1; thence S. 50° W. 400 ft. to Cor. No. 2; thence N. 40° W. 250 ft. to Cor. No. 3; thence N. 50° E. 400 ft. to Cor. No. 4; thence S. 40° E. 250 ft. to Cor. No. 1, the place of beginning. Cor. No. 3 is identical with Cor. No. 3, Sur.

No. 17556B Aztec Millsite, together with all and singular the hereditaments and appurtenances thereunto belonging or in anywise appertaining.

Witness my hand and seal this 6th day of January, in the year of our Lord one thousand nine hundred and ten. James Franklin, SEAL.

Appended hereto are FINAL OATHS of the U S Min Surveyor and his assistants, that they legally made the above surveys, and that the improvements and expenditure on each claim are as stated.

31. MINE SURVEYING (See Section 18, on "Underground Surveying")

32. RAILROAD LOCATION

Preliminary steps. RECONNAISSANCE is an examination of the country through which proposed railroad is to run, for purpose of choosing the general route (or routes) deemed worthy of further investigation. PRELIMINARY SURVEY is a detailed mapping of route chosen as a result of reconnaissance. LOCATION SURVEY follows, consisting in staking out the railroad as straight lines and curves. A mining engineer is often called upon to lay out highways, or branch railroad lines and sidetracks. The treatment here given will, therefore, refer to branch-line work, although the same principles apply to main lines. If reconnaissance be carelessly made, subsequent surveys will not rectify such defect.

The first step is to obtain best available map of the country. For this purpose the U S Geol Surv maps are particularly valuable, because they give elevations as well as flat topography. They can be obtained for 10¢ each from office of U S Geological Survey, Washington, D C. On these maps, a paper reconnaissance can be made in the office, probable grades and alinement can be studied, and a very good general idea obtained without going into the field. Such paper reconnaissance ought always to be followed by an examination of the ground, to determine character of excavation, bridge crossings, and other important details. Large portions of the country have not yet been surveyed by the U S Geol Surv, and it is not uncommon for an engineer to find himself without a map to assist him. In this case he should travel over the most favorable routes, obtaining distances by pacing or stadia, and elevations by aneroid barometer and hand-level; these data are incorporated into a rough reconnaissance map. He should take into consideration not only first cost of road, but also that of maintaining and operating it, noting, therefore, grades, curvature, length of line, earthwork, character of soil, bridging, tunneling, and drainage. MOSAIC MAPS, based upon aerial surveys, are of great value. They may be in form of strip maps of different locations under consideration, and used in addition to or in substitution for existing maps. They are rapidly made, give dependable results both in plan and elev, and may even eliminate necessity for a preliminary survey (Art 28).

A ridge location usually has advantage of affording good drainage. A valley location usually permits better grades, but almost always requires more bridges and culverts, and is more subject to washouts. A sidehill location, where slope is not too irregular, usually has advantage that proposed line can be placed at any desired grade. Often, in mountainous regions, operative grades can not be obtained, even in bottoms of flattest valleys; the road must then be purposely lengthened by entering lateral valleys or by looping upon itself. In routes which cross drainage systems, it is important to determine the lowest passes and use these points as governing the location; in some cases it is advisable either to tunnel at the pass or to operate for a short distance over a steep grade, with a helper engine or by breaking the train in halves. In such locations grade is almost always of much greater importance than length of line or amount of curvature, although due regard must be given to sharpness of curvature, for reasons given below.

Preliminary survey is a topographic survey of the belt in which probable location of the railroad will lie; it will first be run as a transit and tape traverse to be used as a backbone for topographic details. An accurate preliminary survey aids greatly in the location survey which is to follow. Distances are measured usually to 0.1 ft, and deflection angles to nearest minute; in some cases distances are read by stadia. This map is usually plotted on a scale of 100 to 400 ft to 1 in. A preliminary party in open country will cover from 5 to 8 miles per day. The level party following the transit party establishes bench-marks and determines the profile along preliminary line.

Topographic details are sketched to scale on cross-section paper in the field, using elevations given by level party as bench-marks; a stadia survey or a plane table is well adapted to this work in open country; in wooded country a hand-level is satisfactory. A 5-ft contour interval is customary. The preliminary map and topographic details should be plotted at close of each day, from which the locating engineer will judge from day to day whether to push ahead on the line adopted, or to retract and run line over different ground. Not infrequently preliminary surveys of more than one line are made. The kinds of material encountered along the route are noted and the characteristics of bridge crossings. Upon the preliminary plan the locating engineer can draw the location line, composed of straight lines and circular curves, and from the contours a profile of proposed

location line can be plotted. From a study of the different lines and profiles the engineer will determine a final line, which will be drafted upon the preliminary map, and run out by the party as center line of the location.

Location line, as laid out on the preliminary map, will be so chosen as to pass through or lie near controlling points, such as bridge sites and passes, and to minimize total amount of earthwork. Due regard is given to grades and characteristics of the alinement. On curves, the rate of grade should be flattened about 0.04% per degree of curvature, to neutralize additional train resistance; called CURVE COMPENSATION.

It is good practice to lay out grade line for a branch connection as nearly level as practicable for about a train length from its junction with main line. Similarly a flat grade should be given for short distance in vicinity of a mine; this permits locomotive to start its train before climbing up-grade, or, if grade is down, it prevents loaded cars from running away. These flat grades are advisable, from an operating standpoint, even if, to obtain them, it is necessary to steepen other grades. Wherever the line involves a cut more than 500 ft long, it is advisable to let this occur on grades as steep as 0.2%, to provide drainage. In passing through low flat land, the line should be built a few feet above meadows or swamps, with ample culverts for brooks, and the grade should be high enough to permit easy construction of such culverts. It is bad judgment to economize on the size of culvert openings. The width of cuts, at the base, is usually made about 6 ft wider than fills, to provide for side ditches.

When the location line has been finally drawn on map, it will probably cross and recross the preliminary traverse several times. Scaled distances along the traverse line will fix points on location line, to be used by the field party in laying out the tangents of location line. Sometimes when the possible location is constricted, as in a cañon, the final location is worked out by the field party and the preliminary survey omitted. While location line is actually being run out, minor changes are often made on encountering physical conditions not recorded on preliminary map.

In staking out the center line of location, each tangent is first defined and run to an intersection with adjacent tangents; this locates vertices of the curves. The curves are then staked out by DEFLECTION ANGLES. Measurements are taken to nearest 0.1 ft, and angles to nearest minute; a stake is driven at every full station point and at the beginning and end of every curve. The beginning of the curve is called the "T C" (tangent to curve) and the end the "C T" (curve to tangent). A level party, immediately following the transit party, obtains data for a profile, reading surface elevations to 0.1 ft, and T P's and B M's to 0.01 ft. When final alinement has been staked out, the level party, using grade line of location profile as a basis, takes cross-sections and sets grade and slope stakes for construction at every station (Art 19).

Circular curves are used to connect tangents of a branch track, the refinement of transition curves not being necessary. The point at which two tangents meet is called the P I (point of intersection). The deflection angle at intersection of tangents is called the intersection angle, and is equal to the angle subtended between radii drawn to the T C and C T. The length of curve is distance from T C to C T, measured by 100-ft chords (a sub-chord may occur at one or both ends of a curve). Many engineers use the letters P C (Point of Curvature) and P T (Point of Tangency) instead of T C and C T. The degree of curvature D is the angle at center subtended by a chord of 100 ft. A chord less than 100 ft is called a sub-chord and its central angle is a sub-angle. The relation between R and D is expressed by formula $\sin {}^1/_2\, R = 50 \div R$.

In the metric system, the degree of curvature has been defined in two ways: (*a*) As the angle at center subtended by a chord of 20 m. (*b*) As the deflection angle of a chord of 20 m.

Fig 39 represents a simple circular curve. Following terms apply to its elements:

ob = Radius = R	cd = Middle ordinate = M
ab = Chord = C	cPI = External distance = E
aPI = Tangent distance = T	hg = Offset from tangent = t

The following relations between these elements are true when I is less than 180°:

$$\sin {}^1/_2\, D = 50 \div R$$

$$D_{X^\circ} = 5\,730 \div R_{X^\circ}\ \text{(approx)} = T_{1^\circ} \div T_{X^\circ}\ \text{(approx)} = E_{1^\circ} \div E_{X^\circ}\ \text{(approx)}$$

$$R = 5\,730 \div D\ \text{(approx)} = {}^1/_2\, C \div \sin {}^1/_2\, I = T \cot {}^1/_2\, I$$

$$C = 2\,R \sin {}^1/_2\, I = 2\,T \cos {}^1/_2\, I = 2\,M \cot {}^1/_4\, I$$

$$L = 100\, I \div D\ \text{(length of curve in R R and some rural highway practice)}$$

$$T = R \tan {}^1/_2\, I = {}^1/_2\, C \div \cos {}^1/_2\, I = E \cot {}^1/_4\, I$$

$$E = R \operatorname{exsec} {}^1/_2\, I = T \tan {}^1/_4\, I = M \div \cos {}^1/_2\, I$$

$$M = R \operatorname{vers} {}^1/_2\, I = R - \sqrt{(R + {}^1/_2\, C)\,(R - {}^1/_2\, C)} = C^2 + 8\,R\ \text{(approx)}$$

$$t = \text{chord}\ bg^2 \div 2\,R$$

In highway location the length of a curve is usually considered as its actual arc, in which case the degree of curvature D_1 is defined as the central angle for an arc of 100 ft. The central angles of any part of the curve are then proportional to the arcs they subtend. In computing such cases, the above formulas for C, T, E, M, and t apply.

Arc $bca = \pi RI \div 180° = R \times$ (length of arc for radius 1) $= C + (C^3 \div 24\,R^2)$ approx

$C =$ arc $bca - (\text{arc}\ \overline{bca}^3 \div 24\,R^2)$ approx

$R = 18\,000 \div \pi D_1$; L (arc) $= 100\,I \div D_1$.

Deflection angle is the most common method for locating simple curves. To lay out a curve by this method, set transit at the T C, the vernier reading 0°, and sight on A

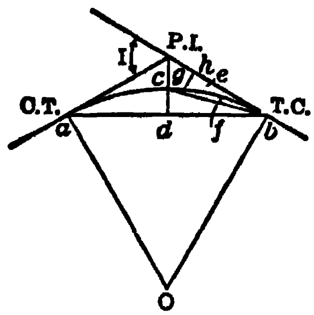


Fig 39. Elements of Circular Curve

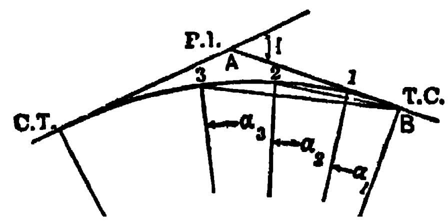


Fig 40. Laying Out Curve by Deflection Angles

(Fig 40). Lay off deflection angles AB 1 and measure chord B 1, thus locating point 1. Angle $AB\ 1 = {}^1/_2\,\alpha_1$, and $\alpha_1 = D \times B\ 1 \div 100$. Leaving lower plate of transit clamped, loosen upper motion, lay off $AB\ 2 = {}^1/_2\,\alpha_2$ and measure chord 1–2, and so on, setting each stake and checking at end of curve on the C T stake. The total deflection angle to any point on curve is evidently half the central angle between T C and that point; where stations are 100 ft apart, the deflections increase by $^1/_2\,D$ for each successive station.

Deflection angle for any sub-chord $= cD \div 200$ (in degrees) $= c \times 0.3 \times D$ (in minutes, when D is in degrees).

Example. Find deflection angles of a 6° curve connecting T C 8 + 41.7 and C T 12 + 73.4, the curve being to right.

$58.3 \times 0.3 \times 6 = 105' =$	$1°\,45'$	to Sta 9
$1°\,45' + 3° =$	$4°\,45'$	" " 10
$4°\,45' + 3° =$	$7°\,45'$	" " 11
$7°\,45' + 3° =$	$10°\,45'$	" " 12
$73.4 \times 0.3 \times 6 = 132' =$	$2°\,12'$	
	$12°\,57'$	to Sta 12 + 73.4

C T 12 + 73.4 − T C 8 + 41.7 = 431.7 = length of curve.
$4.317 \times 6° = 25°.902 = 25°\,54' = I$; and $12°\,57' = {}^1/_2\,I$.

This checks deflection angle from T C to C T. As a rule deflection angles should be computed to nearest half-minute.

Main-line locomotives, as a rule, are limited to curves of 16°–20°, depending upon their length of wheel base and number of wheels having flanges. Some passenger coaches will not go around a 20° curve unless the truck swivel-chains are disconnected. A 4-wheel switching engine will, as a rule, go safely around a curve of 75-ft radius, and a 6-wheel switching engine around one of 90- to 150-ft radius, depending upon its design. A single box car can be pulled around a curve of 50-ft radius, and two box cars around curves of 80- to 100-ft radius if a special long-link coupler be used between cars, and between tender and locomotive. On a curve of about 140-ft radius the corners of freight cars are liable to collide if the equipment has the ordinary M C B coupler. On all especially sharp curves the gage should be properly widened, the rails braced, guard rails set, and curvature maintained uniform. STANDARD GAGE is 4 ft 8.5 in. The widening of gage may be as much as $^3/_4$ in on sharpest curves. It is advisable to lay guard rails on all curves sharper than 14°. It is of importance to lay out a branch line so that a main-line locomotive can run over it; this can be done if the curves are of not less than 300-ft radius.

Cost of branch R R lines varies with amount of earthwork and bridging required. Under ordinary conditions, where cuts and fills average about 5 ft, total cost per mile of single-track branch line is about \$20 000. Ballast and tracks in place will cost from \$2.00 to \$3.00 per linear ft of track. Following estimate assumes ties at 80¢ each, that gravel ballast is obtainable within a few miles, that ballast is 12 in deep under ties, and that relayer rails are used, costing say \$28 per ton (that is, rails previously used on main-line tracks elsewhere, but still serviceable for branch track):

	Cost per ft of track
Gravel ballast @ $1.10 per cu yd in place........	$0.55
Ties, 24 in c-c, @ 80¢ each.....................	0.40
Relayer 85-lb rails, @ $28 per ton..............	0.75
Rail splices, bolts and spikes..................	0.20
Laying and surfacing track, without tie plates....	0.50
Total..................................	$2.40

Cost of turnouts, including rails of both tracks, turnout ties, frogs, switches and guard rails complete, is from $500 to $750 each, for lengths of 70 to 100 ft from point of switch to point of frog.

Wagon roads can be constructed, where excavation and filling is earth, where road follows substantially the ground surface, and few cuts are greater than 3 ft, for $1 000–$4 000 per mile. These costs assume that little material must be hauled and placed upon subgrade, to make a suitable surface for heavy teaming: it also assumes a road with travel way 10 ft wide and occasionally wide enough for teams to pass. To surface a mile of road 10 ft wide with 12 in of gravel, obtained within one mile, will cost from $2 000 to $4 000.

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SECTION 18

UNDERGROUND SURVEYING

BY

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

UNDERGROUND SURVEYING

Introductory. An underground survey is based on precisely the same principles as one on the surface. The methods of reading angles and measuring distances, of computing azimuths, bearings, latitudes and departures, coordinates, and elevations, and of plotting the results, are alike in all essentials; for these principles, reference should be made to the preceding section. In their practical details, however, a number of differences are introduced by certain peculiar conditions under which a mine survey has to be conducted. These conditions are: 1. Working in the dark requires that both the point to be sighted and the cross-hairs of a telescope must be illuminated. 2. Working in constricted places, and in passages through which traffic must not be interrupted, often requires the use of special devices for mounting the instrument. 3. Stations must be placed in the roof, when possible, to assure their permanence and visibility. 4. Steeply inclined sights are more prevalent, requiring that a transit shall be always in perfect adjustment; with very steep sights, an auxiliary telescope is required, involving a correction in the reading of angles. 5. A higher degree of accuracy is usually required than in ordinary land surveying; in this respect, mine surveying is comparable with the locating of bridge piers or the boundaries of city real estate. This is because, in most of the important problems in mine engineering, aside from mere m.pping, an error means loss of time and money in the subsequent mining operations. Furthermore, it frequently happens that a complete circuit can not be made, whereby the error of closure can be determined and distributed, or a mistake located and corrected; hence careful work by refined methods is the only assurance of accuracy. 6. Shaft-plumbing is a special problem, peculiar to underground surveying.

1. STATIONS

Judicious selection of the points to be occupied as stations will facilitate subsequent operations, and extra time devoted to this end is well spent. In modern coal mines, where the workings are regular and long sights possible, it is customary for the head chain man to select and put in the stations as the party proceeds; but in most metal mines, especially if an entire new survey is to be made, it is advisable to select the points throughout the whole workings before beginning the instrumental work. Three men can do this to best advantage. At intersections of workings the point should allow sights to be taken in all directions; intermediate points should allow the sights to be as long as possible. Ordinarily, little advantage is gained by attempting to set a series of points in line, with the idea of reducing office calculations; an exception may occur in the case of a series of chutes along a straight drift, up each of which it is proposed to run short, independent surveys. Sub-stations for this purpose can be set in line with sufficient accuracy by a transit in the drift. When dodging corners in a crooked passage, a longer sight can sometimes be obtained by making an abnormally low or high set-up, at the cost of a little temporary discomfort to the instrument man; this is particularly true of inclined workings, where projections in roof and floor, as well as in the walls, have to be dodged. In passages where traffic is heavy, or where the roof is so low that a projecting spad might be disturbed, stations should be near the wall. In setting a station at which the side auxiliary telescope must be used (Art 4), an additional allowance of space must be left along both sides of both lines of sight.

Fixing stations. In important surveys, and where permanency is desired, stations should be fixed in the rock roof of the workings. For temporary work, stations may be fastened to timbers, or may be set on the ties or rails of a track, or in the floor; but in such cases, permanent roof stations should be placed at frequent intervals. This is best done by drilling a hole in the roof, inserting a wooden plug, and driving a spad into the plug. To save labor of drilling, the hole should be as small as possible; there is no advantage in having it larger than $^1/_4$ in. If the roof is high, or the rock so hard as to require double-hand drilling, a hole of 1 to 1.5 in diameter may be unavoidable. A special drill, 8 in long, $^3/_4$ in diameter, drawn out small at one end and sharpened to a chisel edge $^1/_4$ to $^1/_2$ in wide, used with a 2-lb hammer, is good for this work. In the soft roofs of some coal mines a twist drill will bore rapidly. In any case, the hole should be not less than 1.5 in deep.

In coal mines with thick seams, "jigger" stations are still occasionally employed. A conical depression is drilled in the roof with a pointed steel bar, the other end of which has a device for holding a plumb-bob string exactly at the center of the hole; also a clip holding a paint brush with which to describe a circle around the station.

Plugs should be of soft wood, preferably turned in a lathe, from dry stock, rather than whittled from wet wood; the dry plug is less likely to drop out. Some surveyors soak

plugs in creosote, to prevent decay. Still greater permanency can be secured by using plugs made of soft lead. Plugs should be so proportioned to the size of the holes that they will drive in snugly without mashing, and be flush with the roof.

Spads. The design and composition of spads in general depend upon the importance and permanence of the survey. In a wet mine, especially if sulphur be present, a plain steel spad will not last more than a few weeks; a soft-iron spad will last a little longer, but nothing will resist ultimate corrosion but copper, brass, bronze, zinc, or chrome-steel. A spad should be so designed as to insure that a cord of any size, at any time, will be obliged to hang in exactly the same position. A spad in the form of an open hook is better than one having an eye, because it facilitates the hanging and unhanging of the plumb-bob.

The commonest varieties of spads are (Fig 1): (*a*) Screw-eyes of iron or brass; an iron screw-eye, with a large eye, is objectionable because rust may prevent the cord from hanging always from the same point, while one having a small eye is generally too short on the screw end to hold firmly in the plug. (*b*) Staples and screw hooks; open to the same objections as screw-eyes. (*c*) Finishing nail, with the head bent to form a sharp hook; excellent while it lasts. (*d*) Flat-headed wire nail with a notch filed in the edge; gives accurate position, but it is not easy to adjust quickly the length of a plumb-bob cord when setting up transit. This drawback can be overcome by carrying a short piece of cord with a loop in its lower end, through which the bob cord can be passed; the upper end is wound around the nail. (*e*) Horse-shoe nail, with head flattened, and a hole punched or bored through it; very satisfactory, but subject to corrosion. This spad is improved by making the hole triangular, point down, either by filing, or by using a triangular punch made from a file. The long point of the nail should be cut off, leaving a total length of 1.5 in. (*f*) Flattened horseshoe nail with a notch sawed or filed in edge of head; if sawed, the slit should be made wide enough to allow for rusting. (*g*) Montgomery's spad, sold by survey supply houses, has the advantage of accurate centering and easy manipulation of cord; could be improved by making it of brass, or Cr-Ni steel, instead of soft steel. (*h*) Specially designed spads, as used in the anthracite fields, cast from copper, brass, or zinc, are extra strong and durable, even in acid water. Tinned iron spads of the same design are serviceable. (*i*) Loop of fine copper wire (No 26 B & S) inserted into a $^1/_{16}$-in hole, bored by twist drill, and held up by inserting a shoe-peg; limited to soft roofs, and is not durable nor easily manipulated. (*j*) Floor spad, as used in the Baltic and Champion copper mines, is made from a 3-in wire nail, by drilling an axial hole through the head, large enough to stick a match into. The nail is driven flush into a tie, a match inserted into the hole, illuminated by two candles, and used as a sighting point; subject to the drawbacks of all floor stations, and is specially unreliable when placed in a track-tie. (*k*) A boiler rivet of convenient size and length, split part way by a saw, makes an excellent floor station for chambers so high that roof stations are impossible. A hole is drilled in the solid floor, the rivet is inserted, with a steel wedge to spread it, hammered snug, and the point marked in the head with a center-punch. (*l*) Spad used at Broken Hill South mine, N S W (19), is a strip of brass, $^1/_8$ by $^3/_4$ by 3 in, with notched edges and beveled lower end, through which is a rounded triangular hole, point down; the spad is held in a 1-in hole about 4 in deep by ramming with slightly moistened neat cement, which sets hard enough for use in 15 min; outer end projects only slightly and is well protected.

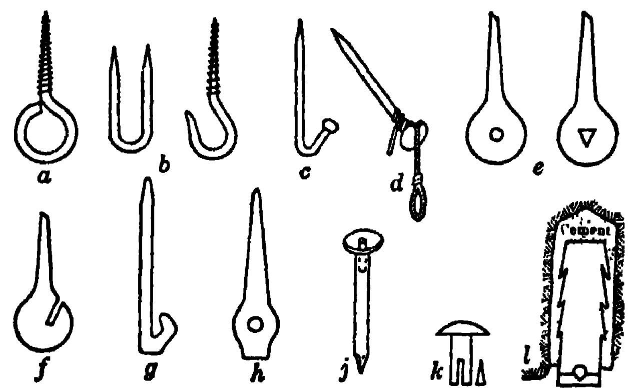


Fig 1. Varieties of Spads

Stations are marked most frequently by red or white paint, applied with a 1-in round brush. A small paint container, holding only enough for one day's work, should be carried, as the paint quickly becomes dirty. The 1-lb carbide container of an acetylene lamp, having a tight screw cover, is excellent for this purpose. Water, mud, or dust should be cleaned from the rock before applying the paint; for this, the flat bristle brush used for cleaning sinks is serviceable. A system of marks will prevent confusion of spads; thus, circles for transit points, squares for benchmarks, crosses for line points, etc. The number of the station may be painted near it, or it may be stamped with steel dies on a tag made of sheet copper, lead, brass, or zinc. When a tag is used, the spad should be driven through a hole in it, rather than to hang the tag from the spad by a wire.

Systematic numbering has many advantages, though some important mines number stations serially in order of setting, and maintain a reference index to identify them. The advantages are: (*a*) Knowing where to go to find a station having a given number. (*b*) Knowing where you are by the number of a station; this also simplifies the notation of samples and assays. (*c*) Being able to identify a station even if its tag or painted number has vanished.

The system must be adapted to the particular mine. In general, it is a good plan to reserve 100 or 1000 numbers for each level; if the workings extend in two general directions, odd numbers may be given to the stations on one side of the shaft, and even numbers on the other. The numbering of stations by their coordinates has been suggested (1), with a letter or Roman numeral to indicate the level. This involves delay until the coordinates have been computed, and the use of unwieldy numbers in the notebooks.

2. ILLUMINATION

Of sighting point. (*a*) By candle or lamp held in back of a translucent screen behind the plumb-bob string. A screen of tracing cloth is good in a dry mine; in a wet mine oiled paper or silk is better. The fabric can most conveniently be stretched smooth by a pair of embroidery hoops, of about 4-in diameter. (*b*) By acetylene lamp, having a 3-in round reflector, or a special reflector with a white enameled face made for this purpose. When using this light, no translucent screen is required, as the reflector gives a bright background. (*c*) Special lamp targets, burning either lard oil or acetylene, used only in connection with the three-tripod method of traversing, described later. (*d*) Plummet lamps, burning sperm or lard oil. These are so constructed (Fig 2) that the point of the bob, and the wick, hang accurately below the station. The hook may be hung directly in the spad, or from a piece of cord, by which its elevation can be adjusted. With long sights in a clear, still atmosphere, the flame affords an excellent sighting point; with shorter sights, the wick tube is bisected. For surveying in gassy coal mines, a plummet safety-lamp has been made on the same principles. (*e*) Specially designed "backsights" of various forms are in use in many mines, their purpose being to economize the services of a man to hold the light. Probably the most satisfactory mechanical backsight is a light tripod, supporting an acetylene lamp, which can be quickly set behind the cord of an ordinary plumb-bob. Instead of a lamp a candle sconce having a translucent screen is often used.

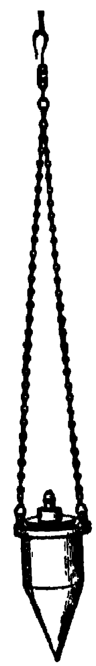


Fig 2. Plummet Lamp

Of cross-hairs. With short sights against an illuminated screen, and even with long sights against an acetylene lamp, no special illumination of the cross-hairs is necessary. When illumination is required, the oblique annular reflector supplied by all instrument makers, for attachment to the end of the telescope, is entirely satisfactory. A similar reflector can be improvised by curling a visiting card and putting it into the front of the ordinary transit sun-shade. Interior reflectors, receiving light through a hollow trunnion of the telescope, are not recommended, owing to their partial interference with line of sight.

Instrument readings are generally made with whatever form of light the surveyor carries. Compass readings must be made with a candle, or a copper lamp. Electric flash lights are convenient for reading verniers. Instrument makers sell a combination electric lamp and magnifying lens for this purpose. If a plain lens is used, it should have a metal case, not celluloid or rubber. For all-round use nothing excels an acetylene lamp, with spherical reflector. The objection of some surveyors that an acetylene lamp, especially of the larger sizes, does not leave the hands free to operate the instrument, can be overcome by hooking the lamp on a tripod leg, or in a button-hole or collar of the coat, or on a strap around the neck.

3. TRANSIT MOUNTINGS AND PLUMB-BOBS

Tripods. For underground work, tripods having extension legs are a necessity. The clamps and rings should be extra loose, to allow for swelling of the extension piece in the mine dampness. The steel points of the legs should be sharpened from time to time, for working on inclined rock floors. Short tripods, allowing the center of a transit to be set comfortably within 30 in of the floor will be found convenient under low roof. For extra low set-ups or in awkward places, as in a timbered stope or a shaft bottom, a trivet (Fig 3) can be set on a firmly fastened plank.

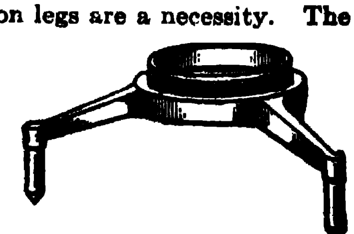


Fig 3. Trivet

Bracket. In timbered drifts, where a tripod would be in the way of traffic, and in constricted places where no tripod can be set, the bracket shown in Fig 4 is useful. A

sound and firm timber must be selected; a hole is bored horizontally at the proper spot, and the bracket is screwed snugly into place with a steel bar fitting through the hole provided for that purpose. The screw-plate is then leveled roughly by the clamp joint, the transit is screwed on, leveled accurately, and brought exactly under the point by the sliding head, the station having previously been fixed with this operation in view.

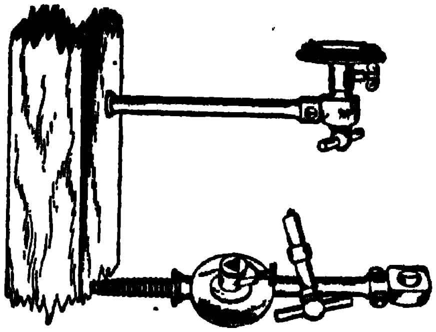


Fig 4. Transit Bracket

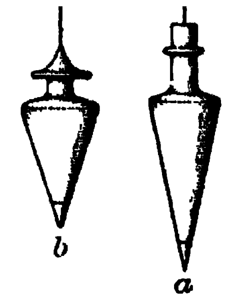


Fig 5. Plumb-bobs

Plumb-bobs. In draughty workings heavy bobs, weighing 14 to 18 oz, should be used. The instrument bob should preferably be of the shape shown at *a*, Fig 5, with a fine point. For sighting bobs the shape shown at *b* is better, as the upper end gives a more definite point to observe on inclined sights when reading vertical angles. Bobs with interior reels are unreliable in underground work, where the cord soon gets wet and dirty; neither are they necessary.

Plumb-bob cord should be braided, not twisted, to avoid spinning of the bob, and raveling at the end; for the latter reason cord having a tubular weave with a core is objectionable. Linen is better than silk for underground work, because a wet knot is easier to untie. When a cord acquires a permanent knot, discard it. Adjusting devices of wood, leather, or buttons, tend towards inaccuracy and serve no useful purpose; a sliding bow knot can be tied in a second, untied by a single pull on the loose end, and readily adjusted with one hand.

4. MEASURING HORIZONTAL ANGLES

By compass. Owing to the omnipresence of steel rails, pipes and cables, trolley wires and electric lighting and signaling circuits, in modern mines, the compass can seldom be relied upon for direct observation of bearings. For accurate work, also, readings to the nearest quarter-degree are not close enough. For less important work the compass can be used for reading an angle by noting the bearings of the two lines converging at the station, and computing it. Local attraction will not affect the accuracy of an angle reading made in this manner. Correct bearings are then computed from the angles, on any desired meridian base. The amount and direction of magnetic declination is immaterial, and need not be set off for underground surveying, unless the compass is being used for supplying details, as in a stope, which are to be tied to an instrumental survey based on true meridian.

The hanging compass (Fig 6) has great utility for traversing through tortuous chutes, or irregular stopes. A strong linen cord is stretched from station to station, which may

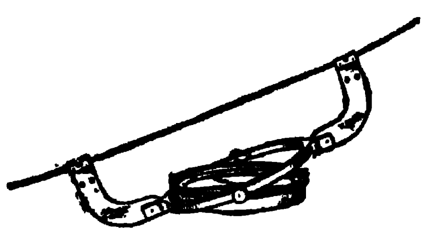


Fig 6. Hanging Compass

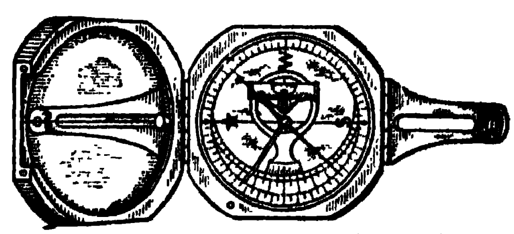


Fig 7. Brunton "Pocket Transit"

be merely of temporary character, like a stick braced from wall to wall. Special brass lugs are provided for screwing into timbers, but a heavy nail answers as well, unless a compass reading is to be taken directly under the station. The compass is then hung at any convenient point on the cord; if the cord is inclined, wooden clips are provided to keep the compass from sliding. For reading angles, where local attraction is suspected, a special hanging appliance is provided, permitting the compass to be brought under the station, to read the bearings of both lines from the same position.

The Brunton "pocket transit" (Fig 7) is a magnetic compass designed for rapid work, usually being held in the hands while taking observations. The cover is hinged perpendicularly to the sighting axis, and has a mirror on its inner side, on which a black line is scratched in continuation of the *N–S* axis; the cover and mirror are pierced with two small

holes on this line, one near the hinge, the other near the opposite edge. On the side opposite the hinge is a folding vane with a sighting slot. In use, the compass is held vertically below the eye, and below the station, the cover opened about 135°, the level tube turned at right angles to the line of sight, and the sight slot lifted to the vertical position. Looking down at the mirror, the operator can then see the reflection of the forward slot and at the same time observe that the instrument is level: the compass is then turned until a beam of light from the other station is seen to pass through the slot and be bisected by the black line on the mirror. The needle is then read. Or the instrument is held on a level with the eye, with the lid open only 45°; sight through the hole near the hinge, and read the needle from its image in the cover. A similar instrument, the Verschoyle "pocket transit," is preferred by some engineers.

The transit is universally employed for accurate work. Any transit having a full vertical circle can be used for mining work involving no vertical angles over 60°, but where underground surveys of magnitude and importance are to be conducted regularly, it is advisable to have an instrument designed for the purpose.

The special features of a mining transit are as follows: (*a*) The construction should be substantial, and not too light in weight, say not less than 12 lb (without attachments). In rough country, for service both above and below ground, and for subordinate surveys in stopes, etc, a light mountain transit is sometimes used, though the small size of its circles and its lack of stability are drawbacks. (*b*) The horizontal circle should be of large diameter, 5 in at least, and should preferably be graduated and numbered from 0° to 360° in one direction only. If graduated in both directions, the figures on the two circles should be inclined to right and left, to indicate in which direction the particular circle is to be read. Graduations should be marked with extra plainness. (*c*) For very precise work, two verniers are desirable on the horizontal circle, reading direct to single minutes; finer graduations are confusing in underground work unless the verniers are provided with magnifiers, which are vulnerable points on a transit subjected to hard usage. Micrometer readers are found on European mine transits, but are not favored in the U S. The verniers should be offset not more than 30° from the *N–S* axis, to avoid necessity of walking around the instrument to read it; also to avoid interference by side auxiliary telescope or its counterweight. (*d*) Compass may well be omitted from a transit intended solely for underground work; it is usually a waste of time to read it, though many surveyors rely upon it to check gross mistakes in vernier readings, and the transit can be made stronger without it. (*e*) Vertical arc should be a complete circle, of same size as the horizontal, and graduated from 0° to 90° and back to 0° in both directions. Two verniers are advisable for the most accurate work. The whole vertical circle should be encased. Edge graduations offer advantages over the more common face graduations on transits likely to be used frequently at difficult or precarious set-ups, as they can be read without shifting position. (*f*) The level tube under the telescope should be extra long and sensitive, and so placed that both ends of the bubble are easily visible when level. (*g*) Striding level on the horizontal axis is recommended for accurate work; its sensitivity should at least equal that of the telescope level, and it should be regularly employed at set-ups involving steep sights. (*h*) Eye-piece should be inverting, to secure better vision where poor illumination is the rule. A prismatic eye-piece is necessary for reading plus vertical angles of 45° or over. (*i*) Stadia hairs are objectionable in a transit intended solely for underground work. If they are retained, oblique hairs intersecting at the center and making an angle of 30° with the vertical hair are advantageous, as serving to identify the middle horizontal hair, and permitting more precise bisecting of a cord on short sights. (*j*) Means of attaching an auxiliary telescope at the side or above the main telescope must be provided, and similar means for attaching the counterpoise. The auxiliary telescope should have magnifying power nearly or fully equal to that of main telescope, so as not to add unnecessarily to the complications unavoidable when using the auxiliary. (*k*) A centering point must be marked vertically above the center of the instrument when the telescope is level; this should be fixed by the maker. (*l*) Illuminating reflectors to fit the ends of the telescopes are advisable. (*m*) An auxiliary lens to be attached outside of the objective, whereby sights as short as 2.5 or 3 ft can be focussed, is often useful in shaft-plumbing operations. (*n*) The interior focussing system with fixed objective, as supplied by certain makers, seems especially advantageous for mine transits; the telescope is permanently water-tight, its balance is not affected by focussing, and collimation errors are more easily avoided. The slight loss in light transmission is compensated by a larger objective.

Importance of transit adjustments. While it is possible, by judicious duplication of readings, to make an accurate survey with a transit out of adjustment, time and worry can be saved by keeping it carefully adjusted; in fact, this is imperative if traverses are to be run by the azimuth method. For methods of adjusting transits see Sec 17, Art 4. For underground work the most important adjustments are: (*a*) Plate bubbles. (*b*) Long bubble parallel to collimation. (*c*) Vertical verniers to read 0° when telescope is level. (*d*) Horizontal axis truly horizontal; this is particularly important because of the prevalence of steep sights. A sensitive striding level attached to this axis is a valuable check on the plate bubbles.

Horizontality of transit axis becomes a most important consideration when taking steep sights. If i is deviation of this axis from horiz and α is inclination of line of sight, the error in the single reading of a horiz angle is closely $\pm\, i \tan \alpha$, which obviously may

be relatively large with steep sights. If lack of horizontality is due only to absence of parallelism between this axis and the upper plate, errors in horiz angles can be fully compensated by doubled readings (telescope direct and inverted); but if it is due to obliquity between the two vertical axes of rotation (upper and lower horiz motions) it can not be fully compensated by doubled readings.

Obliquity between vertical axes can be DETECTED in several ways (14); following are the simplest. (*a*) Level the plate accurately by striding level or telescope bubble, leaving either one of the horiz motions clamped; then loosen this motion and clamp the other; if the plate remains level throughout a revolution, the two axes are either coincident or parallel. (*b*) Sight telescope (not necessarily level) at a distant sharp point and clamp horiz axis. With both horiz motions loosened, slowly rotate the lower motion with one hand, while holding the upper plate stationary with other hand. If line of sight leaves the point, note the approx reading on the horiz circle at which the max vert displacement of line of sight occurs. DETERMINATION of amount of obliquity between vert axes. Clamp upper plate at the reading approx corresponding to max divergence of the axes, as observed by method *b*, above; turning only the lower motion, take reading on a leveling rod held a known distance away and clamp horiz axis of transit; loosen upper motion, turn 180° and re-clamp; turn 180° on the lower motion and read rod again. Difference in rod readings divided by distance to rod is tangent of twice the angle of obliquity between vert axes; it is recommended (15) that this angle shall not exceed 4 or 5 sec in a transit intended for surveying steeply inclined workings.

Setting-up transit. The practice of transferring a roof point to the floor, and then setting over this point, should be condemned. It introduces inaccuracies, wastes time, and is more trouble than setting under the roof point. Hang the plumb-bob so that the cord falls under the center of the spad, not tangent to one side of it. Set the vertical verniers at 0°, to bring centering point into the vert axis of the instrument. Move the tripod under the plumb-bob by shifting and extending the legs, keeping the head of the tripod fairly level and centered approximately under the point, regardless of the transit itself. Level the transit roughly by plate bubbles, adjust the plumb-bob until its point almost touches the centering point on the telescope, and observe the direction and distance by which they fail to coincide. Loosen all leveling screws by the same amount, and shift the transit on the tripod head, without turning it, until the points coincide. Tighten the screws, and level accurately by means of the long bubble and striding level. It may be necessary to repeat the shifting operation once more. If transit has 3 leveling screws, set one plate bubble parallel to 2 screws; this bubble is then controlled by turning these screws in opposite directions, while the other bubble moves when the screws are turned in same direction.

Methods of traversing. The manipulation of the transit, and the several methods of traversing, have been fully described in Sec 17, Art 11. Much uncertainty can be avoided by adopting an invariable rule for observing and recording all angles, whether exterior, interior, acute, or obtuse. It is recommended that angles be read always clockwise, sighting first at the backsight, and invariably recording in the same manner as read. The record that the angle 8–9–10 is 178° can then have but one meaning, namely, that the transit was set on station 9, backsighted on station 8, and turned 178° to the right to station 10. If deflections or azimuths are read, instead of angles, it is understood that the recorded reading refers to the course indicated by the last two numbers. If by force of circumstances it is more convenient to read an angle backwards, that is, in direction opposite to that in which the traverse has been proceeding, the angle should nevertheless be recorded as read; vertical angles, also, should be designated plus or minus, as read, leaving it to the office to compute supplementary angles and transpose signs into conformity with a continuous traverse.

Azimuth traverse. This is the method most commonly employed in underground work requiring only a medium degree of accuracy. For the actual process see Sec 17, Art 11. ADVANTAGES: (*a*) More rapid than other methods. (*b*) Simplifies office calculation and plotting, especially when numerous side shots are taken to locate points off the traverse. (*c*) An error of closure in angles is discovered as soon as final course is read. DISADVANTAGES: (*a*) Accuracy of any one reading is limited to 30 sec. (*b*) Transit must be kept in perfect adjustment by daily inspection, because instrumental errors are cumulative. (This objection can be overcome by taking the backsight with telescope alternately direct and inverted, although, by so doing, each course will be distorted by the amount of instrumental error.) (*c*) If a closing angular error is discovered, there is no sure way to ascertain whether it is cumulative, or occurred at one set-up; nor, if the latter, at which one. The possibility of checking a single error at a station is the only argument in favor of compass readings, but under mine conditions the compass is more likely to be in error than the observer. (*d*) Azimuth traverse is not readily applied in

its simple form where there are steep sights. Since the top telescope can not be sighted either up or down in its transited position, it becomes necessary in a steeply pitching azimuth traverse to use both telescopes at each station. This introduces possible errors of adjustment which would not occur if both sights were taken with same telescope. Also, since faulty adjustments in general produce exaggerated effects on readings at steep inclinations, it is questionable practice to adopt any method for traversing under such conditions that does not permit compensation of instrumental errors so far as possible.

Deflection traverse. The vernier being set at 0°, sight the inverted telescope at the back station with the lower motion; then erect, and bring by the upper motion to point at the station ahead. Record the deflection to R or L. Loosening lower motion, turn instrument back to rear station and clamp. Invert telescope, and turn off the deflection again. Record the final reading, divide by 2, and call this the correct deflection angle. In this manner instrumental errors are compensated, and possible error of each individual reading is reduced to 15 sec. Mistake most likely to occur while traversing by this method is to confuse right and left on very small deflections. If the circle is graduated in clockwise direction only, and deflections to left are recorded as read, the mistake is less likely.

A plan recommended by M. B. Evans (20) combines the azimuth with the deflection traverse and overcomes the chief weaknesses of both methods as follows: In starting a traverse, the vernier (continuous circle) is set at the doubled azimuth of the first course, less 360° when in the 3d or 4th quadrant. At each set-up, the transit is manipulated in same way as for deflection traverse (but with no re-setting of vernier), and the 2 readings, compared with last reading at the preceding station, give 2 values for the deflection angle, of which the average is taken for calculating the azimuth of the course; this should check with half the final vernier reading (plus 180° when necessary). Besides increasing instrumental accuracy of the usual azimuth traverse, this method permits immediate discovery of a mistake in reading.

Angle traverse. By the system of repetition the error of an individual angle reading can be reduced to almost any desired limit. For ordinary mine traversing two sets of four repetitions will give the correct angle with an accuracy greater than can be obtained in the taping of distances, as usually practiced. Method of repeating angles is described in Sec 17, Art 11. Note precautions for recording angles, in previous paragraph on "Methods of traversing." The only drawback to this method for regular and extensive work is that it adds to the burden of office computation, which, however, is often warranted by the higher degree of accuracy attainable.

If a transit is known to have obliquity between its two vert axes of rotation, horiz angles involving steep sights can be read with fair accuracy by following method: Clamp lower motion approx in the position it will occupy when first taking the backsight; center and level accurately by striding level or telescope bubble, while rotating only the upper motion; take a set of repetition readings of the horiz angle, with telescopes erect and inverted. Then turn the lower motion about 90° from its initial position and clamp it; re-center and re-level with the upper motion only, and take another compensating set of readings; average of the two sets should be close to the correct angle. For more precise methods for minimizing or correcting the effects of obliquity see Bib (14).

Three-tripod method requires the use of specially constructed interchangeable transit and two targets. Vertical spindle of the transit is inclosed by a thimble which fits into a socket in the leveling head, and is held there by a clamp and a spring plug engaging a groove on the thimble. Spindles of the targets fit the same socket, and the center of the target is at the same height above the leveling head as the center of the transit. The target standard carries a pair of level bubbles. The method of traversing is as follows: While the transit is being set up, an assistant sets one target accurately under the forward station and one under the back station, the leveling head being set level by the bubbles. The instrument man then reads the angles with as much repetition as he desires, while his helpers are otherwise occupied. Tapings are made from the side center of the telescope to the side center of the target. The transit and the forward target are then interchanged, above the leveling heads, and rear target and tripod are set up at next station ahead. After the interchange, it is advisable to test for centering, and for level. ADVANTAGES: (*a*) applicable to large and high chambers where roof stations are not feasible; (*b*) saves transit man's time in setting-up; (*c*) allows assistants to be engaged in taping, plotting, note-taking, or otherwise, while instrument man is reading angles; (*d*) permits a degree of accuracy in angle reading limited only by the mechanical perfection of the instrument and the personal error of the observer; (*e*) eliminates or reduces liability of error in setting-up, in measuring inclined distance, H I and H S (Art 5), and in reading vertical angles, since all these measurements can be repeated under identical conditions during backsight operations, when a failure to check will be noted. DRAWBACKS: cost of the extra apparatus, and labor of transporting it. For routine surveying in small mines the refinements of this method are unnecessary, and it is used in only a few localities in the U S.

Steep sights. For sighting at an angle exceeding 60° either up or down, when the line of sight of an ordinary transit would be intercepted by the horizontal plate, three expedients are available: Auxiliary telescope; prismatic telescope; eccentric telescope.

Auxiliary telescope may be attached either to the right-hand end of the horizontal axis, or to a lug on top of the main telescope. A counterweight is attached to the opposite point to prevent distortion and unbalancing (Fig 8).

The auxiliary is sometimes described as an "interchangeable" telescope, but this does not mean that it should be interchanged, from side to top or vice versa, at a given set-up, with the idea of saving the calculation of angular corrections. Under conditions requiring its use it is not easy, after setting up, to adjust this telescope with the necessary precision if it is to be depended upon for the single reading of either a vert or a horiz angle, and the difficulties are multiplied if two sets of adjustments have to be made. Therefore, it is much safer to let the auxiliary remain in the same place while reading both horiz and vert angles. Also, because of the exaggerated influence of certain maladjustments in the transit itself upon single observations made at steep inclinations, some method of duplicated readings (direct and inverted) should be followed, in order to compensate unavoidable instrumental errors. This requires that the auxiliary shall be at the side, since a top telescope can not be sighted while in transited position.

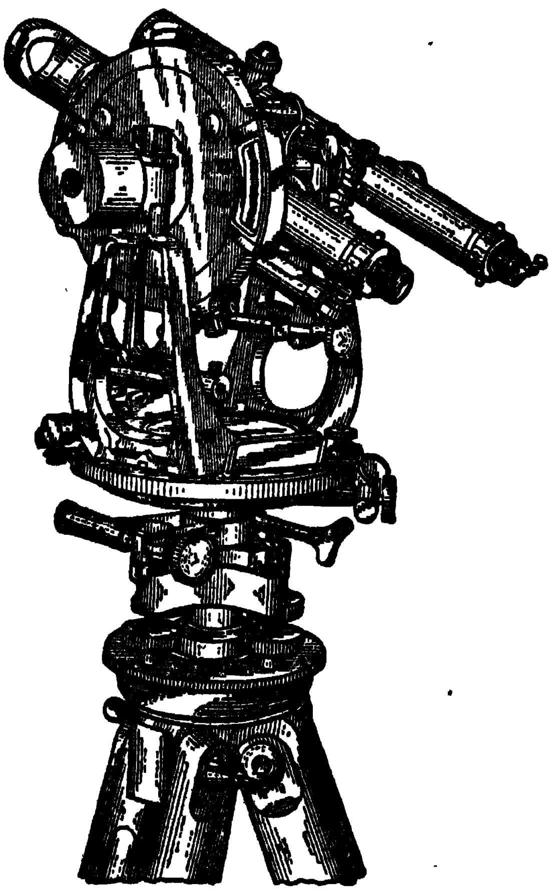


Fig 8. Fully Equipped Mine Transit (C. L. Berger & Sons)

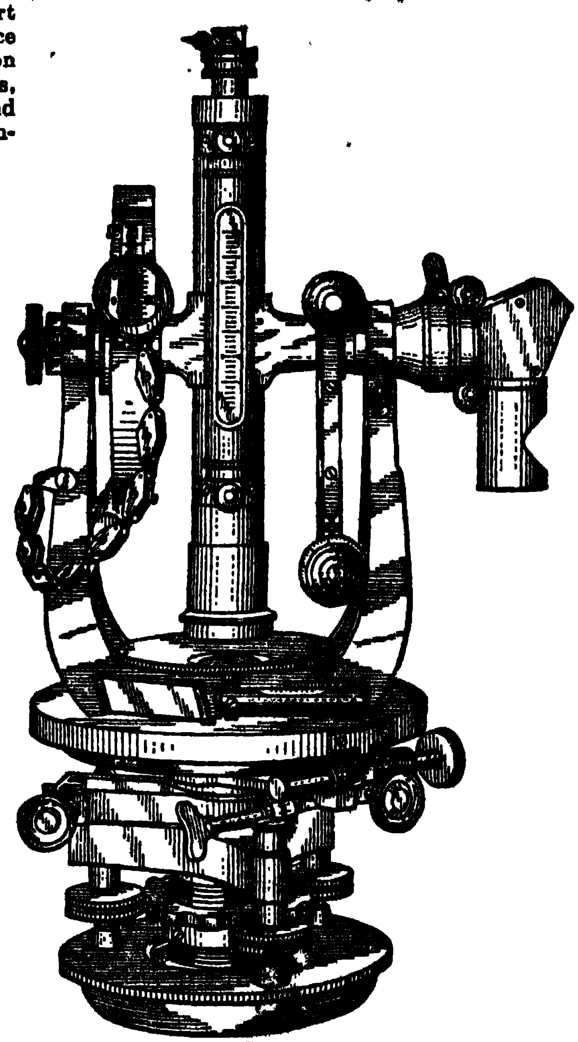


Fig 9. Prismatic and Interior-focussing Mine Transit (Bausch & Lomb)

The following ADJUSTMENTS of the auxiliary telescope must be applied with precision if angles are to be measured by single readings; if compensating observations are to be made, the adjustment becomes largely a matter of convenience only.

To bring the two lines of collimation into the same plane: After screwing on the auxiliary snugly, sight the main telescope at some sharp point and bisect it carefully. Then, with its capstan tangent screws bring the auxiliary to bisect the same point, with its vertical hair if above the main telescope, or with its horizontal hair, if at the side. This adjustment must be made every time the auxiliary is mounted.

To make the two lines of sight parallel: Determine the distance between centers of the telescopes, by direct measuring or by marking the two lines of sight against a wall 8 or 10 ft away. Set a rule, a leveling rod, or a pair of accurately spaced marks, at a distance of 200 or 300 ft, and shift the cross-hairs of the auxiliary until the correct space is intercepted by the two lines of sight. It should not be necessary to repeat this adjustment except at intervals.

Top telescope. When the auxiliary is placed on top of the main telescope, horizontal angles, and azimuths, are read directly on the vernier, as usual. Vertical angles must be

corrected in the manner shown in Fig 10. The amount of the correction, α, is obviously that angle whose sine is equal to the distance between telescopes, c, divided by the inclined distance, d, as taped from the horizontal axis of the transit to the point sighted. If the angle is negative, the correction is subtracted from the angle as read; if the angle is positive, the correction is added.

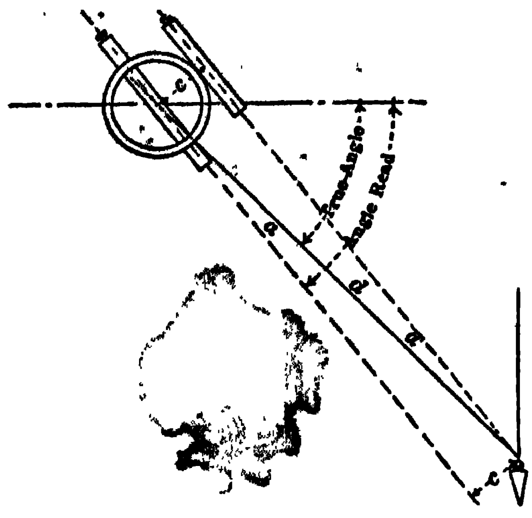


Fig 10. Correction for Vertical Angles

Side telescope. When using this, vert angles are read directly, as usual. In the single reading of a horiz angle, or azimuth, if the backsight is taken by the main telescope, it is obvious by reference to Fig 11 that the correction, β, is that angle whose tangent is c divided by the horiz distance h to the point sighted; this horiz distance, however, is equal to the taped distance d multiplied by the cosine of the vert angle. That is, $\tan \beta = c \div$ (*taped distance* $\times$ *cos vert angle*). If the auxiliary is on the right of the main telescope, and if the angle is read clockwise, the correction must be added to the angle as read. If the side telescope is used for both backsight and foresight in the single reading of a horiz angle, the correction for each sight must be computed separately; from the angle as read (Fig 12) first subtract correction β_1, for the backsight, and then add correction β_2 for the foresight, the auxiliary being on the right, and the angle read clockwise.

The reading of horiz angles with the side telescope can be much simplified, instrumental

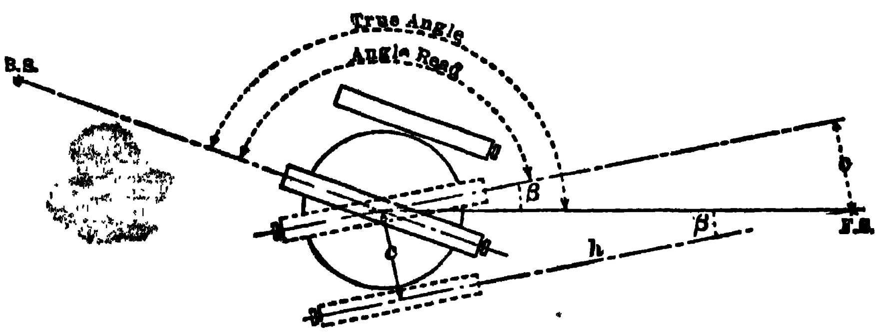


Fig 11. Correction for Single Reading with Side Telescope

errors compensated, and all computation of corrections eliminated in the following manner: Bisect the backsight with either the main or the side telescope, turn the angle and bisect the foresight with side telescope; invert the telescopes and turn the angle again as

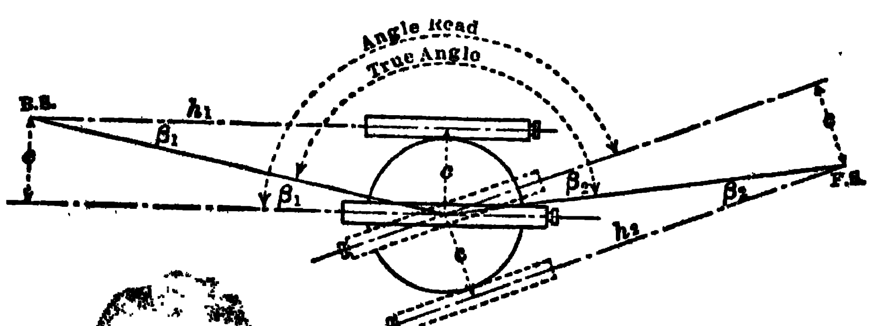


Fig 12. Correction for Horizontal Angles; Both Sights with Side Telescope

before, without resetting the vernier at zero. One-half the final reading will be the true angle. Referring to Fig 12, the first reading = true angle $+ \alpha - \beta$; the second reading = true angle $- \alpha + \beta$; hence first reading + second reading = 2 $\times$ true angle. Instead of making the two readings separately, they are added automatically on the circle of the

transit. There is no object in observing or recording the first reading, since this will not even approximate the true angle unless the horiz distances of the two courses happen to be about equal, that is, unless $\alpha = \beta$ nearly.

If an azimuth traverse is being run, and the use of the auxiliary telescope is only occasionally required, the angles at such set-ups can be quickly determined with the side telescope by the above method, the azimuth computed, and the traverse continued, after setting the vernier at the computed azimuth.

Prismatic telescope. By placing a mirror or prism, which will reflect the line of sight by 90°, on the objective end of the telescope of an ordinary transit, horiz angles of steep

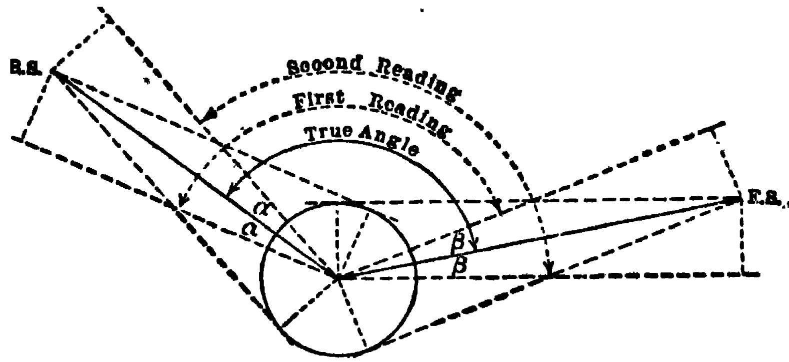


Fig 13. Reading Horizontal Angles with Side Telescope, without Corrections

sights may conveniently be read directly on the horizontal circle. The reading of vert angles by this means involves two difficulties: (*a*) the reflection may not be exactly 90°; (*b*) the movement of the objective, for focussing, alters the degree of eccentricity and thereby the amount of correction to apply to the reading. The Bausch & Lomb prismatic telescope (Fig 14) surmounts both difficulties; first by its "penta-prism," so constructed as to cause an invariable reflection of 90°, and second by its interior focussing device, which renders unnecessary the moving of the objective. The prism may be attached to the main objective, or to a side objective mounted at the right-hand end of the horiz axis, which is hollow. In the latter case the line of sight is reflected a second time by a

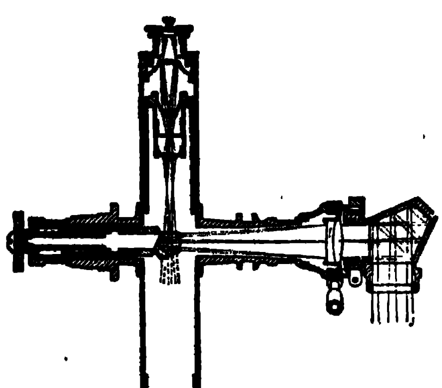


Fig 14. Bausch & Lomb Prismatic Transit. (See Fig 9)

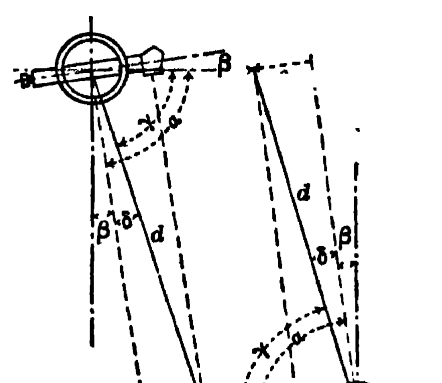


Fig 15. Correction for Vert Angle by Prism on Main Objective

smaller prism which can be inserted, when desired, into the optical axis of the telescope by turning a milled head on the left-hand end of the axis.

Adjustments and corrections. Prism on main objective. Before attaching the prism, sight the telescope on a sharp point at as great an angle as possible above or below the transit point, and bisect accurately with the vert hair; the vert axis of the transit remaining clamped, attach the prism to the end of the telescope, bring the point again into view, and by means of its tangent screw turn the prism until the point is again bisected by the vert hair. Horis angles can then be read directly. The reading of a vert angle by this method involves two computations (see Fig 15). Angle $\alpha = 90° - \beta$, and has the sign opposite to that of the angle β as actually read. True vert angle $\gamma = \alpha - \delta$, and $\sin \delta = e \div d$. If the backsight, for example, is steeply inclined upward from the instrument station, and the foresignt downward, it is more convenient to invert the telescope for

the backsight and read the horis deflection, than to read the actual angle, which would involve adjustment of the prism for each sight.

Prism on side objective. Sight the telescope on a sharp point, and bisect accurately with the horis hair. Attach the prism to the side objective, and bring the interior prism into position. Turn the former by its tangent screw until the point is again bisected. Vert angles can now be read directly. Horis angles are corrected, or corrections may be avoided, in precisely the same manner as with an auxiliary telescope.

Eccentric telescope. Several American manufacturers until recently made special transits on which the main telescope was permanently, or could be temporarily, offset from its central position, though still swinging in same vert plane. They have been discontinued, mainly because, like the top auxiliary telescope, they could not be used for compensating (direct and inverted) readings.

Fig 16 shows two types of transit with permanently eccentric telescopes. *A*, the Whitehead double-telescope theodolite, by E. R. Watts & Son, London, has given excellent service at the Ooregum mines, India, where vert angles of over 80° are common; its usefulness is obviously not confined to steep sights. *B*, by Otto Fennel Söhne, Cassel, Germany, represents a type often

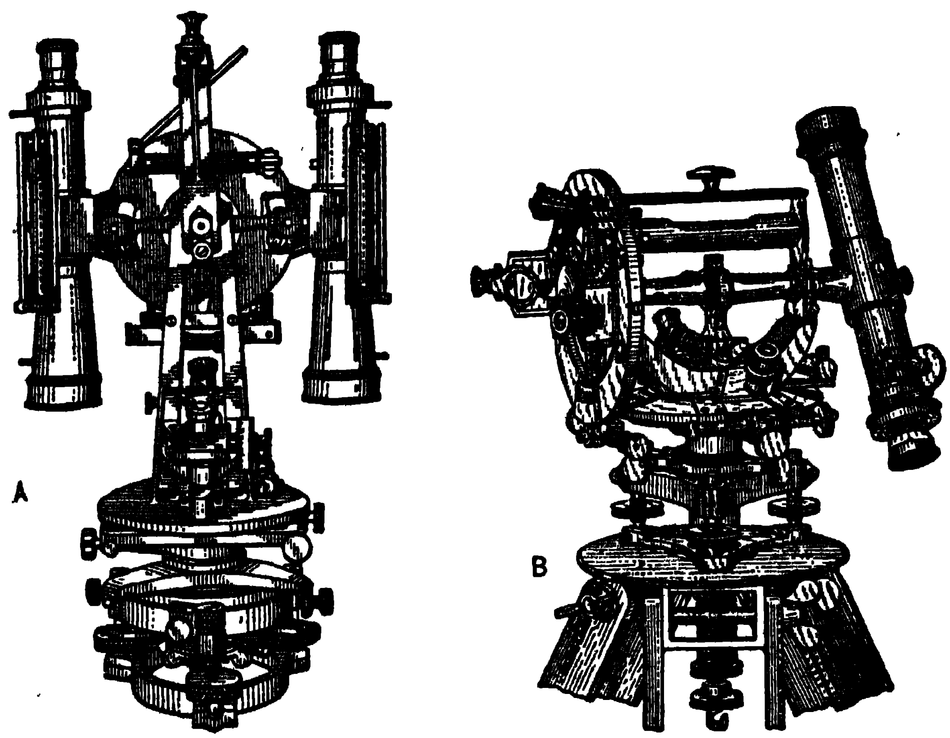


Fig 16. Two Types of Eccentric Transits

employed in Europe, for general work as well as for that involving steep sights. For reading horis angles with type *B* an eccentric target may be used, or the compensation method illustrated in Fig 13 may be followed.

5. MEASURING VERTICAL DISTANCES, LEVELING

In most metal mines, and in many coal mines, vert distances and elevations are obtained by computation on the basis of vert angles; the leveling method is sometimes used, but even then the leveling is often done with a transit. Where much leveling has to be done, it is advisable to use a Y-level, because in important mine problems the respective elevations of various points should be known with no less accuracy than their horizontal coordinates.

Computation from vert angles. Angles below the horis are called negative, and those above are called positive. In Fig 17, if t is the taped distance from the center of instrument at A, to the point sighted B, and α is the vert angle, with its appropriate sign carefully noted in the field, then the vert distance is: $v = t \sin \alpha$, and takes the same sign as α. The height of instrument, $H\,I$, or i, is the vert distance from the center of instrument up to the bottom of the spad, and the height of sight, $H\,S$, or s, is the corresponding distance from the point sighted at up to its spad. The difference in elev between the two spads, d, is thus the algebraic sum: $d = + s - i \pm v$. That is, when the vert angle is positive, add v to s and subtract i; when the angle is negative, add v to i, and subtract s. Sometimes a negative angle will be read, while the actual difference in elevation is positive.

and vice versa, but if the algebraic signs are kept firmly in mind, no confusion will result. Knowing the elev of A, the elev of B is computed by the invariable rule: El B = El $A - i \pm v + s$. In practice it is usually more accurate, and involves but little more office labor, to read vert angles to a definite point on the plumb-bob string (say, the top

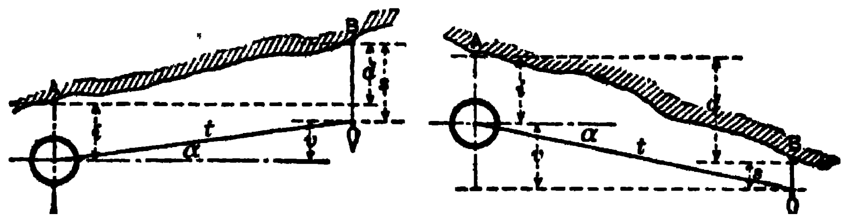


Fig 17. Computation of Elevations from Vertical Angles

of the plumb-bob) just as they come, rather than to attempt to set the sighting point at the same distance below its spad that the instrument is below its point. The $H\ I$ and $H\ S$ are read with a rule, or with a pocket steel tape graduated to feet and hundredths. In measuring the $H\ I$ of a transit it is customary to tape obliquely from bottom of spad to the center of the end of the horiz axis; this does not introduce much error unless the distance is short. The accompanying table gives the corrections to be subtracted from the oblique distance as measured on a transit of which the end of the axis is 0.25 ft from the center of the telescope.

Corrections to Subtract from Oblique Measurements of H I

Oblique distance	Correction	Oblique distance	Correction	Oblique distance	Correction
0.6	0.091	1.1	0.026	1.6	0.012
0.7	0.066	1.2	0.022	1.7	0.011
0.8	0.050	1.3	0.019	1.8	0.010
0.9	0.039	1.4	0.016	1.9	0.009
1.0	0 032	1 5	0 014	2.0	0 008

Clinometer (Fig 18) is a companion instrument to the hanging compass (Art 4), being hung on the same cord before or after the compass has been used. The bob is attached to a fine black hair, which is fastened at the center of the arc. Owing to the catenary sag of the cord, and the weight of the clinometer itself, two readings must be made, equidistant from each end, and as close thereto as possible, and the average taken. If the slope of the cord is not over 10°, a single reading at center of span will give the correct angle. Although not an instrument of precision, the clinometer is useful for subordinate surveys through tortuous, steep, or narrow openings, and in stopes difficult of access, its readings being at least as accurate as those of the compass with which it is usually associated.

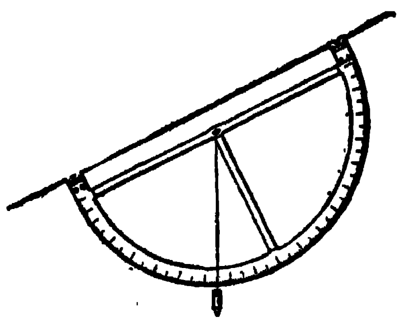


Fig 18. Hanging Clinometer

Brunton "pocket transit" (Art 4), when used to measure vert angles, is operated thus: Open the slotted sighting arm to its fullest extent, turning up its hinged end, containing the sighting hole; also open the lid about 45°. Standing under the station, hold the body of the instrument vertically and sight through the hole in end of slotted arm, and through the round hole in edge of lid close to hinge, to a light at the next station: At same time move the indicator arm carrying the level vial until the bubble, as seen in the mirror, registers horiz. Then read the vernier on the graduated end of the arm. Many other types of hand clinometers, as the Abney, are just as satisfactory for underground as for surface work.

Prismatic and eccentric transits. The use of these instruments for reading vert angles is described in Art 4, in connection with horiz angles, since the correction of one angle usually involves the reading of the other.

Determination of vert angles by transit. Since vert angles can not be repeated, in the same manner as horiz, they must be duplicated and averaged. Every vert angle of a traverse of any importance should be read at least twice, once with the telescope normal and once with it inverted, and the average taken, in order to eliminate zero error of the vert vernier. For important work it is advisable to read both verniers, in both positions, and take the aver of the four readings; this further compensates any eccentricity error in the vert circle.

While traversing by repetition angles, or by double deflections, no time is lost by reading vert angles with the telescope in both positions. On an azimuth traverse, the only check is to read the vert angle on the backsight as well as on the foresight, making two complete sets of readings of taped distance, the *H I*, and the *H S*. This checking of an azimuth traverse can be simplified by adjusting the *H S* at both foresight and backsight to correspond with the *H I* at instrument station, although this practice is not altogether recommended for the more precise methods of traversing. The auxiliary telescope, when attached to the horiz axis, gives vert angles direct; when on top of main telescope, a correction must be applied (Art 4).

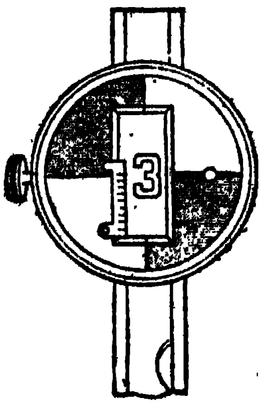


Fig 19. Underground Target

Underground leveling is performed in a manner the reverse of that employed on the surface. The zero end of the rod is pressed upward against the bottoms of the spads, or other desired points; backsight readings are thus negative in sign, and foresight readings are positive. (Compare Sec 17, Art 18.) Elev of a point on the floor is obtained by holding the rod right end up and recording the reading as negative.

Form of notes and method of computing elevations are otherwise exactly like those used on surface work. A self-reading rod can not usually be employed, unless the air is clear, the sights short, and a strong light is available. The target for underground service should have a 1/4-in hole and a 1/32-in slot cut on the horiz line, behind which a light is held (Fig 19). A 3-ft rod, extending to 5 ft, is convenient for mine work. Hooks or other devices for hanging leveling rods from survey spads, to relieve the labor of holding the rod in inverted position, are open to criticism since a spad may be dislodged by a weight for which it is not designed.

6. TAPES AND TAPING

Underground conditions require a tape to be light in weight, because measurements are usually made with the tape unsupported between the two points; strong and durable, to withstand unavoidably rough treatment and sometimes acid water; and its numerals to be as large and distinct as possible, to avoid mistakes of reading in a poor light.

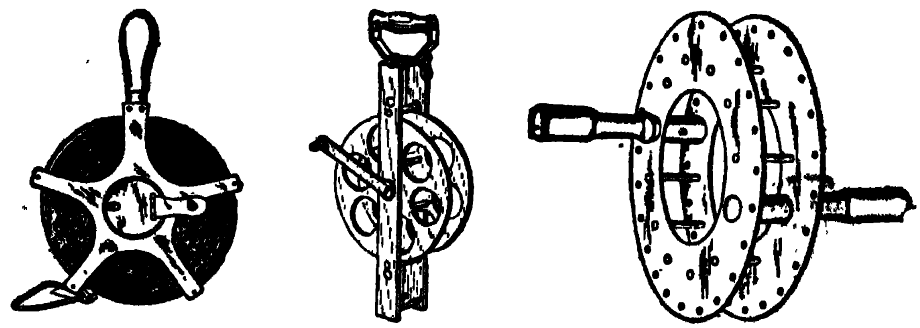
If the graduations are marked on copper or brass sleeves, these should be soldered in place, as well as clamped, to keep the water out. Riveted sleeves are not recommended. Nickel-plated, bronze, and stainless steel tapes are most resistant. A steel tape, plated with white metal and having its graduations, at every foot, stamped on thin bosses of babbitt metal, is on the market. Tapes should be standardized for a constant pull, say 20 lb, and for constant temp, say 62° F. It is seldom necessary to apply corrections for temp, because mine temperatures are fairly constant, although not necessarily corresponding to the standard temp of the tape. Temperature may, however, become a noteworthy factor in correlating surface and underground surveys. For catenary and temp corrections see Sec 17, Art 11. It is rarely possible in mine surveying to tape a distance exceeding 300 ft, and for general purposes a length of 100 or 200 ft is satisfactory. A clamp handle for gripping the tape anywhere along its length adds to convenience as well as to the accuracy of the work, besides avoiding sharp bending of the tape.

Graduations are commonly at 5-ft intervals. Exact distances must then be ascertained by marking on the tape with a pencil, or by holding the thumb-nail on the point, and measuring to nearest 5-ft mark with a pocket tape. This is troublesome, and likely to be inaccurate. The difficulty can be lessened by having the zero mark of the tape 5 ft from the end, the outer length being graduated to single feet. The foresight man then holds the nearest 5-ft mark at his point, and the transitman measures the exact distance with a 6-in rule, graduated by hundredths from 0 to 0.5 ft on one side, and from 0.5 to 1.0 ft on the other, for convenience in reading in either direction to nearest foot-mark. Inaccuracies in reading can be best overcome by using a steel ribbon tape graduated to feet and hundredths throughout. These are more expensive than flat-wire tapes, and require more cautious handling. Practically the same convenience is secured with a cheaper tape graduated only to single feet, and having an extra foot at one end graduated to hundredths backwards from zero point.

Taping practice. Distances should ordinarily be taped along the line of sight, from the center of the horiz axis to the point sighted for reading the vert angle, usually the top of the plumb-bob. An exception is in the case of nearly horiz drifts, where elevations are subsequently to be obtained by leveling; the tape may then be stretched on the floor. At Cerro de Pasco (30), where elevations are determined by leveling wherever possible, the unsupported tape is stretched horizontally by noting, on the cord of the sighted bob, the point level with the transit; this makes measurement of vert angle, *H I*, and *H S* unnecessary, and leaves the telescope in the correct position for the next set-up. For unsupported taping, a steady pull of 20 lb should be given, a spring balance being used

until the operators become expert at judging the pull. For surveys demanding high accuracy, and permitting or requiring taped distances much in excess of 100 ft, the catenary correction for the tape used should be ascertained by referring to accurately established reference points. In general, taping as usually practiced is more subject to error than angle reading and, especially if there are long spans, the error tends to accumulate.

Tape reels for underground work should be larger and stronger than for surface work. If the type shown at *a*, Fig 20, be employed, for a ribbon tape 100 ft long, the reel should



b *c*

Fig 20. Tape Reels

have a capacity of 150 or 200 ft, to prevent clogging by dirt. Type *b* is preferred for long flat-wire tapes. Type *c* is as simple and satisfactory a reel as has been devised. It is made of sheet metal, preferably brass; in the turning of the shoulders on the spacers, plenty of metal should be allowed with which to form the riveted heads. The two wooden handles should be fastened by screws to prevent them from turning inside the ferrules and wearing small enough to drop out. A reel of this type, of 10 in diameter and 2 in width, will hold 500 ft of flat-wire tape. So much of the tape as is not needed for constant use is tied to the reel by cord or leather thongs.

Care and repair of tapes. Tapes used underground should be cleaned, dried, and oiled every day. When a tape breaks, it is not necessary to send it to the maker for repairs. Obtain a few pieces of soft sheet brass, about $^5/_8$ in long and of a width sufficient to wrap around the tape with a double lap on one side ($^1/_2$ in is about right for a flat-wire tape). These pieces are tinned on one side, and then folded as shown in Fig 21. Straighten and clean the broken ends of the tape, apply zinc chloride solution, and tin with a soldering iron for a distance of $^1/_2$ in on both sides of the break. Fit the broken ends closely together inside one of the brass sleeves, close the latter with pincers or by hammering, apply more flux, hold a hot soldering iron on the sleeve until the solder flows, add a little more solder at the ends of the sleeve, and finally wash off carefully any excess of zinc chloride, to prevent rusting. A neater job can be made by cutting out the 5-ft length in which the break occurred, and inserting in its place a 5-ft length cut from an old tape, then stamping the figures on the new sleeves. This plan avoids multiplicity of brass sleeves.

Fig 21. Brass Sleeve for Repairing Tapes

7. UNDERGROUND TOPOGRAPHY

By offsets. This method is best suited to rooms, entries, drifts, and narrow stopes. While the tape is stretched between stations, either before or after the distance has been measured, lay it on the floor in a straight line approximately under the two points, with the zero end preferably at the instrument. At every 5- or 10-ft mark, or at any other intermediate points opposite salient features of the walls, such as corners of pillars, chutes, etc, measure the distance at right angles from the tape to the walls on both sides, using a rod, a rule, or a linen tape.

If the object of this work is to compute excavated volumes, as a basis for payment of wages, for example, the widths should be read to the nearest tenth of a foot, and considerable care should be exercised to measure at right angles to the line of the traverse. For less important work, measurements need be only to nearest foot. If all offsets are measured at about waist height, errors due to irregularity in the walls will compensate themselves in the long run. The readings may be plotted immediately to scale on the right-hand page of the note-book, which should be ruled coordinately, with a center-line corresponding to the line of sight. Or the readings may be recorded to the right and left, opposite their appropriate points on the center line. Another way is to record the readings thus: 50 $^3/_5$, meaning that at the 50-ft point the right wall was 3 ft and the left wall 5 ft from the line. Distances to roof and floor are taken at instrument points by direct measurement, and at intermediate points by a leveling rod sighted from the transit, the target being bisected at each point by the line of sight. Roof and floor notes are best recorded in the form of fractions, as above, but should be distinguished from wall notes by drawing a circle around them.

Radiating offsets. Where more detailed information is desired as to the complete cross-sectional profile of an opening, as for example in estimating for a masonry or concrete lining, the principle of Heller & Brightly's SUNFLOWER is serviceable.

As used many years ago in N Y aqueduct tunnels, the "sunflower" was a brass disk, 14 in diam, graduated around its edge, and having a centrally pivoted arm which served as guide for a measuring stick long enough to reach all points on the perimeter of the section. The disk, mounted vertically on a tripod, was brought into the center line or traverse course and its elevation adjusted or observed by transit at the nearest station; distances to the roof, walls, and floor were then recorded at as many radiating angles in the plane of the disk as desired.

For smaller-scale work in a metal mine the same principle can be adopted in simpler manner, *viz.*: a light board, about 12 by 14 in, is held vertically at an adjustable height by clamping to a leveling rod or other easily portable device, which acts as a supporting column when braced between floor and roof or between walls. A pad of durable paper is tacked to the board and a thin finishing nail driven part way in at the center. After adjusting the board so that its center is on the line of sight between two stations, or otherwise determining its position, a light wooden rod graduated to feet and tenths is rested on the center nail and its zero end brought against the walls at as many points as desired; its position is scribed directly on the paper and the corresponding distance noted. The distances are then reduced to scale and the profile is sketched in somewhat like a plane-table survey, using a fresh sheet of paper at each set-up.

By angles and distances. This method is applicable to broad open chambers, and lends itself particularly to the azimuth method of traversing. Having set up at a station and oriented the lower plate by a backsight, take sights to a light held at as many points as desired, recording the azimuth, the vertical angle, and the distance to each. As these notes are commonly plotted by protractor, duplication of angles is not necessary. After all the sights have been taken, observe the azimuth and distance to the next station, and proceed. If the traverse is being run by deflections, or by angles, the method of taking the side shots is the same, but in plotting the notes the protractor must be separately oriented at each station.

By tape alone. This is suitable only for large open chambers unobstructed by pillars or timbers, in which all of the points are nearly in one plane. From each of two traverse stations measure the distances to each desired point, and plot by swinging intersecting arcs from the plotted stations. This gives the situation of the points, referred simply to their own plane; to project on the horiz plane, an observation must be made on the prevailing slope.

By angles alone. Two traverse stations are required, from both of which all the desired points can be observed. With the vernier at zero, set up at one station and sight the other station with the lower motion. Then read angles, both horiz and vert, successively to each of the desired points. Set up on the other station and read angles to the same points, having the zero directed to the first station. Time is saved by using two instruments, sighting simultaneously at the same point. Plot with protractor, by intersecting lines. Labor of plotting with protractor may be reduced if, instead of observing angles as just described, each transit is oriented to same meridian and azimuths to the several points are recorded. If elevations are required, they can be computed by multiplying the horiz distances, scaled from the map, by the tangents of the vert angles.

If the points in the chamber are inaccessible for the man holding the light, the following expedient has been found successful. Two transits are required, set up at two traverse stations commanding as much as possible of the excavation, and having their zero points oriented. Take the ground-glass plate out of the back of a camera box, and hold in its place an acetylene lamp. This will throw a narrow beam of light through the lens, which can be projected against the roof and walls of the chamber, the bright spot being observed simultaneously by the two transits. Plot and compute as above. A third transit at another station may insure a more complete capture of all the points in the chamber.

The Tonnesen method (2) was devised for estimating stoped areas in the Transvaal gold mines, where the slope is often regular for long distances, and wages are based on area excavated. It aims to reduce the office labor involved in first converting inclined distances to the horizontal basis, and then reprojecting to the inclined plane the areas thus calculated. Tonnesen's instrument, the "stereometer," is based on the same principle as the nautical sextant, and reads angles in the plane of the three points. At least two accurately located points are set in the stope, in convenient positions, and the side shots to the faces of the stope are taken either by angles and distances from one station, or by intersecting angles from both stations. Or, a traverse may be run, in the plane of the stope, close against the face, offsets being measured to the solid ore. If the uniformity of slope is interrupted by faults or folds, the area is divided into panels, each having a uniform slope, and each is separately measured.

8. SHAFT PLUMBING

Wires used for shaft plumbing are of iron, steel, copper, brass, or phosphor bronze. In any case, the size of the wire should be as small as will safely carry the weight of the bob; B & S No 20 or 24 is convenient. Iron wire is easily obtainable, and its ductility permits the bob to stretch out its kinks without breaking. If much plumbing has to be done, at intervals, iron wire is liable to deteriorate by rusting. Steel wire (piano wire)

is much stronger, but is not always easy to procure, must be handled cautiously to prevent kinking, and is subject to rust. Copper wire is easy to get, is ductile and non-corrodible, but as a fine copper wire will not carry much weight, and therefore can not be stretched tight, its position in a shaft is more subject to disturbance by air currents and falling water. Brass wire is stronger than copper, but is subject to kinking, and if it breaks under tension its springiness makes it troublesome to untangle. Phosphor-bronze wire is strong, durable, and not too springy. In general, iron and copper wires are more suitable for shallow shafts free from rapid air currents and falling water, while steel and phosphor bronze are better adapted to deep shafts and troublesome conditions.

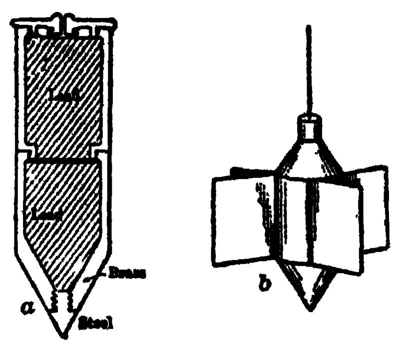


Fig 22. Shaft-plumbing Bobs

Bobs range in character from flat-irons and sash weights to carefully turned and centered instruments, and in weight from 10 to 125 lb. Irregular-shaped weights are objectionable because in twirling they impart a wobbling motion to the wires, which adds to the difficulty in determining their mean position. Cast-iron bobs have been suspected of being susceptible to magnetic influences, tending to deflect them from their true position. This was once observed in a case where a pile of steel rails lay in the magnetic meridian, with their ends close to an iron bob. Ordinarily, magnetic influence on iron bobs is probably negligible. Lead bobs, used in connection with copper wires, are wholly free from this

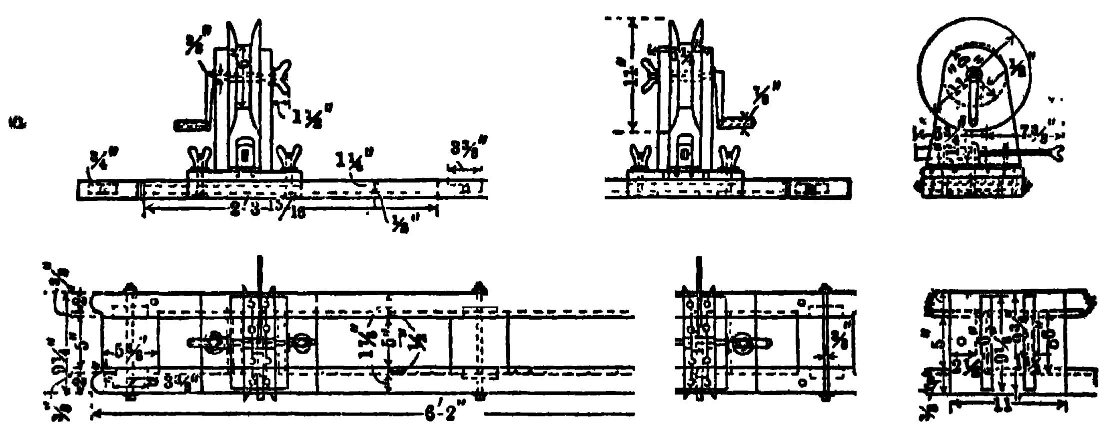


Fig 23. Shaft-plumbing Board, Butte, Mont

suspicion. The Handley (3) sectional bob has a brass shell, accurately turned and centered, with threaded connections, permitting a bob of any desired weight to be put together (Fig 22, *a*). The brass shell is poured full of lead, each section weighing 10 lb. An iron bob fitted with fins to reduce twisting and vibration, when submerged in a bucket of liquid, is shown at *b*. The fins are soldered to a tin-plate jacket closely fitting the bob. An excellent bob can be turned from a piece of steel shafting; additional weight, if desired, can be obtained by making it hollow and filling with lead or mercury.

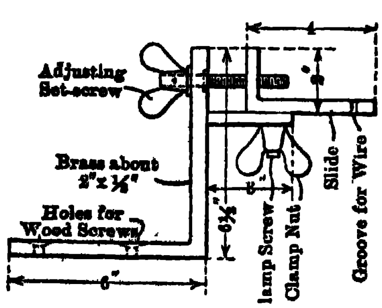


Fig 24. Shaft-plumbing Bracket

Supporting wires at top. If the shaft survey is only an isolated case, the simplest means will suffice for fixing the position of the wires at the surface. The wires may hang over notches in the edge of a board or in the head of a nail, and the excess wire may be kept on a spool. If the plumbing operation is likely to be repeated frequently, it is well to provide better designed and more durable apparatus. It is a convenience to support wires in such manner that their position can be readily shifted to bring them into line, or to vary their distance apart; also to keep the excess wire on a reel, where it can not be injured. The apparatus used at Butte (4), is shown in Fig 23. A type of adjustable bracket is shown in Fig 24. Another type, and the reel used in connection with it, are shown in Fig 25. Still more elaborate forms have been used, but are not necessary for ordinary cases.

When two vert shafts with connecting workings are available, only a single wire in each shaft is needed, preferably hung near the center. Having determined the coordinates of the two wires by a closed surface traverse, set up underground at a station as far from wire No 1 as it can be plainly seen, and sight the mean position of the wire. This operation need not be performed with such extreme precision as when only a single shaft is available, since the base-line for the underground survey may be several hundred or thousand feet long. Assume any bearing for the line from the wire to the first station, and carry the traverse on that basis through the workings to wire No 2. From the underground notes, calculate the length and bearing of a straight line connecting the 2 wires (see Art 9); this length should correspond closely to the similarly computed length on the surface, as a check on the underground work. Compare the computed underground bearing of the closing line with its true bearing as determined on the surface, and correct all underground bearings accordingly.

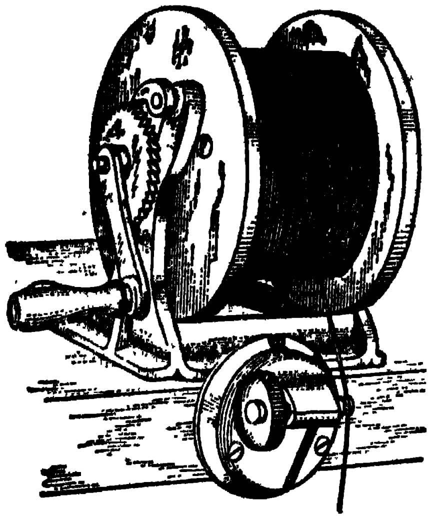


Fig 25. Shaft-plumbing Bracket and Reel

Hanging two wires in one shaft. If only one vertical shaft be available for connecting an underground survey with the surface, two wires must be hung as far apart as possible in this shaft, and their connecting line used as a base for the underground survey. Extreme care is therefore necessary in every detail of the work.

It has been found by experiment (5) that wires are liable to deflection if air is traveling faster than 100 ft per minute in the shaft. The amount of deflection is proportional to the veloc of the current, but whether the direction is up or down the shaft is immaterial. A wire hung in the corner of a rectangular shaft is more strongly deflected towards the long side of the shaft; hence, if two wires be hung in opposite corners, their deflections will be in opposite directions and the bearing of the line connecting them at the bottom will be doubly displaced. This is a more serious source of error than a mere divergence of the wires. It is therefore recommended that in a shaft having a strong current of air the wires should be hung at about the middle points of the ends, rather than in the corners. If possible, the ventilating current should be reduced or stopped while shaft plumbing is going on. If this is not feasible, for important work, board flues can be built from the top to near the bottom of the shaft, and the wires lowered inside of them; these will afford protection also from falling water. A light bob should be used for lowering the wires, and the heavy bobs attached below; a snap hook facilitates interchanging. Freedom of a wire from obstructions can be ascertained: (*a*) by watching, from above, a light slowly rotated around the bob; (*b*) by comparing the swing period of the bob with the correct period computed from the known length of the wire; (*c*) by wrapping a little ring of wire loosely around the plumb wire at the top, and letting it drop; (*d*) by causing the point of support to be moved a definite distance and direction at pre-arranged intervals of time, noting whether corresponding movement occurs at bottom. Adjusting wires in a deep shaft is nervous work; it may be greatly facilitated if telephone extensions can be provided at top and bottom, or a 2-wire cable installed with push-button and buzzer for transmitting signals.

Three-wire system. As a means of detecting angular displacement of the line connecting two plumb-bobs in the bottom of a shaft, it is recommended (6) that three bobs be hung, and that no angles be read until the three wires at bottom are precisely the same distance apart as at the top. If this be the case, since it is highly improbable that all three bobs would have been displaced by exactly the same amount, and in the same direction, the angular positions of the bobs can be accepted as correct. At Hollinger and other deep Canadian mines (21) the third wire is used, not merely for the purpose just described, but as an additional sighting point, both above and below ground, thus permitting a check on position of any one wire by calculations based on 2 or 3 independent sets of observations. Following paragraphs assume that instrumental observations are confined to 2 wires.

Observations at top of shaft. It is usually convenient to bring the transit into line with the wires; if not, their respective positions may be triangulated as described below. Having hung the wires approximately in position, set the transit on a point from which both wires can be seen. If the position of the wires is adjustable, they may be brought into line with the transit by sighting first at one and then at the other; if the wires are

fixed, the transit must be brought into line with them, by the shifting head. The micrometer head shown in Fig 26 is a help in this operation. All being set, measure the distances from wire to wire, from the nearest wire to the transit point, and from the transit point to the last traverse station. Also measure, by repetition, the angle from the last station to the line of the wires. From these data compute bearing of the line of wires, and the coordinates of either or both wires. Before leaving the transit station, both it and some other point accurately in line with the wires should be permanently marked for future reference.

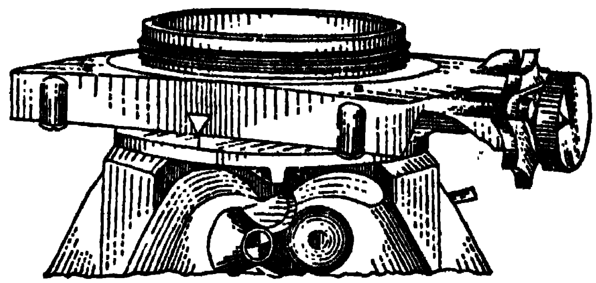


Fig 26. Micrometer Head

Observations at bottom of shaft. If a suitable instrument station can be found at the right spot, the transit may be lined in on the wires; if not, their position must be triangulated. European engineers generally favor the triangulation, or Weisbach, method, with angles as small as 1 to 3 min (preferably not exceeding 25 min) subtended at the transit by the wires, even when the alinement method could be used.

Compensation of instrumental errors by repetition can probably be made more readily when triangulating than when alining on the wires. Methods for increasing the precision of alinement practice are given in Bib (16). For an alinement method permitting check calculations from observations at both top and bottom, see Bib (29).

If the shaft is not deep, and is free from air currents and falling water, sights may be taken on the free wires; otherwise, the mean position of each wire must be ascertained, and the wire held in that position, at least until tapings are finished; for reading angles the ascertained mean positions of the wires can be used for sighting points. In any case an illuminated screen or scale must be fixed behind each wire.

Alinement is obtained by a series of approximations, as at top of shaft. The micrometer head is particularly helpful for this purpose. If distance from transit to the nearer wire is not much greater than the distance between wires, it is possible to focus a sharp image of the farther wire even when both wires are exactly in line with the transit (16). Hence it is not necessary to insert rings or chain links in the nearer wire, unless the transit must be set farther away.

Triangulation. First measure the distance between wires. If this indicates a divergence of the wires, it is highly probable that the line connecting them is also deflected in azimuth, and all necessary means must be used to reduce interference of air currents and falling water. Set transit on a point as nearly in line with the wires as convenient, and as close to one of them as focussing permits. Measure distance from transit to both wires, and to next underground station; also the angle from each wire to that station, and the angle between wires. These measurements are not all essential, but serve to check one another.

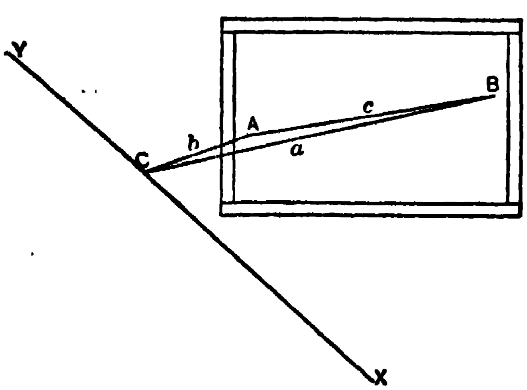


Fig 27. Triangulation at Bottom of Shaft

In Fig 27, if A and B are the wires, C the instrument point, and a, b, c, the sides opposite the angles, the known parts of the triangle are C, a, b, c. Angles A and B can be computed in several ways:

1. $\sin B = b \sin C \div c$ and $\sin A = a \sin C \div c$. As a check, $A + B + C = 180°$

2. $$\tan {}^1\!/_2 (A - B) = \frac{a - b}{a + b} \tan {}^1\!/_2 (A + B) = \frac{a - b}{a + b} \tan (90° - {}^1\!/_2 C)$$

$$A = {}^1\!/_2 (A + B) + {}^1\!/_2 (A - B) \quad \text{and} \quad B = {}^1\!/_2 (A + B) - {}^1\!/_2 (A - B)$$

3. $$\sin {}^1\!/_2 A = \sqrt{\frac{(s - b)(s - c)}{bc}}, \text{ where } s = {}^1\!/_2 (a + b + c)$$

$$\text{Also, } \sin {}^1\!/_2 B = \sqrt{\frac{(s - a)(s - c)}{ac}} \quad \text{and} \quad \sin {}^1\!/_2 C = \sqrt{\frac{(s - a)(s - b)}{ab}}$$

Eq 1 gives more precise results for small values of C; hence the advantage of setting the transit closely in line with the wires; a further advantage is that the error caused by inaccurate measuring of distance is slight. Accuracy is further gained by keeping the ratio of AC to AB as small as possible, preferably not exceeding unity. Eq 2 should be used when it is not possible to set up on a point nearly in line with the wires. Eq 3 is independent of angle C, but inasmuch as the transit must be set at C to read the angle to next station, angle C may also be read. Knowing bearing of $A-B$ from the surface survey, compute the bearing, first of $A - C$ or $B - C$, and then of $C - X$, and finally the coordinates of C and X.

While occupying Sta C (by whatever method determined) and sighting Sta X, it is advisable to set another point accurately on this line, as at Y, particularly if Sta C has not already been permanently marked as a triangulation point; point Y then serves as backsight when continuing the traverse from X.

Sighting on the free wires. Amplitude of oscillation and amount of local vibration can be reduced by immersing the plumb-bob in a pail of water, oil, or molasses. At Anaconda (7) they use a pail 18 in deep and 10 in diam, having a $^1/_4$-in pipe through the side, near the bottom, with its outlet 2 in below top of pail. The pail is filled with water to this level, and $^1/_2$ in of engine oil is poured on top to prevent splashing of falling water. Further protection is given by a split conical lid with a 2-in hole. Fig 28 shows a pail used at Broken Hill South (22) designed to hang from timbers at the corner of a shaft.

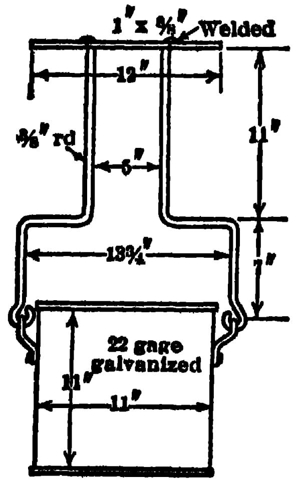


Fig 28. Plumb-bob Damping Bucket, Broken Hill, N S W

Determining mean position of wire. (*a*) If period of swing gives enough time, the wire may be followed with the upper tangent motion of the transit, the extreme right and left position being noted on the vernier. The aver of a series of readings will give the mean position. (*b*) If the transit has oblique cross-hairs, these, in connection with either of the stadia hairs, will serve as a guide in bisecting the amount of max displacement of the swinging wire, after which the mean position can be marked on the timbering. (*c*) Fix a strip of clean wood behind the wire, approx at right angles to line of sight of transit, and mark a succession of extreme positions; bisect these and set a fine nail at the mean position. (*d*) Instead of a wooden strip, a graduated rule may be used, the extreme positions being read through the transit and the mean calculated. (*e*) Draw a graduated scale on a translucent screen placed behind the wire, and read deflections as above. (*f*) A graduated slide for inserting in the eye-piece of the telescope, as in microscopic work, has been suggested.

Note.—In all the above methods involving the use of a scale to calculate the mean position, it is important to take a large and an even number of readings; 100 or more are not too many in a deep shaft, with air currents and falling water. Due to local vibration the bob seldom makes a complete swing. Prof. H. S. Munroe suggested that observations at regularly timed intervals would give the mean position with accuracy. To obtain the exact mean point for purposes of taping, the preceding methods all require 2 sets of observations on each wire, from positions giving lines of sight as nearly as practicable at right angles. One set of readings can be made on each wire from a first position of the transit, which will then be set up at another point for the second set; or, any kind of a telescope can be fixed at the second point, and the readings made simultaneously. Another method (*g*) obviates use of scales and telescopes. Bore a 1 to 2-in hole in a board, and fix it horizontally so that the wire passes through nearly the center of the hole. Cause the bob to swing in various planes and mark points of contact with circumference of hole, indicating which marks correspond. The usual pail of liquid should be omitted. Remove the wire, set a plug flush in the hole, draw lines to connect corresponding points, and set a nail where the majority of these lines intersect. (*h*) An elaborate system is the Roberts' "Shadow Method," used at East Geduld mine, Transvaal (23).

Fixing the wires. It is generally advisable, and, if sights are to be made on them at upper levels of the mine, it is essential to fix the wires in their mean positions. A simple method, under favorable conditions, is to bring the wire gradually to rest against the edge of a board so set in the timbering as to be brought forward gently against the wire. The board is then nailed fast, and the wire tied to it. If the mean position is known, from observations made as above, any simple combination of boards, tacks, and strings will serve to fix the wire.

The Schmidt apparatus (Fig 29) is useful when much work of this kind has to be done.

It is screwed to a board having a 1 1/2-in hole, set in the shaft timbers. The mean position of the wire is calculated by observations on the two scales. The bob is then disconnected, and the wire threaded through a fine hole in the brass plug screwed into the square block; the latter is then placed between the adjusting screws and shifted by them until the wire is seen to occupy its mean position, the bob having been attached again. This operation can be simplified by having a slot sawed to the center of the square iron block, the brass plug being unscrewed and threaded on the wire before the plumb-bob is attached; this saves trouble of removing the bob and then replacing it. Or, if no observations have to be made at higher levels, both bob and wire may be laid aside, and a needle inserted in the fine hole to serve as sighting and taping point after being adjusted to mean position. One advantage of retaining the wires is that all tape measurements can then be made horizontally, from transit to both wires, and some computation thereby eliminated.

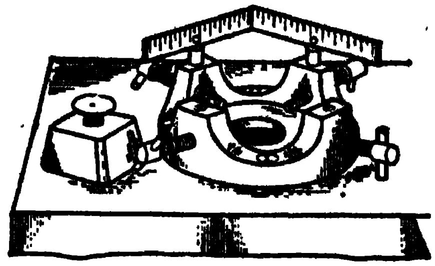


Fig 29. Schmidt Shaft-plumbing Apparatus

A method for bringing a plumb-bob gradually to rest in its mean position and holding it there as long as needed has been suggested (17), though not yet tested in practice; it would obviate both the instrumental determination of mean position and the fixing of the wire by mechanical aids. A can or small pail, only large enough to give the swinging bob clearance on all sides and only deep enough to submerge lower half of bob, is rigidly fastened by cleats and wedges to a plank nailed across shaft timbers. A warm solution of agar-agar (1.5 oz per gal thoroughly dissolved by boiling) is poured into this receptacle; as solution cools it slowly increases in viscosity and at ordinary temperature sets to a jelly rigid enough to hold the bob securely. The receptacle must be protected against falling water. If desired, the jelly can be dried out and used again.

Instrumental methods of plumbing. The side or the top telescope of a transit may be used to project the meridian of a survey with great accuracy down a vertical shaft; special eccentric transits (Art 4) are particularly useful for this. A plumb-bob must also be lowered, to give the coordinates at bottom, but as a slight error in coordinates is not so serious as one in the bearing of an underground base-line, the combination of instrumental observation and a single plumb-bob has great advantages. Observations may be made from either top or bottom.

Transit at top of shaft. Set the transit on heavy timbers in such a way that it can see to bottom of shaft. Sight along a line having a known bearing; then sight down the shaft. Stretch a wire across the shaft on the bottom level, as nearly as can be estimated under the transit, and in same plane as the telescope. The wire should have a weight on one or both ends and at each end it may be laid in the thread of a long screw mounted horizontally on a heavy trestle; by turning these screws manipulate the wire until it is bisected by the transit throughout all its visible length. This wire then gives a bearing on which to start the underground traverse, and a single plumb-bob gives coordinates of a starting point, which may be situated anywhere in the shaft. Instead of a wire, a plank having an illuminated sighting point at each end may be shifted by repeated trials into the plane of the telescope, the two points then being used as the underground base-line (8). In all work of this kind repeated observations should be made, with the telescope normal and inverted.

Transit at bottom of shaft. A method exactly similar to the preceding may be employed, stretching the adjustable wire across top of shaft, the bearing of which will be ascertained later. Or, the wire may be fixed in known bearing, and the transit brought into same plane with it by repeated trials. It is not necessary for the transit to be vertically under the wire, provided the latter is horiz; it may be tilted by its leveling screws until the wire is seen to lie in plane of rotation of the telescope, even though this plane is not vert. If then two points be fixed in the workings *at equal elevations*, whether above or below center of transit, the line connecting these two points will be parallel to the wire at the top, and its bearing can be used for starting the underground survey. Coordinates must be transferred, as before, by a single plumb-bob.

Measuring depth of shaft. (*a*) By direct measurement with tape along the timbers, made by two men, one on the roof of the cage, and one on a seat attached to the hoisting rope about 100 ft above the cage. It is well to have a third man on the cage to give signals promptly. Direct measurement is relatively simple if the shaft has a manway. (*b*) By measuring a plumb-bob wire. Place a small pulley in the head-frame so that the wire can pass over it while hanging freely in the shaft, with plumb-bob attached. Set one mark (*A*) close to the shaft and another (*B*) at any convenient place on the surface, but exactly 100 ft from *A*; the two points should be as nearly as practicable in line with the

pulley. Having noted the elev of the bob with respect to a bench-mark at the shaft bottom, fasten a marker on the wire opposite point *A* and walk with it to point *B*, when another marker is to be attached at *A*. Continue reeling up the wire, measuring off every 100 ft and the final fraction, until the bob arrives at surface and can be referred to a bench-mark. This method compensates for variation in stretch of the wire, due to its diminishing weight as it is drawn out of the shaft.

9. NOTES AND COMPUTATIONS

There is no best form of notes, and every engineer is entitled to his own preference, so long as his system is equally clear to him and to the engineering staff. In general, it is advisable to use a printed form, with headings printed or written in, having no more columns than necessary. The transit man can then tell at a glance whether every item of information pertaining to each set-up is recorded, before proceeding to the next station. If a certain measurement is not required at a given set-up, a check-mark should be made in the appropriate space, to indicate that it was not overlooked. The following headings are suggested for ordinary underground traverses.

Azimuth Traverse

Course	Azimuth	Tape	Vert angle	H I	H S	Remarks

Deflection Traverse

Course	Deflection		Double def	Tape	Vert angle	H I	H S	Remarks

Angle Traverse

Angle	First reading	Second reading	Tape	Vert angle	H I	H S	Remarks

In the form for deflection traverse the narrow column under "Deflection" is for entering initials *R*, *L*. After dividing the doubled deflection (or angle) by 2, the final value may be entered in first column, on line below. If duplicate readings of any measurement are taken they may be entered one below another in same column; it does not pay to economise note-book space. Side-shots, entered one after another in regular order, following the traverse data taken at that station, are preferably lettered to distinguish them from numbered traverse stations. In case of an angle traverse by repetition it is well to devote a whole or a half page to each set-up. Angle readings may be recorded thus:

Angle 7–8–9

	First reading			Fourth reading			Final value		
	°	′	″	°	′	″	°	′	″
Vernier *A* at 0°............	—	—	—	—	—	—	—	—	—
Vernier *A* at 180°...........	—	—	—	—	—	—	—	—	—
	Average...			....			—	—	—

The data relating to courses 8–9 and 8–7 would then be entered lower on the page in their appropriate columns. Sketches and remarks are made on the right-hand pages, which should have coordinate ruling.

Loose-leaf systems. Among the advantages are: (*a*) Pages are kept cleaner, since a particular page need not be kept more than one day underground. (*b*) It is possible to file together all the notes relating to a given section of the operations, though they may have been taken by different engineers at different times. (*c*) A complete record of all notes is constantly on file in the office. (*d*) While working at a distance from the office, the notes can be sent in daily for computation and mapping.

The forms should be printed on bond paper cards, say 5 by 7 in, punched to fit the clasps of a leather or aluminum cover, which should be large enough fully to protect the edges of the sheets. Duplicate copies are sometimes desirable, in order that the engineer may have a complete set as

well as the office. One way to make these is to have white envelopes ruled and punched exactly like the cards, into which the cards fit snugly. One card and a carbon copy are put into each envelope, and a number of envelopes are put into the cover for the day's work. The engineer then retains the empty envelopes, and returns the carbon copies to the office.

Computations. The first step is to convert azimuths, deflections, or angles, into bearings, as in Sec 17, Art 11. Next, reduce inclined distances to their horizontal and vertical components, multiplying by the cosine and sine respectively of their vertical angles. Gurden's traverse tables or mechanical calculators are great helps in this operation. Next compute latitude differences and departures of all courses, or take them from traverse tables. If a closure can be made, compute and distribute the error in proportion to the length of the courses alone. From the balanced latitudes and departures, compute the coordinates of the stations. If no closure can be made the coordinates are calculated from the unbalanced latitudes and departures. Lastly, compute elevations of the stations, from the vertical distances, the H I and the H S, as described in Art 5. The following form will be found convenient for assembling the data and the computations.

Course	Azimuth	Bearing	Inclined distance	Vertical angle	Horizontal distance	Latitudes		Departures		Corrected latitudes		Corrected departures		Coordinates		Vertical distance	Height of instrument	Height of sight	Elevation	Station	Remarks
						+N	−S	+E	−W	+N	−S	+E	−W	Lat	Long						

The whole aim and object of a mine survey is thus to determine the coordinates and elevations of as many points as possible. If the whole system, above and below ground, be based on the same set of coordinate axes, nearly every problem in mining engineering can be readily solved by the application of the principles of trigonometry, or analytical or descriptive geometry. (See Sec 36 and references 9 to 12 in the bibliography.) To minimize the possibility of error in computation it is well to take as the origin of coordinates a point well to the south and west of the entire workings; negative coordinates are thus avoided. For the same reason elevations should be referred to a plane well below the deepest workings.

The following problems are the ones most frequently encountered in laying out new mine workings.

To establish the line connecting any two points. If x_a, y_a, z_a are respectively the longitude, latitude, and elevation, of station A, and x_b, y_b, z_b are the corresponding data for station B:

1. Bearing of the line A–B is that angle whose tangent $= (x_a - x_b) \div (y_a - y_b)$; inspection will indicate in which quadrant the bearing lies.

2. Horizontal distance $A\text{–}B = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$.

3. Difference in elevation between A and $B = z_a - z_b$.

4. Vertical slope of the line A–B is that angle whose tangent $=$

$$(z_a - z_b) \div \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}.$$

5. Inclined distance $A\text{–}B = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2}$.

Starting at Sta A, run a traverse by the shortest available route to Sta B; if this traverse does not form part of a closed circuit, it can be made to close by returning by the same route, but using different stations. A serious error of closure should be investigated, by repeating the work, if necessary, but a reasonable error may be balanced. This gives the coordinates and elevations of A and B. Calculate bearing and slope of the connecting line A–B by Eq 1 and 4, above. Set the transit at either A or B; orient

by sighting on a traverse line with the lower motion; set the horizontal vernier to read the computed bearing or azimuth, and the vertical vernier to read the required slope, and fix two points from which the mine boss can hang sighting cords.

To locate corners of a vert shaft to connect with a winze or raise. From an underground station near the winze or raise, measure bearings and distances to the corners, and compute coordinates thereof. Run a traverse to a station on the surface near the site of the proposed shaft, and compute its coordinates. Calculate bearings and distances from this station to four points having the same coordinates as those observed underground (Eq 1 and 2 above). With the transit set on this surface station turn off successively the computed bearings, and measure the corresponding distances to the four desired points.

To compute point at which a given line will intersect another line, for example, the end line of a claim. The known data (Fig 30) are the bearings of $B-C$, or a, and $D-A$ (which is also that of b), and the coordinates of the points A and B; required, the distance $A-C$, or b. Compute bearing and distance of c by Eq 1 and 2. From the bearing of c compute angle B, by reference to the known bearing of a; also angle C in the same manner. Then $b = c \sin B + \sin C$.

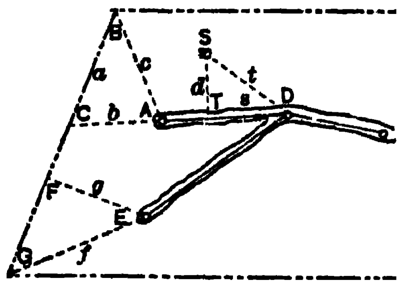


Fig 30

To establish the shortest line from a given point to meet a line of known bearing. The required line will obviously be perpendicular to the given line; hence its bearing can be computed and set off at once by a transit on the given station. If the length of the connecting line is desired, the coordinates of some point on the given line must be known, as G; whence compute bearing and distance $E-G$. In the right triangle EFG, the side f and the angle G are now known; whence $g = f \sin G$.

To determine the point on a given line from which to make the shortest connection with a given point, as, for example, to drive a crosscut from the drift $D-A$ to the bottom of the shaft S. The shortest line will obviously be perpendicular to $D-A$; hence its bearing is known, as are also the coordinates of D and S (the latter from a surface survey). Compute bearing and distance of $D-S$, or t, from Eq 1 and 2; whence compute angle S. Then $s = t \sin S$.

The above are given only as suggestive examples. In general, the shortest solution of a problem will become apparent by the simple method of plotting the known factors.

10. MAKESHIFT METHODS

Circumstances such as (*a*) lack of most suitable equipment, (*b*) absence of a helper, (*c*) difficulties in carrying out an instrumental survey by precision methods, (*d*) need for a quick survey without high accuracy, have led to numerous ingenious applications of trigonometry or descriptive geometry.

One-man survey may be necessary in examining prospects and small mines; it should not be attempted if work requires climbing old ladders or going into unventilated stopes. Brunton "transit" (Art 4) is the usual instrument; a light tripod with telescopic legs aids accuracy but is not essential, since a box or a piece of timber will serve as a mounting when hand-held readings are unsatisfactory. Candle ends serve for station points. Linen tape manipulates easier than steel; at the zero end, tape can be anchored by a cord to a rock, timber, or track tie. A 6-ft folding or semi-flexible rule is better than tape for short measurements.

Traversing steep workings without auxiliary telescope. Method described by C. Ferguson and K. J. Benner (24) is limited to slope lengths of 100–150 ft (steeper the slope, greater the permissible length); it assumes a traverse station with known coordinates and elevation near upper or lower end of the inclined opening.

Stretch a fine wire tightly between permanent or temporary supports at top and bottom, so as to leave as much as possible of the wire visible from the established station; heavy spike with notched head serves well in timbered workings. On this wire, set 2 marks (as by pinching on it short bits of soft copper or solder wire) as far apart as they can be seen from the transit station; read azimuths and vert angles, and tape the inclined distances from transit to both points (head of the supporting spike may serve as one mark); then calculate coordinates and elevations of both. Further calculation (Eq 1 and 4, Art 9) gives azimuth and slope of wire. Tape the distance from either mark to a similar mark at other end of wire, coordinates and elevation of which are now computable. Set transit under the last mark, backsight on the wire or a plumb-bob hanging from it, and continue traverse.

String surveys, requiring no direct angular measurements, have given sufficiently accurate data for plotting steeply inclined and tortuous raises, winzes, and narrow stopes in parts of the North Broken Hill mine, N S W (25).

In Fig 31, A–A' is a traverse course through a drift, details of which are known; plumb-bobs are hung at A and A'. B–B' is a wire or strong cord stretched tightly between temporary fastenings, the position of that at B' being recorded by measurements to roof, floor, and walls. Under conditions as in Fig 31 (a), a plummet is hung from B–B' at C and brought by eye into line A–A'; point D is also marked in line A–A'. Measure (horizontally) C–E, C–D, D–E, and A–C or C–A'; also inclined distance C–B'. Inclination of B–B' is obtained by hanging a plumb-bob at either end (Fig 31 (d)), measuring the length of cord B–H, and distances H–J and B–J (not safe, in this case, to assume that H–J will be measured horizontally). Cases (b) and (c) require setting of 2 temporary points C and D on line A–A', usually also light plummets at E and E'; the dotted lines

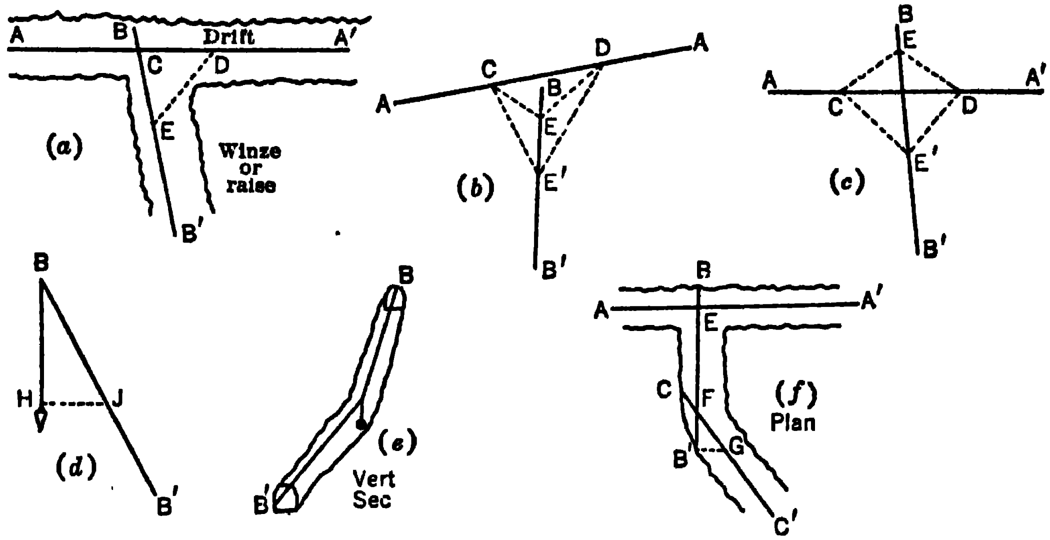


Fig 31. String Surveys, North Broken Hill Mine, N S W

indicate horiz measurements to be made. If the opening turns vertically, as at (e), enough weight is attached to make the cord clear the roof, length and inclination of both segments then being observed. At a horiz turn, as at (f), 2 cords, B–B' and C–C', are mounted, almost touching at F; horiz measurement of B'–F, B'–G, F–G then fixes the horiz angle at F; inclined distances E–F, F–C', with their corresponding vert angles, will locate C', and so on. At the Broken Hill mine, graphic solutions of angles and bearings, at scale of 1 in = 1 ft, have given sufficient accuracy; or, each case shown above can be solved trigonometrically from the indicated data, by Eq 3, Art 8.

Projecting azimuth down inclined shaft. In this case (26) the only available transit had prismatic eyepiece for main telescope, but no auxiliary telescope; hence, could sight only upward. Shaft, 5 ft wide between wall plates, pitched 85°; stations at 80 and 180 ft depth. A short piece of brass angle, with a notch at exact middle of one edge, was fastened to the upper-inner edge of the hanging-wall plate (Fig 32), and a plumb-bob with as long a cord as possible was hung from this notch; bob was damped in a pail of water on footwall side. Surface survey station, 20 ft on footwall side of shaft, was an iron pipe capped with a plug, turned and center-marked by lathe; diam of round plug was same as length of brass angle. Two parallel cords were stretched around this plug and the ends of the brass angle, their azimuth being known from surface survey. At an underground station on hanging-wall side, the transit was then adjusted by trial until the space between the cords was bisected throughout its visible length by the plumb-bob cord; telescope was then in vert plane of known azimuth.

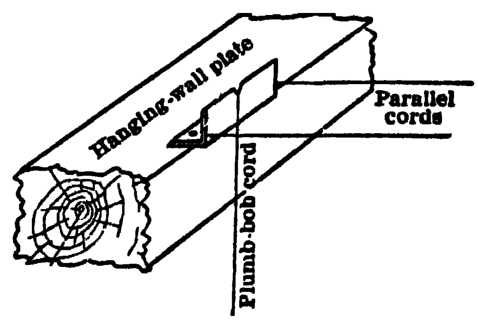


Fig 32. Transferring Azimuth down Inclined Shaft

Graphic solution of inclined raises (A. D. Rood, 27), is useful for alining the inclined and branched raises commonly required for block-caving into chutes (Sec 10, Art 80); for raises up to 125 ft long, accuracy of plotting is commensurate with that of observations by Brunton compass and tape.

Knowing coordinates and elevations of points A, D to be connected (Fig 33), procedure follows: On coordinate paper, and at scale of, say, 1 cm = 5 ft, plot positions of A and D in plan. A protractor then gives bearing A–D. Lay out a vert scale (same scale as horiz) adjusted to show D at

its proper elev. (In Fig 33, coordinates are shown at bottom and right; elevations at left.) Transfer the horiz projection of *A–D* to a vert plane passing through both points by swinging arc *A–A′* and projecting *A′* downward to *A″* at elev of *A*. Inclination of *A″–D*, by protractor, is slope of connecting line *A–D*. If the raise gets off line, a Brunton and tape survey gives bearing, slope, and inclined distance to face *B*. From *A″*, plot this slope and inclined distance to *B″*; project *B″* vertically upward to *B′* and swing arc *B′–B*, to intersect line *A–B* drawn by protractor at its observed bearing. When the face has advanced to *C*, a set of measurements at *B*, with similar construction starting from *B″*, locates *C* in vert and horiz planes. *C–D* then gives the bearing, and *C″–D* the slope and length of the closing line. In this example, note that 3 different vert planes are involved, all containing point *D*.

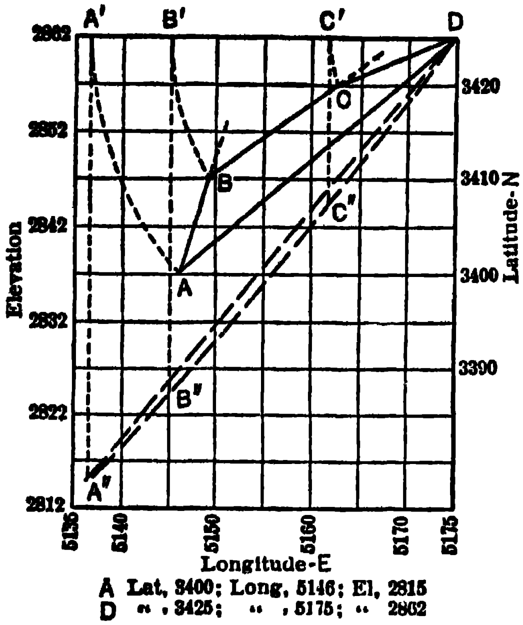


Fig 33. Graphic Solution of Inclined Raise

Survey of glory-holes at Fresnillo, Mex (Sec 10, Art 99) was formerly done by stadia (Sec 17) when walls were at max slope of 45°–50°, and vert angles at transit rarely exceeded 30°.

With increasing depth and steepness, the following methods were applied (28): (*a*) A 4-in steel ball on end of a rope was repeatedly dragged across the pit and its successive horiz and vert positions were observed simultaneously by transits on 2 previously established stations on the rim; position in plan was plotted by intersecting azimuths, and horiz distances to each station were measured by scale; these distances, × *tan* of corresponding vert angles, gave check on elev at each position. (*b*) For slope distances under about 50 m, a steel tape was attached to the ball, in addition to the rope; a transit on a rim station then gave horiz and vert angles, and the tape gave inclined distance to each point, sufficing for its computation. A third suggested plan requires setting of rim stations (not all of which need be monumented) at opposite ends of as many section lines as desired. The ball is brought into positions along each section line by 2 ropes (long enough to reach clear across) manipulated from corresponding stations on rim. Position of ball is observed by horiz and vert angles from a transit on a station at end of pit; positions are plotted in plan by intersections with section line, and elevations computed from scaled horiz distances × *tan* of vert angles.

11. MINE MAPS (See also Sec 19)

The map of a growing mine should be drawn on muslin-backed, tough paper, in a roll long enough to accommodate the most extended workings. The scale 1 in = 100 ft is as small as should be used for a map required to show details; and a larger scale is better if the map can be brought within the available paper.

Sectionalized maps are preferred for mining properties of large extent, to cover which on a scale as small even as 1 in = 100 ft would demand an unwieldy single roll of paper.

At Butte (18), the Anaconda properties are mapped on standard 18 by 24-in sheets, with 1-in margin. An area 20.8 miles E–W by 15.1 miles N–S is divided into 100 sections both ways, each block being plotted on the standard sheet to scale of 1 in = 50 ft. Each level is separately plotted at that scale, these maps being kept closely up to date and serving as the main working diagrams of the respective mines. Additional maps of same size are plotted at scales of 1 in = 100, 200, and 600 ft, with diminishing amounts of detail, the last carrying only triangulation lines and the most important surface features.

The first step is to rule the paper with fine, light-blue lines into accurately constructed squares of 4, 5, or 6 in, depending upon the scale to be used. Number these coordinate lines along margins of the paper. Plot all traverse stations by coordinates only; inasmuch as the coordinates are already computed, no time is saved, and avoidable errors are introduced, by using the protractor for any of this work. Indicate each traverse point by a small red circle, and write its number and elevation in black. It is not advisable to write the coordinates, as the figures would be crowded, and the coordinates are quickly available in the office records; it is advisable, however, to write the elevations to give a better three-dimensional idea when inspecting it. Connect traverse points by fine red lines, stopping at the circles.

Plot the outlines of the workings by offsets from the traverse lines, by intersections, or by angles and distances (reduced to horiz), as the case may be. A protractor is allowable for this work. A paper protractor of 24 in diam, with a semicircular opening cut out of its interior, is nearly as accurate as more expensive instruments. Many special forms of protractor designed for this work are in the market. Those carrying a scale on the rotating arm are excellent where much mapping

on the same scale is to be done. A special device described by J. J. Bristol (13) diminishes the labor of plotting by mechanically reducing inclined distances to horiz. Draw the outlines in black, freehand. If the mine workings lie vertically under one another, on only 3 or 4 levels, colors may be used to distinguish them. If more than this number, it is better to make a separate map for each level, all oriented and coordinated alike. To get a combined view, tracings of the several levels can be superimposed. Conventional designs to indicate various underground features are shown in Sec 19, Art 1.

Finally, a complete map should contain: (I) A title in simple lettering, and in a uniform position (lower right-hand corner recommended) stating: (*a*) name of mine or portion thereof (the words "map," "plan," etc., can be omitted); (*b*) names of surveyor and draughtsman; (*c*) date of survey and plotting; (*d*) scale, indicated thus: 1 in = 100 ft, not 100 ft to the inch. (II) An arrow indicating the true (or assumed) meridian, and another indicating the magnetic declination, if this is of interest. (III) A simple border.

Methods of making maps in glass, wood, etc., are described in Sec 19.

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SECTION 19

MINE GEOLOGIC MAPS AND MODELS

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MINE GEOLOGIC MAPS AND MODELS

1. MINE GEOLOGIC MAPS

Geologic study and mapping are especially useful in complex or faulted veins. Work of this nature, in conjunction with mine development and exploitation, is undertaken chiefly by large mining companies, though similar study is proportionately important at small mines. The surface and underground notes and sketches are transferred to office maps, omitting less important notations from field books, which would tend to overcrowding. Where structure is complicated, geologic plans may be supplemented by vertical sections or by models. For other data on Mine Maps, see Sec 10, 18 and 25.

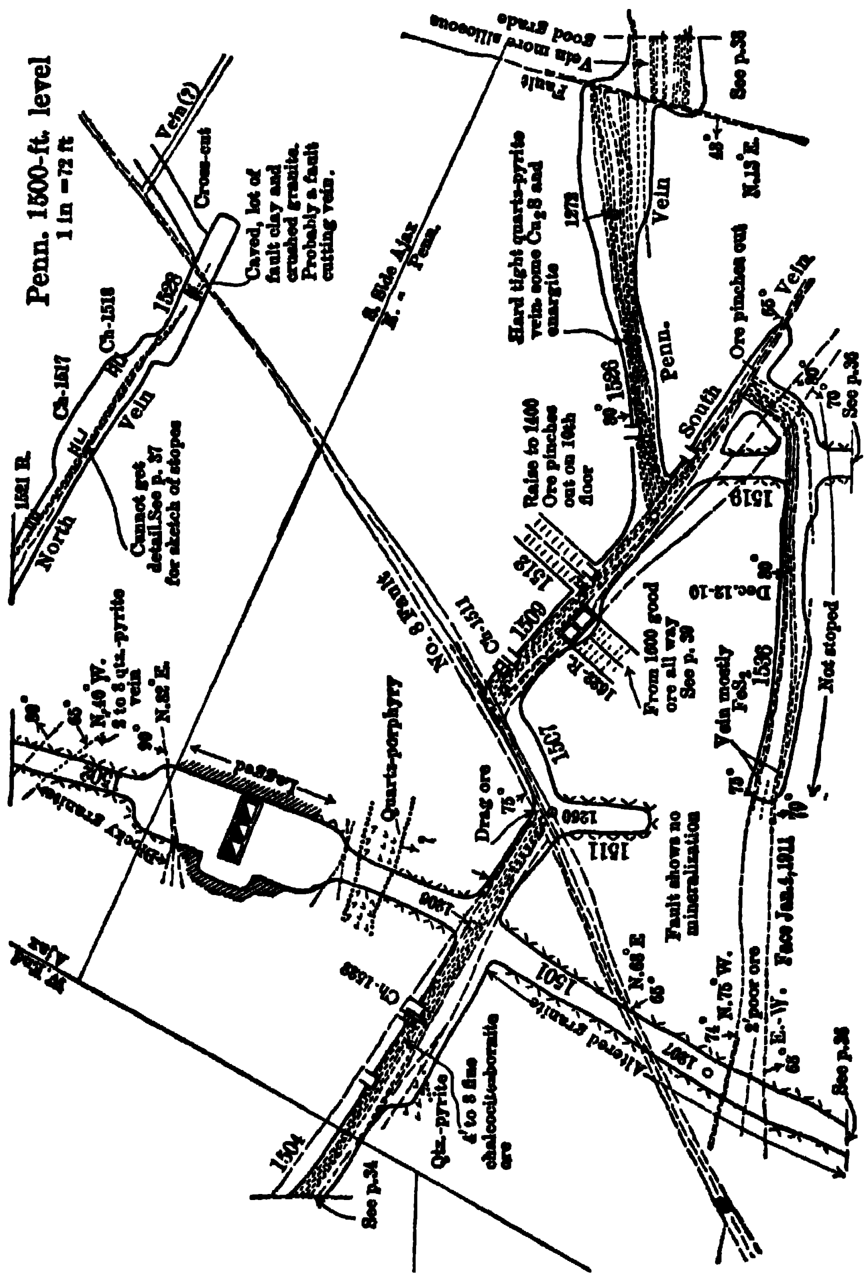

Fig 1. Example of Field Notes

Underground observations are best made in new workings on freshly broken rock and ore. In wet ground, the surfaces soon become coated with black slime, derived from moisture, dust, and products of explosives, which conceals mineralogic characteristics and adds to the difficulty of correct geologic deductions. In shifting or faulty ground, also,

inspection of workings should be made immediately after blasting and before timbering. Important information is frequently found in wet or heavy ground. Prompt geologic mapping is always desirable, to enable those in charge to direct work properly, and avoid the waste of misdirected exploration. Observations should not be confined to level workings, but should cover stopes and all other openings. In vicinity of faults, sill-floor workings in wide veins become heavy, and either cave or are closely lagged. Plans and cross-sections must then be made of the stopes above (Fig 4). Frequently, when engineer or geologist is called on to map mine geology, much of the ground has been stoped and no information as to nature of vein is obtainable.

Each ore-deposit presents its peculiar problems; in taking notes the observer should keep in mind the purpose of his work, and the point of view of the mine owner. In complicated vein and fault systems structural features demand special attention; in other cases, mineralogic associations are paramount. It is important to distinguish essential from non-essential geologic data.

Field notes are best recorded in loose-leaf notebooks, with aluminum covers. Leather will not stand hard underground usage, and sketches in bound books become blurred; with loose-leaf system, only the sheets covering workings visited need to be taken underground. Also, with bound books, growth of the mine soon causes overcrowding of notes and sketches, or scattering of them in several books. Loose-leaf records can be arranged as desired, and made easily accessible in office by a simple filing system. Scale and size of paper depend chiefly on character of deposit; sometimes on scale of mine maps, or the kind of notebook available. If minute details be necessary for proper interpretation of geologic data, use scale not less than 20 ft to 1 in. For sketches, and for use in wet or hot places, a 6 by 8-in page is desirable, ruled with pale blue or green indelible ink in 5-ft squares, with every fourth line emphasized. For more favorable conditions, an 8 by 11-in page is preferable for clear delineation of structural relations. Maps on white-print paper, or cloth, may be used underground instead of notebook, but are unwieldy and liable to be torn or soiled. When possible, begin by transferring the workings direct from mine maps to notebook, orienting them on the pages. See Fig. 1.

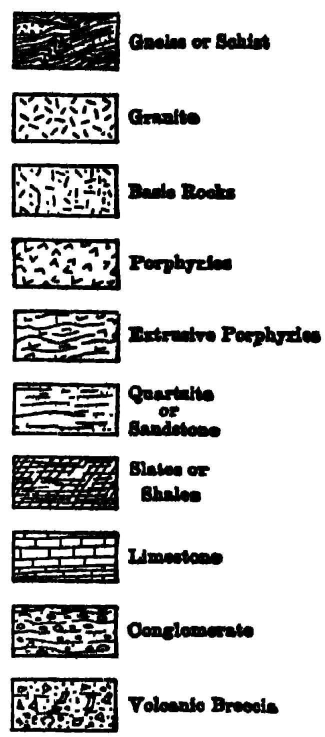

Fig 2. Conventional Signs, Rocks

Underground sketching. Colored pencils are used to denote certain geologic features, thus: red for vein minerals, and blue for faults and other dynamic effects. In representing vein matter by red, distinction between poor and rich ore is left to side notes. Too many colors are confusing, and for mixed minerals differentiation of varieties is impossible. Country rock, except narrow dikes, is best designated by conventional signs (Fig 2). Chief object is to emphasize veins or faults, as contrasted with the less important wall rocks. Vein structure is difficult to represent correctly in notebooks, and given conditions are seldom interpreted alike by two observers; hence careful, accurate sketches are essential. Observer must remember that, as others may have to interpret his notes and sketches, wrong inferences of serious nature may result from inaccuracy. Clear-cut sketches indicate clear understanding of the structure; muddled, mussy sketches denote confusion of ideas.

Facts to be sketched and noted include, in general, everything having present or future bearing on life of mine. Fig 1, 4 and 5 are typical examples of field notes from files of a mining company employing a mine geologist. Conventional signs (Fig 3) are placed along margins of sketches. In crosscuts, drifts, stopes, and raises, all veins, faults, slips, and dikes are plotted, with side notes at intervals respecting strike, dip, and mineralogic character, with their changes, if any; all being accurately located by tape measurement with relation to a mine survey station, or other fixed point (Fig 1 and 4). Pacing is excluded, except as rough check, or for unimportant details. Keep sharp lookout for cross-faults, strike-faults, and splits in vein. At intersections, or unions of veins, note change, if any, in mineralogic character; also, influence of faults and direction of "drag" along the fault. Avoid too copious notes, a common fault of inexperienced observers; note essentials briefly. Base generalities on careful weighing of available facts.

Certain matters are of scientific interest, even if not of immediate practical importance, such as: rock alteration, paragenesis of minerals, depth of oxidation and weathering, and presence of water in vein and country rock.

Instruments are pocket compass and tape. Brunton pocket transit is widely used; Verschoyle transit, Attwood clinometer, and ordinary sighting compass are also good. Metallic linen tape (Sec 17) is accurate enough for most purposes; in dry mines steel tape may be used. For notebook plotting a 6-in transparent celluloid protractor scale, graduated same as office maps, is recommended; or a common scale and protractor will serve.

Safety precautions. Geologic maps are rarely made of a mine in its early development. Hence, the geologist is usually obliged to examine many workings abandoned wholly or in part. Such work is hazardous. A common source of danger is bad air in dead ends, and in old workings lacking ventilation. Enthusiasm in pursuit of information should not lead to taking unnecessary risks. If bad air or deadly gases are known or suspected, get all possible information before entering workings. If air is such that a candle will not burn, do not force matters by using electric or acetylene lamps, both of which will burn in air too poor in oxygen to support life. Other dangers in old workings are: open stopes and winzes, rotten and loose ladders, the latter being frequent in presence of copper or acid water. In freshly broken faces, examine for missed holes or loose slabs before using pick. Always watch for open manways and chutes, loose rock, loose planks in stope floors, trolley wires and trains of cars. Geologists often work singly, but, as their work is hazardous, the practice is unwise.

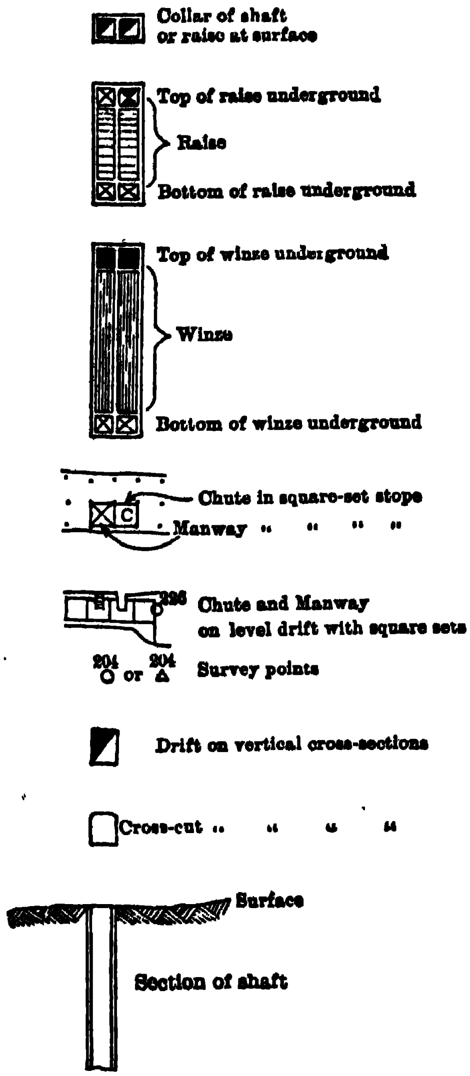

Fig 3. Conventional Signs, Workings

Office records. For a small mine, one set of geologic plans, with one or more cross-sections, or a glass-sheet model, will suffice, except in cases of complex structure. For large concerns, operating more than one mine, maps may be duplicated, furnishing to the superintendent generalized small-scale maps of two or more adjoining mines, and to the foreman detailed maps of mine under his direction. Vertical sections (Fig 5) are essential in conjunction with plans. Sections are taken at regular intervals, and as nearly as possible perpendicular to strike of principal veins; all openings are outlined.

Making the maps. Tracing cloth is in general use, having many advantages over opaque material, like drawing paper or unprepared blue-print cloth. A set of maps comprises surface map and separate sheet for each main working level. Extra sheets for intermediate levels or stopes are added when necessary for correct interpretation of geologic structure. Surface sheet shows property lines, contour lines, and all surface openings; but surface structures are admissible only for reference, and are indicated by light lines. Vein and fault outcrops must be accurately plotted. Approximate positions of concealed outcrops, determined by projection from known points in mine, are indicated by dotted lines. Mine-level sheets (Fig 6) are coordinated, and made to register with surface sheet and with one another. Each horizontal section sheet should show all openings intersected by that section. Claim lines are put on each level sheet, to show relative position of a working or of a certain geologic feature on any level.

Name of mine, with number and elevation of level shown, is placed at lower right-hand corner of each sheet. If mine numbers are used, place them along their respective openings. Raises, winzes, and stope chutes are shown along the drifts. Geologic features are plotted from field books, closely to scale, with colored pencils or ink, corresponding with those used in field work. In map construction, however, wider latitude in colors is permissible, and different veins or faults may be drawn in different colors. Assorted colors are advantageous when veins or faults are not of same geologic age. Veins or faults developed on higher or lower levels, but not intersected by any working on the level under consideration, are projected in their probable positions. Reasonably accurate projections of geologic data on unopened levels assist the superintendent in planning future development.

Scale of map depends on area covered, nature of deposit, and complexity of structure. To avoid necessity of reducing or enlarging, it is convenient to use same scale as for mine survey maps. Except for special uses, tracings should not be larger than 36 by 42 in, though it is advantageous to include on one sheet as much of a mine level as possible, and some allowance should be made for future extension of workings. For simple structure a scale of 100 ft to 1 in may be used; for large areas, 200 ft to 1 in. But, on small-scale maps, accurate drawing is required, little side notation is admissible, and it is difficult to give due prominence to veins, stringers, faults, and slips. The relative importance of

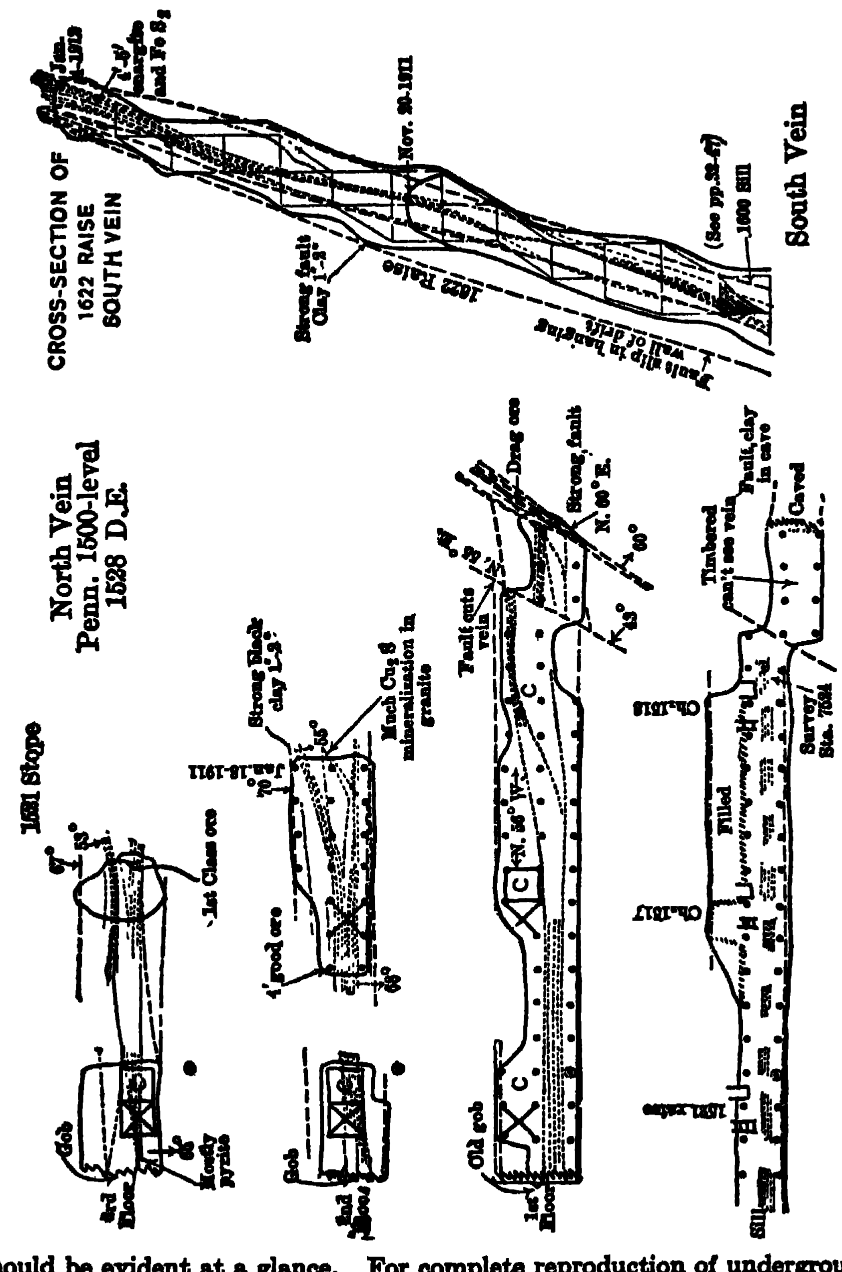

Fig 4 and 5. Examples of Field Notes

veins should be evident at a glance. For complete reproduction of underground notes, a large scale is necessary. If a single sheet does not embrace the whole mining territory, the area may be divided into blocks, with a set of geologic sheets for each. In general, a scale of 40 ft to 1 in is good for average conditions. Accurate field work, followed by careful transfer of notes to maps, is essential.

Filing maps. Wear and tear on geologic maps are excessive. Tracings become wrinkled, dirty, and opaque, and hence less useful. Tearing or rolling of corners and edges may be prevented by a $^1/_2$-in binding of unprepared blueprint cloth; made by cutting cloth in 1-in strips, folding to cover map edge, and stitching with a sewing or carpet machine (Fig 6). Maps should be laid flat in filing cases or drawers, never folded, nor

rolled if it can be avoided. The Shannon spring-arch file serves well for level maps, attached at one edge in pairs to compo board or to thin pine board, cut 1 in larger than map.

Thus mounted, tracing cloth soon tears out where punched. Reinforcement may be supplied by punching the holes in the map binding, or by pasting a patch of blue-print cloth where hole is to be made. Maps are held flat by a light cover board, same as filing board, with slot to go over the Shannon arch. For geologic cross-sections, and other less used maps, the Beck filing case is good for sizes up to 36 by 42 in.

Vertical cross-sections are on planes perpendicular to strike of vein; or, in case of two or more veins or faults, section lines are taken where structural relations will be best brought out. In bedded deposits, section lines may be taken perpendicular to strike of stratification planes, to show relation of deposit to inclosing strata. All workings cut by a section plane are accurately plotted; stope openings are specially important, to exhibit relation of advancing stope to geologic features of level above.

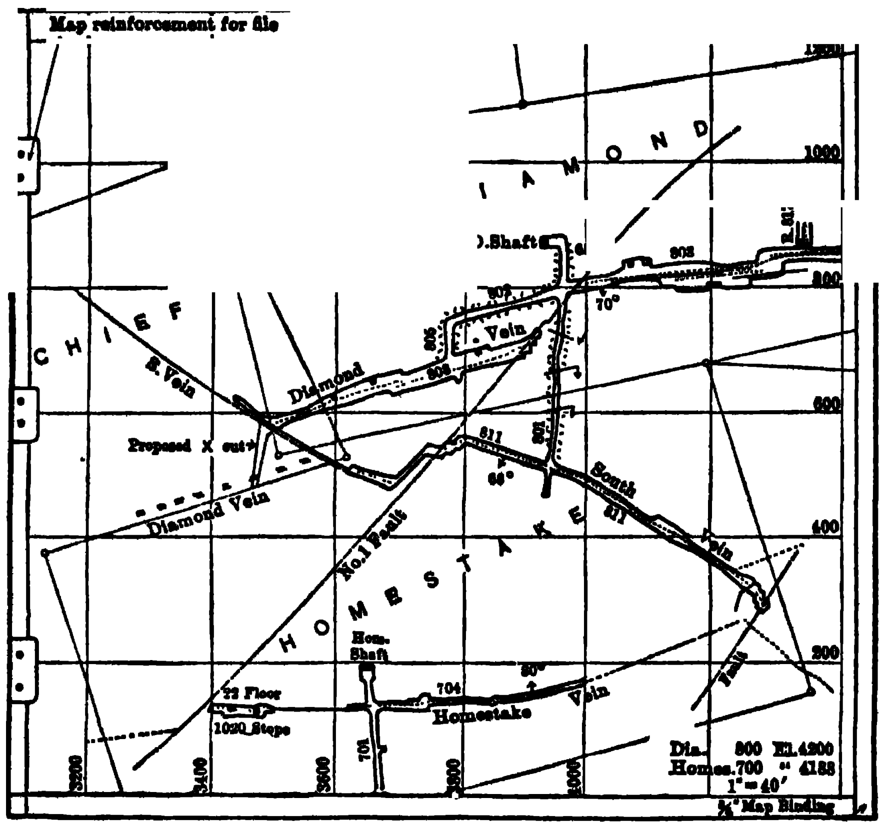

Fig 6. Working Geological Sheet (on Tracing Cloth with Edge Bindings)

In Fig 7, assuming projection of unknown features to be fairly accurate, the prospecting and development of the faulted vein segments are readily planned for minimum cost. Vertical sections are especially useful for projecting veins, faults, dikes, or rock strata, through considerable distances on dip, to determine possible intersections or unions; also to locate vein apices with reference to claim boundaries, under law containing the "extra-lateral right" provision (see Sec 24).

Longitudinal vein projections determine structural relations within plane of vein or fault along the strike. All workings should be shown; whether in vertical projection, or on a plane approximating true dip, must be decided in each case. Principal use of longitudinal sections is to show extent of stopes. In geologic work, intersecting veins, faults, or changes in country rock are the only structural features that can be represented. The pitch or general trend of orebodies occurring in shoots or irregular shapes is well shown. A cross-fault, not exhibiting large horiz displacement on plan, often causes considerable vertical displacement in plane of vein (Fig 8). Longit sections bring out clearly the influence of junctions or intersections of cross-veins, faults, or changes in wall rock.

Practical use of geologic maps: (*a*) to assist in reducing mining costs; (*b*) to lengthen life of mine; (*c*) to preserve a geologic record of mine for future use. A thorough understanding of the mine geology by the superintendent, *in advance of exploration*, is of prime importance for avoiding useless underground prospecting. Where geology is complicated,

it is rarely possible for the mine staff to comprehend intricate structural relations of veins and faults, however familiar they may be with the mine workings. Best results are obtained by combined use of plan and vertical-section geologic maps, possibly aided by glass-section models. Veins and faults are projected on strike and dip, and probable positions indicated by dotted lines.

Such advance interpretation of geologic structure assists superintendent and foreman in planning the larger development of mine, and in searching for new veins or faulted segments of known veins. Level workings intended as permanent openings for drainage, air or haulage ways, shafts, pump stations, and raise connections should, if possible, be laid out to avoid faults and heavy ground. Maintenance costs of mine openings are directly affected by nature of ground encountered; hence the value of carefully prepared maps, showing probable areas of heavy ground. To have the structure well in mind, the geologist must follow closely the advance of mine openings, so that when a vein is cut off by a fault or dike, no money is wasted in search for the lost segment. Mine foreman and geologist must therefore work in harmony, so that, in beginning search for faulted vein, the first round of holes may be properly directed. When fault zone is wide, or composed of more than one fissure, its entire width should be penetrated before search for faulted segment is begun. Owing to irregularities in ore-deposits and frequent tendency of vein fissures to branch, or change dip or strike, the mapping of smaller structural details often assists in keeping an exploratory drift on the important branch or fissure; and the proper subsequent procedure for exploring other branches readily suggests itself. If the deposit be in sedimentary rock,

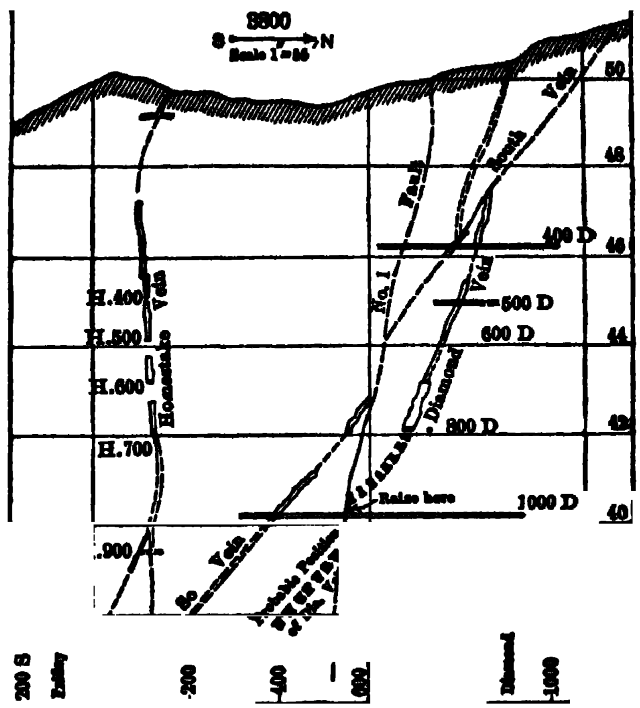

Fig 7. Vertical Cross-section on Coordinate 3800 of Fig 6

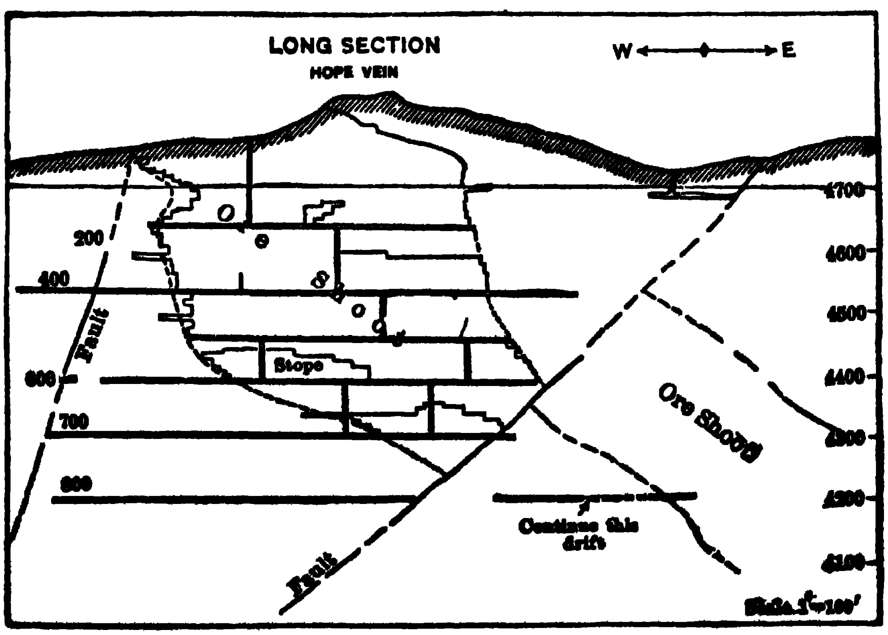

Fig 8. Vein Map, Longitudinal Projection

mapping often discloses structural or genetic relations, if any, between ore and certain rock strata. Relations of ore occurrences to minor fissures, bedding, or cleavage planes may also be revealed by close study of recorded observations.

Comprehensive knowledge of the geology of a mine has a direct bearing on its length of life. The period of profitable operation may be extended by recovery of faulted segments of veins; or new orebodies may be discovered by study of the mine geology and of the workings of contiguous properties. Geologic investigation often reveals the nature of factors controlling ore distribution. When a mine shows signs of failure, through faulting, or impoverishment with depth, those in charge must decide on future course; whether the conditions call for a shut-down, or a further expenditure based more on hope than on sound judgment.

Geologic maps are valuable in case of apex controversies. It may be of the greatest moment to have detailed records of geologic structure, and of mineralogic character of exhausted orebodies and parts of mine which have become inaccessible through caving. Such maps and records, supplemented by models, may constitute the chief court exhibits in mine litigation. When a mine is to be abandoned and allowed to fill with water, accurate maps, brought to date at time of closing, are of scientific value for subsequent geologic study of district, and of vital importance for future work in adjoining properties. Each mine geologic record is also useful in advancing knowledge of the science of ore-deposits.

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2. MINE MODELS

Classification. *A*. Working models, to supplement mine maps and elucidate complex vein or fault structure. *B*. Court exhibits in law suits. *C*. Models for exhibition purposes, or for those who are unfamiliar with underground conditions or lack facility in comprehending maps. A model may be intended chiefly to illustrate a mining method, geologic structure being neglected or broadly idealized; in this case the scale must generally be so large that only a part of the mine can be shown. At the other extreme is the model exhibiting the ore-deposits with reference to boundary lines, as for court use, in which geologic features are made paramount. An intermediate position is occupied by types like the glass sectional models, which combine detailed geologic structure with correct portrayal of the mine workings. According to type of construction, there are 5 classes: (*a*) skeleton working models; (*b*) vein models; (*c*) models of mine timbering; (*d*) solid models; (*e*) glass models.

Choice of design. Important features of a model for mine use and to illustrate geologic structure are: ease and rapidity of construction; facility of making additions to bring model up to date; durability of construction; preferably not requiring services of specially skilled mechanic; reasonable cost. For court use, designer must remember that the model is chiefly for information of laymen, not engineers. Hence, design is simple, to emphasize important features, and on a scale large enough for court and jury to see model clearly. Exhibition models need not be built with regard to future use at mine, nor convenience of making additions. Beauty of design may be given more attention and there is usually more latitude as to cost.

Skeleton working model, based on mine maps, is a small-scale reproduction, in wood or other substance, of shafts, drifts, stopes, etc, which are modeled to scale and supported in true relative position by light iron or wooden trusses. Metal trusses are best, being less bulky and not easily confused with working parts. If shafts extend to base of model they are usually of steel, to serve also as structural supports. The principal workings (other than stopes) are of strong, tough wood, like maple. Stopes are best made of pine, for readily cutting into irregular shapes. Level workings are cut in the rough with jig-saw, shaped with chisel or knife, and fastened together with brads, wooden pins or glue. The trusses are supported by a smooth, wooden base, on which are drawn the claim lines and coordinates of the mine system. Each working is given its true gradient, by previous determination of many elevations, for showing relations to adjacent shafts or mines.

The skeleton model, not intended for geologic study, may be painted one color, preferably light gray. For portraying geologic features, also, the ore-deposits and faults are painted in contrasting colors, like red, blue, and yellow. Trusses and other supports are painted black. Names of shafts and veins, and working numbers are lettered with waterproof ink, or by pasting on printed labels. Surface topography may be shown by: (*a*) iron wires, each bent to represent a contour and fastened at intervals to those next above and below, and all held in position by a light metal frame, attached to supporting trusses; (*b*) ordinary wire window screen, or a sheet of gelatin, shaped to topography. Proper scale of model depends on dimensions of mine: 40 ft to 1 in is good for average conditions; 60 ft to 1 in is too small, and 10 ft to 1 in generally too large.

Skeleton models are not satisfactory for illustrating geologic structure, unless there are many continuous workings in the ore-deposit. They are excellent for court use, if ore-deposit is well stoped out and not excessively faulted. In absence of connections between levels, vein continuity can be indicated by wires, strings, celluloid or gelatin sheets. Cross-faults, or slightly developed veins, the position of which can be approximated, may be shown by glass or celluloid sheets.

Vein models are made to scale from cross-section and level maps, which show vein boundaries. The vein is built of solid wood, beginning with lowest level; or the level sections are made separately and afterward combined. If vein be regular, the whole, or large portions of it, may be shaped from a single piece of wood. Mine workings are outlined with ink on the vein. Topography and claim lines are shown as for skeleton models.

By combining vein model with skeleton model, the position of vein referred to mine openings is shown. Though well-designed vein models may exhibit effects of vein intersections, faulting, and changes in strike and dip, they have little practical value. To build them requires a skilled cabinet maker, and for solving geologic problems they are inferior to sheet-glass models. When much of vein is undeveloped, large portions must be idealized, giving definite thickness and form where these factors are wholly unknown.

"Peg" models are commonly used for tabular deposits like the disseminated coppers proved by vertical drill holes.

Claims (and sometimes surface geology) are drawn to scale on enameled surface of a board or table, in which are small holes corresponding with the drill holes on the ground. In the holes are set 1/8-in pegs. Surface of table represents a certain elev above sea-level, and lengths of pegs are such that the top of each represents the surface elev at corresponding drill hole. Different formations cut by the drill holes are shown by painting the pegs different colors: thus barren capping, white; oxidized ore, green; sulphide, black; sulphide ore below commercial minimum, gray. Strings may be run between transition points in adjacent holes.

Mine timber models are possible only where square-sets are used and accurate stope records kept. Drawbacks: high first cost and cost of keeping model posted up to date, expert workmanship needed and difficulty of making additions. Chiefly for exhibition and court use, but if on large scale, method of timbering is well shown.

Solid glass models consist of superposed plate-glass sheets, in which are cut openings corresponding to those in mine. Accurate stope plans or sections are required at 8-ft intervals, each glass sheet corresponding to a plan or section. Outline of each opening is marked on glass, sawed out as in scroll-sawing, and inner surfaces of openings painted any desired color. Plates are cemented together by Canadian balsam. These models are costly, and can not be altered nor extended.

Solid wood models are of wooden sheets, 1/2–2 in thick, each representing a vert slice through mine. From maps, the workings and geologic structure are drawn on the wood; sometimes openings are cut to correspond with mine workings. Lower edge of each sheet rests on the base, upper edge shaped to represent topog of the slice. After assembling, claim lines, outcrops, buildings, etc, are drawn on upper surface. For ready access to any section of model, the slices may be hung on hinges. To illustrate surface features only, as in RELIEF MAPS, wood, clay, plaster of paris, and composition materials may be used. Concrete has a limited application, for large-scale models of sections of mines, to show methods of timbering; or for topographic models, where but one cross-sec is desired; concrete is too heavy except for permanent exhibits, as in museums.

Glass section models, consisting of horizontal or vertical sections, are the cheapest and most effective for geologic study in connection with mine development. The horizontal-section model is made by transferring the mine level workings to glass sheets, so spaced in

vertical arrangement as to show the workings in true vertical and horizontal relations. Accurate mine maps and geologic maps (or field notes) are prerequisites. The workings are outlined on the glass by a mixture of black oil paint with gold size and turpentine, of a consistency to flow readily from a right-line pen, but not so thin as to spread or make sharp lines impossible. (Waterproof ink does not adhere well, nor stand washing and dusting.) For showing the geology, use tube paints, of colors corresponding to those of the mine maps. Put working outlines on upper side of glass; geologic features on under side, traced

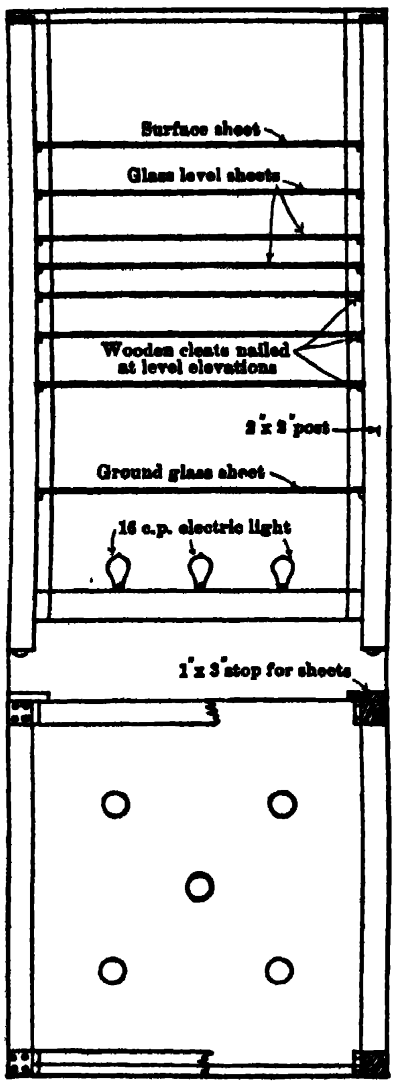

Fig 9. Glass Model, Front Elevation and Plan

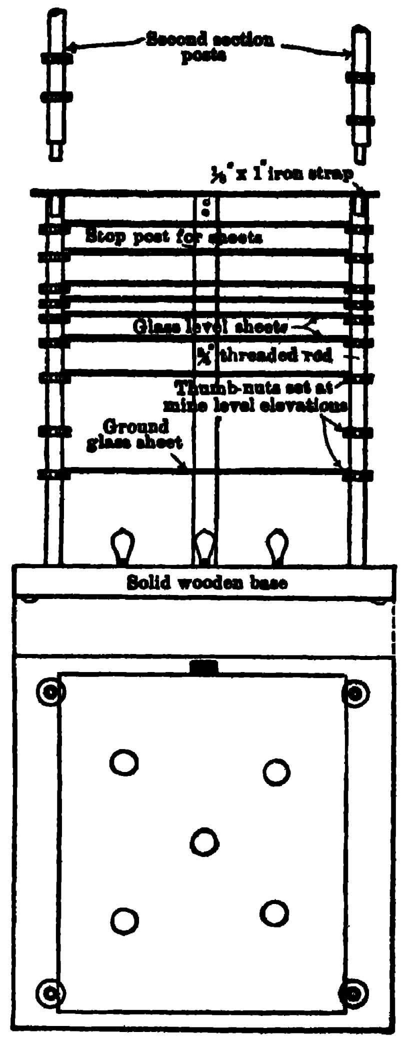

Fig 10. Modified Gl___ Model, Front Elevation and Plan

directly by placing the glass on the geological map. Additions may thus be readily made. Dips of veins and faults may be added, but side notes are omitted. Property lines are drawn on each glass sheet, so that all will show true horiz relations. Put names of claims on uppermost sheet only, together with vein apices, surface openings, and shaft names.

Size of glass. Greatest dimensions should not be over 42 in, on account of weight and danger of breakage. Work is facilitated by using same scale as that of mine maps. Otherwise, where geologic structure is not complex, a scale of 100 ft to 1 in is satisfactory, permitting large area to be shown on a small sheet. Ordinary window glass will do, though picture glass is better, being freer from flaws and more uniform in thickness. Plate glass is heavy and expensive. FOR PERMANENT CONSTRUCTION the sheets are spaced vertically in model frame, to correspond with mine level elevations. Fig 9 shows the older type of construction, the sheets being carried on cleats. In another form, the sheets are supported in grooves; to avoid breakage of glass by possible spreading of sides of frame, metal runners may be nailed flush with bottom edges of grooves. FOR TEMPORARY USE, instead of 4-post frame, sheets may be set up with 2-in wooden blocks between them, of heights equal to differences in elevation of the successive mine levels. When no longer needed, the sheets may be cleaned of paint and used again. A difficulty in using ordinary glass sheets for deep mines

is that, not being perfectly transparent, where there are many sheets, light is cut off from those near bottom. A partial remedy is to set electric light bulbs on base of model, or instead of wooden base use a sheet of ground glass. The geologic details are also confused where there are many levels.

Modified glass model for deep mines. The difficulties mentioned above are successfully met by a design used by the Anaconda Copper Mining Co (Fig 10). Model is built up in sections similar to those of a sectional book case. Sections are of 12-, 18-, and 24-in sizes, each holding a certain number of level sheets. The sections are used independently, or may be built up to any desired height; all are of identical construction, except base section, which provides support and means of illumination.

Mounted vertically on corners of wooden base are four $^3/_4$-in threaded rods. Nuts of 2-in outside diameter are threaded on rods, for supporting the level sheets. Small sheets may rest directly on nuts, but, when larger than 30 by 30 in, large washers are placed on nuts. The rods are held rigidly by 1 by $^1/_8$-in straps across their tops. Alongside of each rod is a $^1/_2$-in square wooden scale, free to revolve, and graduated to different scale on each side. The level sheets are spaced at their respective elevations by the nuts and scales. Each sheet is slid into position from the front to contact with an upright stop at the back, running from base to horizontal straps connecting tops of corner rods. For high and heavy models, an outer wooden frame is added. Succeeding sections are same as base section without the base. The lower ends of the vertical rods of upper section are turned down and threaded to fit into sockets in tops of rods of first section. Additions are also made to the scales, furnishing a continuous graduation to top of model.

Advantages of sectional glass model: (*a*) adaptable to maps of any scale; (*b*) same frame may be used for any mine, new glass only being necessary; (*c*) glass geologic sheets may be filed away like maps, until needed; (*d*) model easily dismantled and stored; (*e*) with slight modification the upper sections can be stood on edge for holding vertical sheets, forming a vertical section model with supplementary level sheets.

Vertical-section glass model supplements the horizontal type, for showing vertical relations of structural features of the deposit. The sheets are spaced according to planes of section, and set in grooves in a wooden frame. Claim lines and intersected workings are shown, as on the horizontal sheets. For a mine more than a few hundred feet deep, a scale of 100 ft to 1 in is satisfactory. Size of glass should be sufficient for making additions, as mine becomes deeper. This type is especially useful for study of deposits having large lateral dimensions, as compared with vertical extent, such as the disseminated copper deposits. Horizontal relations of geology and workings may be shown by drawings on strips of glass, fitted between vertical sheets.

Modifications of glass section-model are generally designed to show geologic and working continuity from sheet to sheet. The following construction is excellent: model in wood the raises, stopes, and other intermediate openings; paint workings same color as that given to vein in which they are made; attach to the glass sheets by "Caementium," which readily adheres to wood or glass. This model, a combination of the glass-sheet and skeleton types, is unsurpassed for indicating vein continuity between levels, where there are raises, but no extensive stopes. THIN GELATIN SHEETS may also be used to represent vein and workings between levels; when slightly warmed, they are readily cut and molded into any desired shape. The gelatin sheets are placed in position between the glass sheets, following variations in strike and dip, and fastened to glass with glue or cement. Thus, width of each gelatin sheet equals distance measured on dip between levels. Areas removed by raises, winzes, and stopes are marked in ink or painted on the gelatin. This construction is not entirely satisfactory for two or more closely spaced veins, or in cases of complicated faulting; otherwise, it combines in a thoroughly practical manner the longitudinal stope-map and the glass-sheet geological model. VERTICAL GLASS SHEETS fitted along section lines BETWEEN HORIZONTAL SHEETS may also be used to show vein continuity and workings between levels. Workings intersected by the plane of section are drawn on the vert sheets, with the geologic facts disclosed. As a substitute for both horiz- and vert-sheet models, this combination is of doubtful value. Though the vert strips furnish some support for large horiz sheets, they restrict the view of observer. For small models, the vert sheets may be continuous, horiz relations being shown on narrow strips between them; but the tendency of narrow horiz strips to sag is not easily overcome.

Dust-proof construction of the supporting framework is desirable for all models, especially for glass-section type, the sheets of which soon lose transparency because of dust. Complete glass cases may be supplied for large permanent models; or a loose wooden box cover of light construction, for models used only at intervals. Cloth covers are not dust-proof.

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SECTION 20

MINE ORGANIZATION AND ACCOUNTS

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Note. Numbers in parentheses in text refer to Bibliography at end of this section.

MINE ORGANIZATION

Introduction. A full discussion of this subject would be lengthy, covering all the relations between the objects of the business and the field of technical knowledge and practice. The aim here is to exhibit methods of mine organisation and management chiefly by means of concrete examples. The examples given are to some extent interrelated. Thus, the daily reports given under Cost-keeping (Art 4) will throw light upon the discussion of Mine Records (Art 5), and both subjects are supplemented and amplified by the extensive series of cost tables in Sec 21.

1. BUSINESS MANAGEMENT

Essentials of efficient operation are: correct understanding of the producing possibilities of the mining property; good technical methods and operating skill; adoption of correct methods of finance. The foundation upon which the working organism rests is the stockholders, who are responsible for the conduct of the business. In the case of the large corporations, now common, actual control is usually exercised through leadership of a few active stockholders. Such a situation sometimes results in abuse and mismanagement, because the body of stockholders feel that their votes have little or no effect on the conduct of the business. Controlling groups sometimes bribe the stockholders by paying unearned dividends. Conduct of the business by a single owner or by a small group of partners is usually more efficient, though not necessarily so. In large corporations the management comes to lie in a practically self-perpetuating committee of stockholders, called the "Board of Directors" (see below). This board usually has power to buy and sell the company's properties and products, to dispose of its funds, and to borrow money; it can expand or contract the business, it is the mainspring of efficiency of the enterprise, and in it lies success or failure.

Mining companies may be divided into two groups: (*a*) those which limit their expectations to working a given property, and when that is exhausted to wind up the business; (*b*) those which contemplate perpetuating the business; these do not depend on a single mine, but by constantly setting aside part of their earnings, or from time to time obtaining new capital, or both, provide for purchase and development of new mines. The second type of corporation has a more aggressive policy, and the Board of Directors is usually composed of men expert in some branch of the mining business, to whom organization of mining operations is a matter of vital personal concern (2). Examples follow.

American Smelting & Refining Co is an example of the second type of corporation. It is the central organization of a group of companies engaged in mining, milling, smelting, refining and selling of copper, lead, zinc, gold, silver and other metals, derived from ores, concentrates and scrap, produced from mines owned, leased or managed by the company, or treated on toll contracts, or purchased outright from other sources. A combined capital of about $175 000 000 is represented by fixed investments and working capital in different forms. There are many thousands of stockholders and probably 100 000 employes. Annual dividends approx $12 000 000 (1938), and large sums are set aside annually to buy new properties to replace old ones. Its general business has steadily grown. The Board of Directors comprise 21 members, of whom 11 might be described as investors or capitalists primarily. The remaining 10 are distinguished employes who have been and still are experts in mining, transportation, metallurgy, accounting, executive work and the law. The Executive Committee consists of 12 of the most active directors. The Finance Committee consists of 2 of the members of the Executive Committee and 8 other directors. There are 17 general officers; President, 8 Vice-presidents, including General Counsel and Treasurer, and a Secretary, Comptroller, Assistant Comptroller, General Auditor, Assistant General Auditor, Assistant Treasurer and Assistant Secretary. Seven of these general officers are Directors. The Vice-presidents are executive heads of the principal departments. There are also 25 minor department heads and consulting engineers. Minor departments comprise sales of metals, purchasing agency, heads of important groups of mines and smelters, traffic and insurance managers. The entire group of 56 business heads, financiers, executives and directors are responsible organizers of the business. Subordinate to them are the managers of mines, mills and smelters.

St Joseph Lead Co, Missouri, is another example of the second type. There are 13 directors, 8 general officers, President and Chairman, Vice-president and Sales Manager,

Vice-president and Treasurer, and two other Vice-presidents, a Secretary and 2 Assistant Secretaries. The operating divisions are in charge of managers and superintendents, with a staff of mining and metallurgical engineers, geologists, and attorneys.

2. TECHNICAL MANAGEMENT

General features of this part of mine organization are illustrated by the description of the American Smelting & Refining Co (Art 1). For smaller organizations, the plan is similar, on a reduced scale, generally with a smaller proportion of staff officers.

Organization data: Total number of employes of an elaborate organization are shown in Table 1. The operations comprise: (*a*) exploration by vertical diamond drill holes from surface; (*b*) development from existing workings to orebody; (*c*) mining without timbering by blasting in horizontal or breast stopes, shoveling ore into cars (between 50% and 60% of the ore is loaded by mechanical shovels), hauling to shafts by trolley and storage-battery locomotives, hoisting by skips in double compartment shafts, there being one shaft at each mill and no surface haulage of ore, all electric pumping; (*d*) primary and secondary crushing by gyratories and rolls, to a maximum of about 7 mm; (*e*) concentration by jigs and tables, with flotation of the slimes and reground middlings; (*f*) transportation of concentrates to smelters; (*g*) roasting, smelting in blast furnaces, refining; (*h*) transportation of pig lead to market.

All the mining and milling divisions are completely electrified and the entire load is carried by one central power station with a capacity of about 35 000 kw. All the steam is generated by large boilers with pulverized coal firing. The load is usually carried by two 12 500-kw turbines, equipped with surface condensers. For condensing purposes the hard mine water is treated by both Booth and Permutit systems before being circulated. The operating cost per kw-hr is, approximately, 0.6¢. This company operates 4 mills and 1 smelter. Normal output is about 15 000 tons of ore per day, and the whole operation is under a Gen Mgr. Each mining and milling division has a supt and there is a supt for each mill. There are also engineering, legal, employes' service and medical departments.

Table 1. Example of Organization Data, for Month Ended Mch 31, 1926

	Employes		Employes
General	108	Prospecting	82
Mines	1 620	Construction	141
Mills	429	Operating	99
Machine shops	195	Power	3
Carpenter shops	35	Heating plants	6
Electric shops	48		
Yards	85	Total shifts for month	2 872
House repairs	8	Total tons " "	15 228
Steam plants	13	Aver tons per man in mines	9.4
		Aver tons, total employes	5.3

Daily mine records. Contrary to a common idea, the daily mine reports are more important in securing results than the permanent records. A mine is a series of working faces, under constant attack, and constantly presenting new conditions to be met from day to day. A manager who waited for monthly costs and production statements as a basis for making changes in his stopes might find that many or all parts of the mine had become unprofitable before he got his information. Since the chief problem is to make the mine yield a profit all the time, any work which has ceased to pay must be stopped immediately. To determine this, the manager must know from day to day the rate of production and the cost of each part of the work (7).

Daily estimates of profit. For these, formal statements are unnecessary. The manager can usually calculate them readily, if he knows the number of men employed and the tonnage produced. Then, total cost of operation, divided by number of shifts of labor employed, gives cost per man-day. In a specific case, the total cost charged for mining, milling, freight, general expense, etc, for one month was $54 302; number of man-shifts, 9 391. Cost per shift was therefore $5.77. So long as the mine continues to work on same basis, this cost is practically constant, and may be counted upon with confidence for daily calculations. With a labor tabulation showing 400 men at work on a given day, the manager of this mine can instantly calculate his expenses: 400 × $5.77 = $2 300. Then, if he can calculate that the ore will yield say 150 oz gold per day, he at once figures his daily income at about $3 100, and his profit, $800. This illustrates the principle underlying the daily records.

Calculation of daily yield, however, is only approximate. For a gold mine, the records may appear somewhat as follows:

Working place	Tons	Grab sample assay, ounces per ton	Total ounces
A............	26	1.06	27.56
B............	19	1.96	37.24
C............	110	0.32	35.20
	155	0 61	100.00

From this tabulation, the ore apparently is worth about $12 per ton. But experience proves that grab samples, as a rule, run high, and that it is safe to figure on only 75% of their value; in this case, $9 per ton. Furthermore, if the 110 tons from working place *C* assays only 0.32 oz ($6.40 per ton, of which 75% = $4.80), and if the working cost is $6 per ton, stope *C* is plainly losing money, and it should immediately be investigated.

Daily Statements. Some simple process of calculation can be established for each mine, and for each important working place in it. Referring to Table 1, the men employed in each subdivision might be entered upon a daily statement, which can be made on a horizontal line of a form ruled with one line for each day in the month. By filling it in daily, the number of men is exhibited in any desired detail; also the daily product from the working places for all the preceding days of the month. Information of this kind is needed also by superintendents and foremen, for their several departments. Though the interpretation of daily reports can be based intelligently only upon the complete periodical cost statement, yet the daily activities of manager, superintendents, and foremen are intimately connected with them (fuller details given in Art 4). The formal monthly statements are regarded as past history by active men, who receive them with only a mild interest and use them chiefly to check conclusions drawn from the daily reports.

Mine maps. Sec 19 gives details of mine maps and models, from the mine geologist's standpoint. Geology of a mine is an indispensable part of the information conveyed by maps; but maps also exhibit engineering features, namely: location of workings; depths, dimensions, and volumes of ore; position of boundaries and approach to areas of disputed title; opportunities for more effective handling of ore, timber, waste rock, filling, etc. A good set of maps is an important aid in attaining efficiency of operation. A system of maps, from which dated blue-prints are made at intervals, and filed for record, furnishes the history of the development of a mine and the best means of forecasting its future. Lacking a comprehensive system, much valuable information may be lost. In working a mine, details accumulate rapidly and are difficult to correlate unless they are recorded at once and made parts of a broad scheme. In general, the maps should be made in following order:

1. Property maps, showing ownerships, boundaries, and monuments.
2. Surface maps, showing topography, roads, buildings, streams, pipe lines, etc.
3. Surface geologic maps.

Note. A single map may cover functions 1, 2, and 3.

4. General maps of workings, showing: (*a*) areas explored; (*b*) areas worked; (*c*) general geologic facts.
5. Detail maps, showing: (*a*) underground geology; (*b*) occurrences of ore; (*c*) diamond-drill records; (*d*) assays; (*e*) depths and elevations; (*f*) other pertinent facts.
6. Detail maps of stopes, developments, workings (drifts, crosscuts, raises and winzes), tracks and ore chutes; also detail assay maps for individual working places.

If necessary, maps can be made conveniently uniform in size, by adopting different scales according to the amount of detail required; and all should be systematized for ready reference from one to another (11, 12, 17). Their utility is determined by the ease with which they yield the desired facts, and display the critical mining conditions.

MINE ACCOUNTS

3. PRINCIPLES

Two methods, broadly speaking, are adopted for keeping accounts of mining companies. In one, expenditures are charged to production at the time they are made; in the other, certain expenditures are carried as "Capital Accounts," to be written off as

depreciation (8, 14, 15, 21, 22). Since enactment of the Federal Income Tax Law (Sec 24) most companies use the latter method, or a modification of it (see following statements).

A. Annual Statement of Mohawk Mining Co, Mich, for 1913

Received from 5 778 235 lb copper @ 15.36¢		$ 887 618.50
Working expense at mine, as per statement	$601 890.77	
Smelting, freight, and N Y and Boston expense	67 263.17	669 153.94
Profit		$ 218 464.56
Expense for construction	$ 66 972.17	
Strike expense	27 652.87	94 625.04
Net profit (balance over all expenditures)		$ 123 839.52
Total surplus, Dec 31, 1912 (balance of quick assets)		897 316.40
		$1 021 155.92
Dividends paid, 1913		500 000.00
Total surplus Dec 31 1913 (balance of quick assets)		$ 521 155.92

B. Mohawk Mining Co, Statement of Income and Profit and Loss for Year Ended Dec 31, 1924

Sales: 17 908 506 lb of copper at 13.5147¢ per lb		$2 420 275.35
Cost of sales:		
Copper on hand Jan 1, 1924	$ 572 139.18	
Operating expenses at mines	1 403 403.06	
Smelting, freight and N Y expenses	281 221.68	
Taxes	61 271.16	
	$2 318 035.08	
Less copper on hand Dec 31, 1924 at cost	204 842 52	
Net cost of copper sold		2 113 192.56
Profit on sales		$ 307 082.79
Miscellaneous income:		
Interest and dividends	$ 21 284.09	
Rents received, etc	32 817.69	
	$ 54 101.78	
Less interest paid	19 330.01	34 771.77
Profit for the year before providing for deprec and depletion		$ 341 854.56

C. Ray Consolidated Copper Company. Operations for the year ended Dec 31, 1925

Operating revenue:		
Copper produced, 142 076 711 lb @ 14.075¢	$19 997 848.94	
Gold " 5 054.7655 oz @ $20	101 095.31	
Silver " 4 116.06 oz @ 70.3¢	2 895.47	$20 101 839.72
Operating expenses:		
Mining, including stripping and development	$ 6 216 944.68	
Ore delivery, mine to mill	906 286.67	
Milling	4 198 532.35	
Treatment, freight and refining	4 183 602.73	
Selling expense	177 588.50	15 682 954.93
Profit from operations		$ 4 418 884.79
Miscellaneous income		215 463.37
Total income		$ 4 634 348.16
Charges against income:		
Depreciation	$ 914 096.64	
Loss on property retired, etc	281 728.06	1 195 824.70
Net income to surplus acct		$ 3 438 523.46
Balance, Dec 31, 1924		$13 478 177.45
Net income for year (without deduction for depletion) as above		3 438 523.46
		$16 916 700.91
Charges against surplus:		
Additional federal income tax for 1918 and adjustments of surplus in connection therewith		1 043 503.17
Balance, Dec 31, 1925		$15 873 197.74

D. Ray Consol Copper Co and Ray & Gila Valley RR Co

Disposition of capital stock, surplus and profits, at Dec 31, 1925

			Increase during yr
Current assets:			
Cash		$ 2 985 281.27	$1 150 236.81
Metals on hand and in transit		8 014 160.06	1 260 021.97
Accounts and notes receivable		883 542.76	246 988.04*
Materials and supplies		1 963 081.37	182 878.11*
Unadjusted debits, prepaid insurance, etc		244 247.43	5 737.20
Current liabilities:		$14 090 312.89	$1 986 129.83
Accounts payable	$ 785 047.26		$ 21 031.32*
Treatment, refining and delivery charges not yet due	1 026 196.56		23 788.96
Reserve for taxes, insurance and other expenses	1 057 889.02		691 891.69*
		2 869 132.84	$ 689 134.05*
Net current assets		$11 221 180.05	$2 675 263.88
Fixed assets:			
Mining and milling property	$11 331 243.41		$ 15 269.62
Construction and equipment, mine, mill and RR, less deprec	11 146 431.84		622 695.36*
Development, stripping and deferred charges	13 286 501.91		534 553.48
Investments	5 236 609.57		32 787.50
Bond deposit account	240 000.00		....
Fixed assets		41 240 786.73	$ 40 084.76*
Total net assets		$52 461 966 78	$2 635 179.12
Total assets are represented by:			
Capital stock		$30 771 790.00	....
Paid-in surplus		5 764 321.09	338 115.55
Surplus from operations		15 925 855.69	2 297 063.57
*Decrease.		$52 461 966.78	$2 635 179.12

E. Ray Consol Copper Co and Ray & Gila Valley RR Co (assets and liabilities)

Assets	Dec 31, 1925	Dec 31, 1924	Increase
Mining and milling property	$11 331 243.41	$11 315 973.79	$ 15 269.62
Construction and equipment, mine, mill and RR, less reserve for deprec	11 146 431.84	11 769 127.20	622 695.36*
Development, stripping and deferred charges	13 286 501.91	12 751 948.43	534 533.48
Total for mining, milling, RR and development	$35 764 177.16	$35 837 049.42	$ 72 872.26*
Investments	5 236 609.57	5 203 822.07	32 787.50
Bond deposit account	240 000.00	240 000.00	....
Unadjusted debits, prepaid insurance, etc	244 247.43	238 510.23	5 737.20
Materials and supplies	1 963 081.37	2 145 959.48	182 878.11*
Accounts and notes receivable	883 542.76	1 130 530.80	246 988.04*
Metals on hand and in transit	8 014 160.06	6 754 138.09	1 260 021.97
Cash	2 985 281 27	1 835 044 46	1 150 236.81
	$55 331 099.62	$53 385 054.55	$1 946 045.07
Liabilities			
Capital stock issued and outstanding	$30 771 790.00	$30 771 790.00	....
Accounts payable	785 047.26	806 078.58	$ 21 031.32*
Treatment, refining and delivery charges, not yet due	1 026 196.56	1 002 407.60	23 788.96
Reserve for taxes, insurance and other expenses	1 057 889.02	1 749 780.71	691 891.69*
Paid-in surplus	5 764 321.09	5 426 205.54	338 115.55
Surplus from operations	15 925 855 69	13 628 792.12	2 297 063.57
	$55 331 099.62	$53 385 054.55	$1 946 045.07

* Decrease.

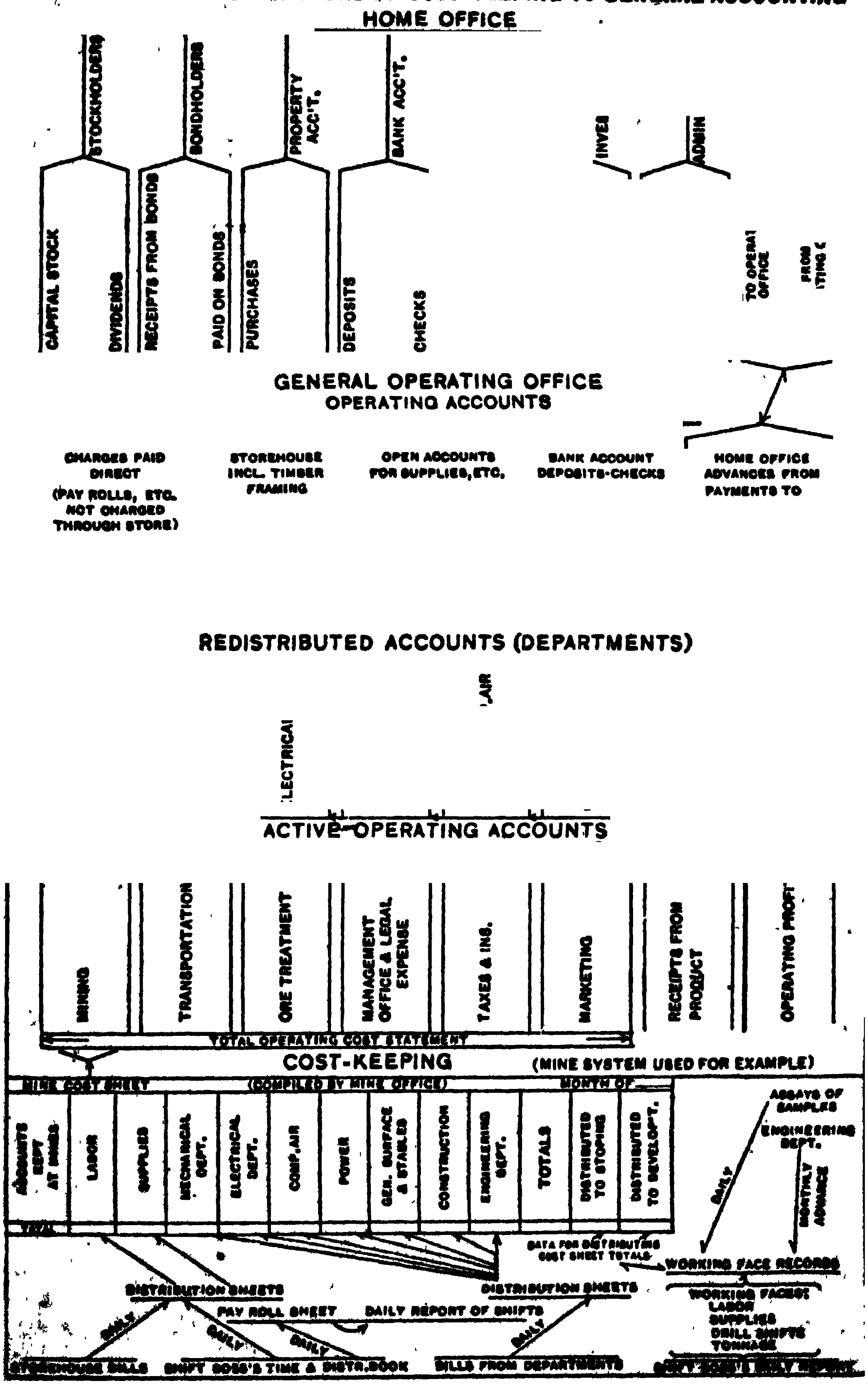
4. COST-KEEPING

General. Table 2 shows that detailed cost-keeping is not essentially a part of the financial records of a mining company, but is a system of costs and data kept separately by each department, the total cost represented on cost-sheets agreeing with total cost shown on operating books. General accounting furnishes a financial record for the management, while cost-keeping provides data for the operating heads, to enable them to regulate expenditures (see Art. 2).

Cost-sheet. Accounts kept by the mining department for its cost records vary according to the mine (14, 21, 22). In any case the final cost-sheet should be brief and deal with totals carried forward from detailed daily records. Many accounts can not be

Table 2

DIAGRAM SHOWING RELATIONS OF COST-KEEPING TO GENERAL ACCOUNTING



directly distributed to working faces each day; therefore, on the mine cost-sheet it is desirable to deal only with general accounts. Referring to Table 2, under the head of cost-keeping, the accounts may be as follows: For DEVELOPMENT direct: miners, muckers, trammers, and timbermen. For STOPING direct: miners, shovelers in stopes, trammers, timbermen, and expense for filling. For GENERAL ACCOUNTS: hoisting, pumping, machine drills, explosives, sampling and assaying, surveying, watchmen, mine office, superintendent, foremen, shift bosses, blacksmithing, repairs and renewals to buildings, repairs and renewals to machinery, mine and shaft repairs, general surface expense, new construction, new machinery, tools, miscellaneous iron and steel, heating. To these accounts the charges are made in the vertical columns of cost-sheet. The total of the mine accounts is apportioned to two general accounts, development and stoping; in some cases a third account, prospecting or exploration, is added. It is not necessary to keep a set of books for a cost system. Best method is a system of daily reports, distribution sheets, and loose-leaf record sheets. To facilitate filing, all reports are preferably in loose-leaf form.

Besides the mine accounts, the following division of costs may be made from WORKING FACE RECORDS (Table 4), after cost-sheet totals have been apportioned to development and stoping. For EXPLORATION: trenching, test-pits, shafts, diamond drilling, churn drilling. For DEVELOPMENT: shafts, drifts, crosscuts, raises, winzes, shaft stations, ore-pockets. For STOPING, the account may be divided according to the different methods of stoping employed. Extending the system further, these accounts may be distributed to each working face, the totals agreeing with cost-sheet total (14, 21, 22).

Table 3. Shift-boss Report

MINE Shaft No 4 DATE Dec 1, 1926 SHIFT Day

Working place		Mach shifts		Timber											
Class	No	H	P	10 × 10	8 × 10	2 × 12	4 × 6	8 × 8	6 × 8	4 × 4	3 × 6	3 × 12			Round
Stope	401	2		4		30									

Explosives		Labor										Hoisting	
		Miners		Shovelers		Trammers		Timbering		Filling			
Powder	Caps	5.25	5.00	5 25	5.00	5.25	5 00	5.25	5.00	5.25	5 00	Cars ore	Cars waste
20	4	2			1		2	1	1		2	35	5

Remarks Please note any new working numbers
New Drift 901, started to-day.
RICHARD DOE,
Shift Boss

5. MINE RECORDS

Working-face records (Table 4) are compiled from the SHIFT-BOSS REPORT (Table 3), which gives for each working face and for each shift: number of miners, muckers, shovelers, trammers, timbermen, and men engaged in filling, drill-shifts; also dynamite and caps consumed (fuse may be averaged per cap used), amount and description of timber used, and tonnage of ore and waste hoisted. In addition to the explosives and timber reported on shift-boss report, any other supplies shown by requisition to be chargeable directly to a particular working face may be entered on this sheet. Assays of ore and face samples are also entered, as reported from assay office. The columns of estimated cost are worked up when desired, by using previous general account averages (14).

The above data give: direct charge for labor (which for cost-sheet may be taken from payroll distribution); direct charge for supplies; production figures, assay records, and advance made. Hoisting is apportioned on basis of tons reported, and total machine drill cost on basis of machine shifts. These are the only charges that may be distributed directly. The remaining general charges of the cost-sheet may be made in proportion to the totals of the direct distribution.

Table 4. Working Face Record

SHAFT 4 CLASS Stope MONTH Dec, 1926

Date	Shift boss report										
	Machine shifts	Timber									
		10 × 10	8 × 10	2 × 12	4 × 6	8 × 8	6 × 8	4 × 4	3 × 6		Rounds
1	{ 2 2	4		30							
2 to 31											

Shift boss report, cont'd

Explosives		Shifts labor										Hoisting	
		Miners		Shovelers		Trammers		Timbermen		Filling		Cars ore	Cars waste
Powder	Caps	5.25	5.00	5.25	5.00	5.25	5.00	5.25	5.00	5.25	5.00		
20 25	4 5	2			1		2	1	1		2	35 40	5

Estimated cost

Mach drills	Timber	Explosives	Total labor	Hoisting	Gen charges	Total

Worked up only when desired during month, or at end of month

Production

Estimated tons	Assay	Total ounces	Tons ore to date	Avg assay to date	Face samples
31.5 36	1.00 1.25	31.5 45.0	67.5	113	

Advance by Eng Dept (Monthly)
Working No, 401 stope

Note. The 4 parts of Table 4 are continuous horizontally on one large sheet; the first column containing days of month from 1 to 31.

Labor records. When a man is hired he should be given an employment slip, stating character of his work. This slip furnishes to the timekeeper proper authority for putting the name on the payroll. A file of identification cards should be kept, giving name, signature, local address, and address of family, for each employe.

Table 5 is a shift-bosses' loose-leaf TIME AND DISTRIBUTION BOOK, found to work satisfactorily at a property where it had been impossible, before its introduction, to get the shift-bosses' daily reports of working faces to balance with time book. By first making the distribution in time book, the daily report is readily filled out. A daily distribution of payroll can be made by the time-keeper from the shift-boss time books. Entries are also made from these books to the monthly PAYROLL SHEET (Table 6), which serves as a full receipt for wages paid. To check the shift-boss time books, brass checks, corresponding to the number in first column of payroll, may be given to the men to deposit

Table 5. Shift Boss's Loose-leaf Time and Distribution Book

Place No 306 Drift	1		Place No 306 Drift	2		Place No 401 Stope	3		Place No 301 Raise	4		Place No Stope Drift Raise	5		Place No 301 Raise	6		Place No Stope Drift Raise	7		Place No Stope Drift Raise	8	
Time			Time			Time			Time			Time			Time			Time			Time		
Day	Night	O'r T*	Day	Night	O'r T	Day	Night	O'r T	Day	Night	O'r T	Day	Night	O'r T	Day	Night	O'r T	Day	Night	O'r T	Day	Night	O'r T
×	—	—	×	—	—	—	×	—	—	×	—	—	*0*	—	—	×	1/8	*Discharged*					
Class work *1*			Class work *5*			Class work *6*			Class work *1*			Class work —			Class work *5*			Class work —			Class work —		

Class of work:

- No 1. Miners, or Drill Runners
- No 2. Muckers and Trammers. Development
- No 3. Shoveling in Stopes
- No 4. Tramming to Chute or Shaft
- No 5. Timbering
- No 6. Filling Stopes
- No 7. Pumping
- No 8. Pipemen
- No 9. Mine and Shaft Repairs

☞ Use these numbers to denote class of work

Shift boss	From	To
Richard Doe	*1*	*3*
Frank Doe	*3*	

OCCUPATION *Miner*

NAME *John Doe*

RATE *$5 25*

MONTH *Dec* SHAFT *4*

PAY ROLL No *106*

NOTE. IN MARKING TIME, MARK—Full Shift, ×; Off, *0*; Hours, *1–8*, *2–8*, etc. *O'r T = overtime.

Note.—The upper part of this form is carried out to provide for month of 31 days.

in a box when coming off shift. Employes may be identified by the paymaster by asking their number and by reference to the identification card. The number in the last column of the payroll is for the pay check, which also serves as a receipt.

Supply records. Distribution of supplies is a comparatively simple matter, if all supplies are kept in a central storehouse, and treated as a separate business. Requisitions should be drawn only for quantities needed for immediate consumption. Requisitions should show the distribution of supplies ordered, and storekeeper makes note of this distribution on the bill, which is sent to the mine office. These bills are entered on a SUPPLY DISTRIBUTION SHEET, to such accounts as are kept for the cost-sheet, the totals of the distributions being checked with a monthly storehouse statement. Slight differences will occur between supplies purchased and supplies shown on shift-boss daily reports, owing to small balances on hand, but these differences should balance themselves from time to time (6).

Table 6. Pay Roll

Number 4 Shaft Month of *Dec*, 1926 Sheet 1

No	Name	Occupation	Days of month, 1 to 31						Total shifts	Rate	Total amount
			1	2	3	4	5	etc			
106	*John Doe*..........	*Miner*	*1*	*1*	*1*	*1*	*1.1*		*5 1/8*	*5.25*	*26.91*
	Total.......... *Daily Total of Shifts*										

No	Name	Deductions		Amount received	Received payment in full of the amount opposite our respective names	Check No
		Store	Hospital			
106	*John Doe*..........	*10.00*	*1.00*	*15.91*	*John Doe*	*1045*
	Total.......... *Daily Total of Shifts*					

Department charges, for services and power rendered, are billed and distributed as for bills for supplies. Detailed costs of power, compressed air, cost-sheets, and department distributions should be kept by the department concerned. At properties where scale of operations does not permit a subdivision into departments, it is desirable at least to keep these costs on separate records.

6. MAJOR AND MINOR OPERATING UNITS

Major units can be divided into smaller, or MINOR UNITS, still more or less independent. Thus, the mining plants consist of the mines, the mills, and the power plants. Carrying subdivision still further, the mines may comprise a number of shafts (with workings), which may be quite independent of one another, but are dependent upon the power plant and a concentrating mill, and each may have a choice of several mills for treatment of its ores (Table 2).

Economy demands that each major unit be run at full capacity. If a mill can treat more ore than is sent to it, part of its equipment and first cost is being wasted. Conversely, if more shafts are operating than are required to supply the mill, they are obviously being worked at reduced capacity, and one or more of them should be shut down. Major units wholly dependent on one another are illustrated by the items of a shaft equipment. Hoisting engine, pumps, haulage plant, shaft timbering, etc, must be kept up and in repair, whether the output is large or small. Therefore none of these elements should be developed out of proportion to the others.

Efficiency engineering consists of a study of units, in order to secure maximum output from each. Hence, ascertain which units are effective and which are only relative.

Example. The object of a machine drill is to mine ore cheaply; the units comprised in its operation are: labor, cost of the machine and repair parts, compressed air, explosives. The total cost per shift, for breaking say 12 tons, is about \$8.60, comprising: wages, \$5; repairs, \$1.48; compressed air, \$0.80; explosives, \$1.32. Economy lies in getting a large output for the cost. It

might be uneconomical to emphasize any of the minor units without reference to the influence of the others. Thus, by reducing wages, inefficient men would produce poor results; excessive economy in repair parts might cause costly delays; cutting down supply of compressed air would result in feeble drilling; reducing quantity of explosive might necessitate an excessive footage of drilling. It would be clearly uneconomical to reduce the cost of operating a drill from $8.60 to $7.30 per shift at the expense of decreasing the tonnage broken from 12 to 10. Conversely, it would be good economy to raise the cost per shift if the output could be increased in greater proportion. Hence, a true unit cost for machine drilling is the total cost per ton of breaking ore, and not merely the cost per drill-shift. This line of reasoning leads to effective management and cost-keeping.

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SECTION 21

COST OF MINING

BY

J. R. FINLAY

CONSULTING MINING ENGINEER

REVISED FOR 2ND AND 3RD EDITIONS BY

ARTHUR NOTMAN

CONSULTING ENGINEER

Introduction

The tables of First Edition of this book, prepared by Heath Steele, along the lines of Finlay's "Cost of Mining," present typical examples of costs, and illustrate systems of account keeping. Three distinct conceptions were adopted: (*a*) Total net expenditure, not including dividends, sums loaned out, nor mere book entries. This includes all money spent on property, construction of plant and for financial and legal transactions; (*b*) Amount spent in operating mines and mills, but omitting expenditures on plant, financial transactions, etc. These costs take the form of averages, calculated for long periods, which, when added to operating expenditure, give total costs. These costs will never, except by chance, agree with expenditure for any one year, but should approximate aver expenditure for a number of years; (*c*) Amount spent on operation, omitting all other expense. This cost will always fall far below total expenditure.

In many of the tables the business as a whole is first exhibited, followed by successive subdivisions, until the details are as minute as practicable. The first table for a given mine gives total expenses by departments. Thus, Example 2, Table 3, shows operating results of Goldfield Consol Mines Co: (*a*) Total expenditures are checked by showing the total outgo, plus a gain, or minus a loss, in quick assets for each year; (*b*) Expenditures in Table 3 are given under 19 different headings, of which only 3 fall directly under mining and milling operations. The cost of Mining is shown as a lump sum (details in Table 5); (*c*) In Table 5 are the mining operations for 3 years, the cost being segregated as labor, supplies, power, and general charges; (*d*) Here reference is made to Table 7, where costs are divided into 32 items at each of 4 shafts, with their averages.

Mining costs per ton differ much less in detail than in total. To do exactly the same thing in different places involves no differences in cost other than those due to procuring labor and supplies. But total costs per ton show great differences, because one ore may require a process of five steps, while another may require fifty. In general, the examples exhibit results obtained on a large scale by successful organizations. It should not be assumed that these costs can be duplicated by small-scale operators.

To enable the reader to compare present-day costs with those of former years, some of the older and fully detailed tables, dated 1909 on, are retained in this Third Edition. To convert pre-war costs to a post-war basis, an approximation can be reached by adding, say, 75% to pre-war costs; or, the latter can be compared with figures for 1924 and 1925, as in some of the original examples, or with costs in the new examples. Much of the information comes from public reports, and all is authentic.

Though the examples are representative of different kinds of mining, the plan has necessarily been modified by the question of availability of information in different fields. Information is freely made public in non-competitive business, like gold mining; but, as competition becomes vital, as in mining low-priced staples like coal and iron, operators are more secretive. This is especially true of the highly integrated organisation of the iron and steel industries. Hence, the number and fullness of the examples are not in proportion to the importance of the individual fields covered.

The Report of the Federal Coal Commission contains an exhaustive analysis of Coal Mining costs; from it are taken Tables 60 to 64. For Iron Mining, no recent representative figures were obtainable. The vertical integration of the iron and steel industry in past years has been so complete that published reports afford little opportunity to segregate costs of producing the raw materials from costs of finished products. Table 55 contains broad facts relating to Michigan iron mines, in form of averages for entire districts. Favored properties in each district do cheaper work. Current costs are not obtainable (see statement preceding Table 55.) For general discussion of iron-mining problems, see Finlay's "Cost of Mining."

Undoubtedly, the proper way to discuss mining costs would be to adopt a unit basis, which has never been done in a broad manner. Having defined and tabulated the steps in mining, milling, and metallurgical processes, the labor, supplies, and power required for each could then be ascertained, the total cost being the sum of those of a given amount of each element at a given place.

EXAMPLES OF COSTS

Example 1. Homestake Mining Co, South Dak

Table 1. Summary of Operations for 1925

	Per ton milled		Per ton milled
Cost of mining	$1.65	Less discount, interest and rents	$0.05
" " milling	0.26	Net cost	$3.42
" " cyaniding	0.19	" Dividend from current earnings	0.35
Freight on bullion	0.01	Bullion yield	$3.77
Indirect costs	0 52		
Total operating cost	$2.63	Total ore milled	1 589 701 tons
Depreciation charged	0 47	Development work	9 765 ft drifts
Depletion charged	0.37		3 105 " raises
Total cost	$3.47	Total development	12 870 ft

Table 2. Operating Results, Homestake Mining Co., 1934-1937 Inclusive

Tonnage mined, 5 598 557; ounces gold produced, 2 129 083, capitalization, 2 009 280 shs, $12.50 per sh

	Totals	Per ton	Per share	
			Total	Annual aver
Bullion revenue	$74 517 938	$13 31	$37.09	$9.27
Other income	1 271 844	0 23	0.63	0.16
Total income	75 789 782	13.54	37.72	9.43
Working costs, administration, etc	22 623 024	4.05	11.26	2.82
Profit before taxes	53 166 758	9.49	26.46	6.61
Taxes	9 091 318	1.62	4.52	1.13
Net income	44 075 440	7.87 (a)	21.94 (a)	5.48 (a)
Dividends	32 188 182	5.75	16.02	4.00
Balance	11 887 258	2.12	5.92	1.48
Decrease in net current assets, Dec 31, 1933 to Dec 31, 1937	733 032			
Balance to be accounted for	12 620 290	2.25	6.28	1.57
Increase of book value fixed assets	865 996			
Increase in consolidated surplus	2 559 407			
Total	3 425 403			
Balance: Capital expenditures and additional taxes	* 9 194 887	1.64 (b)	2.57 (b)	0.64 (b)
Net profits after capital expenditures (a minus b)	....	$6 23	$19 37	$4.84

* This balance was expended for a new 4 000-ft shaft, steel lined, equipped with the two largest hoists ever built in America, a new mill, dismantling of two old mills, a new cyanide plant, refinery, change house, and steam-turbine power plant, additional compressors, and substantial improvements to the townsite; also, certain minor amounts for tax adjustments.

It is unlikely that capital expenditures for next few years will exceed $400 000 per annum (20¢ per share); real earnings should therefore approx $5.25 per share on current tonnage, ore-grade and costs. By including actual capital expenditures over a period long enough to secure a representative annual aver, a more accurate measure of costs is reached than by including the annual write-offs for deprec and depletion. The latter charges are unavoidably affected by exigencies of the tax laws.

The following figures, for the Goldfield Consol Mines Co (Tables 3-12), although out of date, are retained because they illustrate the amount of detail needed in cost accounting to maintain sound, intelligent control over extended operations (see p 02, par 4).

Example 2. Goldfield Consolidated Mines Co (a Gold Mine)

Table 3. Summary of Operating Results

For year ended Oct 31	1909	1910	1911	14 mos to Dec 31, 1912
Bal quick assets forwarded	$1 064 194	$2 225 318	$2 350 576	$2 214 404
Bal quick assets, end period	2 225 318	2 350 576	2 214 404	1 394 909
Gain in quick assets (*Loss)	1 161 124	125 258	*136 172	*819 495
Dividends paid during period	3 201 238	7 118 271	7 118 296	5 694 636
Bal over expenditures	4 362 362	7 243 529	6 982 124	4 875 141
Receipts including selling charges	6 898 571	10 322 115	10 200 858	7 765 152
Total expenditures	2 536 209	3 078 586	3 218 734	2 890 011
General items:				
Mining (Table 4)	$839 178	$1 032 059	$1 109 458	$1 409 001
Transport ore to mill (Table 4)	18 158	36 800	30 629	34 755
Milling (Table 4)	460 646	562 881	626 294	670 427
Concentrate treatment (Table 4)	45 525	82 713	124 315	158 496
Bullion tax	117 463	130 808	126 319	53 635
General expenses	....	....	181 335	188 471
Administration	Sundries	39 684	Gen exp	Gen exp
General and Secretary's office	20 855	26 571	"	"
Legal expense	4 543	36 304	"	"
Corporation expense	Sundries	27 823	"	"
Damages	"	19 334	"	"
Association dues	"	7 338	"	"
Property tax	2 345	15 189	"	"
General maintenance	Sundries	24 022	"	"
Automobiles	"	8 770	"	"
Sundries	62 609	33 270	"	"
Insurance	929	....	"	"
Selling charges on products	299 701	372 130	310 829	246 219
Ore purchased	....	....	....	3 974
Total operating expenditure	$1 871 952	$2 455 696	$2 509 179	$2 764 978
Construction and other expenses	664 257	550 027	682 554	94 679
Federal income tax	....	72 863	27 001	30 354
Total expenditure	$2 536 209	$3 078 586	$3 218 734	$2 890 011
Tons of ore produced (Table 3)	194 480	266 867	330 549	415 786
Operating cost per ton ore	$ 9.63	$ 9.24	$ 7.59	$ 6.65
Total expenditure per ton ore	$13.04	$11.54	$ 9.74	$ 6.96
Average grade of ore mined	$37.98	$40.72	$32.55	$19.97

Table 4. Production from Mines

Year ended Oct 31	1910	1911	1912 14 months
Ore produced from development work, tons	34 699	31 548	Not given
Ore produced from stopes, tons	232 168	299 001	415 786
Total tons of ore produced, all mines	266 867	330 549	415 786
Average grade of ore from development	$25	$22.58	....
Average grade of ore from stopes	$43	$33.60	....
Average grade of all ore produced (gold)	$40.72	$32.55	$19.97
Tons of ore treated at mill	265 352	330 062	403 360
Tons of ore shipped	1 515	487	12 426

The operating costs given in Table 5 cover operations during periods when the 100-stamp mill was the only mill in operation. The company operated a 20-stamp mill up to the end of 1909. Briefly, the operations in this mill consisted of: crushing by 8- by 12-in Blake crusher; conveying by 14-in by 165-ft belt, running on a 20° incline to stamp bins; stamping, 1 250-lb stamps, 109 $6^1/_2$-in drops per min, through 12-mesh screen; amalgamation, tube milling, reamalgamating; separating to sands and slimes; concentrating; leaching sands and agitating slimes; filtering through Butters' filters; zinc pre-

Example 2. Goldfield Consolidated Mines Co (*Continued*)

Table 5. Operating Costs per Ton Mined

Year ended Oct 31	1910	1911	14 mos to Dec. 31, 1912
Mining, including development:			
Labor	$2.08	$2.22	$2.34
Supplies	1.28	0.99	0.89
Power	0.046	0.14	0.16
General charges	0.464	....	....
Total mining (Table 7)	$3.87	$3.35	$3.39
Transporting mill tonnage:			
Railroad operations	$0.079	$0.06	$0.06
Railroad maintenance	0.057	0.03	0.03
Total transporting ore to mill	$0.136	$0.09	$0.09
Milling: (Table 3 for tonnage)			
Labor	$0.558	$0.43	$0.39
Supplies	1.258	1.17	0.94
Power	0.305	0.29	0.33
Total milling	$2.121	$1.89	$1.66
Loss in tailings	....	....	....
Concentrate treatment per ton milled:			
Labor	$0.05	$0.04	$0.07
Supplies	0.236	0.32	0.27
Power	0.026	0.02	0.05
Total concentrate treatment per ton milled	$0.312	$0.38	$0.39
Concentrate treatment per ton conc't:			
Labor	$0.93	$0.63	$1.24
Supplies	0.44	0.40	0.76
Power	4.48	5.09	4.50
Total per ton of concentrate treated	$5.85	$6.12	$6.50
Tons of concentrates treated	14 152	20 306	24 376
Feet of development work performed	41 938	46 739	48 146
Average cost per ft of development	$9.05	$7.51	....

cipitation; amalgam and precipitates to bullion and concentrates to smelter. The ore treated at this plant was practically all oxidized. Table 6 gives an example of the lowest operating costs obtained at the plant, excluding depreciation of plant.

Table 6. Operations, 20-Stamp Mill

Tonnage treated, 28 464. Average value per ton, $37.15. Net recovery, 93.80%, as follows: amalgamation, 41.40%; concentration, 18.20%; cyaniding, 34.20%.

Year '08, '09	Nov	Dec	Jan	Feb	Mch	Apr	May	June	July	Aug
Labor	$1.77	$2.06	$1.73	$1.93	$1.65	$1.73	$1.67	$1.65	$1.65	$1.58
Supplies	1.45	1.74	1.59	1.28	1.38	1.45	1.11	1.17	1.16	1.21
Water	0.23	0.31	0.18	0.22	0.18	0.19	0.18	0.20	0.18	0.18
Power	0.49	0.61	0.48	0.63	0.53	0.67	0.66	0.63	0.42	0.42
Departments	0.24	0.24	0.23	0.28	0.26	0.36	0.21	0.35	0.08	0.08
Admin and gen	0.29	0.36	0.26	0.27	0.23	0.34	0.37	0.36	0.36	0.36
Total per ton	$4.47	$5.32	$4.47	$4.61	$4.23	$4.74	$4.20	$4.36	$3.85	$3.83

To the end of 1912, it had cost this company approx $5 per ton of annual capacity to construct and maintain its plants. Total charge for plant, equipment and maintenance, to end of 1912, was about $1.50 per ton of ore produced.

In Table 3 construction and other items for 1909 were charged to capital account and not shown in costs reported. Of this amount for 1910, $445 864 was for construction; in 1911, $137 831, the greater part of the remainder being evidently spent on outside properties; in the 14-month period, construction costs were $87 395.

The following data, to the end of Example 2, furnished by Albert Burch, Gen Mgr Goldfield Consol Mines Co, cover working conditions and costs during 1913.

The orebodies occur very irregularly as to value, shape, and persistence, in fractured zones in dacite and latite. The dip of the fracture is usually about 40°; in a few instances dip is as flat as 20°, and in a few practically vertical. Country rock is soft, frequently

Example 2. Goldfield Consolidated Mines Co (*Continued*)

Table 7. Details of Mine Operating Cost per Ton Mined

For Four Months Ending Feb 28, 1911	*A* Shaft	*B* Shaft	*C* Shaft	*D* Shaft	Average
Development Direct:					
Miners	$0.115	$0.256	$0.346	$0.177	$0.216
Muckers, trammers, and cars	0.044	0.206	0.334	0.211	0.196
Timber and timbermen	0.042	0.094	0.152	0.105	0.096
Stoping Direct:					
Miners	0.342	0.245	0.187	0.268	0.263
Shovelers, ore to chutes	0.139	0.303	0.235	0.207	0.228
Trammers, chutes to shaft, and cars	0.142	0.149	0.222	0.218	0.182
Timber and timbermen	0.394	0.635	0.419	0.617	0.551
Filling	0.018	0.104	0.157	0.150	0.112
Gen Accounts (to be proportioned to stoping and development):					
Hoisting	0.157	0.229	0.497	0.368	0.303
General blacksmithing	0.007	0.023	0.038	0.010	0.018
Pumping	0.007	0.022	0.125	0.087	0.057
Air drills, including air	0.089	0.169	0.228	0.201	0.173
Explosives, including labor	0.129	0.231	0.329	0.224	0.223
Sampling and assaying	0.056	0.098	0.135	0.074	0.088
Watchmen	0.051	0.049	0.084	0.048	0.055
Mine office and change rooms	0.045	0.033	0.078	0.028	0.041
Supt, foremen and bosses	0.084	0.082	0.151	0.086	0.094
Repairs and renewals, buildings	......	......	0.007	0.011	0.005
Repairs and renewals, machinery	......	0.103	0.100	0.188	0.112
General surface expense	0.114	0.102	0.169	0.147	0.129
Construction	......	0.011	1.005	0.016	0.161
Lighting	0.029	0.040	0.070	0.035	0.048
Miscellaneous iron and steel	......	0.005	0.004	0.003	0.003
Tools	0.005	0.010	0.012	0.009	0.009
Miscellaneous supplies	0.005	0.003	0.006	0.003	0.004
Miscellaneous repairs	......	0.003	0.003	0.005	0.003
Heating	......	0.001	......	......	......
Surveying	0.066	0.046	0.142	0.047	0.064
Mine and shaft repairs	0.023	0.097	0.038	0.116	0.077
Wages for miscellaneous items	0.022	0.020	0.024	0.022	0.020
Geology	0.015	0.031	0.043	0.025	0.027
Ventilation	......	......	......	0.002	0.002
Total cost per ton ore mined	$2.14	$3.40	$5.34	$3.71	$3.56
Tons of ore mined	14 821	24 416	11 084	26 657	77 698

swelling, and most drifts and crosscuts outside the vein require frequent retimbering. Drifts in the vein rarely require timbering, but walls in all stopes must be supported. A few narrow stopes on one vein are worked with stulls; but usually with sets 5 ft square and 7 ft 10 in high, of 8 and 10-in sq timber. As ore is removed, the sets are filled floor by floor with waste from development, and from waste raises run to surface. During the period covered by Table 12, ore was mined from about 60 stopes, some in pillars left in stopes already mined, and most of them were small extensions toward the walls, or out from ends of old stopes. Probably 40% of filling charges was for old stopes, insufficiently filled previously. The mines produce about 280 000 gal of strongly acid water per 24 hr, which, after neutralizing with 1 200 lb of lime, suffice for milling purposes.

Hoisting is done through 6 shafts, 300–1 400 ft. Transport is by rail from mines to mill, and no returns are made for cost of transport system. Operating costs, about 9¢ per ton for aver haul of 2.6 mile. Trammers on main levels handle about 32 ton per shift.

Table 8. Average Power Consumption per Day

Compressors	402 hp
Hoisting	170 "
Pumping	76 "
Machine shop, saws, lights on surface	77 "
Total mine power	725 "
Milling	1 755 "
Total power consumption	2 480 "

Diamond drilling was unsatisfactory, owing to its slow progress in the brecciated quartz in which the ore is found. Cost of diamond drilling averaged about $4.28 for following items: labor, $2.36; supplies, 58¢; carbons, 88¢; and power, estimated at 20 hp, 46¢ per ft (Table 8).

Elec power is used, costing $78.36 per hp per annum. Power is estimated at 90% of aver daily peaks exceeding one min in duration, and is probably in excess of actual hp which would be shown by meters.

Example 2. Goldfield Consolidated Mines Co (*Continued*)

Table 9. Employes in Mining Department, 1913

	No of men	Aver daily wage		No of men	Aver daily wage
Mining:			**Machine shop and surface:**		
Machine drill men	155	$4.00	Master mechanic	1	$7.50
Shovelers	159	3.75	Blacksmith	1	5.50
Timbermen	30	4.18	Blacksmith helper	1	4.00
Trammers	30	3.75	Blacksmiths and drill sharpeners	6	4.83
Topmen	21	3.77			
Stationmen	11	4.00	Compressor men	3	4.00
Watchmen, underground	1	5.00	Machinists	6	5.00
Samplers	3	4.61	Machinists' helpers	4	3.88
Hoisting engineers	14	5.00	Carpenters	3	4.67
Pipe and trackmen	10	4.10	Watchmen	5	5.70
Pumpmen	3	4.00	Sawyer and yard foreman	1	4.00
Nippers (powder and tool men)	11	3.75	Laborers	26	3.56
Foreman and shift bosses	12	5.64	Dry house	3	4.00
Underground supt	1	Salary	Surface supt	1	Salary
Total	461	.....	**Total**	61	.....
Offices:			**Assay dept:**		
Timekeepers	3	4.42	Day foreman	1	6.67
Storekeeper	1	6.67	Night foreman	1	5.33
Storehouse clerk	1	4.17	Balance men	2	4.62
Bookkeeper	1	6.67	Parter	1	4.00
Stenographer	1	4.17	Fluxman	1	4.00
Purchasing agent	1	Salary	Furnace men	3	4.75
Janitor	1	3.50	Clerk and helper	1	4.50
			General	10	4.00
Total	9	.....	**Total**	20	.....
Electrical dept:			**Diamond drill:**		
Chief electrician	1	6.67	Foreman	1	5.83
Sub-station attendants	2	4.00	Drill runners	2	5.00
Sub-station attendant and repairman	1	5.00	Drill runners' helpers	2	4.00
Repairmen	2	5.00			
Total	6	.....	**Total**	5	.....
Surveying:			**Mill water supply:**		
Chief surveyor	1	10.00	Pumpmen	2	4.00
Surveyor	1	5.83	Pipe lineman	1	4.00
Surveyor's assistants	2	5.00			
Total	4	.....	**Total**	3	....

See Table 10 for mill employes.

Ore mined per annum	356 000 tons
Aver development work per annum	40 000 ft
Drifts and crosscuts about 5 by 7 ft in clear. Raises aver 5 by 9 ft in clear	
Cost of mining plant per ton of annual production	$1.826
Total cost of mining plant	$650 000

Example 2. Goldfield Consolidated Mines Co (*Continued*)

Table 10. Employes in 100-Stamp Milling Department, 1913

See Table 9 for mine employes	No of men	Aver daily wage	See Table 9 for mine employes	No of men	Aver daily wage
Crushing plant:			Concentrate treatment (*con*):		
Crushermen	2	$4.50	Solution men	3	$4.50
Conveyer man	1	4.00	Solution men helpers	3	3.75
Chilean and tube mills:			Laborers	2	3.87
Millmen	6	4.50	General:		
Screen man	1	4.00	Foremen	3	6.11
Repairmen	2	4.50	Master mechanic	1	8.33
Solutions:			Storekeeper	1	5.83
Solution men	3	4.50	Time-keeper	1	5.00
Helpers	3	4.00	Electrician	1	5.00
Pumps:			Chemist	1	5.00
Pumpmen	3	4.00	Carpenters	2	5.25
Concentrate roasting:			Carpenters' helper	1	4.00
Roastermen	3	4.50	Machinists	2	5.25
Refinery:			Machinists' helper	1	4.00
Foreman	1	5.83	Blacksmith	1	5.00
Helpers	2	4.50	Blacksmith helper	1	4.00
Batteries:			Sampler	1	4.00
Batterymen	3	4.50	Pipe fitter	1	5.00
Repairmen	2	4.75	Pipe-fitter's helper	1	4.00
Concentrators:			General repairmen	2	4.25
Tablemen	3	4.50	Laborers	10	3.55
Table cleaners	2	4.00	Railroad:		
Repairmen	2	4.50	Conductor and foreman	1	6.50
Filters:			Engineer	1	5.00
Filtermen	3	4.00	Fireman	1	4.00
Washers	1	4.00	Hostler and repairman	1	4.50
Repairmen	2	4.00	Brakemen	3	4.00
Concentrate treatment:			Track foreman	1	4.00
Foreman	1	5.83	Track laborer	1	3.50

Table 11. Milling Costs and Data, 1913

Labor, cost per ton treated		$0.457
Supplies, " " "		1.105
Power, " " "		0.414
Total mill operating cost		1.976
Tons milled per annum	346 000	
Cost of operating milling plant	$1 148 000	
Cost of operating milling plant per ton of annual capacity	$3.318	

Principal mill supplies per ton of ore used: cyanide, 1.92 lb; lime, 13.62 lb; zinc dust, 0.61 lb; lead acetate, 0.95 lb; steel shoes and dies, 0.47 lb; steel, chilean mill, 0.29 lb; pebbles, 3.92 lb; water, 316 gal; power per ton day, 1.19 h p.

Cost of principal supplies laid down in Goldfield, Nev

Explosives:		Round Timber:	
Blasting caps....XXXXX		5 to 6-in small end	$0.095 per ft
$35.65 per case	$ 0.72 per 100	8 to 9-in " "	0.17 " "
Fuse..........Comet..		10 to 12-in " "	0.24 " "
$22.80 per case	0.37 " " ft	Shovels, Kentwood, long handle, round point	0.93 each
Fuse..........Eclipse..			
$26.75 per case	0.45 " " "	Shovels, Kentwood, long handle, square point	0.93 "
Powder, E L F 40% 1 1/8-in	13.90 " " lb		
Powder, Judson F 10% NGL	10.90 " " "	Drill steel, Phœnix	0.077 per lb
Candles, Granite, extra hard (240 in case)	3.45 " case	" " Corona	0.072 " "
		" " Midvale	0.061 " "
Timber:			
Posts and caps framed	28.70 " M	Drill hose, 3/4-in 5-ply Falcon	0.275 " ft
Ties	27.70 " "	" " 3/4-in 5-ply Diamond	0.258 " "
Lagging	26.30 " "	" " 1-in 5-ply "	0.353 " "
Miscellaneous lumber, mining grade, Oregon pine	26.55 " "	Mine track: rails, ties, plates, bolts and spikes	0.30 " "
Miscellaneous lumber, native	23.50 " "		
Wedges	7.50 " "		

Scale of wages: Shift bosses, $5.50; carpenters, $5.00; blacksmiths, $5.50 and $5.00; hoisting engineers, $5.00; cagers, $4.00; pumpmen, $4.00; top trammers, $3.75; miners, $4.00; muckers, $3.75; shaftmen, $5.00 and $4.50; timbermen, $4.50 and $4.00; nippers, $3.75; pipemen $4.00; trackmen, $4.00.

Example 2. Goldfield Consolidated Mines Co. (*Continued*)

Table 12. Operating Costs, All Mines, June, July, August, Sept, 1913

	Stopes			
	Quantity		Cost in dollars	
	Total	Per ton	Total	Per ton
Supplies:				
Caps	60 400 pc	0.544	$434.91	$0.004
Fuse	376 400 ft	3.387	1 505.61	0.013
Powder	97 227 lb	0.875	13 611.75	0.123
Candles	153 360 pc	1.380	1 948.45	0.018
Timber, sawed	165 000 bd ft	1.485	45 793.09	0.412
Timber, round	43 338 ft	0.390	8 234.25	0.074
Wedges	93 740 pc	0.843	703.05	0.006
Drill parts	....	......	2 150.55	0.019
Picks	90 pc	0.001	77.72	0.001
Shovels	386 pc	0.003	359.37	0.003
Drill steel	4 275 lb	0.040	342.05	0.003
Drill hose	573 ft	0.005	160.41	0.002
Pipe and fittings	....	......	424.75	0.004
Tracks	....	......	....	......
Oil and miscellaneous	....	......	4 671.09	0.042
Cars and repairs	....	......	155.88	0.001
Total supplies	....	......	80 572.93	0.725
	Shifts			
	Total	Per ton		
Labor:				
Drill shifts (2-man)	....	......	....	......
Drill shifts (1-man)	6 402	0.058	25 608.00	0.231
Other miners	2 576	0.023	10 304.88	0 093
Muckers and trammers	13 447	0.121	50 427.57	0.453
Timbermen	3 636	0.033	14 546.86	0.131
Filling	3 292	0.029	12 344.35	0.111
Repairs	983	0.009	3 814.68	0.034
Total labor	30 336	0.273	117 046.34	1.053
General:				
Hoisting	....	......	21 273.30	0.191
Pumping	....	......	1 624.16	0.014
Supt and shift bosses	....	......	7 709.11	0.069
Blacksmith and tool sharpener	....	......	2 667.04	0.024
Compressor	....	......	6 746.83	0.066
Mechanical	....	......	4 646.11	0.042
Electrical	....	......	2 936.44	0.026
Surface	....	......	11 148.56	0.100
Office and clerical	....	......	3 351.24	0.030
Assaying	....	......	10 395.55	0.093
Surveying	....	......	2 934.37	0.026
Sampling	....	......	985.20	0.009
Watchmen	....	......	2 529.98	0.022
Nippers (powder and tools)	....	......	3 642.16	0.032
Pipe and tracks	....	......	3 372.55	0.030
Total general (mixed labor and supplies)	....	......	85 962.60	0.774
Total charges	....	......	283 581.87	2.552
Tons ore mined and from development	111 136	......	....	......

Example 2. Goldfield Consolidated Mines Co (*Concluded*)

Table 12. Operating Costs, All Mines, June, July, Aug, Sept, 1912—(*Concluded*)

Drifts and crosscuts				Raises			
Quantity		Cost in dollars		Quantity		Cost in dollars	
Total	Per ft	Total	Per ft	Total	Per ft	Total	Per ft
15 400	1.706	111.07	0.012	4 700	1.306	33.91	0.009
95 400	10.571	381.78	0.042	30 000	8.333	120.09	0.034
24 716	2.739	3 460.22	0.383	7 686	2.135	1 076.10	0.299
41 760	4.627	529.51	0.059	12 000	3.333	152.95	0.042
243 603	26.992	6 699.09	0.743	105 234	29.232	2 893.93	0.804
6 829	7.667	1 297.25	0.144	2 580	0.711	490.29	0.136
15 551	1.723	116.63	0.013	5 800	1.611	43.52	0.012
....	....	660.00	0.073	....	....	250.59	0.070
25	0.003	22.54	0.002	10	0.003	9.06	0.003
106	0.011	98.17	0.011	47	0.031	44.01	0.012
1 266	0.140	101.31	0.012	512	0.124	40.94	0.011
166	0.018	46.51	0.005	66	0.019	18.50	0.005
....	....	116.92	0.013	....	....	50.62	0.014
3 692	0.409	369.19	0.041	....	....	....	....
....	....	1 276.41	0.141	....	....	542.45	0.151
....	....	41.38	0.004	....	....	15.28	0.004
....	....	15 327.98	1.698	....	....	5 782.24	1.606
Shifts				Shifts			
Total	Per ft			Total	Per ft		
408	0.045	1 632.00	0.181	....	....	....	....
2 166	0.240	8 664.00	0.960	1 153	0.320	4 612.00	1.281
848	0.094	3 394.07	0.376	390	0.109	1 558.37	0.433
4 139	0.458	15 521.17	1.720	1 206	0.335	4 522.11	1.256
1 064	0.118	4 257.23	0.471	319	0.089	1 276.72	0.355
864	0.096	3 241.48	0.359	291	0.081	1 090.55	0.303
224	0.025	870.47	0.097	85	0.023	328.22	0.091
9 713	1.076	37 580.42	4.164	3 444	0.957	13 387.97	3.719
....	....	4 416.14	0.490	....	....	1 924.65	0.535
....	....	391.24	0.043	....	....	147.32	0.041
....	....	1 878.16	0.209	....	....	693.14	0.192
....	....	1 062.50	0.118	....	....	527.54	0.147
....	....	2 711.03	0.301	....	....	1 341.95	0.373
....	....	1 861.84	0.207	....	....	946.15	0.263
....	....	1 161.26	0.129	....	....	590.41	0.164
....	....	2 713.83	0.301	....	....	1 000.24	0.278
....	....	815.92	0.090	....	....	301.98	0.084
....	....	2 535.17	0.282	....	....	931.43	0.259
....	....	711.81	0.079	....	....	262.64	0.073
....	....	241.71	0.022	....	....	87.78	0.024
....	....	614.56	0.069	....	....	227.71	0.063
....	....	886.83	0.098	....	....	327.45	0.091
....	....	819.91	0.091	....	....	303.45	0.084
....	....	22 821.91	2.529	....	....	9 613.84	2.671
.... 9 025 ft	....	75 730.31	8.391	.... 3 599 ft	....	28 784.05	7.996

Old track was used almost exclusively

Example 3. Alaska Juneau Gold Mining Co

Table 13. Summary of Operating Results, 1934-1937 Inclusive

Tonnage mined, 12 871 820; gold recovered, oz 547 919; silver recovered, oz 386 527; lead recovered lb, 7 201 060; capitalization, 1 500 000 shares, $10 par value.

	Total	Per ton	Per share Total	Per share Annual aver
Value of bullion recovered	$17 508 668	$1.36	$11.67	$2.92
" " concentrates produced	2 272 036	0.18	1.52	0.38
Other income	332 542	0.02	0.22	0.05
Total	20 113 246	1.56	13.41	3.35
Operating costs: (Per ton)				
Mining $5 670 699 — 0.44				
Milling 3 683 466 — 0.29				
Other operating 976 997 — 0.07	10 331 162	0.80	6.89	1.72
Net operating profit	9 782 084	0.76	6.52	1.63
Administration, other than taxes	325 018	0.02	.22	.05
Profit before taxes	9 457 066	0.74	6.30	1.57
Taxes	1 166 040	0.10	0.77	0.19
Net income	8 291 026	0.64	5.53	1.38 (a)
Dividends paid	7 375 799	0.57	4.92	1.23
Balance of income	915 227	0.07	0.61	0.15
Decrease in net current assets, Dec 31, 1933 to Dec 31, 1937	1 913 248	0.15	1.28	0.32
Cash to be accounted for	2 828 475	0.22	1.89	0.47
Capital expenditures on mine development, plant and equipment	1 380 339	0.11	0.92	0.23 (b)
Net purchases and sales of capital stock and exchange in part of same for mining property	1 448 136	0.11	0.97	0.24
	2 828 475	0.22	1.89	0.47

Note.—Aver annual income, after capital expenditures, was a minus b = $1.15 per share.

Example 4. Alaska-Treadwell Mining Co, Douglas Island, Alaska

Principal Supplies Consumed Per Ton, Over a Period of Five Years

Mining, per ton ore mined

Powder	1.925 lb
Fuse	4.612 ft
Caps	0.922 pc
Candles	0.113 lb
Drill steel	0.125 lb
Steel cable	0.039 lb
Air hose	0.006 ft
Blacksmith coal	0.25 lb
Trails	0.064 lb
Lumber	0.744 ft
Lubricating oil	0.008 gal
Pipe	0.018 ft

Power consumption; h p day per ton ore				
Year	Mining	Milling	General	Total
1905	0.454	0.829	0.160	1.443
1906	0.527	0.729	0.178	1.434
1907	0.755	1.036	0.254	2.045
1908	0.786	0.979	0.265	2.030
1909	0.874	0.946	0.296	2.116
1910	0.952	0.978	0.322	2.252

Milling, per ton ore milled

240-stamp mill:

Shoes, 0.331 lb per ton milled.
Dies, 0.193 lb per ton milled.
Mortar extras, 0.135 lb per ton milled.
Cams, tappets, boss heads, 0.030 lb per ton milled.
Mortars crushed 60 054 tons per mortar.
Quicksilver, 1 flask per 17 158 tons milled.
Cam shafts, milled 90 081 tons per cam shaft.
Stamp duty, 4.59 tons per 24 hr.
Stamp weight, 850 lb per stamp.

300-stamp mill:

Shoes, 0.396 lb per ton milled.
Dies, 0.223 lb per ton milled.
Mortar extras, 0.142 lb per ton milled.
Cams, tappets, boss heads, 0.032 lb per ton milled.
Mortars crushed 536 530 tons per mortar.
Quicksilver, 1 flask per 14 295 tons milled.
Cam shafts, milled 13 167 tons per cam shaft.
Stamp duty, 5.51 tons per 24 hr per stamp.
Stamp weight, 1 050 lb per stamp.

Example 4. Alaska-Treadwell Mining Co (*Continued*)

Table 14. Summary of Receipts and Expenditures, 1912

Receipts for 1912:		Per ton ore
Bullion, amalgam....	$1 152 524.62	$1.2918
Bullion, base bars...	6 876.82	0.0077
Bullion from concentrates..........	1 046 487.43	1.1729
Total net yield of ore treated...........	$2 205 888.87	$2.4724
Received from interest	11 750.74	0.0132
Commercial profits..	36 959.02	0.0414
Sundry adjustments.	5 218.75	0.0059
Total receipts from ore and other sources..	$2 259 817.38	$2.5329

Expenditures for 1912 (Table 15):	
Operating charges..............	$1.1848
Construction charges............	0.0171
Sundry losses for year...........	0.0199
Total charges for year (Table 15)..	$1.2218
Deduct interest, commercial profits, etc, shown in receipts........	0.0605
Total net expenditure............	$1.1613
Depreciation charged off..........	0.6961
Total, including deprec...........	$1.8574

Table 15. Detailed Summary of Expenditures, 1912

	Total cost	Per ton milled
Operating costs:		
Mining, development, stoping (Table 17)........	$760 164.05	$0.8520
Milling, 892 192 tons (Table 18)........	181 561.89	0.2035
Concentrate treatment, 17 397 tons (Table 16)........	79 474.77	0.0891
San Francisco office expenses........	15 963.99	0.0179
London office expenses........	1 748.14	0.0020
Paris office expenses........	227.69	0.0003
Legal expenses........	400.00	0.0004
Taxes........	8 538.04	0.0095
Bullion charges........	9 012.53	0.0101
Total operating charges........	$1 057 091.10	$1.1848
Construction:		
Central shaft........	11 577.67	0.0171
Electric locomotive round house........	1 536.16	
Foundry coke house........	629.17	
Pattern shop store room........	1 472.44	
Loss on dwellings for year........	14 170.50	0.0158
Loss on boarding house for year........	2 905.61	0.0033
Loss on wharf for year........	714 10	0.0008
Total charges against ore production (depreciation not included)........		$1.2218

Table 16. Conc't Treat't Costs

	Per ton
Labor..........	$0.520
Supplies..........	0.096
Train service..........	0.108
Sulphuret car expense..........	0.016
Assaying..........	0.018
Cyaniding..........	3.122
Douglas Island gen expense.....	0.054
San Francisco office, freight, refining, etc..........	0.634
Total..........	$4.568

Tons treated, 17 397

Wages paid in 1912		
Machine drillers...........	$ 3.50	per day
Machine helpers...........	3.25	" "
Mine laborers...........	3.00	" "
Amalgamators...........	120.00	" month
Feeders...........	100.00	" "
Vannermen...........	$95 to 130.00	" "
Machinists and helpers.....	$3 to $7	" day
Blacksmiths...........	5 to 6	" "
Tool sharpeners...........	4.50	" "
Blacksmith helpers.........	3.00	" "
Aver number of men employed...........	745	
Aver wage...........	$3.47	
Ore mined and treated per year per man	1 202 tons	

Example 4. Alaska-Treadwell Mining Co (*Continued*)

Table 17. Details of Mining Costs, 1912 (per ton milled)

Items	Development, 1 258 ft Cost per ft	Stoping, 861 973 tons	Tramming, 892 192 tons	Hoisting, 892 192 tons	Pumping per ton milled	Total per ton milled
Machine drillers	$3.188	$0.143	......	......	......	$0.183
Hand miners	......	0.003	......	......	......	0.004
Laborers	1.551	0.114	$0.026	......	......	0.158
Powdermen	0.072	0.010	......	......	......	0.011
Tool carriers	0.018	0.001	......	......	......	0.001
Engineers, skipmen, oilers	......	......	0.003	0.020	......	0.023
Cagemen	......	......	......	0.004	......	0.004
Chutemen	......	......	0.015	......	......	0.014
Trainmen and trammers	......	......	0.029	......	......	0.030
Trackmen and stable men	0.069	......	0.003	......	......	0.004
Pumpmen	......	......	......	......	0.003	0.003
Foremen and shift bosses	0.177	0.009	0.002	......	......	0.014
Blacksmiths and tool sharpeners	0.153	0.006	0.001	......	......	0.008
Machinists and pipemen	0.056	0.002	......	......	......	0.003
Carpenters and timbermen	0.244	0.003	0.001	0.001	......	0.007
Watchmen	0.022	......	......	......	......	0.001
Contractors	1.241	......	0.003	......	......	0.020
Timekeepers	0.025	0.001	......	......	......	0.001
Total direct labor	$6.816	$0.292	$0.083	$0.025	$0.003	$0.489
Powder	1.239	0.146	......	......	......	0.157
Fuse and caps	0.120	0.016	......	......	......	0.017
Candles	0.078	0.006	......	......	......	0.007
Machine-drill supplies	0.131	0.006	......	......	......	0.008
Iron and steel	0.122	0.003	0.001	......	......	0.006
Steel rope	......	......	0.001	0.008	......	0.009
Lumber and timber	0.095	0.003	......	0.001	......	0.006
Lubricants	0.007	......	0.001	0.001	......	0.002
Blacksmith coal	0.001	......	......	......	......	......
Stable	......	......	0.003	......	......	0.003
Fuel oil	0.031	0.001	......	......	......	0.001
Miscellaneous	0.192	0.002	0.003	0.001	0.001	0.009
Total direct supplies	$2.016	$0.183	$0.009	$0.011	$0.001	$0.225
Repairs to buildings	0.022	0.001	......	......	......	0.002
Electrical repairs	0.049	......	......	......	0.001	0.002
Mechanical repairs	0.169	0.002	0.002	0.008	......	0.015
Power	0.236	0.007	0.001	0.040	0.003	0.054
Train service	0.041	0.002	......	......	......	0.003
Assaying	0.013	0.001	......	......	......	0.001
Surveying	0.052	0.001	......	......	......	0.002
Douglas Island general expense	0.675	0.031	0.008	0.004	0.001	0.051
Compressed air	0.494	0.018	0.001	......	0.001	0.027
Steam heat	0.019	0.001	......	......	......	0.001
Blacksmith shop	0.137	0.005	0.001	0.001	......	0.008
Total labor and supplies	$1.907	$0.069	$0.013	$0.053	$0.006	$0.166
Total cost of all accounts	$10.739	$0.544	$0.105	$0.089	$0.010	$0.880
Less credits	1.803	......	......	0.001	0.002	0.028
Total cost per ft of development and per ton of ore	$8.936	$0.544	$0.105	$0.088	$0.008	$0.852

Example 4. Alaska-Treadwell Mining Co (*Continued*)

Table 18. Details of Milling Costs, 1912 (per ton milled)

	240-stamp mill, 371 308 tons					300-stamp mill, 520 884 tons				
	Crushing	Tramming	Stamping	Concentrating	Total	Crushing	Tramming	Stamping	Concentrating	Total
Laborers	......	......	......	......	......	......	......	$0.002	$0.001	$0.003
Feeders	......	......	$0.025	......	$0.025	......	......	0.017	......	0.017
Amalgamators	......	......	0.015	......	0.015	......	......	0.010	......	0.010
Vannermen	......	......	......	$0.016	0.016	......	......	......	0.011	0.011
Samplers	......	......	......	0.001	0.001	......	......	......	......	......
Foremen	......	......	0.003	0.002	0.005	......	......	0.002	0.002	0.004
Oilers	......	......	0.003	0.003	0.006	......	......	0.002	0.002	0.004
Carpenters	......	......	......	......	......	......	......	0.002	0.001	0.003
Stationary engineers	......	......	0.006	......	0.006	......	......	......	......	......
Watchmen	......	......	......	......	......	......	......	0.001	......	0.001
Shoes and dies	......	......	0.016	......	0.016	......	......	0.017	......	0.017
Cams and cam shafts	......	......	......	......	......	......	......	0.003	......	0.003
Tappets, keys, and mortars	......	......	0.002	......	0.002	......	......	0.002	......	0.002
Mortar extras	......	......	0.004	......	0.004	......	......	0.004	......	0.004
Screens	......	......	0.002	......	0.002	......	......	0.002	......	0.002
Quicksilver	......	......	0.002	......	0.002	......	......	0.003	......	0.003
Lubricants, vanner supplies, etc	......	......	0.002	0.004	0.006	......	......	0.001	0.002	0.003
Miscellaneous	......	......	0.002	0.002	0.004	......	......	0.005	0.003	0.008
Tailings account	......	......	0.001	0.001	0.002	......	......	0.002	0.003	0.005
Ore crushing	$0.025	......	......	......	0.025	$0.024	......	......	......	0.024
Tramming to mills	......	$0.016	......	......	0.016	......	$0.015	......	......	0.015
Pumping salt water	......	......	0.001	0.001	0.002	......	......	......	0.001	0.001
Repairs on buildings and machinery	......	......	0.020	0.003	0.023	......	......	0.010	0.001	0.011
Power	......	......	0.019	0.009	0.028	......	......	0.012	0.005	0.017
Train service	......	......	0.002	......	0.002	......	......	0.001	......	0.001
Assaying	......	......	0.001	0.001	0.002	......	......	0.001	......	0.001
Steam heating	......	......	0.002	0.002	0.004	......	......	0.001	0.003	0.004
Compressed air and lights	......	......	......	0.001	0.001	......	......	......	0.001	0.001
Miscellaneous labor, gen exp	0.001	0.001	0.007	0.004	0.013	0.001	0.001	0.005	0.004	0.011
Total milling cost per ton	$0.026	$0.017	$0.135	$0.050	$0.228	$0.025	$0.016	$0.105	$0.040	$0.186

Example 5. Hollinger Consol Gold Mines, Ltd, Canada

Table 19. Detailed Costs per Ton Milled, 1924

Account	Per ton
General charges	$0.1854
Administration	0.0328
Insurance	0.0096
Clearing surface, roads	0.0037
Camp expense	0.0276
Boarding house expense	0.0002
Alterations to plant	0.0017
Marketing bullion	0.0454
Fire protection	0.0202
Operating hospital	0.0004
Workmen's compensation	0.0568
Interest on bank loans	0.0415
Total general charges	$0.4253
Mining and Development:	
General mining charges	$0.0473
Superintendence	0.0853
Diamond drilling	0.0582
Crosscutting	0.1056
Drifting	0.1332
Raising	0.0169
Shaft sinking	0.0597
Timbering shafts	0.0415
Stoping	0.9466
Scaling walls and roof	0.0790
Timbering stopes and raises	0.2495
Retimbering	0.0138
Back filling	0.0322
Track laying	0.0815
Tramming	0.3273
Electric haulage	0.0860
Extending elec haulage	0.0117
Pipe fitting underground	0.0861
Mine drainage	0.0333
Hoisting	0.1630
Landing and dumping	0.0492
Drill repairs	0.0917
Sharpening steel	0.0709
Distributing steel	0.0586
Mine sampling	0.0468
Change house	0.0106
Mine lighting	0.0150
Excavation	0.0134
Surveying	0.0363
Assaying	0.0422
Mining and Development—(*Continued*)	
Ventilation	$0.0006
Alterations	0.0003
Total mining charges	$3.0933
Milling:	
General milling charges	$0.0629
Superintendence	0.0222
Tailings disposal	0.0613
Heating	0.0167
Lighting	0.0084
Mill power plant	0.0153
Shoveling in bins	0.0019
Crushing	0.0891
Conveying	0.0297
Stamping, rod and ball milling	0.1468
Classification and tube milling	0.1255
Concentration	0.0322
Treat'g and handling concentrate	0.0338
Handling pulp	0.0147
Thickening pulp	0.0078
Cyanide	0.0565
Agitation	0.0252
Continuous decantation	0.0208
Filtration	0.0115
Oliver filtration	0.0360
Neutralizing	0.0189
Refining	0.0198
Clarifying and precipitation	0.0360
Pumping solution	0.0109
Cleaning mill	0.0036
Mill sampling	0.0037
Assaying	0.0211
Maintenance of buildings	0.0035
Experimental	0.0078
Total milling charges	$0.9436
Recapitulation:	
Total general charges	$0.4253
Total mining "	3.0933
Total milling "	0.9436
Total cost per ton	$4.4622

Example 5. Hollinger Consolidated Gold Mines, Ltd (*Continued*)

Table 20. Yearly Aver Operating Costs per Ton Milled

Account	Labor		Supplies		Sundries		Total	
	1924	1925	1924	1925	1924	1925	1924	1925
Mining	$1.9609	$1.7589	$1.4324	$0.9832	....	....	$3.0933	$2.7421
Milling	0.3901	0.3507	0.5535	0.5536	....	....	0.9436	0.9043
Gen charges	0.1959	0.1612	0.0761	0.0382	$0.1533	$0.2107	0.4253	0.4101
Totals	$2.5469	$2.2708	$1.7620	$1.5750	$0.1533	$0.2107	$4.4622	$4.0565

General data	1924	1925
Total ore milled	1 659 475 ton	1 929 988 ton
Aver value per ton	$8.39	$8.51
Value per ton in tailings	0.30	0.33
Solution precipitated per ton ore	1.65 ton	1.55 ton
Cyanide consumed per ton ore	0.368 lb	0.402 lb
Zinc consumed per ton ore	0.081 lb	0.090 lb
Lime consumed per ton ore	2.207 lb	1.912 lb
Lead acetate per ton ore	0.006 lb	......
Lead nitrate per ton ore	0.010 lb	0.017 lb
Aver value pregnant solution	$4.90	$5.27
Ore mined per man-shift	1.65 ton	1.993 ton
Value produced per man-shift	$13.84	$16.28
Expense per man-shift (omitting deprec)	7.23	8.44
Aver number of men employed	2 758	2 668
Development work	46 315 ft	68 926 ft
Ore developed per ft (estimated)	41 ton	38 ton

Table 21. Operations for 1937. Tonnage mined, 1 719 199; ounces gold recovered, 425 082

	Total	Per ton		Total	Per ton
Bullion receipts	$14 877 898	$8.654	General charges	$708 319	$0.412
Miscellaneous income	198 916	0.115	Total	$8 315 152	$4.836
Total revenue	$15 076 814	$8.769	Income before taxes and deprec	$6 761 662	3.933
Working costs:					
Mining	5 972 040	3.474			
Milling	1 131 017	0.658	Taxes	$1 000 469	$0.582
Marketing bullion	165 410	0.096	Depreciation	573 712	0.334
Workmen's compensation	129 486	0.075			
Silicosis assessment	208 880	0.121	Net profit from operations	$5 187 418	$3.017

Example 6. Transvaal Gold Mining Costs

Table 22. Costs per Ton Milled (3 Months Ending Mar 31, 1913)

Name of mine	Aurora West United Mine	Cinderella Consolidated Mine	Meyer and Charlton Mine	New Gooh Gold Mines	Roodepoort United Main Reef	Van Ryn Gold Mines Estate	West Rand Consolidated Mines
Mining	$2.75	$3.87	$2.09	$2.24	$2.75	$1.88	$3.61
Sorting, crushing, and transporting	0.14	0.09	0.09	0.19	0.23	0.15	0.17
Milling	0.42	0.46	0.44	0.43	0.37	0.47	0.41
Cyaniding	0.45	0.41	0.54	0.34	0.35	0.37	0.42
General charges	0.42	0.67	0.60	0.31	0.41	0.31	0.52
Office expense	0.09	0.18	0.45	0.10	0.11	0.14	0.19
Mine development redemption	0.73	0.73	0.23	0.20	0.49	0.45	0.55
Permanent works	......	......	......	0.07	......	......	......
Total working costs	$5.00	$6.41	$4.44	$3.88	$4.71	$3.77	$5.87
Working profit	1.60	......	2.07	1.13	......	3.06	1.40
Revenue per ton milled	$6.60	$6.40	$6.51	$5.01	$4.60	$6.83	$7.27
Expended on capital account:							
Permanent works	$0.141	$1.17	$0.17	......	$0.398	......	$0.392
Excess development	0.065	0.76	......	......	......	......	0.267
Machinery and plant	0.022	0.79	0.054	$0.055	0.004	$0.088	0.074
Furniture	0.004	......	0.005	......	......	0.001	0.001
Buildings	......	0.01	......	0.004	......	0.063	0.085
Live stock, vehicles	......	......	0.004	......	......	0.004	0.011
Total capital expended	$0.232	$2.73	$0.233	$0.059	$0.402	$0.156	$0.83
Less credits	0.025	......	......	......	......	......	......
Total expended per ton	$5.207	$9.14	$4.673	$3.939	$5.112	$3.926	$6.70
Tons ore mined	52 079	64 458	44 214	97 489	96 575	132 962	109 610
Tons waste sorted out	9 980	9 328	2 480	16 579	17 656	8 922	23 410
% discarded as waste	19.1	14.3	5.4	17	18.3	7.2	21.5
Tons ore crushed	41 699	55 170	42 014	81 160	78 891	116 000	86 300
Days running time	84.7	84.28	84.20	81.79	80.31	84.10	75.5
Tons per stamp per day	6.15	8.01	6.65	8.27	19.65	10.22	11.74
Stamps operated	80	80 to 85	75	120	50	135	100
Tube mills operated	......	3	2	4	3	6	4
Feet of development	3 394	3 323	1 245	1 575	3 737	3 626	5 023
Earnings included in revenue stated above	1.49¢	7.3¢	......	1.15¢	1.84¢	1.94¢	1.32¢
Approx stoping width, in	41	40	46	61	38	39	46

Note.—See Table 23 for general data on Aurora West United and the Meyer and Charlton.

Example 6. Transvaal Gold Mining Costs (*Continued*)

Table 23. General Data on the Aurora West United & Meyer & Charlton Mines

Data obtained Oct, 1913	Aurora West United Gold Mining Co	Meyer & Charlton Gold Mining Co
Average thickness of lode:		
Stoping width	42 in	25 in
Reef width	14 in	....
Amount of rock broken annually	216 000 tons	179 731 tons
Proportion of ore from stoping areas	180 000 milling tons	92.11%
Average distance broken ore is moved to tramming level (Note 1)	200 ft	115 ft
Average tramming distance to shaft	750 ft	700 ft
Average hoisting depth	1 079 ft	1 430 ft
Timber used per ton of ore extracted	cost 0.64¢	{ 0.12 ft poles 0.6 ft lagging
Development work performed during 1912:		
Shafts	251.5 ft	473 ft
Drifts	8 346 ft	2 713 ft
Raises and winzes	4 398.5 ft	1 674 ft
Tons of ore made available by development	194 605 milling tons	205 133
Diamond drilling	....	379 ft
Amount of water raised from mine in 24 hr	150 000 gal	65 000 gal
Depth from which water is pumped	1 000 ft	1 650 ft
Amount of power required:		
Pumping	70 h p	100 h p
Development headings	(See Note 2)	140 "
Stoping	"	560 "
Tramming	hand	hand
Hoisting	150 h p	425 h p
Crushing before delivery to mill	35 to 40 h p	55 "
Transport to mill	5 h p	10 "
Crushing at mill	300 "	280 "
Tube milling	....	80 "
Cyanide plant	100 h p	150 "
Lights	40 "	50 "
Men employed:		
White	140	128
Natives	1 345	920

Note 1.— Stopes run on an incline of about 30°; backs average 200 ft, with tramming levels directly below. Note 2.— Greater portion of stoping is done by manual labor, native hammer boys employed. It requires 750 hp to operate compressors used in driving machines in development faces and in stopes where machines are worked. General note. See Table 22 for other cost data for these mines.

Table 24. Summary of Operating Results, 1924 and 1925, at 9 Transvaal Mines

Name of mine	Modderfontein B Gold Mines, 1924	New Modderfontein Gold Mines, 1925	Rose Deep, 1924	Geldenhuis Deep, Ltd, 1924	Nourse Mines, 1925	City Deep Ltd, 1924	Village Deep, Ltd, 1924	Crown Mines, 1924	Randfontein, 1924
Tons of ore crushed ...	796 000	1 479 000	685 100	789 700	602 600	1 174 500	667 700	2 609 000	2 432 000
Gold revenue per ton...	$9 19	$10 55	$5.30	$6.50	$5 84	$8 89	$6.90	$6.87	$6.01
Working cost per ton...	4.61	4.16	4 04	6.06	5.60	4.52	5.76	4.69	4.56
Expended on capital acct.	....	1 08	....	...	0.05	0.42	1.66	0.51	1.40
Total expenses per ton...	4 61	5.24	4.04	6.06	5.65	4.94	6.42	5.20	5.96
Profit on ore per ton....	4.58	5.31	1 26	0 44	0 19	3.95	0.48	1.67	0.04
Miscel profits per ton...	....	....	0 17	0 10	0.07	0.17	0.19	0.09	0.12
Total profit...........	4.58	5.31	1.43	0 54	0.26	4.12	0.67	1.76	0.08
Taxes per ton.	0 76	0 95	0.20	...	0.04	0 52	0.16	0.33	....

Note.—Exchange taken at $4.87. These mines produced approx 3 560 000 fine oz in 1924, or 37% of total output of the Rand.

Example 6. Transvaal Gold Mining Costs (*Continued*)

Table 25. Summary of Operating Results, 1912

Name of mine	Modderfontein B Gold Mines	New Modderfontein Gold Mines	Rose Deep, Ltd	Geldenhuis Deep, Ltd
Tons of ore crushed	388 570	585 900	782 200	627 960
Gold revenue per ton milled	$9.07	$8.38	$7.00	$7.33
Total working costs (see *A*)	4.29	4.54	4.23	6.28
Sundry expenses	0.25	0.01	0.04	0.08
Construction, etc, chg'd to capt acc't	0.28	0.61	0.05	0.18
Total Expenditure per ton milled	4.82	5.16	4.32	6.54
Profit on ore per ton	4.25	3.22	2.68	0.79
Miscellaneous profits	0.07	0.05	0.01	0.09
Total profit over expenditure	4.32	3.27	2.69	0.88
Government income tax on profits	48.6¢	35 6¢	22.6¢	7.2¢

(*A*) Items of Working Cost per Ton Milled

Item	Modderfontein B Gold Mines	New Modderfontein Gold Mines	Rose Deep, Ltd	Geldenhuis Deep, Ltd
Mining (see *B*)	$2.25	$2.78	$2.59	$3.85
Development charged	0.44	0.38	0.24	0.75
Milling expenses (see *C*)	1.03	1.00	1.03	1.15
General expenses	0.44	0.38	0.37	0.49
Renewals and replacements	0.13	....	....	0.04
Total Working Cost per Ton Milled	$4.29	$4.54	$4.23	$6.28

(*B*) Items of Mining Costs per Ton Mined and Other Data

Item	Modderfontein B Gold Mines	New Modderfontein Gold Mines	Rose Deep, Ltd	Geldenhuis Deep, Ltd
Total tons ore mined	437 306	657 806	922 844	776 511
Tons ore from stopes	406 178	601 860	883 349	714 195
Tons ore from development	31 128	55 946	39 495	62 316
Average stoping width in inches	55	57	60	44
Stoping	$0.71	$0 87	$0.81	$1.13
Timbering and rockwalling	0.24	0.14	0.12	0.18
Old workings	....	....	0.25	0.23
Shoveling and tramming	0.71	0.81	0.46	0.51
General and other costs	0.34	0.65	0.53	1.01
Sand filling	....	....	0.02	0.06
Total Cost per Ton Mined	$2.00	$2.47	$2.19	$3.12

(*C*) Items of Milling Cost per Ton Milled and Other Data

Item	Modderfontein B Gold Mines	New Modderfontein Gold Mines	Rose Deep, Ltd	Geldenhuis Deep, Ltd
Tons of ore received from mine	437 306	657 806	922 844	776 511
Per cent waste sorted out of mine ore	11.1%	11%	15.1%	19.1%
Average value of waste sorted out	49¢	59¢	$1.01	30¢
Tons of ore stamped	388 570	585 900	782 200	627 960
Stamps in operation	80	180	300	300
Tons per stamp per 24 hr	14	9.7	7.6	7.1
Tons of pulp cyanided	387 487	586 615	781 735	633 162
Per cent of total yield by mill	57.5%	76.4%	66.5%	68.2%
Per cent of total yield by cyaniding	42.5%	23.6%	33.5%	31.8%
Ore sorting and crushing	$0.08	$0.08	$0.12	$0.16
Transporting from crusher to mill	0.02	0.02	0.04	0.04
Stamp milling	0.20		0.29	
Tube milling	0.18	0.45	0.14	0.49
Amalgamation	0.04		0.06	
Cyaniding	0.51	0.45	0.38	0.46
Total per Ton Milled	$1.03	$1.00	$1.03	$1.15

Table 26. Summary of Operating Results, 1912 (*Concluded*)

Nourse Mines, Ltd	City Deep, Ltd	Village Deep, Ltd	Ferreira Deep, Ltd	Crown Mines, Ltd	Bantjes Cons Mines, Ltd	Durban Roodepoort Deep
61 096	479 630	596 900	559 800	1 920 700	286 453	293 995
$7.41	$8.63	$7.25	$9.70	$7.78	$7.17	$7.27
5.47	5.77	4.84	5.16	4.45	5.85	5.91
0.10	0.07	0.05	0.07	0.13	0.10	0.07
0.26	0.12	0.34	0.02	0.53	0.14	0.25
5.83	5.96	5.23	5.25	5.11	6.09	6.23
1.58	2.67	2.02	4.45	2.67	1.08	1.04
0.09	0.15	0.04	0.18	0.02	0.06	....
1.67	2.82	2.06	4.63	2.69	1.14	1.04
16.5¢	27.2¢	20.5¢	41¢	32¢	11.7¢	11.5¢
(*A*) Items of Working Cost per Ton Milled						
$3.52	$3.52	$2.79	$3.22	$2.94	$3.16	$3.79
0.55	0.77	0.79	0.30	0.28	1.09	0.60
0.98	1.03	0.89	1.15	0.91	1.09	1.03
0.42	0.45	0.37	0.39	0.32	0.51	0.49
....	....	....	0.10	....	....	....
$5.47	$5.77	$4.84	$5.16	$4.45	$5.85	$5.91
(*B*) Items of Mining Cost per Ton Mined and Other Data						
718 128	487 565	698 124	663 084	2 183 305	327 710	357 270
672 349	446 835	645 415	626 044	2 081 222	310 366	319 678
45 779	40 730	52 709	37 040	102 083	17 344	37 592
55	54	60	65	64	41	46
$1.40	$2.09	$1.05	$0.77	$1.01	$1.58	$0.95
0.20	0.24	0.20	0.14	0.14	0.08	0.40
0.04	0.04	....	0.59	0.10	0.02	0.04
0.61	0.71	0.45	0.32	0.57	0.51	0.49
0.73	0.38	0.69	0.77	0.73	0.57	1.24
....	....	....	0.12	0.04	....	....
$2.98	$3.46	$2.39	$2.71	$2.59	$2.76	$3.12
(*C*) Items of Milling Cost per Ton Milled and Other Data						
718 128	487 565	698 124	663 084	2 183 305	327 710	357 270
15.1%	15.6%	14.8%	15.7%	11.9%	12.6%	17.7%
69¢	24¢	26¢	65¢	57¢	63¢	49¢
610 196	479 630	596 900	559 800	1 920 700	286 453	293 995
260	120	180	220	660	80	100
7.6	7.5	9.5	8.0	10.3	11.0	8.6
609 250	477 160	596 860	....	1 915 716	287 270	293 508
74.9%	65.4%	70.4%	68.5%	70.1%	60%	70.5%
25.1%	34.6%	29.6%	31.5%	29.9%	40%	29.5%
$0.08	$0.10	$0.12	$0.14	$0.10	$0.14	$0.18
0.02	0.04	0.02	0.04	0.04	0.02	0.02
	0.22	0.22	0.32	0.31	0.22	
0.49	0.27	0.14	0.18	0.12	0.14	0.46
	....	0.08	....	....	0.10	
0.39	0.40	0.31	0.47	0.34	0.47	0.37
$0.98	$1.03	$0.89	$1.15	$0.91	$1.09	$1.03

Example 6. Transvaal Gold Mines (*Concluded*)

Table 27. Summary of Operations of Eighteen Leading Companies in 1937

	Randfontein Estates G. M. Co	West Rand Consolidated Mines, Ltd	Durban Roodepoort Deep, Ltd	Consolidated Main Reef Mines, Ltd	Crown Mines, Ltd	Robinson Deep, Ltd	Nourse Mines, Ltd	Simmer & Jack Mines, Ltd	Rose Deep, Ltd
Ore crushed, tons	4 657 000	2 050 000	1 209,000	1 916 500	4 258 000	1 350 000	991 000	1 209 500	894 000
Gold recovered, oz	775 472	405 897	245 702	362 801	1 022 572	321 080	201 999	257 611	154 575
Bullion recovery per ton	$5.872	$6.975	$7.146	$6.646	$8.229	$8.354	$7.188	$7.483	$6.083
Working cost per ton	4.144	4.215	5.583	5.042	4.771	4.827	5.709	5.262	5.062
Working profit per ton	1.728	2.76	1.562	1.604	3.688	3.507	1.479	2.221	1.021
Miscel net income	0.032	0.155	0.034	−0.024 (b)	−0.004 (b)	−0.158 (b)	−0.076 (b)	0.091	0.015
Profit before taxes and capital expend.	1.760	2.915	1.596	1.580	3.684	3.349	1.403	2.312	1.036
Taxes	0.486	1.157	0.483	0.538	1.511	1.342	0.382	0.546	0.401
Capital expenditure	0.212	......	1.221 (a)	0.172	0.062	0.372	0.056	0.787	0.109
Dividends	0.982	1.728	1.118	0.895	2.104	2.083	0.889	0.930	0.441
Change in net current assets	+0 080	+0.03	−1.204	−0 025	+0.002	−0 448	+0 076	+0 049	−0.085

	East Rand Proprietary, Ltd	New Modderfontein Gold Mining Co, Ltd	Modderfontein B Gold Mine, Ltd	Government Gold Mining Areas Consol, Ltd	New State Areas	Geduld Proprietary Mines, Ltd	East Geduld Mines, Ltd	Brakpan Mines, Ltd	Sub-Nigel, Ltd
Ore crushed, tons	2 503 000	2 342 000	1 084 000	2 552 000	1 475 000	1 301 000	1 496 000	1 648 000	685 000
Gold recovered, oz	562 422	477 222	184 166	749 391	448 709	326 756	436 337	421 666	496 673
Bullion recovery per ton	$7.896	$7.188	$5.979	$10.112	$10 724	$8 812	$10.229	$9.004	$25.478
Working cost per ton	5.417	3.708	3.479	4.448	4.579	3.604	4.271	4.937	8.356
Working profit per ton	2.479	3.480	2.500	5.914	6 145	4.208	5.958	4.067	17.122
Miscel net income	−0.034 (b)	−0.084 (b)	0.064	......	0.002	1.293	−0.045 (b)	0.073	0.013
Profit before taxes and capital expend.	2.445	3.396	2.564	5.914	6.147	6.501	5 913	4.140	17.135
Taxes	0.890	1.427	1.119	3.332	4.421	2.365	2.699	1.908	8.089
Capital expenditure	0.412	0.028	−0.192 (c)	0.002	0.102	0.153	0.225	0.138	......
Dividends	1.258	1.922	1.615	2.469	1.523	4 211	2.933	2.094	10.162
Change in net current assets	−0 115	0 019	0.022	0.111	0.101	0 226	0 056	.. ..	1.116

(a) These capital expenditures were financed by sale of additional capital shares, and working capital was placed in position to complete a comprehensive program of expansion of operations.
(b) Miscel expenses, net.
(c) Credit from return of unexpended balances previously appropriated.

Note. Exchange taken at $5.00 These mines produced approx 7 851 061 fine oz in 1937, or 68.6% of total output of Rand.

Example 7. Bunker Hill & Sullivan Mining & Concentrating Co, Kellogg, Idaho

Table 28. Summary of Mining and Milling Operations

Receipts	Year ended May 31, 1909	19 mos to Dec 31, 1912	1924	1925
Gross value shipping ore and conc	$3 199 976	$5 396 916	$6 756 343	$7 749 848
Miscel receipts	52 439	96 595	......	......
Other revenue after income taxes	......	. .	174 553	643 223
Total receipts	$3 252 415	$5 493 511	$6 930 896	$8 393 071
Expenditures				
Operating cost	$ 761 374	$1 580 815	$1 229 251	$1 393 923
Freight, treatment and discount on shipments	1 343 043	1 980 231	2 187 853	2 349 660
Exploration	44 152	79 014	40 965	54 786
Miscel expenses and deprec	367 394	393 580	547 268	536 386
Total expenditures	$2 515 963	$4 033 640	$4 005 337	$4 334 755

Note.—In earlier years expenditures for construction, property, etc were charged to current income; in later years such expenses are credited to capital investment, while current income is charged with deprec of these investments.

Balance receipts over expenses and charges other than depletion	$ 736 452	$1 459 871	$2 925 559	$4 058 315
Concentrating ore, tons	341 700	699 160	422 907	453 412
Shipping ore, tons	2 770	3 360	. . . .	. . .
Total tons produced	344 470	702 520	422 907	453 412
Concentrate produced, tons	67 710	96 003	75 229	77 599
Ratio ore to conc	5.05–1	7.26–1	5.62–1	5.84–1
Shipping ore and conc:				
Lead assay, %	46.13	53.78	48.49	48.94
Silver assay, oz	16.35	19.36	16.52	17.55
Aver price of metal:				
Lead, cents	3.947	4.002	8.120	8.987
Silver, cents	51.514	58.329	66.940	69.075
Assay value shipping ore per ton	$44.89	$54.31	$89 80	$100.08
Net cost (before depletion) per ton ore mined	7.15	5.60	9 45	9.56

Table 29. Development Costs per Foot and Ton

	Year ended May 31, 1909	19 mos to Dec 31, 1912	1924	1925
Machine men	Items of costs for 1909 and 1912 are omitted, as segregation was then entirely different.		$ 1.37	$ 1.58
Miners			2.90	2.42
Shovelers			0.72	1.02
Timbermen and carpenters			1.32	1.52
Hoistmen and skipmen			0.07	0.04
Explosives			2.29	2.47
Illuminants			0.04	0.04
Timber and lagging			0.88	0.61
Miscellaneous supplies			1.68	3.17
Machine shop repairs			0.64	0.48
Tool shop repairs			0.29	0.48
Compressed air			1.35	1.09
Surveying			0.22	0.22
Contingent expense			0.36	0.28
Administrative			0.52	0.49
Concreting			.....	0.99
Total costs	$9.507	$7.150	$14.65	$16.00
Less diamond drilling per foot	......	......	5.21	......
Total average cost per foot	$9.507	$7.150	$9.44	$16.00
Total feet of development	4 644	11 050	2 016	3 423
Total feet of diamond drilling	......	......	3 567	......
Development cost per ton ore mined	0.128	0.113	0.097	0.121

Table 30. Cost of Stoping per Ton of Concentrating Ore Mined

	Year ended May 31, 1909	19 mos to Dec 31, 1912	1924	1925
Shift bosses	Items of costs for 1909 and 1912 are omitted, as segregation was then entirely different		$0.63	$0.070
Machine men			0.168	0.169
Chuck tenders			0.006	0.003
Miners			0.220	0.246
Shovelers			0.464	0.502
Carmen and trammers			0.087	0.089
Motormen			0.037	0.042
Timbermen and carpenters			0.267	0.289
Hoistmen and skipmen			0.005	0.005
Nippers			0.020	0.022
Supply men			0.030	0.029
Explosives			0.113	0.123
Illuminants			0.008	0.009
Timber and lagging			0.330	0.356
Miscel supplies			0.106	0.149
Mach shop repairs			0.047	0.058
Electrical repairs			0.012	0.014
Tool shop repairs			0.020	0.019
Electric power			0.002	0.006
Compressed air			0.130	0.112
Surveying			0.006	0.006
Mine ventilation			0.026	0.032
Total cost	$1.555	$1.541	$2.167	$2.342
Tramming cost per ton ore by elec power, including hauling timber supplies and men	$0.083	$0.074	$0.095	$0.100
Hoisting cost per ton ore	......	......	0.105	0.094
Pumping cost per ton ore	......	......	0.085	0.080

Table 31. Concentrating Costs per Ton Ore

	Year ended May 31, 1909	19 mos to Dec 31, 1912	1924	1925
Crushing and conveying:	Items of costs for 1909 and 1912 are omitted, as segregation was then entirely different			
Operating labor...			$0.025	$0.023
Power...........			0.022	0.020
Supplies.........			0.020	0.026
Repairs..........			0.004	0.008
			$0.071	$0.077
Screening:				
Operating labor...			$0.014	$0.014
Power...........			0.006	0.006
Supplies.........			0.021	0.019
Repairs.........			0.003	0.003
			$0.044	$0 042
Jigging:				
Operating labor...			$0.041	$0.039
Power...........			0.052	0.050
Supplies.........			0.026	0.023
Repairs.........			0.004	0.009
			$0.123	$0.121
Grinding:				
Operating labor...			$0 014	$0.014
Power...........			0.018	0.017
Supplies.........			0 025	0.035
Repairs..........			0.003	0.006
			$0.060	$0.072

	Year ended May 31, 1909	19 mos to Dec 31, 1912	1924	1925
Concentrating:	Items of costs for 1909 and 1912 are omitted, as segregation was then entirely different			
Operating labor..			$0.045	$0.042
Power..........			0.032	0.032
Supplies......			0.087	0.072
Repairs.........			0.009	0.007
			$0.173	$0.153
General mill expense:				
Supervision.....			$0.057	$0.052
Heat, light and power........			0.037	0 047
Supplies........			0 017	0.016
Repairs.........			0.080	0.057
Miscel charges .			0.053	0.051
Taxes and insurance..........			0.050	0.047
Administration expense.....			0.028	0 024
Depreciation...			0.072	0.031
Total cost	....	.....	$0.394	$0.295
Total direct cost	$0.386	$0.359	0.471	0.465
Grand total ...	....	......	$0.865	$0.760
Shipping expense per ton.........	$0.227	$0.238	$0.322	$0.306
Tons shipped......	71 522	99 698	75 806	77 394

Table 32. Summary of Operations for 1937

Production from own mines:
- Tons ore....... 386 576
- Lead.......... 59 014 000 lb
- Zinc........... 36 246 000 " in concentrates
- Silver.......... 2 179 684 oz

Total production, including metals from custom ores purchased:
- Lead.......... 106 958 817 lb
- Zinc........... 36 246 000 " in concentrates
- Silver.......... 20 363 308 oz
- Gold........... 4 047 "

Copper, antimony, cadmium and bluestone, all in form of by-products, were also produced.

Gross metal sales..........................		$22 635 991
Direct operating costs...........	$20 726 854	
Depreciation and depletion......	513 486	
Taxes, federal, state and local....	390 783	
	21 631 123	
Less increase in metal inventories..	1 484 036	20 147 087
Operating profit..........................		2 488 904
Net other revenue.......................		101 557
Total profit.............................		2 590 461
Depletion charged.......................		366 193
Profit before depletion...................		$2 956 654

Note. If silver, gold, and all other metals produced, except lead and zinc, are credited to cost of producing lead and zinc, there was a profit before depletion of $62.07 per ton of lead and zinc. As the aver price received for these metals was about $122.20 per ton, the cost, after other metal credits, but before depletion, was about $60.13 per ton.

Example 7. Bunker Hill & Sullivan Mining & Concentrating Co (*Concluded*)

Table 33. Historical Summary of Operations, 1886–1937

Period		Ore mined	Gross assay value of ore recovered	Net smelter returns	Operating profit (before depletion)	All other net surplus (before depletion)	Dividends paid
May, 1886, to Dec, 1912		5 546 787	$ 52 978 903	$ 31 480 913	$17 747 666	$ 2 075 358	$13 911 750
Calendar years	1913	436 060	3 890 139	2 329 888	1 167 317	118 433	817 500
	1914	440 819	3 362 498	2 010 136	973 355	160 257	981 000
	1915	455 205	4 177 819	2 384 543	1 241 031	603 637	1 062 750
	1916	475 784	6 253 048	3 297 827	1 844 758	520 282	1 716 750
	1917	493 030	9 584 963	4 943 782	3 285 827	240 443	2 043 750
	1918	389 027	6 094 285	3 949 878	2 533 888	387 211	1 553 250
	1919	393 698	5 093 825	3 362 191	1 765 992	92 730	1 144 500
	1920	409 986	6 915 140	4 586 976	2 967 683	561 066*	1 962 000
	1921	426 530	4 612 444	2 695 594	874 438	643 303*	981 000
	1922	421 532	5 171 228	3 332 543	1 772 703	274 436	1 062 750
	1923	425 817	5 968 275	3 959 802	2 174 019	70 008*	1 635 000
	1924	422 907	6 755 521	4 592 072	2 751 005	174 552	1 962 000
	1925	453 412	7 766 169	5 439 765	3 415 092	643 222	2 959 209
	1926	459 761	7 202 042	4 949 692	2 858 683	895 496	2 981 709
	1927	456 134	6 910 720	4 705 667	2 567 189	789 961	2 993 628
	1928	452 345	6 711 631	4 366 031	2 156 631	770 718	3 003 962
	1929	451 111	7 188 837	4 659 910	2 170 865	820 629	3 009 042
	1930	455 475	5 921 999	3 646 219	1 184 631	1 032 359	2 109 690
	1921	460 366	4 604 284	2 536 258	265 377	532 329	553 246
	1932	429 880	3 385 038	1 535 357	250 206*	290 486	61 636
	1933	458 565	4 076 743	1 991 217	232 339	726 130	58 901
	1934	362 388	3 578 955	1 711 570	130 186	380 863	56 296
	1935	353 259	4 100 818	2 018 969	260 398	702 845	218 482
	1936	347 603	4 971 124	2 608 327	680 111	992 071	1 525 166
	1937	386 576	6 874 652†	3 913 066†	1 378 344	1 297 238	1 851 659
Totals.....		16 264 057	$194 151 113	$117 008 204	$58 149 332	$13 247 317	$52 216 631

* Deficit. † Includes returns on sale of stored zinc concentrates produced in prior years.

Example 8. Sunshine Mining Company

Table 34. Summary of Operations for 1937

Tons milled	255 800
Metals recovered:	
Silver	12 146 853 oz
Gold	240 "
Lead	347 100 lb
Copper	2 784 000 "

After crediting all other metals produced to cost of producing silver, costs were as follows:

	Per ton	Per oz
Mining	$ 4.68	9.86¢
Development	.45	.95
Milling	.47	.98
Overhead and general	2.58	5.43
Taxes of all kinds	6.53	13.76
Total	$14.71	30.98¢
Net smelter receipts per oz silver	......	75.19
Profit after all charges, including deprec.		44.21¢

Example 9. Burma Corporation

Table 35. Summary of Operations for Year Ending June 30, 1937

Production: 478 806 long tons

Metals recovered:

Refined lead	74 885 long tons
Zinc concentrates	78 213 " "
Copper matte	8 250 " "
Nickel speiss	4 320 " "
Antimonial lead	1 090 " "
Silver	6 135 000 oz

After crediting all other metals sold to cost of producing lead and recoverable zinc in concentrates (39 000 ton approx), the profit from lead and zinc, including cost of treating zinc concentrate, was as follows, after deprec:

		£ = 4.95
Total profit	£1 236 794	$ 6 122 131
Estimated receipts from sale of lead and zinc	2 516 237	$12 455 374
Cost of producing lead and zinc	£1 279 431	$ 6 333 242

Cost of producing lead and zinc, per ton, £11–4–8, per lb, 2.47¢.

Example 10. Lead Mining Costs, Southeastern Missouri

Table 36. General Operating Results, 1925

(a)	Ore mined, wet tons	4 628 746
(b)	Ore milled, wet tons	4 630 036
(c)	Cone produced, dry tons	175 438
(d)	Flotation product, dry tons	71 668
(e)	Total of (c + d), dry tons	247 106
	Lead content of (e), tons	168 074
	Lead per ton ore, lb	72.6
		Cost per ton ore
(f)	Prospecting, $0.115; (g) Development, $0.195	$0.310
(h)	Mining, $0.969; (i) Mine suspense accts, $0.002	0.971
(j)	Milling	0.388
(k)	Shop, yard and stable expense	0.045
(l)	Indirect operating and local general expense	0.156
(m)	Other suspension expense	0.001
(n)	Local electric sub-station	0.009
(o)	Electric line loss and meter difference	0.017
(p)	Total operating cost at mine and mill (f to o)	$1.897
(q)	Mill product transportation	0.035
(r)	Smelting charges	0.498
	Total cost per ton ore, 90% of lead in concentrates being pig lead, f o b shipping point (p to r), excluding deprec, general expense and taxes	$2.430

Table 36a. Wage Scales

	Aug, 1913	May, 1926
Shift bosses	$3.10	$5.90–$5.95
Drillers	$2.15– 2.40	5.05
Blacksmiths	2.55	5.05– 6.60
Hand loaders	2.40	5.00
Electric-shovel operators	....	5.65
Locomotive engineers	2.75	5.05
Trackmen, chutemen, car dumpers	2.40	5.00
Hoisting engineers	2.60	5.05
Compressor men	2.60	5.00
Diamond drillers	2.60	5.05– 5.15
Machinists and electricians	2.10– 3.10	5.00– 6.15
Mill men	1.15– 2.60	3.85– 4.60
Surface, common labor	1.15	3.85

Example 10a. Tri-State Field, Zinc and Lead Mining

(Wade Kurtz, *Bull* Amer Zinc Inst, Apl, 1926)

Table 37. Average Cost per Ton of Concentrates Produced

		1921	1922	1923	1924
Zinc	Labor (all classes)	$ 7.4956	$10.8152	$14.3071	$15.8314
	Salaries	0.8092	0.8490	1.1454	1.0616
	Explosives	1.9039	2.2407	2.6007	3.1568
	Supplies and repairs	2.7663	4.2469	4.7071	5.0576
	Power and power fuel	2.5798	3.0127	2.7257	2.6465
	Liability insurance	0.3457	0.5052	0.7681	1.0362
	Fire and tornado insurance	0.3939	0.2960	0.3241	0.3687
	Taxes	0.2103	0.1388	0.2858	0.2546
	Miscellaneous	1.0654	1.2003	1.6899	2.3383
	Total mining and milling	17.5681	23.3084	28.5539	31.7517
	Deprec and depletion	5.2306	5.8480	5.9192	5.2504
	Royalty	3.0821	4.5884	5.9384	6.0518
	Miscel income (credit)	0.1480	0.1399	0.2762	0.3184
	Aver cost per ton zinc conc	$25.7328	$33.6013	$40.1353	$42.7355
Lead	Mining and milling cost	$17.5681	$23.3084	$28.5539	$31.7517
	Deprec and depletion	5.2306	5.8480	5.9192	5.2504
	Royalty	8.4440	10.7797	14.7642	15.0998
	Miscel income (credit)	0.1480	0.1399	0 2762	0.3184
	Aver cost per ton lead conc	$31.0947	$39.7926	$48 9611	$52.5845

Example 10a. Tri-State Field, Zinc and Lead Mining (*Concluded*)

Table 38. Average Cost per Ton of Ore Mined and Milled

	1921	1922	1923	1924
Labor (all classes)	\$0.7088	\$0.8108	\$0.9635	\$0.9922
Salaries	0.0765	0.0637	0.0771	0.0665
Explosives	0.1801	0.1680	0.1751	0.1978
Supplies and repairs	0.2616	0.3194	0.3170	0.3170
Power and power fuel	0.2440	0.2259	0.1836	0.1659
Liability insurance	0.0327	0.0379	0.5017	0.0649
Fire and tornado insurance	0.0370	0.0221	0.0218	0.0231
Taxes	0.0199	0.0104	0.0192	0.0160
Miscel	0.1007	0.0890	0.1135	0.1466
Total mining and milling cost	\$1.6613	\$1.7472	\$1.9225	\$1.9900

Example 11. Copper Mining Costs

Table 39. Miscellaneous Group; Production and Costs, 1912

Mine	Tons of ore milled	Yield per ton lb Cu	Total pounds of copper produced	Cents per lb of copper				
				Operating cost	Gold, silver credit	Sundry credit	Net cost	Constr, etc, not included
Anaconda	5 069 242	58.0	294 474 161	12.3	2.6	0.24	9.40	0.01
North Butte	425 297	62.0	26 480 123	13.1	3.37		9.73	
Tennessee	444 289	29.8	13 352 634				11.00	0.04
Shannon			16 406 336	12.6	1.05		11.55	
Granby	739 519	17.9	13 231 121	17.3	6.06		11.24	
British Columbia	740 589	15.0	11 146 811	18.5	5.3		13.20	3.1
Calumet and Arizona	159 513	92.1	16 490 229	10.66	1.96		8.70	0.2
Superior and Pittsburgh	288 429	127.0	36 618 399	7.67	1.36	0.05	6.26	0.2
Mason Valley	241 822	66.5	16 058 493	14.8	0 23		14.57	
Old Dominion			16 533 999	9.0	0.51	0.44	8.05	0.59
United Globe	188 524	65.0	12 252 073					
Total and aver			473 044 379	12.16	2.38	0.18	9.6	0.40
Phelps Dodge Co			140 628 798	cost estimated 8.5 to 9¢ per lb				

Example 12. Utah Copper Co, Bingham, Utah

Table 40. Summary of Operating Results for 1925

Operating revenue:		
Copper produced, 214 162 139 lb at 14.069¢		\$30 130 561.89
Gold " 78 152.432 oz at \$20		1 563 168.64
Silver " 629 782.49 oz at 69.02¢		478 152.46
Total revenue		\$32 171 882.99
Operating expenses:		
Mining, including stripping	\$4 282 333.39	
Ore delivery, mine to mill	1 443 082.73	
Milling	6 748 331.44	
Treatment, freight and refining	7 010 871.55	
Selling expense	267 702.74	\$19 752 321.73
Profit from operations		\$12 419 561.26
Miscellaneous income, net		1 118 541.67
Total income		\$13 538 102.93
Charges against income:		
Depreciation	\$1 207 270.38	
Federal income tax and other charges	1 020 894.12	
Loss on plant and retired equipment	206 433.63	2 434 598.13
Net income to surplus account		\$11 103 504.80
" " per lb copper	5.185¢	
" cost per lb copper before depletion	8.884¢	

Example 12. Utah Copper Co, Bingham, Utah (*Concluded*)

Table 41. General Operating Data

Tonnage mined and milled..	12 538 300 tons	Stripping removed..........	7 938 411 cu yd
Copper in ore..............	1.02% or 20.4 lb per ton	Grade of concentrate shipped.	17.47% copper
Aver mill recovery.........	17.78 lb per ton	Production of refined copper..	214 162 139 lb

Operating costs per ton on concentrating ore, including all fixed, general and maintenance charges, 1910 to 1925, inclusive:

Year	Tonnage	Mining	Ore delivery	Milling	Total
1910	4 340 245	$0.4097	$0.2978	$0.4663	$1.1738
1911	4 680 801	0.4479	0.3078	0.4168	1.1725
1912	5 315 321	0.4233	0.2848	0 4158	1.1239
1913	7 519 392	0.3288	0.2797	0.3676	0.9761
1914	6 470 166	0.3232	0.2782	0.3536	0.9550
1915	8 494 300	0.2441	0.2751	0.3402	0.8624
1916	10 994 000	0.2781	0.2792	0.3782	0.9355
1917	12 542 000	0 4446	0.2794	0 6930	1.4170
1918	12 160 700	0.5370	0.2983	0.9277	1.7630
1919	5 538 700	0.4900	0.3040	1.2062	2.0002
1920	5 556 800	0.4823	0.2591	1.2472	1.9886
1921	1 220 700	0.4998	0.1921	1.1679	1.8598
1922	4 364 251	0.3833	0.1612	0.8417	1.3862
1923	11 167 800	0.3488	0.1088	0.6116	1.0692
1924	12 126 600	0.3605	0.1308	0.5990	1.0903
1925	12 538 300	0.3373	0.1151	0.5382	0.9906

Note.—Mining cost for 1925 includes proportional "prepaid" stripping charge of 12.5¢ and fixed and general charges of 4.12¢. Actual direct mining cost of all ores, 21.23¢ per ton.

Example 13. Chile Copper Co, Chuquicamata, Chile

Table 42. Summary of Operating Results for 1925

Operating revenue:		
Copper sold, 207 978 026 lb at 14.273¢		$29 684 407.13
Production cost		11 293 499.32
Operating Profit		$18 390 907.81
Other income		997 100.28
Total revenue		$19 388 008.09
Charges against income:		
Taxes and miscel charges	$2 567 923.25	
Interest and discount on bonds	2 239 958.92	
For deprec and obsolescence of plant and equipment	2 640 975.36	7 448 857.53
Net income carried to balance sheet		$11 939 150.56
" " per lb of copper	5.746¢	
" cost per lb copper before depletion	8.527¢	
" " " " " " " and bond interest	7.450¢	

Table 43. General Operating Data

Ore mined	7 778 910 tons
Copper in ore mined, %	1.592
Waste removed	5 400 221 tons
Ore treated	7 785 875 tons
Copper in ore treated	1.593% = 31.86 lb per ton
Aver recovery	90.36% = 28.789 lb per ton
Aver working force	5 987 men
Copper produced	219 516 420 lb
" " per man per calendar day	100 lb
Ore mined per man per calendar day	3.56 ton

Example 14. Champion Copper Co, Mich (1/2 owned by Copper Range Co)

Table 44. Receipts and Expenditures from Organization to Dec 31, 1925

Capital stock $2 500 000.00

Received from sale of copper:

Yr	Quantity	Amount	Yr	Quantity	Amount
1902	4 165 784 lb at 11.82¢	$492 553.36	1914	15 807 206 lb at 13.38¢	$2 114 468.18
1903	10 565 147 " 13.37¢	1 412 711.43	1915	33 407 599 " 17.40¢	5 814 279.21
1904	12 212 954 " 13.02¢	1 591 109.71	1916	33 601 136 " 25.28¢	8 494 367.18
1905	15 707 426 " 15.56¢	2 444 554.91	1917	27 550 343 " 28.735¢	7 916 569.27
1906	16 954 986 " 19.06¢	3 231 328.71	1918	21 748 514 " 24.757¢	5 384 208.35
1907	16 489 436 " 17.28¢	2 848 838.41	1919	19 886 917 " 18.668¢	3 712 489.67
1908	17 786 763 " 13.39¢	2 381 137.30	1920	13 610 324 " 17.173¢	2 337 318.60
1909	18 005 071 " 13.00¢	2 339 361.62	1921	20 719 307 " 13.0427¢	2 761 792.18
1910	19 224 124 " 12.74¢	2 447 844.73	1922	19 583 806 " 14.3591¢	2 811 069.22
1911	15 639 426 " 12.54¢	1 960 758.13	1923	18 412 630 " 14.86¢	2 736 047.34
1912	17 225 508 " 16.16¢	2 782 457.60	1924	20 061 630 " 13.76¢	2 760 921.91
1913	12 080 594 " 14.89¢	1 798 984.15	1925	17 957 605 " 14 258¢	2 560 419.15

Total production .... 438 404 236 lb copper $77 635 590.32
Aver price received .... 17.708¢

Expenditures:

Real estate (Champion location)	$1 025 000.00	
" " (lands since purchased)	14 095.28	
	$1 039 095.28	
Net cost of construction and equipment, mining operations, smelting and marketing copper, taxes and sundries	45 523 958.06	
Total expenditure		46 563 053.35
Net balance of receipts		$31 072 536.97

Dividends paid:

1903.. $300 000.00	1910.. 900 000 00	1918.. 1 975 720.00
1904.. 200 000.00	1911.. 500 000.00	1919.. 1 280 000.00
1905.. 1 000 000.00	1912.. 1 100 000.00	1920.. 600 000.00
1906.. 1 200 000.00	1913.. 900 000.00	1922.. 600 000.00
1907.. 1 000 000.00	1915.. 3 100 000.00	1923.. 700 000.00
1908.. 500 000.00	1916.. 6 014 540.96	1924.. 780 000.00
1909.. 800 000.00	1917.. 4 480 000.00	1925.. 1 140 000.00

29 070 260.96

Excess of net receipts over dividends $2 002 276.01

Total profit per lb	7.088¢	Percent of total receipts distributed as dividends 37.5%
Dividends paid per lb	6.631¢	
Aver cost per lb	10.620¢	

Table 45. Summary of Operations in 1925

17 957 605 lb copper produced and sold at 14.258¢ per lb		$2 560 419.15
Interest credits		43 778.90
		$2 604 198.05
Expenditures:		
Running expenses at mine and property taxes	$1 803 283.16	
Smelting, freight, cost of marketing copper and gen expense	303 150.46	
		$2 106 433.62
Profit on operations		$497 764.43
Less construction charges for 1925		11 499.79
Excess of receipts over expenditure		$486 264.64
Percentage of total receipts remaining		18%

Table 46. General Data, Copper Range Co, 1915-1925

Rock stamped, 383 746 tons. Refined copper produced, 14 298 916 lb. Refined copper per ton stamped, 37.26 lb.

Average yield of copper per ton of rock from all mines:

1915	32.50 lb	1918	34.42 lb	1921	37.52 lb	1924	40.80 lb
1916	33.07 "	1919	36.14 "	1922	36.32 "	1925	38.38 "
1917	32.97 "	1920	38.70 "	1923	39.74 "		

Example 15. Miami Copper Co, Miami, Ariz

Table 47. Summary of Operations, from Annual Reports

Note.—For full details of 1912 costs, see 1st Edn of this book, pp 1339–1344

	1912	1924	1925	3 mo ended Dec 31, 1925
Mining, per ton ore	$1.203	$1.129	$0.853	$0.437
Milling, per ton ore	0.659	0.637	0.572	0.528
Gen mine expense per ton ore	0.180	0.270	0.195	0.123
Total cost of mining and milling per ton ore treated	$2.042	$2.036	$1.620	$1.088
Cost per lb refined copper:				
Mining and milling and gen exp	$0.0647	$0.0823	$0.0912	$0.0777
Freight on conc to smelter	0.0068	0.0068	0.0094	....
Smelting, refining and frt on blister copper	0.0232	0.0172	0.0191	....
Selling, frt, disc't and comm	0.0013	0.0028	0.0026	....
New York expense	0.0012	0.0034	0.0039	....
Total operating expense	$0.0973	$0.1126	$0.1262	....
Less miscel credits	0.0015	0.0003	0.0006	....
Net operating charges	$0.0958	$0.1123	$0.1256	....
Deprec charged	0.0052	0.0070	0.0082	....
Total cost, including deprec	$0.1010	$0.1193	$0.1338	$0.1203 (est)
Ore milled, tons	1 040 744	2 444 079	2 919 600	893 658
Assay, % Cu	2.393	1.624	1.287	0.972
Concentrate produced, tons	46 683	80 516	97 770	29 457
Copper in conc, lb	34 560 665	63 658 471	54 577 076	13 171 610
Copper extracted per ton ore, lb	33.21	26.046	18.693	14.739
Mill extraction, %	69.39	80.17	72.60	75.79
Net smelter returns, refined Cu	32 832 609	60 475 547	51 851 274	....

Example 15. Miami Copper Co. (*Concluded*)

Table 48. Summary of Operating Costs per Ton of Ore

	1912	1924	1925	3 mo ended Dec 31, 1925
Mining costs:				
Stoping	$0.60636	$0.51873	$0.36433	$0.16214
Develop't charge per ton mined, to extinguish total cost of develop't	0.31000	0.31000	0.24572	0.10000
Hauling	0.09909	0 10623	0.07862	0.05811
Hoisting	0.06327	0.04332	0.04024	0.03978
Pumping	0.00852	0.00346	0.00271	0.00169
General underground	0.04307	0.06165	0.07436	0.04147
Handling men and supplies	0.01586	....	.. ..	....
Underground lighting	0.00439	0.00707	0.00532	0.00251
Engineering and sampling	0.02495	0.02125	0.01781	0.01252
Mine surface expense	0.02776	0.05777	0.02928	0 02329
Total mine operating costs	$1.20327	$1 12948	$0.85839	$0.44151
Milling costs:				
Coarse crushing	See note above Table 47	See note above Table 47	$0.08848	$0.10217
Fine grinding			0.22565	0.18929
Flotation			0.12997	0.10677
Concentrate retreatment			0.00401	0.01239
Concentrate disposal			0.01335	0.01455
Tailing disposal			0.02215	0.03244
Water supply			0.03397	0.02604
General mill expense			0 05404	0.04425
Total milling costs	$0.659	.... .	$0 57162	$0.52790

	1912	1924	1925
General mine expenses:			
Taxes	See note above Table 47	$373 740	$294 008
General expense		147 575	151 433
Mine office		23 364	24 222
General surface		41 854	26 298
General accident		5 352	1 353
Research		188	32
Engineers, Mgr and Supt		68 500	71 500
Total general expenses	....	$660 573	$568 846
Per pound copper produced	Per ton ore $0.180	$0.01092	$0.01097

Construction costs	1924	1925	Construction costs	1924	1925
Mining plant	$31 334	$60 023	Domestic plant	$5 405	$8 908
Concentrator	61 040	416 871	Construction in progress	5 706	120 551
Power plant	18 098	69 110	Total construction expenses	$130 824	$707 342
Water supply	8 112	27 912			
Industrial plant	1 128	3 967			

Total construction for 2 years, $838 166. Deprec charged for 2 years, $849 138.

NOTE.—Mill capac increased from 6 800 to 10 000 tons daily during 1924 and 1925.

Example 16. Nevada Consolidated Copper Co, Ely, Nev

Table 49. Summary of Operating Results, from Annual Reports

	1912	1924	1925
Total tons ore treated	2 887 731	3 418 900	3 832 504
Refined copper, lb	63 063 261	70 237 050	73 653 886
Net yield refined copper per ton ore, lb	21.90	20.86	19.22
Aver copper assay of concentrate, %	1.69	1.20	1.18
Mill extraction, %	68.25	87.09	90.17
Ratio of ore to concentrate	9 09	9 76	.....
Aver copper in concentrate, %	10.49	9.61	.....

NOTE.—Included in 1925 figures are 598 114 tons of material averaging 0.684% copper, removed in stripping, which yielded on concentration 9.63 lb copper per ton and 20¢ gold. Remainder of ore averaged 1.21% copper, of which mill recovery was 92.24%, grade of concentrate 10.70%, ratio of concentration 9.59, and net yield refined copper per ton ore treated 22.32 lb.

Table 50. Nevada Consol Operating Costs

	1912		1924		1925	
	Total	¢ per lb Cu	Total	¢ per lb Cu	Total	¢ per lb Cu
Mining, including proportion of stripping cost	$1 436 366	2.28	$2 763 561	3.93	$2 756 461	3.74
Ore delivery	759 129	1.20	528 986	0.75	564 513	0.77
Milling	1 414 506	2.24	2 058 942	2 93	2 030 263	2.76
Smelting	1 480 089	2.36	1 625 858	2 32	1 684 808	2.29
Rent of Steptoe plant, including deprec	1 204 630	1.91	....	.....	....	.....
Freight and refining	918 151	1.45	1 063 417	1.51	1 106 765	1.50
Selling commissions	103 359	0.16	88 168	0 13	91 859	0 12
Total operating costs	$7 316 230	11.60	$8 128 932	11.57	$8 234 670	11.18
Less receipts for Au and Ag	521 277	0.83	729 333	1 04	798 371	1 08
Cost after crediting Au and Ag	$6 794 953	10.77	$7 399 599	10.53	$7 436 299	10.10
Less other income:						
Dividends on investments	1 459 112	2.44	450 000	0.69	300 000	0.62
Other miscel income	82 808		35 338		153 119	
Net cost of copper on market	$5 253 033	8.33	$6 914 261	9.84	$6 983 180	9.48
Capital expenditure	620 714	0.98	1 394 622	1.99	619 394	0.85
Deprec charged	......	.....	$601 719	0.86	$671 450	0.91

Example 17. Union Minière du Haut Katanga, Africa

Income statement for 1924 shows a profit after all credits and charges, including depreciation, but before interest and depletion, of 93 994 010.40 francs, from a production of 188 370 798 lbs of copper, or 0.4459 franc per lb. With the franc at the value in March, 1926 of 3.91¢, this is equivalent to 1.743¢ per lb.

Table 51. Operating Results for 1925

Total ore mined (excluding fluxes)	1 383 453 metric tons
Copper produced	90 144 " "
Apparent yield, 6.49%, or, 129 8 lbs per short ton (2 000 lb)	129.8 lbs
Total profit after all credits and charges, including deprec, but before interest and depletion	130 521 117.59 francs
Profit as above per lb copper	0.6571 "
With the franc at 3.91 cents, this equals	2.569¢ per lb
Average *E & M Jour-Press* quotation, 1925	14.042¢ " "
Indicated cost per lb after credits and charges as above	11.473¢ " "

Table 52. Union Minière Operations for 1937

Total ore mined (excluding fluxes)	1 628 000 metric tons
Copper produced	150 588 " " or 332 037 139 lb
Apparent yield of copper	9.25%
Total profit after all credits and charges including deprec, but before interest and depletion	420 982 606 Belgian francs
Profit, as above, per lb of copper	1.271 " "
Profit with Belgian franc at 3.375¢ U S cur	4.29¢ per lb
Probable aver price realized (see Note 2 below Example 20)	12.441¢ " "
"Indicated cost"	8.151¢ " "

Example 18. Noranda Mines, Ltd, Hudson Bay Mining & Smelting Co

Table 53. Operations for 1937

Ore mined, tons			3 667 920
Metals recovered, lb:			
Copper	138 160 353	Tellurium	12 850
Zinc	68 972 224		
Cadmium	308 776	Gold, oz	410 220
Selenium	89 733	Silver, oz	2 326 814

Above figures do not include metals recovered from custom ores purchased.

Profits after all charges including deprec	$16 765 290
Applying these total profits to copper production gives	12.135¢ per lb of copper
Price received (see Note 2 below Example 20)	12.400¢ " "
"Indicated cost of copper"	0.265¢ " "

Note: These figures show the great advantage Canadian copper producers have over competitors, because of the by-product metals produced. International Nickel Co, showing similar results, is the largest copper producer in Canada, with Noranda and Hudson Bay second and third respectively in volume of production.

Example 19. Roan Antelope Copper Mines, Ltd, and Mufulira Copper Mines, Ltd, Rhodesia

Table 54. Operations for Year Ending June 30, 1938

Exchange £1 = $4.76	Mined, short ton, and costs	Milled, short ton, and costs	Total cost per ton	Blister copper produced	
				Short ton	Cost per ton
Roan Antelope	3 126 100 at 92.758¢	3 126 100 at 28.56¢	$1.213	84 280	$7.635
Mufulira	1 598 886 at $1.518	1 585 000 at 58.07¢	$2.099	58 101	$9.93

Net profit per lb copper, after all charges, including deprec: Roan Antelope, 4.448¢; Mufulira, 3.966¢.

Example 20. Anaconda Copper Mining Co, Kennecott Copper Corp and Phelps Dodge Corp, 1937

Copper production 2 074 974 125 lb. Net profit, after crediting all profits on other operations to electrolytic copper production: $93 951 029 = 4.528¢ per lb.

Summary of Copper Production, Costs and Profits in 1937

	Production, lb copper	Profit before depletion		"Indicated cost," cts
		Total	Cts per lb	
Canada	138 160 353	$ 16 765 290	12.135	0.265
United States and South America	2 074 974 125	93 951 029	4.528	7.913
Africa, Rhodesia	407 778 560	18 707 335	4 587	6.677
Africa, Belgian Congo	332,037 139	14 208 176	4.290	8.151
Totals and averages	2 952 950 177	$143 631 830	4.864	7.411

Note 1. In calendar year 1937 the above groups produced 58.86% of total World production.

Note 2. The large companies considered here have not reported, in recent years, the aver price received for copper. The aver *Eng & Min Jour* quotation in N Y for 1937 was 13.167¢ per lb. However, certain smaller companies, not considered here, do publish their figures and these indicate a substantially lower price, around 12.400¢ per lb. In arriving at the "indicated cost" by difference between price received and profits realised, this lower price has been taken, except in the case of the Rhodesian coppers, whose gross income represents their receipts from copper fairly closely, because receipts from other sources are insignificant. In the case of Rhokana, a small part of gross income came from sales of cobalt. This would tend to raise the stated aver price of copper and, therefore, the "indicated cost," by perhaps 1/4¢ per lb for that group.

Note 3. The above figures represent results for year ended Dec 31, 1937, with exception of the Rhodesian companies. In the latter case, the results are for the 12 months ending June 30, 1937.

Example 21. Iron Mining Costs

Efforts made to obtain representative current costs of producing iron ore have been without success, for the reasons given in the Introduction to this Section. In the absence of recent costs, it has been decided to retain the following Table. The prewar figures therein may be of some value, as they can be converted roughly into present-day costs by multiplying them by a factor of, say, 60 to 75%, as may seem appropriate in the judgment of the instructed reader.

It may be added that iron mining has become so largely a closely-controlled specialty, that mining engineers in general have less interest in the cost of producing iron than they have in the cost of the non-ferrous metals. Details of the technique of iron-mining methods, in deposits of different kinds are given in Sec 10.

Example 21. Iron Mining in Michigan

Table 55. A Five-year Summary from Records of All Iron Mines Operating in Michigan, 1906 to 1910, Inclusive, Showing Average Cost and Average Value of Ore per Ton, F O B, Cleveland, Ohio

	Gogebic Range, Gogebic County	Iron River District, Iron County	Crystal Falls District, Iron County	Old Menominee Range, Dickinson County	Western Marquette Range, Baraga County	Marquette Hard Ore Mines, Marquette County	Marquette Soft Ore Mines, Marquette County	Swanzy District, Marquette County	Sundry low-grade mines in various parts of state
Mining	$1 35	$1.30	$0 97	$1 13	$1 28	$1 67	$1.33	$1.58	$0 65
Exploration and development	0.10	0 64	0.04	0.09	0.204	0.10	0.08	0.07	0.08
Construction, shafts, mach'y etc	0 16	0 25	0 23	0.10	0.01	0 19	0.11	0.08	0.15
General expense and admin.	0.099	0.09	0 064	0 097	0 065	0.064	0.12	0 08	0.045
Taxes	0.063	0.025	0.02	0.08	0 01	0.12	0.07	0.05	0.015
Total expenditure at mine	$1.772	$2.305	$1 324	$1.497	$1.569	$2.144	$1.71	$1.86	$0.94
Royalties	0.39	0 21	0 24	0.24	0.181	0.14	0.16	0.26	0.12
Rail freights	0.40	0.40	0 40	0 40	0.35	0 40	0.32	0.32	0.40
Lake freights	0.70	0 54	0 57	0.60	0 61	0.60	0.65	0.65	0.60
Commissions	0 045	0 067	0 09	0 07	0 06	0.02	0.01	......	0.10
Total expenditure, f o b, Cleveland	$3.30	$3.52	$2.62	$2.80	$2.77	$3.30	$2.85	$3.09	$2.16
Average loss per ton	......	0.22	......	......	0.02	......	......	......	0.22
Average profit per ton	1 04	......	0.83	0.84	......	1.10	0.85	0.33	......
Average value per ton, f o b, Cleveland	$4.34	$3.30	$3.45	$3.64	$2 75	$4.40	$3.70	$3.42	$1.94

Example 22. Coal Mining Costs

Examples of these costs are limited, few data being given out by the companies for publication. Tables 56 to 66 present some of the main facts.

Table 56. Data on Anthracite Coal Mining in 1913 (Philadelphia & Reading Coal and Iron Co)

Figures are from statements in annual reports, but do not represent all expenditures, as some relate to buying and selling of both anthracite and bituminous coal. Deducting receipts from the sales and amounts for purchase of bituminous coal, it appears that the profit was between 9¢ and 10¢ per long ton of anthracite during year ended June 30, 1913. Company apparently paid about $2.50 per ton for anthracite purchased.

Costs per ton 2 240 lb, year ended June 30

	1912	1913
Tons mined	8 671 013	11 089 742
Mining coal and repairs	$2.12	$2.09
Improvements at collieries, etc	0.097	0.11
Royalty of leased collieries	0.053	0.055
Improvements and repairs of houses	0.0038	0.013
Taxes on coal lands and improvements	0.0680	0.047
Damages on account of coal dirt	0.0007	0.0003
Fixed charges	0.0099	0.0085
Additional wages, account of sliding scale	....	0.0097
Total per ton of 2 240 lb	$2.3524	$2.3335
" " " " 2 000 "	2.10	2.08

During previous years, company has charged about 5¢ per long ton for depletion of coal lands.

Table 57. Bituminous Coal Mining (Pittsburgh Coal Co, from Sept 1, 1899 to Dec 31, 1913)

		Per ton, cts	
Tons of coal and coke produced (2 000 lb)	241 289 912		
Earnings after deducting expenses for operation, taxes, losses, interest on bonds of subsidiaries, and adjustments			24.80
Less: Reserve for depletion of coal lands		4.29	
Depreciation of plant and machinery		4.40	
Interest on bonds		4.65	
Total capital charges			13.34
Apparent profit per ton, including all income			11.46
Paid in dividends			7.15
Record for year ended Dec 31, 1913		**Approx cost per ton**	
Tons of coal produced	24 501 804		
Tons of coke produced	205 400		
Total tons produced (2 000 lb)	24 707 204		
Operating charges: maintenance, repairs, administration, selling, and general expenses		$1.12	
Taxes		0.03	
Interest on bonds, etc, of subsidiary companies		0.031	
Commercial discounts and interest		0.008	
Losses, insurance, and pensions		0.021	
Total operating cost		$1.21	
Total capital charges		0.149	
Net earnings for year			$0.111

Table 58. Average Coal Tonnage Mined per Day per Man During 1912

(Compiled from U S Geol Surv Rep, "The Production of Coal in 1912")

State	Number hr per day	Aver tonnage per man per day	Machine-mined coal †	Aver price at mines per ton 2000 lb
Alabama	8 and 9	2.91	23.2	$1.29
Arkansas	8	2.95	3.7	1.71
California	....	1.14	1.8	‡2.33
Colorado	8 and 10	3.72	23.2	1.49
Illinois	8	3.95	44.9	1.17
Indiana	8	3.88	54.7	1.14
Iowa	8	2.37	1.3	1.80
Kansas	8	2.97	1.1	1.62
Kentucky	8, 9 and 10	3.38	66.4	1.02
Maryland	10	3.11	2.5	1.18
Michigan	8	2.11	52.7	1.99
Missouri	8	2.17	20.7	1.76
Montana	8	4.03	32.3	1.82
New Mexico	10	3.28	8.1	1.42
North Dakota	....	3.46	33.8	1.53
Ohio	8	3.77	87.0	1.07
Oklahoma	8	2.4	7.1	2.14
Pennsylvania:				
Anthracite	9	2.1	....	2.11
Bituminous	8*	3.89	50.8	1.05
Tennessee	9 and 10	2.68	18.6	1.14
Texas	....	1.84	4.8	1.67
Utah	8	3.18	3.8	1.67
Virginia	10	3.6	40.8	0.96
Washington	8	2.69	7.7	2.39
West Virginia	8, 9 and 10	3.68	52.3	0.94
Wyoming	8	3.85	32.1	1.58

* Represents 60% of employes, 40% about equally divided between 9 and 10 hr.
† Average production per year per machine, 13 763 short tons.
‡ Includes Alaska.

Abstracts from Bull No 4, "Coal Mining Practice in Dist No 7," May, 1914, Illinois Coal Mining Investigation: District includes mines operating on bed No 6, west of Duquoin anticline, and covers Bond, Clinton, Fayette, Macoupin, Madison, Marion, Montgomery, Moultrie, Randolph, St Clair, Shelby, Washington, and a portion of Perry, Christian, and Sangamon Counties. Bed No 6 varies from 2.5 to 14 ft thick, average 7 ft. There are 6 slope and drift mines, but shafts are generally used, as bed lies at considerable depth; maximum 707 ft. General system of mining is unmodified double-entry room-and-pillar, but some are worked by panel system. Average recovery of coal, 55%. Output for year ended June 30, 1912, 22 454 672 short tons, or 39.1% of production of state. Mines operating, 196; shipping, 150; local, 46. Tons mined by machines, 13 558 530. Tons mined per 8-hr shift: 140 by chain, 71 by puncher machines. Average days operated by mines, 158. Days work performed, 4 399 826. Employes: surface, 2 354; underground, 25 493; total, 27 847. Aver number underground men for each surface man, 10.8. Tons mined per day per man, 5.1; per surface man, 60.5; per underground men, 5.6; per worker in shipping mines only, 7.3 tons. Tons mined for each man injured, 78 788. In one mine 1 keg powder breaks 30 tons coal in shooting from solid, and 90 tons after puncher undercutting. Tons broken per keg of powder, from 17 to 175, depending on method. Cost of powder, from 1¢ to 10.3¢ per ton coal. Total powder used during year, 426 353 kegs. Timber props range from 1.3 to 7.2 per 100 sq ft of roof, costing from 10.5¢ to 70.3¢ per 100 sq ft of roof, or 0.5¢ to 3¢ per ton of coal. Mines using locomotive haulage, 78; cable, 3; mules, 101; hand power, 14. At one mine a gasolene locomotive hauls to shaft bottom 1 300 tons of coal per shift (consuming 18 gal gasolene) at cost of 0.17¢ per ton. Another hauls 900 tons per shift (consuming 15 gal gasolene) at cost of 0.2¢ per ton; average trip, 25 loads of 4 000 lb each, including cars. Ton mileage in coal of all locomotives ranges from 311 to 2 203 per shift. Cost of keeping mules, from 75¢ to $1 per day. In one mine, on a 2% grade in favor of loads, 2 mules, weight 1 300 lb each, made 75 loaded trips of 700 ft, with 4 cars, capacity 3 500 lb, weighing 1 000 lb empty, the ton mileage for each mule being about 54.67.

Table 59. Cost of Hydraulic Filling at Collieries (prewar)

(U S Bureau of Mines, Bull No 60, to which reference is made for details)

Based on plants of at least 400 cu yd daily capac. Costs in ct per cu yd		Culm	Material used for filling: Culm mixed with crushed breaker and boiler refuse	Local hydraulic sand, loam, gravel, clay, etc	Local crushed sand, loam, gravel, clay, etc	Material brought from distance in returning empty cars
Surface transport (mechanical)	Gravity	1 to 1.5	4 to 5.5	8 to 14	30 to 40	Cost of material, loading freight, unloading, preparing and surface transport
	Scraper or conveyer	4.5 to 6	7.5 to 10	11.5 to 18.5	....	
	Pump	2.25 to 4	5.25 to 8	9.25 to 16.5	....	
Intermediate transport	Shaft or slope	0.25 to 0.5	0.75 to 1.5	0.9 to 1.75	1 to 1.5	1 to 1.5
	Bore hole	0.1 to 0.2	0.2 to 0.5	0.3 to 0.7	0.4 to 0.8	0.4 to 0.8
Underground transport	Trough	0.1 to 0.3	0.1 to 0.3	0.1 to 0.3	0.1 to 0.3	add 25%
	Pipe	0.3 to 0.5	0.3 to 0.5	0.3 to 0.5	0.3 to 0.5	add 25%
Distribution	In flat workings	7.25 to 9	7.25 to 9	7.25 to 9	7.25 to 9	add 25%
	In inclined workings	12.25 to 14.5	12.25 to 14 5	12.25 to 14.5	12.25 to 14.5	add 25%
	In steep workings	14 to 27.5	14 to 27.5	14 to 27.5	14 to 27.5	add 25%
Drainage	Incidental	0.4 to 0.7	0.4 to 0.7	0.4 to 0.7	0.4 to 0.7	add 25%
	Special	1.25 to 8	1.25 to 8	1.25 to 8	1.25 to 8	add 25%
Total	Minimum	9.3¢	9.3¢	9.3¢	9.3¢	11.62¢
	Maximum	56.25¢	56.25¢	56.25¢	56.25¢	70 31¢

Table 60. Anthracite Production: Costs per Ton, Sales Realizations, and Margins from Culm-bank Coal of 8 Companies, Jan, 1917-Mch, 1923

Note.—Data in Tables 60–63 are from 1924 report of Federal Coal Comm, pp 867, 881 and 2019

Period	Production, gross tons	Aver monthly production, gross tons	Costs				Sales realization	Margin
			Labor	Supplies	General expense	Total f o b mine		
Jan–Apl, 1917	853 021	213 255	$0.34	$0.15	$0.28	$0.77	$1.82	$1.05
May–Aug, 1917	1 450 684	362 671	0.41	0.21	0.26	0.88	2.38	1.50
Sept–Nov, 1917	1 164 718	388 239	0.42	0.25	0.29	0.96	2.60	1.64
Dec, 1917–Oct, 1918	5 102 326	463 848	0.64	0.28	0.26	1.18	2.97	1.79
Nov–Dec, 1918	1 018 050	509 025	0.89	0.36	0.32	1.57	3.28	1.71
Jan–Mar, 1919	770 352	256 784	1.09	0.41	0.50	2.00	3.31	1.31
Apl–June, 1919	609 729	203 243	1.02	0.35	0.58	1.95	3.41	1.46
July–Sept, 1919	837 730	279 243	0.88	0.29	0.60	1.77	3.37	1.60
Oct–Dec, 1919	994 013	331 338	0.96	0.27	0.54	1.77	3.41	1.64
Jan–Mar, 1920	926 422	308 807	1.08	0.37	0.49	1.94	3.30	1.36
Apl–Sept, 1920	2 484 593	416 099	1.03	0.37	0.51	1.91	3.74	1.83
Oct–Dec, 1920	1 641 545	547 182	1.03	0.39	0.52	1.94	3.97	2.03
Jan–Mch, 1921	1 021 822	340 607	1.17	0.43	0.59	2.19	4.25	2.06
Apl–June, 1921	531 653	177 218	1.23	0.31	0.68	2.22	3.76	1.54
July-Sept, 1921	304 751	101 584	1.19	0.36	0.77	2.32	3.30	0.98
Oct–Dec, 1921	335 001	111 667	1.01	0.41	0.88	2.30	2.85	0.55
Jan–Mch, 1922	388 284	129 428	1.04	0.31	0.56	1.91	2.78	0.87
Apl–Sept, 1922	322 031	53 672	1.36	0.42	0.97	2.75	3.52	0.77
Oct–Dec, 1922	966 211	322 070	1.12	0.33	0.62	2.07	3.82	1.75
Jan–Mch, 1923	986 175	328 725	1.17	0.39	0.63	2.19	3.78	1.59
Jan–Nov, 1917 (11 mos)	3 468 423	315 311	0.40	0.21	0.27	0.88	2.32	1.44
Dec, 1917–Dec, 1918 (13 mos)	6 120 376	470 798	0.69	0.29	0.27	1.25	3.01	1.76
Year 1919	3 211 824	267 652	0.99	0.32	0.55	1.86	3.38	1.52
Year 1920	5 052 560	421 048	1.04	0.38	0.51	1.93	3.73	1.80
Year 1921	2 193 227	182 769	1.17	0.38	0.68	2.23	3.81	1.58
Year 1922	1 676 526	139 627	1.15	0.35	0.67	2.17	3.51	1.34
Jan–Mch, 1923	986 175	328 725	1.17	0.39	0.63	2.19	3.78	1.59

Table 61. Anthracite Production: Costs per Ton, Sales Realizations, and Margins from Culm-bank Coal of 14 Identical Companies, Jan, 1919-Mch, 1923

Period	Production, gross tons	Aver monthly production, gross tons	Costs				Sales realization	Margin
			Labor	Supplies	General expense	Total f o b mine		
Jan-Mch, 1919	829 649	276 549	$1.10	$0.42	$0.54	$2.06	$3.30	$1.24
Apl-June, 1919	718 351	229 450	1.00	0.34	0.64	1.98	3.26	1.28
July–Sept, 1919	1 009 338	336 446	0.85	0.28	0.65	1.78	3.28	1.50
Oct–Dec, 1919	1 149 987	383 329	0.95	0.28	0.60	1.83	3.39	1.56
Jan–Mch, 1920	1 065 225	355 075	1.06	0.38	0.61	2.05	3.33	1.28
Apl–Sept, 1920	2 814 071	469 012	1.01	0.37	0.63	2.01	3.79	1.78
Oct–Dec, 1920	1 827 352	609 117	1.03	0.40	0.62	2.05	3.05	2.00
Jan–Mar, 1921	1 099 992	366 664	1.21	0.44	0.65	2.30	4.17	1.87
Apl–June, 1921	562 167	187 389	1.31	0.34	0.71	2.36	3.73	1.37
July–Sept, 1921	336 641	112 214	1.29	0.39	0.80	2.48	3.30	0.82
Oct–Dec, 1921	429 717	143 239	1.08	0.47	0.98	2.53	2.91	0.38
Jan–Mch, 1922	525 779	175 260	1.03	0.33	0.76	2.12	2.76	0.64
Sept, 1922	365 927	365 927	1.10	0.33	0.70	2.13	3.58	1.45
Oct–Dec, 1922	1 212 897	404 299	1.09	0.35	0.82	2.26	4.01	1.75
Jan–Mch, 1923	1 196 036	398 679	1.14	0.40	0.70	2.24	3.87	1.63
Year 1919	3 707 325	308 944	0.96	0.33	0.61	1.90	3.32	1.42
Year 1920	5 706 648	475 554	1.02	0.39	0.62	2.03	3.79	1.76
Year 1921	2 428 517	202 376	1.22	0.42	0.74	2.38	3.75	1.37
Year 1922	2 130 838	177 570	1.12	0.37	0.88	2.37	3.61	1.24
Jan–Mch, 1923	1 196 036	398 679	1.14	0.40	0.70	2.24	3.87	1.63

Table 62. Anthracite Costs of 48 Companies, 1919-1922, and Jan-Mch, 1923

Year and type of company	No of companies	Gross tons produced	Labor	Supplies: Operating, repairs, maintenance	Supplies: Light and power purchased	Supplies: Total supplies	General expense: Royalty	General expense: Depletion	General expense: Amortization, leasehold and development	General expense: Depreciation	General expense: Taxes	General expense: Insurance compensation	General expense: Officers' salaries and expenses	General expense: Miscellaneous	General expense: Total general expense	Total f o b mine cost
Year 1919:																
R R coal companies	10	47 800 045	$3.32	$0.75	$0.02	$0.77	$0.09	$0.17	....	$0.07	$0.12	$0.05	$0.05	$0.07	$0.63	$4.72
Larger independents	20	9 632 739	3 58	0.69	0 05	0.74	0.30	0 05	$0.01	0.21	0.09	0.10	0.11	0.06	0.93	5.25
Smaller independents	18	1 936 805	3.53	0 64	0.10	0.74	0.27	0.11	0.02	0.20	0.05	0.09	0.10	0.10	0.94	5.21
Total	48	59 369 589	3 37	0 74	0 02	0.76	0.13	0 16	0.00	0.10	0.13	0.06	0.06	0.05	0.69	4.82
Year 1920:																
R R coal companies	10	45 073 903	3 86	0 84	0 02	0 86	0.10	0.19	....	0.08	0.19	0.07	0.07	0.09	0.79	5.51
Larger independents	20	9 436 095	4.15	0.89	0.06	0.95	0 35	0.05	0 03	0.20	0.11	0.09	0.10	0.11	1.04	6.14
Smaller independents	18	1 993 179	4.05	0.88	0.12	1.00	0.38	0.11	0.02	0.19	0.06	0.11	0.11	0.11	1.09	6.14
Total	48	56 503 177	3.91	0 85	0 03	0.88	0.15	0.16	0.01	0 10	0.17	0.07	0.07	0.11	0.84	5.63
Year 1921:																
R R coal companies	10	51 666 453	3.97	0 86	0.02	0.88	0.10	0.20	....	0.06	0.24	0.07	0.07	0.07	0.81	5.66
Larger independents	20	9 817 762	4.37	0.84	0 08	0.92	0 35	0.05	0.04	0.23	0.17	0.08	0.11	0.10	1.13	6.42
Smaller independents	18	1 863 136	4.58	0.86	0.15	1.01	0.31	0.10	0.02	0.20	0.10	0.11	0.10	0.14	1.08	6.67
Total	48	63 347 351	4.05	0.86	0 03	0 89	0.14	0.17	0 01	0.10	0.22	0.07	0.07	0.08	0.87	5.81
Year 1922:																
R R coal companies	10	27 837 404	4.31	0.84	0.03	0.87	0.10	0.21	....	0 10	0.48	0.06	0.12	0.09	1.16	6.34
Larger independents	20	5 867 302	4.49	0 75	0.15	0.90	0.32	0.05	0.03	0.35	0.33	0 09	0.13	0.30	1.60	7.00
Smaller independents	18	1 359 037	4.26	0.79	0.19	0.98	0.35	0.12	0.01	0 30	0.19	0.10	0.16	0.19	1.42	6.66
Total	48	35 063 743	4.34	0.82	0.06	0.88	0.15	0.18	0.01	0.15	0.44	0.07	0.13	0.11	1.24	6.46
Jan-Mch, 1923:																
R R coal companies	10	13 292 211	3 92	0 69	0 02	0.71	0 10	0.20	....	0.09	0.34	0.05	0.08	0.05	0.91	5.54
Larger independents	20	2 862 021	4.02	0.72	0.10	0.82	0.42	0 05	0 02	0.24	0.23	0.08	0.08	0.19	1.31	6.15
Smaller independents	18	761 111	3.76	0.81	0.15	0.96	0.51	0.10	0 01	0.16	0.13	0.10	0.08	0.18	1.24	5.96
Total	48	16 915 343	3.93	0.70	0 04	0.74	0.18	0.17	0.01	0.11	0.31	0.06	0.08	0.07	0 99	5.66

Table 63. Summary of Bituminous Costs per Ton of 1 180 Operators in 83 Fields, 1918, 1921, 1922

Region	1918 Costs: Labor	1918 Costs: Total f o b mine	1918 Sales realization	1918 Margin	1921 Costs: Labor	1921 Costs: Total f o b mine	1921 Sales realization	1921 Margin	1922 Costs: Labor	1922 Costs: Total f o b mine	1922 Sales realization	1922 Margin
Northern Appalachian	$1.38	$1.95	$2.62	$0.67	$1.84	$2.70	$2.89	$0.19	$1.74	$2.64	$2.98	$0.34
Southern Appalachian	1.44	2.06	2.71	0.65	1.94	2.96	3.11	0.15	1.49	2.28	2.65	0.37
Eastern interior	1.45	1.87	2.36	0.49	2.11	2.74	2.83	0.09	1.95	2.54	3.02	0.48
Western interior and southwestern	2.20	2.75	3.01	0.26	3.23	4.16	4.08	0.08*	3.03	3.94	3.88	0.06*
Great Plains, Rocky Mountains, and Pacific coast	1.63	2.18	2.67	0.49	2.27	3.21	3.51	0.30	2.02	2.88	3.30	0.42
United States, total	1 46	2.00	2.60	0.60	2.00	2.83	3.00	0.17	1.84	2.65	3.01	0.36

* Amount by which total f o b mine cost exceeded sales realization.

Table 64. Production of Bituminous Coal by Identical Operators, to which the Comparable Costs in Table 63 Apply

Region	No of fields	No of operators	Production tonnage: 1918	Production tonnage: 1921	Production tonnage: 1922
Northern Appalachian	36	653	216 058 613	149 808 342	128 864 267
Southern Appalachian	15	135	29 488 284	21 418 212	26 045 838
Eastern interior	11	199	77 164 615	58 887 604	52 587 293
Western interior and southwestern	11	114	18 559 876	10 866 800	8 566 111
Great Plains, Rocky Mountains, and Pacific coast	10	79	30 881 021	21 814 290	21 722 198
United States	83	1 180	372 152 409	262 795 248	237 785 699

Table 65. Labor, Supplies and Other Costs of Mines in Selected Coal Fields, 1934 *

District	Labor		Supplies		Other production costs	
	Aver for district	Range for component fields	Aver for district	Range for component fields	Aver for district	Range for component fields
Eastern Pennsylvania	$1.22	$1.13–1.52	$.301	$.266–.449	$.305	$.252–.381
Maryland-Upper Potomac	1.25	1.25–1.26	.235	.225–.248	.277	.238–.307
Western Pennsylvania	1.14	1.00–1.30	.241	.173–.292	.359	.224–.515
Ohio	1.09	1.05–1.22	.219	.209–.270	.260	.224–.307
Northern West Virginia	.93	.89–1.23	.187	.171–.260	.257	.208–.394
Southern Subdivision No 1	1.09	1.02–1.20	.260	.243–.281	.327	.300–.349
Southern Subdivision No 2	1.03	.78–1.21	.231	.196–.266	.290	.245–.352
Alabama	1.33	1.18–1.89	.366	.330–.575	.317	.288–.343

* Compiled by Industrial Research Dept, Univ of Pennsylvania, from Bituminous Coal Code Statistics, 1933–1935 (Division of Research and Planning, NRA)

Table 66. Costs per Ton of Coal Mined by Commercial Operations in Minimum Price Area No 1, 1934 *

Labor	Dollars per ton		Percentage of total
Daymen	0.3784		23.9
Mining	.5264		33.3
Yardage and deadwork	.0560		3.5
Mine supervisory and clerical	.0710		4.5
Total labor cost		1.0319	65.2
Supplies			
All supplies except power and fuel	.1760		11.1
Power purchased	.0760		4.3
Mine fuel	.0097		0.6
Total mine supplies		.2527	16.0
Other production costs			
Salaries and expenses	.0213		1.4
Taxes	.0229		1.5
Insurance	.0073		0.4
Depreciation	.0891		5.6
Royalties	.0523		3.3
Compensation insurance	.0486		3.1
Depletion	.0421		2.6
Company house expense (less income)	−.0023		−0.1
Mine office, code authority, and operators association dues	.0167		1.1
Unassigned credit	−.0018		−0.1
Total other production costs		.2962	18.8
Total production costs		1.5808	100.0

* Compiled by Industrial Research Dept, Univ of Pennsylvania, from Bituminous Coal Code Statistics, 1933–1935 (Division of Research and Planning, NRA). Minimum Price Area No 1 comprises coal fields of Iowa and all states east of Mississippi River, except those of Ala, Ga, and Southern Tenn.

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SECTION 22

WAGES AND WELFARE

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

WAGES

1. "DAY'S PAY" OR "COMPANY" MEN

Day's pay system, an agreed amount of wages for a definite number of hours' work, is open to the objection that it may entail an undue expense for supervision, to avoid waste of time and supplies. But, for many classes of mine employees, as engineers, mechanics, foremen, office force, and miscellaneous laborers, the day's pay or salary system is the only practicable one. For underground work, day's wages are also preferable to contract or bonus arrangements in those mines in which the minerals are irregularly distributed, requiring discrimination in the separate handling of ore and waste, or in which the mining conditions are difficult, demanding slow and painstaking operation; both of these objects are likely to be defeated by any system which aims to promote speed. Day's pay system is preferred at several large open-pit copper mines (Sec 10, Art 96) where work can be standardized and readily supervised. Among 80 (mainly underground) American mines described (2) to 1932, 13 operated on day's pay exclusively, and 20 adopted day's pay except for mucking or development.

Day's wages compared. Data in Table 1 are useful mainly as a basis of comparison for estimates of mining cost, since labor is almost always the preponderating factor in cost of operation (60 to 65% in most American mines, and with no obvious relation to the mining method). Wages for the same class of labor are naturally higher in remote districts, where supply of workmen is limited, living conditions are unfavorable, and food is expensive.

Fair Labor Standards Act of 1938 (effective Oct 24, 1938) requires every employer (with certain exceptions not related to mining) to pay each employee engaged in commerce. or in production of goods for commerce the following MINIMUM WAGES: during first year (*i e*, to Oct 24, 1939) not less than 25¢ per hr: during next 6 yr, not less than 30¢ per hr; after 7 yr (from Oct 24, 1938) not less than 40¢ per hr, unless a lower rate (not less than 30¢) is ordered by the Administrator after examination and report by one of his appointed Industry Committees. During first 7 yr, the Administrator may similarly order a minimum rate not exceeding 40¢, but such order will expire Oct 24, 1945. Employers (with some exceptions not related to mining) are forbidden to work an employee, who is engaged in commerce or in the production of goods for commerce, longer than the following MAXIMUM HOURS: 44 hr per week during first year (from Oct 24, 1938), 42 hr per week during second yr, 40 hr per week after the second year, unless such employee receives compensation for weekly over-time at 50% increase above his normal rate. Regulation as to payment for over-time does not apply in cases where collective bargains certified by N L R B provide a max of 1 000 hr during 26 consecutive weeks or 2 000 hr during 52 consecutive weeks; in such cases, over-time at 50% increase in rate starts at 12 hr in any workday or at 56 hr in any week. An order of the Administrator can be reviewed in a U S circuit court of appeals within 60 days; findings of fact by Administrator, when supported by substantial evidence, are conclusive. Court will not review an order unless it has previously been argued with the Administrator. Initiation of court procedure does not stay Administrator's order unless so stipulated by the court, upon filing of bond by complainant to cover wages in dispute. Administrator and his representatives have free access to plants and employees; all employers subject to the Act must keep such records, and submit such reports, as Administrator may prescribe.

"Commerce" in the above Act "means trade, commerce, transportation, transmission, or communication . . . from any State (or U S possession) to any place outside thereof." The Act does not apply to the Philippine Islands. The Wage and Hour Division has declined to pass on application of the Act to gold mines. Where it is customary to pay wages partly in form of board and lodging, these items are to be figured at actual cost to employer. Piece-work, contracts, etc, are permitted so long as such earnings are equivalent to those stipulated by the Act. The "week" is 7 consecutive working days, but may begin at any hour of any day; loss (or excess) of time in one week can not be applied to a succeeding week in computing over-time; each week is a unit. Waiver of over-time pay by consent of employee does not excuse non-compliance with the Act. Time spent in attending meetings, lectures, etc, sponsored by employer, and related to employee's work (such as first-aid, mine rescue, etc) was once ruled to be working time; wholly voluntary attendance at such meetings, after usual working hours, was later (July 10, 1939) ruled to be not time worked. For current information, rulings, etc, on this Act, address U S Dept of Labor, Wage and Hour Div, Washington, D C. To late 1939, no Industry Committee on mining had been appointed.

State wages and hours regulations. To late 1939, no mining State had a statute fixing MINIMUM WAGES (except for females and minors). As to MAX HOURS (same and other excep-

Table 1. Wages in American Metal Mines (From Bureau of Mines *Information Circulars*)

	Teck-Hughes, Kirkland Lake, Ont	Cortez, Nev	Fed M & S, Page, Idaho	Ground Hog, Vanadium, N M	Rosiclare, Ill	St Joe, Hughesville, Mont	New Idria, Calif	Central Eureka, Calif
Inf Circ No..	6 322	6 327	6 372	6 377	6 384	6 416	6 462	6 512
Date of scale..	1930	Dec, 1929	1929	Feb, 1930	1930	Mar, 1930	June, 1930	Nov, 1930
Shift boss....	$7.00	....	....	....	....	$6.25	$6.00	
Miner........	4.75	$5.75	$5.50	$4.25	$4.50	5.50	4.50	$4.50
Miner's helper	4.25	....	....	....	....	....	....	3.75
Nipper.......	4.25	....	....	....	....	5.00	....	4.50
Timberman...	4.75	5.75	6.00	4.75	4.75	5.50	4.50	4.50
Timber helper	....	....	5.25	4.00	4.00	5.00	....	3.75
Mucker......	4.25	5.25	5.00	3.35	4.00	5.00	4.00	3.50
Trammer.....	4.25	5.25	....	(a) 3.00	....	5.00	4.00	3.75
Pipeman.....	4.75	5.75	....	5.00	....	6.00	....	....
Trackman....	4.75	5.75	....	....	....	....	....	....
Cage or skip tender......	4.25	....	6.00	(a) 3.50	4.00	5.50	....	4.75
Motorman....	....	5.75	5.75	....	4.00	5.50	4.50	5.00
Motor helper..	....	5.25	....	....	....	5.00	4.50	....
Hoist engr....	....	6.00	....	5.00	4.00	6.00	4.50	5.50
Pumpman....	....	....	....	....	....	6.00	....	5.00
Blacksmith...	....	7.00	....	5.50	5.25	6.50	....	....
Carpenter....	....	....	....	7.00	4.00	5.50	....	....
Surface labor..	....	....	....	....	3.00	....	....	3.75

	McIntyre Porc, Schumacher, Ont	Eureka Std., Tintic, Utah	Park City Cons, Park City, Utah	Pilgrim, Chloride, Ariz	Mogollon Cons, N M	Summitville, Colo	Octave, Yavapai Co, Ariz	Judge, Park City, Utah
Inf Circ No...	6 741	6 851	6 880	6 945	6 987	6 990	6 991	7 003
Date of scale..	1933	Sep, 1933	1935	Jan 1, 1937	1937	Aug, 1937	Nov, 1937	Aug, 1937
Shift boss.....	$7.00	$6.25	$6.75	$6.00	$5.00	$5.00	$5.00	....
Miner........	4.80	4.75	5.25	5.00	....	4.25	4.50	$5.75
Miner's helper	4.24	....	....	....	2.64	3.50	....	....
Nipper.......	4.24	....	....	....	3.08	....	4.00	5.50
Timberman...	4.80	4.75	5.25	5.00	3.52	4.50	4.50	5.75
Timber helper	4.24	4.50	4.75	....	2.64	3.50	....	5.75
Mucker......	4.24	4.25	4.75	4.50	2.64	3.50	4.00	5.25
Trammer	4.24	4.25	4.75	4.50	2.64	3.50	....	....
Pipeman.....	4.80	4.75	5.25	....	3.52	....	....	5.75
Trackman....	4.80	4.75	....	....	....	....	....	....
Cage or skip tender.....	4.80	4.75	5.00	5.50	3.52	....	....	6.25
Motorman....	4.80	4.75	5.25	....	....	....	....	5.75
Motor helper..	4.24	....	4.75	....	....	....	....	....
Hoist engr....	5.44–4.80	5.50	5.50	5.50	3.52	....	4.50	6.75
Pumpman....	4.80	4.75	4.75	....	....	....	....	....
Blacksmith...	4.96	5.50	6.00	....	3.52	....	....	....
Carpenter....	....	5.50	....	....	....	....	....	....
Surface labor..	....	4.00	4.75	....	....	....	....	....

	Golden Anchor, Burgdorf, Idaho	Granada, Rouyn, Que	Cresson, Cripple Cr, Colo	Oatman Distr, Ariz	Hog Mt, Alexander City, Ala	Oceanic, Quicksilver, Cal	Gold Hill, Quartzburg, Id
Inf Circ No......	7 024	6 709	6 806	6 901	6 914	6 950	6 985
Date of scale....	Sep, 1937	Oct, 1932	Sep 1, 1933	1934–35	1936 (b)	1934–36	1937
Shift boss........	....	$5.50	....	....	$3.50–4.50	....	....
Miner...........	$5.50	5.04	$4.40	$4.50	2.75–3.25	$4.00	$4.50
Miner's helper....	....	4.24	....	....	2.00	....	....
Nipper..........	....	4.50	....	....	2.00	....	....
Timberman.......	....	5.04	4.40	4.50	2.50	4.00	....
Timber helper.....	....	4.24	....	....	2.00	....	....
Mucker..........	5.00	4.24	3.85	4.00	1.75	3.50	4.00
Trammer	....	4.24	3.85	4.00	....	3.50	4.00
Pipeman.........	....	....	....	....	2.50	....	....
Trackman........	....	....	4.75	....	....	....	....
Cage or skip tender	....	4.24	5.50	4.50	1.75	....	4.00
Motorman........	5.50	....	4.15	....	....	....	....
Motor helper.....	....	....	....	....	....	....	....
Hoist engr........	5.50	5.40	5.25	5.00	2.75	4.00	5.00
Pumpman........	....	....	....	....	....	....	4.50
Blacksmith.......	5.50	....	5.00	5.00	2.75	5.25	5.25
Carpenter........	5.00	....	4.75	....	2.50	4.50	5.25
Surface labor.....	4.40	....	3.85	....	1.75	3.50	....

(a) Plus a bonus. (b) 9-hr shift.

tions not related to mining), Ala, Calif, Ill, Ind, Md, N J, N M, Ohio, Penn, W Va, and Wis had no statute. In Ariz, Colo, Idaho, Mont, Nev, Okla, Ore, Utah, and Wash, 8 hr is legal working day, except in cases of emergency involving life or property; Ariz, Nev, and Wash permit a longer day for changing shifts every 2 weeks; Mo permits persons to contract for shorter or longer hours, and exempts those hired by the month from the 8-hr law. In Mich and Minn, legal working day is 10 hr; latter State permits over-time, on pay.

Federal unemployment compensation law (Title IX of Social Security Act, upheld by 5:4 decision of Supreme Court, May 24, 1937) requires an employer (with certain exceptions not related to mining) who has 8 or more persons in his employ for some portion of each of at least 20 days during the taxable year, each day being in a different calendar week (not necessarily consecutive), to pay a tax (since 1938) of 3% of their total annual wages. Credit up to 90% of this tax may be claimed as covering payments made to individual States under their unemployment compensation statutes (see below), provided such payments are made prior to the dates on which the Federal tax falls due. Employees pay no tax under the Federal law, and receive no benefits directly therefrom; compensation payments all come through State organizations, who requisition the necessary funds from the Federal treasury (where they have previously been deposited) upon conforming to requirements imposed by the Nat Soc Security Board.

State unemployment compensation laws. All 48 of the States, and Alaska and Hawaii, have enacted statutes acceptable to the Federal authorities, providing for collection of taxes and payment of compensation. Regulations vary (see Table 2) as to definition of a taxable employer, exemption of wages, rate of taxation, contributions by employees,

Table 2. Data on State Unemployment Compensation Laws * (4)

State	Employees of taxable employer	Type of fund	Weekly benefit for total unemployment		
			% of wages	Max	Min
Alabama....	8 (a)	P; *mr* after 1940	50	$ 15	None
Arizona.....	3 (b)	P; *mr* " 1940	50	15	$5 (k)
California....	4 (b)	P; *mr* " 1940	....	7–15	....
Colorado....	8 (b)	P; *mr* " 1941	50	15	5 (k)
Idaho.......	(c)	P; no *mr*	50	15	5 (k)
Illinois......	8 (b)	P; *mr* after 1941	50	15	5 (k)
Indiana.....	8 (b)	*Era*; *mr* after 1939	4 (l)	15	None
Maryland....	4 (a)	P; no *mr*	50	15	5 (k)
Michigan....	8 (a)	P; *mr* after Apr 1, 1942	4 (l)	16	7
Minnesota...	1 (b)	P; *mr* after 1940	50	15	6 (k)
Missouri.....	8 (b)	P; *mr* " 1941	4 (l)	15	5
Montana....	1 (b)	P; no *mr*	50	15	7 (k)
Nevada.....	(d)	P; *mr* after 1941	50	15	7 (k)
New Jersey..	8 (b)	P; *mr* " 1941	50	15	5
New Mexico..	(c)	P; *mr* " 1941	50	15	5 (k)
N Carolina...	8 (a)	*Era*; *mr* after 1939	50	15	5 (k)
Ohio........	3 (f)	P; *mr* after 1941	50	15	None
Oklahoma....	8 (b)	P; *mr* " 1940	50	15	8 (k)
Oregon......	(g)	P; *mr* " July 1, 1941	50	15	7 (k)
Pennsylvania.	1 (b)	P; no *mr*	50	15	7.50
S Dakota....	8 (b)	P & *Era*; *mr* after 1939	50	15	5 (k)
Utah........	(h)	P; *mr* after 1940	50	15	7 (k)
Virginia.....	8 (b)	P; no *mr*	50	15	3
Washington..	8 (a)	P; no *mr*	50	15	7 (k)
W Virginia...	8 (b)	P; *mr* after 1940	50	15	5 (k)
Wisconsin...	8 (i)	*Era*	50	15	1
Wyoming....	1 (b)(j)	P; *mr* after 1941	60	18	7 (k)
Alaska......	8 (b)	P; *mr* " 1941	50	15	5 (k)

* With important omissions, impracticable to tabulate, relating to: due dates for information returns and tax payments; exemptions of certain employers and employees; partial exemptions of wages; benefits for partial unemployment; waiting periods; duration and calculation of benefits; eligibility and disqualifications for benefits. Full text of any statute can be secured from State Capitol. *P* = pooled. *Era* = Employer's reserve account. *mr* = Merit rating system. (a) Within each of 20 weeks per year. (b) On any day within each of 20 weeks per year. (c) Any employer who pays wages of $78 or more in any calendar quarter. (d) Any employer who pays wages of $225 or more in any calendar quarter. (e) Any employer who pays $450 or more in any quarter, or employs 2 or more in each of 13 weeks per year. (f) At any one time within current year. (g) Employer of 4 or more on any day within calendar quarter in which total payroll is $500 or more. (h) Any employing unit with wages of $140 or more per calendar quarter. (i) In each of 18 weeks. (j) And when payroll is $150 or more in a quarter. (k) Or 3/4 of weekly wages, if this is less than stated minimum. (l) Of quarterly wages.

manner of accounting for funds, eligibility for benefits, and their amounts. In all States (except Mich) the rate (1939) is 2.7% of the taxable payroll, whence the whole State tax may be claimed as credit against the 3% Federal tax. In Mich, the rate (1939) is 3% on only the first $3 000 of an employee's annual wages. Employees contribute (1%) only in Ala, Calif, and N J (in latter State, only on first $3 000).

Compensation funds may be treated in one of 3 ways: (a) POOLED FUND SYSTEM; contributions are consolidated and benefits are paid to those eligible regardless of their employers; no person has a preferred claim; there is no provision for adjustment of tax rates. In its simple form, this system is now in force in but few States; most of those with pooled funds also provide for separate employer's accounts for purpose of merit ratings (see below). (b) EMPLOYER RESERVE ACCOUNT; contributions from each employer are separately accounted and can be used only for payment of benefits to his own discharged employees. In the few States using this system, a partial pool or balancing account is usually set up, from which benefit payments can be maintained if an employer's reserve should be temporarily exhausted. (c) MERIT RATING SYSTEM has been adopted by many States (encouraged by Federal authorities) and nearly all of the others have it under consideration. By this plan, the tax rate of an individual employer is made proportional to his annual labor turnover, as an incentive towards stability of employment. The plan requires keeping a separate account for each employer; when experience (usually for 3 years) shows that his contributions substantially exceed benefits to his discharged employees, his tax rate may be considerably diminished or his tax even abolished. In such cases, the Federal law nevertheless allows credit up to the full 90% of its 3% tax. In most States, the merit rating plans do not go into effect until 1941 or 1942; their provisions, as also many other features of the unemployment compensation statutes, are too varied and complicated to permit abstracting here; the applicable laws of any particular State can be obtained from Unemployment Compensation Board (or analogous body) at the State Capitol.

2. SLIDING SCALES FOR WAGES

Sliding scales aim to establish parallelism between wages and price of product, where latter is variable, for mutual advantage of workman and operator.

Examples. TRI-STATE FIELD. Wages (varying slightly from mine to mine) are based on previous week's market price of zinc concentrate (60% Zn). In 1929–32, base was $40 per short ton, wages increasing by 25¢ per shift (contract mucking, by 1/2¢ per "can") with each $5 rise in price if maintained for 1 week; corresponding reduction with falling price to min of $40. COEUR D'ALENE DISTRICT, Idaho. Wages vary with preceding month's aver price of lead. In 1930, additions to base wages per shift at Hecla & Star mine (5) were: Pb @ 5.5–6¢, 25¢; 6–6.5¢, 50¢; 6.5–7¢, 75¢; 7–7.5¢, $1; 7.5–8¢, $1.25. WESTERN COPPER MINES. Wages at most of the large copper mines are graduated according to *E & M Jour* quotations on electrolytic copper delivered in Conn valley, and (in 1939) were increased in steps, usually of 5%, for each 1.5¢ rise in price. In most agreements, neither increases nor decreases become effective until expiration of 30 successive days from the date when copper price first enters the specified range, nor unless aver price during that period remains at or above lower limit (at or below upper limit) of the specified range. At mines where SILVER constitutes chief value, agreements with Mine, Mill, and Smelter Workers (Art 10) base miner's wages (around $4.50 per day) on Govt price of 70–89¢ per oz, and reduce them by 25¢ if price falls below 70¢; at prices above 89¢, wages are increased by 25¢ for each 10¢ rise in price. Same agreements provide that if price of GOLD varies by $1 per oz from that of Nov 1, 1937, both parties have guaranteed right to negotiate a new scale of wages.

3. CONTRACT, OR "PIECE-RATE" SYSTEM

Advantages rest on the psychological fact that most men work more efficiently for themselves than for an employer. The system in some form has been applied to almost every phase of mine operation. From standpoint of an operator, contract system tends to promote speed, reduce unit costs, and diminish expense of supervision; and a contractor generally obtains better returns with shorter working hours than a day laborer; adoption of contract system has almost invariably proved mutually beneficial. Operator can count with certainty on ultimate cost of a given undertaking, while contractor is free to reap reward of his energy and skill.

Drawbacks: (*a*) By creating a class of workmen whose interests do not exactly coincide with those of the company, difficulty arises in enforcing standards of workmanship, and rules for safety. (*b*) Discontent may be caused by misunderstanding terms of a contract, or by refusal of manager to renew a contract on terms which he feels are unduly lucrative to contractor. Disputes over accuracy in measurement of work performed are frequent. (*c*) The most expert foreman sometimes fails to foresee changes in character of rock or ore during term of contract. It has been proposed (47) to meet this by elaborate standardization of all operations affected by nature of ore. Cost of such standardization as thus proposed, or as practiced at North Butte mines (48), is too great for any small company.

Uncertainty is probably best avoided by short-term contracts; Anaconda Co's stoping contracts (49) are for 1 week and may be terminated after 1 day by foreman or contractor. (*d*) In stoping, incentive towards large production may reduce grade of output, unless checked by levying a fixed charge for hoisting or milling contractor's ore, or by refusing payment for volume broken outside of vein walls.

Applicability. For above reasons, contracting has been most successful in exploring and developing mineral deposits, as boring, sinking, tunneling, and driving drifts, winzes, and raises, and, in general, wherever size of opening is constant, rock fairly uniform, or character and amount of work readily measured and inspected. Many so-called "contracts" guarantee a basic rate for a day's work and hence are more correctly described (Art 4) as bonus systems. These, as well as straight contracts, have been increasingly adopted in recent years, and for a larger variety of operations, including stoping and tramming; among 80 American mines described (2) to 1932, 32 conducted almost their entire operations on some contract or bonus system. The following examples are confined to straight contracts, with no guarantee of earnings.

Examples. COAL is almost always mined on contract, payment based on short tons of run-of-mine coal delivered at tipple (Art 11); rates in the Appalachian area, 1937–39, were 98¢ per ton for pick mining, and 76¢ for loading; narrow work and room-turning is contracted per linear yard, to specified width, besides ton rates for the coal; "brushing" (taking up bottom or breaking down top slate to afford necessary head room) is contracted per lin yd per in of slate removed, for agreed width. MARQUETTE RANGE, Mich. At one mine (6) entire production of iron ore is on contract, at 75¢–$1.60 per 4-ton car. Company provides a scraper outfit for each gang; also drills and steel, air and electric power, and timber; miners pay for explosives, carbide, hand tools, and small accessories, and do their own blasting and timbering. For mining methods, see Sec 10, Art 71. TRI-STATE district. Hand loading of ore is usually on contract at prices varying with shoveling conditions; at Crestline, Kas (14) in 1929, price was 10.5¢ per can of 0.6 ton; at Barr mine (18) in 1929, hand loading into cars (unusual practice in district), price was 38–41.5¢ per 1.5-ton car. Aver duty of hand loader is about 25 tons per 8 hr. MASCOT MINE, Tenn (22), contracts nearly all of its underground work (Sec 10, Art 37). Mill-hole contractor breaks and delivers ore into cars at a fixed price per ton, supplying labor and explosives. Chute pullers sub-contract to draw their ore into cars at fixed price per ton. Contractor usually allows his men regular company wages (he may increase them); Company pays his men directly, charging their wages to him. Company settles with contractor every week, holding back 30% of his earnings until a definite amount has accumulated, payable at termination of contract. EL POTOSÍ MINE, Chihuahua, Mex (24), employing about 500 men underground in 1934, lets nearly all of its underground work to contractors. Payments for stoping are based on number of car- or skip-loads; for development, on advance or volume excavated. Unit prices are adjusted to location and size of opening, character of formation, tramming method (hand or motor), accessibility, and other variable conditions. Contracts are measured twice a month. Company pays contractor's men legal wages, charging them to him, along with explosives and carbide. Contractor bosses his own crews and thereby saves the company considerable supervision. For mining method, see Sec 10, Art 37.

Contract prices, for similar work, vary as widely as actual costs; hence not instructive to enumerate them here. For costs, see sections relating to the several operations; in general, a contractor should be allowed a profit of at least 20% above actual cost, this being offset by benefits accruing to company by reason of greater speed. Contractor should usually be required to furnish his own blasting and other supplies; arrangements as to supply of powder, use of machines, tramming facilities, sharpening of drills, bits, repairs, etc, are settled by individual preference.

4. BONUS, OR PREMIUM, SYSTEM

Nature of bonus system. From operator's standpoint, two chief drawbacks of ordinary contract system are: difficulty in maintaining standards of mining, and the fact that contractor reaps practically the whole benefit of reduced cost of operation. Bonus system consists in setting a minimum daily or monthly task (or a minimum wages rate based on aver performance); work performed (or wages earned) in excess of the minimum is then paid for at an additional rate, so computed that the employer shares the saving in operating cost, besides profiting from more rapid work. For comparison of several bonus systems, see Bibliog (3). Following examples include several inaccurately described as "contracts."

S E Missouri in 1929. At one mine (25) HAND-LOADING was paid on "contract" at: $5 for 21 tons, $5.55 for 22 tons, 28¢ for each additional ton. Same, on company work: $5 for 18 tons, $4.50 for anything less than 18 tons provided shift-boss certified valid reason. POWER-SHOVEL OPERATOR: 8¢ per ton, but if his earnings exceeded $60 per week, he received only half of the excess. STOPE CONTRACTOR was guaranteed $5.05 per shift, but usually earned more on a tonnage basis graduated to height of stope: 8 ft high or less, 14¢ per ton broken; 8–20 ft high, 13¢ per ton; over 20 ft high, 10.5¢ per ton; prices were reduced by 1¢ per ton if ore was loaded by machine instead of

by hand; if contractor's earnings exceeded $60 per week, he received 1/2 of excess. At another mine (26) day's wages were guaranteed. Payment for STOPING was on basis of tons broken, measured by number of 2.5-ton cars loaded, and varied with height of stope: low ore, 16-18¢ per ton; medium-high ore, 12-14¢; high ore, 8¢ per ton. Contractors furnished only labor and explosives (sold them by company at less than cost) and carried their own steel. HAND-SHOVELERS ordinarily loaded 20 tons for a day's pay, with bonus for each additional ton; task might be reduced to 14-16 tons for a long tram or scattered ore. For mining method, see Sec 10, Art 31.

Magma mine (43). In 1928, 60-75% of all underground work was on "contract," with day's wages guaranteed; prices set by foreman and approved by supt and gen mgr. Prices per ft for DEVELOPMENT by drifts and crosscuts varied with size and other features of opening; price for raises increased with each 50 ft above level; uniform price per set for drift timbers. Payment for STOPING, on basis of volume excavated, varied with mining method (Sec 10, Art 37). Measurements were made twice a month; earnings computed by supt and auditor. Contractor received allowance above wages for his supervising. Excess earnings on a "contract" were divided among all engaged, on basis (only) of shifts worked.

Champion mine (52). STOPING at this mine involves careful sorting (Sec 10, Art 63) which the bonus system is designed to encourage. A miner and 1 or 2 pickers do whole work in a stope, and distribute their bonus in proportion to shifts worked. Monthly measurements give total volume broken, and car count gives tonnage of sorted ore hoisted from each stope. Bonus BASE is subject to 3 variations: (*a*) it is higher for wider stopes; (*b*) lower for a larger proportion of waste rock handled; (*c*) allowance of free explosive increases with width of stope.

Alaska Juneau (53). In 1928, about 75% of all underground labor was on "contract" with base wages assured. ESSENTIAL FEATURES of system, developed by experience: (*a*) division of bonus among all workers concerned in a given contract on basis (only) of shifts worked; (*b*) close supervision and prompt adjustment of prices to changed conditions; (*c*) judicious selection of contractors; (*d*) withholding small part of contractor's earnings, to be forfeited for poor work or drawn upon to maintain standard day's wages if earnings diminish; (*e*) furnishing good equipment and ample supply of steel and comp air; (*f*) fixed price for explosives, regardless of market; (*g*) combining 2 or more working places under same contract, to promote efficiency. Written DEVELOPMENT CONTRACT was let to successful bidder at set price per ft depending mainly on size of opening; rock conditions were usually known in advance and no timbering was necessary. Contractor paid for direct labor (including liability insurance @ 3% of total wages, including his own) and for explosives; these 2 items usually amounted to about 60% of total cost to company. Contractor's earnings were maintained at minimum of $6 per day by advances (when necessary) from future earnings; when his earnings exceeded $11 per day, surplus was divided among his men and himself in proportion (only) to shifts worked. Contract rates for BULLDOZING, LOADING, and HAULING were adjusted at short intervals with reference to season, weather, character of ore (chunky or slabby), condition of the stope (Sec 10, Art 68) or other factors affecting the handling of ore. At end of every month, contract was credited at set price (aver 14¢ in 1928) per ton trammed out; explosives and direct labor, including contractor and timberman (at company base rates), were deducted; balance, if any, was "divided among contractor's entire crew, in proportion to a certain maximum bonus rate per shift." Any further surplus was divided equally, per shift, among all in the mine, including company men, provided a man had worked 26 shifts in the month. If incapacitated by accident, he received a share of bonus proportional to the time he did work. A man quitting within the month was paid at company's base rate only. TRAMMING was a sub-contract, charged to the stope contractor; trainmen received a bonus of 50¢ per shift (above their base rate) whenever the stope contractor earned a bonus.

Britannia Mining & Smelting Co (54) in 1935 employed a true bonus system based on fixed standards of work; rates for excess work were so adjusted that workers and company shared equally in savings of cost. Standards, and any alterations due to changed conditions, were posted in advance; a considerable staff of bonus engineers was required. System proved disappointing when applied to caving into branched raises or to large shrinkage stopes, because the bonus had to be divided among so large a group that each man's share gave little incentive for increased effort. Method was satisfactory for: (*a*) all development work, on basis of standard footage; such work was sometimes let on straight contract; (*b*) square-set stoping, on basis of tonnage broken, as measured by count of new sets; (*c*) cut-and-filled rill stopes, on basis of surveys; (*d*) narrow shrinkage stopes, based on area of new footwall exposed, regardless of width so long as the whole vein could be broken by the same number of holes; (*e*) back-filling, based on measured volume. Provided the standards were correctly set, there was not much difference in cost (to the company) of a given work on bonus or on straight contract; latter system was preferred where speed was essential, since a contractor's profit may increase more rapidly than the earnings of a bonus worker.

Matahambre, Cuba (55). In 1929, all underground work (Sec 10, Art 62) was on "contract," with day's wages guaranteed. STOPE contractors (usually 2 in a stope, on opposite shifts) were paid a fixed price for all work done, itemized separately under cars of ore drawn from chute, sorting, spreading fill, timbering and lagging, and were charged for explosives, carbide, and time of their men, who were paid at fixed rates directly by the company; also for company men's time setting chutes. Contractor's men trammed timbers and all other materials from shaft stations. A contractor was allowed a small increase in day's pay for his supervising, but his earnings above that rate were divided among his men on basis (only) of shifts worked.

Mascot mine, Tenn (22) in 1930 applied a bonus system to men employed in TRANSPORT (for contract stoping, see Art 3). Motor- and car-men, skip tenders and helpers, shared a bonus based on weekly tonnage; day-shift foremen shared a monthly bonus on production of zinc concentrate; bonus for night-shift foremen (occupied mainly with chute-pulling and shoveling) was based on total tonnage hoisted.

Morenci, Ariz (56). In 1928, 72% of all underground work was on "contract" or bonus, standard day's wages being guaranteed in both cases. "CONTRACT" system, at standardised prices, was applied mainly to development, but included drift timbering, track laying, ditching, undercutting and shrinking in stopes (Sec 10, Art 80), and salvaging of materials. Rates were seldom changed, and then only (on authority of supt) before a contract was let. Work was measured twice a month and earnings were computed by engineers. Earnings might be divided in 4 ways, at discretion of foreman: (*a*) equally to all workmen concerned in contract; (*b*) respective day's wages to all concerned, plus equal share of surplus; (*c*) respective day's wages, plus a share of surplus proportional to day's rate; (*e*) straight payment to each man for his individual work, depending upon its nature. BONUS system was applied to workers in stopes and on transport. "Base cost per ton" was aver day's earnings (based on past records) for each type of work, divided by aver tonnage handled per day. Base for each working place was subject to adjustment at beginning of a pay period. Bonus was half of excess earnings above the base. Shift-bosses did not share in production bonus, but were rewarded for 1 000 man-shifts without accident.

United Verde underground mine (57) in 1931 employed "contract" and bonus systems almost identical with those at Morenci, requiring a staff of 8 engineers and a clerk to set standards and compute earnings.

5. COOPERATIVE SYSTEMS

Division of profits is sometimes arranged by permitting employees to buy shares in the company, their employer; additional incentive to participate in such system may be offered in form of a discount from market price of shares, a guaranteed annual bonus, or an installment plan of purchase. U S STEEL CORPORATION, in 1903, instituted a plan by which common shares were offered to all employees upon easy terms, with benefits beyond those enjoyed by ordinary stockholders. Price of stock was the market price, or usually a little less. This plan was discontinued in 1929, at the precipitous fall in stock prices. For details of its provisions, see Edn 2, p 1530.

On the Comstock Lode, in 1908, certain companies divided 10% of the net value of ore produced each month, in excess of $50 000, among all employ es, in proportion to their wages. Another company made similar division of all its net proceeds (7).

At an Indiana coal mine, a legal corporation of miners was organized, to which the owner leased his property for a test period of 1 yr under restrictions aiming to maintain safety of mine. Owner's royalty, 5¢ per ton, and 0.25 of net profits. The arrangement resulted in increased earnings by miners, and steadier output during dull season (8).

6. LEASING, OR TRIBUTE, SYSTEM

Mining lease, as term is used in U S, is a simple contract by which the LESSOR (mine owner) grants to LESSEE ("leaser," or "tributer"), under certain conditions, the exclusive right to do a certain piece of work, for which the lessee agrees to accept payment in form of a proportion of net returns. Unlike an ordinary contractor, lessee thus shares risk as well as profit (R. W. Raymond, *Trans* A I M E, Vol 21, p 916). A clause called BOND AND LEASE is sometimes included, by which owner promises to sell his mine and deliver a valid deed therefor to lessee for a stipulated price at any time before termination of lease.

Leases may be granted for exploring and developing newly discovered ore deposits, as at Goldfield (9); to reopen a seemingly exhausted mine, as at Aspen and Eureka (10); or during the height of a mine's prosperity, as at Leadville, Cripple Creek, and other Colorado camps (11). In latter case, advantages of leasing are most pronounced when the deposit exists in form of stringers or pockets of high-grade ore. Many such mines return large profits under leasing system, while unable to operate profitably on their own account, for reasons stated below.

Advantages of leasing system, from OWNER'S point of view, are: (*A*) Development accomplished at small cost, since lessees are usually willing to take a risk in performing dead work which a cautious mine manager might shun. (*B*) When developed, ore is produced with small working capital. (*C*) Profits to owner are absolutely net, since lessee usually pays for power, pumping, and hoisting, besides his own working expenses. (*D*) Less ore is lost during mining, tramming, and sorting, whether by carelessness or by stealing. (*E*) Mine is not liable for lien. (*F*) Lessee is responsible for damage to mine and for accidents to workmen. (*G*) Slight supervision required. From viewpoint of LESSEE, advantages are: (*a*) He frequently has keener perception of ore occurrence in the particular deposit than even a skilful foreman can acquire (company miners sometimes deliberately conceal information leading to discovery of new pockets). (*b*) No heavy initial outlay required for machinery or expensive development. (*c*) No expense for office force, interest, insurance (of property), and other customary fixed charges. (*d*) By more careful personal supervision, loss of time and waste of explosives and supplies are reduced, hence affording cheaper mining than an owner can usually attain on his own account. (*e*) Even under adverse conditions, a leaser's returns are usually larger than ordinary day's wages

(as is right and proper considering his extra diligence and skill); excessive dead work is sometimes compensated by an allowance for "footage" (say, $2 per ft), or by stipulation of a smaller royalty.

Royalty rate is usually based on a sliding scale which varies with gross value of ore, while its amount is computed on net returns from smelter or reduction works, after deducting haulage, freight, and treatment charges (sometimes also sampling and assaying fees). Shipments are ordinarily made in name of mine owner, who first deducts his royalty, then his charges (if any) against lessee for supplies or services rendered; balance going to lessee.

At Goldfield, during first stages, royalty rate was 25% on ore over $50 per ton, and 20% on less valuable ore. At one Colo mine (12) rate was numerically equal to 1/3 value of ore; $51 ore, 17%; $150 ore, 50%; special provisions for $300 ore. At a mine in Gilpin county, Colo (13), royalty was 33 to 75%, if drift had already been run under leased block; 25 to 33%, if drift had to be run by lessee At Leadville: under $10, 10%; $10 to $15, 15%; $15 to $20, 20%; over $20, 25%. Table 3 gives royalties in recent leases at Cripple Creek, Colo, and Oatman, Ariz. For mathematical discussion of a rational basis for levying royalties, see (15).

Table 3. Examples of Lease Royalty

Cripple Creek, Colo 1935 (58)		Tom Reed, Oatman, Ariz 1934 (59)		Pioneer, Oatman, Ariz 1935 (59)	
Ore Value	Royalty, %	Ore Value	Royalty, %	Ore Value	Royalty, %
To $15	10	To $10	5	To $15	10
$15–20	15	$10–12	6	$15–25	15
20–30	20	12–15	9	25–40	20
30–50	25	15–20	15	40–50	25
50–100	30	20–25	20	50–60	30
$100 +	35	25–30	25	60–70	35
......	..	30–35	30	70–80	40
......	..	35–40	35	80–90	45
......	..	40–45	40	$90 +	50
......	..	45–50	45		
..	.	$50 +	50		

Other covenants of a mining lease. A lessee can void his contract at any time without penalty, while a lessor has no means of ousting a lessee if the latter fulfils the letter of the contract. For this reason, and because most lessees are not financially strong, it is particularly important that mining leases be carefully drawn, and clearly understood.

Two covenants most likely to cause trouble are: (*a*) What constitutes ABANDONMENT and FORFEITURE? Usual stipulation is for a minimum number of days' work (200 man-shifts per 90 days, at Cripple Creek; 60 shifts per mo, at Goldfield), or a minimum amount of new development (300 ft per yr, at Leadville). Forfeiture results from violation of any covenant of contract (not attributable to casualties, strikes, or litigation) and owner may thereupon resume possession without formality of a lawsuit. (*b*) Ownership of PLANT erected by lessee and disposal of ORE-DUMPS remaining on property at expiration or forfeiture of lease. At Goldfield, all improvements underground, and all broken ore not hoisted before expiration of lease, become sole property of mine owner. At Cripple Creek, all structures and dumps remain property of mine owner, but lessee must remove his machinery within 60 days. It is advisable to include a definite stipulation as to the length of time, after termination of lease, within which owner must market ore on dumps, or beyond which the lessee's equity in that ore shall cease; neglect of this provision has been a fruitful source of lawsuits. Additional stipulations, not already mentioned, relate to: (*c*) Exact boundaries of leased block. (*d*) Date of expiration of lease. (*e*) Work to be done "mine-fashion" to yield maximum output with due regard to development of orebody and safety of both miners and workings. (*f*) Adequate timbering to be placed and maintained. (*g*) Minimum dimensions of openings: shafts commonly 4 by 8 ft, and drifts 3.5 by 6.5 ft clear. (*h*) No underhand stoping; no stoping within 10 ft of shaft; no levels within 50 ft of one another. (*i*) Workings to be kept clear of ore, rock, rubbish, and water. (*j*) Owner or his agents to be admitted to mine at any time for any purpose. (*k*) Bills incurred by lessee to be settled promptly, and vouchers to be shown to lessor on demand. (*l*) No sub-leasing without owner's consent. For verbiage of typical Colo leases, see (16).

Leasing at Cripple Creek (58). In 1935, probably 75% of all mining in the district was being done by leasers. Success of the practice requires continuous and strict supervision by owner; where this has been relaxed, results have often been highly unsatisfactory. On straight leases, royalties have been fairly uniform at the rates shown in Table 3. The larger mines, with managers and superintendents on the ground, frequently adopt the SPLIT-CHECK system. At the CRESSON MINE, such leases run for 1 yr, but are terminable at option of management. Owner retains full authority over safety and sanitation of mine, removal of pillars, etc. Leaser must work minimum of 50 man-shifts per month.

Lease blocks are usually 200 ft square and 1 level (usually 100 ft) high. Owner supplies equipment (except drills), comp air, explosives and all other supplies, and provides hoisting service, but charges 10¢ per ton for transport of shipping ore via aerial tram and loading into RR cars. Leaser supplies labor of miners, trammers, and sorters, and pays their compensation insurance. Net returns, after freight and treatment, are divided equally, but leaser contributes 1% of gross value of shipment as his share of taxes. Owner of a large property sometimes lets a straight royalty lease (at rates usually lower than those in Table 3) to a leasing company, which may then sub-lease to individuals on the split-check system.

7. METHODS OF PAYING WAGES

Interval between pay-days. MONTHLY pay-days reduce amount of clerical labor in preparation of payroll, but often work hardship on individual miners; this arrangement is commonest at mines which are remote from head office, and from banking facilities. BI-MONTHLY pay-days are the rule in anthracite district, and are common elsewhere; involve more clerical labor, but are generally considered more equitable by the miners. WEEKLY pay-days are feasible only with companies having large clerical force, and where there are ample banking facilities. Pay-day intervals are fixed by statute (with penalty for violation) in all mining States except Idaho, Wash, and in the Philippines (60); legality has often been disputed, but generally upheld though for varying reasons. Law in Ala applies only to public-service, and that of S Dak and W Va only to RR employees. Semi-monthly pay-days prevail in most mining States; N J prescribes 2 weeks; Minn, 15 days for public-service and transitory labor (no other regulation). Weekly payment is required in Puerto Rico, and (if requested) in Ind. Alaska and Ore permit monthly pay-days; also Utah, to employees on annual salary (semi-monthly to others). Wyo and Nev allow departure from semi-monthly rule in contracts. Legal lapse between end of pay period and pay-day varies from 5 days in Mont to 10 days in Ind, N M, and Utah, 12 days in N J, 15 days in Md, Mich, Nev, Ohio, and 18 days in Ill. Mo requires 30-day notice of any proposed reduction in wages.

Wages are paid: (*a*) By check. This is often preferred by operators, as it avoids necessity for filling out individual receipts; objection of miners is that cashing checks requires opening of a bank account or visit to store or saloon, sometimes involving a discount; method not recommended for remote districts, and is specifically prohibited by laws of Ariz, Cal, Colo, Ill, Ind, Md, Nev, N J, N M, Okla, Ore, Utah, Wash, unless checks are payable in cash, at sight, without discount. Nev prohibits payment of wages in saloons. (*b*) In currency. This is preferred by miners, and the only serious disadvantage to operator is the risk of theft of cash during transit from office or bank. (*c*) In orders on company store. In an isolated locality, able to support only one store, and when this belongs to mining company, payment of part of wages in this form may not be objectionable; elsewhere, the plan is liable to abuse by company, and may lead to serious discontent among miners; it has been abandoned in the anthracite field, after long protest from the organized workers. Company scrip is debarred by law in Ariz and Colo; permitted (at option of employee) in Ill, Mich, N J, N M, Ohio, and W Va, provided it is redeemable at sight in cash at face value; Wis places no restrictions on methods of paying wages. Compulsory buying at Company stores is prohibited in Ariz, Idaho, N M, and W Va.

Time-books, payrolls, etc. Where day's wages are in force, each foreman has a TIME-BOOK, in which to record the number of hours worked per day by each man under his charge. Some time-book systems are so arranged that the foreman can distribute time of each workman according to nature of his employment, for purposes of cost accounting, but it is unwise to demand too much clerical work from mine foremen. Every day, or once a week, entries are transferred to a time-book in office, where a distribution to different working accounts can be made. At required periods, PAYROLL is compiled from office time-book, setting opposite each name the total hours (shifts, or days) worked, rate per hr (which may be different for different kinds of work by same individual), and total wages earned. In contract work, amounts due are computed from returns of surveyors, foremen, weighmen, etc. DEDUCTIONS, comprising all charges of company against each man, such as store supplies, rent, board, cash advanced, contribution to hospital and doctor's or benefit funds, wages of check-weighman, and Union dues (in some cases), are compiled from the appropriate sources and entered. A copy of this payroll, showing total amount in cash needed for following pay-day, as well as amounts credited to the several accounts, to balance deductions, is sent to main office, which then sends required amount to mine office. Meantime, at best organized mines, an individual "DUE-BILL" is delivered to each man, showing amount earned, itemized deductions, and net wages coming to him. These statements are preferably distributed a few days before pay-day, to give opportunity for rectifying mistakes and for dividing proceeds between partners, in case of a contract. For details of an elaborate payroll system at Britannia Beach, B C, making extensive use of mechanical office devices, see Bib (61): for accounting of payrolls in general, Bib (17).

Anthracite agreements (Art 11) require a contractor to notify company as to the amount due to each of his laborers, so that they can obtain their wages direct from paymaster. If wages are paid in cash, it is convenient to have a coupon at bottom of statement, which can be signed and delivered to paymaster, as receipt for wages.

8. ACCIDENT COMPENSATION AND INSURANCE

Development and purpose. Under the COMMON LAW, an employer, when sued for damages by an employee injured in his service, was allowed to plead the defenses of contributory negligence, negligence of fellow-servant, and assumption of risk by employee. EMPLOYERS' LIABILITY LAWS abolished or modified the defenses of fellow-servant and assumption of risk, but an injured employee was still under burden of proving personal negligence on part of someone as proximate cause of his injury. An injured employee was thus under necessity of incurring both expense of law-suit and enmity of his employer; an employer, on other hand, was liable to be held for excessive damages, based on mere conjecture of a sympathetic jury, or must insure his liability at a high premium, the rate of which was commonly based upon the worst rather than on the best or the average conditions surrounding the employer's particular line of industry. COMPENSATION LAWS seek to diminish injustice to both parties by: (*a*) making obligation to pay damages independent of fault (except criminal or wilful negligence of injured employee); (*b*) making measure of compensation automatic and predetermined (based on aver weekly earnings); (*c*) permitting employer to insure his compensation liability at more equitable rates. Such laws are in force in all mining States, usually under jurisdiction of a special Compensation Board; in Ala, Alaska, N M, Tenn, and Wyo, law is administered by county or State courts. Decisions of a Board are usually final as to facts, but may be appealed to courts on questions of law.

General character of compensation laws. DIRECT, or simple, compensation law places obligation for indemnity immediately upon person in whose employment an injury occurs. INDIRECT, or State insurance, law requires employer to pay an annual tax graded according to nature of his business and proportional to his payroll; an injured workman is then indemnified out of this fund by State officials. COMPULSORY compensation law places absolute obligation on employer, either to pay direct indemnity or to contribute to State fund. ELECTIVE law permits employer to choose whether to abide by State compensation provisions, or to operate under a liability statute which deprives him of all three common-law defenses; employees are similarly required to state their choice; except in Mich, Mont, Nev, and W Va, election is presumed (as to both employer and employee) in absence of positive rejection. Purpose of elective system is to circumvent the usual constitutional provision against the deprivation of property without due process of law, which the N Y Court of Appeals has held to be an inherent defect in any compulsory compensation statute.

Nature of injuries. Existing compensation statutes have adopted practically a uniform wording: "all personal injuries from accidents arising out of and in course of employment, unless due to intoxication or to wilful or intentional misconduct." Necessity of proving responsibility for accident, and degree of fault, is thereby eliminated, except in case of intoxication and wilful misconduct, different forms of latter sometimes being expressly indicated by statute.

Amount of compensation. There is wide diversity in statutory provisions on this subject; common aim is to make amount of compensation, in any case, mathematically determinable. In almost all States, payments are calculated on a variable percentage (66²/₃% is frequent upper limit) of previous aver weekly wages, payable over a limited period of weeks; max (frequently \$16–\$20) and minimum (commonly \$5–\$8, or actual wages if below stated minimum) weekly rates are usually stipulated. MAXIMUM TOTAL PAYMENTS under 4 conditions are given in Table 4; details impracticable to tabulate can be secured from appropriate State authorities.

Insurance of compensation liability. In all mining States except Alaska, employers are required to insure their liability, and in all but Nev, Ore, Puerto Rico, Wash, and Wyo, they are permitted to choose method of insurance. In the most liberal statutes 4 methods are allowed: (*a*) Self-insurance, on satisfactory proof of solvency. (*b*) Mutual companies. (*c*) Regularly organized stock liability-insurance concerns. (*d*) Organization administered by State. In any case, an injured employee usually has direct claim against insurer, for an amount which in no case may be less than that stipulated in compensation act. An indirect benefit of scientifically applied insurance is the influence that it exerts in promoting safety of workmen, since an employer's premium will properly be in direct proportion to accident rate at his works.

Status as of July 1, 1938 (62). Among numerous recent changes, most frequent has been provision of compensation for occupational diseases, now compensable in 27 States (18 on Jan 1, 1936); some statutes enumerate specific diseases; others provide for disability resulting from occupational disease; a few States make "injury" from occupational disease synonymous with "accident" and hence compensable. Many compensation statutes include safety regulations and provide for their enforcement; most of the important mining States accomplish this purpose through other channels.

Table 4. Workmen's Compensation in Mining States (*Monthly Labor Rev*, Sep, 1938, p 566)

State	Compensation is—	Ins req'd in: State fund	Ins req'd in: Private Co or Self	Exemptions (a)	Occupational diseases compensable	Com-law defenses	Waiting time, days	Retroactive, weeks (b)	Max Total Payment (c): Death	Max Total Payment (c): Perm total disab	Max Total Payment (c): Temp partial disab	Max Total Payment (c): Temp total disab	Limits on medical care
									$	$	$	$	
Ala.	Elec (e)	....	Either	16	....	(f)	14	4	5 400	7 950	7 200	5 400	(nn)
Alaska	" (e)	....	(h)	5	....	(f)	1	....	9 000	9 000	7 200	....	(ll)
Ariz	Comp (g)	Compet	Either	3	....	(d)	7	2	(k)	(p)	(dd)	(ii)	(ll)
Calif	Compul	"	"	....	Yes	(d)	7	....	5 000	6 000(q)	6 000	6 000	(kk)
Colo	Elec (e)	"	"	4	....	(d)	10	....	4 375	(r)	2 912(ee)	....	(nn)
Idaho	Compul	"	"	....	....	(f)	7	4	4 800	6 400(s)	3 792(ff)	6 400(s)	(kk)
Ill	Comp (i)	....	"	....	Yes	(f)	7	30 da	5 500	(t)	4 500(gg)	4 000	(kk)
Ind	Elec (j) (e)	....	"	....	Yes	(d)	7	....	5 000	5 000	5 000	5 000	(ll)
Md	Comp (i)	Compet	"	....	....	(d)	3	....	5 000	6 000	3 750(hh)	3 750	(mm)
Mich	Elect	"	"	....	Yes	(d)	7	6	5 400	9 000	3 600	9 000	(ll)
Minn	Compul	....	"	....	Yes	(d)	7	4	7 500	10 000	9 000	6 000	(kk)
Mo	Elec (e)	....	"	11	Yes	(d)	3	3	6 000	6 000 (u)	4 640	8 000	(nn)
Mont	Elect	Compet	"	....	....	(d)	7	3	8 400	10 500	4 200	6 300	(nn)
Nev	Elect	Excl	....	....	....	(d)	7	1	(k)	(v)	3 600(hh)	7 200(ff)	(ll)
N J	Elec (e)	....	Either	....	Yes	(f)	7	7	6 000	8 000	4 600	6 000	(mm)
N M	" (e)	....	"	4	....	(d)	7	....	5 400	9 900	3 240	9 900	(mm)
N C	" (e)	....	"	5	Yes	(d)	7	4	6 000	6 000	3 600	6 000	(ll)
Ohio	Compul	Excl	Self	3	Yes	(d)	7	....	6 500	(w)	4 000	3 750	(mm)
Okla	"	Compet	Either	2	....	(d)	5	....	....	9 000	4 500	5 400	(ll)
Oreg	Elec (e)	Excl	....	....	....	(d)	0	....	(l)	(x)	2 400(hh)	....	(mm)
Penn	" (e)	Compet	Either	....	Yes	(f)	7	4	(m)	(y)	4 770	9 000	(nn)
P. I.	Compul	....	"	....	Yes	(f)	7	....	P3 000	P3 000	P3 000	P3 000	(kk)
P. R.	"	Excl	....	4	Yes	(d)	7	....	3 000	3 000	2 000	1 040	(kk)
S Dak	Elec (e)	....	Either	....	....	(d)	10	6	3 000	3 000	3 000(hh)	3 000	(nn)
Utah	Compul	Compet	"	3	....	(d)	3	....	7 500	(z)	3 200(hh)	6 250	(mm)
Va	Elec (e)	....	"	11	....	(d)	7	6	5 000	6 000	3 200	6 000	(ll)
Wash	Compul	Excl	....	....	Yes	(d)	3	....	(n)	(aa)	3 000	....	(kk)
W Va.	Elect	Excl	Self	....	Yes	(d)	7	3	(o)	(bb)	5 440	1 248	(mm)
Wis	Compul	....	Either	3	Yes	(f)	3	10 da	15 000	(cc)	10 500	....	(kk)
Wyo	"	Excl	....	....	....	(d)	7	3	8 000	10 000	2 500(hh)	....	(mm)

(a) Employer of fewer than stated number is exempt. (b) If disability lasts for stated number of weeks, compensation is retroactive to date of accident. (c) These payments are *maxima* allowable; for graduated scales, consult compensation board of particular State (or county courts in Ala, Alaska, La, N M, Tenn, Wyo). (d) Employer forfeits common-law defenses on neglect or lapse of insurance. (e) Election presumed (as to both employer and employee) in absence of positive rejection. (f) No suits permitted after employer and employee have accepted comp act. (g) As to employers. (h) No security required, but if a beneficiary files notice of death claim, employer may deposit $9 000 with court or give bond for that amount; in other cases, claimant is entitled to writ of attachment, unless employer files undertaking for double the amt sued for. (i) As to public employees and hazardous occupations only (j) Compulsory as to public employees and coal mining. (k) $18.46 per wk during widowhood, or specified minority age of children. (l) $30 per mo for widow, plus $8 per mo for each dependent child, during widowhood or specified

In most States, both self-insurance and insurance in private companies are permitted; only Nev, Ore, Puerto Rico, Wash, and Wyo compel insurance exclusively in a State fund; Ohio and W Va maintain such exclusive funds, but permit self-insurance in certain cases. Some States maintain insurance funds competing with private companies, and permit employers to choose. Data on these and other provisions in the compensation statutes of (only) the important mining States are compiled in Table 4. About 2/3 of the States allow compensation for accidents occurring outside of the State, provided the employee was hired within it, or is a resident of it, or his employer's place of business is within the State; courts of other States have often ruled similarly, in absence of statutory provisions. Utah affords same protection to an employee (of a Co in Utah) even though hired elsewhere. Where both parties have accepted an elective compensation act, suits for damages are usually forbidden. If his employer has accepted, an employee who has rejected may bring suit, but employer retains his common-law defenses. In a large majority of the States, an employer who fails to secure his liability, to provide insurance, or to pay premium when due, may be sued for damages and forfeits his common-law defenses.

Total amount of compensation in a given case depends upon: (*a*) the weekly rate, usually a percentage of normal wages; (*b*) period over which payments are made; (*c*) in most States, a fixed max weekly or total payment (minimum rates are also usually stipulated). These factors are variable, according to nature of the injury: (1) death; (2) permanent total; (3) permanent partial; (4) temporary total; (5) temporary partial disability. "Permanent partial disability" is variously interpreted: sometimes by specific description, sometimes by reference to lost earning power. In general, permanent total disability is treated as a more serious loss than death. No State (except Ore) pays compensation for the first few (most commonly 7) days after a disabling accident, but if disability lasts longer than a specified time, payments in many States become retroactive to date of accident. "SECOND INJURY" compensation receives special provision in all mining States (except Alaska, Philippines, and Puerto Rico). When an employee, already partially disabled and receiving appropriate compensation, is injured a second time and becomes eligible for increased compensation, his then employer is responsible only for the increment, the difference being paid out of "second-injury" fund, which is maintained (among the mining States) in Idaho, Ill, Minn, N J, N C, Ohio, Penn, Utah, Wis; a convenient source of such funds is the compensations awarded in fatal cases where no one proves legally entitled to receive them. All States require MEDICAL AID to be furnished an injured employee, free in nearly all States; in Alaska, an employer may deduct \$2.50 per mo from wages, for medical fund; in Ariz and Nev, 1/2 the cost (but not over \$1 per mo) may be deducted; in Wash, workman pays 1/2 cost. In 16 States, there is no limit on time or amount of medical care, 12 States limit amount but not time, 12 limit time but not amount, and 13 limit both (Table 4). In many States with statutory limits, courts or Compensation Boards have discretionary powers to extend them.

No compensation act discriminates between U S citizens and resident aliens, but 38 out of 53 such acts do now (more than formerly) restrict to some extent the benefits to non-resident alien dependents. Commonest restrictions are: (*a*) reduced benefits; (*b*) limitations on beneficiaries; (*c*) commutation of weekly benefit to a lump sum figured at a lower rate. Among mining States, only Minn, Ohio, Tenn put non-resident alien dependents definitely on same footing as those of citizens; Ind, Mo, N J, and Philippines make no special provision. Several States exempt Canadian dependents from such restrictions.

Compensation statutes of Ariz, Colo, Mich, Mo, Mont, Nev, N M, Ohio, Okla, Ore, Penn, Puerto Rico, S Dak, Utah, Va, Wash, Wyo, demand REPORTS on all industrial accidents; Cal, Idaho, Philippines, on those causing disability of 1 day; Ind, Iowa, Kans, Ky, more than 1 day; Md, N C more than 3 days; Ill, more than 1 week; Ala, more than 2 weeks. In N J, insured employers must report all accidents; uninsured, those causing permanent total disability, or disability of more than 1 week. Minn requires reports on all accidents causing death or serious injury, and on others causing disability for more than 1 day. In W Va and Wis, Compensation Board has discretion as to reports. Alaska's compensation law demands no reports, but accidents in coal mines are required to be reported under a separate enactment.

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minority age of children. (*m*) \$18 per wk for 500 wk, and thereafter until remarriage. (*n*) \$55 per mo to widow with 2 children, plus \$5 for each other child, during widowhood or specified minority age of children. (*o*) \$30 per mo to widow, plus \$5 for each child, during widowhood or specified minority age of children. (*p*) 65% of wages, for life. (*q*) Plus 40% of wages thereafter, for life. (*r*) 50% of wages, for life. (*s*) Plus \$8 per week thereafter. (*t*) 50–65% of wages for 416 weeks, plus life pension of 8–12% of previous payments. (*u*) Plus 25% of wages thereafter, for life. (*v*) 60% of wages, for life. (*w*) 66 2/3% of wages, for life. (*x*) \$8.08 per week, plus \$8 per mo for each dependent child, during period of disability. (*y*) \$18 per wk for 500 weeks, plus \$30 per mo thereafter, for life. (*z*) 60% of wages for 260 weeks, plus 45% of wages thereafter, for life; increased 5% for each dependent child to max of 5. (*aa*) \$60 per mo during period of disability. (*bb*) \$16 per week, for life. (*cc*) \$21 per week, for life. (*dd*) 55% of wages for 260 weeks. (*ee*) In addn to allowance for temporary total disability, at same rates. (*ff*) Plus \$10 per mo for dependents. (*gg*) In addn to payments for total disability for period up to 64 weeks. (*hh*) In addn to payments for total disability. (*ii*) 65% of wages for 433 weeks. (*kk*) No limits. (*ll*) No limit on amount. (*mm*) No limit on time. (*nn*) Both amount and time limited.

9. PENSIONS AND BENEFIT FUNDS
(See also Art 8, "Compensation")

Federal old-age and survivors' benefits. Social Security Act (Titles II and VIII, as amended Aug, 1939) levies an income tax on employees within the U S (with certain exceptions not related to mining) who are under age of 65 yr; also an equivalent excise tax on all employers of such persons, regardless of their number. Excess of an employee's wages above $3 000 per yr is exempt from both taxes, which began to accrue Jan 1, 1937. Rates: 1937–42, 1%; 1943–45, 2%; 1946–48, 2.5%; 1949 and thereafter, 3%. Employer must withhold employee's tax from his wages (giving a receipt therefor) and remit it quarterly, together with his own tax, to collector of internal revenue. Returns are required from each separate employer; consolidated returns of parent and subsidiary organisations not permitted; sundry information returns are also required. Employees are identified by numbers assigned by local branches of the Security Board.

Retired employee is ELIGIBLE FOR MONTHLY BENEFITS if he: (*a*) is 65 yr old; (*b*) is fully insured; (*c*) files a claim. A worker is FULLY INSURED for life when he has received (*a*) as much as $50 for taxable employment in each of 40 calendar quarters; or (*b*) $50 in each of a number of calendar quarters equal to one-half of those elapsed between Dec 31, 1936 (or his 21st birthday, if later) and the quarter in which he reaches age of 65 (or dies), but in not fewer than 6 quarters. MONTHLY BENEFIT (beginning Jan 1, 1940): 40% of first $50, plus 10% of next $200 of aver monthly wages, plus 1% of above total for each year in which worker earned $200 or more in taxable employment. Minimum benefit, $10; max under any conditions, $85. Benefit is forfeited for any month in which worker earns $15 or more in taxable employment. AVER MONTHLY WAGES in above formula is total accumulated wages (excluding amount above $3 000 received in any 1 yr), divided by number of months claimant could have worked between Dec 31, 1936 (or his 22nd birthday, if later) and the quarter in which he becomes eligible for benefits, or dies. SUPPLEMENTARY BENEFITS for dependents: 50% of claimant's personal benefit for (*a*) wife 65 yr or older; (*b*) each unmarried dependent child under 16 yr, or under 18 yr if regularly attending school. SURVIVORS' MONTHLY BENEFITS of a fully insured worker: 75% of worker's benefit for a widow over 65 yr, or for any widow with dependent children; 50% for each unmarried dependent child under 18 yr, or for each dependent parent if there is no widow or dependent child. Amended Act makes provision for widows and minor children surviving a worker, not fully insured, if he has received as much as $50 in each of at least 6 quarters during preceding 3 yr. If no survivor is eligible for monthly benefit, a lump sum equal to 6 times the worker's monthly benefit may be paid to nearest relatives or (up to that amount) to anyone who defrays funeral expenses.

Note that the above is exclusively a Federal affair, as to both taxation and disbursements; no credit against the tax can be claimed on account of payments under any State statute.

Pensions are granted by many of the most progressive companies, and paid from a fund to which employees may or may not be asked to contribute. BENEFIT FUNDS are more frequently maintained by the employees alone, with occasional assistance from company's accounting and clerical force; in some cases, the company also contributes financially.

Homestake Mining Co began in 1917 to pension old employees retiring voluntarily or at suggestion of foremen (21). Annual pension, 25% of last full yr's pay, plus $10 per yr of service; total not exceeding $600. To mid-1931, 163 employees had been pensioned, of whom 52 were then receiving benefits. Co also pensions widows of employees killed by accidents before State compensation law became effective; monthly rate, $30 plus $5 per child under 16 yr. HOMESTAKE AID FUND, organised 1910, with Co supt as treas, chief clerk as secty, and 5 elected directors. Employees each contribute $1 per mo, and up to adoption of State compensation laws the Co contributed $1 000 per mo; fund formerly paid benefits for both sickness and accidents; now only for sickness, since Co must pay accident compensation. If sickness lasts over 3 mo, fund pays $1.50 per day for 9 mo; $800 for death or $200 for suicide or insanity. Co still pays $1 per day lost by accident, up to 10th day, when State compensation begins.

Copper Queen Employees' Benefit Assoc (consisting of Phelps Dodge employees at Bisbee and Douglas, Ariz) provides benefits for death and disabling accidents even though occurring while not on duty (in which case State compensation would not apply); also for sickness, not compensable by State. Admission to membership is limited to employees under age of 60 yr, but benefits for death due to sickness are limited to $200 for an employee admitted at age above 45 yr, against a maximum of $1 500 for those admitted at earlier age. A member contributes, in advance, 2.2% of his pay, to max of $3.30 per mo; contributions are waived during a period of disability. Company contributes $18 000 annually, provided Assoc numbers 50% of total employees (excl Mexicans) at the plants mentioned. Benefits at 50% of a member's wages, but not exceeding $2 per day, are paid during sickness or for accidents occurring off-duty, excepting the first 5 days in either case. For a dis-

abling accident while on duty; (*a*) no benefit unless disability exceeds 5 days; (*b*) if disability lasts between 5 and 14 days, benefit is paid for only 2 days; (*c*) no benefit for disability exceeding 13 days. (If disability from an accident on duty continues for 2 weeks, Ariz State compensation is payable from date of accident). Death benefit of a member is equivalent to his daily wages times the number of calendar days in last previous year, but not to exceed $1 500 (see above); Assoc may advance $200 out of benefit for funeral or other urgent expenses. Accidental total disablement while off-duty receives same benefit as death. Lump-sum settlement at option of member is permitted in case of off-duty injury or chronic sickness.

"Keg funds" are often maintained at collieries. Miners collect and bring up their empty powder cans; company provides storage for them, and periodically sells them to powder companies for about 8¢ each, the money being used for sick benefits and funeral expenses of employees. D L & W company pays out, from this source, about $1 700 monthly.

For additional data, see Bibliog (23).

10. LABOR RELATIONS IN GENERAL

Recent years have yielded a flood of legislation controlling employer-employee relationships. Federal law ("Wagner Act") rests on the Interstate Commerce clause of the Constitution. Among the mining States, only Mich, Minn, Penn, Utah, and Wis (to late 1939) have enacted such laws for regulating their own industries, usually patterned after the Wagner Act, but (in every case, except Utah) with addition of clauses defining "unfair labor practice" by employees, the most criticized omission from the Wagner Act. Mont and Ore permit their State Labor Boards to cooperate with National Board. Statutes of all mining States (except Calif and N M) provide channels for ARBITRATION of labor disputes; for examples, see Art 11. "YELLOW DOG" CONTRACTS (stipulating membership or non-membership in a labor union as a condition of employment) are specifically prohibited in Ariz, Calif, Colo, Idaho, Mich, Minn, Nev, N J, Ore, Penn, Utah, and Wis. Blacklisting is unlawful in Mont, N M, Okla, and Wash. Financial or other material contributions to COMPANY UNIONS are prohibited to employers by Federal law and by State law in Mich, Penn, Utah, and Wis. This last prohibition relates only to collective bargaining functions of such organizations; it does not outlaw an employer's support or encouragement of local organizations purely for social, educational, or benevolent activities. Decisions by two Fed Circuit Courts of Appeal (late 1939) upheld individual plant unions as legitimate collective bargaining units, provided evidence fails to show that such union is "tied to the management."

National Labor Relations ("Wagner") Act, approved July 5, 1935.

Sec 2: Term "employee" includes all employees, and any individual whose work has ceased as a consequence of, or in connection with, any current labor dispute or because of unfair labor practice, and who has not obtained other regular and substantially equivalent employment. Term "labor organization" means any organization, or any agency or employee representation committee or plan, which exists for the purpose of dealing with employers concerning grievances, labor disputes, wages, rates of pay, hours of employment, or conditions of work. Term "commerce" means trade, traffic, transport, or communication among the several States, or any Territory of the U S, or between any foreign country and any State, or between points in the same State, but through any other State, or any foreign country. Term "affecting commerce" means in commerce, or burdening or obstructing commerce, or having led or tending to lead to a labor dispute burdening or obstructing commerce.

Sec 7: Employees have the right to self-organization, to form, join, or assist labor organization, to bargain collectively through their own representatives, and to engage in concerted activities, for collective bargaining or other mutual aid or protection.

Sec 8: It is unfair labor practice for an employer: (1) to restrain, or coerce employees in the exercise of rights guaranteed in Sec 7; (2) to dominate or interfere with the formation or administration of any labor organization or contribute financial or other support to it; provided, that subject to rules by the Board (N L R B) an employer is not prohibited from permitting employees to confer with him during working hours without loss of time or pay; (3) by discrimination in regard to employment or any term or condition of employment to encourage or discourage membership in any labor organization; provided, that nothing in this Act, or in any other U S statute, precludes an employer from making an agreement with a labor organization (not established, maintained, or assisted by unfair labor practice) to require as a condition of employment membership therein; (4) to discourage or otherwise discriminate against an employee because he has filed charges or given testimony under this Act; (5) to refuse to bargain collectively with the representatives of his employees.

Sec 10 (*a*): The N L R B is empowered to prevent any person from engaging in unfair labor practice affecting commerce. This power shall be exclusive, and is not affected by any other means of adjustment or prevention that may be established by agreement, law, or otherwise.

Sec 13: Nothing in this Act shall be construed to interfere with the right to strike.

Applicability of Wagner Act to an employer appears to be measured by the extent to which either his raw materials or his product enter interstate commerce. "The question is necessarily one of degree" (U S Supreme Court). N L R B has taken an expanded view of its jurisdiction, but has been upheld by Federal courts in several cases within mining industry where the employer's dependence upon interstate commerce was fairly obvious (Jones & Laughlin, Eagle-Picher, Alaska Juneau); also in a few cases where as little as 25% of employer's product went outside of his State. In the Idaho-Maryland case (Calif) the Fed court vacated a cease-and-desist order from N L R B, on showing that the company's entire operations, including purchase of supplies, were confined to Calif, its bullion being sold to San Francisco mint and its concentrates shipped to Selby. The fact that some of its supplies and equipment (though purchased in Calif) were manufactured elsewhere was held immaterial to the case. On lapse of legal period for filing an appeal, this decision is now binding in the 9th Fed circuit, including Calif, Oreg, Wash, Ariz, Idaho, Mont, and Nev, and would also carry weight in other circuits dealing with similar cases.

Unfair labor practices by employees. Federal law is silent on this subject. Four of the 5 mining States which (to late 1939) have enacted "little Wagner Acts" have added provisions defining unfair practice by employees as well as by employers. PENN (as amended and approved June 9, 1939) makes it unfair practice for a labor organization: (*a*) to intimidate or coerce any employee with intent to compel him to join or refrain from joining any labor organization, or to influence his choice of bargaining representatives; (*b*) to join a sit-down strike, or to seize or damage an employer's plant with intent to compel him to accede to terms of employment, including a demand for collective bargaining; (*c*) to intimidate or coerce any employer by threats of bodily force or violence with intent to compel him to accede to demands. MICH LABOR RELATION ACT (June 8, 1939) makes it punishable as a misdemeanor: (*a*) to take or withhold possession of property against the will of its owner or rightful occupant; (*b*) by intimidation, to force any person to become or remain a member of a labor organization, or to refrain from engaging in employment. MINN LABOR RELATIONS ACT (Apl 22, 1939) includes in its definition of unfair labor practice both of the misdemeanors recognized by Mich, and adds: (1) to institute a strike in violation of a valid collective agreement with which the employer is complying, or without serving the prescribed 10-days' notice upon employer and State Conciliator; (2) for any person to picket a place of employment of which he is not an employee, unless a majority of persons picketing are employees at the place. WIS EMPLOYMENT PEACE ACT (May 3, 1939) makes it an unfair labor practice for an employee individually or in concert with others (or for any other person): (*a*) to intimidate an employee in enjoyment of his legal rights (including those granted by other sections of the same Act); (*b*) to intimidate an employer into interfering with those rights; (*c*) to violate terms of a collective bargaining agreement (including an agreement to accept an arbitration award); (*e*) to refuse or fail to accept the final determination of any tribunal having competent jurisdiction; (*e*) to engage in picketing or boycotting unless, by secret ballot, a majority of the collective bargaining unit has voted to strike; (*f*) to hinder or prevent by force or coercion the pursuit of any lawful work; (*g*) to engage in secondary boycotts; (*h*) to take unauthorised possession of employer's property, or engage in concerted effort to interfere with production, except by peaceably leaving the premises.

Internat Union of Mine, Mill, and Smelter Workers, an outgrowth of the old Western Federation of Miners and now affiliated with Congress of Industrial Organization, with over 50 000 members (Sep, 1938), is predominant labor organization in U S metal mining districts; in 1937, it signed 97 agreements with employers. Following data (63) refer to MINING AGREEMENTS; smelter and refinery agreements are similar in principle, but differ in some details. CLOSED UNION SHOP is usually stipulated; Co is usually required to furnish the Union with a complete list of its employees within 1 week after the end of each month, and to notify any employee whose good standing is questioned; such employee, if still delinquent after 10 days, is not permitted to work. Check-off collection of Union dues is not usually required (1 such case in 1937). Several Mont mines agree to employ only local men, so long as they are satisfactory to Co. WAGES are usually on a sliding scale with metal prices (Art 2). Over-time draws 1.5 times regular rate; double-time on holidays; 8-hr day is standard, including 30 min for lunch. In some mines, 50¢ extra is paid for wet work. Work between regular shifts is usually paid on guarantee of 4 hr at regular rate. At some mines, engineers and pumpmen are permitted to work 7 days a week, with privilege of arranging for annual vacation.

Some agreements limit night-shift work to 2 weeks at a stretch. Nearly all require provision of change-houses, baths, and first-aid equipment. Some agreements forbid a miner to work alone at actual mining underground; in 1 case, use of machines is required when practicable. Seniority is the usual rule governing employment. All agreements arrange for SETTLEMENT OF GRIEVANCES. A dispute which can not be settled with the foreman may be referred to mine manager or higher official. On disagreement as to facts, a Union committee may make an examination jointly with a Co representative. Men who have been discharged without cause are guaranteed back pay. LOCKOUTS AND STRIKES are prohibited before all means of settlement have been exhausted. It is usually agreed that a strike shall not affect pumpmen and other maintenance men so long as Co makes no attempt to resume production; Union usually reserves right to call out such men if negotiations are not completed within 15 days. At each plant of ANACONDA COPPER MINING Co, there is a local industrial-relations committee, with 5 representatives of each party; if 7 members can not agree, the dispute is referred to an executive industrial-relations committee composed of 1 representative from each party from the Butte, Anaconda, and Great Falls plants; agreement by 4 members of this body is final.

Cons Mining & Smelting Co of Canada (64) offers a notable example of harmonious relations maintained since 1918 under auspices of a cooperative committee composed entirely of employees; earlier complete affiliation with Western Federation of Miners has since been almost wholly abandoned, though membership in outside unions is not restrained.

Committee members, 1 representative from each department, or each shift in a large department, are elected by secret ballot every 6 mos for term of 1 yr (to allow overlapping). Committee elects its chairman, vice-chairman, and secretary; chairman appoints 5 standing sub-committees and others when necessary. Out of aver 5 000 employees, over 300 had served on committee to 1935. No rigorous by-laws have ever been adopted. Committee meets twice a month, about 1/2 on its own and 1/2 on company time; no representative of Co management attends meetings, unless invited. Committee assists management in adjustment of nearly all grievances, and, in case of a complaint deemed not well founded, assumes responsibility for adverse decision. It also co-operates with, but does not control, the employment department. Applications for EMPLOYMENT are received only at Trail (to ensure uniform consideration); Canadians, and sons or relatives of present employees receive preference; applications are not accepted when opportunity to work is likely to be unduly delayed. Applicants must read English, and pass medical examination. Except skilled tradesmen, all new employees enter a general "yard" department, from which other departments draw when necessary; "yard" also absorbs those temporarily not needed elsewhere. Foreman may discharge a man for a serious offense, but must file statement of case with employment manager; latter may uphold foreman or alter the discharge to an "open transfer." Foreman also may issue an OPEN TRANSFER to an unsatisfactory employee, who may then be re-employed by another department; a third transfer within 2 yr from the first means automatic discharge; such discharges amount to less than 2% of the annual turnover. Minimum WAGES are set in relation to local costs of living, subject to bonus for better than aver work; bonus also is on sliding scale with metal prices. In all wage agreements, Co reserves right to increase the rate of a workman above his group scale. For promotion, ability is first consideration; of 2 equally capable men, senior in service receives preference. Every employee, at end of 3-yr service, receives a share of company's stock; free, if he is married, or for $12.50 (1/2 par) if single. HEALTH INSURANCE covers hospital and doctor's care for men and their families; employee pays $2.50 and Co contributes about 90c per man per mo. For LIFE INSURANCE, including total and permanent disability, Co pays for a group policy of $1 500 per man; employee may pay for additional $1 000 at group rate. BENEVOLENT SOCIETY is conducted exclusively by employees; dues of $1 per mo draw benefit of $1.50 per day for max of 6 mos in any one year. Co maintains a revolving fund of $750 000 for HOUSING LOANS on 5% mortgages, principal and interest deducted from monthly pay. Applicant deposits 10% of combined value of house and land, and gets clear title in 8–10 yr. About 1 000 houses have been financed thus; applicant has free choice of plans, under advice from Co's building inspector. PENSIONS, administered by 2 Co officials and chairman of co-op comm, are payable at age of 55 yr after 25-yr service, or at 60 yr after 15-yr service; rate, 1% of aver earnings during preceding 10 yr for each year of service; minimum pension, $240 per yr. APPRENTICESHIP system, primarily for sons of employees, starts with pay at 25% of base rate, increases every 6 mos, and reaches full scale in about 4 1/2 yr.

11. ARBITRATION AND CONCILIATION BOARDS

Conciliation Service of the Federal Department of Labor, established Mch 4, 1913, and still functioning as a separate Bureau, authorizes the Secretary of Labor to act as mediator in labor disputes, and to appoint conciliators. It attempts to adjust disputes in industries over which the Federal gov't has no mandatory jurisdiction; its jurisdiction is not exclusive. It intervenes only on request of either party to a dispute or of State authorities, and is confined to mediation and offering suggestions as to proper settlement. It is not a board of arbitration and can not force either party to confer with its agent.

State-sponsored arbitration. Channels for arbitration and settlement of labor disputes are provided by statute in all mining States except Calif and N M. Powers assigned to

authorized conciliation or mediation officers show considerable variation; arbitration is nowhere compulsory (though practically so in Minn), but an agreement to arbitrate is enforceable in Nev, N J, Utah, and Wis. In nearly all States, the designated mediatory officials have court-supported authority to make complete examinations, compel attendance of witnesses, and take sworn testimony. In Colo, Ill, Md, Mo, Wis, decisions of an arbitration tribunal are binding if parties previously agree thereto: in Penn, only upon agreement after the award. In Ill and Mont, an arbitrated decision is binding for only 6 mos or until revoked on 60 days' notice; in Nev, a decision binds for 3 mos, subject to 30 days' notice of withdrawal. Strikes, lockouts, or changes in working conditions subject to dispute, are expressly forbidden in Ala, Colo, Mich, Minn, Mont, and Nev, so long as arbitration is proceeding or about to begin; Ore does not recognize a jurisdictional quarrel between rival unions as a labor dispute subject to its Board of Conciliation. Following examples illustrate procedure in important mining States (65).

Colorado. (1935 stat, ch 97, sec 29–33). "Industrial Commission shall promote the voluntary arbitration and conciliation of labor disputes, and may appoint temporary arbitration boards, prescribe their rules, conduct hearings, and publish reports The Commission may subpoena witnesses and administer oaths. Its jurisdiction shall extend to all disputes between employer and employee and shall continue until entry of final award. Neither party to the dispute shall alter the conditions of employment until after final determination by the Commission. Findings of the Commission shall bind the parties only when they have, in writing, so agreed" (sec 133): "The Commission shall inquire into labor disputes and, if agreement is not reached, shall attempt to have the parties consent to arbitration by 3 arbitrators selected, 1 by employer, 1 by employees, and the third by these two or by the Industrial Commissioner. Findings by the arbitrators shall be final."

Montana. (Revised code of 1935, secs 3055–58). "The state board of arbitration and conciliation shall, on application of the employer, employing 20 or more, or the majority of the employees in the department in which the controversy arises, visit the dispute, hear all interested persons, advise the parties as to adjustment, and make a written decision which must be made public at once. The parties to the application must agree to continue in business without any strike or lockout until the decision of the board, if made within 4 weeks of the filing of the application. Any decision of the board shall be binding on the parties for 6 months, or until either party has given the other written notice of his intention not to be bound at the expiration of 60 days therefrom".

Nevada. (Compiled laws of 1929, secs 2763–69). "Whenever a labor dispute arises between an employer and his employees, threatening to interrupt the employer's business, the governor shall, on request of either party, attempt to settle the dispute by mediation and conciliation. If unsuccessful, he shall attempt to bring about an arbitration. If the parties consent thereto, an arbitration board of 3 members shall be selected, 1 by the employer, 1 by the labor organization of the employees, and the third by these two or the governor. The submission to arbitration shall contain a stipulation that the award shall be filed within 30 days from the appointment of the third arbitrator, during which time there shall be no change in the status of the parties, and the award shall be final, unless set aside for error of law. Employees dissatisfied with the award shall not withdraw therefrom before 3 months from the making thereof, without 30 days' written notice; nor shall employees be dismissed due to the employer's dissatisfaction before 3 months from the making of the award, without 30 days' written notice. The award shall be in force for 1 year." (Sec 510): "Written agreements to submit any controversy existing at the time of the contract to arbitration are valid and enforceable, and may not be revoked without consent of the other party to the agreement, except upon grounds existing in law or equity for the revocation of any contract."

Pennsylvania (1931 stat, title 43, ch 13, secs 691–701), provides that judges of the courts of common pleas shall, upon petition of 50 or more persons employed as workmen by 5 or more separate firms or persons, or at least by 5 employers, each employing at least 10, or by the representatives of a firm employing at least 75, authorize a tribunal for settlement of disputes between employers and employees in the iron, steel, glass, textile, and coal trades. If a suspension of work has occurred, or is probable, the judge shall hear testimony as to the representative character of the petitioners and may deny the authorization if it appears that the petitioners do not represent at least one-half of each party. Tribunal exists for 1 year. An umpire shall be chosen by the 2 employer and 2 employee representatives on the tribunal, and shall act only after 3 disagreements; his award shall be final as to matters submitted in writing and signed by the tribunal or by the parties, and on questions affecting the prices of labor. Award of the umpire shall only bind employers or employees as they agree after the award. Tribunal may administer oaths and sign subpoenas, and attorneys-at-law shall not appear before it. Award shall be made within 10 days of submission of dispute. Labor Mediation Act (No 177, of 1937) permits the Department of Labor and Industry to offer its services for conciliation of disputes, but failure or refusal of either party to submit to arbitration is not a violation of the Act, nor can it be used as basis for other action in law or equity.

Michigan. Public Act 176 (effective June 8, 1939) creates a labor mediation board of 3 members appointed by the governor, selected without regard to political affiliations, for term of 3 years; 2 members constitute a quorum, but all official orders require concurrence of a majority of the board. Sec 8: "It shall be lawful for employees to . . . form, join or assist in labor organization, to engage in lawful concerted activities for the purpose of collective negotiation . . . or to negotiate . . . collectively with their employers through representatives of their own free choice." Sec 9: "If a dispute arises, which the parties thereto are unable to settle, no strike or lockout shall be put

into effect, unless either party shall serve a notice upon the board of such dispute, with a statement of the issues involved. Sec 9a: "For a period of not less than 5 days after the above notice is served, or until the board undertakes the adjustment and settlement of the dispute (if within 5 days) it shall be the duty of both employees and employers to use their best efforts to avoid a cessation of employment or a change in the normal operation of the business, and during said period the parties shall undertake a mediation thereof. Violation of this section shall be a misdemeanor and punishable as such." Sec 10: "After the board has received the above notice, or upon its own motion, in an existing, imminent, or threatened labor dispute, the board may, and upon the direction of the governor, the board must take such steps as it may deem expedient to effect a voluntary, amicable and expeditious adjustment. To this end, it shall be the duty of the board: (*a*) to arrange for, hold, adjourn or reconvene a conference between the disputants; (*b*) to invite the disputants to attend such conference and submit the grievances; (*c*) to discuss such grievances; (*d*) to assist in negotiating and drafting agreements for the adjustment of such grievances and for termination or avoidance of the labor dispute. The board may designate one of its members to act in its behalf, with all the powers hereby conferred upon the board." Sec 11: "The board and each member (or designee) shall have power to subpoena witnesses and compel their attendance, administer oaths, and receive evidence. Subpoenas may be issued only after the mediation of a dispute shall have been actually undertaken. In case of contumacy or refusal to obey a subpoena, the circuit court of any county within the jurisdiction of which the inquiry is carried on, upon application of the board or commission, shall have jurisdiction to issue an order requiring such person to appear before the board or commission, to produce evidence or to give testimony. Failure to obey such order may be punished by the court as a contempt." Sec 12: No member or officer of the board having financial interest or membership in or affiliation with any labor organization in a trade in which a labor dispute exists, or is threatened, and of which the board has taken cognizance, shall be qualified to participate in the acts of the board in connection with settlement or avoidance thereof." Sec 15 declares sit-down strikes, however accomplished, to be misdemeanors punishable as such. Sec 17: It shall be unlawful (punishable as a misdemeanor) for any employee or other person to attempt to force any person to become or remain a member of a labor organisation, or to refrain from engaging in employment.

Minnesota. The Labor Relation Act (laws of 1939, chap 440) closely parallels that of Mich, except that, instead of a board of 3 members, the governor appoints a permanent labor conciliator, and may appoint other conciliators, all with powers like those in Mich; also, the period following the service of notice, during which strikes or lockouts are unlawful, is 10 instead of 5 days. Powers of the conciliators are upheld by district courts. Labor organizations, as well as employers, must file sworn statements giving names and addresses of their responsible officers.

Anthracite strike commission was appointed by President Theodore Roosevelt in Oct, 1902, when a strike in the anthracite districts, already in its fifth month, threatened to cause hardship to consumers during the winter. Operations were thereupon resumed, pending investigation by the commission, whose report (1), Mar 21, 1903, still retains historical interest as underlying many current labor practices in the anthracite field.

Anthracite conferences. Since expiration, in 1906, of Strike Commission's 3-yr award, wages and other details affecting labor have been adjusted at successive conferences between operators' committees and officials of the United Mine Workers, always the preponderating and now the exclusive bargaining agent of the workers. For accounts of the proceedings and their outcomes, see Bib (27, 28, 50, 75), also Edn 2 of this book, Art 11.

Conferences have been held at irregular intervals of 2–5 yr, and were often accompanied by suspensions of mining, sometimes lasting several months; no interruption occurred in 1936. WAGES of contract miners and their laborers were increased 10% by the 1903 award, another 10% in 1912, and an additional 7% in 1916. Successive increases during the World War increased the 1920 rate up to 65% above that of 1916; a further increase of 10% in 1923 fixed a rate since maintained by agreements in 1926, 1931, 1932, 1936, and June, 1939 (the latter to expire in 1941). No code for anthracite wages was adopted (minimum of $4.62 was proposed) during the life of N I R A. WORKING HOURS were reduced from 10 to 9 by 1903 award. to 8 hr in 1916, and to 7 hr in 1937. CHECK-OFF became obligatory (previously general) in 1936 agreement. SLIDING SCALE for wages (based on N Y price of white-ash coal), established by the 1903 award, was abolished in 1912.

Anthracite board of conciliation, created by fourth award of the 1902 strike commission, and still functioning, consists of 6 members, 3 being appointed by mine operators and 3 by any "organization representing a majority of the mine workers" in the 3 anthracite districts (Wyoming, Lehigh, Schuylkill). Full membership of the board is to be maintained, and either operators or workmen may change their representation on the board at any time when a controversy is not pending. Parties to a controversy may be represented by other persons at all hearings before the board. A decision by a majority of the board is final; if a majority is unable to agree, the board requests a U S circuit judge of the third circuit to appoint an umpire, whose decision is final. Suspension of work by strike or lockout, pending a decision by the board, is forbidden.

Board of conciliation, organized at Wilkes-Barre, June 25, 1903 (29), adopted following RULES OF PROCEDURE: (*a*) Aggrieved workman must first attempt to adjust dispute with mine foreman. (*b*) If unsuccessful, he must request interview with company's supt or mgr, for purpose of adjust-

ment. (*c*) If not satisfied with solution offered, workman may then submit his case to the 2 representatives of the conciliation board in that district, accompanied by proof that effort had been made to settle dispute with supt or mgr. (*d*) If supt or mgr has refused an interview within 10 days, the board or its two representatives in the district, will endeavor to secure an interview for aggrieved workman. (*e*) If dispute still remains unsettled, it comes formally before the full board, which then calls upon the operator to give his reasons for not offering an adjustment. Board may then request presence of both parties for a full hearing. (*f*) Grievance of operators against workmen can be submitted at once to board, through its representatives, the board then calling for explanation from the workmen, or requesting presence of both sides for a hearing. (*g*) Board will not consider a grievance of workmen unless they remain at work, with the understanding that awards by the board will be retroactive, dating from time dispute is first opened.

Grievance committee. In the 1912 anthracite agreement, it was stipulated that a committee of 3 employees might be elected by the workmen of each mine, with power to consider complaints which individual workmen had not been able to settle with mine foreman. It was made allowable for the miners' district representative on the conciliation board to confer with this committee. In case of failure of negotiations between committee and mine officials, complaints were then to follow prescribed course before full conciliation board (see above). It was supposed that the committee might add weight to the grievance of an individual workman, thereby increasing probability of a satisfactory interview with mine officials and diminishing the number of complaints to be carried before the full board, which was felt to be too remote and deliberate a body.

Bituminous interstate joint conference, for adjustment of wages and working conditions, was in sustained operation in the central competitive bituminous territory (Ohio, Ind, Ill, and Western Pa) from 1898 until Apr 1, 1927; after lapse of 6 yr, its functions were resumed, Sep 22, 1933, under protection of the N L R A, by a similar conference representing the Appalachian area, extending from central Pa to Ala (29, 75). The following conditions have become firmly established: (*a*) Privilege of workmen to organize and authority of union (U M W) to negotiate agreements on behalf of mine workers are recognized by employers. (*b*) Basing point is designated in each state, thus compensating operators in one state for natural advantages (distance to market, character of coal seams, etc) enjoyed by those in a more favored state. For example, the basic minimum rates for 8-hr work established in 1933 were (75): Northeast, \$4.60 (increased, 1937–39, to \$6 for 7 hr); south, \$4.20; midwest, \$4.575–\$5; southwest, \$3.75; northwest, \$4–\$5.63; deep south, \$3.40–\$3.84. (*c*) Ton of 2 000 lb is standard. (*d*) Run-of-mine basis of payment, formerly optional in some cases, has universally displaced the screened-coal basis since 1913. (*e*) Differentials are established between machine and pick mining, and between thick and thin coal seams. (*f*) Uniform working day of 8 hr, prevailing 1898–1933, was reduced in 1934 to 7 hr for all classes of mine employees.

Conferences are held in Mch every 2 years, at a previously selected city, and are attended by delegates from miners' unions (usually national and state officials of U M W) and by representatives of coal mine operators selected in any desired manner. Committee on credentials, rules, etc, comprise 2 members of each side from each state; scale committee includes an equal number of members from each side, but numbers vary for different districts; unanimous vote is required to adopt an agreement. Miners' demands, having usually been formulated at a shortly preceding annual convention of U M W, are presented in form of resolution, which customarily fails of unanimous acceptance. Resolution then goes to scale committee, where it is discussed seriatim; agreement is seldom reached on any clause, and being reopened in full conference, resolution is further debated and generally fails to pass. It is then referred to a sub-scale committee, the meetings of which are strictly secret. At this point, a counter-proposition is usually offered by the operators, and argument then ensues until a compromise is effected. Report of sub-scale committee is then approved by scale committee, and is finally adopted by the convention; it is then signed by 2 representatives of each side from each state.

Consistent TREND OF MINERS' DEMANDS has been towards: (*a*) Uniform scale of payment for coal in all states within a competitive territory. (*b*) Adoption of run-of-mine basis alone (finally effected in 1914). (*c*) Decrease of premium for pick-mining over machine mining. (*d*) Uniform wages for outside employees.

Bituminous state agreements (29) are in force in all states represented in the interstate conferences, and also in many others, nearly 100% of the coal-mining territory of the U S being governed, as to wages and working conditions, by formal contracts between operators and organized employees. State conventions follow the interstate conferences and are usually held prior to Apr 1; they are attended by officials of U M W and by delegates from the operators.

In those states in which a basing point has been designated by interstate conference, principal work of convention consists in establishing a system of COAL MINING RATES, ranging both above and below the basic rate, and so adjusted that miners in a given district (those who break down or produce coal) shall receive as nearly as practicable the same returns for a day's work. Factors involved in adjustment of rates are: (*A*) Pick *vs* machine mining; bonus in favor of former is intended to compensate for slower and more laborious nature of operation, but miners constantly

advocate raising the machine-mining rate, for 2 reasons: (a) To discourage use of machines and thereby add to number of miners employed. (b) Chain undercutters produce less slack coal than pick mining and thereby add to operator's profits, to a share of which the miners consider themselves entitled. (B) Thickness of seam; higher rate per ton for thin seams, to compensate for smaller tonnage per unit area undercut. OTHER SETTLEMENTS effected by state conventions relate to: (a) Prices for narrow work and room turning. (b) Wages of inside workmen other than miners; these wages are practically fixed on a uniform minimum basis by the interstate conference, and are accepted, usually unchanged, by state convention in that territory. (c) Wages of outside workmen. (d) Frequency of pay-days, and details as to issuing of wages, statements, etc. (e) Hours of labor; a 7-hr day is generally considered to mean 7 hr spent at working place, exclusive of lunch and traveling time. (f) Provisions for timbering working places, disposal of mine refuse, removal of waste from loaded coal, etc. (g) Loading and firing of shots; generally stipulated that practice must follow legal requirements of the state. (h) Disciplining of miners for sending out inferior coal, for ignoring or disobeying rules, for absenting themselves on working days, etc; in this connection, the Union usually promises its support and cooperation in enforcement of discipline. (i) Procedure for settlement of disputes; this is closely analogous to the present practice in the anthracite fields (see Conciliation Board, and Grievance Committee, above), except that final resort is to a conference between officials of the company and state officials of the U M W. (j) "Check-off" system, withholding Union dues or fines from a workman's wages, is generally accepted by the operators in states where agreements are in force. (k) Prices and quality of powder, oil, tool sharpening, etc.

WELFARE

12. WASH AND CHANGE HOUSES

Change houses, usually provided with bathing facilities, have become a recognized necessity at all mines employing more than a few men. Some companies have hesitated to incur expense of such building, through apprehension that its privileges would be abused or unappreciated; statistics covering several thousand miners provided with such facilities indicate that 85% use the wash house regularly. Use of change and wash house is commonly offered gratis, but one company found that the levying of a small charge, 50¢ per mo, induced better appreciation of the benefits afforded.

Structural and operating details (30, 66). In designing a composite change and wash house, following points deserve consideration: (a) Should be situated as closely as convenient to mine opening; in a cold climate, a tunnel or covered passageway from top of shaft is advisable. (b) Structure must be fireproof, commonly of brick, tile, concrete, or sheet steel on wood frame. (c) Floor space, of changing room only, should be 15 to 20 sq ft per man to be accommodated. (d) Floor and inside walls free from obstructions, the former preferably of concrete sloping 0.25 in per ft towards drain, so that entire interior can be washed with hose. (e) If lockers are used, they should be not less than 12 by 16 in by 3 to 5 ft high, of open metal construction and provided with combination instead of key locks; tops should slope steeply, for ease of cleaning, and to prevent piling rubbish on them. (f) Drying racks standing on floor, or hooks and lines suspended from pulleys spaced 30 in apart in ceiling, are recommended for hanging mine clothes, but should not be used for street clothes; if lockers are used for mine clothes, advisable to give every man 2 lockers. At Noranda, Quebec (66), wire-netted lockers for mine clothes are partly depressed below dressing floor, their hinged and locked wooden-covered tops serving as seats between rows of street-clothes lockers; forced ventilation passes down through these lockers into ducts on lower floor. (g) Ample heat must be provided for drying mine clothes; usually in steam coils running underneath lockers or drying racks. (h) Adequate ventilation is essential because odors and dust are abnormally oppressive; natural ventilation through cupolas is usually sufficient, but artificial and automatically controlled ventilation has been installed in some modern dry-houses; in latter case, it is advisable to admit warm air at ceiling and exhaust through floor, to avoid raising of dust. In the Eureka change house (Fig 1) 8 000 cu ft of fresh air per min displaces entire volume of "dirty" side in 3 min. (i) Desirable for wash room to be separated from change room, preferably by a short passageway having spring door at each end. Some recent change houses are divided by a longit wall into "clean" and "dirty" sides (Fig 1), communicating only through shower rooms; men in mine clothes not permitted on the "clean" side (19). (j) Shower baths most advantageous, but wash basins should also be provided; tubs and swimming pools are unhygienic in this connection; 1 shower booth, 4 by 5 ft, for every 20 lockers is considered adequate if men do not all use wash house at same time. Where a large number of men come off shift at once, as at McIntyre Porcupine mine, Ont, congestion can be avoided by installing continuous sprays in a long, narrow passageway, starting with warm water at one end and finishing with a cold dash at the other; before entering here, men have

opportunity for soaping and scrubbing in a larger space, also supplied with warm water (20). (*k*) About 40 gal water per bath should be provided, but can be reduced by careful supervision; if hot and cold water are separately brought to the booths, pipes should be of ample size to avoid fluctuation in temp when an adjoining bath is opened or closed; mixers for

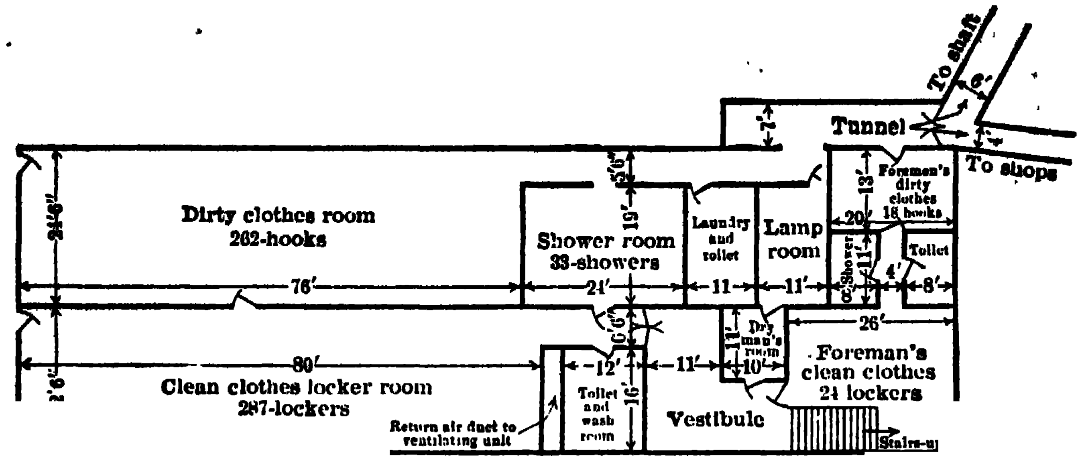


Fig 1. Plan of Eureka Mine Change House, Ramsay, Mich (not to scale)

hot and cold water should be carefully designed; Fig 2 shows one such installation. (*l*) A capable attendant, constantly on duty, will readily justify his wages. TOILETS, specifications for, see Art 15.

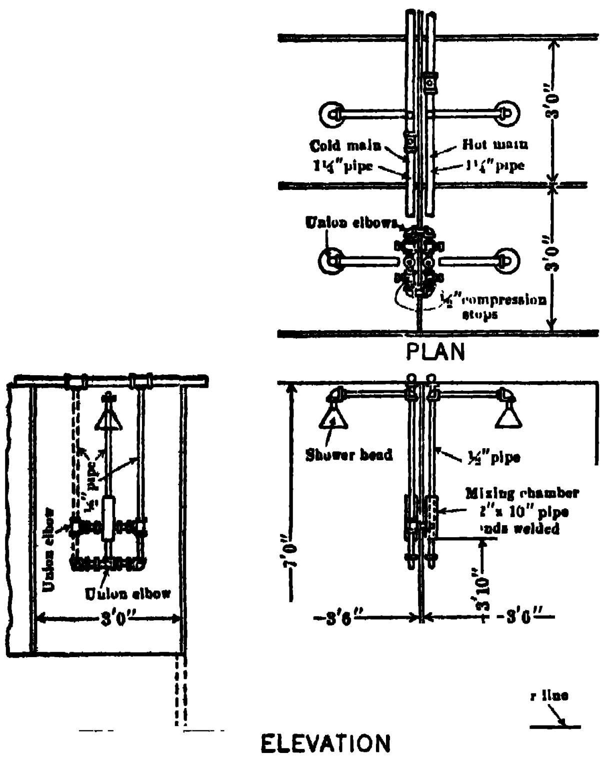


Fig 2. Fittings for Shower Bath, Tenn C, I & RR Co

Detection of "high-grading." Stealing of rich ore by miners has been a notorious source of loss at Bendigo, Kalgoorlie, Kimberley, Cripple Creek, Cobalt, Goldfield, and elsewhere; rich ore is brought out, a few lb at a time, in clothing and dinner pails, among other schemes, and is sold to illegitimate assayers or to prospectors who may operate a small worthless mine simply as a blind. Courts and statutes have proved incapable of stopping this practice. Only practicable method is to require all miners to change clothes as they come off duty, preferably keeping mine clothes in one room and street clothes in another. This procedure has been strenuously opposed by organised labor, and led to a strike at GOLDFIELD in 1907. A compromise was there effected on following basis: Each miner was allotted 2 adjoining lockers; coming from mine, working clothes were removed and placed in one locker; other locker was then opened and street clothes put on; company's time-keeper and other designated watchers allowed to be present, but no strangers. At McIntyre Porcupine mine, Ont, a miner coming off-shift puts his lunch box on a belt conveyer passing through inspection room, and recovers it on leaving the change house (20).

13. MINE COMMUNITIES AND MINERS' DWELLINGS

Advantage of judicious planning of a mining community, to promote healthfulness, cheerfulness, and convenience, is as important from the financial as from the sociologic point of view; a healthy and contented population is both more efficient and less transient. TOWN SITUATED NEAR MINE OPENING has following advantages (31). Shorter distance to walk, entailing less loss of energy in going to work, a serious factor in many mountain dis-

tricts, and less exposure in wet clothes after work, in absence of change house. Having homes, stores, and pay-office nearby is convenient for housekeepers. Responsible officials are within easier reach at all times. Where mining company owns the townsite, proximity to mine plant permits some economies in cost of haulage, lighting, heating, and fire protection. TOWNSITE AT DISTANCE FROM MINE can usually be selected with better regard to economy in construction as well as desirability of situation. Wider spacing of houses pro-

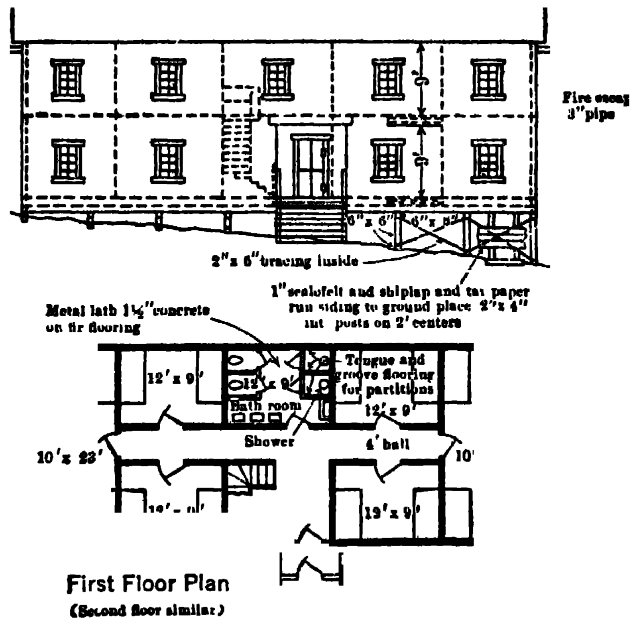


Fig 3. Two-story Bunk House, Flin Flon, Manitoba (67)

motes health and reduces fire risk. Gentle slopes reduce cost of grading streets and erecting houses. Absence of noise, smoke, and dirt is genuine asset. Permissible distance from mine depends upon facilities for conveyance; electric cars in many districts have greatly enlarged the radius of daily travel; some mines operate work trains, and some steam railroads run special cars at reduced fare for mine workmen.

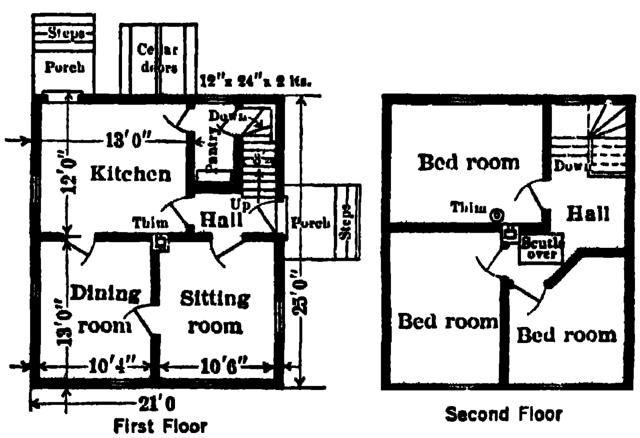


Fig 4. Six-room House, Trimountain Mine, Mich

In locating a new townsite, the following features deserve attention, though all may not be practicable. (*a*) Accurate contour map is indispensable for efficient planning. (*b*) Principal thoroughfares should be approx parallel to contours, involving less expense for grading, bridging, sewers, and surface drainage. (*c*) Narrow but well surfaced street, with gutters and sidewalks, is more suitable for requirements of an ordinary mine town than wide street in poor condition. (*d*) Trapezoidal house lots, fronting on a diagonal street, afford better air and light to each house than rectangular lots with houses all at same distance from street. (*e*) Deep, narrow lot involves less expense for street improvements and permits better utilization of land than a wide shallow lot of same area. (*f*) On closely spaced, deep lots, houses should not all be at same distance from

street; 2-story houses should be farther back than single-story. It is desirable to alternate single and 2-story houses. (g) Houses on both sides of a narrow street should not be directly opposite one another. (h) Along a contour street on a hillside, 2-story houses should be placed on down-hill side and should be as narrow, from front to back, as convenient. Lots on down-hill side may be graded up to level of street, by setting retaining wall under front of house instead of at front of lot. (i) Houses on steeply sloping lots are always expensive and usually inconvenient. Steps and verandas of hillside houses should be at ends rather than at front or back.

Principles of design. A company should have a number of different house plans for: variety of appearance, better utilization of land, diversity in rental. Number of houses of each size can be estimated by investigating size of families in neighborhood. Most work-

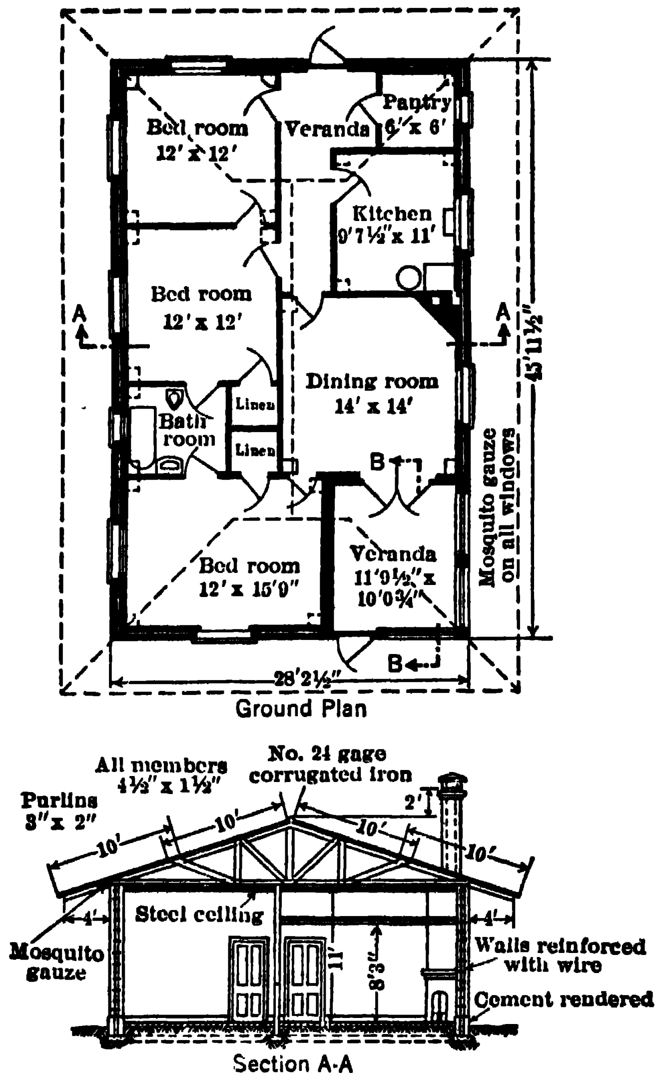


Fig 5. Five-room Brick House, Roan Antelope Mine, Luanshya, Nor Rhodesia (67)

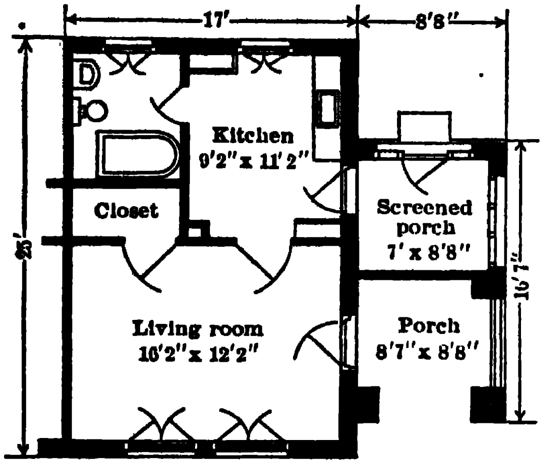
men's houses are too small; probably 40% of foreign workmen in U S use all but 2 rooms of house for sleeping. Kitchen, being most important room in workman's home, should be larger than commonly accepted, and situated on pleasantest (south or east) side of house. Houses of more than 4 or 5 rooms should be of 2 stories, to reduce area of roof and foundation and permit better lighting.

Materials of construction. WOODEN FRAME houses have following advantages: (a) variety in design more easily attained; (b) more readily enlarged; (c) more cheaply constructed; (d) better decorative effects. Certain lumber firms specialize in lumber ready out for erection according to standard patterns, economizing in carpentry and waste. POURED CONCRETE, now widely used (notable examples, Gary, Indiana, and "Concrete City," at Truesdale mine, near Nanticoke, Pa), has advantages of being durable, fireproof, weatherproof and relatively vermin proof. Unless precaution is taken to leave air space in walls, they are likely to condense moisture. MOULDED CONCRETE BLOCKS can often be made cheaply from mill tailings. Variety of appearance attained

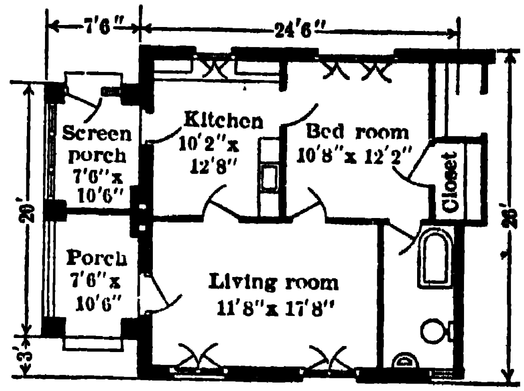
by facing mould with tailings of different coarseness. BRICK AND HOLLOW TILE are excellent for better class of houses, if cheaply obtainable.

Owing to great variety of equally desirable plans, it is not practicable to include here details of either design or construction; see Bib (32, 67, 69).

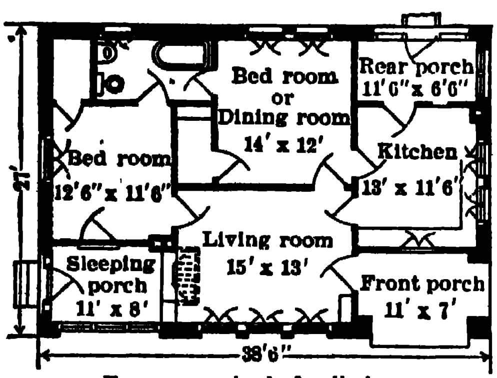
Cost of erection. Table 5; for more complete specifications on these examples, and on several others for which costs are not given, see Bib (67).



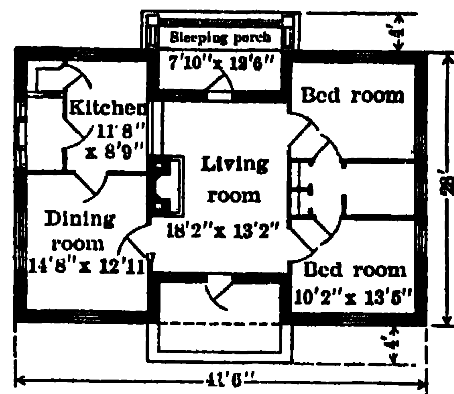
Half of double house having two two-room apartments



Half of double house having two three-room apartments



Four-room single-family house



Five-room one-family house

Fig 6. One-floor Houses, Phelps Dodge Corp, Tyrone, N M (67)

Keast and Jackson (68) give data in Table 6 on preliminary CAMP STRUCTURES erected in Feb and Mar, 1930, to accommodate 60 men developing the Central Patricia claims in northern Ontario; lumber was sawn on the property but all other supplies were hauled 100 miles by tractor and sleds; all buildings framed with 2" × 4" studs, sills, and joists, and covered with a good grade of roofing.

Housing in cold climates. Following observations by Clement and Govan (69) are based on experience in northern Ontario. Benefits anticipated from wall and roof insulation may be largely nullified by poor, leaky construction around doors and windows. As defined here, thermal "resistance" is the time, hr, required for 1 btu to pass through 1 sq ft of wall area when temps on opposite sides differ by 1° F. Resistance of good commercial insulators is 2.5–4 hr per in of thickness, or 10–12 hr for customary 3 to 4-in thickness. Resistances of some common, uninsulated walls (Am Soc Heating and Ventil Engrs) are as in Table 7. Addition of 1 layer of thin-board insulation to such walls decreases heat losses by 20–25%; but, as, in such houses, fuel consumed to supply heat losses through walls and roof is about 60% of total consumption, actual saving by such insulation is only 12–15% in fuel. By more suitable and readily available insulators, properly applied, savings of 50–70% in heat losses are economically possible, reducing fuel consumption by 30–40% or more. As fire retarders, the loose varieties of mineral insulator are worthless.

Table 5. Cost of Erecting Mine Dwellings. Data from A. H. Hubbell (67)

Locality and date	Ground area, ft	Rooms (a)	Cost: Per sq ft	Cost: Per cu ft	Specifications
Luanshya, No Rhodesia	46 × 28	5	$3.40	......	No cellar; walls local brick; floors concrete; inside walls plastered brick; ceilings pressed steel; Oregon pine joists; roof corrug galv iron.
Achotla, Guer, Mex	45 × 28	4	1.60	......	No cellar; stone foundations; adobe walls; concrete floor; tile roof.
Monterrey, Mex	43 1/2 × 31	5	1.60 (b)	......	No cellar; stone foundation; outside walls slag-brick; inside, hollow brick; double wood floors; ceiling beaver board; roof 3-ply asphalt.
Matahambre, Cuba	28 × 30	4	3.57	......	Hardwood posts and sills; pine studs and floor; sides 24-gage iron; partitions battened boards.
	29 × 44	10 (2 fam)	1.20	......	
	80 1/2 × 38	20 (bunkh)	0.73	......	
Pecos, NM (c) 1925–1926	26 × 12	2	1.66	......	Mud sills; double wood floors; 2″ × 4″ studs; walls sheathed and covered with Slate Cote roofing; partitions wallboard on 2×4 studs; ceilings same on 3 × 4 joists; brick chimney single thick; elec wiring; all screened.
	34 1/2 × 14	3	2.13	......	
do	96 × 56	5	0.80	......	
do	48 1/2 × 22 1/2	8 (bunkh)	1.56	......	
Tyrone, N M, 1915–16 For Americans	34 × 25	4 (2 fam)	3.15	......	Small cellar; concrete foundations; walls 8-in hollow tile stuccoed outside; wood studding, metal lath and plaster inside; flat roof with 3-ply asbestos; plumbing and elec fixtures; porches screened.
	49 × 26	6 (2 fam)	2.86	......	
	41 1/2 × 28	5	2.90	......	
	38 1/2 × 27	4	2.45	......	
Tyrone For Mexicans	27 × 24	3	1.76	......	About same, except no cellar; floors wood or cement
	31 × 24	4 (2 fam)	1.84	......	
FlinFlon, Manit	58 × 24 (2 stories)	12 (bunkh)	5.75	$0.32	No cellars; framed with BC fir; walls shiplap outside 2″ × 4″ studs, roofing material inside, and space filled with Insulex; roof asbestos on BC fir boards over Celotex; plumbing and elec fixtures.
do	28 × 23	3	5.75	0.64	
do	36 × 28	5	5.15	0.57	
Chuquicamata, Chile, 1925	112 × 35	18 (bunkh)	2.84	0.28	No cellars; concrete foundations; frame Douglas fir; outside wall metal lath and cement stucco; inside walls and ceilings sheet iron; roofs corrug iron; water and elec fixtures in all but the 3-room apts.
	433 sq ft	3 (d)	2.35	0.24	
do	26 1/2 × 23	4 (e)	4.46	0.44	
do	125 × 32	20	5.00	0.56	
do	43 1/2 × 30	7	5.30	0.49	
Copperton, Utah 1926–30	46 1/2 × 30	5	4.00	0.63	Full basements; walls brick, or stucco on hollow tile; metal work copper or brass; Flintkote copper-clad shingles; hot-air furnaces; water and elec fixtures .
	35 1/2 × 28	4	5.10	0.64	
do 1929	48 × 24	8 (bunkh)	3.00	0.39	
Ruth, Nev, 1924	96 × 29	12 (bunkh)	1.22	......	Frame; 2″ × 4″ studs, 2″ × 6″ joists; inside walls compo board; wood floors; water, elec, and steam heat.
1929	33 × 27	4	2.80	......	
Conda, Idaho, About 1927	32 × 20	4	2.81	......	Concrete foundations; frame Douglas fir; roof and siding pine or cedar; shingles copper-clad compo; inside walls gypsum board or plaster; water, elec, sewage fixtures.
Arvida, Quebec, 1926	26 × 20	6 (f)	9.00	0.37	Full concrete basements; frame BC fir; cedar siding; asbestos or aluminum shingles; floors fir or birch; inside walls plaster board and plaster, painted; water, and elec fixtures; roof and outside walls thoroughly insulated. Variations in cost due principally to inside trim and heating equipment.
do	26 × 26	6 (f)	8.58	0.31	
do	25 × 20	6 (f)	10.00	0.42	
do	36 × 30 1/2	9 (g)	12.75	0.48	
do	26 × 24	6 (f)	16.00	0.49	

(a) Excl bathrooms, closets, and porches. (b) At 1 [illegible] 50¢. (c) Inexpensive construction for expected short life. (d) In blocks of 9 apartments, one of which has 4 rooms. (e) In blocks of 6 apartments. (f) On 2 floors. (g) On 3 floors.

Table 6. Camp Buildings at Central Patricia, Ont.

	Floor area, ft	Cost Total	Cost Per sq ft		Floor area, ft	Cost Total	Cost Per sq ft
Bunkhouse 1	18 × 20	$218	$0.60	Residence	24 × 24	766	1.33
Bunkhouse 2	20 × 30	875	1.46	Storehouse 1	20 × 40	701	0.88
Bunkhouse 3	16 × 18	155	0.54	Storehouse 2	18 × 24	252	0.58
Bunkhouse 4	32 × 20	316	0.49	Meat-house	24 × 30	613	0.85
Cookhouse	50 × 24	1 694	1.41				

unless confined on both sides by incombustible materials such as gypsum or transite board, or asbestos paper on wire mesh.

Table 7. Thermal "Resistance" of Uninsulated Walls

9-in solid brick, 1/2-in plaster on furred wood lath	4.8 hr
8-in concrete " " " " " "	4.8 "
20-in stone " " " " " "	5.0 "
8-in hollow tile, stucco outside, 1/2-in plaster on furred metal lath	5.3 "
Hollow-concrete block, 1/2-in plaster on furred wood lath	5.0 "
Brick veneer on wood frame, wood lath and plaster	4.6 "

Type of small house most economical to build and maintain in cold climates has 1 1/2 stories, without dormers or other breaks in roof; objection to stairs can be largely overcome by putting bathroom on first floor. Cost of such houses in northern Canada (1928), complete with plumbing and electric fixtures, was: for 4 rooms and bath, $1 600–2 200; 4–6 rooms and bath, better finished, $2 600–3 200. Largest saving in cost is by omitting cellars or making them less than full size; foundations may be concrete piers or creosoted cedar posts, latter surrounded, within their pits, by broken rock or coarse cinders. Floor, well insulated, is high enough above ground to permit free access beneath; wooden side walls are carried to ground. For interior partitions, Clement and Govan prefer 3/8-in mineral wall-board to lath and plaster, because of its superior rigidity; the interior bracing afforded by such partitions may permit omission of outside sheathing. For wall and roof insulation, same authorities recommend a gypsum product, containing ingredients which, when wetted, evolve bubbles; the mixture is poured wet, solidifies in about 30 min to a porous mass weighing (when dry) about 8 lb per cu ft, but requires 3–4 mos for thorough drying. Successful use of this poured insulator may involve slight modifications in design of wall-plates, door and window frames, to insure complete filling between studs or joists. With any form of insulation, it is specially important to avoid air pockets on the cold side, where moisture may condense.

Above ideas governed erection of bunkhouse and cottages at a mine 475 miles north of Toronto. In both designs, 2″ × 3″ studs replaced usual 2 × 4's, and 2″ × 4″ joists replaced 2 × 6's. Inside walls and partitions of 3/8-in gypsum board; outside walls ship-lapped only (no sheathing). Wet-poured Insulex was 2.75 in thick in outer walls of cottages and partitions of bunkhouse; 3.75 in thick in outer walls and 4 in under flat roof of bunkhouse. COTTAGES, 1 1/2 story, with 6 rooms, storage room, and bathroom, cost $3 000 (wages of carpenters and electricians, 72¢ per hr; other labor, 53¢ per hr; lumber, $36–44 per M). A single stove or heater in living room kept whole house (9 100 cu ft) comfortable (outside temp sometimes −42° F), with 1/3 to 1/2 the fuel consumed in neighboring non-insulated houses of same size. Two-story BUNKHOUSE, 103.5 × 27.25 ft, with 8-ft ceilings, accommodating 50 men in 32 bedrooms, 2 common rooms, and 2 bathrooms, cost $12 000. All heat for bedrooms came through doors or transoms from longit central halls (1 on each floor) having combined radiator area of 660 sq ft. For plans and some structural details, see Bib (69).

14. PURIFICATION OF DOMESTIC WATER SUPPLY

Impurities in domestic water (33) are not all equally objectionable; probably many supplies which are physiologically wholesome, or could easily be made so, are rejected for purely æsthetic reasons. Most objectionable features are turbidity, color, taste, odor, and pathogenic bacteria; ordinary hardness is objectionable for laundry and boiler supply, but not hygienically. TURBIDITY, most noticeable in river water, can be removed by storage, coagulation, or filtration. COLOR is characteristic on water-sheds comprising swamp areas; produced also by decomposition of vegetable matter in bed of reservoir and by growth of algæ on water surface; seldom deleterious, but may become obnoxious at time of spring and autumn "overturn," when bottom layer of water, highly charged with organic extracts, mixes with whole body of water in reservoir. Color may be partly corrected by filtration, and entirely eliminated by chemical means. TASTE results from same causes as color and also from presence of iron in excess of 1 part per million; iron can be removed by aeration followed by filtration. ODORS arise from same causes as color, and from presence of sewage and industrial wastes; they can be corrected by storage, aeration,

or chemical methods; except for its offensiveness, water having tastes and odors may produce only negligible physiological effects. HARDNESS is known as "temporary," calcium or magnesium carbonate held in solution by presence of CO_2, and precipitated when latter is eliminated by boiling or by addition of quick-lime; and "permanent," mainly sulphates and chlorides of calcium and magnesium, requiring more elaborate chemical processes for their removal.

Contamination by sewage, from hygienic standpoint, is not necessarily objectionable of itself, but possibility is always present that domestic sewage may contain pathogenic germs, notably those of typhoid fever and cholera. These germs, when carried by water, are less tenacious of vitality than BACILLUS COLI, which is always present in enormous quantities in human intestines and in those of some animals; hence, substantial absence of bac coli is considered a sure indication that no pathogenic germs are present. Presence of CHLORINE, beyond that normally contained in domestic water near seashore or salt-bearing springs or strata, is strong presumptive evidence of contamination by sewage. NITROGEN in various compounds may result from vegetable decomposition, but is generally considered proof of sewage pollution, and the nature of its combination indicates the probable length of time elapsing since introduction of sewage; the probable virulence of pathogenic germs can then be deduced. Stages in decomposition of organic matter are: albuminoid ammonia, free ammonia, nitrites, nitrates. Hence, a high proportion of albuminoid ammonia in a colorless water indicates recent pollution; high nitrites, free ammonia, and chlorine occurring together are strong evidence of continuous pollution by sewage. Following figures, in parts per 100 000 (Mass State Board of Health, 1913) show difference between pure and polluted water:

Potable and Polluted Waters of Massachusetts

	Ammonia		Nitrogen as		Chlorine
	Album	Free	Nitrite	Nitrate	
Boston, surface origin	0.0145	0.0015	0.0000	0.0062	0.35
Springfield, surface, filtered	0.0080	0.0009	0.0000	0.0039	0.15
Lowell, tubular wells	0.0064	0.0377	0.0001	0.0162	0.38
Blackstone river, at Worcester	0.1286	0.9320	0.0084	0.0158	4.49
Hoosic river, below North Adams	0.0489	0.0638	0.0024	0.0053	0.88
Merrimac river, above Lawrence	0.0224	0.0245	0.0007	0.0167	0.57

Interpretation of nitrogen determinations should be made by a sanitary chemist, and cover a considerable length of time; remember that principal purpose of these determinations is to indicate probability of presence of pathogenic germs.

Total number of BACTERIA is determined by microscopic count in 1-cc sample, careful note being made of the number of times bac coli is observed in a series of such samples. Bac coli may appear in 1 cc as often as once in 20 times (Cincinnati, 1912) without arousing apprehension of typhoid, but the possibility of typhoid and cholera germs in polluted water must always be considered. In 1912, N Y water supply showed bac coli once in every ten samples (1 cc) but death rate from typhoid was 10 per 100 000; at Baltimore, same year, bac coli was found once in every 100 samples, typhoid death rate being 22. Bacteria, both pathogenic and harmless, can be nearly eliminated by storage and filtration, and completely devitalized by ozone or hypochlorites.

Storage serves 2 purposes: (*a*) equalization of supply; (*b*) purification. For discussion of storage requirements under different conditions of rainfall, run-off, and consumption, see Sec 38; also Bib (34).

Opinions differ as to necessity for stripping all vegetable mould from bed of a reservoir under construction, but this precaution probably diminishes opportunity for color and odor. Reservoir should preferably be narrow and deep, rather than wide and shallow, so that fluctuation in water level between spring and autumn will expose less area for growth of weeds. Supply from reservoir should be drawn from just below surface; if excess water can be wasted, it should be from bottom.

Purification by simple storage is effected by: (*a*) giving sediment (either natural or produced by coagulant) an opportunity to settle, taking majority of bacteria with it; (*b*) diminishing bacterial life by action of sunlight and air (bacteria can be almost completely devitalized by storage in open reservoirs for 48 hr); (*c*) expelling odors by action of wind, and color by action of sunlight.

Growth of ALGÆ is a drawback to extended storage, although its effect is limited to non-deleterious production of color and odor; algæ can be prevented by treatment with $CuSO_4$ (1 part in 4 to 10 million parts of water), or, in case of small reservoirs, by covering with light-proof roof. For recognition of the varieties of microscopic organisms causing tastes or odors, the proportions of $CuSO_4$ required to suppress them, and methods for applying it, see Bib (70). Simple but effective method for reservoirs is to tow from a boat a burlap bag containing $CuSO_4$ crystals.

Aeration, accomplished by fountains, cascades, or flowing over dam, is useful for expelling odors; also, when followed by filtration, for removing iron.

Coagulation is a valuable help for curing turbidity and diminishing bacteria, when followed by storage or filtration. Coagulant commonly used is alum, 1 gr per gal, added to stream of water entering first storage reservoir. Coagulating reaction of alum requires presence of lime, which must be added if not originally present in the water.

Temporary hardness (35) can be reduced by addition of slaked lime in quantity sufficient to neutralize the CO_2, which holds $CaCO_3$ and $MgCO_3$ in solution (say 1 lb per 1 000 gal water of 10 B hardness). Water which is both turbid and hard, requiring an excessive amount of alum, can be more cheaply clarified and softened by lime.

Filtering (36), when properly conducted, will remove all turbidity, practically all bacteria, but less than one-half of color. Two general methods are: (*a*) slow sand filtration; (*b*) pressure or mechanical filtration.

Sand filters. Construction: a shallow reservoir, with water-tight sides and bottom, is made of clay, or more advantageously of concrete, sloping towards one corner. Area may vary from 10 000 to 40 000 sq ft; small bed requires more frequent cleaning, but can be more cheaply protected against freezing; advisable to build at least 2 beds, for alternate cleaning. Radiating system of tiles with open joints is laid down, leading to discharge point. Tiles are next covered by 6-in layer of coarsely broken stone; 5 successive 2-in layers of crushed and screened stone or gravel, of diminishing coarseness, are then placed, surmounted by a layer 30 to 40 in deep of sand. Best sand for upper layer is of clean, sharp quartz, 0.3 to 0.4 mm diam, and of nearly uniform size. Water to be filtered is allowed to fill reservoir to depth necessary to cause percolation at the desired rate of flow, say, 4 in per hr descending velocity; the slower the percolation, the more complete the removal of bacteria. When upper layer of sand becomes clogged, its surface is scraped and removed to depth of 1 or 2 in; when thickness of sand remaining is reduced to 12 or 16 in, fresh sand may be added or the removed portions may be washed hydraulically and returned to bed. Clogging of sand filter increases in proportion to amount of turbidity to be removed and with excess of algæ; in extreme cases, it is desirable to apply preliminary sedimentation, coarse filtration, or coagulation.

Mechanical filters are more commonly used after preliminary treatment just mentioned. By admitting water under pressure up to 10 to 12 lb per sq in, percolation is much more rapid, up to 150 in descending veloc per hr, and, if properly controlled, is no less efficient. Installation of mechanical filters should be supervised by specialist in hydraulic engineering, and their operation must be continuously under careful control.

Disinfection (37) is most advantageously applied to water which has previously been clarified by sedimentation, coagulation, or filtration, for purpose of additional security against pathogenic bacteria.

Liquefied chlorine has almost completely replaced other means for chlorination of public water supplies in U S (51) at about 6 000 plants treating water supplied to 70% of the country's population, at cost estimated at 1¢ per capita per year.

Liquefied chlorine costs (1939) 5.25¢ per lb, or 7¢–21¢ per million gal treated. Annual cost for repairs and maintenance of chlorinating equipment in 36 large cities (1924) was 3.6% of investment; at 71 smaller units, 7.6%. A small public-works chlorination plant costs about $1 000, and still smaller and cheaper units for domestic or transient use are on the market. Most waters, even after aeration and filtering, contain organic or mineral matter the oxidation of which consumes chlorine, the reaction being 90% complete within 10 min at 20° C; this "chlorine absorbed" or "chlorine demand" must first be satisfied before bacteria can be effectively attacked, but the additional quantity of chlorine required for the latter purpose is relatively small. Chlorine is usually added in such amount that rarely more than 0.2 part per million of water is present 10 min after the application. Odors and tastes in disinfected water are probably due to chlorinated organic compounds, and can be avoided by increasing the dose of chlorine to, say, 0.3 part per million of water, tested after 10 min.

Other chlorination methods (51). ELECTROLYTIC CHLORINE, obtained by electrolysis of sodium chloride, has displaced liquefied chlorine in some localities favored by cheap salt and low power costs. CHLORIDE OF LIME (bleaching powder), averaging 30% available chlorine, is now mainly confined to small and isolated plants. The powder is first emulsified in water, kept in agitation, and fed automatically into the stream of water to be treated. Chlorine supplied in this manner is just as effective as liquefied chlorine, but the unavoidable addition of lime may sometimes be objectionable. CHLORAMINE is prepared by mixing ammonia water with bleaching-powder solution, in proportion of 1 lb free NH_3 to 3–4 lb free Cl; or NH_3 and Cl_2, both in gaseous form, may be injected separately (NH_3 first) into water headers. Addition of NH_3 permits more effective application of chlorine, with less danger of producing tastes and odors resulting from chlorination of organic compounds. SODIUM HYPOCHLORITE is convenient for sterilizing water on a small scale, as in camps, fresh-water steamboats, etc.

For domestic and camp service, a useful device (38) is a water-tight canvas bag, 20 in diam by 28 in deep, having loops at top for suspension and one or more nickeled faucets at bottom. Chloride of lime is put up in sealed glass tubes holding 14 to 16 gr each (30 to 32% free chlorine). One tube will kill all pathogenic germs in 300 lb of water in 30 min, and water is safe to drink in 10 to 15 min.

Desirable to filter water through a cloth before treatment. A neutral or slightly acid water is more rapidly and completely sterilized by chlorine than an alkaline water.

Activated carbon (71) is an efficient adsorber of impurities giving rise to tastes and odors. It is most often applied in powdered form, either wet or dry, requiring special equipment due to lightness of the carbon and its resistance to wetting. When applied to an open reservoir, preferably within a day following a $CuSO_4$ treatment, service should be interrupted for 24–48 hr while the carbon is settling. When used in connection with filters, carbon may be added: (a) after coagulation; best where required treatment is slight or of infrequent occurrence; (b) before coagulation, giving the carbon more time to operate; desirable where raw water is high in organic matter, and occurrence of taste or odor is frequent or continuous. Proportion of carbon consumed varies widely; aver about 2 parts per million, or 16 lb per million gal. Granular carbon, supported on sand and gravel within a pressure filter, is convenient at small water systems for removing last traces of taste and odor from previously filtered water.

Boiling is last resort to insure sterility of drinking water. All pathogenic germs are killed by boiling 5 min; some sporiferous vegetable germs require higher temp, but it is not certain that they are dangerous. Taste of boiled water is improved by aeration.

Distilled water, though safe pathogenically, is physiologically unwholesome for continuous use. A still can readily be improvised by a tinsmith from copper pipes and pans, preferably tinned on surfaces which come in contact with water (39). Evaporator can be heated by live steam.

15. SANITATION

Wastes requiring disposal are: garbage and domestic refuse, excreta, wash water. Under favorable conditions, such as: porous gravelly soil, sloping topography, small opportunity for stagnation of water, not too dense population, cool, dry climate, and water supply obtained under reasonable precautions against contamination, no serious difficulty is likely to arise from following well known and archaic methods. In absence of any of these conditions, disposal demands attention, and may become an engineering problem of importance.

Garbage should be placed in covered and water-tight pails, and collected at least once a week in covered wagons. It may then be buried in trenches, burned on surface, at distance from town, or incinerated. Usually, enough inflammable waste is available to provide fuel for latter purpose. Descent of garbage into hearth of incinerator must be retarded by baffle rods, giving it a chance to dry partly. Bib (66) shows a well designed incinerator.

Excreta. Only 2 methods of disposal should be tolerated in a settled community: dry closets, or a system of sewerage, individual or communal. Latter system is expensive to install but cheap in operation. In many mining camps a communal sewerage system is impracticable, because: (a) water supply may be inadequate for flushing; (b) extra liability of ground to settle makes difficult the maintenance of both water mains and sewers; (c) ignorance of many classes of workmen induces expensive repairs.

Dry-closet, an improved form of privy, may be the only available means of disposal in many mining towns. Where water supply must come from shallow wells, as at Mineville, N Y (40), the old form of earth vault must be abandoned. Essential details of construction: (a) Receptacle, galvanized iron, preferably cylindrical for ease of cleaning, must be water-tight. (b) Seat should be only slightly higher than top of receptacle. (c) Lid of seat so designed as not to permit it to stay open, to admit flies. (e) Receptacles removed through door in back, so designed as to fall tightly shut. (e) Adequate ventilation through screened aperture. (f) Door and window screened. Cost of erection at Mineville, N Y, $15 for structure, $3 for receptacle. OPERATION: Once a week in warm weather, twice a month in winter, full can is removed, covered with tight-fitting lid, and a cleaned, empty can is returned to place. Full cans are then loaded on low wagon passing along alley in rear of houses, and taken to disposal grounds for burial, septic treatment, or incineration. Cans are emptied, washed, and dried on a rack. At Mineville, incinerator for disposing of excreta from 238 families cost $500 to build; total cost of disposal averaged $1 per family per mo. Same plan has been adopted at Docena, Ala (Tenn C, I & RR Co) and elsewhere (41).

Sewage disposal (42). Where water supply is adequate for flushing and cost of laying sewers is not prohibitive, a system of sewerage is most desirable. Sewage can be disposed of by dilution, irrigation, septic treatment, or activated sludge process.

Dilution. If mixed with sufficient volume of water, organic matter in sewage decomposes into solids and gases, under combined action of sunlight and of oxygen absorbed from atmosphere. If flow of a stream exceeds 6 cu ft per sec per 1 000 persons discharging sewage, objectionable conditions are unlikely to result; this refers only to creation of a nuisance and does not mean that a stream so polluted would be safe for water supply. If flow is variable between seasons, or if its water has opportunities to stagnate, objectionable conditions may arise. Roughly speaking, a stream will purify 0.02 of its volume of sewage, but not 0.05.

Irrigation with crude sewage has been practiced for centuries in Europe, and has proved successful in many parts of western U S. Prime requisite is a porous, sandy soil, nearly level in topography; less porous soil requires preliminary removal of solid matter in sewage, by sedimentation or septic treatment (see below).

Sewage farm must be underdrained, preferably by tile pipes, 3 to 4 ft deep and 30 to 50 ft apart. In aver soil, 1 acre will absorb raw sewage from 300 persons, or 30 000 gal per day. Land must be so arranged as to take sewage in rotation; it commonly flows for 4 to 10 days, followed by at least an equal period for drying. Berries or salads, eaten uncooked, should not be raised on sewage farms.

Septic treatment involves 2 steps: (*a*) putrefaction, in absence of oxygen, whereby suspended solids are disintegrated and reduced in volume, and most of the organic matter is decomposed into NH_4OH, CH_4, and CO_2; (*b*) oxidation, or "nitrification," in presence of air, whereby NH_4OH is converted into HNO_3, which immediately combines with alkalies, normally present in sewage, to form nitrites and nitrates.

Both processes depend upon bacteria, and considerable delay may occur before their operation becomes well established. Putrefaction is stimulated by temp of 55° to 75° F, and absence of acidity; nitrification proceeds best under following conditions: abundance of air, best afforded by intermittent flow through porous bed; free ammonia not exceeding 0.05%; presence of alkaline base, preferably lime and magnesia, but not in excess; rapid drainage of final products; temperature, the warmer the better. Putrefaction and sedimentation are conducted in "septic tank," the effluent of which is led away for nitrification on natural soil, or in artificial filter or contact beds.

Septic tank for a small system is shown in Fig 7. Capac of sedimentation chamber should be equal to a minimum of 6-8 hr flow of sewage, while more than 24-hr capac is undesirable, because bacterial activity is retarded by accumulation of decomposition products. Flow should enter and leave this chamber with minimum disturbance of sludge and "mat." If sewage contains much mineral matter, as pavement washings, a preliminary "grit" chamber is advisable, and if grease is likely to be abundant, as from a camp cook-house, a grease trap should be placed ahead of the septic tank. Univ of Kansas Eng'g Sta (74) recommends the dimensions in Table 8; those for the siphon compartment or "dosing tank," discharging at a depth of 17-23 in of liquid, refer to a tank from which the effluent is distributed by a line of open tile (see below); for a sand filter (recommended for more than 20 persons) discharge capac of dosing tank should be such as to cover the filter to a depth of 2 in at each discharge, at intervals of 4-6 hr.

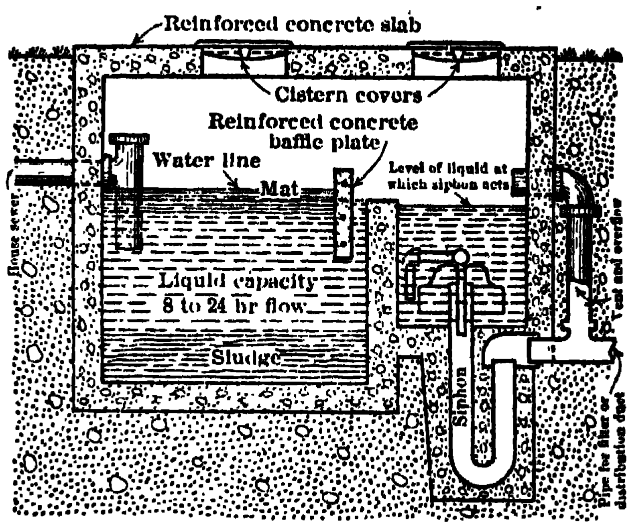


Fig 7. Septic Tank and Siphon Pit

Table 8. Recommended Dimensions of Septic Tanks (74)

Persons	Capac receiving compt, gal	Width	Liquid depth (*a*)	Length: Receiving compt	Length: Siphon compt	Lin ft of distributing tile	Area of sand filter, sq ft
-10	500	3'-0"	4'-0"	6'-0"	....	150-200	....
11-15	730	3-6	4-0	7-0	2'-6"	295	....
16-20	950	4-0	4-0	8-0	3-0	380	....
21-25	1 160	4-6	4-3	8-6	3-0	465	580
26-30	1 360	4-6	4-6	9-0	3-6	550	680
31-40	1 700	5-0	4-9	9-9	3-6	680	850
41-50	2 000	5-0	5-0	11-0	3-6	800	1 000

(*a*) Total depth should allow 12 to 18-in air space above liquid.

Sludge constitutes at most only 30 to 40% of organic solid material entering with sewage, averaging (in U S) 15 to 20 tons (carrying 90 to 95% water) per million gal sewage. It is removed

at long intervals, by bailing (or through blowout, in larger installations), and disposed of by drying, burying, or burning. Its fertilizing value is small.

Nitrification. Effluent from septic tank may be well used for irrigation, on porous sandy soil. SAND FILTER BEDS, where natural soil is suitable, are made by trenching (4 to 6 ft deep, 50 ft apart) and tiling an area of about 1.5 acre per 1 000 persons contributing sewage. Surface requires occasional raking, to give uniform distribution of effluent over whole area. Where soil is not suitable, artificial sand filter, resembling water filter (Art 14) can be made, preferably with concrete walls and bottom, with radiating open-joint tiles for discharging filtered sewage. Tiles are covered with 12 in coarse gravel, 12 in fine gravel, 12 in coarse sand. Area computed on basis of 1 acre per 400 000 gal per day or 0.5 sq ft per gal per day. Aver cost of maintenance (Mass), $7.50 per million gal sewage treated. With sand filters, intermittent treatment is essential, to permit bacteria to obtain their necessary oxygen. In cold climates, filters must be covered with boards and earth, and have ventilators. Removal of sewage bacteria by sand filtering will be over 99% complete, under ordinary care. CONTACT BED is a concrete pit filled with coarse stone, pebbles, coke, etc, with tile drains on bottom, leading to discharge. Septic effluent admitted from open-joint tiles in upper layer. During treatment, organic matter is decomposed by bacteria adhering to a gelatinous film surrounding stones. Best depth is 3 to 4 ft; volume 1 to 1.5 times daily flow. To maintain

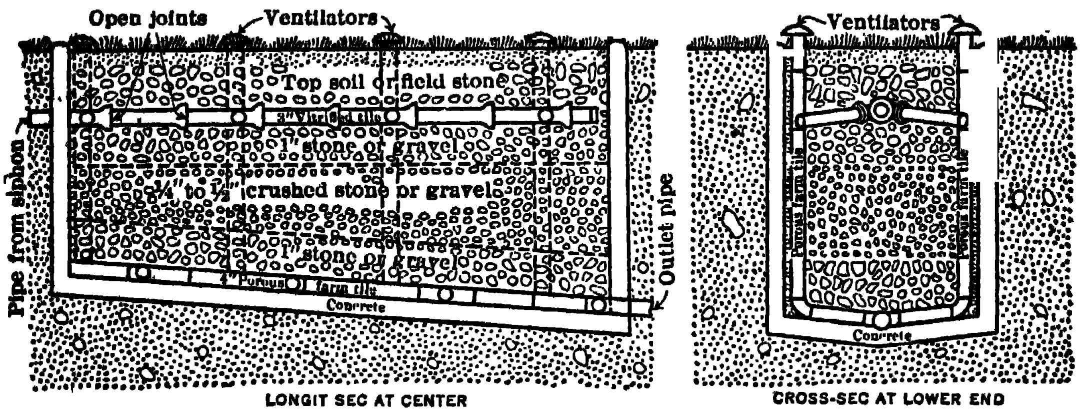


Fig 8. Percolating Nitrification Bed

bacteria in best condition, bed is allowed to fill for 1 hr, stand full for 2 hr, drain for 1 hr, and stand empty for 4 hr, making 3 cycles per day. PERCOLATING BED is similar in construction to contact bed, but flow of septic effluent is continuous; in small installations it enters through open-joint tiles (Fig 8); in larger ones through variety of sprays and nozzles. Filling is 1.5 to 4-in broken slag, coal, or cinders; depth 4 to 5 ft. Percolating bed is cheaper to operate than contact bed, and will dispose of over twice as much effluent per unit area; sand filter is more efficient than other types in removal of bacteria. Where soil is porous, and no wells are closely adjacent, concrete walls and bottom (Fig 8) may be omitted, and bed made in form of a long trench. Such a trench may be 2 ft wide and 26 in deep, and, on sloping ground, should approx follow a contour so that tiles may be laid on grade of not over 2–3 in per 100 ft. Bottom should be filled about 6 in deep with coarse stone on which 3 or 4-in tiles in 1-ft lengths are laid with $^1/_8$-in spaces; tiles are then covered with a few in of coarse stone and the trench finally back-filled. Recommended lengths of trench are given in Table 8. Close approach to trees should be avoided to prevent clogging by roots; if unavoidable, tight-jointed CI pipe should be substituted at that place.

Activated sludge process (73) reverses the septic-tank procedure in that nitrification by ærobic bacteria comes first; system has been widely adopted for large and medium-sized communities, but exceptional circumstances, as in examples below, may justify the cost for plant and its operation on a smaller scale. Raw sewage, from which all solids coarser than about $^1/_{16}$ in have been extracted, and to which has been returned a portion of the sludge recovered by the next subsequent step, is vigorously ærated in a tank or a continuous channel, either mechanically or by diffusion of comp air through a porous medium; air required is 1–2 cu ft per gal of sewage; power, 20–40 hp to compress air to treat 1 000 000 gal per day.

Some mechanical aerators operate with less power, but diffusion gives faster reaction (73). After aeration for 4–6 hr, depending on strength of sewage, mode of agitation, and proportion of sludge returned (the more of the latter, the shorter the time), colloidal constituents become flocculated and, upon settling, collect suspended fine solids, together forming a sludge strongly enriched with aerobic bacteria. Overflow from settlers is innocuous water. Of the sludge here abstracted, 10–20% is returned to the raw sewage entering the plant, either directly, or after receiving added aeration to increase its bacterial activity. Remainder of the sludge, containing up to 99% water, may be spread out to dry, in case of a small installation, but is best transferred to a digestion tank, holding 3–4 months' output, where anaerobic decomposition occurs, stimulated by seeding with partially decomposed sludge or introduction of especially active types of bacteria. Sludge thus decomposed is greatly diminished in vol, dries readily when spread on sand beds, and is good

fertilizer. Gases evolved during digestion of sludge are rich in CH_4; at some large English installations, this gas supplies the whole operating power.

Examples (72). NORANDA, Quebec, is on a lake which affords the only practicable source of domestic water and also the only natural outlet for sewage; extreme precautions are therefore necessary. Water is treated, at 1 000 000 gal per day, by coagulation, gravity filtration, and chlorination. Sewage, 750 000 gal per day, is treated by activated sludge system. A bar screen with 1.5-in spaces catches about 1 cu ft of coarse refuse per million gal, which is removed by hand and buried. A 3 by 3-ft self-cleaning drum screen (Dorrco) with 1/16 by 2-in slots removes about 4 cu ft per million gal, or max of 40 cu ft per day of finer refuse; also buried. Liquid sewage is then aerated and nitrified in activated sludge channels, supplied with comp air through porous tiles in bottom, taking 6 hr for treatment; it is then settled in a 30 by 16-ft clarifier, the overflow running into the lake. Of the sludge, 20% is returned as seed to the nitrifiers; remainder, after digestion in another clarifier, is spread on sand beds to dry, and then goes to dump. FLIN FLON, Manitoba, has 2 lakes, drawing water from one, and applying activated sludge treatment before discharging sewage into the other. Initial plant, treating 150 000 gal of sewage per day (pop, 2 500), was similar to that at Noranda, but sludge was dried on sand beds without further digestion. Latter step was to be included in a proposed larger plant; gases from digestion, carrying 70% CH_4 and with fuel value of 650 btu per cu ft, were to be utilized for warming sewage to best temperature for nitrification.

Toilets. Following standard specifications for use in connection with sewerage systems have been adopted by the U S Steel Corporation's committee on sanitation:

(*a*) Closets should be located as close as possible to the work. (*b*) There should be a number of small closets rather than a few large ones. (*c*) Each toilet room must open to outside light and air. Minimum window space for a toilet room containing one fixture must be 4 sq ft, and for each additional fixture 2 sq ft. Windows must be designed to open. Each toilet room to have not less than 10 sq ft floor space and not less than 100 cu ft air space for each fixture. (*d*) Closets should be separated from lockers and wash rooms. (*e*) Unless wash rooms are nearby, each closet should be supplied with at least one wash basin. (*f*) Adequate urinals should be provided in each closet. (*g*) Number of seats, one to every 15 persons, based upon max number of employees in a shift in departments using the unit. (*h*) Closets should be of bowl type, of porcelain or china, not enameled iron. (*i*) Every bowl should be separately vented and trapped. (*j*) The seat of closet should be of wood or other non-heat absorbing material and varnished or painted. Seats should never be ironware or porcelain. (*k*) Opening should be at least 7 in wide and 11 in long. (*l*) There should be partitions between seats, 6 ft high and 12 in off the floor. (*m*) Distance between partitions should be not less than 30 in, and from front of seat to door not less than 30 in. (*n*) Partitions and bowls should be so arranged that all spaces can be easily cleaned. (*o*) Toilets should be heated in cold weather. (*p*) Floors should be of glazed tile or concrete, with smooth surface and cove corners. (*q*) Regular cleaning is necessary; disinfectant not to be relied upon. (*r*) Toilet paper should be furnished. For underground toilets, see Sec 23.

16. DISEASES ENCOUNTERED IN MINING

Ankylostomiasis, an intestinal disease, is transmitted by germs contained in excreta; germs will survive for long periods in mud and water underground. PRECAUTIONS: (*a*) prohibit indiscriminate pollution of mine workings; (*b*) wash hands in clean water before eating. Symptoms and treatment, see Sec 23.

Silicosis, a laceration of the lungs induced by breathing fine siliceous dust, is now believed to be due to solubility of silica in fluids present in the lungs. Dusts of coal, shales, and practically all rocks not having a vitreous quartz nature, have less direct harmful effect, but serious danger arises through increased liability to contract other pulmonary diseases, even where acute silicosis may be absent. PRECAUTIONS: (*a*) adopt wet drilling methods, or (*b*) require workmen to wear respirators. Symptoms and treatment, see Sec 23.

Malaria (44) prevalent in tropics is transmitted by sting of a few varieties of mosquito (genus anopheles). These mosquitoes breed most freely in clean, running water open to sunlight, less so in stagnant or marshy water, and practically not at all in muddy water, as in dredge or tailings ponds. The most virulent anopheles swarms just before dark and for 2 or 3 hr thereafter, is notoriously persistent in search for an opening in screens or nets. It can be distinguished from other mosquitoes by its attitude of " standing on its head " while feeding. According to Watson (44) it rarely travels more than 1/2 mile (unless carried by a breeze).

Malady is slow and cumulative, and a sufferer usually has sufficient warning to enable him to escape to more healthful climate. Regularity in recurrence of chills is not so characteristic of tropical malaria as of other types. Recognized preventive is quinine, which should be taken regularly on going into a malarial district; sulphate of quinine is more insoluble and therefore less effective than bisulphate or hydrochloride.

Prevention: 3 gr quinine before each meal; avoid constipation, by fruit diet or laxatives, if necessary. Build houses on highest ground available, remove foliage in immediate vicinity, screen carefully, and drain mosquito-breeding pools if possible. SYMPTOMS: fever or chill, sometimes accompanied by violent vomiting; in former case, induce active perspiration with hot lemonade, acetanilid or other antipyretic; in latter case, empty the stomach with 30 gr ipecac and much tepid water, followed by 5 gr soda bicarb. TREATMENT: 15 to 20 gr quinine 3 times a day; more quinine than sufficient to induce dizziness is not necessary. Promote movement of bowels by 10 gr calomel mixed with equal weight sodium bicarb, taking 0.25 of mixture every 20 min. Six hours after calomel, take 1 oz magnesium citrate in water, followed 2 hr later by 20 gr quinine. Patients should remain in bed at least 1 week, taking 30 to 35 gr quinine per day, in 2 or 3 installments, and continued until 25 days pass without return of fever.

Typhoid (45), an intestinal disease, is transmitted by a bacillus in food and drink, but (unlike typhus) probably not by contact.

Precautions: Doubtful water should be boiled, or distilled, and milk sterilized on least suspicion. All refuse should be incinerated, especially excreta from a typhoid patient, all of whose clothing, bedding and utensils must be disinfected. General and individual cleanliness, particularly in connection with preparation and eating of food, is important. SYMPTOMS: headache (frontal), chilliness, languor, increasing in severity; followed by dry hot skin and great thirst; tongue dry, with brown or dark yellow coating; pulse weak and rapid; later, pale red isolated spots, like flea-bites, appear on chest or abdomen; fever 102° to 104° F, usually entailing coma or delirium. SPECIFIC TREATMENT unknown. Liquid diet, principally milk, followed by meat broth after middle of second week; stimulants if necessary. Cold water or tea may be taken in large quantities. Cool baths or sponging with alcohol, or wrapping in wet sheet, helpful for reducing fever, except in case of complications or weakness.

Yellow fever (45), usually an epidemic, is a mosquito-borne disease (carried by Stegomyia fasciata) particularly likely to attack newcomers into countries where it occurs. Promoted by high temp, low barometer, lack of cleanliness, and impure water. One attack generally brings immunity.

Symptoms: invasion usually sudden, denoted by chills or convulsions with severe headache, pain in back, constipation, and vomiting. Urine scanty. These symptoms last 2 to 6 days, when convalescence may begin, or the disease proceed to third stage marked by vomiting of disorganized blood (black vomit). Blood appears in stools and urine and may exude from mouth and nose. Stupor becomes marked and skin yellow. Recovery from third stage is rare. TREATMENT: quarantine and thorough disinfection are necessary. At beginning, complete rest and hot mustard foot-baths advisable. For fever, cold sponging and application of ice. Liquid food in small quantities at frequent intervals. Black vomit is restrained by entire rest for stomach, counter-irritation of abdomen and fixing muscles by broad pad.

Cholera (45) is an epidemic caused by bacillus infesting intestines, which enter only with food or drink. Contributing causes: gastric derangements, excessive heat, and lack of cleanliness.

Symptoms: invasion is always sudden, consisting of painful griping and spasms of muscles of abdomen and calves of legs, with coldness of surface and extreme collapse. Diarrhœa is pronounced, becoming more frequent and consisting of whitish or brownish fluid. Temp sub-normal, pulse weak and rapid, drawn appearance of face. Urine partially or wholly suppressed. Duration of disease may be as short as 2 hours, but in fatal cases death usually ensures on second or third day. TREATMENT: in first stage, enemas of opium and tannin, with stimulants; wrap in hot blankets, bathe several times daily in disinfectant solution. Diet, thin porridge exclusively. During convalescence, careful nursing necessary. Isolation and disinfection of dejecta essential and persons in infected area should use extreme care about cleanliness and moderation in food and stimulants.

Scurvy (45) is a derangement of the blood, most commonly encountered in arctic explorations, supposed to be caused by lack of fruit and vegetable acids and potash salts. Indicated by blood clots under skin, ulcerations on skin or mouth, tongue and gums swollen and ulcerated. Hemorrahages frequent, fever usually absent. Symptoms can be modified by increasing ration of fresh vegetables or fruit; onions and lime or lemon juice are particularly beneficial.

Tropical complaints, in general, may be alleviated by observance of following suggestions (46): (*a*) Before making a first visit to tropics, consult physician; those afflicted with chronic weakness of stomach or liver are most liable to diseases. (*b*) On arriving in a malarial district, begin taking quinine (see "Malaria") for its tonic and prophylactic effects; (Warburg's Tincture, liquid or in capsules, is excellent). (*c*) Build houses on highest available spot; raise a few feet from ground; cut grass and underbrush from immediate vicinity, and screen houses against all insects. Around a permanent camp, drain pools and other breeding places of mosquitoes. (*d*) Diet should be light, and composed largely of fruit and vegetables, fresh if possible; meat may be reduced to small amount, and fresh killed meat should be secured at any reasonable cost; small refrigerating plant is a justifiable expense where practicable. Variety of food and reasonable supply of table luxuries

are important, because appetite is likely to be weak. (*e*) Water should be boiled or distilled. (*f*) Omit liquors entirely; this is particularly important, because alcohol retards normal prophylactic action of the blood. (*g*) Protect top of head and back of neck from direct heat of sun; arrange working hours to permit siesta during middle of day. (*h*) Provide enough light-weight clothing to permit frequent changing, and do not allow wet clothing to dry on the body. (*i*) In short, the care of one's health, under conditions which are known to be unsalubrious, and where failure of an entire undertaking may result from incapacity of one individual, deserves more serious attention than it usually receives in the temperate zone.

Snake bites. Tie ligature as quickly as possible around limb above the bite. Cut deeply enough to start flow of venous blood from the bitten spot, aided by sucking, if possible, which may be done safely unless mouth has an open sore. Apply strong solution of potassium permanganate or bichromate. Purpose of alcoholic liquor, usually recommended, is to stimulate heart action and counteract narcotic effect of the poison.

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SECTION 23

MINE AIR, GASES, DUSTS, HYGIENE, EXPLOSIONS, AND ACCIDENTS

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

MINE AIR, GASES, DUSTS, HYGIENE, EXPLOSIONS, AND ACCIDENTS

1. ATMOSPHERIC AIR

Table 1. Composition of Pure, Dry, Normal Atmospheric Air

	Symbol	By volume	By weight	
Oxygen	O_2	20.93	23.024	Percentages of these constituents are lower when water vapor is present, as is always the case under normal conditions
Carbon dioxide	CO_2	0.03†	0.040	
Nitrogen *	N_2	78.10	75.499	
Argon	A	0.94‡	1.437	
		100.00	100.00	

* Includes small amounts of rare gases behaving, as does argon, like N; hence, ordinarily grouped under N.

† Haldane (*Jour of Hygiene*, Vol 2, p 241) states percentage of CO_2 at about 3 ft above the ground, as varying from 0.025 by day to 0.035 by night; mean about 0.030%. In large towns there is more CO_2. In a London fog Russell found 0.14% CO_2.

‡ According to Sir William Ramsay.

Weight of dry air. At sea-level press (14.7 lb per sq in), 1 atmosphere = 760 mm or approx 30 in mercury barom; wt of 1 cu ft, 0.086354 lb at 0° F, and 0.070914 lb at 100° F (Sec 39, Art 9). Dry air is subject to the general laws of gases. At constant temp its density increases directly as the press.

Humidity. Aqueous vapor is always present in the atmosphere, the quantity depending on climatic and local weather conditions. The percentage of saturation, except in arid districts, is rarely less than 35%, ranging to 100%.

Presence of moisture in a given space is independent of presence or absence of air in that space. Therefore it is not technically correct to say that air is moist, or dry, but this is the common mode of expression, and in mining practice it is convenient to say the ventilating current or the mine air has a certain relative humidity, or carries a certain wt of water vapor per 100 000 cu ft of air in the current (1, 64*a*). Quantity of moisture present is chiefly dependent on temp; press and, other things being equal, the higher the temp the greater the quantity of water vapor, if water is present to furnish vapor.

Absolute humidity is the quantity or wt of aqueous vapor present in a unit of space. Aqueous vapor is saturated when, at the vapor press, the space contains the maximum quantity. If vapor is partly saturated, the percentage of saturation is termed RELATIVE HUMIDITY, which is the ratio of the wt of vapor present to that necessary for saturation. It may be measured by hygrometers (psychrometers) which have 2 thermometers, one termed "dry bulb," the other "wet bulb"; latter has tight thin muslin cover over bulb, kept wet. Evaporation by air current or by whirling cools and "depresses" "wet bulb" thermometer in geometric proportion to decrease of relative humidity of atmosphere.

Mine air, beside normal atmospheric constituents, contains moisture, gaseous impurities and dust (Art 2).

Relative humidity varies widely in different mines and different parts of a mine, and is important in connection with ventilation (Sec 14). Relative humidity of intake air varies with changes in humidity of the outside atmosphere. When intake air is colder than the mine walls, it becomes warmed and its humidity decreases; when warmer than the walls, as in summer, it is cooled and its humidity increases, usually above the saturation or dew-point, in which case moisture is deposited. This effect may extend far from the mine entrance.

In some coal mines water sprays are used in warm weather to cool and thus condense moisture in the intake air; and in some deep, hot metal mines, as at Butte, Morro Velho, Brazil and in South Africa, intake air is dehumidified to improve working conditions (Sec 14). As the temp of mine walls generally varies but little from summer to winter, the relative humidity changes in different parts of a mine with dryness or wetness, and the extent to which spraying is applied.

Dew-point is the temp at which drops of visible moisture form from saturated vapor.

Psychrometric formula, employed by the U S Weather Bureau for determining relative humidity, and the temp of dew-points, is:

$$e = e' - 0.000367\,P\,(t - t')\left(1 + \frac{t' - 32}{1571}\right)$$

where t and t' are the temps of the dry and wet-bulb thermometers; P is the corrected barom press, in; e' is the max or saturation press of aqueous vapor at temp t' of the wet bulb; and e is the press of aqueous vapor corresponding to observed temps t and t'. Weather Bureau tables (3) are too extensive for insertion here. Fig 1 shows curves of equal wts of water vapor per cu ft and curves of equal depressions of the wet bulb for varying temp and relative humidities.

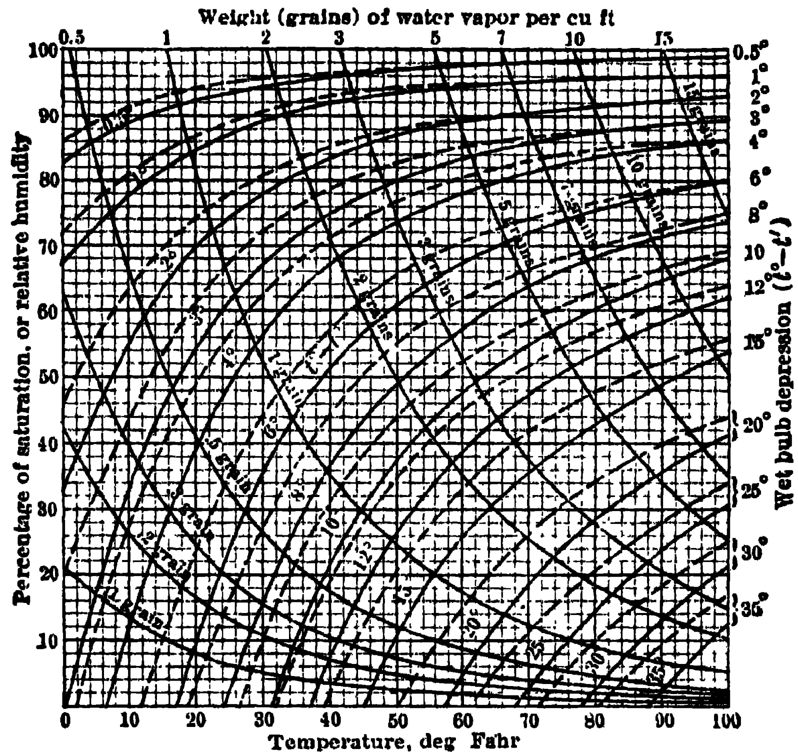


Fig 1. Psychrometric Chart. (Full lines for 30-in; broken lines for 23-in barom)

Example of use of chart. Required relative local humidity and wt of aqueous vapor at barom press of 30 in. Take temp readings of dry bulb t and wet bulb t' (if sling psychrometer is used, readings are made after whirling). If $t = 58°$ and $t' = 50°$, $t - t' = 8° =$ wet-bulb depression. Follow the full-line wet-bulb depression curve of 8° from the right until it intersects vertical temp line 58; then, relative humidity, 56%, is read in left-hand margin opposite point of intersection. Wt of vapor per cu ft is found by noting the curve (or interpolated curve of equal wt) which passes through point of intersection; in this case, 3 grains. At a high altitude, where barom press is, say, 23 in, temp 58°, and wet-bulb depression 8°, the broken line curve of 8° is followed until it intersects temp curve 58°. Relative humidity in this case is 61%; weight of vapor, 3.2 grains. For other barom pressures it is necessary to interpolate, or refer to complete psychrometric tables (2). Barom readings for different altitudes are given in Sec 37. For wt of dry air and water vapor, see Sec 39, Art 9, 10; for wt of mixtures of gases, see Sec 14.

Psychrometer. To determine the difference $t - t'$, between wet- and dry-bulb temp, the sling, or whirled psychrometer, is the most accurate instrument. It is swung to give the wet bulb a veloc of not less than 15 ft per sec. It is easily used, but is fragile, and therefore less suitable for mines than those having protective frames. A form designed by Bur of Mines (Fig 2) has thermometers attached to an aluminum frame. It has a leather case, with aluminum liner and shoulder strap. In whirling the psychrometer, observer stands sideways to air current, and if in open air should shade instrument from sunshine. Tests are repeated several times to insure lowest reading of wet bulb. When temp is below freezing, whirling must be continued for a time after wet muslin cover of the wet bulb freezes, as vapor from ice is given off slowly. STATIONARY PSYCHROMETERS (hygrometers), while not so dependable as the whirling type, are useful; those having a wet bulb are fairly accurate in a strong current of constant veloc. **Recording hygrometers:** one type utilizes changes in length of a hair corresponding to changes in humidity, another

employs differential expansion of large metal bulbs, wet and dry, to operate a scribing pen. Stationary hygrometers, if checked by whirling psychrometer and adjusted periodically, are useful to determine hygrometric condition of an artificially humidified ventilating current.

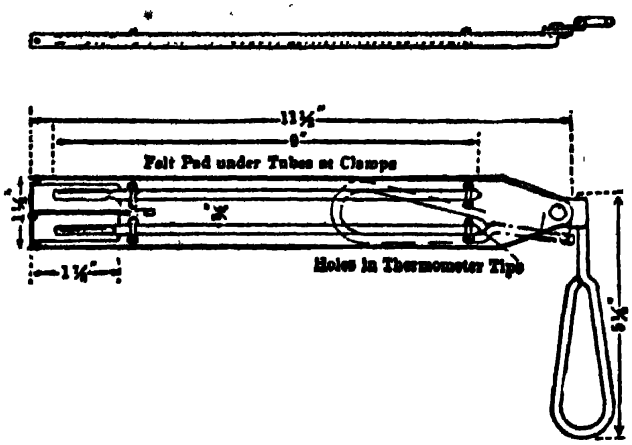


Fig 2. Bureau of Mines Psychrometer

Kata thermometer, for measuring cooling power of the atmosphere, gives a better index of physiological conditions of atmosphere than the wet- and dry-bulb thermometer.

It is a large-bulbed spirit thermometer, graduated between 100° and 95° F. The bulb is heated in hot water (thermos flask may be used), then dried, suspended, and time of cooling from 100° to 95° taken with a stop watch. The number of seconds divided into a factor number (approx 500, and determined for each instrument) gives the cooling power for convection and radiation on the surface of the Kata. Operation is repeated with a muslin fingerstall on the bulb, thus obtaining the wet Kata cooling power due to evaporation, radiation and convection. The difference is the EVAPORATIVE COOLING POWER (3). The Kata is widely used in Great Britain and its colonies, but in the U S is not preferred to the wet- and dry-bulb thermometer, with record of air veloc.

2. CONSTITUENTS OF MINE AIR

Mine air differs from normal atmospheric air in the extent to which it is contaminated by mine gases, or normal percentage of O is depleted. O is absorbed by breathing, rapid and slow combustion or decay, and by the mine walls. Impurities come from exhalations of men and animals, blasting, fires, or explosions, bacteriological action, and gases from strata.

Quality of mine air may vary widely in different parts of same mine. In a mine not artificially ventilated, variations in temp, and influx of gases and water from the walls, cause air currents. Ordinarily the composition of mine air is largely controllable by systematic ventilation (Sec 14). Recent laws in Great Britain and elsewhere require a certain minimum percentage of O, and limit the maximum of deleterious gases in the air (Art 6), as opposed to the older coal mine regulations, which specify certain quantities of fresh air per min to be circulated.

Normal mine air is air considered not injurious to breathe, and which does not contain dangerous proportions of inflammable gases (Art 6). Its composition varies but little from that of the open atmosphere (Art 1).

Oxygen (O) is vitally important for animal life; it forms about 20.93% of the atmos (dry basis) and is slightly heavier than air. One liter of O at 0° C and 760 mm weighs 1.429 gm. At −118° C it may be liquefied at 50 atmos (735 lb) per sq in. Liquid O boils at −183° C, at 760 mm. O has strong affinity for most elements. Slow combination is called oxidation; rapid oxidation due to flame or heat is termed combustion. Self-generation of flame by rapid oxidation is called spontaneous combustion. When air is mixed in certain proportions with carbonaceous or other inflammable dusts or gases, and a spark, glowing piece of metal, or a flame is introduced, an explosion will probably take place. O has neither color, taste, nor odor, and can not be distinguished by the senses as different from air. In well-ventilated mines O rarely falls below 19%. In some collieries, oxidation of coal and roof shale may be so rapid that spontaneous fires result. Pyrite, generally present in coals that fire spontaneously, may assist, but spontaneous combustion may occur in its absence. Coal and carbonaceous shale absorb O without formation of a molecular equivalent proportion of CO_2. This action is more rapid with coal and shales in mines especially subject to "gob fires." In these, if the ventilating current is slow, the O content often falls to 18%. For physiological effect of O and of O deficiency, see Art 4.

Nitrogen (N) in volume forms nearly $^4/_5$ of the earth's atmos. It is colorless, odorless, and tasteless; has little chemical affinity for other elements. At ordinary temp it is inert, and will not support life; acts as a diluent of other gases. An important feature is its faculty of promoting the growth of certain bacteria, thus enriching soils. It boils at −195° C, whereas O boils at −183° C; hence, when liquid air is exposed to the atmos, the first portions that evaporate are richer in N than in O. This fact is taken into account in using liquid-air rescue apparatus (Art 19).

Argon forms 0.94% of the atmos. It is heavier than N (sp gr = 1.38), and behaves essentially like N, being grouped with it in ordinary analysis.

Table 2. Components of Normal and of Impure Mine Air

(b = molecular wt, when O = 16; d = sp gr with air as unity, at 0° C (32° F) and 760 mm or 29.921 in mercury; e = boiling point at 760 mm.)

Name	(a) Chem symbol	(b) Molec wt	(c) Gm per liter	(d) Sp gr	(e) Normal boiling temp, deg C	Usual source in mines
Oxygen	O_2	32.0	1.4292	1.1055	−183	In atmosphere
Nitrogen	N_2	28.02	1.2514	0.9673	−195	" "
Carbon dioxide	CO_2	44.00	1.9768	1.5291	− 78	In atmos., in excess of normal 0.03%, from blasting, oxidation, mine fires, emanations
Carbon monoxide	CO	28.00	1.2505	0.9672	−190	Blasting, mine fires
Nitrogen dioxide	NO_2	46.01	2.0549	1.5895	.....	Burning of nitro explosives
Methane	CH_4	16.03	0.7159	0.5545	−161	Emanations from coal measures
Ethane	C_2H_6	30.05	1.3567	1.0494	−89.3	Component natural gas
Acetylene	C_2H_2	26.02	1.1621	0.9056	.....	From carbide lamps
Hydrogen	H_2	2 016	0.0898	0.0695	−253	Mine fires, explosions
Hydrogen sulphide	H_2S	33.98	1.5392	1.1906	− 10	Explosions and rarely from springs
Sulphur dioxide	SO_2	64.07	2.9266	2.2638	− 10	Combustion of pyritiferous ores or native sulphur
Aqueous vapor	H_2O	18.02	......	......	100	In atmos and from evaporation
Component elements of some of the gases named above						
Solid elements	(a)	(b)		(d)	(e)	
Sulphur	S	32.07	....	1.98 (x)	440	From pyrite in ore or coal
Carbon	C	12.00	......	2.1–2.3 (y)	(z)	From coal, methane or timber

x = sp gr with reference to water at unity; y = sp gr of graphite with reference to water at unity; z, carbon is known only in solid form. Wt of 1 liter of dry air at above temp and press is 1.2932 gm.

Gaseous impurities in mine air. Common impurities (Table 2) and sources of pollution are as follows:

Carbon dioxide, or carbonic acid gas (CO_2), occurs in the earth's atmos in remarkably constant proportions, except in and near cities and manufacturing plants; kept constant by plant life, which takes the C for its tissue, and gives off free O. In mine air, when in excess of the 0.03% in normal atmospheric air (Table 1), it is a product of both slow oxidation and rapid combustion of carbonaceous substances. Hence in closed places and in mines the percentages of CO_2 vary. It is colorless, odorless, and will not support life or combustion. (For physiological effect, see Art 5.) It is a variable constituent of blackdamp, which sometimes has a musty smell due to decay of timbers and to fungous growth. BLACKDAMP is due to slow oxidation and to absorption of O by coal, causing excess of N, and hence is composed chiefly of N, with 1 to 20% CO_2 (rarely over 12%, as it is absorbed by mine water; never CO_2 alone, as sometimes assumed). CO_2 is an important constituent of AFTERDAMP, produced by fires or explosions of gas or dust. "Blackdamp," "Afterdamp," "Firedamp" (miners' terms) are of variable composition (Art 15, 17).

CO_2 is given off in some metal mines, by the dissolving of limestone, or from carbonate minerals. The latter may be the cause of CO_2, which sometimes gives trouble in mines at Tintic, Nev, and Cripple Cr, Colo, requiring bulkheading and increased air press to force the gases back into crevices. Violent CO_2 outbursts, blowing out highly pulverized coal dust, occur in collieries near Alais, France, and in lower Silesia (12). Origin of the gas, though not definitely known, is probably in limestone strata below the coal measures, which have been intruded by igneous magmas that heat the carbonates and form cracks, through which CO_2 has entered so-called "nests" in the coal. CO_2 is a stable gas, but can be reduced to CO by burning coal, or charcoal. Its percentage changes through absorption by percolating water, by disappearance of O of the air, by reactions between O and coal, or timber (rarely by inflow of CO_2 from strata), by supplanting of original atmos, by ingress of CH_4, and to a slight extent by absorption of CO_2 by coal. Water at 0° C absorbs 1.71 of its vol of CO_2. Sp gr, 1.529; boiling point, −78° C; may be liquefied at 0° C by a pressure of 35 atmos. Critical temp, 31° C; critical press, 77 atmos. If the liquid is allowed to escape freely, evaporation cools the remainder to a solid which passes directly to gaseous state.

Carbon monoxide (CO). Appreciable quantities are produced by imperfect combustion. Some CO_2 is reduced to CO by incandescent C at temp above 500–600° C. The

higher the temp in presence of excess C the more rapidly is CO produced. It is incidentally formed to considerable extent in furnace firing. With bad firing or wrongly designed furnace it is in excess in chimney gases. It is the important constituent of artificial fuel gas, of water gas, and of blast-furnace gas.

CO is colorless and inodorous. Mixed with air in proportions of 15.5 to 75%, the mixture will explode if ignited. Combustion formula: $2\ CO + O_2 = 2\ CO_2$. IGNITION TEMP is about 600° C. CO burns in air with blue flame, but in mines is never found separated from other gaseous products of combustion; and, as unconsciousness would ensue after a few respirations when over 0.5% is present, the old idea that a miner unprotected by oxygen breathing apparatus could detect its presence by a flame "cap" in a safety lamp, is fallacious. Probably its association with steam and light-colored smoke in fumes from mine fires led to its being termed WHITEDAMP. As CO may be present in quantities dangerous to life, when there is no steam or visible fumes from a fire, or in afterdamp of an explosion, the term is misleading (Art 5). A portable CO detector, much used by rescue crews, is graduated from 0.05 to 1%, and has a bell alarm which rings when CO reaches 0.2%.

Nitrous fumes (NO_2,N_2O_3) not normally found in mines, are sometimes produced when nitroglycerin explosives accidentally burn instead of detonating; then reddish fumes are produced, very dangerous to life (Art 5). They have the smell of fuming nitric acid, even when greatly diluted.

Methane or marsh gas (CH_4) is the most important of the dangerous gases. By miners it is termed FIREDAMP, or simply " gas," but as these terms are used whether or not other gases and air are present, they are not synonymous with the term methane. It is commonest in coal mines; also found in tunneling in carbonaceous shales and sometimes infiltrates into metal mines at contacts or near carbonaceous rocks (Art 3). It is colorless and odorless; its occurrence in old workings of collieries, where air is musty from decaying timber, has given the idea that it has an odor; it burns with bluish flame. Even when air contains as little as 1% it is visible in flame of a safety lamp, when wick is drawn down, as a faint conical blue cap (Art 9); at higher percentages the cap becomes larger and more distinct. Effect on a flame of normal size is to cause it to " spire," or lengthen. When above 5.0% of the mixture and less than 13.9%, the mixture will explode if ignited. Max violence is attained when CH_4 is 9.4%. (Table 2a.)

J. S. Haldane states a mixture of air with 7.5% of CH_4, which explodes violently at atmos press, can not be exploded if the press is reduced to 200 mm (8 in of mercury). G. A. Burrell found that increasing the press to 5 atmos did not affect the low limit, 5.0%; also that if the mixture of CH_4 and air be heated to 500° C, the limit is lowered to between 3.75 and 4%. This lowering of explosive limit may be an important factor in the propagation of a firedamp, or coal dust and firedamp, explosion. Formula of combustion: $CH_4 + 2\ O_2 = CO_2 + 2\ H_2O$. Volumetrically, 1 vol CH_4 requires 2 vols O or about 10 of air, producing when ignited 1 vol CO_2, 2 vols aqueous vapor, and the unchanged 8 vols of N. Temp of ignition is 650° to 750° C. If the burning gas is cooled below ignition temp, the flame will go out. On this principle is based the construction of safety lamps.

Ethane (C_2H_6), **propane** (C_3H_8), **butane** (C_4H_{10}), and other heavier gases belonging to the CH_4 series, are components of " natural " gas, and are not normally found in collieries. They have a pronounced crude oil or natural gas odor, which, apart from the analytical information, gives warning that there is probable leakage from a neighboring gas or oil well. At the Bruceton (Pa) experimental mine it has been observed that the odor is very strong on entering a place containing 1% " natural " gas, but a person breathing it for some time no longer notices the odor. Ignition temp is 520° to 630° C; hence less than that of CH_4.

Ethane is found in gases contained in certain coals, as in B C and Belgium, and in much smaller proportions than the accompanying CH_4. Due, probably, to its greater density, it diffuses less rapidly through coal in situ than CH_4 or even CO_2 (12).

Acetylene (C_2H_2) is not found in mines under natural conditions, but is sometimes given off in small quantities from unlighted or extinguished acetylene lamps, or from accidental spilling on wet ground of calcium carbide (from which it is produced by agency of water). When mixed with air in percentages between 2.5 and 73%, the mixture is explosive. It ignites at a temp of 483° C (900° F), and can be ignited by a glowing pipe or cigarette. 1 lb of pure carbide yields 5.5 cu ft C_2H_2; commercial carbide about 4.6 cu ft. It is colorless and tasteless. Combustion formula: $C_2H_2 + 3\ O_2 = 2\ CO_2 + H_2O$.

Hydrogen (H) is not found in mines under normal conditions. It occurs in afterdamp from coal-dust explosions, as a distillation product of the hot excess dust, after the explosion flame has passed, and from coal fires as a product of destructive distillation. H is the lightest gas, 1 liter weighing 0.0895 gm. Ignites at 550° C, and burns in air with a blue flame. Mixed with air it is explosive between 4.1 and 74.2%. Combustion formula: $2\ H_2 + O_2 = 2\ H_2O$.

Hydrogen sulphide (H_2S) has been found, though rarely, issuing with CH_4 from gas "blowers" or "feeders" in coal mines. Being soluble in water, it is sometimes given off by stagnant water. Occasionally detected by analysis in small quantities in afterdamp from coal-dust explosions, and in fumes from fires in sulphurous coal. It has the odor of rotten eggs, is very poisonous, and burns in air with a pale blue flame.

Sulphur dioxide (SO_2) is found in coal mines only as a lesser component of afterdamp, but in sulphur mines and in mines having rich sulphide ores in which fires have started, SO_2 gas becomes a serious factor. Very soluble in water, colorless, and has a suffocating smell.

Diffusion of gases is a fundamental principle in the ventilation of mines. Gases diffuse into air at rates in inverse ratio to the square roots of their densities.

Density of CH_4 is about 8 and of CO_2, 22; rate of diffusion of CH_4 as compared with CO_2 is as 4.7 to 2.8, or nearly twice as rapid. Prussian Firedamp Commission made some tests, admitting methane through a top inlet into a compartment of a gallery in quantities corresponding to 3, 4, and 5%; diffusion proceeded slowly, was nearly complete in 3 hr, but not entirely complete until 4 hr. When diffusion is complete gases do not again separate. Diffusion is greatly assisted by mechanical air currents. If gases pass through a fan, as in case of an underground fan, the mixing is immediate, judging from tests at the Bruceton mine. When CH_4 is given off at the face, it immediately rises to the roof owing to its lightness, and if the outflow continues, a body of gas is likely to collect in high places, unless in a strong current of air. In such case, while gas is diffusing into the air along a bottom plane, if it flows in faster than the rate of diffusion the high place will continue to be filled with pure CH_4. A somewhat similar condition exists in pillar workings or old gobs where the roof strata emanate firedamp and the presence of bodies of gas in old workings constitutes a menace when the atmos press suddenly falls (Art 3). Tests by British Safety in Mines Research Board on movement of CH_4 in a mine passage showed that a stream of it will flow along the roof if inclined upward at 1 vert to 10 horiz, against an opposed air current flowing slowly along the floor. This indicates the slow diffusion of CH_4 in absence of agitation of the air and gas currents (88).

Table 2a. Inflammability Limits of Methane, Ethane and Natural Gas (87% CH_4) in Air Mixture (4)

	Lower limit	Upper limit
Mixtures tested in tubes, ignited at bottom:		
Methane, tube closed at upper end, mixture dry	5.24	14.02
" " " " " " mixture saturated	5.33	13.80
" " " " " " 2% water vapor	5.3	13.9
" " " " lower " 2% water vapor	5.0	15.2
Ethane, " " " upper " mixture dry	3.22	12.45
Natural gas (*a*), tube closed at upper end, mixture dry	4.80	13.46
" " " " " " " " " 2% water vapor	4.9	.....
" " " " " " " lower " 2% water vapor	4.7	.....
Mixture in horiz gallery, ignited by black powder (*b*):		
Natural gas, partial inflammation	4.6	.....
" " explosion	5.1	.....

(*a*) 87% CH_4. (*b*) Crawshaw. Fan agitation slightly raised the limits.

3. SOURCES OF IMPURITIES

Blasting. Gaseous products of explosives vary widely with type of explosive, kind of wrapper, and whether used in coal or other oxidizable mineral. Table 3 shows results of tests with a Bichel gage, using 200-gm charges with paraffined wrappers, including analysis when ordinary dyamite is burned instead of detonated.

Table 3. Gaseous Products of Explosives, in Percentages of Volume (U S Bureau of Mines) (5, 6, 92)

(See Sec 4, Art 2)	CO_2	CO	N_2O_2	H	CH_4	N	H_2S	Vol of gas in liters
								(*a*)
30% "straight" nitroglyc dyn	22.9	28.4	....	20.6	0.7	27.4	....	85.8
40% " " "	27.3	26.9	....	18.0	0.4	27.4	....	88.5
50% " " "	24.4	31.2	....	20.7	0.7	23.0	....	105.5
60% " " "	22.2	34.6	....	23.2	0.8	19.2	....	128.9
60% low-freezing dyn	8.9	47.4	....	31.0	0.6	12.1	....	169.5
40% ammonia dyn	41.4	3.8	....	3.1	0.8	45.5	5.4	65.6
40% gel dyn	50.8	3.0	....	1.8	0.8	39.5	4.1	60.3
40% gel dyn, when *burned*	19.4	13.7	11.9 (*a*)	0.4	1.4	53.2	....	(*b*)
5% granulated nitroglyc powder	51.3	2.7	....	0.9	0.7	28.7	15.7	61.6
FFF black blasting powder, in gage	49.7	10.8	....	1.8	0.6	28.4	8.7	67.8
FFF black powder, when used in coal	19.2	28.2	....	10.0	0.4	35.4	6.8	(*c*)

(*a*) Incl 0.6% NO_2. (*b*) Not determinable under conditions of test. (*c*) Tested in a colliery.

Table 3 shows that the NO_2 red fumes, which in smallest amounts are dangerous to breathe, are produced when dynamite is burned instead of detonated. Also, in blasting coal with black powder, the products differ from those in the testing gage; more combustible gas is produced by presence of carbon and hydro-carbons of coal. In this test, CO, H, CH_4, and H_2S totaled 45.4% by volume. If gases thus produced are supplemented by CH_4 in the air and coal dust, they may aid in starting explosions. Smoke from the black powder can be ignited before diffusion by an open light. 1 lb black powder produces about 5 cu ft of gas; this in blasting coal gives about 2.3 cu ft inflammable gas. If the 5 cu ft be mixed with about 7 times its vol of air, the 40 cu ft of mixture may be explosive if there is coal dust or firedamp in the air. Besides gases some Na_2SO_4 and Na_2CO_3 are always present in form of smoke. Explosion of 1 lb 60% dynamite produces 10–13 cu ft gas, with considerable variations in percentage of CO, CO_2, and N. CO is produced from dynamite in proportions and volumes dependent on kind and quantity of explosive, and whether or not it is properly detonated. It is the principal poisonous gas in powder smoke, and, being dangerous to breathe (Art 2, 5), the practice of returning quickly to a working place after blasting may be injurious (5, 6, 92).

In using PERMISSIBLE EXPLOSIVES in a gaseous or dusty colliery, a requirement (U S Bur Mines) is that not more than 158 liters poisonous gas may be produced per 1.5 lb, the permissible CHARGE LIMIT. About half of all permissibles (Jan 1, 1925) give less than 53 liters (92). LIQUID-OXYGEN EXPLOSIVES were used largely in Germany during the world war, except in gaseous collieries (as they give a flame like dynamite). L.O.X., as commonly termed, has never been used in U S coal mines, because its long flame may ignite gas or coal dust (7). Its use in metal mines in U S, Mexico and Perú was stopped by accidents from premature firing due to ignition by miners' lamps of the carbonaceous powder in the cartridge, which was blown out by vaporizing O. Now used underground only in iron mines of Lorraine, France, but widely used in open-cut work in various countries, and in stripping shallow U S coal seams. Premature blasts have occurred in open cuts, quarries and stripping, from frictional sparks in tamping, in deep holes, where wooden tamping bars were not used; also by impact, when the combustible absorbent was too sensitive. They produce CO when delays occur, as in firing rounds of shots; may also produce SO_2 and H_2S, when fired in sulphide ores.

Oxidation and oxygen depletion are important factors in the condition of mine air, not only in collieries, but also in metal mines containing carbonaceous shales, as in some Michigan iron mines, or those in which large amounts of timber are used.

While all coals oxidise slightly, some, like those of central Illinois and Iowa (including carbonaceous roof shale) oxidise rapidly when broken into small pieces. Hence, spontaneous fires frequently occur in old gobs. Furthermore, oxygen is absorbed by coal. From these two causes the depletion of O in the air circulating through old workings often amounts to over 2%. If a mine room is tightly sealed up, the O is rapidly absorbed, so that, in a few weeks or months, practically all of it may have disappeared. This condition assists in extinguishing thoroughly sealed mine fires.

Timber decay is due to fungus growth, requiring some O; bacteria play a secondary role. The action is hastened by hot, humid air, and probably by the crushing of timber, as in the timber mats employed in various caving systems, in the Lake Superior iron mines and Ariz copper mines. Adjacent to crushed masses of timber the depletion of O, and production of CO_2, accompanied by heating, is so marked as to cause discomfort to the miners.

Exhalations of men and animals in well-ventilated mines are of small importance relative to other sources of gaseous impurity, but are a large factor in metal and other mines without artificial ventilation, especially in headings or stopes unconnected near the face with other workings. An aver man under different conditions, from complete rest to extreme hard work, consumes 0.25 to 4 liters of O per min and exhales, together with N and unused O, 10 to 20% less CO_2 than the O consumed, or 0.2 to 3 liters. For aver working conditions in mines the aver exhalation per man is between 2 and 3 liters of CO_2 per min, and he inhales 30 to 60 liters of air per min. If the CO_2 content of adjacent mine atmosphere is to be kept to 1% or less, then 7 to 10 cu ft of pure air per man per min are required, and about 3 times this amount for each draft animal (Art 5).

Combustion of lights, other than electric, has important effect in inner workings of mines where ventilation is slow, particularly in metal mines with only one outlet. According to J. W. Paul (Bureau of Mines tests): a sperm candle, burning 1 gm, consumes 0.614 cu ft O per hr; a miner's brass lamp, burning 4 to 6 gm oil, consumes 0.8 to 1.2 cu ft; a driver's lamp, burning 8 gm oil, 2.16 cu ft; a driver's lamp, burning 6 gm "sunshine," 2.20 cu ft; ordinary type of miner's carbide lamp, using 12 to 14 gm carbide, 0.36 cu ft O per hr. These tests show that a lamp consuming 1 cu ft O per hr uses up all the O in 5 cu ft of air; and, to keep the depletion of O within the safe limit of 1%, such a lamp requires 100 cu ft air per hr. The amount of gaseous impurity given off by large open lights in close places is so considerable that, where air currents are feeble, the use of miner's electric lamps is advisable.

Mine lighting has greatly changed in the last 2 decades, especially in coal mines, where open lights may ignite CH_4. Moreover, permissible cap-lamps give better light, and many are used in metal mines, as they are safer than carbide or oil lamps.

Emanations from strata. CH_4 (Art 2) is the most important and dreaded gas encountered in collieries. It is probably derived from coal-forming material by distillation due

to heat and long-continued pressure. There is no apparent relation between the amount of gas retained in the strata and the geologic age of formation; the quantity seems to depend more on geologic structure. From Carboniferous to Eocene the coal measures when opened are liable to give off CH_4.

Highly flexed regions, especially the contorted Appalachian anthracite folds, produce more gas than the flat practically undisturbed beds of the middle interior fields. Also, in the sharply-folded beds of the Rocky and Cascade Mts, more gas is found than in the gently-folded beds of central Wyoming. But this rule is not invariable, and seems to be less important than depth of impervious cover, giving vertical pressure and less chance for escape of gases.

Pressure of CH_4 in the strata appears to bear a rough relation to the press of a water column equal to depth of bed below water level, but this is subject to wide variations according to permeability of strata. CH_4 is held in the strata in the pores or adsorbed in the coal, from which it is only slowly liberated on exposure in mining, and more rapidly as it is broken or crushed; also in crevices, fault zones, and porous strata above or below the coal bed, which liberate gas as " blowers " or " feeders," lasting minutes, days or years.

Violent outbursts of CH_4 have thrown out large quantities of coal and dust from the face, usually in folded seams, though Rice (12) reports severe "bumps" in B C and Nova Scotia, unaccompanied by gas outbursts. To prevent the latter, 3 or more radiating holes 10 to 20 ft long have been kept in advance of the heading, but this remedy sometimes fails. Practice in mines near Alais, France, where instantaneous outbursts of CO_2 occur, is not to pick or cut into the face, but to drill and prepare shots, and, at end of shift when all men are out of mine, to fire simultaneously by electricity from the surface. The object is to induce blowouts from any possible high press ahead, by concussion due to blasting. In recent years this safer method, termed "shock blasting," was adopted in mines in Belgium, the Ruhr and Silesia (12), subject to outbursts of CH_4 and CO_2. One theory is that the coal is under great press, and upon release flies into powder, liberating gas. Samples of Belgian coals at ordinary temp yielded on successive crushing 69.1 to 84.65% CH_4, remainder being chiefly N and air. Vol was 2.5–5.5 times that of coal crushed. Coal from Saarbrücken, Germany, is stated to have given off 2 to 2.5 its vol of gas. Differences in laboratory methods make direct comparisons difficult. Tests on U S coals (8, 9) show that the total gas from crushed coals at ordinary temp is from 1 to nearly 2 times the vol of coal. Most gas is given off slowly, but with fine crushing, more rapidly. Monongah coal (W Va), crushed to 30 mesh, gave off CH_4 in a few hr equal to 0.86% of the vol of coal, while that crushed to 10 mesh gave off only 0.4% (60). By allowing the gas to drain off in vacuum for 26 weeks, the Monongah coal gave off 1.83 volumes. In tests of American coals without heating, nearly all gas is CH_4; amount of CO_2 is from 5 to 10% of total gas. Samples from a very gaseous mine, Coalfield, B C, and from the Pittsburgh seam, were ground by Fieldner in vacuo, at room temp. Former gave 200 cc occluded gas per gm; the latter, 49 gm. Of the former, 18.8% was methane, 62.0% ethane; Pittsburgh gas, 13.5% methane, 5.1% ethane. Total paraffins gave 2.12 and 0.12 vol. respectively, of vol of coal crushed (12).

Quantity of CH_4 when liberated from the coal. If ventilation is effic, CH_4 is given off too slowly to be dangerous. The greatest menace is the gas stored in open crevices, porous strata, and goaves of unsealed old workings, which may be released by roof falls, or sudden fall of barom press.

Analyses of the return current of many mines show a little less total CH_4 when a mine is idle than when working; in others, no appreciable difference; indicating that the quantity of gas from coal broken in mining is small compared with that issuing from the solid face and contiguous strata (Sec 14). Total steady inflow of CH_4 in individual mines (Table 4) is obtained by multiplying vol of current by % of CH_4 determined by analysis, and reduced to a common basis of cu ft of pure CH_4 per min, irrespective of vol of ventilating current (9). Table 4 does not exhibit aver conditions, which are difficult to obtain; those for the Liévin mines show wide variations in the same mine. It is uncertain whether flow of CH_4 into a gaseous mine is naturally affected by advance of the faces; it appears to depend on structure of the coal basin and origin of gases. In some mines the flow changes little on idle days; in others, it changes much (12).

Haas (9) suggests that gases of occlusion, adsorption, or even volatile matter in the coal itself, are insufficient to explain the enormous volumes of gas exhausted from our mines; that most of the hydrocarbon gases are natural gas, and that the coal seams are simply the reservoir and path of flow from other strata. His graphs of flow indicate an increase with size of mine, but are independent of the ratio of coal production. As anthracite dust is not explosive in itself, the highly gaseous conditions prevailing in many Penn anthracite mines are less serious than in less gaseous bituminous mines, where pure coal dust is very explosive (Art 14). A bituminous mine giving off 200 cu ft pure CH_4 per min, even if ventilation is strong, would ordinarily fall in the class of gaseous mines, requiring safety lamps because concentrations of gas are likely to be found at the face or in old workings. Often, not over 25% of the air reaches the last break-throughs; the rest lost by leakage (81). Bituminous mines making over 500 cu ft CH_4 per min are considered very gaseous; few in the U S make as much as 1 000 cu ft per min. Multiplying by 18, the figures of Table 4 for pure CH_4 gives vol of firedamp per min at lowest explosive limit, 5.0%.

Table 4. Inflow of Methane into Gaseous Collieries in Europe and America

European mines	District	Country	CH_4, cu ft per min	Authority
Neu-Iserlohn No 2	Westfalia	Germany	623	Prussian Firedamp Comm
Kaiserstuhl Shaft	"	"	350	"
Albert Shaft	Saarbrücken	"	337	"
Friedenshoffnung	Lower Silesia	"	178	"
Liévin No 1, 1901	Pas-de-Calais	France	507	Léon Morin
" 1908	"	"	292	"
Liévin No 2 and 5, 1903	"	"	62	"
" 1905	"	"	248	"
" 1908	"	"	523	"
Colliery No 25		England	522	Royal Comm on Mines '07
" 38, 1 seam		"	983	"
A Yorkshire mine, 1912		"	3 315	Robert Clive (8)
" 1926		"	1 888	"
" (after standing 20 weeks), 1926		"	610	"

American mines	Kind of coal	State	CH_4, cu ft per min	Authority for sampling
Lance Mine	Anthracite	Penn	3 701	Darton, N. H. (8)
South Wilkes-Barre	"	"	1 226	"
Dorrance mine	"	"	3 308	"
" (after 1 mo suspension)	"	"	3 212	"
Truesdale	Anthracite	Penn	2 037	Darton, N. H.
Nottingham	"	"	2 500	"
*Zeigler	Bituminous	Ill	181	"
*Dering No 11	"	"	204	"
*(*A*) mine	"	Wash	73	Wolflin, H. M.
(*B*) mine	"	Penn	450	Burrell, G. A.
*(*C*) mine	"	"	700	Rice, G. S.
*(*D*) mine 9/15/10	"	Ala	193	Morrow, S. L.
" 9/23/10	"	"	282	"
" 4/3/11	"	"	200	"
(*E*) mine, Fairmont	"	W Va	479†	Haas, Frank (8)
(*F*) mine, Pocahontas	Semi-bit	"	2 080†	"
(*G*) mine	Bituminous	Mexico	2 830	Denny, E. H.
Michel Colliery	"	B C	1 524‡	Rice, G. S. (44)
" (2 mo suspension)	"	"	929	"
Coal Creek No 1 East	"	"	2 906‡	"
" on idle day	"	"	1 247	"

* Disastrous explosions, with firedamp probably a contributory cause.
† From diagrams in "Occurrence of Firedamp in Bituminous Coal Mines," by Frank Haas (using max amounts) (8).
‡ Max recorded.

Effect of barometric press on issuance of CH_4 is a debatable question. British Royal Commission on Mines deprecated "colliery warnings" of low barometer, on the ground that explosions were generally caused by carelessness or other conditions, rather than by temporary increase of firedamp, and that between such warnings care might be relaxed. Observations by the Bur of Mines, in gaseous Penn and Ala mines, indicated that falling barometer was generally followed by increase of CH_4 in the return current, but the correspondence was not exact. When a gaseous mine has extensive old workings, poorly ventilated, especially if the roof be broken, much gas may accumulate, and when the atmos press falls the gas diffuses into the air currents. It is not generally believed that gas inclosed in the pores of coal, or stored under high press, and issuing in blowers, is affected by atmos press changes, since these rarely reach 0.5 lb per sq in, as compared with the far greater press under which gas exists in the coal.

Outbursts of CO_2 from the strata have been of importance in certain coal basins of Central France; miners smothered, and masses of coal and dust dislodged, and even blown to the surface. As outbursts are especially liable to result from blasting, shots are sometimes fired electrically from the surface; refuge chambers are also provided. Outbursts are due either to heat of igneous intrusions or to acid waters acting on limestone; the latter is probably true of CO_2 sometimes found in fissures and crevices, in the S W Wisconsin lead and zinc district. Cripple Creek mines have had

difficulty with CO_2 filling fractures of volcanic rocks after drainage, until it could be kept back by bulkheading and blowing in air under pressure (9).

Nitrogen occasionally emanates from the strata. A well in Washington and some in Texas gave off almost pure N. The natural gas used in Fort Worth, Dallas, and other Texas cities, contains about 14% N. Generally an excess of N in mine air is due to absorption of the O by coal or carbonaceous material.

Gases from coal mine fires. After the incipient stage, fires in collieries are too serious to permit men, other than fire-fighters, to work; but in the non-gaseous mines of central Ill, Iowa, and Mo, spontaneous fires are common, and when shut off by fire walls are apt to be disregarded, though some fumes often escape through or around the walls. These gases are chiefly CO_2; frequently accompanied by CO which would cause trouble if ventilation were not properly managed. Similar conditions prevail in some thick-bed anthracite districts, where fires in abandoned mines threaten adjacent active mines. Spontaneous fires are so common in the thick coal bed of So Staffordshire, that fire-stopping material is kept on hand at entrance to panels.

Gases from fires in metal mines, as in the carbonaceous and sulphurous roof shales of some Michigan iron mines, may be serious when the fires can not be extinguished by ordinary means. Such gases consist of CO_2, SO_2, and some CO and H. In rich copper sulphide ores of Ariz and Mont, fires have sometimes started spontaneously in old stopes, producing much SO_2 from burning ores, and CO_2 and CO from burning timber.

Hydrogen sulphide (H_2S) may be produced by the heating of sulphurous coal in a mine fire, by action of acid waters on sulphide ore, or by the reducing action of bacteria on sulphates in stagnant water. It may also occur in the products of explosion of black powder. H_2S is very soluble, but may be liberated in dangerous quantities by stirring up stagnant water in traversing old mine workings.

Gases from internal-combustion engines, as gasolene-driven pumps, hoists and locomotives. CO_2 and CO are produced in amounts varying with carburetor adjustment and size of engine. O. P. Hood (14) states, as result of tests on 2 types of gasolene locomotive, that the CO in the exhaust under worst conditions is about 13.5%, while under normal running conditions it seldom exceeds 6%. For safety and health the max should be used in estimating the quantity of air required to dilute the gases to a safe point; or conversely the size of engine may be limited to the max air current possible to maintain. Engines driven by inflammable liquids should not be used underground unless a definite vol of air current is maintained, while engine is running, sufficient to dilute the max output of CO under worst conditions, down to 0.02% or less (Art 5). In Europe disasters therefrom have led to their replacement by storage-battery locomotives, or by the effic Diesel locomotives, which give off little CO (15). Diesels for mine use have devices to spray the exhaust, and prevent back-firing flame from igniting explosive mine gases.

Solid impurities in mine air. ROCK DUST fine enough to remain suspended for a time in air currents (26, 27), is produced by: (*a*) drilling dry holes or "uppers," with ordinary machine drill; (*b*) blowing out holes with compressed air; (*c*) squibbing, or enlarging the bottoms of holes to hold large charges; (*d*) adobe shots (Sec 4, Art 9) or shallow blockhole shots; (*e*) loading dry ore by shoveling or from chutes. Causes *a*, *b*, *c*, and *d* may produce much dust.

To keep rock dust out of the air (referring to above items): (*a*) Water sprays alone are inefficient. In collaring, use full bore of the hose to supply water. Wet the face thoroughly before drilling begins. Water-injection drills are now generally used. Devices to cover the hole while drilling, the bits passing through them, have had limited success. Those connected by a suction hose to dust traps for filtering out dust, though good, are usually limited to headings. Respirators (wet sponge or cloth) catch only part of the dust (tests of Bur of Mines). Fine dust, less than 10 microns diam, remains suspended for hours and is dangerous to breathe (26, 27). Respirators throttle breathing and may cause rebreathing some CO_2 of exhaled air. Masks (developed from war masks) are better, but do not intercept finest dusts and are cumbersome. Recently they have been improved in filtering effic and 9 types are approved by U S Bur of Mines. Latest practice in So Africa, Australia, and most metal mines in U S: 1. Use water drills. 2. Wet working places by hose. 3. Turn on "water-blast" atomizers after blasting (46). 4. Use positive ventilation, with strong air currents to sweep away fine suspended dust. (*b*) Do not blow out dry drill holes; use scraper. (*c*) Do no squibbing and blockhole shooting while men are working nearby; do it at shift end, or midshift; or, after wetting down and cleaning dry dust out of hole; for adobe shots, dynamite should be covered with damp clay or dirt; with no tamping. (*d*) Dust from shoveling is prevented by sprinkling. Coal dust comes from: (*a*) cutting coal by pick or machine; (*b*) blasting, especially "blasting-off-the-solid"; (*c*) loading, especially when by loading machine; (*d*) conveyers and chutes; (*e*) in haulage by grinding of coal jolted under car wheels. REMEDIES: for (*a*), wetting the face before cutting, and spraying cutter bars; (*b*) wetting face before blasting, having the face well

undercut or sheared and firing one shot at a time; (c) wetting broken coal in mechanical loading; (d) use of suction dust-collectors at loading chutes; (e) automatic car sprinkling at gathering sidings. Greater recent use of mechanical loading has increased dust. Coal dust is distributed by ventilating currents; hence need of using water at face, and rock-dusting with limestone or other non-silicious dust in haulage ways (Art 14). Coal dust in light clouds in the air, unaccompanied by silicious dust, is not injurious (Art 5).

Inflammability of coal dust. Bituminous and lignite dusts, when stirred into the air by blasting, CH_4 explosion, wreckage of cars, or fall of roof, may be ignited by flame of a non-permissible explosive, or an electric arc, or even by a torch flame (Art 14). Inflammability increases with percentage of CH_4; decreases when mixed with non-combustible dust. Anthracite dust is not explosive. Remedies to prevent bituminous dust explosions are similar to those cited in preceding paragraph. Inert content of mixed coal and rock dust should exceed 60%, and 5% more for each per cent of CH_4 present (Art 14).

4. TEMPERATURE AND HUMIDITY

Temperature of air in mines approximates the temp of adjacent mine walls. A continued lower or higher temp of air current, even if rapid, modifies that of the walls only a few degrees, to a max of about 10° F.

Seasonal changes of temp affect the temp of the strata for a comparatively short distance below surface (rarely more than 20 or 30 ft, Art 2), unless open fissures admit air currents or streams of water. The Alaska gold-bearing gravels remain frozen the year round, below a few ft in depth, though vegetation in summer is abundant.

Temperature due to locality. Though frost goes to considerable depths in polar regions, geologic conditions have much greater effect on temp of the strata, at depths over 200 ft, than climatic conditions. In the Lake Superior copper mines, the walls are cool at much greater depths than in most mining districts.

Changes of temperature with depth. Rate of increase of temp varies widely.

In Great Britain (25) the temp gradient (depth per deg F rise in temp) is 54 to 95 ft; in Belgium, 56 to 93 ft (isolated case, 22 ft); in Germany, 63 to 67 ft. In Alpine tunnels the temp gradients, measured from mountain crests, are: Simplon, 72 ft; St. Gothard, 84.7 ft; Mont Cenis, 80.4 ft. In Comstock silver mines, Nev, the gradient is stated as only 29 ft per deg, due to hot springs; in Ohio-Tonopah mine, Nev, 33 ft per deg; Grass Valley mines, Cal, 107 ft; Rand mines, Transvaal, 167–220 ft per deg. Deposits due to action of thermal waters, in districts where this action is still going on, will have higher and more variable rock temp than where such changes have long since ceased, or in uncontorted sedimentary strata where increase in temp is due to thickness of covering or former igneous intrusions. Increase of temp is often the controlling factor as to depth attainable (Table 4a). At great depths, even where the temp gradient is low, mining difficulties increase. In coal mining the seams are in sedimentary formation and, while the temp gradient is often high, intrusions have been rare. The deepest mines in Belgium and England are about 4 000 ft, with rock temp below 100° F; conditions readily met by strong ventilation. In the shallower coal mines of the U S, the temp problem is not yet serious.

Table 4a. Mine Temperatures and Rock Temperature Gradients

Mine	Country	Mineral	Depth of temp reading, ft	Rock temp, deg F	Rock temp gradient, ft per 1° F
(a) in Lancashire	England	Coal	3 783	113	59.6
(b) Morro Velho	Brazil	Gold	6 426 8 051	118 130	120
Village Deep	Rand, S A	Gold	7 062	97	220
(c) Robinson Deep	Rand, S A	Gold	3 848 6 214	80.2 93.0	185
			7 349 7 833	99.8 102.7	167
(d) Crown Mines	Rand, S A	Gold	8 500	108.4	190
Champion Reef	India	Gold	6 194	119	141
(e) Calumet & Hecla	Mich	Copper	3 562 5 679	74.8 95.3	108.5
(f) Magma	Superior, Ariz	Copper	4 000	140	53

(a) Lancashire mine, unnamed. Observations reported 1933 stated this was highest temp yet found in British mines. In 1935, a temp of 115° F was recorded in a deep Lancashire mine.

(b) Surface cooling plant (See 14), installed 1921, reduced wet-bulb temp 10° F. Another cooling plant at 6 000-ft level in 1929.

(c) Surface cooling plant installed 1937 (W. H. Carrier, *Trans* A I M E, Feb, 1938). Wet-bulb temp in main airway at 8 000 ft, which had been 85° F, was lowered 8 to 9° F (See 14). For temp in deep bore holes, see *Jour* Chem, Met & Mining Soc S A, Mch, 1936.

(d) Descriptive article, *Mine & Quarry Eng* (London), June, 1938.

(e) Fisher, Ingersoll and Vivian, A I M E *Tech Pap* 481, Feb, 1932.

(f) C. B. Foraker. *Jour*, Min Cong, Nov, 1937. Cooling plant installed at 3 600-ft level.

Temperature of mine atmosphere may locally rise above the wall temp because of: (*a*) inflow of hot waters; (*b*) active fires of broken sulphurous ores, or of coal, carbonaceous shales, or timber; (*c*) oxidation of sulphurous ores, coal or shales, too slow to raise the temp to point of ignition of the material; (*d*) decay and oxidation of timber, due to fungi and bacteriological action, which is very active in crushed timber, as in the block-caving system, and where the air is warm and humid. Tests in the Monongah colliery, W Va, showed that in traversing 4 000 ft of double entry, 30 000 btu per min were transferred to the ventilating current. In other cases intake air averaged 27° F, aver velocity 453 ft, volume, 66 432 cu ft per min; at 1 400 ft from the entrance the aver temp of the air was 45° F, and at 3 800 ft 50°, or within 5° of the temp of the return current (Sec 14).

Seasonal temperatures have little affect on the temp in large mines. Tests lasting 1 year, in 20 Illinois collieries, showed in one a max fluctuation in temp of return current of only 1.5° F, due partly to relative tightness of stoppings and doors (1). In any climate, workings below 50 ft under solid strata, and over 1 000 ft from openings, are not sensibly affected, unless there is a strong current.

Table 5. Temperature and Humidity of Intake and Return Air Currents, Illinois Coal Mines (Highest and lowest weekly averages for 1 year, 1912–13)

Name of colliery	Depth of shaft, ft	Week ending	Temperature, deg F			Relative humidity, %		
			Outside air	Intake air	Return air	Outside air	Intake air	Return air
Oglesby (*a*)	464	Mar 3	15	28	71	74	97	90
		Sept 8	80	76	77	70	85	100
LaSalle (*a*)	440	Mar 3	15	40	74	74	82	85
		Sept 8	80	77	78	68	93	96
Big Muddy No 9	114	Feb 9	16 1/2	43	51 1/2	80	81	100
		Sept 8	79	75	62	75	91	100
Sherrard No 2	210	Feb 9	16 1/2	39	68	76	95	96
		July 14	78	74	68 1/4	74	93	96
Assumption No 1 (*a*)	1 004	Feb 9	15	40 1/2	75 1/2	74 1/2	97	96
		Sept 8	82 1/2	75	75	64	98	84
Empire No 2	105	Mar 3	15	32 1/4	63 1/2	74	88	97
		Apr 27	57	60	63	40	72	97
Peerless	230	Mar 2	19	20 1/2	59 1/2	76 1/4	100	94
		July 28	76	74 3/4	70	57	94 1/2	98
Saline Co No 3	100	Dec 15	27 3/4	38 1/2	60	63	86 1/2	94
		Sept 8	78 3/4	78 1/2	68	75	89	98
Hart & Williams No 1	632	Feb 9	12 3/4	25 1/2	61	73 1/2	100	89
		July 28	78	82 1/2	68 3/4	76	100	100

(*a*) = longwall mines; others are room and pillar.

Hot metal mines, Comstock Lode, Nev. According to Young (25), air entering at 46° attained a temp of 71.6° at 5 200 ft from the Sutro tunnel entrance, 91.9° at 10 000 feet, and 95° at 13 000 ft, in a swift air current. Aver water temp entering the tunnel was 94 to 95° F, which presumably represented aver wall temp at that level. Locally the water temps were much higher.

Adiabatic compression of intake air, produced by wt of the column of descending air, causes increase in temp of 5.5° F per 1 000 ft depth; independent of veloc of air-current. Conversely, uptake air loses heat (see Sec 14). If rock temps were the same at all depths of a shaft, the heat due to this compression would exceed that of the rock and flow into the strata, which acts as a heat reservoir with constant temp (for any specific locality) at any given depth. But rock temp generally rises faster with increasing depth than the temp of air current. In absence of a hot igneous mass, or hot water, virgin-rock temp gradient below 20 or 30 ft from surface is nearly constant for considerable depths, but varies widely in different regions, and normally increases slowly with depth (Table 4a).

As adiabatic compression of dry air theoretically causes a temp rise equivalent to a rock temp gradient of 182 ft, heat flows from the strata into the air current in most deep mines; the rock walls gradually cooling to a thickness depending on their thermal conductivity, which is low (for conductivity of rock, see Smithsonian Physical Tables). In a mine 6 000 ft deep the adiabatic compression-increase of temp is 33° F; hence, if the aver surface temp is 65°, that at 6 000 ft would be (theoretically) with dry air 98° F. Besides the influence of temp of the walls on the air temp, is the effect of cooling the air current by vaporizing the moisture encountered in its passage. In most districts the intake air is less than 50% saturated, and, at 65° F, carries about 4 grains water vapor per cu ft. If the current descended through a dry shaft and attained a temp of 98°, it could take up about 14 grains more water in becoming saturated, and evaporation of this water would lower the air temp about 10°. This would further cool the mine walls, but cause bad physiological conditions (19) (Art 5).

Reduction in temp of mine air can be effected locally by spraying with cold water (with the disadvantage of increasing the humidity), or by active circulation of cool air. At long distances from the intake the latter plan will reduce the temp of working places only a few degrees, rarely as much as 10°. But, if certain places in a mine are heated by fires or by crushing and pressure, as in the caving system, the air can be improved by diverting a current of cooler air from a neighboring working. This is done by brattices or pipes, sometimes aided by supplementary blowers or "booster" fans (Sec 14). The return air from a hot area should pass direct into the main return current. This method has not been as much employed in metal mining as it should be. In coal mining, intermittent use of auxiliary fans or blowers in gassy or slightly gassy mines is dangerous, as CH_4 accumulated during a stoppage may later be blown to a point where there is a source of ignition. Auxiliary fans driven by open motors have also caused explosions; European practice permits them in headings, if driven by compressed air and operated continuously.

Refrigeration of mine air in deep, hot mines has been much discussed for use in mines too hot to get material advantage from such ventilating currents as can be forced through extensive workings or restricted passages. As mentioned above, the limiting depth of mining is determined by the air temp and humidity obtainable (for details and Bib, see Sec 14).

Humidity of air in metal mines varies more than in collieries. Absolute humidity (wt of water vapor per unit of volume) of an intake current cooler than the mine temp never diminishes, and unless already saturated is increased by presence of water. As the temp of mine walls and the contiguous air increases with depths, relative humidity decreases unless water is encountered.

The great variability in relative humidity possible in a metal mine is shown in the Comstock mines, Nev (25). In one tunnel, where intake air did not come in contact with drainage water for first 6 800 ft from portal, the moisture probably came mainly from walls. Rate of evaporation was 43.8 grains per hr per sq ft of exposed surface. From 6 800 to 10 000 ft, the air current was in contact with drainage water and became almost saturated. On basis of 6 ft aver width of exposed water surface, the accession of water vapor was 798.5 grains per hr per sq ft. Relative humidity of working places, 22 to 100%.

Control of relative humidity, when approaching saturation, is comparatively easy. In coal mines this has been used to prevent drying out of inflammable coal dust. VENTILATING CURRENT may be humidified by spraying, separately or combined with artificial heating of the intake by steam pipe coils, or steam jets (64). This method has been employed in So Wales and certain American coal mines to make coal dust less explosive, but as the effect of wetting was limited to a short distance from the intake entrance, and failed to prevent explosions, rock-dusting was successfully substituted. Heating the intake by steam coils at some Middle West mines, where coal is hoisted in intake shaft, has improved winter conditions for men working in shaft bottoms.

Increase in temp *gradient* with depth, found in the Robinson Deep (Table 4a), is confirmed by tests with special recording apparatus in U S oil and gas wells, probably representing original rock temps better than tests in mine workings, where wall heat is lost through constant ventilation. See Table 5a and Bib 19, 20.

Table 5a. Ground Temperatures in Oil Wells (*a*)

Depth ft	Johnstown, Pa		Carlsbad, N M		Bakersfield, Cal		Signal Hill, Cal	
	Observed temp, F	Gradient constant from 100 ft (*b*)	Observed temp, F	Gradient constant from 100 ft (*b*)	Observed temp, F	Gradient constant from 100 ft (*b*)	Observed temp, F	Gradient constant from 100 ft (*b*)
0	46.0	.....	62.1	.....	64.6	.....	60.8	.....
100	48.0	.....	66.1	.....	79.2	.....	71.4	.....
1 000	56.2	108.8	71.3	183.5	94.5	59.1	88.4	53.1
2 000	69.1	89.2	77.5	177.0	119.2	46.0	106.1	55.8
3 000	80.9	85.4	85.2	156.3	140.2	45.6	124.7	55.3
4 000	91.4	86.5	91.7	154.0	161.2	46.0	146.1	53.3
5 000	105.8	85.1	101.8	144.6	180.5	46.8	166.3	51.9
6 000	124.3	79.6	111.8	131.8	198.3	47.8	186.2	51.2
7 000	.....	.....	.....	.....	213.9	49.1	207.5	50.5
8 000	.....	.....	.....	.....	228.9	50.7	226.8	50.3
9 000	.....	.....	.....	.....	241.2	53.0	245.0	50.4

(*a*) Estimation of Temperatures at Moderate Depths in the Earth's Crust. C. E. Van Ostrand, *Trans* Amer Geophysical Union, 1937.
(*b*) Figures for different depths are based on the least-square adjustment of the straight-line equation, $y = a + bx$, where y is the Fah temp and x the depth.

Lowering relative humidity is difficult, and in most cases the only practicable way is to force cool air to working places through pipes, so it may not absorb moisture en route; or, by refrigerating mach'y, to condense the moisture in air current.

Most coal mines in the U S are comparatively shallow, and the temp of working places rarely exceeds 75° F, at which saturation is neither uncomfortable nor objectionable (Art 5). At depths exceeding 3 000 ft, as in certain coal mines in Lancashire, Belgium, France and Germany, high air temp occurs, and the miners' shifts are shortened when wet-bulb temp exceeds about 85° F. On the Comstock lode the temp in some drifts reached 130° F (25).

MINE HYGIENE

5. EFFECTS ON HEALTH OF IMPURITIES AND CONDITIONS OF MINE AIR

Quantity and quality of air for breathing. For good health the percentages of O, N, and CO_2 should not vary widely from normal, though in the matter of wt or density of air, the lung action adapts itself readily to variations produced by changes in press from 0.5 atmosphere at high elevations to 3 atmos in some caissons.

In deep diving a press of 6 atmos has been reached (Art 16). In case of low press, since there is less O in a unit of vol, the lung action must be quicker. N is inert and a diluent only; the same vol and wt is exhaled that was inhaled, when both are measured at same temp. O is the life-supporting element in air, and by its combination with the C of food, through agency of lungs and blood, maintains bodily heat and energy. The irritating effects of O are found only after 48 hr continuous exposure to an atmos containing over 80% O_2 (30).

Table 6. Quantity of Oxygen Consumed and Given off During Resting and Working, and Number and Volume of Respirations, at 760 mm Barom and 32° F. (J. S. Haldane)

	O_2 consumed per min, liters	CO_2 exhaled per min, liters	$CO_2 \div O_2$, %	Air breathed per min, liters	Av vol of each breath, liters	No of breaths per min	Inhaled air consumed, %
Rest in bed	0.237	0.197	0.83	7.7	0.457	16.8	3.08
Rest, standing	0.328	0.264	0.81	10.4	0.612	17.1	3.14
Walking 2 miles per hr	0.780	0.662	0.85	18.6	1.270	14.7	4.17
" 3 "	1.065	0.922	0.87	24.8	1.530	16.2	4.28
" 4 "	1.595	1.395	0.88	37.3	2.060	18.2	4.25
" 4.5 "	2.005	1.788	0.89	46.5	2.520	18.5	4.38
" 5 "	2.543	2.386	0.94	60.9	3.140	19.5	4.15

U S Bur of Mines made tests closely agreeing with above figures. The O consumed is about 20% of that inhaled. Tests of breathing nearly pure O under different conditions of work showed a reduction of 2.11 to 5.44% O, indicating consumption of O is not affected by enriching the air with O. The proportion of CO_2 exhaled to O consumed (the respiratory quotient) increases from 0.8 for a person at rest to 0.95 for a person at hard work, and when over-exerting may rise above 1.00. Ordinarily a person uses only the upper portion of the lungs; but during violent exertion the full lung capacity is required, increasing the respiratory quotient. Though Table 6 indicates that less than 3 liters of O per min is generally required, tests with O rescue apparatus show that to take care of especial exertion 3 liters of O per min should be supplied. This is equivalent to 14.5 liters of air, or about 0.5 cu ft per min, of which the O has been completely consumed, but this is never the case, as indicated above. If in a closed space, like a mine, the O is to be maintained with a loss no greater than 1%, the air supply per man must average about $0.5 \times 20 = 10$ cu ft per min; for draft animals, about 30 cu ft per min.

Classification of conditions and impurities, as to effect on human system:

(*a*) Conditions due to altitude, high temp and humidity; enervating, but not dangerous unless extreme.

(*b*) Impure air, not toxic, as air depleted of O; not dangerous unless O deficiency is great.

(*c*) Gaseous impurities having toxic effect; not serious unless from long exposure to large quantities.

(*d*) Gaseous impurities acutely poisonous, and difficult to recover from.

(*e*) Solid impurities leading to lung diseases.

Effect of high altitude: deep and rapid breathing and marked increase in number of red corpuscles. As much coal and metal mining in the U S is done at elevations between 5 000 and 11 000 ft, and some metal mining up to 13 000 ft, the consequent decrease of O

per unit of volume to 1/3 or nearly 1/3 that at sea level reduces working effic. Men should be examined to see that their heart and lungs are sound before giving them employment.

Effect of high temp and high humidity within certain limits of temp. Either condition alone is bearable, but when combined, discomfort or injury to health results. At moderate temp, say under 70° F, relative humidity up to 100% is unobjectionable, and a nearly saturated atmos is not serious at a temp of 80°, in a good current of air; otherwise it is oppressive and a day's work rapidly diminishes with a further rise in temp and saturation.

Haldane states: "At temp of 93° in still, saturated air I found that, though I was stripped to the waist and doing practically no work, my temp rose 5° in 2 hr, and was still rising rapidly when I found it necessary to come out." Church (monograph on Comstock mines, 1880) reports that "the immediate results of high temp and hot vapors are absent-mindedness, dizziness, fainting, vomiting, and as graver results, insanity and death." Casualties in the Comstock mines at that time positively traceable to heat were 12% of the whole. Probably heat increases the bad effects of powder fumes and natural gases. Lord, referring to the same mines (1883), states that the ultimate effect of extreme heat is not easily noted. Power of recuperation appears to be extraordinary; and unless the strain is intense and frequent, there may be no lasting injury. Young states (1909) (25): "Local physicians say that the aver working life of Comstock miners approximates 25 years, and that they show no greater susceptibility to disease than the town residents." Miners stripped to the waist should keep out of cold drafts; on passing from a hot to a cold place a heavy coat is used. Wet clothes are removed before going to surface, or trousers and coat are worn over them. Miners take hot and cold showers after coming off shift. Frequent drinking of, and bathing hands, wrists, arms, and head, in ice water are resorted to. Temp of 95 to 105° F, with 50 to 70% relative humidity, and air moving 200–300 ft per min, does not prevent effic work; temp of 110 to 115°, with above conditions, considerably reduces effic. Same temp, at high humidity and moderate air veloc, greatly impairs effic; higher air veloc renders workings more bearable, but shifts must be short. Physiological tests of effect of high temp and humidity, with and without movement of air, made at Butte by Sayers and Harrington, gave further precise data (19). They found that the bad effects of almost saturated air, with temp between 90° and 98° F, are much less in moving than in still air. But, at 98.6° to 100° F, there was no benefit from saturated air in motion, even at high veloc; and apparently some disadvantage. It would thus appear that the economic LIMIT OF DEEP MINING is reached when the wet-bulb temp reaches blood-heat (98.6°).

In work at high temp and humidity, the Safety in Mines Committee, England, found that sweating removes salt from body secretions and weakens the subject, beside causing the effects previously mentioned. Total chloride given off from the skin may be a measure of amount of sweat. To supply the deficiency, salt may be taken into the system through liquids and foods. In the Crown Mines, Rand, barrels of a saline solution are distributed through the workings, containing 1 oz lime juice and 10 oz salt to 60 gal water (17). In some U S mines, tablets of salt in pocket containers are furnished the men.

Gaseous impurities without toxic effect, but causing oxygen deficiency, include N and CH_4. EFFECT OF OXYGEN DEFICIENCY near sea-level or below it is the same as reduction of O due to lower atmos press; as in mines at high altitudes.

A press of about 1/5 less than at sea-level, say 5 000 ft, is not noticeable; but at 1/3 less (10 000 ft) violent muscular exertion causes rapid breathing and heart action; continuous exertion causes faintness. The O present in a unit of volume at Leadville (10 000 ft) is the same as that in a sea-level mine, where the O has been reduced from 20.0 to 13.8%. At top of Pike's Peak (14 147 ft) the barometer registers 17.4 in, and the O present corresponds with but 12% O in air at sea-level. People become acclimated at these and higher altitudes. In Perú, Bolivia, and Northern Chile, there are many inhabitants above 15 000 ft.

Haldane states that a person not exerting himself will rarely notice anything unusual until the O falls to 10%; breathing then becomes deeper and quicker, pulse faster, and the face somewhat dusky. At 7% there is usually distinct panting, with palpitations, and the face assumes a leaden blue color; the mind becomes confused, and the senses dulled, though the subject may be unaware of the fact. IN AIR CONTAINING NO O, loss of consciousness occurs within 40 sec, without warning symptoms. Miners accidentally putting their heads up into a roof pocket of CH_4 have almost instantly become unconscious. In a test made at the Pittsburgh station of U S Bur of Mines, on one of the rescue corps, WHEN O WAS REDUCED TO 7%, in a mixture of air and N, the subject lost consciousness without much warning. The test being immediately stopped, the subject at once recovered consciousness in the open atmosphere, feeling no real distress until the next day, when he was decidedly unwell. Haldane says: "Loss of consciousness in air deprived of O is quicker than in drowning; not only is the supply of O cut off, but that previously in the lungs is rapidly removed; loss of consciousness is quickly followed by convulsions, then by cessation of respiration." In the case of cats and dogs, the heart continues to beat from 2 to 8 min; in men this period is probably much longer. SO LONG AS THE HEART BEATS, HOWEVER FEEBLY, ANIMATION MAY BE RESTORED BY ARTIFICIAL RESPIRATION. This may have to be long continued, and the respiratory center may not soon recover. Whereas CO_2 in excess carries its own warning (distinct panting) before danger is imminent, O may be so deficient as to imperil life before a person realizes it (32).

Since an ordinary oil-fed lamp flame is extinguished at about 17% O, the diminishing and flickering of the flame warns that the O is below normal. An acetylene flame is not a good guide to a condition of the local atmos dangerous to life, as it will burn when only 12 or 13% O is present.

Gaseous impurities having toxic effect, but not serious unless with long exposure. Excess of CO_2 is frequent (Art 2, 3). Slight distress begins when it exceeds 1 or 2% of the atmos, but is not serious until over 3%. WITH 3 TO 4% CO_2 BREATHING IS LABORED; with 6%, marked panting and increased frequency of pulse or palpitation of heart; throbbing head and flushed face; headache is particularly noticeable on return to fresh air. With 8 to 10% there is narcotic effect.

Haldane states that AT 11% CO_2 UNCONSCIOUSNESS OCCURS, but death does not result unless there is exposure to this amount for some hours. When CO_2 drifts out from old workings, owing to its being heavier than air, it sometimes lies along the floor, diffusing but slowly in weak air currents; hence a person overcome should have his head held up until he can be taken to fresh air. But, since irrespirable gases from fires may include much CO_2, and being heated and containing aqueous vapor generally drift along the roof, persons not protected with breathing apparatus should keep their heads down. CO_2 in itself has little effect on a lamp flame, but extinguishment of a flame indicates the deficiency of O which always accompanies presence of CO_2. Remedy for effects of CO_2 is prompt removal to fresh air, in serious cases administering O, or first aid treatment or both (Art 20).

Acutely poisonous gases. For their nature and sources, see Art 2 and 3; their hygienic effects are as follows. CARBON MONOXIDE (CO) is the commonest of the deadly mine gases. It is daily encountered in mines having weak ventilation, when blasting is done during shifts; also in fighting mine fires, and in return air currents from imperfectly sealed fire areas. The latter conditions are more prevalent in coal mines in which gob fires occur. CO always exists in dangerous amounts in the afterdamp of explosions (Art 14).

Oxygen necessary to maintain life is absorbed in the lungs through the agency of HÆMOGLOBIN (red-colored matter of the blood); O forms an unstable chemical compound with hæmoglobin, and is thus carried to the tissues. Hæmoglobin has much greater affinity for CO than for O (250 times greater, Haldane says), and when it has taken up CO can no longer take up O. It becomes more or less saturated in a ratio proportionate to the percentages of CO and O present, taking into account the relative affinities for these gases. When the atmos contains 0.04% CO, hæmoglobin finally becomes $^1/_3$ saturated; with 0.08%, $^1/_2$ saturated; with 0.16%, $^2/_3$ saturated. An average man has about 3 liters of blood, which is capable of combining with 600 cc of CO. If the atmos contains 0.2% CO, and the man is at rest, he breathes about 7 liters of air per min, of which 5 liters reach the lung cells. As he absorbs not more than $50 \times 0.2 = 10$ cc CO per min, it will be 0.5 hr before his blood becomes half saturated. In an experiment in which 0.2% CO was present the hæmoglobin did not reach 50% saturation for 70 min.

A percentage of CO far too small to form a "cap" in a safety lamp, or to be at all noticeable (Art 9), *may be very dangerous*. Men often become unconscious without warning.

In recovery work following an explosion in Alabama, an entire party waiting at a base was nearly overcome, and barely rescued by diverting a fresh current of air to them. Though the safety lamps were burning properly, and no one had noticed the effect on himself, several finally sank back unconscious. With 20% saturation there is tendency to dizziness and shortness of breath on exertion; with 50%, it is scarcely possible to stand, and slight exertion causes unconsciousness. Symptoms resemble those caused by alcohol. If death approaches gradually, the hæmoglobin is usually about 80% saturated with CO. The percentage of CO that is fatal if breathed for a long time varies with individuals. Anything above 0.15% is dangerous, and 0.4% may quickly cause death. For continued exposure 0.03% or over is unsafe, as the effects of CO are to an extent cumulative because of the greater affinity of hæmoglobin for CO than for O.

In case of persons rescued after becoming unconscious, recovery is often slow due to injury to the tissues, and serious ailments sometimes ensue. A person may not realize he has been exposed until he reaches fresh cool air, when he may suddenly collapse. Saturation of hæmoglobin with CO gives the blood a pink color, the greater the saturation the deeper the color, and this continues after death; so that a person killed by CO has bright red blood instead of the dark colored blood of one dying from suffocation or violence. In N Y–N J vehicular tunnels under Hudson River, extensive tests were made to determine safe limits of CO (from motor vehicles) in ventilating current. It was found that 0.04% for an exposure of 1 hr was safe (30). Tests on men and animals at Bur of Mines Experiment Station, Pittsburgh, Pa, showed that CO saturation up to 30% was disposed of by the body only after several hr, varying with activity of the man (33).

Remedy for CO poisoning is prompt removal to fresh air, in a warm place, and administration of pure O. If patient is breathing faintly or not at all, give first aid treatment and administer O.

Immediate indications of small percentages of CO are given by the behavior of canaries and small animals. Canaries are so sensitive and certain that they have been officially approved for rescue work in Great Britain, and in the U S by the Bur of Mines. Burrell found that canaries are much more

Table 7. Burrell's CO Tests on Canaries (54)

CO, %	Effect
0.09	Very slight distress at end of 1 hr
0.12	Weaker at end of 1 hr than after exposure to 0.09%
0.15	Distress in 3 min. Fell from perch in 18 min
0.20	Distress in 1.5 min. Fell from perch in 5 min
0.29	Fell from perch in 2.5 min

sensitive than white mice (Table 7). A canary will drop from its perch in an atmos in which a man could remain for some minutes without serious effect, though such warning should cause immediate withdrawal from the place, unless breathing apparatus is used. In a chamber containing 0.25% of CO, a canary showed distress in 1 min and fell from its perch in 3 min. Burrell remained in this atmos 20 min, which caused a slight headache. After withdrawal to fresh air he had nausea and headache for several hours.

Hydrogen sulphide (H_2S) is more acutely poisonous than CO, but is rarely found in mines. According to Lehmann, air containing 0.07% will cause death after 1 hr exposure; in an experiment on a man, alarming symptoms were produced within a few minutes in breathing air containing 0.05%. He found 0.2% would kill dogs and cats in 1.5 min. Haldane says it acts locally as an irritant, and after absorption affects the brain and other parts. It makes no characteristic change in the blood. With very small percentages the first symptoms are irritation of the eyes and air passages; with more, eye inflammation results, accompanied by intense pain, particularly the next night; with 0.05%, giddiness, vomiting and catching of breath are soon caused. Men are quickly stricken on entering an atmos containing much H_2S, as in sewers.

Oxides of nitrogen or " red fumes " (chiefly NO_2) must be carefully guarded against in tunnels or mines using large quantities of dynamite, since they may result from a weak detonation of a blasting cap, causing the dynamite to burn.

In a western tunnel, 13 men exposed to NO_2 by a change in the ventilating current experienced a slight choking, nausea, profuse perspiration and headache, but revived on reaching open air. But they soon began to cough up bloody mucus, and showed NO_2 poisoning. In 3 days 9 men died. The others, including the rescuers, were ill for months. As many as 20 lives have been lost at one time from NO_2 fumes produced by the burning of a box of dynamite. Air containing enough NO_2 to produce irritation in the nose or air passages is very dangerous. The after-effect is often an intensely acute bronchitis. Hence avoid going back too soon into dynamite smoke, if there is any characteristic smell of NO_2. Men who have been exposed to either NO_2 or H_2S, should, after removal to good air, be placed under a physician's care.

Siliceous dust (Art 3) causes phthisis (miners' consumption or silicosis), whereas dusts from limestone, shale, coal and some metallic ores do not. Mortality from siliceous dust is far greater than from all mine accidents. It is very high in the cherty " sheet-ground " of the Joplin zinc mines, where studies by U S Bur of Mines and U S Public Health Service were instituted and marked improvement made. The siliceous-dust problem in Butte copper mines is complicated by poor ventilation, high temp and humidity.

Physiological effects of breathing SiO_2 dust. By inhalation, the coarser particles are caught by mucus in nose, throat and pulmonary passages, and expelled by coughing; but particles of less than 10 microns diam enter lung cells and cause lesions. Fibrous nodules form, increasing in number and size with exposure, producing general fibrosis, which seals cellular lung tissue and checks aeration of the blood. Breathing becomes increasingly difficult, with susceptibility to pneumonia or tuberculosis. Following terms are used for lung diseases (26): PNEUMOCONIOSIS, covering all dust diseases, fibrous or non-fibrous; SILICOSIS, caused by free SiO_2 or quartz dust; SILICATOSIS, by silicates (distinct from the sharply defined, coarse, nodular fibrosis due to SiO_2 dust); ANTHRACOSIS (anthraco-silicosis), a coal miners' disease, is silicosis modified by presence of coal dust; SIDEROSIS, a dust disease found in some metal workers; ASBESTOSIS, caused by breathing asbestos dust (silicate of magnesia) (27, 28).

Dust-disease symptoms are arbitrarily divided into 3 stages by Amer Public Health Assoc, " for convenience of description and possible compensation purposes ": **Stage 1**, man may look well, but has slight shortness of breath on exertion, dry cough and recurrent colds, some indication in radiograph of fibrous nodules; Stage 2, definite shortness of breath, pains in chest, dry morning cough and sometimes vomiting, recurrent colds, chest expansion decreased, movement sluggish, capacity for work reduced, radiograph shows mottling in lung fields; Stage 3, distressing shortness of breath, cough more frequent, expectoration generally slight (sometimes copious), capacity for work permanently impaired, forced inspiration, pulse rate increased, heart may be dilated, radiograph appearances accentuated, mottling more intense, nodules larger, large shadows corresponding to areas of dense fibrosis (27).

Quantities of rock dust in mine air. Thomas and MacQueen (Dolcoath mine, Cornwall, 1904) found the amount of dust in the air, from dry, machine-drilled holes, to be from 0.05 to 1.14 mg per liter, aver 0.46 mg; in wet holes, aver was 0.22 mg; in hand-drilled dry holes, 0.01 to 0.41 mg; in filling cars with dry ore, 0.06 to 0.34; at a rock breaker, when sprinkler was not used, 0.92; in dead ends and raises, within 3 to 5 min after blasting, 0.06 to 0.34 mg (39). U S Bur of Mines found that in certain iron mines 208.85 mg dust per cu met of air were produced by dry drilling, as compared with 12.88 mg by wet; and dry drilling in " chert " produced 74.2 mg of dust as against 7.78 mg from wet.

Coal dust mixed with siliceous dust. Although it is generally agreed that anthracite and bituminous dusts do not produce lung disease, they do cause asthmatic conditions

when breathed over a long period. But, as anthracite seams are much folded and interbedded with sandstone, siliceous dust is often mixed with anthracite, especially in driving cross-headings and slopes, or brushing roof and floor (27). See also Art 3, under "Solid impurities in mine air."

In three anthracite mines the dusts contained the following percentages of total silica: all men in face workings, 11.1%; rock workers, 63.2%; all others underground, 33.7%; breaker workers, 13.5%. Size-frequencies, in microns, of dusts suspended in mine air: 0–0.49, 11%; 0.5–0.99, 60%; 1–1.49, 17%; 1.5–1.99, 7%; 2–2.49, 3%; 2.5–3.49, 1%. Number of particles per cu ft, to which certain inside workers were exposed was 6.9 millions; those at faces, an aver of 480 millions. Of the 2 711 workers in these mines, 22.7% were diagnosed as having anthracosis; in Stage 1 of the disease, 18.8%; Stage 2, 3%; Stage 3, 0.9% (46, 77). No cases found in a control group of miners, whose dust exposure aver less than 5 million particles per cu ft of air. Its prevalence in the entire working force was about 23%; among all except rock workers, less than 2% were affected when duration of employment was less than 15 years, regardless of amount of dust in the air.

Remedies. Strong ventilation and water sprays, and not using explosives for squibbing and pop shots, without first withdrawing the men or wetting down until the dust settles; also, use of masks, filter respirators, and dust traps (29). See also Art 3, under "Solid impurities in mine air."

Poisonous metallic dusts require special treatment. Precautions must be taken in case of $PbCO_3$ dust, to prevent lead poisoning, either by inhalation or through the mouth. In mining cinnabar (Hg_2S) great care must be taken to protect workmen from poisoning. Arsenical ore dust, said to cause cancer of the lungs, must also be guarded against.

Tests for quantity and character of mine air dusts: (*a*) filter through a tube, or granulated sugar in a tube; (*b*) count dust particles in a certain microscopic field (adopted in So Africa and elsewhere). Bur of Mines used this and also the IMPINGER method, by which dust is collected in water or other liquid from a unit vol of air (28). Dust over 10 microns is determined separately from dust under 10 microns. Sampling dust in mine air has been studied by Haldane, Sayers, and So African investigators (28). By the konimeter the particles of dust are rapidly estimated. A small quantity of the air is forced at high veloc upon a greased slide, on which the size and number of adhering particles are estimated by microscope. Modifications have been developed; such as a microscope attached to an instrument operating on this principle, so that the inspector can make determinations on the spot.

6. PERMISSIBLE PERCENTAGES OF IMPURITIES

Volume of air circulated. The older method, still general, is to increase the ventilating current by artificial means until the air in working places seems good. In coal mines in the U S, practice now requires circulation of a definite volume of air, based on number of men and draft animals. Under most state laws the air may be measured near the mine intake; in other states the more intelligent requirement prevails of measuring it in the crosscut or breakthrough nearest the face. The volume is generally 100 to 150 cu ft per min per underground employe, and 500 cu ft for each animal. In the anthracite region, owing to large amount of firedamp, 200 cu ft per man are required. In most collieries these limits are exceeded at the intake, but even in best practice, owing to leakage of stoppings or doors, only 65 to 70% reaches the last crosscut; in some mines, not over 15 or 20%. The stipulated volumes exceed breathing requirements, which are about 10 cu ft per min per man, and 30 cu ft for a mule or horse (Art 5). The added amounts are considered advisable for sweeping away firedamp, and to provide for leakage and slow oxidation of coal and shale. When firedamp is present, enough air must be circulated to prevent accumulations of gas.

Federal requirements for coal mine lessees (34) (made by Dept of Interior for Alaska, 1916, and modified in 1921 for public coal lands, chiefly in Utah, Colo, Wyo, Mont, N and S Dak, and Wash), not in conflict with Territorial nor State regulations: FOR INTAKE AND RETURN-AIR SYSTEMS, mech vent, reversible fans, housing and connections of noncombustible construction, and fans offset from mine opening and with explosion relief doors are required. GASSY MINES must have a duplicate engine or motor for the fan. In MINES EMPLOYING MORE THAN 200 MEN PER SHIFT, shafts must be fireproofed. When part of a mine employing MORE THAN 100 MEN PER SHIFT is over 4 000 ft from nearest airshaft, entries and airways reaching it shall be not less than 4 in number. Crosscuts or breakthroughs must not be over 100 ft apart. No rooms or branches are to be turned ahead of last crosscut. Stoppings in entry crosscuts to be of incombustible material and as airtight as possible.

Worked out and abandoned areas that can not be ventilated to prevent gas accumulations must be sealed off by strong fireproof stoppings.

When gas in sealed areas is under high press, vent holes must be bored to the surface; or, if not feasible, a pipe line made through a stopping and conducted into the return airway beyond any live working or haulage or traveling way.

Quantity of air to be not less than 100 cu ft per min per man, measured at the last entry crosscut of a split, and there shall be not more than 75 men on a split.

Quality of air shall be fit for work or travel. Air must not contain more than 1.25% CO_2, and not less than 19% O_2, as determined by sampling and analysis by U S Mining Supervisor.

A mine or portion of a mine shall be deemed gassy, as defined by State Inspector or by Mining Supervisor, if on 3 occasions not less than 2 days apart, 2% or more CH_4 is found in working places by safety lamp or chemical analysis, or if return air in any split shows 0.5% CH_4.

Electric-driven auxiliary or booster fans may be used in NON-GASSY MINES, where there is no inflammable material within 10 ft of motor. IN GASSY MINES these fans may be used only on intake air, the motor and switch being explosion-proof, and hourly inspection made. When auxiliary or booster fans are driven by compressed air, they may be used in either gassy or non-gassy mines. In a gassy mine, or one in which more than 0.25% CH_4 is present in the air current, trolley or non-permissible storage-battery locomotives may be used only in intake air, not receiving the return air of rooms or abandoned areas.

Classification of non-gassy, slightly gassy and gassy coal mines, recommended in 1926 by U S Bur of Mines (35, 90).

Class 1: a practically non-gassy mine, in which inflammable gas in excess of 0.05% can not be found by systematic search.

Class 2: a slightly gassy mine, in which (*a*) inflammable gas has been found* but in amount less than 2% in still air in any active or unsealed abandoned workings; or (*b*) inflammable gas can be found,* but in amount less than 4%, in some place from which the ventilating current has been shut off for a period of 1 hr; or (*c*) inflammable gas can be found, † but in amount less than 0.25%, in a split ‡ of the ventilating current; or (*d*) inflammable gas enters a split at a rate § of not more than 25 cu ft per min.

Class 3: a gassy mine containing more inflammable gas than specified for Class 2.

Modern practice in European coal mining countries has been to analyse the air in mine workings and return current. This has been extensively done by the French Mine Inspection service. In later FRENCH MINING REGULATIONS, no minimum vol of air is stipulated. The practice is about 3 cu m (106 cu ft) per min per man, but the return air of an advance heading must not contain more than 1.5%, and any other return, 1% of CH_4. IN GREAT BRITAIN, the vol of air is not specified, but certain max limits for impurities are set which require analysis for determination. No lamp or light other than a locked safety lamp (including permitted portable electric lamps) can be used when the air has normally over a 0.5% inflammable gas. If the percentage is 2.5 or over in the general body of air in a working place, workmen shall be withdrawn from that place. Other limits are that "a place shall not be deemed to be in a fit state . . . if the air contains less than 19% O or more than 1.25% CO_2." IN THE U S many collieries regularly analyse their return currents, but no chemical limits are set in state regulations for either coal or metal mining. IN SOUTH AFRICA, AUSTRALIA, AND NEW ZEALAND, mining commissions have recommended the establishment of certain max limits of impurities, temp, and humidity (Table 8).

Table 8. Limits of Impurities in Mine Air, Established Abroad by Analysis

	Minimum % O	Maximum %		Maximum temp	
		CO_2	CH_4(*g*)	Dry bulb	Wet bulb
Coal, French regulations of 1911	....	....	2.00 (*e*)	95° F (*d*)	86° F (*d*)
" British act of 1911	19	1.25	2.50 (*f*)	....	....
Metal, Transvaal	....	0.20	....	....	....
" New South Wales (*a*)	19	1.00	....	....	....
" Victoria	20	0.20	....	87°	80°
" West Australia	....	0.25 (*c*)	....	....	....
" New Zealand (*b*)	19	1.00	....	....	80°

(*a*) Proposed. (*b*) All operations in Thames and Waihi gold fields below 800 ft, 200 cu ft air per min per man: 500 cu ft per horse. In other districts, 150 cu ft per man; 500 cu ft per horse. (*c*) Except for 30 min after blasting. (*d*) Men forbidden to work when wet and dry-bulk readings are higher, except in case of necessity. (*e*) Return air of a working place must not contain over 1.5% CH_4 for advance headings, and not over 1% for all other currents. (*f*) Open lights must not be used when the return air contains over 0.5% CH_4. (*g*) Causing withdrawal of men.

* By employing an approved safety lamp, with flame drawn low, or approved gas detector, or by sampling and analysis with an approved gas analytical apparatus.

† By sampling and analysis with an approved gas analytical apparatus, or employing an approved gas detector.

‡ If there is but one continuous ventilating current, this shall be considered a split for purpose of this definition.

§ Determined by sampling, analysis, and ventilating-current measurement.

7. SPECIAL MINERS' DISEASES

Ankylostomiasis, or miner's anemia, practically the same as "hookworm disease," is due to a thread-like, blood-sucking worm. It attaches itself to the walls of the intestines by hooklets and sucking disks on its head. Length varies in different species and stages of growth; hookworm of southern U S (Uncinaria Americana) attains a length of 0.3 to 0.6 in (36).

The larva, from a microscopic egg, hatches in 1 or 2 days under favorable conditions of air, heat, and moisture, into an active worm-like embryo. Lacking opportunity to enter the human body, it may live a year in soil with suitable environment. HOOKWORM LARVA usually enters the body through the skin. If a person treads on infected ground with bare feet, or gets particles of it on hands or other parts, the larva begins to bore, especially in tender places, as between toes or fingers. This causes local inflammation, called "ground itch." The larva enters a blood vessel, and is swept through the heart into the lungs; but, being too large to pass through the lung blood vessels, bursts out into the air spaces. It wriggles along these and up the windpipe to the throat; then it is swallowed, entering the stomach. Or it may enter directly with infected food. From the stomach it passes into the small intestines, where it grows and becomes a hookworm. A person seriously affected will have several thousand worms clinging to the intestines. They bite into the tissues, suck blood, and more is lost by bleeding. A poisonous fluid given off by the worm causes anemia and catarrh of the bowels. Fortunately, it does not multiply in the intestines, but mates and the eggs pass out with the stool. If a person is not reinfected the worms ultimately die off, but their lifetime in the body is 10 to 15 years.

The disease is widespread in agricultural districts of southern states, where investigations have been made by Rockefeller Sanitary Comm. First observed in Europe about 1880, in driving the St Gothard tunnel. In 1896 many cases occurred in German coal mines, and the disease spread so rapidly that drastic gov't measures were enforced. By 1910, only 1% of miners were affected. Stringent precautions have been taken elsewhere in Europe, where the disease became epidemic.

Some coal mines in the southern states have been affected, but most U S mining districts are free from it. For more details, see 2nd edn of this Handbook.

Symptoms: pain in stomach, capricious and perverted appetite (as dirt-eating), obstinate constipation followed by diarrhoea, palpitations, weak and unsteady pulse, and, in fatal cases, dysentery, hemorrhages, and dropsy. A person seriously affected looks bloodless; skin is tallow-like, and body appears starved. Distended abdomen, sometimes with bloating of face, body, or legs, is characteristic among children, causing stunted growth, an old-appearing head, and other abnormalities; hence the importance of eradicating the disease, whether in a mining or other community. MILD CASES are more common and show less definite signs; positive knowledge is obtainable only by examining stool with a microscope for hookworm eggs.

Preventive means: use of shoes or boots without cracks or holes and leather gloves, cleaned frequently with antiseptic solutions; rigid sanitary precautions in and about mines and homes; neither children nor grown persons to go barefooted; compulsory use underground of portable sanitary latrines, daily cleaned and disinfected (Art 8). If the disease is suspected at a mine, there should be bacteriological inspection of defecations. Malefern, santonine, thymol, and other vermifuges are prescribed; epsom salts and thymol, taken alternately, are common remedies. The disease yields easily to treatment and mild cases are quickly curable.

Nystagmus, peculiarly a miner's disease, affects the muscles and nerves of the eyes, and causes oscillation of the eyeballs. In severe cases the victim is unable to walk straight, owing to an apparent rotary motion. Except in severe cases it is considered curable (37). ALLEGED CAUSES: insufficient light, and work of bottom-holing, or undercutting, while working in a strained position, with eyes fixed on one point. Another cause is said to be working in thin seams, where the head is thrown to one side, and looking diagonally upward to observe the roof as the miner walks along. Consensus of medical opinion is that the chief cause is poor lighting, as by safety lamps, particularly the older types, which have less than 0.6 c p. A German investigation showed that 91% of those affected used safety lamps; in 2 mines using open lights, only 0.35%.

Since the Workmen's Compensation Act became effective in Great Britain, nystagmus has been brought forcibly to the attention of the British mining industry. In 1909 compensation was allowed in 631 cases; 1910, in 956 new cases. General use of the pick in undercutting coal, and necessity in most European mines of regular use of safety lamps, made the disease serious in the past, but the improvement in portable electric lamps has brought improvement. It has been stated that men with nystagmus can not see a "gas cap" in a safety lamp, and hence were a menace in a gaseous mine. Dr. J. Court tested 106 miners; though 58 had nystagmus, all but 4 could detect 2.5 to 4% of CH_4 (15).

Remedy is better illumination. At Cockerill colliery, Belgium, twenty miners were each given 2 benzine safety lamps, instead of one, and examinations at end of a month showed general decrease

of nystagmus. To decrease danger of igniting firedamp by a broken safety lamp, it is better to provide only 1 lamp and a permissible storage-battery light, having reflectors that give a conical angle of lighting of at least 130°, and have aver strength of illumination greater than 1 c p (Sec 42, Art 15). See discussion, *Coll Guard*, June 6–Oct 31, 1930.

In U S few thin coal seams (say less than 36 in thick) are worked, and except in long-wall workings in Northern Illinois little hand undercutting is done. Safety lamps are used in few collieries, except by "fire-bosses" or "gas examiners;" open lights were the rule until supplanted by miners' permissible cap lamps with strong illumination. In gaseous mines, fire-bosses generally use elec cap lights, which are turned off while testing for gas with safety lamp. No extended investigation of nystagmus in U S has been made, but if there were many cases, they would have been noted. In Illinois, 1911, 500 miners were examined, without finding a case.

Color-blindness, though not specifically a miner's disease, is of serious importance in gaseous mines. The faint blue firedamp cap in the safety lamp, when the fuel flame is drawn down, is indistinguishable by those seriously afflicted with color-blindness; hence the advisability of examining for this defect all applicants for positions intrusted with detection of firedamp, as "fire-bosses," mine examiners, underground managers, and State mine inspectors, rejecting those who can not clearly detect the small firedamp cap.

8. UNDERGROUND AND SURFACE SANITATION (see also Sec 22)

Requisites underground, (*a* and *b* being summarised from Art 5):

(*a*) Good air, containing not less than 19% O, not over 1.25% CO_2, and no CO or other poisonous gases. Air temp should be below 80° F, if possible. In coal mines, for safety rather than sanitation, the body of air should contain not over 0.5% CH_4. These conditions are usually attainable by having good ventilation.

(*b*) In mines producing siliceous dust the air should be sprayed. Drilling at the face should be done with water-injection drills, or accompanied by spraying. Dry holes should be cleaned with scrapers and not blown out with compressed air. Adobe shots should be covered with wet stemming material, and dry ore wetted with hose before shoveling.

(*c*) Sanitary latrines should be provided (38), especially in metal mines, and should be so located that one may be reached in not over a 5-min walk, climb or hoist from the working places; where possible the stations should be lighted by incandescent lights. The latrines should be taken daily to the surface, thoroughly cleaned and fumigated. An excellent latrine is a water-tight iron body mounted on a mine truck so that it may easily be taken in and out on cages. Where skips are used tub latrines are preferable, if of porcelain with tight-fitting covers. The Butte latrine, when taken into the mine, is partly filled with water to keep the excrement fluid; it has valve fittings, for flushing out thoroughly by connecting it to a water main. After flushing, latrines should be rinsed with an antiseptic solution. Water-flushed closets sometimes installed at shaft stations, the excrement being flushed into the sump and pumped out with the mine water. If the quantity of water be large, and is not discharged into ponds near habitations, the plan is unobjectionable. In a Canadian mine, the effluent from closets serving 150 men runs into an underground septic tank, the clear water being discharged into the sump. Destructible latrines are sometimes used (wooden boxes about 10 by 18 by 20 in deep); at each latrine station there is placed dry dirt and a scoop, also disinfectant for sprinkling over excrement. When the box is nearly full, the cover is nailed on and it is sent out to be burned in a furnace. In coal mines, near gathering points, as at haulage sidings or on main levels, latrines are badly needed to prevent dangerous pollution in vicinity, but at the "face" in most collieries there is more or less shale or clay refuse, which is stowed in the "gob." In this case the fecal matter can be buried deeply, and thus safely cared for, if inspection is rigid.

(*d*) Underground cabins, pump rooms, engine rooms, stables, and other places where men may gather, should be frequently whitewashed or painted.

(*e*) Miners should be required to take their lunch leavings out of the mine in their dinner buckets; in some cases they may be permitted to throw the scraps into cars of waste rock or refuse to be dumped on the surface. Such measures prevent feeding of rats, which may come in with hay and other supplies, and are liable to transmit diseases.

(*f*) In mines where ankylostomiasis (Art 7) is found, lime should be sprinkled along gangways, or entries, and the resting places where lunches are eaten, whitewashed. In collieries, calcium chloride answers same purpose, and helps to compact coal dust.

(*g*) Good lighting aids sanitation, and haulageways, manways, shaft bottoms, and landings should be whitewashed. Electric lights should be installed, except where not permitted, as in return air of gaseous collieries. They should have wire cages.

(*h*) Shelter should be provided for heated and perspiring men, while waiting at shift ends at landings in down-cast shafts, and cages for hoisting men in such shafts should be inclosed.

(*i*) Drinking water must be of good quality, and provided preferably by fountains not requiring the touch of the lips, as in case of a faucet. As it is rarely feasible to provide running water for drinking throughout a mine, tank cars, kept in sanitary condition, should be provided, each man should have his own drinking cup, and the faucet so placed in a recess that a man can not use his lips, but must use his cup.

Requisites on surface are same as the elemental sanitation underground. In addition, miners coming from a warm mine must avoid prolonged exposure to cold air, while in light-weight working clothes. Hence, within 100 ft of the mine entrance, there should be a warm building, where the miner may be checked in and out, and which adjoins or connects, by covered passage, with the bath

and change house (Sec 22). Men coming up wet with perspiration from a deep, hot mine into cold outside air, are subject to colds and pulmonary troubles. Well-managed mines now have bath and change houses under state supervision. In Great Britain this is done through the Welfare Fund, which, with the Mine Safety Research, is maintained by tax on mineral production. In or close to the change house there should be a safety station, with mine-rescue oxygen breathing apparatus, gas masks, oxygen inhalators, stretchers and "first-aid" equipment and nearby, fire-fighting equipment, portable extinguishers, hose, rolls of brattice cloth and bags of limestone dust (Art 17).

MINE LAMPS AND APPARATUS FOR DETERMINING IMPURITIES IN MINE AIR

9. SAFETY LAMPS

Purposes: (*a*) in gaseous mines to provide means of illumination which, properly used, will not ignite firedamp; (*b*) a quick means of testing for firedamp (by characteristic appearance of flame), whether in gaseous or in " open-light " mines. It is possible but dangerous to test with an open flame. Among the numerous safety lamps the following are representative.

Davy lamp, invented in 1815 by Sir Humphry Davy, was the first safety lamp; now superseded. Its oil flame is surrounded and separated from the external air by a wire gauze mantle (6 in high, 1.5 to 2 in diam, 28 wires per linear in, 784 apertures per sq in) which by its rapid conduction cools the hot gases inside the lamp below the ignition temp of CH_4, thus preventing spread of ignition from the lamp interior to gas outside. The behavior of a safety lamp in a firedamp mixture depends chiefly on its arrangement of air inlets and of outlets for hot gases. In the Davy, air may enter or gases escape from any part of the gauze, making the lamp convenient for quick testing, but introducing an element of danger, since if the lamp is put quickly into an explosive atmos, a strong explosion occurs inside the gauze and may pass flame to the outside. Also, the lamp is dangerous in a current of explosive mixture if veloc exceeds 300 ft per min; at 450 ft per min, flame passes through the gauze in less than 2 min; at 600 ft, in 10 or 15 sec. In a quiet atmos if gas is flaming in the lamp, and it is dropped, flame may pass the gauze. It is therefore not a safe lamp, and in most countries it is not permitted in gaseous mines. Illumination given by the ordinary Davy is small, 0.16 to 0.20 c p.

Stephenson or Geordie lamp, invented by George Stephenson, followed the Davy. Though now obsolete, its cylinder and under air-feed are used in some modern lamps.

Clanny lamp has a glass cylinder around the flame, surmounted by a gauze cylinder, and the air enters lower part of gauze, passing downward to the flame. This causes baffling, which tends to smoke the glass.

Marsaut lamp has 2 or 3 concentric gauzes above the glass cylinder, with small air spaces between them, which widen toward the top by reducing the diam of the inner gauzes. The gauzes retard escape of the burned gases, which form a blanket of non-explosive gas restricting internal explosion. Fig 3 shows a BONNETED MARSAUT.

Parts of the lamp: *a*, steel hood; *b*, steel bonnet; *c*, outer iron gauze; *d*, inner iron gauze; *e*, upper brass bushing; *f*, copper rings attached to gauze; *g*, asbestos washer; *h*, 5 brass standards; *i*, glass globe; *j*, flat cotton wick; *k*, copper wick adjuster; *l*, copper wick holder; *m*, lower brass ring; *n*, brass ring; *o*, brass ring; *p*, brass oil fount; *q*, asbestos washer; *r*, brass ring for washer; *s*, key lock; *t*, vertical openings for air inlet. From his experiments, Marsaut concluded that: (*a*) a small-diam lamp, like the Davy, does not readily pass an explosion, as the volume susceptible to explosion is small; (*b*) a lamp without a glass is safer in case of internal explosions, because the glass confines the gases and acts like a cannon (hence, it is best to reduce diam and height of glass); (*c*) a conical mantle is safer against transmission of internal explosions than a cylindrical mantle of same capacity; (*d*) retention of gases of combustion tends to prevent internal explosions, and it is not advisable to guide them by a chimney. An iron body is better than brass, a brass lamp giving only 70% of the luminous intensity of an iron lamp of same design. Brass has higher conductivity, becomes hot, and makes oil viscous; it is easier to cast, and therefore generally used.

Improved Marsaut lamps. The original Marsaut has the Clanny fault of a baffling air-feed, which is very marked in a strong current; hence the glass may be smoked, giving poor illumination. But it is good for testing gas.

Combustion and illumination have been improved by various devices, and modified Marsauts are today the most favored safety lamps in Great Britain. Among them are the DEFLECTOR and PROTECTOR TYPES. These are similar in principle, the air feed being separated from the gases of combustion. In the earlier modifications the lower part of the gauze is shielded, and an angle diaphragm

ring above top of shield extended to the gauze. There is a perforated air inlet ring over the glass. The entering air is deflected upward, then strikes the ring, and is thrown downward through the gauze and along the glass cylinder. Burned gases pass out through upper part of gauze. Most of the later modifications use the full bonnet, with holes around the top for escape of burned gases, and either 1 or 2 gauzes, 2 being safest. In some, a diaphragm or angle ring separates upper and lower part of the gauze for outlet and inlet, air entering under the bonnet through slotted holes in the ring to which the standards are affixed.

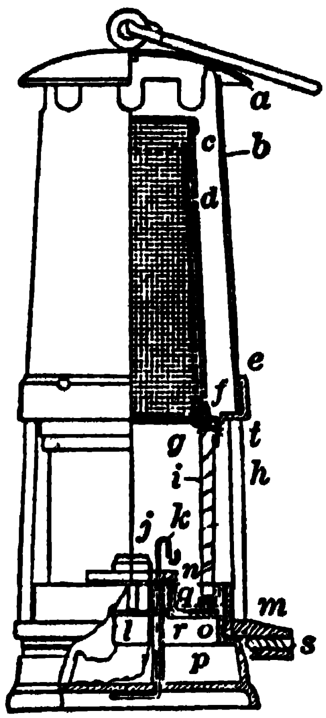


Fig 3. Bonneted Marsaut Lamp

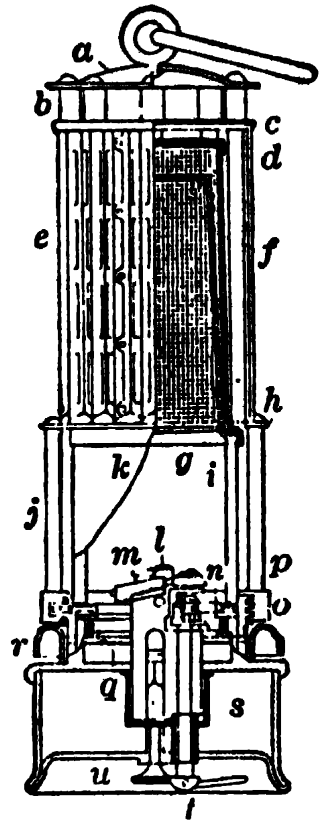


Fig 4. Wolf Safety Lamp, Type 1 B

Wolf lamp (Fig 4) uses benzine, naphtha, or gasolene, has bottom feed, and a wick-feed screw, instead of the pricker used on heavy oil lamps, which require frequent removal of crust forming on wick.

Parts of the lamp: *a*, steel hood; *b*, 5 brass covers on standards; *c*, steel ring, upper; *d*, 5 steel standards; *e*, brass bonnet corrugated; *f*, iron gauze, outer; *g*, iron gauze, inner; *h*, steel ring for standards, middle ring; *i*, asbestos washer; *j*, 5 brass standards; *k*, glass globe; *l*, cotton wick; *m*, wick holder; *n*, expansion ring; *o*, brass ring, lower; *p*, asbestos washer; *q*, brass gauzes, inlet; *r*, brass baffle ring; *s*, brass naphtha fount; *t*, ignition turnbar; *u*, wick adjuster. Air enters slotted holes level with top of the oil vessel, passing thence through a narrow gauze ring to the flame. The wick, now usually flat, about 0.5 in wide, to give more light, is in a metal tube extending about 1 in above the base, with an adjacent tube containing the screw for raising and lowering a clip which holds top of wick. This adjustable bottom is advantageous in testing for firedamp, as the lamp flame is readily drawn down until the yellow has disappeared. The light oil is advantageous for relighting, as internal relighters can be used. The relighter consists of a coil of paper matches, with clasp and striker operated by a rod projecting through the bottom. An alternative arrangement is a flint rubbed by a milled wheel. Care must be taken that the wheels do not contain cerium or other substances which remain hot longer and make lighting easier, because tests show that hot particles from some composition wheels pass through the gauze, and may ignite firedamp. The fuel vessel is packed loosely with wool to prevent gasolene from slopping. The glass rests on a flexible ring to permit expansion, with asbestos gaskets above and below. There are 2 wire gauzes, of 30-mesh, and a plain or corrugated bonnet. The miners' lamp is of brass, weighing 3 lb 4 oz; made of aluminum, 2 lb 3 oz. The round wick lamp gives illumination estimated at 1 to 1.5 cp; flat wick, 1.2 to 1.8 cp.

The Wolf type is excellent both as a working lamp and for testing. With low flame a trained observer can detect 1% firedamp, and an ordinary observer easily detects 1.5 to 2%. This lamp has not been popular in Great Britain, as use of naphtha is disliked, and individual relighters in gaseous mines are not approved. It is more used in U S than any

other. Other lamps of this type in U S, are the Shenk, Seipel, and Koehler. Some Wolf lamps for mine officials have special devices to measure the height of CH_4 " cap."

British Mines Dept report for 1937, 209 539 flame safety lamps used in British mines, of which 102 662 were Marsaut; 105 981 had an inner metal chimney. There were also 407 318 electric safety lamps.

Kind and quantity of oil for safety lamps. At first sperm, seal, and lard oils were used; then vegetable oils, from rape, cabbage or cotton seed; finally mineral oils have been increasingly used; more recently, benzine, naphtha, and gasolene. British authorities require oils which do not vaporize at temp to which they are subjected in the oil-vessel. In the U S, for Davy, Clanny, and other British lamps, high-test gasolene is commonest; sometimes railroad signal oil (a low-proof mineral oil, with little incrusting residue). For the Wolf type, gasolene of 70 to 76° is usual. QUANTITY OF OIL BURNED in safety lamps in 10 hr generally ranges from 500 to 700 gm. A lamp of the Wolf type with flat wick uses about 738 gm benzene.

Table 9. Characteristics of Representative Safety Lamps

Name	Oil used	Air entering with reference to flame	Combustion chimney	No of gauzes	Bonnet orig'l lamp	Candle power (b)
Davy.........	Heavy oil	Below or above	None	One	No	0.15 to 0.25
Geordie.......	"	Below	"	"	No	0.1 to 0.2
Clanny........	"	Above	"	"	No (a)	0.3 to 0.4
Mueseler......	Heavy oil or naphtha	"	Chimney	"	No (a)	0.4 to 0.5
Marsaut.......	Heavy oil or naphtha	"	None	2 or 3	No (a)	0.3 to 0.5
Evan Thomas..	Heavy oil	Essentially below	"	One	Yes	0.4 to 0.5
Ash-Hep-Gray.	Light mineral oil	Below	Essentially	"	Yes	0.5 to 0.6
Wolf (type)....	Naphtha or gasolene	Below	None	2	Yes	0.6 to 1.0

(a) Now frequently provided with bonnets. (b) Candle power horizontally without reflector; simple and improved types. The great variation in candle power obtained by different investigators is due partly to non-standardization of photometric methods, but chiefly to differences in size of lamp, burner, height of flame, and kind of oil used.

Details of safety lamps. LOCKS should be on lamps in all gaseous mines; best type is the magnetic, which can be opened only in a special magnet holder. In Great Britain a lead seal is common; it passes through lugs to prevent unscrewing and can be stamped with a date. In " open-light mines," examiners often use a lamp locked with a key. BONNETS are advisable in rapid air currents, but in Germany, where air veloc is kept low, the bonnet is often omitted, so that gas flaming up in the lamp may be quickly seen, and condition of gauze easily noted. GAUZE mesh was formerly 28, now in Great Britain may be 20 wires per linear in. Two gauzes are best, one serving as reserve if there are holes in the other; also the space between tends to retain gaseous combustion products. An unbonneted single-gauze lamp is unsafe in a current of over 300 ft per min; at 450 ft, flame will pass through gauze in less than 2 min.

Permissible-flame safety lamps approved by U S Bur of Mines (1937) are the Koehler and the Wolf, each with flat or round wicks, magnetic lock, double gauze bonnet, and aluminum frame (or Koehler with steel and Wolf with brass frame).

Lamp house. As a rule, safety lamps are supplied and kept in order by the mining company. They should be systematically cared for in a fireproof building, well lighted by windows, and by protected incandescent lights. The house should be near the mine entrance, but at least 50 ft from it, to be safe from explosions. No one but lamp tenders should be permitted to enter lamp room. Miners, on coming from the mine, must deliver their lamps in this house, where they are taken apart, well cleaned (especially the gauze), the oil vessel filled, and new wicks, asbestos washers, and other parts supplied if needed. The lamps are reassembled, tested by breath or by a compressed-air blowing device, and hung up in order. Each man should be furnished the same numbered lamp each day. Lamps are delivered to men entering the mine by a checking system, which, in case of accident, insures identification by lamp or check number.

Relighting safety lamps. In a gaseous mine lamps should be magnetically locked and no one allowed to carry matches. Even in a so-called non-gaseous mine, lamps must not be opened and relighted with a match. Lamps are relighted in 3 ways: (a) INTERNAL IGNITER in the lamp itself (see Wolf lamp). Not favored in Great Britain, because an explosion inside the lamp while being relit might be communicated to explosive gases outside. (b) ELECTRIC RELIGHTER, consisting of a steel box into which the lamp may be

shut, with a lighting pin extending through the oil reservoir nearly to edge of wick tube; current from a secondary coil passes through the pin and causes sparks at top of wick tube, relighting the lamp. The coil is excited by storage battery, dry cells, or magneto with external crank, the whole being enclosed in the box. Batteries generating even minute amounts of hydrogen are objectionable, as an internal explosion has been known to rupture the box. (*c*) RELIGHTING STATIONS (in charge of "fireboss") located on intake air course. As they are few in number, men may have to go some distance for relighting.

Testing for firedamp (40). Firedamp (chiefly CH_4) increases height of a normal flame, and this increase, proportional to percentage of gas present, furnishes a quick indication known as testing with FULL FLAME; it is not reliable, as height of flame may vary with temp of oil and condition of wick, and wick may have been adjusted in air containing gas. In percentages from 1% to just below explosive limit (5.5% or slightly less, especially if mixed with ethane), CH_4 burns with pale blue flame, and if the wick is lowered until no yellow light shows, will form a faintly luminous pale blue cap over, but not separated from, the primary flame. The latter should be not over $^1/_8$ in high, if lamp is clean and the gasolene not too light. The cap takes some minutes to form, as the lamp must first fill with the mine gas. Nearby lights must be extinguished or screened; height of cap is then estimated by eye, with allowance for normal cap. Wolf lamp permits estimate of as little as 1% firedamp, the cap of which is about $^3/_{16}$ in high. As the proportion increases, the cap becomes more distinct in color.

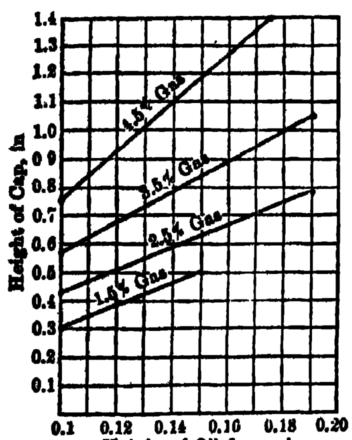


Fig 5. Increase in Height of Cap with Increase in Size of Testing Flame

In advancing rapidly through workings suspected of containing gas, length of full flame should be observed from time to time. If flame has not been adjusted for 0.5 hr prior to leaving on intake current, any increase in its height, or elongation of the normal flame into a high slender one (spiring), is due to CH_4, which should then be tested with flame drawn low. Natural gas, occasionally found in U S mines near gas or oil districts, behaves like CH_4 in a safety lamp.

Height of cap (Fig 5) ranges from about 0.20 in to more than 1 in, varying with different lamps and oils, and with height of test flame. As the latter is increased, Winstanley (87) found that height of cap increased faster when more than 3% firedamp was present. He believes that density or visibility of a cap is a more reliable indication than mere height, and that to estimate percentages correctly requires familiarity with variation in height and visibility for different percentages of gas and sizes of flame.

Tests of firebosses and examiners in a Scotch mine (42) showed: (*a*) that ordinary quick inspection did not detect less than 3.19% CH_4, in some cases much greater percentages; (*b*) careful testing detected minimum percentages varying, with the individual, from 1.06 to 4.42, aver 2.65%; (*c*) smallest percentage detected with a luminous flame, 2.11%; (*d*) max percentage escaping detection varied from 1.31 to 3.05, aver 2.16%.

Effect of deficiency of O and presence of CO_2 on safety-lamp flame is shown in Table 10, of tests run in a closed chamber until the atmos became extinctive. Since several per cent CO_2 may replace an equal percentage of N without appreciable effect on a flame, the resultant loss of illumination was due to deficiency of O, showing importance of keeping the O content of mine air above 20% to get good illumination from fuel-burning lamps.

Table 10. Effect of Deficiency of O and Presence of CO_2 on Safety Lamp Flame

Time	Light, in cp	Per cent of		Temp, deg F		Calculated % of moisture by vol
		O	CO_2	Dry bulb	Wet bulb	
11.50	0.405	20.90	0.05	65.3	56.3	1.15
12.13	0.370	20.66	0.25	69.0	59.2	1.25
12.28	0.315	20.34	0.52	70.7	65.3	1.80
12.43	0.270	19.88	0.88	72.5	67.1	1.80
1.3	0.170	19.34	1.26	74.3	69.0	2.08
1.23	0.11	18.92	1.71	75.6	70.2	2.20
1.45	0.045	18.28	2.17	77.0	73.2	2.50
2.3	0.00	18.01	2.40	77.0	75.2	2.80

Special lamps for testing firedamp, like the Clowes, Pieler and Chesneau, have been used by officials and investigators, but are of little value as compared with analytic methods, and are somewhat dangerous as working lamps. They have been superseded by gas detectors and indicators, as the Burrell, U C C and M S A (51); or by sampling and analysis.

10. ELECTRIC LAMPS AND LIGHTS

Miners' electric lamps for gaseous mines were introduced about 1895 in Belgium. They have been developed largely in conjunction with mine rescue work, but are replacing fuel-burning cap lamps for ordinary use.

In the U S, mine lamp makers report that there are in use (1938) 379 867 elec cap-lamps (about 80% with alkaline batteries), 756 hand elec lamps (used by officials), 4 000 elec signal or haulage trip lamps, and 1 500 animal haulage lamps. Of 616 857 safety lamps used in Great Britain in 1936, 66% were elec; of the latter, 13% were cap lamps.

Permissible electric lamps. Tests show that though there is no danger (at voltage and amperage used) of enough outside sparking in use to ignite firedamp, yet if a bulb is broken, the glowing filament may ignite. Hence lamps must be designed to break the elec circuit the instant the outer protecting glass is broken.

U S Bur of Mines now requires a 12-hr aver lighting intensity on 1 charge of at least 0.4 candle power (42, 43). Permissible cap-lamps now used are the Edison and Wheat, and there are 15 types of hand elec lamps on the permissible list, including a flood light and a pneumatic elec semi-portable lamp (91). Hand elec lamps are used in the U S for special service only, like rescue work. There are now permissible "trip" (car) lamps and one permissible flashlight.

Bur of Mines has recommended that: (*a*) in all coal mines portable lamps for illumination be permissible electric lamps; (*b*) where firedamp or blackdamp may occur, a permissible magnetically-locked flame safety lamp for gas detection, or equivalent permissible device, be supplied to at least one experienced employe in each such place; (*c*) any employe, before being supplied with a permissible flame safety lamp, be examined by a competent official to assure the man's ability to detect gas; (*d*) all coal mines, even if classed as non-gaseous in any part, be supplied with magnetically locked, permissible, flame safety lamps, in sufficient number for all inspection purposes.

Elec cap-lamps have in general been approved in Europe, except in thin and pitching deposits, where the battery hung at the miner's back is inconvenient in narrow workings. 804 Ceag elec lamps in an English colliery showed aver of 1.03 cp at beginning of shift and 0.76 at end (87); cost per lamp per week for 4 275 Ceag lamps in 1-yr period, 3.1¢. British requirement, effective Jan 1, 1916, is 1 cp at beginning and 0.6 cp at end of day.

Electric lamps with gas-indicator attachments. In Great Britain the Ringrose automatic alarm indicator and Thornton automatic elec indicator were approved by Mines Dept, but have not passed the trial stage.

Advantages of electric lamps compared with safety lamps: (*a*) Improved illumination. With cap lamps the illumination is where most needed, in direction towards which the person looks. (*b*) No danger of igniting firedamp through a defect or improper cleaning, or through breakage. (*c*) Using cap lamp, both hands are free for work. (*d*) No gases given off in burning. (*e*) In an atmos containing less than 20% O, the effic does not fall off as with safety lamps. DISADVANTAGE: no means of testing for firedamp. But this is rarely serious for a working lamp, as foremen and fire-bosses make tests periodically with flame safety lamps.

Flood lighting; especially applicable to collieries having concentrated mining systems, involving mechanical coal cutting and loading, and use of face conveyers. Two types of portable flood lights were approved by U S Bur of Mines in 1931, but have not yet been widely adopted.

Dry-cell electric lights (flash lights) except one special, do not meet requirements of permissibility for efficiency, or for safety in the matter of not opening out or cutting the circuit if the bull's eye be broken. But, this is unlikely to occur under conditions of use, and they are otherwise safe, except those having exposed metal parts of opposite potential, the use of which, where electric shot-firing is done, introduces the hazard of premature firing by accidental contact with the wires. This would not apply during rescue work, for which they are very useful. As they cost about 25¢ per day of 9 hr, they can not compete with storage-battery lamps for every-day use (87).

Electric lighting from circuits. Incandescent lights, from power or special circuits, are widely used in metal mines, open-light collieries, and to a limited extent in the intake gangways of gaseous mines. They are of greatest advantage along main haulage entries, slopes, in stables, at shaft bottoms, and on sidings; also for signaling purposes on electric trolley haulageways. They are unsafe on return airways of gaseous mines, since if a bulb is broken the filament will ignite firedamp; or, even if insulated, the feed lines may be short-circuited, due to wear, deterioration, or fall of roof, and an arc thus caused would ignite firedamp.

Where fixed elec lights are permitted by state regulations, they should be installed as prescribed in the National Electric Code, approved by the Amer Standard Assoc "for Class 1 hazardous locations" (in explosive atmospheres of dust or gas); the lamp has a heavy glass globe, sealed to the fixture, surrounding the bulb and protected by metal guards (46). For further protection the globe might be filled with an inert gas.

Electric lighting from compressed air lines is being widely introduced in European collieries, especially in Germany. The air, carried by a rubber hose, drives a tiny turbo-

generator, furnishing current for the lamps; one hose may serve 8 or 10 lamps. If the hose ruptures, the generator stops, as do the lights. The assemblage is easily moved as the face advances. One pneumatic elec light of British make has been approved by U S Bur of Mines. In 1936, 1 607 such lights were in use in Great Britain.

Reflector signs, lighted by headlights of oncoming vehicles and widely used on highways and automobiles, now find use in mines. Ordinary signs (as at switchstands and turnouts), which can not be seen far enough, may be made visible for sufficient distances by studding the letters with facet-cut reflector buttons. Also, a movable lighted stand is set up where repairing or timbering is being done (47). A man traveling haulage ways for switching or inspection might well wear a small reflector button on his belt in case his light goes out or can not be properly seen.

11. APPARATUS FOR DETECTION OF GASES, FOR USE WITHIN MINES

Safety lamps (fuel-burning) for detecting firedamp (see Art 9). Other appliances for detecting or analyzing mine gases are as follows.

Firedamp detectors, which are of a number of types:

(*a*) Those depending on diffusion of gases, or *osmosis*. Example. Webster firedamp indicator (52), consisting of a porous inverted pot, the bottom closed by a sensitive diaphragm, movement of which deflects a pointer. Pot is surrounded by porous, non-conducting and drying material; outside of this are gauzes between which is a material which will absorb CO_2. A tight cover surrounds the whole; put on in pure air, and taken off for making a test. Due to its lighter density CH_4, if present, diffuses through the porous material faster than the inclosed pure air, causing pressure which acts on the diaphragm and deflects the pointer.

(*b*) **Haber firedamp whistle,** brought out in Germany (1913), records presence of CH_4 by the difference in density of air and CH_4 mixtures, which causes a difference in sound waves. There are two whistles having the same note; one is in a pure-air container, the other is exposed to the entrance of the air to be tested. Both are blown simultaneously by mechanical means. If CH_4 is present the whistle exposed to it gives the higher note.

(*c*) **Shaw gas-testing machine** (52) is not portable, and must be used on the surface. It has 2 cylinders, one containing pure illuminating gas, the other the mine air to be tested. Measured quantities, obtainable from either cylinder by mechanical means, are mixed and passed into an exploding chamber, where the mixture is ignited. Force of the ignition causes a gong to strike. If the air contains some CH_4, less of the gas is needed to make the mixture explosive. It is a laboratory apparatus, is expensive, cumbersome and inefficient, compared with the more accurate quick-analysis apparatus.

(*d*) **Detectors employing catalytic agents,** as platinum and palladium. If these are brought into the presence of gases like CH_4, the so-called "catalytic" actions cause a slight rise of temp on their surfaces; this effect is made more noticeable with sponge platinum or palladium. It is utilized to heat adjacent substances which give indications in various ways. In the SUSSMAN ELECTRIC LAMP DETECTOR (50) by heating a mercurial thermometer the rising column of mercury touches sealed terminals, thus completing an electric circuit that lights a red bulb. In the HOLMES-ALDERSON FIREDAMP CUTOUT the heating of a compound strip of two metals, of different expansion coefficients, causes it to curve and complete an electric circuit. This energizes a magnetic solenoid which opens a switch cutting out the power circuit beyond. In the HOLMES-RALPH PORTABLE ELECTRIC LAMP ATTACHMENT (50), by the self-heating of a platinum wire in the presence of CH_4 a change in electric resistance is caused, indicated by a galvanometer needle. The amount of heating in these devices is so small that adjustments are very delicate; and as the degree of catalysis decreases with use, none of these detectors has yet been found practicable for everyday use.

(*e*) **Liveing's firedamp indicator** (49, 87) depends on increase of heat due to CH_4 burning on surface of a glowing coil. Two Pt coils carry current from a hand-operated magneto; one coil is in an air-tight tube the other exposed to atmos in a gauze cylinder; tube and cyl are on a common axis, their near ends enclosed in glass, with a photometric screen between. In a CH_4 mixture the exposed coil glows more brightly, and the screen is adjusted to obtain equal lighting; the extent of adjustment on a calibrated scale shows % of CH_4. The indicator is too delicate for ordinary use.

(*f*) **Léon and Ralph detectors** (50) use a hot wire for surface burning of CH_4, which raises temp of wire and increases its elec resistance. The resistance, measured with a Wheatstone bridge against that of a wire isolated from the mine air, indicates per cent CH_4.

(*g*) **Recent portable firedamp detectors and indicators.** None of the foregoing types has proved satisfactory, and new ones are being brought out, especially for attaching to miners' storage-battery lamps (50, 51).

A distinction has been made by U S Bur of Mines between a DETECTOR which merely shows presence of gas, and an INDICATING DETECTOR, which indicates with some precision varying percentages of CH_4 in the air. Many of these are not sufficiently precise or reliable for official approval. METHANE DETECTORS: 1. Wolf, for attachment to hand storage-battery lamps, is a very small flame safety lamp; 2. M. and K. (German) depends on difference in density of CH_4 and air (small and portable); 3. Martienssen (German) has a hot-wire loop, with a chemical catalyst adhering to parts of the wire. It has a dry cell, and is small enough to be carried in a coat pocket; 4. Gulliford

(British) (51), using the heated wire principle, is attached to storage-battery lamps. When current is switched on, the fine platinum wire is heated dull red; if fire-damp is present, the wire glows more brightly; if exposed to 3% CH_4, it fuses and breaks the circuit. A fresh fuse-wire is then inserted. It has been found safe for gaseous mines by the British testing station; 5. Burrell indicator (51) is a portable volumetric gas-analysis apparatus, carried like a safety lamp. CH_4 is drawn into it by suction, is burned off by a hot wire through a connection to a miner's electric lamp battery, and the contraction of volume is indicated by height of water in a tube, with a parallel scale for direct reading in tenths of 1% CH_4. It is accurate within 0.2 of 1%, has been approved by U S Bureau of Mines, but is less used than later permissible indicating-type detectors (51) as follows: U C C; M S A model 3, type AP with dry-cell battery; M S A model 5, type AP with Edison models H, J or K elec cap lamp; M S A type W-8 with Edison model K cap lamp.

(h) **Ringrose firedamp alarm and Ringrose detector** are connected to hand elec lights. The alarm uses a porous pot as combustion chamber, into which a continuous current of mine air passes through gauses, the CH_4 being burned off by a hot wire and leaving a partial vacuum in the pot. The vacuum, acting on a diaphragm, lights a red bulb as a warning when CH_4 exceeds a minimum previously fixed by adjustment. The detector uses the same principle, but warns by dimming the working lamp. Approved by British Mines Dept and is of limited use in Great Britain, but not approved by U S Bur of Mines.

Hot-wire, fixed, underground station indicator. A loop of current-carrying wire, inserted in a safety lamp, is heated by the turned-low flame of the lamp. The change in electrical resistance of the circuit may be indicated by a voltmeter at a distant point, as in the foreman's office. The objection to leaving an unattended light, even though a safety lamp, makes it unwise to use the device underground in this way. But such indicators are advantageous for laboratory testing of explosion-proof mining appliances.

Continuous methane recorders, enclosed to prevent ignition of CH_4 and with both remote and local signals to give warning of dangerous percentages of CH_4, are much needed; one American maker has produced trial types, but they require close attention and expert servicing, and are not yet officially approved.

Blackdamp detectors. The behavior of an ordinary lamp flame gives an approx idea of proportion of blackdamp present, up to extinction of the flame by a 3 to 4% deficiency of O, which may mean 15 to 18% blackdamp (Art 2). The Haldane flame-test apparatus (41) consists of a graduated glass tube, 7 in long, 0.75 in inside diam, into which is inserted a lighted wax taper, 1/16 in diam, in a thin slit brass holder. Held vertically, the taper induces an updraft, which becomes stronger as the taper is lowered in the tube, and if more than 1% blackdamp is present, will finally blow out the flame. As per cent of blackdamp increases, less draft is needed to extinguish and the taper is not lowered as far. The device does not detect CO dangerous to health, nor firedamp, nor show relative proportions of CO_2 and N, and is affected by humidity, but is valuable for daily use in open-light mines. The tube is graduated for an aver mine atmos condition of saturation at 64° F, at which water vapor is 1.2% of the wt of mixed air and vapor, or 2% by vol at 760 mm press.

Gas analysis. Ordinary devices for detecting one constituent may fail in the presence of other gases. Thus, a good safety lamp, capable in experienced hands of detecting as little as 1% CH_4 in otherwise good air, will have its indications masked or the flame extinguished if 3 or 4% of blackdamp is present, or 0 is deficient. MANY DANGEROUS CONDITIONS OF MINE AIR CAN BE DETERMINED ONLY BY ANALYSIS (55.56). Analyses of adjacent atmos are of particular value in fighting mine fires, especially when these are being sealed off; also in recovering fire areas, when it is not known whether the fire is out or only smouldering. Analyses of air from a fire area have sometimes indicated grave danger of explosion; so that the men were withdrawn from the mine or workings, to be out of danger, and the workings sealed to prevent entrance of air (Art 15, 19).

Control of ventilation in gaseous mines by daily analysis of the return currents from each ventilating area has been adopted by several large coal companies in the U S (56), and by the Ronchamp collieries, France, since 1891. Other French collieries constantly sample and analyze the air from the various working places. In German and Belgian collieries, control by analysis is widely employed. Great Britain, in 1911, established regulations requiring that CH_4 and CO_2 shall not exceed certain percentages, thus practically compelling analysis. Similar requirements are made from time to time in other countries (Art 6). U S Bur of Mines, in regulations for coal mining on the public domain (90), recommends systematic mine-air analysis.

Methods of gas analysis. For general methods, see books on chemistry (55, 56), but exact determinations require services of experienced chemists, and well-equipped laboratories. In recent years quick volumetric methods have been devised, by which determinations of ordinary gases can be made by men without chemical training after a little instruction, to within 0.2 or 0.3% of the exact percentage. This is sufficiently accurate in practice, except in case of CO. When CO is present there is always enough CO_2 accompanying it to be readily determined by field analysis, and to give warning of dangerous conditions. CO in dangerous amounts is never found in coal mines except in afterdamp of explosions and mine fires, when it is nearly always present in dangerous, if not fatal, quantities. In these conditions, particularly if there is much CO_2, the presence of CO would always be suspected, and a canary bird, which is very sensitive to CO, should be carried in a cage by men entering such atmos (54) (Art 14, 15, 17, 18, 19).

CO detector, a pocket-size instrument, is valuable for rescue and investigation crews. The tube is filled with Hoolamite (trade name for activated iodine pentoxide) by drawing air through it by an attached bulb (after breaking the sealing tip). If more CO is present than 7 parts in 10 000, color of the Hoolamite changes from gray-white to green, color intensity varying with % of CO. Comparison with a color chart shows approx % present. PYROTANNIC DETECTOR (portable), for finding the % of CO in blood of a person exposed to air containing CO, is also useful for rescue and first-aid crews.

Portable gas-analysis apparatus is of the volumetric-determination type. That first used for mines in the U S was the Orsat, widely employed for analyzing flue gases. Similar apparatus has been used in Europe. Burrell designed a modified Orsat (56); extensively used for mine rescue and fire-fighting.

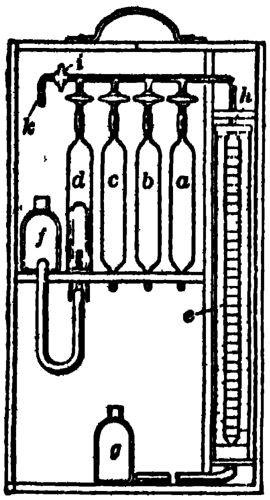


Fig 6. Modified Orsat Apparatus for Complete Analysis of Mine Air

Of the 4 pipettes in Fig 6, *a* contains KOH solution for absorbing CO_2; *b* contains alkaline pyrogallate solution for absorbing O; *c* contains ammoniacal cuprous chloride solution for absorbing CO; *a*, *b* and *c* contain glass rods to increase absorbent surface; *d* is a slow-combustion pipette for burning off CH_4, ignited by a Pt wire connected through sealed wires to a miner's elec light storage or dry-cell battery, the pipette being initially filled with distilled water slightly acidified with H_2SO_4; *e*, burette of 50 or 100 cc capac, graduated to 0.2 cc; *f* and *g*, leveling bottles; *h*, zero mark; *i*, stopcock; *k*, open end of glass tube leading to *e* and connected by branches to the pipettes. The apparatus is housed in a box 12 in high by 6.5 in wide and 6 in deep for carrying; front and back covers being removable.

Operation. By manipulating *f* and *g*, level the solution in each pipette in turn to a mark on its capillary tube connection, then close individual stopcock. By means of *g*, raise water level in *e* to zero mark *h*, stopcock *i* being open to the air. Connect a rubber tube at *k* and insert its free end in sample bottle, which has been inverted and uncorked under water By lowering *g*, draw a measured vol of mine air into *e*, usually all it will hold. Manipulating *g* and opening the proper stopcocks in turn, pass this sample successively into *a*, *b* and *c*, each time returning it to *e* for measurement of loss of vol by absorption. Then pass remaining gas to *d*, burn off CH_4, if any, and repass through *a* to determine CO_2 produced by the combustion (Table 11).

Table 11. Example of Results of Gas Analysis

	cc		cc		cc
Initial sample	50.0	After pass through *c*	40.4	After combustion	45.3
After pass through *a*	48.7	CO absorbed	0.4	Loss by combustion	4.1
CO_2 absorbed	1.3	Pass into *d*	40.4	After repass through *a*	43.4
After pass through *b*	40.8	Oxygen added	9.0	CO_2 by combustion	1.9
O_2 absorbed	7.9	Total vol in *d*	49.4	$H_2 = 2/3\ (4.1 - 2(1.9)) =$	0.2

Results: CO_2, 2.6%; O_2, 15.8%; CO, 0.8%; CH_4, 3.8%; H_2, 0.4%; N_2 by difference, 76.6%.

The small apparatus specified can be used within the mine, provided the work is done where the air is good and the safety lamp shows no dangerous amount of firedamp. A larger apparatus, of 100 cc capacity, is preferable for close determinations, but is more bulky to carry. The larger size is used in the U S Bur of Mines' rescue stations and cars. Haldane has a compact apparatus for determining only CO_2 and CH_4.

MINE ACCIDENTS AND THEIR PREVENTION

12. ACCIDENTS IN COAL MINES

Proper precautions greatly reduce accidents, as shown by comparing the rate of accidents in mines where special care is taken, with the aver rate in same district.

Comparisons may be based on: (*a*) number killed or injured annually per 1 000 employed; (*b*) number killed or injured annually per million tons coal produced. Basis *b* is sometimes preferred, but neglects the influence of local conditions on output per employe, and the question of compulsory mining out of all coal in thin and thick beds and back-filling excavations to prevent surface subsidence, as required in France and Belgium. Men engaged in back-filling operations are exposed to mining hazards, but produce no coal. Many authorities believe that, while basis *a* is not ideal, it affords a better means of comparison; moreover, basis *b* is not practicable for comparisons with metal mining (57).

U S Bur of Mines basis (until 1930) for its statistical comparisons is the number of accidents per yr per 1 000 300-day workers; thus assuming that the mining risk, which might be for only 100 or

200 days, was for 300 shifts in the year. This gives a truer comparison between different years, states or countries, and kinds of mining in which number of working days and kind of mineral mined might widely differ.

Table 12 shows a favorable reduction in accident rates, but more striking in the killed per ton produced, than in killed per hr of employment, chiefly due to greater productivity per man from increasing mechanization.

Table 12. Production, Number of Employes, Days worked, Man-hours, and Number Deaths per Million Hrs and Million Tons Produced, in U S Coal Mines (a)

Year	Production 1 000 tons	Employes	Active days	1 000 Man-shifts	1 000 Man-hours	Killed	Killed per million man-hr	Killed per million tons	Ton per man-hr
1921	506,395,401	823,253	173	142,358,691	1,145,738,000	1,995	1.74	3.94	0.442
1922	476,951,121	844,807	144	121,516,822	979,995,000	1,984	2.03	4.16	.487
1923	657,903,671	862,536	195	168,193,738	1,356,089,000	2,462	1.82	3.74	.485
1924	571,613,400	779,613	192	149,968,980	1,207,475,000	2,402	1.99	4.20	.473
1925	581,869,890	748,805	192	144,068,232	1,160,334,000	2,234	1.93	3.84	.501
1921–25	2,794,733,483	4,059,014	179	726,106,463	5,849,631,000	11,077	1.89	3.96	.478
1926	657,804,437	759,033	221	167,827,732	1,352,840,000	2,518	1.86	3.83	.486
1927	597,858,916	759,177	199	150,919,350	1,219,079,000	2,231	1.83	3.73	.490
1928	576,093,039	682,831	206	140,604,141	1,135,543,000	2,176	1.92	3.78	.507
1929	608,816,788	654,494	221	144,463,453	1,168,551,000	2,187	1.87	3.59	.521
1930	536,911,136	644,006	192	123,893,697	1,002,691,781	2,063	2 06	3.84	.535
1926–30	2,977,484,316	3,499,541	208	727,708,373	5,878,705,000	11,175	1.90	3.75	.506
1931	441,750,978	589,705	168	99,264,019	804,394,130	1,463	1.82	3.31	.549
1932	359,565,093	527,623	149	78,745,344	636,391,330	1,207	1.90	3.36	.565
1933...	383,171,877	523,182	171	89,225,732	719,148,559	1,064	1.48	2.78	.533
1934...	416,536,313	566,426	184	103,940,220	769,430,678	1,226	1.59	2.94	.541
1935...	424,632,005	565,202	180	101,571,654	732,607,581	1,242	1.70	2.92	.580
1931–35	2,025,656,266	2,772,138	171	472,746,969	3,661,972,278	6,202	1.69	3.06	.553

(a) Compiled by Adams, Geyer and Parry. Bur Mines, *Bull* 409 (1938).

Non-fatal accidents in U S coal mines (Table 13). Until 1930, non-fatal statistics were unreliable due to variations in mode of gathering them in different states; but, from 1930, the Bureau obtained data directly from mine operators. Tables 12 and 13 show ratio of total injuries to fatalities in 1930 was about 50 to 1; in 1935, 53 to 1. The British Mines Dept, 1935 report, shows 155 injuries to 1 fatality, but allowance must be made for differences in defining minor injuries. Note.—A table of accidents in the different states of U S in 1914 is omitted here (see p 1542 of 2nd edn of this book).

Table 13. Number of Injuries and Accidents of Coal-mine Employes in U S (a)

Year	Number of injuries				Rate	
	Permanent total disability	Permanent partial disability	Temporary disability	Total	Per million man-hr	Per million ton
1930	122	2 606	101 093	103 831	103.5	193.4
1931	98	1 773	78 478	80 349	99.9	181.9
1932	79	1 448	57 445	58 972	92.7	164.0
1933	51	1 290	59 972	61 313	85.3	160.0
1934	94	1 610	66 304	68 008	88.4	163.3
1935	81	1 793	63 701	65 575	89.5	154.4

(a) Compiled by W. W. Adams, U S Bur of Mines.

Table 14 shows a higher fatality rate per 1 000 employes in U S than in Europe, but lower per ton, due to large amount of dead work done in the deep and pitching European coal beds, to meet requirements of extracting both thin and thick beds and back-filling excavations.

Prevention of coal mine accidents. Probably 90% are preventable; at least half are due to recklessness, absent-mindedness, or ignorance of the victim; the other half to careless acts of fellow workmen, and to deficiencies in equipment and organization. Following analysis, covering 1930 to 1935, is in the order given in Table 15.

(1) **Falls of roof** caused 461 to 905 deaths per year, or 47 to 51% of total underground fatalities. Most of them were due to insufficient propping near the face. The number would be greatly reduced by SYSTEMATIC, REGULAR AND CLOSE SPACING OF TIMBERS, regardless of apparently safe condition of roof. This is general practice in European col-

Table 14. Fatalities in Coal Mines of U S and Foreign Countries (W. W. Adams and E. E. Getzin, Bur of Mines)

Year	Production 1 000 short ton	Employes	1 000 man-days	Killed	Deaths per 1 000 employes (a)	Death rate per million short ton	Aver prod per man-day, short ton	Prod per death, in 1 000 short ton
				United States (all coal mines)				
1930	536 911	644 006	123 894	2 063	5.00	3.84	4.33	260
1931	441 751	589 705	99 264	1 463	4.42	3.31	4.45	302
1932	359 565	527 623	78 745	1 207	4.60	3.36	4.57	298
1933	383 172	523 182	89 226	1 064	3.58	2.78	4.29	360
1934	416 536	566 426	103 940	1 226	3.54	2.94	4.01	340
1935	424 632	565 202	101 572	1 242	3.67	2.92	4.18	342
				United States (bituminous)				
1930	467 526	493 202	92 326	1 619	5.26	3.46	5.06	289
1931	382 105	450 274	73 349	1 080	4.47	2.83	5.21	354
1932	309 710	406 380	59 260	958	4.85	3.09	5.23	323
1933	333 631	418 752	69 882	833	3.58	2.50	4.77	401
1934	359 368	458 044	81 648	958	3.52	2.67	4.40	375
1935	372 369	462 354	82 292	968	3.53	2.60	4.52	385
				United States (anthracite)				
1930	69 385	150 804	31 568	444	4.22	6.40	2.20	156
1931	59 646	139 431	25 915	383	4.43	6.42	2.30	156
1932	49 855	121 243	19 486	249	3.83	4.99	2.56	200
1933	49 541	104 430	19 344	231	3.58	4.66	2.56	214
1934	57 168	108 382	22 292	268	3.61	4.69	2.56	213
1935	52 263	102 848	19 280	274	4.26	5.24	2.71	191
				Great Britain (all mines under Coal Mines Act)				
1930	281 090	943 442	234 828	1 013	1.29	3.60	1.20	277
1931	251 898	877 141	211 566	859	1.22	3.41	1.19	293
1932	239 022	827 439	197 638	881	1.34	3.69	1.21	271
1933	237 256	797 294	192 243	820	1.28	3.46	1.23	289
1934	253 705	797 699	201 073	1 073	1.60	4.23	1.26	236
1935	255 902	779 502	199 550	861	1.29	3.36	1.28	336
				France				
1930	60 689	292 540	79 713	274	1.03	4.51	0.76	221
1931	56 268	276 540	70 789	218	.92	3.87	.79	258
1932	52 115	253 200	60 576	180	.89	3.46	.86	290
1933	52 889	242 840	57 851	179	.93	3.38	.91	295
1934	53 635	230 340	57 029	182	.96	3.39	.94	295
1935	51 940	219 610	54 386	187	1.03	3.60	.96	278
				Belgium				
1930	30 219	155 397	47 631	195	1.23	6.45	0.63	155
1931	29 809	152 713	45 717	151	.99	5.07	.65	197
1932	23 615	138 316	35 170	133	1.13	5.63	.67	178
1933	27 888	134 933	38 152	129	1.01	4.63	.73	216
1934	29 089	125 705	35 907	177	1.48	6.08	.81	164
1935	29 218	120 613	34 207	125	1.10	4.28	.85	234
				Germany (except lignite mines) (b)				
1930	153 215	400 654	115 559	1 191	3.09	7.77	1.33	129
1931	127 152	307 366	88 294	629	2.14	4.95	1.44	202
1932	111 850	250 284	72 215	456	1.89	4.08	1.55	245
1933	117 355	253 396	73 129	471	1.93	4.01	1.60	249
1934	133 929	285 601	82 812	459	1.66	3.43	1.62	343
1935	153 874	343 106	97 779	471	1.45	3.06	1.57	327

(a) Adjusted to year of 300 workdays per employe. (b) Includes Saar district since Mch 1, 1935.

lieries, and the higher cost is justified by low accident rate from falls of roof, especially in France, Belgium and Germany (Table 16). Roof should be FREQUENTLY TESTED by VIBRATION METHOD (striking roof with bar or sledge with one hand and feeling with fingers of other hand to see if portion struck vibrates). This supersedes the more uncertain mode of judging by sound. Important factors for preventing falls: SYSTEMATIC DELIVERY OF

Table 15. Men Killed at Coal Mines in U S, 1930–1935, Classified by Causes (U S Bur of Mines)

Underground

Causes	1930	1931	1932	1933	1934	1935
Falls of roof (rock, coal, or draw slate)	905	690	531	461	497	537
Roof fall due to car or machine knocking out post	34	12	4	12	19	7
Falls of face or rib	144	134	77	91	146	98
Rush of coal, rock, or gob	16	22	15	13	27	22
Other falling objects (not handled by worker)	8	7	4	3	10	7
Falls of persons	9	13	6	9	7	7
Handling materials	6	4	2	11	15	5
Hand tools	3	2	2	1	3	3
Striking or bumping against objects	3	....	2	4	2	4
Haulage, cars and locomotives	325	237	179	194	197	228
Explosions of gas or coal dust	264	88	169	40	52	49
Explosives (not including explosions of gas or dust)	78	40	36	34	36	50
Electricity (not resulting in explosions)	71	65	47	53	56	45
Machinery	46	22	28	25	31	35
Suffocation from natural gases (not from fires or explosions)	11	6	5	3	2	5
Mine fires (burns, suffocations, etc)	7	4	...	1	8	10
All other accidents underground	9	9	8	5	11	9
	11	23	15	16	13	23
Total underground	1 950	1 378	1 130	976	1 132	1 144

Surface, at underground mines

Causes	1930	1931	1932	1933	1934	1935
Mine cars and mine locomotives	17	18	9	14	19	16
Railway cars and locomotives	17	15	13	10	5	11
Handling materials	3	....	3	1	6	3
Hand tools	....	1	1	2	....	1
Falls of persons	10	9	7	12	11	12
Falling objects	4	4	9	4	7	2
Machinery on surface	13	9	13	17	16	9
Electricity	13	12	2	7	6	6
Explosives	1	....	....	1	1	3
All other surface accidents	17	10	5	7	11	9
Total surface	95	78	62	75	82	72
Total at mines	2 045	1 456	1 192	1 051	1 214	1 216

Open-pit mining and stripping

Causes	1930	1931	1932	1933	1934	1935
Falls or slides of coal or overburden	2	3	3	5	3	5
Mine cars and locomotives	3	....	4	1	1	....
Railway cars and locomotives	....	1	....	....	2	1
Explosives	3	1	1	1	2	5
Electricity	4	....	2	2	1	4
Hand tools	...	....	....	....	....	1
Stumbling, slipping, etc	2	1	1	1	....	3
Falling objects, other than coal	1	....	1	....	1	1
Power shovels, scrapers, or buckets	....	....	1	3	1	1
Power drills of all kinds	....	....	....	....	1	1
Stepping on or striking objects	....	....	1	....	....	....
All other	3	1	1	....	....	4
Total	18	7	15	13	12	26
Grand total	2 063	1 463	1 207	1 064	1 226	1 242

The classification of causes of accidents in Table 15 is complex, and varies in different countries, except as to major causes: (*a*) largest group in every coal district is due to falls of roof; (*b*) next are explosions of gas or dust; (*c*) major cause in the U S is haulage accidents.

TIMBERS at each working place; ENOUGH FOREMEN, to inspect and instruct miners, to see that orders to put up props are carried out, and to withdraw men when necessary.

Table 16. Fatalities in Coal Mines in 1935, with Rates per 1 000 Workers
W. W. Adams and E. E. Getzin, U S Bur of Mines

Cause	U S		Gr Britain		France		Belgium		Germany	
	Total	Rate	Total	Rate	Total	Rate	Total	Rate	Total	Rate
Falls of roof or coal	664	1.96	458	0.69	90	0.49	52	0.46	200	0.61
Haulage accidents und'g'd.	228	.67	186	.28	14	.08	16	.14	131	.40
Gas and coal-dust explosions	49	.15	37	.05	...	...	19	.17	25	.08
Blasting accidents	50	.15	16	.02	2	.01	2	.02	18	.06
Electricity	45	.13	3	.01	...	....	...	....	5	.01
Other causes underground	85	.25	58	.09	14	.08	6	.05	29	.09
Shaft accidents	23	.07	16	.02	29	.16	14	.12	18	.06
Surface accidents	98	.29	87	.13	38	.21	16	.14	45	.14
Total from all causes	1 242	3.67	861	1.29	187	1.03	125	1.10	471	1.45

(2) **Roof falls due to knocking out props** by car or machine. Similar in effect to "Falls of roof," but causes different. Fatalities, 4 to 34. In pitching beds (Penn anthracite mines) and where large mining and loading machines are used, cars and machines must be carefully blocked.

(3) **Falls of face or rib.** Fatalities, 77 to 146. Remedy, better side supports and spragging or blocking thick undercut beds, especially in pitching beds. In very thick seams all loose coal should be barred off, starting at top after blasting.

(4) **Rush of coal, rock or gob.** Fatalities, 13 to 27. This is a special hazard in pitching beds. Remedies, better layout of workings, and timbering or walling to support overhanging gob.

Above 4 causes accounted collectively in 1935 for 664 fatalities (59% underground and 53.4% of all), or 1.96 killed per 1 000 employes. The rates abroad are far below those of the U S.

Other causes of accidents in U S (1930-1935). Mine cars and locomotives caused from 179 to 325 deaths, or 16 to 21% of the underground fatalities. In level workings men may be run over or rolled between cars and walls on main haulage roads. These accidents are largely preventable by having GOOD MANWAYS and compelling their use; AMPLE CLEARANCE for a pathway along one side; WHITEWASHED REFUGE HOLES, always on same side of track, and at regular intervals not exceeding 50 ft.

Where haulageways are used as manways in a gaseous mine, only approved elec signal lamps should be used. In non-gaseous mines "protected" incandescent LIGHTS at regular intervals are advisable. Locomotives should have good HEADLIGHTS, and where possible a BLOCK-SIGNAL system (easily installed for elec haulage). Many accidents caused in "spragging" cars are preventable by providing BRAKES. Accidents often occur in mule haulage by riding in front and standing on the tail-chain, instead of on rear of train; if necessary to ride in front, have a MOVABLE SEAT hooking over the car end. Accidents while coupling cars, due to short bumpers, are frequent; bumpers should project at least 10 in, to provide 20 in clear space between car bodies. In pitching workings, in which cars are hauled to the face by mule, locomotive or rope, AUTOMATIC HINGED CAR-BLOCKS will prevent runaways. GOOD TRACKS and CLEAN ROADWAYS are important.

Gas explosions, burning gas and explosions of coal dust in 1918–25 caused 5 to 22% of total fatalities, and in 1930–35, 3.8 to 14.0% (Table 15). In 1935, 49 deaths (4.3% of underground or 0.15 per 1 000 employes) compare favorably with European statistics (Table 17), due principally to adoption of rock-dusting and permissible explosives and machinery.

Firedamp accidents are mostly preventable by care on the part of mine officials. The chief factors are: FREQUENT INSPECTION by competent firebosses and foremen and GOOD VENTILATING METHODS (including large capacity fans; good stoppings in room breakthroughs or crosscuts; use of line brattices, and of overcasts, where possible, in place of doors; use of double and triple doors; and keeping airways cleaned). Abandoned but open workings should be well ventilated, and regularly inspected, or else securely walled off. Walled areas giving off firedamp under pressure should be vented by drill-holes from surface. In gaseous mines, regular analysis of the separate splits is valuable in controlling ventilation. In mines where firedamp is often found (whether classified as gaseous or not) SAFETY LAMPS OR PERMISSIBLE MINER'S ELECTRIC LAMPS should be used (Art 9, 10). Mixed lights in the same mine or district are a menace to safety. PERMISSIBLE EXPLOSIVES (Sec 4, Art 9) should be used instead of long-flame explosives (black powder or dynamite). Machine-mining in gaseous collieries should be done by EXPLOSION-PROOF MOTORS. Electric-trolley locomotives are not

safe to use in working places or return ventilation entries of gaseous mines. Explosion-proof storage-battery locomotives have recently been introduced in the collieries of U S; also explosion-proof storage batteries on mine car trucks for feeding, undercutting, and other machines. This is not only safe, but, by preventing electric peak-loads, is economical. Ignition of gas has occurred from use of electric signalling wires in return air courses.

Accidents from explosives, other than those leading to gas or dust explosions, in 1930–35 caused 2.8 to 4.7% of total fatalities. They are chiefly produced by: (*a*) premature blasts, due to accidental ignition of black powder by open lights, cutting fuse too short to allow time to get out of danger, forgetting a lighted first fuse while lighting others, or getting tools out of the way; (*b*) use of metal tamping bar. Only wooden bars should be permitted. Even copper-tipped bars are unsafe; (*c*) returning too soon to investigate apparent misfires; (*d*) in electric firing, a lead wire may come in contact with a coal-cutting machine under power, or a power line, or with an unprotected dry cell or storage battery; (*e*) opening powder kegs with a steel wedge, or even a wood wedge, if there are sand grains embedded, may cause a frictional spark. PELLET BLACK BLASTING POWDER, made in cylindrical cartridges with central perforation, and wrapped in paraffin paper, is safer in handling and charging than granulated powder, but has a long flame and should not be used in gaseous or dusty mines. PERMISSIBLE EXPLOSIVES should always be used in coal mining, as required in Europe. In "shooting off the solid," they are not effic, being too quick in action, but that method is now barred in most states. Added cost of permissible blasting is insignificant per ton of coal.

Electric firing is safer than fuse, provided that, where electric coal cutters are employed, ends of the leads are kept in contact and covered with insulating tape until power is shut off the machine and its leads. If electric firing is by battery, the latter (if dry cell or storage) should be so constructed that the binding posts are not in circuit until a recessed spring-opening button or lever has been pressed to close the circuit. HAND-OPERATED MAGNETO BLASTING MACHINES are generally preferred. Use of power lines or connections to trolley lines is dangerous. For electric firing from the surface, there should be separate insulated circuits, and switch cutouts at mouth of every entry and working place, to prevent accidental stray currents entering firing circuit, and the switch should be kept open until closed by a foreman as the men leave the mine before blasting (59).

Stemming of explosives. For proper methods, see Sec 4, Art 8. Coal dust should never be used. Mechanical stemming devices have not yet been approved by the U S Bur of Mines, because of the flame hazard. Extensive tests in Great Britain showed that 3 parts sand with 1 part clay, and 3 to 5% $CaCl_2$ to retain moisture, make a good mixture.

Sheathing of explosives. The safety of permissible explosives is relative. Although their flame is short and of small duration, they have ignited firedamp in fractured coal, or in fractured roof when being "brushed" in a gaseous mine. Sheathing cartridges with an $^1/_8$-in layer of $NaHCO_3$, to cool the flame, has found favor in Great Britain; but not yet in the U S, due to increased cost of explosive, and necessity of drilling slightly larger holes.

Substitutes for explosives. "CARDOX" consists of a metal cylinder containing liquid CO_2 and a combustible to be ignited by an elec cap; the liquid vaporizes on heating, and the resultant gas, bursting a thin steel disk at the inner end of the cartridge, issues at high press, blasting the coal without flame. Efficient tamping is necessary to prevent the cartridge from being blown out of the hole. With approval of the U S Bur of Mines, certain types of Cardox find considerable use in mechanized mines, and in Great Britain the device is found well adapted to longwall mining. More recently "HYDROX" employs a steel tube with sealing disk, filled with a powder which, when detonated, generates nitrogen and water vapor. It was approved (1935) by the British Mines Dept. "AIRDOX," developed in the U S for coal mining, employs neither explosive nor combustible. Air under high press from a small compressor on a mine truck, is forced into a metal cylinder like that for "Cardox," and, bursting the inner disk, breaks the coal. HYDRAULIC BLASTING: the "hydraulic cartridge," or wedge, has been used in a few British longwall mines, where conditions are favorable and the coal well undercut. It is a steel tube containing a small hydraulic piston; water press is produced by a hand pump connected by hose to the cartridge. It requires a large borehole, is slow in breaking the coal and will not work in tight corners. A recent American device employs a rubber tube, instead of a steel cylinder. It is still in the experimental stage. Hydraulic cartridges, though safe, are too slow to compete with permissible explosives.

Hangfires and misfires are most prevalent in metal mining, where shots are fired in rounds and where fuse and caps are favored over electric firing (except in shaft sinking). Electric shot-firing failures are nearly always due to poor connections, rarely to imperfect detonators or to explosives impaired by age, heat, or wetting, causing non-ignition or, in extreme cases, burning of the explosive instead of detonation. Failures in use of fuse or delay-action detonators are due to: (*a*) sharp bending of fuse in handling; (*b*) wet fuse; (*c*) imperfect fuse (rare); (*d*) imperfect detonators; (*e*) fuse being cut or outer part of explosive charge cut and thrown off by previous shot in round; (*f*) sticks of explosive

left in stub-end of hole or loose sticks thrown into blasted material. **Remedies**: care in selecting and handling explosives; in using fuse, to count shots carefully, and if one fails or the count is in doubt, to wait at least 4 hr (better 8 hr) before entering the place. In blasting coal and brushing roof or floor, electric firing is much safer (unless firing is done from the surface), firing 1 shot at a time and inspecting before and between shots. In case of hangfire or misfire, disconnect battery leads, and wait 15 min before returning.

Fuse should not be used in blasting coal, as specified by Bur of Mines for "permissibility" of explosives.

Extracting misfires may be necessary in coal mining, because if a parallel hole is drilled close by, the first charge might be set off by concussion or flame and, having less burden, blow out and ignite coal dust. It requires care, especially if a cap is in outer end of charge. If another hole is drilled, everything for 100 ft outbye should be wet down or rock-dusted before shooting. Although careful extraction of tamping and charge is favored by many (especially if it can be wetted or washed out by a water jet), the usual practice is to drill a parallel hole 1 to 2 ft from the first. After firing, examine the broken coal for loose sticks of explosive.

Suffocation by gases from explosives in a well-ventilated colliery is rare, and unlikely to occur unless coal or a pocket of firedamp is ignited, thus adding afterdamp to powder smoke; or unless explosives are burned instead of detonated, by using too weak a cap (Sec 4, Art 8, and Sec 6, Art 10). REMEDY is to delay entering the working place until the smoke is cleared out.

Accidents by flying pieces of rock or coal from blasting are due to carelessness on part of victim in not getting to a place of safety, or firing a shot in adjacent working without notification, or not guarding entrances to working place after lighting a fuse, or while connecting the lead wires at another point. Where workings have several entrances it is well to furnish miners with PAINTED WARNING BOARDS, to be placed across or on floor of entrances not guarded by shot-firer.

Suffocation from mine gases caused 5 deaths in 1935. Except for explosions and fires, which are not included, there is little excuse for such accidents. They are generally due to entering dangerous places unnecessarily. To prevent this, such places should be properly ventilated or permanently walled off.

Accidents from electricity (shock or burns), not including ignitions of firedamp or coal dust, caused 51 to 88 deaths annually in 1930–35. Some were from touching bare conductors, trolley or power lines, some from improper insulation and installation of cables and motors.

Many of these accidents might be prevented by: trolley guardboards, when the wire is less than 6.5 ft above roadway, guarding by boards or props of bare power cables or preferably by rubber covering and sometimes armoring; good installations to prevent grounding of wires. In working places or traveling ways, trolley and powerline potentials should not exceed 275 or 300 volts. Electric coal cutters should be so designed and installed that a strong shock is not received on touching the frame. Cable leads should be of the single, concentric-cable type, the return wires being outermost to avoid shocks from grounding. RULES FOR INSTALLATION AND USE OF ELECTRIC EQUIPMENT in mines, as formulated by the Bur of Mines, should be observed. Animals killed have been fewer since the substitution of conveyers and gathering locomotives for animal haulage. Properly installed underground stables, with passageway in front of as well as behind the animal, would prevent some accidents; but above all, PROPER HANDLING and DRIVING WITHOUT ABUSE would prevent animals from becoming vicious.

Accidents from coal cutters, causing 22 to 46 deaths annually in 1930–35, are due to men falling on the moving parts of machines, or in handling them. Such accidents have increased with increasing mechanization at the face. The first-mentioned dangers are lessened by using inclosed machines; the latter are casual, to be guarded against by the individual.

Fatalities from mine fires (Art 15), by burning and suffocation, cost 26 lives in 1918, but were relatively few in later years; only 10 in 1935. Suffocation, rather than burns, is the primary cause, because fires progress slowly in restricted passages.

The smaller accidents occur near the face; generally due to recklessness or ignorance in entering the return current or afterdamp from fires. In colliery workings such fires are often caused by long-flame explosives, which may set fire to coal or gas feeders. Permissible explosives, properly used, rarely cause fires. In a large Illinois mine, 15 "fire-runners," formerly employed to extinguish fires after black-powder blasts, were dispensed with by introducing permissible explosives.

Falling down shafts or slopes caused 11 to 23 deaths annually in 1930–35. Man cages should have gates hinged to cage; also gates and fences around shaft mouths, and at landing places. Gates should be self-closing.

Surface accidents caused 62 to 95 deaths yearly in 1930–35. Of these 9 to 17 were by mine cars and locomotives in drift slope mines, and 5 to 17 by RR equipment. Practically all such accidents are preventable, either by the individual or by proper installation and safety guards.

Protective clothing for miners. HATS OR CAPS are common in both coal and metal mines. SAFETY SHOES or boots prevent injuries from falls of rock, coal or timbers, or being pinched by a car wheel. GOGGLES are advantageous for protection from particles of mineral in picking, drilling and loading. Some companies have goggles ground to fit eyes with defective vision. DUST MASKS should be worn in dust-making work, especially if dust is silicious; the finer the particle size, the more likely to cause silicosis or other lung troubles. LEATHER GLOVES prevent cuts and abrasions, which may lead to infection. KNEE PADS are needed only in low-roof workings, where crawling is necessary. Improvised pads are much used in thin coal seams in Great Britain. An American maker supplies sponge-rubber knee-cap protectors, which, by expansion and contraction of the cellular spaces, provide skin ventilation under the pad. SHIN GUARDS are useful when working around conveyers or other machinery. In Penna, use of hats or caps reduces compensation rates 14¢, use of safety shoes 8¢, and use of goggles 8¢ per $100 pay roll.

13. ACCIDENTS IN METAL MINES AND QUARRIES

Statistics in following tables are from reports of mine operators (except in a few states where they came from inspection departments). For equal periods of operation or exposure to risk, fatality rate at coal mines is usually higher than at metal mines. Metal mines ordinarily work more days per year, except Mississippi Valley lead and zinc mines, and gold placers. In metal mines single disasters, except fires, rarely kill large numbers, as in coal mine explosions. Tables 18 to 22 show fatalities and injuries for each kind of accident. Falls of roof, etc, caused the largest number of deaths, but the proportion of these to the total is greatest in coal mines. Fatalities in metal mines from explosives are much higher; those from handling and transportation are in far greater proportion in coal mining; and those from "falls of person" in metal mines are large enough to offset losses in coal mines from explosions.

Table 17. Men Employed and Number Killed at All Mines and Quarries in U S, 1931–35
(W. W. Adams, U S Bur Mines)

	Metal mines			Coal mines			Quarries			Total, mines and quarries		
	Number employed	Number killed: Total	Number killed: Per million man-hr	Number employed	Number killed: Total	Number killed: Per million man-hr	Number employed	Number killed: Total	Number killed: Per million man-hr	Number employed	Number killed: Total	Number killed: Per million man-hr
1931	80 940	158	1.01	589 705	1 463	1.82	69 200	61	0.46	739 845	1 682	1.54
1932	53 288	107	1.16	527 623	1 207	1.90	56 866	32	0.34	637 777	1 346	1.64
1933	57 016	95	1.01	523 182	1 064	1.48	61 927	59	0.67	642 125	1 218	1.35
1934	66 645	116	1.00	566 426	1 226	1.59	64 331	60	0.63	697 402	1 402	1.43
1935	92 314	164	1.02	565 202	1 242	1 70	73 005	51	0.46	730 521	1 457	1.45
Aver	70 041	128	1.03	554 428	1 240	1 69	65 066	53	0.51	689 534	1 421	1.48

Table 18. Fatalities, Metal Mines, Coal Mines, and Quarries in U S, 1933–35
(W. W. Adams)

Kind of mine	Falls of roof or mineral	Explosives	Haulage and handling	Falls of persons	Electricity	Mach'y	Explosions	Other causes
	Cause of accident (percentages of total)							
1933 Metal mines	30.53	7.37	11.58	17.89	5.26	7.37	....	20.00
1933 Coal mines	54.70	3.38	21.71	3.01	5.83	4.23	3.76	3.38
1933 Quarries	44.07	5.08	8.48	8.48	6.78	15.25	....	11.86
1934 Metal mines	37.07	12.93	8.62	18.97	1.72	4.31	....	16.38
1934 Coal mines	56.45	3.18	19.98	1.71	5.14	4.00	4.24	5.30
1934 Quarries	25.00	23.33	6.67	15.00	1.67	25.00	....	3.33
1935 Metal mines	31.71	14.63	12.81	14.02	6.10	3.66	....	17.07
1935 Coal mines	53.86	4.67	21.26	2.17	4.43	3.70	3.95	5.96
1935 Quarries	21.57	11.76	13.73	11.76	5.88	19.61	....	15.69

Table 19. Killed and Injured in Metal Mines in 1935 (W W. Adams and M. E Kolhos, U S Bur Mines)

Kind of mine	Number of mines	Men employed				Man-days of employment				Aver hr work, per man per day			Man-hr of work (in thousands)			
		Under-ground	Surface	Open-cut	Total	Under-ground	Surface	Open-cut	Total	Under ground	Sur-face	Open-cut	Under-ground	Surface	Open-cut	Total
Copper	94	6 203	2 355	1 630	10 188	1 700 447	622 248	464 388	2 787 083	8 00	8 00	8 00	13 603	4 974	3 715	22 293
Iron	174	7 691	3 137	3 213	14041	1 676 333	716 297	684 138	3 076 768	8 00	8 01	8 10	13 404	5 735	5 543	24 682
Lead and zinc (Mississippi Valley)	190	5 924	626	178	6 728	1 032 684	123 013	36 704	1 192 401	7 99	8 09	9 41	8 251	994	345	9 591
Gold, silver, and miscellaneous	9 866	32 089	17 543	3 386	53 018	7 415 192	3 256 295	538 302	11 209 789	7 88	7 95	7 93	58 418	25 877	4 271	88 566
Gold, silver lode	5 417	29 183	6 952	970	37 105	6 768 648	1 735 439	158 777	8 662 864	7 88	7 91	8 07	53 311	13 730	1 281	68 323
Gold: placer	4 224	1 066	10 250	1 698	13 014	194 401	1 440 414	288 507	1 923 322	7 91	7 99	7 80	1 538	11 513	2 251	15 302
Miscellaneous	225	1 840	341	718	2 899	452 143	80 442	91 018	623 603	7 89	7 87	8 12	3 567	633	739	4 940
Nonmetal	495	2 498	2 360	3 481	8 339	574 048	748 950	763 333	2 086 331	7 90	7 06	8 31	4 534	5 287	6 345	16 168
Total	10 819	54 405	26 021	11 888	92 314	12 398 704	5 466 803	2 486 865	20 352 372	7 92	7 84	8 13	98 212	42 870	20 220	161 302

Kind of mine	Aver days active			Average hr per man per year			Number killed			Number injured			Widows	Orphans	Rates per million man-hr							
															Killed				Injured			
	Under ground	Sur face	Open-cut	Under ground	Sur-face	Open-cut	Under ground	Sur-face	Open-cut	Under ground	Sur face	Open-cut			Under ground	Sur-face	Open-cut	Total	Under-ground	Sur-face	Open-cut	Total
Copper	274	264	285	2 193	2 113	2 279	14	4	1	1 276	131	59	11	11	1 03	0 80	0 27	0 85	93 80	26 33	15 88	65 76
Iron	218	228	213	1 743	1 828	1 725	18		4	344	24	72	11	10	1 34		72	89	25 66	4 19	12 99	17 83
Lead and zinc (Mississippi Valley)	174	197	206	1 393	1 589	1 940	8	1		607	53		6	22	97	1 01		94	73 56	53 28		68 81
Gold, silver, and miscellaneous	231	186	159	1 821	1 475	1 261	88	15	4	5 587	1 088	152	43	95	1 51	58	90	1 21	95 64	42 04	35 59	77 88
Gold, silver lode	232	250	164	1 827	1 975	1 321	75	14	2	5 038	660	43	42	93	1 41	1 02	1 56	1 33	94 50	48 07	33 57	84 03
Gold. placer	182	141	170	1 443	1 123	1 326	8	1	2	154	392	91	0	1	5 20	09	83	72	100 10	34 05	40 42	41 63
Miscellaneous	246	236	127	1 939	1 857	1 029	5			395	36	18	1	1	1 40			1 01	110 71	56 84	24 35	30 88
Nonmetal	230	317	219	1 815	2 241	1 823	4	2	1	303	264	246	7	18	88	38	16	43	66 82	49 93	38 77	50 38
Total	228	210	209	1 805	1 648	1 701	132	22	10	8 117	1 560	529	78	156	1 34	51	49	1 02	82 65	36 39	26 16	63 27

Table 20. Killed and Injured in Different Branches of Mineral Industries, 1935

Industry	Aver days active	Men employed	Man-days	Man-hr	Weighted aver length of shift	Man-hr per man per year	Killed	Injured
1. Coal mines	180	565 202	101 571 654	732 607 581	7.21	1 296	1 242	65 575
Bituminous	178	462 354	82 291 724	578 511 200	7.03	1 251	968	47 529
Anthracite	187	102 848	19 279 930	154 096 381	7.99	1 498	274	18 046
2. All metal mines	218	83 975	18 266 041	145 134 364	7.95	1 728	157	9 393
Copper	274	10 188	2 787 083	22 293 255	8.00	2 188	19	1 466
Gold, silver, and misc metal	211	53 018	11 209 789	88 566 720	7.90	1 671	107	6 827
Iron	219	14 041	3 076 768	24 682 644	8.02	1 758	22	440
Lead and zinc (Miss Valley)	177	6 728	1 192 401	9 591 745	8.04	1 426	9	660
Nonmetallic mineral	250	8 339	2 086 331	16 168 307	7.75	1 939	7	813
3. All quarries	200	73 005	14 623 303	110 033 341	7.52	1 507	51	4 152
Cement rock	227	24 416	5 546 183	39 243 018	7.08	1 607	12	362
Granite	202	6 877	1 386 029	10 555 416	7.62	1 535	6	570
Limestone	187	30 973	5 804 752	45 197 391	7.79	1 459	24	2 412
Marble	210	2 441	512 481	4 016 819	7.84	1 646	1	176
Sandstone and bluestone	167	2 739	457 217	3 688 135	8.07	1 347	.....	243
Slate	184	2 063	379 385	3 097 339	8.16	1 501	2	168
Traprock	154	3 496	537 256	4 235 223	7.88	1 211	6	221
In and about quarry	177	32 629	5 762 015	44 267 391	7.68	1 357	35	2 712
In outside works	219	40 376	8 861 288	65 765 950	7.42	1 629	16	1 440
4. Metallurgical plants	291	36 493	10 631 513	83 923 699	7 89	2 300	28	1 961
Ore-dressing plants	238	11 841	2 817 005	22 577 689	8.01	1 907	7	631
Smelters	324	14 675	4 752 380	37 160 291	7.82	2 532	14	821
Auxiliary works	307	9 977	3 062 128	24 185 719	7 90	2 424	7	509
5. All coke ovens	321	16 125	5 175 328	40 941 173	7.91	2 539	10	325
Beehive	182	1 075	196 177	1 370 478	6.99	1 275	.....	62
Byproduct	331	15 050	4 979 151	39 570 695	7.95	2 629	10	263
Total	195	783 139	152 354 170	1 128 808 465	7 41	1 441	1 495	82 219

Fatal and non-fatal accidents. Tables 21, 22 and 23 show wider variations in non-fatal than in fatal accidents. Apparent inconsistencies are probably due to variations in mode of reporting slight injuries. Death rates are reasonably consistent.

Table 21. Number of Employes, and Number Killed and Injured at All Mines (Except Coal) in U S (1926–35)

Year	Aver days active	Men employed: Actual number	Men employed: Equivalent in 300-day workers	Total shifts	Number killed: Total	Number killed: Per thousand 300-day workers	Number injured: Total	Number injured: Per thousand 300-day workers
1926	291	127 823	123 870	37 160 978	430	3.47	30 350	245.01
1927	284	119 699	113 447	34 033 963	352	3.10	25 133	221.54
1928	288	113 866	109 345	32 803 610	273	2.50	22 483	205.61
1929	292	118 735	115 394	34 618 120	350	3.03	23 092	200.11
1930	270	103 233	92 900	27 869 982	271	2.92	15 594	167.86
Aver for 5 years	285	116 671	110 991	33 297 330	335	3.02	23 330	210.20
1931	231	80 940	62 405	18 721 486	158	2.53	8 709	139.56
1932	208	53 288	36 984	11 095 167	107	2.89	5 014	135.57
1933	204	57 016	38 807	11 642 113	95	2.45	5 925	152.68
1934	221	66 645	49 077	14 723 215	116	2.36	7 892	160.81
1935	220	92 314	67 841	20 352 372	164	2.42	10 206	150.44
Aver for 5 years	219	70 041	51 023	15 306 871	128	2.51	7 549	147.95
Aver for 10 years	271	128 482	118 677	35 603 019	411	3.46	26 328	221.85

Prevention of accidents in metal and miscellaneous mines, considered in order of causes in Table 23 and referring to year 1935:

Table 22. Metal-mine Accidents, Grouped by Mining Methods, for Year Ended Dec. 31, 1935, for Selected Companies (a)

Method of mining	Number of mines	Aver days active	Man-days	Men employed*	Number killed	Number injured	Rate per million man-hours: Killed	Rate per million man-hours: Injured
Open stope, including room-and-pillar and sub-level stoping	137	229	2 700 612	11 776	29	1 645	1.36	77.28
Shrinkage	22	261	372 780	1 427	5	377	1.68	126.41
Cut-and-fill	25	291	879 572	3 021	15	732	2.13	103.92
Square-set	41	304	1 447 041	4 764	16	1 526	1.40	133.90
Block caving	7	210	258 477	1 231	4	324	1.93	156.66
Sublevel caving	16	224	370 876	1 654	3	75	1.01	25.23
Top slicing	20	231	591 952	2 562	2	96	.42	20.27
Open-cut, with power shovel	38	261	1 157 565	4 442	2	154	.21	16.40
Open-cut, hand loading only	5	173	39 352	227	1	7	3.21	22.45
Total	311	251	7 818 227	31 104	77	4 936	1.24	79.38

(a) Underground and open-cut only. No reports used where less than 25 men were employed.

Falls of rock or ore from roof or wall caused nearly 30% of the deaths and 15% of the injuries; far more than any other source. Remedies: barring down loose pieces of ore or hanging wall after blasting, and timbering where necessary before work is resumed. Accidents from falls of ground are largely a question of method of mining. If deposit is extensive laterally, pillars must be properly proportioned to prevent falls of roof. Roofs of untimbered gangways should be arched.

Falls of rock or ore while loading at working face were a prolific source of injuries; evidently due to neglect of timbering or forepoling in weak ground, before mucking or drilling.

Timber or hand tools were an important source of injuries underground. Many such accidents are preventable by selecting careful timbermen. Hand tools on surface and in open-cuts caused over 2% of all injuries.

Explosives accidents, underground and in open-cuts, caused 14.8% of all deaths and 1.1% of all injuries; the proportion of fatal to non-fatal is high. Such accidents are commoner in metal than in coal mining; most of them are readily preventable by: care in storing and handling explosives and detonators on surface and underground; using wooden tamping bars only; use of electric firing in sinking; not returning to face too soon after firing; selection of explosives producing a minimum quantity of poisonous gases (5) (Art 3); proper thawing if necessary.

Haulage accidents, underground and in open-cuts, caused 6.7% of fatalities and 9.2% of injuries. Underground, best safeguards for mechanical and mule haulage, in drifts with small clearance, are: keeping refuge holes well whitewashed; having brakes on cars, and, on heavy grades, blocking wheels of standing cars; good lighting of traveling ways; and strong, not dazzling headlights on locomotives.

Falls down chute, winze, raise or stope are an important cause of both deaths and injuries. Safeguards: grizzlies over chutes, fences and gates around shafts, winzes and chutes, and proper warning boards.

Runs of ore from chute or pocket: accidents are difficult to prevent in steep veins, but well-designed chutes and gates are important.

Electrical accidents. Proportion of fatal to non-fatal is high. Carrying of trolley wires in constricted passages of metal mines is a special danger. Wires should be guarded by side boards, especially opposite chutes, where if possible wires should be placed on far side of drift, as a man in loosening the ore may strike the wire with his bar. Elec lights should be placed near chutes. Power cables should be insulated or thoroughly guarded, and voltage limited to 275 volts. Electrical rules by the American Eng'g Standards Comm and U S Bur of Mines should be followed. Storage-battery locomotives are being extensively substituted in metal mines to prevent accidents and fires caused by trolley wires. Electrical accidents underground, on surface and in open-cuts caused 6.1% of all deaths and 0.5% of all injuries.

Machinery accidents underground, on surface and in open-cuts caused 3.0% of the deaths and 4.2% of the injuries.

Mine fires in metal mines, though costly, rarely cause fatalities.

Suffocation from natural gases is rare, as inflows of noxious gases are infrequent in U S metal mines. Strong mechanical ventilation is the best preventive.

Inrush of water, a minor cause except in occasional disasters, such as flooding of the Milford mine in 1924, which caused 41 deaths.

Falling down shafts is prevented by good fences and automatic gates at every landing.

Objects falling down shafts. Accidents are largely preventable by: smooth-running cages or skips; not overloading cars or skips; dumping clear of shaft mouth; tight chutes, bins and floors in shaft and shaft house; tight or wire-mesh fences around landings; and placing fences and gates far enough back from shaft to prevent men standing dangerously near. Falling of objects on men on cages or buckets can be prevented by good bonnets on cages, and on crossheads over buckets or skips used for hoisting men. In inclined shafts, man cars or cages should have doors

Breakage of hoisting ropes is a minor cause, considering the great number of man hoists in use.

Table 23. Accidents in All Mines Except Coal, 1935 (W. W. Adams, U S Bur of Mines)

		No of accidents		% of total from all causes		Per cent fatalities in total from this cause
		Fatal	Non-fatal	Fatal	Non-fatal	
Underground	Fall of rock or ore from roof or wall	48	1 572	29.3	15.4	2.96
	Falls of rock or ore while loading at working face	2	803	1.2	7.9	0.25
	Hand tools	1	635	0.6	6.2	0.16
	Explosives	23	99	14.0	1.0	18.85
	Haulage	9	897	5.5	8.8	0.99
	Falling down chute, winze, raise or stope	12	535	7.3	5.2	2.19
	Run of ore from chute or pocket	2	323	1.2	3.2	0.62
	Drilling	1	768	0.6	7.5	0.13
	Electricity	7	34	4.3	0.3	17.07
	Machinery (other than locos or drills)	2	181	1.2	1.8	1.09
	Mine fires	...	12	....	0.1	.....
	Suffocation from natural gases	...	19	....	0.2	.....
	Inrush of water	...	4	....	0.0	.....
	Stepping on nail	...	170	....	1.7	.....
	Handling materials (other than rock or ore)	1	870	0.6	8.5	0.11
	Other causes	2	1 027	1.3	10.1	0.19
	Total, underground	110	7 949	67.1	77.9	1.36
In shafts	Falling down shaft	9	21	5.5	0.2	30.00
	Objects falling down shaft	...	29	....	0.2	.....
	Breaking of cables	...	7	....	0.1	.....
	Overwinding	...	1	....	0.0	.....
	Cage, skip or bucket	12	80	7.3	0.8	13.04
	Other causes	1	30	0.6	0.3	3.23
	Total, shafts	22	168	13.4	1.6	11.58
On Surface	Mine cars and locomotives, or aerial trams	3	66	1.8	0.6	4.35
	Railway cars and locomotives	2	22	1.2	0.2	8.33
	Run or fall of ore in or from ore bins	...	17	....	0.2	.....
	Falls of persons	2	247	1.2	2.4	0.80
	Stepping on nail	...	42	....	0.4	.....
	Hand tools	...	153	....	1.5	.....
	Electricity	2	16	1.2	0.2	11.11
	Machinery	2	205	1.2	2.0	0.97
	Handling materials	2	358	1.2	3.5	0.56
	Other causes	9	434	5.6	4.3	2.03
	Total, surface	22	1 560	13.4	15.3	1.39
Open-cuts	Falls or slides of rock or ore	4	51	2.5	0.5	7.27
	Explosives	1	8	0.6	0.1	11.11
	Haulage	2	45	1.2	0.5	4.26
	Power shovels	...	25	....	0.3	.....
	Falls of persons	...	76	....	0.8	.....
	Falls of derricks, booms, etc	...	4	....	0.0	.....
	Run or fall of ore in or from ore bins	...	1	....	0.0	.....
	Machinery (other than locos or power shovels)	1	45	0.6	0.4	2.17
	Electricity	1	3	0.6	0.0	25.00
	Hand tools	...	69	....	0.7	.....
	Handling materials	...	117	....	1.1	.....
	Other causes	1	85	0.6	0.8	1.16
	Total, open-cuts	10	529	6.1	5.2	1.86
	Grand total	164	10 206	100.0	100.0	1.58

Overwinding accidents have been lessened by the growing use of elec hoists with automatic speed regulators (see Sec 12).

Cage and skip accidents, due to faulty construction, are entirely unnecessary. Absence of fences and gates at landings may allow men to be struck by passing cages or skips; or, when on the cage, some part of the body may extend beyond the edge. Excessive crowding on cages when changing shifts should be prohibited. Man cages should have strong bonnets, inclosed sides and proper end-gates and safety catches. It is notable that, though hoisting by bucket without guides is common in the Joplin zinc district, no deaths from this cause occurred in 1935.

Stepping on nail, underground and on surface, caused 2.1% of all injuries.

Handling materials other than rock or ore, underground, on surface and in open-cuts, caused 1.8% of the deaths and 13.1% of the injuries.

Surface accidents, distinct from those in open-cuts caused one fifth as many deaths and one fifth as many injuries as accidents underground. In open-cuts, proper methods of mining, care in examining ground and efficient supervision are of prime importance.

14. COLLIERY EXPLOSIONS

The number of deaths annually from coal mine explosions in the U S rose steadily until 1907, when 956 men were killed, including 361 in the disaster at Monongah, West Va, the worst that has occurred in this country. The most destructive of all explosions was in the Courrières colliery, France, in 1906, resulting in 1 100 deaths.

Classification of explosions into firedamp and coal-dust explosions and windy shots is attempted by different State Inspection departments. But there is no agreed basis in this classification; some inspectors attempt to classify an explosion by its originating cause, others by its means of propagation (usually coal dust). European statistics also do not usually distinguish between explosions of gas and dust, and both are often concerned. Only in the Penna anthracite district are explosions definitely from firedamp, because anthracite dust does not itself propagate an explosion. Though the logical basis of classification appears to be the mode of propagation, rather than the cause, no distinction is made in Tables 23a and 24, but the Bureau of Mines is now separating " cause ignition " from " means of propagation." Colliery explosion disasters from CH_4 and coal dust, acting together or alone, in which more than 100 men were killed in each, are listed in Table 24.

Table 23a. Killed in Gas and Dust Explosions in U S Coal Mines, Contrasted with Deaths from All Causes (W. W. Adams and L. E. Geyer, U S Bur Mines)

Year	Total killed					Rate per million man-hr	
	Explosions			All causes	% of total from explosions	Explosions	All causes
	Major	Minor	Total				
1921	21	105	126	1 995	6.3	0.11	1.74
1922	269	42	311	1 984	15.7	.32	2.02
1923	296	76	372	2 462	15.1	.27	1.82
1924	458	78	536	2 402	22.3	.44	1.99
1925	261	84	345	2 234	15.4	.30	1.93
1926	348	74	422	2 518	16.8	.31	1.86
1927	155	92	247	2 231	11.1	.20	1.83
1928	326	50	376	2 176	17.3	.33	1.92
1929	146	49	195	2 187	8.9	.17	1.87
1930	217	47	264	2 063	12.8	.26	2.06
1931	56	32	88	1 463	6.0	.11	1.82
1932	145	24	169	1 207	14.0	.27	1.90
1933	7	33	40	1 064	3.8	.06	1.48
1934	17	35	52	1 226	4.2	.07	1.59
1935	22	27	49	1 242	3.9	.07	1.70
1936	28	29	57	1 342	4.2	Not available	
1937	95	23	118	1 467	8 0	Not available	

Causes of explosions can not always be determined, because of conflicting evidence of forces, direction of movements and heat effects. It is safe to assume that almost all major explosions were propagated by coal dust. Rock dusting, except at a very few mines, did not begin until 1924. In the years 1925 to 1934 inclusive there were 1 678 killed, or 168 per year. There was a slight decrease from 1925 to 1930, but more improvement after 1930, and in 1935–6–7 the aver was 48, due probably to wider adoption of rock dusting. Blown-out or overcharged blasts of long-flame explosives, or the discharge of these in the open, igniting gas or coal dust or both; electric flashes; open lights, or defective safety lamps; mine fires, which may ignite gas and in turn coal dust; and lastly, but rarely, CH_4 may possibly be ignited by sparks from flinty rocks falling from roof or running down steep breasts or chutes, striking on each other, or on iron. Some explosions have been attributed to electric ignition of coal dust stirred up into a cloud by derailment and wreckage of trains of cars; others to ignition of bodies of gas by mine fires, where ventilation has been cut off by falls of roof or erection of stoppings (Art 15).

Table 24. Explosions in Coal Mining, Where Over 100 Men were Killed

Date	Mine, location	Killed
United States		
1892, Jan 7	No 11, Krebs, Okla	100
1900, May 1	Winter Quarters No 1 & 4, Schofield, Utah	200
1902, May 19	Coal Creek, Fraterville, Tenn	184
1902, July 10	Rolling Mill mine, Johnstown, Pa	112
1903, June 30	Hanna No 1, Hanna, Wyo	169
1904, Jan 25	Harwick mine, Cheswick, Pa	179
1905, Feb 20	Virginia, Virginia City, Ala	108
1907, Dec 6	Monongah No 6 & 8, Monogah, W Va	361
1907, Dec 18	Darr mine, Jacobs Creek, Pa	239
1908, Nov 28	Marianna, Pa	154
1911, Apr 8	Banner, Littleton, Ala	128
1913, Oct 22	Dawson, N M	263
1914, Apr 28	Eccles, W Va	181
1915, Mch 2	Layland, W Va	111
1917, Apr 27	Hastings, Colo	121
1923, Feb 8	Dawson, N M	120
1924, Mch 8	Castle Gate, Utah	172
1924, Apr 28	Benwood, W Va	119
1928, May 19	Mather No 1, Pa	195
Canada		
1891, Feb 21	Springhill, N S	125
1902, May 22	Fernie, B C	125
1914, June 19	Hillcrest, Alberta	189
Great Britain		
1835, June 18	Wallsend	102
1856, July 15	Cymmer, Wales	114
1857, Feb 19	Lund Hill	189
1860, Dec 1	Resea, Wales	142
1866, Dec 12	Oaks Colliery	361
1867, Nov 6	Ferndale, Wales	178
1875, Dec 6	Swaithe Main	143
1877, Oct 22	Blantyre	207
1878, June 7	Haywood Wood	189
1878, Sept 11	Aberdarne, Monmouth	268
1880, July 15	Resea, Wales	120
1880, Sept 8	Seaham	164
1880, Dec 10	Naval, Wales	101
1885, June 18	Clifton Hall	178
1890, Feb 6	Llanerc, Wales	176
1892, Aug 26	Park Slip, Wales	112
1893, July 4	Combo, Thornhill	139
1894, June 23	Albion, Wales	290
1905, July 11	National Colliery, Wales	119
1909, Feb 17	West Stanley	168
1910, May 11	Wellington, Whitehaven	136
1910, Dec 21	Pretoria, Boeton, Hulton	344
1913, Oct 14	Senghenydd (Universal), Wales	439
1918, Jan 12	Podmore Hall (Minnie Pit), Staffordshire	155
1934, Sept 22	Gresford, Denbighshire	265
New South Wales		
	Killingworth	130
France		
1876, Jan 4	Jabin	186
1889, July 3	Verpillieux	207
1890, July 29	Pélissier	113
1906, Mch 10	Courrières, Pas de Calais	1110
Belgium		
1887, Mch 4	La Boule, Quaregon	113
1892, Mch 11	No 3, Bois de la Haye	160
Germany		
1867	Fundgrube, Saxony	101
1869	Iserlohn, Ruhr	101
1869	Burgher-Schaechte, Saxony	268
1876	Karlingen, Lothringen	147
1894	Camphausen, Saar	181
1898	Karolinenglück, Ruhr	119
1907	Reden, Saar	148
1908	Radbold, Ruhr	360
1912	Lothringen, Ruhr	114
1923, Jan 31	Heinitzgrube, Upper Silesia	145
1925, Feb 11	Minister Stein Ruhr	136
1930, Oct 21	Anna II Alsdorf, Aachen	271

Propagation of explosions. Firedamp was formerly believed to be the only cause of explosions, but bituminous and lignitic dust are now known to be more important means of propagation. CH_4 rarely exists in explosive proportions throughout large areas in mines, even in gaseous districts, because of the generally efficient ventilation. Therefore, though some Penna anthracite mines are among the most gaseous in the world, explosions are localized; while in some nearly non-gaseous bituminous mines explosions have swept through miles of entries.

Firedamp and coal dust make a dangerous combination, each aiding the other in starting and propagating explosions; but, BITUMINOUS AND LIGNITE DUSTS are highly dangerous in themselves, and if ignited when in suspension and in certain densities of dust cloud, will propagate an explosion as violent as a gas explosion; in fact, finely-divided coal dust (and other dry carbonaceous dusts, of sugar, soap, paper and cereal) when mixed with air, behave essentially like explosive gases (61,63,64). Pure coal dust has no definite upper limit of explosibility in air, like CH_4 (14%). Hence, it is more certain to propagate an explosion as far as it extends in sufficient density in air, where not treated with rock dust, or the passages are not well sprayed.

Ignition of firedamp. In still air, by side ignition, CH_4 is just explosive at $5^1/_2$%. In rapidly-moving currents the ignition limit is lowered, and at 1 000 lineal ft per min it is 0.5%. But, such high velocities with explosive proportions of CH_4 in strong ventilating currents are found only in great outbursts. If firedamp is detected in a ventilating current, men are usually withdrawn from that part of the mine. This should always be done (except in emergency work), when there is enough CH_4 to show a cap in a safety lamp in a moving current, or when by analysis there is over 2% CH_4, or, if open lights are used, when the current contains 0.5% CH_4 (Art 6).

CH_4 comes from blowers in the face, floor or roof, especially where there have been falls. In still air it is usually given off faster than it diffuses. Hence, it stratifies at roof level, or in a roof cavity. Then, if ignited, it burns rather slowly unless the body of gas has considerable lateral extension, when it flashes with increasing violence. Pressure theoretically generated by the max explosive percentage of pure CH_4 (9.4%) is about 125 lb per sq in. Through loss of heat, not over 70 or 80 lb press is actually obtained without pre-compression.

Prevention of firedamp explosions: (*a*) Strong air current carried to face by stoppings and line brattices; (*b*) prohibition of open lights and smoking of tobacco; (*c*) obligatory use of safety lamps or permissible electric lights; (*d*) use of permissible explosives only; (*e*) tests for CH_4 before and after blasting; (*f*) if undercutting is done by machine, the motor should be explosion-proof; (*g*) prohibition of trolley locomotives and bare power wires in gaseous parts of a mine; (*h*) daily analysis of return air, for control of ventilation; (*i*) rigid inspection of all parts of mine, by fire boss, inspector or foremen; (*j*) sealing off abandoned parts of mine, difficult to ventilate or inspect, and if these areas produce much CH_4, issuing under high press, a vent hole should be drilled from surface. If too deep for drilling a large enough hole, the gas may be piped into a return airway.

Auxiliary fans (Sec 14) and piping (metal or cloth) are used in both coal and metal mines for headings and rooms. Auxiliary fans are dangerous in gaseous mines. They tend to recirculate the air, so that a gaseous place accumulates CH_4. As they usually run only during working shifts, CH_4 collects between shifts. If driven by non-explosion-proof motors, they may spark and ignite gas. They should be used only for emergencies; and, in gaseous mines, should be driven by compressed-air or explosion-proof motors, for 24 hr per day, as in European coal mines.

Coal dust explosibility investigations, begun by Faraday in 1844, have been carried on in recent years at Liévin Gallery and the Pas de Calais coal field in France; at a testing gallery first at Altofts, later at Eskmeal and Buxton, Great Britain (12, 75); and by U S Bur of Mines at Pittsburgh and the Bruceton Experimental Mine (61, 62, 63). INTERNATIONAL COOPERATION ON MINE SAFETY RESEARCH, begun by an agreement with Great Britain (88, 75) referring especially to mine explosions, standardisation of samples and instruments, and interchange of information, was extended in 1931 to other European nations (88).

Definition of Terms Used in Explosion Experiments

Coal dust means particles which will pass a 20-mesh sieve (Bur of Mines empiric standard), produced in mining and handling coal, or artificially by grinding. To obtain dust of uniform composition and size at different testing stations, coarse coal is ground in ball or roller mills, or other pulverisers. As the inflammability of coal dust increases with fineness, samples are rated by the percentage passing a 200-mesh sieve, all having passed through 20-mesh.

Rock dust includes all inert dusts, from clay, shale or sand, found in mines. For electric haulage quartz sand is used to prevent slippage. While this dilutes the coal dust and gives some protection, since siliceous dust is injurious to breathe, it should not be used as a substitute for shale or limestone dust for general rock-dusting.

Ignition of gas or coal dust (as the term is employed by Bur of Mines) is the starting of inflammation by some initial source of heat. In case of CH_4 the inflammation will pass rapidly into an explosion if there is enough CH_4 in proportion to the adjacent mine spaces. In case of coal dust, premixed with air which has been ignited, combustion may not produce sufficient pressure or concussion to raise more dust, beyond the influence of the agency that brought the ignited dust into suspension; hence the flame will die away and it is not considered a propagation or explosion.

Propagation is the continued inflammation of gas or dust through the mine passages, as far as the gas or dust zone extends.

Explosion, while practically synonymous with propagation, usually connotes considerable dynamic force, advancing as a moving zone of combustion.

Inflammation in gas mixtures is transmitted from point to point by transference of heat by radiation and convection, and by such rapid increase in temp and press of expanding gases that each layer in succession of unburned gas ahead is compressed adiabatically and raised to ignition temp. When this stage is reached, the propagation attains high velocity. This phenomenon was discovered by Berthelot (1881), who termed it "l'onde explosive" (explosion-wave).

Detonation-wave is the term given by Dixon to the explosion-wave of Bertholet. He states "the rate of the explosion-wave is a definite physical constant for each gaseous mixture; the wave travels with the velocity of sound in the burning gas, which itself is moving rapidly *en masse* in the same direction."

Retonation-wave (Dixon), "l'onde retrograde" (Le Chatelier), is a wave of compression traveling backward through the spent gases.

Reflection-wave (Dixon), "l'onde reflechie" (Le Chatelier), is a compression-wave reflected from a closed end or other contraction or reflecting surface of a tube. In irregular galleries these waves are numerous, and travel forward or backward through the spent gases.

Collision-waves (Dixon), "l'onde prolongie" (Le Chatelier), are compression-waves produced when 2 explosion-waves, generated simultaneously in different parts of a gaseous mixture, meet and extinguish each other, each compression-wave continuing in the direction of its originating explosion-wave.

Secondary waves, like the last three named, do not depend on chemical action to sustain them, and therefore gradually die away; they pass through one another, or are so merged as to be unrecognizable. Their velocities approximate those of sound waves, varying with densities of the gases traversed and the movement *en masse* of these gases relative to the walls of the passageways.

Dixon states, if CH_4 and air are ignited at the closed end of an open tube, the speed of flame increases rapidly to 1 000 meters per sec, without great fluctuations, but does not approach the rapidity of a detonation-wave.

Coal-dust explosion-waves resemble those from gas inflammations, and retonation and reflection-waves are probably identical with those in gas mixtures; but, since uniform dust clouds can not be obtained, laboratory experiments, like those for determining gas explosion-wave velocities, are not possible. In the large testing galleries and Bruceton experimental mine, dust clouds have not yet been made sufficiently uniform to obtain constant acceleration of an explosion flame; while velocities of 2 000 to 4 000 ft per sec have been recorded, it may be as low as 50 ft per sec.

Table 25. Velocities of Explosion-Waves (Le Chatelier)

Kind of wave	Velocity, meters per sec		
	$C_2H_2 + O_2$	$2\,CO + O_2$	$C_2H_2 + 2\,NO$
Detonation-wave...	2 990	1 900	2 850
Retonation-wave...	2 300	....	1 140
Reflection-wave....	2 250	1 000	1 350
Collision-wave.....	2 050	....	....

Shock-wave is a wave of compression which travels in gas as fast as sound, the velocity varying with density of the gas and character of the passage traversed. It may be produced by any violent concussion, as from a blast, and hence is important in raising ignitible dust. Successive shock-waves may advance ahead of the slower-moving explosion. At the Bruceton mine a shock-wave set up in still air by a blown-out shot inside the mine, started at about 1 200 ft per sec, its velocity gradually decreasing to less than 1 100 ft per sec in a distance of 1 300 ft.

Advance air-wave (British, "pioneering-wave") is a compression-wave or a series of them, pushed ahead of the flaming zone of an explosion. It makes possible the propagation of a coal-dust explosion by bringing the dust into suspension.

Return-wave, or wave of depression, is the flow of air at atmos press to fill the partial vacuum produced when the expanded after-gases of an explosion cool. Depressions of 6 to 7 lb per sq in have been recorded in the Bruceton tests (63), and in a larger mine greater depressions might occur. Return-waves, being thus limited in press, are much less violent than primary or secondary waves, but are prolonged. It is possible that in an explosion in a large mine an inward rush may begin in one entry before the heated gases cease discharging from another, provided the two entries are not connected, except in the interior of the mine.

Present (1938) knowledge of coal-dust explosions, based on many investigations of disasters and at testing stations, is summarized as follows:

(*a*) **Coal dust** with a ratio of volatile to total combustible of more than 10 is EXPLOSIVE in air if the percentage of ash in the coal is not too high.

(*b*) **Character and fineness of dust.** Other things being equal: the higher the percentage of volatile-combustible the more explosive is the dust; the finer the dust the more explosive; but, in presence of much fine dust, say less than 200-mesh size, dust coarser even than 20-mesh may enter into the explosion; the higher the ash and moisture the less explosive the dust. Pittsburgh coal dust, 70% passing through a 200-mesh sieve, if mixed with 60% of inert or non-combustible dust, will not ignite from a blast of 4 lb black powder; the stronger the source of ignition the more readily will the dust ignite; thus, if a 50-ft zone of pure coal dust is ignited by a blast, it will in turn ignite an adjacent dust zone mixture of 40% Pittsburgh coal and 60% inert dust, which will then propagate the explosion. Under the above conditions, it requires 75% inert dust to prevent propagation. With low-volatile coal dusts less of the inert mixture will prevent ignition and propagation.

(*c*) **Low-volatile Penn anthracite dust** containing less than 6% volatile combustible is not explosive, either alone or in presence of CH_4 below the explosive limit of the latter. This conclusion is supported by immunity of the Penn anthracite district to widespread explosions. A certain variety of anthracite, of softer structure and volatile-combustible ratio of about 10%, would not alone propagate an explosion, but would do so in presence of 1% or more CH_4.

(*d*) **Presence of CH_4** in the air, in percentage below its own explosive limit, INCREASES EXPLOSIBILITY OF COAL DUST (except low-volatile anthracites). An increase of each 1% CH_4 seems to offset 3 to 5% or more increase in inert matter; hence the increased danger of dust explosions in gaseous workings.

(*e*) **High humidity of the atmosphere**, even to saturation, or wet walls and floor DO NOT PREVENT IGNITION OR PROPAGATION of a dry dust explosion, for, while presence of moisture is retardant, the quantity present even at saturation is insignificant as compared with the heat energy of sufficient coal dust to propagate an explosion. Moreover, the specific heat of water vapor is little more than that of the inert nitrogen of the air, which must also be raised to ignition temp.

(*f*) **High moisture content** of the dust IS FAR MORE IMPORTANT than HIGH HUMIDITY OF THE AIR. Moisture must be converted into vapor or steam, and then raised to ignition temp, replacing some of the air and O present. Moisture content of dust is thus more influential than ash content. But, pure inflammable dust is ignitible by a violent source of ignition, unless mixed with enough moisture to prevent the mechanical raising of a dust cloud. In tests, pure fine Pittsburgh dust, with about 20% moisture, propagated an explosion; to be safe required 30% moisture. Normal moisture of Pittsburgh dust is about 2%. Roughly it requires an amount of water equal to about 1/3 of the weight of dust, to prevent mechanical raising of dust into a cloud (1).

(*g*) **The quantity of Pittsburgh dust that will consume all the O present if completely burned,** is 0.123 oz per cu ft of air at atmos press; but, in a violent explosion, when the press reaches say 120 lb, if the additional dust is present it would require 8 × 0.123 = 0.984 oz, to consume all the O, with corresponding increase in energy. But, since coarse unburned particles are always present, more than the theoretical weight is necessary to obtain max explosive violence.

(*h*) **Excess dust in the air retards explosion,** but this is not of practical importance, for, even if the explosive wave is retarded, the excess dust drops to the floor and the explosion flame, though momentarily continuing at a lower velocity, is equally destructive of life.

(*i*) **Minimum quantity of pure fine (200-mesh) dust that will propagate an explosion.** A test reported by Taffanel, on the most inflammable dust from Courrières, showed 0.023 oz per cu ft of air, and in tests by Rice, on Pittsburgh dust, 0.032 oz per cu ft. These are merely indications for particular cases; 0.12 oz of Pittsburgh dust will theoretically consume all the O in 1 cu ft of air at atmos press. It will make a strong explosion, although not all the dust is actually burned. In a heading 6 by 9 ft (54 sq ft), containing 0.12 oz dust per cu ft, there would be 54 × 0.12 = 6.5 oz of dust per linear ft. If spread evenly on ribs, roof and floor, the thickness of the dust film would be only 0.003 in. There is usually much more than this in the cleanest colliery.

(*j*) **Velocity of a coal-dust explosion.** It starts with comparative slowness, taking 0.5 to 1 sec to traverse the first 100 ft, or possibly several sec with impure or high-ash dusts. With pure dusts the second 100 ft may be traversed at 500 ft per sec, and the third 100 ft at over 1 000 ft per sec. Beyond this the veloc may exceed 3 000 ft per sec. Velocities are usually higher when there is CH_4 in the dust-air mixture. There is not a max flame veloc. Flame passes through the moving air, and coal dust is forced ahead of combustion zone.

(*k*) **Pressures of explosions** are to some extent functions of the velocities. If the origin is a blown-out shot (of say 4 lb black powder), the shock-wave may give a momentary local press of 10 to 15 lb per sq in, falling to less than 5 lb at 150 ft from origin; then succeeds a slight depression, followed by slowly increasing press, which may be only 5 lb at end of first sec. In a pure-coal dust explosion, there is then a rapid rise in press. At the Bruceton mine, 350 ft from the origin, 63 lb per sq in has been registered, at 550 ft, 73 lb and at 750 ft, 119 lb. Higher pressures have been reported in the Altofts and Liévin gallery tests. Press does not rise steadily, but by violent pulsations. When it rises, retonation-waves move backward through the spent gases toward the origin, and beyond it if the passageway provides a continuous circuit into another part of the mine. Such waves may be very violent, throwing loaded cars and other objects toward the origin. Then follows a period when the gases cool and produce a depression, causing inflow of air from distant points. This is a slow wave of low press, probably rarely over 8 lb per sq in, which may also move light objects toward the origin or beyond it, following the main explosion (61).

(*l*) **Coked coal dust.** An explosion of bituminous dust of a coking coal usually leaves much coked material in its path. This is loose near the origin, and is thrown against the faces of timbers or projecting surfaces of walls. As the explosion gains headway coke may be found both facing and on opposite side of salients; at very high veloc but little is deposited, and then only on surfaces facing outward, or in recesses. Variations occur, due to retonation-waves and subsequent scouring action of the inrush of dust-laden air. Globular coke is produced by the coking of particles in transit; usually found only in a *cul de sac*, or stub heading. Coke *in situ* is produced when the flame has lingered in gassy workings. Thin layers of coal on roof or ribs are also coked.

(*m*) **Tracing origin and course of an explosion.** No one feature or piece of evidence is infallible. In extensive explosions, all data must be plotted. When gas ignition is the prime cause it is difficult to determine the precise origin; for if the gas is stratified along the roof, as in still air or a very slow current, it may burn several hundred feet along the roof before mixing with the air sufficiently to come within the explosive limits of CH_4 in air.

(*n*) **Afterdamp from an explosion.** Explosion of gas or dust is the rapid combustion of complex hydrocarbons, under complicated and varying conditions of press, with deficiency or excess of carbonaceous matter. In violent explosions there is some pre-distillation of gas by radiant heat or adiabatic compression. The former is more likely to occur in slow-moving explosions; in rapid ones the fine dust probably burns as a whole, though coarser particles continue to burn until the O is exhausted, after the fore-front of the explosion has passed.

Table 26. Typical Analyses of Gases Taken by Automatic Samplers at the Bruceton Mine, 350 Ft from Origin of Explosion (63)

Test No	No of bottle	Time interval, sec	CO_2	CO	O_2	CH_4	C_2H_4	H_2	N
89	1	0.00*	8.15	5.30	6.51	1.10	0.60	3.15	75.19
	2	0.31	4.44	3.23	12.43	1.01	0.10	1.62	77.17
	3	0.83	7.71	7.11	5.55	1.51	0.25	3.16	74.71
92	1	0.00*	13.56	5.36	1.03	1.15	1.31	2.38	75.21
	2	0.55	7.93	5.14	7.05	0.97	0.12	2.45	76.34
	3	3.00	9.98	7.38	2.60	1.51	0.25	3.52	74.76
101	1	0.00*	9.02	7.34	2.84	2.05	0.55	3.51	74.69
	2	0.37	9.61	9.59	1.03	2.35	0.55	5.45	71.42
	3	1.57	8.64	6.53	3.52	2.01	0.20	3.42	75.86

* With operating foil opposite sampler, the first sampler took 0.03 to 0.04 sec to close from burning of foil. The second and third samplers operated by clockwork started by the foil burning and thus breaking an electric circuit.

Prevention of coal-dust explosions. All the rules for prevention of firedamp explosions (Art 14) are equally applicable to coal-dust explosions, because many of them are caused by ignition of a pocket of CH_4, or more generally by presence of a small percentage of CH_4 in the air. The bad practice of stopping the fan at shot-firing time is no longer employed, as CH_4 may accumulate in the interval before firing. If a mine makes any CH_4, even less than can be detected by casual safety-lamp inspection, the danger of igniting dust is increased; tests at the Bruceton mine show no difference in originating an explosion in still air or in a current. Most experiments have been in practically still air.

In absence of other treatment, very little dry pure dust is required to propagate an explosion, and gangways can not be sufficiently cleaned of dust to insure safety. It is advisable to remove frequently from haulageways both coarse and fine coal, to prevent the traffic making more dust. There are two methods of treatment: wetting and rock-dusting.

Wetting method consists in laying dust by watering, so that it will not be raised by a concussion (*Trans* A I M E, Vol 71, p 1185).

Generalised watering has proved to be a failure. Many explosions have occurred in mines considered to be well-watered; similar disasters took place in Great Britain prior to 1924, and in Germany until 1926. Water dries too quickly in a well-ventilated mine, and, unless coal dust is wet enough to be muddy, watering is of little use. But rock-dusting, now approved in all countries, where efficiently done, has not yet failed.

Rock-dusting method (66, 67) is to spread 3 parts shale or preferably limestone dust for each 1 part of coal dust, as checked by analysis. Haulageways are cleaned thoroughly and the rock dust thrown on roof, ribs, and floor, about 5 lb per lineal ft being required for first treatment in entries about 9 ft wide by 6.5 ft high. Advantages: rock dust is visible, does not change from day to day, and degree of protection afforded is readily checked by rough analysis. It is best applied by dusting machines, which blow off any coal dust remaining in high places where most dangerous, as on roof, timbers and ribs, and replace it with coating of rock dust.

In recent years rock-dusting has been widely adopted. Sir William Garforth in 1907 conducted tests with rock dust at Altofts. Following the Courrières disaster, France established a coal-dust gallery at Liévin, in 1907. The U S Gov't began coal-dust gallery testing (conducted by G. S. Rice in 1908) at Pittsburgh and the Experimental Mine (Bruceton) in 1910. Results of all tests were favorable. In 1913, rock-dusting was recommended in preference to watering by U S Bur of Mines. In 1920, the British Gov't made it mandatory in all dry bituminous mines, and in 1924 in all coal mines (except anthracite) not having 30% water in the coal dust; the rock dust to be so spread that the mixed dust on floor, roof, and sides of workings shall nowhere contain over 50% combustible (66). In 1919, at a mine in Illinois, rock-dust barriers were adopted to limit explosions. In 1924 many U S mines began generalized rock-dusting, the U S Bur of Mines issued specifications therefor, and by 1925, it became widely adopted. Many explosions starting in gas at mine faces are said to have been prevented from propagation by the dust. France, after the World War, officially approved the method. In Germany, Apl 1, 1926, rock-dusting in the Ruhr mines was made obligatory (66, 67).

Specifications for rock-dusting, sponsored by A I M E, and concurred in by U S Bur of Mines, were adopted in 1925 by the Amer Eng'g Standards Comm (66). Essential points: all coal mines except anthracite shall be dusted throughout active entries and workings; entries without tracks and old workings shall have at their entrance ROCK-DUST BARRIERS (see below), of types approved by U S Bur Mines; road, rib, and roof dusts systematically sampled; and, if the incombustible anywhere falls below 55%, that part shall be re-dusted. It was expected that the aver dust mixture will then have over 65% incombustible (68).

Character of rock dust. It should be light colored, like limestone or gypsum, should all pass through a 20-mesh sieve and at least 50% through 200-mesh, and must contain less than 5% free SiO_2, so as not to affect health of miners. To prevent accumulation of coal dust, water sprays should be used on the cutter-bars of machines, and loaded cars sprayed in transit.

Distributing rock dust. In European mines the dust is generally applied by hand, though compressed-air funnels or injectors are used in France, and sometimes in England, where, also, mechanical dust-distributing cars operated by hand or from an axle drive are employed in some mines. In the U S, mechanical distributers are usual. Distributers use permissible elec motors for air-blowers and stirrers or mixers, for obtaining large capacity. One common type with a movable pipe for discharging in any direction has advantages for dusting roof and sides of passages; another has a discharge tilted from side to side of an entry or room; a third has a fixed fan-shaped discharge. Certain distributers have a large-diam air hose on the discharge, to extend through holes in stoppings to the parallel entry, for dusting the latter when it has no track or is the return airway.

Road, roof, and rib-dust sampling should be systematic, a certain number of samples being taken weekly, to determine condition of the dust present as to its explosibility in air.

Samples should be taken separately of: (*a*) road dust; (*b*) rib dust; (*c*) roof dust (including that on timbers). In absence of overhead timber, (*b*) and (*c*) may be combined. In sampling, weigh and sise the dust and analyse separately for (*a*), (*b*) and (*c*). Bur of Mines tests demonstrate that (*a*) is the most dangerous, because of position and fineness, (*b*) next in danger, and (*c*) least of the three, as to ignition and propagation in initial stages of an explosion. Samples may be gathered by a Bur of Mines dust-sampling scoop, with lower receptacle covered by a 10-mesh screen. Oversize is rejected; that through 10-mesh placed in sampling cans, and on reaching the laboratory it is screened on 20-mesh, that passing 20-mesh constituting the sample. Sampling strips should not be over 50 ft apart, if the dust along entry or room appears to be approaching the danger limit (35% combustible). If light-colored rock dust is used, the color as darkened by coal dust is an important indication. ANALYSIS OF SAMPLES need be made only for moisture and ash, except in case of limestone or other dust giving off CO_2. When a mine has no chemist, and always for speed in determining noncombustible material, the Taffanel volumeter should be used; it can be handled by a non-technical man. For a modified form, by Bur of Mines, see (66). Results are entered in a record book, and on a skeleton map of the mine, showing dusted and redusted zones duly dated.

Quantity of rock dust required. For first dusting after cleaning an aver entry, use 2–4 lb of dust; for a room, 4 lb or over. Subsequent amounts depend upon rapidity of deposition of coal dust. In some mines 0.5 ton rock dust to 1 000 ton coal produced may suffice; in old, extensive mines, twice as much may be needed for safety. COST of dusting varies, if done thoroughly, from 0.5¢ to 2¢ per ton of coal produced. It is cheaper than systematic watering, and for rock-dusted mines there is now a deduction in the liability insurance premium. Limestone or gypsum dusting improves illumination and thus tends to lessen accidents along roadways. As gypsum cakes rapidly in moist air, it will not be dispersed in the air by the advance waves of an explosion.

Rock-dust barriers are used to extinguish a coal-dust explosion started in unprotected workings. They consist of pockets of rock dust, arranged to be dispersed by an explosion wave directly in the path of the wave (61, 63, 64, 68). The original TAFFANEL BARRIER, consisting of 10 to 15 overhead shelves, placed across the gangway and piled with rock dust, has proved effective in almost all tests. The RICE BARRIER may be inclosed to prevent contamination by drifting coal dust and to keep rock dust dry; it is operated by wind vanes set for certain air velocities (too low to blow dust from open shelves) and discharges 2 to 3 tons of rock dust in a dense cloud. MODIFIED FORMS OF BARRIER have been developed at various mines, but most of them failed in tests, either because they were too sluggish for fast explosions, or dump the dust in masses instead of dispersing it, or hold too little rock dust, or have no covering to prevent contamination by coal dust or to protect rock dust from humidity, or no guard to prevent personal injury by accidental tripping.

Bur of Mines does not advocate barriers as a substitute for generalised rock-dusting, but recommends that mines be sectionalised and barriers placed at entrances to panels, or inaccessible workings, and at other strategic points. When not supplemented by general rock-dusting, a barrier should contain about 100 lb of dust per sq ft of cross-sec of the passageway; less in large passages, more in small (Rice). Taffanel barriers are used in some French mines as supplemental devices at critical points, and at entrances to separate ventilating splits.

Water barriers, consisting of easily overturned water troughs and tanks, have been tried; but, if tripped by an advance air wave, may discharge the water uselessly before the flame reaches them.

Salts for explosion prevention. $CaCl_2$, a deliquescent salt, when strewn on roadways in granulated form assists in packing the dust. It helps in a naturally dry mine, but is not sufficient in itself to allay the dust. Common salt is not effective, as pure NaCl is not deliquescent. Neither salt prevents ignition or propagation of explosions. Fresh coal dust collects on the crusted salt surfaces faster than it becomes wetted.

15. MINE FIRES

Fires in mines containing much timbering are, as disasters, next in importance to mine explosions. Though few mines are entirely free from danger of fire, risk is minimised by care, good design of plant, and fireproof construction at exposed points (70, 71). In coal and metal mines, where only a few or no lives were lost, the aggregate damages have reached millions of dollars. In the Lake Superior iron district, from 1890 to 1913, 31 fires caused 22 deaths.

Causes of fires: OPEN LIGHTS, igniting timbers, cloth brattices, wood stoppings, hay in stables or en route, lubricating or other oils and saturated cotton waste. Ignition of oil refuse, sawdust or chips, or oil-soaked boards, by MATCHES, CIGARETTES, AND BURNING TOBACCO. Ignition of timbers by DEFECTIVE ELECTRIC WIRING; incandescent lamps in contact with timber. BUILDING SMALL FIRES UNDERGROUND for heating lubricating oil or dinner buckets. Explosion or flame of KEROSENE OR

GASOLENE TORCH igniting timbers, floor, or débris. DUMPING HOT ASHES in open pits connected with underground workings, which may contain carbonaceous material or timber. Fires from this cause in the Penn anthracite region have cost vast sums. It is a source of danger in outcrops of coal beds, and where the caving system is used in metal mines. LIVE-STEAM PIPES in contact with dry timber or carbonaceous material. EXPLOSIVES ignited in handling, or by weak detonators. FRICTION OF WIRE ROPES on timbers, or on wood rollers that have stuck. OVERHEATED BEARINGS of continuously-running machinery like underground fans. Fans should have fireproof settings. LIGHTNING causing fire in wood structures at top of shaft, in timber lining, or at bottom of shaft. SPARKS FROM LOCOMOTIVES passing near shaft mouth. INCENDIARISM. COAL-DUST EXPLOSION throwing hot coked dust on timber or loose coal so that, on reëstablishing air current, fires start up. SPONTANEOUS FIRES are likely to occur in underground rooms containing oily waste. In coal mines they may be caused by slow oxidation in gobs and working places; in metal mines, where there are rich sulphide ores, fires may originate by the sliding of ore in steep stopes; friction on footwall, grinding together of the ore aided by slow oxidation, may heat the sulphides to temp of rapid oxidation. A prolific cause of fires in collieries is ignition of CH_4 by longflame explosives; or coal may take fire from flame of overcharged shots. In metal mines, fires were formerly caused by lighted CANDLE ENDS dropping onto timbers or inflammable material; particularly dangerous at shaft landings or chutes. DROPPING LIGHTED PAPER into ore chutes, to ascertain amount of ore, sometimes ignites the wood lining.

Fire prevention. Most fires are preventable. Irresponsible persons are a constant menace. Table 27 shows the commonest causes are CARELESSNESS IN HANDLING LIGHTS, or in throwing away lighted candle ends or smoldering lamp wicks; preventable by using electric lights, or inclosed lamps or lanterns, at landings, stables, and sidings. At the working face, portable electric miners' lamps in coal mines and acetylene lamps in metal

Table 27. Colliery Fires Causing Loss of 10 or More Lives, and Some Metal Mine Fires

	Date		Name of mine	Place	Lives lost	Origin of fire
Colliery fires in U S	1869	Sept 8	Avondale	Plymouth, Pa	179	Not reported
	1890	June 16	Hill Farm	Dunbar, Pa	31	"
	1901	Feb 25	Diamondville	Diamondville, Wyo	28	"
	1908	Aug 26	Hailey-Ola, No 1	Haileyville, Okla	29	Barrel black oil set on fire in shaft bottom
	1909	Nov 13	St Paul, No 2	Cherry, Ill	256	Hay set on fire at foot of intake shaft
	1910	Dec 14	Leyden	Leyden, Colo	10	Underground elec hoist room
	1911	Apr 7	Price-Pancoast	Throop, Pa	73	Floor of underground engine room
	1918	May 20	Villa	Charleston, W Va	13	Fire in surface fan house
	1919	Oct 29	Amsterdam No 2	Amsterdam, Ohio	20	Probably overheating of booster-fan elec motor underground
Colliery fires abroad	1896	Mch 3	Kleophas	Upper Silesia	104	Wooden material in brick-lined shaft
	1898	Apr 19	Whitwich	Leicestershire, Eng	35	Goaf fire breaking thro' stopping
	1908	Mch 4	Hanastead	Staffordshire "	25	Box of candles in shaft bottom
	1913	Aug 3	Cadder	Scotland	22	Near downcast shaft: cause unknown
Metal mine fires	1869	Apr 7	Kentuck—Yellow Jacket—Crown Point	Gold Hill, Nev	41	Fire in timbers, probably from candle
	1895	Sept 7	Osceola	Calumet, Mich	30	Cause unknown
	1901	Nov 20	Smuggler Union	Pandora, Colo	31	Fire in bunkhouse at mine entrance
	1907	Nov 30	Fremont	Drytown, Calif	11	Fire in foot of shaft: cause unknown
	1911	Feb 23	Belmont	Tonopah, Nev	17	Probably from candle end in pile of timber at bottom of winze
	1916	Feb 14	Pennsylvania	Butte, Mont	21	In fan station in air shaft, cause unknown
	1917	June 8	Granite Mountain	Butte, Mont	163	Carbide light ignited insulation of cable in shaft
	1922	Aug 27	Argonaut	Jackson, Calif	47	Probably elec short circuit igniting shaft timber

mines are safer than oil lamps or candles. Permissible elec miners' cap-lamps are safest and give best illumination.

Lubricate cars on the surface, where feasible; if underground, keep oil in a fireproof room, with cement floor and steel fire door. To heat oil, use hot-water coils. Keep a supply of dry sand in buckets. In metal mines, candles or other combustibles should not be stored on shaft landings, but in metal containers at a distance.

Underground pump, engine, and transformer rooms, in all mines, should be arched, and floored with non-combustible material. There should be fire doors opening outward, for quick closing, and sand or rock dust kept in the room and close outside, for extinguishing electrical fires.

Underground stables (Sec 11, Art 11), should be built of non-combustible material. Straw and hay should be dampened or covered with wet tarpaulin, if handled by men using open lights.

Explosives. Only a day's supply should be STORED UNDERGROUND; and while such explosives should be kept in wood-lined magazines, the exterior walls and doors should be of non-combustible material. No OPEN LIGHTS permitted IN MAGAZINES. Thawing of explosives should be done at a distance from shaft, protected by an offset.

Incandescent electric lighting, properly installed and insulated, at shaft landings or bottoms and in stables, is a great measure of protection. Lamp sockets should be fixed, not hung loose from cords, and the bulbs have wire-guards. Elec lights (except permissible portable lamps) and wiring should not be used in coal mines where the air current may contain as much as 0.25% CH_4.

Fireproof construction. Fires often start in colliery tipples, and in the headframes or shaft houses of metal mines, the burning embers dropping down the shaft. Such structures should be of all-steel or concrete. Where wood is used the members should be massive and the space between shaft collar and landing floor not inclosed. Mine entrances should never be covered with inflammable structures. Shaft collars should be concreted where possible. Permanent shafts less than 1 000 ft deep can at reasonable cost be lined throughout with gunite or concrete; it is even more important to fireproof shaft bottoms, and landings. When the escape shaft is near the hoisting, or main shaft, there should be triplicate ventilating doors in connecting passageways, one of which opens in a direction opposed to the other two, and all close automatically by counterweights. This prevents a fire from short-circuiting, and burning out both shafts.

Power lines. Steam pipes in shafts are now obsolete. ELECTRIC LINES must be insulated or armored, and not in contact with coal or timber; bare lines in a passageway should be strung on side opposite to that along which men travel; in levels of pitching veins, on side opposite the chutes. Guard boards are necessary at all crossings where lines are less than 7 ft above track or floor. Lines passing through wooden door frames or brattices must be insulated. Fixed power lines are best put in buried conduits; when in gaseous collieries, should always be installed in intake airways. In collieries, stationary electric machinery should be only in intake airways; portable electric machinery, of permissible type, with approved trailing cable and junction boxes when at working faces, in return airways, or where there may be inflammable gas. See "Safety Rules for Elec Equipment in Coal Mines," adopted, 1926, by Amer Eng'g Standards Comm.

Fires in caved workings of metal mines are sometimes caused by blasting adjacent to broken timbers with dynamite, which has a flame of sufficient length and duration to ignite wood, especially if imperfectly detonated by weak caps. For such work permissible explosives, as for coal mining, lessen danger of ignition.

Spontaneous fires in goaves, rooms, or chambers of COLLIERIES are preventable by hydraulic filling with sand or culm. In METAL MINES having large bodies of heavy sulphide ores, spontaneous fires may occur and are difficult to cope with; where possible, apply hydraulic filling. Means of dealing with such fires is by use of fire stoppings, and pressure ventilation in adjacent live workings.

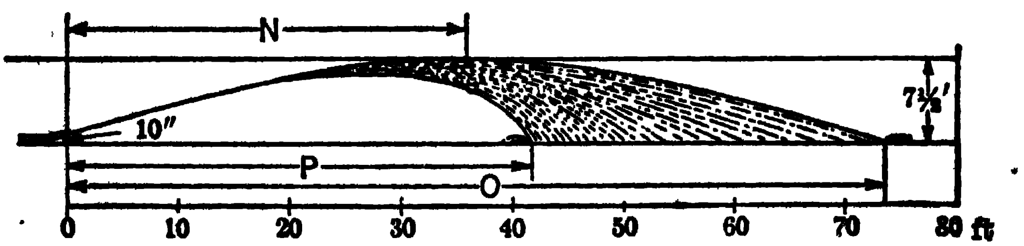


Fig 7. Max Trajectory of Water Jet in an Entry 7.5 ft high. Press at hydrant, 40 lb; length of hose, 50 ft; diam of hose, 1.5 in (71)

Fire-fighting equipment. All mines containing timber should have ample water supply and pipe lines laid around mine openings and surface plant, and to shaft landings. In some cases pipe lines extend throughout the mine, as in many collieries, for both fire protection and wetting down coal dust at the face. Small CO_2 and water fire extinguishers are useful. Hydrants should be not over 100 ft apart; all hose taps uniform in size, with standard threaded connections. FIRE HOSE in 50-ft lengths, on portable reels, and with nozzles attached, should be kept in boxes along entries, timbered drifts or levels, at intervals of 600 to 1 000 ft. For underground handling by one man in emergencies, 1.5-in hose is best size, with 0.5 to 0.75-in cone nozzle; best operating pressure, 30 to 40 lb per sq in.

Fig 7 and Table 28 give results of tests in the Bruceton mine (71) and indicate limitations of stream trajectories in entries or drifts of ordinary height.

Table 28. Water Thrown by Nozzle (71)

(Orifice of nozzle, 10 in above floor; curve of stream tangent to roof, which is 7.5 ft above floor; 1.5-in rubber hose, 50 ft long)

Press per sq in at hydrant	Diam of nozzle	N(a)	P(a)	O(a)	Water thrown per min	Press per sq in at hydrant	Diam of nozzle	N(a)	P(a)	O(a)	Water thrown per min
Lb	In	Ft	Ft	Ft	Gal	Lb	In	Ft	Ft	Ft	Gal
10	7/16	16 1/2	28	34	19 1/4	25	5/8	28 1/2	35 1/2	56 1/2	55 1/4
	1/2	18 1/2	31	37 1/2	23 1/4		3/4	28	35	56	73
	9/16	17	29 1/2	35 1/2	30 1/2	30	7/16	31 1/2	36 1/2	62 1/2	32 3/4
	5/8	15 1/2	27	32 1/2	34 1/2		1/2	33 1/2	41 1/2	65 1/2	40 1/4
	3/4	15	26 1/2	32	48 1/4		9/16	32	38 1/2	64	52 1/2
15	7/16	21 1/2	32	43 1/2	23 3/4		5/8	31	36	61 1/2	60 1/2
	1/2	23 1/2	35 1/2	47	28 3/4		3/4	30 1/2	36	61	79 1/4
	9/16	22	34	45	37	35	7/16	33 1/2	37	66 1/2	35
	5/8	20 1/2	31 1/2	42	43		1/2	35	42	70	43
	3/4	20	31	41 1/2	58 3/4		9/16	34	39	68	56 1/4
20	7/16	26	34 1/2	51	27		5/8	33	36	65	65 1/4
	1/2	27 1/2	38 1/2	54	33		3/4	32 1/2	36	64 1/2	85
	9/16	26 1/2	36 1/2	52 1/2	42 3/4	40	7/16	35	37	70	37 1/4
	5/8	25	34	50	49 3/4		1/2	36	42	74	45 1/2
	3/4	24 1/2	33 1/2	49 1/2	66 1/4		9/16	35 1/2	39	72	60
25	7/16	29	36	57 1/2	30		5/8	34 1/2	36	69	69 1/2
	1/2	31	40 1/2	60 1/2	37		3/4	34	36	68	90 1/2
	9/16	29 1/2	38	58 1/2	48						

(a) N = horiz distance from orifice of nozzle to point where curve of stream is tangent to roof; P = horiz distance from orifice to point where lower part of stream strikes the floor; O = horiz distance from orifice to point where upper part of stream strikes floor (Fig 7) (71).

Water pressure may be regulated in deep mines by automatic differential pressure valves; or by water tanks in offsets from shaft at different levels, with float-operated filling valves. Ample water supply should be kept in elevated reservoirs or tanks, to be independent of fire pumps in case of emergency. To PREVENT FREEZING, carry pipe-lines down the upcast and through return airways, or heat the water in a circuit connected with the water column.

Automatic sprinklers are sometimes used for surface plants and in shafts and underground stables. They are too expensive for use throughout the mine. Where fire-lines are not used, water-filled FIRE BUCKETS and WATER BARRELS painted red should be placed at intervals, particularly at shaft landings.

Emergency fire-doors at critical points are effective for shutting off fire areas. Manipulation of ventilating fans and air currents is of great importance in fighting fires; colliery fans should always be reversible (71).

Fire-fighting organization. Trained crews on each working shift should have monthly practice at assumed points of fire. Copies of SPECIAL FIRE MAPS, showing plan of ventilation, all water lines and taps, and position of pumps, hose, and other apparatus, should be placed in each fire-hose box throughout the mine and on surface. Where pipe lines do not run, FIRE EXTINGUISHERS should be placed at such intervals that no more than 5 min will be required to reach any point in the tributary district. In some collieries large CHEMICAL FIRE EXTINGUISHERS are placed on mine trucks for rapid transport. Underground TELEPHONE SYSTEM is valuable in fighting fires. SMOKE HELMETS, in which the air is renewed by pumping, are good in fighting fire, but as the user can not go more than about 200 ft from the base, they are not well adapted for emergency work. OXYGEN BREATHING APPARATUS (Art 16, 19) is used more for fighting fires than for rescue work after explosions. CO gas masks, approved for use where safety lamps will burn, are now widely used; but where O is deficient, oxygen breathing apparatus is necessary (77).

Fighting colliery fires. It is best to attack directly by streams of water, but if the air current is toward the point of attack and coming past the fire, water may turn to steam and make it difficult for the firefighters. In this case, throw on large quantities of limestone dust or sand to blanket the fire; the former is best if it is possible to approach close enough. In bituminous mines bags of limestone dust are stored at convenient points (66). The dust checks the flames, but, if there is a large mass of burning coal and timber, water

must be applied to cool and extinguish it. If the fire still makes headway or, in case of a gob fire, is not approachable because of roof falls, it must be inclosed by stoppings.

In presence of gas, as closing by stoppings is dangerous, the last stopping which shuts off intake air should be a door, with weights and pulleys for automatic closing. A counterweight is also attached, consisting of a bucket of water which is allowed to trickle away, and when empty the main weight shuts the door. Or, the door can be held open by cords, which are broken by the discharge of caps or a few ounces of permissible explosive covered with clay or rock dust to prevent ignition of possible gas. The danger caused by shutting off the air current is from the backing up to the fire of CH_4 from blowers, or distilled gases from the coal. Explosive gases may be generated by fires in metal mines having shale walls. Generally, gases from timber fires are not explosive. In collieries, if after the temporary stoppings or brattices have been erected for some hours, no explosion results, permanent stoppings should be begun. A pneumatic dam (Art 17) may help as an emergency stopping. A practically air-tight emergency stopping is a light steel or wood frame across the passage, covered with wiring or burlap, thickly coated with gunite. This was used in a Butte mine, to enclose a pyritic fire in a stoped area, and prevent air reaching the fire through joints in the walls.

Direction and volume of ventilating current. The current should not pass over the fire, but a line brattice should be brought up the entry, with a cross-brattice so erected as to prevent air from feeding the fire. The question may arise whether it is better to shut off the air by a stopping on the intake or the return side. Where stoppings can not be erected simultaneously, due to gas and smoke, it is best to place the first stopping on intake side, so the men can work in fresh air. Placing it on the return side may be dangerous, as it backs up distilled and other inflammable gases over the live fire and may result in explosion. Hence, the first stopping should be on the intake side; then, by carrying in air by deflecting brattices, build the stopping on the return side, completing with an automatically closing door. Before erecting fire stoppings in bituminous mines, the vicinity is rock-dusted to prevent a coal-dust explosion from gas.

Reversing ventilation promptly is important. At one disastrous mine fire, which started at foot of intake shaft, when the fan was finally reversed (after lapse of about half an hour), the fire had reached the other shaft and many lives were lost. A fan should never be reversed without knowledge and approval of the underground foreman; hence the importance of an underground telephone system, connected with the surface.

Sealing fire area. Stoppings should be as close to the fire as possible. If the sealing is perfect, flame is soon extinguished in the inclosed area by depletion of O, also in collieries by absorption of O by the coal (Art 3); then slow combustion continues until the O is practically exhausted; with tight fire stoppings and a small inclosed area, this may take from 1 week to several months. Where CH_4 evolves rapidly in the area of a blazing fire, do not place the seals too close to the fire, but 500–1 000 ft away, so an explosive mixture will not occur before the seals are built. In such case CO_2, liberated from tanks on the intake side of the fire, lessens danger of gas explosion (71).

Perfect sealing is impossible. Though the stoppings be airtight, air percolates through fractured strata, or along bedding planes, and most rocks are porous. A change in barom press of 0.5 in above or below normal is equivalent to a difference of 1.3 in of water-gage on one side or other of the inclosing pillars and stoppings. It is then a question whether the O is consumed faster than it is renewed by leakage. This is tested by taking samples of air through pipes with valves, placed in each stopping. Where the nature of strata or thinness of pillars permits rapid leakage, INJECTION OF INERT GAS (CO_2 or SO_2) is useful, not because these gases have materially greater extinctive effect than excess N, but to reduce the proportion of O below the 10 or 12% which permits active combustion. If the area is small LIVE STEAM may be used with advantage (71), serving to keep fresh air from entering, by maintaining higher press than atmospheric in the fire area. If the area is large, as of an entire mine, injection of gases or of steam is of little value; steam is rapidly condensed, and the inert gases are absorbed by the water. Moreover the volume of gas available for injection is usually insignificant as compared with the spaces to be filled. SO_2 GAS, made by burning sulphur, has been tried; results, inconclusive. In burning sulphur, it is difficult to keep the O in the gaseous products down to 10 or 12%. CARBON DIOXODE, produced by action of H_2SO_4 on limestone, was tried at a large fire in a Colo mine, which had been surrounded by fire walls, but failed due to roof breaks admitting air (71). If properly controlled, PRODUCTS FROM BURNING NATURAL GAS, or PRODUCER GAS, might be suitable for injection. But, a fire furnishes its own extinctive gases, and it rarely pays to inject artificial gases. Liquid CO_2 was recently successfully used in a German colliery to extinguish a persistent gob fire in steep-dipping workings, when other means had failed due to leakage of air. After covering the gob with sand, the CO_2 was forced through pipes into the burning mass, cooling it by expanding and also excluding O (71). After combustion stops, the HEAT OF A LARGE FIRE, surrounded by its own ashes, DISSIPATES VERY SLOWLY. The Monarch mine fire, Ill (1910), was sealed for 2 months, and when the area was explored by men with oxygen breathing apparatus, no excessive heat was observed; but, on admitting a ventilating current, smouldering fires under heavy roof falls immediately revived and had to be dug out.

Flooding the mine with water is usually the last resort in fighting fires, since it is slow and costly, first to flood, then to pump out and repair damage. In collieries damage may be considerable, owing to falls of roof, or to squeezes where the bottom is soft shale or

clay. An UNDERGROUND CONTOUR MAP of the mine is of utmost importance in deciding for or against flooding. To LIMIT THE FIRE AREA dams may be necessary, and must be strongly built to resist water press, if on steep dips. Preparatory to flooding, plans must be made to liberate entrapped air or gas above the fire level by drill holes; otherwise, the water level may not reach high points of the fire. In one mine, after attempting to extinguish fire by sealing, water was run into the shafts until it rose above the main working level, but when pumped out, hot coals were said to have been found in an upper portion of the workings, not reached by water on account of entrapped gases.

16. INUNDATIONS AND COLLAPSE OF MINES

Mine inundations in foreign countries, where collieries are sometimes worked under the sea, have been diastrous. In a Japanese colliery, water from the sea broke through a fault plane in sandstone extending up 155 ft to the sea bottom, which consisted of 82 ft of sands and clays; 237 men were drowned. Inundations with large loss of life have been rare in American mines. Except in the Lake Superior district, little mining is done near large bodies of water. In coal mining, of which the records are more complete than for metal mining, to 1915, 7 inundations caused 149 deaths. Four of the 7 inundations were due to inrushes of water from old workings. Sept 25, 1923, the Redding Colliery, Scotland, tapped old water-filled workings, and 40 men were drowned. In metal mining, the loss of life has been light. Feb 5, 1924, an inrush killed 41 men in the Milford iron mine, Crosby, Minn. Nov 3, 1926, an inrush from a swamp killed 51 men in the Barnes-Hecker iron mine, Ishpeming, Mich.

Prevention of inundations. Surface flooding is prevented by proper location of shafts and proper grading. To prevent future flooding from abandoned workings, plans and cross sections of excavations that are to be abandoned must be made. In approaching old workings long holes must be drilled in advance (by diamond drill or equivalent) through which the water is drained. In working large deposits, where the width of orebody prevents adequate solid support of the water-bearing ground, the latter must be kept well drained, or completely excavated down to the rock or ore. Water and sand inrushes in shaft-sinking through water-bearing ground, or driving headings under stream beds in loose formation, is sometimes a problem (Sec 8). For effects of caisson disease, see Sec 15, Art 28, also Bib 83. BARRIER PILLARS left around old workings must be carefully calculated to carry the load due to overlying strata. This is a more important factor in estimating the required strength of barrier pillars than the hydraulic head, for, if the pillar begins to crush, water will percolate through the pillar, and cause its failure. (For data on barrier and other pillars, see Sec 10). This problem has been important in the anthracite district of Penn, where upper workings have filled with water; also in the bituminous field, wherein steep-dipping beds there is different ownership of workings to the rise and those to the dip. The question of required strength of barrier pillars is so important that a state commission was appointed. They reported that a barrier pillar strong enough to resist crushing by the overburden was sufficient to act as a dam (74).

Collapse of mines. "Squeezes" in coal mines, causing the "top" and "bottom" to come together, are usually gradual enough to permit the men to escape; but, in 1914, the Adamson No 1 mine, Okla, collapsed so suddenly that 13 men were killed. Cavings may occur when ore or coal has been too completely extracted, leaving insufficient pillars for roof support. Timbering, also, at considerable depth below surface, may be inadequate to support the superincumbent weight. In coal mining, squeezes are especially apt to occur when the clay underlying the seam is softer than the pillar of coal, which is then forced downward, with side flow and heaving of the clay. PREVENTION OF CAVING in coal mining: (*a*) in room and pillar work, take out in advance or first mining 20% to not over 60%, depending on depth of seam below surface; (*b*) pack excavations with rock débris, or fill by sand flushing; (*c*) or, work the seam by longwall, with stowing (Sec 10). Similarly in metal mining, size of pillars must be proportionate to the strength of rock or ore composing them, and the weight to be supported. In deep metal mining, besides leaving strong pillars, empty stopes should be filled with waste or sand, or with rock blasted from hanging- or foot-wall. Sand from tailing piles is often used for filling. Back-filling with crushed mine waste, blown through a large pipe to a hose at the face, has recently begun in European coal mines, for level workings or even those to the rise, but is less efficient than sand-filling.

"Bumps" in coal mines are primarily due to wt of overlying strata. They throw slabs of coal violently from ribs or sides, break down roof, or burst up the floor with a loud report that sounds like the word "bump." A violent bump will smash down or dislodge timbers and may cause an air-blast.

Tremors, like a local earthquake, caused by a severe bump in 1916 at Coal Creek mine, B C, were felt over 5 miles away (12). Gas is sometimes given off simultaneously. Rice (12) suggests 2 kinds: PRESSURE-BUMPS, in which a pillar or adjacent stratum is overloaded and bursts, like a rock in a crushing machine, and SHOCK-BUMPS, where sudden breakage of a rigid stratum above and spanning a subsidence cavity, produces a hammer-blow on mine roof. Conditions causing pressure-bumps: (a) wt of overlying strata too great for unit strength of coal acting as a pillar; (b) strong roof over the coal bed; (c) fairly strong floor, not flowing or squeezing under press. Then, a weak pillar spalls off violently; or the floor may burst upward and scatter the props. To cause shock-bumps, besides conditions named above, there must be: (a) strong, rigid rocks above the coal seam and immediate roof; (b) subsidence of some kind under these rigid rocks; (c) excessive roof span, so that overlying rock breaks and falls, striking on the immediate roof and causing a shock-wave, imparted to pillars and floor. In certain mines, the roof is so elastic that, after bending a few inches, it springs back, leaving a wedge-shaped space over the edge of pillar. This may displace timbers. Bumps occur only in deep mines (say, 1 500 ft or more), and under strong strata. Severe bumps have occurred in Springhill No 2 mine, Nova Scotia, which Rice believes were pressure-bumps, and those in Coal Creek mine, 1916, shock-bumps. Bumps have been noted in England and elsewhere in Europe (11, 12).

Remedies. Since the chief cause of danger to men is overloaded pillars, or, in case of shock-bumps, false or loose top, and as bumps are rarely dangerous in longwall mining, longwall should obviously be used under conditions causing bumps, and, where suitable, retreating longwall. This refers to severe cases; lesser pressure-bumps are usually preventable by leaving large, evenly spaced pillars on the advance, and drawing them systematically to prevent concentration of wt on weak blocks. Tests on strength of coal in pillars, made for Penna Anthracite Mine Cave Commission, 1912–13, on small blocks taken from the mine, may not give true results owing to effect of cleat, or face joints (74). Coal is semi-plastic when pressure at right angles to bed exceeds 2 000 or 3 000 lb per sq in or more, depending on character of coal (74). The British Safety in Mines Research Board has recently made tests of convergence of roof and floor in various mines, both on longwall faces, and in narrow headings into the face. One example cited (1936) was in a 4 1/2 ft bed, 2 100 ft deep, where slow descent of roof extended 68 ft ahead of the face, and forward slanting breaks in the roof shale were observed 50 ft in advance (88).

Rock-bursts and air-blasts in metal mines resemble bumps in coal mines, being marked by sudden bursting off of masses of hanging wall or pillars. Sometimes individual rock blocks are under strain and easily fly apart (termed "explosive rock"). Since vein matter and walls of metal mines are usually stronger than in coal mines, rock-bursts are not serious until greater depths are reached than where bumps occur in coal mines; rarely giving trouble at depths less than 4 000 ft.

Rock-bursts might be classified like bumps into PRESSURE-BURSTS, SHOCK-DISTURBANCES and, with very local effects, EXPLOSIVE-ROCKS. Pressure-bursts have caused anxiety in the Lake Superior copper conglomerate vein, at vert depths of 5 000 ft or more; especially in drawing pillars between stopes, when the diagonal line of extraction approaches shaft or level pillars, and sometimes have caused air-blasts. Shock-waves also have been caused by ground movements in hanging wall, producing severe tremors felt more on the surface than underground. Previous to 1924, the Calumet & Hecla mine adopted retreating longwall without leaving pillars, at depths of 3 500 to 4 900 ft. Only trouble from bursts was in approaching shaft pillars, which are in the vein (72 *a*, *b*).

In the Kolar mine, India, below the 5 000-ft level, rock bursts began when 60% of vein was extracted; granite blocks quarried on surface, placed in packs to support 40% of hanging wall, gave favorable results, as they compress 10%, compared to 23% compression of packed pigstyes (72*c*). A later study (1937) of the Kolar outbursts illustrates doming and effect of different supports. Pigstyes, pronounced unsuccessful in the Rand mines, are valuable if filled with granite blocks, after the initial squeeze, whereas waste filling is apt to displace the timbers (72*h*).

As mining becomes deeper, rock-bursts have been increasingly numerous on the Rand, with many earth tremors, and in 1924 a commission, appointed to study them, reported that from 1918 to 1924 rock-bursts had killed 193 and injured 498 men.

Remedies. Sand filling of stopes was formerly considered the best remedy; but at great depths though preventing large movements and collapse of walls, it may fail to check small movement and merely lessens but does not prevent bursts. Longwall mining is regarded favorably (72*d*, *e*). In deep iron mining in the U S, and in the Lake Superior copper district, the problem will be of increasing importance. On the Rand, the danger has been stressed (72*e*) of leaving pillars and remnants, as between approaching faces. The Assoc of Mine Managers, 1933 (72*f*), studied rock-bursts, reporting on use of the "sag meter" to record convergence of hanging and footwalls and give warning of impending bursts, and on relative effic of pigstyes, wood cribs filled and unfilled, circular concrete blocks and circular steel supports filled with sand. For a study of ground failure in deep metal mines, formation and shape of pressure arches and invert arches, effect of pillars and dip of strata, see Bib 72*g* (1937).

RESCUE AND RECOVERY WORK

17. MINE RESCUE APPARATUS AND RESCUE STATIONS

Mine rescue apparatus (76–79) may be classified as: oxygen or air-breathing portable apparatus; resuscitation apparatus, for reviving men overcome by gas or smoke; communication appliances, as telephones, signals and guide ropes; lighting apparatus, portable electric lights, safety lamps; gas indicators, canaries for testing, safety lamps, gas analysis apparatus; first-aid-to-the-injured appliances, as stretchers, medical supplies, underground refuge and first-aid chambers; bratticing materials for restoring ventilation; fire-fighting appliances, as smoke helmets, portable extinguishers chemical engines, hose, water buckets and limestone dust (77, 83, 84). A rescue apparatus room should be adjacent to the mine entrance, with a room equipped with first-aid supplies and medicines.

Portable breathing apparatus for rescue work has long been used. In it the contained air is resupplied with fresh oxygen, and the CO_2 fixated by a chemical substance.

In the U S (1908), after the Marianna mine explosion, Pa, the only living man was saved by a party equipped with Draeger outfits. In 1909, at the Cherry mine fire, exploration with breathing apparatus through a sealed shaft led to reopening the mine, and recovery of 21 men 7 days later. Lives have sometimes been lost due to defects in apparatus, or insufficient training. Though breathing apparatus is far from perfect, it has in general been successful in preventing losses of life. Helmets attached by hose to air pumps, though good for distances to about 200 ft, are useless for general rescue work. They are properly fire-fighting equipment, and are called SMOKE-HELMETS. Self-contained breathing apparatus for rescue and exploration should have at least 2 hr working capac. SELF-RESCUE apparatus, that may be carried in the pocket, is sometimes used (79); they are good for 0.5 hr, against CO_2 and small percentages of CO, but do not supply O.

Gas masks, developed during the World War, led the Bur of Mines to devise masks for mining and other industries. They are now part of the equipment of all well equipped mine rescue and fire fighting stations. The usual gases from explosions, blasting, or mine fires, are CO_2 and CO; sometimes nitrous fumes and H_2S. For chemical works in which special poisonous gases or dusts may occur, particular kinds of gas or dust masks must be used (78, 29).

The most difficult gas to break up or render harmless is CO; finally done by making a chemical compound called HOPCALITE, which catalyzes CO to CO_2. Mask designs differ for various gases. Military masks are unsafe in concentrated poisonous gases, and useless against CO; hence, should never be used in mines during fires or after explosions. For this service, certain masks are approved if used only where an oil lamp, or in coal mines a safety lamp, will burn (requiring presence of 17% O); as they do not supply O, they must be regarded as auxiliaries in mine rescue and fire-fighting work (78).

Characteristics of portable oxygen-breathing apparatus:

(*a*) Apparatus employing O compressed to 120 atmos and in some recent apparatus 135 atmos; CO_2 absorption chemicals (chiefly caustic soda for European apparatus; in U S, soda-lime or cardoxide) in can or box, loose, or on wire trays; a rubber breathing bag, with automatic relief valve, to provide for fluctuations in breathing needs; and forced, circulatory system. The DRAEGER and WESTFALIA apparatus, widely used in different countries, are of this type. Both use either helmet or mouth-breather. Wt, about 40 lb, when charged. The WEG outfit, made in England, is of the same type, but has a mouth breather only and oxygen feed is controlled by an automatic valve, controlled in turn by lung movement, whereas all other kinds of compressed oxygen apparatus have fixed feeds of 2 or 2.5 liters of O per min at atmos pressure.

(*b*) Employing compressed oxygen, with CO_2 absorption chemicals in loose or stick form in a flexible breathing bag, which must be shaken from time to time for better absorption of CO_2; emergency valve on by-pass for O flush-out. The FLEUSS or PROTO outfit (Fig 8), made in England, has a mouth breather only; wt, about 40 lb, fully charged. It has a by-pass for extra supply of O, if needed by wearer. Its parts are: *a*, reducing valve; *b*, by-pass; *c*, main valve; *d*, skull cap; *e*, smoke goggles; *f*, nose clip; *g*, mouth piece; *h*, inhaling valve; *i*, pressure gage; *j*, saliva trap; *k*, relief valve; *l*, exhaling valve; *m*, pressure-gage tube; *n*, pressure-gage valve; *o*, oxygen-supply tube; *p*, caustic-soda spaces; *q*, breathing bag, with inhaling and exhaling compartments; *r*, oxygen containers.

(*c*) Employing compressed O, with CO_2 absorber in liquid form. It is represented by Tissot apparatus, made in France, unique in having in combination with the mouth breather a device for insertion in nostrils, to permit breathing either way. Breathing bag and other parts are contained in a wood-covered knapsack. Wt, 14 kg (31 lb).

(*d*) Employing O liberated from $KNaO_3$, through reaction produced by absorption of moisture and CO_2 of the breath. Not having the heavy oxygen bottles of (*a*), (*b*), and (*c*), it is the lightest and promises success, though a difficulty not yet solved is the high temp of the air supplied and the irregular oxygen feed. Represented by the Pneumatogen, made in Austria; used also in England; mouth breather is employed, 1911 type weighs 22 lb. Reactions for liberation of O and fixation of

CO_2: $KNaO_3 + H_2O = NaOH + KOH + O_2$; $KOH + NaOH + CO_2 = KNaCO_3 + H_2O$; $KNaO_3 + CO_2 = KNaCO_3 + O_2$

Self-contained oxygen breathing apparatus had considerable development during and after the World War. General principles have not changed, but the helmet has been abandoned because of the spaces that may fill with exhaled air and danger of leakage. In the U S the Draeger, Fleuss and other foreign apparatus fell into disuse, and in general have been supplanted by American apparatus like the Gibbs and the Paul; which, with the Fleuss-Davis Proto (British) and the Draeger, 1924 type, are on the Bur of Mines permissible list. The Gibbs and the Paul have automatic oxygen feed-regulating valves, instead of a fixed rate of feed, like the Draeger and Fleuss-Proto, and have a slight outward or positive press throughout the air-circulatory system. This protects against slight leaks when using in poisonous gases or those containing insufficient O. GIBBS APPARATUS (Fig 9): O from bottle passes through a tube with closing and reducing valves to cooler; thence, by admission valve and tube to exhalation side of cooler. Mixture of O and exhaled air passes to regenerator, and up into inhalation side; thence into breathing bag through spaces around admission valve; thence by inhalation tube to lungs; thence through valve to exhalation side of cooler. By-pass valve furnishes O to user independently of main closing valve. The McCaa and Paul outfits are similar to the Gibbs, with minor changes. The McCaa has a 2-hr capac; some are made for 1 hr and 1/2 hr, weigh less and permit quick inspection. For details of these types, see Bur of Mines handbook on "Self-contained Oxygen Breathing Apparatus" (revised 1933), which also describes equipment of rescue stations and trucks (77). In France, the present leading oxygen breathing apparatus is the Fenzy, which also has automatic oxygen-feed regulation (77).

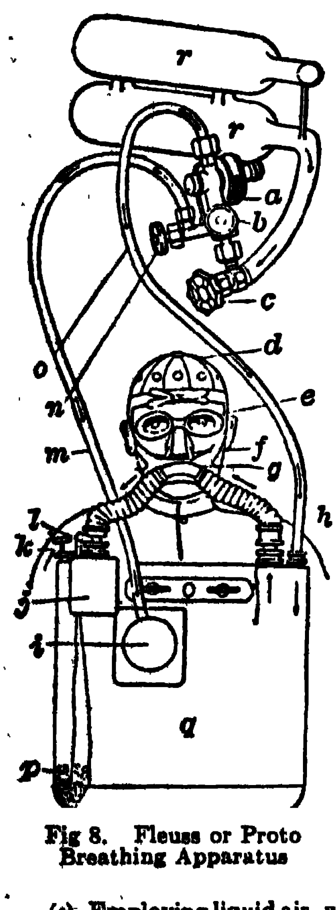
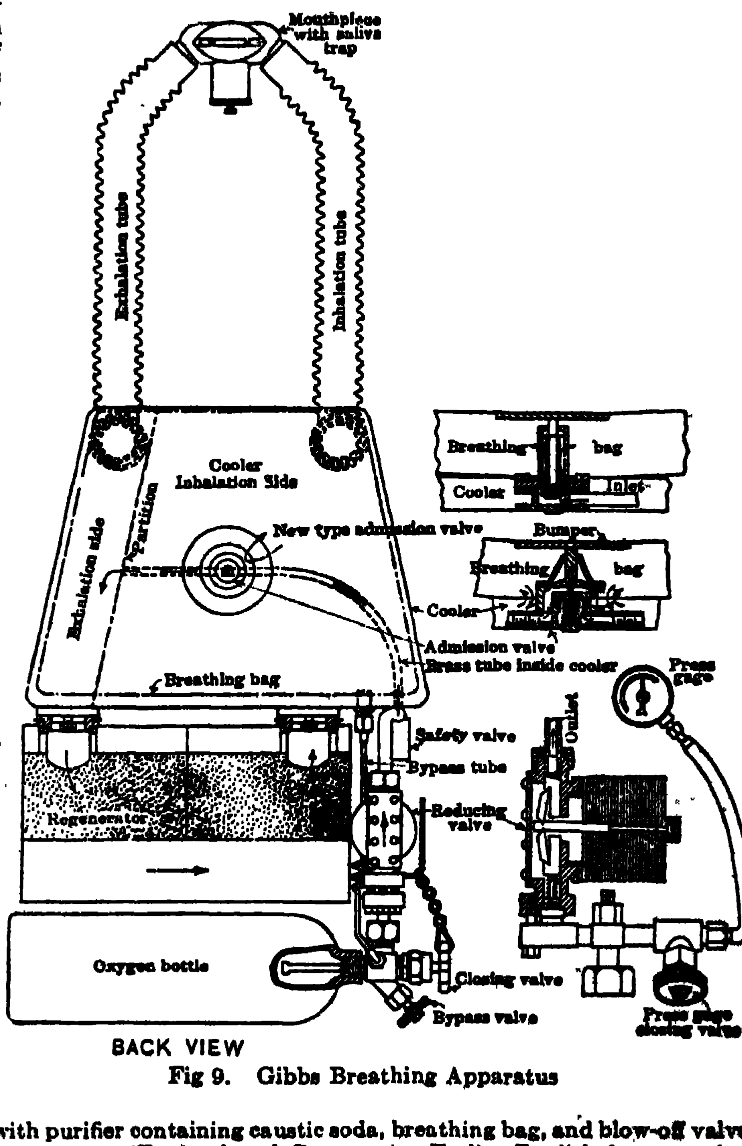


Fig 8. Fleuss or Proto Breathing Apparatus



BACK VIEW

Fig 9. Gibbs Breathing Apparatus

(e) Employing liquid air, with purifier containing caustic soda, breathing bag, and blow-off valve. Represented by the Aerolith apparatus (England and Germany). Earlier English form, used at Newcastle Fire and Rescue Station, weighs 40 lb, fully charged with 8.5 lb liquid air. It is contained in a nickel knapsack. Liquid air supply must be constantly renewed by the plant of the rescue station. Air is liquefied at $-191°$ C, and occupies about 0.08 of its vol at sea-level press. The difficulty of having constantly to renew the supply of liquid air, which evaporates whether the instrument is in use or not, prevents general application of this apparatus. IMPROVED LIQUID O APPARATUS. The Brown and Mills, and the Rotherham, have regenerative features (essentially an absorbent for CO_2); have not yet been introduced into the U S.

Portable breathing apparatus used in the U S in 1938 comprise 1130 Gibbs, 869 McCaa and about 300 Paul, Fleuss-Davis, Proto and Draeger, all on Bur of Mines permissible list.

In recent years the number of available sets has not increased: due to concentration of rescue apparatus at central stations in mining districts and greater use of gas masks for supply and construction parties, following rescue and fire-fighting crews wearing oxygen-breathing apparatus.

Care and use of apparatus (77). It must be kept in perfect condition. Every station should have at least 5 sets, better 10. A rescue party should consist of at least 5 men, so if one becomes unconscious there will be 4 to take him out. Men should be thoroughly trained for not less than 6 days, 2 hr per day, and be able to take apart and put together the apparatus. There should be a leader, paid to care for apparatus and train men.

Crews should practice in apparatus 2 hr per week, and be examined periodically by a physician as to their fitness. No one should be permitted to use apparatus under dangerous conditions, without a certificate of training from the Bur of Mines, State training station, or joint or private rescue station giving equivalent training. Apparatus should be kept in a room by itself, dust-free, dry, well ventilated, and not overheated, so the rubber parts will not be injured. It should be cleaned immediately after use, and hung in its proper place ready for emergency calls (77, 80).

Tests of rescue apparatus at high altitudes made (1915) by Bur of Mines on Pike's Peak, Colo, at from 7 000 to 14 000 ft above sea-level, show that standard types of apparatus are efficient, without reference to altitude and at the higher levels more work can be done by wearing apparatus than without it, owing to the liberal oxygen supply.

Resuscitation apparatus. Besides first-aid methods (Art 20), there is special apparatus for reviving rescued persons, by mechanical movement to assist restoration of respiration, and flush bad air out of the lungs by means of oxygen.

Bratt's Resuscitator draws the air and gases from the lungs by suction caused by an ejector, in conjunction with the compressed O, and alternately forces in by light pressure, pure O, or a mixture of O and air. The change from suction to forcing movement is made by a hand lever. PULMOTOR, made by the Draeger Co, accomplishes this movement automatically by pressure of the oxygen. In both of these, the movement may be made by hand, the O being carried in an iron tank or bottle (like those used in rescue apparatus). When the Pulmotor acts automatically it furnishes a mixture of O and air, the proportions of which may be varied to about 35% O. A committee on resuscitation from mine gases, appointed by Bureau of Mines (82), reported adversely on automatic apparatus; on the grounds that, if improperly used, the suction may cause collapse of the air passages in upper part of lungs; and that, in forcing O into the lungs, injury may be caused by the press rising too high. The Committee favored manual methods of artificial respiration, supplemented, in case of gas poisoning, by giving O by an "inhalator," in conjunction with the "prone" manual method (Art 20); or, in exceptional cases, by the Sylvester manual method. Some physiologists favor using 7% CO_2 with 93% O while administering the manual method, as CO_2 stimulates breathing.

Communication apparatus in rescue work. Portable telephones are helpful, for communication between the temporary underground base and the surface, and between base and party using breathing apparatus. The former can employ ordinary telephones, though preferably of iron-clad mine type. They should be placed in intake air; in return air of a gaseous mine, magneto sparks might ignite CH_4. To keep the advancing rescue crew in touch with the base is more difficult. When helmets were used with breathing apparatus, a satisfactory telephone attachment to the helmet was employed; but, since mouth-breathers are now required, a special phone is necessary, requiring no lip movement.

The Western Electric Co make a phone to be strapped to the head, so that it presses against the throat of man using the breathing apparatus. He "talks" while the breather is inserted in his mouth. This takes practice, but sounds transmitted by an experienced man are readily intelligible to a trained man at the base. Field receiver is of ordinary type, held to the ear by a spring. The wires are made into a light-weight, comparatively cheap, cable, to be abandoned in case of retreat. It is on a special reel, to maintain a circuit when paying out or winding up. The chief at the base is therefore constantly in touch with the advance party. This apparatus has been rarely used. Wireless has been tried with little success. Geophones have some value in shallow mines for locating imprisoned men who may rap signals.

Pneumatic horns are good for signaling, especially in descending shafts on cages or buckets. Gongs are inconvenient. The leader of the crew should have a horn, and the chief at base one of different note, or a bell, a code of signals being agreed upon. The first 3 regular hoisting signals should be employed: 1 bell, go ahead, or lower, or stop when in motion; 2 bells, retreat, or hoist; 3 bells, man to be hoisted; on return signal he will get on cage or bucket, and signal 1 bell. Special signals can be arranged.

Guide ropes. When a rescue party is to enter a smoky atmosphere, so that the mine walls are indistinct (especially in crooked passages of an unfamiliar mine), a light manila rope should always be dragged by advancing party, and a code of signals by numbers of tugs on rope is arranged.

Lighting apparatus for rescue work should consist of portable electric lights of permissible type (Art 10).

If these are not available, charged ready for use, ordinary electric flash lights, with dry cell batteries, must be employed. Outside sparking will not ignite firedamp, but if the outer glass and

bulb are broken so as to expose the glowing filament, firedamp will be ignited; hence they must be carefully handled and regarded only as emergency lights, unless of the types free from this objection and approved by the Bur of Mines. SAFETY LAMPS (Art 9) should be carried by at least 2 in each party, to test the air.

Gas indicators. In recovering a mine the rescue party should have safety lamps, Burrell gas indicators, CO detectors; also canaries (Art 5), carried separately in small cages. The Haldane cage is a light aluminum box with mica windows, tight closing door at one end, a bird perch within, a small oxygen bottle above, which serves as a handle, with valve for oxygen admission. Canaries, though more sensitive to CO poisoning than men (Art 5), are less used than formerly, but the Bur of Mines rescue crews often carry them (54). Their advantage over CO detector is that they can be easily watched as the party advances; if one falls it is quickly noted, whereas the CO detector causes a stop to make tests.

Gas-analysis apparatus (Art 11) should be kept in every rescue station. In case of a mine fire, it is important to know if the atmosphere near it approaches explosive limits, so that men can be withdrawn. Also, in fighting fires, the presence of CO in the return air indicates an active fire, as otherwise CO is rapidly diffused and absorbed by moisture.

First-aid-to-the-injured packets and boxes, including simple medical supplies, stretchers, etc, should be in every rescue station and field party (Art 20) (77). A few mines have been equipped with an underground ambulance car, hand pushed or drawn by mine locomotive.

Bratticing material for restoring ventilation. It is impossible to keep enough bratticing at any rescue station to be of importance in recovery work after an explosion, but every mine should have an ample supply; and a joint rescue station, if in the neighborhood, should keep a reserve of brattice cloth.

Smoke helmets are similar to those for diving, being connected by rubber hose to a hand pump; or, if the mine has compressed-air plant, to an air-pipe line, with an automatic pressure-reduction valve and a relief valve between pipe and helmet, set to give not over 6 or 8 oz press per sq in in the helmet.

Return hose to pump is unnecessary, as air escapes around edge of helmet, thus preventing entrance of bad air, and helping to keep helmet cool. If the wearer has a special coat with flaps at neck for fastening around edge of helmet, the escaping air circulates under the clothing and keeps the body cool; important in fighting fire. Haldane found that a man with smoke helmet could do 4 000 ft lb work per min in an atmos at 93 to 94° F, with wet bulb at 84°, and continue for an hour without discomfort.

Two smoke helmets can be connected to a single air hose by using a "Y"; not advisable unless hose is large, and there are other helmets and ample compressed air to effect a rescue if one man be overcome. For safety, work with helmets should not be carried on farther than 200–300 ft from a point where air is pure. Their use is therefore limited to building fire stoppings, or handling hose and directing the stream on the fire. Many helmets are on the market, differing but little. They have been used advantageously at Anaconda mine, Mont, in fighting stope fires. Masks and O apparatus have supplanted smoke helmets, except for erecting fire stoppings or other construction near a fresh-air base, in which case gas masks or airhose masks are less fatiguing to workers (29).

Portable fire extinguishers should be in every rescue station, for dealing with incipient fires after explosions, even in mines having fire-fighting water lines, as these may be damaged by the explosion.

There are numerous good extinguishers, including those requiring simple overturning to empty a vial of H_2SO_4 or other acid to act on Na_2CO_3 for generation of CO_2, thus producing pressure for throwing a stream of water. Not all are adapted to mine use. As such apparatus will be used by trained men, it is better to have a positive and protected means of breaking an H_2SO_4 bottle, and a strong outfit to stand rough handling. Chemical fire engines on mine trucks would rarely be useful for rescue work after explosions, as mine tracks are apt to be temporarily impassable. SPECIAL CHEMICAL EXTINGUISHERS giving off gases are not good for fires in overhead timbers, which require a stream of water. Carbon tetrachloride extinguishers are effic in fires where there is electric arcing, as this liquid is a non-conductor; but are applicable only where air current is strong and all the men are on the intake side, since phosgene and other toxic gases may be produced. Sand or rock-dust are generally best in electrical fires in confined places; water should not be used until elec current in the vicinity has been cut off, to avoid shocks to men handling the hose.

Fire buckets should be a part of rescue station equipment; painted red and labeled "For fire-fighting only."

Water hose. 1.5-in rubber-lined cotton hose, in 50-ft lengths, with standard pipe threading and hand couplers, should be kept at stations. Even if such hose is maintained for use in the mine, it may not be in condition for an emergency. It should be on light reels, easily carried by hand and have special tees of several sizes, with gate valves attached, for clamping on water pipes, a hole being drilled in pipe by a ratchet drill, extending through the open valve. This permits quick connection, without interrupting flow of water.

Pneumatic fire-dam is a German fire-fighting device, of rubber with iron braces, and when filled with air is shaped like a mattress. It has a mica window to permit inspection of the fire, and is intended for quick erection in mine passages, pending building a permanent fire wall. Used with breathing apparatus it has merit for inclosing a fire area quickly, but is not used in U S.

Mine rescue organization and stations. A mine management should not merely purchase apparatus, and hang it up in a dusty, poorly ventilated room. Organized crews and suitably prepared stations are necessary. A man without training is a hindrance, not a help. Apparatus must be properly assembled and charged. Rescue crew unit consists of 5 men. The number of units should be proportioned to number of employes, preferably a certain percentage of each shift. At least 5% of employes should be trained (70, 84).

Five sets of apparatus are recommended for a station, with a spare set for a single crew of 5 men. The station should be near the mine entrance, but not in line with a slope or drift, nor near enough to be destroyed in a fire or explosion. It should comprise a cool, fire-proof, well-ventilated apparatus room; a repair room, with small tools; and a training gallery, which should be tight, to permit testing of apparatus in irrespirable gases; showers or bathroom, a toilet, and a room for lectures and first-aid training.

Joint rescue stations, of which there are several hundred in the U S, are best where mines to be served are near enough for any one to be reached in 30 min by auto or wagon. Advantage of a central station, aside from the larger stock of equipment, is that it permits employment of a foreman living at or near the station and subject to instant call, who is responsible for upkeep of apparatus, and shall train and lead rescue crews. He should therefore be a high type of man. Besides the stated apparatus, the station should contain copies of the mine maps, prepared with reference to possibility of explosions or fires, and showing direction of ventilating currents, hose connections, etc.

State and Federal rescue stations. A few states, as W Va and Ill, have them, and conduct training work. U S Bur of Mines (1938) has 11 stations, 11 rescue RR cars, with equipment for training and rescue; but recently appropriations have been too small to maintain most of them.

Auxiliary equipment for joint rescue stations may include an AUTO RESCUE TRUCK, equipped with 5 sets breathing apparatus, oxygen charging-pump driven by auto motor. 2 folding stretchers, 2 large oxygen tanks, 6 safety lamps, and 10 permissible electric lights; also, masks and other appliances; all on the truck ready for instant departure. If distances to farthest mines are considerable, an auto bus and additional supplies are desirable. For shaft mines, an emergency or collapsible cage, like that designed by Rice, rendered important service in 3 disasters which had wrecked the regular cages. A light hoisting cable should also be provided, long enough for the deepest shaft in the district, also a disk fan driven by an explosion-proof motor, taking current from a storage-battery locomotive, may be useful.

Refuge chambers in collieries are rare in the U S. Cases of men bratticing themselves off after explosions, or when escape was prevented by smoke from fires, indicate that chambers of simple form are best; that is, marking off a finished room, or unused entry in each division of a mine, erecting stoppings in breakthroughs, and providing doors in mouth of the working. Doors need not be strong, merely tight fitting. Prospect holes, if any, may be cased and refuge chambers placed under them, thus securing ventilation and communication. Men should be instructed to go to such chambers in emergencies; to accustom them to this, drinking water, first-aid, and other supplies, may be kept in the chamber, which might also serve as asst foreman's office. Their location should be indicated by signs, and entrances whitewashed, for ready recognition. In case of disaster, rescue parties would search the chambers successively.

Barricades. At the Cherry mine fire, Ill, 1909, 20 men saved their lives by erecting barricades and were rescued after 7 days. In many other similar cases, including metal and coal mine fires and explosions, several hundred men have thus saved themselves (79).

18. RESCUE AND RECOVERY WORK AFTER EXPLOSIONS

Most collieries are exempt from explosions, but explosions are always unexpected; their possibility should be realized, and all means of prevention adopted. The foreman of a rescue station should know instantly what to do, in case of explosion.

First steps after an explosion. Action must often be immediate. Fan should not be reversed, if there is possibility of escape of men through intake. If fan is seriously damaged: (*a*) repair it if possible; (*b*) borrow or buy a small fan, if it can be erected within 48 hr; (*c*) lower a steam jet into the upcast, or, if intake is a slope or drift, erect a wooden chimney say 18 ft high, close to entrance, box it in, with steam jet in its base; (*d*) in extreme emergency, hang a fire-basket in upcast, or at mouth of slope or drift erect a chimney, with fire grate at base. Unless a mine is unusually gaseous the return air at mouth will not be inflammable. It can be tested by a man on guard with a safety lamp, who can admit fresh air to dilute return gases.

Rescue-work organization. State laws are not definite as to who shall be in charge in times of disaster. Generally the state mine inspector takes charge, as a matter of expediency, not legal right, though he has authority to stop underground work, if he sees fit. In other cases, operators' representative assumes control.

Occasionally both sides have disclaimed responsibility, with unfortunate confusion. The Bur of Mines has no legal authority, and its rescue car and station men can be regarded as volunteers only. In any event, SOME ONE PERSON MUST HAVE COMPLETE AUTHORITY, if the work is to be done systematically. An efficient member of the mine staff is usually the best. If question is raised as to his legal authority, he should confer with the state inspector. Notices, signed by state and company officials, should then be posted, naming the person in authority, who may be entitled GENERAL MANAGER OF RESCUE WORK. He at once appoints GUARDS, to prevent entrance into the mine of any except authorized persons, to keep back crowds from mine openings, and to guide and keep volunteers from interfering with the work before they have been assigned places. Guards are marked in some way, as by a colored band pinned around the arm. Ropes are immediately strung about the mine entrances and working yard. The following CHIEF ASSISTANTS are also appointed, each responsible day and night for his part of the work:

(*A*) Chief of surface guards.

(*B*) Man to engage volunteers and assign their duties.

(*C*) Chief of rescue work, with 7 sub-assistants in charge: (*a*) of breathing apparatus and their crews; (*b*) of rescue men without apparatus; (*c*) of bratticing crews. (Usually, *a*, *b*, and *c* work under general direction of state inspectors); (*d*) of keeping safety lamps in order and giving them out; (*e*) of checking men in and out of mine. Each man entering should have a numbered metal tag, to be given up when he comes out, and is checked off. If metal tags are not available, use pasteboard or paper tag put into an upper pocket); (*f*) of stretcher gangs, for removing the injured and dead; (*g*) of marking location of every dead body, with description of how it lay (generally best done by one of the mine surveyors).

(*D*) Doctor in charge of temporary hospital at mine mouth, transport of patients to main hospital, and main hospital itself, in absence of regular organization. If there are enough physicians and surgeons, they are assigned to underground bases.

(*E*) Doctor in general charge of morgue; bodies are taken in charge by coroner.

(*F*) Manager of supplies for rescue apparatus, brattice materials, timber and lumber.

(*G*) Commissary manager, to keep mine village supplied with food, and rescue crews supplied with hot food and coffee at all hours of day and night.

(*H*) Manager of temporary sleeping and living accommodations for the volunteers, such as tents, cots, and bedding near mine mouth and necessary sanitary arrangements, including water for bathing and drinking. The hard-worked and strained rescue and recovery men must have comfortable resting places, at least after the first day. Completely to explore mines after large disasters often requires 1 to 2 weeks or more.

(*I*) Information man, charged with giving out non-sensational, but satisfactory accounts to the press, and to relatives and friends of entombed miners, thus preventing circulation of sensational stories. He should be familiar with mining, and firm but courteous.

Supplies. In great disasters, food and brattice cloth must generally be ordered by telegraph or telephone. A severe explosion may wreck hundreds of stoppings, which must be replaced quickly by temporary stoppings, each requiring 4 yd of brattice cloth, of a width a little more than aver height of seam up to 6 or 8 ft. Each stopping requires at least 3 props and 2 boards.

Manager *F* (above) must immediately estimate supplies on hand, and then order from neighboring mines or supply houses what is needed. UNDERGROUND TELEPHONE supplies are also necessary for communicating with underground bases. Iron-clad box phones are best, but ordinary wall receivers will do temporarily (Art 17, Communication apparatus). Say 100 to 200 suits of miner's overalls should be ordered; also supply of rubber boots, if mine is wet or flooded.

Safety and electric lamps. Even a safety-lamp mine rarely has enough extra lamps (unissued to those entombed) to supply the rescue parties. These must be ordered by wire, with extra glasses, wicks, repair parts, cleaning tools, and gasolene-charging tank. Permissible portable electric miner's cap lamps, with charging board, should be ordered if the mine has electric power for charging; also, electric flash lights, each with 12 extra dry-cell batteries and 2 extra bulbs, are useful in searching for bodies and for general use.

Disinfectants. Chloride of lime, carbolic acid, and other disinfectants should be obtained; if many mules have been killed large quantities will be needed, their bodies being covered with disinfectant and brattice cloth.

Fresh cloth gloves should be supplied on each shift to the men handling bodies, and these and the clothing of the dead should be burned. If bodies are badly decomposed, rubber gloves should be used and disinfected at end of each shift. Outer clothing of men handling bodies should be daily disinfected; if in bad condition, burned and replaced by new clothing.

Underground rescue work should be begun as soon as the parties are organized, but not sooner. Manager of rescue work should secure blue-print copies of the mine maps for the chief of each crew and each state inspector. An engineer, if at hand, should prepare an outline map of entries, slopes, gang-ways, and shafts, with outline scheme of former ventilation. Persons must not be allowed to enter mine singly, but only in groups of 5, under competent mining men. If trained breathing-apparatus crews are at hand they should lead; at least one of the party being acquainted with the mine. Parties without apparatus should proceed cautiously, testing for firedamp with safety lamps (76).

If possible each party should have CH_4 and CO detectors, and for quick indication a canary, to determine presence of CO; if the canary falls (Table 7, Art 5) the party should return to the point where the air was good. Rescue parties should follow the intake air. It may be necessary to wait until the fan or air connections have been repaired, or until temporary hoisting arrangements are made at intake shaft. Such delays are unavoidable, to prevent foolish sacrifice of life due to entering undiluted afterdamp. Sometimes natural ventilation will start by differences in temp and elevation (Sec 14), and thus permit entrance without waiting for fan repairs. Brattice crews should replace temporarily each successive stopping, giving time after each has been set up to clear the gases ahead. The FIRST PARTS OF MINE TO BE SEARCHED ARE NOT THOSE SHOWING MOST VIOLENCE, BUT WHERE THE EXPLOSION HAS WEAKENED OR DIED AWAY, SINCE IN SUCH PLACES THERE MAY BE LIVE MEN. As the U S Bur of Mines has established rescue stations with trucks and there are private rescue stations in different places, trained crews with breathing apparatus will probably arrive within a few hours. These crews should proceed as follows: a crew goes ahead to next open crosscut, if not over say 100 ft; through this and back on the return for some distance, and back to temporary base, examining for smouldering fires, extinguishing them, and noting location of bodies. Then the open crosscut in front of base should be bratticed, and intake air forced ahead for advancing the base. On branch and butt entries, rooms should be successively examined by the crews, to determine when necessary to carry brattices into them. As soon as the bodies are removed from any pair of entries these should be bratticed off. With rare exceptions, BREATHING-APPARATUS CREWS SHOULD NOT HANDLE BODIES; THEY SHOULD RESERVE THEIR STRENGTH AND TIME FOR THEIR SPECIAL FUNCTION OF SEARCHING FOR THE LIVING, EXTINGUISHING FIRES AND LOCATING BODIES They should not work more than 2 hr at a time. Thus, let crew *A* be relieved by crew *B* for 2 hr, while *A* waits at base; then *B* rests while *A* again works 2 hr; *A* then goes out of the mine. As it may take say 0.5 hr to come and go to headquarters outside, each crew, on finishing its second 2 hr and going outside, will have been on duty 7 hr. Allowing 0.5 hr at beginning and end of shift for eating and 8 hr for sleep, the crew will be able to resume its round, being thus 14 hr out of 24 on duty, of which 8 hr will be under oxygen. This is about the maximum, and more rest should be allowed in recovering a large mine. On above basis, continuous advancing exploration would take a *minimum* of 4 crews of 5 men each, but 6 or even 8 crews of apparatus men would be far better. Bodies are labeled by a number tag, and location and condition recorded in a notebook. On rib or roof close to the body its number is marked in a circle. Knowledge of the position and condition of the bodies is often vital in determining the origin of an explosion.

Investigation of explosion. State inspectors are required to make an inspection and report, which is usually done after most of the bodies have been recovered, and ventilation restored. Inspectors often forbid other examinations until they have made their own.

Operators generally make an independent inspection. Bur of Mines engineers also investigate the cause and mode of propagation of the explosion from point to point; their report is not published, but information is furnished to the manager with a view to remedying dangerous conditions. The Bureau publishes data on the various explosions in a district, but, to avoid legal responsibility, without naming the mines.

19. RECOVERY WORK AFTER MINE FIRES

Preliminary procedure (71) in reopening mines or mine areas which have been sealed (Art 15, 17). Before disturbing the seals it is well to obtain mine air samples from behind the fire stoppings, through pipes left in the stoppings, or by drilling holes through them or at one side, and inserting pipe with valve.

In collieries giving off CH_4 freely in the sealed area, its presence is usually indicated by leakage through or around the stoppings, and men engaged in sampling, drilling or reopening must have no open lights. Also, the hole, if through concrete or brick, is drilled slightly downward, and kept filled with water, to avoid sparks struck at moment of finishing.

Absence of CO in fire-area atmosphere indicates that active FIRE IS OUT; but it is not conclusive, for, if the point of sampling is far from a smouldering fire, the CO actually produced may be absorbed by moisture on the walls. Mine-air samples from a hole drilled to or over the seat of the fire give truer results. They must be taken when the movement is outward, as when barometer is falling, though if sampled through a drill hole the air can be pumped out. If the movement is not outward, the sample will be defective, due to air leakage through or around stoppings. If the sample shows any CO, the fire is probably alive, and unless time is vital it is wise to defer reopening, meantime recoating all firewalls.

Reopening a mine or fire area, after a small fire, especially if near the working face, rarely presents difficulty. But if the fire is nearer the entrances, and, from its duration before sealing, was known to be extensive, especially if the whole mine was sealed, the problem is more serious, above all in a gaseous colliery, but also in any mine in which there is extensive timbering or rich sulphide ores. In this case, it is almost certain there are hot coals buried under ashes and roof falls, and if fresh air is admitted it is likely to revive the fire.

Reopening a gaseous mine, in which there is smouldering fire, is always dangerous, since, if air is admitted, it may make an explosive mixture with CH_4 and distilled hydro-

carbons. Men with hose and wearing oxygen or smoke helmets should first enter behind stoppings to extinguish the fire. Build airlocks of flooring material, with 2 doors and space between, in front of the fire wall which seals the drift entrance, or over shaft, if top of shaft is sealed off. Then have seal broken by men using breathing apparatus (Art 17). If seal is of concrete, certain preliminaries are necessary before erecting the airlocks, so that heavy work will not have to be done by men in apparatus.

This preliminary work consists in cutting a groove around the seal with chisels (preferably pneumatic chisels), as nearly through the concrete as possible without bringing it down, having tarpaulin ready to throw over in case a hole is knocked through. Sometimes, at drift or slope entrances, it may be best to tunnel around the stopping, but the airlock is built before breaking through. Because of the difficulty of reopening, temporary firewalls are best made of brick laid in lime mortar, and coated with cement.

Blasting down firewalls in the presence of firedamp must be done with care; only quick-acting permissible explosives should be used, tamped with fine rock dust or dry cement. This will make an inert dust cloud that will prevent ignition of CH_4 or coal dust. A battering ram, of a heavy timber suspended by wires or ropes from temporary framing, is efficient for knocking down brick walls. In shafts, provision is necessary for lowering and hoisting men and supplies through the airlock. If regular cages can not be used, buckets, or an emergency cage (Art 17) is required. After building an airlock a trained rescue crew wearing apparatus begins investigation.

In critical cases a relief crew in apparatus should be in waiting. If water collects on the shaft bottom to a depth preventing exploration, it may be best to fill the shaft with cinders or gravel to a point above the roof level of the entries, to permit taking off the shaft seal and ventilating to bottom. Then pump out the water. Finally, construct an airlock at bottom, and explore the mine with aid of oxygen apparatus. The investigation should extend as far as it is safe to travel, but not exceeding 45 min (1.5 hr round trip) from base, so that if all goes well there would be on returning 0.5-hr oxygen and soda supply available. This provides little enough margin if difficulties are met, and 1 hr round trip from base is a safer limit. Thus, if mine passages are clear and not hot, a distance of 1 mile from base can be traversed; but under adverse conditions, as extensive roof falls or accumulation of water, it is unsafe to advance more than 1 200 ft from base. If seat of fire is beyond these limits, the party should merely take samples of the air, and study where new stoppings can be erected nearer the fire zone (71).

After conditions within the sealed-off area have been determined by one or more trips of the apparatus crews, the problem is: (*a*) whether stoppings are to be removed and ventilation restored; (*b*) if fire is too distant or extensive to be inspected with breathing apparatus, to make new stoppings nearer to fire zone, to permit part of the mine to be recovered for operation, and carry ventilation to the new stoppings, which will serve as a base for further inspection.

Advance fire stoppings, put in by apparatus crews, are usually of brattice cloth and boards nailed to props. These are generally sufficient to prevent serious escape of extinctive gases, while more substantial stoppings are being erected in their rear. Pneumatic stoppings (Art 17) are suitable for temporary use, when the passageway is of uniform section.

Restoring ventilation in a fire area where there may be a smouldering fire or hot buried coals is dangerous in a gaseous colliery. Mine-air samples should be taken and analyzed to determine that the atmosphere on being diluted will not be explosive. If it is likely to be explosive at one stage of dilution it is safer to withdraw all men from the mine as soon as the stoppings have been opened, which, under serious conditions should be opened automatically, or by blasting by electric circuit from surface. Men should be kept out until, by making frequent analyses of the return air, danger is judged to be over. When men can enter, inspection of the fire area is made by a party carrying a canary. If the fire is found to be reviving, water lines are brought in quickly, and placed on the fire. Fallen rock, under which there may be coals hot enough to flame in presence of fresh air, should be drenched before removal.

20. FIRST AID TO THE INJURED. MINE HOSPITALS

"First aid," is the giving of immediate assistance to an injured person with the means at hand, that may prevent injury from becoming permanent, may save his life, tend to prevent dangerous infection of an open wound, and to lessen pain. It is especially important in mining, since a severely injured man at a distant point of a large mine may not be immediately found, and by the time an empty car can be secured, hauled to the shaft and hoisted, an hour or more may elapse, and perhaps another hour before a surgeon can be brought to the mine receiving room or the miner's home. First aid was first taught to miners in the Penn anthracite district, and afterward extended to many mining districts by the Bur of Mines and Red Cross Soc. It has been a marked success; much suffering has been prevented, and many lives saved. It has taught miners the importance of

hygiene in their daily lives, disciplined them by the necessity of team work, and by inter-mine, state, and national first-aid contests, has given incentive for young miners to advance themselves.

First-aid directions for injuries are given in Bur of Mines circular 8 and "Manual of First-aid Instruction," 1935 (82, 83).

First-aid organization should be formed at every mine, to train men and for social betterment and improved living conditions. Officers of the organization should include a medical director. For practice, membership should be divided into teams of 6 men: 1 captain, 1 patient bearer, and 4 stretcher bearers. Joseph A. Holmes Safety Assn has now (1938) 472 Safety Chapters in 28 states, which cooperate with the Bur of Mines in promoting first-aid and rescue training in all branches of the mineral industry.

Equipment needed for practice. Each squad should have: 12 triangular bandages; 12 medium-size safety pins; 6 packages gauze (plain or picric); 6 first-aid outfits; 6 light wood or yucca splints, 3.5 in wide by 18 in long; 12 roller bandages, assorted sizes; 2 tourniquets; 2 rolls cotton, plain or absorbent; 2 blankets (U S army preferable); 1 stretcher (U S army regulation); 6 wooden splints for legs and back fractures; 1 or 2 sets of first-aid charts.

Organizing teams (76). Men working in same part of mine, and on same shift, should be grouped. Each team should have a first-aid cabinet at a convenient point in their part of the mine, and each member always carry at least one first-aid packet, containing a triangular bandage and sterile compress.

In case someone has been hurt, a first-aid man should hasten to the scene, and quickly and carefully examine the victim. If the injury is serious, other members of the team should be collected, and the mine physician notified (by phone, when the mine is equipped with phones) so he may know the nature of the injury. For gathering the team, each member might have a policeman's whistle, a code of whistles being prearranged to signal the locality of an injured person. If there is a flow of blood, it should be stopped by pressure on the right spot. Place patient in as comfortable position as possible, keep him warm, and cheer him up.

Necessity of antiseptic conditions. Bacteria or germs are minute vegetable cells, found in air, water, the ground, clothing, and on and in the body. The mouth and sputum are filled with germs, some harmless, some more or less harmful, besides those which produce specific diseases. For propagation, each requires a certain food, temperature, and environment. Having found a suitable breeding place, which may be in the human body, they multiply rapidly and give off poisonous substances (TOXINS). These may simply be irritants, or may destroy tissue. Under some conditions harmful bacteria may produce blood-poisoning. Bacteria gain entrance to the body through wounds, or the digestive system; first-aid deals with the former source of infection.

Sepsis arises from entrance of harmful bacteria into a wound, causing inflammation; poisonous substances are produced, destroying tissue, forming pus and preventing healing. To avoid sepsis, prevent germs from entering a wound by keeping unclean things from contact with it. Destroy or prevent growth of germs that may have entered at the time of injury.

Antisepsis is the treatment of wounds by DISINFECTION. Disinfectant, or germicide, kills bacteria, and their spores or eggs. Destruction of disease germs in clothing and excreta is called STERILIZATION.

Hemorrhage. Loss of one-third of the blood in the body usually causes death. Compression and position are methods of first-aid treatment. Compression is applied by fingers, compresses, tourniquets, or constricting bands (handkerchief, belt strap, suspenders, or other means). In ARTERIAL HEMORRHAGE, the most dangerous, blood gushes in a bright red stream. Pressure must be applied between the wound and heart. In CAPILLARY HEMORRHAGE, bright red blood oozes slowly; easily controlled by applying a clean compress of gauze. In VENOUS HEMORRHAGE the blood is dark red or blue, and discharges steadily. The compression should be on the side of wound away from the heart. Usually, bleeding can be controlled by applying a compress of sterile gauze directly over the place. Elevating the bleeding part aids in controlling flow of blood. After stopping bleeding, the patient should be treated for shock.

Shock is a sudden depression of vital powers, arising from injury or profound emotion, inducing exhaustion. Symptons are sub-normal temp; irregular, weak, rapid pulse; cold, pale, profusely-perspiring skin; irregular breathing; patient usually conscious, but stupid and indifferent, lying with partly-closed eyelids. Be sure there is no concealed hemorrhage, needing immediate attention. TREATMENT: Lower patient's head, wrap him in blankets, or whatever clothing is available. If possible give an ordinary stimulant like black coffee, to be sipped hot; or, half-teaspoonful doses of aromatic spirits of ammonia every 20 or 30 min. Inhalation of oxygen is often useful; artificial respiration is necessary in some cases. Hot applications over heart and spine should be made.

Contusion, or bruise. Skin is unbroken, or merely injured. Symptoms: tenderness, swelling and numbness, followed by aching pain. Discoloration occurs quickly in surface contusions. TREATMENT: elevate injured part, bandage tightly to arrest bleeding and control swelling; apply icebag or towels wet with cold water.

Wounds are classified as: INCISED wounds, made by a sharp instrument; CONTUSED, by a blunt object, like a fall of rock; LACERATED, by tearing or dragging, as by machinery. Symptoms: Pain, bleeding, and gaping or retracted edges. TREATMENT: Remove clothing from wound at once; arrest bleeding (see above) and put on sterile dressing. Never wash wounds; leave this to physician. If done in the mine, the wound may be infected by washing unclean substances into it.

Fractures are the most important injuries for treatment in mines by first-aid, because the victim is temporarily crippled, and if neglected or ignorantly handled there may be permanent crippling. Kinds of fracture: INCOMPLETE, bone not entirely broken; COMPLETE, bone is broken; SIMPLE, broken bone does not protrude through the flesh; in COMPOUND FRACTURE, one broken end protrudes, or has cut or torn the flesh down to the bone. Compound fracture is nearly always accompanied by loss of blood and severe shock. Symptoms of fracture: pain, swelling, discoloration, abnormal motion of bone, loss of power, and grating together of bone ends. TREATMENT: Examine with gentleness; handle limb as little as possible; draw it into natural position and fix by splints. If nature of injury is in doubt, await the doctor; a sufferer from broken limb should never be moved until the part is properly supported by splints.

Dislocation is displacement of surfaces of a joint. In simple dislocation, articular ends are separated without injury to surrounding tissue; in compound dislocation, the ligaments around joint are torn. In complicated dislocations, muscles, vessels, and nerves are injured. Symptoms: Pain, swelling, discoloration, rigidity, natural position of limb is changed and length altered. TREATMENT: Restore bone to normal position and hold it in place. First-aid men should never try to reduce dislocations, except those of jaw and fingers; some surgical skill is generally needed.

Sprain is twisting or wrenching of a joint, producing a tearing of ligaments and sometimes of surrounding soft parts. It is followed by severe pains, and marked swelling and discoloration. TREATMENT: Let injured person rest. Elevate injured part and fix it in place by splints, or wrapping joint tightly with a roller bandage or adhesive plaster. Give hot or cold applications by placing injured part in hot or cold water, or by towels wrung out of hot water or ice water.

Strain is wrenching or tearing of a muscle or tendon; usually caused by violent exertion, or sudden movement. It causes sharp pain. TREATMENT: Let injured person rest; bandage injured part tightly, or apply adhesive plaster; splinting sometimes necessary.

Burns and scalds. FIRST DEGREE burn is simply a scorching or reddening of outer skin; SECOND DEGREE destroys entire thickness of skin; THIRD DEGREE destroys skin and tissue beneath. Shock caused by severe burns. TREATMENT: Carefully remove clothing from burned surface, from which exclude air at once with clean covering. Best covering is picric-acid gauze (sterile gauze, saturated with 1% solution of picric acid, or 0.5 teaspoonful to 1 pt water). Moisten the gauze with clean water and apply it to burned surface. Place absorbent cotton over the gauze, and apply bandage to hold it in place. Vaseline, sweet oil, olive oil, and balsam oil are all good dressings. If nothing better is at hand, dissolve bicarbonate of soda in sterilised water. Gauze dipped in this and spread over the burn gives relief. Carron oil (a mixture of equal parts of lime water and linseed oil) has been used, but is not recommended.

Electric shock. Electrical contact with a live wire causes dangerous shocks, burns, and sometimes death. Symptoms: sudden loss of consciousness; absence of, or very light, respiration; weak pulse, and burns at points of contact. If victim is in contact with wire, do not touch him without first short-circuiting the current. Do not search for a switch, as time is too important, but lean a drill or iron bar from one rail to the trolley line, preferably at a point between victim and power station. If no bar is at hand, stand on a dry board or piece of paper, and pull him off with a gloved hand; or having no glove, use cap or other article of clothing, so as not to receive a shock. Or, drag victim from the wire with a rope, belt, or bandana, looped over his feet. TREATMENT: As soon as patient has been freed, begin artificial respiration (see below).

Suffocation or asphyxiation may be caused by an object blocking the windpipe, by drowning, or by inhalation of gas and air mixtures, which fill the lungs without supplying sufficient oxygen; or by inhalation of poisonous gas, like CO (Art 5), causing unconsciousness, and possibly suspension of breathing by poisoning rather than absence of oxygen. TREATMENT: Quickly remove patient to good air; apply artificial respiration, and if he has been in poisonous gas give him oxygen treatment.

Artificial respiration. There are 2 principal methods, the Schaefer, or prone, and the Sylvester, in which the patient is placed on his back. The Schaefer is generally approved by physicians.

Directions for its use (82). Quickly feel with finger in patient's mouth and throat, to remove any foreign body (tobacco, false teeth, etc); then begin artificial respiration. Do not stop to loosen patient's clothing; every moment's delay is serious. Lay the subject on his belly, with arms extended straight forward, with face to one side so that nose and mouth are free for breathing (Fig 10). Let an assistant draw forward subject's tongue. Avoid so laying subject down that any injured places are pressed upon. Do not permit bystanders to crowd about and shut off fresh air. Kneel, straddling the subject's thighs, and facing his head; rest the palms of your hands on victim's loins (or muscles of small of back), with thumbs nearly touching each other and with fingers spread over the lowest ribs (Fig 10a). With arms straight, swing forward slowly so that that the weight of your body is gradually brought to bear on the subject (Fig 10b). This operation should take 2 or 3 sec, and must not be violent, lest internal organs be injured. Lower part of chest, and also the abdomen, are thus compressed, and air is forced out of the lungs. Then swing backward, to remove pressure, but leave your hands in place, returning to first position. By their elasticity, the chest walls expand and the lungs are supplied with fresh air. After 2 sec, swing forward again, and repeat the 2 movements 12 to 15 times a min, making a complete respiration in 4 or 5 sec. Having no watch or clock, follow natural rate of your own breathing, swinging forward with each expiration and backward with each inspiration.

While this is being done, an assistant should loosen any tight clothing about the patient's neck, chest, or waist. Continue treatment, if necessary, 2 hr or longer, without interruption, until natural breathing is restored, or until a physician arrives. After natural breathing begins see that it continues; if it stops, resume artificial respiration. During the operation keep subject warm by

proper covering, and by laying beside his body bottles or rubber bags of warm (not hot) water. Keeping subject warm should be done by an assistant. Do not give any liquids whatever by mouth until subject is fully conscious.

It is considered unnecessary to give oxygen in addition to manual treatment unless patient had been "gassed." Mechanically-forced respiration is not approved, because of possible injury to lung cells, and when oxygen is advisable it should be by hand manipulation, keeping time with normal respiration. Inhalation apparatus can be utilized in unison with the manual treatment, as by the Schaefer method, a mask being fastened over the nose and mouth, and the apparatus (Art 17) operated by an assistant.

Bandages are to keep dressings in place, retain splints on broken limbs, stop bleeding by pressure, and as slings. Kinds: triangular, roller, and special bandage of U S army. TRIANGULAR BANDAGE is useful in first-aid work; easy to make and apply. It should be of unbleached cotton, linen or muslin; cheese cloth is too soft and difficult to fold properly; bed sheeting or pillow cases are good. The cloth should be 34 to 40 in sq, and by cutting diagonally 2 bandages are made. Use safety pins and knots for fastening. ROLLER BANDAGE is of muslin, flannel, gauze, or cheese cloth, wound on itself to form a tight roll; requires practice to apply properly.

Tourniquet, to stop profuse bleeding from arteries or veins, consists of a strap around the limb, and a pad under the strap, to press on artery or vein. Many first-aid cabinets contain U S army regulation tourniquet: a strap of webbing, with buckle and catch. An improvised tourniquet may be made of a handkerchief, towel, triangular bandage, pair of suspenders, or belt strap. Tie around the limb, with a pad over artery or vein, pass a stick through the loop, and twist until blood barely oozes, or is stopped.

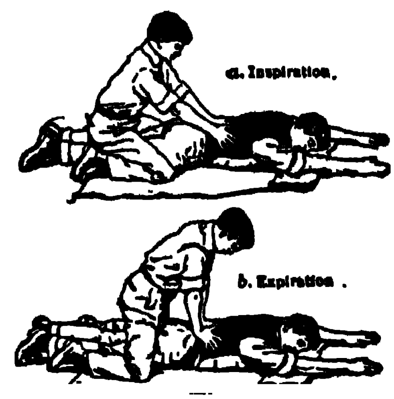


Fig 10. Schaefer Method of Artificial Respiration

Transportation of the injured. In a mine it is often impossible to bring an empty car near the point where the victim lies; he must sometimes be carried long distances. If done in a clumsy manner, much of the benefit of first-aid may be undone. Carry serious cases on a stretcher, sometimes improvised, by using mining drills passed through the sleeves of 3 or 4 coats, the coats being buttoned. These are established first-aid methods of carrying a patient by 1, 2, 3, or 4 men, with and without stretchers.

Ambulance car should be maintained by large mines. It saves much suffering if arranged with springs and suspension supports for a stretcher.

Underground hospital. In extensive mines, the car containing the injured man may be delayed at bottom of a cold downcast shaft. On this account, and to maintain medical and first-aid supplies, some mines have underground hospitals where injured persons can receive supplemental treatment. They should be well-lighted, painted white, and kept clean; have an operating table for first-aid treatment, first-aid equipment, drinking and washing water, etc.

Surface hospital near the mine entrance is advisable, as most mines are distant from a town hospital, and a patient might have to wait in heat, or cold, or a dusty building, for ambulance. A small well-lighted room is sufficient, painted white, with smooth floor, and equipped with first-aid supplies, cot, table, water supply, and disinfectants. Surface and underground hospitals may be inexpensive, but should be absolutely clean.

MINE SAFETY

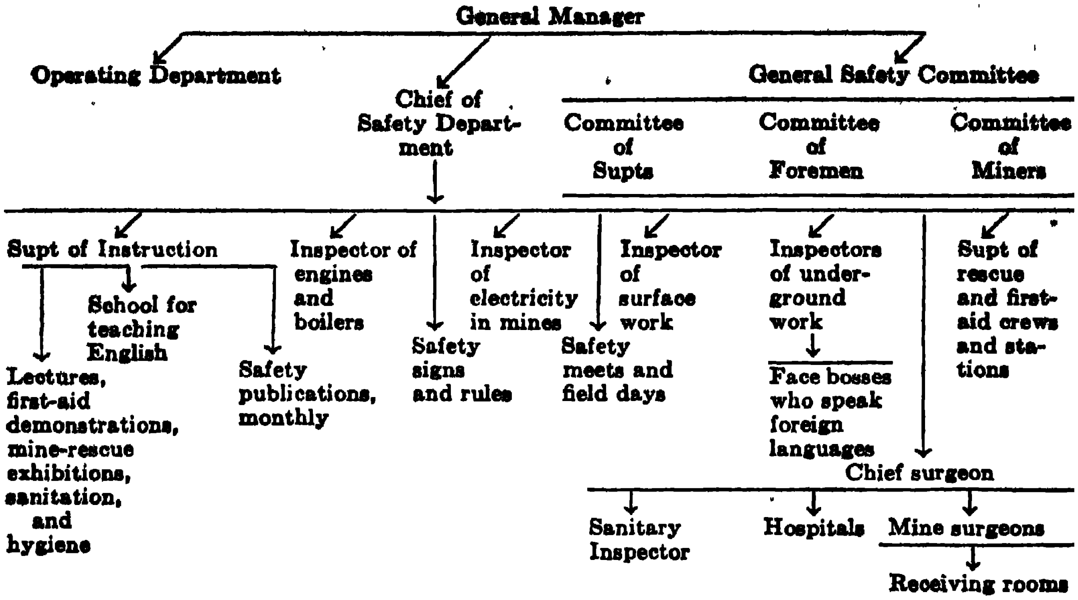
21. SAFETY ORGANIZATION FOR MINING COMPANIES

Safety organization is essential to sound business policy, and rests on humanitarian grounds.

Compensation acts of many states have greatly increased cost of settling death and injury claims; in some states this cost, in percentage of payroll, is 2 to 4 times that which prevailed up to 1910 (Art 24). But compensation now goes more fully to injured persons or needy heirs, and is not as formerly so largely consumed in litigation. Mining companies should care for widows, orphans, and injured, resulting from accidents, instead of depending on public charity. The industry also is benefited by safety improvements, since the cost of accidents must ultimately be added to selling price of product. See Sec 22, Art 8.

Avoidable accidents in U S mines form 60 to 75% of the total. They can be lessened by inspection, education, discipline, safety appliances, and proper organization. Best kind of organization is parallel to, but independent of, the operating department.

Plan of Safety Organization for Large Companies, Suggested by Rice



An energetic manager, in the effort to economise in mining, transport, and loading ore or other product, may overlook questions of safety, and for economy or speed continue to use machinery that should be scrapped. On the other hand, an overcareful, timid manager, constantly fearing accident, becomes inefficient. But, if the organisation includes a competent inspector, responsible to a higher authority than the operating manager, and reports a certain practice unsafe or recommends a change, the manager is relieved that he is not responsible for the cost incurred.

22. MINE SUPERVISION AND INSPECTION

Supervision. Mine accidents are often due to isolation of the miners, one or two in a working place, rendering supervision difficult. While there is usually a foreman, with shift-, timber-, and other special bosses, many miners may be scattered in remote workings, with only a single shift-boss to supervise a large area; especially true under the contract system. As the shift-boss may not be able to visit each working every day, proper timbering and other safety precautions are apt to be neglected. There should be enough assistant foremen or shift-bosses to inspect every working at least twice a day; 4 times a day is the practice in the Yorkshire longwall mines. The best practice is to have one boss to 20 to 25 miners.

Face-boss. Where many underground workers can not speak English, it is important to have a "face-boss," who is a good miner and able to speak several languages, to direct foreigners as to their duties; or, if a written order has been given by the foreman, to translate and explain it. Accidents often happen because the foreman, in giving instructions to foreigners, has supposed they understood when they said "yes, yes," while knowing no English.

Inspection by general company officers every 2 or 3 months, by general supts 2 or 3 times a month, and by mine supt and foreman each day, is vitally important. There must also be detailed inspection by special inspectors. In collieries, especially when gaseous, a FIRE-BOSS must, under some state laws, visit all workings and abandoned places which are not walled off, within 2 hours before entrance of day shift, to test for gas or firedamp, fence off places making gas, and mark dangerous places in the roof for setting timber. On returning to the mine entrance, and before the men are allowed to enter, he shall post information concerning dangerous or gassy places, and enter notes regarding the inspection in a book kept for the purpose. Large mines may require several fire-bosses.

Private mine inspectors, as distinguished from state inspectors, are employed by large companies to report on observed conditions to a higher official of the company, in some cases to the General Manager, in others to the President. This system has resulted in improvement in safety and hygienic conditions; also, through information secured from the miners, it corrects injustice to individuals, and produces better relations between company and employes.

Checking men in and out of mines. At large mines, especially collieries, there should be a complete record of every one underground at any time.

The simplest mode of doing this is to number every employe, keeping his name and place of working either in the pay-roll ledger or in a special book in an office near the mine entrance. Before entering the mine, each man must pass by the office window (the shaft or mine entry being fenced in), and receive a metal check bearing his number, preferably arranged with a safety-pin, for fastening to his clothing, or with a small chain passed through a button hole. Before distribution, as each shift goes on duty, the checks hang on nails on a frame, with a number over each nail. As the number of employes changes, unused numbers are covered by hanging pieces of cardboard over them, the corresponding checks being kept in a drawer or box. As each man comes to the surface and passes by the checking office, he leaves his check, which is hung on the board. Examination of the board at any time shows how many men are underground, and a review of the book, in which each man's working place is recorded, indicates where the men are; an important matter in case of fire or explosion.

Inspection of men and equipment entering a gaseous colliery should be regularly made by a responsible man at the mine opening. He examines lamps, tests safety lamps, and examines clothing of an occasional man and all new employes to see that no one carries matches, smoking tobacco, or pipes. It is possible to light tobacco by heating a wire inserted through safety lamp gauze, or by producing an electric spark through manipulation of an electric lamp or battery; hence possession of smoking material indicates dangerous practices.

23. SAFETY MEETINGS AND PUBLICATIONS

Safety meetings, as an aid to increased care in preventing mine accidents, educate miners and foremen alike, and foster harmonious relations between employer and employe. State and national contests attract attention of miners to the movement and its importance to themselves. SAFETY PUBLICATIONS, issued by mining companies from time to time, do great good, because employes who can read English learn how a certain accident occurred, and how it might have been prevented.

Joseph A. Holmes Safety Assn (see Art 20).

Danger signs and direction signs are useful in preventing accidents, and by adopting uniform symbols, especially the red ball for danger, and the arrow pointing the direction in which to go, non-English speaking miners can understand what to avoid and where to go safely.

Bonus for workmen, as a reward for safety records and suggestions, has distinct value. A large coal Co in W Va, with a very low accident rate, gives a substantial sum monthly to foremen in its half dozen mines. As the mines are sectionalized, the results indicate how large a factor leadership and characteristics of different foremen is in attaining max safety possible under the natural hazards in mining. Other companies have similar practice.

24. MINE-ACCIDENT LIABILITY INSURANCE

Workmen's compensation acts. Until mining states began the enactment of these Acts, the cost of death and injury cases was generally small as compared with payroll cost; widows, orphans, and injured were generously treated by some companies (Sec 22).

To about 1910, coal-mine accident insurance ranged in different states from \$1.50 to \$3.00, and limited insurance was given by one cooperative company in a central state on preferred rates at 75¢ per \$100 of payroll; in the same state rates now range from \$3.50 to \$5 per \$100, and are quoted at \$7 on coal mines in a western state that has had many disasters. Metal-mine compensation costs have been similarly increased by compensation acts of the respective states.

Insurance for meeting liability arising from mine accidents has been of questionable value to operators when flat rates were given in each state by standard insurance companies, regardless of whether safety precautions were observed or not (Sec 22).

Rating of coal-mine hazards to determine premiums has been adopted by some accident insurance companies. They have used a system of credits or discounts for rating organization and for different accident-prevention methods and devices, and employ mine inspectors who not only inspect but give advice. Some states, notably Penna, have accident-liability insurance dep'ts, employing inspectors for rating mines, based on specific valuation of each hazard. They use essentially the plan of certain private Ins Cos, of marking demerits for deficiencies and awarding bonuses for meritorious precautions in each item of hazard rating (86). Placing a money value on safety precautions is important in introducing proper measures and lowering accident rates.

25. STATE AUTHORITY, REGULATIONS AND INSPECTION

Jurisdiction. Each state has police power over the mines within its borders and makes its own regulations, which unfortunately differ widely, instead of being uniform for the same class of mines and mining conditions. Best mode of formulation and administration of mining regulations in each state is probably through a permanent non-partisan industrial commission, to which inspection bureaus and the workmen's liability accident insur-

ance bureau report. Before altering a mining code, public hearings are held, to which interested parties are invited as in N Y, Penna, Wis, Calif, Utah and other states.

Coal-mine regulations have long been on the statute books in the leading coal mining states; one of the best codes is that of the Penna bituminous district.

Operating regulations for coal mining on the public domain (Bur of Mines, 1920), are the most comprehensive of any coal mining codes, in defining mining methods for safety and coal conservation. Respecting mine-fire prevention, the Illinois regulations are the most explicit and stringent, though some features are impracticable if taken literally.

State metal-mining regulations are neither complete nor satisfactory; among the most complete codes are those of Calif, N Y and Utah. A committee of metal-mining men has also formulated a code which has been published as a model by the Bur of Mines (85).

State colliery inspection. Practically every coal-mining state, 25 in number, has a mine inspection bureau. In some states, as Penna and W Va, the head is called Chief of the Department of Mines; in Ohio, the Safety Commissioner of the Industrial Commission is in general charge; in most states, the head of inspection is called Chief Inspector of Mines; in others, simply Mine Inspector.

In the important coal-producing states, as Penna, W Va, and Ill, there are many mine inspectors, each having a specified district, roughly proportioned according to tonnage produced. There is no definite unit, but, in states with large production, there is an inspector for every 2 million to 8 million tons, produced in say 10 to 60 RR shipping mines and many wagon coal banks. Good service requires thorough inspection of every mine in a district 4 times a year; which means, even if mines are close together, that not over 30 large mines can be thoroughly inspected by 1 man. The inspection unit in point of production should not be more than 4 million tons per annum, and, with widely scattered small mines, not over half this. Quality of inspection varies widely. Most important coal-mining states require candidates for inspectorship to take an examination, the inspectors being chosen by the governor from those that pass.

State metal-mine inspection. Latterly, increased attention has been given to this service by many western states, and several have passed regulations for safety; but as yet most state regulations for metal mining are lax as compared with those for coal mining.

County mine inspectors. In some states, the counties have appointed inspectors who may or may not cooperate with state inspectors. Generally, county inspectors have little authority, and are regarded as filling purely political jobs. Occasionally, a strong, energetic inspector has been able to introduce reforms by force of personality.

26. FEDERAL SAFETY INVESTIGATIONS AND PUBLICATIONS

Origin and scope. In 1908, Congress appropriated funds for investigating mine accidents and explosions, and placed the work under the Technologic Branch of U S Geol Surv. In 1910, Congress established the Bur of Mines, transferring to it mine accident and fuel investigations. In 1913, the powers of the Bureau were widened by establishing "in the Department of the Interior a Bureau of Mining, Metallurgy, and Mineral Technology . . . to conduct inquiries . . . concerning mining, and the preparation, treatment, and utilization of mineral substances, with a view to improving health conditions, safety, efficiency, and economic development. . . ."

July 1, 1925, the Bureau was transferred to U S Dept of Commerce; leasing supervision remained under the U S Geol Surv, but the Mineral Statistics division was transferred to Bur of Mines. In Apl, 1934, the Bureau was returned to Dept of the Interior.

Restrictions. Bureau representatives have no right of entrance into mines, except as permission is voluntarily given by mine operators. The Bureau's reports on individual mine accidents are not published, as these might conflict with state inspector's reports, and lead to legal difficulties, but information is given to the mine operator, with recommendations, so that similar accidents may be avoided in future.

Rescue and first-aid equipment. To educate and train miners in first-aid and the use of oxygen-breathing apparatus, there are 11 fully-equipped safety or rescue cars (reconditioned Pullmans, with berths for crews and injured). Crews: foreman miner, first-aid miner, and cook, and, for a fully manned car, a mining engineer, mine physician, and clerk. Recent appropriations permit but few cars to remain in commission.

These cars circulate through different mining districts, giving instruction, and in case of a disaster the nearest car or cars are rushed to the scene. Besides the R R cars, 11 stations are equipped for giving training, at Pittsburgh, Birmingham, McAlister, and elsewhere, the foreman and field mining engineers going immediately to any disaster to assist and investigate the cause.

Testing appliances and conditions for safety in mines. The Bureau conducts, chiefly at the Pittsburgh Experiment Station, testing of explosives, electric appliances, safety lamps, breathing apparatus, mechanical safety appliances, and at the Bruceton Experimental Mine (12 miles from Pittsburgh), tests on gas and coal-dust explosions, strength of mine pillars and timbers.

Federal inspection of mines is conducted only in Alaska, on the segregated Indian lands in Oklahoma, where the Department of the Interior acts as agent for the lessees (Indian nations), and in a few mines on government lands and in the National Parks in western states. This inspection, formerly under the Bur of Mines, has been done since 1925 by the U S Geol Surv.

Publications of the U S Bureau of Mines on mine safety and health comprise: (*a*) BULLETINS of definite data on general subjects, of permanent reference value, also annual accident statistics; (*b*) TECHNICAL PAPERS on specific subjects, of more or less continuing value; (*c*) REPORTS OF INVESTIGATIONS (mimeographed) on special subjects and tests, not always final; (*d*) INFORMATION CIRCULARS (mimeographed) of data from general observation or foreign literature, not from Bureau investigations; (*e*) SCHEDULES OF TESTS FOR PERMISSIBILITY of specific machines and devices; (*f*) HANDBOOKS of rescue apparatus and first-aid methods; (*g*) MINERS' CIRCULARS for foremen and miners on mine explosions, falls of roof, miners' diseases, etc. Subjects treated: character and use of explosives; limits of inflammability and explosibility of gases, coal and other mineral dusts, and metallic dusts; prevention of coal-mine explosions; mine ventilation and air conditioning; allaying dusts harmful to health; miners' occupational diseases; safer mining and shaft-sinking methods; causes of fall of roof or ore and better timbering; hazards in mechanization and use of electricity; fire prevention; mine lighting; inspection methods; safety organizations; permissibility of safety equipment; bursts of gas, bumps and rock bursts in deep mining; safety and health statistics.

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SECTION 24

MINING LAWS

WRITTEN FOR THE FIRST EDITION BY THE LATE

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OF THE CALIFORNIA BAR

REVISED FOR THE SECOND AND LARGELY REWRITTEN FOR THE THIRD EDITION

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MINING LAWS

This Section has particular reference to Federal Mining Laws, and those of the western mining states, largely based upon the text written by the late Horace V. Winchell, of the California Bar. Articles 1–6, on the historical development of the U S Mining Laws, also much of Art 11–13 and 16–20, are changed but little from Winchell's text.

The present edition includes a discussion of extralateral rights by Judge Clayberg; Art 27–32, on Federal Mining Taxes, by Paul Armitage, of the New York Bar; Art 33–42, on the Mining Laws of Canada, by Gordon McMillan, K C, of the Toronto Bar; and Art 43–46, on the Mining Laws of Mexico, by Robert T. Brinsmade, an associate of the law firm of Basham & Ringe, of Mexico City.

The more important Apex cases have been checked by Professor William Edward Colby, of the California Bar. Respecting the Apex Law, the text of the U S Code has been used, rather than referring back to the original statutes. As lack of space prevented verbatim inclusion of the Mining Statutes of the Western Mining States and of Alaska; and because it was necessary to give a summary of the Mining and Depletion provisions of the Federal Revenue Acts, now in force; also the Canadian and Mexican Mining Laws, and forms of perpendicular side-line agreements, with examples of surface mining deeds; it was decided to set forth in some detail the Mining Statutes of California, as illustrative, and to refer briefly to the Mining Laws of the other states subject to the Federal Mining Acts. No reference is made to Mining Laws of states not containing public mining lands.

The U S Code includes a chapter on the Bureau of Mines and one on "Lands Containing Coal, Phosphorus," etc. Parts of the entire Code, and references to other parts by Title, for the information of the mining profession, are presented. The articles on problems of the Apex Law, by Judge Clayberg, are taken from the first editions of this handbook, with brief references to some later decisions.

It must be impressed upon all those who read this Section that its object is only to summarize the mining laws, which, with the statutes, change from year to year. While it is hoped that this cursory review will be generally informative, the writers urge upon all who have legal problems to solve, to consult an able mining lawyer practicing in the State or Country in question.

NOTE.—The reader is particularly cautioned that the information contained in this section of the book is as of Dec 31, 1938, and that, as stated above, changes are frequently made in the mining laws of the several states of the United States and in the other countries cited.

1. INTRODUCTION

Character of mining laws. These laws define the status of the prospector for mineral deposits, establish his methods of procedure, protect him in possession while searching for mines, and give him assurance of title when all required conditions have been fulfilled and valuable minerals discovered. Regulations for the operation of mines, and for the safety and health of employes are also subjects of Federal and State legislation, but are rather in the nature of police and sanitation rules for the daily guidance of mine workers; they are subject to frequent change, and are not considered here. (See Sec 22, 23.)

Importance of an understanding of mining laws is indicated by three considerations:

1. There is always a prescribed course of procedure to be followed in acquiring and perpetuating such title as is granted by the laws, both while a property is in the exploratory stages and after ores have been discovered; and without strict compliance with these statutory provisions there is danger of losing the mines after they have proved of value.

2. The peculiar apex provisions of the U S mining law and the decisions of the Supreme Court are of vital importance to the mine owner in those states where the laws apply. The titles to many mines have been seriously affected or entirely forfeited by inattention to the obvious and unmistakable wording of the statutes.

3. A mining engineer, called upon to place a valuation upon mines in districts where the apex laws are in force, must be familiar with those laws. Possibility of apex litigation hangs over many mining investments, and no engineer can safely recommend the purchase of a quartz (vein) mine without taking it into consideration.

2. THEORIES RELATING TO ACQUISITION OF MINING CLAIMS

Principles of construction of mining laws of the world may be grouped in two classes: 1. That in which the State or a private owner of mining property has the right to grant concessions or leases to individuals or corporations at discretion, or under certain general

restrictions. This is the "Concession" system. 2. That in which any individual, under specified restrictions, generally as to nationality or color, has the right to locate on discovery or otherwise certain limited areas of ground, and under certain conditions to hold, work, or dispose of the same. This is the "Claim" system (Alford, p 1).

Concession system had its origin in the rights of kings and feudatory lords to the mineral products of the ground, and it prevails today in modified forms in all the ancient civilizations. Its advantage is that the State or other owner of mining rights may select its concessionaires, and thus place the mines only in the hands of those who are able to operate them and to pay the required rents, taxes, or royalties. There is incidental economy in the maintenance of order, collection of dues, and the greater stability of mining operations. The system is criticized for placing unduly large powers of property control in the hands of a few men, and for taking away from the poor prospector the chance of sudden wealth; it is also said to destroy competition in the sale of mines, placing inordinately large profits in the hands of a few; and that it conduces to the holding of mining ground unworked, thus reducing mineral production and restricting opportunity for labor. These objections may perhaps be over-balanced by the facts that the terms of concession may and usually do require a certain amount of development; that they reserve to the Government some control of the financial transactions and capitalization of the concession holders, and that the operations of large corporations are generally conducted with greater efficiency and economy, in both prospecting and mining stages, than those of smaller concerns or of individuals.

Claim system originated in the early days of mining in the U S. Prospectors rushed to the newly discovered gold districts of California, later to those of Australia; and to preserve public order, some arrangement became necessary for determining on the spot the area of ground that each man was allowed to work, and the conditions under which he could hold and deal with it. Hence arose the right of the discoverer to a "claim," a term now in general use in legal parlance. Where consolidation of claims is prohibited, the result is waste and many failures; where claims are permitted to be held without systematic development, no good results to the country at large, nor to the claim holder; and it is criticized as being a system which, although it may appear at first sight to encourage individual enterprise, in reality leads to great waste of capital and unorganized energy.

Two fundamental principles form the basis of all mining laws: 1. The right of mine operators to a secure and indefeasible title to their property so long as they fulfil certain specified conditions, compliance with which is absolutely within their own power; 2. The right of the state or other landlord to certain rents, royalties, or taxes on the profits of the mines, and to reasonably constant performance of effective work on the mines, so that capital, represented by property and labor, shall not be unduly idle. These principles are essentially common to the laws of all countries, though in individual cases modified to suit climatic, ethnological, and physical conditions, as well as the history and theories of land tenure of the particular mining region (Alford, p 7).

UNITED STATES MINING LAWS

3. SOURCES OF THE LAWS

Public mineral domain. Since nearly all of the domain in the states and territories subject to the federal mining laws was originally acquired by treaty or purchase from France and Mexico, wherein the civil law was the basis of jurisprudence, and at the time of session both of these nations had well established and defined codes of mining law, it is natural to expect to find the influence of French and Spanish laws in the growth and development of our own system (Lindley, p 6). Traces of the principles of the common law, under which, in general, minerals were the landlord's property and went with the surface, are also found in the old ante-revolution British grants. Some customs and rules adopted in the West are traceable to Cornwall, and from there to Germany. The principles of common law prevail in the U S, and establish in the owner of the surface the *prima facie* title to minerals beneath it, until rebutted by evidence that ownership of the mines and minerals had, from some special cause, become severed from that of the soil and surface. Such separate and distinct title to minerals may arise through legislative acts of Congress or of any state, or through voluntary act of transfer by the individual who once owned both surface and minerals.

Colonial grants. In almost all crown grants to the colonies, there was reserved to the sovereign a certain fixed proportion of the "royal metals" (gold and silver) discovered. Thus the charter of N C (1584), granted to Sir Walter Raleigh by Elizabeth, contained the

following: " Reserving always to us, our heirs and successors, for all services, duties and demands, the fifth part of all the ore of gold and silver that may from time to time and at all times after such discovery, subduing, and possessing, be there gotten and obtained." This form of reservation occurs, with few exceptions, in all succeeding grants.

The U S, in first dealing with its public lands, followed these precedents and made similar reservations, suggested by precedent rather than considerations of public policy. The general policy of the U S, as expressed in the statutes, executive acts, and proclamations prior to 1845, was to reserve mineral lands from sale absolutely. These lands, so far as then known, contained lead, iron, copper and zinc, in the part of the U S territory then called the Northwest or Indian Territory, and comprised the area now embraced by Mich, Wis, Ill, Ia, Mo, and Minn.

Federal leasing system. Within the territory of the Louisiana purchase, acquired from France in 1803, lead mining began about 1720, under patent to Law's Mississippi colony, while that region still belonged to France. On March 3, 1807, Congress passed a law providing: " that the several lead mines in the Indiana territory . . . shall be reserved for the future disposal of the United States; and any grant which may hereafter be made for a tract of land containing a lead mine which had been discovered previous to the purchase of such tract from the United States shall be considered fraudulent and null, and the President of the United States shall be and is hereby authorized to lease any lead mine which has been or may be hereafter discovered in the Indiana territory for a term not exceeding five years."

This legislation inaugurated the policy of the U S of leasing mineral lands. Leases were given under supervision of the War Department, covering tracts at first 3 miles, afterward 1 mile square, and bound the lessees to work the mines with due diligence and return to the U S 6% of all ores raised (Lindley, p 64). No leases were issued under the law until 1822, and but a small quantity of lead was raised previous to 1826, from which time production increased rapidly. For a few years the rents were paid with some regularity; but after 1834, because of the immense number of illegal entries of mineral land at the Wis land office, smelters and miners refused to make further payments, and the government was unable to collect them. After much trouble and expense, it was, in 1847, finally decided that the only way was to sell the mineral land, and do away with all reserves of lead or any other metal, since they had been only a source of embarrassment to the department. Meanwhile, by a forced construction of the same act (afterward declared invalid), hundreds of leases were granted to speculators in the Lake Superior copper region, which from 1843 to 1846 was the scene of wild and baseless excitement. The bubble burst during 1846; the issue of permits and leases was suspended as illegal, and the act of 1847, authorizing sale of mineral lands and a geological survey of the district, laid the foundation of a more substantial prosperity (Hewitt, *Trans* A I M E, vol V, p 180).

Mexican grants. The treaty of Guadalupe Hidalgo, concluding peace with Mexico, was signed Feb 2, exchanged May 30, and proclaimed July 4, 1848.

This treaty added to the national domain more than 500 000 sq miles, embracing Cal, Nev, Utah, Ariz (except Gadsden purchase of 1853), N Mex west of the Rio Grande and north of Gadsden purchase, Colo west of the Rocky Mts, and SW part of Wyo. Gadsden purchase added to the public domain 45 532 sq miles, forming part of the present States of Ariz and N Mex. By the treaty, all property theretofore belonging to Mexico within the limits defined by the compact between the two nations passed to the U S, upon which government was imposed the obligation to protect titles acquired under Mexican rule. A valid right before the cession was equally valid afterward.

Registration of Mexican grants. As to those in Cal, Congress provided for the appointment of a board of land commissioners, to whom all persons claiming lands by virtue of any right or title derived from the Spanish or Mexican government were required to present their claims. Appeals could be taken from decisions of this board to federal courts, even the highest. By acts of July 22, 1854, Feb 28, 1861, and Feb 24, 1863, the surveyors-general of N Mex, Col, and Ariz, respectively, were instructed to report to the Interior Department upon the status of private land claims in their jurisdictions. At present, it is presumed that all rights and claims of every nature to lands arising out of Mexican grants have been finally adjudicated (Lindley, par 117, *et seq*).

4. SOURCE OF THE PUBLIC DOMAIN

The national government acquired no property rights within the present boundaries of the 13 original states, nor in the states of Vt, Ky, Me and W Va; hence, no question of apex rights or other questions of federal procedure or titles arise therein. The first acquisition of national domain which became subject to disposal of Congress was by cessions of territory claimed by 7 of the original states.

These cessions, beginning with that by N Y (March 1, 1781) and ending with that of Ga (April 24, 1802), brought within the jurisdiction of the federal government all that portion of the present

area of the U S now comprising Tenn, Ill, Ind, Ohio, Mich, Wis, those portions of Ala and Miss lying north of the 31st parallel, and all that portion of Minn lying East of the Mississippi River. In 1803, the territory acquired by purchase from France, commonly called the "Louisiana purchase," added over 1 000 000 sq miles to the national domain, embracing parts of Ala and Miss, the states of La, Ark, Mo, Iowa, Kas (except a portion in the southwest corner), Neb, all of Col east of the Rocky Mts and north of the Arkansas River, both Dakotas, the greater part of Mont, part of Wyo, and the Indian Terr. The acquisition of land from Mexico has already been mentioned. Alaska was purchased from Russia in 1867. By treaty of cession between the U S and Hawaii, adopted by Congress July 7, 1898, all lands within the Hawaiian Islands, covered by the treaty, which belonged to the republic as distinguished from private lands, passed to the U S, and became subject to the disposal of Congress the same as other public lands.

As yet, the public-land laws of the U S have not been extended to the new territory of Hawaii, and, until further action by Congress, the land laws of the republic existing at the time of the treaty of cession are continued in force. These laws do not provide for sale or other disposition of lands classified as mineral. By the treaty of Paris, Porto Rico was ceded to the U S by Spain, and all crown lands passed to the U S. By act of Congress, July 1, 1902, all such lands were ceded by the U S to the territory of Porto Rico for the use and benefit of its people, and are subject to sale and disposal under territorial laws of that island. Treaty of Paris also transferred ownership of the Philippine Islands from Spain to the U S. On July 1, 1902, was approved an act containing a code of laws governing disposal of public lands valuable for minerals. This law was amended in 1905 (33 Stats at Large, 692; Comp Stats, Supp 1911, 527; 10 Fed Stats Ann 267). It will be seen later that federal statues regarding the public domain do not apply to all territory controlled by the U S. As to one portion, there was no congressional jurisdiction for lack of title, and as to still other portions there has been specific exception by act of Congress.

5. THEORY OF UNITED STATES MINING LAWS

Character of mining property. Mines in the U S are not ranked as public property, to be worked by the federal government. Mining with us is not a " public utility," but a private industry, to be fostered and regulated like all other economic industries (Lindley, p 120). The regalian doctrine of the crown's ownership of the royal metals, wheresoever found, based upon the theory that these metals were a prerogative of the crown, which prevailed in England, France, Spain, and Mexico, was never recognized in this country. A grant or conveyance by the U S carries all minerals, unless reserved expressly or by implication in the law or instrument purporting to pass the title (Lindley, *loc cit*). The government of the U S does not concern itself with mining lands nor the mining industry after it parts with the title. This title vests in the patentee absolutely. No royalties are reserved, nor is any governmental supervision attempted. Upon issuance of the deed of the government, mining land becomes private property, subject to the same rules of law as other real property (Lindley, par 22). Although quite within its sovereign power, the U S government has never engaged in mining operations.

Minerals in private lands. No provision is made in the U S laws for location of mining claims upon or within privately owned lands. Laws of many countries in Europe and South America reserve this right under the theory that mining is a public utility. But in the U S public mineral lands only may be located and entered under the mining laws.

Reservation of mining rights. Authority to administer public lands was given to the Department of the Interior under act of Congress passed Mar 3, 1849. Prior to 1866 it was the settled policy of the U S government to reserve the minerals in all lands sold or granted to individuals, states, or railways, and other corporations.

The first sale of mineral lands by the government was that of the reserved lead and zinc mines in Mo at $2.50 per acre under act of Mar 3, 1829. By act of July 11, 1846, sales of similar lands were authorised in Ill, Ark, Wis, and Iowa. Under act of Mar 1, 1847, lands in the Lake Superior district containing lead, copper, or other valuable ores, were offered at public sale at $5 per acre; and by act of Mar 3, 1847, the sale of lands in the Chippewa district, Wis, was authorized on similar terms. Excepting these special sales of mineral lands, it was the uniform policy of the government to reserve mineral lands from sale or grant in all cases. Thus, in grants for educational purposes and for internal improvements within the states, there was often a reservation of lands known to be mineral at the time of grant. If, however, minerals were discovered after completion of the transaction, the state's title thereto would not be affected. The same is true in general of railroad grants. Non-mineral lands only were to be granted; but discovery of mineral character after title has legally passed does not invalidate title, because issuance of patent to the land implies that it has been examined by the land department and found to be non-mineral in character, and establishes vested rights which can not be attacked. The important features of the law are presented briefly, in the following pages. For fuller discussion see textbooks listed in the bibliography.

6. FIRST GENERAL MINING ACT

First mining claims to land which afterwards became part of the public domain were staked in Nev and Cal. The first miners made their own laws, unmindful of legislation by territory, state, or nation. Their regulations were framed to cover local conditions, and embodied ideas and knowledge of the subject possessed by miners from Mexico, Cornwall and other parts of Great Britain, and the continent of Europe. In accordance with these rules "the right of property in mines is made to depend upon *discovery* and *development*; that is, *discovery* is made the source of title, and *development*, or *working*, the condition of the continuance of that title. These two principles constitute the basis of all our local laws and regulations respecting mining rights" (Halleck, Introd to De Foos on Law of Mines, p vii).

Law of July 26, 1866, was the first general statute by which title could be acquired to any public mineral lands within what are known as the mining states and territories. This act has been largely superseded or repealed by subsequent legislation, but is said to have established at least 3 important and beneficent principles:

"1. That all the mineral lands of the public domain should be free and open to exploration and occupation.

2. That rights which had been acquired in these lands under a system of local rules, with the apparent acquiescence and sanction of the government, should be recognised and confirmed.

3. That titles to at least certain classes of mineral deposits or lands containing them might be ultimately obtained" (Lindley on Mines, par 54).

7. MINING ACT OF 1872

On May 10, 1872, Congress passed "An act to promote the development of the mining resources of the United States." This is the famous "Apex Law." With a few additions and amendments, it is the law under which Federal mining rights are now acquired, and in accordance with its provisions the vast majority of mining claims in the territory to which it applies have been located. This law is now incorporated in the U S Code Annotated, Title XXX, Chap 2, which is herein quoted in full. It takes the place of Title XXXII, Chap 6, of the Revised Statutes of U S, on pp 1659–1664 of the Second Edition of this Handbook. The Code Annotated may be found in the library of almost any law firm, and in any public law library.

Text of Chap 2, Title XXX, of the U S Code follows:

21. Reservation of mineral lands from sale under general laws. In all cases lands valuable for minerals shall be reserved from sale, except as otherwise expressly directed by law.

22. Valuable mineral deposits open to location. Who may locate. All valuable mineral deposits in lands belonging to the United States, both surveyed and unsurveyed, shall be free and open to exploration and purchase, and the lands in which they are found to occupation and purchase, by citizens of the United States and those who have declared their intention to become such, under regulations prescribed by law, and according to the local customs or rules of miners in the several mining districts, so far as the same are applicable and not inconsistent with the laws of the United States.

23. Length of lode claim. Discovery essential to location. Width of claim. End lines must be parallel. Mining claims upon veins or lodes of quartz or other rock in place, bearing gold, silver, cinnabar, lead, tin, copper, or other valuable deposits, heretofore located, shall be governed as to length along the vein or lode by the customs, regulations, and laws in force at the date of their location. A mining claim located after May 10, 1872, whether located by one or more persons, may equal, but shall not exceed, 1 500 ft in length along the vein or lode; but no location of a mining claim shall be made until the discovery of the vein or lode within the limits of the claim located. No claim shall extend more than 300 ft on each side of the middle of the vein at the surface, nor shall any claim be limited by any mining regulation to less than 25 ft on each side of the middle of the vein at the surface, except where adverse rights existing on May 10, 1872, render such limitation necessary. The end lines of each claim shall be parallel to each other.

24. Proof of citizenship, under this chapter, may consist, in the case of an individual, of his own affidavit thereof; in the case of an association of persons unincorporated, of the affidavit of their authorised agent, made on his own knowledge or upon information and belief; and in the case of a corporation organised under the laws of the United States, or of any State or Territory thereof, by the filing of a certified copy of their charter or certificate of incorporation.

25. Affidavit of citizenship. Applicants for mineral patents who reside beyond the limits of the district wherein the claim is situated, may make any oath or affidavit required for proof of citizenship before the Clerk of any Court of Record, or any Notary Public of any State or Territory.

26. Extralateral and intralimital rights. The locators of all mining locations heretofore made or which shall hereafter be made, on any mineral vein, lode or ledge situated on the public domain, their heirs and assigns, where no adverse claim existed on May 10, 1872, so long as they comply with the laws of the United States, and with State or Territorial regulations not in conflict with the laws of the United States governing their possessory title, shall have the exclusive right of possession and enjoyment of all the surface included within the end lines of their locations, and of all veins, lodes, and ledges, throughout their entire depth, the top or apex of which lies inside of such surface lines extended downward vertically, although such veins, lodes or ledges may so far depart from a perpendicular in their course downward as to extend outside the vertical side lines of such surface locations. But their right of possession to such outside parts of such veins or ledges shall be confined to such portions thereof as lie between vertical planes drawn downward as above described, through the end lines of their locations, so continued in their own direction that such planes will intersect such exterior parts of such veins or ledges. Nothing in this section shall authorize the locator or possessor of a vein or lode which extends in its downward course beyond the vertical end lines of his claim to enter upon the surface of a claim owned or possessed by another.

27. Tunnel rights. Length of tunnels. Where a tunnel is run for the development of a vein or lode, or for the discovery of mines, the owners of such tunnel shall have the right of possession of all veins or lodes within 3 000 ft from the face of such tunnel on the line thereof, not previously known to exist, discovered in such tunnel, to the same extent as if discovered from the surface; and locations on the line of such tunnel of veins and lodes not appearing on the surface, made by other parties after the commencement of the tunnel, and while the same is being prosecuted with reasonable diligence, shall be invalid; but failure to prosecute the work on the tunnel for six months shall be considered as an abandonment of the right to all undiscovered veins on the line of such tunnel.

28. Mining-district regulations by miners. Annual labor on claims pending issue of patent. Expenditure on tunnels considered. The miners of each mining district may make regulations not in conflict with the laws of the United States, or with the laws of the State or Territory in which the district is situated, governing the location, manner of recording, amount of work necessary to hold possession of a mining claim, subject to the following requirements: The location must be distinctly marked on the ground so that its boundaries can be readily traced. All records of mining claims made after May 10, 1872 shall contain the name or names of the locators, the date of the location, and such description of the claim or claims located by reference to some natural object or permanent monument as will identify the claim. On each claim located after May 10, 1872, and until a patent has been issued therefor, not less than $100 worth of labor shall be performed or improvements made during each year. On all claims located prior to May 10, 1872, $10 worth of labor shall be performed or improvements made each year, for each 100 ft in length along the vein, until a patent has been issued therefor; but where such claims are held in common, such expenditure may be made upon any one claim; and upon failure to comply with these conditions the claim, or mine, upon which such failure occurred shall be open to relocation in the same manner as if no location of the same had ever been made; provided that the original locators, their heirs, assigns or legal representatives, have not resumed work upon the claim after failure and before such location. Upon failure of any one of several co-owners to contribute his proportion of the expenditures required hereby, the co-owners who have performed the labor or made the improvements may, at expiration of the year, give such delinquent co-owner personal notice in writing, or notice by publication in the newspaper published nearest the claim for at least once a week for 90 days, and if at expiration of 90 days after such notice, in writing or by publication, such delinquent should fail or refuse to contribute his proportion of the expenditure required by this section his interest in the claim shall become the property of his co-owners who have made the required expenditures. The period within which the work required to be done annually on all unpatented mineral claims located since May 10, 1872, including such claims in the territory of Alaska, shall commence at 12 o'clock Meridian on the 1st day of July succeeding the date of the location of such claim. Where a person or company has or may run a tunnel for the purpose of developing a lode or lodes, owned by such person or company, the money so expended in said tunnel shall be taken and considered as expended on said lode or lodes, whether located prior to or since May 10, 1872; such person or company shall not be required to perform work on the surface of said lode or lodes in order to hold the same as required by this section. On all such valid claims

the annual period ending December 31, 1921, shall continue to 12 o'clock Meridian on July 1, 1922.

29. Patent proceedings. A patent for any land claimed and located for valuable deposits may be obtained in the following manner: Any person, association, or corporation authorized to locate a claim under this chapter, having claimed and located a piece of land for such purposes, who has, or have, complied with the terms of this chapter, may file in the proper land office an application for a patent, under oath, showing such compliance, together with plat and field notes of the claim or claims in common, made by or under the direction of the United States Supervisor of Surveys, showing accurately the boundaries of the claim or claims, which shall be directly marked by monuments on the ground, and shall post a copy of such plat, together with a notice of such application for a patent, in a conspicuous place on the land embraced in such plat previous to the filing of the application for a patent, and shall file an affidavit of at least two persons that such notice has been duly posted, and shall file a copy of the notice in such land office, and shall thereupon be entitled to a patent for the land, in the manner following: The register of the land office, upon the filing of application, plat, field notes, notices and affidavits, shall publish a notice that such application has been made, for the period of 60 days, in a newspaper to be by him designated as published nearest to such claim; and he shall also post such notice in his office for the same period. The claimant at the time of filing this application, or at any time thereafter, within the 60 days of publication, shall file with the register a certificate of the United States Supervisor of Surveys, that $500 worth of labor has been expended or improvements made upon the claim by himself or grantors; that the plat is correct, with such further description by such reference to natural objects or permanent monuments as shall identify the claim, and furnish an accurate description to be incorporated in the patent. At the expiration of the 60 days of publication the claimant shall file his affidavit, showing that the plat and notice have been posted in a conspicuous place on the claim during such period of publication. If no adverse claim shall have been filed with the register of the proper land office at the expiration of the 60 days of publication, it shall be assumed that the applicant is entitled to a patent, upon the payment to the proper officer of $5 per acre, and that no adverse claim exists; and thereafter no objection from third parties to the issuance of a patent shall be heard, except it be shown that the applicant has failed to comply with the terms of this chapter. Where the claimant for a patent is not a resident of or within the land district wherein the vein, lode, ledge or deposit sought to be patented is located, the application for patent and the affidavit required to be made in this section by a claimant for such patent may be made by his, her, or its authorized agent, where said agent is conversant with the facts sought to be established by said affidavit.

30. Adverse claims, filing of, and suit upon. Where an adverse claim is filed during the period of publication, mentioned in Section 29 of this title, it shall be upon oath of the person or persons making the same, and shall show the nature, boundaries, and extent of such adverse claim, and all proceedings, except the publication of notice and making and filing of the affidavit thereof, shall be stayed until the controversy shall have been settled or decided by a court of competent jurisdiction, or the adverse claim waived. It shall be the duty of the adverse claimant, within 30 days after filing his claim, to commence proceedings in a court of competent jurisdiction, to determine the question of the right of possession and prosecute the same with reasonable diligence to final judgment; and a failure so to do shall be a waiver of his adverse claim. After such judgment shall have been rendered, the party entitled to the possession of the claim, or any portion thereof, may, without giving further notice, file a certified copy of the judgment-roll with the register of the land office, together with the certificate of the United States Supervisor of Surveys, that the requisite amount of labor has been expended or improvements made thereon, and the description required in other cases, and shall pay to the Register $5 per acre for his claim, together with the proper fees, whereupon the whole proceedings and the judgment-roll shall be certified by the register to the Commissioner of the General Land Office, and a patent shall issue thereon for the claim, or such portion thereof as the applicant shall appear, from the decision of the court, to rightly possess. If it appears from the decision of the court that several parties are entitled to separate and different portions of the claim, each party may pay for his portion of the claim with the proper fees, and file the certificate and description by the United States Supervisor of Surveys, whereupon the register shall certify the proceedings and judgment-roll to the Commissioner of the General Land Office, as in the preceding case, and patents shall issue to the several parties according to their respective rights. Nothing herein contained shall be construed to prevent the alienation of the title conveyed by a patent for a mining claim to any person whatever.

31. Same; oath of claimant. The adverse claim required by section 30 of this title may be verified by the oath of any duly authorized agent or attorney in fact of the adverse

claimant cognizant of the facts stated; and the adverse claimant, if residing or at any time being beyond the limits of the district wherein the claim is situated, may make oath to the adverse claim before the clerk of any Court of record of the United States or of the State or Territory where the adverse claimant may then be, or before any notary public of such State or Territory. (April 26, 1882 c. 106, Sec. 1, 22 Stat. 49.)

32. Same; findings by jury; costs. If in any action brought pursuant to Section 30 of this title, title to the ground in controversy shall not be established by either party, the Jury shall so find, and judgment shall be entered according to the verdict. In such case costs shall not be allowed to either party, and the claimant shall not proceed in the land office or be entitled to a patent for the ground in controversy until he shall have perfected his title. (March 3, 1881, c. 140, 21 Stat. 505.)

33. Rights under patents issued before Act of 1872. Applications for patents for mining claims under laws existing prior to May 10, 1872 and pending on that date, may be prosecuted to a final decision in the General Land Office; but in such cases where adverse rights are not affected thereby, patents may issue in pursuance of the provisions of this chapter; and all patents for mining claims upon veins or lodes issued prior to May 10, 1872 shall convey all the rights and privileges conferred by this chapter where no adverse rights existed on May 10, 1872.

34. Description of claim on surveyed and unsurveyed lands. Monuments control locations. The description of vein or lode claims upon surveyed lands shall designate the location of the claims with reference to the lines of the public survey, but need not conform therewith; but where patents have been or shall be issued for claims upon unsurveyed lands, the United States Supervisor of Surveys, in extending the public survey, shall adjust the same to the boundaries of said patented claims so as in no case to interfere with or change the true location of such claims as they are officially established upon the ground. Where patents have issued for mineral lands, those lands only shall be segregated and shall be deemed to be patented which are bounded by the lines actually marked, defined, and established upon the ground by the monuments of the official survey upon which the patent grant is based, and the United States Supervisor of Surveys, in executing subsequent patent surveys, whether upon surveyed or unsurveyed lands, shall be governed accordingly. The said monuments shall at all times constitute the highest authority as to what land is patented, and in case of any conflict between the said monuments of such patented claims and the descriptions of said claims in the patents issued therefor the monuments on the ground shall govern, and erroneous or inconsistent descriptions or calls in the patent descriptions shall give way thereto, prior to May 10, 1872.

35. Placers and other forms of deposit not in place may be entered and patented. Claims usually called "placers," including all forms of deposit, excepting veins of quartz, or other rock in place, shall be subject to entry and patent, under like circumstances and conditions, and upon similar proceedings, as are provided for vein or lode claims; but where the lands have been previously surveyed by the United States, the entry in its exterior limits shall conform to the legal subdivisions of the public lands.

Placer locations must conform to public surveys. Homesteads. Where placer claims are upon surveyed lands, and conform to legal subdivisions, no further survey or plat shall be required, and all placer mining claims located after May 10, 1872, shall conform as nearly as practicable with the United States system of public-land surveys, and the rectangular subdivisions of such surveys, and no such location shall include more than 20 acres for each individual claimant; but where placer claims can not be conformed to legal subdivisions, survey and plat shall be made as on unsurveyed lands; and where by the segregation of mineral lands in any legal subdivision a quantity of agricultural land less than 40 acres remains, such fractional portion of agricultural land may be entered by any party qualified by law, for homestead purposes.

36. Subdivisons of claims. Group entries. Maximum extent of placers. Legal subdivisions of 40 acres may be subdivided into 10-acre tracts; and two or more persons, or associations of persons, having contiguous claims of any size, although such claims may be less than 10 acres each, may make joint entry thereof; but no location of a placer claim, made after July 9, 1870, shall exceed 160 acres for any one person or association of persons, which location shall conform to the United States surveys; and nothing contained in this section shall defeat or impair any bona fide preëmption or homestead claim upon agricultural lands, or authorize the sale of the improvements of any bona fide settler to any purchaser.

37. Patents for lodes within placers. Where the same person, association or corporation is in possession of a placer claim, and also a vein or lode included within the boundaries thereof, application shall be made for a patent to the placer claim, with the statement that it includes such vein or lode, and in such case a patent shall issue for the placer claim, subject to the provisions of this chapter, including such vein or lode, upon the

payment of $5 per acre for such vein or lode claim and 25 ft of surface on each side thereof. The remainder of the placer claim or any placer claim not embracing any vein or lode claim shall be paid for at the rate of $2.50 per acre, together with all costs of proceedings; and where a vein or lode, such as is described in § 23 is known to exist within the boundaries of a placer claim, an application for a patent for such placer claim which does not include an application for the vein or lode claim shall be construed as a conclusive declaration that the claimant of the placer claim has no right of possession of the vein or lode claim; but where the existence of a vein or lode in a placer claim is not known, a patent for the placer claim shall convey all valuable mineral and other deposits within the boundaries thereof.

38. Patents obtained on adverse possession. Where such person or association, they and their grantors, have held and worked their claims for a period equal to the time prescribed by the statute of limitations for mining claims of the State or Territory where the same may be situated, evidence of such possession and working of the claims for such period shall be sufficient to establish a right to a patent thereto under this chapter, in the absence of any adverse claim; but nothing in this chapter shall be deemed to impair any lien which may have attached in any way whatever to any mining claim or property thereto attached prior to the issuance of a patent.

39. Deputy mineral surveyors. Expenses of survey and patent. Publication of notices. Designation of newspaper. Fees of officers. The U S Supervisor of Surveys may appoint in each land district containing mineral lands as many competent surveyors as shall apply for appointment to survey mining claims. The expenses of the survey of vein or lode claims, and the survey and subdivision of placer claims into smaller quantities than 160 acres, together with the cost of the publication of notices, shall be paid by the applicants, and they shall be at liberty to obtain the same at the most reasonable rates, and they shall also be at liberty to employ any United States deputy surveyor to make the survey. The Commissioner of the General Land Office shall also have power to establish the maximum charges for surveys and publication of notices under this chapter; and, in case of excessive charges for publication, he may designate any newspaper published in a land district where mines are situated for the publication of mining notices in such district, and fix the rates to be charged by such paper; and to the end that the Commissioner may be fully informed on the subject, each applicant shall file with the register a sworn statement of all charges and fees paid by such applicant for publication and surveys, together with all fees and money paid the register of the land office, which statement shall be transmitted, with the other papers in the case, to the Commissioner of the General Land Office.

40. Affidavits, before what officers to be made. All affidavits required to be made under this chapter may be verified before any officer authorized to administer oaths within the land district where the claims may be situated, and all testimony and proofs may be taken before any such officer, and, when duly certified by the officer taking the same, shall have the same force and effect as if taken before the register of the land office. In cases of contest as to the mineral or agricultural character of land, the testimony and proofs may be taken as herein provided on personal notice of at least 10 days to the opposing party; or if such party can not be found, then by publication of at least once a week for 30 days in a newspaper, to be designated by the register of the land office as published nearest to the location of such land; and the register shall require proof that such notice has been given.

41. Cross-lodes and uniting veins. Where two or more veins intersect or cross each other, priority of title shall govern, and such prior location shall be entitled to all ore or mineral contained within the space of intersection; but the subsequent location shall have the right of way through the space of intersection for the purposes of the convenient working of the mine. And where two or more veins unite, the oldest or prior location shall take the vein below the point of union, including all the space of intersection.

42. Millsites, classes, patents for. Where non-mineral land not contiguous to the vein or lode is used or occupied by the proprietor of such vein or lode for mining or milling purposes, such non-adjacent surface ground may be embraced and included in an application for a patent for such vein or lode, and the same may be patented therewith, subject to the same preliminary requirements as to survey and notice as are applicable to veins or lodes; but no location hereafter made of such non-adjacent land shall exceed 5 acres, and payment for the same must be made at the same rate as fixed by this chapter for the superficies of the lode. The owner of a quartz mill or reduction works, not owning a mine in connection therewith, may also receive a patent for his millsite, as provided in this section.

43. State legislatures may pass supplementary laws. As a condition of sale, in the absence of necessary legislation by Congress, the local legislature of any State or Territory

may provide rules for working mines, involving easements, drainage, and other necessary means to their complete development; and those conditions shall be fully expressed in the patent.

44. Homesteads upon mineral lands. Wherever, upon the lands heretofore designated as mineral lands, which have been excluded from survey and sale, there have been homesteads made by citizens of the United States, or persons, who have declared their intention to become citizens, which homesteads have been made, improved and used for agricultural purposes, and upon which there have been no valuable mines of gold, silver, cinnabar, or copper discovered, and which are properly agricultural lands, the settlers or owners of such homesteads may avail themselves of the provisions of Chap 7, of Title 43, relating to "Homesteads."

45. Secretary of Interior may set apart agricultural lands. Upon the survey of the lands described in Art 44 of this title, the Secretary of the Interior may designate and set apart such portions of the same as are clearly agricultural lands, which lands shall thereafter be subject to sale as other public lands, and be subject to all the laws and regulations applicable to the same.

46. Additional land districts, establishment of. The President is authorized to establish additional land districts, and to appoint the necessary officers under existing laws, wherever he may deem the same necessary for the public convenience in executing the provisions of this chapter.

47. Construction of act, generally. Nothing contained in this chapter shall be construed to impair, in any way, rights or interests in mining property acquired under laws enforced prior to July 9, 1870, nor to affect the provisions of the act entitled "An act granting to A. Sutro the right of way and other privileges to aid in the construction of a draining and exploring tunnel to the Comstock lode, in the State of Nevada," approved July 25, 1866.

48. Lands in certain States excepted. Except as otherwise provided under sections 141–152, 181–194, 201–208, 211–214, 221, 223–229, 241, 251 and 261–263 of this title, the provisions of sections 21–24, 26–30, 33–47, 51 and 52 of this title shall not apply to the mineral lands situated in the states of Mich, Wis, and Minn, which except as otherwise provided in this title are declared free and open to exploration purposes according to legal subdivisions, in like manner as before the 10th day of May, 1872.

The provisions of this chapter shall not apply to the mineral lands situated in the States of Mich, Wis, and Minn, which are declared free and open to exploration and purchase, according to legal subdivisions, in like manner as before May 10, 1872. And any bona fide entries of such lands within the States named since May 10, 1872, may be patented without reference to any of the foregoing provisions of this chapter. Such lands shall be offered for public sale in the same manner, at the same minimum price as other public lands. *Note:* See also Section 49, relating to the exclusion of deposits of coal, iron, lead or other mineral in the states of Mo and Kan. If interested, refer to the original section.

50. Grants to States or corporations not to include mineral lands. No act passed at the first session of the 38th congress, granting lands to States or corporations to aid in the construction of roads or for other purposes, or to extend the time of grants made prior to Jan 30, 1865, shall be so construed as to embrace mineral lands, which in all cases are reserved exclusively to the United States, unless otherwise specially provided in the act or acts making the grant.

51. Vested rights to use water for mining. Right of way for canals. Whenever, by priority of possession, rights to the use of water for mining purposes, have vested and accrued, and the same are recognized and acknowledged by the local customs, laws, and the decisions of courts, the possessors and owners of such vested rights shall be maintained and protected in the same; and the right of way for the construction of ditches and canals for the purposes herein specified is acknowledged and confirmed; but whenever any person, in the construction of any ditch or canal, injures or damages the possession of any settler on the public domain, the party committing such injury or damage shall be liable to the party injured for such injury or damage.

52. Rights subject to vested and accrued water rights. All patents granted, or preemption or homesteads allowed, shall be subject to any vested and accrued water rights, or rights to ditches and reservoirs used in connection with such water rights, as may have been acquired under or recognized by Sec 51 of this title.

8. MINING LAWS EXTENDED TO NON-MINERAL LANDS

Summary of United States Code, Chap and Sec Headings

Reference to the following Chapters and Sections of the U S Code, Title XXX, having no bearing on the apex law, but applying to Federal mineral lands, may be of interest. Chap 3 of Title XXX refers to " Land Containing Coal, Phosphates, Petroleum, Oil, Oil Shale, Gas, Sodium, Potassium, etc., and Building Stone." See sections 71–263 inclusive, together with notes up to 1938 in the Cumulative Section of the U S Code. A copy of these sections is too voluminous for use in this Handbook. Sub-titles of Chap 3 are here given for reference:

Entry on coal lands in general, sections 71–77.

Entry under Non-mineral Land Laws of coal lands, with reservation of coal to United States, sections 81–90.

Entry under Mining laws of lands containing petroleum or other mineral oils or gas, sections 101–104.

Homestead entry of lands in Utah withdrawn or classified as oil lands, sections 111–113.

Agricultural entry of lands withdrawn or classified as containing phosphate, nitrate, potash, oil, gas or asphaltic minerals, sections 121–123.

Locations under Placer Mining Laws of lands, containing phosphate rock, section 131.

Permits to prospect for chlorides, sulphates, carbonates, borates, silicates or nitrates of potassium, sections 141–152.

Entry of building stone or saline lands under Placer Mining Laws, sections 161, 162.

Disposal of lands in Alabama as agricultural lands, sections 171, 172.

Leases and prospecting permits, subdivision 1, general provisions; sections 181–194, subdivision 2, coal; sections 201–208, subdivision 3, phosphates; sections 211–214, subdivision 4, oil and gas; sections 221–236, subdivision 5, oil shale; section 241, subdivision 6, Alaska oil proviso; section 251, subdivision 7, sodium.

Note. All references in Chap 3 apply only to U S lands and the entry upon the same.

9. THE BUREAU OF MINES

(See also Chap 1, of Title XXX, sub-title "The Bureau of Mines," sections 1–11, sections 1 being given in full.)

Section 1. **Bureau of Mines, establishment, director, experts and other employes.** There is established in the Department of Commerce a bureau of mining, metallurgy and mineral technology, to be designated as the Bureau of Mines, and there shall be a director of said bureau, who shall be thoroughly equipped for the duties of said office, by technical education and experience and who shall be appointed by the President, by and with the advice and consent of the Senate; and there shall also be in the said bureau such experts and other employes, to be appointed by the Secretary of the Interior, as may be required to carry out the purposes of this chapter in accordance with the appropriations made from time to time by Congress for such purposes. Sections 2–11 inclusive codify the Acts of Congress relating to the Bureau. An historical note on the Establishment of the Bureau will be found in Chapter I of the above Title, p 27.

10. WITHDRAWALS OF PUBLIC LANDS

The President may, at any time in his discretion, temporarily withdraw from settlement, location, sale or entry, any of the public lands of the U S, including the District of Alaska, and reserve the same for water-power sites, irrigation, classification of lands, or other public purposes to be specified in the orders of withdrawals, and such withdrawals or reservations shall remain in force until revoked by him or by an act of Congress.

By act of Congress approved Aug 24, 1912, the act of June 25, 1910, was amended so as to provide that all lands withdrawn under provision of the act shall at all times be open to exploration, discovery, occupation and purchase under the mining laws " so far as the same apply to metalliferous minerals " (37 Stats at Large, p 497).

11. REGULATIONS OF THE LAND DEPARTMENT

Origin and nature. Establishment of the Department of the Interior was authorised by act of Congress, Mar 3, 1849, and supervision of mineral lands was transferred to the general land office in that department (9 Stats at Large, p 395).

This department, consisting of the Sec'y of the Interior, the Commissioner of General Land Office, and their subordinates, is a special tribunal, vested with judicial powers to hear and determine claims to public lands, and with authority to execute its judgments by conveyances to proper claimants. Decisions of this department as to questions of fact within its jurisdiction are final; there is no appeal to any court, nor may the courts anticipate its action nor take upon themselves administration of public lands (Lindley, p 1644). The department rules, when not in conflict with the law, have the force and effect of laws, and courts take judicial notice of them. The dep't has not power of itself to grant land, but to facilitate and consummate grants authorised by law. When lands are withdrawn by order of Congress or lawful executive order, its jurisdiction is suspended; and when lands have passed into private ownership and title has been transferred from the government, its authority vanishes. Where the land officers have clearly mistaken the law of the case as applicable to the facts, courts of equity may give relief; but "where there is a mixed question of law and fact and the court can not so separate it as to see clearly where the mistake of law is, the decision of the tribunal to which the law has confided the matter is conclusive" (Lindley, p 1 666).

Classification of claims. There are two classes: Lode claims and placer claims.

12. LODE CLAIMS

Dimensions. Under the law of 1872, lode claims may have a maximum length of 1 500 ft, and a maximum width of 600 ft, in no case exceeding 300 ft on each side of the vein at the surface. The owner of a valid location is also given all other veins of which the tops or apices lie within the exterior boundaries of his claim, provided they were not adversely claimed on May 10, 1872.

Initiation of title. No lode claim shall be located until after discovery of a vein or lode within the limits of the claim. Sufficient development work is therefore required by the department to disclose existence of a vein, and if possible its general course and indications that it contains valuable minerals.

Location notice should give the course and distance as nearly as practicable, from discovery shaft on the claim to some permanent, well-known points or objects, such as stone monuments, blazed trees, confluence of streams, prominent buttes, hills, etc, in the immediate vicinity; also names of adjacent claims. A post or monument of stones should be erected at the discovery point or shaft, and at each corner of the ground claimed. Notice posted at the discovery should give name of the lode claim, names of the locators, number of feet claimed and their direction from the discovery, and whether the claim thus staked is wholly or in part on one side of the discovery point. Location notice must be filed for record as required by the state or territorial laws and local miners' rules, if any there be.

Perpetuation of possessory title to a lode claim located since May 10, 1872, is maintained by the performance of at least $100 worth of work upon it each calendar year. This work is not required after entry for patent. Failure to perform the required amount of work subjects the claim to re-location, unless original locator or his heirs, assigns, or legal representatives, have resumed work before re-location. Non-contributing partners may be " advertised out " by publication of notice and summons in the nearest newspaper once a week for 90 days, or by personal notice in writing. If, 90 days after such notice in writing, or 180 days after first newspaper publication of notice, the delinquent partner shall have failed to contribute his proportion of expenditures for assessment work, his interest in the claim passes by law to those co-owners who have paid for the work.

Method of procuring patent. The owner must first have a correct survey made of the claim, under the authority of the surveyor-general of the state or territory containing the claim. One copy of the plat of survey and one copy of the field notes shall be retained in the office of surveyor-general, one copy of the plat given to the claimant for posting upon the claim, one copy of plat and field notes given to the claimant to be filed with the proper register, and finally transmitted by that official, with other papers in the case, to the land office, and one plat to be sent by the surveyor-general to the register of the proper land district to be filed for future reference. The plat is to be posted in a conspicuous place on the claim, together with notice of patent application, containing date of posting, name of claimant, name of claim, number of the survey, mining district and county, and names of adjoining and conflicting claims as shown by the survey plat.

After posting notice, the claimant shall file with the proper register and receiver a copy of plat and field notes, together with affidavits of at least two credible witnesses that such plat and notice are posted conspicuously upon the claim, giving date and place of such posting; a copy of the notice so posted to be attached to and form a part of said affidavit. The claimant must also make affidavit that he has possessory right to the premises described, by virtue of a compliance upon his part and that of his grantors (if he claims by purchase) with the mining rules, regulations and customs of the district and the mining laws of Congress. The affidavit must state the facts constituting such compliance, the origin of his possession, and the basis of his claim for patent. The vein or lode must be fully described, the description to include a statement as to kind and character of mineral, extent thereof, whether ore has been extracted and of what amount and value, and such other facts as will

support the applicant's allegation that the claim contains a valuable mineral deposit. The affidavit must be supported by a copy of each location notice, certified by the legal custodian of the record thereof, and also by an abstract of title of each claim, certified by the legal custodian of records of transfers, or by a duly authorized abstractor of titles. The certificates must state that no conveyances affecting or purporting to affect the title to the claim or claims appear of record other than those set forth. The register will then, at expense of the claimant, publish a notice of the application for 60 days; after which period, upon furnishing an affidavit of publication and another of posting upon the claim, and a certificate of the surveyor-general testifying to the performance of at least $500 worth of work upon the claim, the claimant may pay for the land at rate of $5 per acre, taking a receipt, and receiving in due time a patent, if the papers are found to be regular.

13. PLACER CLAIMS

Nature. As in case of lode claims, a discovery of valuable mineral in place is necessary before location of a placer claim. Proceedings to obtain patents for placer claims, including all forms of mineral deposits, excepting veins of quartz or other rock in place, are similar to those prescribed for obtaining patents for vein or lode claims; but where a placer claim shall be upon surveyed lands, and conforms to legal subdivisions, no further survey or plat will be needed. The price fixed by law for placer claims is $2.50 per acre or fractional part of an acre.

Placer applications should contain, in addition to the recitals necessary in and to both vein or lode and placer applications, such detailed data as will support the claim that the land applied for is placer ground containing valuable mineral deposits not in vein or lode formation, and that title is sought not to control water courses or to obtain valuable timber, but in good faith because of the mineral therein.

This statement, of course, must depend upon character of deposit, and natural features of the ground, but the following details should be covered as fully as possible: If the claim be for a deposit of placer gold, there must be stated the yield per pan, or per cu yd, as shown by prospecting and development work, distance to bedrock, formation and extent of the deposit, and all other facts upon which the applicant bases his allegation that the claim is valuable for its deposits of placer gold. If a building-stone deposit, or deposit other than gold be claimed under the placer laws, applicant must describe fully the kind, nature and extent of deposit, stating the reasons why same is by him regarded as a valuable mineral claim. He will also be required to describe fully the natural features of the claim; course of streams, amount of water carried, and amount of fall within the claim; also kind and amount of timber and other vegetation thereon, and adaptability to mining and other uses.

If the claim be all placer ground, that fact must be stated in the application and corroborated by accompanying proofs; if of mixed placer and lodes, it should be so set forth, with description of all known lodes situated within its boundaries. A specific declaration, as required by section 37 of U S Code (supra), must be furnished as to each lode intended to be claimed. All other known lodes are, by the silence of the applicant, excluded by law from all claim by him, of whatsoever nature, possessory or otherwise.

Further regulations by the Department provide for methods of proof and procedure relative to millsites, citizenship, possessory right, adverse claims, duties of deputy surveyors, charges for publication of notices, conduct of hearings on contests and applications for entry, and various other matters of detail not necessary to be quoted here.

MINING LAWS OF WESTERN MINING STATES AND ALASKA

14. CALIFORNIA MINING ACT

The States of Ariz, Col, Cal, Idaho, Mont, Nev, New Mex, N Dak, Ore, S Dak, Utah, Wash, Wyo and Alaska have all adopted laws for the working of mines, securing of easements, etc, under authority of Section 43 of the U S Code. These Statutes were stated in full in Second Edition of this Handbook, pp 1664–1667 and 1669–1682. As there are constant changes in these Statutes, it is thought advisable to set forth the laws of one typical mining State and refer to laws of other States by title only. The California Acts, as now contained in the Civil Code of 1937, and amended through the 52nd Session of the Calif Legislature of 1937, covering location and methods of location of mining are therefore now given in full.

California Civil Code, 1937 (Deering)

Lode claims, how located. 1 426. Any person, a citizen of the United States, or who has declared his intention to become such, who discovers a vein or lode of quartz, or other rock in place, bearing gold, silver, cinnabar, lead, tin, copper, or other valuable deposit, may locate a claim upon such vein or lode, by defining the boundaries of the claim, in the manner hereinafter described, and by posting a notice of such location, at the point of discovery, which notice must contain: *First*, Name of the lode or claim; *Second*, Name of the locator or locators; *Third*, Number of linear feet claimed in length along course of vein, each way from point of discovery with width on each side of center of claim, and general course of vein or lode, as near as may be; *Fourth*, Date of location; *Fifth*, Such description of claim by reference to some natural object, or permanent monument, as will identify the claim located.

Boundaries and extent of lode claim. 1 426*a*. The locator or locators of any lode mining claim must define the boundaries of such claim so that they may be readily traced, but, in no case, shall the claim extend more than 1 500 ft along the course of the vein or lode, nor more than 300 ft on either side thereof, as measured from the center line of the vein at the surface. On all lode mining claims made after this Act takes effect and within 60 days after the location of the claim, the locator or locators shall erect at each corner of the claim and at the center of each end line, or the nearest accessible point thereto, a post not less than 4 in diameter, or a stone monument at least 18 in high.

Record of location of lode claim. 1 426*b*. Within 30 days after the posting of his notice of location upon a lode mining claim, the locator shall record a true copy thereof in the office of the county recorder of the county in which such claim is situated, for which service the county recorder shall receive a fee of $1.

Placer claim, location of. 1 426*c*. Location of a placer claim shall be made in the following manner: By posting thereon, upon a tree, rock in place, stone, post or monument, a notice of location containing the name of claim, name of locator or locators, date of location, number of feet or acreage claimed, such description of claim by reference to some natural object or permanent monument as will identify the claim located, and by marking the boundaries so that they may be readily traced; *provided*, that where the United States survey has been extended over the land embraced in the location, the claim may be taken by legal subdivisions, and no other reference than those of said survey shall be required, and the boundaries of a claim so located and described need not be staked or monumented. The description by legal subdivisions shall be deemed the equivalent of marking.

Record of location of placer claim. 1 426*d*. Within 30 days after posting of notice of location of a placer claim, the locator shall record a true copy thereof in the office of the county recorder of the county in which such claim is situated, for which service the recorder shall receive a fee of $1.

Discovery shaft. 1 426*da*. On every lode mining or placer claim, located after this Act takes effect, the locator or locators thereof shall, within 90 days after the date of location, sink a discovery shaft upon such claim at the point of discovery to a depth of at least 10 ft from the lowest part of the rim of such shaft at the surface, exposing the deposit upon which discovery and location is based, or shall drive a tunnel, adit, or open cut upon such claim at the discovery point to at least 10 ft below the surface, exposing the deposit upon which such discovery and location is based.

Placer locations, work on. 1 426*db*. On all placer mining locations containing more than 20 acres, located after this Act takes effect, the locators thereof shall, within 90 days after date of location, perform at least one dollar's worth of work for each acre included in such claim. This work may all be done at one place on the claim if so desired, and must be actual mining development work, exclusive of cabins, buildings, or other surface structures. Nothing in this section shall be construed as a modification of the requirements of Sec 1 426*da* of this Code.

Relocation of lodes or placers. 1 426*dc*. The relocation of any lode or placer mining location which is subject to relocation, shall be made in the same way as an original location as herein required by law to be made, except that the relocator may either sink a new shaft upon the ground relocated at the discovery point to the depth of at least 10 ft from the lowest part of the rim of such shaft at the surface, exposing the deposit upon which location is based, or drive a new tunnel, adit, or open cut upon such ground at the point of discovery to at least 10 ft below the surface, exposing the deposit upon which location is based; or the relocator may sink the original discovery shaft 10 ft deeper than it is at the time of relocation, or drive the original tunnel, adit, or open cut 10 ft farther.

Tunnel right, location of. 1 426*e*. Locator of a tunnel right or location shall locate it by posting a notice at the face or point of commencement of the tunnel. which must

contain: *First*, Name of locator or locators; *Second*, Date of location; *Third*, Proposed course or direction of tunnel; *Fourth*, Description of tunnel, with reference to some natural object or permanent monument as shall identify the claim or tunnel right.

Boundaries of tunnel location. 1 426*f*. Boundary lines of the tunnel shall be established by stakes or monuments placed along the lines at an interval of not more than 600 ft from the face or point of commencement of the tunnel to the terminus at 3 000 ft therefrom.

Record of tunnel location. 1 426*g*. Within 30 days after posting notice of location of tunnel right or location, the locator shall record a true copy thereof in the office of the county recorder of the county in which such claim is situated, for which service the recorder shall receive a fee of $1.

Amended notice of location. 1 426*h*. If at any time the locator of any mining claim heretofore or hereafter located, or his assigns, shall apprehend that his original location notice was defective, erroneous, or that the requirements of the law had not been complied with before filing; or in case the original notice was made prior to the passage of this act, and he shall be desirous of securing the benefit of this act, such locator, or his assigns, may file an additional notice, subject to the provisions of this act; *provided*, that such amended location notice does not interfere with the existing rights of others at the time of posting and filing such amended location notice, and no such notice, or the record thereof, shall preclude the claimant, or claimants, from proving any such title as he or they may have held under previous locations.

Record of survey of mining claim is prima facie evidence. 1 426*i*. Where a locator, or his assigns, has the boundaries and corners of his claim established by a United States deputy mineral survey, or a licensed surveyor of this state and his claim connected with a corner of the public or minor surveys of an established initial point, and incorporates into the record of the claim the field notes of such survey, and attaches to and files with such location notice, a certificate of the surveyor, setting forth: *First*, that said survey was actually made by him, giving the date thereof; *Second*, the name of the claim surveyed and the location thereof; *Third*, that the description incorporated in the declaratory statement is sufficient to identify; such survey and certificate become a part of the record, and such record is *prima facie* evidence of the facts therein contained.

Millsite, location of. 1 426*j*. Proprietor of a vein or lode claim or mine, or owner of a quartz-mill or reduction works, or any person qualified by the laws of the United States, may locate not more than 5 acres of non-mineral land as a millsite. Such location shall be made in the same manner as hereinbefore required for locating placer claims.

Record of location of millsite. 1 426*k*. Locator of a millsite claim or location shall, within 30 days from the date of his location, record a true copy of his location notice with the county recorder of the county in which such location is situated, for which service the recorder shall receive a fee of $1.

Annual labor required. 1 426*l*. The amount of work done or improvements made during each year to hold possession of a mining claim shall be that prescribed by the laws of the United States, to wit: $100 annually.

Record of proof of annual labor. 1 426*m*. Whenever a mine owner, company, or corporation shall have performed the labor and made the improvements required by law upon any mining claim, the person in whose behalf such labor was performed or improvements made, or some one in his behalf, shall within 30 days after the time limited for performing such labor or making such improvements make and have recorded by the county recorder, in books kept for that purpose, in the county in which such mining claim is situated, an affidavit setting forth the value of labor or improvements made, the name of the claim, and the name of the owner or claimant of said claim at whose expense the same was made or performed. Such affidavit, or a copy thereof, duly certified by the county recorder, shall be *prima facie* evidence of the performance of such labor or the making of such improvements, or both.

Fee for recording proof of labor. 1 426*n*. For recording the affidavit herein required, the County Recorder shall receive a fee of 10 cents per folio, 20 cents for endorsement, and 10 cents for indexing the name of each claim and each owner.

Forfeiture to co-owners. Contribution by delinquent co-owners. 1 426*o*. Whenever a co-owner or co-owners of a mining claim shall give to a delinquent co-owner or co-owners the notice in writing or notice by publication provided for in section 2 324, Revised Statutes of the United States, an affidavit of the person giving such notice, stating the time, place and manner of service, and by whom and upon whom such service was made, shall be attached to a true copy of such notice, and such notice and affidavit must be recorded in the office of the county recorder, in books kept for that purpose, in the county in which the claim is situated, within 90 days after the giving of such notice; for the recording of which, said recorder shall receive the same fees as are now allowed by

law for recording deeds; or if such notice is given by publication in a newspaper, there shall be attached to a printed copy of such notice an affidavit of the printer or his foreman, or principal clerk of such paper, stating the date of the first, last and each insertion of such notice therein, and where the newspaper was published during that time, and the name of such newspaper. Such affidavit and notice shall be recorded as aforesaid, within 180 days after the first publication thereof. The original of such notice and affidavit, or a duly certified copy of the record thereof, shall be *prima facie* evidence that the delinquent mentioned in § 2 324 has failed or refused to contribute his proportion of the expenditure required by that section, and of the service of publication of said notice; *provided*, the writing or affidavit hereinafter provided for is not of record. If such delinquent shall, within the 90 days required by § 2 324 aforesaid, contribute to his co-owner or co-owners, his proportion of such expenditures, and also all costs of service of the notice required by this section, whether incurred by publication charges or otherwise, such co-owner or co-owners shall sign and deliver to the delinquent or delinquents a writing, stating that the delinquent or delinquents by name has within the time required by § 2 324 aforesaid, contributed his share for the year . . . ., upon the . . . . mine, and further stating therein the district, county and state wherein the same is situated, and the book and page where the location notice is recorded, if said mine was located under the provisions of this act; such writing shall be recorded in the office of the county recorder of said county, for which he shall receive the same fees as are now allowed by law for recording deeds. If such co-owner or co-owners shall fail to sign and deliver such writing to the delinquent or delinquents within 20 days after such contribution, the co-owner or co-owners so failing as aforesaid shall be liable to the penalty of $100, to be recovered by any person for the use of the delinquent or delinquents in any court of competent jurisdiction. If such co-owner or co-owners fail to deliver such writing within the said 20 days, the delinquent, with two disinterested persons having personal knowledge of such contribution, may make affidavit setting forth in what manner, the amount of, to whom, and upon what mine, such contribution was made. Such affidavit or a record thereof, in the office of the county recorder, of the county in which such mine is situated, shall be *prima facie* evidence of such contribution.

Records to be received in evidence. 1 426*p*. The record of any location of a mining claim, millsite or tunnel right, in the office of the county recorder, as herein provided, shall be received in evidence, and have the same force and effect in the courts of the state as the original notice.

Copies of record as evidence. 1 426*q*. Copies of the records of all instruments required to be recorded by the provisions of this act, duly certified by the recorder, in whose custody such records are, may be read in evidence, under the same circumstances and rules as are now or may be hereafter provided by law, for using copies of instruments relating to real estate, duly executed or acknowledged or proved and recorded.

Effect of act on mining districts. 1 426*r*. The provisions of this act shall not in any manner be construed as affecting or abolishing any mining district or the rules and regulations thereof within the state of California.

1 426ra. Failure or neglect of the locator or locators to comply with the requirements of Sections 1 426, 1 426*a*, 1 426*da*, 1 426*db*, or 1 426*dc*, of this Code, shall render such location null and void, and no portion of the area within such location shall be subject to relocation by the same locator or locators within the period of three years from the date of such void location.

Failure to perform annual labor—Relocation void. 1 426*s*. The failure or neglect of any locator of a mining claim to perform development work of the character, in the manner and within the time required by the laws of the United States, shall disqualify such locators from relocating the ground embraced in the original location or mining claim or any part thereof under the mining laws, within 3 years after the date of his original location and any attempted relocation thereof by any of the original locators shall render such location void.

§ 2. All acts and parts of acts in conflict with this act are hereby repealed.

§ 3. This act shall take effect and be in force on and after July 1, 1909.

The California mining law has been quoted in full as being one of the more recent expressions of state legislation on the subject of the location of mining claims, upon the public domain, within the particular states.

Laws of 1927, Chap 128, Art IIj, created a State Department of Natural Resources. A Chief of Division shall be in charge of the Division of Mines and Mining, who shall be known as the State Mineralogist.

Laws of 1929, Chap 280, provided for annual reports by mine owners, and lessees to the State Mineralogist, at his request. The word "mining" includes all mineral, oil or gas bearing property.

Laws of 1931, Chap 480, provide for filing with the Department of Finance annual reports on the number of tons of minerals taken from lakes and streams. A royalty of 25¢ per ton is provided for.

Laws of 1933, Chap 161, require employers of miners to have sufficient assets on hand with which to pay wages of miners before the start of any wage period.

Laws of 1935, Chap 482, provide for recording of grubstake contracts and prospecting agreements.

Other Laws have been and will be from time to time passed, but the preceding statements cover the most important of the California Mining Acts. Special provisions for the location and sale or lease of state mineral lands have also been passed by Calif, Mich, Minn, Nev, N Y, Tex, Wis, and some other states.

15. REFERENCES TO FEDERAL MINING LAWS OF OTHER STATES AND ALASKA

Arizona. Revised Code (1928), Chap 51, Secs 2 266–2 277

Colorado. Annotated Statutes (1935), Chap 110, Sec 168, pp 133 *et seq*

Idaho. Annotated Code (1932), vol 3, Title 46, pp 168–206

Montana. Revised Code (1935), vol 3, Chap 95, Secs 7 365–7 380 (see also repealing Statutes Laws, 1937, Chap 148)

Nevada. Compiled Laws (1929), with 1934 supplement, vol 2, Secs 4 120–4 144

New Mexico. Statutes Annotated (1929), Art I (see supplement, 1938, pp 444–447)

North Dakota. Compiled Laws (1913), with 1925 supplement. See Supplement Index, pp 1884–1888

Oregon. Code Annotated 1930, vol 3 and 5 (containing 1935 supplement)

South Dakota. Compiled Laws (1929), Secs 8 707–8 756

Utah. Revised Statutes (1933), Title 55, pp 664 *et seq*

Washington. Remington's Revised Statutes (1932), vol 9, Secs 8 615–8 635 (see supplement, 1935)

Wyoming. Revised Statutes (1931), with 1934 supplement, pp 1 106–1 114

Alaska. Compiled Laws (1933), Secs 321–367

SUMMARY OF UNITED STATES LAWS

16. HOW MINING RIGHTS MAY BE ACQUIRED

Discovery is the first and DEVELOPMENT the second condition of the right of possession. Halleck says: "Discovery is made the source of title, and development, or working, the condition of the continuance of that act." Just as there can be no valid appropriation of a mining claim without antecedent discovery, so in all of the mining states, except California and Utah, a certain amount of development work is essential to the completion of a location. In addition to this preliminary work, a certain amount of labor and improvement is required as a condition to perpetuation of possessory title. This amount is fixed by the national law at $100 worth annually, and is by different states made to apply to other improvements than actual excavation, or translated into an equivalent in hours of labor.

In the case of Butte Superior Coal Co *vs* Clark-Montana Co, decided Mch, 1919 (249 U S 12) the court held that "priority of right is not determined by date of entries or patents of the respective claims, but by priority of discovery and location, which may be shown by testimony other than the entries and patents," and that "a failure of the earlier location notices to comply with the state law is immaterial if the junior locator, at the time of locating, knew that the earlier locator was in possession and working his claim." See also Cole *vs* Ralph (252 U S 287), decided Mch, 1920, overruling Work Mining & Milling Co *vs* Doctor-Jack Pot Co (194 Federal, 620).

Posting of notice is not specifically required by the federal law. Such formality is, however, made one of the steps preliminary to completion of location by the laws of practically all the mining states; and the place and manner of posting are often prescribed. It is, moreover, in some instances specifically provided that the location shall be void if this requirement be not complied with. It is therefore advisable in each case to read carefully the local or state regulations.

Marking of location on the ground, so that its boundaries can be readily traced, is one of the requirements of the federal statute. This law has been supplemented by most of the mining states; and the number and character of stakes and monuments are specified.

Recording notice of location is not demanded by the national law, but is necessary under the laws of all the mining states. The time allowed in which to record varies from 30 days after date of discovery to 90 days after location. Here, also, care is needed.

17. HOW MINING RIGHTS MAY BE LOST

Forfeiture of a mining claim may be caused by failure of the claimant to perform any of the acts required by the laws for maintenance of possession. (Lindley, p 1 598.)

Abandonment likewise results in loss of mining rights. Judge Lindley remarks: "Abandonment operates instanter. Where a miner gives up his claim and goes away from it without any intention of returning, and regardless of what may be come of it, or who may appropriate it, an abandonment takes place, and the property reverts to its original status as part of the unoccupied public domain." (Lindley, p 1 597.) "Abandonment is always a question of intention. In forfeiture the element of intent is not involved." Resumption of work in good faith before initiation of possessory rights by others operates to prevent forfeiture.

18. PATENT PROCEEDINGS

Patent is the deed or grant of the U S, conveying the title authorized to be given by the mining or other land laws. It establishes the character of the land which it conveys, and is conclusive evidence of performance of all necessary steps leading to its issuance.

Land Department is that quasi-judicial tribunal which has exclusive jurisdiction over the disposition of lands of the public domain under the laws of Congress and the supervision of the Department of the Interior. The department consists of the Secretary of the Interior, the Commissioner of the General Land Office, and subordinate officers, such as registers and receivers of land offices, and surveyors-general. These officials are appointed by the President, the former for the term of four years; the latter for no fixed period. The surveyors-general may appoint deputies who are subject to the approval of the land commissioner.

First step in proceeding to obtain a patent for mining land from the U S is to make application for a survey. The application, addressed to the surveyor-general of the district, must be signed by one of the claimants or legal attorney in behalf of all the petitioners, and should contain:

1. Name or names of the claimants in full.
2. Name of each location embraced in the application.
3. Names of the mining district, state, and county in which the claim is located.
4. Name of the U S deputy surveyor to whom the order is to be issued.

Applicant must also furnish a copy of the location notices of each location to be embraced in the survey, each duly certified by the proper recording officer, and similar copies of each amended location. A deposit must be made to cover estimated cost of fees in office of the surveyor-general. The claimant may employ any deputy mineral surveyor in the district. Fee for the survey is additional to the amount deposited with the surveyor-general, and is adjusted between deputy and claimant by mutual agreement.

In making the survey the following particulars are to be observed:

1. Exterior boundaries of claim, number of feet claimed along vein, and, as nearly as possible, direction of the vein, and number of feet claimed on it, in each direction from point of discovery.
2. Points of intersection of survey lines with those of any conflicting prior surveys.
3. Conflicts with unsurveyed claims, where applicant for survey does not claim the area in conflict.
4. Total area of claim, and also area in conflict with each intersected survey.
5. If claim is situated within two miles of any corner of a public survey, the plat and field notes must show a line connecting the claim with such corner; if there be no such corner within two miles, it must be connected with a U S mineral monument. (Sec 17, Art 16, of this Handbook.)
6. End lines of each claim embraced within the survey must be made parallel.

Where PLACER CLAIMS are upon surveyed lands, and conform to legal subdivisions, no further survey or plat is required. If a known lode exists within the placer, it must be described in the survey, and surveyed as though it were situated elsewhere.

19. ADVERSE CLAIM

Any person asserting an adverse mineral right to all or some portion of the land for which patent is sought may file an adverse claim thereto. "Adverse proceedings are called for only where one mineral claimant contests the right of another mineral claimant." Controversies over the character of the land, *i e*, whether mineral or agricultural, etc, are not proper subjects for adverse claims; but may be heard by the land department under the head of protests. The adverse claim shall show the nature, boundaries and extent of the claim, and all proceedings, except the publication of notice

and making and filing of the affidavit thereof, shall be stayed until the controversy shall have been settled or decided by a court of competent jurisdiction, or the adverse claim waived. Except in Alaska, adverse claims must be filed prior to the expiration of the 60 days allowed by law for publication of the application notice; in Alaska, they may be filed during the time of publication or within 8 months thereafter. The claim must be filed with the register and receiver of the land office in whose district the land is situated. Within 30 days the adverse claimant must then begin action in a court of competent jurisdiction, to determine the question of right of possession, and prosecute the same to final judgment. The jurisdiction to try such cases involving title to real estate is in the state courts; and such actions are almost invariably heard by such courts. Where no action is commenced in court before the expiration of the 30 days, the land department may proceed as if no adverse claim had ever been filed.

20. NATURE OF TITLE CONVEYED BY MINERAL LODE PATENTS

Patent to a lode claim conveys: 1. Exclusive right of possession and enjoyment of all the surface included within the limits of the location, as described in the patent, subject only to pre-existing easements. 2. All veins, lodes, and ledges throughout their entire depth, the tops, or apices, of which lie within the boundaries, the right to pursue the vein in depth outside of such boundaries being limited, however; * * * * 3. *Prima facie*, such a patent confers the right to everything found within vertical planes drawn through the surface boundaries; but these boundaries may be invaded by an outside lode locator holding the apex of a vein under a regular valid location, in working his vein on its downward course underneath the patented surface (Lindley, p 1 897).

Placer patent conveys to the patentee everything within vertical planes drawn downward through the surface boundaries, EXCEPT: 1, such lodes or veins whose tops or apices are within the placer limits, whose existence was known prior to the filing of the application for placer patent, and were not included in the placer application; 2, such segments of veins having their tops or apices elsewhere, as may underlie the placer surface, and which may lawfully be taken by the apex lode locator under a regular valid lode location, pursuing his vein on its downward course. In the last class of cases the question of priority of location is wholly unimportant (Lindley, p. 1 915).

21. EXTRALATERAL RIGHTS

Probable origin. The right given by the mining laws of the U S to follow the vein beneath the surface of land owned by another was aptly termed the " extralateral right " in an early discussion by Dr. R. W. Raymond (*Trans* A I M E., Vol 12, p 387), and has passed into current use in legal phraseology. This provision of the law has given rise to interminable litigation; and in its interpretation volumes of opinions have been furnished by the courts and writers of law books. The manner of its introduction into our system of laws does not seem to be clearly understood. It is at variance with the Spanish and Mexican codes, and, outside of the U S (so far as we have been able to find), exists only in the code of Southern Rhodesia.

However, Thomas T. Read, Professor of Mining at Columbia, under date of June 2, 1939, makes the following comments: "The extralateral rights associated with the law of the apex were not entirely without precedent. In Bolivia in the 16th century, especially at Potosí and Machacamarca, the claim owner was privileged to extend his workings in any direction, provided only he did not cross the workings of any other operator. This led to so many difficulties that the square claim was later adopted. In the tin deposits of Yunnan province, China, a similar arrangement has prevailed since time immemorial, with similar result. In both cases, the deposit must be followed down from the surface; only in the U S was it possible to trace a deposit discovered underground back to a possibly hypothetical surface outcrop and thereby establish a claim to it."

Extralateral rights were also tried for a long period by the Germans in the sixteenth and seventeenth centuries, where a sort of inclined location was allowed, the miner having the right not only to follow the vein beneath the surface, but having also 7 *lachter* (about 25 or 30 ft) on each side of the vein, parallel to its walls, within which to work (R. W. Raymond, "Mineral Resources," 1869, p 195); but it was there abandoned because it gave rise to so much litigation. It was later tried for a short time in British Columbia, but promptly given up, for the same reason. (J. M. Clark, *Trans* A I M E, Vol 43, p 617.) It may have arisen in the U S from the fact that our laws resulted from the local rules of the miners, and that these rules were a product of the evolution and development of the western mines, starting with surface rights only, and at first solely with reference to placers, and being later extended to lodes. Each man located his vein and aimed to work it regardless of its dip or inequalities and without interference with or from his neighbors. In many districts, moreover, there was a general theory, at least in early days, that the veins were all parallel to each other in dip and strike; and such complexities of structure and uncertainty in mining as afterward developed could not have been foreseen.

Extralateral law is now contained in Chap 2, Title XXX, U S Code given in full, Art 7, herein. Section 23 provides that end lines of each claim shall be parallel to each other. It is not believed that this provision of the law operates to defeat extralateral rights, where end lines converge. It has also been held that end lines must not be broken (Lindley, p 863); but the universality of this rule seems to depend somewhat on the definition of " end lines," and relation of the apex of the vein to such portions of the end lines as are parallel. A concise summary of court decisions relative to the vexed questions of extralateral rights was published by Judge Clayberg (California Law Review, May, 1913; *E & M Jour*, Sep 20, 1913, pp 537–543), and is quoted *in extenso* below:

Ideal claim defined. An ideal quartz mining claim is a portion of mineral land, 1 500 ft or less in length, along the vein, and 600 ft or less in width. Fig 1 illustrates the surface of such a claim, showing the apex and course of the vein, and the side and end lines of the claim. Fig 2 illustrates a cross-section of such claim, showing the vertical side lines thereof, and the dip of the vein. The statute, by its terms, seems to refer only to an ideal location, with the vein on its course or strike passing through its two parallel end lines. The planes of the end lines, dropped downward perpendicularly, and extended in their own direction until they intersect the exterior portions of the vein, bound the extralateral rights on such vein. In actual mining, however, an ideal location is the exception rather than the rule. Such conditions result from the inability of the locator to determine the actual course or strike of the vein at the time of completing his location. As a matter of fact, the locator of a mining claim usually judges such course or strike from a small portion of the vein disclosed in his place of discovery. It is almost impossible to determine correctly the true course or strike of a vein from such a small exposure. Veins are irregular in their course or strike, and may be found running through the earth in almost every conceivable direction.

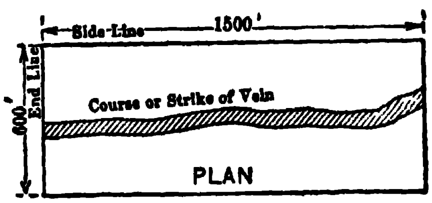

Fig 1. Ideal Location

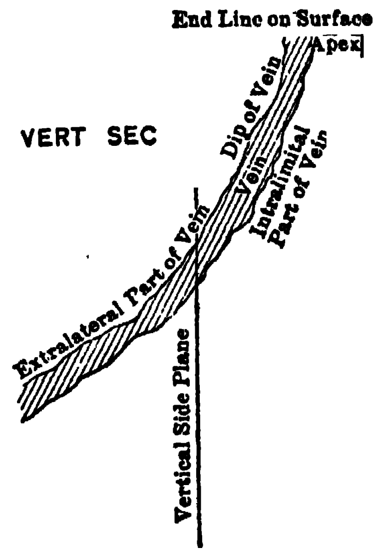

Fig 2. Cross-section, Ideal Claim, Showing Dip

Principles of extralateral rights. The application of the law of extralateral rights, as provided for by the Act, to the varied conditions which have presented themselves, depending on the course or strike of the vein within the surface boundaries of the location, and the form or shape of the location itself, has been the source of prolific litigation in the mining regions of the U S. In applying this statute, certain well-established principles must be always be borne in mind, as follows:

1. The basis of extralateral rights is the EXISTENCE OF THE APEX of vein within surface boundaries of the location, dropped downward perpendicularly, and therefore their extent along the vein should always be limited by the extent of the apex within the claim. This is recognized by Supreme Court of the U S in Del Monte Mining Co *vs* Last Chance Mining Co (U S, 55) in which the court says:

In considering the questions presented, we are dealing simply with statutory rights. There is no showing of any local customs or rules affecting the rights prescribed by the statute, and beyond the terms of the statute, courts may not go. They have no power of legislation. They can not assume existence of any natural equity, and rule that by reason of such equity a party may follow a vein into territory of his neighbor and appropriate it to his own use. If cases arise for which Congress has made no provision, the courts can not supply the defect. Congress having prescribed the conditions upon which extralateral rights may be acquired, a party must bring himself within those conditions, or else be content with simply the mineral beneath his surface. It is true that the primary thought of the statute is the disposal of the mines and minerals, and in interpreting the statute this purpose must be given effect. Hence, when a party has acquired title to the ground within whose surface area is the apex of a vein, with a few or many feet along its course, a right to follow that vein on its dip for the same length should be awarded to him if it can be done under any fair and natural construction of the language of the statute. If the surface was everywhere level, and veins constantly pursued a straight line, there would be little difficulty in legislation to provide for all contingencies; but mineral is apt to be found in regions where great irregularity of surface exists, and the strike of

veins is as irregular as the surface, so that many cases may arise in which statutory provisions fail to secure to a discoverer of a vein such an amount thereof as equitably it would seem he ought to receive. Twenty-one Mining Co *vs* Sixteen to One Mine, 255 Fed 658 (Feb, 1919), held that in following a vein down beyond its sidewalls, the owner of the apex may extend his working beyond the walls of the vein, if necessary for its proper economical working.

2. In order that extralateral rights exist in any location, its END LINES MUST BE PARALLEL (Sec 2 320, U S Rev Stats, Iron Silver Mg Co, *vs* Elgin Mg Co, 118 U S, 196). It is not held that the end lines of a mining location must be parallel in order that such location be valid, but only that they must be parallel in order to give extralateral rights to any vein, the apex of which may lie within the surface boundaries of such location. A prospector has the general right to make his location in such shape as he desires, and it will be held valid if he has otherwise complied with the law. It has also been decided that the parallelism of the end lines need not be mathematically accurate; that a substantial parallelism is all that is necessary.

In the case of Iron Silver Mining Co *vs* Elgin Mining Co, the location involved was one of peculiar character (Fig 3). The lines marked end lines on the location, and which were parallel, were not the lines of the location through which the vein passed on its course, and therefore not end lines governing extralateral rights. The peculiar condition of the apex of the vein was probably occasioned by erosion. The vein lying very nearly horiz, forces of nature had disintegrated certain portions

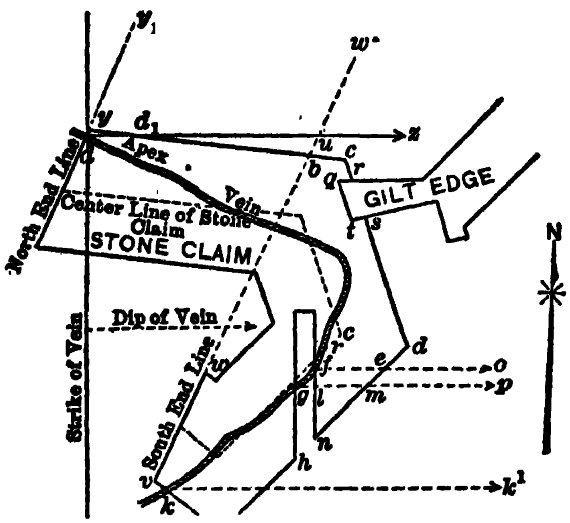

Fig 3. Iron Silver-Elgin Case

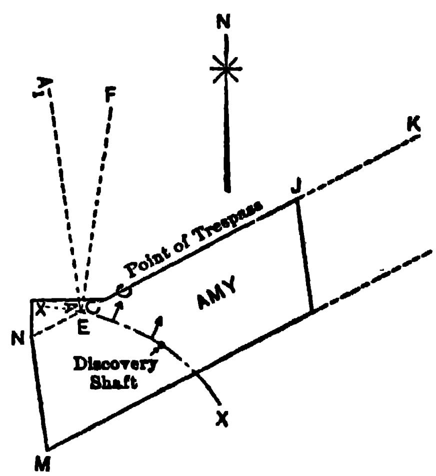

Fig 4. Amy Silversmith Claims

thereof, by wearing out a gulch, which, of necessity, caused a portion of the apex of the vein to recede from the line formerly occupied.

The only method of discussing extralateral rights clearly is by considering the various conditions which may present themselves, and applying the law to each condition, as follows:

1. **Ideal location.** When apex of a vein on its course passes through both end lines of a location, which are parallel to each other, the extralateral rights to such vein are clearly defined by the statute, and are bounded by the vert planes of the end lines extended in their own direction until they intersect the exterior portions of the vein. This condition is illustrated in Fig 1 and 2, and needs no further comment.

2. **Outcrop cutting both side lines.** Where apex of a vein on its course passes through both side lines of a location, which are parallel to each other, the U S Supreme Court has decided that the side lines of the location become end lines, for the purpose of determining extralateral rights to the vein, and such rights are bounded on the strike of the vein by vert planes from such side-end lines extended in their own direction until they intersect the exterior parts of the vein (Flagstaff Mg Co *vs* Tarbett, 98 U S, 463; King *vs* Amy Silversmith Co, 152 U S, 222). This condition is clearly disclosed in Fig 4.

3. **Outcrop cutting two diverging boundaries.** Where apex of a vein on its course crosses two boundaries of a location diverging from each other in direction of the dip of the vein, in our opinion no extralateral rights can exist, because the end lines are not parallel, and one is only entitled to so much of the vein on its dip as he has length of apex thereof within his surface lines. If the apex of a vein within the boundaries of a location is a certain number of feet in length, equitably the owner of such location should be given the same number of feet on the strike of the vein extralaterally, at any depth, and no more (33 Mont, 46). See Fig 5.

4. **Outcrop cutting two converging boundaries.** Where the apex of a vein crosses two boundaries of a location which converge toward each other in the direction of the dip, it has been held, in the case of Carson City Gold Mining Co *vs* North Star Mining Co (86 Fed, 658), that the extralateral rights would exist, bounded by the vert planes of the end lines, projected in their own direction, until they intersect. On this point Judge Lindley states:

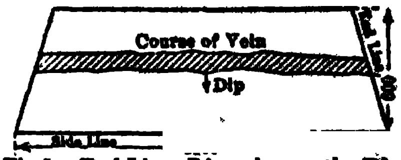

Fig 5. End Lines Diverging on the Dip

There can be little doubt that extralateral rights may be enjoyed under locations made or patents issued under either Act where the end lines converge in the direction of the dip of the vein, giving the locator or patentee less in length on the vein beneath the surface outside of his vert boundaries than he has at the surface.

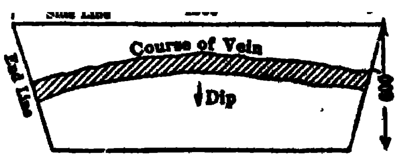

Fig 6. End Lines Converging on the Dip

It is admitted, however, that the decisions of the highest courts are not yet conclusive on these questions, and some lower courts have held the contrary of the opinion expressed above. This case is shown in Fig 6.

5. **Outcrop cutting end and side line.** Where apex of a vein on its course crosses one side line and one end line of a location, the doctrine has been established by the Supreme Court that extralateral rights are bounded by a plane dropped perpendicularly downward through end line of claim through which the vein passes, and a plane parallel therewith, dropped downward perpendicularly through the point where the vein departs from surface boundaries, across the side line. These planes extended in their own direction would bound the extralateral rights on such vein under such conditions (Del Monte Mining Co *vs* Last Chance Mg Co, 171 U S, 55; Clarke *vs* Fitzgerald, 171 U S, 92). See Fig 7.

6. **Outcrop entering and leaving through same side line.** Where apex of a vein on its course enters and departs from the claim through the same boundary line, the Supreme Court of Colo has held that no extralateral rights attach (Catron

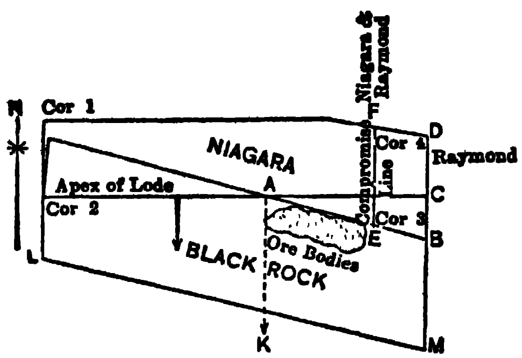

Fig 7. Apex crossing One End Line and One Side Line

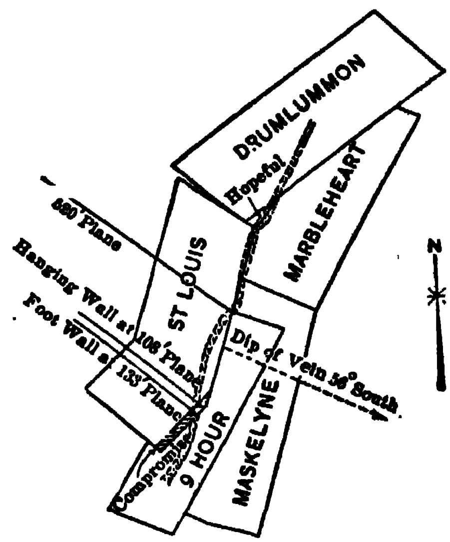

Fig 8. St Louis-Nine Hour or Drummummon Case

vs Olds, 23 Colo, 433). The U S Court of Appeals for the Ninth Circuit, however, holds the contrary (St Louis Mg Co *vs* Montana Mg Co, 104 Fed, 664; C. W. Goodale, *Trans* A I M E, Feb, 1914).

The extent of extralateral rights on a vein in a case such as that last referred to presented an interesting question. The vein in controversy was a wide one, entering and departing from the St Louis claim, at an acute angle, through the same side line. At the point of departure from the St Louis claim, the apex of the vein was conceded to be 25 ft wide, measured along the boundary which it crossed. The St Louis claim was conceded to be the prior location, and the owner thereof claimed that he was, therefore, entitled to all extralateral rights on the vein, so long as any portion of its apex remained within the claim boundaries. The Court of Appeals upheld this contention saying:

The only deduction which can be made from the foregoing rules is that, inasmuch as neither statute nor authority permits a division of the crossing portion of the vein, and the weight of authority favors the senior locator, the entire vein must be considered as apexing on the senior location until it has wholly passed beyond its side line. Fig 8 illustrates this condition, showing properties in conflict in St Louis case.

This decision of the appellate court has been cited by the U S Supreme Court, and thus appears to have received the sanction and approval of the highest court of the land. In a later opinion the Court of Appeals speaks as follows (183 Fed, 61): We are therefore of the opinion that the right of the St Louis Mining Co to extralateral rights in the Drumlummon to the extent that it apexes within the St Louis Mining Co's claim, has been previously determined by this court, and that this determination has been affirmed by the Supreme Court of the U S (204 U S, 204), and that such has become the law of the case.

The vein in question was an incidental or secondary vein, and not the discovery vein; but the rule is the same, because the statute provides that the owner of a lode claim is entitled to follow extralaterally all veins which apex within its surface boundaries. The doctrine here announced gives to the locator precisely the same extent along the course of the vein beneath the surface of the adjacent claims as that which he has upon the surface within his location boundaries.

7. Wide outcrops. Where the apex of a vein on its course is split or divided by a boundary line of a location, or where the vein is wider than the location itself, the U S Supreme Court has decided that the senior location is entitled to the extralateral rights of the entire vein, and the junior location takes it up after the rights of the senior are exhausted (Argentine Mining Co *vs* Terrible Mg Co, 122 U S, 478; Lawson *vs* U S Mg Co, 207 U S, 1; Empire State Co *vs* Bunker Hill Mg Co, 114 Fed, 417). See Fig 9.

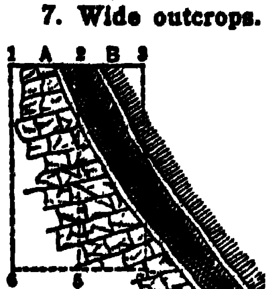

Fig 9. Vein Split by Boundary Line (Vert Sec)

8. Outcrop cutting one end line only, where the apex of an original or discovery vein on its course enters through one line of a location, and ceases before it reaches any other boundary; and

9. Outcrop cutting no boundary line, where the apex of the vein is wholly within the claim, and does not cross any boundary.

These two conditions are clearly analyzed by the U S Supreme Court, in discussing the Del Monte cases (Del Monte Mg Co *vs* Last Chance Mg Co, 171 U S 55), as follows:

The law places a limit on the length of the vein beyond which he shall not go, but it does not say that he shall not go outside the vert side lines unless the vein in its course reaches the end lines. Nowhere is it said that he must have a vein which either on or below the surface extends from end line to end line in order to pursue that vein on its dip outside the vert side lines. Naming limits beyond which a grant does not go is not equivalent to saying that nothing is granted which does not extend to those limits. The locator is given a right to pursue any vein whose apex is within his surface limits, on its dip outside the vert side line, but may not in such pursuit go beyond the vert end lines. And this is all the statute provides. Suppose a vein enters at an end line, but terminates halfway across the length of the location, his right to follow that vein on its dip beyond the vert side lines is as plainly given by the statute as though in its course it had extended to the farther end line. It is a vein, the top or apex of which lies inside of such surface lines extended downward vertically.

The Court of Appeals for the Eighth Circuit is also quoted as follows (Work Min & Mill Co *vs* Doctor-Jack Pot, 194 Fed, 620, 629): It does not follow that to secure extralateral rights the vein must extend from end line to end line, or, for that matter, intersect either end line.

Note.—The rule laid down by Judge Clayberg does not seem to cover the ground in cases 8 and 9. Veins are sometimes faulted or broken by torsional or differential movements which give them a broken and discontinuous apex, but still leave them with continuity beneath the surface. In such case, the extralateral rights might be bounded by vert planes parallel to the end lines of the claim, passed through the ends of the apex on the surface rather than by the vert end line planes themselves.

10. Outcrop departing and returning through same side line. Where a vein passes through both end lines of a location, but in its course departs from the claim through a side line and reënters the claim through the same side line, the case is said to be still a situation without specific ruling by the courts, although discussed in case of Waterloo Mining Co *vs* Doe (82 Fed, 45) as follows: The grant is to lodes having their apex in the ground patented. The fact that a part of the apex might be in the ground patented would not give any right to any part of the vein, the apex of which was not therein, although the apex might be cut by both end lines of the granted premises.

The vein must therefore be deemed, in general, to have extralateral rights within planes parallel to the end lines, for each segment of apex included within its surface boundaries.

—See also discussion as to what constitutes a vein in Utah Con Mg Co *vs* Utah Apex 277 Fed 41 and same *vs* same 285 Fed 249 and cases there cited (1922). See Twenty-one ... o *vs* Original Sixteen to One Mine, 265, Fed 469.

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22. EXTRALATERAL RIGHTS ON INCIDENTAL OR SECONDARY VEINS

Statute gives extralateral rights to *all* veins whose tops or apices lie within the surface lines of the location. The vein on which the location is based is called the "discovery" or "original" vein; others are called "secondary" or "incidental" veins. While all veins have equal and similar rights under the law, yet the location or discovery vein, in its relation to the surface boundaries, is the vein which determines the direction of the planes limiting the extralateral rights for all other veins apexing within the claim.

Decisions of the courts in the Providence-Champion case (63 Fed, 552; 171 U S, 293; 18 Sup Court Rep, 909, 43 L Ed 170), the Drumlummon case (Mont Min Co, Ltd, *vs* St Louis M & M Co, 102 Fed, 430; 104 Fed, 664), the Anchoria-Leland (Jefferson Min Co *vs* Anchoria-Leland M & M Co, 32 Colo, 176, 75 Pac, 1 070), and in Supreme Court of Colo (Ajax Gold Min Co *vs* Hilkey, 31 Colo, 131; 72 Pac, 447; 22 Morr Min Rep, 585) have given rise to much discussion. The weight of authoritative opinion seems to support the views of the Colo court, contained in the following:

The end lines constitute a barrier, beyond which a locator can not follow a vein on its strike, whether it be a discovery or a secondary vein; and they also limit the bounding planes within which his extralateral rights are to be exercised in following such vein on its dip. In exercising such extralateral rights the locator can not in any case pursue the vein on its dip beyond the bounding planes drawn through the end lines. * * * The extent of the right depends upon the length of the apex, and the extralateral rights are measured not necessarily by the end lines, and only so when the vein passes across both end lines, but by bounding planes drawn parallel to the end lines passing through the claim at the points where it enters into and departs from the same. It would seem, therefore, necessarily to follow that the extralateral right depends, *inter alia*, upon the extent of the apex within the surface lines, and, while the end lines of the claim as fixed by the location are the end lines of all veins apexing within its exterior boundaries, the planes which bound such rights of different veins may be as different as the extent of their respective apices, though all such planes must be drawn vertically downward parallel with the end lines. It makes no difference in what portion of the patented claim the apex is. Its extralateral rights under this rule can be easily ascertained.

The same court, summing up the opinion of Justice Brewer in the Del Monte case, speaks as follows:

Our conclusion is that for all veins, both discovery and secondary, of a patented claim, the owner has extralateral rights at least for so much thereof as apex within the surface lines; that such rights as to secondary veins are not confined to such veins as apex within the same segment of the claim in which the apex of the discovery vein exists; and while the end lines of the location as fixed and described in the patent are the end lines of all veins apexing within the surface boundaries, and may constitute the bounding planes for such extralateral rights, and in no case can the locator pursue the vein on its dip outside the surface lines beyond such planes continued in their own direction until they intersect such veins, yet these bounding planes, which in all cases must be drawn parallel to the end lines need not be coincident.

Judge Lindley (p 1 378) remarks that "the modern tendency of decision is to apply to all secondary veins at the points of departure planes parallel to the end lines controlling extralateral rights on the original vein."

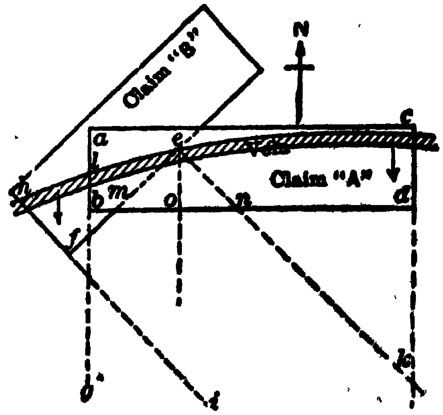

Fig 10. Conflicting Extralateral Rights

Conflicting extralateral rights. Extralateral rights may be curtailed or interrupted by conflict on the dip of the vein with the rights of an older location. In other words, two claims may be so located on the same vein that the planes of their extralateral rights will intersect. In such cases the senior valid location is entitled to those portions of the vein which are in conflict. The junior rights again attach beyond the area of conflict. Thus, in Fig 10, *A*, if senior, owns all ore between its end lines extended in their own direction. If *A* be junior, the vein from *h* to *e* belongs to *B* between the planes of *B*'s west end line and the parallel plane *ek* (with possible exception of triangle *emn* or *emo*).

23. VEINS DIPPING INTO AND BENEATH AGRICULTURAL PATENTS

Whether veins properly located under the mining laws can be followed down beneath the surface of patented agricultural lands seems not to have been decided by the U S Supreme Court. The Ninth District Circuit Court decided (Amador Min Co *vs* S S H Min Co, 36 Fed, 668) that such veins may not be followed by their owners beyond the points where they enter vertically beneath the surface of such prior agricultural grant. With this ruling Judge Lindley takes issue strongly (p 1 462), as follows:

We simply affirm that lodes or veins having their apices outside of the agricultural grant are, to the extent that such lodes or veins on their downward course underlie it, reserved by law out of such grant. Nor does this doctrine militate against the well-established rule of law that the patent is conclusive evidence of the character of the land. The land covered by an agricultural patent is conclusively deemed to be agricultural, but this does not necessarily imply that a lode under its surface apexing outside of it, cannot be reserved without impeaching the patent and changing the legal character of the land. The two classes of grants may exist without conflicting, in a legal sense.

By mutual agreement, the boundaries of extralateral rights, or their entire abolition, may be legally fixed, thus doing away with that dangerous provision of the federal law. Indeed, many such agreements have been made within recent years in many of the copper mining districts of the United States.

24. SOME UNSETTLED QUESTIONS AND RECENT EXTRALATERAL CASES

The opinion is frequently expressed that " all the doubtful points in the law of extralateral rights under the statute of 1872 are practically settled." Its marvelous perfection as a piece of trouble-breeding, perplexing and ambiguous legislation is abundantly illustrated by the fact that, more than 60 years after its enactment, this statement is still far from true. Judge Lindley remarks (p 1 423) that " lack of harmony between courts of different jurisdictions in the construction of the mining laws is a matter of frequent occurrence. For this reason, the law often remains in a state of uncertainty until the questions are finally decided by the Supreme Court of the U S. Even then, courts at times disagree as to the proper construction to be given to the decisions of the court of final resort."

Judge Clayberg, as quoted in Art 21, admits that "the forms of mining locations may be so variant, and the position of veins within the vert boundaries thereof so strange and peculiar, that many complications, now unthought of, may present themselves for adjudication." On this point the late Mr. Winchell expressed himself as follows: "While it is true that the U S Supreme Court has handed down decisions which definitely settle some of the principles of apex law, it is very far from true that no other vitally important questions remain undetermined. There are points upon which the U S Courts of Appeal in different circuits are in direct conflict. There are questions involved in litigation now pending which have never been passed upon by the Supreme Court, and some of these questions contain possibilities which one shudders to contemplate." (Why the Mining Laws Should be Revised. H. V. Winchell, *Trans* A I M E, v 48, p 361.) This is equally true in 1939.

Recent extralateral cases have been summarized as follows by William E. Colby, of the San Francisco Bar.

Whether or not a suit to establish extralateral rights is local in nature, to be tried in the jurisdiction where the mine is situated, depends upon the laws of the state where the question arises.

A suit to quiet title to asserted extralateral rights can not be maintained in the absence of any development showing whether such rights exist. Arizona Co *vs* Iron Cap Co, 27 Ariz, 202, 232 Pac, 549. Certiorari denied 270 U S 642; also see 29 Ariz, 23; 239 Pac, 290.

A deed to a lode mining claim, without express mention, conveys the extralateral portion of a vein apexing therein. Since there can be no priority of title to different portions of the same mining claim, an extralateral plane dividing rights as between grantor and grantee must be passed through the point of intersection of the center line of the apex with the common boundary line. Right to do further work to establish this fact was granted. Midwest-Butte Co *vs* Butte-West Side Mines Co, 32 Fed (2d), 841 (9 C C A).

Burden is on extralateral claimant to prove continuity from apex to point on vein outside boundaries. Facts ascertained by trial judge on inspection of premises are to be considered as evidence. Brugger *vs* Lee Yim, 12 Cal App (2d), 38.

When end lines of a lode claim converge in the direction of the dip of a vein the extralateral right is not destroyed because they are not parallel. Approximate parallelism is

sufficient. As between co-tenants, the purchase by one of an adjoining claim containing the apex of a vein dipping beneath the claim owned in common results in the holding in trust for the benefit of the other co-owner of that portion of the vein situated vertically beneath the claim held in common. Grant *vs* Pilgrim, 95 Fed, (2d), 561 (9 C C A).

The owner of land held under agricultural patent may not convey such land back to the U S and receive a lode-mining patent therefor so as to entitle him to exercise an extralateral right based on such new patent, where the statutory period to set aside the agricultural patent had expired. Such mining patent was void, not voidable, and could be collaterally attacked. Empire Star Mines Co *vs* Grass Valley Bullion Mines, 99 Fed (2d), 228 (9 C C A).

25. VERTICAL SIDELINE AGREEMENTS

The Apex Law of 1872 has led in the past to so much litigation in applying its provisions that many years ago there grew up a practice in some mining districts, to avoid the effect of the Apex Law, by entering into sideline agreements. The application of such agreements in the Bisbee mining district, Ariz, was largely due to the leadership and judgment of the late Dr. James Douglas, where sideline agreements were executed between several of the larger mining companies. Such agreements are now often made, limiting each party thereto to the mining of ore directly beneath the surface of its own claims, and doing away with the provisions of the Apex Law in so far as that law permits following the apex of the vein beyond the sidelines of the claims. A useful form of this agreement follows:

MEMORANDUM OF AGREEMENT, made and entered into this .... day of ...., 19 ...., between ...., a corporation of the State of ...., party of the first part, and ...., a corporation of the State of ...., party of the second part,

WITNESSETH: THAT WHEREAS, the parties to this agreement are the owners of certain mines and mining claims situated in .... Mining District, ... County, ..., some of which claims are adjoining and contiguous to each other, and others located in close proximity to each other; and WHEREAS, under the laws of the United States relating to mining claims, the locator or owner of a mining claim is given exclusive right of possession and enjoyment of all veins, lodes and edges throughout their entire depth, the top or apex of which lies inside of the surface lines of the mining claim extended downward vertically, although such veins or lodes may so far depart from a perpendicular on their course downward as to extend outside the vertical side lines of the surface locations, the right of possession being confined to such portions thereof as lie between vertical planes drawn downward through the end lines of the locations so continued in their own direction that such planes will intersect such exterior parts of such veins, lodes or ledges, which rights are generally known as extralateral rights; and WHEREAS, as a result thereof, disputes and underground conflicts frequently arise involving expensive litigation; and such undetermined rights create uncertainties as to the legal and equitable rights attaching to the ownership of mining claims; and WHEREAS, the parties hereto desire to settle and adjust forever, as between themselves, their respective rights to any veins, lodes, or deposits, or parts of veins, lodes, or deposits, existing within the vertical boundaries of the certain mines and mining claims owned by them respectively, by waiving all such extralateral rights, privileges and ownerships arising under the statutes of the United States, now existing, or which may hereafter be adopted, as against the ownership of the other party hereto,

NOW THEREFORE, in consideration of these presents and the covenants, conditions and privileges herein set forth and the grants, conveyances, relinquishments and releases which are hereby made by and between the parties hereto, it is hereby covenanted and agreed by and between the parties hereto as follows:

(1) That in all cases where any vein, lode, or deposit of mineral bearing rock or earth, having its top or apex within vertical planes drawn downward through the surface boundary lines of any mining claim herein mentioned and referred to and owned by the party of the first part, shall pass on its downward course or otherwise outside of and beyond vertical planes drawn downward through the boundary lines of said mining claim of the party of the first part and into and beneath the surface of any mining claim owned by the party of the second part, and herein referred to, the party of the first part does hereby relinquish and release to, and forever vests in, grants and conveys to the party of the second part that portion of every such vein, lode, or deposit which is beneath the surface of any such mining claim of the party of the second part. This provision is intended to relinquish and release to, convey, grant and vest in the party of the second part only such part of any vein which is beneath the surface of the mining claim of the party of the second part and not to deprive the owner of said apex of extralateral rights against mining claims other than those of the party of the second part, herein referred to.

(2) That in all cases where any vein, lode, or deposit of mineral bearing rock or earth, having its top or apex within vert planes drawn downward through the surface boundary lines of any mining claims herein referred to and owned by the party of the second part, shall pass on its downward course or otherwise outside of and beyond vertical planes drawn downward through the boundary lines of said mining claim of the party of the second part and into and beneath the surface of any mining claim owned by the party of the first part and herein referred to, the party of the second part does hereby relinquish and release to, and hereby forever vests in, grants and conveys to the party of the first part that portion of every such vein, lode, ledge or deposit which is beneath the surface of

any such mining claim of the party of the first part. This provision is intended to relinquish and release to, convey, grant and vest in the party of the first part only such part of any vein which is beneath the surface of the mining claims of the party of the first part and not to deprive the owner of said apex of extralateral rights against mining claims other than those of the party of the first part, herein referred to.

(3) That upon the application of either party hereto for a United States patent for any claims herein referred to, no protest, objection or adverse claim or suit shall be entered, made, filed or instituted by the other party on account of the working or mining of any vein, lode, or deposit of mineral bearing rock or earth or extraction of ores therefrom upon the dip of any such vein or lode.

(4) That in all cases in which United States patents may be hereafter granted to any mining claim, herein referred to, of either of the parties hereto, this agreement shall operate as a covenant on the part of each of the parties hereto that, upon the acquisition by either party of the outstanding title of the United States in such unpatented claims, all the covenants herein shall be deemed immediately applicable to and shall control and determine the rights of the parties hereto in relation to following any vein or lode of mineral-bearing rock or earth in such patented claims beneath the surface of any mining claim of the other party hereto, notwithstanding the terms of the grant and conveyance by the United States to either party hereto or the provisions of the Federal mining laws.

(5) That either party hereto, upon the request of the other and without further or additional consideration, shall and will make, execute and deliver to the other, such further or additional instrument or conveyance as shall, in accordance with the foregoing paragraph, absolutely vest the ownership of the portion of any such vein, lode, or deposit in the other party to this agreement in so far as such vein, lode, or deposit is beneath the surface of any mining claim of the other party, herein referred to.

(6) That the party of the first part for the considerations herein expressed and in consideration of the sum of One Dollar ($1) to it paid by the party of the second part, the receipt whereof is hereby acknowledged, has forever released and discharged, and by these presents does forever release and discharge the party of the second part from any and all debts, claims, demands, damages or suits at law or in equity for or on account of any trespass or injury done or committed in working in or upon any vein, lode, or deposit of mineral-bearing rock or earth within the vert boundary lines of any mine or mining claim owned by the party of the second par' and herein referred to.

(7) That the party of the second part for the considerations herein expressed and in consideration of the sum of One Dollar ($1) to it paid by the party of the first part, the receipt whereof is hereby acknowledged, has forever released and discharged, and by these presents does forever release and discharge, the party of the first part from any and all debts, claims, demands, damages or suits at law or in equity for or on account of any trespass or injury done or committed in working in or upon any vein, lode, or deposit of mineral bearing rock or earth within the vertical boundary lines of any mine or mining claim owned by the party of the first part and herein referred to.

(8) That the mine and mining claims and interest and parts of mining claims referred to in Schedule A hereto annexed constitutes the property of the party of the first part hereto, and the mines and mining claims, interests and parts of mining claims mentioned and referred to in Schedule B hereto annexed constitutes the property of the party of the second part hereto, and said Schedules are annexed hereto, and made a part of this agreement.

(9) This agreement shall extend to, inure to the benefit of, and be binding upon the parties, their successors and assigns.

IN WITNESS WHEREOF

26. DEEDS OF SURFACE RIGHTS IN MINING PROPERTIES

It is often necessary or desirable for a mining company to convey a deed of surface rights, reserving to itself the ores and minerals beneath. The following form of deed, to be made by the owner of the surface to the purchaser of the surface, for such consideration as may be stated in the deed, has been frequently used with advantage.

Quit-Claim Deed .... unto said party of the second part and unto his heirs and assigns forever, all the right, title, interest, claim and demand which the said party of the first part has in and to the following described real estate and property situate in the County of ...., State of ...., to wit: That portion of the surface of the .... Mining Claim situate in the .... Mining District, in .... County, ..., the United States Patent whereof is of record in the office of the County Recorder of .... County, in Book .... of Deeds, at Page ...., covered by that certain lot, piece or parcel of land known and described as Lot Numbered .... (   ), in Block Numbered .... (   ), of the Town of .... County, according to the .... of said Town of ....; together with the improvements thereon situate.

This deed is not intended to convey and does not convey any of the above described premises to a greater depth than .... (   ) feet immediately beneath the surface, nor any of the ores, minerals, or metal contained therein, and said party of the second part, his heirs and assigns, shall not have the right of lateral or subjacent support, as against said party of the first part, ..., its successors and assigns, and said party of the first part, its successors and assigns, shall not be liable for any damages caused by the subsidence or other disturbance of the surface or other part of said premises on account of mining or other operations beneath the premises hereby conveyed, or beneath the adjoining or other premises not hereby conveyed. Said party of the second part will and by the acceptance of this deed does, for himself and for his heirs and assigns, hereby release said party of the first part, ..., from any and all liability for any property

loss sustained upon said premises by reason of any operation or business carried on by said party of the first part, its successors or assigns, or caused by fire, smoke, gas, flood, damages from mining, milling, smelting, or reduction of ores, railroads, electrical transmission lines, or directly or indirectly connected with any of the same owned or controlled by said party of the first part. TO HAVE AND TO HOLD the same, together with all and singular the appurtenances and privileges thereunto belonging, or in anywise appertaining, and all the estate, right, title, interest and claim whatsoever, of the said party of the first part, either in law or in equity, in possession or expectancy, to the only proper use, benefit and behoof of the said party of the second part, his heirs and assigns forever, subject to the foregoing reservations.

IN WITNESS WHEREOF

FEDERAL TAX LAWS RELATING TO MINES

Summarized by PAUL ARMITAGE, of Douglas & Armitage, of the New York Bar

With the advent of Federal Income and Profits Taxes mine valuations became important. Engineers should have a general knowledge of the occasions when such problems may arise under the Federal Tax Laws.

Beginning with the Revenue Act of 1913 provision was made for an allowance for depletion of the mines, and this depletion allowance was enlarged and extended in the various subsequent Revenue Acts. Depletion, like depreciation, has the same basic concept of voiding a tax on capital, and, as in many cases, it is voided on a mine valuation, it is important for mining engineers to be familiar with these laws. Similar questions often arise under state laws in assessing income and inheritance taxes.

27. CLASSIFICATION

The five principal cases in which mine valuation may become essential are in determining: (*a*) depletion; (*b*) profits or losses on resale or disposition of a mine; (*c*) invested capital or paid-in surplus; (*d*) depreciation or obsolescence, including amortization; (*e*) value for estate taxes of a decedent's interest in a mining property.

28. DEPLETION

Basis for determination. As a mine is a wasting asset, provision must be made by owner or lessee for the return of the capital out of operating profits. Beginning with the Revenue Act of 1913, which merely allowed " five per cent of the gross value of the mine on the output for the year " (1913 Act, section b), Federal Tax Laws recognized this, and now allow annual depletion as a deduction from gross income in order to arrive at net taxable income. There have been developed over a series of years various extensions of the method and basis of computing depletion. In the 1938 Revenue Act there are four distinct bases:

(1) If the property was acquired before Mch 1, 1913, the basis of depletion is the fair market value of the mining property on that date.

(2) If the property was acquired after Mch 1, 1913, the depletion base is the out-of-pocket cost of the property to the taxpayer.

(3) If the mine was discovered (but not purchased or acquired as a proven tract) by the taxpayer after Mch 1, 1913, and the discovery value is " materially disproportionate to the costs," then the value on the discovery date, or within 30 days thereafter, forms the depletion base. This discovery value extends not only to the discovery of a new mine, but includes " minerals in commercial quantities " in an existing mine or mining tract, " if the vein or deposit thus discovered was not merely the uninterrupted extension of a commercial vein or deposit already known to exist." Depletion, of course, is only applicable for tax purposes if the discovered minerals can be separately marketed at a profit (Revenue Act of 1938, section 114). However, the annual allowance is limited to 50% of the net income for the current year from the discovered property.

(4) In the case of coal and metal mines and sulphur, depletion is allowed as a percentage of the gross income from the property during the taxable year, 5% in the case of coal mines, 15% in that of metal mines and 23% in that of sulphur deposits. A similar percentage is applied to oil and gas lands, amounting to 27.5% of the gross. However, such allowance shall not exceed 50% of the net income from the property. This percentage depletion first found its place in the Revenue Act as applicable to coal, metal and sulphur mines in 1932, and the taxpayer was afforded a right to elect for 1934 and sub-

sequent years. New elections were granted under the 1936 and 1938 Acts, but only as to newly acquired property. If the operator of a mine has not availed himself of this election, then he must find his depletion on the other bases. Hence, as to all mines acquired or discovered prior to Mch 1, 1913, the value on that date becomes the basis for depletion, and if discovered after that date, but before 1932, the discovery value becomes the basis for depletion. On the taxpayer or his engineers therefore, rests the burden of establishing this value to obtain a proper depletion allowance. The depletion rate is obtained by dividing this value by the estimated units in the mine (ton, lb or oz). The unit depletion rate applied to the annual extraction gives total yearly depletion. For example, on the valuation of a mine as of Mch 1, 1913, at \$50 000, containing by estimate 1 000 000 lb, the taxpayer is allowed a depletion rate of 5¢ per lb. Assuming in a given year a production of 200 000 lb, the total depletion deduction for operating profits for that year would be 200 000 × 5¢ = \$10 000.

Operating owners, lessors, lessees, sub-lessors, sub-lessees, life tenants, purchasers of royalty interests, owners of fractional or over-riding royalty interests, tenants in common and trusts and beneficiaries of the same, are each entitled to their respective shares of the allowance, and the Supreme Court has held that the right to depletion does not depend upon the retention of ownership, or of any other particular form of legal interest in the mineral content of the land. It is enough if the taxpayer has retained a right or share in the mineral content produced. Stockholders are not directly entitled to a depletion allowance, but distributions to the stockholder, made from a depletion reserve based upon cost or other basis of the property, are not considered as being paid out of earnings or profits, but the amount thereof reduces the cost or other basis of the stock upon which distributions were declared, and are non-taxable until the cost or other basis has been returned. However, if such distributions are from a depletion reserve based upon percentage depletion or discovery value, to the extent that the reserve represents the excess of percentage depletion, or the discovery value over the cost, they are taxable as an ordinary dividend. (Regulations 101, Section 115, Revenue Act 1938). But, any distributions are considered as paid out of current earnings until such earnings are all distributed (ibid).

In certain cases where, as a result of operations or development work, it is ascertained that the prior estimate of recoverable units on which depletion was based is in error, a revision of such prior estimate of the units may be made and the depletion base accelerated or retarded accordingly. But the Federal Treasury Dept has declined to allow any revision of the base for depletion, even though the estimate of recoverable units has formed a major factor in arriving at the same (such as a case of valuation).

Where beneficiation of the ore occurs after extraction, knotty problems arise in segregating and detaching from the price paid for the finished product the value added by such process of smelting, refining or concentrating. The Treasury regulations have established extensive rules for accomplishing this, as in the case of copper, gold, silver and coal mines.

Of course, the annual allowance for depletion of the mine and the base thereof is distinct from and in addition to the annual allowance for the depreciation of the plant and equipment used to operate the same.

29. INVESTED CAPITAL OR PAID-IN SURPLUS

Under the FEDERAL REVENUE ACTS OF 1917, 1918 AND 1921, imposing excess and war profits taxes, invested capital, including paid-in surplus, became a material factor. Also, where the capital stock of a corporation was paid up by the transfer of a mining property, its value at date of transfer may be a factor in arriving at the corporation's invested capital and paid-in surplus under the tax laws.

30. GAIN OR LOSS ON SALE OF MINING PROPERTY

Basis for determining gain or loss on sale of mining property is the same as that provided for depletion, except that no discovery value is allowed. That is, if the mining property was acquired before Mch 1, 1913 and subsequently sold, its value on that date becomes the basis for determining the taxable gain. If the property was acquired after Mch 1, 1913, the cost becomes the basis for determining gain or loss.

In some of the earlier Acts there was a special limitation of surtaxes in case of sale of mines, the principal value of which "had been demonstrated by prospecting or exploration or discovery work by the taxpayer." In 1926 this limitation was fixed at 16% (Revenue Act of 1926, Section 211-b).

31. VALUE FOR ESTATE TAX OF A DECEDENT'S INTEREST IN A MINING PROPERTY

On the death of a taxpayer owning an interest in mines or mining property, it becomes part of his estate and taxable both for federal estate taxes and local state inheritance taxes. Thus, a valuation of the mines is required as of the date of death, and similar proof of values of the mine must be made by competent engineers familiar with the property. The engineers of the Federal Revenue Bureau have recognized and adopted standard methods of valuing mines prevalent in the mining community, such as the Hoskold formula, and this has been followed by the tax boards, courts and state taxing commissions.

32. CAUTION TO MEMBERS OF THE MINING PROFESSION

As each successive revenue act has changed the phraseology of prior acts, under both the depletion provisions and those relating to gain and loss, the engineer will find it difficult to follow development of the laws without specialized study of each case, and it is essential to refer to the particular act under which a problem arises, and the decisions thereunder. Hence, the discussion in Art 27–31 gives only a general outline of the different cases in which valuation of a mine becomes essential, and should be read only as an index or guide to examining the statute itself and the decisions and regulations thereunder.

MINING LAWS OF CANADA

Summarized by GORDON MCMILLAN, K C, Toronto, Ontario

Administration of lands. The Dominion Government owns public lands in Yukon and North West Territories, administering them through the Dept of Mines and Resources. Public lands in the Provinces are administered by their respective Depts of Mines.

33. NORTH WEST TERRITORIES

Quartz mining claims. License (which costs $5 for an individual and from a minimum of $25 on a sliding scale to $100 per $1 000 000 of capitalization for a company) entitles holder to stake 6 claims, each 1 500 ft square, for himself and up to 6 each for 2 other licensees; a total of 18 claims.

Representation work amounting to $100 per year must be done, and claims up to 36 in number may be grouped for this purpose. Geol investigation, aerial reconnaissance or other like preliminary operation may be allowed.

Lease for 21 years, renewable for a like period, is granted for a fee of $10 per claim plus $50 rental per claim (renewals $200 per claim for 21 years) upon performing work to the value of $500 surveying claims and finding valuable mineral thereon. Lease lapses on non-payment of rental within 3 months.

Staking is by 4 posts, No 1 bearing name of claim, name and license number of licensee, date and hour of staking, and if claim is staked on behalf of another his name and license number. Witness posts may be erected.

Recording of claim must be within 15 days, unless claim is over 10 miles from Recording Office, when an extra day is allowed for each additional 10 miles. If claim is over 100 miles from Recording Office any 5 licensees may appoint an "emergency recorder" from among their number.

Royalty. On annual profits in excess of $10 000 and to $1 000 000, 3%; on excess over $1 000 000 to $5 000 000, 5%; on excess over $5 000 000 to $10 000 000, 6%; on excess over $10 000 000 a proportioned increase of 1% for each $5 000 000. All mines under same general management may be dealt with as one mine.

Placer claims are of 3 classes: (1) CREEK (on natural water courses with aver width at low water of 100 ft or less) are 500 ft in length along water course and 1 000 ft wide on each side of base line. (2) RIVER (on water courses over 100 ft wide) are on one side of river only, 1 000 ft along stream by 1 000 ft deep. (3) INLAND claims are rectangular, 1 000 ft square. The first locator on any creek is entitled to a discovery claim 1 500 ft long, and on a river or inland area to a discovery claim 3 000 ft long, with provisions for additional claims depending on size of locating party.

Any person over 18 years of age may locate claims. Upon location a grant for 1 year may be obtained on payment of $10 fee, and renewed on payment of the same fee and showing performance

of work to value of $100. Locating is by 2 posts, one at each end of the base line for creek and river claims, and at each end of the side nearest the creek or river on which they front for inland claims. Application for claim must be made within 10 days after location; if over 10 miles from Recorder's Office, an added day for each 10 miles, and if over 100 miles distant any 5 locators may elect an "Emergency Recorder." Transfers and other agreements affecting title must be in writing and recorded with Supervisor of Mines. Special provisions are made for obtaining the right to divert water for working claims.

34. YUKON TERRITORY

Quartz mining claims. Any person over 18 years of age may personally locate a claim 1 500 ft square by setting a legal post at each end of location line bearing appropriate inscription. Claims must be recorded within 15 days if within 10 miles of Recording Office, with one day extra allowed for each added 10 miles. If over 100 miles from Recording Office, 5 locators may appoint one of their number an "Emergency Recorder." If a power of attorney is filed in advance, 2 claims may be staked by proxy. Once located and recorded, a claim is good for 1 year and thence from year to year, provided holder performs work thereon to value of $100 per annum or pays that sum to Recorder. When holder has performed work to value of $500 (or paid that sum to Recorder), found a lode or vein, and had a survey of claim made, he may obtain a lease for 21 years at rental of $50 for the period; renewable for 21 years more at rental of $200.

Iron and mica location up to 160 acres may be recorded, in which case assessment requirements are doubled and rentals are $150 for first 21 years and $600 for subsequent periods.

Placer claims. Any person over 18 years may locate claims 500 by 1 000 ft, and not more than 500 ft in length along creek or parallel to creek on which they front. Staking is by 2 posts, one at each end of base line. First locator on any creek, hill, trench, bar or plain, is entitled to one claim 1 500 ft long; and, if party consists of 2 or more persons, 2 claims 1 250 ft long in length, claims to be of regular size. Recording is within 10 days, with one extra day allowed for each 10 miles after first 10. After location a grant may be obtained for 1 or 5 years at $10 per annum, with right of renewal upon payment of fee and performance of work to value of $200 per annum. Provisions are made for granting rights to divert water for mining purposes. Royalty of 2.5% is charged on all gold shipped from territory. A lease to prospect for 1 year, renewable for 2 years, may be obtained on certain conditions covering an extent of 5 miles in length (if abandoned ground or 1 mile if not previously prospected), at rental of $25 per annum per mile. Lessee must expend $1 000 per annum per mile and may before termination of lease stake out claims in area covered by lease.

Coal mining leases may be obtained in Yukon and North West Territories at rental of $1 per acre per annum; max area, 2 560 acres; royalty, 5¢ per ton of output. Provision is made for granting prospecting licenses where surface rights have been disposed of.

35. ALBERTA

Quartz mining claims are 1 500 ft square, staked by 4 corner posts. Any person over 18 years may locate and record in 1 year 3 claims in his own name and 2 each for 2 other persons, at $10 per claim. Claims must be recorded within 15 days, but if over 10 miles from Recording Office 1 day is allowed for each added 10 miles; provision is made for appointment of "Emergency Recorder." Claims may be retained from year to year upon performing work to value of $150 per claim, or paying that amount to Mining Recorder. LEASE (fee $10) may be obtained after performance of work to value of $750 (in which survey is credited up to $150 for location of vein or lode and survey of claim). Leases are for 21 years; rental, $50 for first 21 years, and $200 on renewals. Surface rights may be leased at $1 per acre per annum. Leases contain a reservation of royalty fixed by Order-in-Council. Transfers must be in writing, and in case of leases consent of Minister must be obtained.

Iron and mica claims 160 acres in area may be granted, work requirements being double those for Quartz Claims; rental, $150 for first 21 years and $500 for subsequent periods.

Placer claims. Regulations are similar to those in North West Territories, except that minimum work requirement in Alberta is $150 per annum.

Petroleum and natural gas leases, with maximum area of 1 920 acres, may be obtained at rental of 50¢ per acre for first year and $1 per acre per annum thereafter; term, 21 years, and leases are renewable. In unsurveyed territory applicant must locate by planting

2 posts, and apply for lease within 30 days, with extra day allowed for each 10 miles over 100. Machinery and equipment satisfactory to Minister (but not exceeding $10 000 in value) must be on property within 1 year and drilling operations must be commenced within 15 months. Royalty rate is fixed by Order-in-Council.

Coal mining rights are leased at annual rental of $1 per acre for 21-year period. Maximum area is 2 560 acres on unsurveyed territory and 640 acres in surveyed territory. In unsurveyed territory there must be a location by applicant. Operations must be commenced within 1 year from notification by Minister, and a specific amount of coal must be produced. Royalty of 5¢ per ton is payable in addition to rental.

36. BRITISH COLUMBIA

Miner's certificate. Holder of a FREE MINER'S certificate (fee for an individual $5; fee for a joint-stock company varies with its capital) may prospect on Crown lands, stake out claims and mine. He is restricted to staking 1 mineral claim on same vein or lode, but may acquire others by purchase. A free miner may locate 1 placer claim or leasehold in his own name, and one for each of 2 free miners for whom he acts as agent, on any separate riverbed, bar or dry diggings, but other claims may be acquired by purchase.

Mineral claim is a rectangular (unless it has one or more boundaries of previously recorded claim) piece of land not exceeding 51.65 acres in area. It is located by erecting 3 legal posts: discovery post where mineral in place is discovered and posts No 1 and 2 on line of ledge or vein and marking ends of claim. Each post must bear name of claim, name of locator and date of location; on No 1 post an inscription indicating direction of No 2 post and number of ft on right and left of line between them. Location line between posts 1 and 2 must be distinctly blazed if in a timbered area, or, if in bare country, marked by earth or rock monuments. 15 days are allowed for recording, plus 1 day for each 10 miles from recording office, after first 10 miles. Until a Crown grant is issued the claim is held practically on a yearly lease; minimum assessment-work, $100 value yearly, or equal sum paid to Mining Recorder, and assessments must be recorded within year. When $500 assessment-work is recorded and survey made, owner of claim is entitled to a Crown grant on paying $25 fee and giving notices required by Act.

Placer claims are of 4 classes: creek, bar, dry and precious stone diggings. CREEK DIGGINGS: claim to be 250 ft in direction of general course of stream and 500 ft wide on each side of middle of same. BAR DIGGINGS contain up to 250 ft on any bar covered at high water, or 250 ft long at high-water mark, and width extending from high-water to extreme low-water mark. In DRY DIGGINGS a claim is 250 ft square. Discovery claims, varying in number and length with the number of free miners in discovery party, have same width as ordinary placer claims. Discovery to be established to satisfaction of Gold Commissioner, and thereafter no discovery to be permitted within 5 miles measured along water-courses.

Placer claims are as nearly as possible rectangular, marked by 4 legal corner posts with name of locator, number and date of issue of free miner's certificate, date of location and name given to claim marked on each. In timbered areas boundary lines must be blazed and marked by posts not over 125 ft apart; in bare areas monuments of earth and rock may be substituted for all posts except at corners. Provisions for recording are the same as for mineral claims, but placer claims must be recorded before expiration if to be held for more than one year. A claim must be worked by owner or person on his behalf continuously as far as practicable during working hours; if work is discontinued for 7 days, except during close season or for other reason to Gold Commissioner's satisfaction, claim is deemed abandoned. Gold Commissioner may declare close season for all claims in his district, or a lay-over if water supply is insufficient. He may also grant tunnel and drain licenses, and easements over other claims. Mining partnerships are authorised for both mining and placer claims.

Placer-mining leases. Gold Commissioner of district may grant leases of unoccupied Crown lands approx 80 acres in extent, after location is made by staking along a location line not more than 0.5 mile in length (2 640 ft). Only an initial and final post are required. Width of leasehold is not to exceed 0.25 mile (1 320 ft), and locator, on both initial post and in notice of intention to apply (posted at Mining Recorder's Office), must state how many feet in location to right and left of location line. Annual rental is $30; $250 to be expended annually on development work.

Iron and steel bounties. By act of 1929, Lieutenant-Governor in Council may agree to pay bounties on pig-iron and steel shapes, when manufactured within the Province, not exceeding certain amounts per ton. A bounty, as on pig-iron, may also be paid on molten iron of a particular character.

Phosphate mining. By Act of 1925, tricalcium phosphate is removed from "Mineral Act" for purpose of administration, so as to permit staking of phosphate claims 1 mile square. There are special provisions relating to prospecting, staking, licensing and renewals.

Coal, petroleum and natural gas. For 640-acre blocks, with N–S and E–W boundaries and no side exceeding 80 chains in length, a license to prospect may issue for 1 year, subject to renewal for second or third year. Annual fee, $100. Discovery of coal being made during or within 30 days of expiry of license, land being surveyed and license conditions fulfilled, a lease may issue for 5 years at rental of 15¢ per acre, subject to renewals for 5 successive 3-year periods on payment of $100 per lease in addition to annual rental. After bona fide working, lessee may purchase the lands at $20 per acre where surface is available, or at $15 per acre if surface rights are not available. Royalty in addition of 2.5¢ per bbl (35 Imperial gal) of crude petroleum.

Taxes. Special provisions are made for leasing and subsequently purchasing Crown-granted mineral claims, reverting to Crown through non-payment of taxes. These claims are taxed 25¢ per acre per year. Output tax (payable quarterly), of 2% on gross value of mineral less transportation and treating costs, is imposed on all mines except coal, but refunded if ore shipments of mine do not realize market value of $5 000 in any year. Income from all mines subject to income tax are subject to exemptions and allowances in "Income Tax Act"; but, if output tax is payable, only the excess of income tax over output tax is payable. Coal is subject to tax of 10¢ per ton of 2 240 lb (payable monthly), except where shipped to coke ovens in Province. Coke and coal land are also taxed.

37. MANITOBA

Gold and silver. The holder of a miner's license (fee for an individual $5; for a Company, from $25 on a sliding scale to $100 per $1 000 000 capitalization) may stake 3 claims for himself and 6 for other licensees. Claims are 1 500 ft square in unsurveyed territory, and 40 acres in surveyed territory. Staking is by 4 posts with name, license number and date on No 1 post. Recording must be within 15 days of staking, but if over 10 miles from Recording Office an added day for each 10 miles; and if over 200 miles, 3 licensees may appoint one of their number an Emergency Recorder.

Assessment work consists of 125 days work in 5 years, at rate of 25 days per year. Diamond or core drilling counts 5 days for each 4 ft, and drilling by compressed air 2 days for each man employed. Survey counts 50 days, if made in first 2 years, and 25 days if made thereafter.

Lease, which is obtainable after performing assessment work, surveying claim and furnishing security for damage to owners of surface rights (if any), is for 21 years and costs $10, plus rental of $1 per acre per annum. Leases are renewable, but at double the original rental. Surface rights may be leased at $1 per acre per annum. Transfer must be in writing, and in case of lease must be approved by Minister.

Income tax on mining companies is as follows: Income up to $100 000, 5%; from $100 000 to $400 000, 6%; from $400 000 to $700 000, 7%; from $700 000 to $1 000 000, $8%; from $1 000 000 to $1 300 000, 9%; over $1 300 000, 10%. There is also a tax of $5 per claim.

Placer claims are as far as possible governed by the above regulations.

A Mining Board of 3 members has power to hear and determine disputes between licensees.

38. NEW BRUNSWICK

Prospector's license, fee $10, entitles holder to stake ten 40-acre claims, which must be recorded within 30 days. 25 day's work must be performed by Dec 31, unless claim is staked after Oct 31.

Mining license, at $10 per claim, may be obtained on showing that work has been done, claim surveyed and bond posted to indemnify owners of surface rights (if any), and is renewable on same terms. Licenses are in effect up to Dec 31 in year following year of issue; renewable upon performing 25 days' work per claim on payment of $25 per claim. The license gives right to mine.

Mining Lease. Holder of mining license, who has opened a mine and operated it for 6 months, may obtain a lease at $10 per year for each 40 acres, but rental is reduced by amount paid as royalty. Lease is for 20 years, renewable to 80 years and is for specified minerals. Minister may cancel lease for failure to operate for over 6 months.

Transfer of claims must be in writing and registered, and consent of Minister of Lands and Mines is necessary in case of leases.

Royalties. Coal and oil shale, 10¢ per long ton; petroleum and natural gas, 5% of value at well's mouth; other minerals, as fixed by Order-in-Council.

39. NOVA SCOTIA

Gold and silver. Mining claims are 40 acres. Application may be made for Prospecting License covering any number of mining claims, at 25¢ per acre and good for 1 year. Special licenses for 3 months may be obtained covering areas to 10 sq miles for $10. Within limit of license, a lease may be obtained good for 40 years' fee, which is 50¢ per acre with annual rental of 50¢ per acre. Work requirements are 2 days' work per acre of the claim, with diamond drilling and surveys allowed. Special provision is made for forcing owner to sell undeveloped areas either for cash or on a royalty basis and for rendering assistance by the Province to operators in development of mines.

Minerals other than gold and silver. Prospecting licenses (fee $30) and mining leases (fee $50) are granted on areas up to 1 sq mile, with a max length of 2 miles. License is good for 1 year, and 100 days' work per area of 1 sq mile or less must be performed, one-half thereof within 3 months. Diamond-drilling counts as 2 days per ft and surveys at 1 day for each $4, with max of 30 days; geological, laboratory or chemical work also accepted. Rental is $30 per annum per sq mile.

Royalties. Coal 12.5¢ per long ton; copper per unit (1%) per long ton, 4¢; lead or zinc per unit, 2¢; iron ore, 5¢ per ton; gold 35¢ per oz.; silver 2¢ per oz; other minerals, 5% of value. Lessee may offset against rent any greater amount paid as royalties in any year. Prospecting on private lands is permitted, subject to compensation to land owner for damages.

40. ONTARIO

Miner's license is necessary to stake out or acquire Crown lands for mining purposes; fee, $5 per year for an individual; for companies, $25–$150, depending on the share capital. Holder may stake out and apply for 9 claims in any mining division in any year on his own license, including up to 3 each for 2 other licensees.

Claim in unsurveyed territory, is a square, 20 chains on a side, of 40 acres, with lines N–S and E–W astronomically. In surveyed territory claims have an area of approx 40 acres, and not exceeding 50 acres, and constitute subdivisions of lots of varying sizes. Claim is staked by placing post No 1 at N E angle, No 2 at S E angle, No 3 at S W angle, No 4 at N W angle, and by blazing the lines connecting the posts if on timbered land, or erecting mounds of earth or rock in bare country.

Application for claim must be filed with Mining Recorder within 15 days after staking, one extra day being allowed for every additional 10 miles from Recorder's office. Fee for filing, $5. Within 4 months after recording, metal tags bearing number of claim (provided by Recorder) must be affixed to posts, and 30 days' work done in sinking, trenching, etc, except in winter (Nov 16–Apr 15). Including first 30 days, a total of 200 days of 8 hr is required, and not less than 40 days yearly within 5 years from recording date. Contiguous claims held by same owner may be combined in groups of 6 for assessment work. In computing assessment work credit is allowed for diamond drilling at rate of 1 day for each foot drilled; for survey by a recognized geophysical method, at rate of 1 day's work for each man necessarily employed in such survey; and for survey of claim by Ontario Land Surveyor, when required, at rate of 40 days per claim. Affidavits proving performance of work must be filed with Recorder within 10 days after expiration of each period, and when work is completed the holder obtains a grant in fee simple on payment of $2.50 per acre in unsurveyed, or $3 in surveyed lands; or, if in a Provincial Forest, a 10-year lease, renewable, rental first year $1 per acre; subsequent years, 25¢. If in unsurveyed land, an Ontario land surveyor's plan and field notes must be filed. On timbered land, permission to work claim may be withheld until timber is removed.

Provision is made for dredging leases, but no special rules are provided for placer workings, these not being a feature of mining in Ontario.

Petroleum, natural gas, coal and salt may be prospected for in Northern Ontario under authority of a Boring Permit, on staking out a block of 640 acres and paying a fee of $100. Permit for 1 year; renewable. If a workable deposit is found, a lease may issue for 10 years; annual rental, $1 per acre, subject to expenditure of not less than $2 per acre per year.

Mining Court of Ontario has power to hear and determine all disputes relating to mining lands, whether patented or unpatented. It has power to grant such rights and easements as necessary for proper operation of a mine.

Taxes. A graded tax is collected on annual profits exceeding $10 000 and up to $1 000 000, at rate of 3%; on excess above $1 000 000 and up to $5 000 000, 5%; on excess above $5 000 000, 6%. Where there is no municipal taxation, an annual tax of 5¢ per acre is levied on mining lands. Natural gas is taxed at 2¢ per thousand cu ft; a rebate of 75% being allowed if the gas is used in Canada.

41. QUEBEC

Mining lands may be acquired: (*a*) as a concession by purchase; (*b*) by occupying and working same under a mining license. In either case, a Miner's Certificate ($10) is necessary; holder may stake for himself 1–5 claims, and same number for each of 2 other certificate holders. Claim is a square of 40 acres, 20 chains to a side, running N–S and E–W; numbered stakes planted at corners, as in Ontario. In subdivided lands a claim is restricted to a half or quarter lot; where the lot contains more than 120 acres, size of claim depends upon area of the lot. Claim must be registered with the Dept of Mines or nearest local agent within 15 days; if 50 miles from a RR, an extra day is allowed for each added 10 miles. Metal tags bearing claim number must be affixed to stakes within 3 months. Claims are valid for 1 year (2 years if over 100 miles from RR, and assessment work is done in first year). After the year (or 2 years) a DEVELOPMENT LICENSE may be obtained on (*a*) payment of $10 and rental of 50¢ per acre, and (*b*) showing performance of 25 days' work per claim; licenses valid for 1 year; renewable. Provision is made for extending time for renewing and in certain cases for paying added rental of 50¢ per acre in lieu of assessment work. Five contiguous claims (subject to increase by Dept) held in one name may be grouped for assessment work. If a mine is opened on private lands, owner must be compensated.

Mining concession (grant) of the land may be had at any stage on payment of $5 per acre for SUPERIOR METALS and $3 per acre for INFERIOR METALS, conditions of the grant being that mining shall begin within 2 years and that during that time there be expended for each 100 acres at least $1 000 in case of superior metals and $500 for inferior metals. LETTERS PATENT may issue after granting of concession and performance of conditions. COMPANIES, unless wholly constituted under laws of Quebec, may not acquire any right in hydraulic power, land, forest, or mine forming part of the public domain of the Province on Mch 15, 1937, or which may form part thereof at any time after such date.

Taxes on profits: between $10 000 and $1 000 000, 4%; on excess of $1 000 000 to $2 000,000, 5%; on excess above $2 000 000 to $3 000,000, 6%; on excess above $3 000 000, 7%; exploration and development work may be deducted in determining profits.

42. SASKATCHEWAN

Quartz mining claims. Holder of a miner's license (fee for an individual $5; for a company from $25 on a sliding scale to $100 per $1 000 000 capitalization) may stake 3 claims for himself and 3 each for 2 other license holders. Claims are 1 500 ft square on unsurveyed territory and 40 acres in surveyed territory. Minister may grant iron and mica locations of 160 acres. Staking is by 4 posts, No 1 bearing name of claim and staker, date and hour of staking and license number. Claims must be recorded within 15 days, but if over 10 miles from Recorder's Office, one day is allowed for each added 10 miles; and if over 300 miles from Recorder's Office, 5 licensees may appoint an Emergency Recorder from among themselves.

Assessment work of $100 per year must be done in each of first 5 years. Diamond or core drilling counts $5 per ft; drilling by compressed air counts $7.50 per ft; survey work is allowed at cost and a geophysical survey may be counted as 1 year's work.

Lease, obtainable after performance of work, survey of claim and discovery of mineral, is for 21 years, and costs $10 plus rental of $5 per claim per annum; $15 in case of 160 acres claim; renewable for further period at double original rental. Security for damages to surface rights (if any) must be given before beginning operation. TRANSFERS must be in writing and for leases, consent of Minister must be obtained.

Royalty on precious metal ores, with aver value of $5 per ton or less is 2.5¢ per ton. Over $5 per ton, royalty rate is obtained by dividing aver value per ton by 10 in the case of precious metals and by 20 for other ores; the quotient is then multiplied by the royalty rate to obtain royalty per ton of ore. Mines under same general management or control may be considered as one mine. All royalties and penalties constitute a special lien on properties and machinery.

Placer claim regulations are same as those in North West Territories.

Petroleum and natural gas leases cover a minimum of 40 acres and a maximum of 1 920 acres in unsurveyed territory, and a minimum of 40 acres and a maximum of 19 200 acres in surveyed territory. In unsurveyed territory, no more than 3 leases may be held except by assignment. Rental, 50¢ per acre for the first year, $1 per acre for subsequent years. Machinery and equipment for prospecting satisfactory to the Minister must be on property within 1 year. Boring operations must begin within 15 months and be continuous, or at least $2 000 must be expended per year (provision for extension by Minister). Bond must be furnished before drilling begins, and all

drillers licensed. Royalty is payable at rate fixed by Order-in-Council, being not less than 2.5% or not more than 5% for first 5 years and not less than 5% or more than 10% thereafter.

Coal mining rights. An area up to 640 acres may be leased for 21 years at rental of $1 per acre per annum; licenses renewable. For mining operations, license to operate must be obtained from Coal Administrator. Royalty of 5¢ per ton is payable. Operations must begin within 1 year and at least 5 tons per acre mined in each succeeding year, subject to waiver by Minister. Surface rights may be leased at $1 per acre.

Alkali mining. An area up to 1 920 acres may be leased at 25¢ per acre per annum. Lessee must expend $10 000 per lease or group of leases; $2 500 first and second year and $5 000 third year, unless sooner expended. Royalty, 12.5¢ per ton. Surface rights may be leased at $1 per acre.

MINING LAWS OF MEXICO

Summarized by ROBERT T. BRINSMADE, Associate of Basham & Ringe, Attorneys, of Mexico City

43. HISTORY

The history of Mining Rights in Mexico comprises 3 periods: (a) Pre-Colonial; (b) Colonial; (c) the period following the Independence of Mexico. In the pre-Colonial (prior to 1516) and the Colonial (1516–1821) periods, the ownership of all subsurface rights was vested in the Spanish Crown and was inalienable and imprescriptible. The " Ley de Partidas " (1265), " Ordenamiento de Alcalá (1348) and the Law of John I (1387), also recognised that ownership of all mines in the Spanish Dominions was in the Spanish crown, but Spanish subjects could work mining properties under license from the Crown, by paying the Crown a royalty of two-thirds of the profits. The first law affecting mining concessions in New Spain (Mexico) was enacted in Dec, 1526, and known as the "Real Cedula"; it contained the principles of the earlier Spanish laws, and the "Ordenanzas de Minería," enacted 1563 and 1584, both in the reign of Philip II, retained the principal provisions of the earlier laws; the second "Ordenanzas" being more liberal, in permitting the King's subjects to work mines without a special license.

In 1783 the last "Ordenanzas de Minería" were enacted. These referred specifically to mines in Mexico, and after its independence they became the basis of its mining legislation. Under these " Ordenanzas " a mining claim was acquired as follows: (a) make a denouncement; (b) register the same; (c) pay the required tax to the Royal Treasury. Articles 1 and 2, Title 5 of these " Ordenanzas " provided that the owner of a mining claim could sell, rent or assign it.

After Mexico became independent, mining continued to be a public utility and all subsurface rights were vested in the Mexican Nation and like the ownership during the Spanish Crown was inalienable. In 1857 a new Constitution was adopted, and power to legislate on mining matters was by implication granted to the Mexican States, most of which adopted in toto the substantive provisions of the " Ordenanzas " of 1783. In 1883 the Federal Constitution was amended, and mining legislation was entrusted to the Federal Congress; and in 1884, the first complete Federal Mining Law was enacted. In 1892, the Mining Law of Mexico was amended for the purpose of encouraging exploration and exploitation of mining claims. The Mining Law of 1909, without modifying the principles established in the laws of 1884 and 1892, reaffirmed their essential features. The Law of extralateral rights governing claims in the U S is not recognized in Mexico.

44. BASIC PRINCIPLES

These are set forth in the Constitution of Mexico (1917) now in force. Paragraphs 4 and 6 of Article 27 of the Constitution are as follows:

" In the Nation is vested direct ownership of all minerals or substances which in veins, layers, masses, or beds constitute deposits whose nature is different from the components of the land, such as minerals from which metals and metalloids used for industrial purposes are extracted; beds of precious stones, rock salt and salt lakes, formed directly by marine waters, products derived from the decomposition of rocks, when their exploitation requires underground work; phosphates which may be used for fertilizers; solid mineral fuels; petroleum and all hydrocarbons, solid, liquid or gaseous."

" In the cases to which the foregoing paragraph refers, the ownership of the Nation is inalienable and may not be lost by prescription; concessions shall be granted by the Federal Government to private parties or civil or commercial corporations organised under the laws of Mexico, only on condition that said resources be regularly developed, and on the further condition that the legal provisions be observed."

45. MINING LAW OF 1930, NOW IN FORCE

This law, promulgated Aug 7, 1930, consists of 137 Articles, divided into 14 Chapters and is too extended to be included here. However, the first 7 Articles, also Articles 9–13 under the subtitle of " Mining Claims," are quoted in full, and a brief reference is made to some other Articles that are of particular interest. No attempt is made to give the procedural steps in acquiring concessions, and many other important sections are only briefly summarized. (The engineer or mining company desiring further information is advised to consult competent mining attorneys in Mexico. Editor)

Art 1. Scope of the law. This law shall regulate the exploitation, extraction and treatment of all natural mineral substances except petroleum and its derivatives, and those enumerated below, which will be regulated by special laws or by the Civil law: I. Soils suitable for agriculture or forestry. II. All rocks not commercially useful in the mining or petroleum industries. III. Products derived from the decomposition of the rocks mentioned in the foregoing paragraph, when their exploitation does not require underground workings. IV. Substances contained in suspension or dissolution by underground waters not having their source in any mine. V. Materials used for constructions purposes. VI. Products from salt deposits (when not formed directly by sea water), and " tequesquite " (saline surface crust).

Art 2. Minerals subject to this law are divided into: I. Metallic minerals; II. Nonmetallic minerals, including guano and amber; III. Coal and graphite.

Art 3. Exploitation and treatment of minerals subject to the provisions of this law are of public utility, and are therefore to be given preference over any other form of land utilisation.

Art 4. Concessions in general. The right to exploit and treat any minerals hereinbefore mentioned is originally acquired from the Nation, through concessions granted by the Dept of National Economy.

Art 5. Concessions are of 3 classes. I. Prospecting concession (Cateo), which authorizes and protects the person obtaining same to discover and work minerals susceptible of exploitation. II. Exploitation concession, which authorizes the person obtaining same to exploit and treat all minerals extracted from the land covered by the concession. III. Concessions for smelting plants (Plantas de Beneficio), which authorize and protect the construction and operation of smelters and mills.

Art 6. Concessionnaires. Only Mexican citizens and Mexican companies or associations have the right to obtain prospecting and exploitation concessions; foreigners may be granted the same right, provided they first comply with the provisions of Article 27 of the Federal Constitution and its reglamentary laws. Foreign companies, Governments, and Sovereigns cannot under any condition obtain such concessions.

Art 7. Rights derived from a concession can not be transferred in whole or in part to foreign governments or Sovereigns, nor can these be admitted as partners, co-partners or shareholders nor hold any right under the concession. Consequently, any act or contract in violation of this prohibition is null and void.

Art 9. Mining claims. The unit of a concession is the mining PERTENENCIA (claim), which is a solid of indefinite depth, bounded on the surface by 4 vertical planes corresponding to a horizontal square of 100 meters on each side.

Art 10. A mining claim is an isolated pertenencia or group of adjoining ones, although they only touch at one point, and which are protected by a single title of concession.

Art 11. When because of adjoining mining properties, it is not possible to reduce the claim to entire pertenencias, the resulting fractions may be added to it, in which case the claim will be deemed to be formed of as many pertenencias as there are hectares comprised in its horizontal projection, each fraction of hectare being considered as one entire pertenencia.

Art 12. If, because of the circumstances set forth in the preceding Article, it is not possible to form one entire pertenencia, the lot or claim may consist of a fraction of a pertenencia, but the area shall be computed in the manner provided for in that Article.

Art 13. In order that the division, amplification, reduction and unification of an exploitation concession may be legally effective, petitions requesting the same must be filed, and the procedure prescribed in this Law and its Regulations for the issuance of new titles and the cancellation of former ones must be followed.

Art 16. provides that surface owners have a preferential right to a concession.

Art 17. Assignment of a mining concession may be only to persons or companies who, according to the provisions of this law, have the power to obtain the same from the Nation. Assignments will only be effective against the Department or third persons, from the date they are recorded in the Public Mining Registry.

Art 20. Prospecting (Cateo), concessions, area: I. Claims must consist of 9 pertenencias included in a square of 300 by 300 meters, with positions fixed astronomically from N to S and from E to W. The starting point of the survey shall be in the center of the square, but such portions as invade titled or pending claims, and any fractions not contiguous to the one containing the starting point, shall be deducted from said square. II. Concessionaires may dispose of ore obtained from working the above mentioned concessions and erect mills for the treatment of such ore. III. They shall be granted for 2 years. IV. The beneficiaries shall have the exclusive right to petition for exploitation concessions, to replace the prospecting concessions in whole or in part, provided said petitions are filed while the latter are free. If a concession which has been requested is pending when the above term expires, the prospecting concession, with its rights and obligations, shall be extended until the new concession is issued or a final decision is reached.

Art 21. Location of the prospecting concession on the land shall be limited to fixing the starting point and to the erection of the corresponding identifying monuments. The prospector will not be required to erect boundary monuments.

Art 22. Mining agents shall grant to the applicant, prior to the issuance of the respective title, permits to do mining work and to dispose of the products so obtained, subject to the approval of the Department.

Art 24. No person or company can be the owner at one time of more than one prospecting concession.

Art 25. Exploitation concessions shall only be granted for the purpose of exploiting one of the following: (1) metallic minerals; (2) non-metallic minerals; (3) coal and graphite. They shall be granted for an unlimited period and their surface area shall not exceed 100 pertenencias, except in the case of coal, when their surface area may be 1 000 pertenencias. This Article also provides that the concessionaire must prove annually the regular work required by Article 27 of the Federal Constitution and shall pay an annual surface tax.

Art 27. Rights of a concessionaire. The concessionaire may during the life of his exploitation concession construct within or without the area of his concession means of transportation, storage stations, aqueducts and pipe lines, pumping plants, power transmission lines for his exclusive use, metallurgical and treatment plants and all other installations necessary for the purposes of the concession. All the foregoing shall be subject to the provisions of the Regulations of this Law.

Art 28–30 provide that proof of regular work must be made annually.

Art 33. Forfeiture of concessions. Exploitation concessions shall be forfeited in certain cases.

Art 34, 35, 36 provide for concessions for mills, the character of the concession for Public Service or private service and method of application therefor.

Art 37, 38, 39 deal with rights and duties of the mill concessionaires in treatment of ores delivered by the public and the rates therefor.

Art 40, 41 provide that a concession for a mill shall be forfeited in certain cases.

Art 42. Right of expropriation. A concessionaire under this law has the right to expropriate, after proper indemnification, such land as is absolutely essential, in the opinion of the Department, for offices, installations and other fixtures necessary to operate the concession, for dumps, tailing deposits, waste, etc, and to utilise mine and other waters.

Art 43. If the concessionaire is a foreigner, the expropriation requested by him will not be considered unless he can prove that he has complied with the provisions of Article 27 of the Federal Constitution and its Regulatory laws.

Art 44–51 deal with rights and duties of the concessionaire and his limitations in expropriation, and the rights of the landowner.

Art 52–73 refer to procedure for obtaining a concession from the Mining Agency and the Department, with reference to applications therefor, and in application for a concession to erect a mill and oppositions thereto.

Art 74, 75 provide for filing Articles of incorporation, concessions, lease or other contracts, relating to the concession, with the Public Mining Register. Only documents and contracts which affect mining concessions will be recorded in the Register.

Art 80. Documents originating in a foreign country, and which in accordance with the laws of Mexico should be in form of a public document, must be protocolised within the Republic before being recorded.

Miscellaneous provisions. Chapter XII includes Articles 98–125. The pertinent provisions are: mining enterprises, contracts for the purpose of prospecting, exploiting or transferring a mining concession or disposing of ores derived therefrom are considered mercantile acts and therefore subject to provisions of the Commercial Code when not otherwise provided for by this Law. The beneficiaries of exploitation and mill concessions must submit annual reports to the Department,

setting forth the details of operation, nature of work executed and quantity of ore extracted. Reports must be accompanied by plans of the projected horizontal and vertical workings.

Concessionaires must permit students of the Mining Schools in the country to study and to acquire practice in the mines and plants operated by them, and provide said students with facilities and data for their advancement.

The concessionaire must keep a duly authorised representative or agent in the Republic of Mexico, with full power to receive and execute all instructions issued by the Department and decide any questions which may arise. Concessions granted in accordance with the Mining Law of May 3, 1926, or prior laws may be changed for exploitation concessions authorised by this Law, and thereby become subject to all rights and obligations established by the latter. Guarantee deposits made in accordance with the Mining Law of 1926 shall be returned to the beneficiaries upon the issuance of the new title, provided they prove due compliance with all obligations inherent in the former concession.

Art 130–137. Commission to promote mining. This among other purposes has for its object the following: to establish public mineral exchanges for buying and selling ore, to establish supply warehouses for miners, install local and regional metallurgical plants to treat ore from the public or the Commission, buy and sell concentrates, and give financial and technical aid to mining cooperative societies. The Commission shall also be charged with the exploration and exploitation of National Reserve zones, and supervision of mining operations carried out by concerns with which the State may enter into contracts for such exploitation.

Regulations of the mining law, enacted Sept 24, 1930, comprise 162 Articles, outlining the procedure to be followed in making application for mining and mill concessions, and also the procedure for effecting expropriations and registering documents in the Public Mining Register. MINING POLICE AND SAFETY REGULATIONS were enacted Oct 17, 1912.

46. MINING TAX LAW

A Mining Tax Law was enacted Aug 31, 1934, imposing taxes on mining properties, on the production of metals and metal compounds, and the production of non-metallic minerals. Charges dependent upon the monthly fluctuation in the price of ore *in New York*, are also imposed on the following: Sampling, assaying, smelting, minting and inspection. Taxes are also imposed on mining concessions based on the area, and on non-metallic mineral concessions. There is likewise an export tax of 12% on all exports of minerals. Most of the cooperative mining societies have been exempted from payment of this tax by a special Presidential decree.

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Morrison, R. S. Mining Rights on the Public Domain. San Francisco, 1917. Concise summary

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Note.—Due to frequent changes in the Mexican Mining Laws, no very useful list of publications can be given.

SECTION 25

MINE EXAMINATIONS, VALUATIONS AND REPORTS

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Note.—Numbers in parentheses refer to Bibliography at end of this section.

MINE EXAMINATIONS, VALUATIONS AND REPORTS

1. GENERAL CONSIDERATIONS

Object of an examination is to form the engineer's judgment; that of a report is to set forth this judgment in such form and detail as will be most intelligible and legitimately useful to the client.

Diversity of temperament and circumstance of engineers and clients, coupled with that of the character, location, size, value, and state of development of properties, occasions great differences in method and scope of both examinations and reports. Examinations of developed mines that have been worked for considerable periods tend to become standardised in proportion to growth of standard conceptions of operating principles and practice. But, an examination even of such a property, while giving due weight to all data obtainable, must be treated as an original problem if the client's needs are to be met.

Standardization of method is sought and practiced by examining engineers whenever feasible, but no method capable of general application is likely to be developed.

Since the value of a mine is its resources for producing future profits, that value is no more susceptible of accurate determination than any other expectancy, even in the ideal case of an absolutely developed mine. Fluctuation in market value of any product may change a profit into a loss or the reverse; and it is seldom, if ever, determinable whether or not further discoveries, laterally or in depth, will resuscitate an apparently exhausted deposit, or whether improvements in treatment, transport facilities, or labor conditions, will render profitable a seemingly valueless mine. The engineer must bear in mind these uncertainties, use his best judgment to determine their probable combined effect and, though it is clear that he should bring his conclusions to as definite a "yes" or "no" as is possible, the uncertainties in the bases of these conclusions should be so expressed in the report as will indicate to the client the degree of importance they occupied in the engineer's judgment.

In this section an attempt is made to set forth generally recognized principles and practice, to describe methods found useful under certain conditions, and to cite illustrations which may help practitioners to meet the problems confronting them.

Theory of procedure in an examination is to secure, carefully weigh, and assign to its proper place in the engineer's judgment, all obtainable information bearing on the enterprise requisite to make the property profitable, or which will tend to prove that profitable operation is impossible under attainable conditions. In examination work, practice departs from theory so far as to eliminate investigation which the engineer judges unlikely to yield a gain to his client proportionate to the outlay of time and money involved.

Conservatism, though as essential to sound judgment in mining as to any other undertaking, should not be made an excuse for timidity or indifference. To attain the closest approximation to fact (not the largest possible discount of favorable data) is the engineer's obligation and the client's due, and is necessary to progress. An exact forecast being impossible, and error on the side of conservatism being usually less disastrous to the client than a mistaken optimism, the inevitable error should be kept on the conservative side. Proper margins of safety are essential, but a serious underestimate of value is as great an error as an overestimate; it may be as disastrous to the client, and it is but little less blameworthy, though it usually involves less conspicuous censure. As close an approach as possible to the property's true value must be made, and the engineer must do this fearlessly, without concealing either favorable or unfavorable aspects.

Relation and responsibility of engineer to client are similar to those of the lawyer. No honorable means of serving the client's interest should be neglected, and every effort should be made so to present data that their meaning to the client shall be that which the engineer's judgment indicates as most nearly correct. Though the engineer is justified in charging such fee as he believes the examination to be worth to his client, or in contracting to do the work for less if he believes this to be to his own interest, it is improper and ultimately unprofitable to regulate the extent and thoroughness of the examination (or the care in preparing the report) according to the amount of the fee. Within the scope of the examination (which should be thoroughly understood and preferably embodied in a letter or other writing before the work is undertaken), the client has a right to expect that the engineer will give the best of which he is capable.

Client is the individual, individuals, or organization, by whom the engineer is employed and to whom he is responsible, makes his report, and looks for his fee.

In dealing with organizations, the engineer has both a right and an obligation to assume that the legally authorized officers who employ him properly represent their organization and will use his services solely for its advantage. But it is obviously the duty of the engineer, and to the ultimate advantage of his reputation, that he should sufficiently acquaint himself with the character and conduct of these officials to be assured that his services are not being used to the detriment of his true client, the organization as a whole.

Engineer is the individual or individuals responsible for the examination and report. ASSISTANTS are here included to the extent of their responsibility for either function.

2. LOCATION, GEOLOGY, MAPS, EQUIPMENT, ACCOUNTS, AND REPORTS

Location. Distance from sources of SUPPLIES and from the MARKET for the output, also cost per unit, safety and reliability of existing means of TRANSPORT, should be ascertained. If transport improvements are essential or desirable, data concerning means, costs, and advantages should be secured.

Mines producing much high-grade ore have been successful in unfavorable locations and with primitive means of transport, as, for example, the backs of men; but hard-rock, low-grade properties (the output of which has a gross value of say $5 per ton of crude ore) are workable only when on rail or water transport lines. Difficulty of access to the property from the FINANCIAL CENTER furnishing development capital may retard development of small or low-grade deposits. Location with respect to supply of LABOR, WATER, and FUEL, or of HYDRO-ELECTRIC POWER, is of prime importance; if these are lacking, the cost of supplying them should be estimated, and an adequate amount added to capital required. CLIMATE and its effect on operation of the property should be noted. Nature of the GOVERNMENT of the country in which the property is located is important, if there is question as to its stability or ability and inclination to give proper protection to industry. Tendency to levy excessive taxation, to pass restrictive laws, or to obstruct operation of the property by permitting illegitimate interference should be considered.

Local geology is studied for the light it may throw upon the probability of presence or absence of economic quantities and values of the minerals sought, upon the relation of exposed values to quantity of assured mineral (Art 11), upon the probable persistence laterally and in depth of values and characteristics already revealed, and upon the best methods of exploitation. See Table 1.

Strikes, dips, pitches, pinches, stratifications, and other rock characteristics, faults, horses, and primary or secondary enrichments of possible influence (Sec 2), should be considered in deciding upon the amount of ore proved by existing or likely to be proved by proposed development. These data, particularly when scanty, should be supplemented by similar information regarding neighboring properties or districts; especially those having the same values, or those of similar character with different values. Government or private geological maps and reports of the property and district, or of neighboring properties or districts, should be investigated for such information. U S Geol Surv folios, which include topographical and geological maps, geological sections, and descriptive matter, are valuable. They may be had by writing the Survey office, Washington, D C, ordering by number as in Table 1, and enclosing price in currency or P O order, payable to Director of the Survey. Maps and folios by the geological surveys of the respective states are frequently available at state capitals or universities.

Original geologizing, including geophysical surveying, should be undertaken so far as is likely to throw light on the value of the property with expenditure of available time and money. For details of geophysical methods see Sec 10 A. It should include a thorough examination of surface and underground exposures, and study of existing drilling records or cores (Sec 9). Character and position of wall rocks, and of the deposits themselves, should be considered as to their influence in determining the best method of exploitation.

Examples. Method and cost of working flat-lying seams, as many coals, clays, and phosphate beds, may be influenced by the strength and permeability of roof and floor. Depth, character, and relation to topography of the overburden of flat-lying secondary enrichments, as the copper porphyries, determine the utility of the caving system of mining, or of the removal of overburden and open-cut work (Sec 10). The workability of a placer deposit by dredging, hydraulicking, ground sluicing, or drift mining, is determined almost as much by the character, condition, and slope of bedrock, occurrence, disposition and size of boulders, and depth and character of overburden as by the engineering features of quantity and head of water and débris disposal (Sec 10).

Maps of mine, property, and general topography, both private and government, should all be investigated and their reliability determined (as far as concerns data of

Table 1. U S Geological Survey Folios (available 1938)

Name	No	Price
Alabama:		
Bessemer-Vandiver	221	50¢
Arizona:		
Bisbee	112	25¢
Ray	217	25¢
Arkansas:		
Eureka Springs–Harrison (Mo)	202	25¢
Hot Springs	215	25¢
California:		
San Francisco;		
Tamalpais		
San Francisco		
Concord	193	75¢
San Mateo		
Haywards		
Colorado:		
State map	...	$2.00
Apishapa	186	50¢
Castle Rock	198	25¢
Colorado Springs	203	25¢
Raton–Brilliant–Koehler (N Mex)	214	50¢
Delaware:		
Coatesville–West Chester (Pa)	223	50¢
Elkton–Wilmington (Md, Pa, N D	211	25¢
Georgia:		
Ellijay	187	25¢
Illinois:		
Gillespie–Mount Olive	220	25¢
Murphysboro–Herrin	185	25¢
Tallula–Springfield	188	25¢
Belleville–Breese	195	25¢
Galena–Elizabeth (Iowa)	200	25¢
Colchester–Macomb	208	25¢
New Athens–Okawville	213	25¢
Carlyle–Centralia	216	25¢
Iowa:		
Galena–Elizabeth (Ill)	200	25¢
Kansas:		
Cottonwood Falls	109	5¢
Joplin District (Mo)	148	50¢
Leavenworth–Smithville (Mo)	206	25¢
Syracuse–Lakin	212	25¢
Maine:		
Eastport	192	25¢
Maryland:		
Choptank	182	5¢
Tolchester	204	25¢
Elkton–Wilmington (Del, Pa, N J)	211	25¢
Michigan:		
Ann Arbor	155	25¢
Detroit:		
Wayne		
Detroit		
Grasse Point	205	50¢
Romulus		
Wyandotte		
Minnesota:		
Minneapolis–St Paul:		
Minneapolis		
St Paul		
Anoka	201	25¢
White Bear		
Missouri:		
Herman–Morris:		
Herman		
Barrett	210	25¢
Chokio		
Morris		
Missouri:		
Joplin District (Kans)	148	50¢
Eureka Springs–Harrison	202	25¢
Leavenworth–Smithville (Kans)	206	25¢
Montana:		
Phillipsburg	196	25¢
Nebraska:		
Camp Clarke	87	5¢
Scotts Bluff	88	5¢
New Jersey:		
Raritan	191	25¢
Elkton–Wilmington (Del, Md, Pa)	211	25¢
New Mexico:		
State map	...	$1.50
Raton–Brilliant–Koehler (Colo)	214	50¢
Deming	207	25¢
Silver City	199	25¢
North Carolina:		
Ellijay	187	25¢
Gaffney–Kings Mountain (S C)	222	50¢
North Dakota:		
Bismarck	181	5¢
Ohio:		
Columbus	197	25¢
Oregon:		
Riddle	218	25¢
Pennsylvania:		
Barnesboro–Patton	189	25¢
Claysville	180	5¢
Coatesville–West Chester (Del)	223	50¢
Elkton–Wilmington (Del, Md, N J)	211	25¢
Fairfield-Gettysburg	225	50¢
Somerset Windber	224	50¢
South Carolina:		
Gaffney–Kings Mountain (N C)	222	50¢
South Dakota:		
Olivet	96	5¢
Parker	97	5¢
Mitchell	99	5¢
Alexandria	100	5¢
Huron	113	5¢
De Smet	114	5¢
Aberdeen–Redfield:		
Northville		
Aberdeen	165	5¢
Redfield		
Byron		
Newell	209	25¢
Central Black Hills:		
Deadwood		
Rapid City	219	$1.00
Harney Peak		
Hermosa		
Tennessee:		
Ellijay	187	25¢
Texas:		
State map	...	$2.50
Van Horn	194	25¢
Wyoming:		
State map	...	$1.50

interest) by comparison with each other and with the reality. They are studied for information as to dimensions and positions of boundary lines, existing or possible transportation, drainage, topography, outcrops, underground exposures, and drilling developments.

Whenever practicable, copies should be made of all maps of interest. If the original are on linen, silver negatives are recommended as furnishing a record from which copies can be cheaply made when desired; new titles drafted on small pieces of linen, and black bordered to conceal the linen edge, may be added in the printing frame in making these negatives. Otherwise, tracings or photographic negatives should be made, which, though more expensive, are readily reproduced. If insufficient or lacking, maps should be supplemented by original sketches, or made from original surveys to the extent justified by the circumstances (Sec 17, 18, 19). U S Geol Surv topographical maps

an advanced price, those of their district.

Mine equipment should be examined as to its future utility, and whether its alteration, enlargment or replacement would improve operating conditions. Except in cases of obvious inutility, a complete INVENTORY is desirable.

If much of the property value is in equipment, if property is to be acquired on purchase or lease, or if the engineer assumes responsibility for subsequent operations, a fairly complete inventory is essential. It should state the condition as well as items of all realty, machinery and its foundations, grading, costs of erection, tools, supplies, and general merchandise. Items of plant having value if moved should be distinguished from those which are valueless except in place (as grading, foundations, and erection costs), the two classes being designated as MOVABLE and FIXED PLANT. Amount and value of securities held, cash and balance of good accounts receivable and payable, should be ascertained. Character and cost of improvements and additions necessary for further operation should be noted with such detail as accords with the engineer's responsibility in the matter, respecting machinery required for mine, mill, smelter or transportation system, and general equipment, including offices, laboratories, houses for management, staff, foremen and general labor and commissary, also water supply and sanitation (Sec 22, 23).

Operating methods in use should be studied to determine whether advantages could be gained by modification or entire change. If so, an estimate, in such detail as is justified by circumstances, should be made of cost of change and of probable saving to be effected.

Accounts and reports; operative and special. All available periodic reports of enterprises that have been based on the property should be examined for the light they may throw upon total and average outputs and values, equipment and operating costs, profits and losses, capital employed and authorized, expenditures and resources, the skill shown and the difficulties encountered in operating with the equipment used.

These reports may range from daily to annual, and include records of development, production, assays, financial transactions and conditions, costs, and profit and loss. Those of sufficient importance to justify it under the scope of the examination, are best studied by tabular compilation (below). By this method periodic summations, averages and rates per ton, oz, lb, ft, or other unit, can be made as desired, and checks secured against the ledger accounts, which in turn can be checked against the bank accounts. Reports by superintendents, managers, consulting engineers or directors, should also be checked against the results of these compilations. Much important information of determinable reliability can sometimes be thus secured, together with important suggestions as to points requiring investigation. Data of neighboring or similar properties are sometimes available and can also be profitably studied. If the examination and records are of sufficient importance, and the ledger accounts are numerous and detailed (Sec 20, 21), the services of an expert accountant may be advisable.

Tabulation of Monthly Costs During 19—(Blank columns for unexpected additions)

Month	Tons mined or treated	Mining		Transport		Smelting		General		Misc				Total	
		Amount $	Per ton ¢	Amount $	Per ton ¢	Amount $	Per ton ¢	Amount $	Per ton ¢	Amount $	Per ton ¢	Amount $	Per ton ¢	Amount $	Per ton ¢
Jan															
......															
......															
12 mo															

Terms of acquisition. The engineer should assure himself whether he is to be responsible for the effect the terms may have on the success of the enterprise. If so, he should familiarise himself with these terms; if not (and the nature of the case fails to make this obvious) he should bring out the fact in the report in an inoffensive manner.

3. TITLES, LEGAL ASSISTANCE, AND FORMS

Titles. A mining engineer may not be well qualified to pass upon questions of title, but tenure and ownership are of such vital importance that he should inform himself thereon as fully as possible. Terms of tenure, such as operating concession, working bond or lease, or ownership in fee simple, and the chain of title on which the tenure is based, should be thoroughly understood, unless the client prefers to take all responsibility, or has specifically limited the scope of the examination so as to eliminate these questions. In the latter cases the report should definitely but inoffensively state the situation. Otherwise the engineer should investigate along the above lines until he can make definite statements.

To illustrate, the engineer may have occasion to make one of the following statements in his report: A. In view of the client's assurances, all questions of title have been ignored; or the titles have been assumed to be perfect. B. The titles have been examined, or the engineer has had them examined by competent attorneys (whose names should be given), and found perfect; or certain defects, (which should be specified) have been found. C. In case of defects which attorneys and engineer agree are remediable, recommendation should be made as to incurring further expenditure on the property, or stopping it pending perfection of title. In case of irremediable defects, a joint opinion should be given as to whether the chances of gain are enough greater than chances of loss to justify proceeding regardless of these defects.

Legal assistance (preferably that versed in local laws and practice when of sufficient caliber and reliability) should be secured not only on questions of title, but in making any important agreement, as a deed, working bond, or escrow agreement. In absence of legal advice, following forms (2) may be used:

Deed

This Indenture, Made the .... day of ...... A.D., 19.., between ...., the part... of the first part, and ...., the part ... of the second part;

Witnesseth: That the said part ... of the first part, for and in consideration of the sum of...... ...... dollars, lawful money of the United States of America, to .... in hand paid by the said part ... of the second part, the receipt whereof is hereby acknowledged, ha... granted, sold, and forever quitclaimed, and by these presents do .. grant, sell and forever quitclaim, unto the said part ... of the second part, and to .... heirs and assigns, all the following described real estate, situate in .... mining district, county of .... State of ...., to wit: (Here follows description)...... Together with all and singular the mines, minerals, lodes and veins within the lines of said claims and their dips and spurs, and all dumps, plant, fixtures, improvements, rights, privileges, and appurtenances thereunto in anywise belonging. To have and to hold the lands, tenements, and hereditaments hereby conveyed unto the said part ... of the second part .... heirs and assigns forever.

In Witness Whereof, the said part ... of the first part ha.. hereunto set .... hand .... and seal ...... the day and year first above written.

Signed, Sealed and Delivered
in the Presence of .... Seal

....

.... Seal

(To be signed, witnessed, sealed, and acknowledged before a Notary or other proper commissioner, to meet the requirements of the State or County in which the property is situated. Provisions for recording should also be observed.)

Mining Lease

This Indenture, Made this .... day of .... A.D., 19.., *between*................., of......................, lessor, and................., of................ , lessee;

Witnesseth: That the said lessor, for and in consideration of the royalties hereinafter reserved, and the covenants and agreements hereinafter expressed, and by the said lessee to be kept and performed, ha.... granted, demised and let, and by these presents do . grant, demise and let, unto the said lessee, all the following described mine and mining property, situated in .... mining district, county of ...., State of ...., to wit: (Here follows description) .... Together with the appurtenances to have and to hold unto the said lessee for the term of .... from the date hereof, expiring at noon on the .... day of ...., 19.., unless sooner forfeited or determined through the violation of any covenant hereinafter against the said tenant .... reserved.

And in consideration of the said demise the said lessee does covenant and agree with said lessor as follows: to wit: To enter upon said mine or premises, and work the same in the manner necessary to good and economical mining, so as to take out the greatest amount of ore possible, with due regard to the development and preservation of the said premises as a workable mine, and to the special covenants hereinafter reserved. (Here insert Special Conditions and Agreements of Lease, Work to be done, etc.)

In Witness Whereof, etc. (See deed, except that usually the lessor will require the signature of the lessee, with corresponding alteration of form.)

Working Bond

(A lease and option, also known as a working bond, is a common and preferable form of purchase. At end of the mining lease above outlined, and preceding the final paragraph, proceed as follows:)

And in consideration of the foregoing lease and the expenditures to be made thereunder and the well and faithful keeping of the covenants thereof, the said lessee shall have the right to purchase the said premises, together with all improvements, etc, for the sum of .... dollars ($............), to be distributed in the following payments: On or before the .... day of ...., 19.., the sum of .... to be paid (designate manner and place of payment), and on or before the .... day of .. .., 19.., the sum of .... (Here insert further conditions of payment.)

Time being of the essence of this contract as to such payments, and upon the tender of such payments, the lessor will execute, acknowledge and deliver at his own cost, good and sufficient deeds to the lessee, or such person or company as the lessee shall nominate, conveying the said premises clear of incumbrance. (Deeds to the property are generally placed in escrow with some bank or responsible party, to whom payments are made when due under the agreement, and who delivers the deeds when payments are completed.)

The forefeiture, surrender, or termination of the above lease for any cause shall render the option void, and the above mentioned payments may not thereafter be tendered.

It is expressly agreed and understood that this Agreement shall be considered as an option to purchase only, and not as obligating the said lessee to purchase said property.

In Witness Whereof, etc. (See close of Mining Lease.)

Notice of Right to Water

The undersigned claims the water running in this............stream to the extent of.......... inches for mining purposes to be conveyed by (ditch or flume) from this point to the.............. placer claim.

Dated............, 19... ....

Locator

(This notice to be posted near the stream outlet, and the following form duly recorded in the district or county recorder's office.)

Pre-emption of Right of Way for Ditch and Location of Water

To whom these presents may concern, know ye, that I,............, of the county of.........., in the State............, a citizen of the United States, do hereby declare and publish as a legal notice to all the world, that I claim, and have a valid right to the occupation, possession and enjoyment of all and singular, that tract or parcel of land lying and being in the county of.......... in the State of.............., for the exclusive right of way for the purpose of constructing a flume or water ditch from................stream to.................placer claim, more particularly described as follows: Commencing (here describe the exact route for ditch or flume).

I also claim, and have a valid right to the enjoyment and use of............inches of water from said stream for mining purposes, to be conveyed through such flume or water ditch to said claim, together with all and singular, the hereditaments and appurtenances thereunto belonging, or in anywise appertaining.

Witness my hand and seal this............day of.........., A.D., 19..

(Name)

....

Notice posted on the stream.........., 19...

Ditch commenced at claim or at stream.........., 19...

............of..........County of.............., ss.

On this,....... day of ...., 19.., before me, a .... in and for the county aforesaid, in the state aforesaid, personally appeared........................, to me personally known to be the person who executed the foregoing written instrument, and acknowledged that he executed the same for the uses and purposes therein set forth

Witness my hand and official seal

....

Escrow Agreement

The enclosed deed of the .... lode is hereby placed in the .... Bank of ....in escrow. If A.B. shall place, or cause to be placed to the credit of C.D. and E.F., in said............Bank of......................, or on before.............., 19..,

the full sum of................dollars, then and in that case the said Bank is hereby authorised to deliver the enclosed deed to A.B., or his order. In case the said A.B. shall fail to deposit the said sum of..............dollars, as above provided.................., then the said Bank is hereby authorised to deliver the enclosed deed to the said C.D. and E.F. or their joint order.

(Signed) C.D.
E.F.
.................., 19... (Place and date) A.B.

(When the option for purchase of a mine is desired by a third party, it is safest and best for the owner to put a deed in escrow. It saves incumbering the record, and averts questions that might arise concerning payment of money. The deed should be a warranty, quitclaim, or mining deed, as agreed, fully executed and acknowledged, ready for delivery, put in a sealed envelope and placed in a bank, or left with a responsible person, with an agreement written upon the envelope, as above.)

For information as to location notices and patent proceedings, see Sec 24.

4. THEORY OF SAMPLING; METHODS OF CRUSHING, CONING, AND QUARTERING

Sampling is the process of securing more or less representative samples of ore for the purpose of gaining information as to the composition of the whole by investigation of the part (see also Sec 29, "Definition"). It is necessary unless the conditions are such that more reliable information can be gained from records of operation. The process is essentially one of approximation: only by special care can the heterogeneous masses with which the mining engineer usually has to deal be made to yield samples more than roughly representative. These should approach the truth as closely as practicable, and when used with full knowledge of their limitations may be valuable. But, if the number of points from which equal and sufficiently large parts of the sample are taken is great enough in proportion to the irregularity in composition, and if these points are uniformly distributed, it is possible to draw a sample as exactly representative of the average composition of the mass as may be desired. The greatest difficulty in securing accurate samples is due to the inaccessibility of most of the interior of the mass. Dumps, or superficial deposits easily penetrated, can usually be sampled by sinking drill holes or pits. In case of mineral masses in place, especially those exploited by underground workings, sampling is generally limited to EXISTING EXPOSURES. These are made in the ordinary course of mining, and not to furnish uniformly distributed and convenient points for taking samples.

Sampling is thus usually reduced to securing as accurate samples of the different exposures as circumstances admit and justify. The average contents of the deposit are then determined by calculation of combinations made according to judgment, after studying the geology, and the relative position and size of the exposures (Art 10). Methods of sampling vary in detail according to size, character, accessibility and general condition of the deposit, the degree of accuracy required and the means available.

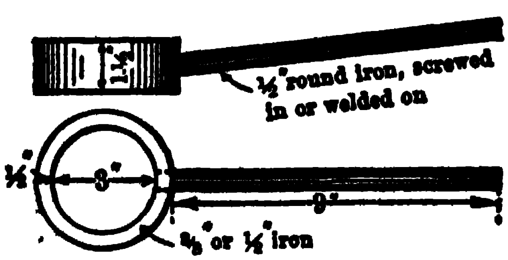

Fig 1. Ring for Crushing Samples

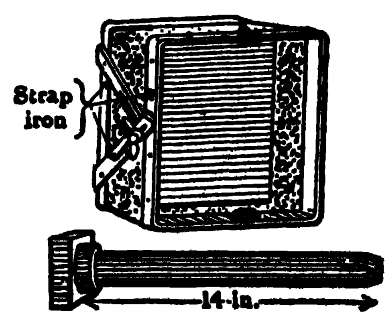

Fig 2. Coal-sampling Mortar and Pestle

Crushing the sample. Unless a mass is composed entirely of relatively fine particles, or is practically homogeneous, one of the most important features of sampling is crushing. It is a laborious process, and power crushers should be employed when feasible (Sec 28, 29). But much of the crushing necessary in mine sampling must be done on the spot, where power crushers are not available. Therefore, the miner's hammer and an anvil consisting of an old stamp head, large sledge, or piece of hard, barren rock, are usually employed. Iron rings with handles (6) (Fig 1) are useful to hold the piece on the anvil, so that few chips fly and the fingers are not exposed.

Hand-hammer crushing should be done on a sampling sheet or quartering floor, to prevent loss of material or introduction of foreign matter. An iron mortar, with pestle operated through a hole in a canvas cover, to prevent loss by flying chips, is good. Soft materials like coal may be crushed in a light mortar, with iron-covered wood bottom and collapsible canvas sides (7) (Fig 2).

In working down a sample, the largest piece must always be so small in proportion to the total volume that its retention or rejection will have little effect on the result, even though such piece contain the maximum amount of impurity or value possible in the circumstances. Crushing to a maximum size of piece weighing 0.001 of the mass to be quartered (as a 1-oz piece in a 62.5 lb lot) is usually satisfactory. The extreme case of a 1-oz piece of chalcocite (containing say 75% Cu) would raise or lower the sample assay by 0.15% Cu, if it occurred in the last quartering, while compensating errors in the whole series of quarterings would probably reduce this error considerably. If care is taken that very rich or impure pieces are crushed comparatively fine, and thoroughly mixed throughout the mass during the work, accurate results can be secured by quartering masses containing ordinary pieces several times the above size limit; and when it is known that no pieces of erratic value are present, the maximum size may be considerably increased. But, unless the work is in well-trained hands, fine crushing before reduction is preferable when erratic values are present. Under prescribed conditions proper crushing can be insured by using a screen as is done by the U S Bureau of Mines, which provides a $^3/_8$-in mesh screen with its coal-mine sampling outfit. But trained observation can determine and regulate the suitable size with sufficient accuracy (see also Sec 31).

Coning and quartering (see also Sec 29, Art 3). The crushed mass is worked into a cone by shoveling all the material to one point on the quartering floor, in such manner that the particles roll down in all directions from the central point. Irregularities in composition of the mass are thus distributed as concentric layers of a cone. The top of the cone is then flattened with the edge of the shovel, by spreading the material equally in all directions until a disk is formed, thickness of which is about 0.1 the diam. This disk is marked into quadrants, and the diagonally opposite quarters are cut out, care being taken that all material in the rejected quadrants is removed, but no other. The mass now contains half the original quantity and, after reducing the large pieces proportionally, coning and quartering are repeated, thus securing one-quarter of the original. The cycle of crushing, coning, and quartering, is repeated until the sample is of the desired size. With sufficient care that the cones are made up of layers of each shovelful, evenly distributed about a fixed center, and that the cones are flattened by being worked out evenly from the center in all directions (coning about a fixed vertical rod (12) insures accuracy in these respects), results of any desired accuracy are obtained, provided sufficient crushing is done either before or during the process.

Unless the sample is corrosive, the best quartering floor is a piece of boiler plate, large enough to hold the disk of material prepared for the first quartering. When available, the turn-plates at shaft landings make good quartering floors. A well-laid floor, preferably of hard wood, is the next best. When neither is available, a platform may be made by laying boards as solidly and evenly as possible on the ground and covering them with a sampling sheet. Cloth covering is harder to keep free from residual fines than a plate or wood floor, but thorough brushing with a whisk, or scrub brush, usually suffices, and the cloth yields better results than when working on a floor containing cracks in which fines can lodge and be lost or inopportunely reappear. The whole process, including preparation, is often referred to as quartering.

Rolling. When using a sheet in reducing lots of 100 lb or less, time is saved by rolling instead of coning. It consists of mixing the mass by drawing one corner of the sheet forward towards the diagonally opposite corner, until the material has been rolled over and over on itself to the limit of the capacity of the sheet. The first corner being laid back in place and the diagonally opposite corner pulled over the sample, the process is reversed, and so back and forth. The material then being left in a long mass across the center of the sheet, rolling is done at right angles to it, by working with the other diagonally opposite corners of the sheet.

Two cycles, of two rolls each back and forth with each set of corners, give good mixing and leave sample ready for final rolling into the center by all four corners preparatory to quartering. Mixing may be imperfect unless the folded-over corner of the sheet is held horizontally as it is pulled forward; otherwise the material may slide on the sheet instead of being rolled over on itself. Also, if the mass is wet and sticky it will adhere to the sheet and the mixing will be imperfect.

5. SPLITTING, DECIMATING, TRENCH, PIT, DRILL, SMALL-PARCEL, DUMP, AND LARGE-SCALE SAMPLING

Splitting is the reduction of a mass by some type of splitter, or series of parallel troughs which alternately retain and reject a series of equally broad sections from a stream of crushed material poured over them (Sec 30, Art 3).

For this the Jones sampler (9) is recommended, as giving results equal to the best quartering. In using it, care should be taken to insure the usual proportion between the size of the largest piece and that of the mass about to be split (Art 4); to maintain a steady, full-width flow from the shovel; to see that the entire lot is put through and that the apparatus is kept clean. The form shown in Fig 1, Sec 30, is best, as it is readily cleaned with an ordinary brush. Splitting should not be used on wet,

sticky samples. Quartering in such case is less objectionable than other methods of reduction, but, unless unavoidable, reduction of wet, sticky samples should not be attempted by any process.

Decimating. A mass in process of shipment or other movement is most conveniently sampled by decimating; that is, by selecting for the sample every tenth car, barrow load, shovelful, or other unit of handling.

For accuracy, the number of units taken for sample must be large in proportion to irregularity in composition of the mass, and if the units of handling are large compared with the whole, every 5th, 3rd, or even 2nd unit may be required for the sample. With fairly uniform materials, such as the product of a good coal seam, 10 units in a sample give results accurate enough for most purposes. Ores of not very high grade and free from large masses of rich material may be satisfactorily sampled by taking 100 units in sample. Very rich, spotty ores may require 1 000 units, and it is difficult to secure a good sample from them without carefully working down the entire mass. Unless already fine enough, the whole sample secured by decimation is crushed and reduced by further decimation, quartering, splitting, or by use of a MECHANICAL SAMPLING equipment (Sec 29).

Trench sampling consists in cutting a series of trenches in a dump or superficial deposit, and decimating or otherwise reducing the product, or interval-sampling the sides of the trenches. The samples will be satisfactory if all classes of material composing the mass are cut in their true proportions.

Pit sampling. A number of pits or vertical shafts of small section are sunk in a manner similar to trench sampling. In dumps and superficial deposits, where large pieces usually accumulate at the bottom, variations in composition are more marked with depth than with horizontal extent. A good sample can therefore be secured if a sufficient number of pits are so regularly distributed as to cut uniformly across this greatest variation.

Pits in dump material must be supported by timbering, or other means, for retaining the vertical walls necessary to secure a uniform yield from top to bottom. In stiff clay, and with short-handled tools, 36-in diam round shafts can be sunk 100 ft or more without any support. Small elliptical shafts (13) only 28 in on largest diam, have been sunk in dumps to depth of 50 ft, lining with corrugated iron sheets, driven as poling, and supported on $^3/_8$ by 1.5-in elliptical iron rings, spaced by $^3/_8$-in round iron hooks.

Drill sampling. Though a single pit yields better samples than a single drill hole, where drill sampling is practicable it is preferable because, at same cost, many more points can be tested by drilling than by sinking pits. This is especially true in sampling flat deposits beneath rock overburdens, as many coal seams, secondary enrichment "porphyry" copper deposits of western U S, flat-lying lead deposits of S E Missouri, and zinc deposits of Oklahoma or Wisconsin. Drill holes should be located with the same regularity as pits, close enough together to average the irregularities of the deposit and the drilling must be done with care (Sec 9, 10).

Small-parcel sampling. Lots of a few tons or less can be sampled by quartering, splitting, decimating, or mechanically at a sampling works (Sec 29). With care any of these methods will give accurate results; though decimating, the cheapest and commonest method, is essentially the least accurate unless it is modified by taking every alternate shovelful. The value may be fairly approximated by careful grab sampling, particularly if the lot is first well spread out (Art 4, 5).

Dumps containing more than a few tons may be sampled as above when their value and irregularity justify it. Decimation of rich dumps by shoveling is quite usual. If the contents are not exceptionally irregular, dumps may be sampled with sufficient accuracy by trench, pit, or drill; the latter, however, only when there are not enough large pieces to interfere with hand drilling, or when the dumps are large enough to justify installation of power drill.

Large-scale samples of many tons, or even thousands of tons, may be taken to determine the value of erratic ores; especially those containing metallics, as native gold, silver or copper, or very rich minerals, the average occurrence of which can be determined only by handling large amounts. Large samples are also taken to determine amenability to treatment by mill or smelter tests.

Difficulty of maintaining efficient surveillance in taking and handling large samples so reduces their usefulness that they are taken only where other methods fail, as in the Lake Superior native-copper deposits, or where full-scale treatment is desirable. Such samples are usually secured by mining from the entire exposure a sufficient, uniform thickness to furnish the desired amount. If this would give too great a bulk, miniature stopes may be made, of fixed size, and at regular intervals; or the tonnage may be further reduced by making blasts at regular intervals, and taking equal quantities from the average of each blast. Judgment and care must be exercised in this work, and rules are of little service.

6. INTERVAL-CHANNEL, CHIP, AND GRAB SAMPLING

Interval-channel sampling is done by cutting channels at regular intervals along the length of the exposure. To secure an accurate average, the intervals must be small enough, relative to the irregularity in composition and value of the exposure, to insure proper representation in the sample of any considerable irregularity. The amount from each cutting must be sufficient to counteract the effect of erratic occurrence of rich spots. Thus, the metallics of the Lake Superior copper deposits require mill-runs instead of assays. Usually a few lb or even less per ft of channel will suffice. Channels cut in a flat surface for combination into one sample must be equidistant, of equal length (or of full width of the exposure), and of uniform depth and width, to yield a constant quantity per linear ft of channel.

To sample a curved or oblique surface, which for calculation must be considered as flat and normal either to the dip or to the vertical, the channel lengths are assumed to be on the projection of the curved surface upon such normal flat surface, and the depth and width of channel must be decreased to yield the uniform quantity per ft of projection, instead of per ft of actual length of channel. Thus, the curved back of a drift in a banded gold vein, sufficiently rich and well-developed to warrant such detail calculation, should be measured and sampled in the regular way only between *B* and *C* (Fig 3). *AB* and *CD* are measured on their projections *AB′* and *C′D*, and the size of the channel reduced so as to furnish the regular yield per ft of projected distance (5). It is generally undesirable to make these adjustments in the field. Separate samples are usually taken from each channel, and if great diversity in composition occurs, or adjustment for curvature is necessary, from each fraction of channel requiring such adjustment. Adjustments are made on basis of measurements taken at time of sampling. Channels in well-banded masses should cut across the banding. Otherwise the direction of individual channels may bear any relation, from normal to parallel, to that of the line of interval measurement, provided they do not so converge as to emphasize or neglect any particular sampling area.

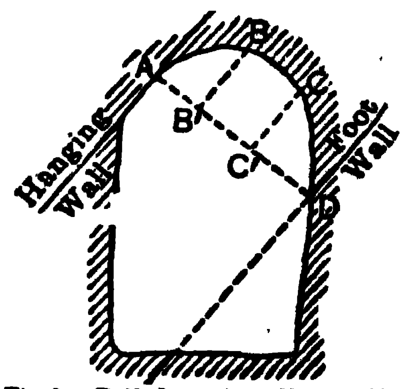

Fig 3. Drift Sampling; Narrow Vein

Mode of cutting. In soft, comparatively uniform material, as coal or clay, channels are readily cut with a prospector's or a miner's pick. In hard ore, picks are almost useless, and in mixtures of hard and soft it is almost impossible to pick down proper proportions of each. Hence, in all but uniformly soft materials, the channel should be cut with moil and hammer, either single or double-hand according to hardness of the ground.

1, 2, or 3 men may thus be required, depending on whether a single-hand hammerman holding his own moil works on a sample sheet, or a sample box has to be held, or a double-hand hammer is used besides holding moil and box. A small pneumatic drill and moil steel may be used if compressed air is available. In any event, there should be an ample number of hands and eyes, to insure careful, watchful work. Moils are usually of 0.75-in octagonal steel, forged to a diamond point on a taper about 2 diam long. The point for about 0.5 in from tip is at a larger angle, to increase its strength. Moils vary from 6 to 18 in long; most of them, 10 to 12 in. For hard ground, at least a dozen should be kept ready, to avoid delay caused by using dull points. Hammers are double or single-hand, of weight suited to the strength of the hammerman. They vary from 3 lb for single, to 7 lb for double-hand work and should be as heavy as can easily be swung by the sampler on all-day work, but, if too heavy, the worker suffers unnecessary fatigue.

Precautions. After determining where the channel is to be cut, care should be taken that the ore surface at this place is free from dust, foreign matter, and any efflorescence or decomposition, which may change the character of the minerals since first exposed.

Dirt or dust can usually be removed by brushing with a whisk or wire brush, or washing with water, but efflorescence and other altered matter must be chipped off down to a solid surface. In firm ground, where there is little danger of jarring off loose pieces outside the channel, the sample may be caught on a canvas sampling sheet, say 6 ft square, or large enough when spread on the floor under the channel to catch all material cut, including flying chips. When using a sheet, extraneous matter may drop on it without the engineer's knowledge, and enter the sample. Hence, cuttings are often caught in a dynamite or candle box, in a gold or a round-cornered rectangular bread pan, in a canvas bucket with round bottom, or a sack the mouth of which is kept open by being bound on a wire ring. Chips are then prevented from flying by the sample catcher placing his gloved hand over the point being cut. Objections to this method are the possibility of spill from the cutting point down past the receptacle, and the added attention required. Since a sample sheet will serve to prevent contamination or loss while hammer crushing,

samples may be immediately reduced on it to a few pounds weight, or small enough to be readily protected from salting (Art 9). Before beginning to cut a channel, it is well to mark its delimitations (usually by making soot streaks with the lamp flame) along each side and for the entire length. To lessen opportunity for salting the sample in place, the channels should not be marked until they are actually to be cut. Considerable ingenuity may be demanded in cutting samples from exposures difficult of access. In high-backed drifts and backs of old stopes it may be necessary to build platforms on light timbers wedged like stulls between walls, or, in a wide stope, to erect light scaffolding on "horses."

Sampling shafts. UNTIMBERED SHAFTS and WINZES are usually best sampled from platforms on light stulls. If the material is easy to sample, the work in a vertical shaft or winze may be done while seated on a short plank suspended through the center on a windlass rope, and held as close against the side as is desired by a light cross pole hinged to the plank and extending obliquely up and back to the far side of shaft (5). The sampler, seated astride the rope, fixes his position as rigidly as desired by pressing his feet against one side of shaft, thus acting against the pole, which points back and a little up against the opposite side. The sampler's hands are thus free for cutting the sample which he can receive on a sheet spread over his legs or on the shaft bottom.

To sample CLOSE-LAGGED SHAFTS part of the lagging must be cut out, and if care is taken to expose but a small area at a time and promptly to replace the lagging, no injury to the shaft should result.

In lagged shafts running down with the banding of the deposit, the full width of exposure is made accessible by cutting out entire panels between timbers; when necessary this can be done by cutting out, sampling back of, and replacing, one piece of lagging at a time. Shafts in the midst of a mass, as test pits of a placer deposit, may be sampled by removing part of the lagging between timber sets, and cutting a uniform channel of such size as the dimensions of the opening admit. If values vary greatly between sides of the shaft, channels should be cut on both sides and handled separately. Otherwise it should suffice to cut a single channel between sets, locate it on the side next to that in which the channel just above was cut, and so work around the shaft as depth is gained (Art 7).

Chip sampling. Some unstratified masses, as the pyrrhotite deposits of Ducktown, Tenn, are so hard, tough and close-grained, that cutting channels is extremely difficult. Fortunately these qualities produce a broken surface, the irregularities of which are independent of composition, so that chip sampling gives satisfactory results. The work is like channeling, except that the sample material consists of chips moiled off along an assigned line. As in channeling, the chips must be so broken as to represent equal distances by equal quantities. A proper proportion of fines and coarse must be secured from each point of chipping, and the number and size of chips determined in accordance with the irregularities of the exposure.

Grab samples are taken by "grabbing" pieces at random. Information as to general characteristics can thus be gained quickly, and by careful adaptation of interval sampling a method of some precision is developed. The value of a dump surface, for example, may be determined with as much accuracy as desired by making the "grabs" at regular and closely-spaced intervals, securing at each point the proper proportion of coarse and fines.

7. UNDERGROUND AND PLACER SAMPLING

Underground mineral exposures are usually best sampled by interval-channeling or chipping (Art 6). As stated, deposits the values of which are largely in native metals or other rich minerals can not be sampled and assayed in the usual way, and large-scale sampling followed by mill or smelter runs is requisite. Such work must always be checked against previous operating records and a general estimate of value based on careful inspection of ore exposures. In fact, before undertaking any sampling, the peculiarities of the deposit should be determined as far as possible by such inspection. For this purpose fresh fractures should be made at frequent intervals, by breaking off pieces with a pick, or, in tough material, with moil and hammer.

Time and fatigue will be saved, and the engineer's efficiency increased, if this work be done by 1 or 2 good hammermen, at points indicated by the engineer, who is thus left to devote his entire attention to studying the exposures. The sample interval is usually 5 or 10 ft; preferably 5 ft, which is short enough to afford means of check by calculating two sets of results on alternate samples. Precious-metal deposits should be studied carefully for rich streaks and shoots, and sample interval made short enough to include all exceptionally high or low values. In case of streaks, it is often desirable to divide the channel across the banding into two or more samples, to insure proper proportioning and

make it possible to exclude (when revealed by assay) low-grade or barren streaks occurring so that in subsequent mining they could be left untouched (Art 6).

Placer sampling must be done in a manner to show much smaller values than are considered in solid deposits. Large-scale dredging, favorably located, as that on American or Yuba rivers, Cal, gives high percentage of recovery at pre-war cost of about 5¢ per cu yd, and in such cases 10¢ ground is profitable when in quantity sufficient to keep a dredge at work for a number of years (Sec 10).

Under favorable conditions (properly sloping bedrock, abundant water under sufficient head, and ample dump room) (Sec 10), hydraulic mining costs even less than dredging. 10¢ per cu yd is between 7¢ and 10¢ per ton (0.0035 to 0.005 mg per assay ton, or about the minimum an assayer can report with any assurance of accuracy), but placer samples are reported to the nearest 0.1¢ and averages calculated to nearest 0.01¢. For this the samples are measured and concentrated by rocking and panning before being assayed. Fig 4 gives details of a practical rocker (11).

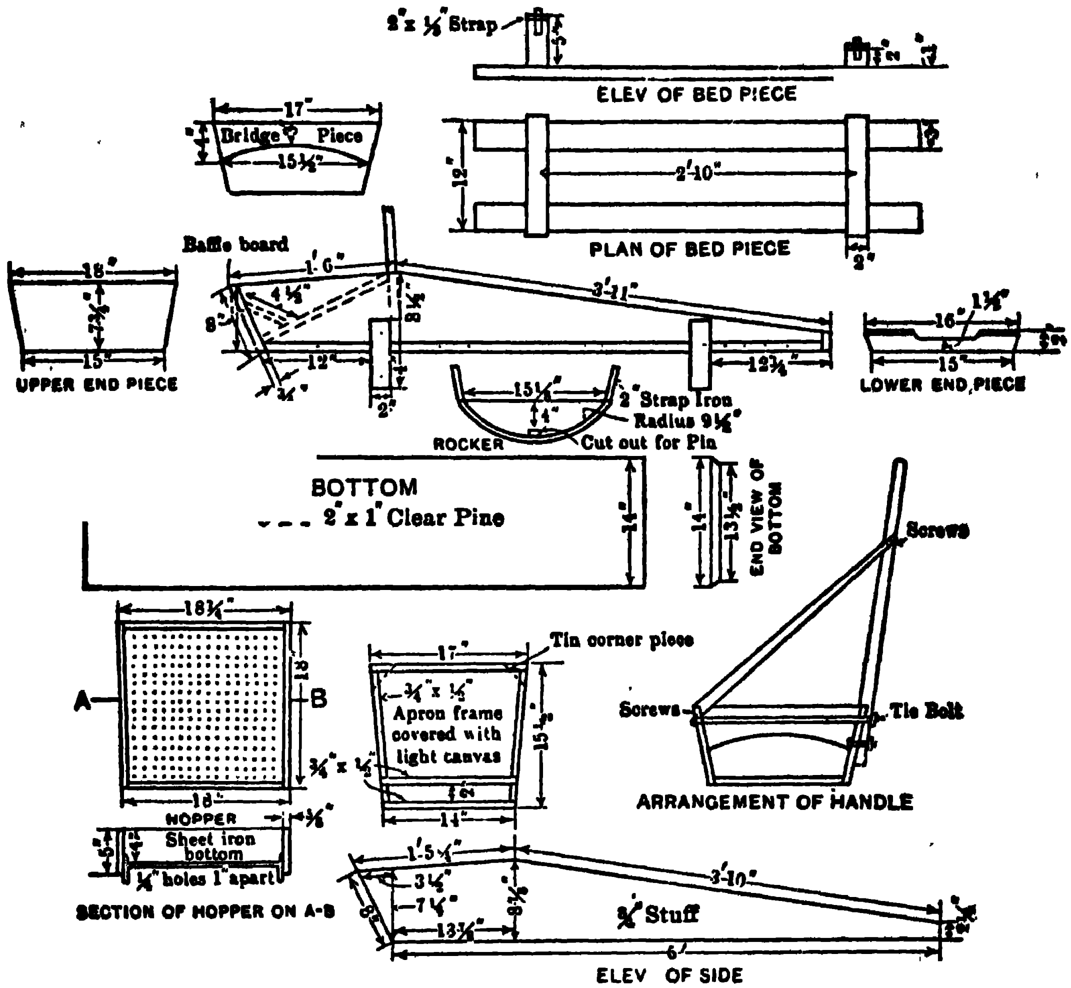

Fig 4. Placer Gold Rocker

Besides determining amount, condition (whether coarse or fine), grade and distribution of the gold, other features should be noted at time of sampling. If the property is to be dredged, it is desirable that the gravel (material up to size of a man's head) be free from boulders. Strongly-built dredges, with buckets of a few cu ft capacity, will handle, at some additional operating cost, pieces up to 2 ft diam, while 15 and 16-cu ft bucket dredges will handle boulders weighing over a ton. But, for a low operating cost, few large boulders can be present. Cemented gravel increases cost as does sticky clay, which clogs the entire equipment, resists washing, and carries off values (Sec 10).

Maximum dredging depth has thus far not much exceeded 150 ft in regular, average cost practice. Of this as high as 40 ft has been carried in the bank and the balance below water level. Experienced dredge builders, such as Yuba Manufacturing Co. of San Francisco, Calif, however, feel that for a deposit of sufficiently large scale and high value to justify initial cost, a dredge could be designed and built which would operate satisfactorily at considerably greater depth to bedrock, which must be cleaned to prevent loss of gold.

Minimum depth to bedrock must exceed that required to float the dredge, and clear the sag of the bucket chain, with the ladder in any necessary operating position. Cost of removing surface timber, including stumps, must be determined. Presence of any considerable amount of buried timber prohibits dredging.

Provision for handling débris is largely a question of dirty water, which can not be settled to a clear overflow; 50 miner's inches flowing into a dredge pond will usually keep water clean enough for gold saving and offset seepage losses. Rapid-running streams in narrow valleys subject to unmanageable freshets preclude dredging, because of danger to the boat. Very hard, rough bedrock, containing crevices in which gold value would be lost, is equally impracticable for dredging. But, what shows in a stream bed as hard rock may be covered beneath the gravel by a soft, decayed, but sufficiently firm layer, suitable to cleaning by dredging (Sec 10).

Methods of sampling placers. The only exposures available in a virgin placer are those made by freshets and streams. But some prospecting will probably have been done before the engineer's examination, usually consisting of pits for mining by hand methods, or of shafts or drill holes. Each accessible point must be sampled, and note made whether BEDROCK has been reached. If possible, bedrock should always be reached, to inspect and sample the gravel lying immediately upon it. Character, strike, and dip of bedrock can then be determined, comparison with similar observations elsewhere precluding possibility of deception by a large boulder. The exposures, if large enough, should be interval-channel sampled and each sample panned (Sec 10 and 31) or "rocked" independently. After recording, all the "colors" from a single pit are usually combined for final weighing and assay (Art 5). Table 2 is useful for calculating valu from fineness (2). The value of a mg of gold 800 fine is 0.09¢, a factor which may be used when weighing colors from placers where the average fineness of gold has not yet been determined.

Table 2. Fineness and Value of Gold Basis, $35.00 per oz

Fineness	Value per oz	Value per grain	Value per mg	Fineness	Value per oz	Value per grain	Value per mg
950	$33.25	$0.0693	$0.00107	750	$26.25	$0.0547	$0.00084
925	32.38	0.0674	0.00104	725	25.38	0.0529	0.00082
900	31.50	0.0656	0.00101	700	24.50	0.0510	0.00079
875	30.63	0.0638	0.00098	675	23.63	0.0492	0.00076
850	29.75	0.0620	0.00096	650	22.75	0.0474	0.00073
825	28.88	0.0602	0.00093	625	21.88	0.0456	0.00070
800	28.00	0.0583	0.00090	600	21.00	0.0438	0.00068
775	27.13	0.0565	0.00087				

A deposit which has been proved by drilling can be CHECK-SAMPLED only by fresh drilling or shaft sinking. When the water flow is not enough to prevent shaft sinking, the sinking of pits on say half a dozen holes of representative location and varying value is the best checking method. Measured representative amounts of the material from each 5 ft of sinking can be rocked, and after the shaft is complete these results may be checked as many times as desired by interval-channel sampling of the sides of the shaft. To be successful the shaft must reach and thoroughly clean the bedrock, particular care being taken in measuring and rocking gravel close to and on bedrock, where most of the value frequently occurs. All the gold in a measured amount of gravel in place should be cleaned off the bedrock into the sample, but the rich gold-bearing sand surrounding but outside of the measured sample must not be allowed to run in and contaminate the sample.

When, because of wet ground or other reason, shaft sinking is difficult and costly, and a drill and crew are available, adequate check can be secured by re-drilling holes instead of sinking shafts on them. That is, a second hole is bored within a few ft of the original hole, or as close as possible without risk of entering ground disturbed by previous drilling. A portable churn drill, such as "Keystone No 3," is usually employed (Sec 9). This machine, with 6-in tools and properly used, yields 1 cu yd of gravel from approx 100 ft of hole. When a new outfit must be purchased for boring only a few holes, so that first cost is a large item, or when labor is cheap and unskilled, or when transportation is difficult, a light hand-power outfit, like the "Empire" (Sec 9) is useful. Bedrock must be penetrated far enough to determine its character, and that it is genuine and not simply a large boulder. The character and amount of ground drilled and the number of colors secured should be noted ft by ft, and the ground-water level recorded (Art 5).

Whether the work be done by pits or drill holes, a close check of individual holes or pits is largely accidental, and whether the results of a certain amount of drilling are to be accepted (though with modification) is a matter of judgment.

The engineer must determine if the average of the checking holes is in sufficient agreement with the average of the holes or shafts checked, or if the divergence is too great in

proportion to the possible operating profit to permit use (after discount) of the original data. Methods of combining data are discussed under Conclusions, Art 10. 75 to 80% of the prospecting value is usually considered to be recoverable by dredging, while 100% has occasionally been secured in practice.

8. CARE OF SAMPLES, NOTES, AND ASSAY MAPS

Sample tagging. A good method is to write the number (and any descriptive matter) with soft lead or indelible pencil in heavy lines on strong bond paper, like that of ordinary loose-leaf notebooks. The sheet is folded close, to enclose and protect the writing, and placed on top of the sample just inside the container. This marking may be confirmed, in case of sack containers, by marking the number with pencil on the sack, either within the throat just outside the tying, or on the outside if there is no objection to showing the sample number.

Some prefer thin strips of soft wood, about 0.1 by 1 by 2 in, on which the sample number is written in heavy pencil marks which can not be obliterated by a moist sample. Others use small paper or linen shipping tags, one inside, the other tied outside the container; or brass number tags placed inside. The marking should always be in duplicate, so that the sample can not become valueless through loss of a single mark.

Field notes should show date of sampling, name, and connection with the samples of every one concerned in or present at time of sampling, and other information regarding the specific day's work. As each sample is taken its number and the data regarding it are recorded, including a full description of the sample itself; as, exact location referred to an established point (preferably a survey station), length and character of cut, and any peculiarities of deposit or workings noted at or near the point sampled.

In an examination involving a considerable amount of sampling, the notes may be systematised by using a tabular form arranged for specific conditions. Fig 5 shows a form suitable for underground interval-sampling.

Office record. To provide against loss, and for convenience, it is well to copy field notes into a permanent office record. Fig 6 is a form of office record adapted to field notes in Fig 5. Since field notes are made as samples are taken, samples of work in progress become scattered over a number of pages, but by keeping a separate sheet in the office record for each working (preferably in a loose-leaf binder) samples will appear in the order of their position underground.

Obverse						Reverse
Mine.................Level.........						Date..............
Working Place........						Sampler..........
Loc dist from	Section		Sample		Assay value	Remarks
	Formation	Width	No	Width		

Fig 5. Field Sample Note-book Headings

Sampling													Record
Mine..............Level..........Working Place...............													
Date	Loc dist from	Section		Sample		Assay value	Width by assay	Progressive totals			Field book		Remarks
		Formation	Width	No	Width			Commencing with sample No	Width	Width by assay	No	Page	

Fig 6. Office Record, Sample Note-book Headings

Sample containers. Samples of ores, the moisture or other volatile contents of which need not be considered, are best sacked as taken, or immediately after such reduction as is made at the point of sampling. If, as is often the case, interests exist which might be served by deception, the sacks should immediately be sealed or placed under lock and key. The latter can best be done in the field by using a locked leather mail sack, in which the samples are in little danger of being tampered with, even by injection of values in solution (Art 9). But, given sufficient time and motive, salting can usually be accomplished, and even the best protected samples should as little as possible be left accessible to others than the engineer's confidants.

When moisture or other volatile contents is to be determined, samples should be taken on an enameled cloth or rubber sheet, and should completely fill the container. They are best carried in screw-top cylindrical galvanised or tin cans, which can be hermetically sealed by screwing the top

down on a rubber gasket, or by binding the joint with several layers of adhesive tape. Corrosive solids are best carried in glass fruit jars with glass tops; liquids and gases in glass or earthenware bottles, or demijohns with cork, rubber or glass stopper, according to the character of the sample.

Assay maps are the best method of permanently recording sampling results, of studying these results in relation to each other and to the property as a whole, and of submitting these data in a report. They are made by tracing a map of the mine, entering the sample positions thereon by dot or line, and writing the number, assay, or value of each sample alongside its location (Sec 19, Fig 1).

For entry of a single value for each sample, the scale of 1 in = 50 ft is as satisfactory for assay plans as for most other work. If several constituents are to be entered for each sample, a scale of 1 in = 20 ft is preferable. If several entries per sample are necessary, and only small-scale maps are available (1 in = 50 ft or smaller), the sample number only may be written at the point of location, and a table of all data desired placed in a corner of the map sheet. This is not so good as the direct entry of the data at the location, as the variation in value in the different workings is less obvious. Variations in assays for a single element, or currency values of several elements, are sometimes indicated by intensity of shading, variety in coloring, or design symbols, the fluctuations being thus shown at a glance. This method has gained favor on the Rand, So Africa.

Besides the assays, assay maps should show the tonnage calculation blocks, in amounts and average values. For steeply-dipping deposits, these blocks are best shown on longitudinal elevations or on slope plans. For flat deposits, plans are most instructive.

9. SALTING

Salting, the process of intentionally or unintentionally raising the value of a sample above that of the exposure sampled, must be continually guarded against. The possibility of intentional salting being attempted must be borne in mind even when circumstances seem to give little occasion for it.

Example. *An engineer examining a copper mine, taken on operating lease with privilege of purchase, recommended that the smelter be operated on average ore to be obtained by taking a slice of uniform thickness from the floor of each level. The mine foreman, however, extracted all the ore for the test run from a particularly rich lot which he knew to exist in the floor of one of the levels and, in absence of the engineer (who had been employed only for the single examination), the lessees received the impression from the smelter returns that workings actually averaging 3% contained between 4 and 5% copper. The foreman had no connection with the owners of the property, was solely the employe of the lessees, and had no apparent object in the deception. But he was a resident and property owner in the district, and when subsequently tasked with the fraud explained that he feared that if the lessees had known the truth they would not have completed the purchase, the property would not have been regularly worked, and the expected rejuvenation of the district dependent on operation of the property would not have been realized.*

Since the commonest cause of salting is the desire of vendors to effect a sale, the only safe practice for the engineer is to take every precaution against salting, even if the vendor's reputation and the general circumstances are such as to allay suspicion. Though it is not the examining engineer's province to analyze the motives of those with whom he deals, he is responsible for so conducting his examination as to assure himself that deception has not been practiced upon him. On the other hand, unnecessary obtrusion of the precautions essential to such assurance is not only a breach of courtesy, but may cripple or retard examination work, since the sympathetic assistance of those who are showing the property is of advantage in promptly calling the engineer's attention to such features as might otherwise be noted only late in the examination, or possibly be entirely overlooked.

Salting has been effected by preparing the ore exposures from which samples were likely to be drawn, or by limiting the exposures to those showing high values. The latter is usually done by timbering and close lagging, or by heavy blasting so as to cause a cave (or the appearance of one) in barren sections. Rich ore may be placed in piles throughout the mine, as though blasted from adjacent faces, in the hope that samples will be taken from them rather than the faces. The practice of stopping each heading while it happens to be in good values is also employed. But in such case there is a fair chance that the vendor himself may be deceived by having discontinued development just as large values are about to be encountered.

The commoner method of contaminating the sample itself may be practiced upon the engineer at any point where he allows unprotected samples to be handled by others than himself or his confidential assistants. Local miners employed to cut samples can readily salt them by adding valuable mineral, by breaking into the sample an undue proportion of the richer streaks through which the sample is being cut, or by picking out pieces of waste during the work. Also, misleading results may be produced unintentionally.

Example. The early history of one of the great copper properties in southwestern U S was checkered by incorrect conclusions based upon hundreds of progress samples averaging 4% copper, from ore actually containing only 2%. These samples were taken by the operating company for its own information, by a system of interval-channel sampling laid out by a reputable European engineer. The actual sampling, however, was left to the casual Mexican miner who happened to be least needed on other work and he, minerlike and wholly innocently, broke down and retained in his samples the best ore showing at the points sampled.

Samples may be salted by syringe injection of valuable solutions through the cloth of sack containers. Exposure of samples during the quartering process affords excellent opportunity for salting by adding highly concentrated values shaken from the hair, finger nails, or clothing, or by introducing them as expectorant, tobacco ashes or what not. By manipulation of the reduction process, an undue proportion of barren material may be thrown into the reject. In fact, the methods which ingenuity can devise for salting a sample, even though it be in a sealed sack, are innumerable. The only safety for the engineer is to check constantly both sampling and assaying, and so far as possible, keep his samples out of the hands of all except his confidential assistants. A sample known to be barren, and purposely carried for assay with the others, may catch some of the salter's values if the lot is tampered with, and the sampler will be thus placed on his guard. (See also Sec 30, Art 9.)

10. SAMPLING CHECKS AND CONCLUSIONS

Checks on accuracy of both sampling and assaying should be made in sufficient number to give assurance that results are as reliable as circumstances admit or make desirable. When possible, ERRATIC SAMPLES should be retaken with special precaution against inaccuracy or contamination. When assays can not be made prior to leaving the property, it is well to duplicate 5 or 10% of the sampling, selecting the samples for duplication at random, but so as to include those which appear to be of high, low and average value.

Assaying should be checked by sending for assay a small percentage of duplicates (the reject of the last reduction in sample preparation), and a few blanks of known barren rock, all under new numbering. Also, a few duplicates should be sent to a different assayer. These may not always agree in detail, but the average of the duplicates by either assayer should not vary from that of their originals by more than a small percentage of the values contained. Provided the assaying is accurate, the DUPLICATE SAMPLING should check in a similar manner; that is, actual duplicates may vary considerably, but the AVERAGE of duplicates and originals should show reasonable agreement. If sampling has been done on a short interval, compared to the total length of exposure (say 5 or 10 ft on large work), the average of alternate samples should closely agree with that of the intervening samples; the greater the number of samples, the closer should averages agree.

Conclusions from results of sampling should be based upon proportioned averages; that is, the quantity of material represented by an assay should be as much a factor in calculating an average as is the assay itself. Thus, in a well-defined vein, exposed over its full width and sampled at regular intervals, each assay is multiplied by the sample width, and the sum of the products divided by the sum of the widths to determine the true average. If sample intervals are irregular, the width is first multiplied by the interval to the succeeding sample (using the last interval on both the last and next to last sample) and this product multiplied by the assay. The sum of the interval-width-assay products divided by the sum of the interval-width products then gives the proportioned average.

In valuing ore exposures containing large and irregular proportions of heavy minerals in light gangues (as galena in quartz), a SPECIFIC-GRAVITY DETERMINATION is sometimes made on each sample, and the interval-width product multiplied by it before multiplying by the assay. The sum of the interval-width-gravity-assay products divided by the sum of the interval-width-gravity products then gives an average assay, which includes allowance for the specific gravities of different parts of the ore exposure. This refinement is unusual. Cases are rare in which the engineer's knowledge of the mining value of the deposit would be sufficiently increased by gravity determinations to justify the added cost; for, since the value usually follows the heavy mineral, the probable effect is to raise the average assay. This, unless subsequently discounted, seems more likely to increase than to decrease error in determining the mining value. Owing in part to the more friable nature of the valuable minerals, and particularly to the inevitable inclusion, when mining, of some of adjacent low-grade or barren material (which can not properly be added to the sample), the yield per ton of a carefully sampled deposit is generally lower than the calculated average, and the tonnage higher. An allowance (say 10%) should therefore be made for this in reporting values.

A precaution sometimes taken against over-valuation is the ELIMINATION of all HIGH

ERRATIC VALUES, substituting for them the average of the two adjacent samples, but this practice is questionable. If there are enough erratic values to effect the result seriously, there may also be enough high values to justify inclusion of all. The proper disposition of erratic samples calls for careful study of each case, and for the engineer's best judgment.

When the deposit is wider than the exposure in drift, upraise or stope, and samples of uniform widths have been taken at regular intervals, the width and interval factors disappear and the mathematical average of the assays applies to the area sampled. But, unless the workings sampled are uniformly distributed throughout the deposit, mathematical average should be applied only to the immediate and smallest possible area, and area assays should then be combined with their respective area factors. That is, the average of the whole is found by dividing the sum of the assay-area products by the sum of the areas. This avoids the common error of applying to the whole the average of samples secured chiefly from a small, well-developed portion of a comparatively large but little developed area.

Calculation of average values is facilitated by using a form like Fig 7. From sample numbers, assays, measurements, and their products, the sums and averages can be recorded, and final summations and combinations made, from which to secure proportioned averages of different blocks and of the whole.

Sample No	Width			Gold			Silver			Lead						Total values		
		Prog total			Width by assay			Width by assay			Width by assay			Width by assay			Width by assay	
	of sample	Including sample No	Inclusive total	Assay	of sample	Prog total	Assay	of sample	Prog total	Assay	of sample	Prog total	Assay	of sample	Prog total	Assay value	of sample	Prog total

Fig 7. Form for Calculating Average Assays. (Blank columns for unexpected additions)

A pocket adding machine (as the "Ve-Po-Ad") and a slide rule assist in making field calculations. More elaborate adding and calculating machines (as the "Monroe") are excellent for office work.

In drawing conclusions from results of sampling, it should be borne in mind that the development of a deposit, or even a portion of it, is never so complete that an accurate, mathematical description is possible. The value of a mass of ore can not be accurately known until it is mined and treated, and allowance must always be made for contingencies.

A single block of ore may be sufficiently developed to give results by sampling and calculation closely approaching those ultimately secured by actual mining. But the engineer should scrutinize the data of each block, to determine whether its development is of this thorough nature, or whether allowances must be made in the light of his knowledge of the characteristics of the deposit. This knowledge should have due weight in determining his declaration of assured values of the entire deposit, which of course are based primarily upon his sampling and calculation. The report should include all important data bearing upon the conclusions reached.

11. DETERMINATION OF QUANTITY AND VALUE OF ASSURED MINERAL

Assured mineral is the amount existing of which the engineer can give definite assurance. It is necessarily a minimum, not an ultimate quantity, though in case of mineral well blocked out relatively to the known irregularity of the deposit, there may be masses of proved mineral which can be quite accurately calculated. The term ORE IN SIGHT has been widely employed, but has been so variously used and misused that it has ceased to have a precise technical meaning. It has been suggested that ore exposed on at least 3 sides be termed "in sight;" but, in irregular deposits ore so exposed may still be uncertain in quantity and value, while in regular deposits the ore may be assured, although not exposed on 3 sides. Hence, the use of the term is being abandoned by those who wish to convey a definite idea. An engineer's statement of assured mineral is essentially a conclusion based upon judgment, not merely the result of mathematical calculation.

Methods of calculation, though simple, must be carefully applied in each case, so that the relation of amount and reliability of data to extent and importance of conclusions may be fully realized. Assay maps (Art 8) best show this relation, especially when studied in three dimensions (plan and elevations), and form one of the prime sources of information

as to size, shape, value and relationship of the different ore exposures. Structural geology of the deposit must also be studied, so that isolated exposures will not be considered consecutive and intervening barren volumes calculated as of value (Art 2). The average assay value of the exposures of a given block being determined, volumes are calculated by multiplication of average area by average thickness, by inclined distance between exposures, or by depth between levels, according as the block is in a flat-lying seam, bed or zone of enrichment, in an inclined vein, seam or lens, or in a deposit of vertical extent or development (Art 10).

In a well-defined, strong vein of fairly evenly distributed values showing no tendency to concentrate into shoots, the value areas, measured normally between walls, and exposed at consecutive levels close enough together to preclude unexpected irregularities, are multiplied by the dip distance between levels to determine the volume of mineral, and the values of the exposures, combined proportionately to their respective areas, may be taken as the value of the block.

In large masses, developed by well-defined and sampled cross-sections, the accuracy of calculation may be increased by using the prismoidal formula. A mid-section is constructed by plotting adjoining sections, connecting their definitive points, and forming an area by connecting the mid-points of the connection lines, as in railroad earthwork practice (Sec 17). Masses that are well developed by drilling cross-sections are advantageously so calculated, the volume being found by multiplying $^1/_6$ of the distance between consecutive sections by the sum of the areas of these sections plus 4 times the area of the plotted mid-section.

Unless the development and sampling of a deposit is so uniform that the size, exposure, and number of samples of each block are practically the same, the blocks should be as small as the data will permit, so that a minimum number of assays will be grouped into an arithmetical average before being combined with the volume factor which they represent (Art 10). General averages are then obtained by including the volume factors.

An adaptation of Fig 7, by substituting tons" for "feet," facilitates determination of totals and average values of any desired combination of ore blocks. In laying out blocks for calculation, care must be taken that all available data are included, and that such as are available suffice to assure the presence of the full calculated volume and value; or, if only a portion, what proportion is assured. Possible influence on each block of unexposed faults, pinches, horses, low-grade spots, shoots, rich masses or other inclusions, should be decided only after careful investigation of the development of the deposit in question, and similar deposits. Minable ore of less than stoping width must either be diluted in the calculation by enough barren rock to make workable width, or the cost of separate mining of the necessary waste included in the cost estimate. The engineer's trained judgment should be brought to bear on each block.

Though the practice is regrettable, flat-lying deposits (especially placers) are often tested by irregularly placed pits or drill holes, so that the mathematical average of values shown gives an erroneous idea of the average value of the deposit, even though the depth factors are included. As the tendency is to sink the greatest number of pits or drill holes in the richest spots, the error almost always exaggerates the true value.

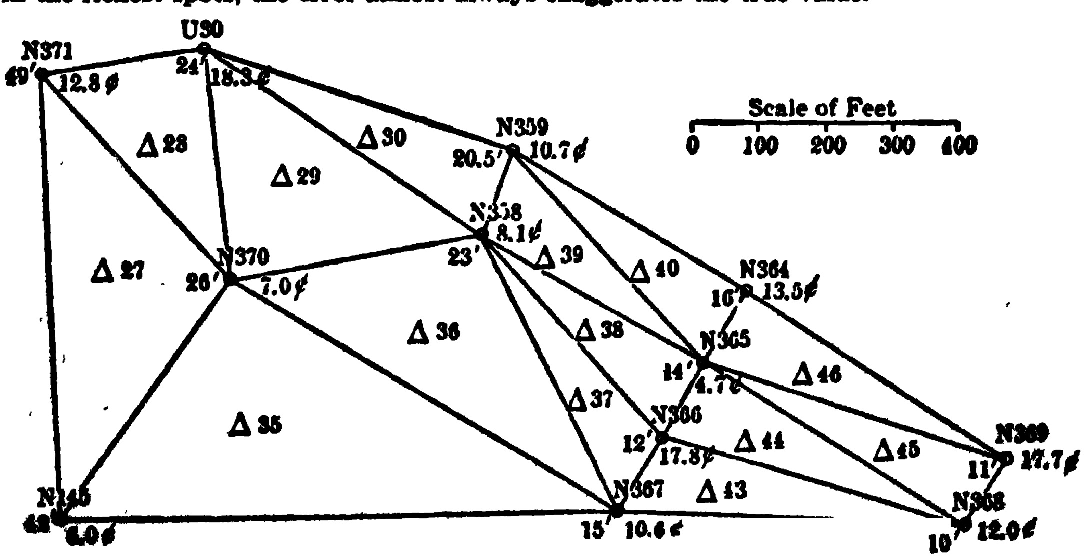

Fig 8. Example of Irregularly Spaced Drilling

To minimise this error, divide the area of values on the map into as many triangles as possible, having their angles at the pits or holes as in Fig 8. Avoid over-emphasizing any hole by centering an

unusual number of triangles about it. Number triangles and assign to each the average depth and value of its corner holes. In computing average value, include respective depth factors. These data may be arranged as in Fig 9. Another table (Fig 10) should show the scaled base and half altitude, and the calculated product or area of each triangle. Add to this (from Fig 9) the depth and value of each triangle, and calculate the volume and total value of each. Dividing total of the values by the total volumes, gives a value per volume (¢ per cu yd, in case of gold placers), in which theoretical errors or proportioning irregular developments are much reduced, provided the triangles have been so laid off as to avoid overemphasizing any hole. Equilateral triangles are ideal (each angle = 60°), and by introducing the factor of the number 60° units (or decimals) in each angle of each triangle, greater theoretic accuracy is attained (24). But, good judgment in using the simpler method often gives as close an approximation to the actual values present as is attainable with the unsystematic exploration under consideration. Regular exploration is always best.

Triangle No	Corner holes: Hole No	Depth: Ft	Depth: Yd	Depth: Aver 3 cor holes	Value, ¢ per cu yd	Product, value by depth	Aver value triangle, ¢ per cu yd
28	N 371	49	16.33	....	12.8	209.02	....
...	N 30	24	8.00	....	18.3	146.40	....
...	N 370	26	8.66	....	7.0	60.62	....
Total & aver		...	32.99	10.99	....	416.04	12.6
29	N 370	26	8.66	....	7.0	60.26	....

Fig 9. Sample Holes and Values

Triangle No	Scaled dimensions, ft: Base	Scaled dimensions, ft: Alt	Base, yd	Half alt, yd	Base by half alt = area, sq yd	Aver depth of 3 corner holes	Area by aver depth = volume in cu yd	Aver value, ¢ per cu yd	Aver value in $ by vol in cu yd = total val in triangle, $
28	410	210	136.66	35.00	4 783.10	10.99	52 566.27	12.6	6 623.35

Fig 10. Volumes and Values

Besides natural variations from apparent values, artificial variations may occur. Abandoned workings may be concealed by timbering or muck, intentionally or by accident. The latter is common in mines which have passed through many managements, and may reveal unlooked-for conditions in apparently well-defined blocks. A case is on record in which a rich ore-block, defined by excellent exposures in backs of levels above and below and in upraises at either end, had been almost completely gutted through a winse in floor of upper level, which had been timbered over and covered with tram-road muck (4). When the decision hangs on presence *in toto* of a few well-developed blocks of ore, care must be taken to insure that neither natural peculiarities nor vicious intentions deceive; possibilities which must be recognised and guarded against (Art 10). But such deception can rarely be practiced on sufficient scale materially to affect conclusions. After being shown over the property, the engineer should put directly to the executive in charge (and, as opportunity offers, to each subordinate independently) the question whether the engineer may be assured that he has seen every known opening or exposure, and whether any such exposures are incorrectly shown on, or omitted from, the maps submitted. Unintentional oversight will usually be thus discovered, and, in case of intentional deceit, some one reply may be sufficiently evasive or contradictory to arouse suspicion and lead to discovery.

Tonnage factors. The factor by which the volume in cu ft should be divided to reduce it to tons is determined by dividing 2 000 lb (or 2 240 lb for the long ton) by 62.5 lb (wt per cu ft of water) × sp gr of the mineral mass (Sec 1, Art 3).

To determine sp gr take the aver of a number of sp gr bottle-tests on aver crushed samples. Fill a narrow-neck bottle to a mark with distilled water at 60° F and weigh (a). Introduce a weighed amount (b) of sample, restore water level to the mark after insuring removal of all air accompanying the sample (done by boiling, if ore is sufficiently insoluble) and take final wt (c). Then, sp gr of the mineral = $b \div [(a + b) - c]$. (See Sec 1, Art 3.) After estimating aver ore composition, the aver tonnage factor may be computed from Table 3 (1).

Example. A ton of ore in place, composed of 70% pyrrhotite, 7% chalcopyrite, and 23% schist, will occupy a volume = (70% of 7) + (7% of 7.6) + (23% of 11.9) = 8.2 cu ft; or, in practice, call tonnage factor 9 cu ft.

As in the example, it is advisable to allow for inaccuracies by using a larger tonnage factor than calculated, regardless of basis of calculation. For porous mineral masses,

1, 2 or more cu ft may have to be added, to get correct tonnage factor. In important cases, where uncertainty exists, the weight of an accurately measured excavation should be taken to determine the proper factor. If deposit is large enough for planimeter measurement of reserve areas, and if production records are sufficient, a tons-per cu ft factor

Table 3. Weights of Minerals and Rocks

Metal	Mineral	Wt, lb per cu ft	Cu ft per ton (2000 lb)	Metal	Mineral	Wt, lb per cu ft	Cu ft per ton (2000 lb)
Antimony	Native	417.8	4.8	Lead	Pyromorphite	436.4	4.6
	Stibnite	286.8	7.0	Manganese	Pyrolusite	299.3	6.7
Arsenic	Orpiment	218.2	9.2		Psilomelane	262.0	7.6
	Realgar	218.2	9.2		Wad	218.2	9.1
Barium	Barite	280.5	7.1		Rhodochrosite	224.4	8.9
	Witherite	268.1	7.4		Rhodonite	224.4	8.9
Bitumen	Carbon	93.5	21.4	Mercury	Native (aver)	897.9	2.2
Calcium	Calcite	168.4	11.9		Cinnabar	505.1	4.0
	Aragonite	187.1	10.7	Molyb-			
	Gypsum	143.4	13.9	denum	Molybdenite	293.0	7.0
	Fluorspar	199.4	10.0	Nickel	Millerite	349.2	5.7
	Apatite	199.4	10.0		Niccolite	467.6	4.3
Coal	Anthracite	93.5	21.4	Platinum	Native (aver)	1091.2	1.8
	Bituminous	81.0	24.6	Silver	Native (aver)	654.7	3.5
Cobalt	Linnaeite	305.5	6.5		Argentite	455.1	4.4
	Smaltite	405.3	4.9		Hessite	530.0	3.8
	Cobaltite	386.6	5.2		Petzite	561.1	3.6
	Erythrite	187.1	10.7		Sylvanite	498.8	4.0
Copper	Native	554.9	3.6		Pyrargyrite	361.6	5.5
	Chalcocite	355.4	5.6		Stephanite	392.8	5.1
	Chalcopyrite	262.0	7.6		Proustite	342.9	5.8
	Bornite	311.8	6.4		Cerargyrite	336.7	6.0
	Enargite	274.4	7.3	Sulphur	Native (aver)	130.9	15.3
	Tetrahedrite	305.5	6.5	Tin	Cassiterite	424.0	4.7
	Atacamite	236.9	8.4		Stannite	280.5	7.1
	Cuprite	374.1	5.3	Tungsten	Wolframite	455.1	4.4
	Chalcanthite	137.2	14.6		Scheelite	374.2	5.3
	Malachite	243.1	8.2	Zinc	Blende	249.4	8.0
	Azurite	230.7	8.7		Zincite	355.4	5.6
	Chrysocolla	137.2	14.6		Goslarite	124.7	16.0
	Dioptase	205.7	9.1		Smithsonite	274.3	7.3
Gold	Native	1184.7	1.7		Calamine	212.0	9.4
Iron	Pyrite	318.0	6.3		*Rocks*		
	Marcasite	299.3	6.7		Quartz	162.1	12.3
	Pyrrhotite	286.8	7.0		Andesite / Basalt	181.0	11.1
	Arsenopyrite	374.1	5.3				
	Hematite	311.8	6.4		Diabase / Diorite	187.0	10.6
	Menaccanite	299.3	6.7				
	Magnetite	311.8	6.4		Granite	168.0	11.9
	Limonite	236.9	8.4		Limestone	168.0	11.9
	Siderite	236.9	8.4		Porphyry	162.1	12.3
Lead	Galena	467.6	4.3		Rhyolite / Sandstone	149.6	13.4
	Cerussite	405.3	4.9				
	Anglesite	393.0	5.1		Schist	168.0	11.9
	Crocoite	374.1	5.3		Shale	162.1	12.3

Note.—For compositions and specific gravities of minerals, see Sec 1.

may be used for computed volumes and known tonnage produced. That is, if vol of extraction V yielded T tons, how many tons will reserve-vol V^1 yield?

Roughly, cu ft of anthracite coal may be reduced to long tons by dividing by 24; bituminous coal to short tons by dividing by 25; quartz and feldspar masses by dividing by 13 (or by factors down to 10, if sufficiently sulphided); sulphide and iron-oxide masses by 10, 9, 8, or 7, according as they consist chiefly of limonite, ZnS, FeS, FeS_2, Fe_2O_3, Fe_3O_4, or PbS, or contain more or less gangue, or are dense or porous in structure. Less than 5 cu ft of pure PbS make 1 ton. In dumps, the volume usually ranges from 1.6 to 1.9 times the volume of same material in place. Calculation of combined tonnages and values, as of individual exposures, are best made in tabular form (see Fig 3.)

12. PROSPECTIVE VALUES, PERCENTAGE OF RECOVERY, COST OF PRODUCTION

Prospective quantities and values. This subject requires more highly trained judgment than, and is of as much vital interest to the client as, any other part of an engineer's report. Having collected all available data as to past yield and present showing of values, geological conditions in which these values occur and records of similar conditions and deposits in vicinity or elsewhere, a careful sifting, balancing, and digestion of these data must be made in order to form as definite an opinion of the prospects as possible. To crystallise this opinion in practical form, the engineer may propound to himself the following questions: 1. What are the geological and artificial limitations, as folds or change of formation, well-determined faults, and property lines (including apex-law limitations), and to what extent do these preclude possibility or probability of further development of values in depth or horizontal extent? 2. Are there any highly probable continuations or indications of independent values on the property? If so, what is the least development and cost which will yield definite information regarding such values, and what development and cost is necessary to prove thoroughly these values, for establishing a profitable industry thereon? 3. In addition to highly probable values, what hopes may reasonably be entertained as to extent and value of the ultimate mineral resources of the property, what scale of outlay is likely to be involved and justified, and what, if any, definite limitations do local conditions place on such hopes? In general, deposits not definitely limited are likely to contain at least 25 to 100% more ore than can be actually assured.

Example I. (Question 1) A deposit was limited at one end by a property line across which the orebody passed to an adjoining mine, while the structural geology showed plainly that the main workings bottomed in and were limited by a synclinal trough. (Question 2) Comparatively small outlying developments on what could be traced as the downturn of an anticlinal fold succeeding the syncline, exposed a small but excellent ore showing at a lower level than the bottom of the syncline. The property being in regular operation and equipped for diamond drilling, $1 000 spent in boring in the floor of the bottom ore was likely to add many times this amount to the value of assured ore reserve. (Question 3) The drilling was also likely to trace ore to sufficient depth to justify sinking a winze, so as to render a new level accessible to drilling. The winze, including drilling at the lower level, would cost $5 000. Granting the first favorable diamond-drill showing, the general features of the deposit and the reputation of the district warranted the hope that enough ore could be proved and indicated to justify opening this outlying deposit direct from the surface (replacing the long crosscuts and hoisting winzes through which previous development had been made) by an efficient main shaft to cost about $100 000. This shaft in operation, a low-grade outlying deposit on the strike (of which there was hope, because of the 25 to 50% increase in value with the depth of the deposit being developed) could readily be proved and exploited in depth. This outlying body once proved valuable, in addition to the main deposit, the mine would have an indefinitely great future, notwithstanding the limitations of property line and fold (Question 1). Since the mine was equipped and in profitable operation, its purchase at the full profit value of all assured ore, figured on the highest likely average market, was attractive for those skilled in the business and looking for promising, mining ventures, and it was a serious question whether a cash purchase was not advisable at a much higher price. Ultimately, all expectations were so largely fulfilled that the true value proved to have been but slightly forecasted by the value of the assured reserves.

Example II. (Question 1) An obviously secondary enrichment, lying between a clay overburden and underlying limestone, was found exposed by an open-cut working supplying a small concentrating mill, the operation of which investigation proved could be made profitable. Test pits to bedrock at different points about the open cut showed ore extending out on 3 sides and proved considerable value. At no point on the property were commercial values found in the underlying limestone. Property lines were at considerable distances in all directions from the open cut. Post-hole-digger holes to bedrock, the records of which in many cases showed ore, were found scattered about the open cut, but as many of them were reported barren, it was doubtful whether the deposit extended over any considerable area of the property. (Question 2) Though the records of holes within the area proved by test pits showed the values to be exaggerated, continual recurrence of records of barren holes suggested that the exaggeration was due rather to error in judgment than to intention, and that the presence of ore could at least be presumed in holes having a record of ore value. Therefore, systematic prospecting could be expected to add considerably to the values already assured. The facility with which both holes and pits had been sunk through clay and ore to the limestone proved the practicability of cheap exploration, and it was obvious that for $1 500 the entire property could be tested in the following simple manner. The area was laid out in 500-ft squares and a hole bored at each corner; then holes were bored at corners of 100-ft squares laid out around each hole showing ore indications. The 100-ft square system was then extended until all ore areas developed were completely surrounded by barren holes. Ore areas thus defined were finally proved by test pits at corners of 200-ft squares. (Question 3) Though expectation of a definite increase in reserves beyond those actually assured at time of examination was fully justified, the barren hole record showed that such increase would be moderate, and that the present assured reserves must be considered as forming a large proportion of the ultimate reserves, at least in so far as the secondary enrichment was concerned. Primary values in the limestone being lacking in the exposures, they

received little consideration and the property was prospected for secondary-enrichment values, as described. The above inferences being found substantially correct, the cost of equipment and scale of operations were proportioned to the known limitations of the secondary-enrichment values, and the highly profitable operations thus inaugurated were not overburdened with necessity for returning large equipment outlay.

Percentage of gross value recoverable and cost of production. Estimates of recovery and costs are based on past and present accomplishment on the property; on results of investigations of the ore by field, laboratory, or ore-testing works; on the operation of a small but practical preliminary plant, erected for the purpose; and on practice in mining and treating similar ores elsewhere. ACTUAL ACCOMPLISHMENT on the property can best be determined by investigating the records, and checking them against each other, against tailing-dump samples and, if feasible, against test runs of the equipment (Art 2). Equipment should be carefully inspected to determine its adaptibility, condition, and whether general appearances, coupled with available records, indicate that the management could be improved. PANNING AND SIZING TESTS often assist in suggesting possibilities of treatment by concentration. ORE-TESTING WORKS treatment of larger lots may be of still greater value (Sec 31). MUFFLE-ROASTING AND LEACHING TESTS in the laboratory may suggest possibilities of treatment other than by concentration. On large, low-grade properties, demanding heavy capital outlay for equipment before profitable operation can begin, it is customary to erect a PRELIMINARY OPERATING PLANT, of sufficient capacity (sometimes hundreds of tons per day) for ore treatment by full-size machines, under working conditions. Such a mill is operated for a number of months at least, even though at a loss, simply to secure data and to test various modifications of treatment, preliminary to designing a large equipment for regular operation. Its design is as flexible as possible, so as to include all promising modifications of treatment which preliminary tests in laboratory or testing works have suggested. ACTUAL PRACTICE in mining and treating similar ores elsewhere is important for estimating operating costs and percentage of recovery.

Conclusions drawn from practice must be based on study of the similarity of general conditions, and of the mineral to be treated. Relative cost of labor and supplies and the scale of operations at successful properties, as compared to those feasible at the property in question, should be thoroughly understood. SCALE OF OPERATION often determines operating costs. Thus, concentration costs in the great porphyry-copper mills in western U S, each treating thousands of tons per day, have been as low as 50¢ per ton treated. The same treatment in a 100-ton mill might cost 80¢, $1, or even more per ton (Art 14). In general, the important estimates of percentage of gross value recoverable and cost of production should be made only after analyzing all information obtainable. In estimating total costs, ample allowance must be made for general overhead charges, selling and sampling costs, freight to market and contingencies, the last usually being put at 10% (Sec 21).

13. MARKET PRICE OF MINERAL PRODUCTS

Theories and difficulties. Gold, because of its relation to currency, has been held by the U S gov't, since 1934, at $35 per Troy oz. Prior to 1933 the statutory price had been (with the exception of the Civil War specie payment suspension) $20.67+ per Troy oz since 1837. During 1933 the price, with fluctuations, continued to rise to the present (1938) value, fixed Feb 1, 1934. This price, made by presidential proclamation, is to continue until altered or modified by proclamation "as the interest of the United States may seem to require;" under present law, between $34.45 and $41.34+ per Troy oz. Fluctuations in market price of all products form one of the most uncertain elements of mine valuation. Being essentially a commercial question, business men having experience in dealing in the product in question can often estimate more closely than can the engineer the average price it will command in the future. But, as opinions may conflict, the engineer himself is often forced to choose the standard of price for the basis of his estimates.

Current prices may be adopted, but may lead to false impressions, and conclusions that become absurd before the property is developed. Hence, the engineer should consider at least one other price, or preferably several others. In case of metals or coals having a standard market, it is questionable whether an engineer should recommend purchase of a property which he estimates is incapable of making a small profit, when mining its choicest reserves and selling its product at the MINIMUM PRICE of any the 48-yr record shown in most items in accompanying table; unless the property itself, or one very similar, has already achieved considerable success and continues to do so. The value of a property which would fail to make an attractive profit under normal operation, at the AVERAGE PRICE for a like period, is even more doubtful. In case of mineral products having no standardised open market value, especially when their prices depend on empirical gradings as to color, texture, etc, there should be similar hesitation in acquiring a property which, though meeting other requirements, fails to promise an output comprising enough of the highest grade product to insure a profit regardless of out-

Table 4. Mineral Products of the U S (Yearly Quantities, Prices, and Averages)

mils lb = millions of pounds; sh t = short ton	Highest price			Lowest price			1937		Aver 48 yr, 1890–1937	
	Year	Quantity	Price	Year	Quantity	Price	Quantity	Price	Quantity	Price
Metals										
Aluminum, mils lb (except '90 in thous) & ¢	90	61	100	11	46	18	293	19	92	25
Antimony imports, hund thous lb & ¢ (90–34 Chinese, 35–37 Amer smelter price)	15	212	30	21	296	5.0	335	15	164	12
Bauxite, thous long t & ¢	34	158	715	96	18	258	420	582	197	574
Chromite, hunds long t & $	18	804	48	24	3	4.0	23	6.4	43	30
Copper, tens mil lb & ¢	17	189	27	32	54	6.3	167	12	101	15
Lead (refined), tens mil lbs & ¢	25	131	8.7	96	38	2.8	89	5.9	76	5.5
Manganiferous ore, 06–37, tens thous long t & ¢	18	117	481	14	10	222	134	288	57	304
Manganese ore, hunds long t & $	21	135	37	95	95	7.5	402	26	317	23
Mercury, thous flasks (76 lb) & $	16	30	128	94	30	31	17	90	22	62
Molybdenum, 15–37, hunds thous lbs & ¢	18	9	146	20	0.3	49	301	68	43	68
*Nickel, metal & alloys imports, 95–37, hunds thous lb & ¢	12	2	46	24	256	20	817	25	159	25
Pig iron, hunds thous long t & $	20	357	32	94	67	9.8	352	21	232	19
Platinum imports, 95–37, thous troy oz & $	25	106	106	99	188	7.8	149	40	98	43
†Silver, hunds thous troy oz & ¢	94	495	129	32	240	28	717	77	569	74
Tin imports, 03–37, mils lb & ¢	18	165	64	32	78	21	197	53	130	43
Tungsten ore, 00–37, tens sh t & tens $	16	592	204	03	29	15	350	117	143	90
Zinc, tens mils lb & ¢	16	113	13	32	41	3.0	110	6.5	62	6.2
Non-Metals										
Bituminous coal, mils sh t & ¢	20	569	375	98	167	80	442	178	360	172
Natural gas, 11–37, tens billion cu ft & ¢ per M	22	76	29	11	51	15	237	22	121	22
Penn anthracite coal, hund thous sh t & ¢	26	844	562	98	534	141	519	381	705	250
Petroleum, tens mil bbls (42 gal) & ¢	20	44	307	92	5	52	128	120	41	120
Structural Materials										
Cement, mils bbl & ¢	20	97	202	94	8	60	116	148	75	138
Glass sand, 02–37, tens thous sh t & ¢	20	217	219	02	94	86	280	170	173	148
Gypsum, crude at domestic mines, tens thous sh t & ¢	20	313	355	90	18	102	306	190	217	210
Lime, 04–37, tens thous sh t & $	20	357	11	05	298	3.7	412	7.3	344	6.7
Sand, 05–37, mils sh t & ¢	20	80	76	14	78	29	187	50	109	51
Abrasives										
Corundum and emery, tens sh t & $	92	177	102	14	49	5	32	8.7	243	23
Tripoli, 17–37, hunds sh t & $	21	123	17	19	243	7.5	349	13	273	14
Garnet, 96–37, tens sh t & $	21	305	86	96	269	26	486	79	461	56
Pumice, 02–37, hunds sh t & ¢	30	568	591	06	122	137	710	425	371	396
Chemicals										
Arsenious oxide, 01–37, hunds thous lb & ¢	23	285	9.8	11	63	1.2	353	1.5	155	4.3
Bismuth imports, thous lb & ¢	19	77	298	96	124	73	67	80	113	160
Borates (crude), thous sh t & $	93	4	150	04	46	15	359	20	87	26
Bromine, hunds thous lb & ¢	16	7	131	08	8	9.7	262	20	32	24
Calcium magnesium chloride, 09–37, hunds sh t & $	21	24	22	09	13	4.9	102	13	55	16
Fluorspar, thous sh t & $	19	138	25	03	43	5.0	181	20	82	15
Phosphate rock, tens thous long t & ¢	90	51	630	97	104	257	396	328	228	381
Pyrites, thous long t & ¢	19	421	608	32	190	263	584	304	261	379
Salt, tens thous sh t & ¢	21	498	493	05	364	168	924	261	509	290
Sulphur, 91–37, tens thous long t & $ (except '94, units)	94	446	45	15	29	13	247	18	82	18
Pigments										
Barite (crude), thous sh t & ¢	20	228	939	97	26	224	356	625	123	589
Cobalt oxide imports, thous lb & ¢	21	164	209	12	32	48	843	126	190	141
Miscellaneous										
Asbestos, hunds sh t & $	20	16	412	13	11	10	121	29	23	56
Asphalt, ten thous sh t & $	20	90	15	90	4	4.7	433	9.2	106	9.3
Feldspar, thous long t & ¢	99	22	979	98	12	270	269	515	98	637

Table 4. Mineral Products of the U S *(Concluded)*

mils lb = millions of pounds sh t = short ton	Highest price			Lowest price			1937		Aver 48 yr, 1890–1937	
	Year	Quantity	Price	Year	Quantity	Price	Quantity	Price	Quantity	Price
Miscellaneous *(Continued)*										
Fuller's earth, 95–37, thous sh t & $	20	128	20	04	29	5.7	226	10	114	12
Graphite (96–29 production; 30–37 imports), hund thous lb & ¢	18	260	5.9	07	586	0.5	592	1.3	187	1.9
Magnesite (crude), 91–37, thous sh t & $	98	1	15	03	4	2.8	203	7.3	73	8.4
Mica (scrap), 96–37, hunds sh t & $	99	15	34	98	40	6.9	252	14	56	15
Mica (sheet), 96–37, tens thous lb & ¢	96	5	133	10	248	11	169	17	11	23
Potassium salts (K_2O), 15–37, thous sh t & $	16	10	436	35	225	22	267	34	68	64
Silica (quartz), hunds sh t & ¢	26	277	989	11	879	176	130	508	464	376
Talc & soapstone, thous sh t & $	02	27	20	17	144	9.8	230	11	104	13
Tungsten ore, 00–37, tens thous sh t & tens $	16	592	204	03	29	15	350	117	143	96

Notes: * Nickel: domestic refined '95–'99, foreign refined '00–'37. Pure rolled nickel (small part of gross imports, including ore, matte, oxide, etc) about 10¢ higher price than aver of metal & alloys.
† Silver : U S gov't price for domestic produced silver only.

put of other grades. This is due to the uncertain market for low grades of unstandardized products, both as to price and to ability to absorb at any price. Profit from such a mine is in producing the highest grade, the lower grades being produced and sold largely as unavoidable by-products.

Question of SIZE OF MARKET, available within a radius of feasible freight rates, is important, since the opening of a large new property in a restricted market area may demoralize existing prices. Facilities for sale of product demand serious consideration. Marketing a peculiar product may constitute a large item of cost; sometimes the market can be secured only through channels available on no other terms than practical surrender of control of the property itself. For preliminary use of those not in touch with sources of more detailed information, Table 4 has been prepared, largely from the U S gov't reports.

Valuation for tax depletion, being an effort to hold an even balance between rights of buyer and seller, is a problem in itself, in which estimation of future market price is a crucial point, perhaps requiring even more elaborate investigation than a physical appraisal (Sec 24).

Example I. Aver price of copper for 48 yr ending 1937 was 15¢ per lb on annual aver output of 1 010 000 000 lb. Minimum price (1932) was 6.3¢ on output of 540 000 000 lb. Aver for 1937 was 12¢, on output of 1 670 000 000 lb. In view of these facts, an engineer might hesitate to recommend purchase of an inoperative property, capable of producing copper only and unlikely to be able to maintain an output from best ore reserves for say a year, at gross cost of less than 6.3¢ per lb unless success of this or entirely similar property were so great as to well overbalance possibility of the copper market again dropping to 6.3¢. Again, such a property would be undesirable unless it could make good profit while mining its aver ore and selling at 15¢. Finally, the fact that in 1917, during the War, the aver price was 27¢, on an output of 1 890 000 000 lb, should be recalled in considering possibilities of a copper mine of long life; providing future governmental regulation allows war profits.

Example II. Aver price of barite for 48 yr ending 1937 was $5.89 per ton on aver annual output of 123 000 ton. Minimum price (1897) was $2.24 on output of 26 000 ton. Aver 1937 was $6.25 on output of 356 000 ton. An engineer might therefore hesitate to recommend purchase of a barites mine unlikely to be able to maintain an output from best ore reserves, for say a year, at a gross cost of less than $2.24, or which would fail to make good profit on aver ore at $5.89 per ton, unless this or entirely similar property were in highly successful operation at the time. In view of the small production of barites, of the physical and chemical qualities required by the trade in this mineral, and of the high relative transport cost, due to its low value per ton, the engineer should also hesitate to make recommendation before determining detail character and location of available markets. The fact that 228 000 ton were sold at $9.39 per ton in 1920 might have speculative interest were a deposit under consideration of sufficiently large indicated tonnage to insure unusually long life.

14. OPERATING SCALE AND PROFITS, CAPITAL REQUIREMENTS, AMORTIZATION

Scale of operation is determined, on the one hand, by the minimum rate at which the property can be worked at a low enough cost per unit to insure a profit. On the other hand, the maximum scale of operation is limited by the assured mineral and probable ultimate mineral resources from the exploitation profits of which the capital, interest, and ultimate profit must be secured; also by the capital available for development and equipment, and by available markets. These factors being determined as far as practicable, and estimates made of operating costs on the proposed scale of operations, the profit per

unit is estimated by deducting the cost of production from the assumed market price or prices (Art 12, 13). The profit per unit, multiplied by the proposed annual production (on the usual basis of 300 days actual operation per year), gives the estimated annual OPERATING PROFIT (Sec 21).

Capital requirements. Estimates should first include the probable cost of prospecting the property sufficiently to give a safe amount of assured mineral (Art 11). That is, there should be ASSURED MINERAL RESERVE sufficient to produce profits that will more than return the capital required to put the property into successful operation. Assured reserve should be so large that its extraction at the proposed rate can not exhaust it in a reasonable maximum length of time necessary to prove and block out additional reserves.

Sufficient assured reserves being secured, there should be ample capital to develop them to an extent which will insure regular production at the desired output. Capital must also be provided for EQUIPMENT necessary for proper handling of output to the point of sale as raw material, concentrate, metallurgical product, or whatever form conditions require. Milling plant, smeltery, employes' houses, etc, must be considered, and capital provided therefor when necessary (Art 2). There must be sufficient OPERATING CAPITAL to carry stocks of general and commissary supplies, mineral in process of treatment, and finished output prior to sale and settlement. In absence of more exact data, 3 months' cost of output may be allowed for these purposes. In estimating total capital requirements, the cost of securing the capital (underwriters' commission, brokers' charges, and incidental financial expenses) should be included. Under favorable conditions, such as an ownership associated with strong financial interests, this may not exceed 10%. Promoters' profits are usually included in the cost of the property.

Amortization of capital is its return with interest, at or before the time of exhaustion or "death" of the property. If the profit is to be genuine, total operating profits must include an ultimate profit over and above amortization.

Amortization of equipment is usually effected by annual or monthly depreciation or write-off on equipment capital account, which amount is charged to operating expense; 10% of the gross cost is a not unusual annual allowance on ordinary mine, mill, or smelter plant. If this charge is maintained as a surplus accumulating at 4% compound interest, funds will be on hand in a little over 9 yr for complete replacement of equipment.

Table 5. Amortization

Annual div rate	Years life required to yield ..% interest, and to furnish annual instalments which, if reinvested at 4%, will return original investment at end of period (See also Sec 45)					
%	5%	6%	7%	8%	9%	10%
6	41.0	....	....	....	....	....
7	28.0	41.0	....	....	....	....
8	21.6	28.0	41.0	....	....	....
9	17.7	21.6	28.0	41.0	....	....
10	15.0	17.7	21.6	28.0	41.0	....
11	13.0	15.0	17.7	21.6	28.0	41.0
12	11.5	13.0	15.0	17.7	21.6	28.0
13	10.3	11.5	13.0	15.0	17.7	21.6
14	9.4	10.3	11.5	13.0	15.0	17.7
15	8.6	9.4	10.3	11.5	13.0	15.0
16	7.9	8.6	9.4	10.3	11.5	13.0
17	7.3	7.9	8.6	9.4	10.3	11.5
18	6.8	7.3	7.9	8.6	9.4	10.3
19	6.4	6.8	7.3	7.9	8.6	9.4
20	6.0	6.4	6.8	7.3	7.9	8.6
21	5.7	6.0	6.4	6.8	7.3	7.9
22	5.4	5.7	6.0	6.4	6.8	7.3
23	5.1	5.4	5.7	6.0	6.4	6.8
24	4.9	5.1	5.4	5.7	6.0	6.4
25	4.7	4.9	5.1	5.4	5.7	6.0
26	4.5	4.7	4.9	5.1	5.4	5.7
27	4.3	4.5	4.7	4.9	5.1	5.4
28	4.1	4.3	4.5	4.7	4.9	5.1
29	3.9	4.1	4.3	4.5	4.7	4.9
30	3.8	3.9	4.1	4.3	4.5	4.7

Amortization of entire property is usually effected by disbursement of dividends. It is then the stockholders' responsibility to reinvest a sufficient proportion of the dividends to amortize the investment cost. Nevertheless, the engineer must make allowance for amortization in drawing conclusions as to value of the property. In cases where sufficient mineral values have been determined, and the price at which the product will be sold can be forecasted with sufficient likelihood to justify, amortization may be calculated by use of Table 5 (3). In cases where other uncertainties are sufficiently eliminated to justify, such as those dependent on assumed fixed conditions in making tax assignments, it should be noted, and allowance made for the fact, that this table continues the higher risk rate of interest on the entire amount throughout the entire period, notwithstanding that the sinking fund, presumably invested safely, is steadily growing and throughout the second half period constitutes the bulk of the investment (25, 26).

Example. A thoroughly proved gold placer deposit in successful operation by dredging at rate of 1 000 000 cu yd per year, at cost of 6¢ per cu yd, contained 10 000 000 cu yd, averaging 13¢ per cu yd recovery value. Annual operating profit was therefore $70 000 and the life 10 yr. Table 5 shows that an income having a 10-yr life will yield 5% per yr (besides a sufficient amortization

charge, accumulated at 4%), when the investment is such a sum that the annual dividend is between 13 and 14% thereon (in column 1, opposite to "10.3 to 9.4 yr" in column 2, will be found "13 to 14%"). To secure 10% return the dividend rate must be between 18 and 19%. Dividing $70 000 by 13, 14, 18, and 19 respectively, shows that the property will yield 5% net, besides amortisation, if between $500 000 and $538 000 is paid for it, or about 10%, at a cost between $369 000 and $389 000.

In absence of assured mineral reserves, the whole question of value is one of inference, and amortization tables have little application. But, when part of the property's value lies in assured mineral, it is often instructive to carry out the amortization idea as far as possible on the assured reserves, and then note the years of work, tons of ore, etc. required beyond the reserves to return the valuation placed upon the property with appropriate profit.

Example. A copper mine, having assured reserves of 750 000 tons ore yielding, at 14¢ copper, $2 per ton profit, is operating at rate of 150 000 ton per annum, and its securities sell in open market at a price aggregating $2 000 000. At the annual profit of $300 000, the yield is 15%. Assured life, 5 yr. Table 5 shows that a 15% dividend rate requires 8.6 yr to amortize the principal, if 5% per annum return is to be had, or 15 yr, if the risk is such that a net 10% return is required. The $2 000 000 market valuation of the property therefore assumes a life beyond the assured reserves of between 3.6 and 10 yr, a sufficient rise in the aver price of copper above 14¢, or a combination of the two. The additional ore required, independent of change in price of copper, would be 540 000 to 1 500 000 tons. At an aver ore yield of 2 000 tons per ft of depth, these added tonnages would involve the continuation of the deposit with average strength for a further depth of 270 and 750 ft, respectively.

For determination of present value of well-developed mines, where total sales value of product, costs of extraction and marketing, are known, the probable life and investment risk may be estimated with considerable confidence (Sec 45).

15. VALUATION OF MINES HAVING NO ASSURED MINERAL, OR IN WHICH VALUE OF ASSURED MINERAL IS UNPROVED

To value mines of this kind is simply an attempt to discount the future, and is therefore almost entirely a matter of personal judgment. A great variety of method and conclusion, according to temperament and viewpoint of the engineer, must be expected. But the engineer should not make this an excuse for failing to give his client the benefit of his best judgment on this vital question, and statements in Art 12 as to prospective values and quantities apply here with equal force. All possible information should be secured regarding the property in question, neighboring properties and their relative location, the district in general, and results secured elsewhere from apparently similar properties. The strength and character of outcrop; extent of, and values shown by underground development and drill records or cores; apparent tendency of values to pinch or swell, to become enriched or impoverished, or to assume a simpler or more complex character with depth or with horizontal extent; likelihood of finding a feasible treatment process, and the total values per ton in the mineral mass, whether known to be recoverable or not, should all have weight in reaching an opinion.

General geological conditions should be considered, as to whether they are favorable to the deposition of quantity and value, or the reverse; and also respecting their similarity to those of successful properties elsewhere. As to geographical location, see Art 2.

All these points having been studied and digested, the engineer should allow himself to be influenced by his general opinion, or his instinctive feeling in the matter. Analysis of reasons for this opinion should be worked out and recorded, to enable the engineer to explain and defend his position to others and steady his own resolve during periods of doubt and uncertainty in subsequent developments. But an instinctive sense of conviction, rightly developed by exhaustive study and restrained by realization of its limitations, is of great value to the client, and should be cultivated by the engineer as a source of strength rather than suppressed as a weakness.

Example I. An enrichment of a highly acid rock (locally known as quartz-porphyry) was exposed by underground workings, near the center of a property, to the extent of about 500 000 tons of ore. Property included an area covered by the porphyry rock, about 0.75 mile wide by 1.5 miles long, generally iron-stained at the surface. Presence of value similar to that of the proved ore was revealed at nearly every point where shafts of a few hundred ft depth or less had been sunk. Bottom level of the principal workings and the main shaft sunk below it showed distinct tendency to impoverishment with depth, and the workings as a whole showed an enriched zone about 150 ft thick, lying beneath about the same thickness of leached overburden. Though a small, dark-colored basic dike was noted in one side of the underground workings and at the surface, study of this dike in connection with showings of value elsewhere on the property where no dike could be found, discouraged the theory that the dike was the origin of values. Altogether, the indications pointed to the existence of a secondary enrichment zone of fluctuating value at varying depths beneath the entire porphyry area. The apparently flat-lying position of the values quite near the surface suggested cheap prospecting by systematic drilling from the surface, and the rock being both hard and badly shattered, power churn-drilling promised to be best. Judging by the thickness and depth

beneath the overburden of the exposed enrichment zone, probably millions of tons further reserves could be proved. But, the assured ore was too poor for profitable operation by any process then known, and none of the outlying shafts promised that further development would yield materially richer ore. The further objection had to be met that those who were treating elsewhere richer deposits of same character doubted the possibility of making a success of so low a grade. Altogether the property could be given no definite value. But, success had been achieved elsewhere on even poorer ore (though differently constituted mineralogically) by large-scale work employing extensive equipment and large capital outlay. The promise of vast tonnage provable by drilling, at a fraction of a cent per ton, was therefore an attractive feature. If successful treatment could be devised, there would be ample justification for venturing the comparatively small amount of money necessary to prove the tonnage required to justify capitalization for large-scale equipment. Balancing one consideration with another, the engineer was finally convinced that, in spite of great uncertainties, the chance of large profit was sufficiently good to make the property a highly attractive venture. The client was so advised, but the large capital which would ultimately be required was not available. The engineer, therefore, advised the client to devote available funds to investigating processes, to proving the practicability of cheap tonnage development by drilling, to meeting the considerable expense of maintenance of the property, to interesting heavy financial interests, and to maintaining a cash reserve which would give such independence of position as would secure satisfactory terms when final negotiations for the transfer of the property to strong hands became desirable. Some 8 years elapsed between the forming of a small company on this basis, and the beginning of regular dividends. But in half that time the client had realized many times his investment from sale of part of the securities received on transfer of the property to large financial interests, and, the property becoming a marked success, his remaining holdings were eventually valued at a much greater amount.

Example II. A gold placer property on a small stream was limited at each end across the stream by property lines, while outcrops and drill holes on the side hills showed that the bedrock cut off the gravel on each side. Records of churn-drill holes, made by an experienced placer prospector, showed yardage and values which should leave an operating profit of several times the cost of a dredging equipment. But, as no trained engineer was responsible for the drilling, the records were of uncertain value. Further, there were too few holes, and they had been placed too irregularly to be conclusive, even if correctly drilled and recorded. Value calculations could not therefore be considered conclusive, but indicative only. The likelihood that the ground was valuable for dredging was emphasized, however, by the fact that the valley below had been successfully dredged, while drift-mining had been done above. As the company had contracted with a leading builder for a well-designed dredge, which was nearing completion, the most direct and cheapest mode of checking the drill records was to await beginning of dredging operations. Meanwhile the property had no proved value, and the engineer felt the enterprise could scarcely be considered a good gamble, even at the cost of the dredge. The client (owner of a minority interest) was therefore advised to consider further provision of funds only on the basis of a very limited valuation of the property, and then only as a pure speculation and in case satisfactory assurance could be given of proper financing and operating of the company. These conditions could not be fulfilled and, as the engineer had advised was probable, a receivership and ultimate sale of the property took place. To avoid losing his original investment the client bid in the property at a fraction of the cost of the dredge. Sufficient value was ultimately proved to cover the client's outlay, plus a reasonable profit; but only a trifle of the original hopes of the enterprise were realized, and only those willing to contribute to the purchase at the receiver's sale avoided total loss.

16. CONDUCT OF EXAMINATIONS AND OUTFIT REQUIRED

Preliminaries. All information furnished by the client (which may range from a mere statement respecting location and supposed value of the property, to reports and maps by skilled engineers) is reviewed, to gain an idea of the scope and character of the examination, and the time and facilities required to make it. The extent of development of the property, especially that of the exposure of mineral values, largely determines the time and equipment for sampling, and the importance of sampling results in reaching a valuation.

Upon the location and accessibility from the engineer's headquarters depend the time and expense required to reach and return from the property. Technical periodicals and transactions, and books on economic geology, sometimes yield useful information, through their own or special mining indices. The relations of the client to the property are usually such that free access is taken for granted; if not, the limitations must be understood and the scope of the promised report restricted accordingly. If any considerable outlay is required to gain access to underground workings, the question of providing for this cost should be understood.

Ideal examination, regardless of time or money, would be one made wholly by the engineer personally, who would thereby become thoroughly familiar with all conditions affecting the problem. But reports must be made within a reasonable time, and are generally needed as quickly as possible. Moreover, a competent engineer is too expensive a man to devote time to unimportant details, and some assistance is always required, if only for clearing out old workings. In case of a large, operating property, the whole mineral exposure of which must be sampled, and its records and operations reviewed, a corps of assistants is necessary for preparing a reliable report with reasonable outlay of time and money.

Assistants must be selected with great care. Their integrity must be beyond suspicion, the capacities of each well understood, and none should be taxed with responsibility that he is not well able to carry.

An assistant should be given work that is new to him only under supervision of an experienced superior. Bright young technical graduates make excellent raw material to train for sampling, surveying, drafting, and similar work. But an inexperienced graduate should never be sent into the field alone, except to gather preliminary information, his field work being checked by an experienced man before any important decision is reached.

Sampling. As this is usually the most tedious part of the examination it should be begun promptly; though not until enough information has been gained (preferably by inspecting the mineral exposures) to determine if the cost of thorough sampling is justified.

If only one trained assistant is required, he usually does the sampling, thus allowing the engineer to devote his time to more general investigations. For extensive sampling the assistant may himself require trained assistance, or at least unskilled help. The latter is generally obtainable at or near the property, though it is sometimes desirable to bring it in from the outside to lessen possibility of salting (Art 9). But the chief sampler can alone usually retake enough samples to detect fraud, thus making it generally feasible to employ local labor.

Expert accountants sometimes assist to a thorough understanding and analysis of the mine accounts, though the engineer's knowledge of bookkeeping may be sufficient for him to understand and check the local bookkeeper's explanations, who can, when so checked, make analyses or segregations.

Legal assistance is sometimes required, in dealing with complicated legal situations arising in connection with titles, leases, or contracts, though the explanations of the owner's legal adviser are usually sufficiently clear and final to make unnecessary the expense of independent advice. If the engineer be unable to grasp the full purport of any important item, he should secure a competent attorney whose interest in the matter is identical with his own (Art 2).

Estimation of time and expense for making an examination and report of an unknown property is difficult. There is great diversity in scope of investigations, and in state of development of properties. Also, an engineer frequently requires much time to bring his own experience into line with the conditions, and to get himself into such a definite and well-thought-out position that he can submit a reliable and convincing report.

In general, the examination of an unfamiliar property, and a report thereon requiring careful consideration, can rarely be made in less than 10 days' time. Hence, as a minimum fee, it is customary to charge 10 times the daily rate which the engineer's standing commands. As time spent in the engineer's own office is subject to the interruption of his routine work which he has the advantage of being able to keep in hand, many engineers count only half to three-quarters time for office work.

Contingent fees, dependent on or proportional to certain eventualities, though just and desirable in cases where the engineer's interests are identical with those of his client, are undesirable in examination and report work, because the engineer can not maintain the position of impartiality so important in making valuations. But, if a client unprepared to make direct payment offers a satisfactory contingent fee, the engineer may accept such fee provided his report shows that he is not disinterested (Art 1).

Specific features to be noted in examining different classes of properties: ANALYSIS OF SAMPLES for other than the principal value sought is usually limited to a few composite samples, or to special samples the appearance of which suggests that more complete analysis might yield useful information. COAL. Volatile, fixed, and total carbon, moisture, heat value, coking quality, sulphur, and any impurities objectionable to probable metallurgical uses, should be determined (Sec 30, Art 15). Presence or absence of gas in the deposit, character of roof and floor rocks, water problems, character of overburden (particularly quicksands) through which shafts may have to be sunk, and the practicability of mining several seams simultaneously, if present, must be studied in their bearing on operation of the property. IRON AND MANGANESE. Determinations should be made of Fe, Mn, S, P (for Bessemer ore, P should not exceed 0.1% of the quantity of Fe present), SiO_2, and Al_2O_3; also, if present in any quantity, moisture, CO_2, CaO, MgO, Ti, Cu, Zn, etc. OTHER METALS. Precious metals, Cu, Zn, Pb, Sn, Ni, Co, Fe, Mn, As, Sb, S, SiO_2, CaO, MgO, and Al_2O_3 should be determined according to circumstances. If several economic metals are present, the possibility and cost of separation by concentration or smelting must be considered. Penalties imposed and bonuses offered by the smeltery where the ore will be treated usually regulate the necessity for earthy, ferrous, and zinc determinations. PRECIOUS METALS IN SOLID-ROCK FORMATION. Au, Ag, and the proportion of value recoverable by free milling, concentration, cyaniding, or direct smelting are the principal determinations. Many of the determinations listed above may be made for their indicative value regarding probable method of recovery (Sec 30, 31, 32, 33). PLACER DEPOSITS. Au and Pt fineness and the assay of concentrates of Sn or monazite should be determined according to character of deposit. Amount and head of water supply; elevation and situation of available dump room; character, depth, and slope of bedrock; quantity and disposition of boulders or buried timber, cost of removal of surface vegetation, climate and governmental restrictions, must all receive especial consideration in examining placers (Art 7, 9).

Outfit required varies greatly, but the following suggestions may be found useful:

General

Pocket aneroid barometer
Brunton pocket compass
50-ft "metallic" tape in leather case
Pocket lens
Prospector's pick
Telegraphic code
Drawing instruments, with pencils, paper, india and red ink and tracing linen
"Art gum," ink, and pencil erasers
Soft and indelible pencils
Red, blue, and white crayon pencils
Fountain pen, and ink in traveling bottle
Loose-leaf note books (quadrille-ruled sheets)
Ruled stationery for drafting reports
Triangular scale of 6 scales, 1 in = 20 to 60 ft
1-ft steel folding rule
Pocket adding machine
Photographic camera

For Sampling

6 by 12-in and 14 by 20-in sampling sacks
Twine for tying above
Sample envelopes, with flexible metal-strip mouth
Seal and sealing wax
6 by 6-ft canvas, ticking, or enameled cloth (for moisture samples) sampling sheet
Jones sample-splitting outfit
Large iron mortar with pestle and canvas cover, or collapsible mortar outfit for coal (Fig 3)
Galvanized iron screw-top cans or glass jars for moisture samples
Adhesive tape for above
Leather mail sack, with lock and keys
Whisk, clothes, or scrub brush
Single and double-hand hammers
Miner's pick
3/4-in moils, 6 to 18 in, diamond points
Gold pan and vanning plaque

Additions for Placer Sampling

100-ft steel tape on reel
Level and rod
Carpenter's pencils
Magnet
1 by 3/8-in bottles, with corks, for gold colors
Portable gold balance, sensitive to 0.5 mg
Mercury
Annealing cups and nitric acid
Alcohol and lamp
Gummed labels
Short-handle shovel
Galvanized-iron panning tubs
Retort stand with alcohol stove

17. WRITING OF REPORTS

Report forms. The form of a report depends largely upon circumstances (Art 1). One form is that in which an introduction, consisting of the mere formalities of submission, is followed by a summary and brief statement of conclusions, followed in turn by the details of facts, conclusions, and advice. This form places the gist of the situation immediately before the reader. A busy man gets the information he wants in the minimum time, and the interest of any reader is more promptly aroused than when the entire report must be read before its conclusions are revealed.

The older still used form leads up through description to conclusions. This is advantageous in a report which can not be brought to an unequivocal conclusion, and where a correct impression can be given only by the atmosphere created by detailed description. For convenience of reference, the subject matter is divided by headings, usually capitalized, underscored and centered on the line, with sub-headings at the side, also capitalized and underscored. Below is a simple form of introduction, giving desirable information as to date and duration of the examination. This is followed by headings which have been found useful under various conditions. These, and the article and paragraph headings of this section of the book, furnish suggestions for deciding on the best form for a specific case.

..............................Esq., President, New York, June...., 19...
..........................Copper Co.,
......................Broadway, New York

Dear Sir:

Pursuant to your request, I have carefully examined the............property, spending ....days, from..............to.................., on the ground, and herewith submit my report thereon.

Summary and conclusion.

Location of the property, names and positions of claims or of different blocks, areas, and titles.

History, including shipments, profits, and difficulties met.

Economic geology, topography, vegetation, climate, transportation facilities, claim and topographical maps.

Governmental conditions (when below standard), taxes, laws and illegitimate influences.

Sampling methods, checks, limits of accuracy, and assay maps.

Tests on character and extractable value of the mineral.

Assured mineral, calculations, and amount.

Prospects and limitations of the property.

Present equipment (photographs), mine development, underground maps, methods of working and of treatment.

Supply and cost of labor, water, power, fuel, timber, explosives and miscellaneous materials.

Operating costs, general conditions, and profits. Financial condition of the enterprise.

Market (for mine product) conditions, and assumptions.

Advice, respecting development, scale of operations, equipment, and capital required;

Estimated returns. Acknowledgments (of courtesies received and to assistants employed).

18. ESTIMATING STANDING TIMBER (By Prof C. H. Burnside)

Estimates of timber tracts are based on amount of standing timber, cost of logging and of transportation, and value of land after logging. Depending on the degree of accuracy required in an estimate of STAND in a belt of timber, the work of the CRUISER, or timber estimator, varies from a rapid preliminary inspection to an exhaustive and detailed tree count. PRELIMINARY COUNT is made by running once through each 40 acres ($^1/_{16}$ section), called a "forty," to size up the timber in a general way or estimate number of trees in any selected strip. TOTAL COUNT is made by running through each forty 8 to 12 times, and actually counting trees on either side of a compass line for a distance sufficient to cover the whole area (23).

Cost of "cruising" is from 10¢ to $1 per acre, depending on accuracy of estimate.

Rules observed by timber-cruisers are generally crude, varying with the individual, and with the proportions of different kinds of timber. Thus, in southern U S, much timber runs only 3 to 5 M ft per acre. It is fairly uniform in stand and without heavy undergrowth, making it possible for a cruiser to go through on horseback, counting an acre at intervals, taking the aver result, and multiplying by number of acres in the tract for total count. In the northwestern U S, much timber land is a succession of ridges and canyons; undergrowth is heavy, making cruising difficult; timber is not uniform in stand (10-acre spots carrying little timber may adjoin an acre having 300 M ft or more); also the trees vary greatly in size, scaling from 5 to 25 M ft per tree. Under these conditions, a cruiser must count every tree, or as nearly so as possible.

Methods of estimating the stand are: (*a*) Counting trees in selected strips or circles, computing the contents of the aver tree and multiplying by number of trees to obtain stand per area covered. To obtain total stand per 30 acres, multiply the stand per area covered by a factor depending on percentage of acreage actually estimated. (*b*) Determining contents of each individual tree in selected strips or circles, and multiplying the total by a factor as in method (*a*). (*c*) Roughly estimating entire acreage by much criss-crossing of an area between compass (practiced mostly by men of long experience).

(*a*) and (*b*) differ but slightly; in the one case the number of trees is multiplied by an aver tree. in the other the trees are estimated separately and added for the total. There is little difference between circle and strip methods of counting, but strip method is always used for total tree count.

Volume of individual trees is found from a volume table, based on an ideal tree of uniform taper. The table gives board measure contents of trees ranging from 12 to 90 in

Volume Table, Showing Contents of Logs of Uniform Taper (23)

	Butt diam, 4 to 6 ft above ground, in	Diam at top of 32-ft log or fractional log, in							No of logs	Aver taper, in	Volume, bd ft	
		1st	2d	3d	3 1/2	4th	4 1/2	5th			From scale of individual logs	Volumes for same aver taper
Hemlock	23	18	16	11	...	...	...	...	3	4	879	889
	23	20	17	13	11	...	...	...	3 1/2	3 1/2	1 180	1 082
	23	19	17	15	...	12	...	...	4	3	1 294	1 281
	23	20	18	15	...	14	12	...	4 1/2	2 1/2	1 563	1 596
Douglas Fir	21	18	15	11	...	...	...	...	3	3 1/3	831	789
	21	20	18	16	...	15	12	...	4	2	1 657	1 406
	40	29	27	24	...	20	...	...	4 1/2	5	3 656	4 596
	44	37	34	31	...	27	...	19	5	5	6 650	6 476
	50	44	38	34	...	29	...	24	5	5	8 762	9 398
		Diam at top of 16-ft logs										
		1st	2d	3d	4th	5th	6th	7th				
Yellow Pine	18	16	15	14	11	...	...	...	4	1 3/4	475	451
	18	17	16	14	13	11	10	...	6	1 2/3	670	556
	18	17	16	15	14	12	10	...	6	1 2/3	727	556
	32	30	29	28	25	23	...	...	5	2	2 662	2 466
	32	31	30	28	25	23	20	15	7	2 1/2	3 164	2 537
	36	35	33	33	32	29	25	...	6	2	4 298	3 718
	36	34	34	32	31	30	27	23	7	2	4 699	4 059
	36	33	32	31	31	31	27	26	8	2	4 908	4 335

or more in butt diam, and carrying a different number of logs and a varying taper for each diam.

Diam of standing trees is determined by a DIAMETER TAPE, length and taper being found by measurement. Due to natural variations, the sum of the scale of the individual logs does not usually correspond to the volume given in the table for the aver taper of the tree; a tree rarely has same taper for each log; trees of same diam vary greatly in length and taper; and top and butt logs usually have a much heavier taper than those in middle of the tree. Estimator must decide upon taper and number of logs in a tree.

Details. A "two times run" through a "forty" is made at intervals of 2 tallies or 660 ft, estimating or counting trees for a distance of 25 paces (about 66 ft) on each side of the compass line, and multiplying the amount of timber by 5 for the total. A "four times run" is made at intervals of 1 tally or 330 ft and multiplying amount of timber by 2.5 for the total. In an "eight times run" (intervals of 164 ft), the estimator sends a survey crew ahead to locate corners, set tally stakes along the section to keep the compass-man in alinement, and do the pacing, while he counts trees.

If two or more runs through a forty are made, the estimator gathers data for a map, covering such topographic details as will give general characteristics of the whole tract. Elevations for contours are based on barom readings. As cost of logging a tract is as important to know as the amount of timber on it, a good map is essential for solving logging problems.

A cruiser's report may vary from a simple sketch of the section divided into forties, and the total amount in bd ft inserted on each, to an elaborate form reporting for each forty the number of trees of each species, their total contents, aver length, diam, and contents per tree, and percentages of different grades, with brief comments on the different species and logging conditions, all compiled from field notes. For large tracts, the detailed reports on forties or sections are usually supplemented by a descriptive report on the tract as a whole, describing character of timber, fire hazard, topography and accessibility of the country, general plan of logging and facilities for transportation.

A buyer wishing general information as to character of a stand of timber does not care to spend much money, and so sends an estimator on a preliminary cruise. If the report is satisfactory, a more detailed estimate is made. Where stumpage is $3 or $4, or higher, the buyer will probably want a complete tree count. An owner wishing to put his timber on the market needs an estimate and description on which to base his price.

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SECTION 26

AERIAL TRAMWAYS AND CABLEWAYS

BY

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MINING ENGINEER

Note.—Numbers in parentheses in text refer to Bibliography at end of this Section.

AERIAL TRAMWAYS AND CABLEWAYS

Definitions. An AERIAL TRAMWAY transports loads in carriers suspended from wire ropes forming the tracks, between fixed points, usually a long distance apart. Tramways are divided into 3 classes: (1) bi-cable (formerly called double rope), (*a*) continuous, (*b*) reversible. (2) twin-cable; (*a*) continuous; (*b*) reversible. (3) mono-cable (formerly called single-rope), continuous.

On continuous tramways, a series of loaded carriers travel in one direction on a track cable, and empty carriers return in the other direction. On reversible tramways, one carrier travels back and forth on a cable. Bi-cable tramways have a fixed track cable, along which the carriers are hauled by a traction rope. Twin-cable tramways are similar, except that carriers run on a pair of track cables. Mono-cable tramways have a single running rope to support and move the carriers. A CABLEWAY transports a load for a short distance, in a single carrier traveling back and forth on a single cable, or on multiple parallel cables, a hoisting operation being combined with the transfer of the load; the operation is intermittent.

GENERAL FORMULAS

1. CABLE FORMULAS

Catenary *vs* parabola. Theoretical curve taken by a cable of uniform weight when suspended from its ends is a catenary; it is the curve of the funicular polygon formed by a series of equal weights hung equidistant along the ARC. Its equation is complicated and difficult to apply.

A parabola is the curve of the funicular polygon formed by a series of equal weights hung equidistant along the CHORD between 2 supporting points. It nearly coincides with the catenary for the flat arcs occurring in tramways, its equations are simple, and results by its use are practically correct.

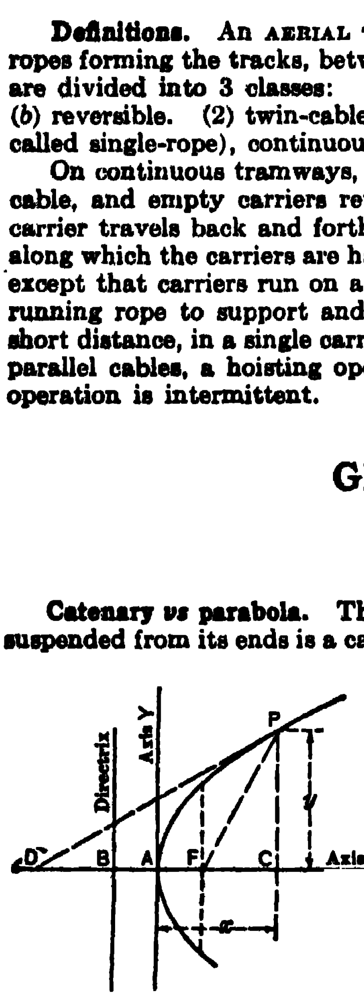


Fig 1. Parabola with Rectangular Axes

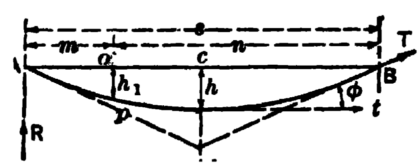


Fig 2. pty Cal Span

EQUATION OF PARABOLA (Fig 1) is: $y^2 = 2px$ (1)

Empty cable; level spans. In Fig 2, let: s = span, ft; h = deflection at center, ft; h_1 = deflection at any point of curve, as p, ft; t = tension in cable at center, lb = horizontal component of tension at any point; T = tension in cable at any point, lb; w = weight of cable per ft, lb; W = total weight of cable in span, lb = ws; R = total vertical reaction at a tower; for cable only, it is $\frac{1}{2} W = \frac{1}{2} ws$; m and n = sections into which point p divides span; ϕ = inclination of tangent to the curve at any point, as B.

Deflection at center. Take moments about point c of forces to left of center, then: $\frac{1}{2} W \frac{1}{2} s - \frac{1}{2} W \frac{1}{4} s = th$, or $\frac{1}{8} Ws = th$. But $W = ws$, hence, $h = ws^2 \div 8t$ (2)

Deflection at any point, as p. Take moments about point o of forces to left of it, then: $\frac{1}{2} wsm - wm \frac{1}{2} m = h_1 t$, but $wsm = wm(m + n)$, hence, $h_1 = wmn \div 2t$ (3)

Tension in cable at any point is: $T = t \div \cos\phi$ (4)

It is also equal to tension at any other point + or − the product of vert distance

between the 2 points and the wt of cable per ft. Thus, if V = vert distance between points B and p, the tension at p is: $T' = T - wV$ (5)

Bib 10a gives many cable formulas. Simplified working equations may be obtained by substituting numerical values for w and t in Eq 2 and 3; for with any one kind of cable w and t bear fairly constant ratio to each other. Thus, with locked-coil track cables, the wt of all sizes is practically 3.7 lb per ft per sq in of cross-section. The breaking strength of cast-steel grades is 121 000 lb per sq in, and with factor of safety of 3.5, the working tension is 34 600 lb per sq in. These values, substituted in Eq 2, give:

$$h = \frac{ws^2}{8t} = \frac{3.7s^2}{8 \times 34\,600} = 0.000\,012s^2 \qquad (6)$$

Table 1 gives wt and strength of cables used for track; by substituting the values for different cables in Eq 2 and subsequent formulas, a simplified equation suitable for any case is obtained.

Table 1. Weight and Strength of Track Cables

Diam, in	Locked-coil cables					Smooth-coil cables				
	Wt, lb per ft	Cast steel		Special steel		Wt, lb per ft	Cast steel		Plow steel	
		Breaking strength, lb	Working strength, lb	Breaking strength, lb	Working strength, lb		Breaking strength, lb	Working strength, lb	Breaking strength, lb	Working strength, lb
1/2	......	......	......	......	......	0.55	25 200	5 600	30 600	6 800
5/8	......	......	......	......	......	0.86	38 400	8 550	44 600	9 900
3/4	1.41	50 000	14 300	58 000	16 600	1.24	55 200	12 300	65 000	14 450
7/8	1.92	64 000	18 300	81 000	23 300	1.69	75 200	16 700	88 800	19 700
1	2.50	84 000	24 000	108 000	30 800	2.20	98 400	21 850	116 000	25 800
1 1/8	3.16	108 000	30 800	132 000	37 700	2.70	120 000	26 650	141 400	31 400
1 1/4	3.91	130 000	37 100	162 000	46 300	3.23	143 600	31 900	169 200	37 600
1 3/8	4.73	156 000	44 600	200 000	57 200	4.01	177 600	39 400	210 000	46 700
1 1/2	5.63	186 000	53 200	241 000	68 900	4.88	216 800	48 000	255 000	56 600
1 5/8	6.60	216 000	61 700	280 000	80 000	5.63	248 000	55 000	292 000	64 950
1 3/4	7.66	250 000	71 500	330 000	94 200	6.59	291 600	64 700	342 000	76 000
1 7/8	8.79	276 000	78 800	375 000	107 000	7.28	322 000	71 500	378 000	84 000
2	10.00	316 000	90 300	430 000	123 000	8.40	370 000	82 200	436 000	97 000
2 1/4	12.50	......	......	560 000	160 000	10.36	466 000	103 600	532 000	118 000
2 1/2	15.20	......	......	690 000	197 000	13.10	570 000	126 700	670 000	149 000
2 3/4	18.30	......	......	840 000	240 000	......	......	......	......	......
3	22.20	......	......	1 000 000	286 000	......	......	......	......	......

The above weights and breaking strengths are given by Amer Steel and Wire Co (Bib 10a). The working strengths were calculated by author, using safety factor of 3.5 for locked-coil cable, and 4.5 for smooth-coil; these low factors are reasonable for reliable material like steel wire and where working stresses are nearly all tensile and can be determined with a high degree of accuracy. On tramways, it is customary to apply tension by means of weights, and hence overloading is improbable; also the cables are elastic and can absorb occasional shocks without injury. Where the tramway or cableway has a single span and track cable is anchored at both ends, there is a possibility of overloading; hence, it is advisable to increase the safety factors, but not above 4.5 and 6 respectively, for locked- and smooth-coil. On reversible tramways with a single heavy carrier, the cable can be treated as though counterweighted, even if anchored at both ends, because the slack will shift as the bucket travels. Track cables are also called track strand, and smooth-coil type, round-wire track strand. For aerial tramways, track cables have not exceeded 2 in diam (Art 7); for cableways all sizes are used (Art 23–27).

Slope of tangent to curve of empty cable. The tangent at end of span, when chord is horizontal, has a slope (Fig 3) of $\tan\phi = 2h \div \frac{1}{2}s = 4h \div s$ (7)

Substituting value of h from Eq 2: $\tan\phi = ws \div 2t$ (8)

but $ws \div 2$ is the reaction R, hence: $\tan\phi = R \div t$ (9)

In any one case, w and t are constant; hence Eq 8 can be simplified by substituting numerical values. Thus, for cast-steel locked-coil cable, w = 3.7 lb and t = 34 600 lb, giving $\tan\phi = 0.000054\,s$.

Concentrated loads. Let Fig 4 be the funicular polygon resulting from 3 concentrated loads, g_1, g_2, g_3, and a, b and c be their respective distances from right-hand end. Then the reaction at left-hand end is: $r_1 = \frac{g_1a}{s} + \frac{g_2b}{s} + \frac{g_3c}{s}$.

Taking moments about o gives: $r_1m = h_2t$, or $h_2 = r_1m \div t$ (10)

If wt of cable be included, moments about point o are:

$$R_1m - \tfrac{1}{2}\,wm^2 = Ht, \quad \text{or} \quad H = \frac{R_1m - \tfrac{1}{2}\,wm^2}{t} \tag{11}$$

In Eq 11, R_1 is reaction at left end, when both cable and concentrated loads are considered, and equals $\frac{g_1a}{s} + \frac{g_2b}{s} + \frac{g_3c}{s} + \tfrac{1}{2}\,ws$; and H is deflection at point o. Hence, deflection at any point for any loading equals moment, M, of all vert forces on one side of o, divided by the horiz tension, or $H = M \div t$ (12)

Experience shows that in spans carrying 3 or more equal loads, uniformly spaced, deflection at any point is practically the same as for a uniformly loaded cable of which the wt per ft

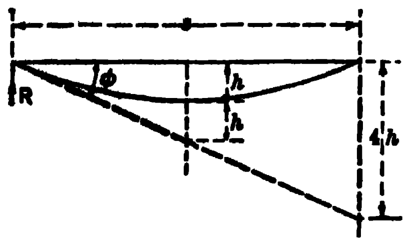


Fig 3. Tangent to an Empty Cable

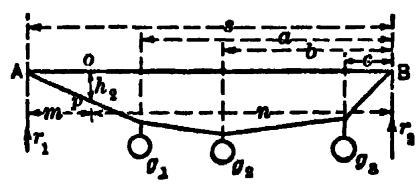


Fig 4. Funicular Polygon Due to Loads Only

is equal to the wt of cable plus that of one load divided by the spacing in ft. If value of R_1 be substituted in Eq 11:

$$H = \frac{R_1m - \tfrac{1}{2}\,wm^2}{t} = \frac{r_1m + \tfrac{1}{2}\,wsm - \tfrac{1}{2}\,wm^2}{t} = \frac{r_1m}{t} + \frac{\tfrac{1}{2}\,wm}{t}\,(s - m)$$

but $(s - m) = n$, whence $H = \frac{r_1m}{t} + \frac{wmn}{2t}$. Also $\frac{r_1m}{t} = h_2 =$ deflection at p due to concentrated loads (Eq 10), and $\frac{wmn}{2\,t} = h_1 =$ deflection at p due to cable (Eq 3). Hence, total deflection at any point is the sum of the deflection due to cable alone and that due to load alone.

Deflection due to one concentrated load. Three cases commonly occur: One load AT ANY POINT. Deflection at that point, by Eq 11, is:

$$H = (ws + 2\,g)\,\frac{mn}{2\,st} \tag{13}$$

One load AT CENTER. Deflection at center is found from Eq 13, by making $m = n = \tfrac{1}{2}\,s$, whence:

$$H = (ws + 2\,g)\,\frac{s}{8\,t} \tag{14}$$

One load AT DISTANCE y from one end. Deflection at point distant m from same end is found by applying Eq 11. The reaction R at opposite end of span for 1 load plus cable is $gy \div s + 0.5\,ws$. Taking moments at R gives: $th = Rn = 0.5\,wn^2$. Substituting above value of R, $H = (wms + 2gy)n \div 2\,st$.

$$H = (wms + 2\,gy)\,\frac{n}{2\,st} \tag{15}$$

Inclined Spans. Parabola with inclined axes. Let Fig 5 be such a parabola; then by analytical geometry:

$$y_1^2 = \frac{2\,p}{\sin^2\beta}\,x_1 \tag{16}$$

Empty cable. Let Fig 6 represent such a case, l being length of chord. Applying principle of moments:

Deflection at center, $$h = \frac{wls}{8t} = \frac{ws^2}{8t\cos\alpha} \qquad (17)$$

Deflection at any point, $$h_1 = \frac{wmn}{2t\cos\alpha} \qquad (18)$$

Cable carrying concentrated loads; chord inclined. Total deflection at any point is the sum of deflection due to cable alone and that due to loads alone. Deflection due to cable alone is given above. Deflection at any point due to loads alone equals moment

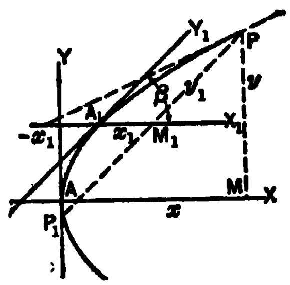


Fig 5. Parabola with Oblique Axes

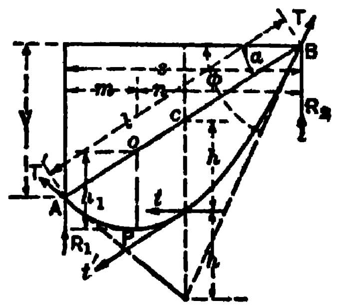


Fig 6. Empty Cable on an Inclined Span

of vert forces due to loads on one side of that point, divided by horiz tension (similar to Eq 10), thus:

Load at any point of an inclined span. Deflection at load is:

$$H = (wl + 2g)\frac{mn}{2ts} \qquad (19)$$

Load at center of an inclined span. Deflection at load is:

$$H = (wl + 2g)\frac{s}{8t} \qquad (20)$$

Load at distance y from one end of an inclined span. Deflection at a point distant m from same end, with moments taken about opposite end (see analysis for Eq 15):

$$H = (wml + 2gy)\frac{n}{2ts} \qquad (21)$$

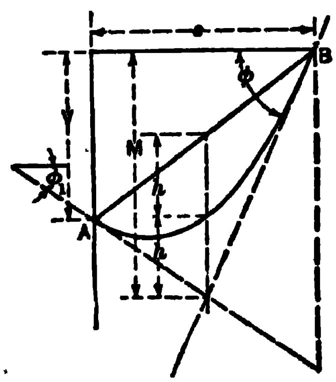


Fig 7. Tangent to an Empty Cable

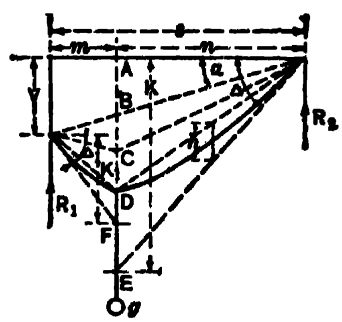


Fig 8. Tangent to a Loaded Cable

Slope of tangent to inclined empty cable. Let Fig 7 be inclined span. Then slopes of tangents to the two ends are:

$$\text{At } B,\ \tan\phi = \frac{M}{{}^1/_2\,s} = \frac{4h + V}{s} \qquad \text{At } A,\ \tan\phi_1 = \frac{M - V}{{}^1/_2\,s} = \frac{4h - V}{s}$$

Hence, slope of any tangent is: $\tan\phi = \dfrac{4h \pm V}{s}$, in which V = vert distance between ends of span, ft. (22)

Loaded cable. Let Fig 8 be an inclined span with concentrated load at some point. Slopes of tangents to the two ends are: $\tan\Delta = K \div n$ and $\tan\Delta_1 = K_1 \div m$. But K and K_1 are made up of a series of known parts. Thus, $K = AB + BC + CD + DE$, and

$$AB = \frac{V}{s}n;\quad BC = \frac{gmn}{st};\quad CD = \frac{wmn}{2t\cos\alpha};\quad DE = 4h = \frac{4wn^2}{8t\cos\alpha}$$

Substituting these in above, and simplifying:

$$\left.\begin{aligned} \tan\Delta &= \frac{gm}{st} + \frac{ws}{2\,t\cos\alpha} + \frac{V}{s} \\ \tan\Delta_1 &= \frac{gn}{st} + \frac{ws}{2\,t\cos\alpha} - \frac{V}{s} \end{aligned}\right\} \tag{23}$$

Length of parabolic curve. The length of a symmetrical parabolic curve, L, when the rise is small, is closely approximated by equation (Weisbach's "Theoretical Mechanics," trans by E. B. Coxe, p 299)

$$L = s + \frac{8h^2}{3s}, \quad \text{or} \quad h^2 = \frac{3}{8}s(L - s) \tag{24}$$

If chord be inclined, the length, for taut curves common in tramway practice, may be taken as equivalent to a symmetrical parabola having same chord length and same deflection *normal* to the chord. The normal deflection is nearly equal to: vert deflection $\times \cos\alpha$. Bib 10a treats of length of curve when stretch of cable is considered.

Horiz tension (t). In tramway construction, cables are stretched by tension applied to one end of span (usually by a weight), equal to working tension of cable. This tension is used in formulas for the value of t; it makes the actual horiz tension in cables a little less than that used in the calculations, and hence the sag will be greater than calculated. Owing to flatness of the curves, this error is not serious, when the chord is at a low angle. But on inclined spans it may make considerable difference; the error can be partly compensated by calling the applied tension t', and assuming that its direction is parallel to inclined chord; then the approx horiz tension will be $t = t' \cos\alpha$.

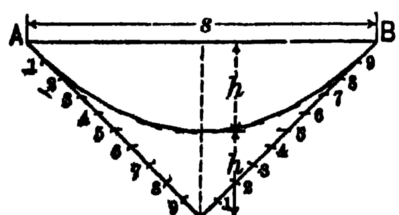


Fig 9. Plotting a Parabola, Axes Rectangular

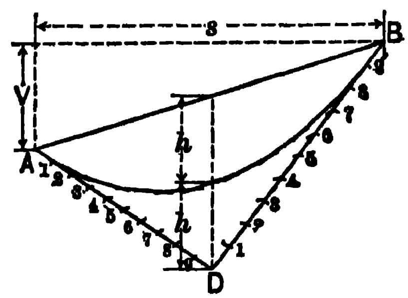


Fig 10. Plotting a Parabola, Axes Oblique

Plotting of parabolic curves may be done by points or by tangents. BY POINTS: Series of offsets is calculated by Eq 1 or 3, if chord be horis; or by Eq 16 or 18, if chord be inclined. These are plotted, and a curve is passed through their lower ends. BY TANGENTS: Chord of span and center deflection are plotted to desired scale, as in Fig 9 or 10 (same method applying whether chord is horis or inclined); deflection is prolonged to a total length of 2 h; lines drawn from ends of chord to this point are tangents to the curve. These tangents are then divided into same number of equal parts, numbered as shown. Lines are next drawn connecting corresponding numbers; these are tangents to the curve, and the curve can now be drawn tangent to them. By making a sufficient number of divisions in the two end tangents, the sides of the polygon formed by the tangents will be short and practically form the curve, as shown in Fig 9 and 10, where for clearness only the end tangents have been drawn.

2. OPERATING FORMULAS

Spacing of carriers. For a system operating a continuous series of carriers, let: t = time interval between carriers, sec; d = distance between carriers, ft; l = load in one carrier, lb; n = number of tons (2 000 lb) to be carried per hr; v = velocity of tramway, ft per min. Then, $n \times 2\,000 \div 3\,600$ = lb per sec to be carried by tramway, and this divided into l, gives time interval in sec, reducing to $t = 1.8\,l \div n$. (25)

Distance between carriers is, $d = tv \div 60$, reducing to $d = 0.03\,lv \div n$. (26)

Tension in traction rope. Tension in a rope holding any body at rest on an incline, neglecting friction, is $T = W \sin\alpha$. Friction of a body moving on an incline is, $F = f'W \cos\alpha$; in which: T = tension parallel to the plane, in same unit as W; W = total weight of bodies on the plane, preferably in lb; α = inclination of plane; f' = coeffic of rolling friction = 0.02 with plain bearings for sheaves and carriage wheels, and 0.01 for ball or roller bearings; F = total friction, in same unit as W.

When the body extends whole length of incline, as a rope lying on it, then total weight is $W = wL$, where: w = wt per ft, in same unit as W, preferably in lb; L = inclined length

of the plane, ft. Substituting this value of W in general equations for T and F above, gives:

$$T = wL \sin \alpha = wV \tag{27}$$

$$F = f'wL \cos \alpha = f'wH \tag{28}$$

in which: V = vert distance between ends of plane, ft; H = horiz length of plane, ft.

Tension in traction rope of a double-rope tramway, or in endless rope of a single-rope tramway, at its upper end, taking friction into account, will be: $T = wV \pm f'wH$ (29)

Sign of last term is $+$ when rope is being pulled up the incline and $-$ when it is moving down. If rope be empty, w is the wt per ft of rope alone; but if carriers be attached, w must be increased by wt of the carriers including their load, if any, uniformly distributed over whole length of incline, or else the effect of carriers must be calculated separately and added to value of T. Tension T is further increased by pull exerted by tension weight applied to the floating sheave; half the pull on this sheave being added to traction rope tension on each side of tramway.

3. TRANSMISSION OF POWER BY BAND DRIVES

This discussion is applicable to any type of band or rope drive, or to band brakes where tension is known.

In a band drive, tension on slack side must bear a certain relation to tension on taut side, to develop sufficient friction to prevent slipping. The formula for this relation, from analytical mechanics, is: $T = Se^{f\pi n}$ (30)

in which T = tension on taut side; S = tension on slack side, both in same unit, preferably lb; e = base of Napierian logarithms = 2.71828; π = 3.1416; f = coeffic of friction (values given in Table 2); n = number of half laps on drum (number of turns of 180° each).

The useful effect, or force transmitted by the drive, is the difference between tensions on the two sides, that is, $T - S$. If S be subtracted from both sides of Eq 30, this becomes, on reducing: $T - S = S\,(e^{f\pi n} - 1)$ (31)

which is an expression for force transmitted by a band drive, and is useful in determining S for a given drive when force to be transmitted is known. Table 3 gives values for $e^{f\pi n}$ for a number of combinations, for saving time in applying Eq 30 and 31.

Table 2. Coefficients of Friction, f, for Band Drives. By William Hewitt

Condition of surfaces	Iron on iron	Iron on wood	Iron on rubber and leather
Dry	0.170	0.235	0.495
Wet	0.085	0.170	0.400
Greasy	0.070	0.140	0.205

Table 3. Values of $e^{f\pi n}$. By William Hewitt

f	n = Number of half laps about sheaves or drums					
	1	2	3	4	5	6
0.070	1.246	1.552	1.934	2.410	3.003	3.741
0.085	1.306	1.706	2.228	2.910	3.801	4.964
0.100	1.369	1.875	2.566	3.514	4.810	6.586
0.120	1.458	2.125	3.099	4.518	6.586	9.602
0.130	1.504	2.263	3.405	5.122	7.706	11.593
0.140	1.552	2.410	3.741	5.808	9.017	13.998
0.150	1.602	2.566	4.111	6.586	10.551	16.902
0.170	1.706	2.910	4.964	8.467	14.445	24.641
0.200	1.875	3.514	6.586	12.346	23.140	43.376
0.205	1.904	3.626	6.904	13.146	25.031	47.663
0.235	2.092	4.378	9.160	19.166	40.100	83.902
0.250	2.193	4.810	10.551	23.140	50.637	111.318
0.265	2.299	5.286	12.153	27.941	64.239	147.693
0.300	2.566	6.586	16.902	43.376	111.318	285.680
0.350	3.001	9.017	27.077	81.307	244.152	733.145
0.400	3.514	12.346	43.376	152.405	535.488	1 849.140
0.410	3.626	13.146	47.663	172.814	626.577	2 271.775
0.450	4.111	16.902	69.487	285.680	1 174.480	4 828.510
0.495	4.716	22.425	106.194	502.881	2 381.400	....
0.500	4.810	23.140	111.318	535.488	2 575.940	....

Most calculations for tramway drives and brakes are based on the coeffic of friction for wet ropes, as this often occurs; neither rope nor brake should ever be greasy.

Grip sheave, as in Fig 29, has a series of jaws about its circumference which are tightened by the rope's press; these multiply the press between rope and sheave by the ratio of lever arms of the jaws, and hence proportionately increase the friction. For a grip sheave with iron or steel jaws, having leverage of 1 : 3, the value of $e^{f\pi n} = 2.228$, and where the leverage is 1 : $3^1/_2$, $e^{f\pi n} = 2.5$, in both cases with rope for which $f = 0.085$.

BI-CABLE TRAMWAYS

4. GENERAL CONSIDERATIONS

Elements. A bi-cable tramway consists of: 1, two track cables stretched at required tension; 2, an endless traction rope for moving the loads; 3, numerous carriers for the loads, each fitted with carriage or trolley to run on track cable and a clamp or grip for seizing traction rope; 4, a station at each end to operate or control the traction rope and provide places for loading and unloading carriers; 5, intermediate towers for supporting track cable and traction rope.

In operation, each load is placed in a carrier while standing on a track in loading terminal; the carrier is then attached to traction rope, and hauled to discharge terminal; there it is released from traction rope and contents are discharged. Empty carrier is then attached to traction rope on return side and is hauled back to loading terminal.

Capacity. Economic limits of capac for ordinary bi-cable tramways are approx 10 and 100 ton per hr. If less than 10 ton per hr, it is more economical to operate tramway fewer hours or on alternate days, than to reduce carrying capac.

Demand for tramways for traffic exceeding 100 tons per hr has recently developed special heavy equipment and structures, permitting construction of lines carrying up to 300 tons per hr.

Length. It is seldom wise to use bi-cable tramways for distances less than 1 000 ft, because terminal machinery costs as much as for a longer line and so makes first cost excessive. Max limit, under favorable conditions, is about 4 miles; the resulting 8 miles of traction rope is as long as can be operated successfully; the tension in traction rope becomes excessive; an exceptional case occurred on the Benguet tramway, which is 49 500 ft long (Art 17). For very long distances, a series of tandem tramways can be built, and the carriers switched from one to another; but each division is practically an independent tramway. Thus, a tramway at Chilecito, Argentine, 21 miles long (Art 17) has 8 divisions. In any tramway, the track cables are divided into sections of 3 000 to 5 000 ft each, one end of each section being anchored and tension being applied to the other end; each section of cable may be made up of as many pieces as desired, provided the couplings do not come close to tower saddles. As the carriers pass through anchorage and tension stations without interruption, lengths of cables limit neither operation nor possible length of line or division.

Loads. Net weights of 500 and 2 000 lb are the usual lower and upper limits for loads on ordinary tramways; below 500 lb there is no saving in cost of carriers, and for over 2 000 lb the wear on cables is great. The time interval between carriers is conveniently about $^1/_2$ min. Then 2 000 lb every $^1/_2$ min = 120 tons per hr, and is near the maximum capac of bi-cable tramways, using two-wheel carriages. In recent practice, larger and stronger track cables permit use of higher cable tensions, and 4-wheel carriages are used to keep the wheel load down. Net loads to 4 000 lb are carried where the necessity for large tonnage warrants the extra cost of plant (Art 6a).

5. TRAMWAY SURVEYS

Survey of proposed route, with surface elevations, must be made and a profile of center-line must be plotted before a tramway of any type can be designed. At summits, or wherever sudden changes in profile occur, elevations should be determined at intervals of 25 or 50 ft. The stations staked out may be numbered 1, 2, 3, etc, for distances of 100, 200, 300, etc, ft from the origin, as in R R surveying; intermediate distances are noted as + ft from preceding station. In very rough country, distances and elevations of important points on summits may be determined by stadia measurements, and the position of additional points on each side by taping and leveling from the stadia points. In any case, the data collected must be such that an accurate profile can be plotted.

Location line of tramway should generally be straight, as this is the shortest distance between terminals, and cheapest to build and operate. An exception occurs where tramway passes over a ridge, with a sharp summit in profile; then, for a large tonnage, or where the summit is high or slopes are steep, it is advisable to search for a lower pass, even if off the direct line; a second profile is made from loading terminal straight to this pass and thence straight to discharge terminal, making a horiz angle between the lines at the pass. It may be found, on laying out the two routes, that the one with the angle is the better. If the ground has a steep cross slope, say over 15° (or 1:4), cross-sections should be made at intervals, so as to describe the conditions.

Additional data. Besides profile of center-line, the survey should include plan of site of both terminals, with contours; and show position of all buildings to which tramway must be connected. These plans should cover an area considerably larger than the proposed terminal, say 50 ft on each side of center-line and 300 ft along it. Similar plans should be made of sites along the route where intermediate stations are likely to be located, in order to place all conditions before the designer. Relation of plans and cross-sections to points on profile must be given by reference marks on each, and permanent marks left on the ground so that the line can be located when construction is begun. It is important to preserve survey points near terminals, and if these are likely to be destroyed by excavation for the structure, other reference points must be set. Survey should be plotted by profile, cross-sections, and plans, before leaving the locality, to insure that all necessary data have been secured; but survey notes giving distance to, and elev of, each point should be sent in with the plots, so that the latter can be redrawn, if desired, without having to scale the surveyor's drawings.

6. DESIGN OF BI-CABLE TRAMWAYS

Proportioning of interrelated parts of tramways depends on many variables and conditions. Values in Table 4 apply to lines with easy grades, and are useful for preliminary layouts. For conditions which increase difficulty of design, see Art 6*a*.

Table 4. Relations of Cable Sizes, Loads, Etc, to Tonnage

Tons per hr	Diam of cast-steel Cables, in		Loads			Gage, ft	Carriage
	Loaded side	Empty side	Wt of material, lb	Wt of carrier, lb	Total load, lb		
5 to 10	1	7/8	500	300	800	6	Light
15 to 20	1 1/8	7/8	700	400	1 100	6	"
25 to 30	1 1/4	7/8	900	425	1 325	6 or 8*	Heavy
40 to 50	1 3/8	7/8	1 200	450	1 650	8	"
60 to 75	1 1/2	1	1 600	500	2 100	8	Extra heavy
80 to 100	1 5/8	1	2 000	550	2 550	8 or 10*	"

* Wide gages are needed for long lines, where driving machinery must be large.

These combinations are based upon the fact that the size of load for a given tonnage determines the number of carriers on the line; if load be small, the cost of rolling stock is increased, due to the number of carriers, since part of cost of each unit is constant; if load be large, the size of track cable increases in proportion to the individual weight carried, although the loads may be few in number. An empirical rule sometimes used to determine SIZE OF CAST-STEEL TRACK CABLE is to permit a gross wt of carrier = to 1 200 lb per sq in of cross sec of cable on small sizes, and increase permissible load to 1 400 lb per sq in of section on larger cables, assuming sectional area equals that of solid bar of same diam. The GAGE (distance apart of track cables) may be 6 ft for small carriers, but must be 8 ft for large ones, and may need to be 10 ft to suit bulky materials.

Tramway speeds must be high to carry large tonnage without increasing weight of moving parts, nor the stresses due to their weight, but when speed is too high it is difficult to handle the carriers at terminals. The practical speed is about 8 ft per sec, with 10 ft per sec as max with favorable conditions and careful designing.

Let Fig 11 be a typical profile on which is to be built a tramway to carry 60 000 lb broken ore per hr, weighing 100 lb per cu ft. By Table 4, the cables will be 1 1/4 and 7/8-in diam. Let carrier capac be 800 lb, and gage 8 ft. If speed be taken at 480 ft per min, the spacing of carriers will be (Eq 25 and 26): time interval $t = 1.8\ l + n = 1.8 \times 800 + 30 = 48$ sec, and distance apart $d = tv + 60 = 48 \times 480 + 60 = 384$ ft. An 8-cu ft bucket weighs 420 lb, making gross weight of one loaded carrier = 420 + 800 = 1 220 lb.

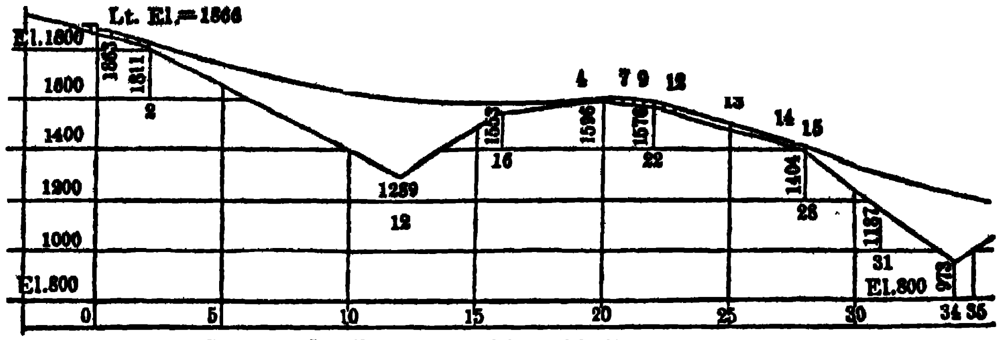


Fig 11. Profile of Typical Bi-cable Tramway

Locating the line. The profile is plotted to a natural scale, say 1 in = 40 ft on "Plate A" profile paper, 20 in wide, ruled 4 by 20 lines to the inch. If difference of elev between ends of line be great, several strips may be joined together to get required width. If line is too long to be operated as one division the junction points are determined and each division is laid out as a separate line; later, the junction stations are designed (Art 16).

By inspecting the profile, points where a rope stretched from one terminal to the other would first touch the ground can be seen; in Fig 11, these will be at Sta 21 and 50. The profile is then tacked to a long table, exposing one portion of the line, say from loading terminal to the first high point; and the terminal structure is plotted in outline to fit roughly the conditions imposed (Art 16). A pin is set in the terminal at point where cable leaves the horiz rail, another pin is set at the minimum height of tower at the ridge (say 12 ft if snow does not lie there over 3 ft deep); a silk thread is looped over one pin, passed around the other and led over the edge of the table where the spool's weight (a hitch having been taken in thread to prevent unwinding) draws it tight. If the profile paper be ruled in green, and red thread used, there will be no confusion with black ink or pencil lines that may be added. The thread represents a chord, from which the cable curve can be plotted by laying off deflections at as many points as necessary. If it develops that the chord is not in best place, it is shifted by moving the pins. It can be subdivided into shorter chords by holding it with intermediate pins. Tramways are usually laid out for the heavy cable; the light one carries only the empty carriers and will fit same supports as the loaded one, though sometimes it may be necessary to lay it out for separate study.

Towers. The location and height of towers must be such that: *a*, track cable will lie firmly in the saddles under all conditions of loading; *b*, angle made by cable over any saddle at a crest will not be excessive. These requirements are expressed by EMPIRICAL RULES: 1. The tops of towers are brought up to a CONSTRUCTION CURVE, the tension of which is 1.5 times the working tension of track cable. 2. The deflection angle α, Fig 12, made by empty cable over any saddle, shall not exceed 2° 52′; that is, an angle the tangent of which is 0.05. As these angles are conveniently laid out by their tangents, they are referred to as deflections of 0.05 or 5%. But, the tower at end of a long span should have no deflection over it when cable is empty. Angle α is the deflection between chords only for short spans; on others, it is the deflection between tangents to adjacent empty cable curves.

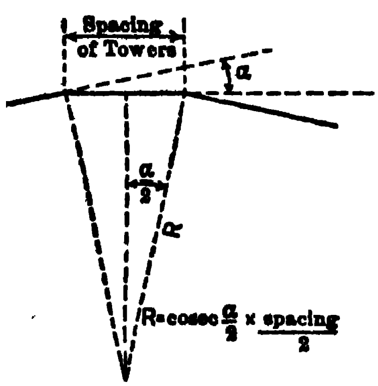


Fig 12. Derivation of Table 5

After preliminary layout is finished, different points on the line are studied separately (Art 6a), the position of cable is determined mathematically and the design modified to fit the specific conditions.

Construction curve has a tension 1.5 times that of the working curve, hence its equation for cast-steel cables is:

$$h = \frac{ws^2}{8t} = \frac{3.7\,s^2}{8 \times 34\,600 \times 1.5} = 0.000\,009\,s^2 \qquad (32)$$

in which 3.7 = wt per sq in of steel in cable 12 in long, and 34 600 is working tension per sq in of steel (Eq 6).

To plot construction curve, points are located at intervals by deflections from the chord, as marked by the thread, and construction curve is then drawn through them, by wooden curves. These curves should be 18–24 in long, made by laying out deflections from a chord for a number of points, by Eq 3, using proper constants, and must be con-

structed on same scale as the plot. The towers are then plotted in outline at convenient points, with their tops at the construction curve and the curve of the cable at working tension (with or without loads) is drawn in between them, thus forming a slight angle at every tower.

On long spans, the loaded position of the cable should be calculated and plotted, and its relation to towers and ground investigated. If there are 3 or more loads on a span,

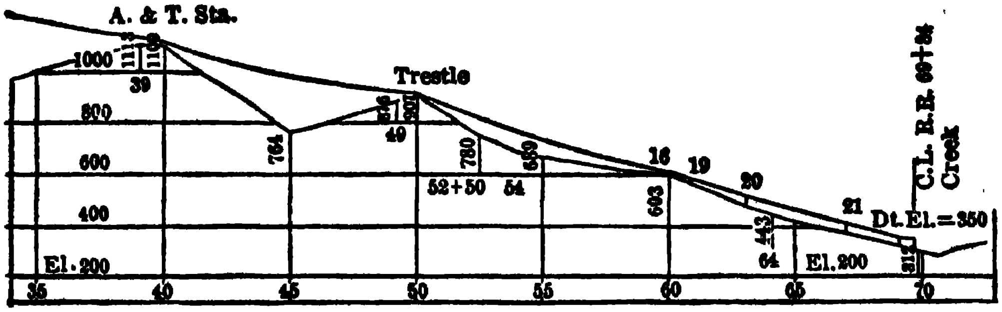


Fig 11. Profile of Typical Bi-cable Tramway—*Continued*

their effect is practically the same as an equivalent load uniformly distributed; hence the wt per ft w, in equation giving the deflection, will be:

$$w = wt\ cable\ (lb\ per\ ft) + \frac{gross\ wt\ of\ carrier\ (lb)}{spacing\ (ft)} + wt\ of\ traction\ rope\ (lb\ per\ ft) \qquad (33)$$

In Eq 33, spacing is found by Eq 26, and the traction rope, for preliminary determination, may be taken as 0.75-in diam, weighing 0.89 lb per ft. With only 1 or 2 loads on a span, the deflection at load points is found from Eq 11. Having determined deflection at one or more points, a smooth curve connecting them with the end points can be drawn. If spans are inclined, the formulas for deflections of inclined spans must be used (Art 1).

Application of rule 1 for locating towers is shown in Fig 11, from Sta 50 to the discharge terminal. A pin is set 15 ft above the ground on profile at Sta 50, another at end of rail in the terminal, and the thread is stretched. As the construction curve would strike surface at Sta 60, a pin is placed here about 10 ft above ground, giving 2 chords. From Sta 60 to terminal, the chord is not far above ground; hence the construction curve is drawn and towers 20 and 21 are place under it, to divide the space about equally and avoid a tower in the hollow.

Exceptions are sometimes made to the rule requiring towers to reach up to the construction curve: 1, if spans are long, causing a great difference in sag between empty and loaded conditions of cable, involving sharp bends over the towers; 2, if the line crosses a ravine, where it could not be operated without towers, but where the towers would be unduly high if brought up to construction curve. In both cases, the tops of towers are placed along a slacker curve, and the empty cable is prevented from lifting out of saddles by steel plates bent over the cable and fastened to saddles. The curve through tops of towers should be high enough to cause cable to bear in the saddle when line is loaded, and so relieve the hold-down straps. When this construction is used, the position of traction rope must be investigated, as it may be taut enough to lift and foul track cable or tower saddles, if there are no buckets on the span to hold it in normal position. Designing on basis of loaded cable is further discussed in Art 6*a*, under Broad Valleys.

Construction at crests is a difficult problem. The principal wear on track cable occurs near towers, and this is aggravated at crests by the downward pressure due to pull of traction rope, added to weight of carrier. Downward pressure is minimized by making the curve over the crest as easy as possible, by introducing a series of towers at short intervals and distributing the bend among them.

Illustration of this occurs at Sta 60, Fig 11. A pin having been set at Sta 60, a tower is placed equidistant on each side of it; the one on upper side is so placed that the cable will make a 5% deflection over it when span is loaded, and the end tower on lower side is brought up to curve of the empty cable, with no deflection. Total deflection angle between tangents to these two curves is 8°. A deflection of 5% per tower corresponds to an angle of 2° 52′, which, divided into 8°, indicates that 3 towers are needed. A fourth tower, No 19, is added and brought up to construction curve drawn from No 18, to take the bend due to a loaded carrier in the span from No 19 to No 20. The profile makes it convenient to place the towers 40 ft apart.

The location of towers at a crest is expedited by using a set of circular curves, finding by trial the one that fits between tangents to the two cables and is best suited to the profile, and then building the towers up to this curve. If the towers are lined on a circular arc, and an equal deflection occurs at each, they will be spaced at equal chord-lengths along the arc, the radius of which can be calculated for any chord. Radius of the curve through

towers, for different deflections and tower spacings, is given in Table 5, the data on which it is based being shown in Fig 12.

Table 5. Radius of Curve Through Towers (All dimensions in feet)

Deflection	Tangent	Angle α	Cosec $\frac{1}{2}\alpha$	Spacing of towers, measured on the curve									
				10	20	30	40	50	60	70	80	90	100
1/2 : 20	0.025	1° 26′	79.95	400	800	1 200	1 600	2 000	2 400	2 800	3 200	3 600	4 000
3/4 : 20	0.0375	2° 09′	53.81	270	540	810	1 080	1 350	1 620	1 890	2 160	2 430	2 700
1 : 20	0.05	2° 52′	39.98	200	400	600	800	1 000	1 200	1 400	1 600	1 800	2 000
1 1/4 : 20	0.0625	3° 34′	32.13	160	320	480	640	800	960	1 120	1 280	1 440	1 600
1 1/2 : 20	0.075	4° 18′	26.66	133	267	400	533	667	800	933	1 067	1 200	1 333
1 3/4 : 20	0.0875	5° 00′	22.93	115	230	345	460	575	690	805	920	1 035	1 150
2 : 20	0.10	5° 42′	20.11	100	200	300	400	500	600	700	800	900	1 000

At crest at Sta 21, a similar group of towers occurs, and Fig 13 gives graphic solution of problem. A point A, at Sta 21 and El 1 600, is taken as a trial point and a chord drawn to Tower No 3. It is

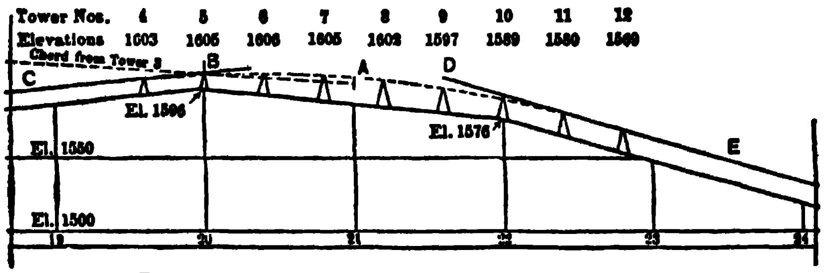


Fig 13. Details of Construction at Sta 21, Fig 11

evident that a curve drawn to this chord will strike the ground, hence Sta 20 is a more suitable terminus for the chord. A point B is therefore set at Sta 20 and El 1 605, and a new chord drawn to Tower No 3, giving a span from Sta 2 to Sta 20 = 1 800 ft horizontally. Next, the center deflection of loaded cable is computed and the tangent B to C is constructed. On the right of summit it is evident that the cable will follow slope of ground to Sta 28, hence a line DE is drawn parallel to and 15 ft above ground. A trial of several circular curves shows that one of 20-in radius (which, at scale of 1 in = 40 ft, corresponds to 800 ft radius), laid tangent to the lines BC and DE, conforms to the profile. By Table 5, the 800-ft curve corresponds to towers 40 ft apart, for deflections of 5% on each. This arc is then drawn in, towers are set near tangent points, with a distance between them equal to a multiple of 40 ft, and other towers are plotted up to the curve with a spacing of 40 ft, thus giving 9 towers for the vert curve.

Tramway trestle is a structure with a series of saddles mounted at intervals of 5–20 ft, with a slight deflection of cable at each saddle, and is used on a summit where a group of towers would be objectionable.

Fig 14 shows an application of this construction, for conditions at Sta 50 (Fig 11). The figure also gives, for comparison, the graphic solution for a series of towers, similar to arrangement at Sta 21.

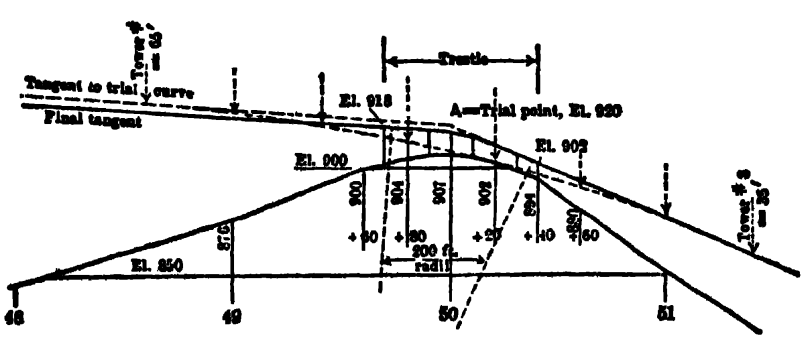


Fig 14. Details of Construction at Sta 50, Fig 11

A trial point A is set at Sta 50 and El 920 and, as the spans are long, the tangents to loaded cables are drawn on both sides, AB and AC. A curve of 800-ft radius is drawn tangent to these lines, and towers are spaced 40 ft apart, giving the 8 towers, shown by dotted lines and inverted arrows; but these are undesirable, owing to their heights (upper one being 65 ft high) and to necessity of cutting the summit of ridge. Next a trestle with saddles 10 ft apart and a deflection of 5% on each is

investigated. Under these conditions, the saddles will lie on a curve of 200-ft radius (Table 5). This curve, placed tangent to the lines *AB* and *AC*, is unnecessarily high at the left, hence line *AB* is lowered, which moves point of intersection of tangents to the right. The structure adopted is shown by full lines.

In making final design of a trestle, the elevs of end points are scaled from profile, the inclinations of tangents to loaded cables are computed and the difference is the angle of deflection between the two tangents; this divided by the angle corresponding to desired deflection, gives number of saddles required. From this the slope of chord from saddle to saddle is found and the elev of each saddle calculated to nearest 0.1 ft. A similar process may be used to calculate elev of individual towers of a group, after locations of end towers have been determined by scaling.

Rail trestles are similar to type just described and have the same application, but a rail is added to take the rolling load and relieve the cable (Art 13).

Track cables are tightened by a weight or some tension device at one end, the other end being anchored (Art 11). On short lines, the anchorage is at one terminal and the tension is applied at the other. A section from 3 000–5 000 ft is as long as can be controlled from one tension point, hence long lines must be divided into sections of approx 4 000 ft. Exact lengths of sections are controlled by choosing, along the line, suitable sites for stations. It is customary to place anchorage at the high end and apply tension at low end of section; then the wt of cable aids in working the slack down to the tension weight, and erection is also facilitated. Intermediate stations are necessary for long lines (Art 12). The typical tramway (Fig 11), being nearly 7 000 ft long, requires one intermediate tension station, placed on the ridge near Sta 40. It is constructed as a combined anchorage and tension station (see end Art 12).

6a. MORE COMPLICATED PROBLEMS OF DESIGN

When a tramway is to carry heavy individual loads, or large tonnage, or the difference in elev between ends is great, serious problems are presented. Heavy loads require a large cable, or a carriage with more than 2 wheels, or both, to distribute rolling load and reduce bending stresses in the cable. Large tonnage or great difference in elev creates heavy traction stress (computed by Eq 29), which requires rope of large size or high tensile strength. The vert component (Fig 18) of a heavy rope stress augments the rolling load, which in turn injures the cable. Hence, when an unusually strong traction rope is needed, say over 0.75 in of cast-steel grade, it should be taken as a danger signal and the design carefully reconsidered. Heavy rolling loads also affect the design of structures and machinery, increasing bending stresses in saddle beams on towers and stations, and in the members of rail trestles (Art 13).

Determination of rolling load. A profile to scale of 1 in = 40 ft (or 1 to 500 metric measure) is generally convenient. Humps in the line can be located, and these replotted to twice the above scale for detailed study; choosing first those points, near upper end, where tension is greatest and conditions likely to be most severe. As the exact position of the cable can not be plotted in the early stages of design (some conditions being unknown), an approximation must be made.

For a given make or type of tramway certain mechanical parts have fixed and known dimensions, viz: the distance at carrier of traction rope below the carriage and distance of traction rope carrying sheaves or rollers below the grip. On these bases, a diagram is drawn to scale of 1 in = 20 ft (or 1 : 250 metric), with locus of tower saddles as an arc, the path of grip or clip as a similar arc, and a third arc representing the locus of traction rope rollers. These arcs are concentric, and their distances apart are determined by the above mechanical features. Fig 14*a* shows such a set of curves (exaggerated). For any position of a carrier, lines drawn from its grip tangent to the roller curve will be chords of the traction rope, as lifted by a passing carrier. This is true unless the curve is so sharp that the chord of the traction rope to next carrier does not touch a roller, when a different construction will be needed, as in Fig 14*b*, discussed later. With traction rope chords drawn to roller arc (Fig 14*a*), a parallelogram of forces is laid out on them, and the vert component determined. If the traction rope stress is laid off as 100, the vert component is a percentage, which, multiplied by the actual tension in the rope (Eq 29) gives the vert stress due to tension. This can also be found analytically (see calculation below Fig 14*a*). Total rolling load is the sum of: wt of carrier and its load, if any, and vert stress found from the chords.

Radius of arc over saddle for any case should be the one best suited to shape of the crest, taking into account the sharpness of the ridge and breadth of the hill, and be as large as possible without making the ends too high from ground. Sometimes on a sharp ridge a cut or even a tunnel is needed to allow flattening of curves. Spacing of towers to fit the saddle arc is found from Table 5 for a desired deflection angle over individual saddles.

When radius of saddle arc can not be increased enough to reduce the rolling load to a desired amount, a rail structure can be built over the crest in place of several towers, thus carrying the load on solid track instead of on the cable. Such a structure often costs no more than a group of towers; sometimes it can be combined with an anchorage or a tension station, at little added cost. The rail must be long enough at each end to keep carriers off the cable, so as to prevent heavy vert stresses on cable when carriers are near the crest. See Fig 14*b*, where carrier *P* on tangent to the summit curve has a heavy vert stress due to traction rope tension, which, if on the cable, would cause serious bending. If the rail is extended down the tangent, so that a carrier, as *Q*, is on the

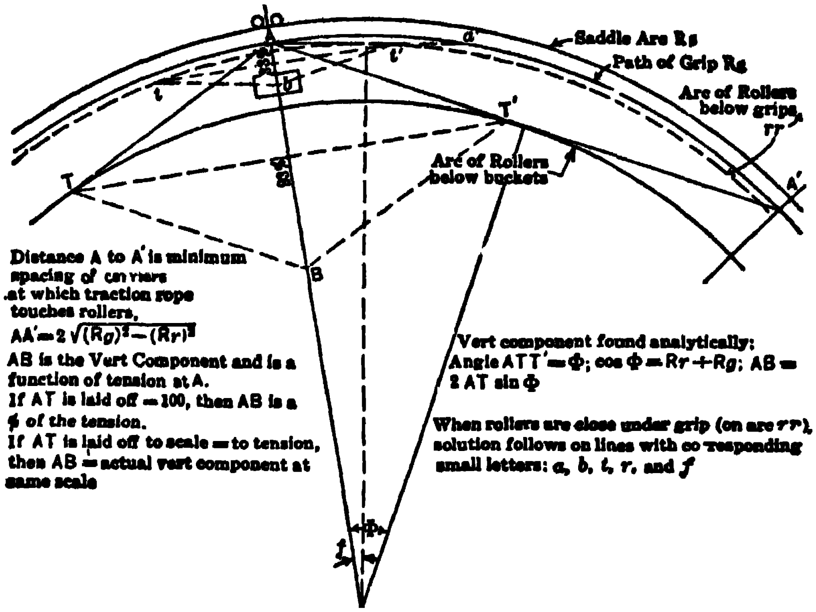


Fig 14*a*. Stress Diagram of Vertical Component

rail, the rolling load does not come on the cable. Use of supporting sheaves for the traction rope, close under grip or clip, makes the vert stresses much smaller than when rollers are below bucket. Fig 14*a* gives solutions of these stresses for the two types of construction.

When saddle arc has a large radius, is short, or carriers are close together, the traction rope may be supported by the carriers, not touching the rollers; this often occurs where traction rope rollers are below the buckets, but less frequently when rollers are close under the grips. When traction rope does not touch rollers, the position of next carrier on each side is plotted, and a force diagram plotted on the chords drawn between grips, as for carrier at *N* (Fig 14*b*). Note that the diagram would be the same if chords were drawn between the points on cable where the carriers stand, instead of from grip to grip.

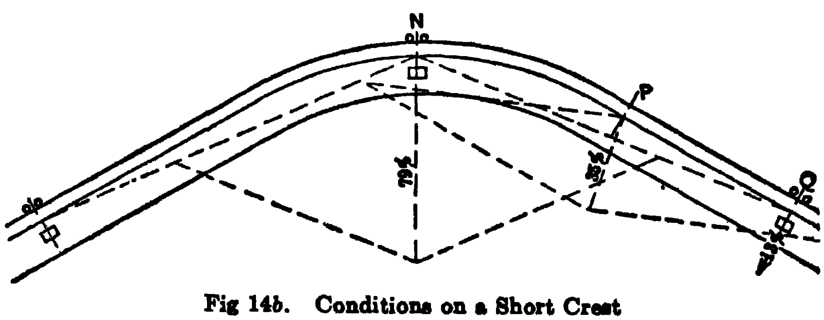


Fig 14*b*. Conditions on a Short Crest

Permissible rolling loads. Table 4, showing net loads that can be carried on cast-steel cables of different sizes, is useful in beginning to study a problem. The total rolling load must be considered; it consists of net load of material carried + wt of carrier + vert component of the traction rope tension; and should not exceed 1 200 lb per sq in of cable section, when a 2-wheeled carriage is used. The metallic area of cable is found by dividing the wt of cable per ft (Table 1) by 3.7, and the working tension is 34 600 lb per sq in of area.

Bending of the cable under the traveling load decreases with increased tension. Where a cable had been injured, investigations indicate that a ratio of rolling load to horiz cable tension was too great, and that a ratio $W \div t = 0.035$ is about the limit of load that can be carried without undue wear. This ratio corresponds to a deflection angle, between the tangents to the cable on the two sides of the load, equal to 2°, with carriages having two

10-in wheels, 18-in centers. If the ratio exceeds 0.035, a larger track cable, or one of stronger wire, may be used; either will increase value of t by permitting the use of higher tension in the cable and so decrease the deflection angle at the load. If the tension can not be so increased, the design of the carriage may be altered by using four 12-in wheels on 20-in centers, mounted in an equalising frame. The load can then be doubled and still give a deflection angle under each pair of wheels not exceeding 2°; that is, the total deflection angle for a 4-wheeled carriage may reach 4°, the tangent of which is nearly 0.07.

Where the gross load is too great for the cable at a few points, and a larger cable throughout is not warranted, the deflection angle under a load can sometimes be decreased by building a section of the tramway with a larger cable, thus permitting a higher local tension, with the same effect as reducing the rolling load. This device can also be used on a long span, to reduce sag of the track cable.

Angle over towers. In Art 6 it was stated that the deflection angle of the empty cable over towers should not exceed 2° 52′, or a change of direction of 5%; this rule is applicable to many lines using fixed saddles, especially for capacities up to 60 tons per hr. But conditions may occur, especially if duty is high, where the rule is not applicable, as the difference between loaded and empty position of cable on long spans is so great, that the loaded position of cable must control the design.

Example. If a span is 1 000 ft long, empty cable weighs 3 lb per ft, total uniformly distributed wt of loaded cable is 9 lb per ft and cable tension 24 000 lb; then, from Eq 8, the slope of the empty cable at the end tower will be 6.25% and slope of loaded cable 18.75%. If the tower at end of span is set with relation to the next one, so that there is no angle over it when cable is empty, there will still be a 12.5% deflection when span is loaded. No construction will permit so great a change in position of cable. In such case, the towers at ends of the span must be so set that there is an upward deflection on end tower when cable is empty; then, when loads come on the span, the cable will sag and deflect downward over that saddle. Thus, if the empty cable had a deflection of 6.25% acting upward, the final deflection when span was loaded would be 12.5 − 6.25 = 6.25% downward. To hold the cable in the saddle when it is empty, the saddle must then have a strap (hold-down saddle), and movable points to permit the passing carriers to leave and return to the cable. In trying to reduce the angle over a tower at the end of a long span, the position of cable is determined by gravity and can not be altered, and to get an easy approach the positions of towers back of the end tower are the only conditions susceptible of change.

For the heavy lines discussed in this Art, saddles are preferably of the rocking type, some in use have arcs 10° long, with radii of 15 ft for $^7/_8$–$1^3/_8$-in cables and 25 ft for 1.5–2-in cables, supplemented by arcs of smaller radius at each end. They may be used where the cable deflection over them is as much as 8°, or 14%. When a load approaches such a saddle, the depression of the cable rocks the saddle toward the load and, after passing over it, the depression of the cable rocks it in the opposite direction. The rocking saddle is essential where tower spacing is long, as the deflection angle between tangents to the cable is then large; if it exceeds 8°, a double tower with 2 saddles for each cable may be used.

To determine type of saddle and construction needed at any location, the exact slope of the track cables must be found. The worst condition is when a load approaches or leaves a saddle and is a short distance from it; this distance varies with the saddle design and length of carriage, but may be assumed as 6 ft for a general study; the positions of the other loads are determined by the load spacing. The inclination of the cable at a saddle is the algebraic sum of: (1) inclination of the chord found from equation $\tan \alpha = V \div s$ (Fig 7), which may be up or down; (2) inclination of the empty cable with respect to its chord, from Eq 8, is $\tan \beta = ws \div 2t$; (3) inclination of the funicular polygon due to the loads; thus, in Fig 4 (from Eq 9) at the end A, $\tan \phi = r \div t$, where r, the reaction at opposite end B is due to the concentrated loads.

When one load is close to a saddle, the next one on opposite side will be nearly a load spacing away and the others will be multiples of the load spacing from it. Deflection angle of the cable over the saddle can be determined from the inclination of the 2 cable tangents, and with the rocking saddles described above would be limited to 8°. The deflection can be figured in percentage by adding the tangents, as they are practically proportional to the angles when the angles are small; the limiting deflection would then be 14%. If the next saddle, on the side opposite the one where the load is close to the saddle, is less than a load spacing away, the inclination on that side will be due to the chord and the empty cable only; this condition may occur where towers are closely spaced, or where 2 saddles are mounted on a double tower.

Broad valleys present another case where construction must be based on the loaded cable, especially if the valley is too deep to use ordinary towers, or is shallow enough for the loaded cable to touch the ground on a clear span. Loaded cable deflection would then be computed for a point in the valley where a tower could be built, using a tension

25% or more greater than the working tension. If a tower is built up to this position of cable, the empty cable must be held down on it, but the loaded cable will bear upon it with only a moderate angle. Several trials may be necessary before a solution is found. In such case, the traction rope must remain below the track cable, so as not to foul, nor have a lifting action that will raise an empty carrier off the cable. The position of traction rope is determined from its wt per ft and its tension at this point. It may be possible to hold it down at towers by rollers placed above the grip.

Example. Suppose the valley is 3 000 ft wide, and chord is horiz; empty cable weighs 3 lb per ft, loaded cable has a total uniformly distributed wt of 9 lb per ft and cable tension is 24 000 lb. Then, by Eq 2, the deflection of empty cable at center of span is 140 ft, and end slope is 18.67%; when loaded, deflection is 420 ft and end slope is 56%. For a construction curve at 1.25 times the working tension, deflection will be 336 ft, and topography may be such that a tower can be built up to the cable at this point. The end slopes of this construction curve will be 45%. Assuming that the surface on the flanks of valley has about this slope, towers can be placed under the cable at both ends of the 3 000-ft span, and so reduce the half spans to say 1 000 or 1 200 ft from the flank towers to tower in mid-valley. The flank towers will probably need hold-down saddles. Several trials will be necessary before final solution is found, as the introduction of towers changes conditions. The tower in the center of valley will also need a device for holding down the cables. Sometimes hold-down saddles can be used; otherwise a short rail station should be built. An anchor or tension station, if needed, may be located in the valley and so hold down the cables. Where the traction rope does not naturally hang below the cable, it may be possible to hold it down by sheaves above the grip, but care must then be taken not to produce an upward stress that will lift an empty carrier off the cable. If the traction rope can not be held below the cable, the valley tower must be raised to hold the cable above the curve of traction rope.

Anchored spans, having a take-up to produce tension in the track cables, are sometimes feasible. They require more judgment in erecting and in keeping the proper tension in cables than where weights are used, but this construction is usually cheaper than weighting, as the structures can be lower than those used with weights and the equipment is cheaper. The tension in cable is produced at one end of a section by a wire rope tackle, or a turn buckle, or rods with long threads, forming part of the permanent equipment.

On Riblet tramways, cable tension is nearly always produced by wire rope tackle between end of cable and an anchorage. The tackle blocks have sheaves set tandem between 2 long steel plates; the sheaves in each block are of different diam, the one nearest the connection being the largest; this places the sheaves and ropes in one plane, but the ropes clear each other due to the different sheave diameters. The pull on the loose end of the tackle rope is obtained by winding it on a drum driven through worm gearing operated by hand. The fixed tackle block is attached to the drum frame, and the whole is anchored to a block of concrete or masonry.

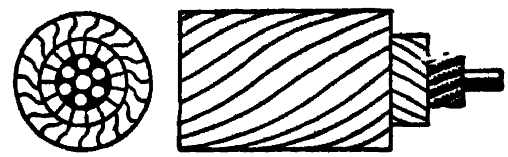
In erecting a line with anchored spans, tension should be applied to empty cable to bring it a little higher than the computed position of the loaded cable when fully loaded, say with a center deflection 75% of that of loaded cable. Then, when loads are added and cable stretches, it will occupy approx the calculated position. The layout of tramway with anchored spans should be such that a concentration of loads on a long span (due to conditions in loading or stripping the line with buckets, or accidental uneven spacing during operation, causing the span to sag and pull up adjacent spans) does not lift the empty cable off of its saddles. This also occurs in a reversible tramway with anchored cables (Art 20, 21), where there is a single heavy carrier. In such tramways the slack of the cable will move from span to span with the carrier, and adjacent spans will be taut. Bib 10a gives examples of anchored spans and calculations of effect of stretch of cable.

A condition identical with an anchored span is obtained by placing blocking under the tension wt, after the cables have been loaded. This holds cable in same position as if anchored, and prevents it from lifting when unloaded. The wt affords a safeguard, however, in that it can lift if the span is overloaded. This relief is absent from an anchored span; when load is increased the cable stretches somewhat, which increases the sag and counteracts some of the increased stresses of moderate overloading. Anchored spans and blocked weights require careful design. The operating conditions are easily misunderstood, and tightening of the cable, as it stretches and becomes slack, is neglected; hence, such spans should be used cautiously, and only under skilled supervision.

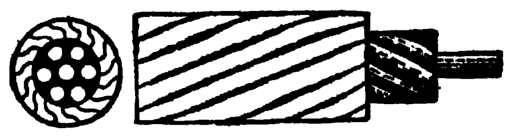
7. TRACK CABLES

Track cables for all types of aerial tramways are usually smooth-coil or locked-coil cables. They are made of large wires (Fig 15) in order to have long life under the surface wear of the carrier wheels. Ordinary haulage ropes, with 6 strands of 7 wires each, may be used on light tramways, where low first cost is desired; but usually the small wires wear so quickly that they are uneconomical. See Sec 12.

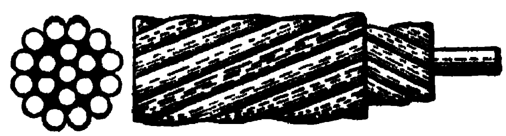
Locked-coil cables have a smooth exterior, the shape of outer wires resembling an angular oblique figure 8, so that they interlock. This locking holds one broken wire, but if several break near each other the outer layer will unravel. The center of a locked-coil cable is a 7-wire strand, and is surrounded by key and locked wires. In small sizes the key wires are omitted (Fig 15 *b*); larger cables (Fig 15 *a*), have one layer of key wires; very large ones, over 2 in diam (used on cableways), have 2 layers of key wires. For ordinary tramways, the cables are of C S wire, but for heavy capac lines the wire is of a special grade. For example, if conditions require a 2-in C S cable (Table 1), a stronger cable can be had in the 1.75-in size of special steel, making a saving of 23% in wt. A 2-in cable is considered as large as can be economically used, because couplings would be so large in diam as to require very wide carriage wheels to travel over them, which would increase the size of other equipment.



(*a*) Locked-coil Cable, 1 1/8–2 in diam



(*b*) Locked-coil Cable, 7/8 and 1 in diam



(*c*) Smooth-coil Cable

Fig 15. Track Cables

Smooth-coil cables, entirely of round wires, are not so smooth as locked-coil cables, but become fairly smooth by wear. As the wires do not interlock, a broken wire is apt to work out and repairs must be made promptly by applying a sleeve-like clamp, permitting carriage wheels to pass. They are easier and cheaper to make than the L-C. As they are of high tensile strength wire, their wt (Table 1) and cost are reduced; but they must be replaced oftener than L-C cables. One center wire is surrounded by 2 to 5 layers of wire, all of same size laid in opposite directions. The number of wires may thus be from 19 to 91; their diam, 0.15–0.22.

Couplings. Length of a piece of cable is limited by length of wire that can be manufactured. Sections required for long tramways are joined by couplings to form a continuous cable from anchorage to tension station. Each coupling (Fig 16) is in halves, one of which is wedged to end of each section, the halves then being connected by a plug having right- and left-hand threads. Couplings are as small in diam as consistent with strength, and the carrier wheels are wide enough to pass over them. At the ends, the cables are attached to sockets similar to that used for couplings, and designed so they can turn when cable is rotated. Couplings should not come within 15 ft of a tower, or within 10 ft of each other, so that the cable will be free to sag as a carrier passes over a coupling, and not receive sharp bends.

Attaching couplings and sockets. Cable is seized with wire for several inches from end to prevent distortion; then sawed off square; a clamp is put on beyond where fitting will come; the seizing is removed and wires are cleaned with gasolene. Cable is inserted in small end of the taper hole in fitting, the rows of wires are separated by annular wedges driven between them, spaces between wires are filled with slender steel wedges and any small spaces remaining are filled with slim shoe pegs. This wedging grips each wire and makes a solid steel knob on end of cable which prevents it from pulling through the taper. Wedges and thimbles may be driven with a hammer and punches, but a screw press is quicker and better. With a press, the fitting is placed on cable as above, thimbles and wedges are inserted and driven part way down; then the fitting is placed in the press, the entire set of wedges forced home, and the bulged end of cable pushed firmly into the taper fitting.

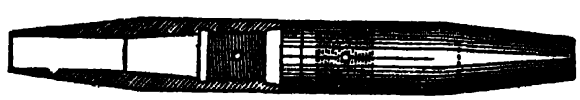


Fig 16. Coupling for Track Cables

Rotation of track cable. Every week or so the cable should be turned through 1/8 of a revolution. This distributes wear over its entire surface, prevents flattening of cable and displacement of wires. The turning should be in such direction as to tighten the outer wires of cable; thus, if wires have a left-hand lay (Fig 15), the cable should be rotated to the right, or clockwise.

Maximum wear on a track cable occurs in a length extending about 5 ft on each side of a summit saddle. Directly over the saddle the rolling pressure is greatest, due to pull of traction rope combined with wt of carrier; just off the saddle the cable gets a reversed bend between carrier wheels and saddle, especially on the side where loaded carriers approach the tower. The downward pull of traction rope can be reduced by proper location of towers and traction-rope rollers (Art 10), and bending minimized by stretching cable as taut as its strength will permit.

Cables are oiled about once a month with a heavy oil, to prevent rusting, to provide internal lubrication, and reduce friction of carriers. Oiling may be done by men riding slowly over the line in a carrier and slushing oil on cable with a handful of waste; or with a mechanical oiler, hauled over the line like a carrier. Saddle grooves should be greased to facilitate rotation of the cables, and to permit them to slide so as to maintain their tension.

To repair a cable the tension is taken off the end, tackle attached to each side of break, the blocks are drawn together, and break is cut out. If break is short, half a coupling is wedged to each end, and they are united with the right- and left-thread plug; if worn part is long, a new piece of cable is inserted. Inserted piece should never be less than 10 ft long, or wires will break at ends of couplings, due to stiffening of cable by the couplings.

8. ROLLING STOCK

Each piece of rolling stock (carrier) consists of: carriage, container, hanger, and clamp or grip for attaching to traction rope.

Carriage runs on track cable; its wheels, 8–12 in diam, are mounted in a frame with a connection for the hanger supporting the container. For heavy loads, carriages have 4 wheels; they are practically 2 ordinary carriages, connected by a swivelled equalizing frame, from which the load is suspended. This distributes the load and reduces wear of the cable due to bending.

Container for bulk material is a bucket; for other materials it may be a tank, platform, or cage. For timber, the hangers terminate in hooks, to which timbers are attached by slings; these are used in pairs, one at each end of load. Sometimes workmen are transported by a tramway during certain hours of the day. Long buckets, each holding 2 men, are best. One man sits in each end with his legs between those of the other man.

Hanger is of bar steel, with C-I or steel parts to facilitate connection with carriage and container, and hold clamp for traction rope. Its form is largely governed by the parts to which it connects.

Clamp for traction rope may be a GRIP, which can be applied to or released from the rope at terminals, thus allowing carrier to be shunted to loading or discharge point; or it may be a CLIP, permanently attached to both carrier and traction rope.

Grips differ in design and often in principle on each make of tramway, and are shown in maker's catalogs. Devices used to obtain gripping pressure on traction rope are: (1) Levers and toggles. The latter, by passing a center closes and locks the grip, as in the Webber grip (Maker 1, Art 29) extensively used in the past; obsolete for new work. Its motion was prescribed and it would not grip the rope where its size varied beyond the rope's compressibility. (2) Springs. By cushioning one jaw, grip can take ropes of varying size; levers are used to compress the spring and produce initial press; holding power depends on strength of spring and its leverage on grip jaw. Used formerly in modified Webber grip, and now in Wico grip (Maker 1, Art 29). (3) Screws. Grip jaws are brought together by a screw as in a machinist's vise; the gripping force is large and jaws will seize ropes of considerable difference in diam and are locked wherever the screw stops turning. This type was originated by Otto in Germany, and in slightly different detail is now made by Makers No 8, 3, 9 (Art 29). (4) Wedges. A slim sliding wedge is used to force jaws together on the rope; the taper being less than coef of friction there is no tendency for grip to release. They will hold on ropes whose diam varies due to stretch or wear. Made by 2 and 9 (Art 29). (5) Gravity. Wt of container, acting through levers, gives the holding force on traction ropes. The closing motion stops when jaws have seized the rope; this permits gripping of ropes of varying sizes. The grip is usually attached to hanger below the carriage (under-hung grip); for special purposes it may be built into the carriage (over-head grip). Made by 5, 6, 7 (Art 29).

Most grips can be attached by hand, but are usually closed by an attacher; they are always released automatically when carrier enters terminal. A grip depending on the weight of container for its holding force requires a special system of rails for lifting the container, in order to open grip preparatory to inserting traction rope on outgoing side, and a similar device on incoming side to open grip and release traction rope.

Some grips occupy most of the space, not needed for clearance, between carriage and carrier, so that the rollers supporting traction rope at towers must be placed below the carriers. This construction is good on most lines. As the carrier is free to swing sidewise through considerable range, guards are often used to fend the rope onto the rollers. Other grips allow supporting sheaves to be placed close under their jaws, so that the traction rope is lifted but little as carrier passes the sheave. This requires carriers to be guided, to prevent colliding with the sheave, and to make the rope drop back on sheave after grip has passed. Guides are objectionable when applied to every tower, but to have the sheave close under the grip is advantageous in passing over crests, as it reduces the downward press. Guides can be easily constructed on summit structures. (See discussion regarding Fig 18, Art 10.)

Clip is a U-shaped band of steel, bent around traction rope and permanently clamped to it by bolts. As clip can not be released from rope while tramway is in operation, terminals must be so arranged that carriers can pass around end sheaves; the buckets are loaded while in motion by an automatic loader, and dumped while in motion by a tripper. Tramways with clips are obsolete. Disadvantages: wear on traction rope due to gripping is concentrated at clips; the non-stop feature makes terminals inflexible; speed of the line is limited by centrifugal force developed by carriers passing around sheaves.

Dimensions of 3 sizes of ore carrier accompany Fig. 17. In a preliminary design these may be used for several sizes of buckets smaller and larger than those noted. The distance *A* differs considerably; its minimum is about 3.5 ft, when the traction rope is carried on rollers below carriers. *A* must be increased to clear sheaves supporting traction rope if close under the grips, to admit special latches and tripping mechanism, for clearance in terminals, and for loading. Distance from track cable to grip is kept constant on each system, at about 2 ft, so grip will pass under the cable saddle and end of its supporting beam.

Dimensions

	Capacity of bucket, cu ft		
	5	10	20
A	3′–6″ with traction rope roller below carriers; 5′–0″ with traction rope sheave under grips		
B	1′– 3″	1′–10″	2′–5″
C	2′– 7″	2′–11″	3′–2″
D	1′–11″	2′– 3″	3′–0″
E	1′– 5″	1′– 9″	2′–3″
Weight, lb	300	420	55

Above are approximately applicable to buckets of 4 to 6, 7 to 15, 16 to 25 cu ft, respectively.

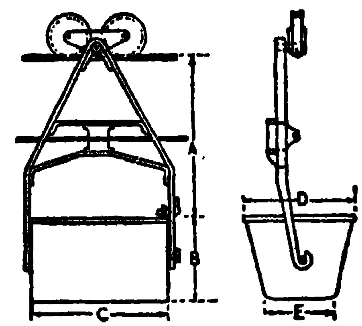


Fig 17. Ore Carrier

9. TRACTION ROPE

This is usually Lang lay, of as large wires as consistent with flexibility and to offer max wearing surface. For sizes of 0.75 in diam and less, there are 6 strands of 7 wires each, all of same diam, laid about a hemp center. For larger diam, the strands have 19 wires; 1 center and 9 outer wires of about 25% of diam of strand, and an intermediate layer of 9 wires half the diam of outer ones; this combines flexibility with good wearing surface. For moderate tensions the wire is of cast steel, to make a rope of such diam that the gripping surface is ample. For higher tensions (ropes over 0.75 in diam), plow-steel is used, to keep down size and wt and give the flexibility due to smaller wires. Factor of safety, 5. See also Sec 12.

Lubrication. Traction rope should be oiled frequently, to lubricate the interior. Lubricant must be carefully chosen, so as not to reduce holding force of grips. A thin oil may be used in abundance, to penetrate the interior, the surplus being wiped off to leave surface dry. It can be applied mechanically by an oiler on the outgoing empty side of a terminal. Some operators prefer a thick sticky lubricant, which works well into the interior. Admixture of linseed or other drying oils in the lubricant is advantageous, in causing exterior surface to harden, and enabling grips to take a firm hold.

10. TOWERS

Stresses in towers have 3 sources: 1. Downward press on saddles, due to vert component of tension in track cables, wt of loaded carriers, and vert component of tension in traction rope. 2. Racking motion parallel to line of tramway, due to changes in direction of press from the cable when a load approaches and leaves the tower. 3. Side press, due to wind.

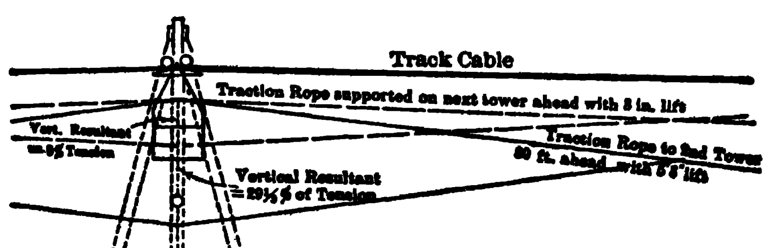


Fig 18. Stresses in Saddle Beams; Towers 40 ft Apart; Cable Deflected 5% on Each

1. Downward press on saddles is often so great that their supporting beams must be of oak or steel to resist bending stresses and crushing by the saddles; steel is preferable. At towers along the line, downward press from traction rope can be determined only when exact conditions are known, as the next point of support for traction rope is generally the next carrier on each side, and the positions

of these depend on their spacing and the sag of the track cable under them. For mode of determining vert stresses, see Art 6 a. Fig 18 shows the vert stresses in percentages of the tension for a single tower. The full lines show the vert stresses when traction rope is supported on rollers below the carriers, and dotted lines show stresses when traction rope is supported on rollers 8 in below its position when in the grip. In practice a vert resultant of 29.5% (Fig 18) would be prohibitive, unless traction rope tension is very small.

2. Racking. The problem of supporting the saddle beams and providing rigidity against racking is made difficult by necessity of maintaining a clear passage for the carriers.

3. Wind. Tower base is spread parallel to center line of tramway, so resultant of press of cables on saddle falls well within base, even when cables are moderately inclined; hence will usually resist overturning by wind parallel to line. High towers may be narrower in other directions and should be investigated for resistance to wind at right angles to line; they may need to be guyed.

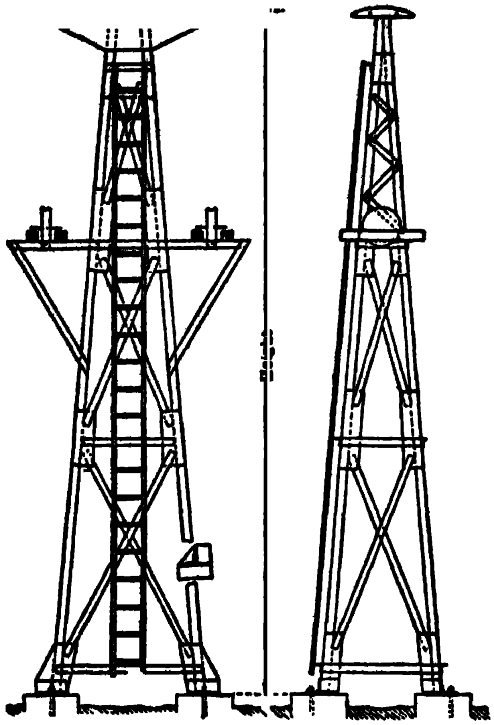


Fig 19. Tramway Tower: Pyramid Type in Steel

Material. Steel with concrete or masonry foundations is best for permanent plants and where wood is costly. Riblet uses steel fabricated near tramway site and pre-cast concrete blocks for foundations (11). Wood is usually framed at the point nearest tramway site to which timber can be delivered; main posts are 6 by 6 to 8 by 8 in; rarely 10 by 10; saddle beams are usually 8 or 10-in steel channels In remote districts, towers have been of round timber with hewed bearing surfaces, but timber can usually be whipsawed or ripped in a portable saw mill; squared timber reduces cost of framing, distributing and erecting. Foundations should be concrete or masonry, but are often of such material as is procurable: logs, cribs with or without rock filling, or piers of stone laid up dry; such foundations will not resist up-lifting stresses and many towers must be guyed. Concrete and masonry towers are sometimes used, especially when tramway serves a cement plant; they are bulky, and attractive designs have not been worked out.

Pyramid tower (Fig 19) is economical of materials. When of steel, a rigid head can be designed with shapes and plates securely fastened to the angle posts. When made of wood, the small area at top of posts makes it almost impossible to attach saddle beams so that they will not rack and work loose. In this type, whatever material is used, clearances between a large carrier and the side of a wooden tower are too small; the base

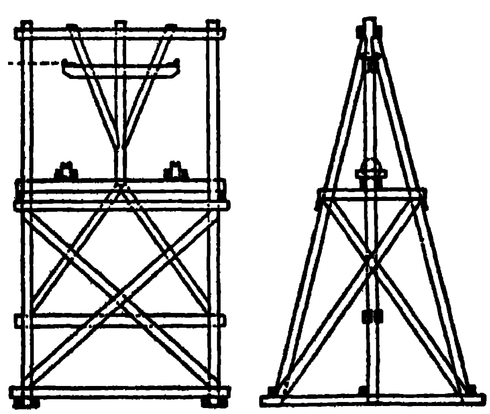


Fig 20. Tramway Tower: Through Type

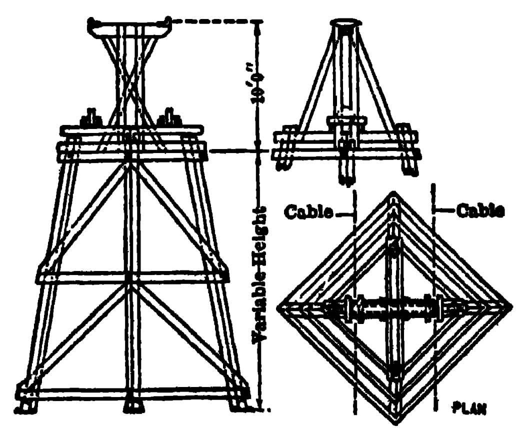


Fig 21. Tramway Tower: Composite Type

also is small and provision must be made for wind stresses, and on steep lines for cases where resultant of cable stresses falls outside the base.

Through tower (Fig 20) is rigid and efficient, but contains about 100 bd ft of timber per ft of height up to 60 ft, which is nearly double the amount for pyramid towers. Sometimes, on light lines, the outside center posts are omitted below roller deck, especially on high towers.

Composite tower (Fig 21) has a top identical for all towers, the base being varied to give desired height. Base stands with its diagonal on center line, hence its long dimension is in line with main stresses. Its top is rigid against racking. Framing is simpler than it seems in a drawing. These towers contain less timber than those of through type.

Side-hill tower (Fig 22) is of through type and designed for steep lines to ensure that line of press shall fall within base, and slope of inclined side is made to suit direction of press.

The principle of the through tower can be used for steel towers with good results.

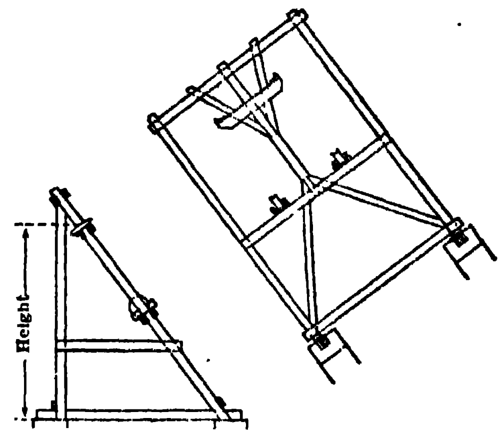


Fig 22. Tramway Tower: Side-hill Type

11. ANCHORAGE AND TENSION OF TRACK CABLES

Track cables must be taut enough to support the load and minimize bending under the carriage wheels, both on spans and when carriers approach tower saddles.

Tension weights to keep cables taut are wooden boxes filled with stone or scrap iron, or are moulded concrete blocks. It would be desirable to weigh the tension wt, but, this being usually impracticable, the wt of the box is computed and added to that of its contents, taken as 100 lb per cu ft for broken stone or 350 lb for packed scrap iron; concrete blocks are taken to weigh 140 lb per cu ft. Form and dimensions of weight are influenced by the allotted space. A tension wt can keep taut 3 000 to 5 000 ft of cable depending on profile of line and the care with which saddle grooves are kept clean and well greased. The weight usually equals the working tension of the cable, unless the slope is so great that the added wt of cable would overstrain the upper end; then the tension wt should be reduced.

Anchorage at opposite end of cable from tension point must be designed to cover fact that slope of line may increase tension due to the wt alone. Sometimes the cables terminate in station framework. For steel construction this is simplest; for timber it is often difficult to design the structure to take these stresses, and it is usually economical to deflect cables to the center, carry them through station and attach them directly to a masonry anchorage. See terminal stations (Fig 33, 34), also Fig 23.

The anchorage block must be a monolith, equal in wt to the vert component of the tension, allowing 140 lb as wt per cu ft of masonry or concrete, and the front must have sufficient area to give a bearing resistance against the earth equal to horiz component of tension. Horiz resistance of aver earth may be taken as 1 000 lb per sq ft for a deep block, but on a down-hill slope the anchorage must be carried into the hill by a tunnel. For an anchorage in rock, a T-shaped trench is excavated and the anchorage bars set in it, the space around them being filled with concrete. Metal parts of the anchorage must be designed for required tension, and also to allow the cable to be turned (Art 7) while under tension. Anchorage bars must be enclosed in masonry to a point above water level, and extend above dampness before coupling to cables, to avoid injury to the wires from rust.

12. INTERMEDIATE STATIONS

These are needed to apply working tension to track cables which are too long for terminal apparatus to be effective throughout (see end of Art 6). There are 3 types: Double-anchorage; double-tension; anchorage and tension. In all, the track cables are deflected to center of structure, out of the way of carriers, and pass downward to anchorage or to tension weights. Space between sections of track cable is bridged by rails, to allow carriers to pass through without being detached from traction rope.

When both pairs of cables are in place and under full tension, the pull of those on one side of station will counteract the pull on other side, but the structures must be so designed as to be secure if cables on either side are released while the others remain under tension, a condition which may occur during erection, while making repairs, or in case of accident. These stations are similar on all tramways, but must be designed to suit conditions on each profile. They may be of wood or steel; if steel, the bents can be simplified and all posts placed between the cables, somewhat like the steel tower, Fig 19, or wooden summit station, Fig 26. Cable sockets are designed to allow cables to be turned (Art 7)

Double-anchorage stations can be as low as profile conditions will permit, provided there is ample clearance for carriers above the ground. Where there is no snow, they

need not be over 10 ft high. Fig 23 shows essential features, with cables anchored to the structure. It is preferable, however, to continue the cables to masonry anchor blocks set in the ground, as shown in terminal, Fig 33.

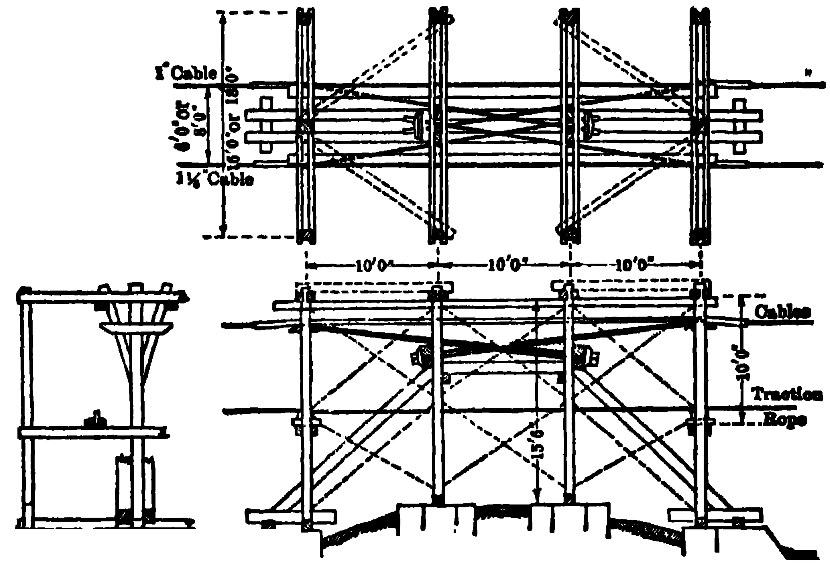


Fig 23. Double-anchorage Station

Double-tension stations must be built high enough to allow tension weights a vertical motion of 6 to 8 ft, and still have their tops below path of carriers. Even with squat weights, this makes stations 20 to 30 ft high to the rail (Fig 24).

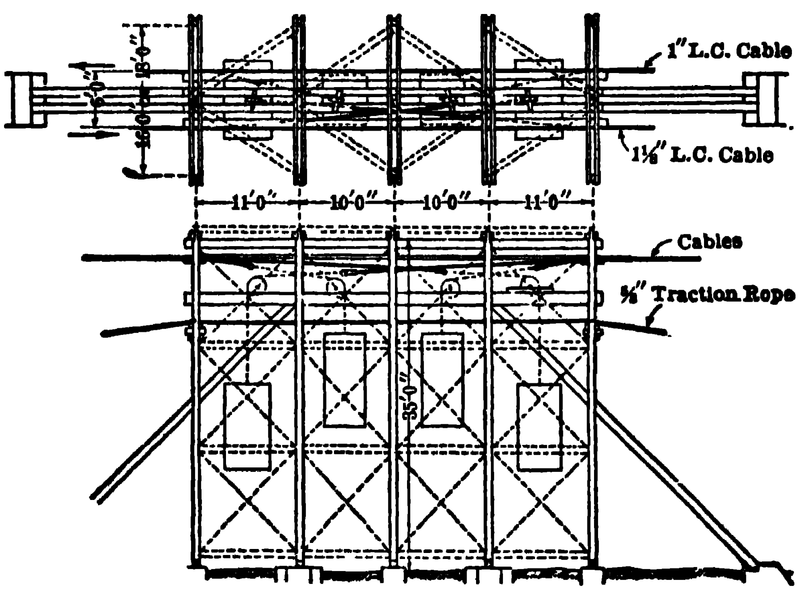


Fig 24. Double-tension Station

Tension weights are attached to chains or wire rope, passing over sheaves on short axles, resting in bearings. Horis pull is transferred to main beams by gains in the timbers, into which the bearings fit. Main beams support the vertical load and transfer horis stresses to inclined braces, which in turn transfer them to foundations. Inclined braces may develop an upward thrust when design is such that the full tension box does not come on end of inclined brace, or when all cables are not under tension. These upward forces are resisted by giving the foundations a mass equal to the uplift, and bolting the posts to them. Tension stations may be no higher than anchorage stations when take-up tackle replaces floating weights (see end of Art 6*a*).

Anchorage-and-tension stations provide for anchoring the cables of one section and applying tension to those of the other. They are practically half of a double-tension station, combined with half of a double-anchorage station of same height.

13. ANGLE AND SUMMIT STATIONS

Angle stations are required for horiz deflections. Main cables are terminated, traction rope passes through and is deflected by sheaves; carriers are detached from incoming traction rope, run over a rail bridging gap between cables, and reattached to outgoing traction rope. Fig 25 shows a simple station, with both pairs of cables anchored; in it, the carriers detach automatically, but 2 men are required to reattach them to traction

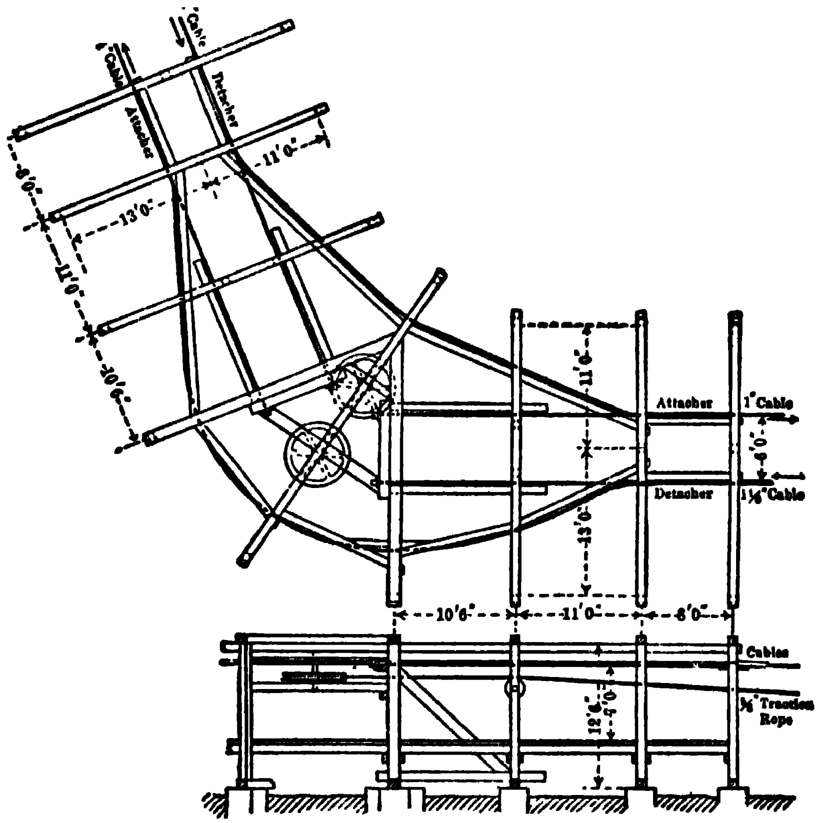


Fig 25. Angle Station

rope. Occasionally, angle stations are designed so that carriers detach and reattach automatically, or are arranged so that they make the turn without being released. Either arrangement complicates details and adds to first cost of the tramway, but reduces operating expenses by saving attendants at angle station. See end of Art 16 for similar problems. An angle station may be introduced at a junction, between two divisions of a tramway, without increasing first cost or operating expense (Art 17).

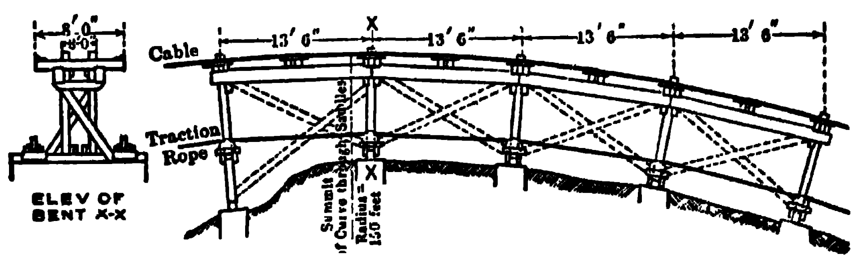


Fig 26. Summit Station, Traction Rope under Carrier

Summit trestles are introduced at crests in a line to save erection of a group of towers (see near end of Art 6), and carry a series of saddles with a slight bend of rope on each (Fig 26); they have limited application; a rail station (see below) being preferable.

Rail stations are similar in purpose to summit trestles with saddles. The main cables are carried through the station, but rails are mounted above them, on which the carriers run, and so relieve cable wear. Each end of the rail has a point, to lead the carriage from cable

to rail and let it down from rail to cable (Art 16, par 3). Rail stations are used where rolling press is heavy, due to heavy loads, limited space for structure, heavy tension in traction rope, or where conditions prevent a reduction of vert component.

Traction rope, in both forms of summit station, is supported at a uniform distance below the track. The larger the radius of track cable, or rail, the less will be the deflection of traction rope at the grip, and consequently both press and wear of carrier wheels on the track are reduced. Fig 26 shows a trestle with saddles set on an arc of 150-ft radius, with rollers below the carriers. Downward stresses due to traction rope could be reduced for this trestle by increasing the radius of the curve through the saddles; or, if the design of grips and carriers permit, by supporting the rope on rollers close under the grips (Art 6*a*). Vert stresses are found by similar methods to those in Fig 14 *a*, *b*, 18.

14. POWER REQUIRED OR DEVELOPED BY TRAMWAY

Tensions. Given: wt of individual load = l; wt of empty carrier = e; spacing of carriers = d; wt per ft of traction rope = r. Then the weights per ft of uniformly distributed load corresponding to wt of carriers plus wt per ft of traction rope, are:

For loaded side, $w_l = \dfrac{l + e}{d} + r$ (34); for empty side, $w_e = \dfrac{e}{d} + r$ (35)

If w be the total uniformly distributed wt per ft for any case, then tension in traction rope at upper end of line (Eq 29) is $t = wV \pm wf'H$. If load is descending, the sign of last term is minus; if ascending, plus. Total tension equals the tension thus found plus that put into the rope by tension wt applied to a sliding sheave. The tension added to each rope by tension wt = t_w.

Power. Let gross tension on taut side = T; and gross tension on slack side = S. Then $(T - S)$ is the force to be applied to move the traction rope, or to be opposed to traction rope, to restrain it. This force multiplied by veloc of rope in ft per min (v) gives the work done on or by the traction rope per min; hence, power required or developed (omitting friction of terminal machinery) is

$$\text{H p} = \frac{(T - S)\,v}{33\,000} \tag{36}$$

Friction losses. Power thus computed will be increased when power is required, or decreased when power is developed, by friction of driving machinery at terminals. If friction coeff of this machinery be 0.33 $^1/_3$% of its wt, the power to overcome friction will be

$$\text{Friction h p} = \frac{0.00\,^1/_3\,Wv}{33\,000} = 0.000\,000\,1Wv \tag{37}$$

in which W = wt of moving terminal machinery in lb, and v = veloc of rope in ft per min. Combining Eq 36 and 37, gives total power of tramway as

$$\text{H p} = \left(\frac{T - S}{33\,000} \pm 0.000\,000\,1\,W\right)v \tag{38}$$

Inertia. At instant of starting, inertia of the line is to be overcome, and moving resistance is higher than when running. To insure sufficient starting power, the following empirical rule may be used: in determining the power developed by a gravity line in starting, divide wt of individual load l by 2; or for the power required to start a power-driven line, multiply l by 2, before substituting in Eq 34.

Return freight may be carried on a tramway. This at times increases tension on slack side, a condition which must be recognized in power calculations. On a gravity line considerable up freight can be carried when line is running at full capac of descending loads, but when there is little descending material, driving machinery must be supplied, for use at such times.

To solve Eq 36 and 38, T and S must be known. But when a drive is first investigated, these factors are unknown, since each is composed of the stresses due to loads, plus half the tension wt, the amount of which has not yet been determined; but, as half the tension wt is applied to each side, the value of $(T - S)$ will be equal to the difference of the stresses on the two sides due to the loads, as determined by Eq 29, without considering tension wt.

Tension weight. A sheave which will furnish friction necessary to drive or restrain the traction rope is found by Eq 31, by substituting the value of $(T - S)$ as above and the value of f and n suitable for the friction surface, and number of half laps under consideration, and then solving for S, the tension on slack side. If S be less than the tension

for this side as found by Eq 29, no tension wt need be applied by sliding sheave to provide the necessary friction. If S be greater than the tension on empty side, the deficiency must be supplied by a tension wt equal to twice this difference, applied to a sliding sheave. Trial calculations may indicate several possible types of drive with their respective tension weights, and the merits of each can be considered. If S be less than the tension produced by loads on slack side, a sliding sheave is needed simply to take up slack in traction rope, and the wt applied need be sufficient only to keep the rope taut near sheave. If the resulting tension, after tension wt is added, is greater than necessary, it shows that the power sheave is capable of doing more than is required. Having found the wt to be applied to sliding sheave, T is found by adding t_w to tension on taut side, due to the loads as found by Eq 29. This is the stress that finally determines size of traction rope required.

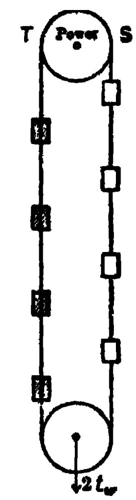


Fig 27. Power Machinery at Upper End

Application of these principles to the typical tramway, Fig 11, illustrates method of computation. Fixed data for this line are: wt of load = l = 800 lb; wt of carrier = e = 420 lb; spacing of carriers = d = 384 ft; wt of traction rope = r = 0.89 lb per ft; horiz length of tramway = 6 900 ft; difference in elev of ends of tramway = 1 866 − 350 = 1 516 ft. As carriage wheels have plain bearings, f' = 0.02.

Let Fig 27 represent CONTROLLING MACHINERY AT UPPER END, the most probable arrangement for this tramway. Calculation of forces developed by descending loads and required for ascending loads, to accompany the figure, is as follows:

LOADED CARRIERS DESCENDING (By Eq 29, $t = wV \pm wf'H$)

Loaded side; descending.	Empty side; ascending.
$w = w_l = \frac{l+e}{d} + r = \frac{1\,220}{384} + 0.89 = 4.07$	$w = w_e = \frac{e}{d} + r = \frac{420}{384} + 0.89 = 1.98$
$t_l = 4.07 \times 1\,516 - 4.07 \times 6\,900 \times 0.02 = 6\,170 - 562 = 5\,608$	$t_e = 1.98 \times 1\,516 + 1.98 \times 6\,900 \times 0.02 = 3\,002 + 273 = 3\,275$

$$(T - S) = t_l - t_e = 5\,608 - 3\,275 = 2\,333 \text{ lb} = \text{force developed.}$$

LOADED CARRIERS ASCENDING (By Eq 29, $t = wV \pm wf'H$)

Empty side; descending.	Loaded side; ascending.
$= w_e = \frac{e}{d} + r = 1.98$	$w = w_l = \frac{l+e}{d} + r = 4.07$
$t_e = 1.98 \times 1\,516 - 1.98 \times 6\,900 \times 0.02 = 3\,002 - 273 = 2\,729$	$t_l = 4.07 \times 1\,516 + 4.07 \times 6\,900 \times 0.02 = 6\,170 + 562 = 6\,732$

$$(T - S) = t_l - t_e = 6\,732 - 2\,729 = 4\,003 \text{ lb} = \text{force required.}$$

In above case, for descending loads, $(T - S) = 2\,333$ lb. For a gravity line, controlled by hand brakes, a pair of plain sheaves might be used (stresses being too great for wood filling). This assumption gives $f = 0.085$ and $n = 2$; hence from Table 3, $e^{f\pi n} = 1.706$, which substituted in Eq 31 gives $(T - S) = S(e^{f\pi n} - 1)$, or $2\,333 = S(1.706 - 1)$; hence $S = 2\,333 \div 0.706 = 3\,300$ lb. This practically equals value of t_e; hence t_w might be zero if there is to be no margin of power. For a power margin of 25%, new value of S is 4 125; substituting this in the expression $S = t_e + t_w$ gives $t_w = 4\,125 - 3\,275 = 850$ lb; hence tension wt should be 1 700 lb, which is reasonable.

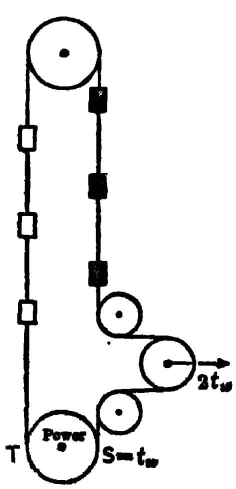


Fig 28. Power Machinery at Lower End

To haul the same loads up the line, a grip sheave with steel jaws (Fig 29) would be necessary to develop required pull. Let the jaws have a ratio of 1 : 3; then $f = 3 \times 0.085$, $n = 1$ and $e^{f\pi n} = 2.228$ (Art 3). Substituting these values in Eq 31 gives $(T - S) = S(e^{f\pi n} - 1)$, or $4\,003 = S(2.228 - 1)$ and $S = 4\,003 \div 1.228 = 3\,250$ lb, which is greater than t_e. Hence, with no margin of power, a tension wt of $2(3\,250 - 2\,729) = 1\,042$ lb is essential for power purposes. It is well, however, to add 25% and make up the increased tension on slack side by adding to the tension wt.

Another combination occurs with CONTROLLING MACHINERY AT LOWER END of tramway (Fig 28). Then the tension in slack side due to loading will be zero, and the only tension on slack side of driving sheave will be that given by tension wt attached to sliding sheave; that is, $S = t_w$. Tension on taut side will be $T = (w_l - w_e)V \pm (w_l + w_e)f'H + t_w$; the plus sign being for ascending loads, the minus for descending. From these equations the force to control movement of loads $(T - S)$ can be determined. For ascending loads, power is always required, but for descending loads there are two combinations: (1) on a steep slope, first term may be the larger and the value

of $(T - S)$ positive, indicating that force is developed; (2) on a gentle slope, second term may be the larger and $(T - S)$ negative, indicating that force is required to pull the loads down. With large powers, either required or developed, it is not practicable to place power machinery at lower end, because all tension on slack side must be obtained artificially, and by so doing tension on loaded side is increased prohibitively. Value of $(T - S)$ is substituted in Eq 31, together with trial values of f and n, and the value of S is determined. Since $S = t_w$ for this arrangement of driving machinery, the required tension wt is known at once.

15. POWER MACHINERY

Rope drives on plain sheaves may have one or several half laps on the sheaves. A C-I sheave with a single groove, giving a half lap on the sheave, makes a satisfactory drive, but the pull exerted is small. Two or more grooves in a C-I sheave, where the grooves are positively connected, is objectionable, because the rope in first groove has more tension; hence this groove wears faster and becomes of smaller diam than the others, producing a differential stress of large but unknown amount. Multiple-groove drives are satisfactory if the grooves can slip or creep to compensate for this differential stress. Elliptical-grooved sheaves with several laps about them, as used on capstans, are not applicable to tramways, as the rotation of the rope down the slope in one direction will twist it and give trouble by tilting empty carriers.

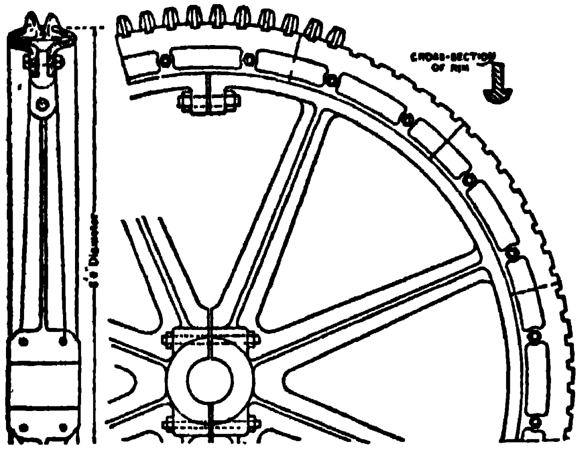


Fig 29. Hallidie Grip Sheave

Wood-filled sheaves can be used for transmission of considerable power, where $(T - S)$ is large, but actual tensions are small. If tensions are heavy, the wood will be cut out. RUBBER AND LEATHER-FILLED SHEAVES give max friction for rope drives, but are easily cut and not applicable to tramways.

Grip sheaves give a strong hold on rope for a single half lap on sheave, and do not produce differential stresses in rope. The Hallidie pattern (Fig 29) is well adapted to transmit considerable power, and does not injure traction rope. The jaws multiply the rope press by 3 or 4, increasing the friction. This increases f in Eq 30, 31 by same amount, whence the power transmitted by a single grip sheave is equivalent to that of a 3 or 4-grooved iron sheave. The grip sheave costs about the same as the multi-grooved sheave and its idler.

Band brakes may be used with machinery running constantly in one direction. The force absorbed by the brake is $(T - S)$ and ratio of T to S is given by Eq 30. Brake should be so arranged that the greater tension T will come at fixed end; then S is the pull exerted on other end in applying the brake. If the conditions are reversed, the max stress comes on movable end and the brake tends to release itself. The bands are about $^1/_8$ in thick, lined with blocks of maple to increase friction. Frequently operated brakes must not absorb too much power per unit of area, or they get excessively hot and destroy the blocks. A good rule is to allow 72 sq in of brake surface for each hp absorbed.

Automatic speed regulators for gravity lines save brakeman's wages and the tramway runs smoothly; on power-driven lines they assure constant speed. The best controller is an elec motor, belted or geared to the tramway drive sheave. It furnishes operating power when conditions prevent the tramway from running by gravity, and turns current back into the power line when being driven by the tramway. It will run about 5% under speed when acting as a motor, and 5% over speed when generating current. Current may be either d c or a c. If a c, the motor may be of constant-speed type to say 30 h p; above that, the variable-speed type avoids excessive line loads or starting shocks. Motor should have an ELECTRIC BRAKE capable of stopping the line if current fails; without a brake, the line would run away when the motor ceases to offer resistance. Hand brakes are also required, to shut down and hold the line, or for use when the motor is not acting. The starting device should admit current to the elec brake and release the brake when it connects the motor to power line. A d-c motor can be made to generate current under any line condition, but an induction motor acts as a generator only when connected to a line supplied with current by a separate a-c generator. Current turned back into the line can be utilized elsewhere.

Other automatic regulators having limited application: (*a*) hydraulic controllers, offering resistance by pumping water or oil against a pressure, which is automatically changed in proportion to the power absorbed; (b) fans running at high speed in the air, with blades arranged to swing out, like governor balls, thus acting with longer radius and more resistance as speed increases. They can be adjusted to keep tramway speed constant; (*c*) water may be pumped or air compressed to absorb surplus power; the water or air can be utilized or run to waste; (*d*) where mill or other machinery is adjacent, a tramway can be connected to the line shaft; the governor for the mill machinery then controls tramway speed and power generated by the tramway aids in driving the mill.

Brakes may be operated by levers, handwheels and screws, or combination of the two. Levers are quicker to operate, but not so powerful as handwheel and screw. The operating stand may be on the terminal floor or on an elevated platform. When brakes are operated by levers, the latter can be extended by rods to any point.

Power-driven tramway. When using less than 20 h p, connection to the vert shaft is conveniently made by bevel gears placed either at top or bottom end of shaft. With a gear ratio of 1:5, the horiz shaft runs at convenient speed for belting to motor or engine. When power exceeds 20 h p, torsion in vert shaft is considerable. This is avoided by bolting brake wheel to one side of main sheave and a spur gear to the other side and driving latter by a pinion mounted on a parallel vert shaft, the latter being driven through bevel gears from a horiz shaft. On heavy drives the sheave will usually be a grip sheave. Bevel gearing may be avoided by deflecting traction rope over a pair of vert sheaves, and causing it to pass under main driving sheave mounted on a horiz shaft. Sheave is bolted to a spur gear, driven by a pinion on a parallel shaft to which motor or regulator is geared. As the ropes tend to lift main sheave, the downward press of the shaft in its bearings is reduced. When the traction rope tension is sufficient to lift the driving sheave, with any gear or brake wheel attached to it, the bearings should be inverted, with lubricating grease well below the shaft, like a R R car wheel, the bearings being placed on under side of supporting beam. The connection between motor and tramway sheave may include a belt drive (not as reliable as gearing). Silent-chain drives and speed reducing gears have also been used. A machine-moulded spur gear attached to main sheave, followed by cut gears with rawhide pinion on motor, give a good drive at reduced cost.

Gravity lines. Where power is absorbed by automatic regulator, or utilized for any purpose, connection to main sheave is made by gearing, as above, for power driving.

Tension sheave at lower end may slide in a horiz or slightly inclined plane (Fig 32), or the rope may be deflected so as to make it travel vertically (Fig 40), or in any other direction.

16. TERMINAL AND JUNCTION STATIONS

Purposes. Every terminal station must be designed for 3 distinct functions: 1. To secure track cables. 2. To operate traction rope. 3. To transfer carriers from cables to rail, lead them to loading or discharging point, and return them to the cable.

1. **Cables are secured** in same manner as at anchorage and tension stations (Art 12), with changes in details to suit conditions.

2. **Traction rope is operated** by power drive, or controlled by a brake system at one end, usually the upper, and is tightened by passing over a sliding sheave, usually placed at lower end. Occasionally power machinery and tightener are at same end. Type and arrangement of controlling and tension mechanism depend on local conditions, and are subject to considerable modification (Art 14 and 15).

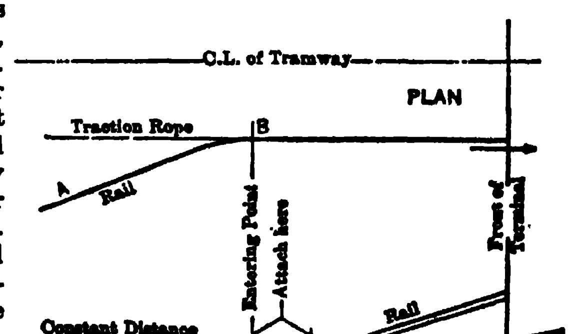


Fig 30. Attaching Point for Grips Opening Sidewise (Dotted lines show alternative position of rail and traction rope)

Manner of deflecting and guiding traction rope in terminals is influenced by the process of getting the rope into or out of the grip, and of attaching or detaching carriers. For a grip OPENING AT SIDE, the rope must be brought through a point where it will be opposite the grip jaws at the instant that grip comes into the vert plane through the rope, as the carrier moves toward rope in running along rail. Thus, in Fig 30, the carrier moves along rail from *A* to *B*, the grip travels in horiz plane *abc*, and when carrier reaches *B*, the traction rope must be at *b* so as to enter the grip, which is then closed, after which the carrier supports the rope. A second case, where rail and traction rope slope downward, is shown by dotted lines. In detaching, these operations are reversed.

For a grip OPENING DOWNWARD, the carrier moves so as to bring grip into plane of rope, but somewhat above it, then advances toward inclined rope, or is lowered onto rope, if it is horiz, until it bears in the grip when jaws are closed. Thus, in Fig 31, carrier moves from *A* to *B*, with grip above rope, then advances until point *C* is reached, when rope will be in jaws of grip, which can be closed. In detaching, the process is reversed. If grip jaws OPEN UPWARD, the grip will pass under the rope before taking it.

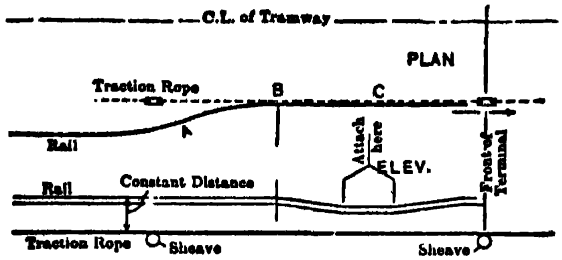


Fig 31. Attaching Point for Grips Opening Downward

3. Transfer of carriers, from cables to rail, or vice versa, must be done as smoothly as possible, to prevent lunging of carriers and hammering of cable where carrier leaves the rail. Lunging of the carrier is reduced by accelerating it on entering the attacher, to approx speed of traction rope. Experience shows that wear on cable is least when carriers run on to cable outside the terminal saddle, where cable is free to deflect when load comes on it. This requires a terminal point movable vertically. When carriers take the cable at a rigidly supported point, the cable is hammered and the wires are displaced and worn.

At terminal saddle, rail is placed parallel with track cable, but back of terminal point the rail is curved vertically so as to change from the inclination of cable to the horiz before reaching point

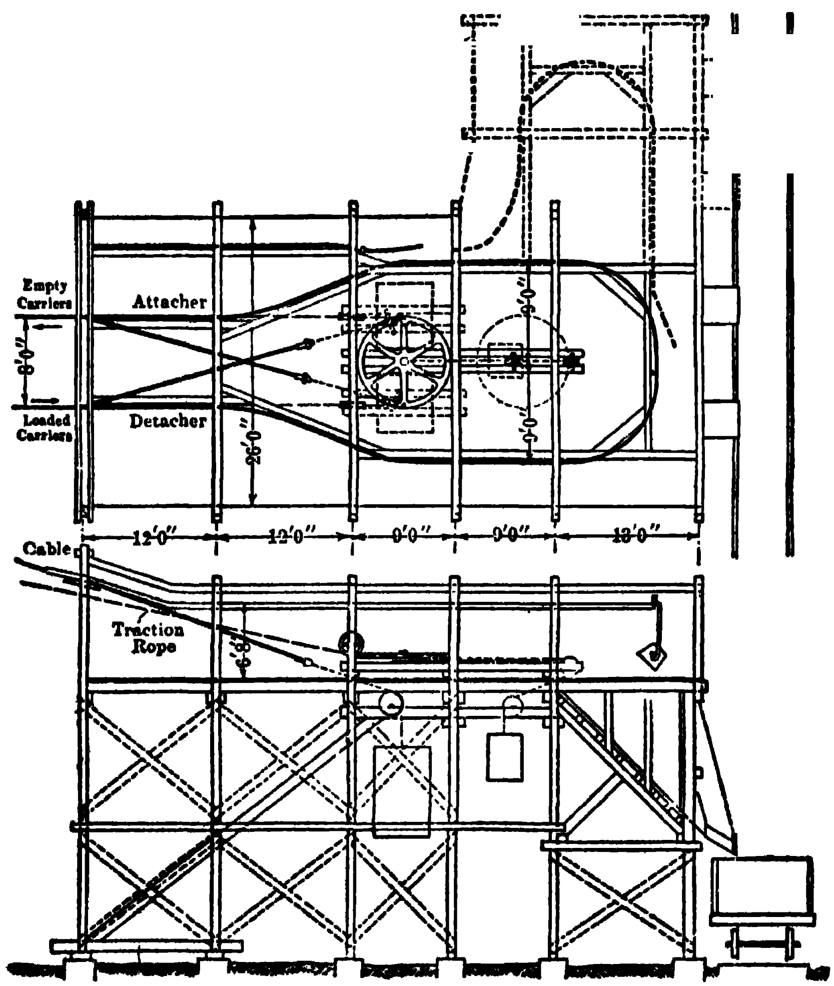


Fig 32. Discharge Terminal for Sidewise-opening Grip

where grip is released from traction rope. Where cables slope downward from a terminal, rails are bent on convex vertical curves at the front of a terminal, resembling a rail station at a crest, until they are parallel to slope of cables; the curved rail saves cable from downward press of carriage, and reduces wear on cable. When cables leave a terminal on an upward slope, the rails are bent on concave curves, and provision must be made for holding traction rope down, to make it change its direction without lifting carriers off rail.

Back of detaching point, the rail is bent horizontally on easy curves so that the incoming bucket, running at high speed, will take curves smoothly. On outgoing side, the curve leading to attaching point may be a little sharper, as the carriers are pushed by hand or move by gravity and travel at a slower speed. The rail must form a loop large enough for carriers to clear the terminal machinery and the framing used for its support. Rail can be extended by switches and loops into as elaborate a system as desired, to reach loading and discharge points (see dotted rails in plan of Fig 32). Rail

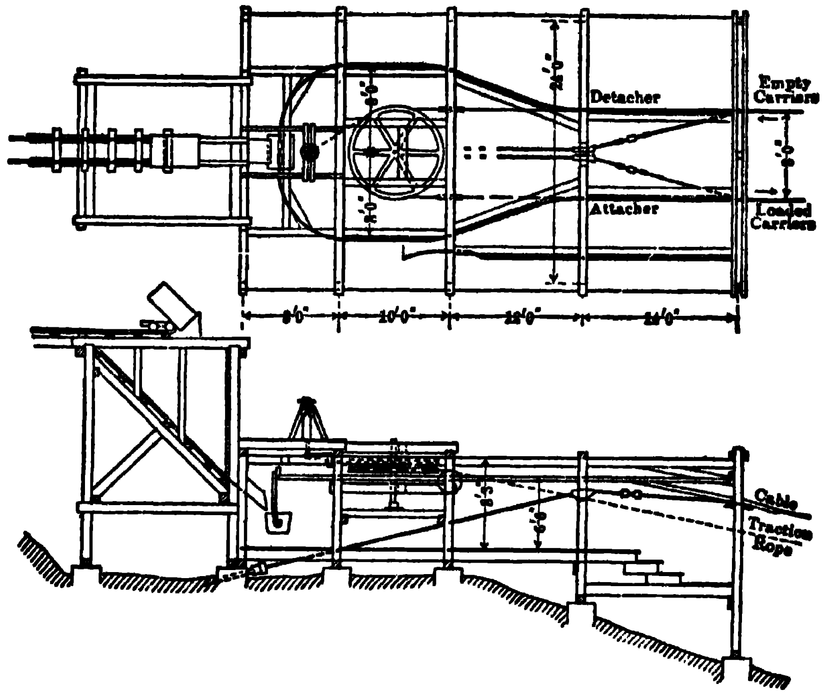


Fig 33. Loading Terminal for Sidewise-opening Grip

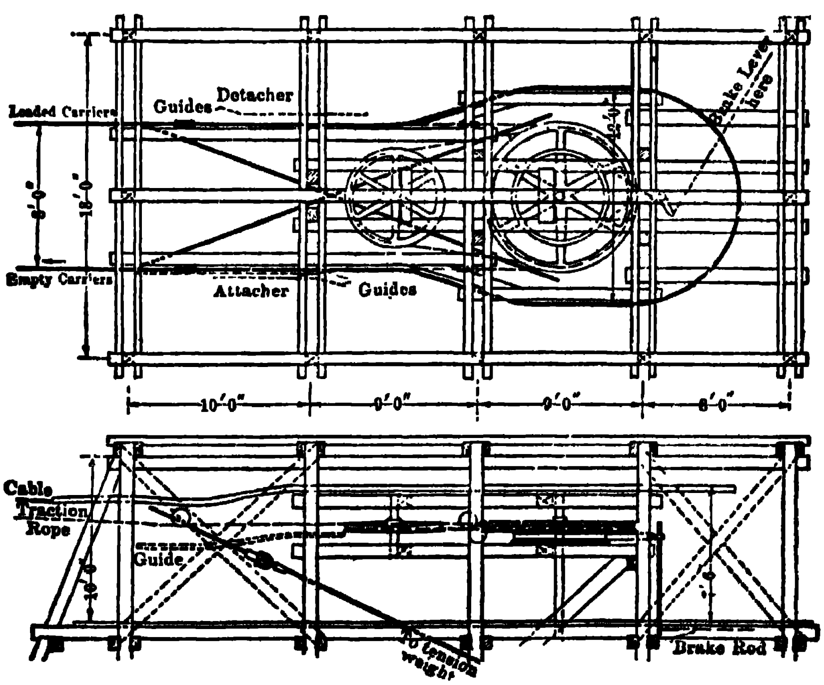


Fig 34. Terminal for Downward-opening Grip

is often graded, so that parts of the run can be made by gravity. At a loading terminal for bulk materials, carriers are filled from bin chutes, and at discharge end are run over pockets into which they are dumped. Fig 32 and 33 show typical arrangements of terminals for tramways with grips opening at side, with horiz curves to release traction rope from grip. Fig 34 shows a terminal for grips opening downward. In this the horiz curves are not necessary for gripp'ng operations, but are required for the carriers to clear the center posts.

Quarry loading terminals may be designed to transfer buckets from tramway hangers to flat cars, for conveyance to quarry face. Larger rocks than those that will pass through bin gates, or material too sticky to handle in chutes, can thus be loaded directly into buckets. The loaded cars are hauled back to tramway terminal and the buckets replaced in hangers (tramway makers 1–8, Art 29). Buckets can be transferred from hangers to cars by running the car track under tramway rail and grading track or rail so that the hanger hooks or unhooks from the bucket as required, when car and hanger are moved along together.

Transfer by a mechanical lift (maker 5, Art 29); see *The Engineer*, London, Oct 9, 1936. At loading terminal of a tramway transporting clay to brick works at Reading, England, buckets are hauled to the pit on cars and filled by an excavator. The cars have a platform movable vertically about 6 in by a lever. To transfer an empty bucket to a car standing beneath it, the operator raises the platform under the bucket and lifts it out of the hooks on tramway hanger. After loading, the platform is lowered so that the bucket trunnions again ride in the hanger hooks (Maker 6, Art 29).

Underground tramways and terminals. Where entrance to mine is by tunnel, with its mouth in steeply sloping ground, the rear part of the loading terminal has been built in an underground chamber below the tunnel, with an ore pocket at tunnel level.

At San Francisco del Oro mine, Mex, a bi-cable tramway extends 4 715 ft from discharge terminal at mill to a point where tramway makes a deflection angle of 67°, enters an adit, 3.5 m high by 4 m wide, and runs underground 3 000 ft to a terminal from which loops extend to 2 loading points at bottoms of old stopes lined with concrete to form storage bins. At one loading point the

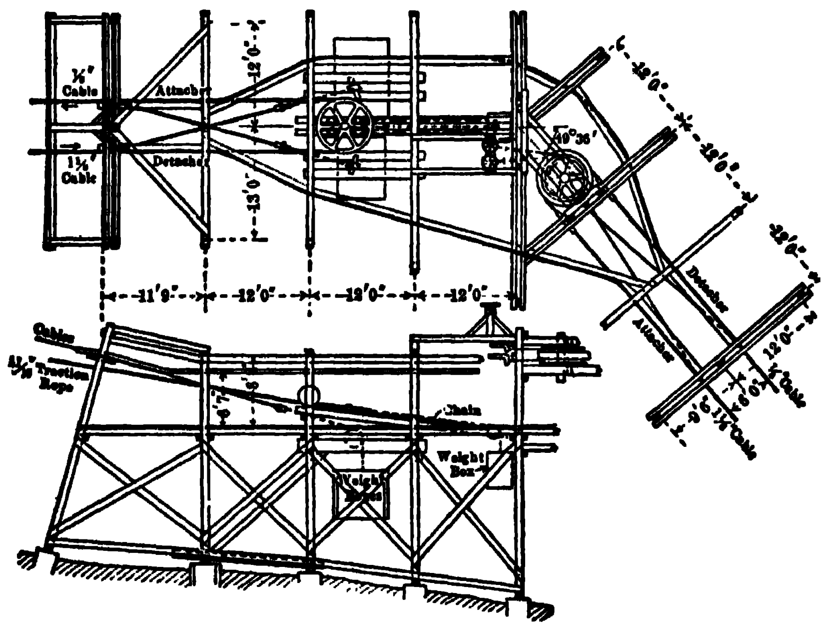


Fig 35. Junction Station

rails are hung from bottom of the bin; at the other, supported from the floor. From the terminal, the track cables are carried on rocking saddles, 30 m apart, resting on hangers suspended from 8-in channels concreted into hitches in the adit walls. The traction rope, free for full length of adit, is supported entirely by the buckets. Tramway cables are locked-coil, 1.5 and 7/8 in; carriers weigh 200 kg and hold 800 to 1 000 kg of ore, giving a capac of 100 to 125 metric tons per hr. Traction rope is 5/8 in, plow steel; speed, 150 m per min. Buckets are filled by hand-operated chutes and coast to the terminal, where they are dispatched at proper intervals, travel to the angle station where they detach; then coast through structure and re-attach automatically. From mill terminal, buckets coast to bin, where a tripper is set, dump load, and run to return side. For a round trip, a bucket is handled by a loader and a dispatcher at each end terminal. Cost of operating the above and a feeder tramway for more than a year was 2.3¢ U S cur per metric ton per km; covering labor, power, supplies and rope replacements (14). Maker 1, Art 29.

Automatic discharge terminals can be built to return carriers without detaching them from the rope and without attendance. They contain a horis tail sheave, usually 12 ft diam, around which the traction rope travels and, parallel with it, a curved rail on which the carriers run. At the terminal, buckets leave cable and run on a rail, dumping in transit either when on terminal loop or when on cable outside of terminal; the carriers, preferably empty, pass around the loop and run off the rail onto the return cable. This eliminates labor at the terminal, but requires a large diam sheave, to which the speed of travel for whole line must be proportioned, to keep down the centrifugal action of the carriers in making the end turn. Tramway speed is 300 ft per min with 12-ft sheave and 500 ft with 16-ft sheave. Loading terminals are of the usual type (Fig 33, 34), including

release of the carrier grips. **Angle stations** can be designed so that carriers will pass around them at low speed without being detached from the rope; but more carriers are needed than for the ordinary tramway, thus increasing direct stresses on the line. The grips must be designed to pass around the sheaves. These conditions increase first cost, but decrease operating costs. Makers No 1–8, Art 29.

Junction stations are practically 2 terminals placed back to back where divisions of a long tramway meet. An angle may occur between center-lines of the divisions without increasing cost of operation, and with only a slight complication of design. Fig 35 shows a junction station at an angle, which contains the driving machinery for one division and the tension sheave for traction rope of the other. The cables from one right-hand division are deflected and anchored; those from the other being stretched by tension weights.

17. TRAMWAY INSTALLATIONS

Ordinary tramways. Until about 1920, bi-cable tramways had capacities under 100 tons per hr; net loads seldom exceeded 2 000 lb; carriages had two 8-in, or 10-in wheels; many are still being built. Following examples are illustrative; see also Bib 13.

Argentine Government tramway, built about 1910, connects a mining field in Mejicana Mts with smelter and RR at Chilecito. Length, 34.3 km (21 miles); fall, 3 528 m (11 600 ft). As there are 8 divisions, each with its own drive and traction rope, the line consists of 8 indeoendent tandem tramways, connected by junction stations; at these the buckets, detached from incoming traction rope of one division, coast on rails to outgoing rope of the next and are there attached automatically. At several junctions the line makes horiz angles. Control machinery for divisions is at junction stations, each division being subdivided into sections by suitable anchorage and tension stations for the track cables. Country is rugged, requiring rail trestles on crests, and long spans over ravines (2 of 258 m and 540 m); through one sharp crest is a tunnel 150 m long, 4.5 m wide, by 4 m high. All structures are steel. Capac of line, 40 tons per hr of ore down from mine, and 4 tons per hr up freight. Buckets hold 1 125 lb ore; speed, 510 ft per min. Track cables are both locked- and smooth-coil, 35–30 mm diam on loaded side and 26–20 mm on empty side (approx 1 3/8–1 3/16 in and 1–0.75 in). Cables are stressed by weights hung on wire ropes passing over sheaves. Cables oiled by traveling oiler (Maker 7, Art 29) (15). See Bib.

Spanish Peak lumber tramway transports lumber from saw mill in forest to planing mill at Gray's Flat, Calif. Length, a little over 5 miles. From loading terminal line rises 1 200 ft in 3 miles and then drops 2 100 ft to discharge terminal. Loads consisted of packages of lumber not over 16 by 32 in by 32 ft, containing 300–600 bd-ft, weighing about 1 250 lb; hung by chain slings from a carrier near each end. Nominal capac, 10 000 bd-ft per hr, about 15 tons. Loads were sent out every 3 min, spaced 1 242 ft apart. Lang-lay traction rope, 5/8 in diam, runs normally at 414 ft per min. Cables are locked-coil, 1 1/8 and 7/8 in. Running normally, line develops about 10 hp, but requires 20 hp when loading up empty line. It is controlled by two 24-in Pelton water wheels opposing each other; a governor admits water to forward-turning wheel when tramway slows down 5% below normal, and to reverse wheel when speed exceeds 5% above normal.

Tramway was built in 1916 by maker I (Art 29); in 14 years carried over 150 million bd-ft of lumber; idle 5 years and put to use again in 1935, by Meadow Valley Lumber Co, Quincy, Calif (16).

Consol Coal Co, W Va, installed a 3 400-ft bi-cable tramway for mine waste disposal. The first half passed over a summit and across a ravine with a clear span of 1 700 ft, from which buckets were dumped. Tramway operated by 1 man at loading terminal; the empty buckets detach automatically, coast to loading chute, and are filled from an air-operated under-cut rotary gate. Loaded buckets coast to an automatic dispatcher which released them at intervals to coast into an attacher, gripping them to traction rope. On the long span, the rotating self-righting buckets passed through a frame which dumped them automatically; they then went to outer terminal, passed around a horiz sheave without detaching and returned to loading terminal, which contains driving and traction rope tension machinery. Outer terminal contained return sheave and tension weight for track cables. Buckets, hung from 4-wheel carriages, held 0.75 cu yd (2 000 lb). Traction rope, 7/8-in; driven 350 ft per min by 75-hp motor. Track cables, locked-coil, 1.5 and 1 1/8-in; tensions, 62 000 and 34 000 lb (Maker 1, Art 29) (17). Line had capac when built of 30 cu yd per hr, to be increased to 50 cu yd by adding more buckets and reducing the spacing from 525 to 315 ft, with 54 sec time interval.

Benguet Consol Mining Co, Philippine Is, has a 49 500-ft tramway with fall of 873 ft, in a straight line, driven by power as one unit; 0.75-in endless traction rope is nearly 19 miles long; buckets, 6 cu ft; capac of line 15 tons per hr; 8 spans are 2 770–5 180 ft long. Flat profile, grade in favor of loads and light tonnage, give a low traction-rope tension (Maker 1, Art 29).

Riblet Tramway Co has built bi-cable tramways containing unique features. NORTHERN PERÚ MINING Co has 4 interconnected tramways, connecting 3 mines with mill and smelter, so that ores, concentrates, smelter products and supplies can be sent to any point without reloading buckets. The system, 30 miles long, was put in operation in 1927. One division, 9 000 ft long, feeding smelter with coal and ores, has capac of 50 tons per hr; the others, 15 tons per hr. Track cables are 1 1/8 and 1 in plow-steel smooth-coil, and 1 3/8 and 1 1/8-in cast-steel locked-coil. Steepest incline, 35°; longest span, 4 351 ft. Traction ropes, all 0.75-in plow-steel Lang lay have speed of 500–550 ft per min; 10 motors, totaling 510 hp, start the line, but when running only 7 are needed, with total capac of 410 hp (11, 12). In BOLIVIA, a 6.5-mile tramway starts at mine at elev of

14 000 ft, crosses summit of Andes at 16 000 ft and ends at concentrator at 9 000 ft. One span of 3 000 ft crosses a chasm so rugged that a 7-mile detour is required (11). At GLORIETTA, N Mex, a tramway with capac of 80 ton per hr, has 2 divisions at an angle; junction station contains the controls for both; 1 span is of 4 000 ft. HYDER, Alaska, tramway is 11 miles long, in 1 unit. Unusual features: (1) a continuous traction rope; (2) 3 angles in the line, all deflecting in same direction; combined deflection, 171°, so that the direction of travel at the ends is nearly reversed. Line has speed of 600 ft per min; was designed for capac of 15 tons per hr, which has been greatly increased (12a).

Heavy-duty tramways. Since 1920, there has been demand for tramways of larger capac; an important recent application is in hauling aggregate for concrete at dam sites, the concrete being carried and deposited by cableways (Art 26). Bi-cable lines with capac to 300 tons per hr, have been built to carry 4 000-lb loads suspended from carriages with four 12-in wheels. FOR RAPID LOADING, a measuring hopper holding 1 bucket load, may be filled by a chute with under-cut arc gate; or, if material is coarse, by pan conveyer or vibrating feeder; hopper discharges quickly into the buckets, which can be dispatched at rate of 3 or 4 per min.

A typical plant was used on the PARDEE DAM, for water supply for San Francisco Bay cities. Length of line, 18 255 ft, total rise, 389 ft. One summit was 456 ft above loading terminal; near discharge terminal a river was crossed by a span of 1 320 ft. Carriers were 32-cu ft end-dump buckets, holding 3 000 lb, hung from 4-wheel carriages; speed, 470 ft per min; spacing, 192 ft; time interval, 24.5 sec; capac, 220 tons per hr. Track cables, locked-coil, 1 $^5/_8$ and 1 $^1/_8$ in, except for river span, where they were 1.75 and 1 $^3/_8$-in Cables of first division, anchored at loading terminal and the angle station, were stressed by concrete weights at a double-tension station about the middle of the section. Cables of second division, anchored at the angle station and a double-anchorage station, were stressed at a tension station near the middle. The river span, with larger cables, was stressed independently from discharge terminal. All structures of wood. Traction rope, $^7/_8$-in; that for the first division was driven from angle station; for second division, from discharge terminal; each drive comprised a grip sheave and spur gear, driven by a 125-hp motor. Traction-rope slack was taken up by floating sheaves at ends of each division. At loading terminal, empty buckets detached from rope coasted to the loading chute, were filled in 4 or 5 sec, and coasted to the dispatcher, which held the bucket until the one ahead had traveled the length of the spacing; then released it to coast down grade to acquire the traction-rope speed; the attacher closed the grip and the bucket departed. At the angle station, empty and loaded buckets, detached from traction rope, coasted on rails between the divisions, and took the out-going rope of the other division. At discharge terminal, buckets coasted across the bins, dumped while in motion by trippers, rounded the loop, gripped the outgoing rope, and began their return trip. All operations were automatic, supervised by 1 man (18) (Maker 1, Art 29). At CONCHAS DAM, N Mex, a bi-cable tramway was built in 1937–8 to transport concrete aggregate. Region rather flat, favoring a truck road, yet contractor decided low cost of tramway operation more than offset its greater cost. Line crossed river with 1 085-ft span. High winds are common, but caused only slight delays to operations. To prevent freezing of wet aggregate in zero weather, steam was forced into load at lower terminal, preventing freezing in transit. The line was unloaded at night during cold weather; the bucket loads being reduced 1.5 cu ft successively until all were empty. Operating crew per shift: foreman, 4 men at terminals, and a lineman. Details of tramway are: length, 9 963 ft; rise, 235 ft; capac, 224 tons per hr; 36-cu ft end-dump buckets, with 4-wheeled carriages, were spaced 264 ft; net load, 3 600 lb. Track cables, 1 $^5/_8$-in and 1-in locked-coil. Traction rope, $^7/_8$-in 6 by 19 Lang lay plow steel; speed, 550 ft per min; interval between buckets, 28.8 sec. Driven from lower terminal by 150 hp motor. At discharge terminal buckets turn around a 16-ft sheave without detaching (Maker 1, Art 29) (19). For details of other dam construction, see Bib 20–22.

Heavy-duty tramways have recently (before 1938) been built by Maker I (Art 29) as follows. U. S. GYPSUM Co, Alabaster, Mich. Line 6 800 ft long to dock in Lake Huron, to carry 260 tons per hr, with provision to increase capac to 300 tons, is carried on 8 towers 750 ft apart, built on cribs in lake. PENNSYLVANIA-DIXIE CEMENT Co, 1 mile long to carry 250 tons per hr. Buckets loaded and dumped automatically after line is started. One man oversees operation. CANYON COAL & COKE Co, Morgantown, W Va. Line 3 412 ft long transports 200 tons coal per hr in 62-cu ft end-dump buckets, hung from 4-wheel carriages. Having a fall of 150 ft, line runs by gravity, and develops 20 hp, which is absorbed by a 25-hp induction motor. Cables, locked-coil, are anchored at both terminals; stressed by weights at a double-tension station on the Cheat River, whence they make a clear span of 1 500 ft to discharge point and thence to anchorage. Buckets are loaded at a chute with air-operated under-cut arc gate, and are automatically attached to traction rope, hauled over the line and back to loading terminal where they coast to loading chute. Enroute they are dumped at the discharge point and returned around a tail sheave. Line speed, 450 ft per min; spacing of buckets, 195 ft = 26 sec time interval. Operated by 1 man, with another available for relief and chores at loading terminal (23).

18. COST OF EQUIPMENT AND OPERATION

Cost of a tramway comprises: 1. First cost of machinery. 2. Freight to the tramway site. 3. Cost of timber and foundation materials. 4. Distributing machinery and material along the line. 5. Framing and erection. 6. Installing machinery, including

cables, saddles, terminals, etc. 7. Superintendence and contingencies. If structures are of steel, cost of fabricated steel replaces cost of timber and its framing in items 3 and 5 above. Attempts to give costs are almost futile, as conditions and prices vary greatly, but the following discussion (repeated from 2nd Ed) will aid in estimating by indicating the items to be considered. See also the 1938 costs given below.

1. **First cost of machinery** for a bi-cable tramway of grip type, for aver conditions and ordinary equipment, in 1926, was approx as follows:

Table 6. Price and Weight of Machinery

	Capac, tons per hr	Cables, diam, in	Carriers, capac, cu ft	Spacing of carriers, ft	Price per ft	Wt per ft, lb
Line machinery, including tower equipment, cables, ropes and carriers. Towers taken as 250 ft apart; line speed, 500 ft per min	10	1 and 7/8	5	750	$1.60	8.0
	20	1 1/8 and 7/8	6	450	2.00	10.0
	50	1 3/8 and 7/8	12	360	2.40	12.0
	100	1 5/8 and 1	20	300	3 20	16.0

	Price	Wt, lb
Machinery for 2 terminals, including all metal-work, but no motor or automatic regulator	$4 000 to $6 000	24 000 to 36 000
Intermediate station machinery	$1 667	10 000

Motors, with solenoid brake, controller, transformer, oil switch, 500 r p m and 1 000–1 200 v, cost per h p; 25 h p, $50; 50 h p, $32; 100 h p, $24; 150 h p, $20.

2. **Freight** by R R, boat or team from maker's factory can be estimated from above weights and prevailing traffic rates. Hauling by motor truck on highway costs roughly 5–10¢ per ton per mile.

3. **Timber and foundation materials** are usually obtained locally. Timber needed: 2-in planks, 6–12 in wide; timbers 3 by 6 to 10 by 10 in; a few beams and struts, for terminals and intermediate stations, 10 by 12 to 12 by 16 in. No timber need be over 30 ft long. Foundations should be of masonry or concrete.

Average conditions require an intermediate station every 4 000 ft, which may be assumed to contain same material as an anchorage and tension station.

Table 7. Quantities of Materials in Structures

	Timber, ft B M	Foundations, cu yd
Loading terminal, no bins	10 000	50
Loading bin	8 000	10
Discharge terminal, no bins	15 000	50
Tower	80 per ft of height	Under 20 ft high = 4
Average height, 30 ft	2 400	Min = 6
Intermediate station:		
Double anchorage station	8 000	25
Double tension station	16 000	45
Anchorage and tension station	12 000	35

4. **Cost of distribution** is a local matter. Wt of machinery and quantity of timber are stated above. Length of haul to destination depends on topography, and these data, with local charge for hauling, form basis for estimate; roughly, say 50¢ per ton mile, from freight terminus to last points accessible to motor trucks or wagons.

5. **Cost of framing and erecting timber** depends on local wages and quality of labor. An approx figure is $50 per 1 000 ft B M for terminals, bins, intermediate stations and towers. Foundations, including excavation, approx $15–$20, per cu yd. STEEL STRUCTURES and towers cost in 1921–22, f o b factory, 3.5–4.5¢ per lb fabricated ready for erection.

6. **Cost of installation** varies greatly. Rough estimate is: for heavy cables, 3.5¢ per lb; light cables and traction rope, 5¢ per lb; machinery, 1.5¢ per lb. These figures include cost of distribution of machinery along the line, by dragging or sledding from end of truck or wagon haul.

7. **Superintendence and contingencies** may be figured as 20% of TOTAL ESTIMATED COST. If a quotation is obtained on the machinery, the allowance for this item may be put at 30% on the REMAINING ITEMS of the estimate.

On a 4-mile tramway in Tenn, erected 1903, in rolling country, where all parts of the line were accessible for teams, and discharge terminal was near the R R, costs per 1 000 lb were very low (see accompanying table).

Material	Unloading at R R and haul	Installing	Total cost per 1 000 lb
Cables	$2.76	$3.64	$6.40
Traction rope	Unloaded, pulled through towers, spliced and installed in one operation		7.00
Machinery	$1.02	$2.22	3.24

In general, cost of timber, framing and erecting the structures and installing machinery, on lines carrying 40 tons per hr and over, is a little under the price of machinery; on lines of less than 40 tons

per hr, a little over that price. Conditions under which tramways are built are so variable that it is difficult to specify averages; hence all the above prices must be taken as approx, and may easily be changed 25% by local conditions.

Quantities of material in structures, weights and costs of equipment and cost of erecting in 1927, were discussed and conclusions tabulated by Carstarphen (7).

In 1938, a tramway engineer for an important maker furnished following figures. Tramway costs: lumber, per M bd-ft erected, \$80–\$100; structural steel, per ton (2 000 lb) erected, \$160–\$200; installing cables and machinery, per ton, \$60–\$70. Quantities of concrete in foundations: 25 to 30-ft towers (higher towers take more), 4 cu yd; anchorage and tension stations (min), 25–30 cu yd; terminals (min), 30–40 cu yd. For costs of construction and operation of tramways, see Bib 8a, 12a, 19, 24.

Cost of operating consists of labor, repairs, interest on investment and a sinking fund allowance.

Operating labor for a line carrying 50 ton per hr or less will be: 1 brakeman, 1 laborer at each terminal and an oiler half time if line is not over 2 miles long. On heavier lines, 2 laborers may be required in each terminal and the oiler would work full time. Oiler should travel over the line daily, inspecting machinery, tightening loose parts and making petty repairs; when his whole time is given, he may be the line foreman. Automatic regulators, or a power drive, will save the brakeman; on light lines, loader may act as brakeman; sometimes buckets can be dumped automatically and all labor at discharge terminal be dispensed with.

Repairs for first 2 or 3 yr will be light, due mostly to accidents; in 4th and 5th yr, all parts will show wear, and renewals may amount to 2% per yr on first cost; after 5 yr, repairs may be 5% per yr on first cost.

Carstarphen discusses comparative costs of hauling by truck and by aerial tramway, and concludes that only on short hauls (say 1 mile) and moderate tonnage (as 100 tons per day) are trucks cheaper than tramways (24).

OTHER TYPES OF TRAMWAYS

19. TWIN-CABLE TRAMWAYS

These, invented by W. C. Lawson, have carriers running on parallel track cables on same level. Carriers have 2 or more sheave-wheels on each side and are hauled by a single traction rope (Maker 3, Art 29); they resemble a system of cars running on rails and hauled by a wire rope. If tramway is continuous, there are 2 pairs of track cables, one pair over the other; loaded carriers travel on the upper cables and empties return on the lower. At each end, traction rope passes around a vert sheave 6 or 8 ft diam, and carriers leave track cables and travel on rails, which are curved at the ends to parallel the sheave circumference, and having a radius about 18 in greater than the sheave radius.

In passing from upper to lower pair of cables, carriers are inverted for discharging; they return inverted to starting point and are righted in passing around the vert sheave at that end. As carriers are permanently attached to traction rope by clamps or sockets, they are filled while in motion by a mechanical loader, automatically drawing material from bin and measuring proper load. Speed of ropes must be slow enough to permit loading without spilling, and maintain a moderate centrifugal force at the curved ends. Fig 36, 37 show diagrams of terminals of typical automatic tramways (Interstate Equipment Corp).

Carriers may be attached to traction rope by grips; by detaching them, loading of other than bulk material is facilitated; but this eliminates some automatic features, and few lines have been thus designed. By detaching, return cables may be on same level and to one side of outgoing cables; carriers then return right side up, and may be used to haul back freight.

Towers are open through the center at their tops, and track cables are supported on rocking saddles attached to insides of tower posts, the carriers passing through towers and between saddles. Traction rope is supported on wide-faced sheaves, a little below clearance line of carriers.

Horiz angles can be turned by ending cables at a structure with curved track rails on which carriers travel and are hauled by traction rope as in a rope haulage system on the ground. After leaving the curve, carriers again run onto the cables. The curve must have a radius of 57 ft or more. As curve is horiz, the line speed must be reduced, or radius of curve must be large enough to prevent excessive centrifugal force.

Carriers for bulk material are usually shallow rectangular steel cars, of 10-60 cu ft capac for different hourly tonnages and weights per cu ft of load. In dumping, a baffle plate causes the material to flow instead of being dropped; this prevents breakage in case of coal.

Track cables are either locked- or smooth-coil, 0.5-1 5/8 in in diam, and are stressed as on bi-cable tramways (Fig 15 and Table 1). On short lines, the stressing devices are at

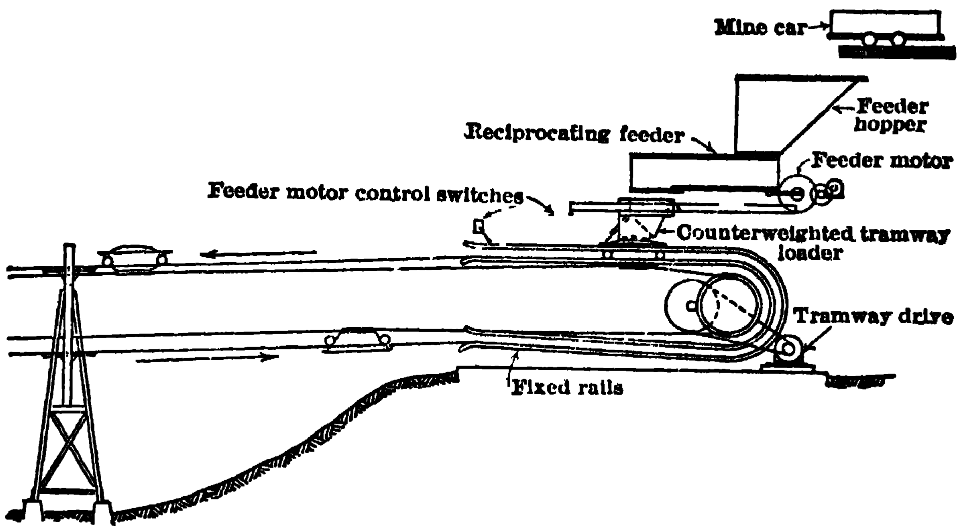


Fig 36. Loading Terminal, Automatic Tramway

one end; on long lines, the cables are anchored or stressed at intervals of 2 000-3 000 ft, to preserve the desired tension.

Traction ropes are 6 by 19 hoisting rope, their size and grade depending on working conditions; 0.5 in to 1 in are common; safety factor of 5 should be used. The rope is made up of lengths equal to spacing of carriers with sockets on both ends of each length; sockets are then attached to bottoms of carriers by pins, and tension is applied by equipping an occasional carrier with special splicing devices. The splice carriers are spaced not more than 5 000 ft apart, and have means for holding the traction rope in any position

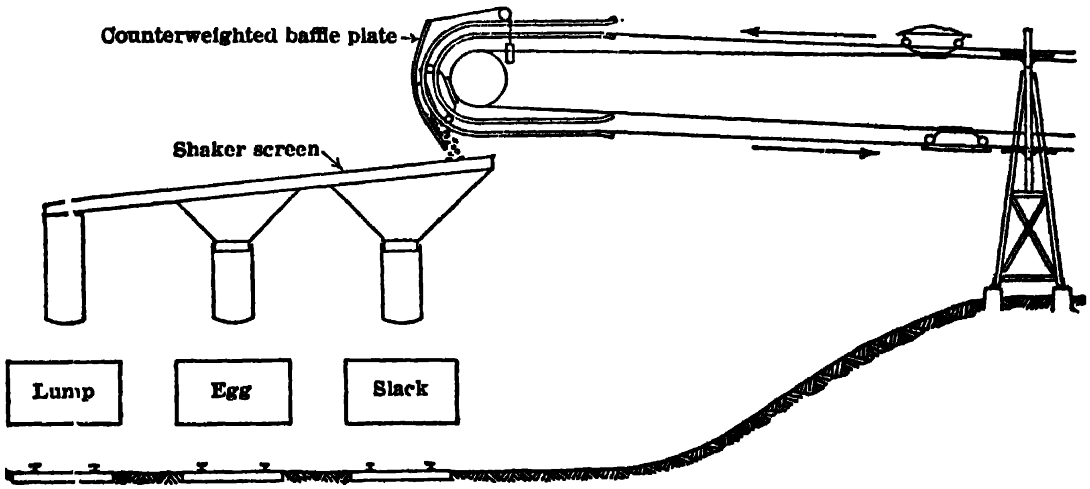


Fig 37. Discharge Terminal, Automatic Tramway

after being stressed to desired tension. When the rope stretches, the slack is taken up at a splice. When carriers are attached to traction rope by grips, traction rope is endless and slack can be taken up at a terminal by a sliding tension sheave. If loads cause very heavy tension in traction rope, it may be necessary to divide the tramway into sections, each dumping its loads into the bin of the following section.

Capacity. Lines have been built to carry 300 tons coal per hr, using 60-cu ft carriers running on 0.5-in locked-coil cables. Speed of travel directly influences capac: it is usually about 400 ft per min (Table 8).

Table 8. Recent Twin-cable Installations

	Horiz length, ft	Material	Capac, tons per hr	Fall, ft	Carriers, cu ft	Speed per min, ft	Traction rope, in	Track cable,* type & diam
Cold Spring Mining Co, Cold Spring, Va	13 000	Mine run kaolin	30	650	9	380	5/8	SC 5/8 & 1/2
Blue Diamond Co, Arden, Nev......	3 600	Mine run gypsum	80	700	20	400	3/4	SC 1 1/8 & 3/4
Nephi Plaster & Mfg Co, Nephi, Utah..	9 500	Mine run gypsum	75	700	18	400	5/8	SC 1 1/8 & 3/4
Keystone Mining Co, East Brady, Pa...	2 200	Coal	170	300	18	400	5/8	LC 1 1/8 & 3/4
Anchor Coal Co, Highcoal, W Va...	1 500	Coal	180	240	20	420	5/8	LC 1 1/8 & 3/4
Brule Smokeless Coal Co, Otsego, W Va.	4 000	Coal	150	100	40	450	5/8	SC 1 1/8 & 3/4 Plow steel
Stowe-Fuller Refract Co, Alexandria, Pa	5 000	Crushed stone	30	800	9	400	5/8	SC 3/4 & 1/2
Koppers Coal Co, Kopperston, W Va	600	Mine refuse	100	300 Rise	20	400	3/4	LC 1 1/8 & 3/4

* SC = smooth-coil. LC = locked-coil.

20. REVERSIBLE TRAMWAYS

These, sometimes called " jig-back " or " to-and-fro " tramways, consist of: 1. One or two track cables. 2. Traction rope for carriers. 3. One carrier on each track cable, moved to and fro by traction rope. 4. Station at each end for operating machinery, and filling or dumping carriers. 5. Sometimes intermediate towers are used to support track cable and traction rope, but are disadvantageous, as they limit speed to 1 000 ft per min, while without them carriers may be run at 2 000 ft per min. Carriers dump automatically at discharge terminal; the machinery is then reversed and carrier returns on same cable. One man fills the carrier and operates the line. Made by all tramway builders.

Limitation. Cost of erection and operation is low, but capac is limited by the reciprocating movement. With ordinary construction this limit occurs, with a 2-cable system, when product of tonnage per hr multiplied by distance in ft approximates 50 000; that is, a tramway 1 000 ft long can handle 50 tons per hr. With special construction much larger tonnages are carried. With a single track-cable, capac is a little less than half the above.

Sizes of carrier and cable are governed by same conditions as other bi-cable tramways; the load must not cause excessive bending in the cable (Art 6, 6a), but, as loads pass at longer intervals than on a continuous tramway and therefore cable receives fewer bends per hr, heavier loads may be used, subjecting cable to more bending than on a continuous tramway. When loads must be heavy to secure tonnage, carriages may have 4 wheels. For a cheap plant, the track cables may be smooth coil, or, for temporary use, standard 7-wire strand rope.

Track cables are seldom weighted, but are made taut by turnbuckles or wire-rope tackles. The cable tension is judged by deflection of empty cable at center of span. For a single span with anchored cable and single heavy load, see discussion at Fig 45, Art 27.

Carriers, when there are 2 track cables with 1 carrier on each, are clamped to traction rope so that one will be at loading terminal when the other is at discharge terminal; thus one is loading while the other is discharging. If there are no intermediate towers, the carrier hangers can be extended down from both sides of carriage, thus preventing carriers from jumping off the cable, and permitting higher speeds. At each end, in any case, carriers stop just before terminal saddles are reached, for loading and discharging. When loads are descending, the system will often operate by gravity like a gravity inclined plane. A reversible tramway may be operated with 1 track cable and 1 carrier. This is practically half a double system, and always requires power.

At loading terminal, when there are 2 track cables, they are sometimes brought close together, one above the other, so that a carrier on either cable can be loaded from one chute. Between the terminals the cables are far enough apart to permit carriers to pass each other. They may be brought close together again at the outer end.

Installations. The principle of reversible tramways can be utilized to transport very light loads to loads of several tons. In general, this tramway is the simplest and cheapest for moderate distances. Tonnage per hr is limited by long time interval between loads,

due to the intermittent action, and by size of the load. The time is fixed; size of load is the only variable, and when increased, the size of cable and strength of construction must be increased also, and an economic limit is reached if more than 100 tons per hr are carried.

Table 8a. Typical Installations of Reversible Tramways

Capac, tons per hr	Material	Length	Net load, lb	Loads per hr, cap + wt	Time interval, min	Loading time, min	Speed, ft per min
25	Coal waste	755	2 500	20	3.0	0.5	650
25	Coal	1 200	2 700	18.5	3.25	0.5	873
40	Coal waste	1 320	4 000	20	3.0	0.5	1 056
75	Coal waste	440	4 200	35.7	1.68	0.4	687
90	Coal waste	1 250	6 000	30	2.0	0.5	1 667
100	Coal	1 600	6 667	30	2.0	0.4	2 000
100	Coal	1 600	8 000	25	2.4	0.4	1 600

21. DESIGN OF REVERSIBLE TRAMWAYS

Fig 38 is a profile of a typical reversible tramway, with cables anchored at both ends; 1 tower; horiz length, 610 ft; fall, 228 ft. Assume line is of double-cable type, to carry 20 tons per hr, aver 30 loads per hr. This requires a load every 2 min. Allowing 0.5 min for loading, running time per load, is 1.5 min. On the steep grade shown (37.4%), the inclined length of 651 ft, divided by time of 1.5 min, gives running speed of 434 ft per min, which is reasonable for this length of line. Net wt of load is capac per hr + number of loads, or 1 333 lb. Assume nearest size bucket is 15 cu ft, and that it might be loaded with 1 500 lb. Its empty wt (Table 9) is 500 lb. Assume traction rope weighs 0.75 lb per ft.

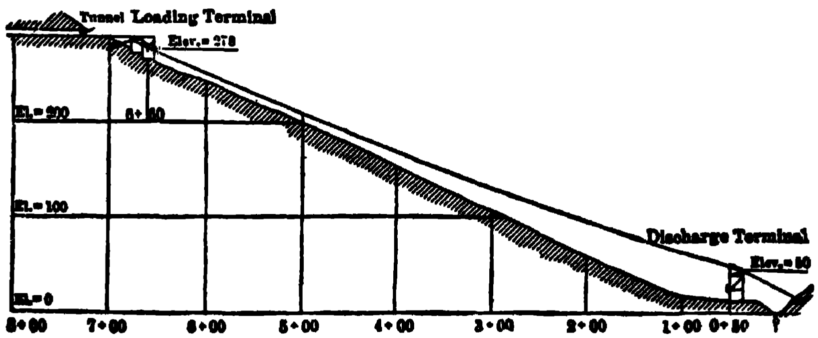


Fig 38. Profile of a Double Reversible Tramway

Track cables. Having chosen a load suitable for desired tonnage at a speed practicable for the length of line, the gross load = wt of carrier + net load + wt of traction rope hanging on carrier. As these can be closely determined and are not subject to increase, the size of track cable can be based on a load of 1 500 lb for each sq in of section for cast steel cable; then $W + t = 1\,500 + 34\,600 = 0.0435$; this is 25% more than is allowable for continuous tramways (Art 6, 6a), but this work is intermittent. Then, gross load + 1 500 = cross-sec of cable; this multiplied by 3.7 = wt per ft of whatever type of track is used, and by Table 1, the size of suitable track is found. If a light-weight rope is desired, one of higher grade steel but equal strength may be used. In this example, load is 2 250 lb; hence, $2\,250 + 1\,500 \times 3.7 = 5.55$ lb per ft of cable, which corresponds to 1.5 in C S locked- or $1\,^5/_8$ in smooth-coil cable.

Table 9. Weight of Rotating Buckets for Reversible Tramways

Capacity of bucket, cu ft......	6 to 8	10 to 15	18 to 20
Weight of empty bucket, lb...	400	500	700

Larger buckets, for coal, weigh about 30 lb per cu ft capac.

Path of load must be investigated. When cable is weighted, the position of loaded cable at different points is determined by Eq 13 or 19, but, when stretched by turnbuckle or tackle, and not weighted, position of loaded cable must be determined by method used for cableways (Art 27, Fig 45), because cable has fixed length, and tension is neither constant nor fully known.

Max stress in traction rope occurs when loaded carrier is on steepest part of line, which will probably be when it is at the top. Its size is determined by: Tension = (*wt loaded carrier* $\times$ *sine inclination of cable*) + wV + tw; in which wV comes from Eq 27,

and t_w = tension due to wt applied to sliding sheave. If the sheaves are large, the wear on rope will be mostly on its surface, and a 7-wire strand rope is suitable; for small sheaves, 19-wire strand should be used.

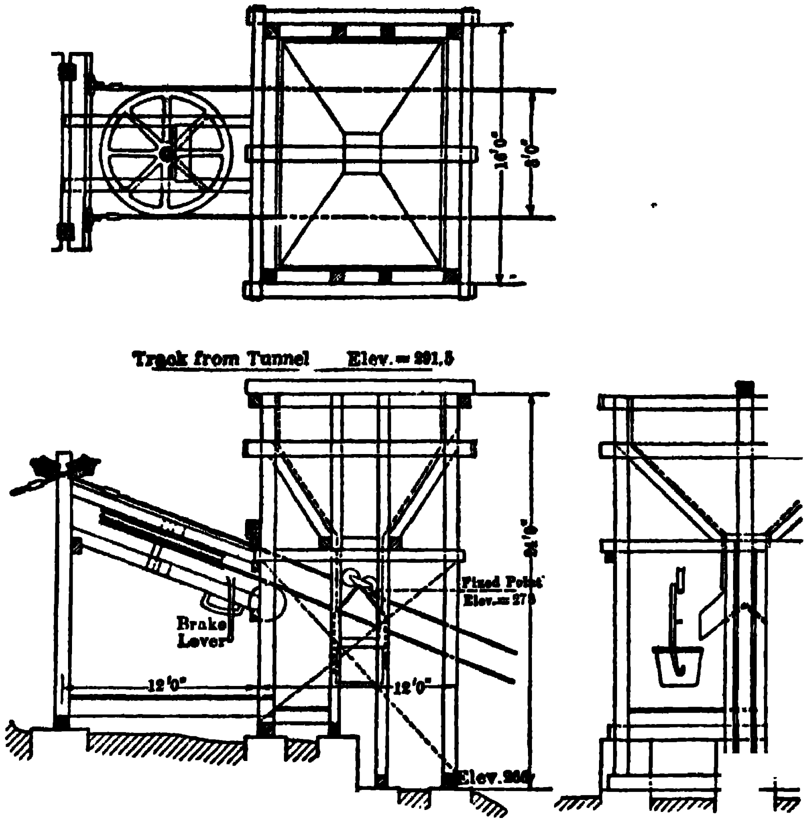


Fig 39. Loading Terminal, Double Reversible Tramway

On gravity lines, relation of operating force to resistance must be investigated at all points, with especial attention to conditions at discharge end, for there the slope of cable is flat and little power is generated, while the other carrier is climbing steepest part of its track and offers max resistance. Momentum of carriers may complete the trip after loaded carrier has ceased to generate power, but this should be counted on with caution, as it requires a skillful operator to run loaded carrier with sufficient speed to take it into its terminal and stop without undue shock. On flat inclines, or with a heavy carrier and slack cable, the lowest point in path of carrier may be at a distance from the discharge terminal, and momentum of carrier may not be sufficient to complete the trip. In such circumstances, this type of tramway should not be installed, or power should be provided to complete the trip of carriers; or, a large track cable should be used, so it can be stressed at higher tension and give a track more nearly approaching the chord.

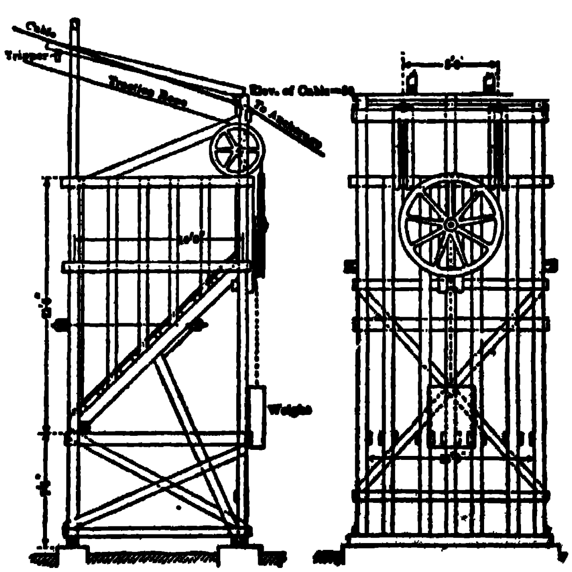


Fig 40. Discharge Terminal, Double Reversible Tramway

Controlling machinery may be either brakes or a power drive. The drive must provide for stopping and for running in either direction.

Shifting belts are satisfactory for this reversing mechanism. They wear well, are simple and easily repaired. Band brakes may be double, with bands applied in opposite directions, so that one

will be efficient, whatever the direction of rotation, and the most advantageous one will be used for main control, while the other is left set throughout run. Power calculations are analogous to those in Art 14. Fig 39, 40 show typical terminals.

22. REVERSIBLE TWIN-CABLE TRAMWAYS

These may have either a single or a double pair of cables (Art 19). Twin track cables permit heavy loads. Carriers for bulk material are usually steel cars, with sloping bottoms, swinging end doors, and 2 or more wheels on each side, traction rope being attached near the wheel line; they dump into a bin at discharge terminal, or are dumped along the line (Art 25). Use of end doors gives a gradual discharge and avoids the upward snap of the cable caused by sudden dropping of the load.

Lines are in use with carriers to 125 cu ft capac, traveling at 1 500 ft per min over towers on pairs of 1 $^5/_8$-in locked-coil track cables and hauled by a 0.75-in 6 by 19 plow-steel rope (Maker 3, Art 29).

A heavy twin-cable type of single reversible tramway was built across the American River, Calif (13). Span, 2 600 ft, with approx horiz chord; four 2-in cables, 2 on each side of a cradle, carried a standard-gage freight car, which loaded with lumber weighed 58 000 lb. The cradle had 8 wheels on each cable, and was hauled at 1 500 ft per min by an endless rope (Maker 1, Art 29).

23. MONO-CABLE TRAMWAYS

These are also called ropeways and single-rope tramways. They comprise: (1) a single endless rope running around large sheaves at each terminal and over small supporting sheaves at towers; (2) carriers suspended at intervals from running rope; (3) terminal machinery for driving the rope and loading and unloading carriers. They are of 2 forms: (*a*) carriers permanently attached to running rope by clips; (*b*) carriers suspended from running rope by "box-heads" (see below) which ride on the rope and are detached at the terminals for loading and unloading.

Clip type. The clip consists either of a thin steel strap encircling the rope, or a steel forging inserted in the rope by slightly separating the strands. Due to this permanent connection, the carriers must be loaded while in motion; where they are small, this may be done manually with scoop shovels, but usually by a mechanical loader; at discharge end, the carrier latch is tripped automatically and load is dropped. One man attends to loading, and operates the control, if this is at the loading end.

The control, by brakes or a power drive, is attached to a grip-sheave at upper end (Fig 28). At lower end, the rope passes around a plain sheave with means for applying tension to the rope. These terminal sheaves are horiz and usually 8 ft diam; carriers passing around them develop centrifugal force, which is kept low by limiting speed to 180 ft per min. Towers have a sheave, 24-in diam at each end of top cross timber, on which the running rope travels. Where downward press of rope is heavy 2 or more sheaves mounted on a rocking beam are used to reduce bending of the running rope. To hold the rope down, sheaves can be placed above the running rope, but, as this adds a load to the rope, bending stresses should be small, and avoided if possible.

Angles in the line are made by deflecting the rope around sheaves of the same diam as terminal sheaves, but without releasing clips from the rope; the sheaves being arranged so that the carrier hanger never comes between it and the rope; the clip projects outward from the sheaves at all times. The rope on the convex side of the angle can be deflected by a single sheave, but that on the inside requires 2 sheaves; these are placed beyond the angle point and at different elevations, so carriers can pass over the other ropes.

Junctions between 2 tramways, where the running ropes turn around terminal sheaves, are made by the carriers of one line dumping into bins, from which the material is reloaded into carriers of the second line. The structure has practically 2 terminals, one above the other. The 2 lines may be at an angle without materially affecting the construction. Fig 41 shows the profile of a short line, and Fig 42, a general standard terminal; with slight changes, it can be adapted to nearly all conditions at loading or discharge end of line, or at junctions.

Limitation. These tramways give good service to a capac of 15 tons per hr, a length of 2 miles, and a difference in elev of 2 500 ft. For greater capac, length, or inclination, running rope becomes heavy and wear is excessive. The limited field for this type has resulted in its being dropped by makers in the U S, although for a light line it is cheaper, in both first cost and operation, than any other tramway. They are still made by English firms 4, 5, 6 (Art 29).

Box-head type of tramway. A box-like casting, a few inches square and 10–20 in long, rides on the running rope and from it the carrier is suspended. There are 2 designs: (1) with a lug at each end, having corrugated V-shaped grooves in their lower ends to increase friction and prevent length-wise slipping (Makers 4 and 5, Art 29); (2) with a grip operated by wt of the carrier; one style (Maker 6) has a tong-like action which closes the jaws about the upper two-thirds of the running rope and gives a positive hold. Another (Maker 8) has a pair of rollers at each end of the box-head, the axes of which are parallel to the rope; the rollers are grooved to match the strands of the rope, each pair being contained in a chamber with tapering sides. When the box-head is set on the rope, the

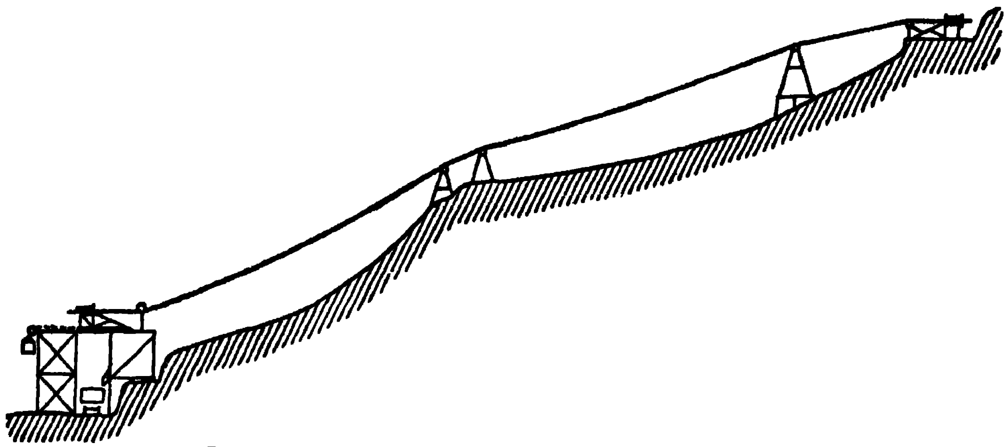


Fig 41. Profile of a Mono-cable Tramway, Clip Type

rollers mesh with the rope strands and the tapered surfaces cause them to grip the rope. Both styles are released at terminals, when the side wheels take the load. The BOX-HEAD, of whatever design, has 2 wheels on one side. At a terminal, an angle or a junction station, a shunt rail is placed parallel to the running rope, a few inches from it and at such a grade that when the box-head wheels engage the rail, the box-head lifts off the rope and runs on the rail. The rail system can be arranged to suit conditions as for bi-cable tramway stations, and lead the carriers to loading or discharge points; or it may simply overcome a gap where the running rope is deflected or interrupted. The rail leads the carrier to the point where it is replaced on the running rope, by depressing the rail with respect to the

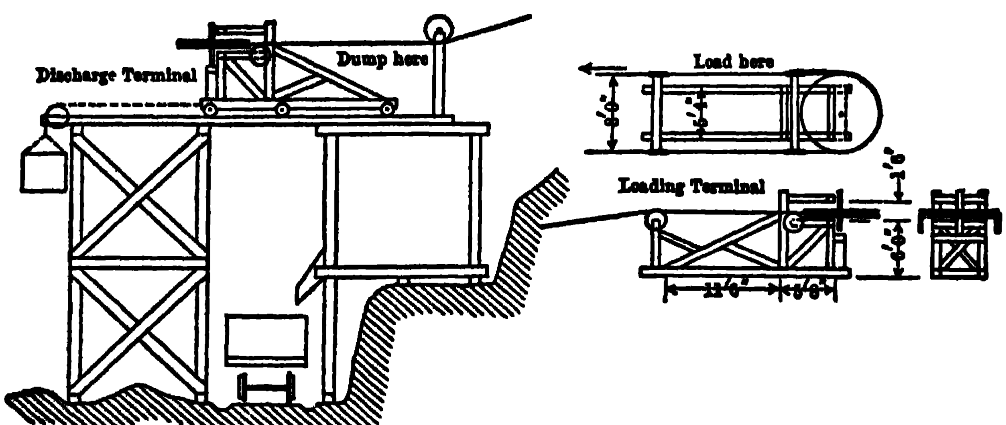


Fig 42. Terminals, Mono-cable Tramway, Clip Type

rope (4). By grading the rail the carrier coasts to or from its stopping points and may sometimes attach itself to the running rope automatically. The rail, if long, may be replaced by a moving chain, which conveys the carriers at about one-third the rope speed across the gap to the attaching point on the running rope (Maker 4, Art 29).

Controlling machinery for the running rope is best located at highest point of the line and preferably includes a grip sheave (Fig 29); to which, for manual control, brake wheels are attached. For power drives or automatic control, the grip-sheave shaft is geared, or a bull gear is bolted to the sheave. See Art 14, 15 for data on power machinery applicable to mono-cable tramways

Towers are usually pyramidal, of steel, with a cross beam at the top to support the sheaves. For light lines, 3-legged towers may be used; with 2 legs on a line normal to center line of tramway and 1 on the center line. The cross beam for the sheaves is attached to the first 2 legs. On moderately heavy lines, 4 sheaves in an equalizing frame are used on the loaded side and 2 on the empty side. In case of a sharp angle over a summit, the tower is replaced by a structure carrying several groups of 4 sheaves each. On light lines, 2 sheaves may be used on the loaded side and 1 on the empty side.

Running rope is usually Lang lay, 6 strands of 7 wires and hemp center; a more flexible rope is sometimes used; rarely, one with wire-strand core. For severe service, plow-steel rope may be needed. Rope sizes are $^5/_8$ to $1\,^3/_8$ in. Initial safety factor should be 6, so that the factor will not fall below 4.5, after rope is worn. Bending stresses are reduced by high rope tension, and those due to large angle over a tower bv multiple sheaves.

Installations. Some of the longest tramways yet built are mono-cable, made up of a series of sections operating in tandem (Makers 4 to 8, Art 29).

Examples. Hollinger Consol Gold Mines, Canada, has a 3.75-mile line, over flat country, driven by 100-hp motor; it holds the capac record for mono-cable, of 190 tons per hr; buckets, 19 cu ft, run at 527 ft per min. During 1934, the tramway averaged over 162 tons per hr while operating crew was on duty, including repairs. From 1927, when started, to June 17, 1935, it carried 5 756 634 tons. During a representative year, transport cost was (cents): loading buckets, 1.23; unloading, 1.56; operating (including power and maintenance), 2.1; repairs and replacement of cable, 1.6; total, 6.49¢ per ton. The running rope, replaced 1938, carried 2.5 million short tons (26). Maker 4, Art 29.

Tilmanstone Colliery, Kent, England, hauls coal to bunkers in Dover harbor (28). Line is over 7 miles long. Loading terminal is at elev of 198 ft; line rises to 357 ft, and drops to 89 ft. Capac, 120 tons per hr. From bunkers, ships are loaded by belt conveyers at 1 000 tons per hr. Buckets hold 1 450 lb coal; running rope, 1.25 in; speed, 390 ft per min; bucket spacing, 138 ft; time interval, 21.4 sec, requiring 60–65 hp. Line is in 2 sections: the first, 18 090 ft, is straight, from loading terminal to a control station at 261 ft elev; the second, to discharge terminal, 20 222 ft, contains 2 angles and passes through twin tunnels about 0.25 mile long. Two divisions were necessary, due to wt and length of the line. The control station contains the driving machinery and tension mechanism for both divisions. At this station and the 2 angle stations, the box-heads leave the running rope at incoming points and coast on rails to the outgoing point, taking the rope automatically. Total cost of tramway, about £120 000, or $15 per ft. Started, Feb, 1930. Operating cost at full capac, 9.8¢ per ton (Maker 4, Art 29).

Indian Copper Corp, Chota Nagpur, India, has a 31 050-ft tramway, over rolling country, with 1 horiz angle to make a crossing normal to river with span of 975 ft (25) (Maker 4. Art 29). A single endless running rope, at light tension, serves entire line. When built in 1928, capac was 40 tons ore per hr and return freight, delivering 100 loads of 800 lb per hr; bucket spacing, 201 ft; time interval, 29 sec; running rope, 1 in, 6 by 7 Lang lay; breaking strength, 70 000 lb; speed, 420 ft per min. Capac was increased to 70 tons per hr in 1933, with larger rope. First rope carried 743 000 tons; second, 712 000 tons; replacement cost, 1.3¢ per ton. Operating cost for 1 month in 1931 (after line had been running 3 yr, single shifts), for labor, power, stores, repairs and supervision, 5.25¢ per ton, for 16 000 tons. Adding 1.3¢ for replacing first rope, total cost was 6.55¢ per ton. A later company report gives cost of 5.5¢ for the entire line. Labor, all native at low wages, was approx 32¢ per man per day, 13 men and foreman being required.

Maker No 4 has also built tramways for Central Provinces Manganese Ore Co, India (97 650 ft), and Rahman Tin Mines, Malay States (7 600 ft).

Cost of materials for a mono-cable tramway, steel construction, f o b England, 1938, is stated by one engineer as approx: (*a*) capac, 20 tons per hr; line material, $1.83 per ft; terminal machy and frame, $3 640. (*b*) capac, 100 tons per hr; line, $4.40 per ft; 2 terminals, $6 800.

Operating cost (stated in general terms by Harrison Roe, *So Af Min & Eng'g Jour*, Aug 22, 1936) for either mono- or bi-cable tramways, about 2 miles long and carrying 15–150 tons per hr, is between 2¢ and 4¢ per ton-mile, at English wages. On short lines, cost is higher, as same labor is required and its cost distributed over fewer miles. On long lines, direct cost may be lower, but more obstacles may tend to increase cost.

General data. Longest mono-cable tramway yet built in one unbroken line is 31 200 ft (nearly 12 miles of running rope); capac, 55 tons per hr. In rugged country, practical length of a tramway, or a section, is about 3 miles. Spans to 3 000 ft can be operated where capac is moderate and tension in running rope is high, but these favorable conditions seldom occur together. Usually, a span of 1 000 ft causes severe stresses and is near the practical limit. In general, spans on bi-cable tramways can be longer than on mono-cable. Capac is limited to about 150 tons per hr by size and wt of running rope needed; max size in use is about $1\,^3/_8$ in; max speed, 400 ft per min. As heavy loads increase bending of running rope, the max is about 1 800 lb. Grades of 50% are possible, but 40% is about the practical limit.

24. DESIGN OF BOX-HEAD MONO-CABLE TRAMWAYS

The economic size of individual loads, determined by experience, is about 1% of hourly tonnage; with small capac they may exceed and with large capac may be under it. For lines carrying 10 tons per hr, loads are 350–400 lb and ratio is nearly 2% of hourly tonnage; for 100–150 tons per hr, loads are about 1 800 lb and ratio is 0.8–0.55%. Speed of running rope is about 400 ft per min; distance traveled per hr divided by number of loads gives the carrier spacing; see Eq 26 (Art 2), which reaches same result by different analysis.

For a tramway of moderate gradient, over rolling country where towers can be placed at short intervals, the running-rope tension at highest point determines size of rope; for tension due to loading, see Eq 29 (Art 2). As the wt per ft in the equation must include that of the running rope, the rope size is found by assuming a wt, say 1 lb per ft for lines carrying 20 tons or less per hr, and 2 lb for heavier tonnage. Total tension equals the above, plus half the wt applied to tension sheave at lower end, which may be taken as 10 000 lb, and a cast-steel rope with a breaking strength 6 times this tension is chosen. The actual wt of this rope is then used in Eq 29, giving a closer value for tension. By a few trials a suitable rope is found; if a rope of cast steel is too large, use a plow-steel rope. The tension wt at lower end must be such as to prevent excessive sag; it may be the only source of tension in the running rope for some distance from lower end. Here the need for large tension can be reduced by making the tower spacing less than at higher parts of the line. Tension varies from a max at the highest point to a minimum at the lowest. If rope is too small, there may be steep slopes at upper ends of inclined spans, exceeding those at which box-heads will hold. These slopes may be reduced by shortening the spans and holding the rope nearly up to its chord. Attempts to decrease sag by increasing the tension wt at lower terminal sheave adds to rope tension throughout the line, which must be considered. On clip tramways, steep slopes offer no difficulties from endwise slipping.

After the proposed profile has been plotted, the tramway can be laid out by methods analogous to those in Art 5, 6. The process is simpler than for a bi-cable tramway, as the running-rope tension is less for the wt carried than that of track cables; hence the rope can follow the topography more closely. The running rope is supported on each end of top beams of towers by 24-in sheaves, and, to limit bending stresses, the deflection over any sheave must be moderate. On clip-type mono-cable tramways, where running rope is 0.625 to 1 in, and tension is low due to light loading, one 24-in sheave is set on each end of tower beam; for larger tensions, one 36-in sheave, or a pair of 24-in, may be used on loaded side. On box-head tramways, the running rope and its tension are often large, so that the rope approximates the curve of sheave, if only one is used, giving high bending stresses. Hence a series of 4 to 6 sheaves with equalizers is set on loaded side to distribute bending; fewer are needed on empty side. If a tramway makes large vert angles over summit towers, 2 series of sheaves may be used; or, in a severe case, a summit structure with several clusters of sheaves may be substituted for a tower. Such structures are like Fig 26, except that the running rope and sheaves would occupy the top instead of cable and saddles. In making a large deflection angle over a sheave the rope tends to conform to the sheave, causing great bending stresses. But, with a small deflection the rope curve has a much larger radius than that of the sheave (see Sec 12). Thus, in a series of sheaves at 36-in centers, and a chord deflection of 2° at each, if the rope is assumed to bend to a uniform curve like a solid steel bar, the radius of rope curve will be 86 ft (Fig 12 and accompanying discussion). The total deflection angle between tangents to rope on both sides of a summit, divided by chord deflection angle between sheaves, gives the number of sheaves required.

Long spans. If the country is rough and a long span is needed to cross a depression, the tension required by this span influences the total tension. The tension in this span is found by transposing Eq 2 (Art 1) thus: $t = ws^2 + 8h$, in which w = wt per ft of running rope, plus the uniformly distributed wt per ft of loads on span (see discussion of Eq 33, Art 6). If the long span has a steeply inclined chord, the tension formula must be modified as under "Horiz tension," Art 1. If the long span is near the upper end, its tension may not exceed that due to difference in elev between that point and the lower terminal; but, if near the lower end, the required tension must be supplied by the tension weight; this affects the whole line and may need too large a rope. If so, the line may be in 2 sections, one including the long span, the other having a tension due to its gradient and a moderate tension weight; or, it may be best to change to a bi-cable tramway.

Example illustrating design of a mono-cable tramway for 150 long tons per hr (4): horiz length, 8 125 ft; fall, 623 ft; max horiz span, 690 ft; permissible sag when loaded, 52 ft; assumed position of long span's lower end, 4 200 ft from lower terminal and 300 ft above it; gradient of span chord, 1 in 15; loaded carriers, 2 464 lb, spaced 140 ft, giving uniformly distributed load of 17.6 lb per ft;

assume 1.25-in plow-steel running rope, at 2.7 lb per ft, breaking strength, 115 600 lb; then, total load is 20.3 lb per ft. These data, substituted in Eq 22 (Art 2), give slope of tangent to rope at lower end of long span, 0.2348, and at upper end, 0.3681, corresponding to grades of 13° 13′ and 20° 13′ respectively. The horiz tension at any point of a rope curve, from Eq 17 transposed, is: $t = ws^2 + 8h \cos \alpha$, in which $\alpha = 3° 49'$ inclination of chord, and $\cos \alpha = 0.9978$. Solving, the horiz tension in long span is 23 284 lb. Rope tensions at ends of this span, by Eq 4 (Art 1), are: lower, 23 900; upper, 24 800 lb. From upper end of span to upper terminal, the rise is 277 ft in 3 235 ft horiz; this gives rope tension due to loading (Eq 29, Art 2) of 4 400 lb, which added to upper span-end tension gives max rope tension at upper terminal of 29 200 lb. As the rope's breaking strength is 115 600 lb, the safety factor is 3.96. For these conditions, the tension wt at lower terminal = 2 × (23 900 − 4 400) = 39 000 lb. **Note.** In this example, of a tramway built in 1912, the author had to assume some data not given by Blyth (4), but the results are close to his final figures. The author believes the conditions are too severe for a single-rope line, and that a bi-cable, in which max traction-rope stress will not exceed 10 000 lb, would be preferable.

25. SPECIAL APPLICATIONS OF TRAMWAYS

In the U S, tramways are chiefly for long distance transport over rough country. In Europe, where industrial plants may be cramped for space, aerial transport leaves the ground free for other purposes. This fact has emphasized certain features less common elsewhere. The first cost of such installations is secondary to low operating cost, safety or effic.

Industrial tramways must provide for loading and discharging at fixed points, which often results in angles, steep grades, and automatic loading and discharge.

Examples. (*A*) A complicated bi-cable system was installed at Imperial Chemical Industries, Billingham, England (29), comprising a main line, feeder tramway and several minor lines. Main line carries 170 tons per hr, plus 35 tons per hr from the feeder. Locked-coil cables are 2 1/16 and 1.25-in respectively; breaking strengths, 300 000 and 110 000 lb. Traction rope is 15/16-in, flattened strand; breaking strength, 58 000 lb; driven by an 8-ft triple-groove sheave, geared to a 60-hp motor. Buckets weigh 1 200 lb and carry 3 500 lb; they rotate for dumping when latch is released by tripper, and are righted by a finger on the bottom, contacting with a spiral rail. Carriages have 4 wheels; wheel base, 3 ft 8.5 in (Maker 6, Art 29). At another part of the plant, 104 tons ore per hr are brought from a mine, and dumped at treatment points by a bi-cable tramway (Maker 5). Traction rope is driven from loading terminal, and the line contains 2 angle stations which the buckets pass without detaching. The cables terminate at the plant and the carriers travel on rails, though still hauled by the traction rope. The rail section is at an elev of 60–85 ft, to leave the ground unobstructed. To limit the number of structures, and to protect men working below from spill from buckets, the rails are supported by floored bridges of light trusses. Buckets dump automatically at a series of bins. (*B*) At a dynamite plant, a complicated tramway was installed to deliver explosive "dope" to any of 3 mixing houses (13). A carrier took the siding at the desired point, as determined by a "selector" on its carriage, set by the dispatcher at the loading point. As the dope is highly inflammable, the loads are spaced far apart on the rope, to prevent fire from spreading in case of accident in any one building (Maker 1, Art 29).

Aerial dumping provides for dumping loads at any point along the line, by continuous or reversible tramways. On continuous BI-CABLE LINES, a tripping device, set at any desired dumping point, is suspended from the track cable, has a finger to release the bucket latch and allows the load to drop. If the load is large, the tripper may be a sizeable frame, guyed to the ground to prevent the supporting cable from snapping upward when the load is suddenly released and so throwing the empty bucket off the line. When the tripper is to be shifted to a new position, a man goes out in a bucket, releases its clamps, slides it along cable and reclamps it. Maker 5 (Art 29) attaches a small wire rope to the frame, for moving it along the cable by a hand winch at one of the towers. On REVERSIBLE TRAMWAYS, a tripper like the above may be used; or a mechanism in the carriage trips the latch: (1) when a carrier has traveled a predetermined distance, a slow-moving disk with a crank pin pulls the latch, the disk being driven through gearing by a carriage wheel; (2) a device on the carriage comes into action when direction of travel is reversed, and releases the latch (Makers 1, 5, Art 29). The device is well adapted to single-rope lines. The reversal can be made manually, or automatically by switches controlling the current driving the motor. When done manually, a load can be discharged at any point, permitting the material to be stacked in different piles. With double reversible lines, the travel of loaded carrier harmonizes with that of the empty; the loaded carrier being at the outer end when the empty is at the loading chute. The dumping point can be at one point for a period and then moved (4). Maker 5, Art 29. With the MONO-CABLE SYSTEM, the tripper is suspended from a stationary rope stretched above the tramway, between 2 adjacent towers. The tripper frame has a supporting sheave to keep the rope and tripper in the proper relation for dumping. The towers carrying the stationary rope are extended

upward, and from them the rope goes to dead men or to a winch; the tripper can then be moved by shifting the rope.

Stocking ore. A Scottish iron plant has a bi-cable tramway (30, 31) (Maker 6). Ore from R R cars goes to a hopper, is conveyed to 2 storage areas, dumped into piles and reloaded by steam shovel into cars going to furnaces. The piles are 35 ft high and 90 ft base diam. One area is served by outgoing cable, the other by the return cable. From the loading terminal the cables rise steeply to two 45-ft towers, 125 ft apart, a cable going to each dumping area; there they terminate in 2 angle stations, connected by a cross cable. These stations have large sheaves to deflect the traction rope, with track rails for the buckets, which are dumped by trippers. Buckets hold 4 000 lb; lock-coil track cables, 2-in; traction rope, 1-in; capac of line, 300 tons per hr. Buckets are filled and pushed into the running-rope attacher by 2 men; after dumping, they return, are detached automatically, and coast to filling chute.

Dumping of waste material, as for building embankments, resembles stock-piling, except that the pile is constantly growing. If on level ground, guyed steel towers, slim or mast-like, as high as 250 ft, have been used. When they are buried in the pile, the line is extended on new towers (32). As such tramways usually run up hill, a power drive is needed (Art 14). DUMPING TRESTLES make very high towers unnecessary. A tramway is built to the dumping terminal; the buckets run onto rails on a lattice truss trestle, inclined upward at angle of repose of the waste (say 35°). At end of trestle is a large return sheave for the traction rope, the buckets dumping as they pass around it. The trestle is extended as required (Makers 5 to 8, Art 29). Dumps have thus reached heights of 500 ft. For waste disposal, several types of tramway are suitable: twin-cable (Art 22), single- or double-reversible (for moderate tonnage and distance) (Art 20), and mono-cable (Art 23).

Loading ships with bulk material. Where ships can not be docked, tramways are sometimes used. The discharge terminal may be built on bins on an island-like wharf, where water is deep enough for ships; the intermediate towers being on pile foundations. From the bins, the ship is loaded by belt conveyer or other means (Makers 1 to 8, Art 29).

Freight tramways for bulky materials, built as adjuncts to RRs, to carry goods over difficult topography, must be designed for necessary clearance over summits. Carriers are buckets, crates, tanks, hanging platforms, or chain slings for barrels, lumber and pipe (Makers 1 to 8, Art 29).

Passenger tramways. On ordinary tramways, 2 persons are carried in long buckets, sitting facing each other. Some Rocky Mtn mines have no other means of transport in winter.

Special passenger tramways. About 50 have been built from 1912 to 1936 by Makers 7 and 8, for various services, chiefly in mountainous regions. Most of them are of double reversible type (Art 20), with 1 car suspended from each track cable; cars hold 20–35 persons; grades, up to 50%; carriages have 8 or 12 wheels, with equalizers to distribute the load; speed, 100–300 ft per min. A double reversible passenger line of Bleichert design (Maker 1, Art 29), began operation in 1938 on Cannon Mtn, N H; horiz length, 5 000 ft, with 3 towers; rise, 2 022 ft; cars, hung from 8-wheel carriages, hold 27 persons; running time, 5.5 min; locked-coil track cables; terminal sheaves for running rope, 13 ft diam; shock absorbers reduce fore and aft oscillation and brakes seize running rope in case of accident. There is an auxiliary rope and mechanism to land passengers from a stalled car. Similar Bleichert and Pohlig tramways have been built in the Tyrol and Swiss Alps.

CABLEWAYS

26. GENERAL DESCRIPTION (see also Sec 5)

A cableway comprises: (*a*) fixed track cable; (*b*) carriage running on cable; (*c*) hoisting or fall rope, to raise and lower load by tackle suspended from carriage; (*d*) hauling rope to move carriage back and forth; (*e*) 2 towers; (*f*) hoister at head tower; (*g*) carriers of various types. Fig 43, 44 show typical designs.

Track cables are up to 3 in diam. locked coil, cast-steel, or plow-steel where load stress is great. At each end is a swivel socket, for turning the cable to distribute the wear from carriage wheels. For cables 2 in diam and over, ball-bearing sockets are used. At one end, cable is anchored; at other end, a take-up tackle provides tension.

Carriages for light loads have 3 track wheels; for heavy loads, as many as 8, mounted in equalizing frames, which also carry the head-block sheaves for hoisting and dumping tackles, provide connections for the hauling rope, support the button rope, and carry the tower horn for collecting the fall-rope carriers. On large cableways, 6-wheel carriages operate in tandem, with considerable space between them, one being rigged with the hoisting tackle, the other with the dumping tackle; this distributes the load on the cable, and checks the swinging of the carriage when brought to a stop.

Hoisting ropes are cast- or plow-steel, $^5/_8$–1 in, with 6 strands of 19 wires; wear on rope due to bending is kept low by using large tower sheaves. The rope runs from drum to the head-tower sheaves, thence through the fall-rope carriers, over the carriage sheaves, around fall-block sheaves, and ends at the carriage. The tackle is usually reaved, so that 4 parts of the rope carry the load. The fall-rope carriers support the rope at intervals, to

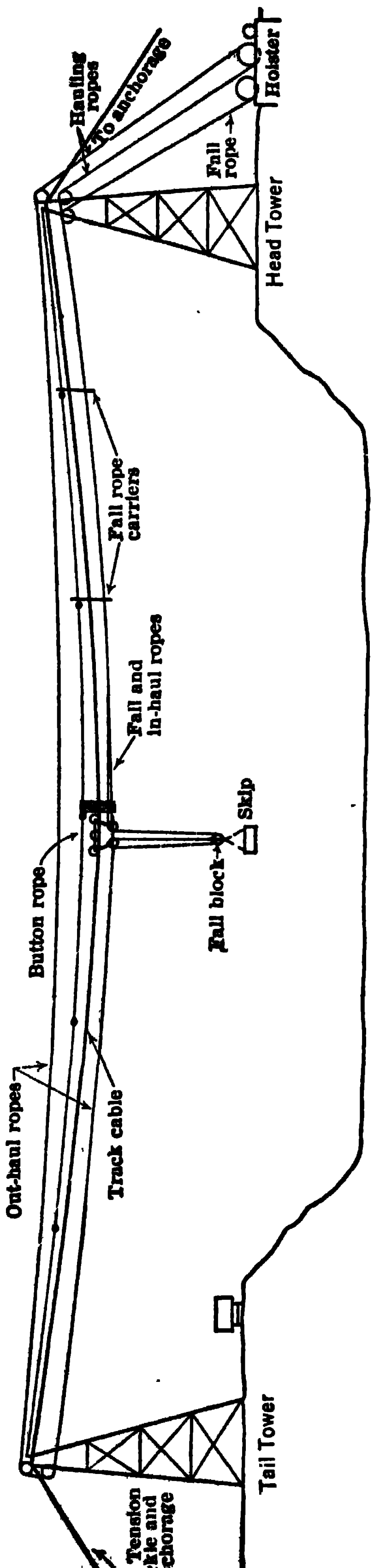


Fig 43. Cableway with Fixed Towers

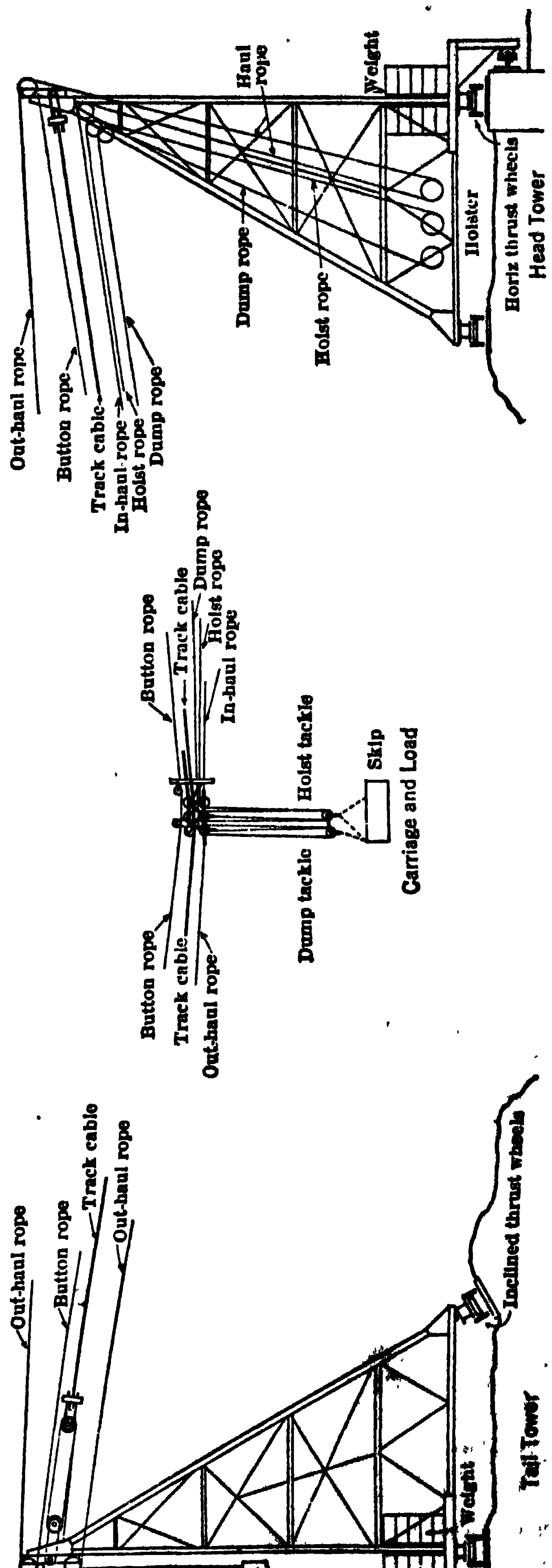


Fig 44. Cableway with Movable Towers

prevent undue sagging when there is a light load. The rope-supporting system consists of a stationary button-rope, on which are placed buttons of different diameters, increasing in size toward the tail tower; the carriers being fitted with rings of correspondingly increasing sizes, through which the button-rope passes. These carriers are stored on the horn on the tower end of the carriage, the one having the largest ring being next to the carriage. As the carriage travels from the head tower, the first button passes through all the rings except the outer one, which is smaller than the button; this pulls the carriers off the horn one by one and sets them on the cable. On the return trip, the carriers are picked up by the horn, and kept ready for next trip.

The entire load sometimes hangs from the fall block and is dumped or detached manually; or, when the skip is dumped mechanically, the main fall block is attached to a chain bridle on the open end of skip, with a second block on the rear end, which is raised by a dumping rope. In the latter case, the dumping rope goes back to a third drum on the hoister. Drums, ropes, and tackles for hoisting and dumping are identical, and operate in unison until a skip is to be dumped, when the speed of one line is slowed by a clutch and brake on that drum, and skip is tilted. The 2 fall blocks permit handling extra heavy loads, by attaching both to a single load.

Hauling rope has one end linked to near end of carriage, passes to sheave on head tower, then down to make several turns on a concave-faced drum on the hoister; returns to head tower sheave, passes above the track cable to top of tail tower, thence around a sheave and back to the carriage. The rope is usually the same size as the hoisting rope, but has less bending stress, and may be of larger wires, to withstand surface wear due to slippage on drum.

Towers at each end support the entire equipment and must be high enough for loads to be hoisted to the elev desired at any point. Stationary towers are often 150 ft and movable towers 100 ft high; usually of steel. Contractors prefer steel, with bolted connections, so the towers can be taken down and re-used. For STATIONARY TOWERS, the cables are anchored back of the tower (Fig 43). When the back cable has the same slope as the normal main-cable slope, the resultant stresses are nearly vert, and a 4-leg pyramid tower is suitable. Sometimes guyed A-frames or masts are sufficient. MOVABLE TOWERS are used when the cableway must be moved at intervals, to serve a large area. They are usually steel, mounted on platforms on railroad-like trucks, with wheels on multiple tracks. As they can have no anchored back stay, the cables terminate at the towers, which are designed to take the horiz tension. This has developed a tower design (34) with vert back legs in tension and inclined front legs in compression; a massive weight being placed close to the vert legs (Fig 44). Towers are moved by a winch in lower parts a rope, anchored at both ends, is given a few turns on the winch drum, and the tower pulls itself along the rope. For heavy cableways (as at Conchas Dam, with 1 650-ft span) each tower has 2 motors, geared to 4 of the supporting wheels.

Movable cableways are of 2 kinds: (*a*) both towers travel on straight tracks; (*b*) 1 tower is stationary, the other moving on a circular track, with the stationary tower as center of arc. When both move their winches are operated in unison by a switch in head tower; rate of travel, 50–150 ft per min. At Norris Dam, 2 cableways (spans, 1 925 ft) cover an area 500 ft long in their travel (36).

Hoister comprises: (*a*) hoisting drum, holding the rope in one layer; (*b*) hauling drum, having wide enough face for the rope, with several laps, to travel to and fro across it (for light work; a capstan-like drum is used); (*c*) dumping drum; (*d*) small auxiliary drum for moving towers. Hoisters are elec-driven; controlled like elec hoisting engines (Sec 12).

Carriers. Skips are used for excavated material; chain or wire-rope slings for large rocks, quarried stone, or heavy equipment; bottom-dump buckets, sometimes as large as 8 cu yd, for depositing concrete.

27. DESIGN OF CABLEWAYS

Data required: 1, plan and cross-sections, or contours, of volume to be excavated, the sections showing probable max depth of excavation; 2, position and elevation of discharge points; 3, kind, size, and wt per piece, or per cu ft, of material to be handled; 4, quantity handled, tons per hr; 5, nature of power to be used.

Weight of load sometimes depends on size of individual pieces or units. In stripping it is usually determined by quantity of loose material to be handled per hr, although wt of single blocks of stone may be the controlling factor.

Time between loads is time to hoist load, transport and dump it, return empty skip to pit and change connections to a loaded skip. Speed of hoisting is usually 300 ft per min,

but may be up to 400 ft; lowering time is usually less than for hoisting; with regenerative braking, speed can be one third greater. Max speed of carriage along cable, 1 800 ft per min; usually, 1 200 ft. Max distance hauled is about 80% of span; aver distance, rarely more than 60%. Hence, time per load $= \dfrac{2 \times \text{depth of pit}}{\text{hoisting speed}} + \dfrac{2 \times \text{distance hauled}}{\text{hauling speed}} +$ **0.5 min, the last item being allowance for changing and dumping. Round trips are often made in 2 or 3 min. As cableways are intermittent conveyers, with limited max speed, large tonnage can only be handled with large loads.**

Size of cable depends on gross load, span and allowable deflection. In beginning a design, assume a deflection of 0.05 of span, when load is at center; if this deflection gives towers of reasonable height, it is adopted and tension in cable is found by solving Eq 14 (Art 2) for t; but, as both w and t are unknown, see Table 1 for the relation between them; w can be expressed in terms of t. For 1.5 to 2-in locked-coil, cast-steel cables, $t = 9\,300w$ (approx); for 2 to 3-in plow-steel, $t = 12\,600w$, when the minimum safety factor used is 3.5. If the 5% deflection requires undesirably high towers to clear obstacles, the tension is increased by using: (a) larger track cable; (b) cable of higher grade steel; (c) a smaller safety factor, unless the minimum factor of 3.5 has already been used in the trial calculation. This factor is small, but sufficient where the max load is definitely known, because the strength of cable is accurately determined and not subject to shock loading. Also, a taut cable suffers less from bending under the wheel load than a slack one. On long spans, the deflection increases faster than the span, due to wt of cable (see Eq 14), and it is often 0.06 or even 0.065 of span; on short spans, it may be less than 0.05, if load is light and cable taut.

Tension tackle for track cable is of wire rope; it must be long enough to connect socket on end of cable with the anchorage, when the cable has been drawn near to final position with ordinary tackle. At first the tension rope is passed over only part of the tackle sheaves; then, as cable becomes tighter, the rope is threaded over the others for the final pull, and is clamped; it may be pulled by a winch-head on the hoister, or by a special drum.

Path of load. Tension is max when load is at lowest point, and decreases as load moves either way. This variation is unknown, hence the equations of Art 1 for deflection, which involve tension, can not be used for determining deflection with load at various points. Path of load may be found approx by assuming that the two supports A and B (Fig 45) are the foci of an ellipse. Then lines $AC + CB$ equal the major axis, and the ellipse can be calculated. If origin O be at center of span, and OC be deflection due to wt of cable and concentrated load at center, then $OC = H$ (Eq 14, Art 2) = the semi-minor axis, while CB = the semi-major axis of ellipse. The ordinates of any point P are x and y. The equation of an ellipse referred to its center is:

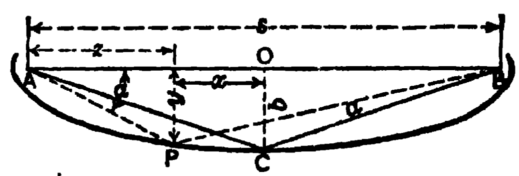


Fig 45. Path of Load on Cableway

$$a^2y^2 + b^2x^2 = a^2b^2, \quad \text{or} \quad y^2 = b^2(a^2 - x^2) + a^2 \tag{39}$$

But $b = H$ and, from Fig 45, $a^2 = H^2 + (s^2 + 4)$; hence, by substituting a series of values for x, corresponding values of y can be calculated and path of load determined for a number of points. If z be distance of load from end of span, the deflection at the load is $y = z \tan \alpha$, which gives slope of chord AP when load is near tower, and this is practically the slope of cable. From last expression, or from path of load, it is seen that the cable becomes so steep, as load approaches either tower, that it is not feasible to run the carriage within 10% of the span from either tower.

This method of finding path of loaded point assumes that the sum of the 2 chords AP and PB is constant for all positions of load, which is not quite true. As load approaches tower, the long section of cable will sag and raise the load a little higher than position indicated by Eq 39. The deflection is further decreased by the pull of hoisting and hauling ropes, which take a varying part of load in holding carriage on the inclined part of cable. These errors are all on safe side, as path of load is usually ascertained to make sure that the load will clear some object. The same problem occurs with single heavy loads on reversible tramways (Art 21). Bib 10a treats of cable spans and length of curve when stretch of cable is included.

Towers on stationary cableways are designed chiefly to resist the vert resultant of the cable tensions; those for movable cableways have a base, parallel to line of cableway, of approx half the height. Hence, to overcome the pull of the track cable and other ropes at top of tower, a balance weight of more than twice these tensions is required at rear of the tower to resist rotation about lower end of inclined front leg. Towers must also be designed to resist a moderate side pull, if cable system is out of line, and to resist wind press acting on the tower and on the half span of cable and ropes. For towers of movable cableways, these side forces require additions to the weights on the bases. These weights are usually precast concrete blocks, which can be removed when towers are dismantled.

Examples of cableways. Advances in practice began about 1910, with 2 plants at GATUN LOCKS, Panama Canal: spans, 800 ft; cables, 2.25-in; load, 6 tons; 85-ft steel towers on parallel tracks; elec hoisters, 150 hp; traveling speed, 1 800 ft per min: hoisting, 335 ft. U S RECLAM SERV, N Mex, 3 plants: spans, 1 420 ft; loads, 10–15 tons; elec hoister, 300 hp. BUFFALO FILTER plant, N Y, built 1925. Two spans, 900 ft; load, 15 tons; 85-ft towers, on parallel tracks, traveled 150 ft per min by engines in towers. MADDEN DAM plant, Canal Zone, built 1932, marked further progress in heavy-duty work: span, 1 265 ft; gross load, 25 tons; buckets for concrete, 8 cu yd, 20 tons gross load; rock skips, 10 cu yd; cables, 3-in locked-coil; 2 carriages ran tandem, each on 6 wheels, with equalizers; 3-drum hoister, driven by 400-hp motor; traveling speed, 1 200 ft per min; hoisting, 300 ft. Steel towers, movable 410 ft by 100-hp motor, were on 32 wheels, in 4 groups, with equalizers, running on 2 tracks, 80 ft apart, at 66 ft per min (20, 21). All the above installations were built jointly by Makers 1 and 12 (Art 29).

From 1932 to 1938, 9 other plants were built for 25-ton loads; and 6 for 15 to 20-ton. Largest of these were 2 spans of 2 575 ft for HOOVER DAM, Ariz. Hoisters driven by 500-hp motors; 3 tandem drums, 53-in by 70-in face. Traveling towers moved at 50 ft per min by a concave-face winch drum in each tower, geared to 100-hp motor, and controlled from head tower to run in unison. On upper deck of each tower were 2 50-cu ft compressors for operating brakes and clutches. Tandem 6-wheel carriages, 40 ft apart, were used, preventing oscillation of buckets due to the long drop of 700 ft; hoisting and dumping lines were $^7/_8$-in plow-steel rope, running through a 4-part tackle; roller bearings for all tower sheaves and fall blocks. Track cable, 3-in locked-coil, had 6% deflection with load in middle of span; it terminated at tail tower in swivel sockets for turning, to distribute wear, and was tautened by a 10-part take-up tackle, with 1.5-in rope. On this dam were 3 other similar cableways, with shorter spans (34, 35). NORRIS DAM, Tenn, has twin cableways, each for 18-ton load, but can be combined to lift 36 tons; spans, 1 928 ft; 3-in cables; towers movable (36). BOULDER DAM, Ariz, cableway was designed for 150-ton loads (largest on record). Span, 1 256 ft; loaded RR cars can be lowered from canyon rim to the bottom, about 600 ft below dam. Track cable comprises 6 super plow steel, 3.5-in Lang lay ropes (6 × 37), with wire centers, placed at 18.5 in centers. Carriage has 48 wheels, 24 in diam, 8 on each cable. Hoisting is done by 2 super plow steel 1 $^1/_8$-in ropes, with hemp centers, each reeved through twin fall blocks, to make two 8-part tackles to an equalizing frame from which load is suspended. Hoisting ropes wind on 2 drums, 13 ft diam by 17 ft face, each driven by a 175-hp dc motor, but operating in unison. Two hauling ropes, same as hoisting ropes, wind on a third drum, geared to a 400-dc motor. Speeds: traversing, 240 ft per min; hoisting, 120 ft for loads under 40 ton and 30 ft for heavier. Towers: a 100-ft 4-leg steel tower at one end, with tension take-up; at other end, a steel and concrete saddle; at both ends, the cable is anchored in concrete, which fills a 60-ft tunnel. Cableway is a permanent equipment for the power plant (37).

All of above installations were supplied jointly by Makers 1 and 12, Art 29.

28. OTHER FORMS OF CABLEWAYS

Light cableways are used for sewers, subways, and similar long, narrow, shallow excavations (Fig 46). Towers are single A-bents, 25–40 ft high; whole plant designed for portability. After an excavation has been made for length covered by 1 span, plant is moved ahead for another section. Spans about 300 ft long and fall rope carriers usually dispensed with, as empty bucket keeps hoisting rope taut. But if loads are hoisted near center of span and run to tail tower, a single-fall carrier may be run out on track cable and

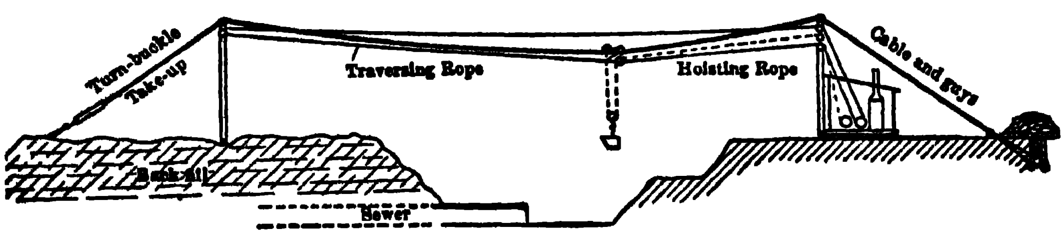


Fig 46. Diagram of a Small Cableway

fastened by a chain dropped to the ground. It is set where it will not interfere with carriage, but will hold up center of hoisting rope when carriage is at tail tower.

Equipment. Machinery for a portable cableway for a net load of 2.5 tons on a 300-ft span consists of: 1.5-in cable, with anchorage and take-up fittings; $^5/_8$-in hoisting and hauling ropes; carriage with 12-in wheels and 16-in sheaves; fall block with 16-in sheave, to give a 2-part tackle; 2 fall rope carriers; 5 1-cu yd self-dumping and self-righting tubs, 48 in wide by 45 in long by 31 in deep; 30-h p reversible hoister, cylinders 8.25-in diam by 10-in stroke, drum 24 in diam by 26 in face, hauling drum 25 in diam in concave face; vert tubular boiler, 42-in diam by 90 in high; engine car with 10 by 16-ft timber platform, housed and mounted on four 15-in wheels; 180 ft of 25-lb T rail; 2 towers of 8 by 10-in timbers, 30 ft high, with 1-in galvanized guy ropes. Engine and boiler can be replaced by elec hoister.

Capacity. These plants will hoist net load of 2.5 ton at 225 ft per min, and convey it at 450 ft per min. If tubs are loaded in a trench, 30–40 loads can be handled per hr.

Operating cost. Labor: 1 engineman, 1 fireman (if steam-driven), 1 signalman, and 2 dumpmen, when loading wagons or dumping into a hopper. One man can dump buckets when machine is back-filling rear end of trench. These cableways can be moved in 2 or 3 days with good organization, if new anchorages (deadman and slings) are set in advance. Force: 5 laborers, 1 handy man, and part of foreman's time.

Semi-permanent cableways have towers, A-frames, or masts of wood or steel. Track cables, 6 by 7 standing rope or locked-coil cable; anchorage is weighted with rock or concrete blocks. Hauling rope is 0.5 in or larger, with fall-rope carriers of button type. Carriage may have 3 track wheels, and, with fall-block, make a 2 or 3-part tackle. Buckets are bottom-dump, or skips with knock-off at open end; tripped by hand, or mechanically by a clamp on hoisting rope above the fall-block, or by an arm on the carriage; either device engages a lever on the skip and releases the load. Hoisters are run by steam, oil or elec (Makers 12, 13, Art 29). These inexpensive cableways are applicable where heavier plant is not warranted.

Inclined cableways ("Blondins") have a high head tower at discharge end and a low one at other end. The carriage is held at loading point by an endless rope, passing around a brake sheave at head tower; this rope merely holds the carriage in position. When loaded skip is hoisted to carriage, the brake on the holding rope sheave is released and the hoisting rope pulls carriage up the inclined cable. At discharge point, the carriage is held by a "gate," while the load is dumped and empty skip hoisted to the carriage again. On raising the gate, the carriage returns by gravity. As one rope does both hoisting and hauling, resistance to hoisting with a 3-part fall must be less than hauling resistance; hence, the fall-block stays against the carriage during hauling. This requires a grade of about 25% in the cable, and limits this type to short spans and favorable conditions.

Excavating cableways are adapted to cases requiring mechanical excavation before transport. GRAB-BUCKET type (with clam-shell or orange-peel bucket) operates with regular equipment (Art 26), except that an extra hoisting rope and drum are needed, with extra sheaves in towers and fall-rope carriers. As resistance to lifting the bucket out of the soil after filling adds to the load, heavier cables and operating ropes are needed than when digging stresses are absent (Maker 12, Art 29). Another type of grab bucket operated by one fall rope has been adapted to cableways having grade of 10°–15°; fall rope hauls carriage and load up grade, the return being by gravity. This outfit is limited to about 500 ft max span. The carriage is latched to movable stops, attached to cable at loading and discharge points and the fall block is hooked to carriage when fully raised. Dumping is automatic. After dumping, the bucket is hoisted until fall block hooks to carriage and unlatches latter, when it is free to return by gravity, latches to stop which unhook fall block which descends for a new load. Carriage thus shuttles between 2 stops, the position of which may be altered.

Stocking and reloading can be done with cableways and grab buckets. Usually, one man can operate the cableway. With stationary towers, a long narrow pile can be stocked; if tail tower travels on a curved track, so cableway can take various radial positions from head tower, a triangular pile can be made; if both towers travel on parallel tracks, a rectangular pile of any length can be stocked. These cableways usually have spans of 500–1 000 ft (Maker 12, Art 29).

Existing plants have following features: (*a*) Span, 420 ft; 3.5-cu yd clam-shell bucket; capac, 350 ton per day of hard and soft coal; max 1 000 ton; both towers travel. (*b*) Span, 996 ft; 3.5-cu yd clam-shell bucket; capac to 225 ton per hr, when stocking and 175 ton when reclaiming; max, 2 200 ton coal in 9 hr. Head tower is stationary, and 246 ft high to permit bucket to clear obstructions; tail tower, 90 ft high and travels on a curved track. Due to conditions, operator is at a distance from head tower and runs elec hoist by distant control. During 1924–25, it handled 283 000 net tons, cost for labor, power and lubrication, 3.5¢ per ton. (*c*) Span, 725 ft; 3.5-cu yd clam-shell bucket; aver capac, 222 net tons per hr. Towers travel on parallel tracks. Total cost for handling first 200 000 net tons, 1.77¢ per ton.

Slack-line cableways are used to strip orebodies or coal seams near the surface (Sec 10), reclaim mill tailing for retreatment, build earth dams, and dig canals (Sec 3). They comprise a track cable, a carriage from which a skip-like scraper is suspended, hauling rope, high head tower and low tail tower, and a drum on hoister for rapidly varying the track cable tension. Spans, usually 300–1 000 ft. Towers may be fixed; or tail tower movable, for a triangular area; or both towers movable, to serve a rectangular area. Speeds: for digging, 100–200 ft per min; hauling, 300–600 ft. Economical loads: $^1/_3$ cu yd for 300-ft span, to 3.5 cu yd for 1 200 ft; digging depth may be 80–100 ft below water line, with total lift of 150 ft or more. Some large plants handle 10 to 15-cu yd loads (38, 39).

In operation, by slackening the cable, the carriage runs down by gravity until the scraper reaches the loading point; the hauling rope then pulls scraper toward head tower, and when filled the cable is tautened to lift the scraper and simultaneously the hauling rope pulls it to dumping point, where the carriage engages a stop on the track cable. Thus, one operation digs, conveys, elevates and dumps, and at less cost per cu yd, than any other means; all controlled by 1 man at head tower (40,41). These cableways are long-distance excavators, useful where material is to be delivered at a high point, for which a drag-line (Sec 27) is not applicable. By adding an out-haul rope, they can work with a flat span, the load being dumped near tail tower.

29. MAKERS OF TRAMWAYS AND CABLEWAYS

1. American Steel & Wire Co, 350 Fifth Ave, New York: bi-cable tramways, passenger tramways, wire rope, track cables
2. Riblet Tramway Co, W 29 Main Ave, Spokane, Wash: bi-cable tramways
3. Interstate Equipment Co, 18 W Jersey St, Elizabeth, N J: twin-cable tramways (Lawson type), bi-cable tramways (successor of Broderick & Bascom)
4. Ropeways, Ltd, 152 Great Portland St, London, England: Roe mono-cable tramways, bi-cable tramways, cableways
5. British Ropeway Engineering Co, 14 High Holborn, London: bi-cable tramways (Bleichert type), mono-cable tramways, cableways (taut- and slack-line)
6. R. White & Sons, Widnes, Lancashire, England: waste disposal plants, mono-cable tramways, bi-cable tramways, movable dumping trestles
7. Bleichert Transportanlagen, Leipzig, Germany: bi-cable, mono-cable and passenger tramways, cableways (taut- and slack-line)
8. J. Pohlig Aktiengesellschaft, Köln, Germany: bi-cable, mono-cable and passenger tramways, cableways
9. Ceretti & Tanfanisa, Milan, Italy: tramways and cableway (in use, but present manfg activity uncertain)
10. John A. Roebling's Sons Co, Trenton, N J: tramways and cableways designed for specific projects, wire rope
11. Most wire rope makers build occasional tramways or cableways of moderate capac
12. Lidgerwood Manfg Co, 775 Lidgerwood Ave, Elizabeth, N J: taut-line cableways, hoisters
13. Sauerman Bros, 438 S Clinton St, Chicago: slack-line and taut-line cableways, hoisters

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SECTION 27

UNDERGROUND MECHANICAL LOADING, CONVEYING AND HANDLING

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

UNDERGROUND MECHANICAL LOADING, CONVEYING AND HANDLING *

1. INTRODUCTION

Mechanization of work formerly done by hand has greatly increased the effic of mine labor, thus reducing production costs to meet the long downward trend of the all-commodity price index (1). Mechanization has also tended to offset the pressure of depletion. The U S Bur of Mines began investigation of mechanical loading in metal mines in 1919 (2). Output figures for mechanization in coal mines were first compiled by the Bureau in 1923, but the data by type of units employed were not available until 1926 (3). For a more detailed statement, see Bib (4).

Early loading machines in metal mines followed the design of surface plant for handling earth and rock, as scrapers, belt conveyers and RR shovels. The initial design of coal-loading machines incorporated more original ideas, to meet the relatively simple mining conditions in flat seams. In both fields, the first attempts to design loading machines for operating within the prescribed spaces for minimum clearances resulted in many construction weaknesses and failures. These have been largely overcome, so that the present-day loaders are flexible, economically priced, of low heights and narrow widths, and have speeds and tonnage capac far in advance of current mining requirements.

Metal-mine loaders comprise scrapers, shovels and conveyers, operated by compressed air or elec; coal machines are mobile-loaders, scraper-loaders, pit-car loaders, and other conveyers; most of them having elec drives. All machines of shovel type involve the principles of digging, lifting, swinging or backward transfer, for loading cars; also "crowding" and gathering broken material by rotating arms or blades onto a self-discharging conveyer; scrapers have the same cycle of loading, transferring to dumping point and discharging, by use of ropes, actuated by a separate power unit. Sometimes scrapers are used for conveying over long distances.

Importance of mechanization. Primarily, increased wage rates are responsible for mechanization in mines. IN COAL MINES, the influence of wage rates on mechanization has been exemplified in Ill, Ind and the Rocky Mountain regions, which early adopted mechanical loading to reduce costs and maintain their competitive position, usually against hand-loading mines having lower wage scales. Further evidence of the wage influence is furnished by rapid increase in mechanization in Eastern and Southern coal fields, following the upward trend of wages beginning with the bituminous code. The industry-wide problem of reducing costs to combat the use of oil or elec is another factor in recent progress of mechanization (4). IN METAL MINES, the period since 1879 has been marked by progressive depletion, which in some fields has largely offset the discovery of new deposits (5); the necessity for cost reduction in utilizing lower-grade ore has been shown by the adoption of mechanization in the iron mines of Minn and Mich, and lead mines of S E Mo and the Tri-State zinc area. Faced with increased wages, shorter working hours and higher taxes, all branches of mining are realizing the need of further savings in cost.

Trends in mineral technology are best measured in terms of output per man: from 1880 to 1929, iron mines recorded an increase from 234 to 2 560 ton; coal (anthracite and bituminous), 422 to 930 ton; copper, 9 290 to 44 900 lb of metal; phosphate rock, 90 to 1 206 ton (6). Though iron, lead and zinc mines have long experimented successfully with mechanical handling and loading, machines have not been adopted as widely as in coal mining, and there are no statistics to show the number of units in use, or tonnage produced.

For coal, statistics are available from 1926 to date, showing that mechanically loaded coal increased from 1 879 000 ton in 60 mines in 1923, to 47 177 000 ton in 317 mines in 1935 (5). In 1935, 13.5% of the underground production of bituminous coal and 21.2% of anthracite was loaded mechanically. In 1936, the combined coal production mechanically loaded or mined by stripping was 112 309 958 ton, or 22.75% of total output of both classes of coal (7). Table 1 shows trends in mechanical loading, conveying and handling, in terms of sales of equipment in 1937, compared with number of machines in preceding years (8). A survey based upon reports from 11 makers of shovels and 8 makers of scrapers and scraper hoists, shows yearly number sold for use underground in metal and nonmetallic mines, as given in Table 2 (1). Many earlier types were installed before 1923.

* Due to the great changes in equipment since the Second Edition of the book was published, most of this Section has been rewritten by Mr. Dake. Incorporated in the Section are data on the subjects hitherto contained in Article 92 of Section 10. Articles 9 to 13 herein, originally written for the First and Second Editions by Lincoln de G. Moss, are included in this revision.

Table 1. Sales of Mechanized Loading Equipment in 1937, Compared with Number of Machines in Use in Preceding Years

	Number of machines in use, as reported by mine operators								1937 Machines sold, reported by 28 makers
	1929	1930	1931	1932	1933	1934	1935	1936	
Bituminous mines;									
Mobile loaders	488	545	583	548	523	534	657	980	292
Scrapers	126	150	146	128	93	119	78	106	13
Pit-car loaders	2 521	2 876	3 428	3 112	2 453	2 288	2 098	1 851	32
Conveyer equip, duckbills and other self-loaders	99	140	165	159	132	157	179	234 } (2)	835
Hand-loaded conveyers	(1)	(1)	(1)	(1)	525	574	670	935 } (2)	
Anthracite Mines (Penn):									
Mobile loaders }	350	384	5	11	18	14	1	483(3)	16
Scrapers }			457	479	455	517	507		
Pit-car loaders	....	....	28	24	19	25	22	1 679(3)	260(2)
Conveyer equip, duckbills and other self-loaders	355	421	1	17	12	13	30		
Hand-loaded conveyers	....	....	547	818	940	1 338	1 563		

(1) Number of units not reported in these years. (2) Reported as face conveyers (hand-loaded) shaker drives, and duckbills. Sales in 1937 are not exactly comparable with number in use in 1936, due to uncertainties in defining what constitutes a conveyer. (3) Penn Dept of Mines.

Table 2. Sales of Underground Shovels and Scrapers in Metal and Non-metallic Mines

Figures show sales in continental U S, not including exports to contractors on construction projects (subject to revision)

Year	Scraper loaders (hoists or complete units)	Shovel loaders	Year	Scraper loaders (hoists or complete units)	Shovel loaders
1923	254	57	1931	126	2
1924	341	18	1932	104	14
1925	373	15	1933	62	12
1926	284	36	1934	67	23
1927	414	39	1935	135	44
1928	363	32	1936	249	70
1929	645	35			
1930	335	22	Totals	3 752	419

As effected by geographical distribution of mechanical loading, equipment differences, wage rates and physical conditions, the coal areas where mechanization advanced farthest to 1935 were the northern Rocky Mountain states, Ill, and Ind, where seam conditions were favorable and wage rates, from 1923 to 1933, compared with the southern and eastern fields, were relatively high. In 1936, 92% of the deep-mined output of Wyo was mechanically loaded; in Mont, 83.2%; in Ind, 65.6%; and in Ill, 63.6%. In 1935, the proportion of the W Va output mechanically loaded was 2.1%, and in 1936, 7.4%, while Ky records 1.4% in 1936. Recently, market conditions and higher wages have stimulated mechanization in the Appalachian region, and a large part of the equipment sales reported by makers in 1937 went to the bituminous fields of the east and south. The largest installations of mechanical loading equipment seem to have been in W Va in 1936 and 1937. Types of mechanized equipment sold from 1933 to 1937 are shown in Table 3 (8).

Table 3. Mechanized Equipment Sold to Coal Mines, 1933 to 1937 (*a*)

	1933	1934	1935	1936	1937	Percent increase (+) or decrease (−)	
						1937 over 1936	1937 over 1935
Mobile loaders	41	55	115	344	292	− 15.1	+153.9
Scrapers (*b*)	65	34	22	28	29	+ 3.6	+ 31.8
Conveyers (*c*)	396	610	681	994	1 095	+ 10.2	+ 60.8
Pit-car loaders	18	26	28	11	32	+190.9	+ 14.3

(*a*) As reported by 29 makers, 28 of whom were canvassed each year from 1933 to 1936. (*b*) Reported as scrapers, or scraper haulers and hoists. (*c*) Includes hand-loaded conveyers, duckbills and other self-loading heads. A number of these, in 1936–1937, were used with mobile-loading machines.

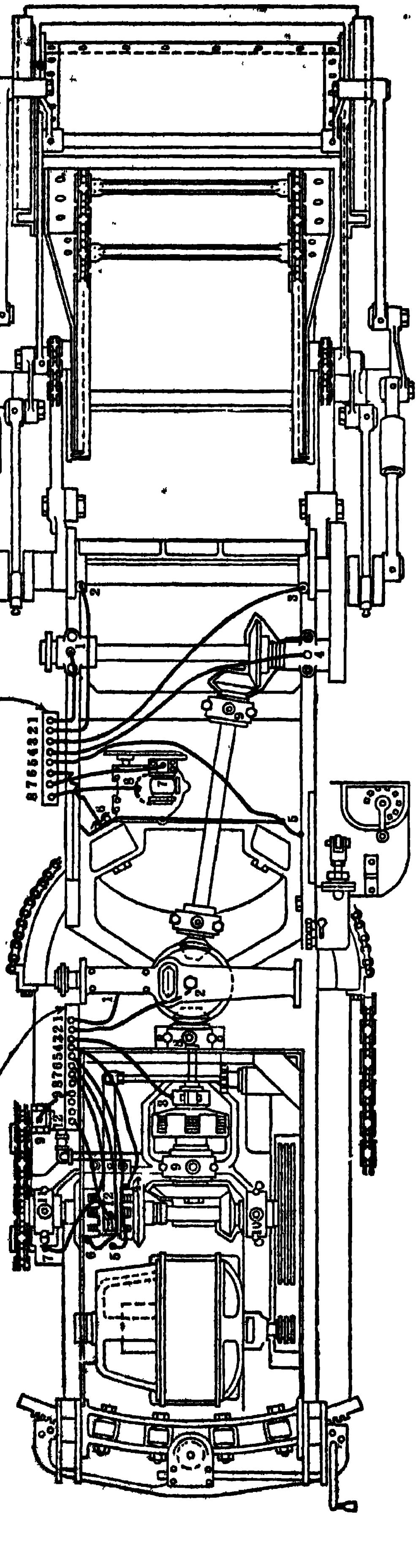


Fig 1. Myers-Whaley Shoveling and Loading Machine

2. TYPES OF MECHANIZED COAL MINING EQUIPMENT

Though many types have been invented since the introduction of the Stanley Header from England in 1888, progress has been measured in terms of evolution or elimination, and only the major units now being used commercially are herein described.

Mobile Loaders

Mobile loaders are those using: (*a*) shovels; (*b*) gathering devices. The Myers-Whaley and others of class (*a*) force the shovel forward to enter the broken coal; then the loading head is tilted, and discharges backwards onto a receiving plate and into the conveyer. Machines of class (*b*), as the Jeffrey, Clarkson and Goodman, have a continuous chain on each side of a conveyer; the chains moving in opposite directions, and carrying "flights" running horiz on the floor to undermine the face and gather broken coal into the conveyer. Joy machines have 2 gathering arms on opposite sides of the loading head, which pull down and sweep the standing coal into the conveyer, for delivering to cars. Myers-Whaley, Jeffrey, Clarkson, Goodman and Umeco machines are track-mounted; the Joy, caterpillar mounted. Mobile loaders comprise scrapers, self-loading conveyers, pit-car loaders, and types within each class, and are described here in chronological order of their development.

Myers-Whaley shoveling and loading machine (Myers-Whaley Co, Knoxville, Tenn). The first was installed 1908, by Windrock Coal and Coke Co, Tenn. An automatic shovel (Fig 1) is mounted on a swinging jib, pivoted on rear end of main frame. The jib carries a conveyer, delivering from the shovel to a second conveyer on a rear frame, also pivoted for lateral movement. The truck wheels are driven by a reversing clutch, worm and chain drive, for moving forward or backward. The shovel itself is wide and short, supported in inclined guides at the rear and moved by driving rods; it has side pieces, connected to the rods for tilting. The two sets of driving rods are synchronized, so that the shovel lip travels in an orbit, its rear end traveling in a plane inclined to the horiz. When front end is lifted, the material slides onto a receiving plate below and between the guides, and thence to the conveyer. The machine is driven by a hydraulic motor on the chassis; 4 levers

control the major movements, one for forward and backward movement of the entire machine, another operating the motor which swings the jib section, a third raises or lowers the rear conveyer and the fourth adjusts height of the shovel. By lateral adjustment of the rear conveyer, the material is loaded directly behind, or into cars standing on side tracks. These machines are of 2 types: (*a*) for rock work (Fig 1); (*b*) the "Automat," for car loading in coal mines. The No 3 Automat is designed for 4-ft headroom, from top of rail to roof, and 45-in traveling clearance; the No 4 for 5-ft headroom and 4.5-ft clearance. Both have permissible (inclosed) motors, for use in gaseous mines (Table 4).

Table 4. Whaley Automats

	No 3	No 4
Weight, lb	17 500	19 500
Length (approx to suit mine car), ft	29	30
Width (overall track gage to suit), in	68.5	68.5
Height (parallel lift, rear conveyer), in	44	53
Reach of shovel (each side center line of track), ft	11	12
Width of shovel, in	44	36
Power consumption (approx)	1/4 kw-hr per ton loaded	
Motor (continuous rating)	7 ton per min total time	
Aver loading rate	3 ton per min, actual loading time	

Joy loaders (Joy Mfg Co, Franklin, Pa). An experimental loader was installed in Sommers No 2 mine, Pittsburgh Coal Co, Pa, in 1916. The successful forerunner of present types was installed in same mine, 1917. The loader is mounted on a self-propelling chassis, and comprises: (*a*) gathering head with rotating arms, actuated through multiple-disk clutches set to slip at overloads, which sweep the coal into a conveyer delivering to mine cars; (*b*) caterpillar traction, having gaged lugs to act as wheel flanges for moving about on mine track, and allowing free movement off-track, when loading from floor at working faces. These loaders have 2 kinds of conveyers: types 8 BU, 7 BU and 11 BU have a single-strand flexible conveyer from gathering head to end of discharge boom, which swings through an arc of 45° on each side of center line; and types 5 BU and 10 BU, 2 double-strand conveyers, one from gathering-head to a discharge hopper, which is a part of the second conveyer and is mounted on a boom swinging through an arc of 90° on each side. The gathering head moves vertically, and discharging conveyer is adjustable vert and horiz by hydraulic pressure, controlled through a multiple-acting valve, placed close to clutch levers and motor starter. All the above loaders have a single motor for tramming, loading and conveying, and are operated by 1 man. The Joy Junior, built especially for thin seams and conveyer transport behind the machine, has 2 motors in tandem, one for the hydraulic system and one for each caterpillar, for locomotion and steering. Its total height is 26 in. A few models are made with uninclosed elec equipment, but are approved by U S Bur of Mines. For lowering through mine shafts of small section, the loaders may be disassembled into 3 parts: gathering head, chassis and discharge conveyer. Typical units are shown in Fig 2, 3; specifications, for thickness of coal seam from 30 in up, are given in Table 5.

Table 5. Specifications of Joy Loaders

	Joy Jr	8 BU	7 BU	5 BU	10 BU	11 BU
Rated capac, ton per min	3/4	1 1/2	2	2	4	4
Maximum capac, ton per min	1 1/2	2 1/2	3 1/2	3 1/2	6 1/2	6 1/2
Total wt, lb	8 000	9 500	14 500	15 600	19 000	19 000
" height, in	26	35	40	53	54	54
" width	4′ 4″	4′ 6″	6′ 0″	6′ 0″	7′ 0″	7′ 0″
" length	17′ 9″	20′ 5″	23′ 9″	24′ 6″	25′ 0″	25′ 0″
Speed of caterpillar, low, ft per min	40	55	37	37	54	54
Speed of caterpillar, high, ft per min	120	170	114	114	178	178
Speed front conveyer, ft per min	170	167	175	191	220	264
Speed rear conveyer, ft per min	.....	.....	.....	258	240	.....
Speed of gathering arms, strokes per min	35	42	37	38	37	37
Max reach of gathering arms	4′ 10″	5′ 4″	6′ 8″	6′ 8″	7′ 4″	7′ 4″
Number of motors	5	1	1	1	1	1
Motor rating, hp (4 motors)	20	20	35	35	50	50
Crank-pin digging force, lb	3 300	4 200	5 750	5 750	7 900	7 900

Jeffrey loaders (Jeffrey Mfg Co, Columbus, Ohio). This Co, one of the first in the field, now offers 2 types of mobile loaders: the 44 class, C, CC, D, DD, E and EE, was

brought out in 1929. In the self-propelling, track-mounted 44-DD and 44-EE machines (Fig 4), the gathering conveyer has a jib with continuous chain "flights" running partly on mine floor to sweep the coal upward, through a trough, into the discharge conveyer. The latter is raised or swung right or left for loading by rope drums, which also feed it into the

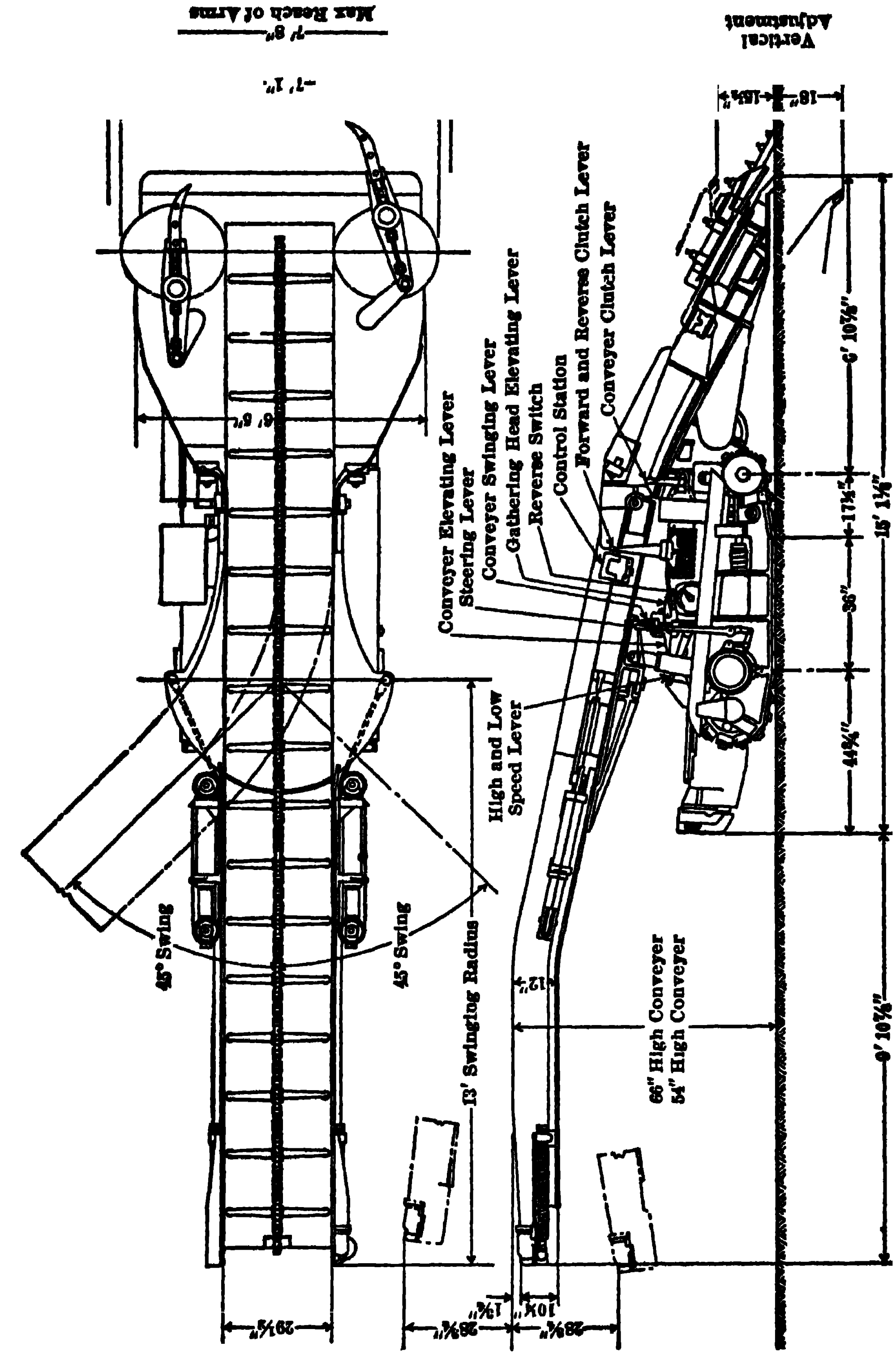


Fig 2. Joy 11-BU Loader

coal. The chassis carries the controls for all movements. There are 2 motors, in both non-permissible and government-approved types, one in gathering head and one in the chassis. Machines are in 3 sizes, high, standard and low (Fig 4), for rated loading capacities of 1 to 2 ton per min. Max heights above rails, 43–60 in; overall lengths, 37.5–47.5 ft.

The track-mounted L-400 machine, rated at aver of 3 ton per min (max 6 ton in loose coal) is single-motored; height, 40 in above rails. The gathering head carries a double-chain conveyer, one chain running clockwise; the other, counter-clockwise, on which are mounted "flights" meeting to form an upward-drag conveyer. Flights run horiz at level of mine floor to undermine standing

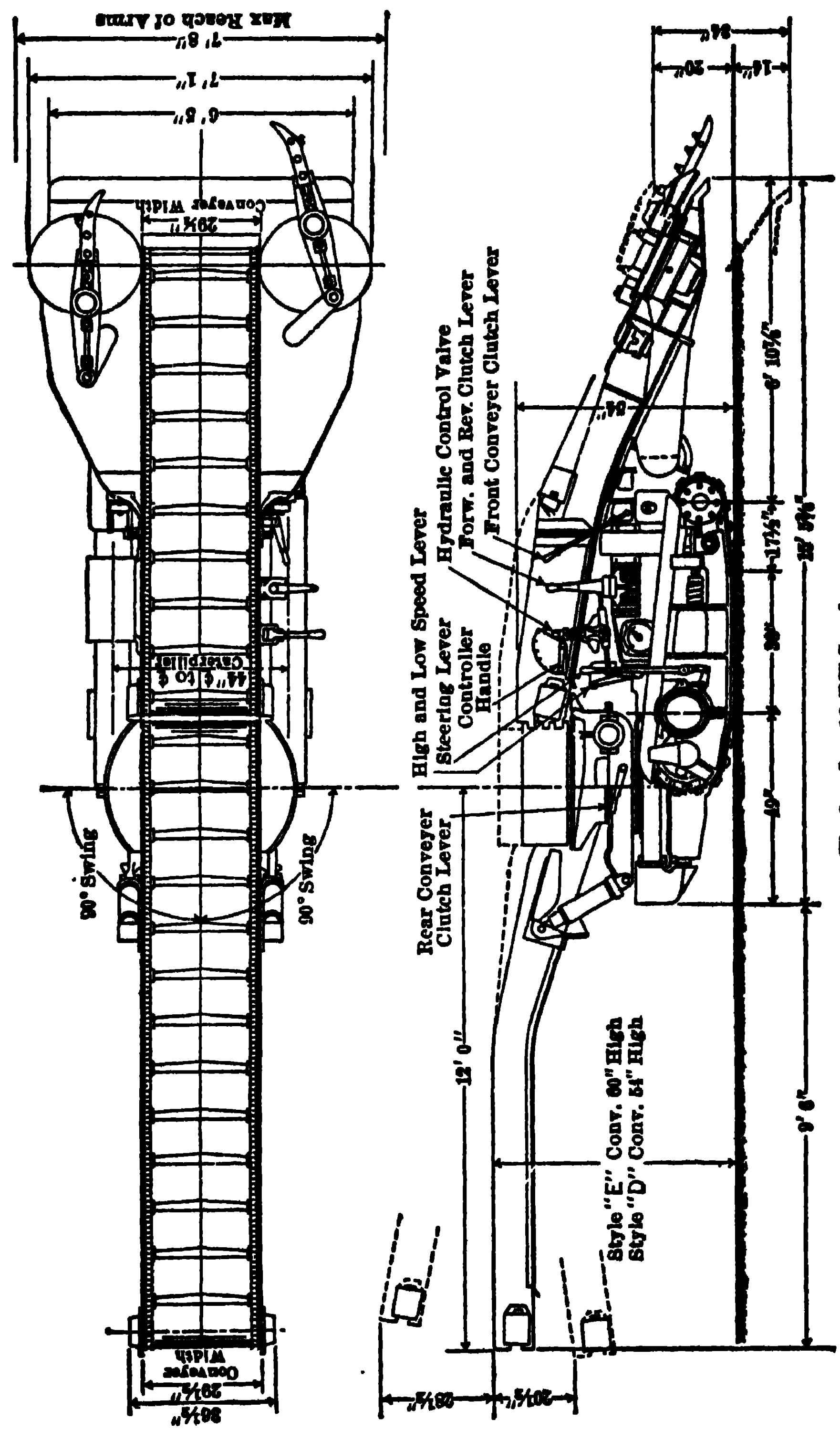


Fig 3. Joy 10-BU Loader

coal, and gather it with minimum crowding action. Discharge conveyer is flexible single-strand, adjustable horiz through 40° on each side of center line, and 18 in vert above and below normal level line of the boom, which is pivoted on main chassis. The chassis carries the motor and power take-off for all movements, and supports gathering and discharge conveyers at the same pivot points. Hydraulic cylinders on the chassis guide movements of both conveyers, with a central

starter and "finger-touch" control, for one man operation. The L 400 has 2 tramming speeds, slow "sumping" and fast traveling, an automatic cable-reel, trolley pole being optional.

Clarkson loader (Clarkson Mfg Co, Nashville, Ill) is track-mounted with 3 main units: gathering head, main frame and discharge conveyer (Fig 5). The gathering head, of 24-in

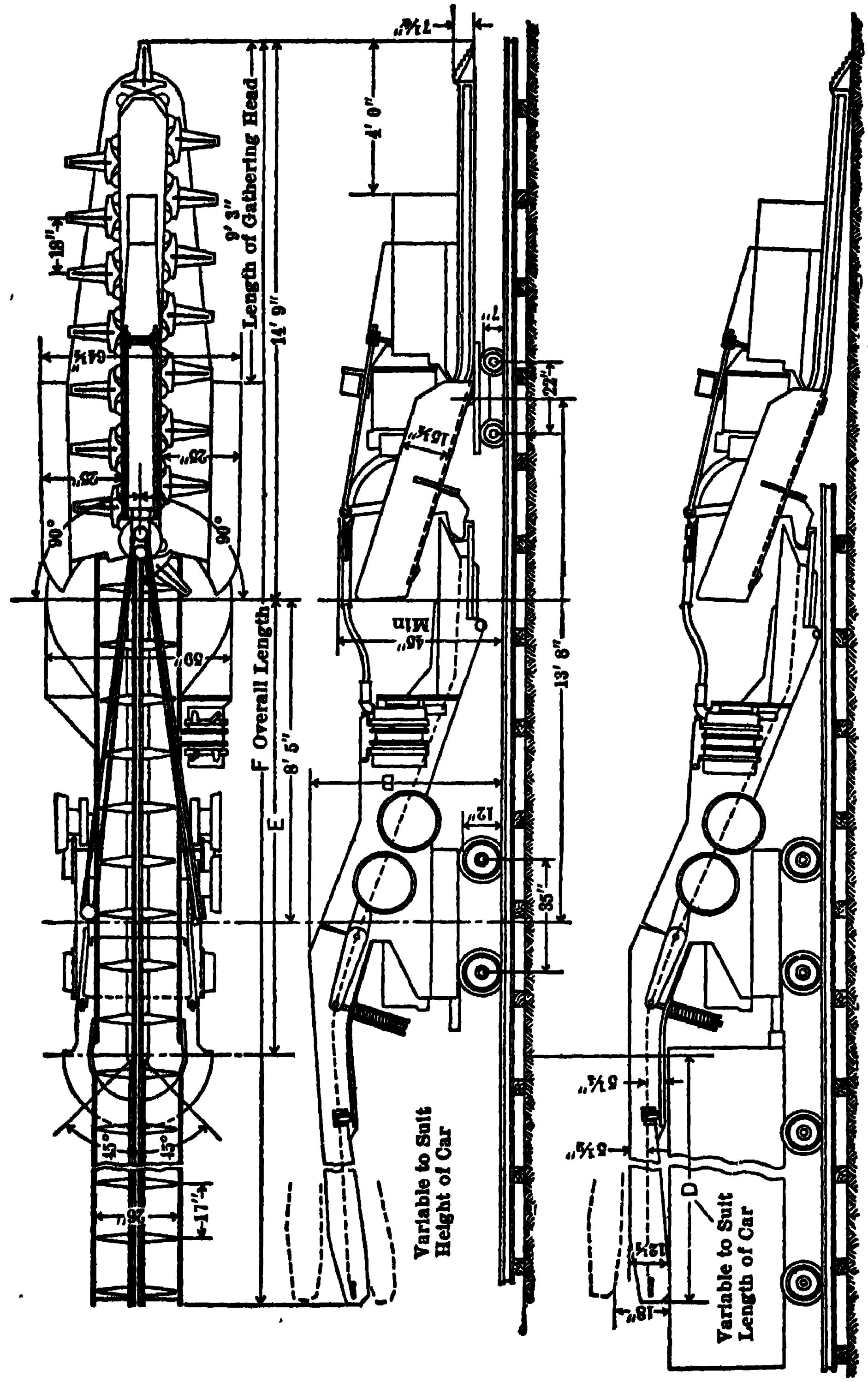


Fig 4. Jeffrey Loaders, High, Standard, and Low Types

I-beams, serves as a conveyer trough and as a base for attaching side plates and angle irons to form the foundation for the 20-hp gathering-head motor, placed under rear end of conveyer. Two deck plates form the loading head proper, which carries on either side gathering chains revolving in opposite directions. To the chains are attached large, pointed picks

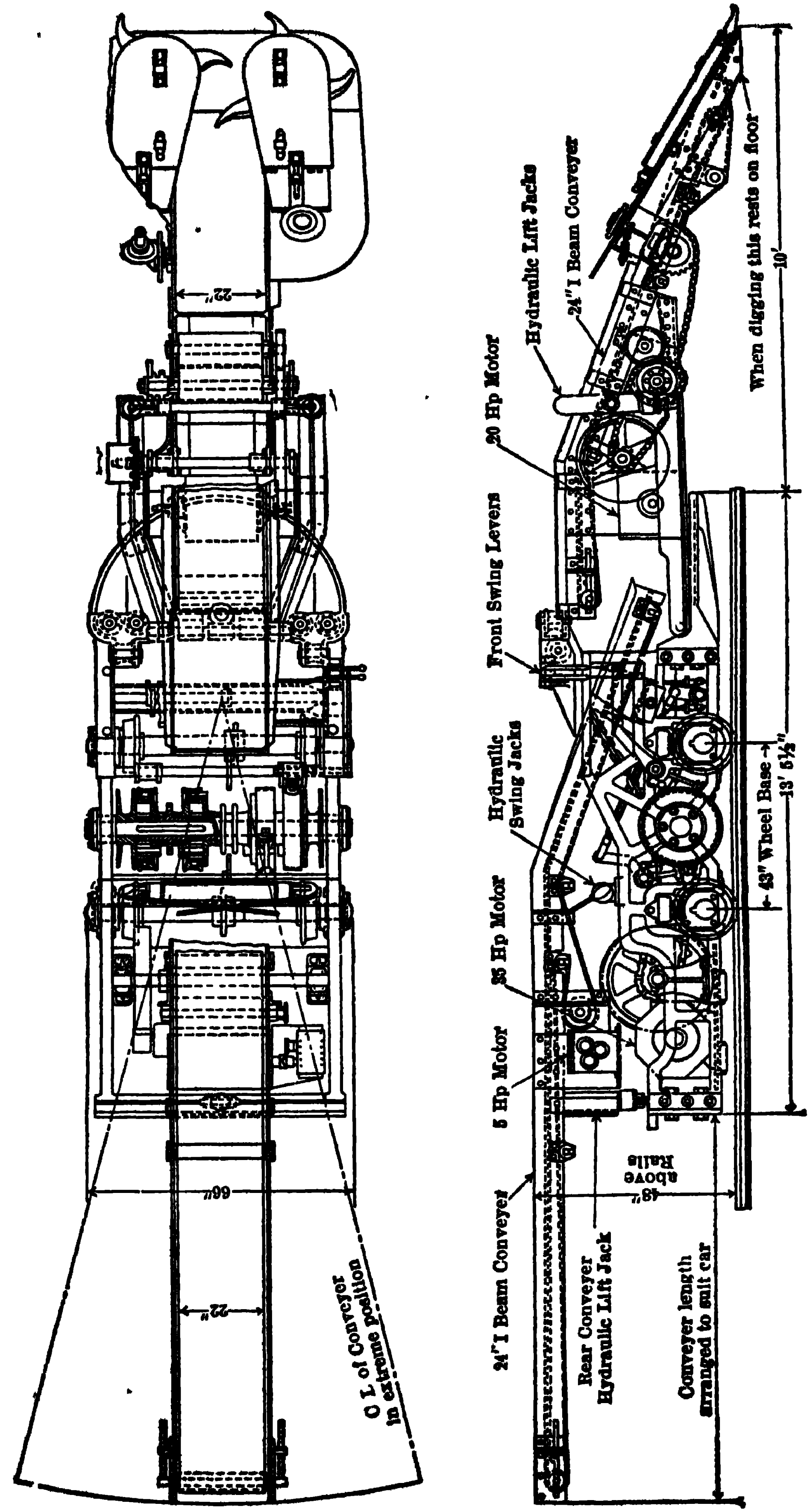


Fig 5. Clarkson Loader

to loosen and carry the coal upward to the belt conveyer in the I-beam trough. The gathering head is adjustable by hydraulic jacks, and is moved mechanically to either side of the center line. Main frame is supported by heavy axles in a 4-wheel truck, a front bumper acting as a swing support for the gathering head, and a rear bumper as a special support for the rear conveyer. The 35-hp motor is in the main frame, with one-man controls for all movements. The discharge conveyer, having a 22-in belt and driven by a separate 5-hp motor, is adjustable both horiz and vert; wt, 18 500 lb; length, 13 ft; width, 66 in; height, 48 in; capac, 2 ton per min.

Goodman loader (Goodman Mfg Co, Chicago) installed their first shovel for coal loading in 1924 and in 1933 the first track-mounted conveyer unit was operated in Ill and Ind mines. Type 260 loader (Fig 6) has a gathering head and front and discharge conveyers.

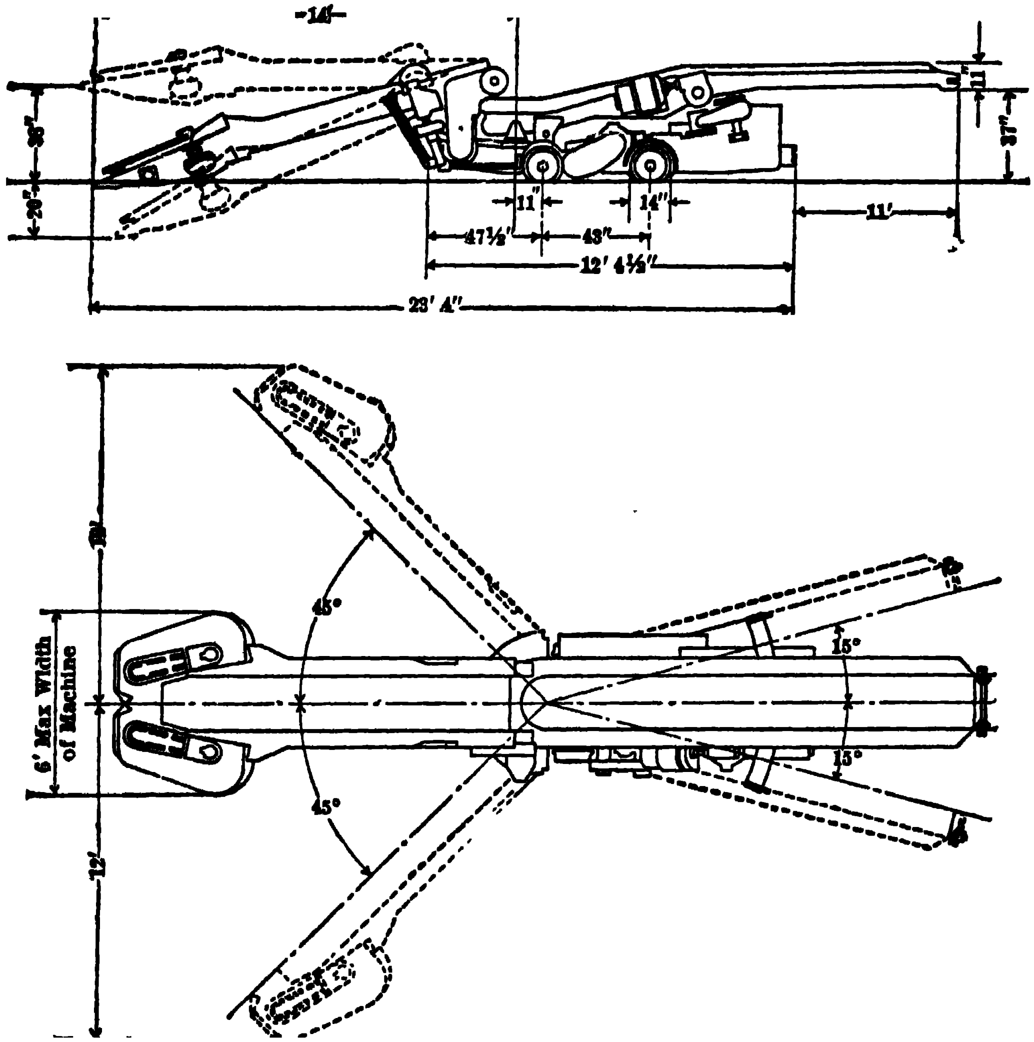


Fig 6. Goodman Loader, Type 260

The head carries 2 chains running in opposite directions, with 4 digging arms and 4 snubbing arms, equally spaced and alternated on each chain; is adjustable vert from 20 in below to 38 in above the track, and can swing 45° on each side of center line. Gathering mechanisms are mounted on each side of the main drag-flight conveyer, all protected from excessive shocks by a slip-clutch sprocket geared to main motor. The chassis, supported at 4 points by floating axles, carries the transmission gearing for low and high tramming speeds, the 50-hp main motor, and the discharge-conveyer elevating motor, with all controls for one-man operation. The discharge conveyer, driven by a separate motor, is also of the drag-flight type, and is adjustable vert from 11 in above to 37 in below the level line of the boom, which swings 15° on each side of center line. This machine has a max clean-up width of 24 ft, and can work in coal seams to 60-in thickness. Wt, 13 ton; height, 54 in. A new Goodman 360 loader, similar in general to type 260, and designed for seams as thin as 4.5 ft, appeared in Jan, 1938.

Umeco loader (Utility Mine Equipment Co, St Louis) installed their first mobile loader in 1933. The latest type, placed in the St Louis and O'Fallon Coal Co's mine, Ill, in 1936, is track-mounted, with 2 continuous-chain gathering mechanisms, one on each side of the conveyer. Arms extending ahead of the gathering apron sweep the coal upward and into a drag-flight conveyer, delivering to the discharge conveyer, pivoted at rear of the chassis. The loading head is adjustable 54 in vert and swings through an arc of 90°; the rear conveyer has a vert adjustment of 24 in and swings through 60°. A 15-hp motor drives the gathering head, a 10-hp the chassis, and a 5-hp motor the rear conveyer. Height above rails, 33 in; overall length, about 30 ft; width, 5 ft; approx wt, 11 ton; loading capac, 4 ton per min.

Scraper Loaders (See also Sec 3)

The first anthracite scrapers were used by Lehigh Valley Coal Co in 1914 (10). In 1916, the Evans scraper (Goodman Mfg Co) and others were installed by the Hudson Coal Co, Scranton. Evans and Goodman scrapers were first used in bituminous fields by the Penna Coal and Coke Co in 1917. The Goodman 23-B scraper was the forerunner of several types and sizes by that Co. The number of scraper loaders in coal mines declined from 1930 to 1937, according to makers' reports (11), due to the facts that effic scraper operation demands very favorable mining conditions, adequate roof span and roof control (12); and because improvements in mobile loaders and conveyers have widened their fields of use since 1934.

Table 6. Specifications of Goodman Scraper Hoists

	Type	Motor hp	Arrangement of drums, Fig 6	Rope speed and pull				Rope capac of each drum					Wt with motor, but no rope	
				Max speed, ft	Min pull, lb	Min speed, ft	Max pull, lb	3/8"	1/2"	5/8"	3/4"	7/8"	A-c	D-c
Two drums	1 136	7 1/2	(a)	300	825	125	1 975	700	400	......	......	......	1 880	1 825
		10	"	300	1 100	125	2 650	700	400	......	......	....	1 920	1 880
		15	"	300	1 650	125	3 950	700	400	......	......	.....	2 070	2 025
	336-A	15	(b)	300	1 650	155	3 200	725	435	275	......	......	2 410	2 365
		20	"	300	2 200	155	4 250	725	435	275	......	......	2 520	2 455
		25	"	300	2 750	155	5 325	725	435	275	......	......	2 610	2 455
	336-B	15	(a)	300	1 650	142	3 475	1 100	650	425	....	......	2 335	2 290
		20	"	300	2 200	142	4 650	1 100	650	425	......	......	2 445	2 380
		25	"	300	2 750	142	5 800	1 100	650	425	......	......	2 535	2 380
	936	15	Tandem	600	825	325	1 525	2 000	1 150	775	520	......	3 175	3 300
		25	"	600	1 375	325	2 525	2 000	1 150	775	520	......	3 225	3 590
		35	"	600	1 925	325	3 550	2 000	1 150	775	520	....	3 275	3 650
		45	"	600	2 500	325	4 575	2 000	1 150	775	520	.....	3 425	3 900
		50	"	600	2 750	325	5 050	2 000	1 150	775	520	......	3 450	3 950
	36	50	Tandem	500	3 300	250	6 600	....	1 300	850	600	......	4 200	4 500
Three drums	736-A	15	(a)	300	1 650	175	2 825	900	500	325	215	......	3 075	3 200
		25	"	300	2 750	175	4 700	900	500	325	215	......	3 200	3 700
		35	"	300	3 650	175	6 225	900	500	325	215	......	3 250	3 825
	636	50	Tandem	500	3 300	300	5 500	......	1 300	850	600	......	5 700	6 000
		75	"	500	5 000	300	8 250	......	1 300	850	600	......	6 500	6 780
	536	75	(c)	400	6 200	325	7 600	......	......	1 400	950	520	12 150	13 000
	436	125	(c)	300	13 750	300	13 750	......	......	2 600	1 800	1 275	22 800	23 700

Scrapers are primarily for use on long faces, though sometimes applied to entry and room loading. The equipment comprises a movable hoist, driven by compressed air or elec; ranging from 5 to 125 hp, with 2 or 3 drums, independently controlled, which carry the lengths of wire rope for traversing the working face. One rope fills and hauls the scraper to discharge point; another, attached to the back of scraper and threaded through a sheave anchored at far end of the face, pulls the scraper for re-loading. Scrapers are usually "hoe" and "box-shaped" (Fig 7, 8); or modified "Hoe" type (Fig 9) and are bottomless. Table 7 shows their capacities (13). They sometimes have teeth, to dislodge standing coal when pulled along the face. Due to their shuttle-like movements, scrapers are well suited to very thin seams, where cost of removing enough roof or floor material to permit use of mobile loaders would be prohibitive. However, the horiz clearance needed for scrapers to pass between face and props prevents their use except under favorable mining conditions, or where longwall mining can be adopted. In coal mines, the scrapers usually discharge into a feeder or elevating conveyer to the mine cars; or, for development work, utilize a car-

loading slide. Depending upon seam thickness, face length, operating speed of scraper, and overall effic and coordination of work, scraper-loading output ranges from 25 to 300 ton per shift. Coal weighs 50–75 lb per cu ft. For wt of soils and other substances, see Sec 3. Number of men per crew depends upon above factors, plus variables in seam characteristics, which may or may not demand heavy timbering and other non-productive work. Computation of mucking capac of a scraper involves: (*a*) size of body; (*b*) aver rope speed; (*c*) distance; (*d*) overall effic. The relation of these factors is expressed by $\frac{Cx+Tx+Sx+E}{2D}$ = cu ft handled, where C is scraper capac, cu ft; T, gross time, min; S, aver rope speed, ft per min; E, overall effic; D, distance scraped, ft.

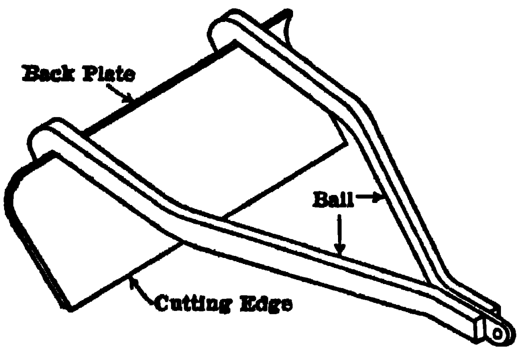


Fig 7. Hoe-type Scraper

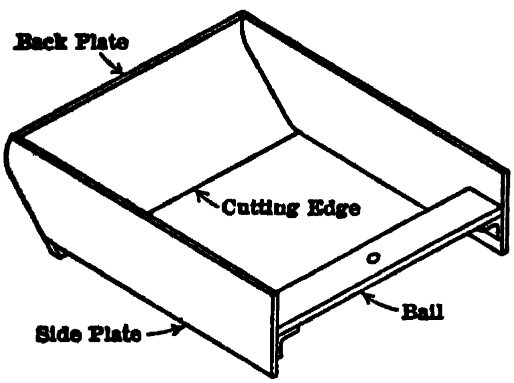


Fig 8. Box-type Scraper

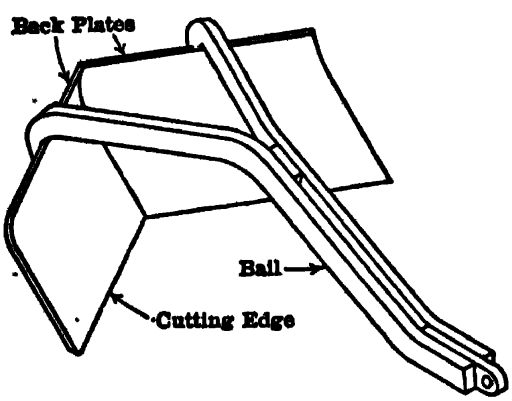


Fig 9. Modification of Hoe-type Scraper

Table 7. Widths and Capacities of Scraper Bodies

Width, in	Approx capac, cu ft		
	Hoe	Hoe-box	Box
	Scraper body, 20 in high		
30	.....	6	7
32	.....	6 1/2	7 1/2
34	6	7 1/2	8 1/2
36	6 1/2	8	9 1/2
38	7 1/2	9	11
40	8	10	12
42	9	11	13
44	10	12 1/2	14 1/2
46	11	13 1/2	16
	Scraper body, 22 in high		
48	13	16 1/2	19
50	14	18	20
52	15 1/2	19	22
54	17	21	24
56	18	22	26
58	19	24	28
60	21 1/2	26	30
62	22	27 1/2	32
64	23 1/2	29	34
66	25	31	36
68	26 1/2	33	38
70	28	35	40
72	29 1/2	37	43
74	31	39	45 1/2
76	32 1/2	41	48
78	34	43	50 1/2
80	36	45 1/2	53
82	38	48	55 1/2
84	40	50 1/2	58 1/2
86	42	53	61
88	44	55 1/2	64
90	46	58	67

Self-loading duckbill conveyers (Goodman Mfg Co) have a flat, funnel-shaped loading head which moves upon the floor and to which is attached one or more troughed pans, forming a continuous conveyer from face to point of discharge. The pans, mounted on rollers in cradles, and linked together are actuated by a compressed-air or elec motor, geared eccentrically to give a slow forward-thrusting motion and quick-lifting return to the conveyer. Loading-head is a telescoping double-trough, swiveled at its connection with main conveyer to allow movement along the face, and has a manually controlled ratcheting device to extend the duckbill into the prepared coal, or to retract it after the face has been cleared to depth of cut; a conveyer section is then added to continue work on a freshly prepared face.

Self-loading shaking conveyers have limited application, due to increased maintenance costs if the max distance for which they are designed is exceeded. They operate best in flat workings, or

where gradients are slightly in favor of load. Handled by 2 or more men; tonnages per shift vary widely with different seam conditions.

Hand-loaded conveyers, most used for coal, comprise: shaking, chain-and-flight, and belt conveyers. All are portable and have a compressed-air or elec motor, to which is attached a series of standardized conveyer sections as the face is advanced. In length, the sections usually conform to the aver depth to which coal cutters work, extensions being added as face advances. In room loading, conveyers are moved into position after the

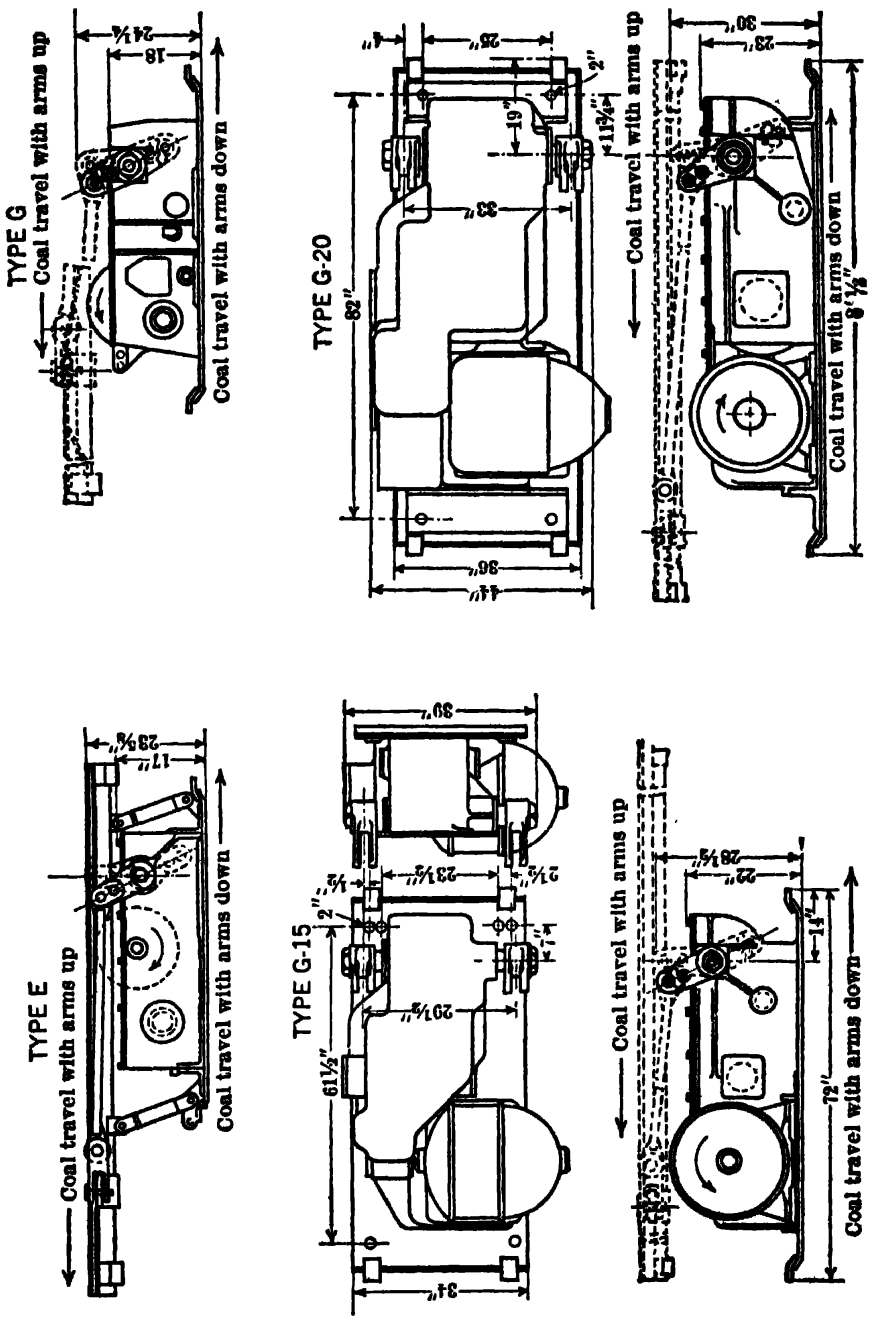


Fig 10. Goodman Shaker Conveyers

undercut is finished, and the shot-holes are drilled and fired; hand shovelers load the coal on to the conveyer.

Face conveyers discharge into the room conveyer, which is placed along either room-rib, and extended as work advances. The room conveyer delivers direct to mine cars, or, in multiple-conveyer workings, to a cross conveyer and cars at a central loading point; or to a main-line conveyer belt, for transport to shaft-bottom or to surface. Though the work of loading onto conveyers is less than for hand loading cars, the conveyer method is classed as mechanical loading only insofar as it reduces the height of shoveling, and provides continuous transport, thereby increasing production per man. But, its effect in

initial cost reduction over hand loading into cars is always worth consideration, aside from its tendency towards later replacement by complete mechanization. The saving shown by aver hand-loaded conveyer installations is reduction in amount of manual labor, or cost saving due to increased effic of working cycles and improved transport facilities.

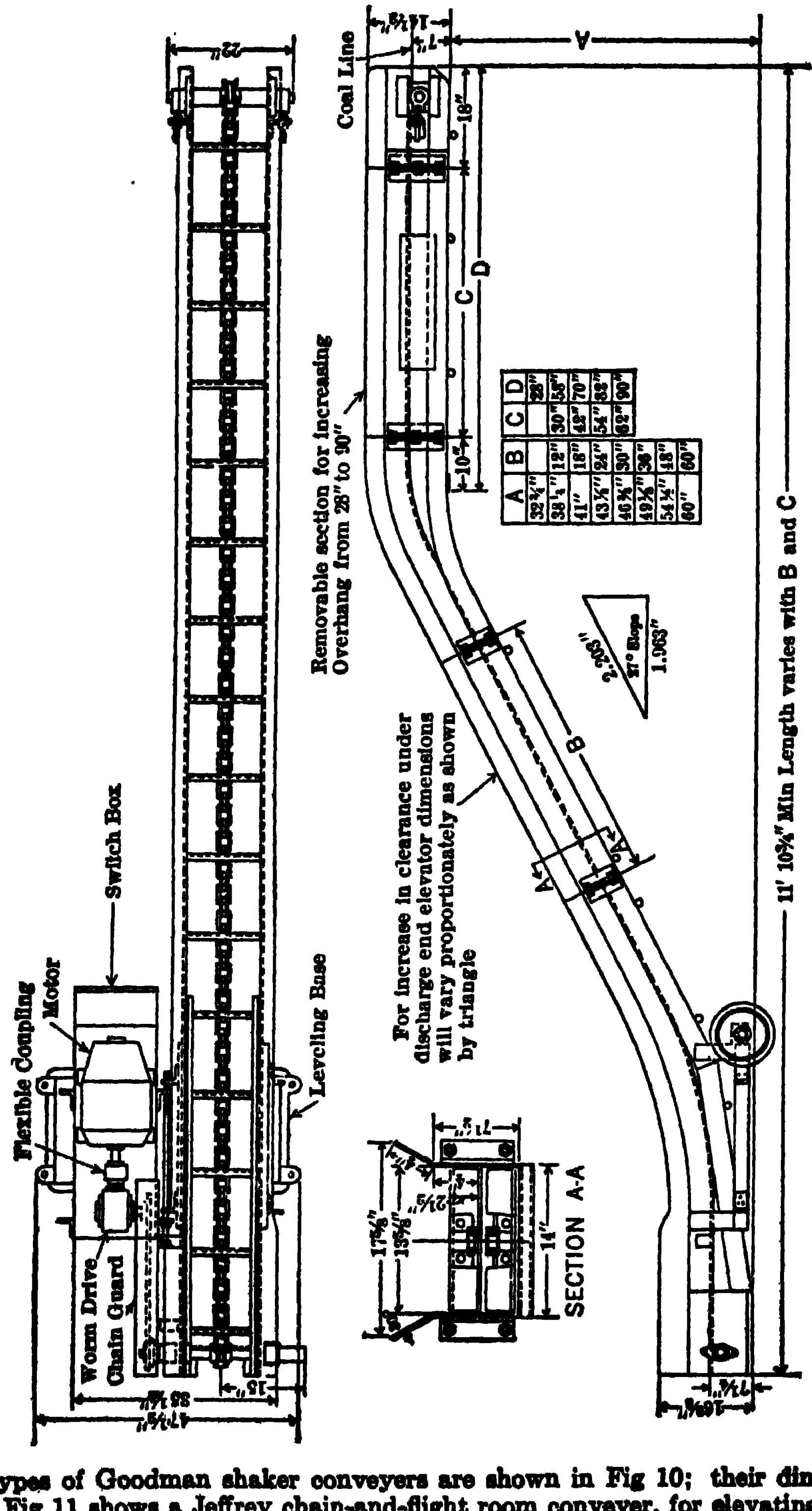


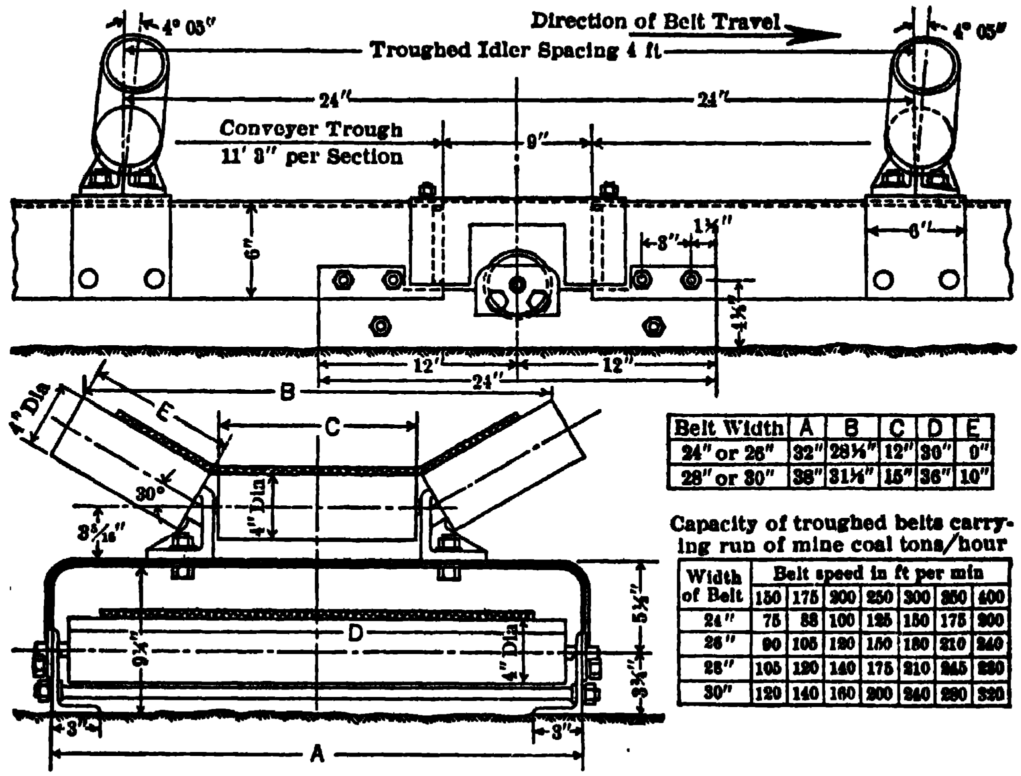
Fig 11. Jeffrey Chain-and-flight Room Conveyer

Four types of Goodman shaker conveyers are shown in Fig 10; their dimensions in Table 8. Fig 11 shows a Jeffrey chain-and-flight room conveyer, for elevating coal from room to mine cars. Weights: head end, 450 lb; tail end, 220 lb; standard section, 133 lb (all less power unit and chain); 10 hp motor, 1 050 lbs; chain and flights per ft, 5.2 lb. Fig 12 shows a typical Joy main-haulage belt conveyer; Table 9 giving weights of parts. Nearly all hand-loaded conveyers now have larger power units, greater widths and higher

speeds for coordination with high-speed car loading machines; or, in some flat-seam mines, for discharging at the surface (16).

"Rubber-tired haulage" is a recent development; a combination of mobile loader and storage-battery tractor, with trailer traveling on the mine floor from loader to conveyor, which discharges into mine cars.

Pit-car loaders, designed to decrease height of lift in hand-loading, were first introduced by Jeffrey Mfg Co in 1916, though it was not until 1928 that large-scale pit-car loaders were installed in Ill and Ind mines. They are low conveyers, on which the coal is shoveled at almost floor level, for elevation and delivery to a car (Fig 12). From 2 to 4 men pick down and shovel the coal on to the loader, which is moved from face to face as required. Output per man averages 20-24 ton, in 5- to 6-ft seams.



Belt Width	A	B	C	D	E
24" or 26"	32"	28 1/8"	12"	30"	0"
28" or 30"	38"	31 1/8"	15"	36"	10"

Capacity of troughed belts carrying run of mine coal tons/hour

Width of Belt	Belt speed in ft per min						
	150	175	200	250	300	350	400
24"	75	88	100	125	150	175	200
26"	90	105	120	150	180	210	240
28"	105	120	140	175	210	245	280
30"	120	140	160	200	240	280	320

Fig 12. Joy Main-haulage Belt Conveyer

Table 8. Specifications of Goodman Conveyer Drives

	E	G-12 1/2	G-15	G-20
Height, drive arms up........	23 5/8 in	24 1/4 in	28 1/2 in	30 in
Height, drive arms down....	17 in	18 in	22 in	23 in
Width without motor........	38 in	38 in	39 in	44 in
Length.....................	67 in	67 in	72 in	96 1/2 in
Size of trough..............	0	0-1	0-1-2-3	0-1-2-3
*Conveying capac............	15-20 ton per hr	21-38 ton per hr	18-67 ton per hr	19-70 ton per hr
*Length of trough line.......	Up to 350 ft	Up to 350 ft	Up to 350 ft	Up to 350 ft
Weight, without motor......	1 850 lb	2 150 lb	2 620 lb	4 500 lb
Strokes per min.............	70, 76 or 82	75-79-86	74-82	70-77
Length of stroke............	4 7/8 or 7 1/8 in	7 1/8-4 5/8 in	8-5 1/4 in	9-6 5/16 in

* For level grades and aver conditions.

Table 9. Joy Troughed Belt Conveyers

24-26-in belt conveyer	Wt, lb	30-in belt conveyer	Wt, lb
Drive unit, including 20-in drive pulley, 12-in snub pulley, 15-hp motor.......	3 800	Drive unit, including 30-in drive pulley, base, 40-hp motor and controller.....	6 130
Pan assembly.........................	259	Pan assembly.........................	340
Troughing idler assembly, including 2 idler brackets, 2 end and 1 center roller.....	54	Troughing idler assembly, including 2 idler brackets, 2 end and 1 center roller.	58
Return roller..........................	28	Return roller..........................	32
Cradle assembly.......................	47	Cradle assembly.......................	48
Take-up unit..........................	1 300	Enclosed tail end.....................	1 500

3. VARIABLE FACTORS AFFECTING TYPE OF INSTALLATION

Local conditions must be studied carefully, profit realized depending upon: (*a*) mental attitude of management and men towards machines and knowledge by mine foremen of the principles, and the selection of crews who will endeavor to modify old practices to fit the machines; (*b*) for coal, the minimum, aver and max thickness, and pitch of the seam, determine size and power of equipment for given conditions, and to handle loads at top speed on adverse gradients; (*c*) character and texture of coal affect mode of drilling and blasting and manner of preparing faces to load out; (*d*) size and shape of undercut block, and sizes of coal desired, affect the choice of loader capac per min and power required; (*e*) depth of cover overlying the seam and character of roof limit the width of working places, and determine the method of mining; (*f*) character of floor. Soft fireclay may absorb enough moisture to bog down machines and track, or scale off and mix with the coal while being loaded; hard floors reduce the grade on which caterpillar machines can maneuver, and increase the cost of maintaining rails for track-mounted machines. Use of mechanical loaders is not much affected by timbering, as systems now employed, even under adverse conditions, afford protection, while assuring enough clearance for loading and moving; (*g*) types of loaders suitable for advancing or retreating pillar and long-wall methods are important; the mechanical gathering of coal can be slow or rapid, with consequent high or low cost per ton. Changing from hand to complete mechanization requires an aver increase of 1-kw hr of power per ton of production.

Kinds of equipment for mechanized handling of coal from working face to surface preparation plant must be compared for different outputs, to determine loader output and cost per ton. Also, a proper balance is necessary between the capacities of coal cutters and loading machines, to avoid overtime labor rates. Bottom-, center- and top-cutting methods affect results of drilling and blasting, as well as of mechanical loading. An added cost of removing banded impurities by careful cutting may reduce the cost of surface picking or cleaning plants. Mechanical loading profit depends on net reduction in total costs, not in reducing certain phases of operation. For example, while "overshooting" may reduce face-loading costs, the resulting increase in fine sizes of coal lowers the sales receipts, and might mean a net loss on mechanized operation, as compared with hand loading. Tonnage per working face and concentration of working places determine the most economical capac, size and type of mechanical loader. Development in wide workings can be done by all loaders, but it is as impractical to apply slow-speed, low-capac units under favorable thick seam conditions, as to use high-speed, heavy-duty machines for thin seams or other adverse conditions. Thus, a loader of given capac can not be applied as economically to double-entry development in a thin seam, where shooting on the shift is prohibited, as in 5 parallel headings where blasting is permitted at any time. Small tonnage from isolated parts of a mine can be profitably loaded mechanically if the size of crew, plus the added productive capac of loading machines, is balanced against all limiting physical conditions. This is also true where adverse conditions impose more deadwork cycles, which, in turn, curtail available loading time. Tonnage produced per total employes, not the tonnage per machine-loader shift, determines the percentage reduction of mining costs. Also, the dimensions and aver capac of cars serving the loader, or the peak capac of conveyers from machines to central loading points, or to the surface, affect the aver tonnage per shift. Track arrangements for quick car changes behind loading machines, and the type of conveyers used, relative to the time for adding or taking off sections of conveyer also affect tonnage per shift. The number of cars in relation to length of haul, the round trips per shift, and the effic of dispatching loads and distributing empties, are often factors in economical mechanization. Conveyer transport, from working face to a central loading point, has largely replaced small-capac cars; and multiple conveyers, for continuous movement of coal from face to surface, have reduced the investment for transport in new mines. Tipple and preparation capac also affect the decision as to introducing mechanization.

In changing from hand to mechanical loading, where the seam has banded impurities, the waste may be separated underground or the entire product delivered to the tipple, thus requiring more pickers; in either case, this cost must be deducted from the savings due to mechanization. The saving by mechanical loaders also depends upon contract tonnage and day-wage rates, relative to output per man. As labor approximates 60% of total mine costs, an analysis of all operating conditions is necessary in comparing hand loading with expected results from mechanization. Since most of the variations are in development and tonnage production, all handling costs should be segregated, from face to main-line transport, and thence to preparation plant; supplemented by cost of mine maintenance, preparation, engineering, management, overhead charges, supplies and power. Table 10 contains an itemized list of these variables.

Table 10. Mechanization Variables

1. Attitude of men and management towards mechanisation
2. Operating organization effic
3. Kind of mine: shaft, slope or drift
4. Thickness of seam, minimum and max
5. Character and texture of coal
6. Pitch of seam
7. Wet or dry operation
8. Character of floor and roof
9. Seam impurities, banded or inherent
10. Depth of cover
11. System of mining
12. Panel dimensions
13. Width of entries and rooms
14. Longface or longwall
15. Timbering systems
16. Pillar extraction, advancing or retreating
17. Surface cleaning
18. Power and distribution system
19. Hoisting and transport
20. Rail weights, gage and condition of track
21. Conveyers and their capac
22. Aver length of main haulage and tonnage per trip
23. Type, number and capac of cars
24. Dispatching system and effic
25. Cutting methods and costs
26. Drilling and blasting costs
27. Loading methods and costs
28. Yardage and deadwork costs
29. Contract and day-wage rates

4. COMPARISON OF MECHANIZED COAL-MINING METHODS

Mobile loaders. Their application to multiple-entry development (see Sec 10, Coal Mining Methods), where break-throughs and room necks are turned at right angles, causes difficulty in laying track on curves of a radius permitting easy access of large machines. The difficulty may be met by driving these openings at angles less than 90°; or increasing the number of entries to 3 or 5, with tracks through crosscuts for flexible car transport. The multiple-entry systems simplify movements of cutting and loading machines, give more working places, cause less interference with mining, timbering, track extension and loader operation, and increase production.

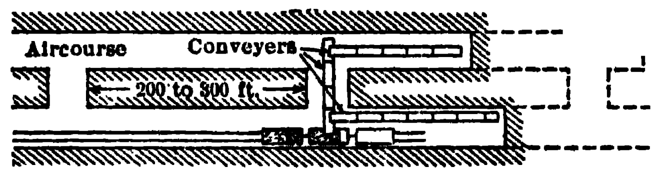


Fig 13. Conveyer Setup for Driving Haulageway and Aircourse Simultaneously

Conveyers may be used in development work for advancing a haulageway and air courses alternately, or driving both simultaneously (Fig 13). Although the latter plan increases equipment cost, it gives faster development, probable lower operating cost, and better timing of the working cycle.

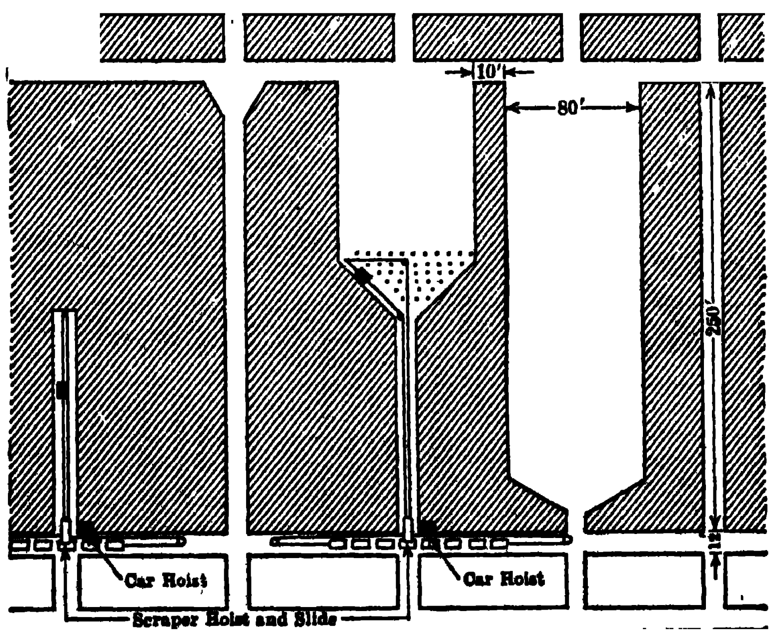


Fig 14. Scraper Loading in Rooms on the Retreat System

Scrapers, with inclined slide for loading cars in a single heading, and rooms mined on the retreat by a V-system, may be used in development (Fig 14). Scraper loading is also used for block and longwall work.

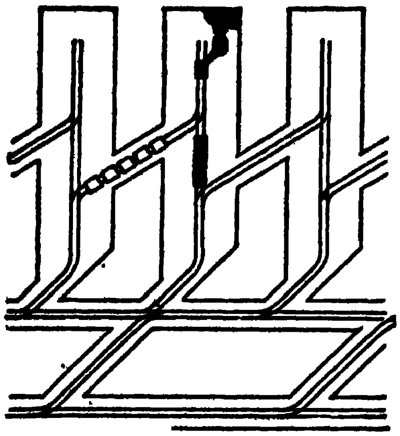


Fig 15. Mechanical Loading for Room-and-Pillar

Modified systems of mechanical loading, resulting in greater concentration of work, and more flexible trackage for cars behind the loader, are shown in Fig 15–18. The layouts in Fig 16, 17 are adapted to weak roofs close to the face. The "funnel system" of timbering uses 3 or more steel rails or H-beams, on temporary props, set to allow working clearance for the loader and which are moved forward after each cut, permanent timbering being set behind (Fig 17). Where roof is strong enough to allow driving wide rooms, the props may be centered, with a curved track along the face, for loading into several cars while moving past the machine (Fig 18). In the thick coal seams of Ill, a modified room-and-pillar system has triple main and double panel entries, all break-throughs and room-necks being at 45°. Two mobile loaders, 2 coal cutters and

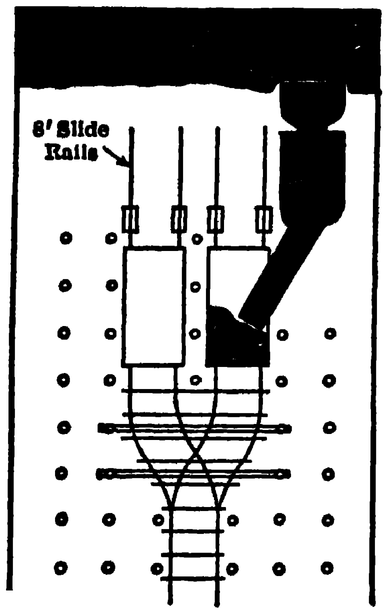


Fig 16. Mobile Loader with Portable Track

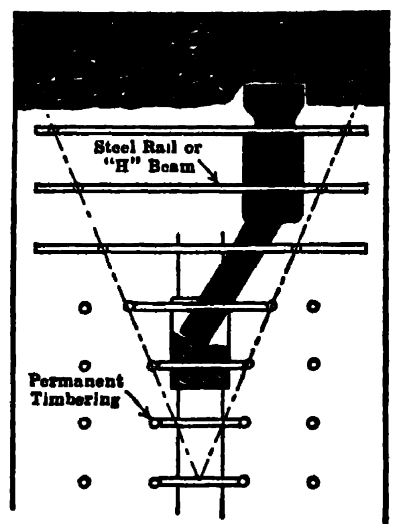


Fig 17. Loading with "Tunnel Timbering" and Steel Beams for Weak Roof

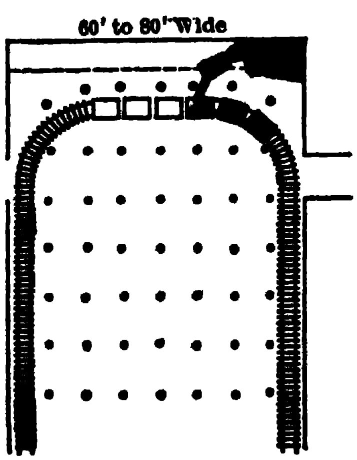


Fig 18. Loading on Circular Track

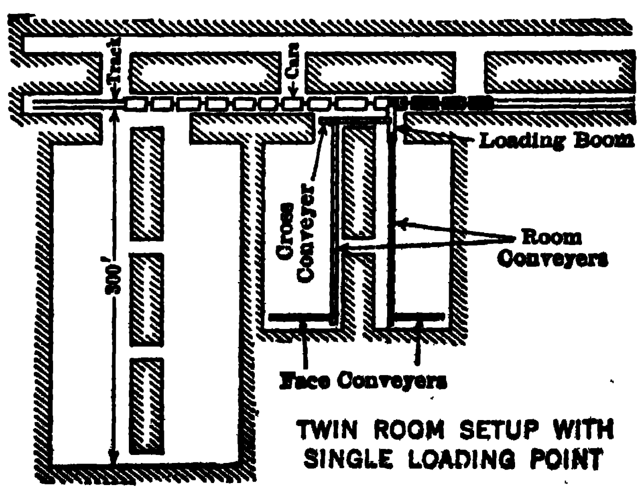


Fig 19. Face Conveyers for Rooms

2 drilling crews rotate work in 9 rooms on each side of panel entries, one development loader, double-shift, keeping pace with 4 room loaders on single shift.

Room production by conveyers (Fig 19). A twin-room set-up, with one loading point, requires a cross conveyer to the car loading boom. Fig 20 shows combination Joy chain-and-flight conveyer, with trough-belt for transport to surface (first installed in Goose Creek mine, Ky, in 1937). In the thick coal seams of Ind, the advancing rooms are double-tracked and break-throughs cut half through the pillars on both sides to give more faces for concentrated loading (Fig 21). High-tonnage production is made by mobile loaders in 6.5-ft seams in Ind and northern West Va; and complete pillar extraction by mobile loader and conveyer in a 42-in seam, southern West Va (Fig 22). Ideal face preparation for mechanical loading requires a completely loosened face, without excessive breakage of coal.

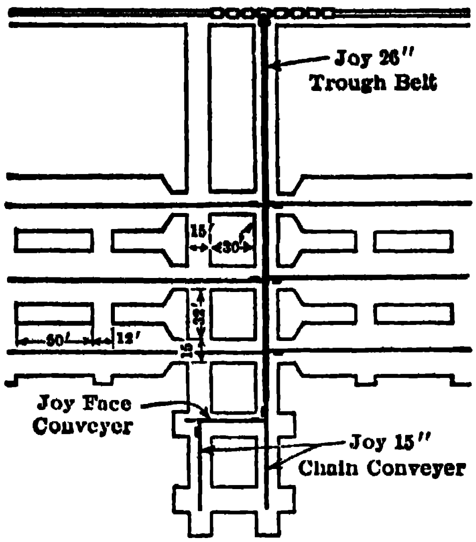


Fig 20. Joy Chain-and-flight and Trough Belt Conveyers

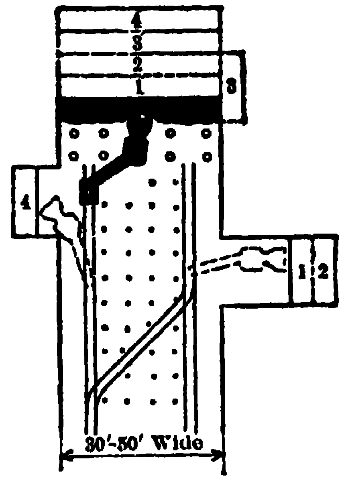


Fig 21. Double-tracked Room in Thick Indiana Coal Seam

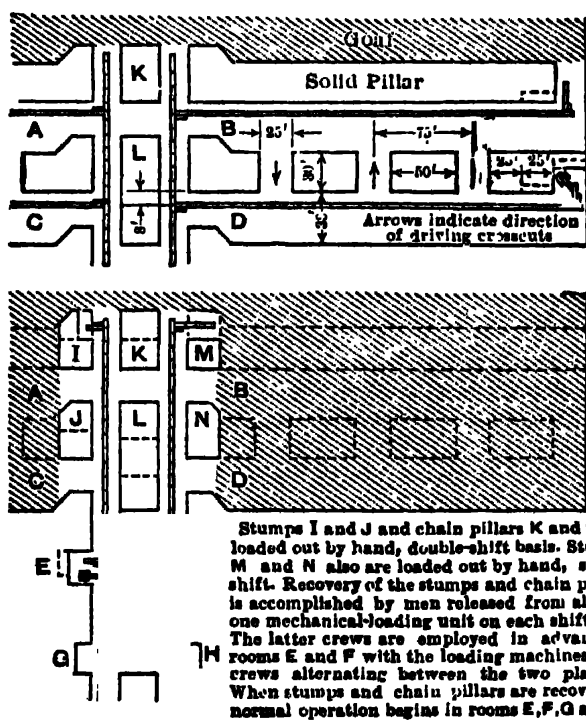


Fig 22. Pillar Extraction by Mobile Loader and Hand Conveyers

5. TRANSPORT FOR MECHANIZED MINING

Both locomotive haulage and machine loaders require heavy rails, up to 70 or even 90 lb on entries, instead of the former 30–40 lb. Cars also are larger; in 1938, an Ill mine introduced a 10-ton car (14-ton capac with sideboards). In room-and-pillar mining with mobile loaders, ideal car servicing requires trackage for continuous trips of empties pulled past the loader. Recent conveyer transport behind loaders permits continuous movement of coal from face to car-loading point or to surface. Typical mobile-loader installation, with conveyer transport is shown in Fig 23. Fig 24 shows hand loading onto multiple conveyers, discharging into a main-line unit delivering to cars at the entry, with one loading machine for the panel. Rubber-tired haulage recently installed, comprises tractor-trailer units running from loading machine to a central dumping point, for delivery to surface. For mines already having mechanical loaders, this equipment affords flexibility of haulage at relatively small capital cost. The added equipment cost at a mine of Ingle Coal Co, Ind, in a 6-ft seam, was about $23 000, of which $10 750 was for 5 tractors, $3 410 for 5 trail cars, $2 000 for dump hopper-feeder-conveyer, $4 440 for 2 transfer trucks, $1 200 for 30-kw battery-charging generator, and sundries $900. Already in use were 2 cutting machines, 2 loaders, 2 elec drills and the tractor batteries. With this equipment a force of 23 men delivered 900–1 000 ton of mine-run (720–800 ton clean coal) to the slope bottom every 7 hr. Max product per shift, 1 035 ton of mine run (830 ton clean coal); aver, July 1938, 750 ton prepared coal.

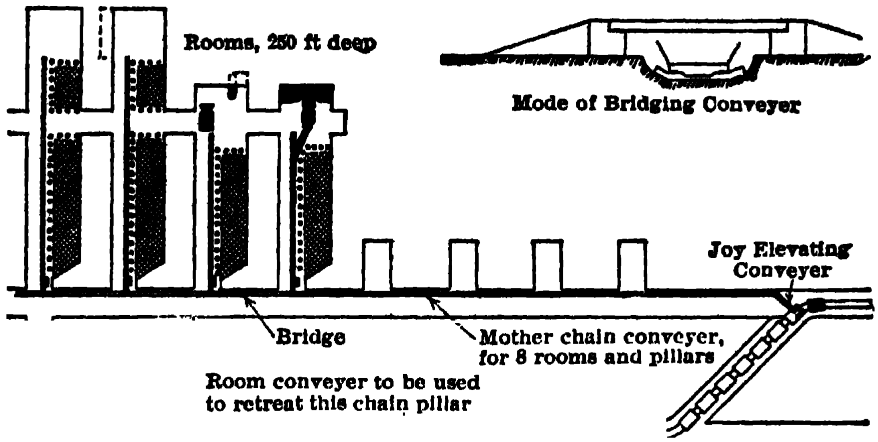


Fig 23. Chain Conveyer System, Pittsburgh Seam

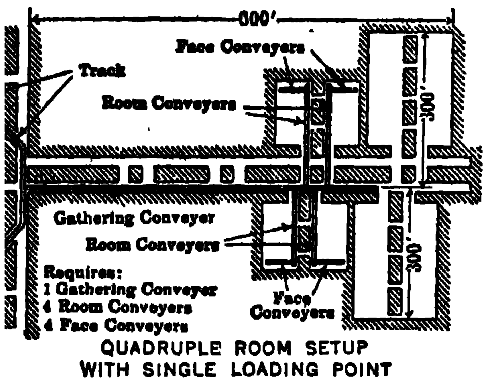


Fig 24. Quadruple Room Setup for Hand-loaded Conveyers, with One Machine Loading Point

6. COORDINATION OF MEN AND EQUIPMENT

Operating cycles are: (*a*) face preparation, cutting, drilling and blasting; (*b*) loading and transport; (*c*) clean-up, timbering, track or conveyer extension. Loader crews are 12–18 men for large machines in thick seams, to 5 or 6 men for small units in thin seams; correct number depends on character of seam, mining method, and kind of auxiliary equipment.

Profit possibilities. The following example is based on conditions in certain mines east of Mississippi River, and aver wage rates. The assumed type mine has favorable operating conditions, including good roof and a 5-ft coal seam; output, on deciding to change from hand to machine loading, 2 500 ton per shift. Development plan, room-and-pillar panels, with triple main entries, was continued with mechanical loading. To maintain same tonnage, 2 loaders were used in narrow work; 4 for wide work. Table 11, 12 show men and wages for delivering output to the main-line parting. Based on aver time distribution for

Table 11. Crew Makeup; 1 Man for 2 Loaders

Narrow work	Number of men	Total cost per day
Foreman ($8.00 per day)...	1/2	$4.00
Cutting and drilling men ($6.86)................	4	27.44
Shotfirer ($6.86)...........	1/2	3.43
Loader operator ($6.86)....	1	6.86
Loader helper ($6.00)......	1	6.00
Motorman ($6.16).........	1	6.16
Brakeman ($6.00).........	1	6.00
Track and timber men ($6.00)	2	12.00
Face laborer ($5.76).......	1	5.76
Total...............	12	$77 65

Table 12. Crew Makeup; 1 Man for 2 Loaders

Wide work	Number of men	Total cost per day
Foreman ($8 per day)......	1/2	$4.00
Cutters ($6.86)............	2	13.72
Driller ($6.16)............	1*	6.16
Shotfirer ($6.86)...........	1	6.86
Loader operator ($6.86)....	1	6.86
Loader helper ($6.00)......	1	6.00
Motormen ($6.16).........	2	12.32
Brakemen ($6.00)........	2	12.00
Relay motorman ($6.16)...	1/2	3.08
Trackmen ($6.00).........	2	12.00
Timbermen ($6 00)........	2	12.00
Facemen ($5.76)...........	2	11.52
Total...............	17	$106.52

* Pro rata share of total drillers.

Table 13. Loading Rates

Narrow work	Minutes	Shift time, %
Loading, 1.5 ton per min...	193	45.95
Car changing, 1.5 min per car	90	21.43
Moving loader, 5 min per move...................	60	14.29
Safety factor, delays......	77	18.33
Total shift time........	420	100.00

Table 14. Loading Rates

Wide work	Minutes	Shift time, %
Loading, 2.5 ton per min....	192	45.72
Car changing, 1 min per car..	100	23.81
Moving loader, 4 min per move.................	40	9.52
Safety factor, delays........	88	20.95
Total shift time........	420	100.00

Table 15. Time Study after Installation

Operation	Minutes	Percent	Direct cost
Loading...........	172	40.95	$43.62
Car change........	156	37.14	39.56
Moving loader.....	35	8.34	8.88
Car shortage.......	27	6.43	6.85
Late start........	30	7.14	7.61
Totals........	420	100.00	$106.52

Aver shift, started, 8 a m; aver shift, finished, 3 p m; aver ton per car, 4.4; aver cars loaded per shift, 78; aver ton per shift, 343.

Table 16. Loader on Development

Operation	Minutes	Percent	Direct cost
Loading...........	155	36.90	$28.66
Car change........	96	22.86	17.75
Moving loader.....	45	10.71	8.32
Car shortage.......	84	20.00	15.53
Delays............	40	9.53	7.39
Total........	420	100.00	$77.65

Aver time shift started, 8.10 a m; aver time shift finished, 3 p m; aver tons per car, 4.5; aver cars loaded per shift, 48; aver tons per shift, 217.

the loaders after partial coordination was effected, optimum estimates were made of loading rates and time factors for delays (Tables 13, 14). With 23.2% (580 ton) of total product from 2 crews on development, and 76.8% (1 920 ton) from 4 crews on room faces, an aver output of about 27.25 ton per man-shift, at cost of 23.23¢ per ton, was expected. Two weeks after installation, time studies on one wide-work machine for 4 shifts, showed (Table 15) that 86.43% of time, or $92.06 in wages, went to loading, car changing and moving machine from face to face, and 13.57% ($14.46) to time lost waiting for empties, and late starts at face. Output was 8 tons less per unit crew, and labor cost 9¢ more per ton than forecast. It might seem that, if lost time could be saved, 57 min would be gained for loading, car and machine moves, or about 13.5 more tons loaded per shift than in Table 14. But, total possible output would still be 124 ton below original estimate; and further study shows actual loading rate is 2 ton per min (not 2.5); that car-change time is double the estimate, and that the loader moves take 20% more time than warranted by distance covered. Also, it was found that the cutting machine men were not making the cuts for best blasting results and rapid loading. The shot-holes were not drilled horiz as planned, and the breaker, rib and top shots not placed high enough. Excessive time for car changing and moving the loader was partly due to lack of crew experience, and to the fact that at first only alternate crosscuts were tracked.

Table 16 shows aver of 3 consecutive-shift studies of 1 loader, which moved from face to face as per estimate. Face preparation gave loading rate of 1.4 ton per min, only 0.1 ton

Table 17. Results of 108 Time Studies

Operation	Minutes	Percent	Direct cost
Loading	181.3	43.17	$45.98
Car change	86.7	20.64	21.98
Moving machine	43.3	10.30	10.98
Faces not prepared (*a*)	25.1	5.98	6.37
Transport (*b*)	42.8	10.19	10.86
Power (*c*)	11.2	2.67	2.84
Supplies (*d*)	7.0	1.67	1.78
Maintenance (*e*)	22.6	5.38	5.73
Total	420.0	100.00	$106.52

Aver time shift started, 7.30 a m; aver time shift ended, 3.00 p m; aver tons per car, 4.802; aver cars loaded per shift, 96.312; aver tons per shift, 462.5. Delays, due to: (*a*) cutting, drilling, blasting, track and timbering; (*b*) wrecks, derailments and lack of cars; (*c*) no power, low voltage, cable and other repairs; (*d*) lack of supplies at faces and distributing system; (*e*) mechanical failures of all equipment.

Table 18. Avoidable Delays, Referring to Table 19

Classification	Losses indicated per shift, by 108 time studies		
	Minutes	Percent	Direct cost
Faces not prepared	25.1	5.98	$6.37
Transport	42.8	10.19	10.86
Power	11.2	2.67	2.84
Supplies	7.0	1.67	1.78
Maintenance	22.6	5.38	5.73
Total	108.7	25.89	$27.58
Operating delays	**Aver delays per shift**		
	Minutes	Percent	Direct cost
Preparation:			
cutters	2.1	.0050	$0.53
drillers	4.6	.0110	1.17
shotfirers	13.0	.0309	3.30
trackmen	4.3	.0103	1.09
timbermen	1.1	.0026	0.28
Transport:			
main-line wrecks	4.7	.0111	1.19
derailments	16.0	.0381	4.06
no empties	16.2	.0386	4.11
blocked loads	5.9	.0141	1.50
Power:			
power off	1.3	.0031	0.33
low voltage	0.9	.0022	0.23
loader cable	2.2	.0052	0.56
locomotive cable	6.8	.0162	1.72
Supplies:			
lack of timber	1.2	.0029	0.31
lack of rails	3.0	.0071	0.76
lack of ties	2.0	.0048	0.51
lack of lubricant	0.8	.0019	0.20
Maintenance:			
cutter failure	2.5	.0060	0.63
drill failure	0.7	.0016	0.18
loader failure	0.9	.0021	0.23
locomotive failure	18.5	.0441	4.69
Total	108.7	0.2589	$27.58

less than estimated, but car-change behind the loader was 25% slower, due to light locomotive used, requiring 3 two-car trips to load out 1 cut; heavier locomotive saved a third of the time. Faulty dispatching of empty cars called for more storage partings near working places. Idle time of development crew, due to car shortage, cost $46.59, or 20% of wage charge. Records covering narrow- and wide-work machines for the first 6 months of mechanical loading are in Table 17. These machines loaded about 74% of daily output in 6% less time than estimated; total tonnage, about 4% less. But avoidable delays showed a corresponding increase, totaling 109 min. Table 18 classifies avoidable delays, and, using the direct cost factor for each operation, the dollar losses for the 108 shifts in Table 17 were: faces not prepared, $687.96; transport, $1 172.88; power, $306.72; supplies, $192.24; maintenance, $618.84; total, $2 978.64. The 25.89% avoidable lost time for 1 loader is $11 914.56 for the 4 wide-work loaders. About 78% of room preparation losses were due to delays by cutters, drillers and shotfirers, and 22% to incomplete tracklaying and timbering; hence, it was decided that cutting and blasting, and distribution of supplies be done on night shift, track and timbermen on day shift to follow the loading cleanup of wide faces; also, because transport losses accounted for more avoidable delays than other operations, all main and secondary trackage was realined, regaged, ballasted, and curve radii lengthened and banked for higher speed. With an indicated loss of $5.25 per machine shift for wrecks and derailments, about $2 168 could thus be saved in labor cost. As maintenance losses were about $2 475 for the 6 months, all equipment was tested daily. Table 19, 102 time studies for first 6 months of mechanical loading, shows a nearly perfect production cycle, only 3.86% of time being lost by delays. This high effic is due to concentration of working places

Table 19. Development-machine Performance

Operation	Minutes	Percent	Direct cost
Loading	212.0	50.48	$39.19
Car change	110.6	26.33	20.45
Moving machine	81.2	19.33	15.01
Transport (*a*)	12.0	2.86	2.22
Miscellaneous (*b*)	4.2	1.00	0.78
Total	420.0	100.00	$77.65

(*a*) Delays due to wrecks, derailments and lack of haulage coordination. (*b*) Delays due to power, mechanical failures, etc.

Aver time shift started, 7.30 a m; aver time shift finished, 3 p m; aver tons per car, 4.7; aver cars loaded per shift, 69.149; tons per shift, 325.

in the triple-entry system, where 3 headings and at least 2 crosscut faces are always available for the machine loaders on the advance, and 2 headings, 2 crosscuts and 2 room-necks in panel development. Loss in the period, $712 for the 2 loaders (20).

Examples of Mechanized Coal Mining

Knox Consol Coal Co, Bicknell, Ind, operating in 2 veins about 6.5 ft thick; the output of 7 mobile loaders, 3 car loaders, and hand loading (149 machine-men working 1 039 hr), was 3 250 ton (Table 20).

Table 20. Knox Consol Coal Co, Average Daily Performance, by Joy Loaders

	Wage per shift	No 1 mine (4 Joy loaders)		No 2 mine (3 Joy loaders)	
		Men employed	Hours work	Men employed	Hours work
Day work:					
Machine-men	$6.75	4	28	3	21
Helpers	6.75	4	28	3	21
Drillers	6.15	4	28	3	21
Helpers	6.15	4	28	3	21
Loader operators	6.75	4	28	3	21
Helpers	6.75	4	28	3	21
Clean-up men (*a*)	4.57 1/2	4	28	3	21
Tracklayers	4.57 1/2	4	28	3	21
Timbermen	4.57 1/2	4	28	3	21
Gathering motormen	5.14	4	28	3	21
Trip-riders	4.69	4	28	3	21
Loading-unit bosses	6.75	4	28	3	21
Cagers	4.57 1/2	1	7	1	7
Couplers	4.57 1/2	1	7	1	7
Car greasers	4.57 1/2	1	7	1	3 1/2
Main-line trackmen	4.57 1/2	1	7	1	7
" motormen	5.14	2	14	2	14
" trip-riders	4.69	2	14	.....	....
Relay motormen	5.14	2	14	2	14
" trip-riders	4.69	2	14	2	14
Electricians	5.64	1	7	1	14
Helpers, repairmen	5.64	3	21	2	14
Wiremen	4.57 1/2	2	14	2	14
Pumpers	4.57 1/2	1	7	1	7
Firebosses	5.50	3	21	2	14
Night work:					
Blasting	6.15	4 (*b*)	28	3 (*b*)	21
Loading spilled coal	6.75	2	14	2	14
Recovering steel	4.57 1/2	2	14	2	14
Supply men	5.14	1	7	1	7
Road cleaners	5.14	2	14	1	7
	4.69	.....	....	1	7
Repairmen	5.64	1	7	1	7
Greasers on machines	5.64	1	7	1	7
Total	.....	83	581	66	465.5
Tons produced by mobile loaders per day		1 325		1 100	
Total tons produced per day		2 000 (*c*)		1 250	

(*a*) Prepare places for cutting. (*b*) Blasting for all loading units. (*c*) Including 675 ton by hand-loading.

Kathleen mine, Union Colliery Co, Dowell, Ill. In 1935, daily output of 4 600–4 700 ton was by 16 machines, in a 7.5–8-ft seam. Rooms, 26–28 ft wide, on 45-ft centers; depth, 250–300 ft (23).

Robinson Run mine (24), Christopher Mining Co, West Va, in a 6.5–8-ft seam, produced 15.6 ton per man-shift by mobile loaders on pillar work (Table 21).

Talleydale mine, Snow Hill Coal Co, Ind, in 1936 installed 7 machine loaders in a 5–6.5-ft seam. Product in one day was 2 268 ton, by 144 men underground (Table 22).

Table 21. Robinson Run Mine, One Month's Work, 1936

Number of loading machines	2
Character of work	Pillar
Tons shipped	31 385*
Number of loading-machine shifts	60
Man-shifts underground	2 014
Tons per man-shift underground	15.6
Number of cars dumped	6 716
Places cleaned up	1 268
Tons per place	24.7

* Shipments were approx 90% of output

Pemberton Coal Co, West Va, hand loading onto conveyers in a 32–40 in seam gave aver output of 10.6 ton per man, delivered to main haulage, for 10 months in 1935 (Table 23).

Jewel Ridge Coal Co, Va, by triple shifting of 6 hand-loaded face conveyers (2 hp each), 6 room conveyers (10 hp each), and one 1 500-ft belt conveyer (25 hp), product was 18 000 ton per month, from a 38-in seam, with 33 1/3 men per day; shift tonnage per loader, 8.75.

Eureka Coal Co, Paris, Ark, working a 18–24 in seam, the normal product from about 1 600 ft of longwall face, with 4 conveyer units, was 345 ton per 7-hr shift.

Table 22. Talleydale Mine, Underground Employes and Output, Sept 11, 1936; Seven Loading Machines Operated

Occupation	Loading crews, No of men	General crew: Day shift	General crew: Night shift	Total	Occupation	Loading crews, No of men	General crew: Day shift	General crew: Night shift	Total
Mine foreman....	..	1	..	1	Couplers.......	..	2	..	2
Night foreman...	..	..	2	2	Tracklayers....	18	..	2	20
Face foreman....	4	..	..	4	Timbermen......	19	..	2	21
Loading machine.	16	..	..	16	Pumpmen........	..	1	..	1
Coal cutters.....	12	..	..	12	Firebosses.......	..	1	..	1
Drillers..........	7	..	..	7	Bratticemen.....	..	1	..	1
Bugdusters......	7	..	..	7	Seals............	..	3	..	3
Shotfirers........		..	4	4	Deadwork.......	..	..	6	6
Motormen.......	7	4	3	14	Electricians......	..	..	..	10
Trip-riders.......	7	4	..	11					
Cagers..........	..	1	..	1	Total.....	97	18	19	144

Mine cars loaded, 477; tons washed coal loaded, 2 268. Washed coal represents approx 85% of material hoisted.

Table 23. Pemberton Coal Co, Equipment for Conveyer Units

	No per unit	Make	Type	Intermediate section: Height	Intermediate section: Length	Total length, ft	Motor rating, h p	Motor make	Tons per min
Face conveyers	4	Jeffrey	61–HG	6 5/16 in(a)	5 ft	30	5	W	1
Room conveyers......	4	Jeffrey	61–AM	9 in.	6 ft 1 3/4 in	300	10	W	3/4
Gathering conveyers..	1	Jeffrey	61–W	10 1/4 in	6 ft 1 in	300	15	W	1.2
Elevating conveyers..	1	Jeffrey	61–EW	....	....	15 (b)	5	W	....
Drills..............	4	Jeffrey	A–7, 1-man, permissible, 38-lb	....	....	....	1 1/2	....	....
Blowers............	4	Jeffrey	12-in	....	....	....	1 1/4	R–M	....
Hoists..............	1	Brownie	HKC	....	....	....	5	....	....
Coal cutting machines	4	Goodman Shortwall	12–AA	....	....	....	50	....	....

(a) Height without sideboard. (b) Used without intermediate section, thus reducing length to 11 ft. W, Westinghouse; R–M, Robins & Myers.

Due to varying costs, mining conditions, and fluctuation in coordination of manpower and equipment, no general formula for savings by mechanization is possible; but some light is obtained from Table 24, from statistics of National Bituminous Coal Comm, covering mines producing more than 50 ton a day in Ill and Ind.

Table 24. Costs of Mechanized Mining in Bituminous Coal Mines

Mine classification	District No 10, Ill 9 mos, ended Dec, 1937: Tonnage	District No 10, Ill 9 mos, ended Dec, 1937: Av cost per ton	District No 11, Ind 9 mos, ended Dec, 1937: Tonnage	District No 11, Ind 9 mos, ended Dec, 1937: Av cost per ton
Strippings........................	7 979 424	$1.4319	4 998 417	$1.4631
Mechanical loading, underground......	16 742 458	1.7457	4 600 134	1.7636
Hand loading, underground...........	6 747 216	2.1793	910 217	1.9572
Total, commercial mines.............	28 044 423	1.7574	10 417 029	1.6326
Total, captive mines................	3 424 675	1.8612	91 739	2.1791
Total, deep mining..................	23 489 674	1.8703	5 510 351	1.7962

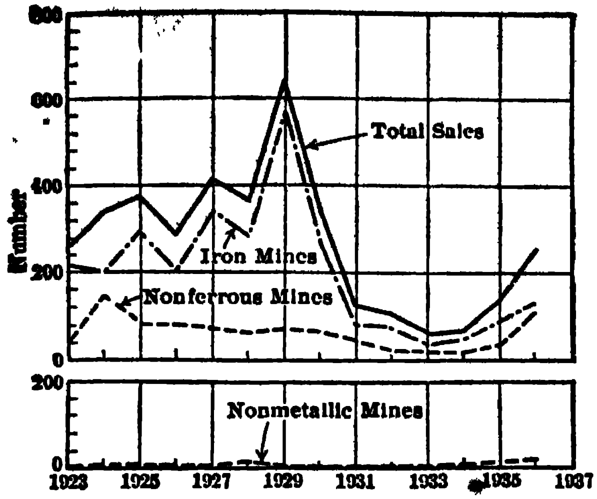


Fig 25. Installations of Underground Scraper Loaders, 1923–1936 (U S Bur Mines, from reports of 8 makers) (28)

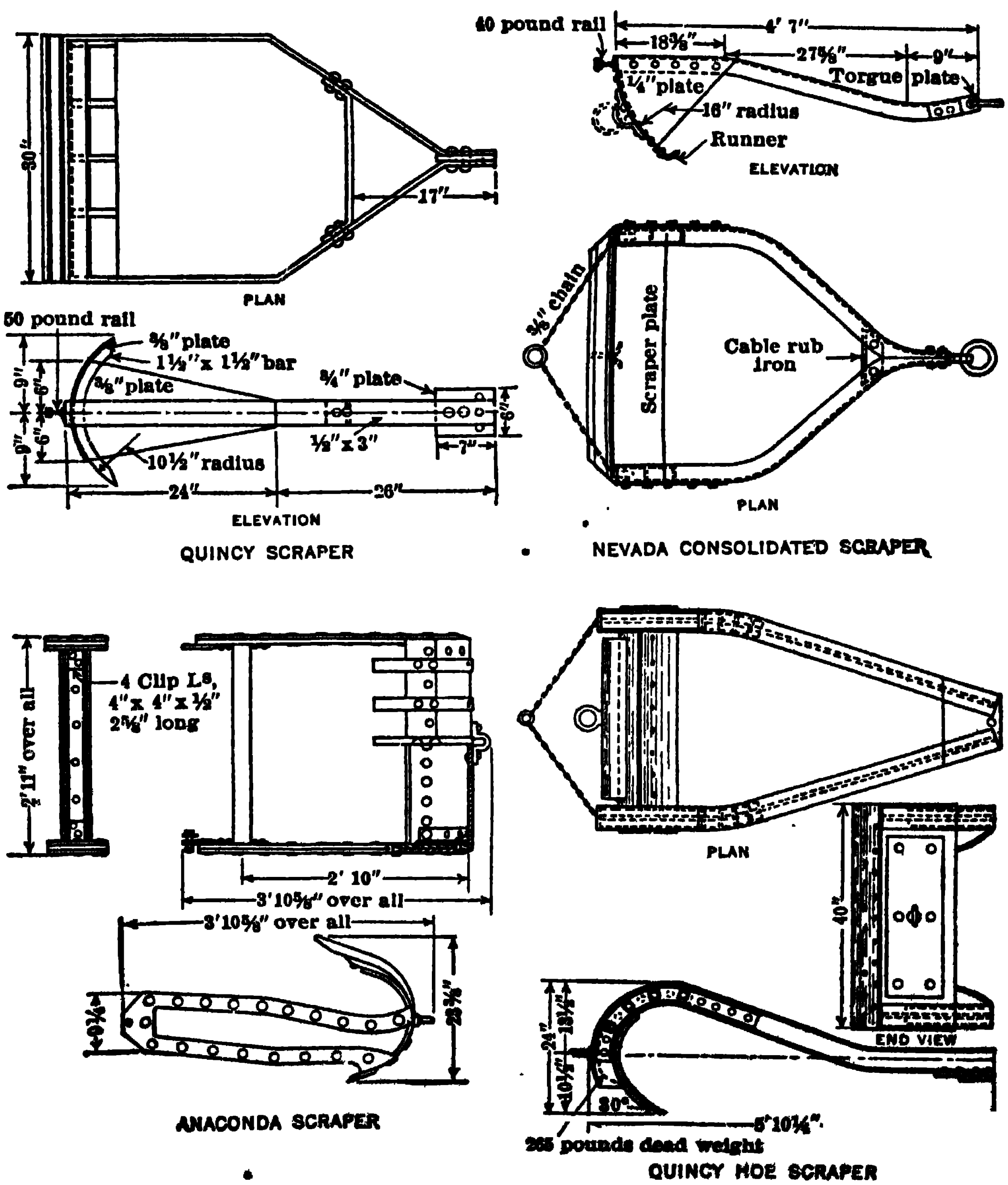


Fig 26. Types of Scraper Loaders with Slusher Hoists

7. TYPES OF MECHANIZED METAL-MINING EQUIPMENT

Underground metal-mine loaders were at first of the steam-shovel type, requiring large clearances; latterly, more compact designs have been evolved, as the Armstrong, Myers-

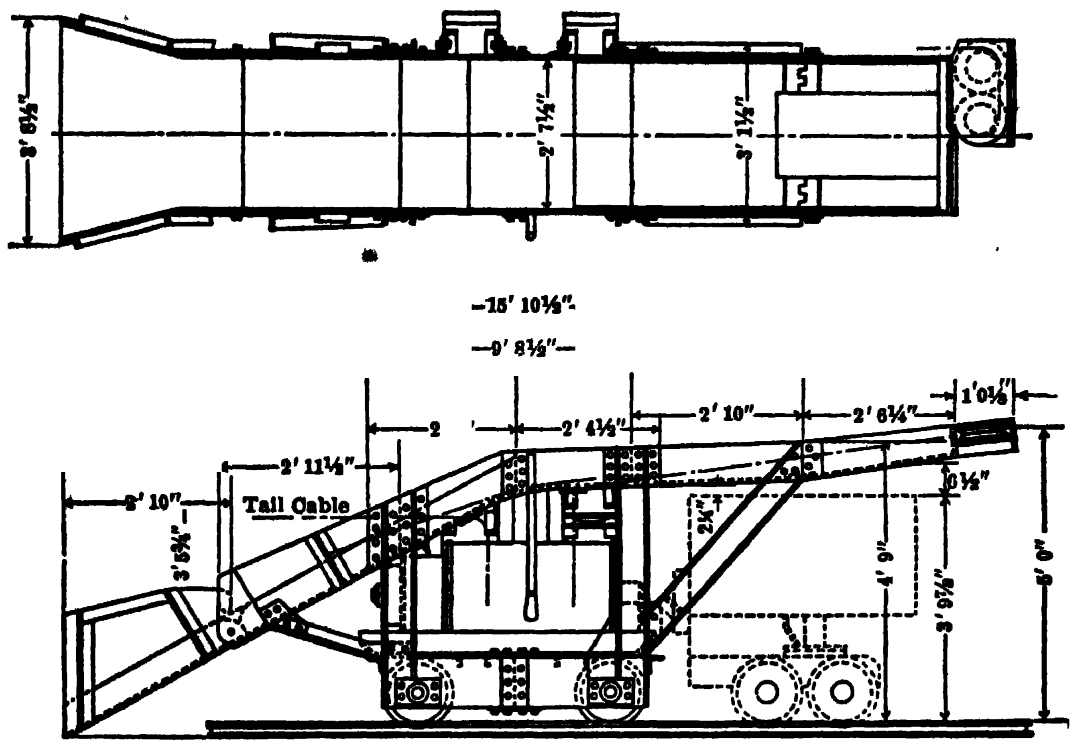


Fig 27. Slide for Scraper-loading of Cars

Whaley, Nordberg-Butler, Conway, Hoar, Eimco-Finlay, Gardner-Denver, and Sullivan. Meanwhile, scraper (slusher) loading was developing in several districts in U S and Canada, and is now common in the Lake Superior iron and copper mines, iron mines of the south, Tri-State lead and zinc districts, copper deposits of Ariz, Utah and Nevada, and wherever workings are too flat for gravity loading. The history of mechanical loading to 1923 in mines other than coal was published jointly in 1924 by the U S Bur of Mines and the Missouri School of Mines (27).

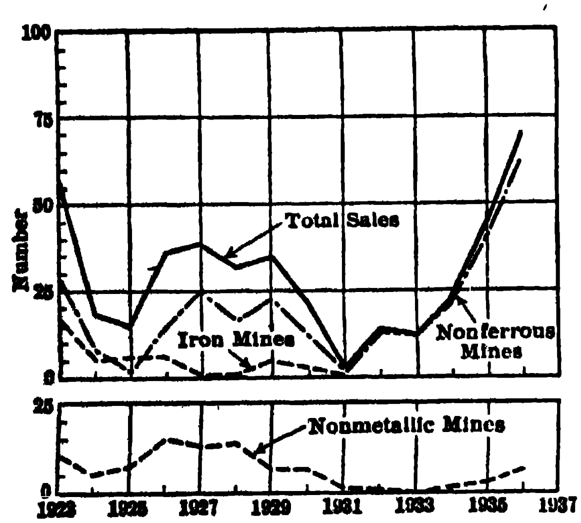


Fig 28. Installations of Underground Shovel Loaders (U S Bur Mines, from reports of 11 makers) (28)

Scraper loaders were first used in the Mich copper mines and iron ranges from 1915 to 1917, and in the Tri-State area in 1919, and proved their possibilities when shortage of manpower and rising wages made labor-saving devices necessary; and, following the war, declining prices of metals further stimulated cost reduction. From 1923 to 1929 sales of scraper equipment were large (Fig 25), but dropped during the depression, with some recovery since 1933. From 1929 to 1933, scraper sales to metal and nonmetallic mines totaled 3 752 hoists or complete units, made by: Sullivan Mach'y Co, Gardner-Denver Co, Lake Shore Engine Works, Goodman Mfg Co, Vulcan Iron Works Co (Denver), Vulcan Iron Works (Wilkes-Barre), Lidgerwood Mfg Co, and Sauerman Bros. Ranging in motor size from 7.5 to 150 hp, scraper and slusher haulers and hoists of double-drum type, have rope capacities of 165–1 000 ft per drum;

weights, 1 450-10 315 lb. Fig 26 shows types of scrapers, as used with slusher hoists; a typical slide for scraper loading of cars is in Fig 27.

Mobile loaders. The earliest of these were the Marion, Thew, Hoar, Osgood, Nordberg-Butler, Conway, Harnischfeger and Myers-Whaley, followed by the smaller Gardner-

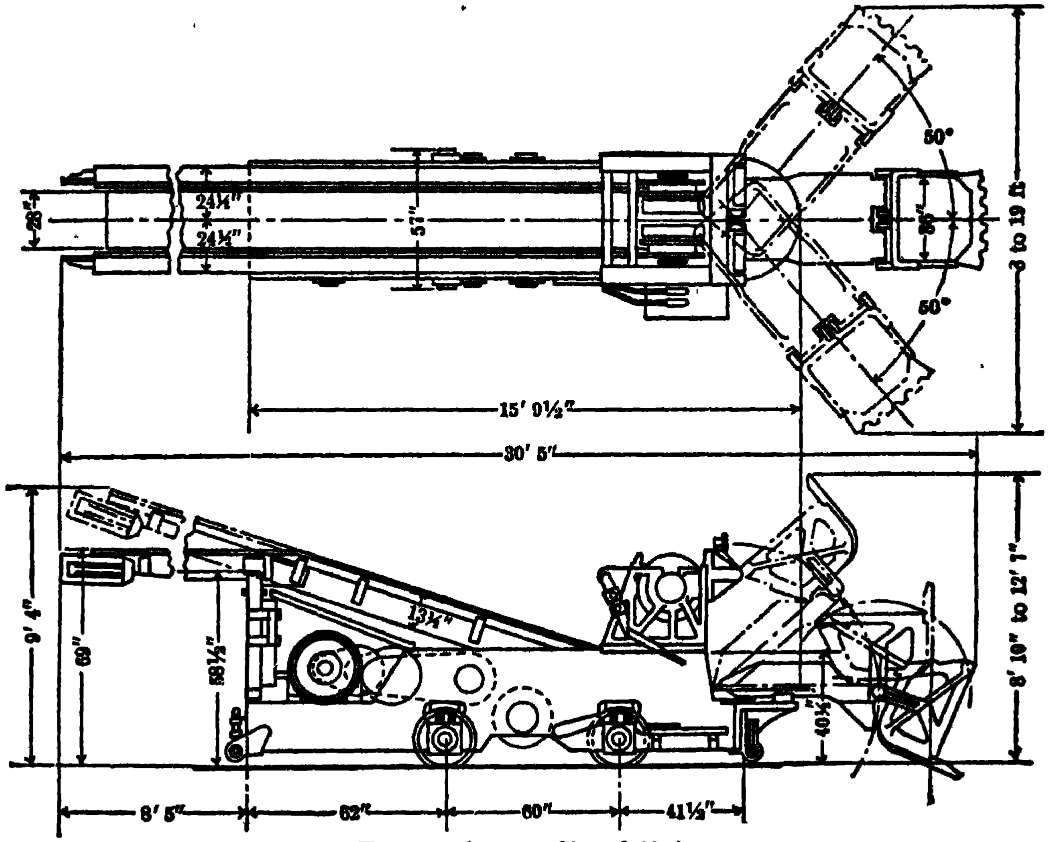


Fig 29. Conway Shovel 60-A

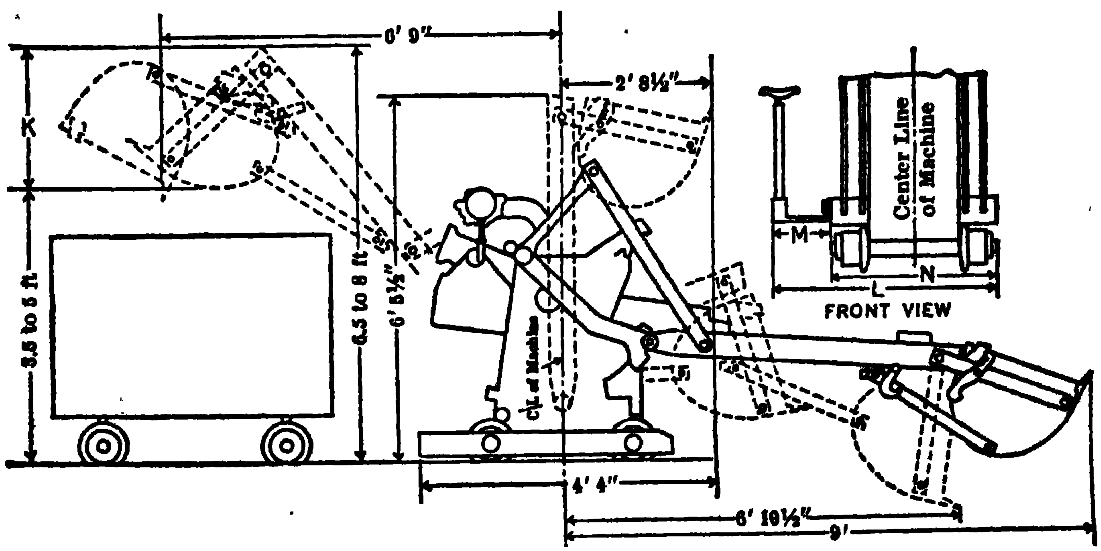


Fig 30. Nordberg-Butler Shovel

Denver, Eimco-Finlay and Sullivan. These 11 makers furnished the data in Table 2 and Fig 28. From 1923 to 1931 mobile loaders were designed chiefly for open stopes; since 1932, smaller units, for headings or tunnels, have predominated. Unlike the gathering machines used in coal mines (Joy, Jeffrey, Clarkson, Goodman and Umeco), loaders for metal and nonmetal mines employ the shovel movement only. This principle, embodying crowding,

lifting and discharging, is used in all machines, from the Marion and Thew shovels to the arc-swing loading head of the Myers-Whaley and Conway (which discharge on to conveyers delivering to cars), and the smaller later Eimco-Finlay, Gardner-Denver and Sullivan machines, in which the bucket is swung completely over, for discharge into the mine car. The larger machines are generally powered by elec motors; the smaller are air-driven (Fig 28).

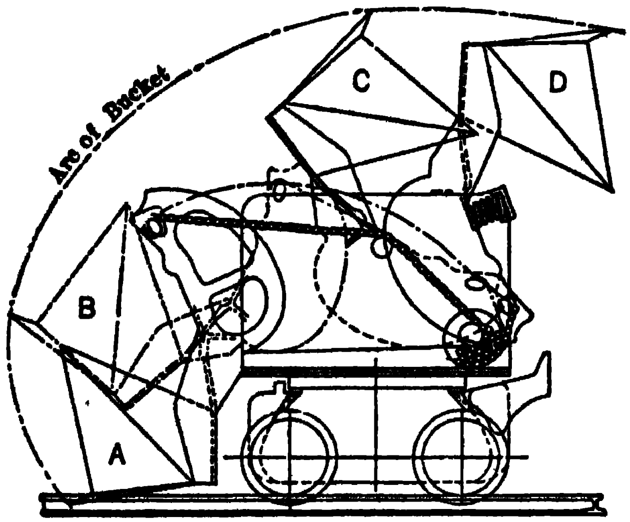


Fig 31. Eimco-Finlay Loader

Examples. Myers-Whaley shovel, for rock and tunnel work, has some resemblance in the chassis design to the coal-loading "Automat" of the same makers (Art 2), but embodies dipper movements similar to those shown in Fig 29. Conway shovel, sold by the St Louis Power Co, and built by the Goodman Mfg Co, has a dipper on a swinging boom, discharging on to a belt, and thence to the car. The digging mechanism is swung horiz, and raised and lowered by a hoist at front end of machine, and is driven through 2 planetaries, each with a drum and hoist chain. The propelling unit is similarly actuated. Made in 8 sizes. Dipper capac of the 60-A shovel, 13.5 cu ft; 5 loading cycles per min; belt widths, 28 in; hp, 60; wt, 15 ton. Nordberg-Butler shovel (Fig 30). The dipper is operated by 2 pistons of a direct-thrust air cylinder, for taking its load, raising to the revolving position and dumping ahead, to the side, or to the rear; revolving power is from a 2.5-hp air motor. Speed of work, 3–3.5 dippers per min. Shovel is moved by hand, and will operate in a tunnel 5.5 ft wide. Domestic shipping wt, about 5 170 lb. Gardner-Denver loader is a small machine for tunneling and mining, using about the same volume of air, at 45–90 lb, as an ordinary rock drill; wt, 4 000 lb; overall width, 33.5 in; length, 69 in; height, with bucket lowered, 52.75 in; minimum headroom, 76 in; clean-up width, 96 in. Eimco-Finlay loader is in 2 sizes, weighing 3 950 and 5 900 lb; operated by 2 special Ingersoll-Rand air motors, one for propelling and crowding, the other for digging, lifting and dumping. The patented rocker arm action in its different positions is in Fig 31. Sullivan loader, for narrow headings, has a rocker-arm action similar to the Eimco, but with a positive centering device for dumping into middle of car when on a curve (Fig 32).

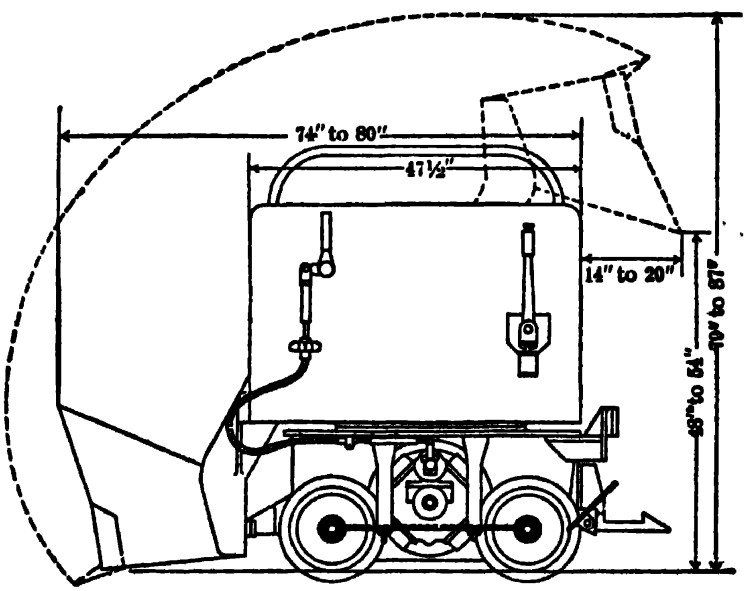


Fig 32. Sullivan Loader, for Narrow Headings

8. COMPARISON OF MECHANIZED METAL-MINING METHODS

The same mine may profitably employ mechanical loading in one section and gravity or hand loading in another (28). But, although most ore and country rock will probably continue to be handled by gravity systems, the improvements in design and effic of mechanical

loaders point to their wider use as wage rates tend to rise. In general, loading machines and scrapers are recommended for development headings, haulageways and flat workings, for delivery to mine cars, ore pockets, or to conveyers; scraper haulers, for inclined shafts or winzes, or stopes up to 30°, to conveyers or cars (35).

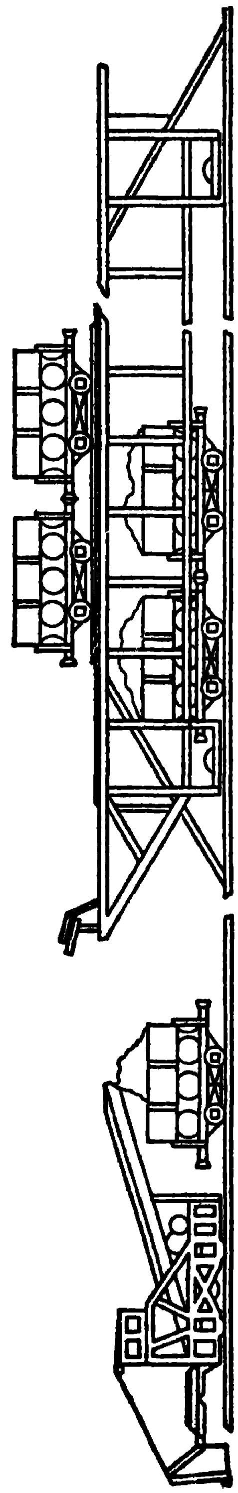


Fig 33. King Car-passer, for Machine Loader

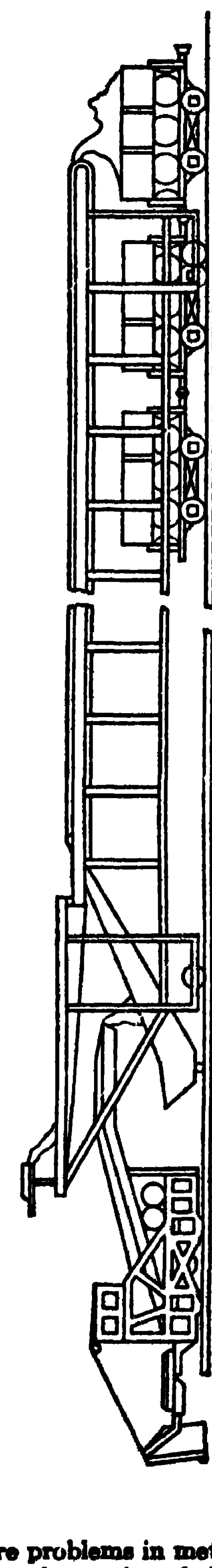


Fig 34. Dixon Conveyer for Machine Loader

Car servicing for track-mounted loaders presents more problems in metal mining than in coal; adits and drifts are generally narrow, double-track openings being driven only where increased effic of loading machine will warrant added cost. Simplest "side switch" for light cars behind a loader in narrow headings is a TURN SHEET, the empty being "kicked

off" the track onto the plate to allow the loaded car to pass. Cars are also changed in narrow workings by the "CHERRY PICKER," a movable frame carrying an air hoist, which raises an empty from the track, to allow the loaded car to pull away from the loader. KING CAR-PASSER (Fig 33) is a steel frame about 150 ft long, traveling on rails on each side of main-line track, with a hinged ramp at each end operated by an air hoist. Track is laid up the ramps and along the deck, for cars to pass to the loader. Both ramps are raised when loaded car is to be pulled out (36). DIXON CONVEYER (Fig 34) has a long belt on a frame like that of the King. A hopper-like hinged trough at front end receives material from the loader, and transfers it to the conveyer which loads car at the rear (36). CALIFORNIA SWITCH, widely adaptable for car changing behind loading machines, consists of a portable combination of siding and switch superimposed on the main track, with tapered end rails and a spring switch at each end, for cars to pass on either side from the permanent track. The empties are pulled from one siding to the loader by a small hoist, the loaded cars being pulled away on the other track. The entire siding is moved forward, as a unit, by the loader or the locomotive (36).

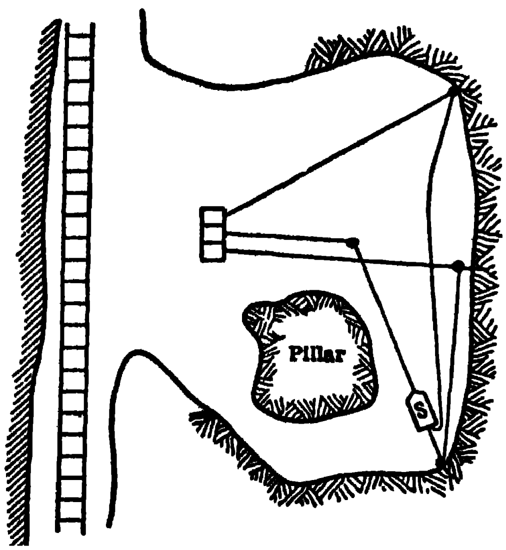


Fig 35. Scraper Loading from Stope; 2 Tail Ropes

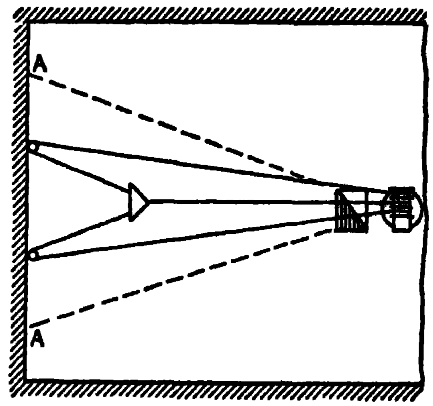


Fig 36. 3-drum Hoist for Mucking a Large Area

Scraper loading by 3-drum hoists. Fig 35 shows setup, with 2 tailropes, in a large flat stope; Fig 36, a setup for mucking horiz cut-and-fill stopes, and spreading the waste where ore passes are obstacles to free spreading. Work in cut-and-fill stopes can be done by placing the mucking hoist in one panel, the filling hoist in another (13).

Examples of Mechanized Metal Mining

Colorado River Aqueduct (36), 91 miles of 16-ft tunnel, were driven with machine loaders at aver speed per heading of 18 ft per day of three 8-hr shifts; aver cost per cu yd of excavation, \$3.90. Crews were from 18–20 men; skilled wages, \$5–\$6; unskilled, \$3.20–\$3.80.

United States mine (U S Smelting, Ref and Min Co, Bingham, Utah). Cost of hand mucking, 35.8¢ per ton; with Eimco-Finlay loaders, 28.7¢; a saving of 20% (38).

Suyoc Consol Mining Co (Philippines) drove a 12 by 12-ft tunnel 9 450 ft. Speed with hand mucking averaged 250 ft per month; afterward, with a Conway shovel, 1 327 ft in 1 month at cost of \$14.74–\$17.73 per ft (39).

Butte mines, Mont. Mechanical loading can replace hand work and effect a saving in 7 by 5-ft drifts or larger (40). The small machines are fast, effic and rugged in construction. From 1933 to 1938, machine loading increased from 6 700 to about 500 000 tons, some 50 small loaders now being used. Aver loading time, 4 min per 4-ton car. Track arrangements for car servicing are shown in Fig 37.

Scraper loading on the MICH IRON RANGES showed a marked increase of output per miner per day, from 2.9 ton in 1923 to 5.96 ton in 1929; distances scraped, 20–100 ft (13). RAIMUND NO 1 MINE, Birmingham, Ala (Republic Steel Corp) uses both hand and scraper loading (41). Double-drum elec hoists, carrying 500 ft of rope, are driven by 35-hp motors. Box scrapers, weighing 1 900 lb, capac 1–1.5 ton, handle 100–150 ton per 8-hr shift in heading work. At the PARK-UTAH MINE (42), scrapers have been used in square-set stopes, handling 2 000 ton in 1 month with 4 men on each of 2 shifts, the setup being on the timbering. LA RUE MINE, Mesabi Range (42), uses both underground and surface conveyers, for ore dipping about 15°. Belt haulage with elec shovels, first installed for mining large

rooms, were abandoned, as the roof was too treacherous for the 16-ft span required for the shovels. Scrapers and elec hoists are now used for top-slicing and sublevel caving. Scrapers of 1 cu yd are hauled from the face by 20-hp hoists, for delivery to a 1.5-cu yd scraper and 40-hp hoist, which in turn delivers to a surface conveyer by which the ore goes to RR cars or to stock pile. This system, involving use of 4 440 ft of 30-in belt conveyer, of which 2 135 ft are underground and 2 305 ft on surface, handle up to 400 long tons per hr.

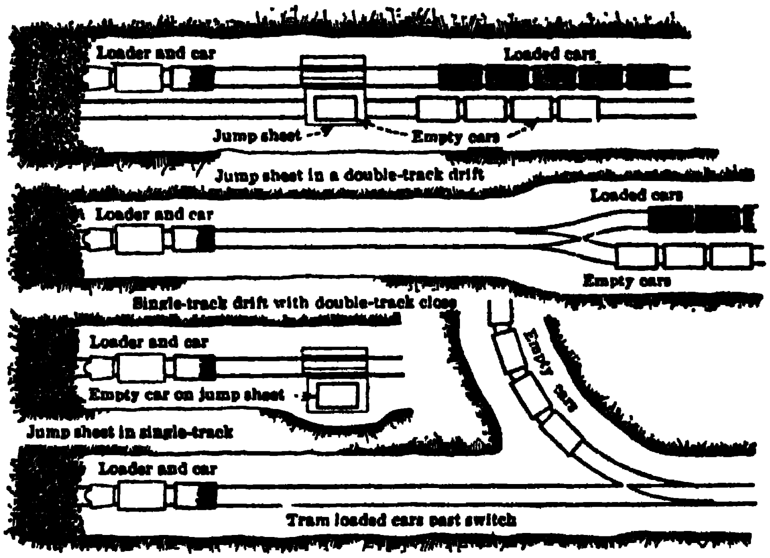


Fig 37. Car Servicing for Small Machine Loaders in Butte Mines

Within the past decade, mechanical handling of ores and rock has embraced all forms of loading equipment. In general, as shown by the preceding examples, the saving in cost over hand loading ranges from 10 to 50%, and furnishes ample incentive to study the problem for every set of local conditions.

Details of Conveyers and Elevators (L. de G. Moss)

These find their widest application in coal mines, but some are used in metal and non-metallic mines (30, 31, 32).

9. CHAIN AND BUCKET CONVEYERS

Buckets or pans bolted to chain links form a continuous moving trough, delivering at the head sprockets by inverting (Fig 38). The kind of material determines the shape of buckets. The ends must be shaped to permit buckets to pass around sprockets: for sticky materials, all sides are sloped. For intermediate delivery the buckets are hinged at one end, with guide wheels at the other, which drop on a depressed curved track to spill the load (Fig 39). The dumping ends must be open, and, owing to shock of dumping, speed is reduced to about 50 ft per min; with fixed pans, speed may be 80–100 ft per min. Roller chains with closed and bushed bearings are best. Capacities reach 1 000 tons of stone per hr. Power required, 2 to 4 hp per hr per ton-mile on level track. Clean tracks and well lubricated

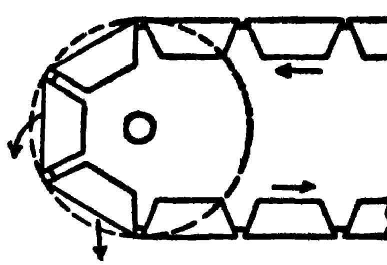


Fig 38. Bucket or Pan Conveyer

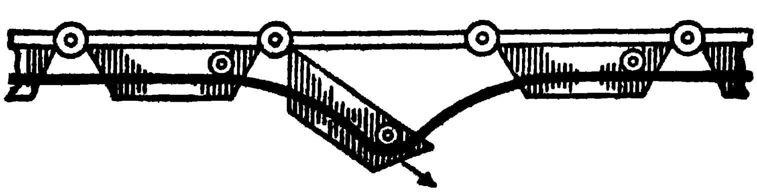


Fig 39. Mode of Dumping

rollers are essential. Conveyers on steep upgrades are uneconomical, because a large part of dead wt pulls on the head-shaft bearings. Grades may be up to 30°. Inclines do not cause decrease in capacity, as in flight conveyers.

10. CHAIN AND BUCKET ELEVATORS

Types. CENTRIFUGAL-DISCHARGE elevator has 1 or 2 strands of chain with outstanding buckets (Fig 40). Linear speed of buckets = speed of chain, except when turning over sprockets. For proper relations of chain speed and head sprockets, see Fig 41. For very gritty material, one strand of chain can be used, driven by a toothless traction wheel. They may run vertically. Faults of centrifugal elevators: high veloc of discharge breaks up lumps which should be kept intact; chute lining wears rapidly; high speed reduces life. It is the cheapest form of elevator. PERFECT-DISCHARGE elevator (Fig 42) runs between 80 and 200 ft per min. The chains are deflected inward by 2 choke or snub sprockets, to secure better delivery. This principle can not be applied to single-strand elevators, nor to belt and bucket elevators. CONTINUOUS-DISCHARGE elevators (Fig 43) run 60 to 80 ft per min, and are preferred for stone, sand, and minerals. The buckets, with no spaces between them, have lips to serve as small chutes. GRAVITY DISCHARGE (Fig 44) gives a clean delivery by using 48° V buckets, bolted to chain, with a horiz run at top. Roller chains are needed, owing to horiz portion of delivery track and chute.

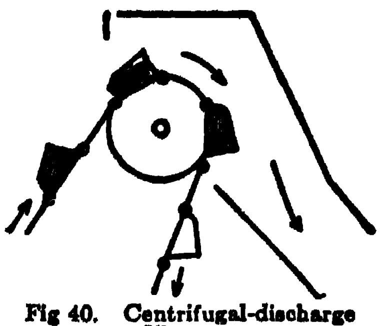


Fig 40. Centrifugal-discharge Elevator

Buckets are commonly of sheet steel, No 8–16 gage. A better design has C-I ends, with renewable steel body. Malleable-iron buckets are most durable. Standard elevators have not yet been made so that the bodies of the buckets form the links, as in dredging elevators, with obvious advantages for large units.

Chains. Short-pitch chains give the more uniform speed; long-pitch causes swaying due to rhythmic variations in speed. Large conveyers should have enclosed pins with bushings. The Ewart detachable-hook chain is satisfactory for temporary or light service. SLACK must be prevented by screw-geared take-ups, having a gape of more than length of 1 link. They should be placed at the foot of the elevator, where all slack accumulates. It is sometimes necessary to put them under the head-shaft bearings,

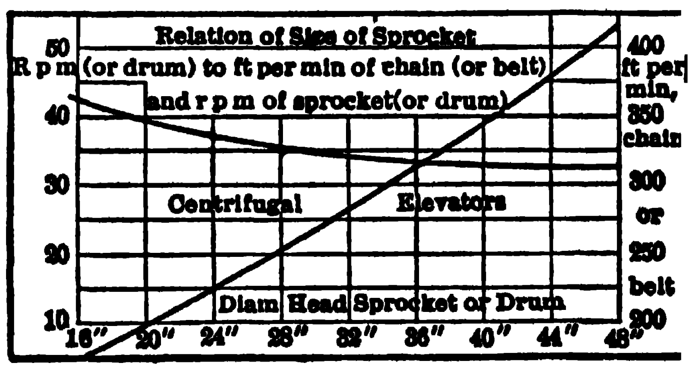


Fig 41

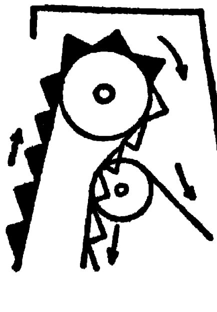


Fig 42. Perfect-discharge Elevator

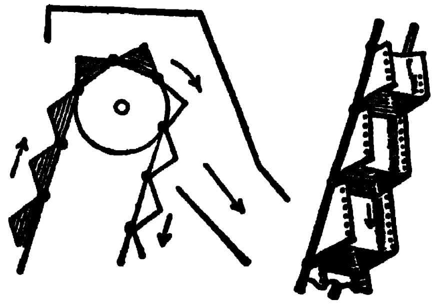


Fig 43. Continuous-discharge Elevator

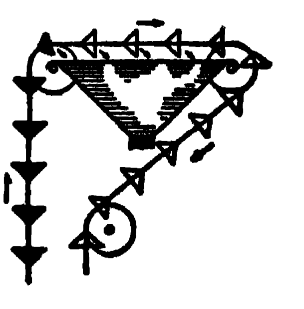


Fig 44. Gravity-discharge Elevator

making them strong enough to carry the concentrated pull of the entire conveyer. Unless very strongly built, trouble will follow from vibration and loosening of bolts. It is desirable to cross-connect the 2 take-up screws, to rotate together and keep shafts parallel; sprockets and chain are cheaper than bevel gears and shafts for this purpose.

Boots must receive material on rising side of chain. They can not be used for stuff harder than coal, excepting fine granular material. Timber boots, lined with sheet steel, are not so good as those made of C-I sides and sheet-steel body. In this case the take-ups are built into the boot. For ore and stone, the arrangement in Fig 45 works well. The elevator is preferably inclined at 60° from horiz. The delivery is high enough to keep one empty bucket below the chute to catch fines and lessen cleaning up. This design is cheap, and will not clog, jam, or freeze. There should be plenty of room around boot or loading chute for ease in repairing, and for safety.

Delivery chutes should not have their bottom line extended higher than the head shaft; 4-6 in lower is better for a clean delivery. If closed on top, the upper sheets should be hinged for quick access in case of a jam. Generous spacing of buckets (pitch) is desirable. Anthracite, if clean, will slide on a 15° polished chute. A little dirt, or worse, dry snow, necessitates 30°–35°; bituminous requires 45°. If the chute is hoppered, the corner angles of a 45° hopper will be flatter than 45°, and material will hang there. Some dirty and sticky ores require 60°. Bottom sheets should be bolted for easy renewal. Channel sections for sides are frequently the most economical, giving minimum of shop work with economy in metal. The cheapest and most durable chute bottoms are of hard tank plates.

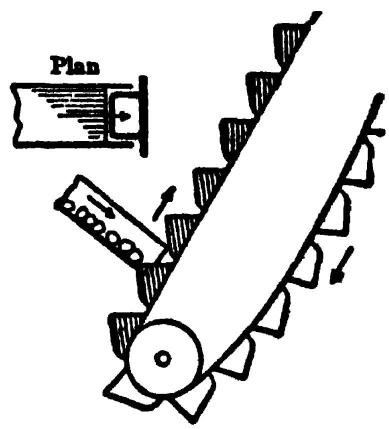


Fig 45. Bucket Elevator

Frames and casing. Modern construction, and because of fire risk of flames rapidly climbing, as in a flue, is justified. Good bracing is essential, but tension rods are undesirable. All braces should be angles. Drive supports should be proportioned for deflection and stiffness, regardless of unit stresses. Covering, if of black sheets, must be kept well painted; galvanized muck-bar iron is better. Panels on one side should be bolted for access, and lead washers under bolt heads will prevent rusting at holes. GUIDE WHEELS on the rising side should be set 5–7 ft apart, depending on size of elevator and its angle; short vert elevators require no guides. Number of guides on slack side depends on path of the chain, which approximates a parabola; 10 ft centers is close enough.

Power required is 2 to 3 times the net work of lifting the unbalanced load; or, motor hp = ton per hr × ft lift ÷ 990 × 2 or 3. Large capac, low speed, and short lift tend to economy. In estimating power, assume all buckets to be filled; but, in estimating capac of buckets, consider them not more than $^2/_3$ full for ordinary conditions of feeding. Elevators have been built for 120-ft lift. Exceptional care in unit stresses and loads is needed for all high-lift elevators.

11. BELT AND BUCKET ELEVATORS

Buckets (Art 10) are bolted to belt with special flat-head bolts.

Belts are canvas and rubber, 4 to 12 ply. Canvas should be 32-oz duck. For the best belts, the adhesion of the rubber covering, for a strip 1 in wide, should be 16 to 20 lb; 10–14 lb is good. Thickness of rubber is $^1/_8$–$^1/_4$ in. Splices are best made with metal hooks. Pierce all holes first with an awl, instead of driving the hooks through the solid belt. Width of belt should be 2 in more than width of buckets; width of drums, 2 in more than belt. Working stress on belt, per in width per ply, should not exceed 22 lb for high-grade belts having 30 to 32-oz duck plies. Reduce stress to 20 lb for 28-oz duck.

Drums. Coeff of friction is ordinarily 0.2 to 0.3 for a finished C-I pulley. Rubber-belt lagging, bolted on, will raise the coeff to 0 35 or 0.4. Certain kinds of dust raise the coeff, others lower it; sulphur dust is the worst. Oil and grease must be kept away from belt, as they dissolve rubber. The great advantages of belt and bucket elevators are that they are simple to install and maintain, and have no chain and sprocket troubles in gritty materials. They usually cost a little more than chain elevators. ARC OF CONTACT between drum and belt is 160°–180°, depending on angle to horiz. Slack side is permitted to hang in an easy parabolic curve, which is modified by the tension of lower take-up screws. When laying out, give ample clearance in the boot for the slack side, which at times may also oscillate a foot. The actual curve is more like a parabola than a catenary.

Frame should be well braced for transverse stiffness; but may be more flexible under beam load. As a column, 90 ($l \div r$) is a safe limit. Besides the dead load, the live load on head shaft and pillow blocks is the sum of the taut and slack tensions of the belt. The slack tension is usually 0.3 the taut tension, and may be 0.5 with take-ups set up. Put GUIDE IDLERS under taut side of belt, 5–7 ft apart; none under slack side.

Ratio of drum diam to belt thickness is important as affecting life of belt, the most expensive item in total cost. It is best stated in terms of belt plies; 5 in diam per ply of belt is advisable for permanent plants. Tendency is to use a smaller, cheaper drum, which revolves faster and requires less gear reduction. As drums and gears do not wear out in the life of an elevator and belts do, real economy lies with the high ratios. Lower drum is usually smaller than driver, because driving tension in belt is almost nil at that point, with only bending tension remaining. Guide idlers are often too small, do not turn easily, and drag on the belt.

For chain-bucket and other elevators for excavating earth, trenching, etc, see Sec 3; for gold dredging, see Sec 10; for tailing "sand-wheels" and ore concentrators, see Taggart, Handbook of Ore Dressing.

12. HELICAL CONVEYERS

Characteristics: low first cost, compactness, small capacity, large consumption of power for useful work done. To friction loss of load sliding through the trough, must be added the friction of screw turning within the mass. Low capacity is due to the need of keeping load line below the shaft center and bearings. For grain, a 30-in screw can deliver 100 tons 100 ft per hr, with 40 hp. This is about 20 hp per ton-mile, or 20 times as much power as for a belt conveyer. The disparity increases with wt of materials, or higher coeff of friction: for bituminous coal, about 25 hp-hr per ton-mile; ground cement, 30 hp; for rock and ore (with clean, lightly loaded trough), 25 to 50 hp per ton-mile. When finely-ground stuff is to be carried up an inclined belt (100 mesh, or less, like cement), a short length of screw conveyer is required as a feeder, to expel air and compact the material. Hot material is sometimes cooled in a screw conveyer. Conversely, some substances are heated in transit; or the screw may be cut into paddles for mixing heavy materials. Ordinary grades of concrete can be cheaply mixed; first-class concrete requires a batch mixer. Shafts are preferably tubular, for transverse stiffness due to large diam. Bearings should be babbitt lined, with grease cups. Spacing depends on wt of material and lateral bending of shaft, and may vary from 7 to 15 ft. Conveyers in series, if in line, should have flexible couplings; otherwise, angle or bevel gears. Gates are plain slides. Screws are usually of heavy sheet steel; for minerals, concrete, cement, and ashes, smoothly molded C I is more durable. Troughs are of sheet steel, curved to conform to screw. Generous clearance between screw and trough permits accumulation of a layer of fines, which protects the bottom from wear. Plank boxes, of rectangular section, lined with sheet steel, are not so good as half-round steel troughs. Careful lining up, and stiff supports are essential. Thrust bearings are plain shoulders on end of shaft, with C-I bearings; they require constant lubrication and are generally too small for long life. Long screw conveyers are apt to get out of line on timber supports, where beam loads cause changing deflections, just as this condition affects the good running qualities of long overhead line shafts.

13. FEEDERS, GRIZZLIES, GATES AND CHUTES

Feeders for continuous conveyers are essential when delivery is intermittent, as from cars, skips, and buckets. They act as equalizers, to smooth out the peak loads, and are also advantageous in feeding crushers and trommels, to prevent clogging, steady the motor load, and reduce size of motors. TYPES: (*a*) reciprocating troughs, inclined and level; (*b*) swinging plates, single and double; (*c*) plungers, with stationary troughs; (*d*) continuous aprons;

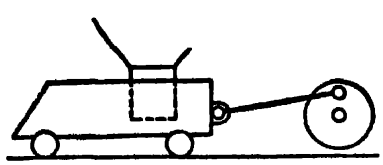


Fig 46. Reciprocating Feeder

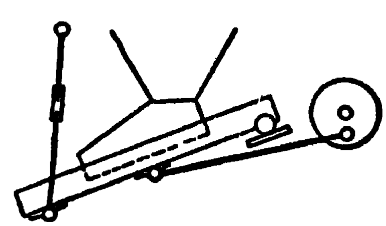


Fig 47. C. K. Baldwin Feeder

(*e*) automatic timed gates; (*f*) rotary plates; (*g*) revolving grizzlies; (*h*) roll feeders. SIMPLE INCLINED TROUGH, inclined 10°–15°, suspended by 4 adjustable rods, is apt to sway in an oval orbit and strain itself. LINK-BELT FEEDER obviates this by placing a reciprocating flat or inclined chute on 4 wheels (Fig 46). If inclined 15°, it can be made to empty itself. C. K. BALDWIN FEEDER has 2 rear wheels, with adjustable rods at front. It is always inclined to insure self-cleaning (Fig 47). In all of these, there is considerable vibration,

requiring stiff supports. Crank shaft should not rotate more than 80 r p m. All bearings should have bronze bushings. Long connecting rods are desirable; best made of pipe with forged ends. Throw is usually 12 in. Crank disks can be tapped for 3 radii of crank pin, to vary stroke. Stresses on moving mechanism and crank-shaft bearings are doubled

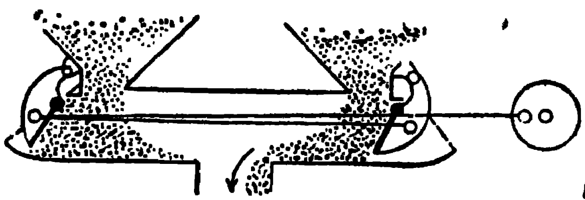


Fig 48. Duplex, Balanced Feeder (L. de G. Moss)

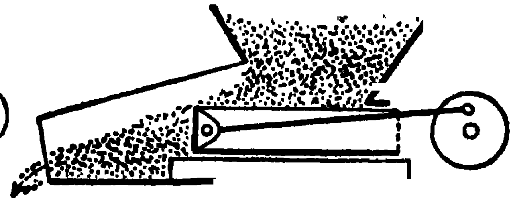


Fig 49. Dempcey's Plunger Feeder

for impact. Counterbalances in crank disks lessen vibration. Gear teeth may be cut on rims of disks to keep torsion out of crank shaft and save 1 gear wheel. Single and double swinging plates have been used by the writer, to reduce vibration of large masses. Fig 48 shows a duplex balanced form for a track hopper. Horiz chutes should be as short as possible, just so that material will not fall beyond angle of repose; speed, 100 r p m. Dempcey's plunger feeder (Fig 49) may be horiz or inclined. It can be made duplex under a large hopper. Plunger is hollow, preferably of C I, and slides on rails, which should be easily replaced, as they lie unprotected in the material; speed, 100 r p m. The throw of feeders can be varied while running, by a vibrating link-arm with constant throw, using a shifting slide on link to vary travel of reciprocating pan.

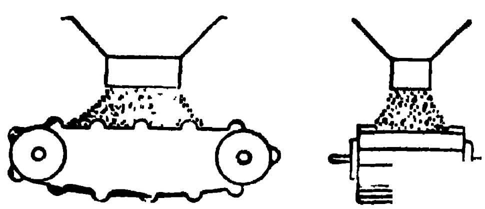


Fig 50. Apron Feeder

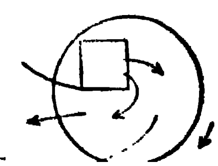


Fig 51. Action of Plate Feeder

Capacity can be calculated by displacement, main factor being breadth. Reciprocating feeders, if made as shown with a back plate, are really stationary pusher feeders. The incline does not increase capacity, but lessens work

Apron feeders make an excellent gate for ore (Wellman-Seaver-Morgan Co) and may be power-driven (Fig 50). No skirt-boards are used. Rotary plates, as in the Challenge feeder and Trump mixer, consist of circular plates, level or slightly inclined, with a plow. They will not sweep off the theoretical volume, owing to slip backwards on the plate (Fig 51).

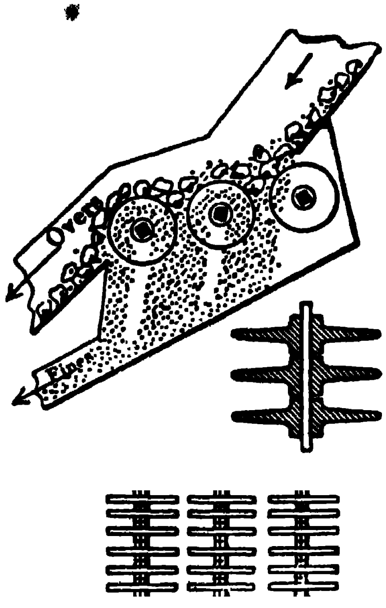


Fig 52. Stockwell Feeder

Revolving grizzlies were introduced by E. V. Stockwell, chief engr, Braden Copper Co (Fig 52). A single set or a series of rolls may be used. The C-I plates are tapered to prevent wedging of lumps, and loosely fitted on a square shaft to aid in freeing lumps. With sticky materials, stationary finger bars between disks will clear them. The spaces drop out the fines to a by-pass chute. These grizzlies now made by Robins Conveying Belt Co.

Roll-feeders, introduced by Hoover & Mason for ore and limestone, have a single hollow roller with C-I spiders and plate barrel (Fig 53); they also serve as gates. They feed too unsteadily for conveyers, but are good for filling cars. Variable-throw feeders are also used.

Gates of the undercut type are occasionally used for feeders; driven by shafting and cranks or eccentrics. Cranks are preferable, giving less trouble and using less power.

Gates of sliding type, though cheap, are often troublesome. They rust, jam with fines, and freeze tight in winter, often requiring a sledge to open them. Their length should be at least 1.5 × width. The single undercut gate is very satisfactory in sizes less than 20 in diam (Fig 54). Duplex undercut gates work well with fine material, but, for ore, should have offset jaws (Fig 55) to avoid jamming. The Briggs folding flap neither jams nor freezes, and is easily moved. Clearance between flap and sides must be small to avoid lodgment of spalls.

The flap may be external, as in Fig 56. Fig 57 is a combined gate and chute for large stone, subject to much freezing. Stationary grizzlies in chutes, for by-passing fines, are best made of tapered bars now in the market. Clogging at lower end of bars by gradual accumulation

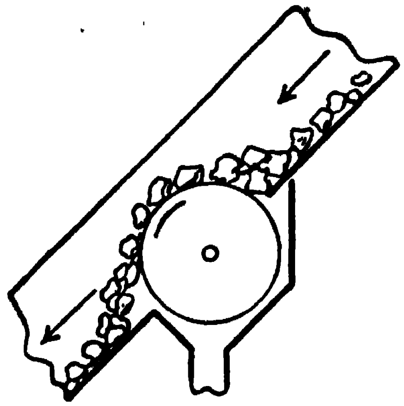


Fig 53. Roll Feeder

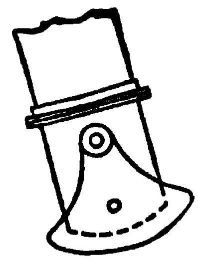


Fig 54. Single Undercut Gate

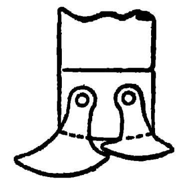


Fig 55. Hunt Undercut Gate

of spalls is avoided by curving the ends downward. C-I slotted spacing bars are preferred by many to through bars and washers, which offer more obstruction. Chutes and hoppers, subject to heavy falls of ore or rock, last well if lined with old rails instead of plate.

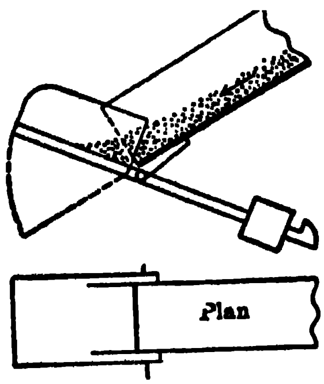


Fig 56 Briggs Gate

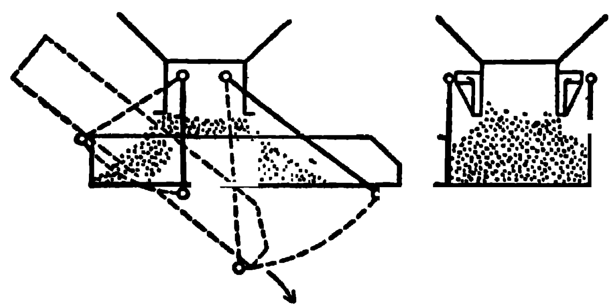


Fig 57. Combined Gate and Chute

			Tons per hr				Tons per hr
Reciprocating feeders require	1 hp	for	15–20	Roller grizzlies require	1 hp	for	15–20
Plunger and push-plate "	1	" "	20–30	Apron feeders (short) "	1 "	"	12–20
Rotary plates "	1	" "	10–15	Roll-feeders "	1 "	"	25–35

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SECTION 28

BREAKING, CRUSHING, AND SORTING OF ORES

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

BREAKING, CRUSHING, AND SORTING OF ORES

Ore is crushed at the mine to aid subsequent transport to the mill, or because a certain amount of fall is available, or to facilitate hand sorting. For some one of a variety of reasons, it may be desirable to perform these preliminary operations at the mine rather than at the treatment plant.

The usual crushing plant, whether located at mine or mill, is designed to produce feed to the grinding mills. As such it comprises jaw and gyratory crushers, classed as BREAKERS; and reduction gyratories, cone crushers, and rolls, called INTERMEDIATE CRUSHERS. Steam stamps are used only in the Lake Superior native-copper plants, as intermediate crushers; gravity stamps are found in many gold mills. It is unlikely that steam stamps will ever be generally used, and the gravity stamp is approaching obsolescence.

Size reduction, except in the smallest plants, involves two or more stages. Aver reduction in max size of lump in primary breakers is 4 : 1, in cones 5–6 : 1 and in rolls 4 : 1 in closed circuit and 2 : 1 in open circuit. Hence, to reduce 24-in run-of-mine ore to minus 1 in ball-mill feed, a primary breaker with 24-in effective gape followed by a cone with 6-in effective gape would serve; if minus 10-mesh (0.065 in) ball-mill feed were demanded, 3 or 4 stages would be necessary, for example, primary crusher to 6 in, standard cone to 1 in, short-head cone to 0.25 in, and rolls in closed-circuit with vibrating screens 0.25–0.065 in.

1. BREAKING BY HAND HAMMERS (24)

Hand sledges, 10-lb for soft rock and work at high altitudes, 12-lb for aver service, 16-lb for hard rock, are used to break ore through grizzlies in the mine and on ore-bins small enough to pass through chutes and gates; also to break large lumps so that they can be nipped by the primary breakers when these are too small for their service. CAPACITY figures are unreliable; but it is certain that sledging is almost always uneconomical, and that ore should either be broken smaller at the face, or that gates, chutes and primary crushers be made large enough to take the run-of-mine.

2. JAW CRUSHERS (24)

Jaw crushers. Best known type is the Blake, with movable jaw pivoted at top. The Dodge, with movable jaw pivoted at bottom, has relatively small capac. Single-toggle crushers with conventional plane jaw plates, or with either the swing-jaw plate or both jaw plates convex outward, are coming into use, principally as secondary crushers. Primary jaw crushers are usually of the Blake type. Table 1 gives essential data respecting Blake crushers (from makers' catalogs). For many sizes the figures have been checked against operating data, and found to be conservative.

Main frame, swing jaw and flywheels are of cast iron, semi-steel or cast steel; pitman, cast steel; toggles, cast steel with chilled ends, or alloy steels (life, 90 days to several yr); toggle seats, high-carbon steel or alloy steels (life, 90 days to several yr); shaft, high-carbon or alloy steel forgings, heat-treated, tempered, turned and polished; jaw plates, chilled iron or manganese steel (consumption of manganese steel 0.01–0.06 lb per ton crushed; of chilled iron, 0.1 lb per ton aver); cheek plates of same metal as jaw plates (consumption of manganese steel, about 0.01 lb per ton; of chilled iron, 2–3 times as much for same service). Modern trends are toward lighter crushers made of high-strength alloys. Too light a crusher will shake to pieces either itself or its foundation.

Reduction ratio in crushing is ratio between size of feed and of product; in coarse crushing max sizes should be considered. NIP ANGLE is the angle between jaw faces. Aver reduction ratio for jaw crushers of all sizes in Table 1, with minimum setting, is about 4.5, estimated on the basis of max feed size of about 85% of gape, and max product size 1.7–2 times open setting; for max setting, 2–2.5. Larger ratios are allowable in the smaller crushers, because the corresponding nip angle is smaller. Max nip angle for above ratios is less than 24°. In mills, the aver ratio for 20 crushers, ranging from 6 by 20-in to 66 by

84-in size, was about 3.7; max, 9; min, 1.5. Corresponding nip angles, 18–23° (max angle in largest crusher).

Capac of Blake-type crushers depends primarily upon character of ore, size of feed, and discharge opening; it is affected also by throw, speed and angularity of jaws, and shape and character of surface of jaw plates. Table 1 gives average capacities (from makers' catalogs) based on rock like limestone, easy to crush. But, capac figures are generally given in terms of a feed, none of which will pass through the crusher without breaking, whereas ordinary crusher feed contains much material that requires no breaking, but passes directly through.

Quartz, quartzose ores and firm brittle ores can usually be crushed at a rate equal to catalog figures. Tough ores, as basic silicates, traps and diorites, crush less readily. Hersam (1) shows that if the capac of a crusher on quartz is taken as 100, its capac on a granite such as he tested would be 89.5 and on trap, 83; mfrs usually drop these figures to 80–85 and 75 respectively. Minerals of low sp gr, as coal, fibrous minerals like asbestos, and clayey materials, can be crushed only at rates much below catalog figures. Choking by clayey materials may be lessened by a trickle of water run into the crusher with the feed; this lubricates the crushing faces, so that the compressed material slides forward as the jaw recedes, and thus works through the crusher. With slate, which cleaves easily and tends to pass through the crusher in slabs much larger in one or two dimensions than the jaw opening, the capacity may be much reduced, due to the measures necessary to prevent such discharge.

Table 1. Blake Crusher Data *(from Makers' Catalogs)*

Size of receiving opening, in	Approx capac, tons per hr to crushed sizes stated, in*										R p m	Hp	Weight, lb
	Size	Tons	Size	Tons	Size	Tons	Size	Tons	Size	Tons			
7×10	0.75	1.5–2	1.0	2.5	1.5	4	2	5–6	.....	....	250–300	7–8	6 800– 8 400
9×15	1	4–6	1.5	5.5–11	2	8–10	2.5	10–12.5	.....	....	250–300	10–15	7 500– 16 900
10×20	1.5	10–15	2	15–17	2.5	17.5–22	3	20	.....	....	250–300	14–20	8 800– 22 300
12×24	1.5	15	2	20	2.5	25	3	30	4	35	250–300	20–25	22 600– 24 000
15×24	2	22	3	32.5	3.5	40	4	45	.....	....	210–250	30–35	25 000– 30 000
15×30	2	20–27	3	35–40	4	45	5	50	.....	....	250	35–55	17 000– 40 000
18×30	2	25–35	3	37–45	4	45	5	50	7	80	240–275	40–53	41 500– 55 500
18×36	2	40–45	2.5	40–60	3	50–70	3.5	60–80	.....	....	225–275	40–65	59 000– 63 000
24×36	2	25–50	2.5	38–55	3	41–70	4	60–90	5	75–105	180–250	60–80	56 000–100 000
24×72	2.5	110	4	200	.....	....	.....	....	.....	....	150	150	160 000
30×48	4	100	5	120–125	6	150–190	8	225	.....	....	175–225	100–125	92 000–180 000
36×42	4	76	5	108	6	144–175	8	235	10	290	175–225	90–115	93 000–131 000
36×48	4	100–130	5	130–165	6	150–200	8	260	10	325	175–225	125–150	95 000–215 000
42×48	5	118–140	6	150	8	240	10	320	12	250	150–225	110–150	155 000–218 000
42×60	5	175–185	6	225	7	245–260	9	320	16	350	125–225	140–165	180 000–327 000
48×60	5	175–180	6	175–235	8	230–450	9	290	10	320–713	125–200	90–200	205 000–265 000
48×72	6	210–240	7	315	8	280–360	10	350–450	.....	....	125–150	150–215	240 000–257 000
60×84	6	380	7	285–375	9	360–500	10	450	11	495–625	80–100	100–300	415 000–588 000
66×86	8	330–510	9	420	10	415–798	12	495–1 110	.....	....	80–90	275–300	460 000–680 000

* With straight jaws. With curved jaw-plates, convexed toward each other and approx parallel near the throat, speeds may be higher, with higher capac, but at expense of some reduction in size of largest particle that will be nipped.

Crusher capac increases progressively with decrease in reduction ratio, and with increase in speed to a certain limit, although not proportionately. Increase in nip angle decreases capac somewhat, but the effect is small for angles near those common in practice. With granite, Hersam (1) found that greater tonnage could be crushed with smooth than with medium or rough jaw faces, tonnage with medium faces being greater than with rough. Increase in jaw throw causes marked increase in capac, if same minimum opening be maintained. Curved jaw plates tend to permit crushed material to spread out in the fine-crushing zone, which decreases tendency to choke and increases capac.

Empirical formula for capac, relatively accurate for all except smallest and largest crushers, is $T = 0.6\,A + R = 0.6\,LS$; where T = ton per hr; A = area of receiving opening, sq in; R = reduction ratio; L = length of receiving opening, in, and S = width of discharge opening (SET), in. For small crushers the result will be high; for large crushers, low.

Power consumption per ton crushed is considerably greater in small crushers than in large. Tons per hp-hr ranges from 0.7 in smaller sizes to 1.0 in crushers with 30-in width of receiving opening (GAPE). For 36–60-in gape, tons per hp-hr increases to about 1.3–1.8. For 66-in gape, 2.5 tons per hp-hr is fairly conservative. These figures are all based on reduction ratio of 6 : 1 and on a continuously busy crusher. For smaller reduction, tons per hp-hr increase, roughly in inverse ratio. REDUCTION TONS PER HR = hourly tonnage × by max-particle reduction ratio. Reduction tons per hp-hr for crushers in Table 1 range from 2 for 7-in gape to 10 for 66-in gape. Practice shows variations of about 50% on both sides of these aver figures.

Fall through crusher itself is about twice the gape; to which must be added the drop necessary to get feed to the crusher and product away from it.

Lost time in mills, due to causes chargeable to the crushers, as repairs, renewals, clogging and its attendant difficulties, is less than 1%. Renewing jaw and cheek plates and re-babbitting bearings are chief causes of lost time. When crusher is planned to run only 1 or 2 shifts per day, as in a majority of mills, renewals and repairs are done in off shifts; in which case practically no delay is chargeable to the crushers.

Attendance. Usual practice is 1 man per crusher; rarely, 1 man to 2 crushers. Principal duty of attendant is to regulate feed and pick out powder, steel and waste. When crusher is fed by dumping car- or skip-loads directly into the jaws, no picking is possible, but attendant must see that the ore does not bridge, and break up such jams as occur.

Feeding. Jaw crushers should be fed regularly and up to capac, if possible. But they should not be " buried," since bridging at the mouth often occurs and necessitates digging out to start the flow of ore again. A power-actuated hook, or similar device, for removing lumps from crusher jaws is an economy. Ample provision should be made to prevent pieces of ore from dropping into the actuating mechanism.

Breaking points are provided in most coarse crushers, to take care of excessive loads caused by entrance of steel or other foreign material. In Blake crushers the usual breaking point is a toggle. Without this provision, the usual effect of stoppage under full working load with power on is to crush the pitman babbitt, or throw off the belt, or both. If the belt does not throw off, and there is no satisfactory overload circuit breaker on the driving motor, the motor will burn out.

Size of product. With straight jaw plates, product will all pass a square-mesh testing sieve having aperture equal to 1.7–2 times open setting; 65–85% will pass an aperture equal to open setting, the lower percentage for tough or slabby rock, the higher for relatively friable rock that breaks granularly; 25–40% will pass an aperture equal half the open setting, the lower perçentage with sized feed, the higher with run-of-mine rock. The size showing max wt is usually at or near the open setting. With curved plates and higher speeds, apply the above percentages to the mean of the open and closed settings.

Cost of jaw crushers (1938) was 10.5–12¢ per lb for small machines; 9–10¢ for large, when fitted with chilled-iron; add 10–20% if fitted with manganese steel. OPERATING COST is best estimated from data given on power consumption, attendance and wear; these items making about 90% of total cost. For rough estimates, 8–10¢ per ton for small crushers, to 2–3¢ for large, are outside figures.

3. GYRATORY CRUSHERS (24)

Gyratory crusher consists of a fixed crushing surface, in form of a frustum of an inverted cone, around the axis of which gyrates a movable crushing surface, having the shape of a conical frustum in erect position. The feed passes into the downwardly converging annular space between the crushing surfaces, and is crushed when the surfaces approach; falling through when they recede. There are 3 types: suspended-spindle, supported-spindle, and fixed-spindle type. The first is most used; the second is fast disappearing. The third is the newest form, and has been satisfactorily installed as a primary or secondary

coarse crusher in many plants. Its relatively small height lends itself to rugged construction and the short spindle reduces the clear height above the crusher required for convenience in repair work. The fact that the length of stroke is the same for both large and small pieces of ore is advantageous for crushing soft, tough ore, but this is unnecessary and may be disadvantageous for hard and brittle materials.

Sizes are indicated by the gape of receiving opening, in inches. Length of receiving opening is the circumference of the outer edge of receiving opening (between adjacent faces of the spider arms) × number of spider arms. This length is approx 8 times the gape in lever-type crushers in sizes below 18-in, and 7 times the gape in larger sizes.

Details. SHELL is of C I; high-test iron or cast steel; SPIDER, C I in small crushers and high-test iron or cast steel in large; SPINDLE, hammered open-hearth steel, heat-treated, turned and polished; SUSPENSION BEARING varies in detail, the principle being to bring the suspending surface as near the point of no-movement as possible; ECCENTRIC SLEEVE should be large and lubricated by forced feed; GEARS, cast high-carbon steel, or forged steel with cut teeth; COUNTERSHAFT BEARING, long and rigid; OUTBOARD BEARING, ball-and-socket; DRIVE PULLEY, of such size as to transmit 1 h p per in width, at 600-750 ft per min; CONCAVES and MANTLES are usually manganese steel (life 90 days-2 years). Modern practice inclines toward roller bearings and tex-rope drives.

Adjustments: (*a*) WIDTH OF DISCHARGE OPENING is changed by raising or lowering the breaking head. Range of this adjustment is limited, because the nip angle is markedly increased by wear of the breaking head. This increase in nip angle causes great decrease in discharge capac. If more than a small adjustment is required, it must be got by making the concaves and breaking head thicker and shorter. THROW, less than in jaw crushers, is adjustable only by changing eccentric sleeves. It should be greater in large crushers than in small; greater for relatively soft and tough ores than for hard and brittle ores. SPEED may be varied between wide limits. Increased speed does not cause the marked increase in vibration and shock that occurs in jaw crushers. Lowest speed compatible with required capac is most economical within certain operating limits. A tendency to clog when crushing sticky ores may often be overcome by increase in speed, thus increasing the sharpness with which the head recedes from the concaves.

Speed, throw, capac, reduction ratio, and power consumption are closely related. Reduction ratio increases when crusher is working near max capac. Crusher speed must be increased to keep up capac; this being accompanied by considerable increase in power consumption. If the crusher is sufficiently over-motored, so that the change brings no perceptible strain on the driving equipment, it is well to watch for heating of the eccentric, as any marked increase in speed and capacity over the figures recommended by the makers is likely to cause burned-out bearings. Increase in reduction ratio, without mechanical troubles, should be accompanied, with straight liners, by decrease in speed and decrease in amount of material in the crushing zone. With curved liners, ratio may

Table 2. Catalog Data on Suspended-spindle Gyratory Crushers (short-shaft type) Fitted with Standard Heads and Concaves (*c*)

Receiving opening, gape × approx total length, in (*a*)	Approx capac, ton per hr, with open setting stated, in (*d*)				Approx wt, lb	Rpm	H p (*c*)	Fall, ft and in (*b*)
	Minimum setting	Tons	Max setting	Tons				
2 1/2 × 28	3/8	1/2	1/2	3/4	550	700	4	1-9
8 × 70	1	14	2 1/2	47	16 000-20 000	450	15-25	5-6
10 × 80	1 3/4	39	3 1/2	93	30 000	400	25-40	6-9
12 × 90	2	39-63	3 1/2	128	28 500-45 000	375	45-75	6-9 8-2
14 × 110	2 1/4	60	3 1/2	95	39 500	335	75	7-7
16 × 120	2 1/2	100	4 1/2	200	60 000	350	60-100	8-9
20 × 148	3	150	5	245-275	94 000-104 000	330	75-150	10-6
26 × 200	3 1/2	225	6	400	160 000	320	200	13-0
30 × 210	4	235	6 1/2	450	175 000	325	125-175	13-6
36 × 262	4 1/2	370	7	600	260 000	300	175-250	15-3
42 × 284	5	410	7 1/2	700	285 000	300	200-275	16-6
48 × 332	5 1/2	1 100	9	1 890	520 000	250	350	19-6
54 × 360	6 1/4	875	9 1/2	2 100	625 000	225-250	225-400	26-6
60 × 400	6 1/4	990	10	2 400	725 000-1 000 000	200-250	225-500	31-0
72 × 484	9	2 500	12	3 400	1 400 000	175	500	32-0

a Crusher size is designated by gape. *b* Top of hopper to lip of discharge chute. *c* Usual gear ratio is approx 1 : 2.5. *d* Run of mine rock. *e* Convexly-curved concaves and a concavely-curved mantle can be fitted to any of the crushers listed, and have (1938) been used on sizes to 42-in gape, with capac increases from 25% to 60%, according to makers' claims.

be increased without decrease of speed; in fact, speed and capac may both be increased somewhat without corresponding increase in power consumption, due to non-choking characteristics of curved liners.

Reduction ratio, based on max sizes of feed and product, ranges, according to makers' ratings from a minimum of 1.5 to max of about 4; general aver, 2.5–3. A greater ratio is recommended for large crushers than for small. These are just about the range and aver for reported practice.

Angle of nip in gyratory crushers is 21°–23°; aver very near 22°.

Capacity depends primarily upon character of ore, size of feed, and discharge setting. Throw, speed, angularity of jaws, and shape and character of crushing surfaces have material effect. Table 2 gives capacities for different sizes, according to makers' catalogs. See Art 2 for discussion of applicability of these figures to different ores.

The discussion (Art 2) of effect of various factors upon capac of jaw crushers holds also for gyratories. No simple formula expresses results of gyratory crushing, as does that given for jaw crushing. Capacities in makers' tables are often far exceeded in mills, although the excess capac usually results from feeding material much smaller than the max that the crusher will take, with consequent freedom from bridging and clogging, or from the use of curved liners, or both.

Power consumption. Reduction tons per hp-hr range from 1 for smallest machine to 28 for a 42-in machine, according to makers' ratings; reported field performances tend to run 10–20% below these figures.

Fall through crusher proper is about 10 times the gape, up to 12-in gape; for larger crushers, the fall decreases with increase in gape from 8 to 6.

Lost time in mills, due to causes chargeable to the crushers, as repairs and renewals, clogging and its attendant delays, averages less than 1%. Renewal of mantles, concaves, and chute liners, and re-babbitting the eccentric bearing are the chief causes stated. Most of the crushers reported were planned to work 1 or 2 shifts per 24 hr, thus leaving 1 shift free for minor repairs. With good planning and ample supply of repair parts, such operation practically eliminates lost time.

Attendance. Usual practice, 1 man per crusher to 2 or 3 crushers per man. Where the gyratory is the primary crusher, there should be 1 man for each to remove waste; where it is secondary, little or no attendance, apart from oiling and watching for trouble, is necessary.

Feeding should be regular, and as nearly as possible up to capac. If feed contains pieces near the largest the crusher will receive, it is wise not to bury the crusher, as bridging easily occurs and may require laborious digging out. But bridging is much less likely to occur in gyratories than in jaw crushers, and many gyratories are fed so as to be buried. At most plants crushers are fed by chutes or over stationary grizzlies; belt or pan conveyers, drum feeders and shaking grizzlies are also used. Pan conveyers are probably best for feeding initial crushers. Chain feeders are useful to prevent "bounding" with bouldery feed from bins. Chute-fed crushers mostly occupy a secondary position in the mills, taking feed from primary crushers.

Breaking points are usually omitted in modern gyratories. Large gyratories will pass almost any piece of steel that enters, without stalling or breakage; small ones will generally slip a belt before breaking, and, if motor is properly safeguarded against overload, this is fairly satisfactory. For direct-connected crushers, overload circuit-breakers are probably best for providing against damage.

Size of product. For primary crushers with straight liners, all the product will pass a square-mesh testing sieve of aperture equal 1.7–2.2 times the open setting; 70–90% will pass an aperture equal the open setting; and 30–50% will pass an aperture equal half the open setting; the higher percentage in both cases corresponding to relatively friable run-of-mine rocks that break granularly. The size showing max wt is at or slightly above the open setting. For curved liners in primary service, substitute the mean of open and closed settings for the open setting in the above text.

Cost of gyratories in 1938 was 8.5¢ per lb for a 42-in machine; 10.5¢ for a 16-in, and 16¢ for an 8-in. These prices are for crushers chilled-iron fitted; add 10–20% for manganese-steel fitting. Over 90% of OPERATING COST is for power, labor, and wearing parts. If crushers are run to capac, cost should range below 6–8¢ per ton for small crushers and 1.5–2.5¢ for large.

4. COMPARISON OF JAW AND GYRATORY CRUSHERS

Table 3 compares crushers of the two types of one maker. It shows that, when worked to capac, the price of a gyratory is 27–77% that of a jaw crusher capable of same reduction in particle size; also, that in same circumstances, the gyratory will crush

Table 3. Comparison of Jaw and Gyratory Crushers

Crushing from......to......inch	6-1		9-1.5		12-2		18-3		24-4	
Type of crusher	J	G	J	G	J	G	J	G	J	G
Size receiving opening, in.	7×10	8×74	10×20	12×92	15×24	14×110	20×24	20×160	28×36	26×200
Capac, ton per hr	4	16	10	28	17	52	34	157	66	310
Wt, ton	3.5	7.75	10	14	14.9	19	18.9	46.5	36	76.5
Installed h p.	7	12	15	25	25	40	35	90	70	135
Price, ¢ per lb (a)	16	18	15	17	14.2	16	13.6	11	11	10.6
Fall through crusher, ft	2.33	6.25	2.5	7.75	2.75	8.6	3.5	12.33	4.8	15
Hrly capac ÷ ton wt	1.14	2.06	1.00	2.00	1.14	2.74	1.80	3.38	1.83	4.05
Hrly capac ÷ installed h p	0.57	1.33	0.67	1.12	0.68	1.30	0.97	1.74	0.94	2.30
Price, $ per ton hrly capac	280	174	300	170	249	170	151	65	120	52.4
Relative capac ÷ ton wt, G to J	1.8		2.0		2.4		1.9		2.2	
Relative capac ÷ h p, G to J	2.3		1.7		1.9		1.8		2.4	
Relative price per ton hrly capac, G to J	0.62		0.57		0.68		0.43		0.44	
Relative price, G to J	2.5		1.6		1.4		2.0		2.0	
Relative capac per $ of price, G to J	1.6		1.8		1.5		2.3		2.3	
Price per ton hrly J capac*	......	696	......	476	......	520	......	300	......	246
Approx actual h p, full load	3.5	7.2	7.5	15	12.5	24	17.5	54	35	81.0
Approx actual h p, idling	1.2	2.3	3.8	4.5	6.2	7.2	8.8	16	18	24
Tons per hp-hr, based on J capac	1.1	1.1	1.3	1.2	1.4	1.3	1.9	1.4	1.9	1.8

Crushing from......to......inch	30-5		36-6		48-8		60-10		72-12	
Type of crusher	J	G	J	G	J	G	J	G	J	G
Size receiving opening, in.	36×42	36×272	42×48	42×306	56×72	60×420	66×86	72×484	84×120	72×484
Capac, ton per hr	108	660	150	1 015	475	1 920	778	3 000	1 970	3 432
Wt, ton	58	150	77.5	180	155	390	230	625	437.5	625
Installed h p.	105	150	115	200	200	320	300	400	500	500
Price, ¢ per lb (a)	13	10	12.5	9.8	10.9	8	9	8	8.5	8
Fall through crusher, ft	6.25	18.1	7.0	19.5	9.25	29.25	11	36	14.5	36
Hrly capac ÷ ton wt	1.86	4.40	1.93	5.64	3.06	4.92	3.38	4.80	4.50	5.50
Hrly capac ÷ installed h p	1.03	4.40	1.30	5.08	2.38	6.00	2.59	7.50	3.94	6.86
Price, $ per ton hrly capac	140	45.5	129	34.8	71.2	32.4	53.2	33.3	37.8	29.1
Relative capac ÷ ton wt, G to J	2.4		2.9		1.6		1.4		1.2	
Relative capac ÷ h p, G to J	4.3		3.9		2.5		2.9		1.7	
Relative price per ton hrly capac, G to J	0.33		0.27		0.46		0.63		0.77	
Relative price, G to J	2.0		1.8		1.8		2.4		1.3	
Relative capac per $ of price, G to J	3.1		3.7		2.2		1.6		1.3	
Price per ton hrly J capac*	......	278	......	235	......	131	......	128	......	51
Approx actual h p, full load	52.5	90	57.5	120	100	192	150	240	250	300
Approx actual h p, idling	26	27	29	36	50	58	75	72	125	90
Ton per hp-hr, based on J capac	2.1	2.8	2.6	3.1	4.8	5.2	5.2	6.8	7.9	9.4

J = Blake-type jaw crusher. G = Suspended-spindle gyratory. * Compare sixth line above, where price per hourly ton is computed on G capac.
(a) Prices are as of 1925. Add about 1¢ per lb for 1938. Comparative price relationships in remainder of table are substantially unchanged.

1.7–4.3 times as much as the jaw crusher per installed h p. But, when the quantity of rock to be crushed per hr is within capac of 1 jaw crusher of proper gape, price of a gyratory for same work is 1.3–2.5 times that of a jaw crusher. Power for a jaw crusher when idling is about 50% of that at full load, and full-load consumption is about 50% of installed h p; corresponding percentages for the gyratory are 30 and 60%. Applying these figures to Table 3, it appears that for machines crushing to 4 in or less, power consumption per ton is less in the jaw crusher. In coarser crushing to 5–12 in, power per ton crushed is less for gyratory than for jaw crusher. Repairs, cost of installation, and loss of head are all greater for gyratory than jaw crushers for all sizes. Hence, it may be concluded that, for quantities that can be handled by 1 jaw crusher, the jaw crusher is the more economical on all counts; also that, for coarse crushing, the gyratory consumes less power per ton, but costs more to buy, install and keep in repair, and for elevating the material. The gyratory is advantageous in allowing expansion in capac, it can be set higher than a jaw crusher, due to absence of vibration, and can be fed from all sides. But, if the rock is clayey or fibrous or does not break freely, it is more likely to clog. The shape of the gyratory's crushing zone prevents discharge of large unbroken slabs, such as can pass through a jaw crusher; hence gyratory product is more uniform than that of a jaw crusher.

Following empirical relation, based on analysis of good practice, indicates roughly the proper crusher to use, considering tonnage and size only: if hourly tonnage to be crushed, divided by the square of the gape in inches, yields a quotient less than 0.115, use a jaw crusher; otherwise, a gyratory.

5. GRIZZLIES (24)

Grizzly bars are best of taper section, to prevent choking; sometimes of special cross-sec, such that when worn out there will be minimum waste. Bars are threaded on rods and distanced by thimbles. Their slope must exceed the angle of friction (usually greater than 32°); when less than 40°, the ore must sometimes be helped forward by hand. They are generally used for separating fines from breaker feed. Grizzly dimensions are determined more by mode of feeding and using, than by tonnage treated; capac should greatly exceed demands. An aver figure for capac is 125 ton per sq ft per 24 hr per inch of aperture, with proportionately greater (or less) capac with greater (or less) bar spacing.

Heavy, coarse woven-wire screen or punched plate is often used instead of bars; especially for flat, shaking grizzlies. Roller grizzlies, consisting of a number of spaced circular disks on a horiz revolving shaft, or rings on a cylindrical cage; and traveling grizzlies, composed of parallel bars, spaced between corresponding links of parallel chains, making a continuous apron running over head and tail sprockets, are also used as combined feeders and screens ahead of coarse crushers.

6. REDUCTION GYRATORY. CONE CRUSHER

Reduction gyratory, a new type, for intermediate crushing, is made both in the suspended-spindle and the pillar-shaft (fixed spindle) types. The latter is similar to the pillar-shaft breaker, except that the flare of the breaking head is greater in the reduction gyratory, and the concaves flare downward, instead of converging, to maintain the required nip angle. Suspended-spindle machines are fitted with convexed bowl liners (concaves) having chords vertical or flared downwardly. Head has straight surface elements, or is concaved near bottom (bell-head). Speeds are higher than for primary machines. Combination of high speed, long discharge opening and substantially parallel-sided fine-crushing zone tends to eliminate choking and to size the discharge nearer to closed setting of crusher. Machines are rated according to the bottom diam of crushing head. Performance data, as estimated by mfrs, are given in Table 4. Structural materials correspond to those described for primary gyratories.

Size of product. Machines in typical fine-reduction service (fine bowls, closed settings of 0.5-in or less and small throws of 0.25–0.38-in), make products all of which will pass a square-mesh screen of 2–3 times the closed setting; 50–65% is finer than closed setting and 25–35% finer than half the closed setting. This construction has advantage of giving a large discharge area for a given set of the crusher, and consequently a larger capac. Usual sizes are 4-, 6- and 10-in gape. Capac of this new machine is not established; as given by one maker, it ranges from 24 ton per hr to 0.75 in, to 48 ton to 1.5 in, for a 6-in crusher, and from 80 ton per hr to 1.5 in to 135 ton to 2.5 in for a 10-in crusher.

Cone-type crusher is of the flaring-bowl gyratory-type of intermediate crusher. The crushing head, flat-conical or hemispherical in shape, is supported on a large bearing of special design (hemispherical, roller, etc) and is gyrated by a long, ball-supported eccentric, driven by bevel gearing. The stationary crushing surface (bowl) is attached to the main

frame by bolts acting against a nest of springs, which compress and allow the head to lift when an uncrushable body enters. Set may be adjusted while running, by loosening lock nuts and revolving the bowl. Sizes, capac and power consumption, as stated by one maker, are given in Table 4.

Reduction ratio, based on max sizes of feed and product, ranges, on some 20 field reports, from 3 to 18, with the aver between 5 and 6. High reduction ratios are possible only with coarse and medium bowls, annularly corrugated near mouth; short-head machines work at lower ratios in open circuit, but can be pushed to 8 or 10 by closed circuiting, in which case circulating loads may be upwards of 100% and new-feed capac will be somewhat reduced.

Angle of nip at mouth in short-head machines is about 10° or less, and decreases in fine-crushing zone; in the standard machine, the coarse bowl apparent angle in coarse-crushing zone is about 35°, but this is decreased to between 25 and 30 by annular corrugation of bowl.

Capacity is a function of area of discharge opening and of crushing surface. A relationship, reasonably safe for purposes of estimate for standard crushers taking primary crusher discharge with a max particle size 75–85% of crusher gape, is

$$C = 7 + \left(\frac{S - 0.05}{0.11}\right) d^2$$

in which C = ton per hr, S = set, in and d = rated size of crusher (diam of head at bottom in ft). With sized feeds, smaller reduction ratios (max feed particle about 50% of gape), and settings of 0.5 in and under (usual short-head practice) add 25–50% to formula capacities, the larger figure for the larger crushers. Nordberg Mfg Co call attention to the fact that cone-crusher capac on a given ore may vary 50%, according to moisture and clay content, and the like, and they therefore do not approve the use of a formula for their machine.

Power consumption. Reduction tons per hp-hr range from about 2.5 for 2-ft cones with fine bowls to 8 for 7-ft cones with coarse bowls, or roughly a little more than the cone-size rating in ft.

Size of product. In open-circuit crushing the product will all pass a square-mesh screen of aperture 1.8–2.2 times the closed setting of the crusher; 20–25%, on the aver, is coarser than the closed setting; range is 10–65, being greater, in general, with coarser settings; about 35–40% is finer than half the closed setting. These figures are based on sizing tests from some 20 random field performances. In closed-circuit work the limiting size may be substantially equal to the closed setting, with upwards of 60% of the product finer than half the closed setting.

Table 4. Cone-crusher Data from Manufacturers Catalog (Nordberg Mfg Co)

Size of crusher	Effective gape, in (a)		Minimum setting, in		Capac, tons per hr, with closed settings indicated, in									R p m	H p	Wt, lb
	Fine	Coarse	Fine bowl	Coarse bowl	1/8	3/16	1/4	3/8	1/2	3/4	1	1 1/2	2			
20-in	1 1/2	3	1/4	1/2	...	...	10	15	20	30	35	...	...	700	20–25	7 100
2-ft	1 5/8	3	1/4	1/2	...	...	14	20	25	35	45	60	...	575	25–30	10 500
3-ft	3	4 1/2	1/4	1/2	...	...	25	35	40	70	80	90	95	580	50–60	21 000
3-ft SH (a)	1 1/4	2	1/8	1/4	15	20	30	40	50	...	...	...	...	580	60	22 500
4-ft	4 1/2	6 3/4	3/8	1/2	...	...	...	60	80	120	150	177	185	485	75–100	35 000
4-ft SH	1 3/4	2 1/2	1/8	1/4	20	35	50	75	100	...	...	...	...	485	125	45 000
4 1/4-ft	6 3/4	9 1/2	1/2	3/4	...	...	...	...	100	140	160	185	190	485	125–150	45 000
5 1/2-ft	7	9 3/4	3/8	5/8	...	...	...	100	130	200	275	340	375	485	150–200	85 000
5 1/2-ft SH	2 1/4	3 1/4	1/4	3/8	...	...	100	135	175	...	...	...	...	485	150–200	88 000
7-ft	9	14	1/2	3/4	...	...	...	...	225	330	450	600	800	435	250–300	137 000
7-ft SH	3	4	1/4	3/8	...	...	150	240	300	...	...	...	...	435	250–300	143 000

(a) Short-head. Capacities given are for closed-circuit operation.

Feeding should be regular, and as nearly as possible up to capac at the speed at which the machine is operated. Reduction of speed, from catalog figures, will save some power in consistently underloaded crushers. Symons cone has a circular feed plate, mounted on a pedestal on top of crushing head, from which plate the material spills off evenly all around, if feed stream is brought to it vertically and near the center. Side-directed entry of feed stream overloads one side and underloads other; much oversize is discharged on underloaded side. If bridging does not occur, due to slabby oversize, cone-type crushers may be buried, but this practice involves much digging after emergency shut-downs.

Wear on head mantles and bowl liners is remarkably uniform; as a result manganese-steel fittings may sometimes be run down to 0.5 or 0.38 in before scrapping, the residue then comprising only 20–25% of the original wt. Reported lives of Mn-steel fittings range from 50 000 to 850 000 ton; steel consumption, 0.005–0.05 lb per ton crushed, aver near 0.02 lb per ton.

Attendance. Special attendance, such as is required for primary crushers, is not needed. In general, one attendent can take care of the secondary-crushing section of a plant of any size, except where screens require constant attention due to moisture, or to an excess of wood chips or other fibrous waste.

7. ROLLS (24)

Rolls are of two general types, rigid and spring. RIGID ROLLS, the older, are now rarely used. Their bearings are rigidly fixed on the frame, so that the rolls must stall, or their shafts bend or break when an uncrushable particle enters.

Spring rolls. Table 5, generalized from catalogs of the principal makers, gives sizes, weights, speed and power consumption of standard rolls. Weight of a given size of roll, as offered by different makers, varies widely. The lighter rolls should be chosen only for small tonnage, or very easy crushing, or where first cost is paramount. Heavy rolls for heavy service repay the greater first cost in a short time, in lower repair costs and continuity of operation.

Shells are of high-carbon steel, rolled like locomotive tires; of forged chrome-steel, rolled or bored to size and taper; or of manganese-steel, ground to proper shape. For light service, rolled high-carbon shells are excellent; it is claimed that, because the surfaces are not extremely hard, their nip is more assured than with harder shells; also, cracking less readily, they can be worn very thin before replacing.

Life of shells is in general 20 000–50 000 tons crushed for small (less than 36-in) fine-crushing rolls, of chrome or high-carbon steel, and 300 000–1 000 000 tons for large rolls in coarse crushing, tonnage in both cases including circulating loads. Manganese and high-carbon steel are best for coarse crushing. Thickness of new shells is 2.5–4 in for rolls to 36-in diam, and 3.5–6 in for larger; thickness when discarded is generally 0.5–1.5 in. Shells are rarely worn as thin as $^3/_8$ in, which can be done only in fine crushing and with very tough steel. Consumption of shells, including waste, is 0.01–0.1 lb per ton.

In a mill where rolls crush to different sizes, partly worn shells from fine rolls may be transferred to coarse rolls, if of the same size, since the loss of effic in coarse rolls, due to slight pitting, is much less than in fine. Flanging and pitting of shells are prevented or materially lessened by proper attention to lateral adjustment. Emery bricks held constantly against the back of the rolls diminish corrugation. At Van Roi mill (3), use of emery bricks tripled life of fine-roll shells. In S E Missouri, flanged shells are taken off and ground or turned down in a lathe, according to whether they are manganese or rolled steel. A 54- by 20-in shell can be turned down in 20–30 hr, dependent upon extent of corrugation (4).

Adjustments possible in well designed rolls are: (*a*) distance between roll faces (set); (*b*) lateral adjustment of one or both roll shafts. ROLL SETTING is done by pinning the nuts at one end of tension rods, and so arranging the nuts at other end that they are both moved equally and dependently by the adjusting mechanism. This mechanism also provides for backing the movable roll away from the fixed roll, to free them in case of clogging. LATERAL ADJUSTMENT. To prevent FLANGING, the range of this adjustment must be such that either edge of both rolls can be made to run, a part of the time, on the face of the other roll. To prevent CORRUGATION, rolls should be shifted about 0.6 the diam of largest particles in feed. Lateral adjustment is manual or automatic. Objection to manual adjustment is that it may be forgotten or purposely neglected by roll operator; a short period of neglect may ruin the surface of the shells. The objection to automatic adjustment is the difficulty of making a simple, durable and certain shifting mechanism.

Angle of nip is angle formed by tangents to roll faces, at their points of contact with particles to be crushed; it rarely exceeds 25°. The range in a number of mills was 5° 36′

to 35° 30′; averaging 23° 21′ for feeds coarser than 2 in.; 19° 26′ for feeds of 1–2 in, 14° 38′ for feeds of 0.5–1 in and 11° 25′ for smaller than 0.5 in. The variation is due to use of large rolls with small feeds for getting capacity, and the angles with coarse feeds may be taken as a safe aver.

Table 5. Summary of Catalog Data Concerning Crushing Rolls

Size, diam × face, in	No of makers (*a*)	Wt, lb		Speed recommended, r p m		H p recommended		Approx overall dimensions, in		
		Min (*b*)	Max (*b*)	Min (*c*)	Max (*c*)	Min (*d*)	Max (*d*)	Length	Width	Height to top of hopper
12×12	1	3 700	....	.....	230	6	.....	.....	.....	.....
18×10	1	6 500	....	250	300	8	.....	65	71	29
20×12	1	6 000	....	150	225	7	.....	.....	.....	.....
24× 8	1	10 500	....	100	160	7.5	.....	.....	.....	.....
24×10	2	10 600	11 000	90	160	8	11	.....	.....	.....
24×12	3	10 900	11 600	90	230	8	11	82	78	36
24×14	3	11 200	12 000	90	160	8.5	11	.....	.....	.....
26×15	1	16 500	....	75	125	.....	.....	.....	.....	.....
27×14	1	10 800	....	125	200	10	.....	.....	.....	.....
30×10	2	16 000	16 500	66	130	10	15	.....	.....	.....
30×12	2	16 900	17 000	66	130	10	15	.....	.....	.....
30×14	5	11 300	19 200	66	190	10.5	15	100	94	46
30×16	2	18 600	19 600	75	180	10.5	15	100	98	46
36×12	3	21 200	23 100	50	100	12	18	.....	.....	.....
36×14	3	23 000	31 000	51	150	12.5	20	118	106	54
36×15	1	33 000	....	40	100	.....	.....	.....	.....	.....
36×16	5	18 500	31 500	50	175	13	25	118	108	54
40×15	1	38 000	....	50	100	.....	.....	.....	.....	.....
40×16	1	23 000	....	80	100	25	.....	.....	.....	.....
40×20	1	42 000	....	50	100	.....	.....	.....	.....	.....
40×30	1	50 000	....	50	100	.....	.....	.....	.....	.....
40×36	1	52 000	....	50	100	.....	.....	.....	.....	.....
42×12	1	35 200	....	41	.....	20	.....	.....	.....	.....
42×14	2	36 100	37 000	41	100	20	30	131	107	59
42×16	4	35 000	62 000	41	100	20	50	147	123	72
42×18	1	63 500	....	95	120	55	.....	147	126	72
48×12	1	49 000	....	70	90	28	.....	.....	.....	.....
48×14	2	50 500	57 600	33	90	23	31	.....	.....	.....
48×16	3	51 800	75 000	33	100	23	31	.....	.....	.....
48×18	2	60 000	75 000	33	105	23	60	176	135	74
48×20	3	50 000	80 000	33	105	23	65	176	136	74
54×16	2	51 800	88 000	50	90	35	.....	.....	.....	.....
54×18	1	101 000	....	28	.....	25	.....	.....	.....	.....
54×20	5	55 000	103 000	28	95	25	70	189	141	79
54×24	5	60 000	136 000	28	95	25	75	189	145	79
60×20	1	85 000	....	50	60	60	.....	.....	.....	.....
60×24	2	90 000	150 000	50	85	70	90	208	173	91
60×30	1	165 000	....	65	85	100	.....	208	179	91
72×20	4	118 000	220 000	40	120	90	100	.....	.....	.....
72×24	4	133 000	230 000	40	120	100	125	233	194	111
72×30	2	205 000	235 000	50	75	125	.....	233	200	111
78×20	2	242 000	305 000	100	.....	300(*e*)	.....	.....	.....	.....

(*a*) Catalogs of 5 principal makers are summarized. This column gives the number of these makers who manufacture a given size. (*b*) lightest and heaviest of the size listed. (*c*) Not usually by same maker. Low speeds usually correspond to light-weight rolls. (*d*) The lower figure responds to light rolls at low speed and *vice versa.* (*e*) Two motors.

Angle of nip varies with diam of rolls, diam of particle and set of rolls. For large particles, large diam rolls must be used, or the reduction ratio must be small. When feed of given size is not being nipped, it is usual to install larger rolls or increase distance between them, and consequently the size of product. An expedient at one plant is to cut a transverse groove in one shell; which, of course, increases the nip angle at that point. At American Graphite Co, smooth shells would not nip; overcome by drilling 8 sets of four 1.5-in holes at equal angular distances around the faces (5). Shells with transverse corrugations are sometimes used for coarse crushing, but the practice is not well established.

Diam of rolls (Table 6). Largest commercial roll is 78 in diam; sis naller than 24-in are rare except in laboratories.

Speed should be lower for hard, tough rock than for soft and brittle rock, less for dry than for wet feed, less for coarse feed than for fine, and less for a large reduction ratio than for a small, nip being the controlling factor in each case.

Table 6. Diameter of Rolls for Different Sizes of Feed

Diam largest feed particle, in	Minimum diam of roll, in (a)				
	Reduction ratio				
	6:1	5:1	4:1	3:1	2:1
6	....	....	....	....	121
5	....	....	....	....	100
4	....	....	....	....	80
3.5	....	....	....	95	70
3	....	....	....	81	60
2.5	....	....	77	68	50
2	68	65	61	54	40
1.75	60	57	54	48	35
1.5	52	50	46	41	31
1.25	43	41	38	34	25
1	35	33	31	27	20
0.75	26	24	23	20	15
0.5	17	16	16	14	10
0.38	13	12	11	10	8
0.25	9	8	8	7	5
0.12	4	4	4	3	2

(a) Allowing 25° nip angle.

Reported speeds are from 382 ft per min for 24-in rolls, to 2 060 ft for 72-in. More or less independently of the other factors, practice tends to keep below 900 ft per min for rolls to 36 in diam, below 1 000 ft for 42-in, and not above 1 500 ft for 56- and 72-in; higher speeds are dangerous to springs, shafts, frames and foundations. At Miami Copper mine, 55-in rolls taking −3.5-in feed run 100 rpm; same size with −2-in feed, at 115 rpm.

Capacity of rolls is, theoretically, the wt of a ribbon of ore, the length of which is the peripheral travel per unit of time; breadth, the width of face; thickness, the set, or distance between roll faces. With open setting, actual capac never reaches the wt of "theoretical ribbon;" which is more nearly approached the smaller the set.

For rolls set coarser than 1 in, about 5% of theoretical ribbon is to be expected; for sets between 0.25 and 1 in, aver performance is 15–20% of theoretical; for sets less than 0.25 in, aver is 20–30% of theoretical, with free feeding. With choke feeding, in closed circuit with a screen, from 100 to 250% of theoretical ribbon is to be expected, the set being assumed as the screen aperture × 0.3. Rolls may be in actual contact, or with a small space between faces; mean cross-sec of ribbon is, of course, greater than the set, since the rolls recede against the spring pressure at short, irregular intervals. Percentage of theoretical ribbon that can be crushed is about 50% greater for soft, easily crushed rock than for hard, tough rock.

Reduction ratio, based on max sizes, in open-circuit crushing averages about 2; when circuit is closed with screens, ratio may run as high as 12, but averages near 4. Circulating load with high ratios may run to 300–400%, against 50–100% with aver ratio; in general, large circulating loads mean choke crushing.

Power consumption, based on amount of new feed per h p-hr, depends upon kind of rock, size reduction, size of product, and the circulating load, if any. Averages: 0.4 ton per h p-hr to less than 0.25-in size; 0.6 ton per h p-hr to sizes between 0.25 and 0.75 in; 0.75 ton per h p-hr to sizes between 0.75 and 1 in; 1.6 ton per h p-hr, for sizes between 1 and 1.5 in, and 2.8 ton between 1.5 and 2 in. Reduction tons per h p-hr aver 1 in open-circuit and 2 in closed-circuit crushing, ranging from 0.3 for 24-in rolls to 3.5 for 78-in in open-circuit, and from 0.3 to about 5 in closed-circuit work for same size range (30 mills reporting).

Feeding. For max capac and effic, rolls must be fed at a constant rate and with stream distributed over full width of face. In FREE-FEED there is freedom of movement between particles resting in the V of the rolls prior to nipping; in CHOKE-FEED, the particles in the V are piled up to such depth that no free movement exists. In free feeding each particle is broken substantially individually, and crushing is almost continuous; in choke feeding masses of material pass through intermittently, the roll faces springing apart to permit their passage; there is much abrasion between particles, resulting in less granular product than with free feeding. Except for the largest rolls, choke feeding is practicable only with material already crushed to 0.25-in or less. Rolls are ordinarily run dry.

The lower limit of size for effic crushing is not clearly established. If a product passing 10-mesh is all that is desired, it is economical to complete the crushing in rolls. For a finer product, ball- or rod-mills are more economical. Preponderant practice (1938) is to feed ball mills with material of max size, ranging from 0.25-in to 10-mesh, and to use rolls in closed circuit with vibrating screens to produce this material. Much test work is being done with short-head cones to displace rolls in this service, because of the larger reduction ratio possible, and results of some tests are promising. At Nevada Consol (6) rod mills taking minus 0.75-in feed are reported to show an advantage of 2.5¢ per ton over rolls for making minus 10-mesh feed for ball mills. Rolls can crush to 20-mesh or

finer, but to do so must be set close and choke-fed. This is very uneconomical, due both to wear and tear and to power consumption.

Graded crushing reduces particle size by a series of crushers, each with a smaller discharge aperture than the preceding, and material fine enough to pass the next crusher is removed between the crushing steps. This minimizes production of slimes. Size reduction in the successive steps is usually small; of the order of 2 or 3, based on aver size. The alternative extreme is to break down with as big steps in reduction ratio as size and strength of the crushers permit; with no removal of fines between the crushers, except that the last one is in closed circuit with a limiting screen.

Before the application of flotation processes to base-metal milling, when minimum sliming was essential to max recovery, graded crushing was assumed necessary, apparently with little experimental evidence. But, an exhaustive investigation by N J Zinc Co, in crushing by rolls a sphalerite ore with granitic gangue from 1 in to 0.1 in max size, showed that the amount of −0.025 in size produced was the same, within a range of about 2% of the weight crushed, irrespective of number of steps or presence or absence of intermediate screening. Tests in Columbia School of Mines laboratory have shown that the sizing test of product of rolls with a given set is practically the same with a given ore, irrespective of size of feed; provided only that the rolls are free crushing, that they nip the particles, and that the feed contains no undersize. The significance of the last restriction lies in the fact that, if different feeds contain different amount of undersize, these will affect the screen tests of products, even though they pass through the rolls without breaking. These facts seem to establish definitely that, in free crushing in rolls, there is no advantage in graded crushing and intermediate screening.

Character of roll product. Two cases arise: (*A*) rolls are set with a definite distance between faces; (*B*) faces are set close. In case (*A*) the feed is generally more than 75% coarser than the set (aver of 19 random cases, 83%); in the cases investigated, the product ranged from 4% coarser than the set to 78% coarser (aver 45%) which gives 45% aver reduction in percentage of material coarser than the set. In the same operations, the percentage of material finer than half the roll setting averaged 30, and ranged from 2 to 66% when there was an aver of less than 5% of such material in the feed. In case (*B*) the aver reduction in max particle is close to one-half, this aver applying as well where max feed size is 20 mm as for 1.5 mm. Aver percentage of material in the product smaller than half the max particle is about 60; range, 30–80; smaller than half the set, aver 20% in open-circuit crushing and 30% in closed-circuit. In the cases investigated this aver represented an increase in such material over that present in the feed of about 2.5 times. Applying these generalizations to specific problems: (*a*) For a feed containing 75% of +1-in material to be crushed in rolls set 1 in, the aver product would contain 41% +1-in and 30% −0.5 in. (*b*) For a feed containing 5% of +20-mm, to be crushed in rolls set close, the aver product would contain about 5% of +10-mm and about 60% of −5 mm size.

Applicability. Rolls are the most widely used intermediate crushers for feeds smaller than 1.5 in, and delivering products down to 0.1 in. In such service they have large capac, low power consumption and relatively low repair costs. They are rugged, reliable, simple in construction and easy to repair.

Cost of roll crushing. The elements are power, labor, repairs and lubrication. For power consumption see Table 5. One man can attend to 3–12 sets of rolls; aver in 20 plants was 6, where roll tender had no other duties. Repairs may be estimated at about twice the cost of shells. Consumption of lubricant is 2–30 lb per 24 hr. On these bases, cost of crushing to −0.25-in should not exceed 7¢ per ton in small rolls (36-in or smaller), nor 4.5¢ in large. Coarse crushing costs considerably less, due to smaller power consumption and labor cost.

8. GRAVITY STAMPS (24)

Gravity stamps are now used only in mills in which they are already installed and their discard and replacement are not economically justified on the basis of expected life of the mine, or in very small gold mills where plant capac is insufficient to justify installation of a 4-ft diam ball mill. In large mills, stamps are now run as an intermediate crusher, with open front or 0.75 to 1-in screen; in small mills, as intermediate and final crusher, with fine screen and primary-crusher product as feed.

Weight. Stamps are rated on the wt of the falling part: in American mills, 1 250–1 500 lb; in South Africa, 1 500–2 000 lb are more usual; old California practice, 850–1 050 lb, many of which are still found.

Life of parts. Dies and shoes are of chilled C I, semi-steel, forged steel, chrome steel and manganese steel. Consumption of C-I dies ranges from 0.15 to 0.30 lb per ton crushed; for

chrome-steel dies, 0.10–0.20. Corresponding consumption of shoes is 2 to 3 times as great. STEMS are of hammered iron or mild steel, turned and polished, and tapered both ends to make them reversible when broken. Amount of breakage depends upon length, position and condition of guides, and wt of tappet. Broken ends may be turned down and the stem again used, if not too short. Annealing before turning down defers subsequent breakage. SCREENS are subject to considerable wear; their life depends upon hardness of ore, acidity of water, the screen material, type of screen, and kind of perforation. Discharge through screen depends upon kind of perforation, and percentage and size of opening. Present practice, where large capac with coarse discharge is sought, is to use woven wire, which has a high percentage of opening. Brass, copper, bronze and steel wire are common. Steel wears best, but fails quickly in acid water. Screens for fine-crushing stamps are usually renewed before failure, due to increased size of aperture from wear; hence, for fine crushing, heavy plate and coarse wire are not very economical, especially because they reduce the area of opening, and therefore the capac; they are also more subject to clogging. Capac requirements dictate screens of moderate wt, even if more frequent replacement becomes necessary. Percentage of opening varies but little for medium-weight wire-cloth screens, irrespective of aperture. It is greater for fine cloth than for fine punched plate, but the latter is stronger. Life is extremely variable, ranging from 2 or 3 days to perhaps 2 weeks for fine screens, and 2 weeks to 2 months for coarse.

Height of drop is variable within small limits with a given cam, by changing position of tappet on the stem. Amount of variation is small if the tappet is to be picked up at the point on face of cam for which it was designed. Height of drop is usually 6 to 8 in; it increases as dies and shoes wear; it is kept as nearly constant as possible by changing position of tappet on the stem to compensate wear.

Drop sequence. Cams are spaced equally on cam shaft, with their arms 36° or 72° apart, depending upon whether the shaft carries 10 or 5 cams. Sequence of drop in any one mortar has marked effect on the work done. Rules governing sequence: (*a*) no two adjacent stamps should fall in succession; (*b*) when one stamp is falling its neighbor should be rising.

Common sequences aimed to satisfy these rules are: HOMESTAKE, 1, 3, 5, 2, 4, which, stated backward, is 1, 4, 2, 5, 3; CALIFORNIA, 1, 4, 2, 3, 5 = 1, 5, 2, 4, 3; and modifications of the latter, as 1, 5, 3, 4, 2 and 1, 5, 3, 2, 4. The Homestake sequence comes nearest to satisfying theoretical requirements, but many operators hold that the California distributes the pulp better on the dies, and its swash of pulp in the mortar best facilitates discharge through the screen. When a 10-stamp battery is used, the sequence 1, 3, 5, 2, 4 becomes 1, 7, 3, 9, 5, 2, 8, 4, 10, 6 and 1, 5, 2, 4, 3 becomes 1, 6, 5, 10, 2, 7, 4, 9, 3, 8.

Height of discharge is the vert distance from top of die to top of lower rail of screen frame. It increases as the dies wear. To keep it nearly constant, as should be done, CHUCK BLOCKS of different heights are used, which vary height from die to bottom of screen opening; for closer regulation, SLATS 1–1.5 in thick are placed between bottom of screen and top of chuck block. Height of discharge may also be varied by a false bottom under the dies, but this has an unfavorable effect on character of impact of the blow. High discharge tends to produce slime. If this is undesirable, a fine product can be made by lowering the discharge, and using a finer screen.

Duty is tonnage crushed per stamp per 24 hr. Representative figures: 1.8 ton for 750-lb stamp and 0.022-in screen; 21.1 ton for 1 550-lb stamp and 0.2–0.28-in screen; 30 ton for 2 000-lb stamp and 3/8-in screen.

Conditions affecting duty: character of ore, size of feed and product, wt of stamps, drops per min, height of drop, drop sequence, shape of mortar, mortar foundations, and condition of shoes and dies. Increase in capac with increase of FALLING WEIGHT is substantially uniform. Tests on a 1 050-lb stamp indicate max duty with FEED SIZE between 1 and 1.5 in; but, while this is true in general, the max point is not definitely fixed. Several plants using coarse battery screens and finishing in tube-mills report an increase in tonnage per h p-hr due to placing a by-pass screen ahead of the battery, with the same aperture as the battery screen. Clark (11) says a 900-lb stamp can handle 3-in feed, and that no gain follows decreasing size below 2 in.

Speed. Aver is close to 100 6 to 8-in drops per min with heavy stamps and somewhat longer drop with light stamps.

Power required varies with wt of stamp, height of drop, and number of drops per min. Theoretical power per stamp may be calculated by formula: $Hp = W H N \div (12 \times 33\,000)$; where W = wt of stamp, lb; H = height of drop, in; N = number of drops per stamp per min. Total theoretical power for a battery is the above figure × number of stamps. Actual power exceeds the theoretical by 16–70%. An allowance of 25–30% excess is safe for estimating. Aver tonnage per hp-hr is 0.074, with screen aperture finer than 0.05-in; 0.138 for apertures from 0.05 to 0.25; 0.25; 0.164 for apertures coarser than 0.25-in. Truscott states (12) that So African stamp-milling averages 0.05 ton per hp-hr,

from −2-in through 30- to 40-mesh screen, 0.1 ton through 12- or 16-mesh, and 0.2 ton, through 3- or 4-mesh.

Water. Current practice is confined to wet crushing. Reports from 6 mills show moisture in pulp discharged through the screen to range from 75–94%. Aver Rand practice is about 85%; range 50–90%. Quantity of water per ton is less with coarse screens than with fine, and decreases with height of discharge.

Feeding is always automatic, to secure proper regulation. Irregular feed decreases capac, and increases breakage of stems and cam-shafts. Challenge feeder is the commonest and most satisfactory, especially on wet and sticky ores.

Lost time is due to breakage of stems, shoes, cams and cam shafts; dropping of boss heads or shoes; slipping of tappets, pulleys, or cam-shaft collars; renewal of shoes, dies and screens; adjustments of height of discharge and of drop; and dressing or cleaning up amalgamating plates. The latter cause is not directly chargeable to crushing. The other losses range from 1 to 11%.

Cost of crushing by gravity stamps is from about 15 to 50¢ per ton, depending upon size of product discharged.

Fine crushing is done almost exclusively in ball-, tube- and rod-mills (see Taggart, Handbook of Mineral Dressing; also, Sec 33).

9. HAND SORTING (24)

General. Sorting or HAND-PICKING is the manual removal of selected grades from broken ore. The picked material is usually high-grade or SHIPPING ORE, or WASTE, or both. Due to its coarseness, sorted ore may be worth more per ton than mill concentrate; and, being eliminated thus early, is not subject to danger of loss in mill treatment.

Advantages of sorting are increased output from a given mine and mill equipment, and reduced wear on milling plant. On the other hand is the question of allowable production. Even if the increased mill capac is not utilized, effic may be increased, due to the reduced load on the mill. Shipping ore and waste are often picked simultaneously, the residue being MILLING ORE. High-grade complex ores may be separated into several classes; as many as 16 have been made at Clausthal (14). Such close work requires breaking with hammers, besides actual sorting. Breaking with heavy long-handle hammers is called SLEDGING; further breaking with light long-handle hammers, SPALLING; final breaking with light short-handle chisel-peen hammers, COBBING. Due to high wages, these operations are rarely practiced in U S, except at prospects and very small mines, but they are common in many foreign countries.

When the ore-treatment process is chemical, sorting serves to remove deleterious substances, that consume chemicals, hinder settling and filtration, adsorb and carry valuable solutes into tailing, etc.

Apart from necessity for removing refuse from mill feed, and deleterious substances from feed to chemical processes, the decision as to advisability and extent of sorting is purely economic. The cheaper the labor, and the more ineffic and expensive the mechanical treatment, the farther can sorting be carried and *vice versa*. Undoubtedly sorting could be introduced advantageously at many plants, but it is equally true that it is sometimes practiced where economy demands its discard or curtailment. For investigation of economics, see Art 11.

Sorting of some kind is a part of every mining and ore-treatment operation. In narrow orebodies with distinct walls, much country rock unavoidably broken is sorted out underground and may be used for filling. In mines containing segregations of pure valuable mineral, as in some Lake Superior native-copper deposits, the valuable mineral is picked out underground and sent to the surface. But underground sorting is usually uneconomic, because of restricted working places, poor light, poor presentation of material and obscuring effect of the fines present. Some sorting to remove wood, rope ends, powder and tramp steel is done ahead of the primary crusher in practically all mills; but this work is incidental to crushing, not to hand sorting proper.

If valuable mineral occurs in coarse aggregates, or if considerable waste is mined with the ore, and ore and waste are readily distinguishable by eye, the economics of sorting should always be investigated.

10. SORTING SURFACES AND OPERATIONS

Sorting is done on floors, stationary tables and grizzlies, and various kinds of moving surfaces, as revolving tables, pan or belt conveyers (Sec 27), shaking feeders, shaking screens or grizzlies (Sec 34, 35). In modern U S practice and on the Rand, material fed

to sorting surfaces is prepared mechanically, with little or no breaking during sorting, but in European and some Latin-American mills much spalling and cobbing is done.

Sorting floors are used where labor is cheap, or spalling and cobbing are practiced. In its crudest form a "floor" is a level surface that can be thoroughly swept, on which ore is dumped and picked. Contract work is customary. Each contractor is assigned a certain space; delivery of ore and collection of products being made by the company. Sorting floors have reached their highest development at some of the Rand gold mines. DISADVANTAGES: all material must be moved manually, and sorters work in a stooping, tiring position. ADVANTAGE is thorough inspection, because every piece of material must be turned over and pickers are not unduly hurried.

Tables for sorting permit pickers to sit or stand comfortably. They remove whichever separable component of the feed is present in smallest bulk, and drop it below them into proper receptacles, finally scraping the residue through an opening in the table surface.

Fixed chutes and grizzlies for sorting are of the usual types, with the limitations that their slope must be near the sliding angle of the ore (15°–25°), and width must not exceed that which is readily inspected and worked (about 24–30 in when worked from one side and 48 in when worked from both sides). If slope is less than sliding angle, material is moved along with rake or hoe; if greater, flow is stopped as desired by a board, hoe or shovel inserted into stream. These devices are not used when much of the total material is to be separated, nor for close sorting. They are chiefly useful when rope, wood, powder and tramp steel are being removed from the primary crusher feed to obviate mill trouble. A grizzly expedites selection, because of removal of fines; but, if particles are tabular or wedge-shaped, grizzlies clog badly at the low speeds at which the material passes and it is therefore difficult to control movement of material.

Moving surfaces for sorting (see beginning of this Art) eliminate manual handling of reject material. But, as this reject is not turned over by or for the picker, material that should be removed is overlooked; also, all material passes at a uniform rate, irrespective of the components that should be removed, with the result that pickers are sometimes unduly hurried and sometimes underworked, if aver speed of travel is right. Nevertheless, much hand sorting is now done on moving surfaces because of the advantage of mechanical transport of the reject.

Belt conveyers (Sec 27, 35) are the commonest picking surfaces. Width, 24–30 in for one row of pickers; 48 in for double row.

Stations for pickers are 3–6 ft apart, with a chute at each for the reject. These CHUTES are placed beside the picker or on opposite side of belt; the latter being probably best for sizes to 3 or 4 in, that can be thrown by a flick of the wrist; but pieces needing two hands are best drawn toward the picker, and it may be less tiring to draw one-hand pieces larger than 4 in to the picker's side than to throw them away. Chute mouths should be large, so that accurate throwing is not necessary, and so shaped that pieces will not bound out. SPEED of belts is 10–80 ft per min; aver, between 30 and 40 ft. The smaller the pieces and the greater the amount of reject, the slower the speed and the longer the belt. Tuttle (15) states that in coal picking (Sec 34) belts are usually 4 ft wide and run at 30–60 ft per min. For picking oversize of 1.5-in screen at 30 ton feed per hr he recommends a length of 15 ft, plus 10 ft for each 3% of waste removed. On 0.75 to 1.5-in sizes he recommends 30 ft per min travel and, for a feed rate of 20 ton per hr, 15 ft of belt for every 1.5% of material removed. For more than 4–6% impurity, washing is preferable to picking. Belt should be only slightly troughed, to prevent heaping-up in the middle. A wide, flat belt, with feed carried not nearer than 6 in to the edges, is often used. Belts are suitable for any size of feed that can be handled by pickers, but they will not stand much sledging, and wear excessively with feed coarser than 6–8 in, especially when, as should be the case, fines have been screened out.

Pan conveyer, used for coarse material, resembles the belt. Speed is usually slower, for mechanical reasons and because larger lumps are handled. Pan conveyer stands sledging and wears less than belt with large lumps. Best form is a shallow trough, with stationary sides; it permits better removal of large lumps than when articulated sides form part of the moving mechanism. Slope of belt and pan conveyers should not exceed 20°; they may be extended to elevate and convey as well as to provide sorting surface.

Revolving table is an annular iron picking surface, 16–25 ft outside diam, usually inclined toward one edge, and supported on a frame that also carries a circular track resting on wheels. It is revolved by gearing, at 20–40 ft per min. Feed enters at one point. Pickers stand or sit around the outer and inner peripheries and throw sorted material into chutes or boxes. Reject is moved by a scraper into another chute.

These tables are common in So Africa and Europe. Some are supported by ribs from a central spindle, thus preventing picking stations on the inner periphery. Another design has 2 decks, the upper being about half the width of lower and 6 in higher, and receiving the selected material; thus both reject and selected material are presented to the inspector. ADVANTAGES of revolving tables: compactness, with consequent ease of supervision and collection of products. DISADVANTAGE, as compared to rectilinear conveyers, is loss of elevation suffered by reject in passing over the table.

Shaking surfaces are widely used in collieries, but, except as primary-crusher feeders, not to any extent in metal-treatment plants. They are essentially chutes with perforate or imperforate bottoms, set on a slope of about 10° in direction of flow and shaken at 100–250 2- to 6-in throws per min by an eccentric.

With the Ferraris mode of suspension, the surface may be horiz. For careful work, these are the least satisfactory type of moving picking surface, but when screen bottoms are used, they are justified in serving the triple purpose of screens, conveyers and sorting surfaces.

Washing of feed is essential to rapid and accurate sorting. It is usually done in the screens removing undersize, but may be done by hose or sprays on floors or moving surfaces. Sprays on a troughed inclined belt, just above the feed point, will wash fines down the incline and over the tail pulley, where they can be collected.

Table 7. Performances in Hand Sorting at Different Mills

Plant	Kind of picking surface	Width, in	Length, ft	Slope, in per ft	Speed, ft per min	Size picked, in
Copper Range, Mich (*a*)	Sloping chute	.....	......	6	.....	6–12
Phelps Dodge Co, Morenci, Ariz (*b*)	Belt	36	......	2 3/8	27	0.75–3
Butte and Superior, Mont (*c*)	Pan conveyer	.....	......	3 7/8	30	3–12
Morning Mine, Idaho (*d*)	Belt	36	88	1.25	43	1– 6
Witherbee Sherman Co, N Y (*e*)	Flight conveyer	.....	......	0	20	4–16
Elko Prince, Nev (*f*)	Belt	30	8	.....	20	2– 7
Tonopah-Belmont, Nev (*f*)	Steel belt	.....	......	1.25	45	2– 9
United Eastern, Nev (*f*)	Pan conveyer	42	......	0	1.5	......
N J Zinc Co, Franklin, N J (*f*)	Revolving table	.....	......	0	30	3–18
" " " " " (*g*)	Belt	.....	......	2 7/8	122	......
" " " Ogdensburg, N J (*f*)	Revolving table	.....	......	0	28.5	2.5–24

Plant	No of pickers	Spacing of pickers, ft	Distance picked material thrown, ft	Ton per man per hr	% of total feed removed	Kind of ore
Copper Range, Mich (*a*)	1	......	......	0.055	0.001	Copper
Phelps Dodge Co, Morenci, Ariz (*b*)	4	6	6	0.33	0.7	"
Butte and Superior, Mont (*c*)	1	......	1	0.62	1.0	Zinc
Morning Mine, Idaho (*d*)	10	4	1.5	0.34	20	Lead
Witherbee Sherman Co, N Y (*e*)	7	3	2	5	20	Iron
Elko Prince, Nev (*f*)	1	......	1.5	0.16–0.25	10	Gold-silver
Tonopah-Belmont, Nev (*f*)	7	4	1	1.5	15	"
United Eastern, Nev (*f*)	1	......	5	0.75	1.75–2.25	"
N J Zinc Co, Franklin, N J (*f*)	6	8	2	1.5	4	Zinc
" " " " " (*g*)	1	......	2	......	......	"
" " " Ogdensburg, N J (*f*)	4	8	2	1.25	10	"

(*a*) Native copper. (*b*) Smelting ore. (*c*) Waste, wood and steel. (*d*) Rich ore and waste. (*e*) Lump ore and waste. (*f*) Waste. (*g*) Refuse (wood, steel).

At Britannia M & S Co (16) the ore is cupriferous pyrite in schist, carrying about 2.7% Cu, 8% Fe, 1.5% Zn, 6% S, 70% SiO_2, 25¢ Ag and trace of gold. Sizes from 1.5 to 3.5-in are picked on a belt; 4 men per shift on 2 belts pick shipping ore (and hard country rock for tube-mill pebbles) from the oversize of 600 ton per 24 hr. Shipping ore, assaying 10–18% Cu, comprises about 10% of total product, which includes concentrate from jigs, tables and flotation. Handy states (17) that the labor in a typical Coeur d'Alene sorting plant, handling 800 ton per day of 1.5 to 4-in size, yielding 50 ton shipping ore and 150 ton waste, consists of 20 sorters, with 5 bosses and repair men. Normal cost is 16¢ per ton of run-of-mine, or 65¢ per ton sorted out. Louis (18) gives data on coal picking on belts in England. At 2.1–17.5% picked out, each worker handled 0.03–0.35 ton per hr, the highest tonnage corresponding to largest percentage removed, adherence to this rule being general, though not exact. Huntoon (19) gives the cost at Tonopah-Belmont, 1911, of sorting 15–20% of waste on belts as 68¢ per ton of waste, and at Tonopah Mining Co, 80¢ per ton removed, when this comprises 11% of the feed.

Amount removed by sorting varies according to ore treated. With native copper ores of Lake Superior, the amount at Copper Range is as small as 0.001%. At Federal Mining and Smelting Co lead mines and at Witherbee Sherman magnetite mines, 20% total high-grade ore and waste are removed. At Alaska Juneau (20), 7.73% of total mill feed was picked as milling ore assaying $5.06, and 34.64% (the remainder of 2-in trommel oversize) rejected as 7-cent waste. On the Rand (21), material from 8 in to 1.75 in constitutes 50–70% of total hoisted. Waste picked out forms 10–30% (aver about

16%) of total hoisted, being about 50% of total waste hoisted. In IDAHO mills, treating coarsely disseminated lead ores, shipping ore picked out ranges to 60% of total concentrate produced.

Labor for sorting is generally unfit for heavier work: boys, girls, women, or old or crippled men. Boys and girls are the quicker and, if properly supervised, do best work.

Size of material sorted usually ranges from 2.5 to 12 in.

Smelting ore as fine as 0.75 in is picked at Morenci plant of PHELPS DODGE Co, and as coarse as 24 in at Ogdensburg plant of N J ZINC Co. The number of movements required to make tonnage on small sizes is so great that the pickers' capac is low, and difficulty in handling and judging 24-in lumps is likely to retard work below the rate on intermediate sizes. Wiard states (22) that best size for sorting is between 1 and 3 in; Richards (23), that max rate is on 3 to 4-in lumps; but Table 7 shows that the max per man-hr corresponds to feed averaging 6-12 in. Comparison between FEDERAL M & S Co and WITHERBEE SHERMAN MINES is especially instructive, since at both mines 20% of the feed is picked as shipping ore and waste, and the number of pickers indicates sorting to be the sole responsibility of the workers. At Witherbee Sherman mine 5 ton per man-hr of 4 to 16-in material is picked, against 0.34 ton per man-hr of 1 to 6-in material at Federal M & S plant.

Lighting. Daylight is best; but, as picking must usually proceed on all shifts, artificial light is necessary. Since luster and color are the chief guides in sorting, light should be good and steady, and placed to keep shadows off the material picked. Diffused or flood lighting is preferred, but incandescent lights, directly over the feed and shaded from pickers' eyes, are sometimes used. Experiment is the guide. Ore moist from recent washing is probably in best condition for quick and ready selection.

11. ECONOMICS OF SORTING (24)

Following formulas are useful for determining monetary saving to be expected from sorting, roughing or other treatment, in which a finished product, either concentrate or tailing, is to be removed in advance of the place for its removal in the treatment scheme taken as standard.

Removal of concentrate. When the material to be eliminated is finished concentrate: let F and f = wt in tons and assay of original feed; P and p = wt in tons and assay of concentrate to be produced by proposed operation; M and m = wt in tons and assay of residue from proposed operation; T = tons of final mill tailing, without the new operation: T' = tons of final mill tailing to be produced from M tons of residue from new operation, C = tons of mill concentrate under normal operation; C' = tons of concentrate to be produced from M tons of residue from new operation; c and t = assays of C, C' and T, T'; V = value in dollars per unit of metal in concentrate to be produced by new operation; V' = value in dollars per unit of metal in mill concentrate produced by either operation; R = cost of new operation in dollars per ton of concentrate (P) produced thereby; S = cost in dollars per ton of milling H tons of original ore; S' = cost in dollars per ton of milling M tons of residue from new operation.

The assumption that assays of tailing and concentrate from original and sorted ore would be the same is justified by experience; which means that relatively small changes in tonnage or assay of mill feed have little effect on assays of mill products. Values assigned to V and V' should be net, and take full account of penalties, freight and smelting charges.

By the proposed operation, the net return from P tons of concentrate produced is its value, less cost of production $= PpV - PR$. Return from milling M tons of residue $= C'cV' - MS'$, and total return $= P(pV - R) + C'cV' - MS'$.

If this operation is omitted and the total feed undergoes same treatment that M is subjected to above, the return $= CcV' - HS$. Saving in dollars (or loss, if sign is negative) to be expected from adopting the proposed operation $= [P(pV - R) + C'cV' - MS'] - (CcV' - HS)$.

$$\text{Saving per ton on original feed} = \frac{P}{H}(pV - R) - cV'\left(\frac{C}{H} - \frac{C'}{H}\right) - \frac{M}{H}S' + S \qquad (1)$$

But
$$P \div H = (h - m) \div (p - m); \quad C \div H = (h - t) \div (c - t);$$
$$M \div H = (p - h) \div (p - m); \quad C' = M(m - t) \div (c - t).$$

Then: $\dfrac{C'}{H} = \dfrac{M}{H}\left(\dfrac{m - t}{c - t}\right) = \left(\dfrac{p - h}{p - m}\right)\left(\dfrac{m - t}{c - t}\right)$; whence, by substitution of above values in equation (1), the saving per ton of original ore to be expected from the proposed process

$$= \frac{(h - m)(pV - R) - (p - h)S'}{p - m} - cV'\left[\frac{h - t}{c - t} - \frac{(p - h)(m - t)}{(p - m)(c - t)}\right] + S \qquad (2)$$

Removal of waste. If, instead of concentrate, W tons of waste assaying w are discarded, at cost of R dollars per ton; then, when the new operation is employed, the loss due to discarding waste is $= WwV' + WR$, and the return from milling the remainder $= C'cV - MS$. Net return $= C'cV' - MS - W(wV' + R)$. If the proposed operation is omitted, the net return $= CcV' - HS$; whence, saving (loss, if of negative sign) to be expected from installing the new operation $= C''cV' - MS' - W(wV' + R) - CcV' + HS$.

$$\text{Saving per ton of original ore} = S - cV'\left(\frac{C}{H} - \frac{C'}{H}\right) - \frac{M}{\mathrm{H}}S' - \frac{W}{\mathrm{H}}(wV' + R) \quad (3)$$

Substituting assay values, as above, the saving per ton of original ore

$$= S - cV'\left[\frac{h-t}{c-t} - \frac{(m-t)\,(h-w)}{(c-t)\,(m-w)}\right] - \frac{(h-w)\,S' + (m-h)\,(wV' + R)}{m-w} \quad (4)$$

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SECTION 29

ORE SAMPLING

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

ORE SAMPLING

Definition. The correct sampling of a lot of ore is the process of obtaining from it a smaller quantity which contains unchanged percentages of all the constituents of that lot. The commercial object is accomplished when the ultimate sample meets these conditions within an allowable limit of error, has been obtained at a reasonable speed and cost, and is of the bulk, dryness and fineness required for the chemical or other determination of one or more of its constituents. This commercial sample needs not be exact. As there are limits to the accuracy of weighing and determining the constituents it is necessary only that the error in sampling be smaller than the error in assaying.

1. CONDITIONS CONTROLLING ORE-SAMPLING PRACTICE

Segregation of minerals. Constituents of an ore are rarely distributed uniformly, but are more or less segregated. This is especially true of gold and silver ores. At the mine most sulphides and other useful metals break finer than the gangue, and every subsequent handling (in transport, crushing or screening) increases segregation of the finer and richer particles. The ore to be sampled is therefore an imperfect mixture of fine and coarse particles, from 200 mesh or finer to pieces of several inches diam. A perfectly uniform mixture of different-sized particles is impossible, and re-shoveling or re-coning merely decreases the degree of segregation; while most hand-sampling methods, and the inclined chute, barrel-mixer and knee-joint chute, increase segregation. Accuracy of a sampling process depends upon whether it takes from a lot of ore the same proportion of all its different-sized particles. Any process that selects the finer or the coarser particles is incorrect. The success of the older sampling systems depends not so much upon intimate mixing as upon the uniformity of distribution of different-sized particles around the axis of a truncated cone. Later systems use mechanical means for taking frequent small samples from a stream of ore; this is called FRACTIONAL-SAMPLING, since the final sample is composed of many partially mixed portions, any one of which can not seriously affect the value of the sample, but which in aggregate approach the true average and are within the assaying error. (Sec 25, Art 4–10 and Sec 30, Art 3.)

Table 1. Smallest Permissible Wt of Sample for Different Crushing Sizes of Gold Ores

Size, diam, in	Lb
2	10 000
1 1/2	5 000
1	2 000
3/4	1 000
1/2	400
3/8	300
1/4	200
3/16	100
1/8	75
6 mesh	50
10 "	25
18 "	10
30 "	4
50 "	1

Degree of crushing is sufficient when the size of the largest particle does not exceed that permissible for the weight of sample obtained. Insufficient crushing, especially in the later steps of the process, probably causes most of the persistent unintentional errors in sampling. Table 1 shows relative sizes and weights adopted in the best sampling plants. For its mathematical derivation see Bib (2).

Psychology of sampling relates to the personal equation of the sampler, who, to protect his employer, may overlook possible slight causes of salting (Sec 30, Art 9), permits advantage to be taken of segregation, or jockies with assays. It is responsible for use of devices similar to knee-joint and inclined chutes (Art 5), under conditions causing irregularities and inaccuracies evident only to the expert. Improved methods and customs lessen its influence, but its existence must be recognised in any discussion of sampling.

2. REMARKS ON SAMPLING METHODS

General. Ore is sampled by hand or mechanical methods. HAND METHODS, the oldest and simplest, require merely shovel and wheelbarrow. They are applicable to all ores; and are preferable, usually necessary, with wet or sticky ores. In MECHANICAL METHODS, sta-

tionary or moving devices in the stream of ore automatically divert the desired proportion for the sample. Though first cost and maintenance of a mechanical plant are important items, its operation may be continuous and the preliminary sample obtained a few minutes after unloading; mixing the ore is less important and probably unnecessary; and, when correctly constructed and operated, liability to error is much less than with hand methods. Although a mechanical plant may be so constructed that a correct sample is impossible, thorough inspection always discloses any such condition.

Mechanical devices are of 2 kinds; stationary, which continuously divert certain portions of the stream for the sample; moving, which periodically divert the entire stream for the sample. They respectively take part of the stream all the time, and all the stream part of the time. Theoretically and actually, stationary are not as dependable as moving devices. Ore does not flow uniformly, but is composed of many streams of varying proportions of fine and coarse. Thus, corrugated roll shells permit passage of coarse particles at certain points, and in an inclined or angled chute, or barrel-mixer, the finer will seek the lower points. Therefore one or more compartments of a stationary device may steadily receive an undue proportion of coarse or fine. With properly constructed moving devices, passing across the entire stream, the manner of delivery is unimportant, even if the ore was previously sized, because the sample spout is under all sections of the stream for equal periods of revolution or oscillation.

Preliminary and final sample. Whether sampling is continuous, from first crushing to final grinding, or is a series of unrelated processes, the operation of obtaining the preliminary sample from wet original ore is distinct from that of obtaining the final sample for assay. The preliminary sample may be taken by either hand or mechanical methods.

3. PRELIMINARY SAMPLE: HAND METHODS

Grab and pipe sampling are the simplest hand methods, being similar to those used in mine examinations (Sec 25). They may be used in sampling plants on low-grade concentrates or for rough checking.

Coning and quartering is applicable to all kinds and grades of ores, needs no expensive plant, and keeps the sample constantly in sight (for details, see Sec 25, Art 4). Chief objections are large labor cost and ever-present opportunities for utilizing segregation to obtain a high or low sample. Even with a perfect cone particles graduate in size, from finest at the apex and around the axis to coarsest at the bottom, the sizing progressing with height of cone. When a cone is built by shoveling, even from rough piles of ore, each shovelful is in itself a rough cone or pyramid with the coarsest particles near the heel and in position to be thrown on the pile where desired. When shoveling is done from 2 or more adjacent cones, opportunities for selection are greatly increased. The apex of the rising cone is frequently shifted from the true center. If flattening is done by dragging ore with the shovel from center to outside, the finest is left as a top dressing and an occasional deeper drag places more fines in any selected sector. If flattening is done by a churning motion of the shovel, fines will work to the top, leaving the coarsest nearer the floor. As each movement of the shovel is a distinct operation, such inaccurate work is difficult to detect, and a watcher hesitates about complaining. In QUARTERING, lines may be drawn at one side of center, or so as to include fines previously placed in a certain sector. In shoveling out the reject, the face of the sample quarter will rest at an angle to the vertical, causing loss of the upper wedge of fines, and a gain of an approx equal amount of the lower coarse. To avoid inaccurate work, it may be requested, after quartering, that the removed quarters be used for the sample. Use of the iron or wooden cross lessens some of the errors during quartering, but does not insure accuracy elsewhere. BENCH or COBB SYSTEM is an excellent modification of the usual practice, lessening segregation and overcoming many inaccuracies of the single large cone. It consists of a series of cones of 50 to 100 lb each, each built on the flattened mass of the preceding cone: quartering is done only after the final cone has been flattened.

Fractional-shoveling is applicable to all ores, but is principally used as incidental to unloading ore at destination. During unloading every 2nd to 10th shovelful or any proportion desired, is thrown into a barrow or pile, to be removed later to the sampling room. Thus the bulk of the ore is promptly placed in storage bins, and sampling cost is reduced in proportion to the quantity discarded. As the reject is immediately thrown with other ores, preventing any check, this sampling is of equal if not greater importance than any later work. Much of the shoveling is done in open cars, with loss of dust and fine ore. With cheap labor the count for the periodic sample shovelful may be unreliable, whether left to the shoveler or to a signal from the foreman. Variations from the 3rd to the 11th, and from

the 6th to the 29th, shovel have been noted when a 0.1 sample was intended (7). This method should be used only on low-grade and finely-crushed ores.

Fractional-shoveling is also used as part of regular sampling in some mills and smelters, though possibilities of error are greater than in coning and quartering; notably so if the ore has previously been piled by passing over an inclined chute. A workman naturally shovels ore from the point nearest receiving receptacle; hence if one barrow is placed near the back, and another near the front, of a conical pile, the sample will be high or low depending on which barrow is used for the sample. Errors during fractional-shoveling, whether from pile, bin, or railroad car, are difficult to detect and prove, as each shovelful loses its identity as soon as delivered, and the workman may change his system between 2 shovelfuls. A mode of checking is to request that the reject be used for the sample, though this will not often be granted. An effectual protection against use of such unfair methods is to crush a low-grade ore very finely, mix this with coarsely-crushed high-grade ore, and ship as one lot. This is particularly effective in fractional-shoveling, whether the sampling be done from car, rough pile, or cone, and is useful in coning and quartering and even in some mechanical mills. When the buyer realizes the situation, the practice may be reversed. Because of lack of confidence in their own methods some plants screen all ores of suspected mixed values, sampling separately different sizes, and averaging according to the two weights.

4. PRELIMINARY SAMPLE: STATIONARY MECHANICAL DEVICES

Whistle pipe. Fig 1 represents an outline of this type. *A* is the housing, with front removed to show pipe; *B*, the sampler, is a vertical pipe with 5 notched openings cut halfway through, as at *C*, each being 90° horizontally from the one above. In the notches are rectangular steel sheets at 45° to the vertical, the top edges forming diameters of the pipe. Above each notch is a cast-iron liner *D*, narrowed at bottom to collect the ore in a smaller stream before striking the dividing edge. Ore delivered through hopper *F* falls on the first partition, approx one-half being rejected through *R*, while the sample-half continues to the second partition. With 5 partitions, as in Fig 1, the sample leaving the pipe at *S* is $^1/_{32}$ of the original lot. This device is cheap to build and operate, and rapidly and simply produces a small sample. For accuracy, the whole lot must be crushed to fineness necessary for weight of sample (Sec 30, Art 3). For, if the crushing is planned for the average 100 000 lb lot, and no change is made, the safe maximum size may cause serious error on a 20 000 lb lot. The apparatus is tightly housed, so that frequent examinations are inconvenient, and, as the openings are invisible during sampling, improper working may be unnoticed. With dry, free-running, and properly-crushed ore, an accurate sample is obtainable by this device; but with sticky ore one or more openings may become clogged and cause the final sample to be 0.1 or 10 times the calculated weight. Sample and reject openings may alternately become clogged and produce a sample of proper weight, but with too little of the first and too much of the last part of the lot. It is not suited for the general run of ores, and is used in but one part of the U S.

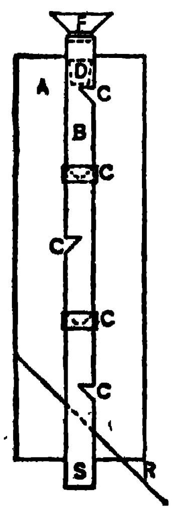

Fig 1. Whistle-pipe Sampler

Bank or combination riffles consist of 5 riffles set in a frame, the upper being placed over 2 lower ones, which are followed by 2 still lower sets. Fig 2 shows lines of flow. Ore fed to first riffle *A* is divided by vertical partitions into 20 or more streams, the odd and even-numbered falling on opposite sides of the riffle. These streams impinge on inclined iron aprons *B*, and are diverted to riffles *C*, similar to *A*. From these 2 riffles flow 4 sets of streams, each representing 0.25 of the lot. Two are rejected through *R*, and 2 diverted as above to aprons *D* and riffles *E*, and are again divided into 4 streams. Two unite and are rejected and 2, *SS*, each representing 0.125 of the lot, are united or treated as original and duplicate samples (Art 8). When the ore is slightly damp, rough cones form on the aprons. These may remain some time or be promptly dislodged; meanwhile the coarser ore is diverted to succeeding riffle divisions on each side, while the finer drops vertically. A small error on the first may be a serious accumulated error after passing 3 riffles. This danger is lessened by regular motion of the riffles across the stream; but to be effective this motion should be caused mechanically and not left to the workman's whim. Because of the tendency of the small divisions to clog, the device is applicable only to the drier and finer ores.

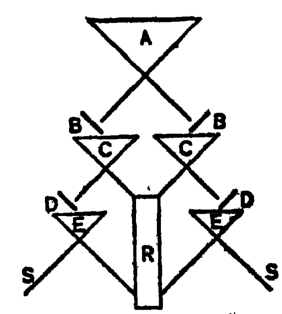

Fig 2. Bank or Combination Riffles

5. PRELIMINARY SAMPLE: MOVING MECHANICAL DEVICES

These are so constructed and operated that during 0.05 to 0.20 of a period of rotation or oscillation a division called the sample spout diverts the entire stream for the sample.

Snyder sampler (Fig 3) consists of a cast-iron plate *A*, revolving in a vertical plane on axis *B*, with an inclined sample spout *C* passing through it. As the spout passes through the stream it deflects for the sample such proportion as the arc of the spout bears to the circle of revolution. At other times the ore strikes the plate and is thrown back into the reject. These machines being of cast iron have no easily bent or twisted parts, and are accessible at all times. With one spout about 20 samples per min are taken, but this may be increased by constructing with 2 or more spouts. Sides of sample spout must be at right angles to the plate and in planes passing through center of revolution. Thus, as the edges wear, the spout will cut the same arc. With the usual inclined delivery chute any other construction will cause error. As the device revolves in a vertical plane, wet ore may stick to the plate for half a revolution and then drop into the sample spout, or from spout to the reject.

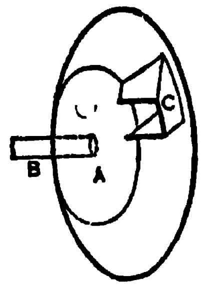

Fig 3. Snyder Sampler

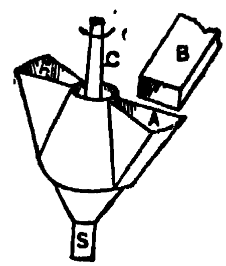

Fig 4. Vezin Sampler

Vezin sampler is used quite generally and in combination with many methods of sampling. There are many varieties, the type form being shown in Fig 4. Vertical shaft *C*, revolving with the arrow 10 to 40 rev per min, carries sampler spouts *AA* under delivery chute *B*, and diverts sample to center and through spout *S* at bottom. During remainder of revolution the ore falls into a reject receptacle.

Advantages: can be made accessible for cleaning and examination during sampling; frequency or proportion of sample is dependent on number or angular width of spouts; 2

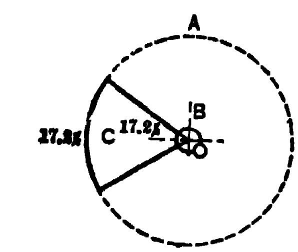

Fig 5. Vezin Sampler, Spout Edges Radial

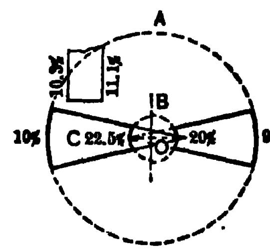

Fig 6. Vezin Sampler, Spout Edges Non-radial

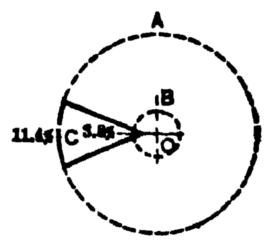

Fig 7. Vezin Sampler, Edges Non-radial

intermeshing machines may be operated for taking duplicate samples. Disadvantages: requires much head room; damp ore collects in or may fill the sample spout; the long upper edges of spouts quickly become bent, uneven, and worn. It is not patented and there are no restrictions on manufacture or use, though the inventor, H. A. Vezin, specified certain conditions as necessary for a correct sample. He insisted that sides of sample spout be in planes passing through center of shaft *C*, that feed chute should incline 58° from the horiz, with bottom edge radial to circle of revolution, and that speed of revolution should be the horiz speed of ore at spout edges, to lessen liability of particles to jump from or into sample spout. Chief cause of error is the position of sample-spout cutting edges, due to faulty construction, or wearing of non-radial sides. A variation from radial position, imperceptible with the machine in motion and proved only by careful measurements, may cause appreciable error, as illustrated in Fig 5, 6, and 7, which are horiz views of Vezin samplers in actual use. Fig 8 is from a photograph of a machine just before installation. If segregating delivery chutes are added, the error may be anything desired, as shown in Fig 9 (7). In the cuts: *B* is inside line of spout travel; *O*, center of carrying shaft; *C*, top of sample spout; *D*, bottom edge of inclined chute delivering ore to sampler; %, proportion taken for sample at points indicated. Fig 5 shows sides of spout radial, with equal arcs cut on *A* and *B*, assuring a correct sample regardless of segregation in the delivery chute. Angle between spout edges is too acute in Fig 6, 9, and too obtuse in Fig 7, 8. Percentages show

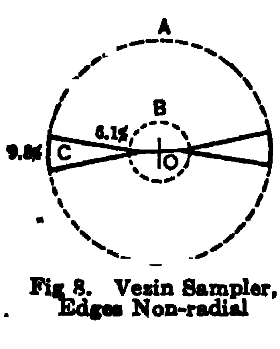

Fig 8. Vezin Sampler, Edges Non-radial

proportion taken for sample at inside and outside limits of spout, and also at extreme points on delivery chute. Even such machines might not cause commercial errors except for use of the inclined, and more especially the knee-joint chute (Fig 9). By adjusting the angle between D and D', and varying their lengths, control of distribution of coarse and fine is possible, as proved by samples taken across the chute. The chute shown in Fig 9 would probably cause a low sample in Fig 6, 9, and a high sample in Fig 7, 8.

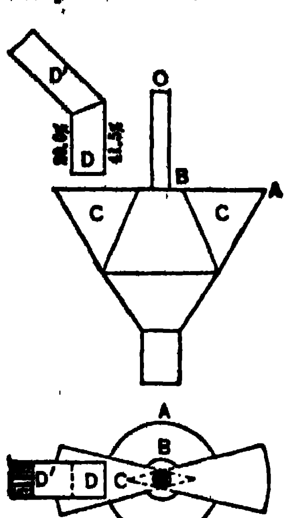

Fig 9. Knee-joint Chute, with Non-radial Vezin Sampler

There are many modifications of spout and chute that will cause this otherwise excellent machine to deliver an inaccurate sample. Previous to year 1900, machines were found with sample spouts intentionally leaky, causing loss of fines; delivery chutes dropped fines just beyond outer circumference of spouts; and sample spouts were made with parallel instead of radial edges, with ore feeding from outer circumference toward the center, and producing samples of 60% or less of the actual gold value. Since 1900, similar but more refined practices have been devised, so that it is wise always to inspect carefully every installation. The machine is generally conceded to work accurately if the spout edges are always radial and feeding chute is vert and at proper distance above the spout.

Chas. Snyder sampler is in principle similar to the Vezin, in that it revolves in a horiz plane and has spouts with radial edges (see Fig 10, lettered like Fig 6). It has a special annular, vertical delivery chute, covering an arc of 90° directly over the sample spout, and becoming narrower at top where it receives ore from elevators, rolls, or shaking trays. Within this spout are a number of short horiz iron rods, to scatter and delay the falling ore and destroy any previous segregation. The machine has 4 sample spouts, assuring continuous flow through them, instead of the intermittent flow of other devices. Errors of construction noted under Vezin sampler apply, but in less degree because of narrowness of spouts.

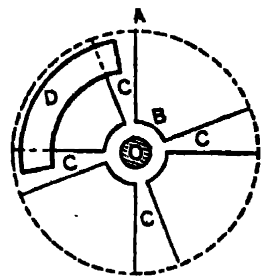

Fig 10. Chas. Snyder Sampler

Brunton vibrating sampler. Fig 11 shows front and side views. Ore is delivered through chute A, narrowing at lower end B. Operating mechanism drives arm C from positions C and C' and back again, diverting ore to the reject and sample sides R and S, respectively. Usual sample taken is 20%, but a simple adjustment permits any desired percentage. Correct feed is necessary for an accurate sample, and, as insisted upon by the inventor, D. W. Brunton, is assured by the narrowing chute or by use of shaking tray.

Possible inaccuracy, through cutting a large stream where particles are segregated, is indicated by Fig 12. As the diverting arm rests at C longer than at C', and takes an appreciable time to change positions, the reject R will receive more ore near C' than near C. Ore from rolls A, falling on inclined side of hopper B, is delivered over full area of chute D, the coarse scattering fairly uniformly while the fine slides to the opposite side. Therefore sample S might contain a fair proportion of coarse, but too much fine, and

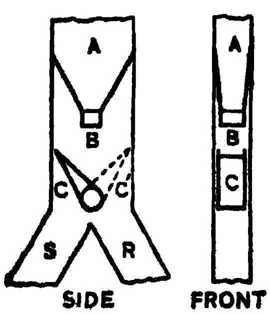

Fig 11. Brunton Vibrating Sampler

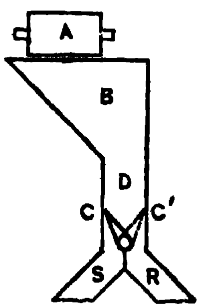

Fig 12. Vibrating Sampler (Incorrect Feed)

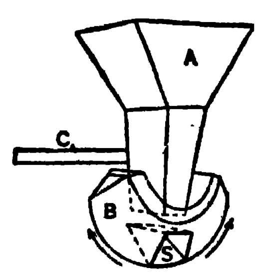

Fig 13. Brunton Oscillating Sampler

would probably be high in value. A change of angle of inclined side of chute, or a different length of portion D, might cause a high sample.

Brunton oscillating sampler (Fig 13). A is delivery hopper, and B is the moving portion suspended from shaft C, and oscillating in direction of arrows. Ore is diverted for the sample through spout S, and for the reject through oppositely-inclined spouts on both

sides of *S*. This machine is free from the undesirable features of earlier inventions. The receiving part of sample spout is rectangular, making it practically independent of segregation; cutting edges are removable and quickly replaced; it is accessible at all times; is operated at speeds (regulated by size of largest particles) sufficient to take 20 to 80 samples per min, with the greater accuracy due to small frequent samples; the rapid oscillating motion tends to free the spouts from wet ore or obstructions; its compactness permits including several machines with intervening rolls and shaking trays in a low-roof mill. Usual sample is approx 20%, though 5% may be taken with a special elliptical drive.

6. FINAL SAMPLE

When diam of largest particles is 0.125 to 0.5 in, and weight is 25 to 500 lb, the sample is taken to the bucking room for further reduction, drying, and preparation for assay.

Further reduction is made by any hand method (Art 3), by Brunton shovel, or by riffles. BRUNTON SHOVEL rapidly and accurately reduces the sample to the quantity desired. Fig 14 shows the 3-compartment shovel, with 1 sample and 2 reject divisions, preferably used only on large samples of medium-grade ores, concentrates or tailings, since, when shoveling from a cone, the center portion tends to select fines. The 7-compartment shovel, with 3 sample and 4 reject divisions, may be used with higher-grade and smaller samples, and for original and duplicate samples. RIFFLING is preferable on small quantities of high-grade ore, and may be done on the flat, Jones, or Taylor and Brunton forms of riffle. The Jones (Sec 30, Fig 1) is similar to the flat form, set at an angle of 45° to the horiz. Taylor

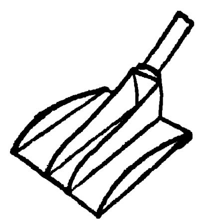

Fig 14. Brunton Shovel

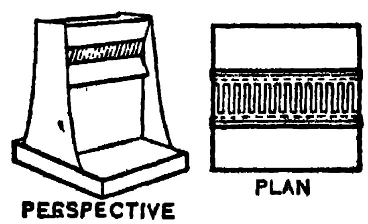

Fig 15. Taylor and Brunton Riffle

and Brunton riffle, Fig 15, resembles a double Jones, with short stiff horiz dividing edges, strongly built to withstand rough usage and to avoid the bending, leaking, and clogging of other forms. Riffling gives good results only if the device is kept in repair, the ore moved across the riffles during feeding, and partitions do not overflow.

Drying. When the sample weighs from 2 to 50 lb, but preferably not less than 20, it is dried by steam or electricity. Steam plates are steel boxes through which steam circulates, having a top drying area of 6 to 20 sq ft, on which the sample is spread. Steam coils are enclosed in a brick or steel closet, the pipes forming shelves for holding pans containing sample. Electric driers consist of resistance coils in a closet containing shelves for holding sample in pans. Temperature is important and should be slightly above the boiling point corresponding to elevation above sea-level. Thermometers are used for steam, and automatic regulating devices for electric driers. Insufficient or excessive drying causes trouble and error in assaying and valuation of an ore (Art 7).

Grinding of the dried sample to 40 to 60 mesh is done by rolls, or grinders of the Englebach cone- or Braun disk-type. The ENGLEBACH TYPE consists of a bell-shaped cone, revolving horizontally within and beneath a stationary iron ring flaring at top and bottom. The cone rests on a vertical driving shaft, and the ring is carried on a hinged frame so that parts are quickly swung open for cleaning; 8 or more channels are cut vertically in the grinding surfaces of both cone and ring. Ore is fed through top of ring and channels. Fineness of grinding is controlled by regulating closeness of contact between cone and ring by an adjusting screw and lever attached to driving shaft. BRAUN DISK-TYPE consists of a circular iron plate, revolving in a vertical plane with its face in contact with the face of a similar stationary plate, hinged at the bottom to permit separating and cleaning grinding surfaces; 6 or more feeding channels are cut in faces of both plates, beginning about 1 in from circumference and deepening at center. Ore is fed into a hopper passing through center of the stationary plate. An adjusting screw on shaft carrying the revolving plate regulates pressure between the plates and consequent fineness of grinding.

Quality of iron used in grinders is important. If hard it soon polishes and loses efficiency; if soft it may lose enough iron to the sample to lower the value appreciably, or may even take up particles of valuable metals from one sample and drop them into a succeeding sample. Imperfect parts must be promptly discarded; filling cavities with babbitt is not permissible, as metals are removed from the sample by the soft filling. The ground sample may be further reduced, if desired, by methods described above.

Screening is done through sieves varying from 80 mesh for low-grade to 200 mesh for highest-grade gold ores, average being 100 and 120 mesh (Sec 30).

Sieves should be free from missed wires and holes, and so constructed that all soldering is on the outside. Sieves with broken or misplaced wires should be promptly discarded, and not repaired with a drop of soft solder which may gather metals from the sample. While it is not desirable, it is accepted custom to use loose iron washers in the sieve to prevent balling of dried sample and to hasten screening. Some plants even use in addition a stiff paint brush. If hastening is overdone, especially near end of process, long wires of copper, gold, or silver may pass through and cause puzzling differences in assaying. Single wires of gold have been found heavy enough to cause 0.5 oz gold difference in check assays (7). With such ores, washers should be discarded and more time taken to grind the pieces of metal. Oversize from the sieve is either reground or reduced on the bucking board (Sec 30, Art 1). Quality of iron in bucking board and hammer is important (see above).

Metallic particles resisting grinding, and consisting of metals from the ore (or copper caps or iron from the machinery), are separately weighed and assayed (Sec 30, Art 3).

Mixing the ground sample, now called PULP, is done by coning, by pouring into a rapidly revolving gold pan, by turning over with a spatula, by passing through an Anaconda mixer, or by rolling (not sliding) on a sheet of heavy paper or oil cloth (Sec 25, Art 4). Dividing and sacking the pulp is done by coning and quartering, by the spatula, by a mechanical mixer and divider, or preferably by riffling (Sec 30, Art 1, 3). Paper sacks prepared for mailing are used for holding this assay pulp.

7. MOISTURE SAMPLES

Moisture samples. Ordinarily 2 to 4 samples, of 2 to 5 lb each, are taken from each carload, while high-grade lots may require 10 or more samples. As ore may gain or lose water rapidly, moisture samples are taken as near as possible to the time of weighing the lot. In case of grab samples, care is necessary to obtain proper proportions of all sizes, as fines usually contain much more water than coarse pieces.

As individual grab moisture samples may vary 25%, many must be taken to assure an average. To avoid this difficulty samples may be taken by grabbing, or with a mechanical device, during or after the sampling and when the ore is reduced to 0.25 in or smaller. As ore becomes drier during sampling, it is customary to add arbitrarily about 10% to the moisture as determined from the crushed sample, regardless of varying atmospheric humidity. While all modes of determining moisture are only rough approximations, the sample taken after crushing is preferable and is commonly used.

Weighing the wet sample requires care and judgment, and the exclusive use of either the finest or the coarsest material for final balancing while weighing should be avoided. Special scales, for weighing wet and dry samples, show percentage loss without calculation. Sample should be dried on same drier and at same temperature used for assay sample. Otherwise, because of differences in moisture or in roasting of oxidizable material, the value of the lot can not be accurately computed.

8. MULTI-SAMPLES

Check sampling. Custom plants usually retain each lot of ore intact until settlement is agreed upon. Owing to limited storage room, mills and smelters bed or store the bulk of a lot, retaining intact rarely more than 0.5 to 0.2 of it, and usually only a few hundred pounds. Local custom or contracts usually determine the amount so held, though the State of Montana provides by statute for retention of 2.5%. It is the practice in many plants to make MULTI-SAMPLES, called ORIGINAL and DUPLICATE, or even TRIPLICATE and QUADRUPLET, the division being made when sample has been reduced to 200 to 500 lb. This exposes gross errors in later work, detects salting, is a guide as to necessity for resampling, and averages errors of faulty methods.

At some plants anything less than a specified percentage difference between assays of multi-samples makes settlement on the average compulsory, while a greater difference automatically calls for resample of the retained portion. This difference varies from 5 to 20%, usually 10%. Close checking of multi-samples indicates uniformity of work in the portions so handled, but does not prove correctness of sampling of the entire lot. On the other hand, large or constant differences indicate general carelessness, or improper methods, and make possible undesirable "jockeying" in reaching a final settlement (Art 11). Many plants are so constructed and operated that the average variation in resampling of any sized lot may be less than 2%, with only an occasional 5% or greater.

9. SYNCHRONISM

Definition. When two or more mechanically-operated sampling machines are arranged in series, there is a constantly recurring cycle in their relative positions. If one horizontally-revolving machine is directly beneath another with same rotative speed, the spouts will have a constant angular difference, and with no intervention the second sample spout may

be arranged to receive all, a part or none of the first sample. At different speeds, one spout will pass the other at regular intervals and, if not obstructed, the sample from the first machine will regularly strike certain points on the second machine in any cycle desired. Table 2, containing data from a large plant, illustrates this. Upper row figures *S* show number of consecutive samples from first machine passing partly or entirely into second sample spout. Lower row figures *R* show number of consecutive samples passing entirely into the reject of second machine. These should be read, *S* 5, *R* 5, *S* 10, *R* 1, *S* 1, *R* 5, etc.

Table 2. Example of Synchronism (record representing 0.2 of complete record)

S..............	5	10	1	6	1	4	1	1	4	3	6	1
R..............	5	1	5	1	5	1	1	7	1	5	1	5
S..............	8	1	4	1	6	1	7	1	4	1	8	1
R..............	1	5	1	3	1	6	1	7	1	5	1	5

The value of any sample portion depends on at least part of it reaching the ultimate sample, its complete loss being equivalent to omitting from the shipment the portion of ore which it represents. Intervening rolls alone do not correct synchronism, but elevators, the continuous delivery of the Chas. Snyder sampler, closed hoppers, barrel-mixers, and shaking trays, lessen or prevent it. The BARREL-MIXER has the disadvantage of segregating coarse material and corrugating the following rolls. The SHAKING TRAY causes ideal delivery, besides being a generally useful device. It delays the ore sufficiently to prevent synchronism, gives uniform feed, can be made to correct uneven wear on rolls and to stop pieces of steel or hammer heads that might injure the machinery.

10. GENERAL CONSIDERATIONS

Weighing. Small lots, or high-grade ores, are weighed on special bullion scales. Carload lots are weighed on railroad scales, preferably housed and with space between platform and coping covered by old belting to prevent jambing by pieces of ore. Railroad scales should be tested monthly by special test cars of approx weight of the empty and loaded car, and daily or weekly by 40 test weights totaling 2 000 lb. Great care is necessary in checking with small test weights, as the 20-lb minimum error of the scale is equivalent to 2 000-lb error on a load of 100 000 lb.

Cars should be uncoupled and stationary when weighed, and time allowed for scale rider to swing both up and down. Rapid balancing with rider slightly over and underweight for gross and tare will cause 100 lb or more error in net weight. Balancing scales by placing fine ore or shot on rider may cause an error of several thousand lb. Wagon scales, too short for both team and wagon, must have roadway at each end level with platform, and brakes and tugs must be loose during weighing.

Incomplete lots. Portions of a lot may be received at widely different dates, necessitating the questionable practice of taking several preliminary samples to be combined later for the final sample. With mechanical methods liability to error is not serious, but with hand methods different sets of workmen taking slightly different proportions may cause errors never discoverable, owing to bedding of the bulk of the lot. A combination may be made of SAMPLING WITH ROASTER-CRUSHING PLANT (see Flow sheet No 3), though this of doubtful desirability. There is danger of loss and salting through excessive dust, and general lack of care when sampling is merely incidental to fine crushing.

Cleaning. Plants should be reasonably tight to avoid strong air currents, and ore chutes tightly enclosed to prevent the too common winnowing-out of fines. In hand methods, cleaning with brush and broom proceeds with the sampling. In mechanical mills, cleaning immediately follows the last of the ore, and should proceed from first to last crushing and sampling process, causing cleanings to pass through all following machinery to insure their proper proportion in the sample. Many mills supplement brush and broom cleaning with a blast of compressed air piped to convenient points. After the regular cleaning, rolls, and fine-grinders especially, are fed with barren rock or slag, and again cleaned before receiving the following lot. It is good and economical practice to discard cones, rings, or disks of fine grinders immediately after grinding a very high-grade ore.

Salting. Deliberate salting by addition of rich ore or barren sand is similar to salting at the mine (Sec 25, Art 9). Devices for salting at sampling works have been quite elaborate, including high-grade samples of the exact weight necessary to cause the desired salting, and provided in duplicate for emergencies or for following a lot from sampler to smelter. This practice, undetected for nearly a year, was largely responsible for the financial failure of a well-known smelting company. Of greater importance is salting by incorrect machinery, producing either a high or low sample. As a precaution the miner may employ trained men, called MOOCHERS, to watch the sampling; and the sampling company may lock sam-

pile-hoppers and erect wire screens around important machines. The most effective preventive is a correctly built mechanical mill, operated according to scientific and unvarying rules.

Confusion of samples. This is a worrying possibility in plants handling many large and small lots, of unknown and varying values, especially when they are begun and finished by different workmen.

Accompanying is a record made at the Taylor and Brunton Ore Sampling Co's plant, Murray, Utah, on a form designed primarily to prevent this confusion. It is of strong cardboard, punched for loose-leaf practice, and in 3 sections to permit prompt reports from each department. Upper margins were cut and cards filed for permanent reference. MILL CARD, signed by foreman Quist, shows that 73.28 tons passed through the mill in 2.5 hr, at cost per ton (wages at 25¢ per hr) of 5.1¢ for unloading and 0.8¢ for reloading into 2 other cars. Had the ore arrived in dump cars, costs might have been halved. Cleaning took 0.5 hr, increas-

Sampling Mill Cards (devised by T. R. Woodbridge)

6266 Date *O* MILL

Mine and Lot Number *Grand Central—55* DAY ~~NIGHT~~ } Shift

IN Cars *15367, 28118* OUT Cars *17171, 27075*

Number of IN Cars Box Dump Wooden Gonds *2* Steel Gonds

Bulk Sacked Coarse Fine Concentrates Wet Dry Frozen

What Cutters Used *4*

Number of UNLOADERS *6* LOADERS *1*

Time Unloading Started } *7:00 A.M.* Finished } *9:30 A.M.*
Time Sampling Started } Finished }

Pounds in Lot *146560* Next Lot *Wyoming, 195* Started *10:00 A.M.*

Lot before this *6262* Signed *Quist*

....

6266 *O* CUTTING ROOM

Mine and Lot Number *Grand Central—55* DAY ~~NIGHT~~ } Shift

Commenced Cutting *9:35 A.M.* Finished Cutting *10:05 A.M.*

Number Cuts before Grinding *3* After Grinding *0*

Weight Taken to Sample Room *22 lbs*

2 Moistures taken by *Davies (4.1, 4.3 = 4.2%)*

Lot before this *6262* Next Lot *Wyoming 195*

Signed *Davies*

....

6266 *O* SAMPLE ROOM

Mine and Lot Number *Grand Central—55* DAY ~~NIGHT~~ } Shift

Received (*date*) Time *10:10 A.M.* By *Spoonley*

Put in Dryer *1* Time *10:20 A.M.* By *Davies*

Put in Grinder No. *1* Time *3:00 P.M.* By *Spoonley*

Sacked (*date*) Time *4:40 P.M.* By *Spoonley*

Number of Pans in Dryer *3* Number of Times Cut before Bucking *3*

Number of Sacks put up *6* Weight of Pulp Bucked *42 oz*

Lot before this *Sand* Next Lot *Sand*

Asst *Davies* Signed *Spoonley*

Note on Reverse Side any Delay, Cause, Presence of Ore Rep, etc

ing cost 20%. (At present-day rates of wages, these costs would be from 40 to 50% higher). 4 cutters (sampling machines) delivered a theoretical wt of 183.2 lb for the preliminary sample. CUTTING CARD shows that Davies (ass't sampler) began cutting (using Brunton half-shovel) 5 min after last of ore was unloaded, and finished 5 min after clean-up. 8 shovel-cuts produced a partially-dried sample of 22 lb, instead of the theoretical 22.9 lb. 2 moisture samples, taken from the reject at this point, averaged 4.2%. SAMPLE CARD shows delivery to sample man, Spoonley, and prompt placing in steam drier, where it remained 4 hr and 40 min. It was then ground in No 1 grinder, riffled 3 times, bucked, and sacked at 4:40 P.M. Finished pulp weighed 42 oz instead of the theoretical 43.8 oz. Grinder was cleaned with barrens both before and after this lot, eliminating danger of salting from lot 6262 or from Wyoming lot 195. Cost of sampling an ore received in different classes of cars, and in varied physical conditions, is instantly available from the card; it also gives a check on efficiency of foremen and workmen. Its use generally settles disputes.

Manner of sale of ore. Local custom or private agreement determines the procedure. In outlying camps a lot may be sold on samples and prices made at the mine by buyer's agent. But ore is generally sent direct to mill or smelter, is transit-sampled on its way to smelter, or is sold to a custom plant. Valuation may be based on sample and assays made by custom plant, on assays made by buyer and seller on the custom sample, or on buyer's sample only. In last case, seller may use the custom sample as a check and a guide as to the wisdom of calling for a resample by the smelter. As this gives the seller an advantage, some buyers request or demand a report of all such checks before even considering seller's objections.

Resamples may be demanded by either buyer or seller. In custom plants, resamples of entire lots are freely granted, usually on basis of an extra sampling charge if the 2 samples check within an agreed limit varying from 0.05 oz gold on low-grade to 2% on high-grade ores. Other mills, especially hand-sampling plants, have various rules depending on exigencies of the occasion and the confidence existing in methods employed. A commoner practice is to resample freely on unchecked ore, but only in case of 10 to 20% difference, if ore has been check-sampled. Several resamples may be demanded if the earlier ones vary by a specified difference. To be of value resamples should be made of the whole lot, rather than of the small portion usually retained.

Most custom plants and some smelters make payment on the average of assays by buyer and seller on same sample. Others pay on basis of buyers' assay only, with or without knowledge of sellers figures, permitting the seller to demand a repeat, an umpire assay, or a resample, with correction in payment if the assay variation so found is sufficiently large. As the seller is unlikely to make demands should his assay be the lower, the buyer, under this arbitrary system, takes a certain risk which he may lessen by having duplicate assays made by the same or a different assayer, reporting either one or an average, as his judgment and conscience may direct. Custom varies concerning settlement assay in cases of repeats, umpires, and resamples. The last result or the "middle" assay may be selected, or the whole series averaged.

11. COMPARISON OF ASSAYS

Comparison of assays of large shipments and over long periods of time are of value, where special assays or selected resamplings are indecisive. Tables 3, 4, and 5 contain assays representative of the entire shipments from 1 mine to 3 sampling plants for approx 1 year, in lots of 30 to 150 tons. The following uniform rules for settlement existed, but were not rigidly enforced. Pulps were assayed by both seller and buyer, and in some cases by an umpire. Settlement was made on buyer's assays, when results were within limits

Table 3. Comparison of Gold Assays on Buyer's Sample

(*O* and *D* are original and duplicate samples; values in oz)

Seller		Buyer		Seller		Buyer	
O	*D*	*O*	*D*	*O*	*D*	*O*	*D*
0.98	0.92	0.87	0.86	2.65	2.67	2.62	2.62
1.67	1.87	1.66	1.78	2.16	2.18	2.12	2.16
1.27	1.11	1.06	1.24	2.32	2.38	2.30	2.32
1.51	1.51	1.46	1.46	2.71	2.77	2.66	2.66
1.22	1.21	1.18	1.18	2.40	2.40	2.35	2.35
1.58	1.58	1.52	1.54	3.28	3.28	3.24	3.24
1.24	1.24	1.19	1.19	4.48	4.46	4.34	4.38
2.70	2.78	2.65	2.64	5.80	5.77	5.72	5.77
Average of entire series				2.37	2.38	2.31	2.34

varying from 0.1 oz gold to 10% of buyer's assay. For greater differences, even after a reassay or umpire assay, a resample was made of the retained portions (Art 8). Original and duplicates were made in all plants. The tables show that seller's assays are generally higher than buyer's (Art 9).

Table 4. Comparison of Gold Assays on Buyer's Sample

(Brackets indicate resamples; *O* and *D* are original and duplicate samples; values in oz)

Seller		Buyer		Seller		Buyer		Seller		Buyer	
O	*D*	*O*	*D*	*O*	*D*	*O*	*D*	*O*	*D*	*O*	*D*
1.26	1.24	1.26	1.16	1.14	1.10	1.16	1.16	2.44	2.48	2.32	2.40
1.54	1.74	1.52	1.66	1.59	1.63	1.72	1.80	2.16	2.14	2.07	2.02
1.22	1.11	1.16	1.06	1.02	1.10	1.00	1.04	2.82	2.92	2.70	2.84
1.25	1.28	1.18	1.25	1.61	1.75	1.58	1.71	3.98	3.98	3.63	3.82
1.50	1.48	1.38	1.38	1.82	1.74	1.64	1.66	3.28	3.66	3.23	3.66
1.41	1.43	1.40	1.38	1.35	1.39	1.28	1.34	3.09	3.11	3.06	3.04
1.23	1.17	1.18	1.08	1.04	1.00	1.02	0.94	3.26	3.24	3.18	3.12
0.92	0.98	0.88	0.94	3.00	2.90	2.92	2.80	4.60	4.68	4.50	4.69
1.03	1.09	0.99	1.02	2.62	2.68	2.56	2.86	5.46	5.42	5.27	5.53
1.37	1.36	1.30	1.33	2.88	2.92	2.79	2.81	5.76	5.90	5.76	5.74
Average of entire series								2.40	2.41	2.32	2.35

Table 5. Comparison of Gold Assays on Buyer's Sample

(Brackets indicate resamples; *O* and *D* are original and duplicate samples; values in oz)

Seller		Buyer		Seller		Buyer	
O	*D*	*O*	*D*	*O*	*D*	*O*	*D*
0.96	1.26	0.90	1.26	2.44	2.44	2.46	2.32
1.06	1.14	1.04	1.14	2.28	2.60	2.20	2.62
1.18	1.18	1.10	1.12	2.39	2.40	2.40	2.46
1.04	1.02	1.16	0.96	2.80	2.72	2.64	2.64
0.86	0.74	0.84	0.68	3.02	2.58	3.04	2.60
1.32	1.50	1.32	1.50	2.80	2.18	2.76	2.24
1.24	1.58	1.28	1.60	3.33	2.71	3.32	2.76
1.05	1.27	1.08	1.28	3.52	3.22	3.47	3.18
1.40	1.33	1.38	1.40	3.11	3.13	3.04	3.04
1.25	1.25	1.20	1.20	3.08	3.34	3.04	3.28
1.29	1.33	1.26	1.24	2.79	3.03	2.84	2.88
1.54	1.58	1.54	1.40	5.34	5.40	5.26	5.34
Average of entire series				2.04	2.05	2.01	2.00

Table 6 shows assays of shipments from 1 seller to 1 buyer for 15 months, and illustrates good sampling and assaying. In Tables 3, 4, 6, resamples were made only on the retained portion of 200 to 500 lb, and were not a check on the whole lot (Art 8). Many of the sampling machines figured in preceding pages are from plants represented in the tables. Table 7 shows assays of shipments from a custom sampler to a smelter using hand methods. Averages in Tables 3, 4, 5, are of much longer series, shown here only in part.

Table 6. Comparison of Gold Assays on Buyer's Sample

(Representative of 400 assays)

(Brackets indicate resamples; *O* and *D* are original and duplicate samples; values in oz)

Seller		Buyer		Seller		Buyer		Seller		Buyer	
O	*D*	*O*	*D*	*O*	*D*	*O*	*D*	*O*	*D*	*O*	*D*
0.46	0.49	0.47	0.47	1.27	1.28	1.33	1.32	2.06	2.07	2.03	2.04
0.67	0.67	0.64	0.67	1.03	1.06	0.98	0.98	2.17	2.24	2.16	2.18
0.81	0.74	0.80	0.80	1.15	1.16	1.11	1.11	2.62	2.61	2.64	2.64
0.76	0.77	0.75	0.76	1.00	1.00	1.01	1.04	3.48	3.42	3.41	3.44
0.62	0.63	0.62	0.62	1.25	1.20	1.28	1.26	3.57	3.65	3.55	3.68
0.50	0.49	0.52	0.52	1.22	1.21	1.11	1.13	3.60	3.62	3.44	3.42
0.42	0.46	0.47	0.47	1.06	1.05	1.00	0.97	3.33	3.31	3.34	3.30
1.05	1.06	0.98	1.02	1.00	1.02	1.00	1.00	3.10	3.02	2.89	2.91
1.66	1.66	1.56	1.56	1.17	1.16	1.16	1.16	3.05	2.95	2.93	2.83
1.87	1.81	1.92	1.89	1.71	1.74	1.75	1.79	4.53	4.56	4.55	4.55
1.05	1.06	1.08	1.06	1.73	1.63	1.74	1.72	3.77	3.72	3.82	3.86
1.63	1.64	1.63	1.63	1.88	1.89	1.88	1.87	4.02	3.91	3.96	3.91
Average of entire series								1.41	1.45	1.41	1.40

Table 7. Comparison of Gold Assays on Buyer's Sample taken by Hand Methods

O, *D*, *T*, and *Q* indicate Original, Duplicate, Triplicate, and Quadruplet Sample, taken on last Quartering; values in oz

Buyer					Seller					Umpire				
O	*D*	*T*	*Q*	Aver	*O*	*D*	*T*	*Q*	Aver	*O*	*D*	*T*	*Q*	Aver
19.65	19.54	20.16	19.96	19.83	19.87	19.69	20.20	19.85	19.90	......	......	......	......	......
20.83	19.98	20.32	20.33	20.36	21.35	20.43	20.76	20.84	20.84	20.98	19.86	20.23	20.52	20.40
19.9[illegible]	19.87	20.38	19.77	19.97	20.17	20.38	20.54	19.97	20.27	19.89	19.88	20.55	19.57	19.97
18.[illegible]8	21.12	19.60	19.89	19.90	18.94	21.42	19.72	20.30	20.10	19.02	21.30	19.78	19.96	20.01
20.83	19.98	20.32	20.33	20.36	21.35	20.43	20.76	20.84	20.84	20.98	19.86	20.23	20.52	20.40
21.43	21.98	21.73	21.29	21.61	21.56	22.34	22.12	21.44	21.87	21.58	21.86	21.85	21.16	21.61
39.74	39.60	38.69	40.39	39.54	39.50	39.58	38.84	40.50	39.58	......	......	......	......	......
41.62	40.64	42.61	40.41	41.32	42.30	41.26	43.04	40.84	41.86	......	......	......	......	......
40.03	38.91	39.52	39.24	39.42	40.70	39.56	39.50	39.38	40.78	40.00	38.72	39.71	39.26	39.42
40.55	40.30	39.38	39.06	39.82	40.90	40.66	39.80	40.12	40.37	40.40	40.43	39.80	39.70	40.08
119.00	121.10	120.50	120.30	120.22	123.10	125.38	123.83	122.68	123.75	120.50	123.00	123.03	122.83	122.34
242.50	246.50	245.10	245.00	244.75	245.25	247.30	248.75	251.82	248.28	......	......	......	......	......
268.10	264.60	270.20	276.30	269.80	276.03	272.15	272.02	275.60	273.95	......	......	......	......	......
273.80	272.60	270.70	275.60	273.17	271.75	267.56	273.61	274.38	271.82	......	......	......	......	......
(*a*) 84.78	84.76	84.84	85.56	85.00	85.89	85.58	85.96	86.33	86.01	......	......	......	......	......
(*b*) 37.69	37.90	37.72	37.53	37.71	38.51	38.57	38.38	38.20	38.60	37.92	38.11	38.15	37.94	38.03

Jockeying with assays. Assays reported by the seller are usually higher than the buyer's, the relation persisting through reassays and resamples (Tables 3, 4, 5, 6). On high-grade ores larger differences occur (Table 7). Eliminating the deliberate falsifying of the mining camp telephone-system of comparison of figures, there still exists the jockeying with assays frequently found in ore-settling practice. Simplest example is the cupeling at an intentionally, but not criminally, high or low temperature, giving correspondingly low or high assays with no particular strain on assayer's conscience.

The principal and more specious excuse for jockeying is the fact that assaying of ordinary pulp is not an exact science. Due to conditions shown in Art 6 (Screening), the most conscientious and skilful assayer may be unable to weigh from the pulp two or more charges containing exactly the same amount of gold or silver. Hence it is customary to assay from 3 to 12 separate charges from each pulp, to weigh all the buttons together, and report the average assay only. But the large differences occasionally occurring between the buttons furnishes a plausible excuse for weighing them separately and discarding, in the average, such as differ excessively from the majority. This elimination may be done by the assayer, or by the settlement clerk, neither of whom may relish the responsibility. It is natural that judgments of buyer and seller will differ as to what buttons should be rejected; but even a conscientious umpire is at a loss what to report when buttons vary by several oz per ton. Possibilities for jockeying are greatly increased when 2 to 4 multi-samples, with repeats, umpires, and resamples are made, involving consideration of perhaps 160 separate buttons for one settlement.

Another opportunity for jockeying is the relation between assays and treatment charges, or the so-called "splitting" points with assays. As-

suming that an ore contains approx 1 oz gold per ton, and treatment charge is \$4 if below 1 oz, and \$5 if 1 oz or more, with gold at \$20 per oz; the buyer prefers a final average of 1.00 to 1.045, and seller anything from 0.995 to 0.955, each preferring the critical assays of 1.005 or 0.995 oz gold respectively. This is shown as follows:

1.045 oz	= \$20.90	0.995 oz	= \$19.90
Less treatment charge	5.00	Less treatment charge	4.00
	15.90 *A*		15.90 *D*
1.00 oz	= 20.00	0.955 oz	= 19.10
Less treatment	5.00	Less treatment	4.00
	15.00 *B*		\$15.10 *E*
1.005 oz	= 20.10	Seller prefers *D* nd *E*.	
Less treatment	5.00	See Sec 32 for ent charges.	
	\$15.10 *C*	Values of 0.955 and 1.045 oz (0.09 oz difference) are equal commercially.	
Buyer prefers *B*, but can stand *A*.			

The question of the splitting point on RR freight classification arises in a similar manner, especially where the ore is bought f o b at shipping point.

NOTE.—In above statement, the value of gold is taken as \$20 per oz (old standard being \$20.67). Present market price (Jan, 1940) is approx \$35, in terms of U S "money on account," a phrase used by the Treasury Dept in its quarterly circulars on Monetary Units. As this assumed value may at any time be changed by Act of Congress, the \$20 standard is retained here and in Tables 3 to 7. In using the tables, assay values can readily be adjusted to existing market price of gold. (See Table 29, Sec 45.)

12. FLOW SHEETS

Flow sheets 1, 2, and 3 are selected from a series of 55 compiled for the U S Bureau of Mines (7). The form is a modification of that used by D. W. Brunton (3). Each line represents a distinct operation. Single spacing indicates that ore passes from the operation shown in one line to that in the line below. Double spacing indicates that the flow of ore divides at that point, different streams taking courses shown by arrow points. Figures after crushers and rolls are indications of standard sizes and fineness of crushing. Percentages taken for sample are shown. At certain stages approx wt of sample is given, based on original lot weighing 100 000 lb. Discard, unless followed by lines or single spacing, indicates elimination of that portion from further sampling.

Hand-sampling Mill (Flow sheet No 1)

Railroad beside storage bins
Shovel sample

Sample 10%, 10 000 lb, left in car ← → Discard
Shovel sample

Sample 50%, 5 000 lb, left in car ← → Discard
Car taken to coning floor
Spread in ring, and coned over wooden cross inside of ring
Quartered

Sample, 50%, 2 500 lb ← → Discard
Spread in ring
Coning and quartering repeated until

Sample weighs 150 to 300 lb ← → Discard
Taken to roll room, and partly dried if necessary
Rolls, 12 by 20 in; about 20 mesh
Riffle, Jones type

Sample, 10 lb ← → Discard
Dried
Grinder, Englebach type, to 50 mesh
Riffle Jones type

Sample, about 2 lb ← → Discard
Hand sieve, 100 mesh ←

Size ← → Oversize
↓ Bucking board
Roll on glazed paper. Put in 6 sacks with spatula; excess discarded

Mechanical-type Sampling Mill (Flow sheet No 2)

Railroad car over hopper. Unloaded into hopper
Crusher, 13 by 24 in; to 2.75 in
Rolls, 16 by 42 in; to 2 in
Shaking tray to elevator No 1
Sampler No 1, Brunton oscillating
Sample 20%, 20 000 lb ← → Discard
Rolls No 2, 16 by 36 in; to 1 in
Shaking tray
Sampler No 2, Brunton oscillating
Sample 20%, 4 000 lb ← → Discard
Rolls No 3, 14 by 27 in; to 0.375 in
Shaking tray
Sampler No 4, Brunton oscillating
Sample 20%, 800 lb ← → Discard
Rolls No 4, 8 by 20 in; to 0.125 in
Shaking tray
Sampler No 5, Brunton oscillating
Sample 20%, 160 lb ← → Discard
Rolls No 5, 8 by 12 in; to 20 mesh
Locked sample car. Taken to sample room
Half-shovel, Brunton
Sample, 20 to 40 lb ← → Discard
Steam dryer
Englebach grinder, to 50 mesh
Riffle, Taylor & Brunton
Sample, 20 to 24 oz ← → Discard
Bucking board ←
Hand screen
Size ← → Oversize
Rolling cloth. All riffles into 4 sacks

Sampling and Crusher-roasting (Flow-sheet No 3)

Track beside mill. Ore shoveled to pan conveyer
Crusher, gyratory No 5; to 1.5 in
Conveyer belt
Hopper
Oxide ores ← → Sulphide ores
Revolving wire screen, 2 mesh ←
Size ← → Oversize
Trommel, 1 in holes
Size ← → Oversize
Rolls, 14 by 30 in. Trommel, 2 in holes
Size ← → Oversize
Rolls, 16 by 36 in — Crusher
Gyratory No 4
Hopper
Sampler, Vezin No 1, 2 arms
Sample 20%, 20 000 lb ← → Discard
Shaking tray
Sampler, Vezin No 2, 2 arms ← → Sampler, Vezin No 5; intermeshing
Sample 20%, 4 000 lb ← → Discard ← → Sample 12.5%, 2 500 lb (Legal reserve)
Shaking tray
Rolls, 12 by 12 in; to 0.125 in
Sampler, Vezin No 3
Sample 20%, 800 lb ← → Discard
Shaking tray
Sampler, Vezin No 4
Sample 20%, 160 lb ← → Discard

Locked sample hopper. Wheeled to bucking room
Partly dried
Coned and quartered
Sample, about 150 lb ← → Discard
Riffled, Jones type
Sample, about 10 lb ← → Discard
Dried
Grinder, Englebach type, to 50 mesh
Riffled, Jones type
Sample, about 2 lb ← → Discard
Hand sieve, 100 mesh ←
Size ← → Oversize
Bucking board
↓
Roll on glazed paper
Sack in 6 pulp samples with spatula, excess discarded

13. COSTS

Table 8 shows operating costs of customs sampling plants in Colorado, Utah, Montana, and Nevada. Costs are affected by price of labor, distance from supply markets, character of ore handled, and service rendered. The probability of receiving even a small percentage of high-grade gold ores necessitates a distinctly better mill and operative care, more time lost in cleanups, more assaying, and generally greater expense than sampling of lower-grade gold, silver, lead, and copper ores. Plants exclusively for transit sampling may or may not furnish assays to customers, and need small clerical force; whereas, if ores are also bought and sold, assaying, clerical force, administration, demurrage, and interest are large items, increasing with value and quantity of ore so interchanged, and distance from smelting points. (See also Sec 32.)

Table 8. Costs of Sampling (1938)

Plant	Period, months	Tons in period	Hr per day	Common labor, wages, dollars	Costs, cents per ton									Footnotes		
					Mill labor	Mill mach'y and supplies	Mill power, light, etc.	Total mill	Assaying	Administration, taxes, ins, etc	Totals, c plants	Buying and selling	Totals, cp plants			
A	3	22 600	10	3.75	54	11.3	2.8	68.1	6.7	12.6	87.4	7.2	94.6	*w*	*g*	*cp*
B	24	111 004	10	4.50	29	2.2	3.2	34.4	10.0	13.4	57.8	11.1	68.9	*w*	*s*	*cp*
C	3	15 433	8	4.50	53	7.1	3.0	63.1	6.2	24.0	91.3	10.8	102.1	*wt*	*g*	*cp*
D	3	35 076	9	4.25	21	3.9	2.3	27.2	0.3	11.7	39.2	....	.....	*m*	*o*	*c*
E	1	9 779	8	4.50	16	3.0	2.3	21.3	0.2	5.6	27.1	....	.....	*m*	*o*	*c*
F	3	52 899	8	5.25	24	5.4	1.6	31.0	3.3	10.9	45.2	....	.....	*m*	*o*	*c*
G	3	49 712	8	5.25	21	6.9	3.2	31.1	2.7	4.9	38.7	....	.....	*m*	*o*	*c*
H	24	199 537	8	4.50	35	6.1	1.9	43.0	5.0	10.1	58.1	8.5	66.6	*m*	*g*	*cp*
I	9	105 252	8	4.85	38	5.8	5.2	49.0	5.7	9.6	64.3	15.1	79.4	*m*	*g*	*cp*
J	3	35 094	8	4.85	35	9.0	2.5	46.5	6.0	13.0	65.5	18.4	83.9	*m*	*g*	*cp*
K	3	17 647	8	5.50	36	6.7	5 3	48.0	13.0	22.4	83.4	27.2	110.6	*wm*	{*hgs*, *hs*} *	*cp*

w = ore transferred by wheelbarrow
t = ore transferred by tram
m = ore transferred mechanically
g = gold ores exclusively
s = silver ores exclusively
h = large proportion of high-grade ore
o = general ores, mostly low grade
c = custom sampling
p = some of ore purchased
* This company had 2 plants

Table 9 shows one month's mill-labor costs, taken from plant *H* in Table 8, and for a day-shift run only. This form was designed for the Taylor and Brunton mills, copies being exchanged monthly between managers of 6 plants for comparison and suggestion. Platting the cost figures for a year's run develops interesting and valuable facts, and tends to efficiency of management.

Hand methods at smelters are usually combined with mechanical methods, and the preliminary fractional shoveling may be charged to sampiing, bedding, crushing, or some other department. Therefore comparative sampling costs are difficult and of doubtful value. Different smelters show costs of \$0.45 to \$2.25 per ton. Assuming a 100-ton lot, crushed to 0.5 in, and labor at \$0.40 per hr, the cost of fifth shoveling this to 20 tons, half-

shoveling to 10 tons, and coning and quartering to 250 lb, will be about $2 per ton, including cost of wheeling or tramming the reject a reasonable distance to bedding floor. This does not include mill supplies and machinery, making the sample, taking moisture, assaying, nor any supervision or administration. At 40¢ per hr, the single operation of half-shoveling 10 tons costs about 8¢ per ton. Were it feasible to handle so large a lot as 100 tons by coning and quartering only, labor cost would be 45 to 75¢ per ton, depending on care required for the grade of ore. In general, the cost of coning and quartering varies from 5 to 10 times the cost of mechanical sampling.

Table 9. H Mill in Table 8. Labor Costs for One Month

9 648 tons	Days, hr	Cost, $	Cents per ton	Remarks	9 648 tons	Days, hr	Cost, $	Cents per ton	Remarks
Weighing....	31	139.00	1.44	*	Clean-ups....	75	337.50	3.50	*
Foremen....	31	248.00	2.57		Repairs......	42 3	274.50	2.85	
Unloading...	190 1	855.90	8.83		Sweeping mill.	6 2	28.10	0.30	
Oiling.......	1 7	8.70	0.13		Sweeping site.	15 2	68.60	0.72	
Crusher.....	24 1	108.60	1.11		Watchman....	31	186.00	1.92	
Loading.....	95	427.60	4.42		Total......	614 2	3 055.55	31.66	30 shifts
Sample room.	71 2	373.27	3.87						

*11.5 cars (321.5 tons) per shift; 48 tons per lot. 200 clean-ups. Mill idle, 6 days, 2 hr.

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SECTION 30

ASSAYING

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

ASSAYING

Introduction. This subject is treated from the standpoint of field work, as to both equipment and methods, with particular reference to local conditions in the more remote districts. Rare minerals, and some others of low unit value, are omitted, because their assay or analysis requires special laboratory facilities. For fuller discussion of assay methods, see textbooks listed in the Bibliography.

In a remote region, the question whether to take in an assay equipment, or to send out the samples, will generally be decided by the number of samples, availability of satisfactory fuel for fire assay, and distance from an established assay office. But, in either case, to secure reliable results, adequate equipment must be provided on the ground for reduction of samples if transport of large samples to place of assay is to be avoided; and, unless individual samples are reduced to small size, their total weight may exceed that of assay equipment.

It is often important, in making mine examinations, to be able to obtain immediate results on certain samples, in order to resample in case of doubt; also, the cost of extensive sampling may be curtailed by advance information. Therefore, when the number of samples is large, or the mine is in a remote district, an assay equipment is almost imperative, notwithstanding that it is rather bulky and liable to damage in transport, and that assay work in the field is often beset with difficulties. In the absence of adequate equipment, simpler and more convenient methods of making determinations are sometimes sought, but there is no satisfactory substitute for the standard methods.

Blowpipe may be used for identification and qualitative tests (Sec 1); quantitative results require much practice and skill, and the results are unreliable. Further, fewer assays can be made per day by blowpipe than by standard methods, and the work is more tedious. An example will indicate the inherent shortcomings of the blowpipe method. Employing the usual charge of 100 mg for a 1-oz gold ore, 0.0034 mg of gold will be present. Combining results of 10 fusions gives only 0.034 mg, a quantity too small to be accurately measured without a microscope, and requiring a delicate balance for weighing. The total wt of ore used, 1 gm, is insufficient for a representative sample, and each assay would require 2 to 3 hours. The blowpipe is more applicable to high-grade silver ores and base metals. The so-called "pocket smelter" gives no more accurate results than the blowpipe.

Pan is useful for testing gold-bearing gravel, or pulverized gold ores, particularly those in which the base metals are oxidized. In experienced hands, it will give better results on spotty ores containing coarse gold than the ordinary assay on 0.5 A T (1 A T, or assay ton = 29.166 gm) charges. (See Art 8; also, Sec 25, Art 7, Sec 31, Art 5.) The pan is useless where the gold is combined as telluride or is enclosed in sulphides, unless the sample is first roasted.

1. EQUIPMENT

Crushing and grinding devices. If power is available, and samples are large and numerous, mechanical devices for reduction of sample should be used. They save time and energy, and diminish tendency to excessively large cuts on coarse ore, which vitiate accuracy of sample. In selecting a JAW CRUSHER, capacity, strength, weight, and convenience in cleaning are important; for ores likely to pack (due to moisture or talcy nature), a crusher with forced discharge is desirable. For fine grinding, the DISK PULVERIZERS are efficient. There are several makes; those having a stationary and a revolving disk give excellent results on hard and granular ores, though for talcy or heavy sulphide ores, they are less satisfactory. Ores of this class are better handled by a PLANETARY GRINDER, in which a revolving disk travels in an orbit, and discharge is positive.

For reduction of coarse ore by manual labor the JAW CRUSHER is most efficient. Relatively small jaws and heavy flywheels of large diameter are necessary. There is a knack in operating these machines, slow feeding and high rotative speed being essential. A simpler device is a STEEL PLATE AND HAMMER (cast iron plate, unless heavy, is apt to break), and a 3 or 4-in section of 10 or 12-in pipe, to prevent the pieces from flying; or, instead of the hammer, a "tamper" with a taller piece of pipe will be found more efficient.

For added convenience, the tamper may have a long handle, projecting through a hole in a board overhead, and the pipe section screwed to a flange for stability. For fine grinding, nothing is better than BUCKBOARD AND MULLER, which, to minimize salting of samples (Art 10) must have a finished surface and be free from pits. Buckboards with rim or flange on 3 sides have an unfinished surface. Some assayers substitute a piece of I-beam or channel iron for the buckboard, using a muller to fit the groove. Mullers should not be over 4 in wide, their greater bearing surface diminishing the unit crushing pressure. The Guanajuato Development Co has long used stone buckboards and cobble-stone mullers, with which thousands of samples have been ground. They are of silicified rhyolite, and it is stated that the pulp is diluted less than 0.5%. (See also Sec 25, Art 4.)

Sampling devices. A convenient and satisfactory sampler is the Jones riffle (as made by Denver Fire Clay Co, Fig 1). It has short spouts, facilitating cleaning. If it is set on the work bench so that its center line coincides with the edge of a rectangular opening in the bench, the rejected portion is delivered directly into a receptacle underneath. The opening is readily closed with a cover plate when both portions of sample are to be saved. The old trough and space-riffle is not recommended for general use; when carelessly used, results are unreliable. (See Sec 25, Art 4, 5, 6, 7.)

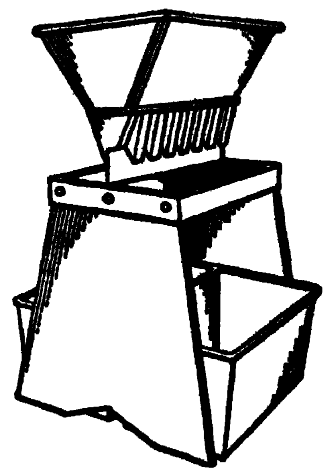

Fig 1. Jones Riffle Sampler

Furnace depends on method of assay, availability and cost of fuel, transportation, and amount of work to be done. MUFFLE FURNACE is equally adapted for all work; gives a more uniform and constant temperature; and is free from effects of furnace gases. It usually gives the best fusions, and crucibles last 2 or 3 times as long as when in direct contact with burning fuel. Soft coal and wood require this type. Disadvantages: Fuel economy is less, size of crucible limited, and destruction of muffles comparatively large. Though the increased cost of muffles is generally offset by saving in crucibles, their size and uncertain life may be the deciding factor when working in remote regions. Advantages of separate crucible fusion and muffle cupellation furnaces are practically the reverse of above. COMBINATION FURNACE attempts to combine the separate furnaces into one compact unit, using one burner for both fusion and cupellation, but it is difficult to maintain both compartments at proper temperature. It is recommended, however, for small amounts of work, or where transport is costly. A stock furnace, obtainable for practically all fuels, is generally preferable. Carborundum muffles are 10 to 1 better than clay.

Furnace built in field, for solid fuel, is rectangular in section, of fire brick laid in a mixture of $^2/_3$ burnt and $^1/_3$ raw clay. Ordinary brick does not stand well and stone is useless. Furnace is bound with angle irons at corners and braced with tie rods, or built in a sheet-iron case. Transverse iron bars at front and back support grate bars, which should project into the draft opening, for convenient dumping of ashes. At 8 to 12 in above grate bars is an opening, through which the muffle is inserted and supported by a block at the rear. Muffle opening is closed by a plug when used for crucible work. Space between muffle and furnace walls must be sufficient to allow fuel to pass.

If old crucibles are available, they may be ground to pass an 8-mesh screen, mixed with water and 10 to 20% of clay, and rammed into a sheet-iron case, containing a core of the shape desired for interior of furnace. Slag adhering to crucibles so used acts as a bond.

Fuel is coal, coke, or charcoal, but the above furnace works best with coke. Charcoal burns rapidly, requires constant attention, and gives fluctuating temperature. Soft coal is apt to coke and form a solid mass, which must be broken up, and the fuel-bed temperature is low until most of volatile products have been expelled. Soft coal gives best results in muffle furnace, thickness of bed being 5 to 8 in, on a grate 15 to 18 in below muffle. Anthracite makes a comparatively slow fire, in which it is difficult to obtain fusions without strong draft.

Oil-burning furnaces are best for field work. When power is available for a blower, heavy oils may be used, at a low fuel cost. In absence of power, gasolene furnace largely compensates for added fuel cost by saving of labor, rapid attainment of any desired temperature, and ready control. Gasolene furnace requires considerably more fuel while heating to required temp than subsequently; hence, short runs should be avoided. Two of the largest manufacturers are: Denver Fire Clay Co, Denver, Colo, and Braun Corp, Los Angeles, Cal. Fig 2 shows a light and satisfactory type, gasolene consumption for which is stated as 0.5, 0.75, and 1 gal per hr, for furnaces taking 6, 8, and 10 20-gm crucibles respectively. The $^1/_8$ or $^1/_4$-in pipe and elbows usually furnished with these

furnaces can be replaced with advantage by small-bore copper tubing, known as hollow wire; it is easier to install and eliminates numerous joints, with their liability to leakage.

Balances. A cheap spring-balance, costing 15¢ to 25¢, with a pan suspended by 3 strings, answers for weighing stock charges. PULP BALANCE should have pans holding at least 1 A T (assay ton = 29.166 gm), and should be capable of being dismounted and packed in the case drawer. They have either all-steel bearings, or steel knives and grooved agate bearing plates, the latter being preferable. For FINE BALANCE, following points should be considered. Since silver beads do not require extreme accuracy in weighing, a very sensitive balance, which is always slow and liable to derangement, should not be chosen for this work alone. Gold determinations are most exacting, and for a large number of weighings, the balance should be of the best. The most sensitive balances are very delicate, and will not stand rough handling. The inverted or "no column" type is recommended. Several makers have special portable balances, which are good, but less satisfactory than the standard types, and should not be used for permanent equipment.

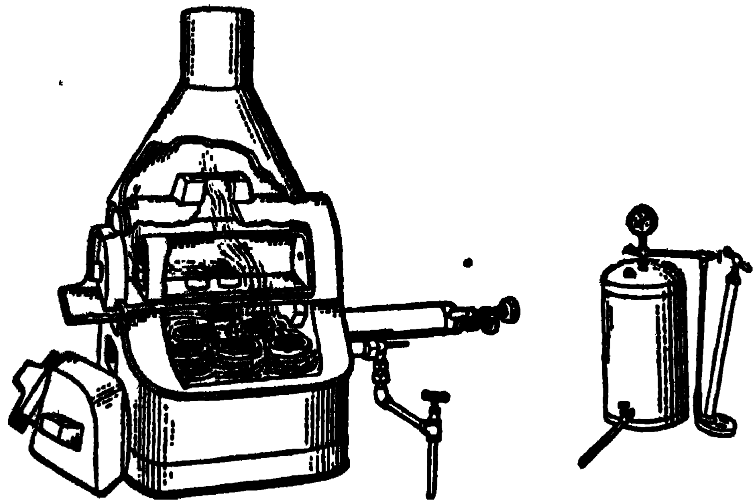

Fig 2. Gasolene Assay Furnace

Weights. For GOLD AND SILVER ASSAY, combination set of 0.01 to 100-gm and 0.1 to 4-A T "second grade" weights is advisable. The ASSAY TON (A T) contains as many milligrams as the avoirdupois ton contains troy ounces; therefore, 1 mg of gold or silver obtained from 1 A T represents 1 troy oz in 1 avoirdupois ton. The short (2 000 lb) and long ton (2 240 lb) will be represented by an A T containing respectively 29 166 + mg (29.166 gm) and 32 666 + mg (32.666 gm). BEAD WEIGHTS comprise 1 mg to 1 gm, and should be of the highest grade. These weights, as furnished by supply houses, are usually inaccurate and should be tested. Slight discrepancies can be disregarded for silver weighings, but a difference of 0.01 mg causes an error respectively of 20¢ on 1 A T, and 40¢ on 0.5 A T charges of gold ore. ANALYTICAL WEIGHTS for base metal and miscellaneous determinations should include 5-mg to 100-gm first grade weights. If both classes of work are to be done, the 5-mg to 1-gm weights may be omitted from the analytical set. It would be possible also to eliminate the gram weights in assay set, but this is inadvisable.

Where ore is weighed in METRIC TONS, gold and silver are reported in grams and kilos. As there are 1 000 kg in a metric ton and 1 000 mg in 1 gm, 1 mg of gold or silver obtained from 1 gm of ore will represent 1 kg per ton. Example: if 10 gm of ore gave 15.5 mg of silver and 0.35 mg of gold, the ore contains 1.55 kg silver and 0.035 kg, or 35 gm, of gold per metric ton. Where metric system is employed the A T weights are useless.

2. REAGENTS

Litharge (PbO) is employed in crucible assay to furnish lead for a button; also, in many charges, as a basic flux, oxidizer, and desulphurizer. It should be tested for silver by fusing 10 A T with silica, borax glass and soda, and enough reducing agent to give a 35-gm lead button. If appreciable bismuth is present, it will be indicated in cupellation by a brown stain at the point where last of the lead is oxidized. Bismuth causes inaccurate results, for which correction can not be made, as in case of silver.

Sodium carbonate (Na_2CO_3). Commercial carbonate or soda ash (Na_2CO_3) is sufficiently pure for assaying and should be employed. Bicarbonate is unnecessary, costs more, and is more bulky per unit of Na_2O. Na_2CO_3 acts as a basic flux and in certain cases is a desulphurizer.

Test lead is a solvent for gold and silver. In scorification assay it is oxidized to litharge, which is the principal flux. It may contain Ag and Bi, and should be tested by

scorifying four 80-gm portions, which are combined and rescorified until sufficiently reduced for cupellation.

Potassium carbonate (K_2CO_3) performs same function as Na_2CO_3; when used with the latter it gives a mixture of lower melting point. It is much more expensive, not essential, and, being hydroscopic, is an unsatisfactory reagent in moist climates.

Borax ($Na_2B_4O_7 + 10\,H_2O$) is a fusible acid flux, useful in assisting to dissolve many basic oxides. It should always be employed in form of borax glass, prepared by fusing commercial borax and pulverizing the resulting glass.

Silica (SiO_2) should be considered as an acid reagent and not a flux, because its melting points, in combination with the constituents of an ore, are higher than those generally used in assaying. When added to a charge an increase of the fusible fluxes is necessary.

Potassium cyanide (KCN) acts as a reducing and desulphurizing flux. It is useful in base metal determinations, but not for gold and silver assays. It should be of good quality. Cyanide containing any considerable amount of cyanate is absolutely worthless for some determinations.

Flour is the most convenient reducing agent; 1 gm will reduce about 10 gm of lead.

Niter (KNO_3) is used to counteract excess reducing action of some ores; it has an oxidizing power of about 4.25 gm lead per gm. Since the power varies, a determination may be necessary, preferably done against the reducing agent occurring in the ore, usually pyrite. Make up two charges, using 2 gm of pyrite, or enough ore to reduce 15 to 20 gm lead, 5 gm silica, 7 gm soda, and 60 gm litharge. In one of these charges introduce 2 gm niter, and fuse both at strong red heat. Reaction should be completed in 15 min. Weigh resulting buttons and divide difference in weight by 2, giving oxidizing power of niter. Niter also acts as a basic flux.

Stock fluxes are advisable when many assays are to be made on one class of ores, using the proportions given below under individual charges. It is often desirable to omit niter or reducing agent, adding these separately to individual assays as required.

Reagents for wet analysis and parting should be chemically pure (c p), unless otherwise stated.

Water. If distilled water can not be obtained, rain water may be used, collected after a portion of the precipitation of a shower has run off. Lacking this, chemically treated water should be used, especially for parting in gold and silver assays, where chlorine, generally present in ground waters, interferes seriously. To prepare water for parting it should be treated with a slight excess of silver nitrate, heated, and allowed to stand for at least 24 hr, then filtered or decanted. This water is used for diluting HNO_3, and for first washing. Another portion of water not treated with silver nitrate should be made slightly ammoniacal and used for second washing.

3. SAMPLING (See also Sec 25 and 29)

General procedure. Working down an original sample to the final stage is often more difficult than making a satisfactory assay, and because of the labor and care involved this operation is frequently slighted. A mine or prospect sample, as usually taken, is apt to be much less accurate than the results of assaying. (For grinding and sampling devices see Art 1.) The original sample should be reduced to small size at place of taking, unless suitable mechanical devices are available elsewhere. Table 1 assumes that the assay office will not have to deal with samples of over 20 lb (preferably 5 lb or under).

Table 1. Minimum Permissible Weight of Sample for Given Size of Ore (1)

Wt of sample		Low-grade and uniform ores		Medium ores		Rich and spotty ores	
Gm	Lb	Mm	Approx mesh	Mm	Approx mesh	Mm	Approx mesh
....	20	3.6	6	2.4	8	1.00	17
....	10	2.5	8	1.7	10	0.71	24
....	5	1.8	10	1.2	14	0.50	32
....	2	1.1	16	0.76	22	0.32	45
....	1	0.8	20	0.54	30	0.22	65
....	0.5	0.57	28	0.38	38	0.16	88
90	0.2	0.36	40	0.24	58	0.10	150
45	0.1	0.25	55	0.17	82	......	....
22.5	0.05	0.18	76	0.12	125	......	....
9	0.02	0.11	150	......	....	......	....

Best method of working down a sample can be ascertained only by experiment, and for a number of similar samples it should be determined. The size of ore particle indicated in Table 1, for a given weight of sample, is much finer than necessary in many cases. The screens used are generally multiples of 10. To save labor, the number of cuts when grinding by hand will be greater than for machine work (Table 2).

Table 2. Working down Samples by Hand and by Machine

Hand work				Machine work			
Original wt of sample	Mesh	Number of cuts	Final wt of sample	Original wt of sample	Mesh	Number of cuts	Final wt of sample
20 lb	4	....	20 lb	20 lb	4	0	20 lb
20 "	8	2	5 "	20 "	10	2	5 "
5 "	20	2	1.25 "	5 "	40	4	5 oz
1.25 "	40	2	5 oz	5 oz	120	assay	....
5 oz	100–120	assay	....	....	....	....	....
1.25 lb	40	3	2.5 oz	....	....	....	....
2.5 oz	120	assay	....	....	....	....	....

To check accuracy of sampling (see Art 9), mix rejected portions thoroughly and repeat operation. If sampling procedure is correct, results will check within limits of assay accuracy. Thorough mixing and mode of dividing sample are as important as size of ore particle. The coning and quartering method, while capable of giving good results, is liable to serious error and the use of riffles (Art 1) is much preferred. Riffles should have an even number of spaces and the sides should be kept in perfect alinement. Ore coarse enough to cause bridging must never be fed on a riffle, and a flat-bottom scoop, over which the ore is uniformly distributed, should always be used. After splitting, the two portions of a sample should differ but slightly in weight.

Metallics in an ore necessitate following procedure. A sample sufficiently large to insure an average value must be taken. This will depend on character of ore, and can be determined only by trying several samples and comparing results.

Assume an ore containing coarse gold to be ground to 20 mesh; weigh 20 A T; grind to 150 mesh and screen. Wt of metallics and coarse ore on screen, 3 gm. Scorify this with 40 gm of lead and cupel; wt of gold = 800 mg. Of siftings, weigh 0.5 A T (in gm) $-\frac{3}{40}$ gm = 14.508 gm = proportionate amount of fines in 0.5 A T of ore. Assay siftings; wt of gold = 200 mg.

$$
\begin{array}{lcl}
\text{Weight of gold in metallics from 1 A T of ore} & = \dfrac{800}{20} & = 40 \text{ mg} \\
\text{" " " " siftings " " "} & = 200 \times 2 & = \underline{400} \\
\text{Weight of total gold in 1 A T of ore} & = & 440 \text{ mg}
\end{array}
$$

or 440 oz per ton.

If metallics on screen will give a bead of more than 2 gm, scorify and sample the product, as indicated below. Ores containing nuggets are extremely difficult, and in some cases impossible, to sample by usual methods. A large sample must be taken.

Assuming it is necessary to take 50 lb of a 6-mesh rich silver ore to obtain an accurate sample: Weigh out 64.3 lb = 1 000 A T (1 lb = 15.5521 A T) and grind to 30 mesh. Metallics on screen weigh 500 gm. Flux with soda, borax, and niter if metallics contain oxidizable constituents such as sulphur, arsenic, or antimony. Weight of bar = 400 gm. Sample the bar and cupel 500 mg. At same time cupel an amount of pure silver and copper, equal in wt to the silver and base metal in 500 mg of bar. Add loss of pure silver in cupellation of latter to bead from bar sample. Say fineness equals 950. Then coarse metallics contain 380 gm silver. Of the 30-mesh product, weigh 10 A T $-\frac{500}{100}$ gm (291.66 − 5) = 286.66 gm, the proportionate amount of fines in 10 A T of original ore, and grind to 120 mesh. Metallics on screen = 8 gm. Scorify with 80 to 100 gm lead to a button of at least 60 gm, since rich gold- and silver-lead alloys are brittle. Roll button and cut sample from different portions of sheet. Cupel 6 gm wrapped in 5 to 10 gm of lead foil. Weight of bead = 450 mg; whence, 450 × 10 = 4 500 mg total silver in metallics.

Of the 120-mesh siftings, weigh 0.5 A T $-\left(\frac{500}{2\,000}\text{ gm} + \frac{8}{20}\text{ gm}\right)$ = 14.583 gm −

(0.25 + 0.4) = 13.933 gm, or 0.5 A T minus the proportional amounts of over 30-mesh and 120-mesh metallics. Resulting bead = 350 mg.

Weight of silver in coarse metallics from 1 A T	= $\frac{380\,000}{1\,000}$	=	380 mg
" " " " fine " " "	= $\frac{4\,500}{10}$	=	450 "
" " " " siftings (120-mesh) "	= 350 × 2	=	700 "
Weight of total silver in 1 A T of ore	=		1 530 mg

or 1 530 oz silver per ton.

Low-grade, spotty gold ores (2), illustrated by some from California, and Black Hills, S D, have their values so segregated that the ordinary 0.5 A T charge of 100-mesh product is unreliable. A sphere of gold 0.147 mm diam (opening in a 100-mesh screen) weighs 0.032 mg. If we conceive a $4 ore in which the particles of gold are of that diam, there would be 3 particles of gold in 0.5 A T, whence the absurdity of expecting to obtain an accurate assay portion of this material is apparent. It is necessary to take 20 to 40 A T of such ore, and as this amount is large to flux, the coarse gold is amalgamated by adding 5 gm mercury, with enough water to make a sludge sufficiently thick to hold fine globules of mercury in suspension. This is ground for 5 to 10 min in a mortar, preferably of porcelain, or a small jar mill, then thinned with water, and panned to recover the amalgam. If the mercury does not collect well, add a small quantity of sodium amalgam (Art 8; see also Sec 31). Place the amalgam in a parting flask or cup, dissolve the mercury in HNO_3, anneal and weigh the gold. The gold may contain some silver and should be inquarted and parted. The tailing is dried, mixed, and assayed, using 1 or 2-A T portions. The two results are combined in proper proportions.

Sample composition tests. Approximate composition of an ore must be known, to make up a proper charge. When ore is in lump form, this may generally be determined by inspection. If pulverized, take 5 to 10 gm in a pan or clock glass, add water and agitate; pour off fine slime and repeat if necessary. Add more water and stratify remaining coarse minerals by panning motion, then determine minerals and their approximate percentage by inspection. Do not attempt to effect a concentration; instead, remove the indistinguishable fine minerals, leaving the coarse particles so segregated that they may be identified and estimated. Treat about 1 gm of ore with HCl; rapid evolution of gas indicates CARBONATES. Allow complete action, noting percentage dissolved, which will be classed as base. Odor of H_2S may give the impression that the action is due to SULPHIDES. Mineral sulphides, however, react slowly with HCl and do not give a rapid evolution of gas. HNO_3 will not answer for this test, as it attacks sulphides vigorously and it is not possible to distinguish between the action of carbonates and sulphides. Evolution of chlorine denotes MANGANESE dioxide, and the amount of this mineral present must be judged by color of sample, remembering that pyrolusite is black. Add strong HNO_3 to about 0.25 gm ore, boil, dilute, make ammoniacal, filter or allow to settle. Blue solution indicates COPPER, and the percentage may be approximated by the depth of color. To test for ARSENIC and ANTIMONY, separate the concentrated metallic minerals from panning test, and heat with blowpipe, or in porcelain capsule, mixed with a little charcoal. Characteristic fumes of arsenic and antimony will denote their presence.

4. CRUCIBLE METHODS FOR PURE ORES

Size of lead button. Crucible method is recommended for field work, as it is applicable to all ores, permits use of large, representative samples, and reduces multiplication factor. While the charges vary for different ores the principle is the same for all. The ore is mixed and fused with such fluxes as will effect complete decomposition and solution at a reasonable temperature, thus liberating the gold and silver. The oxidizing and reducing agents are proportioned to produce 20 to 25 gm of lead, which collects the gold and silver in one button. A lead button of 25 gm is sufficient to collect the values in 1 A T of ore; a larger button is not in itself detrimental, but it must be realized that when a charge is made to yield a button of definite size, all lead reduced in excess of that amount diminishes the quantity of litharge available as a flux, and excessive reduction may thus seriously affect results. Lead buttons of less than 15 gm may or may not collect all the values, and, if adopted, this point should be proved. It is essential that the gold and silver shall be completely liberated before all the lead has settled.

Fusion. Manner of fusion is important and varies with different ores. Close attention should be given to instructions on this point, which accompany the different charges.

As a rule, a fusion should be complete in 25 to 35 min. Fusions at too low a temperature will affect recovery of both gold and silver; too high a temperature usually affects only the silver. Do not place crucibles in the furnace until it is up to working heat.

Slags. A glassy or vitreous slag, which can be drawn into threads when viscous, is generally desirable, since a slag of opposite characteristics may indicate incomplete decomposition, and may also cause poor collection of lead. Glassy slags are produced by having sufficient silica and borax present to form a solid solution with the bases. Character of slag is controlled by these reagents.

Charges for pure ores. Pure ores are composed chiefly of the ordinary rock-forming elements, with small amounts of the elements classed as impurities, viz, S, As, Sb, Cu, Zn, etc. The problem is mainly to proportion the fluxes so that the ore will go completely into solution. The principal fusible fluxes, Na_2CO_3 and PbO, are capable of readily dissolving SiO_2, but their action on the bases is rather limited. To decompose basic ores, enough SiO_2 must be added to combine with the bases in the ore and fluxes used. Different bases require varying amounts of SiO_2 to produce a slag having desirable characteristics. This is indicated in Table 3. In first column is a charge for 100% SiO_2 (acid); in third column, the charge is for 100% of the base; in second column is a charge for what may be called the neutral ores, which have just enough acid to satisfy the base and give a good slag. It is not supposed that ores will have the exact compositions indicated in these charges; but, having 3 points defined for a single base, it is easy to vary the charge according to the ore constituents. If more than one base is present the charges can be combined in proportion to the constituents. There is considerable latitude in composing a charge. With a basic ore, if enough SiO_2, or even a moderate excess, be added to affect complete combination, and then enough flux to give a slag with medium fusing point, good results will be obtained. Flour is the usual reducing agent, yielding approx 10 gm lead per gm. Amount required for a charge is not given, because, since the ore often contains reducing or oxidizing agents, it is possible to state only the total reducing action, in terms of lead; not necessarily the net reducing action, or amount of lead desired. (Remarks following the table should be studied before making use of the charges.)

Table 3. Typical Assay Charges

CALCIUM OR MAGNESIUM CARBONATE

	Acid: SiO_2, 100% (*a*)	Neutral: $CaCO_3$ or $MgCO_3$, 50%; SiO_2, 50% (*b*)	Basic: $CaCO_3$, or $MgCO_3$ 100% (*c*)
Ore	0.5 A T	0.5 A T	0.5 A T
Silica	0 gm	0 gm	15 gm
Sodium carbonate	20 "	15 "	30 "
Litharge	45 "	45 "	75 "
Borax glass	5 "	5 "	10 "
Reducing agent =	20 gm lead	20 gm lead	25 gm lead

IRON OXIDE

	SiO_2, 100% (*a*)	Fe_2O_3, 80% SiO_2, 20% (*d*)	Fe_2O_3, 100% (*e*)
Ore	0.5 A T	0.5 A T	0.5 A T
Silica	0 gm	0 gm	3 gm
Sodium carbonate	20 "	15 "	18 "
Litharge	45 "	45 "	55 "
Borax glass	5 "	5 "	6 "
Reducing agent =	20 gm lead	28 gm lead	30 gm lead

MANGANESE DIOXIDE

	SiO_2, 100% (*a*)	MnO_2, 60% SiO_2, 40% (*f*)	MnO_2, 100% (*g*)
Ore	0.5 A T	0.5 A T	0.5 A T
Silica	0 gm	0 gm	10 gm
Sodium carbonate	20 "	5 "	7 "
Litharge	45 "	40 "	70 "
Borax glass	5 "	15 "	25 "
Reducing agent =	20 gm lead	40 gm lead	55 gm lead

ALUMINUM OXIDE

	SiO_2, 100% (a)	Al_2O_3, 60% SiO_2, 40% (h)	Al_2O_3, 100% (i)
Ore	0.5 A T	0.5 A T	0.5 A T
Silica	0 gm	0 gm	10 gm
Sodium carbonate	20 "	15 "	25 "
Litharge	45 "	45 "	75 "
Borax glass	5 "	5 "	8 "
Calcium fluoride	0 "	5 "	8 "
Reducing agent =	20 gm lead	20 gm lead	25 gm lead

LOW-GRADE ACID ORES AND TAILINGS (k)

Ore	3 A T
Sodium carbonate	120 gm
Litharge	150 "
Borax glass	15 "
Reducing agent =	35 gm lead

Ore plus added SiO_2 is considered as ore for the purpose of composing charge, and fusible fluxes are added accordingly. These charges form silicates and borates with the bases. One-half A T of ore + 15 gm SiO_2 requires same flux as for 1 A T of acid ore. Excess SiO_2 above that required will do no harm, provided sufficient soda and litharge are used. It is therefore apparent that charge (c) may be used for 1 A T of acid ore. Borax glass may be omitted in many charges for siliceous ores, eliminating the more expensive flux. Its use, even when not required, does no harm. In the absence of large amounts of impurity, soda and litharge may be used in these charges to a certain extent interchangeably in ratio of 2 to 3, without objectionable results. Ores frequently contain more or less S, As, Sb, Cu, Zn, which should be oxidized. As this can be done only with litharge, the charges as given will have the greatest general application. There is no special advantage in a charge of the following type, which is in common use: Ore, 0.5 A T; Na_2CO_3, 15 gm; K_2CO_3, 15 gm; borax glass, 15 gm; litharge, 30 gm; flour, 1.75 gm. The use of litharge in above amount makes it impossible to obtain a lead button too large for cupellation, but this is of doubtful advantage. Any saving in cost of litharge is offset by the use of potassium carbonate and increase of borax. The amount of reducing agent required depends on presence or absence of reducing and oxidizing agents in the ore. Pyrite (FeS_2) up to 13% of the ore, or its equivalent, is permissible with these charges; at 13%, it replaces all the reducing agent, giving a 24-gm button. In charges (c), (g), (i), and (k) larger lead buttons are produced because of increase in size of charge. In (d), (e), (f), and (g), a lead button of 20 to 25 gm is desired, but excess reducing agent is necessary to counteract oxidizing effect of Fe_2O_3 and MnO_2. In (f) and (g), manganese as MnO_2 does not go readily into solution; high soda interferes with its reduction; borax glass materially assists dissolution. Charge should be brought rapidly to fusion temperature, the maximum being 1 050° C, or light yellow. Al_2O_3 in combination is a frequent constituent of ores (charges h and i); it slags with difficulty, and interferes with collection of lead. The use of fluorspar (CaF_2) assists in taking Al_2O_3 into solution. If omitted, soda and borax glass must be increased 100%.

5. CRUCIBLE METHODS FOR IMPURE ORES

These include ores containing enough S, As, Sb, Zn, Cu, Ni, Co, and Te, to increase slag and cupel losses, unless their effect is counteracted. For this an oxidizing charge is necessary, and as litharge is the only oxidizing agent which can be used in excess, it is employed freely.

Iron pyrite ores containing less than 13% pyrite have been discussed in Art 4. Those containing over 13% pyrite are treated by either niter or nails method. NITER METHOD gives more reliable results for gold and silver, and admits the presence of other impurities. It usually requires a preliminary assay to determine reducing power of the ore, and this, together with the large flux charge, makes it slow and costly. The NAILS CHARGE is capable of direct application, but only to sulphides comparatively free from other impurities. Gold results by this method are fairly reliable, with possible error of 1%. Silver results, except in the case of small amounts of pyrite (20% or under) are low; the loss increases with percentage of pyrite, reaching 10% with pure pyrite. When in doubt whether an ore is pure or pyritic, the nails charge may be used satisfactorily, even though the pyrite present is less than that requiring this charge.

Niter method. In this the KNO_3 oxidizes all the FeS_2 in excess of that required to reduce the desired amount of Pb, the remaining FeS_2 being oxidized by litharge. To obtain the required definite state of oxidation, a relatively large amount of base is necessary, in which case the S is completely oxidized to SO_3.

Preliminary charge. The reducing power of an ore must be known, to determine the amount of niter required. A knowledge of the ore may furnish the information, or, with sufficient experience, it may be determined by panning. Preliminary fusion is generally necessary, using following charge and calculation:

Ore....................	0.1 A T
Sodium carbonate.......	7 gm
Litharge...............	50 "
Wt of lead reduced.... =	W

$$\frac{(W \times 5) - 20}{4.5} = X \text{ gm } KNO_3$$

or

$$\frac{W - 4}{0.9} = X \text{ gm } KNO_3$$

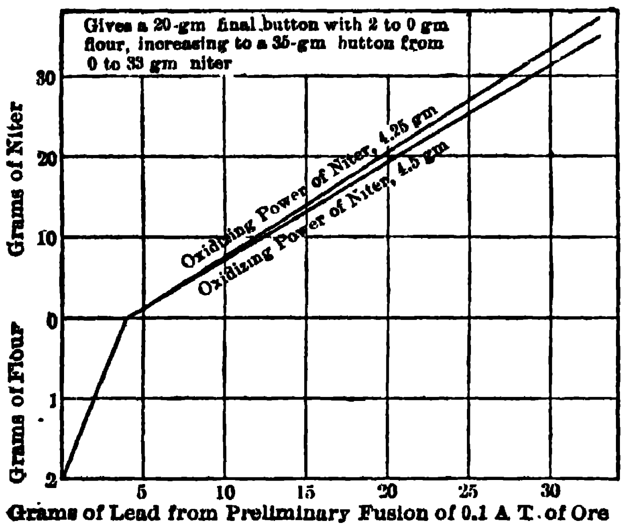

Fig 3. Reduction Diagram for Impure Ores

Charge should be fused in 10 to 12 min, at a light yellow (1 050° C). Slow fusion, or too long a time in the furnace, gives low results. The chart (Fig 3) will eliminate calculations, or HAWLEY'S METHOD may be employed. Weigh out 3.24 gm ore (instead of 0.1 A T) and fuse with the above charge. Place lead button obtained in left-hand scale pan, and suspend a 5.55-gm weight from hook on opposite end of beam (The 3.24 and 5.55-gm wts are based on niter having oxidizing power per gm of 4.5 gm lead.) Balance with niter, thus obtaining the amount required for final charge. The special weights are readily cut from lead; the larger one is suspended by wire from the pan hook. If the lead button from preliminary charge weighs less than 5 gm, or fails to counterbalance the 5.55-gm weight, reducing agent and not niter is required.

Table 4. Final Charges

Over 50% SiO_2 in ore		Under 50% SiO_2 in ore	
Ore..........................	0.5 A T	Ore..........................	0.5 A T
Sodium carbonate..................	10 gm	Silica..........................	6 gm
Litharge..........................	75 "	Sodium carbonate................	15 "
Potassium nitrate..................	X "	Litharge........................	100 "
		Borax glass	6 "
		Potassium nitrate............	X "

Final fusion. Lead button of 20 to 25 gm is desirable for ores containing less than 50% pyrite; for more than 50%, the button should weigh 25 to 35 gm, the size increasing with proportion of pyrite. Success of this charge is largely dependent on manner of fusion, especially in high-pyrite ores. For low to medium amounts of pyrite, a temp of 1 000° to 1 050° C (yellow heat) is satisfactory. For larger percentages, a high initial temp is essential, say 1 050° to 1 100° C (light yellow) maintained for 10 min, or until charge is thoroughly liquid. Temp of furnace is then lowered, and fusion continued for 15 to 20 min. Charge should not be allowed to reach original high furnace temp, or silver losses will increase. This fusion should be made in a high-grade crucible, like the Battersea soft clay pot (not to be confused with the hard clay crucible). There is but little boiling in these fusions, and the dry charge may fill 3/4 of the crucible. From 75 to 100 gm litharge is used to insure complete oxidation of S to SO_3, which combines with Na_2O, forming Na_2SO_4. This floats on slag as a thin liquid, forming a white crust on cooling.

Nails method is essentially reducing, with no attempt at oxidation. Sulphur is absorbed in slag, size of lead button is regulated by adding only sufficient litharge, and

button is desulphurized by nails. It is therefore apparent that impurities can not be oxidised, and have a tendency to concentrate with the lead. Charge is:

Ore	0.5 A T	Borax glass	10 gm
Sodium carbonate	30 gm	Nails (10 d)	4 "
Litharge	25 "	or 1 R R spike	

If ore is high in pyrite, or contains a large amount of base, borax glass should be increased by 5 to 10 gm. Fusion should be at temp of 950° to 1 000° C (yellow), and requires 25 to 40 min. Avoid a strongly oxidizing furnace atmosphere, which produces an unsatisfactory fusion. In withdrawing nails, shake well while their points are immersed in slag, and not simply knock against side of crucible.

Copper ores. Presence of copper up to 5% does not interfere in charges which contain litharge in excess of that which is required for a lead button (Art 4). Copper does not interfere in the fusion of an ore, but has a strong tendency to contaminate the lead button. If the button contains about 8% Cu, or more, it will freeze if cupeled at 700° to 750° C ("feather temperature"). If button contains 5% or less of copper, it will cupel at a lower temperature than pure lead. Even though copper is present in amounts insufficient to cause a freeze, it is detrimental, increasing the loss. While some copper remains in bead, the amount is less than the induced loss, giving low results (especially for silver). Both this loss, and the amount of copper retained, increase with proportion of copper in the lead button. Therefore, copper in excess of 2% of the lead is not advisable. Copper is eliminated from the lead button by large quantities of litharge, either added in the charge or formed by scorification. Amounts of ore and the charges indicated below are intended to give final lead buttons containing less than 2% copper; 2 to 3% copper will stain cupels gray green, shading to a lighter color at center. A blackish-green indicates presence of too much copper. While 175 gm of litharge is given for all charges, it should be reduced when copper is lower than stated. If a scorification is not feasible and ore is rich in copper, the ore charges must be smaller, without diminishing quantity of flux; thus,

Copper 20%, use 0.5 A T		
" 45%, " 0.5 "		and scorify lead buttons + test lead.
" 60%, " 0.25 "		run 4 charges, combine buttons 2 by 2, and scorify.

Note.—In SCORIFYING BUTTONS, add sufficient test lead to make total lead 50 to 70 gm, depending upon amount of copper; 70 gm lead scorified to 15 gm will slag about 75% of the copper. Add about 2 gm SiO_2 to prevent cutting of the scorifier. This does not increase copper in the lead button, for, if not introduced, its equivalent will be taken from the scorifier. Temperature should be low, as there is no mineral to decompose. When lead buttons are not too large for cupellation, and amount of copper not excessive, time is saved by melting the buttons with a cover of 50 to 60 gm litharge for 10 to 15 min in a scorifier, and pouring. About 75% as much copper is removed by this method as by scorification.

Oxidized ores include those which do not contain reducing agents in excess of that required to produce suitable lead buttons.

Acid Gangue		Basic Gangue	
Ore	0.25 or 0.5 A T	Ore	0.25 or 0.5 A T
Sodium carbonate	15 gm	Silica	X gm
Litharge	175 "	Sodium carbonate	15 "
Borax glass	5.0 "	Litharge	175+ "
Reducing agent	= 25 gm lead	Borax glass	5.0 "
		Reducing agent	= 25 gm lead

Note.—It is a disadvantage to produce lead buttons much in excess of required size, especially when scorification follows, because while copper content decreases in percentage it increases in total amount.

For copper ores with basic gangue, see Art 4 to determine amount of SiO_2 to add; then increase soda and litharge in proportion. The minimum of SiO_2 which gives a good fusion should be used, as it directly affects the advantages of large amounts of litharge.

Sulphide ores, or those which have an excess reducing action.

Ore	0.25 or 0.5 A T	
Sodium carbonate	15 gm	
Litharge	175 "	
Potassium nitrate (see above note)	X "	to produce 25-gm Pb button

PRELIMINARY CHARGE:

Ore	0.1 A T	For 0.25 A T charge $\frac{(W \times 2.5) - 25}{4.5} = X$ gm KNO_3
Sodium carbonate	7.0 gm	
Litharge	50.0 "	For 0.5 A T charge $\frac{(W \times 5.0) - 25}{4.5} = X$ gm KNO_3
Lead reduced = W gm		

(See iron pyrite, niter method, for principles and method of fusion.)

SiO_2 or borax glass should not be added to these charges without greatly increasing the litharge. Fusions are made, if possible, in Battersea 20-gm soft clay crucibles, as they withstand the action of charges better and furnish less silica.

Arsenic and antimony ores, being similar in behavior, are considered together. When present in an ore to any great extent these elements occur as stibnite (Sb_2S_3) and mispickel (FeAsS). The quantity of arsenic or antimony in silver minerals is generally insufficient to require special treatment, but should the amounts present require attention the same principles apply as for Sb_2S_3 and FeAsS. The tendency to high slag and volatilization losses, particularly of silver, and the resistance of oxides of As and Sb to moderate amounts of flux, are the principal difficulties in assaying these ores. A strongly oxidizing charge is generally required to decompose the minerals completely. This is not always necessary, but the use of nails is not permissible, because a speiss, carrying values, will be produced when arsenic is present. For ores containing 5 to 10% As or Sb, the charge may be composed as dictated by the principal mineral constituents, possibly increasing the litharge, and varying the amount of reducing agent. Above 10%, a large proportion of litharge should be used.

	(a) FeAsS to 75% Sb_2S_3 to 50% Acid gangue	(b) FeAsS to 100% Sb_2S_3 to 75% Acid gangue	(c) Sb_2S_3 100%
Ore	0.5 A T	0.5 A T	0.25 A T
Silica	....	6 gm	6 gm
Sodium carbonate	15 gm	15 "	15 "
Litharge	75 "	90 "	75 "
Potassium nitrate	X "	X "	X "

Quantity of KNO_3 is determined by preliminary fusion. For principles, and formula for charges (*a*) and (*b*) see pyrite charge (Art 5). For charge (*c*) use,

$$\text{Weight of lead reduced} = W; \quad \frac{(W \times 2.5) - 2.5}{4.5} = X$$

Zinc ores containing no more than 5 to 10% Zn do not affect method of assay. The difficulty in assaying ores high in zinc is to slag the zinc completely after it is oxidized, so as to prevent loss of gold and silver by volatilization with the zinc, and by slag and cupel absorption. To accomplish this, the proportion of flux to ore is usually large. An oxidizing method is necessary where zinc is present as sulphide in gold and silver ores. If the zinc is in an oxidized state, a charge is used similar to those for pure ores (Art 4), the exact composition depending upon proportion of acid and base, but increasing the relative amounts of fusible fluxes. For sphalerite ores containing up to 35% ZnS, having excessive reducing power, use niter charge as for iron pyrite (Art 5). With zinc above that amount, increase soda and litharge, adding SiO_2 and borax glass if ore is of low acidity. MAXIMUM QUANTITIES, for 100% ZnS, are as follows:

Ore	0.5 A T	Silica	5.0 gm
Sodium carbonate	15 gm	Borax glass	10 "
Litharge	120 "	Potassium nitrate	X (see above)

Telluride ores. Tellurium, when present, is nearly always combined or alloyed with gold or silver. The smallest amounts therefore require attention. With only 0.1 or 0.2%, its interference is confined to slag losses; but as it increases there is a tendency to concentration in the lead button, resulting in increased cupellation losses, with a possible spitting of the bead. In extreme cases it may cause complete absorption of gold and silver by the cupel.

	(a)	(b)	(c)
Ore	0.5 A T	0.25 A T	0.1 A T
Sodium carbonate	0.33 "	0.33 "	0.33 "
Litharge	3.5 "	3.5 "	3.5 "
Silica	0 gm	2.5 gm	5.0 gm
Borax glass	10 "	10 "	10 "
Reducing agent	= 25 " lead	= 25 " lead	= 25 " lead

For ores up to 300 oz of gold and silver use charge (*a*); above that amount use (*b*), except for specimens composed largely of telluride, in which case (*c*) is preferable. Charges should be fused at moderate temperature, though it is possible to have too low a temperature, giving low results, probably due to incomplete decomposition of the tellurides. Charge of litharge is high, to give a low melting, highly oxidizing slag, which oxidizes the tellurium, liberating gold and silver. Gangue of telluride ores is generally acid, but if basic, the borax glass must be increased or SiO_2 added. SiO_2, however, should not be added in excess of requirements, as it diminishes the oxidizing effect of litharge and will defeat the desired object. SiO_2 is added in (*b*) and (*c*) to assist in forming a clean slag.

Lead ores are the simplest to assay for gold and silver, because the lead present acts as a reagent. Oxidized ores should be run as acid or basic (Art 4), depending upon the gangue. As the lead increases the required amount of flux decreases. If the ore has excess reducing power, the niter method gives best results, though the nails method may be used (Art 5). With proper equipment, the scorification method is the simplest for lead ores.

Nickel and cobalt ores, carrying appreciable amounts of gold and silver, are uncommon except at Cobalt, Canada, and neighborhood, and in this case the gold is negligible. Ores of this district are often very complex, containing large amounts of As, Sb, and Bi, besides the Ni and Co. Metallic silver in pieces of considerable size is an important feature of these ores. Nickel and cobalt are readily reduced with lead in fusions, and, depending upon amount, interfere with or prevent cupellation, by formation of infusible oxides coating the lead. Experience with these ores indicates a choice of several different methods. If metallic scales are present they must be removed and assayed separately (Art 3). For ores up to 100 oz silver, crucible method seems preferable. The crucible and scorification methods give equally good results on 100 to 500-oz ores, above 500 oz, scorification is more reliable. For ores containing large amounts of impurities, particularly bismuth, it may be desirable to use a combination method (3), treating with HNO_3 either the ore before fusion, or the lead button obtained from fusion of the ore. These statements are general and considerable latitude should be allowed in their application.

	Crucible (*a*)	Crucible (*b*)	Scorification	
Ore	0.5 A T	0.25 or 0.5 A T	Ore	0.1 A T
Sodium carbonate	0.66 "	0.75 "	Silica	0.1 "
Litharge	2.5 "	3.5 "	Test lead	75.0 gm
* Silica	0.3 "	0.33 "	Borax glass	2.5 "
† Borax glass	0.33 "	0.66 "	Note.—SiO_2 not required in very acid ores.	
Reducing agent	= 20 gm Pb	= 25 gm Pb		

* Charge of SiO_2 depends on quantity in the ore. † Borax glass used as a cover.

For the less impure ores charge (*a*) may be used. Very impure ores require charge (*b*), using $^1/_4$ or $^1/_2$ A T, depending upon value and total impurities. In some cases scorification of the lead buttons is necessary or desirable.

6. SCORIFICATION (3)

Limitations. This method is treated briefly here, because, for field work, it is less convenient and has a smaller range of service than the crucible method. It is not suitable for highly basic ores, nor for gold and low-grade silver ores, unless several portions of the sample are run and the buttons combined, to reduce the multiplication factor and the percentage loss, which is usually high due to the small amount of silver present. Good results are obtained on acid ores, but it is best applied to impure ores in which the oxidizing action decomposes the impurities and throws them into the slag.

Charges are closely similar for all ores. They vary from 0.1 to 0.2 A T of ore, 50 to 75 gm test lead and 1 to 3 gm borax glass, depending upon amount and character of impurity present. Ore is mixed with half the test lead in bottom of scorifier and covered with other half, over which the borax glass is spread. Charge is placed in muffle at light yellow heat (1 000° to 1 100° C), and door closed. When charge is thoroughly melted, the muffle door and damper are opened to give a good draft, and temperature is slightly lowered. Slag should be perfectly fluid at all times; it should appear red hot from the lead almost to its surface contact with scorifier, and not merely where the hot litharge from oxidation of lead runs into slag. If sulphur is present, little grease-like patches of Na_2SO_4 will float on the slag, and the furnace should be hot enough to keep these molten, and, as the assay progresses, to cause the slag to run up side of scorifier by capillary action. With an excessive amount of reducing minerals present, the assay will not show

rapid decomposition and solution of the ore, because principal flux is litharge, which can not form to any extent until the reducing agents are oxidized. Oxidation proceeds until the lead is from two-thirds to completely covered by slag; charge is then poured.

7. CUPELLATION

Characteristics of cupels. Separation of lead from gold and silver by cupellation depends upon the high surface tension of the latter metals, which prevents them from sinking into the pores of the cupel, while the fluid PbO is readily absorbed. Any material with a high melting point, not attacked by litharge, and in granular form, may be used for making a cupel. Magnesia, Portland cement, and other substances are used to a limited extent, but bone ash is the most common. Several brands of manufactured cupels are on the market, but there is no special advantage in using them. Bone ash in bulk will stand all conditions of transportation, even to a thorough wetting; and, as cupels can be made with a hand mold at the rate of 100 per hr, the final product is cheaper. A cupel should have a smooth surface, and readily absorb its own weight of litharge without cracking. Cracking is usually due to the cupel being too dense; excessively fine bone ash, too much water or pressure, favor this condition. In making cupels there is no better bonding substance than water, the amount used being so proportioned to the pressure that the cupel will be firm enough to handle, but will break when dropped 8 or 10 in; they become much harder on standing. The cavity of a cupel should approximate the surface of a sphere, with a radius equal to half the diam of cupel and a depth two-fifths of radius; the rim may be angular or preferably rounded (Fig 4). This will give a capacity for lead of 20 to 25 gm, 30 to 35 gm, and 40 to 45 gm, for a 1-in, 1.25-in, and 1.5-in cupel respectively. Weight of the cupel should be greater than its capacity for lead.

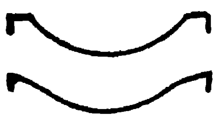

Fig 4. Shape of Cavity in Cupels

Operation. Ordinary bone ash, air-dried cupels should be placed in the muffle close together in transverse rows, sufficiently far back for uniform temperature. Broken rows should always be in the rear, and an empty row, or a fire-clay bar should protect the front row from drafts. After heating 5 to 10 min with furnace at 850° to 900° C (light cherry), insert the buttons. As soon as buttons "uncover," open muffle draft and lower temp to 700° C (below full cherry red), as the burning lead produces considerable heat. When near the end, raise temp to about 800° C. This finishing temp is extremely important; if excessive, large losses will occur and small beads may entirely disappear. A carefully made cupellation will always show a ring of litharge crystals surrounding the bead at a distance of $^1/_8$ to $^1/_4$ in. In case of gold ores, silver should be added before cupellation (Art 8), saving subsequent inquarting and practically eliminating gold losses.

Sprouting is caused by vigorous expulsion of oxygen on rapid solidification of precious-metal button. It is often difficult to prevent, but is minimized by a high finishing temperature and slow cooling. Sprouting does no harm unless particles of the bead are completely ejected or are broken off in cleaning.

Freezing is caused by too low a temp, or presence of impurities. In a low-temp freeze, if remaining lead is less than 5 gm, add test lead to make at least this amount and raise temperature. A piece of wood inserted in muffle will assist "reopening." When cupellation has started, lower the temperature rapidly; best done by placing in the muffle a piece of fire brick or its equivalent. If properly reopened, the results, though questionable, may be satisfactory. Freezes due to impurities may sometimes be recovered by adding considerable test lead and raising temp. Results are unreliable.

Beads are cleaned by squeezing in stout pliers until the base is distorted, and then brushing. Beads need not be weighed closer than 0.02 mg of the exact weight, except when the multiplication factor is large.

8. PARTING AND INQUARTING

Parting of the gold and silver is done preferably in Royal Meissen capsules (Fig 5), 45 mm diam, using first 1 to 12 HNO_3 and heating just below boiling until action ceases, decanting with cup turned well over; then treating with 1 to 1 HNO_3, heating just below boiling for 10 min, decanting and washing once or twice with distilled or treated water (Art 2). Always decant closely. Dry the residue and ignite in muffle or over flame. Take utmost care to prevent gold from breaking up; hence, at first use weak acid and avoid boiling. Flattening the bead to $^1/_3$ its original diameter is helpful, by increasing surface of attack. Parting cup should be free from stains, and the gold should have yellow color after ignition.

If a bead contains less than 4 of Ag to 1 of Au, complete separation may not be affected by parting as described. But, as little as 2 of Ag to 1 of Au may be completely separated by boiling in 1 to 1 HNO_3, if bead is less than $^1/_{64}$ in thick. Beads containing more than 25% Au may be treated directly with strong acid and boiled without breaking up. It is impossible always to tell by appearance whether a bead will part; 60% Au produces very slight color, and, when a bead is near the parting ratio limit, the acid action and color change are practically the same whether the bead parts or not. Pure gold color of the treated bead is no criterion of complete parting. A properly treated bead must have parted if it has lost at least 80% of its wt, without boiling, or at least 66%, with boiling; this is best indication of complete parting.

Fig 5. Parting Dish

Inquartation, or the addition of silver to a bead, is performed by wrapping the bead with silver, in ratio of 1 gold to 4 to 10 silver, in 2 to 5 gm of lead foil, and cupeling.

Temperature must be quickly dropped after starting, else the buttons, being small, will finish at the high initial temp. Silver foil may be added directly to the ore charge, or if silver is to be determined, a measured amount of standard $AgNO_3$ solution may be used, 5 c c being permissible. Since all the silver is not recovered, the value of this solution must be determined by assay of a blank charge.

Weighing gold. Since 0.01 mg Au from 0.5 AT = 40¢ per ton, Au weights must be checked; best by weighing singly and together.

Gold in pannings may be separated from other minerals by blowing, if the sample is perfectly dry. This method is not recommended, and is permissible only in absence of very fine gold. It is better to add a globule of mercury to the gold, either in pan or small mortar, and rub them together. If the gold fails to amalgamate, add 50–200 mg of sodium amalgam. (SODIUM AMALGAM is prepared by adding metallic sodium in pieces, $^1/_4$-in cube or less, to mercury, pressing the pieces beneath surface of mercury and against bottom of container with pestle or rounded stick. If large pieces of sodium are added, objectionable explosions will occur. Continue adding sodium until the amalgam becomes pasty. Keep the amalgam under gasolene or light oil.) There is no objection to adding enough sodium amalgam to the original mercury to cause a slight adherence to iron. The gold amalgam is placed in a parting cup, treated with dilute (1 to 1) HNO_3, washed, dried, and weighed (see Parting). Residue is not pure gold; its fineness is determined by taking 500 mg and inquarting, as above, with silver approximately equal to 2.5 times the actual gold content. Hammer or roll the bead to less than $^1/_{32}$ in, and part in a flask with dilute (1 to 1) HNO_3, giving two separate treatments, boiling each for 10 min. Wash, anneal, etc. If gold is of varying composition, the total product from pannings must be melted together and sampled. The fineness obtained represents the acid-treated product and not the fineness of original gold. A fineness determination made on original gold should not be used in calculations based on the weights obtained from acid-treated amalgam.

9. CHECK ASSAYS AND SALTING

Concordant results on duplicate assays are not conclusive evidence of accuracy, especially when samples are run under the same conditions. Greater weight can be given to checks when varying weights of ore are taken and run at different times. Checks obtained by DIFFERENT METHODS are the best indication of accurate results. When it is undesirable to run duplicates on many samples, the work as a whole may be checked by weighing out extra and equal portions from each ore when the original charge is made up, and carefully mixing and assaying these extra portions. If this work has been carefully done, the composite sample should give results that check to the second decimal the average obtained from the individual samples. (See also Sec 29, Art 8–11.)

Salted assays. To determine whether an assay sample is salted is often difficult. The crude plan of introducing metal filings may easily be detected with a pan and microscope. When fine native metal is employed the problem is more difficult and conclusions less certain. If ore is coarse, the finer material should be screened out and assayed separately from the coarse. The fines will undoubtedly run higher, but if the discrepancy is great there may be reason for suspicion. Where the material used in salting has a different origin than that in the ore, a test of its fineness may bring out this point. Pulverized samples, salted with rich concentrates derived from the same ore, will pass as genuine and can not be detected. If the sample is not ground, the fines should be screened out and the coarse product crushed to produce another portion of fines about equal to the first, which is then separated from the remaining coarse portion. In a normal sample, assay results of the two fine portions should not, as a rule, vary greatly. When salting is

done with solutions, some of the rock components will frequently decompose the original salting, rendering the metal insoluble and frustrating efforts at detection by solution of the metallic compound. The ore may be tested for the acid radical originally associated with the metal. Gold and silver are the only metals likely to be introduced by this method, with a chloride radical for the former, and a nitrate for the latter. Rocks are apt to contain chlorides, and the nitrate may be decomposed. But, a sample which assays high in gold or silver and, even under the microscope, shows no free gold, nor silver minerals, nor minerals which usually contain or are associated with these metals, may be strongly suspected of having received their values from unnatural solutions. See Sec 25, Art 9.

Accidental salting is more frequent; in fact, it occurs to an extent unappreciated by the average engineer. This is particularly true where miscellaneous samples are being treated. The most common source of contamination is in the grinding apparatus, and is at its maximum when small samples are being successively ground, some of which contain metallics. It is almost impossible thoroughly to clean grinding appliances without grinding therein a prohibitive amount of barren material. Pores in the grinding surfaces become filled with ore; particularly when metal is present, which is gradually fed out as the iron surfaces are abraded. Separate apparatus should be used for low- and high-grade samples. Where this is not practicable, barren material is ground after high-grade samples, and in case of gold and silver this material is assayed if the samples which follow are important. Another source of salting is in using old crucibles. This practice is unobjectionable if high- and low-grade pots are kept separate. All reagents, bone ash, and cupels should be protected from dust, especially when rich materials are being handled. Blank assays should be made occasionally on these products to test their purity.

10. PLATINUM METALS (11)

Qualitative tests, To differentiate and determine the metals of this group require considerable equipment and a skilled analyst. Many published methods will not give accurate results, and others only under certain conditions. Platinum, iridium, and palladium are the three most important. The first two come mainly from placer deposits, the last occurring more frequently, though in small quantities, with gold and silver in rock formations. As compared with gold and silver, their important assaying properties are as follows: Melting point: silver, 961°; gold, 1 063°; palladium, 1 550°; platinum, 1 755°; iridium, 2 300° C. If cupellations are finished close to the minimum permissible temperature for gold and silver, 5% of palladium and less of platinum and iridium, will produce a frosted bead. Frosting may also be produced on an ordinary bead by sudden chilling just before the finish, but solid litharge is then generally present on the cupel surface. Larger quantities of the platinum metals produce a rough bead, of leady appearance. HNO_3 dissolves silver and palladium, giving, with the latter, a yellow to reddish brown solution, depending on concentration; 10 c c of dilute HNO_3 solution, containing 0.15 mg of palladium, in a parting cup, has sufficient color to be detected when matched against distilled water. When alloyed with silver, platinum is also soluble in HNO_3, though not completely. The solution is only slightly colored, but may appear dark, due to colloidal platinum. Gold and iridium are not attacked. Hot concentrated H_2SO_4 dissolves silver and slowly attacks paladium; the other metals are unaffected. Aqua regia converts silver to chloride and dissolves palladium, gold, and platinum, but not iridium, except with vigorous treatment or when alloyed with platinum. All these elements, except silver, give colored solutions, which range from yellow through orange to dark red, depending on the combination and concentration of the metals. Oxalic acid precipitates gold from slightly acid solutions, but not the others. Potassium or ammonium chloride in concentrated solutions will precipitate platinum and iridium, but not gold or palladium in its usual condition. Potassium iodide, even in dilute solutions, with gold, gives free iodine; with platinum, brownish red color of potassium platinic iodide; with palladium, a black precipitate soluble in excess. This last reaction is extremely delicate. Dimethylglyoxime precipitates palladium completely, platinum less readily. Silver precipitated as chloride in presence of palladium and platinum is contaminated by these metals. Platinum metals concentrate in lead buttons with gold and silver in the usual method of assay, but unless present in sufficient amounts to affect cupellation or parting are apt to be overlooked. If parting acid is evaporated to small bulk a few hundredths of a mg of palladium will produce a color. When platinum is present the parted residue is likely to be finely divided. Iridium may show as dark spots on the gold.

Methods of assay. For ores and black sands, large samples, 10 to 20 A T, are frequently necessary to yield sufficient metal. Samples are fused in several portions (Art 4 and 5), and lead buttons are combined by scorification. Platinum concentrates are fused

with lead, producing a 1 to 6, brittle, non-homogeneous alloy, which is pulverized for sampling and assay. An approximate knowledge of the metal content is necessary for good results, and the first assay may serve only as preliminary. Alloy for cupellation should contain at least 7 times as much gold as platinum metals, and silver 2.5 times the gold, these being added if necessary. Wrap alloy in sheet lead, cupel, finish rather hot, and allow to remain in muffle 10 min after bead has set. Weigh, flatten to 0.5 mm, and if palladium is absent, part with boiling concentrated H_2SO_4 for 15 min in a flask, inserting a glass rod. (A glass tube 3 mm diam, sealed with a blowpipe flame 2 mm from the end, leaving a small cavity, prevents violent bumping.) (**Note.**—A yellow or orange solution indicates palladium, in which case this procedure is not applicable.) Decant and repeat. Wash twice, transfer to annealing cup, heat, and weigh. If silver ratio has been very high the parted residue will be so finely divided that some may decant with solution; hence, solution must be diluted, filtered on ashless paper, and ignited with main residue. Loss in weight is silver. Inquart with silver 2.5 times the weight of gold, flatten bead and part in dilute (1 to 1) HNO_3, giving two treatments, boiling for 15 min in parting flask, washing twice. Repeat inquartation and parting. Anneal and weigh gold, plus iridium, if present. Loss in weight is platinum. When only small quantity of platinum is present, one inquartation and parting with HNO_3 suffices. To test for iridium dissolve the gold in cold dilute aqua regia (HNO_3 1, HCl 3, water 5, parts); residue is iridium. Results obtained as above are fairly accurate, and if a check of pure metals, duplicating the assay, is run at same time, errors are largely eliminated. When palladium is present, the H_2SO_4 parting is omitted, and the bead treated with HNO_3 as described. In absence of platinum the HNO_3 is nearly neutralized, and palladium is precipitated with dimethylglyoxime, heating for an hour and allowing to stand over night. Filter, ignite, and weigh palladium. When platinum is present, repeat inquartation and parting if necessary, dilute the solution considerably, precipitate silver with HCl, filter, evaporate solution nearly to dryness, add HCl, evaporate again, and repeat. Add a saturated solution of ammonium chloride and allow to stand over night. Filter on ashless paper, wash with 75% alcohol, ignite, weigh Pt. Palladium may be precipitated as described above. Unless errors have compensated, results will be only approximate.

11. COPPER AND LEAD

The following methods for copper presuppose that the metal is soluble in acids. Ores which are not decomposed as described require use of hydrofluoric acid (H F) or a fusion.

"Slop cyanide method" is not suitable for all ores, but where interfering elements such as Ag, Zn, Ni, Co, and certain other elements to a lesser extent, are absent or present in small amounts, the results are remarkably satisfactory. It is most useful when a large number of determinations are to be made on fairly uniform ore, and a high degree of accuracy is not required. Success with this method depends upon a strict uniformity of conditions, as to quantity of chemicals used, volume and temperature of solution, and rate of adding cyanide. Cyanide solution, containing 22 gm c p KCN per liter, or half that amount for low-grade ores, is standardized against an aver sample of the ore to be assayed, a check on the solution being run every few days with the ore samples. Copper content of standard sample is accurately determined by electrolytic or iodide method.

Treat 0.5 to 1 gm of ore with 10 c c HNO_3, of 1 42 sp gr, in a 6-oz copper flask, or 20 c c of dilute (1 to 1) HNO_3 containing 80 gm $KClO_3$ per liter. Boil until all the lower oxides of nitrogen are expelled, add 25 c c of water and enough ammonia to give an excess of 5 to 10 c c, usually 20 c c. Cool, titrate to a faint blue, filter, drain (washing unnecessary), and finish titration of filtrate. The end point is not particularly sharp, and a definite end tint must be selected. For large amounts of work, a dozen burettes will be found convenient, titrating this number of assays at one time, as it is necessary to allow the ferric hydrate to settle when the end point is approached, in order to observe color of solution. Dispensing burettes for reagents are a convenience.

Iodide method, for accuracy, is not surpassed even by the electrolytic method as usually carried out, and, as it requires less expensive and elaborate equipment, is given the preference here. A solution of sodium thiosulphate containing 19 gm per liter, or one-half that strength for low-grade ores, is necessary. STANDARDIZE by dissolving 200 mg of copper, or 100 mg for the weak solution, with 5 c c of (1 to 1) HNO_3, in a 6-oz flask. Boil until lower oxides of nitrogen are expelled, add 5 c c of bromine water and continue boiling until the bromine is eliminated. Add ammonia until the solution just turns deep blue and boil vigorously for a few minutes, or until the ammonia is nearly expelled. Acetic acid is then added, 2 or 3 c c in excess, and if any precipitate fails to dissolve, the solution is boiled. Cool, dilute to 50 c c, add 3 gm potassium iodide and

titrate with hypo until the brown color fades to a light straw. Add starch solution until the solution becomes a decided blue, and finish titration slowly towards the end. (STARCH SOLUTION is made by dissolving 2 or 3 gm soluble starch in 100 c c of boiling water and allowing to cool. Lacking soluble starch, pour an emulsion of ordinary starch into boiling water, boil for a few minutes, and allow to cool. Starch solutions do not keep well, but Low states that a few drops of oil of cassia will preserve them perfectly.)

Decompose 0.5 to 1 gm of ore with 10 to 15 c c of strong nitric acid and 3 to 5 c c of hydrochloric acid in a 6-oz wide-mouthed Erlenmeyer flask, boiling until red fumes disappear. Add 6 to 10 c c of concentrated H_2SO_4 and continue heating until SO_3 fumes appear, then give the flask a circular motion over a free flame until SO_3 vapors are visible only at the mouth of flask. Allow to cool, add 20 c c of water, boil, filter, and wash. The copper may be precipitated from this solution, after diluting to 50 or 75 c c, by boiling with 2 pieces of aluminum foil, 1.5 in square (bend corners 90°, having adjacent corners bent in opposite directions). After complete precipitation, the copper is filtered out, care being taken to prevent oxidation and re-solution. Dissolve copper in beaker, treating aluminum foil with dilute (1 to 1) HNO_3, and titrate as in standardizing.

If a MUFFLE FURNACE is available, the following method is preferred. The filtered sulphate solution is diluted to 50 c c, brought to boil, and a hot 50% solution of thiosulphate is poured in slowly until the precipitate which forms turns black; boil 3 to 5 min, until the supernatant liquid is clear, filter, and wash with hot water. Allow to drain, fold filter, and place in scorifier or clay annealing cup and ignite in muffle at barely red heat until completely oxidised. Transfer to flask, dissolve in (1 to 1) HNO_3, and proceed as in standardizing. The thiocyanate or permanganate (4) method for copper has become deservedly popular and is extensively used on the so-called porphyry ores.

Lead; fire assay. Weigh 10 gm of ore and mix with 25 to 30 gm of the following flux in 10 or 15-gm crucible: Na_2CO_3, 38 parts; K_2CO_3, 38; borax glass, 15; flour, 9 parts. Insert 4 10-d nails, points down, place in furnace at 750° C (cherry red), and hold for 1/2 hour, or until reaction nearly subsides. Then raise temp to 1 050° C (yellow to light yellow), leaving pots in furnace until reaction, which has become more vigorous, ceases, except possibly for slight bubbling around nails. This usually requires about 15 min. The degree and control of heat, varying somewhat for different ores, are of utmost importance. Therefore the temperatures and times given above are more indicative than exact. Results by this method are in the average 1 to 2% low, but in the presence of other reducible metals they may be very high.

12. TIN

Vanning test is unreliable as an assay, on account of the loss of fine tin and the retention of foreign minerals. Shipments supposedly of black tin, from Nigeria, as determined by this method, were found to contain 4% and less of that metal. The test may be of use in ascertaining possible recovery, but this requires a knowledge of the percentage of tin in concentrate produced. SPECIFIC GRAVITY of a tin concentrate is a fair index of the metal content, and when the relation is ascertained for a given product a close approximation may be obtained (5).

Fire assay (6) is applicable only to a high-grade product free from other reducible metals. For ores, this involves a preliminary concentration, with its attendant loss which may reach 30% of the tin. Results on a properly cleaned concentrate are, however, fairly accurate. To assay an ore, crush to 40 or 60 mesh, to liberate the tin minerals, taking care to produce a minimum of slime. Weigh a portion to yield about 10 gm of concentrate, and pan. If the concentrate contains oxidizable material, it is roasted and soluble constituents removed by boiling in strong nitric acid. After drying, mix the concentrate, or a 10-gm portion of it, with 30 gm of pure potassium or sodium cyanide, and place in a crucible having 5 gm of cyanide in the bottom; cover with another 5 gm. Charge is fused for 20 to 30 min at a good red heat (750° C), cooled, and crucible broken. Prills of uncollected tin are recovered by crushing slag and panning.

Volumetric assay. Best wet methods are those of Beringer (7) and Pearce-Low (8).

Pearce-Low method. Ore must be crushed to at least 100 mesh, and portions of 0.2 to 2.5 gm are weighed. Add to 5 times its weight of sodium peroxide, and fuse in a nickel crucible. Should the ore contain readily oxidized material, melted peroxide is allowed to solidify and the ore gradually added to the hot mass. The crucible is then given a circular motion over a burner till a clear melt results. Hold the outer surface of crucible in contact with 50 c c of water in a beaker, till cool; then immerse, and when melt has dissolved remove crucible, and wash. Transfer beaker contents to a 600-c c flask. Add 125 c c HCl, using a portion of this to clean the crucible further, and enough water to

make a total volume of 400 c c. Strips of nickel having a surface of at least 10 sq in are added, the flask closed with a stopper fitted with a small funnel or Bunsen valve, and heated to boiling until tin is completely reduced; 25 min is sufficient for 40 mg and 1.5 hr for 400 mg of tin. Add a crystal of calcite (4 or 5 gm), cool in water. Remove stopper and nickel strips after calcite is completely dissolved, and titrate with iodine solution containing 11 gm iodine and 20 gm potassium iodide per liter for a rich product, or one-half that strength for poor material.

To standardize iodine solution, weigh 0.2 gm pure arsenious acid (As_2O_3), and dissolve in 5 c c of a 20% NaOH solution, dilute, acidify with HCl, neutralise with sodium carbonate, using litmus paper as an indicator; then add 20 c c cold saturated $NaHCO_3$ solution and titrate to a blue color, using starch as indicator. The arsenic standard multiplied by 0.1202 gives value in tin. Iodine solution should be standardised at least once a week.

Interfering elements are bismuth, titanium, and tungsten. BISMUTH is precipitated as metal during reduction and should be filtered out, the reduction being completed with clean nickel. To eliminate effects of TITANIUM, add the weighed ore, small portions at a time, to 5 gm of $KHSO_4$ melted in a platinum or quartz crucible, and fuse for 5 min. Dissolve in 60 c c water plus 7 c c H_2SO_4. Filter, dry, ignite, and fuse residue with sodium peroxide as described. TUNGSTEN, if present, gives a blue precipitate during the reduction with nickel. This is filtered off, and solution again reduced before titration. When tungsten and titanium occur together, acid sulphate fusion is made as described above, and washed residue treated with ammonium carbonate solution, filtered, and washed. This residue is ignited, fused with sodium peroxide, and so on.

13. MERCURY

Whitton's apparatus affords results less accurate than those obtained by wet methods, but is better adapted for the technical assayer. Fig 6 shows apparatus as made by Braun Corp, Los Angeles. Take an amount of ore containing not over 50 mg of Hg (0.15 to 2 gm), mix with 6 gm of iron filings sized between 50 and 80 mesh, freed from grease and dried at a high temp; add 3 gm more as a cover. Clamp in place silver foil and cooling dish filled with water. Heat retort with a low flame for 15 min and allow to cool. Remove silver foil, and weigh it; gain in wt is Hg. The deposit should be white, and, after weighing, may be removed by volatilizing, when the foil is again ready for use.

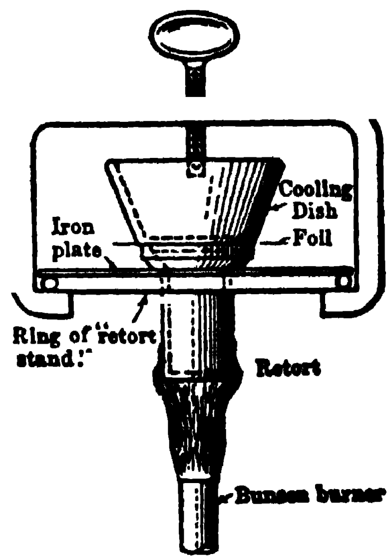

Fig 6. Whitton's Apparatus for Mercury Assay

14. ANTIMONY

Fire method, fusing 10 gm ore mixed with 40 gm KCN and KCN cover, may give approx results by compensation of errors, but is unreliable.

Permanganate method (4). DECOMPOSITION. Treat 0.5 gm ore with 10 to 15 c c HCl, followed by small additions (0.2 gm) of $KClO_3$, heating until all soluble matter is decomposed. Better results are sometimes obtained by following method: Treat ore with 15 c c HNO_3, evaporate to 4 c c, add 20 c c HCl, and evaporate again to 5 c c. If ore contains oxides, insoluble antimony (Sb) is likely to remain in residue. In this case, fuse residue or ore with 15 times its weight of Na_2O_2 in a 2-in iron or nickel crucible, preferably nickel, to complete decomposition (5 to 10 min). Give crucible a rapid circular motion while cooling, thereby causing melt to solidify in thin layer on bottom and sides. Place crucible in beaker of water and heat until melt is dissolved. Remove crucible, wash well, using dil HCl, if necessary, and acidify with HCl. If fusion is made on insoluble residue, add resulting solution to that from original acid treatment of ore.

SEPARATION. Add 3 gm tartaric, dilute to 50 c c, if necessary, filter dilute filtrate to 400 c c, pass H_2S until sulphides are completely precipitated. Filter and wash with H_2S water. Test filtrate by passing more H_2S. Unless precipitate is nearly pure antimony sulphide, wash from filter into beaker without removing paper from funnel and treat twice with hot alkaline sulphide solution, approx 10%, prepared by dissolving the salt in water or passing H_2S through a NaOH or KOH solution. Filter through original

paper, and wash with dilute alkaline sulphide solution. This treatment dissolves As, Sb, and Sn from other sulphides. Acidify filtrate with HCl; filter and wash with H_2S water. If As is absent, treat antimony sulphide as described later. When As is present, wash precipitate from paper into beaker, using a little dilute caustic solution and a minimum of wash water. Dissolve sulphides in HCl and a little $KClO_3$; boil out chlorine and transfer to pressure bottle. (Clamp-top beer bottles are satisfactory.) Add twice the volume of conc HCl (sp gr 1.2), saturate cold with H_2S, cork bottle, place in cold water, bring water to boiling, and allow to stand at least 1 hr. Lacking pressure bottles, ordinary flasks may be used; after saturating liquid with H_2S, close with stoppers and allow to stand cold over night. Filter out arsenic sulphide on double papers (S and S 597). Wash with HCl diluted 2 to 1, and saturated with H_2S. Dilute the filtrate 6 to 8 volumes, and complete precipitation of Sb with H_2S. Filter and wash precipitate with H_2S water. Test filtrate by passing more H_2S. Transfer antimony sulphide to 350-c c flask, removing traces of precipitate from paper with a little dilute caustic solution. Add 5 c c of conc H_2SO_4 and 3 gm $KHSO_4$. Heat over free flame, giving flask a circular motion until water is expelled and precipitate completely dissolved. Cool. Add 25 c c HCl and slowly add 125 c c H_2O, and cool.

TITRATE with $KMnO_4$ solution. The end-point color is not permanent. For ores containing less than 30% Sb, use a solution of approx 2.6 gm $KMnO_4$ per liter; for ores over 30%, use double that strength. To STANDARDIZE SOLUTION, weigh 0.2 gm sodium oxalate for the weak solution, or 0.4 gm for the strong; dissolve in 150 c c water, and 10 c c strong H_2SO_4. Heat to 70° to 80° C, and titrate. Start titration slowly, as the first additions are not decolorized readily and manganese oxide may separate out. Once the reaction has started the permanganate solution may be added as rapidly as desired. Sodium oxalate value multiplied by 0.8963 gives Sb standard. It is more satisfactory to standardize with pure Sb when this can be obtained. Dissolve the metallic Sb in H_2SO_4 and $KHSO_4$, as described for antimony sulphide, and titrate.

Iodide method (10). Final titration is more commonly, and possibly more accurately, done with KI and hypo. Dissolve the antimony sulphide in strong HCl, with small additions of $KClO_3$. Boil off 25 to 50% of solution to remove Cl, and bring acid to a constant boiling strength. Now add enough (1 to 1) HCl to make a total volume of 90 c c and dilute to 600 c c with cold water from which air has been boiled. (The foregoing steps are important.) Add 3 gm KI. Allow to stand a few min, and titrate with hypo solution, adding starch indicator near finish (see Art 11). Reaction between KI and Sb proceeds slowly to completion, and if hypo is added rapidly toward the end the color may be completely discharged only to reappear. Hypo should be added at such rate that the blue color gradually fades to colorless, which condition should hold for several min. Hypo solution is preferably standardized against pure Sb, following the procedure described for antimony sulphide. The solution may be standardized with Cu (Art 11), multiplying results by 0.9549, not by the theoretical factor 0.9454, to obtain Sb standard.

15. COAL (12)

Proximate analysis of coal (9) requires certain standard conditions, as the method is arbitrary and results vary with procedure. Crush sample to 0.25-in size, and if moisture content is likely to change, place a sample in stoppered bottle for a separate moisture determination. The remainder should be air-dried, heating moist sample, if desired, to 70° C, and allowing to cool in the air. Grind a portion to 80 or 100 mesh and place in stoppered bottle.

Moisture. Place 1 gm in crucible and heat to 104° to 107° C for 1 hr. Cool in desiccator, and weigh covered.

Volatile and combustible matter. Heat 1 gm for 7 min in a 20 to 30-gm covered platinum crucible, supported by a triangle 6 to 8 cm above top of a Bunsen burner having a flame 25 cm high.

This test may be made in a muffle at moderate red heat (950° C), using clay or porcelain annealing cup, though results will not be identical. Moisture must be deducted from loss in weight thus ascertained. Fixed carbon = 100 − (moisture + ash + volatile matter).

Ash. Burn off the carbon completely from sample used for moisture determination, using a low heat at the beginning. This operation may be conducted in a platinum crucible over a Bunsen burner, or in a clay or porcelain cup in a muffle.

If, on air-drying, the sample loses an appreciable amount of water, a determination should be made on the original moisture sample, and results corrected. Where this is necessary, and percentage of moisture is not over 10%, correction may be simplified, though not in accordance with the standard method, by weighing out 1 gm minus the

percentage loss of weight in air-drying. Results may then be pointed off in % directly. EXAMPLE: Loss of wt in air-drying, 2.5%. Wt of sample for determination, 0.975 gm. Ash.—Wt of ash, 0.0975-gm; percentage of ash, 9.75%. Moisture.—Loss in wt, 0.0275 gm; moisture in air-dried sample, 2.75%.

16. LABORATORY EQUIPMENT FOR GOLD AND SILVER ASSAYS

Following list includes equipment and reagents for 500 crucible assays. Quantity of fluxes will vary with character of ore assayed and charge used; they are estimated mainly for acid ores on 0.5 A T samples, with excess PbO charge.

- 1 gasolene furnace, with 2-in burner, capac six 20-gm crucibles and 24 1.25-in cupels
- 1 tank outfit
- 4 tongs: for crucible, scorifier, parting cups and cupels
- 1 muffle scraper
- 3 6-hole assay molds
- 1 anvil
- 1 hammer, 2-lb, for Pb buttons
- 1 cupel tray
- 2 bead trays, 24-hole
- 1 button pliers
- 1 button brush
- 1 kerosene or gasolene stove for parting
- 4 asbestos stove mats
- 1 sheet of asbestos, 3/16-in, 40 by 40 in
- 50 porcelain parting cups, 1.5-in
- 2 hand cupel molds, 1.25 and 1.5-in or Iler's cupel machine, dies 1.25 and 1.5-in
- 200 Battersea or other soft-clay 20-gm crucibles
- 50 scorifiers, 2.5-in and 10 3-in, Bartlett shape
- 3 Battersea annealing cups, A
- 3 Erlenmeyer parting flasks, 50-c c
- 1 blowpipe
- 1 alcohol lamp
- 6 5-in roasting dishes
- 1 still and condenser
- 1 demijohn for distilled water
- 3 ft 5/8-in glass tubing, 1.5-mm wall
- 2 ft 3/16-in glass tubing, 1.25-mm wall
- 3 ft 3/8-in rubber tubing
- 2 pinch cocks
- 3 32-oz wash bottles
- 2 6-in and 2 2-in watch glasses
- 24 test tubes, 6 by 3/4-in
- 1 test-tube brush, 9 by 3/4-in
- 1 500-c c graduated cylinder
- 1 nest beakers, 100 to 600-c c
- 2 funnels, 1.5 and 6-in
- 2 5-pint acid bottles, empty
- 1 pkg filter paper, 8 in
- 1 oz c p Ag foil
- 40 lb soda ash
- 50 lb litharge, silver-free
- 10 lb borax glass
- 3 lb niter
- 2 lb silica
- 3 lb test lead; 1 lb lead foil, silver-free
- 40 lb bone ash
- 14 lb nitric acid c p
- 1 lb hydrochloric acid c p
- 1 lb sulphuric acid c p
- 1 lb ammonia
- 1/4 lb charcoal, powdered
- 6 charcoal sticks for blowpipe
- 1/2 lb reddle, for marking
- 1 small ore crusher
- 1 buckboard and muller
- 1 Jones sampler, 8-in, or 9-in riffle sampler
- 1 nest 8-in tin sieves, 20-40-60-100-mesh
- 1 sieve, 10-mesh, 12-in, wood frame
- 2 brushes, 4-in for buckboard
- 2 doz tin sample pans, 6-in
- 500 paper sample bags, 4 by 7-in
- 1 scoop, 4.5 by 5-in, flat
- 2 7-in steel spatulas
- 1 horn or aluminum spoon
- 1 camel's hair brush, flat, 1.5-in, and pencil brush
- 1 yd rubber mixing cloth
- 1 piece of surfaced mixing cloth, 50 by 50-in
- 1 spring balance for weighing stock flux
- 1 button balance
- 1 pulp balance
- 1 set assay and gram weights
- 1 set button weights and riders, 1 mg to 1 gm

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SECTION 31

TESTING OF ORES

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(**Note.**—The text herein on Amalgamation and Cyanidation is retained from original Sec 31, of first edition, by J. E. Clennell and Edward K. Judd.)

Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

TESTING OF ORES

Testing involves the determination of: (*a*) best method for treating a given ore; (*b*) practicability and best method of operating a given process. In general, testing for a process requires the application of several processes to the same ore, while testing of processes involves subjection of different ores to the same process. The work requires painstaking attention to detail, close and accurate observation and record of performance, and incessant inquiry into causes. Correct interpretation of results requires wide experience in testing and mill operation, and a healthy balance between pessimism and optimism. Though the best laboratory results can often be bettered in mill operation, it may be impracticable in the mill to expend the care lavished on laboratory tests; mill operation is seldom economically subject to as close control of quantities treated, and purity and uniformity of substances used, as is the laboratory. This is especially true of hydro-metallurgical processes and flotation.

Time is usually an element in testing work. On the one hand is the demand for quick results; on the other, necessity for thorough investigation before announcing conclusions. Clear recognition of this situation should be a sufficient guide.

Principal tools for testing are: assay, testing sieve and microscope. Others are: the specific-gravity flask, heavy solutions, and small-scale replicas of or substitutes for mill machines, including hammer and anvil for estimating crushing resistance; hand jig, pan, batea and plaque for gravity concentration, and hand magnet.

1. OUTLINE OF PROCEDURE

Steps: (*a*) Obtain a proper sample; (*b*) Determine qualitative mineralogical composition; (*c*) Determine content of valuable mineral (assay); (*d*) Determine distribution of valuable mineral (sizing-assay test); (*e*) Determine aggregation of valuable mineral (microscopic examination and sizing-sorting-assay test); (*f*) Study existing flow-sheets for treatment of similar ores; (*g*) Devise a tentative flow-sheet for laboratory procedure; (*h*) By following (*g*), procure material representing the feed to each machine, and treat these materials in batches on the indicated machines, to determine best possible conditions of treatment and corresponding results; (*i*) Construct a metallurgical balance sheet of results of the batch testing (Art 8, Table 2); (*j*) Make a continuous run to confirm the results of (*i*).

Continuous run. A testing laboratory rarely has sufficient equipment, properly balanced as to size, to make a continuous run on a scale that will give reliable indications of mill performance, except to an experienced interpreter, and the greater his experience the greater his doubt as to his interpretation. Hence, if the size of the mill installation justifies the cost, the continuous run should be made in a pilot mill, built at the mine, fed with freshly-mined ore as nearly as possible representative of what the final mill will get, and run long enough to answer definitely, with minimum interpretation, what will be the performance of each machine and of the whole mill. Cost of a pilot mill is usually a small fraction of that of the main plant, is fully justified as insurance and is quickly saved, if the main mill is thereby put more quickly into regular operation.

When the contemplated mill is too small to justify cost of a pilot mill. the continuous run or runs must be made in the laboratory; in which it is usually impossible to duplicate mill conditions as to middling circulation and water reclamation. But, except for questions of sedimentation and incrustation in the water channels and their effects, reclaimed water has no effect on performance of gravity concentration. Its probable effect in flotation or in hydro-metallurgical processes can be investigated by small-scale tests, supplementing the continuous run. On the other hand, middling must be circulated in the continuous run and eliminated from the final products. If the middling circuit differs, as it does in almost all cases, from that to be expected in the operating mill, its effect must be closely studied, for making allowance in the final design. This is by far the most difficult part of testing and interpretation.

Samples must be correct, and of such wt (50–1 000 lb) as is necessary for all the initial small-scale laboratory tests. The smaller amount usually suffices when enough is already

known concerning the ore to indicate fine grinding and a simple treatment like cyaniding or flotation; the larger amount is necessary when nothing is known or when what is known indicates a complicated flow-sheet.

Mineralogical composition of the ore must be approximately known as the first step in the testing work (1-5). Gold and silver ores usually require an assay; complex base-metal ores demand microscopic examination.

Sizing analysis comprises quantitative separation of a mass of material of various sizes into a number of grades, each having a relatively small size-interval between largest and smallest particles. Means of grading: (*a*) screening; (*b*) differential settling, in water or in air.

2. STANDARD TESTING SIEVES

These are special sets of screens (usually 8-in diam), with pan and cover, or telescopic for field work. Apertures in successive sieves vary according to a regular rule.

The TYLER SERIES (best known) begins at 0.074 mm (0.0029 in), the aperture of each coarser size being obtained from that of the preceding by the multiplier $\sqrt{2}$. Full series in mm is: 0.074, 0.104, 0.147, 0.208, 0.295, 0.417, 0.589, 0.833, 1.168, 1.651, 2.362, 3.327, 4.699, 6.690, 9.423, 13.33, 18.83, 26.67. Proposed ASTM STANDARD series comprises a coarse and a fine sub-series. Coarse screens are the nearest usual fractions to a $\sqrt[4]{2}$ series, progressing from 1 in through the range from 0.25 in to 4 in, thus: 4, 3.5, 3, 2.5, 2, 1.75, 1.5, 1.25, 1.0, 0.875, 0.75, 0.625, 0.5, 0.438, 0.375, 0.312, 0.25. Fine series adopts U S STANDARD SCALE. It progresses on a $\sqrt[4]{2}$ ratio from 1 mm through the range from 5.66 mm to 0.037 mm, thus: 5.66, 4.76, 4.00, 3.36, 2.83, 2.38, 2.00, 1.68, 1.41, 1.19, 1.00, 0.84, 0.71, 0.59, 0.50, 0.42, 0.35, 0.297, 0.250, 0.210, 0.177, 0.149, 0.125, 0.105, 0.088, 0.074, 0.062, 0.053, 0.044, 0.037. Micron designation (1 mm = 1 000 microns) is customary for fine screens. Both of preceding series derive from RITTINGER (base = 1.0 mm, multiplier = $\sqrt{2}$) and RICHARDS (base = 1.0 mm, multiplier = $\sqrt[4]{2}$). INSTN MIN & MET (London) SERIES has no regular basis of variation. The rating is in number of meshes per linear in; the list is 5, 8, 10, 12, 16, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 150, and 200-mesh. Aperture, in, is the reciprocal of twice the mesh designation; thus, 100-mesh screen has 1 ÷ 200-in aperture. Other series proposed are: DEKALB, base = 0.003 in, and, for any other size, add $0.001n$ in (n being the number of screen in the series); HOOVER, base = 1.0 in, constant multiplier (or divider) = $\sqrt[3]{2}$. SIEVES FOR COAL TESTING, round hole, base = 1 in, constant multiplier (or divider) = 2 (Sec 35).

3. METHODS OF SCREEN ANALYSIS

Crude analyses, suitable for ordinary work, are made by placing a weighed dry sample on the top or coarsest screen of a nest, shaking the nest (1-2 min), until most of the undersize has passed the coarse screens; then removing 1 screen at a time, beginning at top, shaking each separately over a pan until the amount passing through in 1 min is less than 1% of that remaining on the screen. Undersize is added to top screen of the remaining nest. On coarse screens (0.75-in or larger) pieces near the screen size may be tested and put through, if possible, by hand. Oversizes and final undersize should be weighed and kept separate until all have been weighed and the total checked against original wt. Weighings should be accurate to within 1%, and total wt of the grades should check original wt of sample within 1%. Screens should be so shaken as to cause the material to travel slowly in a thin sheet over whole surface of the sieve, and jarred at the same time to cause the cloth to vibrate gently in a direction perpendicular to its plane.

When fine dry material is caked, it may be broken by rubbing on the sieve with bristle brush or rubber cork. Cut-metal washers are sometimes placed on the finer sieves while shaking, but this wears and distorts fine screen cloth and, with soft material, produces an excessive amount of the finest size.

For testing coarse material, sample must be large, due to impossibility of cutting down accurately. But, except for very accurate work, a carefully riffled sample of −0.12-in material weighing 250 to 300 gm is large enough. Considerable time can be saved by sifting a large sample roughly on the 0.12-in screen, cutting down the undersize to 250 to 300 gm and screening this on the finer screens, while oversize is re-screened, beginning with coarsest screen. On re-screening, the undersize from 3.327-mm may usually be safely considered as oversize on next finer screen and so treated in calculating the redistribution (24).

Standard method for hand sieving. Abstract of method recommended by ASTM: SIEVES should be 8-in diam with well-fitting pan and cover, free of crevices that will hold particles; sieve-scale ratio, not greater than $\sqrt{2}$; finest sieve, 400-mesh (0.037-mm). PREPARATION OF SAMPLE. Dry at 110° C, mix well and riffle out a sample to within 10% of the accompanying figures.

If largest particle is — mm	Sample wt should be — gm	If largest particle is — mm	Sample wt should be — gm
16.00–11.32	40 000	2.00–1.00	500
11.32– 8.00	12 500	1.00–0.50	250
8.00– 5.66	5 000	0.50–0.25	100
5.66– 4.00	2 000	0.25–0.00	50
4.00– 2.00	1 000		

General procedure. Sieve wet through finest sieve, with water containing little or no dissolved solids. If the material is coarse, wash out all slime by vigorous decantation and put only decanted material on screen. Dry undersize and oversize at 110° C. Add loss, if any, to undersize. Sieve dried oversize first on finest screen and the oversize on successively coarser screens.

In this operation a rough end-point is reached when undersize for one-min shaking is less than 0.1% of original wt of charge. At this point remove oversize, and brush both sides of screen cloth to remove dust and lightly-held particles. Return oversize, and sieve further until a one-minute undersize is less than 0.05% of original wt of charge; remove oversize, brush screen as above and re-screen oversize one minute. If undersize of this screening is again less than 0.05% of original charge, return to oversize; otherwise add to undersize and repeat until two successive one-minute sievings as above yield less than 0.05% of original charge as undersize. MANIPULATION: Work over a smooth paper. Hold sieve with pan and cover in place, slightly inclined in one hand; shake with an amplitude of 6–8 in, about 150 strokes per minute, terminating each stroke by a gentle blow against the other hand. Turn sieve about $^1/_6$ rev every 25 strokes. With 6-in diam sieves, use 200 strokes per minute and turn $^1/_8$ rev every 25 strokes.

Wet samples are best handled by jigging on finest screen at the surface of water in a pail, and washing with a fine jet until practically all the slime has passed. Oversize is then dried and re-screened on a nest including the finest screen, and undersize of finest screen on the dry sifting is added to dried undersize from wet screening. This procedure is quicker and more accurate than preliminary drying and dry sifting, due to difficulty in breaking up slime cake formed in drying. But, as sampling wet pulp for the screen sample is difficult, drying, riffling and wetting to break slime cake may be justified.

Mechanical testing-sieve shakers. It requires 1–3 hr to sift a 200-gm sample containing 30 to 50 gm of −200-mesh material by manual shaking. Mechanical shaking will reduce the time to from 30 to 45 min, and, while the sieves are being shaken, the operator is free for other work. If mechanical sieving is used as a substitute for hand sieving in the proposed standard method, end points for each sieve should be tested by hand as therein prescribed.

Ro-tap testing-sieve shaker (W. S. Tyler Co), for 8-in sieves, consists of a movable cage with base and top plate, between which a nest of 13 half-height sieves, or 7 full-height with pan and cover, can be mounted and subjected to rotary sifting motion, while a lever strikes the top plate once per rev and produces vibration of the screen cloth. A time switch on the motor is useful. Duplicate samples, sifted for equal periods of time on the same or different machines, check well within limits of sampling error. If total amount of dust in the sample is important, the sieves (after removal from the shaker) should be brushed around the inside of the rims with a soft brush and shaken individually for a short time as in hand sifting, as a little dust collects around edges of coarser screens during mechanical shaking and does not pass through.

A simpler and cheaper apparatus comprises a vertical motor with eccentrically loaded flywheel on the rotor shaft mounted between two parallel hardwood sticks about 10 in apart and 44 in long, which are joined at their lower ends by a base plate for the screen pan and carry between this and the motor a vertically slidable recessed plate that fits over the cover of the nest of screens. The whole structure is suspended by a $^3/_4$-in rod from a suitable support. The motor revolves 900 rpm and vibrates the frame and contained screens through a small amplitude.

4. ELUTRIATION

This operation may be resorted to for further sizing of finer-than-sieve sizes. It consists in grading finely divided solid matter according to size, by taking advantage of differences in settling rate of different-sized particles in air or water. There are two general

methods: (*a*) allowing solids to settle freely in still water for varying periods; (*b*) subjecting the solids to rising air or water currents of different velocities.

Decantation is the simplest method. Place a 10 to 20-gm sample in a tall beaker of 300–500 cc capacity, fill with water to a predetermined depth, stir thoroughly, then allow the beaker to stand for such a time that the time in seconds, divided by depth of water in mm, equals a predetermined settling rate corresponding to the largest-sized particle desired. Required time may be read from Fig 1. Next, pour off supernatant liquid with the non-settled solids in suspension, re-fill to the mark and again allow settlement for same time; repeat until the appearance of supernatant liquid indicates that all the finest slime has been removed. Combine decanted portions, take a sample, while wet, for microscopic determination of size (Art 5), then dry at 110° C, cool and weigh. Re-fill the beaker to the mark with fresh water, and repeat above operations, allowing a shorter time for settling corresponding to the next coarser grade desired, and repeating with this settling time until the supernatant liquid, at end of period, shows no solids in suspension. Repeat as above until the desired number of grades is obtained. If, in using tap water, there is obvious chemical action (formation of gelatinous precipitates), use distilled water. If this does not remedy the trouble, weak solutions of acid or alkali may be used, but the presence of the added chemicals or their salts in the dried solid must be recognized. Organic liquids, as kerosene, benzene, acetone, may, with some dry pulps, prevent flocculation of fine sizes, but the chart velocities (Fig 1) can not be used. If microscopic sizing of the different grades is omitted, overlapping of sizes in the different grades must be borne in mind, because not all the solids begin settling from the surface of the liquid in the beaker, and some fine particles that begin to settle from a point below the surface will reach bottom while coarser particles starting above them are still in suspension. Repetition lessens this inaccuracy, but does not eliminate it.

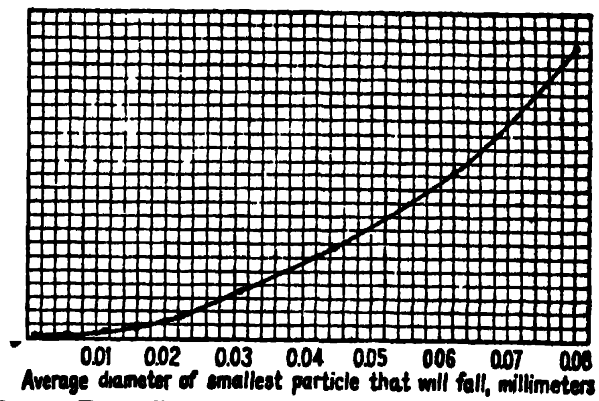

Fig 1. Free-falling Veloes of Quartz (after Richards)

Elutriation by rising currents is done by subjecting the material to be graded to rising currents of different velocities, and collecting the material lifted by each current separately. General form of apparatus is the same, whether the fluid is liquid or gas. It consists of a series of cone-bottomed cylinders of increasing diam, closed at the top and pipe-connected so as to permit continuous fluid flow from the smallest through largest cylinder. Provision must be made to collect solids deposited in successive chambers. When air is the fluid, a ball or the like rests on the bottom inlet in such a way as to give annular entry to the fluid stream. Haultain's infra-sizer, using air for fluid, is the best today; it produces granular grades to 10-micron size that, when magnified, look like screen grades.

Interpretation of elutriation tests. Settling rate of solids in fluids depends upon: sp gr and shape of particles, degree of packing of particles in the sorting column and the uniformity of sorting current. If the sample sorted comprises grains of different sp gr, the grades always contain grains of highly divergent sizes; also, even if all the material is of same sp gr, the coarser grades will contain more or less finer grains carried down mechanically or by eddy currents; the finer grades will contain flat scaly particles that settle much more slowly than their aver "diam" would indicate. Hence, the only sure way to determine aver size of grains in a given grade is by microscopic measurement. If desired, this may be made separately on heavy and light particles and on flat and rounded particles, and the analysis re-cast on basis of these measurements.

5. TESTING WITH THE MICROSCOPE

Microscope is used: (*a*) to aid in mineral and rock identification and classification; (*b*) to aid in study of mineral occurrence and the characteristics upon which the treatment scheme depends; (*c*) in quantitative mineralogical analysis; (*d*) in sizing analysis. The microscope may save much time and money in qualitative determinations, which frequently will demonstrate the uselessness of more elaborate quantitative investigations.

Microscopic mineralogy and petrography are familiar to the geologist and the literature is excellent and large. Principal apparatus required is a petrographic microscope. Samples for examination are prepared either as thin sections or pulverized fragments. For details of methods and interpretation of thin sections, and study of pulverized fragments see Bib 1, 3, 4, 5.

Mineragraphy is the study of opaque minerals, including native metals, most sulphides, certain base-metal oxides and the like, by the metallographic microscope. The methods, including study of polished sections by reflected light, etching and various microchemical tests, hardness tests, etc, are much more effective in identification of opaque minerals than either of the ordinary petrographic methods (6–11). See also Sec 1.

Particle size. Of greater importance in testing than identification and the information concerning ore genesis afforded by mineragraphic work is the knowledge given by the polished section as to particle size and method of occurrence of the valuable mineral. The first fact can be established with coarse-grained ores by a sizing-sorting-assay test (Art 8); for such ores, the second fact is of minor importance, but with finely disseminated and complex ores the polished section quickly gives essential information obtainable in no other way.

Determination of grain size fixes the size to which the ore must be crushed to free the valuable mineral; upon this and the kind of mineral and gangue the chief elements of a tentative flow-sheet can be founded. The appearance of the edges of sulphide-mineral grains and of cracks traversing them will tell whether any alteration likely to affect flotation has occurred. Inclusions of worthless or deleterious substances that would lower the grade of concentrate, as silicates between the laminae of graphite grains, blende in galena and the like, are immediately apparent in a polished section. The degree of admixture of sulphides in complex ores can be readily studied, and one or two properly chosen sections often show the hopelessness of attempts at mechanical separation. Time and money can thus be saved, and ores should never be tested beyond the preliminary assay stage without such microscopic examination.

Quantitative mineralogical analysis is useful primarily in studying the products of concentration. The first step is identification of important minerals and mineral groups. The analytical work is done with low magnification, since identification of valuable mineral, accompanying heavy mineral or minerals, and the gangue minerals as a group, is easy. Failing this, analysis must be preceded by identification, for which Rogers' crushed-fragment method with the petrographic microscope is best (1).

Mill pulps may also be mounted in a matrix of bakelite or similar plastic, by stirring into a viscous mass thereof softened by heat, and, when the mass is resolidified, may be polished for mineragraphic examination. For quantitative applications of this technique, see Bib 16b, 16c.

Microscopic sizing analysis. Sizing by screens below 200-mesh (0.074 mm) is highly inaccurate, because of imperfections in screens and the difficulty of getting material through them. Elutriation alone is similarly undependable, due to differences in settling rates induced by differences in shape and sp gr of particles. Microscopic sizing has of late been highly developed, to supplement or supplant settling methods. With proper equipment and good technique, microscopic sizing can be carried to less than 1 micron (1 micron = 0.001 mm), and the aver size of very fine products stated with a precision of 0.1 or 0.2 micron.

Indirect measurement, described by Green (17), is adapted to very fine materials of greater uniformity than the non-metallic natural fillers measured by Weigel (18). It involves making a slide, photographing one or more fields, projecting the negative on a screen and measuring and counting the grains thereon.

For details of more elaborate methods of screen sizing, elutriation and microscopic analysis, see Taggart, Handbook of Mineral Dressing.

6. AVERAGE SIZE OF PARTICLES

Mineral particles produced by crushing and grinding have an almost infinite variety of shape and size. No accurate numerical expression of the dimensions of a single particle nor of the aver of a group is possible; only an approximation is feasible, ordinarily expressed as a single number, as though the particles were spheres or cubes. This number is called the DIAMETER or SIZE of an individual particle, or the AVER DIAM or AVER SIZE of a group of particles.

Diameter of a particle. The fundamental assumption of particle measurement is that it has three principal axes at right angles and that its dimensions are completely stated when the distances between the intercepts of the surface on the respective axes are given. Starting with this assumption, which is, on its face, only a crude approxima-

tion except in the case of materials with cubical cleavage, averaging of the three principal dimensions into a single figure is attempted by one of several different methods, as follows:

$$d = b, \quad (1); \qquad d = (l + b) \div 2, \quad (2); \qquad d = (l + b + t) \div 3, \quad (3)$$

$$d = \sqrt{lb}, \quad (4); \qquad d = \sqrt[3]{lbt}, \quad (5)$$

$$d = \frac{\sqrt{2lb + 2bt + 2lt}}{6}, \quad (6); \qquad d = \frac{3lbt}{lb + lt + bt}, \quad (7)$$

where d = "diameter," and l, b and t respectively the distances between the intercepts of the surface on the long, intermediate and short axes; in common parlance, length, breadth and thickness of particle. The significance of the first 5 approximations is immediately apparent; the sixth gives the edge of a cube, the total surface of which equals the total surface of a rectangular parallelopiped of dimensions equal to the principal dimensions of the particle. The seventh is the harmonic mean of the dimensions. When screens are used alone, assumption (1) is adopted perforce; for elutriation, without subsequent microscopic measurement of fractions, assumption (5) is necessarily involved; when individual particles are measured, any one of the seven may be adopted, but (2) and (4) are most commonly used.

Aver diam is computed by averaging the mean or equivalent diam of a number of particles. Perrott and Kinney (19) suggest the following:

1. Arithmetical mean $D = \frac{d_1 + d_2}{2}$
2. Geometrical mean $D = \sqrt{d_1 d_2}$
3. Laschinger's mean $D = \frac{d_1 - d_2}{\log_e d_1 - \log_e d_2}$
4. Mellor's mean $D = \frac{\sqrt[3]{(d_1 + d_2)(d_1^2 + d_2^2)}}{4}$
5. Mean of form $D = \frac{4}{5}\frac{d_1^5 - d_2^5}{d}$.
6. Von Reytt's mean $D = 0.435(d_1 + d_2)$
7. Number mean $D = \frac{\Sigma nd}{\Sigma n}$
8. Length mean $D = \frac{\Sigma nd^2}{\Sigma nd}$
9. Surface mean $D = \frac{\Sigma nd^3}{\Sigma nd^2}$
10. Volume mean $D = \frac{\Sigma nd^4}{\Sigma nd^3}$

To these should be added:

11. $D = \sqrt{\frac{\Sigma nd^2}{\Sigma n}}$
12. $D = \sqrt[3]{\frac{\Sigma nd^3}{\Sigma n}}$

where D = mean diam; d_1 and d_2 = max and min mean particle diam; d = successive mean particle diam in a sizing operation, and n = numerical frequency of the corresponding d. For discussion of the above, see Taggart, Handbook of Mineral Dressing.

Table 1. Comparison of Methods of Calculating Average Diam

Microscopic analysis								
Diam, microns (d)...	60	50	40	30	20	10	5	2
No of particles (n)...	87	100	156	660	1 750	6 200	25 600	155 000
Percentages								
Σn ...	0.05	0.05	0.1	0.3	0.9	3 3	13 5	81.8
$\frac{nd}{\Sigma nd}$ ...	0.9	0.9	1.1	3.4	6.1	10.7	23.7	53.2
$\frac{nd^2}{\Sigma nd^2}$ ...	7.8	6.3	6.3	14.9	17.5	15 5	16.1	15.6
$\frac{nd^3}{\Sigma nd^3}$ ...	22.4	14.9	11.9	21.2	16.7	7.4	3.8	1.5

Formulas	Microns
1. $D = (d_1 + d_2) \div 2 = (60 + 2) \div 2$	= 31
2. $D = \sqrt{d_1 d_2} = \sqrt{60 \times 2}$	= 11
3. $D = (d_1 - d_2) \div (\log_e d_1 - \log_e d_2) = (60 - 2) \div (2.303\,[1.7782 - 0.3010])$	= 17.0
4. $D = \sqrt[3]{(d_1 + d_2)(d_1^2 + d_2^2)} \div 4 = \sqrt[3]{(60 + 2)(3\,600 + 4)} \div 4$	= 38.2
5. $D = 4(d_1^5 - d_2^5) \div 5(d_1^4 - d_2^4) = 4(60^5 - 2^5) \div 5(60^4 - 2^4)$	= 48

Formulas, continued — Microns

		Microns
6.	$D = 0.435(d_1 + d_2) = 0.435(60 + 2)$	$= 27$
7.	$D = \Sigma nd \div \Sigma n = (0.05 \times 60 + \ldots 81.8 \times 2) \div 100$	$= 3.0$
8.	$D = \Sigma nd^2 \div \Sigma nd = (0.9 \times 60 + \ldots 53.2 \times 2) \div 100$	$= 7.0$
9.	$D = \Sigma nd^3 \div \Sigma nd^2 = (7.8 \times 60 + \ldots 15.6 \times 2) \div 100$	$= 21.0$
10.	$D = \Sigma nd^4 \div \Sigma nd^3 = (22.4 \times 60 + \ldots 1.5 \times 2) \div 100$	$= 36.4$
11.	$D = \sqrt{\Sigma nd^2 \div \Sigma n} = \sqrt{(87 \times 60^2 + \ldots 155\,000 \times 2^2) \div (87 + \ldots 155\,000)}$	$= 4.6$
12.	$D = \sqrt[3]{\Sigma nd^3 \div \Sigma n} = \sqrt{(87 \times 60^3 + \ldots 155\,000 \times 2^3) \div (87 + \ldots 155\,000)}$	$= 7.6$

Comparison of methods. Perrott and Kinney (19) give the microscopic sizing analysis shown in Table 1. Comparison of the aver diam computed by the different formulas shows clearly the uncertain meaning of this term, and necessity for stating the method of calculation when giving a numerical result. The result by formulas 1 to 6 is unaffected by the amounts of any of the grades. Hence, each will give the same result for any mass of mixed size grains, irrespective of the size composition of the mass, provided the largest and smallest particles are in each case of same size.

This fact condemns these formulas for anything but the crudest work. Formulas 9, 10 place too much weight on the coarser sizes, and, with No 8, give results that are meaningless in terms of the diam of ideal particles that could be substituted for the actual particles. But, No 9 is useful when specific surface is important. No 7 gives the easiest calculations and the physical significance is most readily visualized. It weights the finest particles most heavily and therefore gives an aver that leans toward the fine end. No 12 weights the coarsest particles most heavily, and consequently the results lean toward the coarse end. No 11 is intermediate between the other two and would, on that score alone, seem preferable; it is distinctly so when surface is the valuable property of the material.

7. PLOTTING SIZING TESTS

Sizing tests are best compared and their significance understood by graphs. Common methods are: DIRECT PLOT, particle size against percentage weight; CUMULATIVE DIRECT PLOT, particle size against cumulative weights of the entering and the coarser (or finer) sizes; DIRECT LOGARITHMIC PLOT, weights as ordinates against logarithms of particle sizes; CUMULATIVE LOGARITHMIC PLOT, cumulative weights against logarithms of particle sizes; PERCENTAGE APERTURE PLOT, cumulative weights against apertures expressed as percentages of the aperture passing all of the sample; RECIPROCAL PLOT, weights, cumulative or direct, against reciprocals of particle size; FREQUENCY CURVES, number of particles (or a function thereof) against size (17).

8. SIZING-SORTING-ASSAY TEST

This test affords an excellent basis for estimating the recovery possible to make on an ore, and of the flow-sheet needed for treatment.

A sizing test is made of a sample ground to approx the size at which clean mineral or clean tailing results, separating each grade by approx means into concentrate, middling and tailing, and weighing and assaying the products. Hand picking can be used for separation to the oversize on a 1-mm screen, and panning on the finer sizes; or, if the bulk of the finer sizes is large enough, and the ore amenable, they may be combined into a sample for a flotation test.

Table 2 shows results of a test thus run and the method of calculation. Assuming that the concentrate from retreating middling separately would average 65% Pb and the tailing 0.4%, the middling recovery would be 97.8%. This assumption was justified in the test, because microscopic examination of the middling showed that practically all mineral would be free at 100 mesh, that concentrate from 14% feed would be somewhat richer than that from 5% feed, and that assays of tailings from both feeds would be roughly in proportion to the feed assays. Such middling retreatment would add 3.710 ton concentrate containing 2.4095 ton lead, and 13.386 ton tailing containing 0.0535 ton lead. If doubt exists as to behavior of the middling on retreatment, it should be ground and treated.

Table 2. Sizing-sorting-assay Test on Minus 10-mm Lead Ore

No of line	Screen mm	Feed Wt, gm	Feed Ton per 100 ton	Feed Assay, % Pb	Feed Ton Pb per 100 ton	Feed % of total Pb content	Concentrate Wt, gm	Concentrate Ton per 100 ton feed	Concentrate Assay, % Pb	Concentrate Ton Pb per 100 ton feed	Middling Wt, gm	Middling Ton per 100 ton	Middling Assay, % Pb	Middling Ton Pb per 100 ton	Tailing Wt, gm	Tailing Ton per 100 ton	Tailing Assay, % Pb	Tailing Ton Pb per 100 ton
1	6.680	55.6	2.02	3.14	0.0634	1.20	.....	.....	....	......	19.3	0.702	8.62	0.0605	36.3	1.318	0.22	0.0029
2	4.699	213.0	7.73	3.41	0.2635	5.00	0.44	0.012	86.8	0.0104	59.4	2.164	11.25	0.2434	153.2	5.554	0.18	0.0100
3	3.327	315.5	11.44	3.66	0.4190	7.96	1.09	0.030	86.7	0.0260	81.3	2.959	12.60	0.3726	233.1	8.451	0.24	0.0203
4	2.362	465.5	16.90	3.85	0.6510	12.36	4.80	0.132	80.3	0.1059	104.8	3.814	13.70	0.5227	355.9	12.954	0.17	0.0220
5	1.651	442.0	16.09	4.73	0.7615	14.46	5.93	0.163	82.0	0.1336	91.7	3.329	18.10	0.6025	344.4	12.598	0.20	0.0252
6	1.168	248.5	9.04	4.75	0.4295	8.15	4.33	0.119	81.6	0.0972	46.8	1.701	18.59	0.3163	197.4	7.220	0.22	0.0159
7	0.833	85.0	3.08	4.97	0.1531	2.90	2.11	0.058	82.4	0.0476	14.1	0.513	18.86	0.0967	68.8	2.509	0.35	0.0088
8	0.589	102.4	3.72	6.10	0.2268	4.30	5.02	0.138	81.9	0.1127	15.7	0.571	18.47	0.1055	81.7	3.011	0.29	0.0087
9	0.417	84.6	3.07	7.08	0.2175	4.12	6.55	0.180	81.4	0.1469	11.8	0.430	15.02	0.0646	66.2	2.460	0.24	0.0059
10	0.295	80.7	2.93	8.02	0.2352	4.46	8.55	0.235	79.7	0.1874	10.2	0.371	11.36	0.0421	61.9	2.324	0.23	0.0054
11	0.208	61.2	2.22	9.06	0.2012	3.81	8.47	0.233	76.2	0.1775	6.9	0.252	8.19	0.0206	45.8	1.635	0.19	0.0031
12	0.147	80.0	2.90	10.35	0.3003	5.68	13.73	0.378	74.5	0.2815	8.0	0.290	5.34	0.0155	58.3	2.332	0.14	0.0033
13	0.104	73.9	2.68	9.89	0.2648	5.02												
14	0.074	55.4	2.01	8.00	0.1608	3 04	} 57.80	2.100	63.2	1.3254	....	.....	.....	....	461.5	16.760	0.15	0.0252
15	0.074	390.3	14.17	6.53	0.9250	17.54												
16	Total	2753.3	100.00	5.27	5.2726	100.00	118.82	3.778	70.1	2.6521	470.0	17.096	14.42	2.4630	2164.5	79.126	0.20	0.1567
Distributed middling.......				....	......	.....	......	3.710	65.0 (a)	2.4095	....	.....	.....	....	......	13.386	0.40 (a)	0.0535
Calculated totals...........				....	......	.. ..	......	7.448	67 6	5.0616	. ..	.....	. .	....	....	92.512	0.227	0.2102

(a) Assumed

Recovery, from weights $= \frac{5.0616}{5.2726} = 96.0$

" " formula $= \frac{67.6(5.27 - 0.227)}{5.27(67.6 - 0.227)} = 96.0$

Ratio of concentration $= \frac{100}{7.488} = 13.3$

" " by formula $= \frac{67.6 - 0.227}{5.27 - 0.227} = 13.4$

9. TESTING OF MACHINES

Crushing. Behavior of ore in coarse and intermediate crushing and in grinding should be investigated, because the results of breaking apart mineral aggregates by this crushing may differ entirely from those due to breaking down the individual mineral grains by grinding. The safest method is to make parallel tests on an ore of known crushing characteristics and the unknown ore; these tests being made with machines of the types used in mill treatment of the known ore and to be used for the unknown. Laboratory crushing tests used as a basis of mill installation are, in any case, about 5% manipulation and 95% interpretation based on experience, and an experienced investigator, with a few lumps of the unknown and some known ores and a hammer and anvil, can tell more than an inexperienced man with a complete laboratory equipment.

Accessory apparatus. Testing an ore for performance in hydraulic and mechanical classifiers, thickeners and filters, dryers and the like is usually done in small-size substitutes for the essential parts of the various machines. Experiments require much skill in manipulation, and experience in interpretation. They form part of the general testing campaign for determining a flow-sheet and should not be attempted as part of preliminary field work. For details of methods, see Taggart, Handbook of Mineral Dressing.

10. HAND PICKING

This often affords valuable information, even when not considered a practicable mode of treating a given ore under existing conditions. It may be applied to ore as fine as 0.25 in, and, in investigating screened products, may be carried down to 1.0 or even 0.5 mm with aid of a hand-glass or a low-power microscope. The ore should be clean and sized between rather close limits; washing brings out the distinctive color or luster of the minerals.

Practicability of hand picking on a commercial scale may be tested by attempting to make the following products, or as many of them as practicable: (*a*) pure minerals, fit for market or for metallurgical treatment, as galena, blende and chalcopyrite; also very rich ore, which might be treated better metallurgically than by mechanical means, for example, copper carbonates or silver chloride; (*b*) rich ore, with coarse disseminated mineral; for coarse crushing and jigging; (*c*) fine disseminated ore, usually poor, containing useful mineral in such small particles as to require very fine crushing to liberate it; (*d*) material that does not seem to be mineralized. Several classes may sometimes be made of this, if differences of color, texture, etc. indicate a possible difference in richness. Assays will show which may be thrown away, and which should be included with the ore for mechanical treatment.

Economy of hand picking may be investigated by the formulas in Sec 28, Art 11.

11. HAND JIGGING

This is used for testing ore ranging in size from about 12 to 2 mm. It requires a tub of water and a few 6 to 10-in screens of differing mesh. Before jigging, the ore should be sized fairly closely.

Put about 2 in depth of sized ore into a sieve having a mesh fine enough to retain it, and jig for several minutes under the surface of water, with a long, slow stroke for coarse ore, and a shorter and quicker stroke for fine sizes. A quick down stroke combined with slower up stroke is most effective. Care should be taken to keep the sieve level and to avoid horiz or overturning movements. When tailing appears clean, scrape off upper layer and replace with an equal amount of ore; then resume jigging. Middling and concentrate may be allowed to accumulate on sieve, until the layers become inconveniently thick. Main object of preliminary jigging is to produce a tailing as poor as possible. Concentrate is cleaned by careful jigging, aided by hand picking, if necessary. Skimmings from the concentrate are added to the middling, and this is re-jigged and reduced to the smallest possible bulk, by separating concentrate and tailing. Sometimes two or more grades of middling may be made, their richness being tested separately to determine the effect of combining them in part with concentrate or with tailing. Assays of products, and close scrutiny of middling obtained at different sizes, will indicate the max size at which jigging should be begun. Mill operation will rarely improve hand-jig tailing, but mill concentrate is usually of higher grade.

12. PAN, GOLD PAN, MINER'S PAN

The above are different names for the same device. It is in shape a conical frustum, 10, 12.25, 16 in or more across the top, 2–2.5 in deep, the sides at an angle of 35–75° to the bottom. It should be light, with smooth inner surface, free from grease and rust. Polished steel and granite ware are the usual materials, though the latter is apt to chip. USES: (*a*) to assay gold-bearing gravels in prospecting, and, less frequently, to make a rough assay of crushed vein material for gold; (*b*) to work gold-bearing gravel on a small scale; (*c*) to make gravity-concentration tests on heavy-metal ores.

Procedure. To pan gold gravel, take a panful, and work the material over in water with the hands, rejecting stones free from fines and clay. Continue this until all coarse gravel is removed and the mass thoroughly disintegrated. Then, with the material in the pan submerged, and the pan held nearly horiz, subject it to a rotary motion sufficient to produce suspension of the solids, and settle the heavy particles, leaving the surface metal-free. Next, with pan above water, tilt it slightly forward, and, by rocking it from side to side, cause the water in the pan to wash solid matter to the lower edge; the amount of tilt and speed of rocking being such that only the superficial particles are washed down into a "toe" at the lower rim. The toe is then washed off by alternate lowering and raising through the water surface. These operations are repeated until nothing remains in the pan but heavy minerals and some fine light sand. Further separation can sometimes be made by so moving the pan that a little water therein will course around the trough formed by intersection of side and bottom. This strings out the material with the lightest sand ahead and the heavy material behind, so that more sand can be removed, and particles of gold uncovered. Final separation of gold from the heavy minerals is usually made by amalgamation; or by drying, and separating magnetite with a magnet, or by blowing.

In prospecting, it is important to know the number of pan loads per cu yd of gravel in place. This ranges from 100 to 200, according to size of pan and its load, and should be determined by measurement in each case.

Testing an ore differs from above procedure in that: (*a*) ore must first be ground to a size that will free much of the valuable mineral; (*b*) the wt of the sample should be determined; (*c*) stratification in the pan requires more careful and prolonged manipulation; (*d*) the surface layer removed in each cycle must be thinner. The operation should be conducted over a tub, in which the first tailing is collected for repanning. Concentrate always contains considerable gangue and the tailing some valuable mineral. An experienced operator can make about the same recovery as is possible in mill work, but mill concentrate, especially coarser sizes, will assay higher.

A skilled panner, working steadily, can treat about 100 pans of loose gravel per 10-hr day; proportionately less if cemented matter is present. The same man can not run down more than one-third as many samples of galena-quartz ore; even less of ores in which the difference in sp gr of heavy and light minerals is smaller.

13. VANNING

This resembles panning, but is not applicable to as coarse material and is limited to much smaller quantities, say about 50 gm. Vanning plaque is of enameled iron, in shape of a spherical segment, about 12 in diam by 0.75 in deep at the center. Cornish vanning shovel is essentially a plaque on a 24-in handle. A batea or a large watch glass may be used instead of a plaque.

Procedure. Sample, if dry, is first wet down carefully, avoiding skin flotation. Wet pulp is first swirled vigorously to get slime in suspension, then more slowly to let all granular material settle, after which the slime is decanted. Repeat until all slime is removed. Thereafter, a vanning operation comprises: (*a*) stratification by horiz rotary motion, as in panning; (*b*) throwing the lower heavy concentrate to the edge of plaque, while the upper stratum remains near center; (*c*) washing the upper stratum off the plaque away from the head of concentrate. To throw up the head of concentrate, the plaque, held on opposite edges in the hands, is moved as in swirling with one hand while the other describes, at each rev, a small vertical circle, say 1-in diam, in clockwise direction, moving downward more rapidly than upward. The swirling motion keeps upper stratum in suspension, while the lower hugs the surface; hence, as the plaque moves away from the operator, the heavy material moves with it, and thus moves from under the pulp in suspension. The rapid down stroke drops the plaque from under the heavy mineral, and when the latter again reaches the plaque surface it rests on a point farther away

from operator than *before*. The head of concentrate is thus made to travel up onto the edge of the plaque away from the operator, while the lighter sands remain nearer the center. Horiz separation is continued by imparting a gentle swirl that moves the water only, giving the plaque a smart shake while the swirling water is traveling away from the concentrate. Thus, the sand is washed down toward the operator by a film-sizing action. Some operators manipulate the second phase so as to draw the head toward them, washing tailing off the far side; others throw up the head by simple swirling with one hand and jarring the side of the plaque at each rev against the heel of the other hand. In this case, the mechanics of the horiz separation are different, but result in the same; it is easier to learn than the first, but is tiring, if much vanning is done. With a vanning shovel the head is thrown up by a slight side fillip on the back stroke of the swirl.

Principal uses of the plaque are in examination of finely-ground mill products, and in assaying, as in the Cornish tin-ore assay. Vanning has an advantage over a chemical assay in giving some idea of the amount of middling and the size of the free-mineral grains. It is being superseded in Cornwall tin assaying by chemical methods, but for many years it was the chief, if not the only method there used. It had, of course, the apparent advantage of indicating only recoverable tin (free cassiterite of a size that could be won by gravity concentration), and this was all that most millmen were interested in; but experiment showed that skilful operators reported discrepant results on the same sample, none, of course checking the chemical assay.

Haultain super-panner is a mechanized vanning horn, about 30 in long by 10 in wide, set with a slight longitudinal slope and having a curved bottom of radius increasing toward the lower end. A cam-controlled bump moves settled solids toward the high end, while a side motion of controllably-various magnitude, together with wash water, suspends and moves light material toward and over a transverse baffle at lower end. It may be used on charges as small as 1 gm; making remarkable separations on infra-sized grades.

14. HEAVY SOLUTIONS

Liquids of high sp gr (up to 3.55) may be used to separate minerals more closely than by any other means; but the method is limited to mineralogical testing, except in the use of $ZnCl_2$ solution for coal (Sec 35, Art 11).

15. FLOTATION

Testing flotation processes is more difficult than testing other methods of concentration, because the process operates by means of chemical reactions in solutions so dilute that solutes derived from the ore itself or present in the mill water may exert very great influence.

Fundamentally, in order that bubbles may attach preferentially to a solid surface, that surface must first be water repellent, while all other particle surfaces remain readily wettable by water, as they are in virgin ore (excluding ozokerites and the like). The desired water-repellent surface is always of hydrocarbon nature. In a few special cases, as sulphur, graphite and coal, it can be applied by selective smearing with relatively insoluble oil. This is done by agitating the ore with water and a little oil. In all other cases, coating is effected by chemical reaction between the affected mineral and a collecting agent. This results in an oriented precipitate of a reaction product of mineral and reagent at the mineral surface.

Substantially any mineral may be separated from any other by proper choice of a collector. The prerequisites are: (1) slight solubility of the mineral to be coated (and floated) in the pulp solution; (2) an organic reagent that will form an oriented precipitate of hydrocarbon nature on the desired mineral surface, by reaction therewith, while the other minerals are unaffected thereby. Solubility of the mineral, of the organic collecting reagent and of the reaction product may be controlled by other suitable (CONDITIONING) reagents. In general the silicates, except those containing considerable proportions of heavy or alkaline-earth metals, are non-floatable. Substantially all other minerals are floatable. For more complete discussion of the chemistry involved, see Bib 20—23.

Reagents. Salts of organic thio acids, usually carrying upwards of 4 carbon atoms, are commonly used as collectors for sulphide minerals. Examples: xanthates, thiocarbonates, thiophosphates, mercaptans. Fatty and resin acids and their alkali-metal salts (soaps) are the usual collectors for non-metallic minerals. Added oil is an aid in coating the latter. Pulps are usually kept slightly on the alkaline side (pH 8–10).

Differential reagents are added to modify the surface of certain minerals and thus change their behavior. Copper ion replaces zinc ion on sphalerite surfaces and thus, since zinc does not form insoluble precipitates with many thio-acid ions of low hydrocarbon content while copper does, activates the sphalerite. Alkali cyanides added to a pulp containing copper ion suppress the concentration of this ion. Hence they are added to prevent activation of sphalerite by incidental copper in galena-sphalerite ores. With a high concentration of zinc ion, obtained by addition of zinc sulphate or chloride, cyanide probably forms an insoluble zinc cyanide on sphalerite surfaces and further depresses that mineral. Cyanide probably also forms ferrocyanide coatings on pyrite in alkaline pulps and thus depresses it. These examples constitute a bare outline of possibilities of differential control.

Steam-distilled pine oil and cresol are the most usual frothing agents.

Crude, small-scale tests, by the agitation-froth process, may be made by hand shaking in a test tube or a wide-mouth bottle, or in a milk-shake machine. A motor-driven bar mixer with a small square glass jar is excellent for preliminary tests; froth being removed by overflow by introducing sufficient water through a tube below the pulp level.

Preliminary pneumatic-process tests may be made with a short length (8–12 in) of 1.5 or 2-in tubing, a 1-hole rubber cork and a piece of finely porous cloth, arranged as in Fig 2. Air may be supplied by a bicycle or automobile pump or, at worst, by an atomizer or syringe bulb. It is well to have a receiver made, say, of a 2-liter acid bottle, between the pump and cell, arranged so that the air supply can be regulated by a pinch-cock. Preliminary tests by such apparatus are

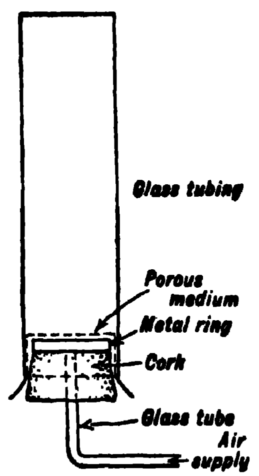

Fig 2. Blowing Tube for Pneumatic Flotation Tests

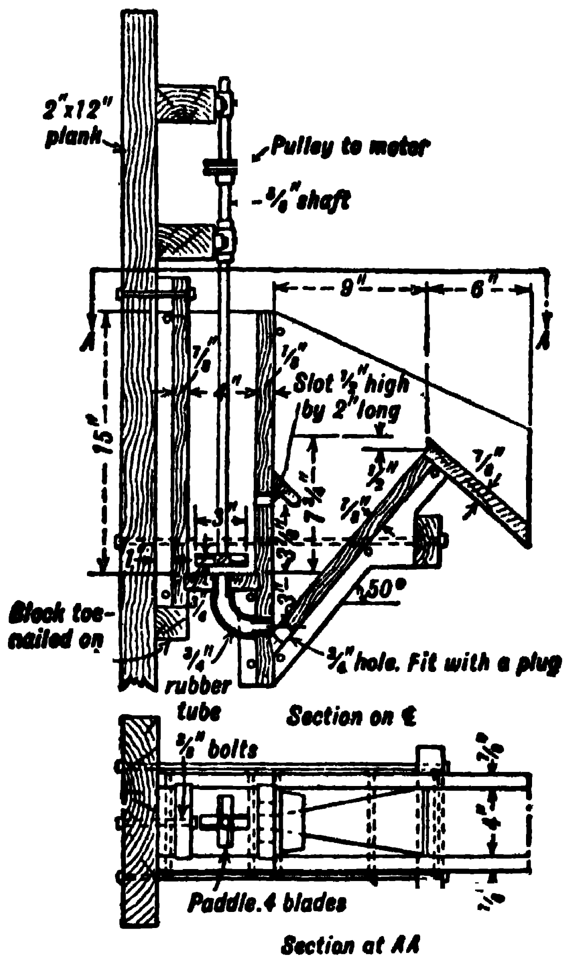

Fig 3. Agitation-froth Laboratory Flotation Machine

valuable chiefly to determine amenability and, perhaps, to give some idea of the grade of concentrate that can be made. They will not yield good recoveries.

The best machines for laboratory work are single-cell models of mill machines, taking charges of 500–1 500 gm, with provision for froth overflow, and arranged for pulp circulation. They will yield results that are usually directly comparable to mill operations. Such models of their commercial machines are made by many mfrs.

Fig 3 shows a satisfactory agitation-froth machine, which is easily made in any mine shop. A 0.25-h p motor, giving the agitator a speed range of 800–2 000 r p m, is desirable. Ore charge, 750 gm. The pneumatic machine in Fig 4 can be built in any good mine shop; arranged for either continuous or batch treatment, according to position of the tailing-discharge slide. Ore charge for a pulp containing 20–25% solids, 1 500 gm. For small-scale batch tests, a machine similarly proportioned, without glass side and arranged for overflow both sides, 2 by 7 in in plan, is easier to handle and can operate on 500 gm

or less. It should be fitted with a small air-lift, of glass tubing, to circulate from the deep end back to the shallow end.

Testing procedure. All apparatus should be clean. Mode of preparing the ore charge depends upon nature of the ore and reagents. Dry grinding in a disk sample-grinder, or a small ball or pebble mill permits preparation of many samples at one time, but the character of the product may not be the same as that of the same ore wet-ground. Wet grinding, simulating mill conditions, is best, unless it is known that dry grinding produces no essential difference in results.

In wet grinding the ore is first reduced to pass 10 or 20-mesh dry. The charge is then weighed into a small batch cylinder mill, water equivalent to 30–50% of the combined wt is added, and the charge ground for the predetermined time required to produce the desired size. Collecting and conditioning agents may be added, so as to be present during grinding. With non-metallic ores it may be preferable to do all fine grinding out of the presence of iron, or to add a precipitant for iron ion, as phosphate or hydroxyl ion. For many ores, preliminary tests of metal content of tailing may be made by vanning, assays being superfluous unless vanning indicates good recovery.

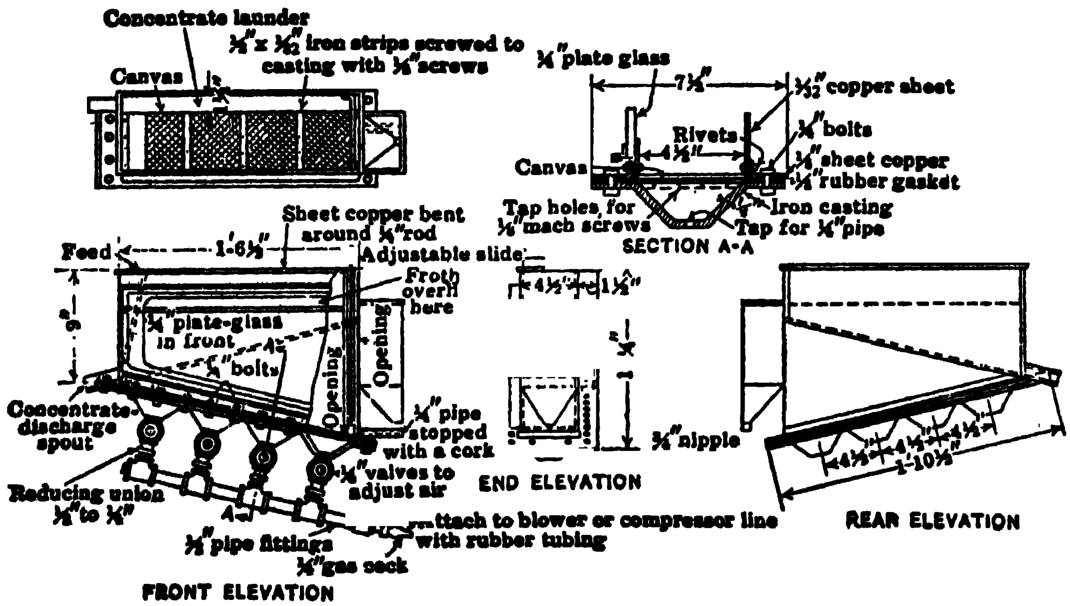

Fig 4. Callow Cell, Laboratory Size

Quantity of reagents. Thio-acid collectors, 0.05–0.15 lb per ton of dry ore; soaps, 0.2–1.0 lb per ton; neutral oils, as petroleums, when needed, 0.25–3 lb per ton; Cu, 0.05–0.25 lb per ton; hydroxyl ion (lime, caustic soda or soda ash) to give desired pH; cyanide, 0.25–0.5 lb per ton (concentration of CN, with a given quantity of cyanide salt present, increases with alkalinity); frothing agents, as pine oil, cresol, sulphonic soaps, 0.05–0.15 lb per ton.

For a bottle test, place the pulp (containing about 25% solids) and reagents in a wide-mouth bottle holding 500–2 000 cc, filling it half to two-thirds full, cork, shake vigorously for 3–10 min, moving the bottle parallel to its axis, so that the mass of pulp acts as a piston or plunger, then allow it to stand for froth to separate. This test requires practice, and an inexperienced operator should not be discouraged by failure.

For a machine test, place the pulp with reagents (as above) in the machine and agitate at 1 000–1 500 ft per min peripheral speed for 10–30 min. If first froth is high-grade, it may be taken as finished concentrate and later overflow as middling; if first overflow is low-grade, take all overflow as rough concentrate and clean in a second operation.

For differential flotation of galena from blende and pyrite, grind the ore with 2–4 lb per ton of sodium bicarbonate, 0.2–0.4 lb coal-tar or wood creosote, 0.5–1.0 lb sodium cyanide and 1–2 lb zinc sulphate; then charge to the machine with 0.1–0.25 lb pine oil and float the lead. Next add about 1 lb copper sulphate and, if necessary, H_2SO_4, sodium carbonate or another inorganic agent, to float blende away from pyrite. Sodium carbonate, bicarbonate, or cyanide alone may serve to separate a blende-pyrite ore or to float chalcopyrite from pyrite. The same procedure as for flotation of galena will frequently float chalcopyrite from blende and pyrite. Sodium sulphide (1–10 lb per ton) may sometimes be used instead of cyanide.

For further details of flotation testing, see Taggart, Handbook of Mineral Dressing.

Interpretation of flotation-test results. Translation of laboratory results into terms of mill-scale operation is generally less difficult in flotation than in gravity concentration, and is always more certain than where there are chemical reactions, as in leaching and precipitation. Any flotation result obtainable in a laboratory machine is obtainable in mill operation, if the essential laboratory conditions are duplicated. The converse of this statement is also true, except that the mill-size machine can handle somewhat coarser feed than the laboratory machine. Considering the essentials of pulp treatment in detail, the translation from laboratory to mill results are as follows:

Average size of feed may be slightly coarser in mill than in laboratory, or, if the mill grinding is carried to the same extent as in the laboratory, a somewhat better result may be expected in the mill, other conditions being equal.

Water may make a considerable difference between laboratory and mill results, and this difference may be either in favor of or against the mill. It will usually be in favor of the mill if part of the water is reused; in which case the flotation agent returned by the mill water may lessen considerably the amount of new flotation agent that must be added, and the froth is more easily obtained with admixture of this reclaimed water. But, if there is much soluble salt in the ore, or if the settling ponds are of considerable area and in an arid region and there is much dissolved salt in the new water, these salts may injure flotation.

Flotation agents in the mill will be the same as in the laboratory, except that in the mill it is generally possible to lessen the proportion of frothing agent in the mixture.

Peripheral speed of agitators is usually somewhat less in mill than in laboratory.

Air consumption per cu ft of pulp treated in pneumatic machines is usually less in mill than in laboratory. Pressure on underside of the blanket must be higher in mill than in laboratory machines, due to the greater head on the pulp side of the blanket.

Time of treatment necessary in the mill will be very nearly the same for a given recovery and grade of concentrate as in the laboratory. Hence, the required volume of mill machines may be calculated therefrom.

Grade of final concentrate obtained in the mill will be close to that in the laboratory.

Recovery will be close to the indicated extraction, computed by the formula (Art 19, Eq 4) from laboratory results, if the figure used for grade of concentrate is that obtained from the cleaner operation; the figure for rougher tailing is that obtained from the rougher operation, and the middling or cleaner tailing obtained in the laboratory is disregarded, provided that the grade of this middling product is not more than twice the grade of the original heads, and that the mineralogical character of the middling is not markedly different from that of the original feed.

Mill tests. It can not be too strongly urged that, before a mill is erected, some testing work be done with mill-size flotation machinery. It should be done in a test mill at the mine, on ore, the prior handling of which corresponds closely with the scheme to be followed in the finished mill; the water also should approximate the character of the water that will be used in the final plant. If such a test merely confirms the laboratory results, it will pay for itself in the information that it gives concerning mill operation on the ore, and the test may reveal conditions which were overlooked in the laboratory. As some of the equipment used in such a test can generally be utilized in the final plant, not all of it needs to be charged to the testing work.

16. AMALGAMATION TESTS

Pan amalgamation. A sample of the ore, previously crushed to 30 or 40-mesh, is ground for a definite length of time in a Wedgwood or an iron mortar, with addition of Hg and such ingredients as would be used in practice; for example, caustic soda, salt, copper sulphate, or H_2SO_4. Following mixture is commonly used: ore, 300 gm; water, 100 cc; caustic soda, 1 gm; Hg, 50 gm. Grind for 1 hr, pan off the Hg, dry and assay the residue, and calculate extraction.

In a small test of this nature, results calculated from DIRECT DETERMINATION of the Au and Ag taken up by the Hg are seldom reliable, owing both to the difficulty of obtaining Hg quite free from these metals, and to the likelihood of loss while driving off the excess Hg by heat, or dissolving it in acid. If the Hg breaks into small globules that will not coalesce readily, the addition of caustic soda, sodium amalgam, or sal ammoniac may assist in causing the Hg to unite in a single globule.

Plate amalgamation. A method often giving good results is to roll a weighed sample backwards and forwards in a large bottle containing Hg and dilute caustic soda solution. Following proportions may be used: ore, 200 gm; solution, 100 cc, containing 1% NaOH; Hg, 10–15 gm. The bottle is kept revolving for 0.5–3 hr, after which the contents are panned to separate the Hg. Extraction is calculated by difference between original and

final assays of pulp. SIZING-ASSAY TEST, made on tailings from this bottle best, tells whether finer crushing would be likely to improve recovery. TESTS ON PLATES may be made by passing small quantities of ore over an amalgamated plate or pan, or by agitating with strips of amalgamated copper in a bottle. Tests simulating plate amalgamation will almost always give lower recoveries than can be obtained in a mill.

17. CYANIDING TESTS (see Sec 33)

Preliminary tests are often desirable to determine whether a given ore is at all suitable for cyanide treatment. The first test may imitate the extreme limits to which such treatment could be carried in practice. No standard test will cover all possible cases, but the following will serve in many instances. Take 100 gm of ore ground to 200-mesh. Add lime or caustic soda until a distinct alkalinity remains permanent on agitation with water. Dilute to 3 parts of liquid to 1 of ore; add cyanide to give strength of 1% KCN, and agitate continuously for 72 hr; wash out on vacuum filter; titrate filtrate for cyanide. Wash residue on the filter thoroughly with water, dry and assay. With certain classes of ore it may be necessary to supplement this treatment by injection of compressed air before or during the agitation, and to wash the final residue repeatedly by decantation, stirring, and settling each time before decanting, with or without vacuum filtration.

Variable conditions in cyanide treatment. If the ore proves generally suitable for cyanide treatment, the next points requiring investigation are: (*a*) fineness to which ore must be crushed; (*b*) alkali necessary for neutralization; (*c*) most suitable dilution of pulp; (*d*) strength of cyanide required; (*e*) time of treatment necessary; (*f*) whether to treat entirely by agitation, or to separate a portion for percolation treatment; (*g*) whether oxidizers or other auxiliary agents are necessary.

Bottle agitation. Most of the above points can be ascertained by simple agitation tests in bottles along the lines of the standard test given above. These are conveniently performed by attaching bottles (glass preserve jars are suitable) to any revolving mechanism, placing the stoppers towards the center of rotation. Bottles may be protected by wrapping in cloth, or may be secured in wooden cases affixed to the wheel, and firmly packed to prevent slipping. Speed should be sufficient to give efficient agitation, but not so rapid that the pulp is held against the bottom of the bottle by centrifugal force; about 20 rev per min are generally suitable. The following scheme may serve as a model, to be varied in detail according to circumstances.

Fineness of crushing. Make 6 bottle tests, using in each case: ore, 100 gm; solution, 300 cc; lime, 1 gm; cyanide, 0.5% KCN. Crush an average sample of the ore to 20-mesh and weigh out one charge of 100 gm for test; then crush remainder to 40-mesh, taking 100 gm for test; and so on successively to 60, 80, 120, and 200-mesh. Agitate in bottles as above for 24 hr, test strength of solution and add enough cyanide to restore solution to its original strength of 0.5%. Continue agitation another 24 hr, make up strength as before, and continue agitation for a total period of 72 hr. Filter, wash, and assay residue.

Alkali strength. On indications of previous series of tests, determine fineness to which the ore should be crushed. Supposing that nearly identical results are given with 120 and 200-mesh ore, crush a sufficient quantity to 120-mesh, and prepare 6 identical tests, using: ore, 100 gm; solution, 300 cc; cyanide, 0.5% KCN. Add respectively the following weights of lime: 0.02 gm, 0.05 gm, 1 gm, 1.5 gm, 2 gm, 3 gm. Agitate simultaneously for 72 hr, and test residue as before; also determine alkalinity and cyanide strength of final solution (see below).

Total acidity of an ore is determined by agitating with an excess of alkali solution of known strength. For example, agitate 100 gm of ore, crushed, say, to 120-mesh, with 200 cc of 1% NaOH solution, without cyanide. After 1 hr, titrate solution (without diluting) with standard acid, and calculate consumption of alkali. Sometimes comparative tests are desirable to determine whether lime or caustic soda is preferable.

Dilution of pulp. Crush a sample of ore to fineness indicated by preceding tests. Add lime or other alkali, also as indicated. Make 6 tests, each on 100 gm ore, with the following volumes of 0.5% KCN solution: 100, 150, 200, 250, 300, 500 cc. Agitate 72 hr, maintaining strength at 0.5%; test final solutions for cyanide and alkali, and assay residues after filtering and washing. Results will indicate the most desirable dilution of pulp, assuming that the ore is to be treated by agitation, on the all-sliming principle (Sec 33, Art 11).

Strength of solution. Supposing that best results are obtained with a dilution of 2 : 1, run a series of 6 identical tests, with 100 gm of ore, 200 cc solution, 1.5 gm lime (or other amount as previously indicated). Agitate continuously for 72 hr, using solutions respectively of 0.05, 0.10, 0.25, 0.5, 0.75 and 1% KCN strength. Test final solutions,

calculate cyanide consumption, and assay residues after washing and drying. In case one or more of these tests show all or nearly all of the cyanide to have been consumed, it is advisable to repeat the series, determining the cyanide strength at intervals of a few hours, and adding sufficient solid cyanide, or strong solution, to bring the strength up to the required standard. This is repeated until successive tests show practically no further consumption of cyanide. Any solution drawn off for such tests should be replaced by an equal volume of water, so as to maintain a constant ratio of solution to ore. Final extraction, when the solution has been maintained at given strength, will often be much higher than that obtained by a test in which most of the cyanide was destroyed early in the treatment.

Example showing method of maintaining strength. Suppose 200 cc of 0.25% KCN solution originally taken; total cyanide contents = 0.50 gm. After period of agitation with ore, 50 cc taken out, after standing to settle clear, shows a strength of 0.19% KCN; remaining solution, 150 cc at 0.19% = 0.285 gm KCN. Cyanide to be added to make up strength = 0.50 − 0.285 = 0.215 gm. Suppose we have a strong solution containing 1.05% KCN; then 20.5 cc of this will contain 0.21525 gm KCN. Measure out 20.5 cc of the strong solution, dilute this to 50 cc with water, and add to the test. This gives 200 cc, containing altogether 0.50025 gm KCN, or practically 0.25%.

Time of treatment. Assuming previous tests to show best result with 0.25% KCN, run tests with (say) 100 gm of ore, 200 cc of 0.25% KCN solution, 1.5 gm lime, stopping treatment successively after 12, 24, 36, 48, 72, or 96 hr. Test solutions and assay residues.

It may sometimes be desirable to vary two or more conditions simultaneously. Thus, if a strength of 0.25% KCN gives the best result with 72 hr agitation, it does not necessarily follow that this strength of solution will be most advantageous with 24 or 96 hr agitation. Where facilities permit, a large number of tests under varying conditions may be made simultaneously, thus shortening time required for investigation.

Percolation tests. Whenever the results show a tolerably good extraction without extremely fine grinding, it becomes a question whether percolation would not be advantageous for at least a portion of the ore, and the following test may be made.

Crush (say) 20 to 100 lb of ore to pass 40-mesh. Separate fine portion by hydraulic classification, or by repeated stirring with water, settling, and decanting. Drain the sandy portion as dry as possible, mix thoroughly, by passing through a coarse screen, and on an average sample determine moisture and value. Add lime to the remainder, in proportion determined by previous acidity tests, and charge into small percolation tank. This may be an inverted glass bottle with bottom cut off, a stoneware jar, or half of a small cask. Filter-bed is required, which may be a perforated wooden disk, or a framework of wooden slats, covered with thin canvas, or a layer of cocoa matting with canvas cover. The edge of the filter-frame must be tightly packed in with rope, waste or similar material. An outlet for the leaching solution is provided below the filter frame, the flow of liquid being regulated by rubber tube and screw-clip or pinchcock. Only a little space must be left between filter-frame and bottom of percolator. Solutions are added at intervals, once or twice a day, in amounts of 0.1 to 0.2 times the wt of ore each time. A good plan is to begin with a weak solution to replace moisture in the ore; follow with strong solution, draining off and adding more until the whole charge is saturated with it; then add enough strong solution to cover the charge, and allow to stand, with outlet closed, for at least 24 hr. After this, a number of washes of successively weaker solution may be given, ending with a water-wash, not exceeding in weight the original moisture of the sand treated. Each solution should be drained off as completely as possible before applying the next, to secure maximum extraction with minimum of liquid. The charge may be sampled at intervals with a sampling tube, taking care to get a correct average from top to bottom, and not to remove too large a proportion of the charge. If all the solutions drawn off are measured, and tested for cyanide and alkali, a very fair estimate may be made of the probable consumption of chemicals on a working scale. Finally, the entire charge should be emptied, thoroughly mixed, sampled, and assayed. A portion may also be tested after extra water-washing on a vacuum filter.

Aeration tests are generally made in small model tanks, imitating the construction of the Pachuca or other form of aeration tank. The regulation of air supply requires careful attention; if insufficient, agitation is ineffective; if too abundant, consumption of chemicals may be excessive. It is instructive to compare results from aeration tests with those from bottle tests made without aeration, but under otherwise similar conditions.

Other tests. A great variety of conditions may occur in different ores, which may necessitate an elaborate research into such questions as the necessity or disadvantage of preliminary water-wash, acid-wash, or alkali-wash, oxidizing agents, roasting, concentration, amalgamation, or other process, and the use of additional reagents during cyanide treatment, such as bromocyanide, lead salts, etc. These tests are not, however, in the general class of field tests. WATER-WASHING is

advisable for partially oxidised ores containing iron or copper sulphides, and ACID WASHING is also frequently beneficial under the same conditions, particularly where basic sulphates insoluble in water are present. ALKALI WASHES may be advisable with arsenical and antimonial ores. OXIDIZING AGENTS improve extraction from certain ores containing easily oxidisable sulphides and certain kinds of organic matter; preliminary treatment with permanganate, with or without dilute sulphuric acid, is sometimes advantageous when these conditions are present. ROASTING is beneficial and sometimes essential with ores containing heavy sulphides, arsenides, sulpharsenides, tellurides, etc, but the cost of the operation, in comparison with the possible increased saving of valuable metals, should always be considered. CONCENTRATION is often advantageous when values are associated with sulphides or other heavy minerals, provided that the latter do not constitute too large a percentage of the ore. AMALGAMATION should always be tried in presence of coarse Au and perhaps, also, of native Ag. BROMOCYANIDE has been commercially successful only with telluride and with mispickel ores.

Precipitation tests following bottle tests or small-scale field tests are not reliable, due to the minute amounts of precious metals to be handled.

Consumption of KCN. In the past, assay of solutions for KCN, after classification, has been depended upon to give an idea of KCN consumption. But methods have been developed for regeneration of KCN, that return important amounts in excess of that obtained by precipitation of the Au and Ag. They involve formation and collection of hydrocyanic acid, and are not amenable to accurate field determination.

Interpretation. Plant recovery will not generally be as high as that obtained in the preliminary test in strong solution, provided the ore is ground to same size in both cases. But mill practice will probably permit use of weaker solutions than the best laboratory work, and KCN consumption will be smaller than is indicated; lime consumption in the mill may also be less.

It is distinctly questionable whether the other tests outlined are justified as field work, because amenability to cyanidation is determined by the preliminary test, and testing on a larger scale than bottle tests is necessary before plant construction is begun. Besides elaborate cyaniding tests, other tests, on grinding, mechanical classification, thickening and filtering, should also be made. For detailed instructions for general cyanide testing, see Bib 24–27.

18. OTHER METHODS

Leaching with different solvents is an important possibility in treating low-grade Cu ores, especially those in which much of the Cu is oxidized.

For certain ores, water alone serves as a solvent; others require chemicals, among which H_2SO_4, H_2SO_3, Fe_2SO_4 and ammonia are most promising. Leached Cu may be precipitated as metal, and floated, all in presence of the original pulp solids. ZnS ores, in form of fine concentrate only, can be leached with H_2SO_4 after roasting. Oxidized Pb ores containing Ag can be leached with strong acidified brines, but leaching has nowhere been commercially successful. Leaching of base-metal ores is not yet sufficiently standardized to permit of specifying methods of field testing.

With copper ores, simple tests may be tried with dilute solutions; and, if these give high extraction, the details of precipitation and plant operation can probably be worked out successfully. For ores containing copper sulphides, aeration is important in the leaching reaction.

Leaching applied to Pb–Ag and Zn ores is not a matter for crude field testing.

Volatilization is a possible method of treating oxidized Pb ores containing Au and Ag. Ores should be free from sulphides. Amenability may be tested by grinding fine and roasting a small charge in a dish in a pot furnace at temperature of incipient fusion. A small amount of a chloride may be added, but it is not generally necessary. Extraction is determined by assay of tailing.

19. FORMULAS FOR MILLING CALCULATIONS

Notation. C = wt of concentrate, expressed in any units, but necessarily the same as F. For use in these formulas, concentrate may be defined as any product of treatment of a given feed that contains more of a given ingredient than the feed.

c = assay of concentrate (see notes to f).

c_a, c_b, c_c. See notes to f_a, substituting "concentrate" for "feed."

E = effic. This term is usually applied to two-product classification, indicating the ratio of wt of classified material in overflow to wt of classifiable material in the feed.

F = wt of feed, expressed in any units. Feed is the material entering for treatment in a plant or individual machine.

f = assay of feed, expressed in any units, as: %, oz per ton, lb per ton, or dollars per ton, when the value is directly proportional to the wt of a given constituent, say Au or Ag, but not when it is the combined value of the Au and Ag, unless the ratio of their weights is the same in all products as in the feed.

f_a, f_b, f_c, etc. When feed contains more than one valuable ingredient, assays are made of each, expressed separately in the formulas, and distinguished by subscripts.

K = ratio of concentration, meaning the ratio of wt of feed in a given operation to wt of concentrate obtained from it; or, the tons of feed required to produce 1 ton concentrate.

M = wt of middling, expressed in any unit, but it must be the same unit as F. Middling, in these calculations, is the product of the treatment of a given feed containing an amount of a given ingredient that lies between those in the tailing and concentrate.

m = assay of middling (see notes to f).

m_a, m_b, m_c. See notes to f_a, substituting "middling" for "feed."

R = recovery, the percentage ratio of the wt of the sought-for ingredient in the finished product of a given operation, to the wt of same ingredient in the feed in this operation. The terms EXTRACTION, INDICATED EXTRACTION, ESTIMATED EXTRACTION and ACTUAL EXTRACTION are sometimes used to indicate recovery. The terminology is so greatly confused that the meaning of none of the terms can be safely assumed without specifying the method of determination. Some writers have attempted to distinguish between extraction and recovery, making extraction mean the value of R computed from assays of feed and products, and recovery, the ratio of the wt of metal in concentrate actually recovered (= actual wt of concentrate × assay of concentrate) to the actual wt of metal in the feed (= actual wt of feed × assay of feed). Others use the term INDICATED or ESTIMATED EXTRACTION to signify R from assays alone, and ACTUAL EXTRACTION to signify the % of total metal fed that is actually recovered.

T = wt of tailing, expressed in any unit, but it must be the same unit as F. Tailing is here defined as that product of treatment of a given feed which is distinctly impoverished in content of a given ingredient as compared to the feed.

t = assay of tailing. (See notes to f.)

t_a, t_b, t_c. See notes to f_a, substituting "tailing" for "feed."

Two-product formulas:

$$C = F(f - t) \div (c - t) = T(f - t) \div (c - f) \quad (1)$$

$$T = F(c - f) \div (c - t) = C(c - f) \div (f - t) \quad (2)$$

$$K = (c - t) \div (f - t) \quad (3) \qquad R = 100c(f - t) \div f(c - t) \quad (4)$$

Three-product formulas, one metal:

$$C = \frac{F(f - t) - M(m - t)}{(c - t)} = \frac{T(m - t) - F(m - f)}{(c - m)} \quad (5)$$

$$M = \frac{F(f - t) - C(c - t)}{(m - t)} = \frac{F(c - f) - T(c - t)}{(c - m)} \quad (6)$$

$$T = \frac{C(c - m) + F(m - f)}{(m - t)} = \frac{F(c - f) - M(c - m)}{(c - t)} \quad (7)$$

$$R = 100\left[\frac{c(f - t)}{f(c - t)} - \frac{Mc(m - t)}{Ff(c - t)}\right] = 100\left[\frac{c(m - f)}{f(c - m)} - \frac{Tc(m - t)}{Ff(c - m)}\right] \quad (8)$$

$$K = \frac{F(c - t)}{F(f - t) - M(m - t)} = \frac{F(c - m)}{T(m - t) - F(m - f)} \quad (9)$$

If F is taken = 1:

$$R = 100\left(1 - \frac{Tt}{f}\right) = 100\left[\frac{Cc(f - t) + Mm(f - t)}{Cf(c - t) + Mf(m - t)}\right] \quad (10)$$

and

$$K = \frac{C(c - t) + M(m - t)}{C(f - t) + M(f - t)} \quad (11)$$

Three-product formulas, two metals, *a* and *b*:

$$C = F\left[\frac{(f_a - m_a)(m_b - t_b) - (f_b - m_b)(m_a - t_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right] \quad (12)$$

$$M = F\left[\frac{(c_a - f_a)(f_b - t_b) - (c_b - f_b)(f_a - t_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right] \quad (13)$$

$$T = F\left[\frac{(c_a - m_a)(m_b - f_b) - (c_b - m_b)(m_a - f_a)}{(c_a - m_a)(m_b - t_b) - (c_b - m_b)(m_a - t_a)}\right] \quad (14)$$

In formulas (12), (13), (14), C = wt of concentrate rich in metal "a," and M = wt of concentrate rich in metal "b."

$$K_a = \frac{(c_a - m_a)\,(m_b - t_b) - (c_b - m_b)\,(m_a - t_a)}{(f_a - m_a)\,(m_b - t_b) - (f_b - m_b)\,(m_a - t_a)} \tag{15}$$

$$K_b = \frac{(c_a - m_a)\,(m_b - t_b) - (c_b - m_b)\,(m_a - t_a)}{(c_a - f_a)\,(f_b - t_b) - (c_b - f_b)\,(f_a - t_a)} \tag{16}$$

$$R_a = \frac{100c_a[(f_a - m_a)\,(m_b - t_b) - (f_b - m_b)\,(m_a - t_a)]}{f_a[(c_a - m_a)\,(m_b - t_b) - (c_b - m_b)\,(m_a - t_a)]} \tag{17}$$

$$R_b = \frac{100m_b[(c_a - f_a)\,(f_b - t_b) - (c_b - f_b)\,(f_a - t_a)]}{f_b[(c_a - m_a)\,(m_b - t_b) - (c_b - m_b)\,(m_a - t_a)]} \tag{18}$$

Four-product formula. Let G, S, M, T be the products, their assays in 3 metals, a, b and c being: c_a, c_b, c_c; s_a, s_b, s_c; m_a, m_b, m_c; t_a, t_b, t_c. If F be the feed, assaying f_a, f_b, and f_c:

$$G = F\left[\frac{(s_c-t_c)(m_a-f_a)(t_b-m_b)-(t_a-m_a)(s_c-t_c)(m_b-f_b)+(m_b-f_b)(t_c-m_c)(s_a-t_a)}{(s_c-t_c)(m_a-c_a)(t_b-m_b)-(t_a-m_a)(s_c-t_c)(m_b-c_b)+(m_b-c_b)(t_c-m_c)(s_a-t_a)} \right.$$
$$\left. \frac{-(m_a-f_a)(t_c-m_c)(s_b-t_b)+(t_a-m_a)(s_b-t_b)(m_c-f_c)-(t_b-m_b)(s_a-t_a)(m_c-f_c)}{-(m_a-c_a)(t_c-m_c)(s_b-t_b)+(t_a-m_a)(s_b-t_b)(m_c-c_c)-(t_b-m_b)(s_a-t_a)(m_c-c_c)}\right] \tag{19}$$

Formulas for S, M and T may be written from Eq (19) by symmetry: $S = F(X \div Y)$, in which X is obtained from the numerator of Eq (19) by substituting f for s, wherever

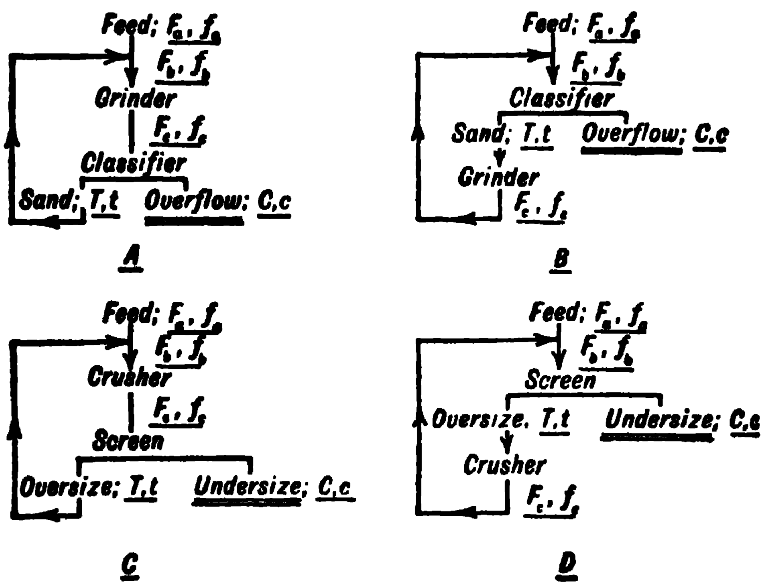

Fig 5. Typical Closed Crushing Circuits

the latter appears, and c for f. Thus the first term of the numerator becomes $(f_c - t_c)\,(m_a - c_a)\,(t_b - m_b)$. By similar substitution, the first term in formula for M is $(s_c - t_c)\,(f_a - c_a)\,(t_b - f_b)$; for T it is $(s_c - f_c)\,(m_a - c_a)\,(f_b - m_b)$. In all cases, Y is the same as the denominator of Eq (19).

Warning. Results from these equations may be seriously in error due to small inaccuracies in sampling and assaying, or even to the inaccuracies inherent in slide-rule calculation. For discussion, see Taggart, Handbook of Mineral Dressing.

Classifier efficiency, $$E = \frac{100(c - f)\,(f - t)}{f(100 - f)\,(c - t)} \tag{20}$$

Tonnages in milling circuits may often be determined by applying the two-product formulas. Fig 5 shows 4 typical closed-crushing circuits.

In Fig 5 (A), $$F_c = \frac{F_a(c - t)}{f_c - t} = F_b \tag{21}$$

$$T = F_c - C = \frac{F_a(c - t)}{f_c - t} - F_a = \frac{F_a(c - f_c)}{f_c - t} \tag{22}$$

$$f_b = \frac{F_a f_a + Tt}{F_a + T} \tag{23}$$

In Fig 5 (B). $$F_b = \frac{F_a(c-t)}{f_b - t} = F_a\left(\frac{c - f_a}{f_c - t} + 1\right) \tag{24}$$

$$T = F_c = F_b - C = F_b - \frac{F_b(f_b - t)}{c - t} = \frac{F_b(c - f_b)}{c - t} \tag{25}$$

$$f_b = \frac{F_a f_a + F_c f_c}{F_a + F_c} = \frac{cf_c - tf_a}{(c - f_a) + (f_c - t)} \tag{26}$$

$$F_c = \frac{F_a(f_a - f_b)}{f_b - f_c} \tag{27}$$

$$T = \frac{F_a(c - f_a)}{f_c - t} \tag{28}$$

$$f_c = \frac{F_a(c - f_a - t) + F_b t}{F_b - F_a} \tag{29}$$

$$f_a = \frac{f_c(f_b - c) + f_b(c - t)}{f_b - t} = c + (f_c - t)\left(1 - \frac{F_b}{F_a}\right) \tag{30}$$

$$R = \frac{c(f_c - t)}{cf_c - tf_a} \tag{31}$$

Specific gravity assay. When an ore is a mixture of two minerals only, or of one useful mineral and a mixture of gangue minerals the relative proportions of which are substantially constant, rapid approx assays may be made of a mixture of useful mineral and gangue by determining the sp gr of the mixture, provided the individual sp gr of useful mineral and gangue are already known.

The usual method is by use of a sp-gr flask. Weigh the flask empty and when full of water. Dry it, introduce the ore sample, weigh, fill with water (taking care to remove all air bubbles) and again weigh. In centimeter-gram units, if S_o = sp gr of ore, S_m of mineral and S_g of gangue; F = wt of dry flask, W = wt of water to fill the flask, O = wt of dry ore, and T = total wt of flask + ore + water; then the wt of water required to fill flask with ore in it = $T - (O + F)$ = volume of water in the flask, and volume of ore in flask = $W - [T - (O + F)]$.

$$S_o = \frac{O}{F + W + O - T} \tag{32}$$

If m = % of mineral in ore = wt of mineral per unit wt of ore, then

$$m = \frac{S_m(S_o - S_g)}{S_o(S_m - S_g)} \tag{33}$$

Cyanidation. Let F, wt of feed = 1; C = wt of concentrate (if made), T = wt of tailing from concentrating (= F, if no concentrate is made), A = wt of sand feed, B = wt slime feed, X = wt sand tailing and Y = wt slime tailing, all expressed as decimal parts of F; and f, c, t, a, b, x and y = respective assays of above products, then,

$F = C + T$ (34). $T = A + B$ (35). $f = cC + tT$ (36). $Tt = aA + bB$ (37)

$C = \dfrac{f - t}{c - t}$ (38). $T = \dfrac{c - f}{c - t}$ (39). $\dfrac{A}{T} = \dfrac{t - b}{a - b}$ (40)

Substituting: $A = \dfrac{(c - f)(t - b)}{(c - t)(a - b)}$ (41). $B = \dfrac{(c - f)(t - a)}{(c - t)(b - a)}$ (42)

$$R = \frac{c(f - t) + \dfrac{c - f}{a - b}[(t - b)(a - x) - (t - a)(b - y)]}{f(c - t)} \tag{43}$$

Pulp consistency. Let p = % solids = wt of solids in unit wt of pulp; D = dilution = water-solid ratio = parts water, by wt, per part of solid, usually written like 6:1; S = sp gr of dry ore; d = sp gr of pulp; then,

$\dfrac{p}{S} + 1 - p = \dfrac{1}{d}$ (44). $d = \dfrac{S}{S - p(S - 1)}$ (45). $p = \dfrac{S(d - 1)}{d(S - 1)}$ (46)

$S = \dfrac{dp}{1 - d(1 - p)}$ (47). $D = \dfrac{1 - p}{p} = \dfrac{S - d}{S(d - 1)} = \dfrac{1}{p} - 1$ (48).

$d = \dfrac{D + 1}{D + \frac{1}{S}}$ (49). $p = \dfrac{1}{D + 1}$ (50). $S = \dfrac{d}{1 - D(d - 1)}$ (51)

If Z = SOLID FACTOR = ton solid per FLUID TON (= 32 cu ft) pulp; G = fluid ton pulp per ton dry solids; and q = % solids by volume; then,

$$Z = pd = \frac{d}{D+1} = \frac{pS}{S - p(S-1)} = \frac{S}{DS+1} = \frac{S(d-1)}{S-1} \quad (52)$$

$$G = \frac{1}{Z} = \frac{1}{pd} = \frac{D+1}{d} \quad (53) \qquad q = \frac{Z}{S} = \frac{d-1}{S-1} = \frac{p}{S - p(S-1)}. \quad (54)$$

Counting assay. If all particles are practically of same shape (that they are of substantially the same intermediate dimension has been assured by sizing), the volume percentages are equal to the number percentages. If there is a distinct difference in aver shape of particles, the number percentages must be adjusted by factors expressing relative volumes of the mean shapes, in order to get volume percentages. If v = % by volume of mineral in sample = volume per unit volume, and S_m and S_g = sp gr of mineral and gangue respectively, the % of mineral by wt is

$$m = \frac{vS_m}{v(S_m - S_g) + S_g} \quad (55) \qquad v = \frac{mS_g}{mS_g + S_m(1-m)} \quad (56)$$

Note that these formulas are applicable to the case of mixtures of solid and water, in which case, if m and v are taken as percentages of solid, S_m is sp gr of solid and S_g = 1.

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SECTION 32

SELLING, PURCHASING, AND TREATMENT OF ORES

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

SELLING, PURCHASING, AND TREATMENT OF ORES

Introduction. This section deals briefly with some of the metallurgical problems relating to the disposal of ores after they have been mined, and which are of interest to mining engineers. An understanding of the terms used in buying ores, and reasons for same, are of prime importance; often making it possible for miners to sort and classify ores so as to obtain max return on a fixed or given schedule (17–23). To cover the subject thoroughly would require citation and discussion of numerous schedules, which would be out of place here. The schedules and forms given will clarify the text.

When an ore is of constant character and value, as is common, for example, in case of iron and zinc, fixed and definite terms can be formulated for sale or purchase. But, for variable or complex ores, special terms and quotations are necessary, which are sometimes difficult to comprehend. Most non-ferrous reduction works receive heterogeneous ores, involving more or less complicated schedules and terms of settlement. Moreover, as local conditions in different ore-producing centers often vary widely, schedules applicable to one district may require radical modification in another. Rates are seldom intentionally made complex, but in certain cases it must be admitted that they could be simplified, for readier comprehension by small shippers. NOTE.—Due to varying local conditions, and governmental regulations of monetary values, exact prices and rates of general application can not be given. Figures on following pages are a fair average over a long period of years (except where dates are given), and will serve as a guide for the seller.

1. TREATMENT OF LEAD AND COPPER ORES

Smelting operations: (*a*) roasting the ore, if necessary; (*b*) furnace treatment; (*c*) refining the metallic product from furnace, and treatment of by-products.

Products of smelting Pb-Ag ores: LEAD BULLION (shipped to refinery) and SLAG (waste). By-products: MATTE, artificial sulphide of a metal or metals; SPEISS, artificial arsenide of Fe, FLUE DUST, COTTRELL, and BAG-HOUSE FUME; these are retreated to recover metals. Products of smelting copper sulphide ore: COPPER-IRON MATTE, converted to BLISTER COPPER, which goes to refinery; SLAG. By-products: SPEISS (seldom produced), FLUE DUST and FUME.

Cost of each treatment step must be considered in fixing the terms for purchasing an ore, and these costs vary with local conditions at the smelting plants.

Mine managers should realize in a general way, the complicated processes necessary to extract metals from ores, and prepare them for market (see Fig 1 and 2). Low-grade ore is often concentrated before being smelted, to eliminate worthless, and/or undesirable, gangue, thus reducing cost of subsequent treatment. High-grade lead and copper ores can be smelted direct.

The sale of small lots of ore while a mine is being developed is of advantage in providing funds for the work. As it is impracticable for the smelter to treat small lots of ore separately and pay for the actual metal recovered, the only alternate for the custom smelter is to purchase the ore outright and settle for it on basis of the analysis as determined by sampling and assaying (23). (See Sections 29 and 30.) This is especially true of ores carrying Au, Ag, Cu and Pb.

Gold ore and silver ore containing little or no Cu or Pb are often treated by the cyanide process (Sec 33). Zinc is recovered from high-grade ore or concentrate by distillation in a retort, or by dissolution and electrolysis.

Many plants now roast ore so perfectly that no matte is produced in lead blast furnaces. The small amount of Cu in the charge passes into the base bullion, and then into the drossing kettle. Dross is treated in a reverberatory, the products being: lead bullion, sent to refinery; copper matte, to the copper smelter; speiss, back to the blast furnace; slag, to waste. This practice often eliminates the copper-matte blast furnace from the scheme and increases direct production of lead bullion.

General basic costs at large plants, per ton of charge, are. for lead smelting, exclusive of roasting and sampling, \$3–\$5; for copper smelting under the same conditions, \$2–\$3. Corresponding costs per ton of lead- or copper-bearing material alone are almost always higher, due to inclusion of barren flux in the charge. For refining lead bullion, \$8–\$12

per ton; for converting copper matte, about $10 per ton of blister copper produced; for refining copper bullion, $15–$23 per ton of copper cathodes produced. Costs at small plants are much higher than for large. Interest and amortization should always be included.

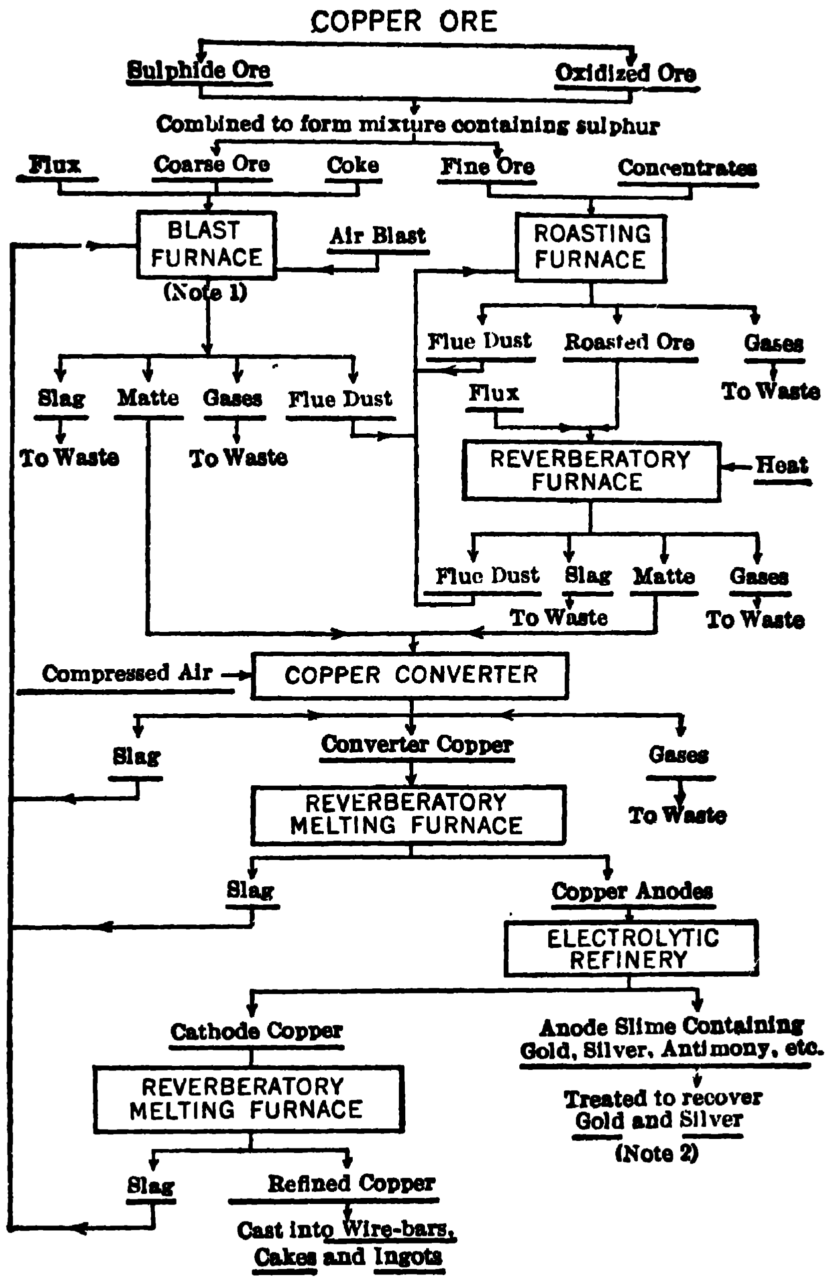

Fig 1. Scheme for Treating Copper Ores.

Note 1. Formerly in the U S copper ore was almost always smelted in blast furnaces; later, both blast and reverberatory furnaces were used. Now (1938) copper ore and concentrate with few exceptions are smelted in reverberatories; a practice due largely to increase in amount of fine concentrate produced and to more economical handling.

Note 2. Metals of the platinum group contained in the gold are recovered by electrolysis.

Metal losses in smelting and refining. In LEAD ORE, loss of Pb varies from 4 to 15%, depending upon percentage of Pb, and upon whether refractory or not. The lower limit is for a charge carrying about 40% Pb; the higher, for a charge carrying about 10–12% Pb. COPPER loss in lead smelting sometimes reaches 30%, as the amount of Cu permissible in a lead-smelting mixture is limited. In smelting COPPER ORE, the loss varies with the

percentage of Cu, but the recovery is always much higher than with lead. Loss in SILVER is small, only about 2%, unless conditions are abnormal. Loss in GOLD is small, and is not usually considered.

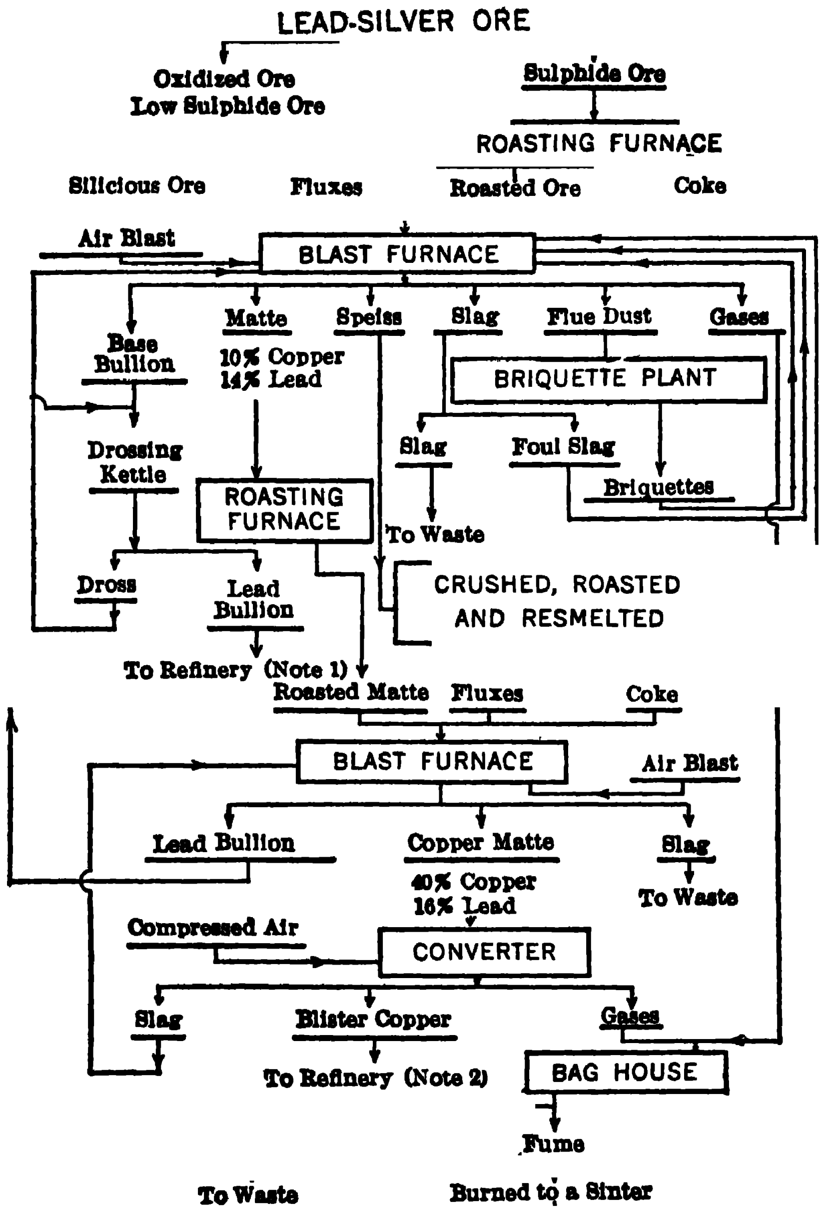

Fig 2. Scheme for Smelting Lead-silver Ores.

Note 1. At the refinery the lead bullion is desilverized, the products being refined lead, silver, gold and antimonial lead, if, as is usual, the ore charge contains antimony. If bismuth is present in refined lead, it can be recovered by an electrolytic process.

Note 2. Blister copper, if any is produced, goes to the copper refinery, for treatment in the reverberatory anode-casting furnace (Fig 1).

2. SALE AND PURCHASE OF LEAD AND COPPER ORES

Principal questions relating to sale and purchase are: (*a*) freight rates; (*b*) method of sampling; (*c*) composition of the ore; (*d*) fluxing requirements to produce a satisfactory slag, and available fluxes at the smelter, either in form of barren material or other classes of ore; (*e*) value of recoverable metals; (*f*) influence of impurities contained in the ore; (*g*) cost of treatment and refining; (*h*) metal losses occurring in treatment; (*i*) interest on value of metals tied up during treatment, and commission for selling metals; (*j*) terms of payment; (*k*) base charge; (*l*) interest and amortization on plant.

Freight charges from mine to reduction works vary with: (*a*) distance; (*b*) value of ore; (*c*) tonnage handled. Custom plants usually pay the freight from the mine to their works, the amount being included under FREIGHT AND TREATMENT. Freight rate on high-grade ore always exceeds that for low-grade, because of the extra responsibility assumed in transporting more valuable material, and to stimulate production of low-grade ore, thereby increasing the railroad's business. Shipping ore should be as dry as possible; smelting returns are on dry ore, and shipper pays freight on moisture present.

Ores and concentrates are usually transported from western mining districts to the smelters by RR or trucks. When the quantities are large and distances long, RR transport is the cheaper. For small lots and short distances, trucking may be more economical. Also less-than-carload lots can usually be shipped cheaper by truck than by rail.

Gardner (26) summarizes the cost of trucking regular tonnages over good dirt roads, when trucks can be steadily employed, and with no shoveling in or out, in conditions existing in gold-mining districts in N W Ariz and Calif in 1935, as follows:

Distance, miles	Aver cost per ton-mile	Distance, miles	Aver cost per ton-mile
up to 1	\$0.35	5 to 10	\$0.09
1 to 2	.22	10 to 20	.06
2 to 5	.12	20 to 100	.05

When shipments are made by rail, a base rate for low-grade ore is quoted, an increase being made for regular increments in the ore values. Aver base rate on western RRs is about 1¢ per ton-mile (1937). Rates for large tonnages of low-grade ore are as low as ½¢; rates on branch RRs may be 2¢ or more per ton-mile (27).

Sampling. For methods, see Sec 29. The moisture sample must be carefully taken, especially in wet or snowy weather, and, as nearly as possible, at time of weighing the ore. It should be dried at not over 212° F, otherwise water of crystallization may be driven off, which is not moisture within the scope of the present definition and the shipper thereby loses weight in settlements for ore. Final fine-pulp sample is usually divided into 4 parts, each placed in a sealed envelope and marked for identification. One sample is used by seller and one by purchaser, to determine the value of the ore. If these determinations agree within certain SPLITTING LIMITS, provided by agreement, the average is taken (Sec 29, Art 11); otherwise, the third sample is sent to an umpire, the fourth being held for emergency use of seller, purchaser, or umpire. The seller or shipper should always have a representative at the smeltery, to watch the sampling, and receive duplicate samples.

Chemical composition. Determinations should be made for: Au, Ag, Cu, Pb, Zn, SiO_2, Fe, Al_2O_3, CaO, S; sometimes for MgO, Ba, As, Sb, and other substances. The analysis reveals the gangue constituents, and indicates whether the ore is self-fluxing, silicious, or basic. A typical analytical smelter report is shown in Form 1.

Form 1. Tacoma Smelter, A S & R Co

Tacoma, Washington 193

Bought of

Material

Smelter lot Mine lot Date received

Car or vessel Entry No Dated

Gold quotation	Silver quotation	Foreign / Copper quotation / Domestic	
Date	Date	Date	Date
\$ per oz	Less	Less	Less

Pb	Zn	As	Sb	Ni	Bi	Sn	Fe	SiO_2	CaO	S	Cl

Lot No	No sacks	Wet weight	H_2O	Dry weight	Assays: Au	Ag	Cu	Contents: Gold, oz	Silver, oz	Copper, lb

(Space of 14 lines)

Notice: If not advised to the contrary within.........days from date we will assume the returns are satisfactory. All prices on ore, not under contract for a specified time, are subject to change without notice. An additional charge of \$10 per lot. for sampling and assaying, will be made on all lots of less than 5 tons, and a reasonable size sample will be held for a period of 30 days.

Few ores are self-fluxing; they are usually either silicious or basic. If silicious, basic material is added to the furnace charge; if basic, silicious material is added. As most of the ore mined in many regions is silicious, basic ore commands a premium, and silicious ore is penalised. The aim is to make a self-fluxing mixture. Allowances for Fe and CaO, and the penalty for SiO_2, are such that they offset each other when the ore mixture is calculated to produce proper slag. The smelter is supposed to obtain no advantage from this system of premiums and penalties; in practice the results may be slightly in his favor.

Fluxes. For proper fluidity of slag, certain percentages of the silicates of FeO and CaO must be present. In copper smelting, slag formerly contained more silica in proportion to bases than in lead smelting. Now (1938) the difference is slight; in some cases lead slag is even more silicious than copper slag. In making up a smelting mixture, CaO usually costs 5–10¢ per unit, and Fe, 8–14¢ (a unit is 1% of a ton, or 20 lb for a short ton); but prices must be accurately known for any given region. The charge for SiO_2 is sufficient to offset payments for Fe and CaO. Penalty for silicious ore may be returned as bonus for Fe and (or) CaO in basic ore.

Example. If smelter pays 5¢ per unit for CaO and 10¢ per unit for Fe, the charge for SiO_2 is determined as follows: Composition of slag desired, 36% SiO_2, 36% FeO(=28% Fe) and 19% CaO. Cost for 28 units of Fe at 10¢ = \$2.80, 19 units CaO at 5¢ = \$0.95; total \$3.75, which is the cost of fluxing 36 units of SiO_2; whence, the penalty per unit of SiO_2 = 10.4¢, say 11¢. Under these conditions, CaO is paid for at 5¢ a unit, Fe at 10¢, and SiO_2 is charged for at 11¢ per unit.

Fe is usually obtained from special ores containing a large Fe excess. But CaO occurs to such a limited extent in most ores that it must be secured by smelting barren limestone, quarried as near the smelter as possible.

3. VALUE OF PRODUCTS; INFLUENCE OF IMPURITIES

Value of recoverable metals is determined by analysis and assay of the samples (Art 2). The principal metals in smelting ores are Pb, Cu, Au and Ag.

Pb and Cu are reported in percentages. The value of Cu is determined by the standard quotation per lb; of Pb by the quotation per lb or unit. Au and Ag are reported in Troy oz per ton of 2 000 lb; their values are now (1938) regulated by the Govt; terms are given in Art 4. In determining the value of Cu, Pb, and Ag, it is specified that the quotation used shall be that of a certain date. The buyer usually pays for the ore at once, making allowance for the time he has to carry the values in the ore. Sometimes it is specified that the quotation used shall be that of a certain number of days after settlement of assay (30, 60, or even 90 days); during which period the metals may be recovered by the smelter and sold at approximately the price that is paid for them.

Influence of impurities has an important bearing on the value of an ore and upon the price paid for it. If the impurities are small in amount, they receive no attention. But, if there is possibility that the ore contains impurities to an extent that will interfere with reduction and recovery of metals, penalties are imposed.

Principal impurities. SULPHUR in copper ores is not detrimental and is not penalized. But sulphur in lead ores, if above a certain limit, makes roasting necessary. As this increases treatment cost, an ore containing more than say 3% S is penalized on the basis of 15 to 25¢ per unit in excess of that amount, up to a maximum of \$2.50 per ton. ZINC, especially as blende, creates irregularities in smelting; part goes into the matte, part into the slag, making it pasty, and part is volatilized, causing trouble in the furnace, and increasing silver losses by volatilization. The usual limit is 8% Zn; at some smelters, all in excess of 5% is penalized. The charge for excess Zn is 15 to 50¢ per unit. ARSENIC is usually penalized in lead, and frequently in copper ore. With lead, it forms SPEISS (Art 2), a troublesome by-product difficult to retreat; with copper, some of the As is reduced in the bullion, increasing refining cost. ANTIMONY is also troublesome, but less so than As. In refining lead bullion from smelting Pb-Ag ore, part of the Sb is recovered as antimonial lead. BISMUTH is objectionable, not being removable in ordinary refining and is found in the refined lead. It can be removed only by electrolysis. PENALTIES for impurities vary greatly, and exact figures are impossible. In lead smelting, As + Sb + Sn in excess of 0.5 to 2 units are often penalized at 50¢ to \$1 per unit. Bi is usually penalized at 50¢ per lb or fraction thereof in excess of 0.1% of the lead content of ore.

4. TERMS OF PAYMENT FOR METALS

In purchasing any class of ore, all the points stated in paragraph 1 of Art 2 must be considered. From (*e*) is found the value of recoverable metals, and from (*j*) the amount paid for them, the difference being the MARGIN ON METALS. From (*c*) and (*d*) result the premium or penalty for basic or silicious ores, and from (*f*) the penalty for impurities. Cost of treatment (*g*) and losses of metals (*h*) are determined from records and experience

in the smelting plant. A base charge (k) is then fixed by considering all the costs and the margin on metals, adding to the margin a charge that will give a total equal to all the costs, plus profit desired. Thus the greater the margin on metals the less the base charge needs to be, to secure a given profit. The base charge, therefore, is an arbitrary figure, acting as a balance wheel.

Gross value of an ore to the smelter is the total metal content, at prices for each constituent arrived at by taking market quotations and deducting from them the cost of freight, smelting, refining and metal losses. The term GROSS MARGIN means the difference between total value of the metals present and the amount paid for the ore; it covers all operating costs, losses of metals, and profit (1).

Gold (typical terms): none paid for if less than 0.03 oz per dry ton. If 0.03 oz or over, entire content paid for at "contract price" as follows: on old standard price for gold, if less than 1 oz per dry ton, \$19 per oz; if 1 oz or over but less than 5 oz, \$19.50; if 5 oz or over, \$20; provided that under the conditions stated in the "Gold Schedule" USAU-A7 (obtainable from the Govt or any large smelter), whenever in any calendar month the "realized price" as defined therein exceeds \$20.67 per oz, payment is made at contract price plus 90% of such excess; and, whenever in such month the "realized price" is less than \$20.67 per oz, then the realized price is paid, less 8%. At present Govt price (June, 1939) these terms are equivalent, for 0.03 oz or over but less than 1 oz per ton, to \$31.818 per oz; for 1 oz or over but less than 5 oz, to \$32.318 per oz; and for 5 oz or over, to \$32.818 per oz. As there is little or no loss of Au in ordinary smelting operations, the difference between value of the Au and amount paid for it more than covers the carrying interest and losses, and therefore adds to the margin on metals in the ore.

Silver (typical terms): none paid for if less than 1 oz per dry ton. If 1 oz or over, entire content less 5% or 0.5 oz per ton (whichever deduction is greater) is paid for at the aver of Handy & Harman N Y silver quotations for the week of delivery at smelter. But if the silver is qualified for delivery to U S coinage mints, as provided in the "Silver Schedule" (USAG, obtainable from the Govt or any large smelter), then, subject to conditions stated therein, the seller shall receive the aver Handy & Harman N Y silver quotation or the "Mint price," whichever is higher on quotation dates. Handy & Harman price (Apl, 1938) was 42.75¢ per oz and the Govt price for U S Silver, 64.64¢. Usually more Ag is recovered in smelting than is paid for, the difference adding to the margin on metals, as for gold (10).

The "Gold" and the "Silver Schedule" (USAU-A7 and USAG) are complicated, but the above summaries give a clear idea of what may be expected.

Lead. Many different payment schedules are in use, the most common deducting 1.5 units from the wet lead assay and paying for 90% of the remainder at some current published price, less freight to refinery and refining and selling charges. A Pb content of less than 5% is rarely paid for. The assay deduction is supposed to cover smelting loss and in making it the variation in Pb content does not affect subsequent terms and conditions. Charge for freight, refining and selling may be about \$30 per ton or 1.5¢ per lb of lead bullion. **Example**: an ore in Colo containing 21.5% Pb, the published base price at N Y being 6¢. Then 21.5, less 1.5 units assay deduction, is 20 units Pb, of which 90% or 18 units (360 lb per ton) is paid for at 6¢, less 1.5¢ for freight, refining and selling, or 4.5¢ per lb. Net return to seller for Pb, 360 lb at 4.5¢ or \$16.20 per ton ore. For old local schedules, see Bib 1, 6, 10, 13, 14. MODE OF ASSAY (Sec 30). WET ASSAY is exact, preferable for seller and now always used. DRY ASSAY (by fire) is usually lower (by 1% to 2%) than wet assay, and is antiquated.

Copper was formerly paid for on basis of the dry or CORNISH ASSAY, estimated at about 1.3 units less than the amount of Cu in the ore. This difference was supposed to cover loss of Cu in old methods of treatment. This assay is now seldom used, Cu being paid for on basis of the wet (electrolytic) assay, less a certain percentage (usually 0.5 to 1 unit) to cover assumed treatment loss. With present methods, this loss seldom reaches 1.3 units. After making deduction, remainder of the Cu is paid for on basis of N Y quotation (¢ per lb), less amount taken to cover cost of smelting, refining and treating, for bringing the Cu to merchantable shapes. The deduction (cts per lb) for Cu in ore sent to a lead smelter is much greater than for ore to a copper smelter. (For aver prices of metals, see Sec 25, Art 13).

These terms seem complicated and the question is often asked, why should not all ore be purchased with an outright and specific base charge? The reason is that, if the terms of payment for each metal do not allow for treatment losses, variations in grade of ore will materially affect reduction cost and losses. Then, if no allowance were made for the Pb loss in quoting for Pb, the base charge would have to be much higher for a rich than for a poor ore, and it would be difficult to frame a sliding scale to fit all conditions. Excessive allowances are often made for losses, but if proper terms are used the miner suffers

no hardship, since the smelter can name a lower charge than would otherwise be possible (5, 7, 8, 16, 23, 25).

Interest and commission are relatively small items. When ore is paid for at time of settlement of assay, interest is allowed on the value of metals, for say 30, 60 or 90 days, depending upon how soon they can be recovered. But, if the metals are not paid for until sufficient time is allowed for their recovery, no interest charge should be made. Commission for selling is usually allowed for in the net price received for refined metals.

Base charge. After arranging terms of payment for metals, premiums or penalties for character of ore, and penalties for impurities (Art 2, 3), a base charge is made. This charge, with other allowances, covers cost of smelting (and roasting, if necessary), treatment of by-products, freight on metals or products, refining of metals and profit to smelter. The base charge is sometimes less than smelting cost, in which case the margin on metals covers difference, and furnishes the profit.

Problem in ore-purchasing. Assume an ore containing: Au, 1 oz; Ag, 20 oz; Pb, 15%; Cu, 2.3%; Fe, 30%; CaO, 3%; S, 16%; SiO_2, 12%; Zn, 5%. Terms which might be used for such an ore are: Au, pay for 100% @ \$31.82 per Troy oz; Ag, pay for 95% at say 65¢ per oz, for U S production; Pb, pay for 90% of wet assay less 1.5 units, at N Y price, less 1.5¢ per lb (say 6¢ less 1.5¢ = 4 1/2¢ per lb) or \$0.90 per unit; Cu, pay for wet assay, less 1 unit at N Y price, less a deduction (say 14¢ less 6¢ = 8¢ per lb); Fe, pay for at 10¢ per unit; CaO, pay for at 5¢ per unit. SiO_2 is penalized at 11¢ per unit; Zn, if over 10%, penalized at 25¢ per unit in excess of 10%. Base charge per ton, \$2. Using these terms, the following statement results:

Method of Calculation

Analysis	Net amount paid miner		Total gross value of metals at smeltery	
Gold, 1 oz	100% @ \$31.82..................	\$31.82	@ \$34.85	\$34.85
Silver, 20 oz	95% @ 65¢ per oz..............	12.35	@ 0.65	13.00
Lead, 15% wet assay	90% of assay less 1.5 units @ 90¢ per unit.......................	10.93	@ 0.045	13.50
Copper, 2.3% wet assay	less 1 unit @ 8¢ net per lb..........	2.08	@ 0.08	3.68
		\$57.18		\$65.03
Iron, 30%	@ 10¢ per unit....................	3.00		
Lime, 3 %	@ 5¢ per unit....................	.15		
Sulphur, 16%		\$60.33		
Less Silica 12% @ 11¢ per unit..........	\$1.32	3.32		
Zinc 5%, no penalty..............				
Base charge....................	2.00			
		\$57.01		\$57.01
			Gross margin	\$8.02

Outcome:					
Gross margin........		\$8.02	Lead loss 10%.......	1.35	
Iron 30% @ 10¢.....		3.00	Silver loss 2%.......	0.26	
Lime 3% @ 5¢......		0.15	Copper loss 20%.....	0.74	
Silica 12% @ 11¢....	\$1.32		Profit to smelter *...	2.00	
Cost of roasting.....	1.50				
Cost of smelting.....	4.00			\$11.17	\$11.17

* In this statement, interest, amortization on plant, taxes, and expense of home office are not included, and may considerably affect the apparent profit shown.

This statement shows that the value of the metals at the smeltery is \$65.03, while the amount paid for them is \$57.18, a difference of \$7.85; which, deducting the Pb, Ag and Cu losses of \$2.35, leaves \$5.50 in favor of the purchaser. Though this is not an actual profit on metals, it enables the purchaser to fix a lower base charge; in this case, \$2 a ton, which is much less than the cost of roasting and smelting. If the margin on metals were reduced, the base charge would have to be correspondingly increased, to obtain the profit shown.

For reasons already given, it is advantageous to allow for losses in quoting on metals, and in so doing the seller suffers no hardship. The premium paid for Fe and CaO, and penalty levied for SiO_2, are considered in the outcome. The ore, because of its Fe and CaO content, has an added value equal to the premium paid, and the cost of SiO_2 which must be fluxed, is considered in treating, this excess being the amount of the penalty for SiO_2 in the purchase statement. When a deduction for Zn is considered in the gross margin, it might seem that it should also appear in the outcome as a cost. But this is difficult to arrange. Excess Zn increases losses and treatment cost, but the amount of increase for different classes of ore is not readily determined. For average ore, the losses and costs are ascertained from the metallurgical results, which furnish the only true data (5, 7, 8).

In certain localities, as in parts of Colo, ores of uniform character, having their chief value in Ag and Au, are sold on a FLAT SCHEDULE, no attention being given to ore composition. This schedule is based on a sliding scale, depending on value of ore (1). Treatment charge on low-grade

ore is such as to stimulate its production; higher-grade ore can stand a higher charge. The charge for a Pb or Cu ore frequently decreases as the value increases, because: (a) an ore of high metal content makes less slag; (b) the Pb or Cu is a valuable collector for Au and Ag.

Different forms are used for making ore purchase calculations. Form 2 is comprehensive, and

Form 2. Terms of Metallurgical Treatment

Marginal calculation for ______ Contract No ______ Plant ______ 193_

Owner ______ Location ______

Class of ore ______ Est monthly shipment ______ Tons ______

Basis of cost used that of 193__ Shipping station

Au	Ag	Pb fire	Pb wet	Cu wet	Insol	SiO_2	Fe	Mn	CaO	Al_2O_3	Zn	S	As	Sb	Ni	Bi

Quotations	Deductions	Payments
Date	Base charge	Gold: oz @ $
Au per oz		
Ag per oz		
Pb per 100 lb		Silver: oz @
Pb (London) per 2 240 lb		
	Insol	
	Zn	
U S equiv per 100 lb	S	
Frt & ins " " "	As	Lead:
N Y parity " " "	Sb	
	Ni	
Cu W B per lb	Bi	
Cathode "	Bricking	
		Copper:
Sterling exchange	Freight charged shipper	
Mexican "	Switching " "	
	Taxes " "	
R & D deductions used		
Au per oz		
Ag "		Total
Pb per lb		Less deductions
Cu "		Net amount paid per dry ton (2 000 lb)
	Total deductions	

Treatment cost per dry ton (2 000 lb)	Value per dry ton (2 000 lb) (R & D deducted)
Freight paid, if for plant account	Gold: oz @ per oz
Switching " " " " "	
Frt on moisture " " "	Silver: oz @ per oz
Roasting:	
Sintering:	
Sulphur:	Lead, domestic:
Bricking:	
	Lead, foreign:
Smelting:	
Converting: lb Cu @	
	Copper, domestic:
Flux: Silica	
Iron	
Lime	Copper, foreign:
Zinc	
	Total value
Impurities:	Less amount paid for ore (above)
	Smelter margin
Metal loss:	
Gold	
Silver	Less net cost of treatment
Lead	Smelter outcome
Copper	Depreciation
Interest % @ days	General administration
Net cost of treatment	Net outcome

includes all the steps required in determining ore or bullion value, margin on metals, premiums and penalties, detailed for treatment costs and profit.

5. SMELTER SCHEDULES

The Govt action in increasing the price of gold, and of silver produced in the U S. has greatly stimulated the mining industry. There are now more small shippers of gold and silver ore and concentrate to the smelters than previously. Tables 1 and 2 are lists of the principal smelting plants, and their purchase schedules.

Table 1. Copper and Lead Smelters in Western States (27)

State	Company	Smelter	Location	Ores treated
Arizona	Phelps Dodge Corp.........	Copper Queen Branch	Douglas	Gold, silver, copper, and lead ores and concentrates
do	do	United Verde Branch	Clarkdale	Gold, silver, and copper ores and concentrates
do	do	Morenci Branch (a)	Clifton	do
do	Magma Copper Co........	....	Superior	do
do	International Smelting Co (b)	....	Miami	do
do	American Smelting & Refining Co.................	Hayden (a)	Hayden	do
California	do	....	Selby	Gold, silver, and lead ores and concentrates
Colorado	American Smelting & Refining Co.................	Arkansas Valley	Leadville	Gold, silver, lead, and copper ores and concentrates
do	do	Durango plant (a)	Durango	Gold, silver, and lead ores and concentrates
Idaho	Bunker Hill & Sullivan Mining & Concentrating Co...	Bunker Hill	Bradley	do
Montana	Anaconda Copper Mining Co	Anaconda Reduction Works	Anaconda	Gold, silver, copper, and zinc ores and concentrates
do	American Smelting & Refining Co.................	....	East Helena	Gold, silver, and lead ores and concentrates
Nevada	Nevada Consolidated Copper Co.....................	....	McGill	Gold, silver, and copper concentrates and ores
Texas	American Smelting & Refining Co.................	....	El Paso	Gold, silver, copper, and lead ores and concentrates
Utah	International Smelting Co (b)	....	Tooele	do
do	American Smelting & Refining Co.................	....	Murray	Gold, silver, and lead ores and concentrates
do	do	....	Garfield	Gold, silver, and copper ores and concentrates
do	U. S. Smelting, Refining & Mining Co..............	....	Midvale	Gold, silver, lead, and zinc ores. Zinc concentrates shipped
Washington	American Smelting & Refining Co.................	....	Tacoma	Gold, silver, and copper ores and concentrates (copper refinery)

(a) Idle June 1936. (b) A subsidiary of Anaconda Copper Co

Table 2. Schedules for Gold and Silver Ores at Lead Smelters (July, 1936) (27)

Plant	Payments												
	Gold [1]			Silver [2]			Lead [3]			Copper [4]			Iron and lime
	Minimum paid for oz per ton	Payments: Classes of ore, oz per ton	Payments: Rate per ounce	Minimum paid for, oz per ton	Minimum deductions, oz per ton	Percent paid for	Deduction: Units	Deduction: Cts per lb	Percent of quotation after deduction	Deductions: Lb per ton	Deductions: Cts per lb	Percent of quotation after deduction	
El Paso	0.03	All	$32.81825	......	0.5	95 [5]	1.5	1.425	90 [6]	8	5 025	95 [7]	
Murray	.02	All	31.81825	1.0	0.5	95	1.5	1.5	90 [6]	15	5.5	100	Iron [10] Lime [11]
East Helena [13]	.03	0 03 to 3 3 to 5 5 to 10 Over 10	31.81825 32.31825 32.6743 33.0304	.....	1.0	95	1.5	1.5	90 [6]	20	6.0	100 [14]	Iron [15]
Selby [16]	.03	Under 5 5 to 15 Over 15	31.81663 32.31663 32.81663	1.0	1.0	95	......	....	0	.....	....	0	
Selby [18]	.03	Under 5 5 to 15 Over 15	31.81663 32.31663 32.81663	1 0	1.0	95	......	....	0	......	....	0	
Leadville [20]	.03	0 to 1 Over 1	31.81825 32.31825	......	1.0	95	1 5	0	90	20	6.5	100	Iron [21]
Leadville [22]	.03	0 to 1 Over 1	31.81825 32.31825	......	1.0	95	1.5	1.5	100 [23]	20	6.5	100 [24]	Iron [25] Lime [26]
Leadville [29]	.03	0 to 1 Over 1	31.81825 32.31825	......	1.0	95	1.5	1.5	90	20	6 5	100	
Midvale	.02	0 to 5 Over 5	31.81825 [30]	1.0	0.5	95	1.5	1.5	90 [31]	15 [32]	5.5	90	Iron [33]
Kellogg, Idaho [36]	.05	Under 5 5 to 10 Over 10	31.81825 32.17431 32.53037	1.0	....	95 [37]	1.25	0	90 [6]	20	8.0	100 [38]	
Kellogg, Idaho [41]	.05	Under 5 5 to 10 Over 10	31.81825 32.17431 32 53037	1.0	......	95 [37]	......	....	0	20	8.0	100 [38]	

Table 2 (continued). Deductions for Treatment Charges and Penalties

	Treatment charge			Penalties														
				Insoluble		Zinc		Arsenic (As) Antimony (Sb) Tin (Sn)			Bismuth		Sulphur			Moisture		
	Gross values	Base per ton	Maximum	Units free	Charge per unit	Units free	Penalty per unit for excess	Element or combination	Units free	Penalty per unit for excess	Units free	Penalty per pound for excess	Units free	Penalty per unit for excess	Maximum penalty	Units free	Penalty per unit for excess	Maximum penalty
El Paso	0 to $25 Over $25	$3.70[8]	$6.70	0	$0.05	5	$0.30	As Sb+Sn	2 1	$0.50 1.50	[9]	$0.50	2	$0.20	$2.00	...	0	
Murray	[12]	2.50	.....	0	0.10	6	0.30	As+Sb	2	0.50	...	0	2	0.25	2.50	...	0	
East Helena[13]	0 to $30 30 " 40 Over 40	6.00 5.50 5.00	6.00	...	0	5	0.30	As Sb	2 1	0.50 2.00	[9]	0.50	...	0	.....	...	0	
Selby[16]	All	6.00	6.00	17	0	...	0	As+Sb+Sn	1	0.50	0	0.50	...	0	.....	...	0	
Selby[18]	0 to $35 Over 35	6.50[19]	10.00	...	0	...	0	As+Sb+Sn	1	0.50	0	0.50	...	0	.....	...	0	
Leadville[20]	All	6.00	6.00	...	0	5	0.50	As+Sb+Sn	0.5	1.00	0.05	0.50	...	0	.....	10	$0.05	0
Leadville[22]	0 to $ 8 8 " 10 10 " 50	4.00 [27] [28]	.....	0	0.05	8	0.30	As+Sb+Sn	0.5	1.00	0.05	0.50	1	0.25	2.50	10	0.05	0
Leadville[29]	All	8.50	8.50	...	0	8	0.30	As+Sb+Sn	0.5	1.00	0.05	0.50	...	0	.....	10	0.05	0
Midvale	[34]	2.50	.....	0	0.10	6	0.30	......	...	0	[35]	.....	2	0.25	2.50	...	0	
Kellogg[36]	[39]	12.00	.....	...	0	5	0.30	As+Sb	0	1.00	0	2.50	4	0.25[40]	2.00	6	0.20	$2.00
Kellogg[41]	0 to $20[42]	6.50	9.00	...	0	5	0.30	As+Sb	0	1.00	0	2.50	4	0.25	2.00	6	0.20	2.00

FOOTNOTES FOR TABLE 2

[1] Payments on gold are based on new mint price of $35 per oz (net $34.9125)
[2] Payments on silver are based on new mint price for new American-mined ore at 64.64¢ per oz
[3] Payments on lead are based on N Y quotation for common desilverized lead
[4] Payments on copper are based on the *Eng & Min Jour* quotations
[5] Less a deduction of 1 1/2¢ per oz
[6] Nothing paid for lead less than 5% wet assay
[7] Nothing paid for copper less than 1/2%
[8] Add 10% to base charge for excess value over $25 per ton
[9] 0.1% of wet-lead assay is free
[10] Pay for all at 6¢ per unit
[11] Pay for all at 5¢ per unit, if 5% or over
[12] Add 10¢ to base charge for each unit of lead under 30% and deduct 10¢ for each unit over 30%
[13] Siliceous ore schedule for ores and concentrates having excess of iron are identical, except that treatment charge is a flat $5 per ton
[14] Nothing paid for copper less than 1%
[15] No credit
[16] Schedule for gold concentrates
[17] Add 10¢ per ton per unit of iron short of 25 units excess over insoluble
[18] Schedule for crude siliceous gold ore
[19] Add 10% to base charge for excess value over $35 per ton
[20] 1934 schedule for irony ores and concentrates applies only to ores and concentrates containing 20% or more of iron excess over insoluble
[21] Excess over insoluble, all at 10¢ per unit, not to exceed $3 per ton
[22] Crude-ore open schedule
[23] When lead is over 9¢ per lb, deduct 25% of excess
[24] When copper is over 15¢ per lb, deduct 25% of excess
[25] Pay for iron plus manganese at 5¢ per unit, but credit is not to exceed charge for insoluble
[26] All at 8¢ per unit, if 10% or over
[27] Add 25% of gross value to base treatment charge, when value is between $8 and $10
[28] Add 10% of gross value to base charge, when value is between $10 and $50
[29] 1934 siliceous-ore special for ores with 50% excess insoluble
[30] On direct smelting ores containing over 5 oz gold per ton, price per oz for the excess will be left to mutual agreement between buyer and shipper
[31] No payment for lead under 3% dry assay
[32] Minimum deduction
[33] All at 6¢ per unit
[34] Based on 30% dry lead assay. Debit 10¢ for each unit under 30% and credit 10¢ for each unit above 30%
[35] Midvale smelter reserves right to reject any shipment containing more than 0.1% bismuth
[36] Lead ore open schedule
[37] Ore over 35 oz per ton, deduct 2¢ per oz
[38] No payment for copper under 1%, or when quotation is 8¢ per lb or less
[39] Based on 50% lead. Add 10¢ per unit when over 50%, and deduct 10¢ per unit when under 50%
[40] Penalty applies only to ore under 20% lead; no penalty for ore of 20% or over
[41] Siliceous-ore open schedule: ores containing no lead, or under 5% for which no payment is made (lead determined by wet method less deduction of 1 1/4 units)
[42] Between $20 and $35, $7 per ton; from $35 to $50, $7.50; from $50 to $75, $8; from $75 to $100, $8.50; over $100, $9 per ton

6. MILLING ORES (1)

Terms used in purchasing ore to be treated by amalgamation or cyanidation (Sec 33) are comparatively simple, the operations being much less complex than in smelting. The gangue composition is far less important, as there is no question of making a proper slag, and treatment losses are easily determined. The percentage of recovery of Au and Ag is less than in smelting, but the treatment cost is so much lower (especially in cyanidation) that many ores can be more satisfactorily milled than smelted. Au and Ag ore adaptable to milling usually contains but little Cu or Pb.

The "Open Schedule," given in substance in the 4 following paragraphs, is a good example of terms for treating milling ores and concentrates in western part of the U S (1938):

Treatment rates per ton for ores and concentrates: of value not over $8 per ton, $2.50; over $8 but not over $10 per ton, $3; over $10 but not over $15 per ton, $4; over $15 but not over $20 per ton, $4.50; over $20 but not over $40 per ton, $5.50; over $40 per ton, $6. Additional charges: for sampling any lot of less than 10 ton, dry wt, $5, and shipper pays for umpiring or check assaying; for re-sample of such lot, $5; when more than one lot is shipped in a car, $2.50 per lot; if ore is sacked, 10¢ per ton; if received frozen, 10¢ per ton; for each 1% moisture over 10%, 5¢ per ton. Minimum moisture deduction, 1%. These rates are subject to change without notice and are exclusive of freight charges, which must be prepaid or guaranteed by shipper.

Not accepted at this plant: ore containing over 0.2% non-sulphide copper, or over 3% total copper, or over 3% zinc. Concentrates are accepted at option of mill manager and payment is made for gold only. New shippers or those whose ore changes in character, or who move from one mine to another, must send a 5-lb sample, charges prepaid, to be tested before shipments are consigned.

To avoid demurrage and enable settlements to be completed, shippers must notify mill promptly of all shipments, by bill of lading or letter stating name of mine, owner, lessee (if any) and car number. Shipper agrees that, after sampling, the shipment (except the reserve sample) may be mixed with other ores, or otherwise disposed of; and agrees to settlement in accordance with standard practice at this plant. To insure prompt settlement shipper must compare assays promptly by telephone, assay certificate or letter, or authorize settlement on mill assays when consigning; without such information within 10 days of receipt of shipment, settlement will be made on mill assays. No umpires or re-samples are allowed on shipment of 0.25 oz gold or less per ton.

Payment. GOLD; no payment if less than .02 oz per ton; from .02 to 0.5 oz inclusive per ton, $32.26 per oz; over 0.5 but not over 5 oz per ton, $32.76 per oz; over 5 but not over 10 oz per ton, $32.76 for first 5 oz and $33.12 per oz for balance; over 10 oz per ton, $32.76 for first 5 oz, $33.12 for second 5 oz and $33.47 per oz for balance. For gold in truck shipments and shipments of less than 10 ton, deduct $1 per oz from above prices. SILVER; no payment if less than 1 oz per ton; from 1 to 5 oz per ton, 50% paid for; over 5 but not over 10 oz per ton, 65% paid for; over 10 but not over 20 oz per ton, 75% paid for; over 20 but not over 50 oz per ton, 85% paid for; over 50 but not over 100 oz per ton, 90% paid for; over 100 oz per ton, 95% paid for. Provided miner's silver affidavits are promptly furnished, as required by the Treasury Dept, payment is made under existing government regulations at 64.64¢ per oz; otherwise at the market quotation for date immediately proceding date of settlement, excluding fractions of cents; except that for ore exceeding 100 oz per ton, 2.5¢ additional will be charged for excess over 100 oz. SULPHIDE LEAD in crude ores only: no payment if less than 3%; if 3% or more, pay for 80% of wet assay less 1%, at N Y quotation for date immediately preceding date of settlement, less 2.5¢ per lb. Non-sulphide lead, no payment. (NOTE. The "Open Schedule" ends here.)

Copper ore. For Cu ore containing Au and Ag, but no Pb, or so little that it is not considered, terms of payment are illustrated by the following schedule, in force 1937:

Gold and silver on the same basis as for lead ore. Copper, pay for 100% of wet assay, less 8 to 15 lb per ton ore, *Eng & Min Jour* quotations, less 2.5 to 3¢ per lb. **Penalties.** Zinc, no penalty at many copper smelters, though some charge 30¢ per unit in excess of 6 units. As, Sb, and Sn are free at most plants, at others 50¢ to $1.50 for all in excess of 1 to 2 units. Bi, no charge, except at El Paso smelter, where all in excess of 0.1 unit is charged for at 50¢ per unit. Treatment charge, $2.50 to $6 per ton, based on gross value of ore up to $100; over $100, $8 to $15.

Example 1. Method of calculating returns on a shipment of ore based on preceding schedule: wet wt of lot = 36.28 ton; moisture = 8.5%; dry wt of lot = 36.28 (1.00–0.085) = 33.20.

Chemical analysis: gold, 1.27 oz per ton; silver, 2.10 oz per ton; copper, 1.2%; zinc, 6.4%; arsenic, .60%; antimony, .72%; bismuth, .08%; iron, 12.4%; sulphur, 19.1%; silica, 49.7%; alumina, 6.0%; total, 96.20%.

Payments: gold, 1.27 × $32.76 = $41.605; silver (2.10 − 0.5) × 64.64 = $1.034; copper @ 14¢ (avg) per lb, 20 (1.2 − 0.75) × (0.14 − 0.025) = $1.035; lead, none. Gross payment per dry ton, $43.674.

Deductions: treatment charge for ore of this value, $5.50. Penalties: zinc (6.4 − 6) × 0.30, .12; arsenic + antimony (0.60 + 0.72) − 2.0, none. Total deductions per dry ton, 5.62. Net payment per dry ton, $38.054.

Total returns by the smelter to the shipper, 33.20 × 38.054 = $1 263.39.

In Canada production of copper has become an important industry. Examples 2 and 3 show typical returns prior to 1938 from a Cu-Au-Ag ore, and a Cu ore from which iron pyrite is recovered and marketed, terms change, but form of computation is useful.

Example 2 (29). A copper mine in B C produced from 100 ton ore assaying 1.08% copper, 0.003 oz gold and 0.116 oz silver: 3.2 tons of copper concentrate assaying 27.63% copper, 0.06 oz gold and 2.45 oz silver, recovering 88.4% of copper, 61.2% of gold and 68.2% of silver. Value of the concentrates, which were shipped to a copper smelter in the U S, is calculated as follows:

Terms: copper assay, less 1.3 units at dollar equivalent of London copper price, less 2¢, 98% of gold at $35, 95% of silver at silver price less 1 1/2¢. Treatment charge $5 per ton. Assuming copper price 15¢, and silver price, 45¢: Copper, 27.63%, less 1.3% = 26.33% = 526.6 lb at 15¢, less 2¢ = 13¢ = $68.46; Gold: 98% of 0.06 oz = 0.0588 oz at $35.00 = $2.06; Silver: 95% of 2.45 oz = 2.3275 oz at 45¢, less 1 1/2¢ = 43 1/2¢ = $1.01; total, $71.53. Freight $1; treatment charge, $5 = $6.00. Net revenue per ton concentrate, $65.53; revenue from 3.2 ton concentrate (from 100 ton ore) at $65.53 = $209.70; revenue per ton ore, $2.10. Gross value per ton ore, 1.08% copper, 0.003 oz gold, 0.116 oz silver: Copper 21.6 lb at 15¢, $3.24; Gold 0.003 oz at $35.00, 10¢; Silver 0.116 oz at 45¢, 5¢; total, $3.39.

Example 3 (29). A copper-pyrite property in Quebec produced from 100 ton ore assaying 3.71% copper, 13.6 tons copper concentrates assaying 25.62% copper, with a recovery of 93.77%, and 51 1/2 ton of iron-pyrite concentrates, assaying 49% sulphur. Value of the concentrates, which were shipped to the U S, is calculated as follows:

Terms on copper concentrate: Copper assay less 1.3 units at dollar equivalent of London copper price, less 2¢. Treatment charge $5 per ton. Assuming Copper price 15¢: Copper, 25.62% less 1.3% = 24.32% = 486.4 lb at 15¢ less 2¢ = $63.23. Freight ($5) treatment charge ($5), $10.00. Revenue per short ton copper concentrates, $53.23. Terms on iron pyrites: 49% sulphur at 13¢ per long ton unit, $6.37; freight, $2.00. Net revenue per long ton pyrite concentrate, $4.37; 13.6 short ton copper concentrates at $53.23, $723.93, 51.5 short ton pyrite concentrates, *ie* 46 long tons at $4.37, $201.02; revenue from 100 short tons ore, $924.95; revenue per ton ore, $9.25.

Based on copper content, the gross value of this ore, assaying 3.71%, 74.2 lb at 15¢, equals $11.13. The revenue in this case compared to the gross value is increased by the sale of iron-pyrite concentrates. To make two concentrates, instead of allowing the pyrite to go with the tailings, entails higher cost of concentration; and the feasibility of such procedure depends on having a market for the pyrite concentrate at a low enough freight cost.

Zinc ores. The purchasing terms are entirely different from those for Pb and Cu. Zn ore is mostly sold in open market at a base rate per ton of sulphide ore (concentrate) containing 56 to 60% Zn, a premium of about $1 being paid per unit over 60, and $1 penalty charged per unit under 56. A charge may also be made for CaO, Pb and Fe. The flat price per ton varies with conditions.

The value of Zn ore is always determined basically by its Zn content, less an allowance for metallurgical loss, less a deduction for freight and treatment. This is expressed by the formula: $[(T - 8) \times 20 \times P] - R$, in which T is the Zn assay in percent, 8 the deduction for loss, 20 the lb per unit (changed to 22.4 if the long ton is used), P the price of Zn in cts per lb at the basing point and R the "returning charge" or deduction for treatment, in dollars per ton.

In the U S the principal source of Zn ore is the Tri-State district, where its value is determined by auction. At present (Feb, 1938) the Joplin quotation is $30 per 2 000 lb for standard ore of 60% grade (1 200 lb Zn per ton). From this the factor $(T - 8)$ indicates a yield of 1 040 lb or 86.67%, which is a fair figure for overall recovery from green ore. As the price for Prime Western zinc at St Louis is 4.75¢ per lb and the freight rate from Okla points to St Louis is 0.175¢, the quotation for Zn at point of origin is 4.575¢, at which the value of 1 040 lb is $47.58; and deducting $30 base payment for the ore leaves the metallurgist a margin of $17.58, which is R in the formula (W. R. Ingalls).

Example 4, showing the settlement price for Zn concentrate in the Joplin district, when base price and Zn content are lower than stated above, and penalties are levied on impurities. Base price is quoted each week per dry ton of 60% Zn concentrate, with flotation product at $1 per ton less than gravity grades. For concentrate of 56% Zn or more, the value of the Zn content equals the quoted base price divided by 60 and multiplied by the actual Zn assay in percent. For lower grades, this value is computed as though for 56% and reduced $1 per ton for each unit below 56% Zn. Lead-free Zn concentrate of 0.1% Pb or less usually commands a premium of $1 per ton above the quoted base price. Allowable Fe 3%, Pb 2%, CaO 1.5%; excess of any one constituent is penalised at $1 per unit.

For a $27 base price and a dry ton of concentrate assaying 55% Zn, 4% Fe, 3% Pb, and 2% CaO, the calculation is as follows:

$$\text{Value of 56\% Zn concentrate} = \frac{27}{60} \times 56 = \$25.20$$

Penalties, 1 unit Zn deficiency.........	$1.00	
1 unit Fe excess............	1.00	
1 unit Pb excess............	1.00	
1/2 unit CaO excess..........	0.50	$3.50
Settlement price........		$21.70

Formulas for purchase of Western ores are more complicated, due to occurrence of recoverable silver and lead. Such ore is usually sold to Au electrolytic refinery on basis of a special schedule prepared by the refinery.

The value of zinc ore delivered to European smelters is theoretically determined in the same way, but with more complications as to variation in the returning charge, according to the price for spelter, etc.

In Canada, the terms for concentrates from a Zn-Pb ore prior to 1938 were as follows:

Example 5 (29). A lead-zinc mine in B C produced from 100 ton ore (assaying 10.9% Pb, 14.7% Zn, 2.08 oz Ag) 12.5 tons Pb concentrate (assaying 80.5% Pb, 3.6% Zn, and 10.02 oz Ag) and 21.6 ton Zn concentrate (assaying 61.08% Zn, 1.09% Pb and 2.54 oz Ag). 92.6% of the Pb and 60.4% of the Ag were recovered in the Pb concentrates, and 89.9% Zn in the Zn concentrates. The value of the concentrates, which were shipped to Europe, is calculated as follows:

Terms on lead concentrates: 95% of Pb content paid for at London price. Return charge, 20 sh gold per gross ton, based on London lead price of £12 gold, with an added charge of 1s 6d for each £1 advance. Full content of silver at London price per standard oz for each fine oz content (as standard silver is 925 fine, this means a deduction of 7 1/2%). Assuming lead price £24, this equals £14.564 gold (with gold at 140s); return charge is therefore 23.846s gold, or 39.295s paper.

Lead: 95% of 80.5% = 76.475% at £24, 367.08s paper. Silver: 10.02 oz at 20d, 16.70s paper. Total 383.78s paper. Return charge, 39.295s paper; smelter payment per long ton, 344.485s paper; exchange at 4.97, $85.60; freight, $16.50; revenue per long ton lead concentrates, $69.10.

Many zinc smelters in the U S and most of those in Europe recover by-products, but their value is compounded in the zinc formula.

Terms on zinc concentrates: Assay, less 8 units at London zinc price. Return charge 40s gold per gross ton, based on London zinc price of £9 gold, with an added charge of 3s for each £1 advance up to £12; then 3s 3d for each £1 advance up to £15, and 3s 6d for each £1 advance thereafter. Assay basis, 52% to 54%, with 9d gold advance in the return charge for each percent above 54%. Assuming zinc price £22.5, this equals £13.654 gold (with gold at 140s); return charge is therefore 59.6855s gold, or 98 354s paper. 61.08% less 8% = 53.08% at £22.5, 238.86s paper; return charge, 98.354s paper; smelter payment per long ton, 140.506s paper; exchange at 4.97, $34.91; freight, $14.00. Revenue per long ton zinc concentrates, $20.91; 12.5 short ton lead concentrates, *ie*, 11.16 long ton at $69.10, $770.15; 21.6 short tons zinc concentrates, *ie*, 19.3 long ton at $20.91, $403.56; Revenue from 100 short ton ore, $1 173.71, = per ton ore, $11.74. Gross value per ton ore, 10.9% lead, 14.7% zinc, 2.08 oz silver. Lead 218 lb at 5.325¢ (£24, exchange $4.97), $11.61; Zinc 294 lb at 5¢ (£22.5 exchange $4.97), $14.70; Silver 2.08 oz at 45¢ (London 20d per standard oz), 94¢. Total $27.25.

7. MISCELLANEOUS ORES AND NON-METALLIC MINERALS

Iron ore is purchased at flat rate, depending on percentage of Fe, with penalty if it falls below a certain grade, and premium, if above. The base ore standard for bessemer (low phosphorus) and non-bessemer ores (high phosphorus) is 51.5% Fe; standard phosphorus content for bessemer ores, 0.045%. Above or below this standard, within small limits, the price paid for iron ore is slightly decreased or increased respectively.

At beginning of the shipping season in 1938, the rates (31) f o b Lake Erie docks, were: bessemer ore $5.25 to $5.10, non-bessemer ore $5.10 to $4.95, for the above standard grades, with penalty for a lower and premium for a higher content. Eastern ore is often sold at a stated price per unit of Fe. In the past (18) about 83% of ore mined in U S came from Lake Superior districts, 12% from Southeastern States (value at mines about $2.13 per long ton), 2% from Northeastern States (value at mines about $2.59 per long ton), remainder from the West and miscellaneous.

Manganese ore. A sliding scale is always used in purchasing. The higher the Mn content, the higher the price per unit; see following schedule, which was in effect by one consumer in 1924 (16). All prices are f o b at furnaces.

Metallic Mn, %	Fe, max %	SiO_2, max %	Phos, max %	Cts per unit per long ton
50 and over	5	12	15	68
48 to 49.99	5	12	15	65
46 " 47.99	6	13	15	60
43 " 45.99	6	14	15	50
40 " 42.99	5	14	15	45

For each 1% of SiO_2 in excess of the maximum stipulated here, there is a deduction of 25¢ per long ton, fractions in proportion. For each 1%, or fraction, of iron in excess of the maximum, there is a deduction of 25¢ per long ton.

The above gives in general the terms used, but prices have changed since 1924, as shown by following quotations for Mch, 1938 (30). Per long ton unit of Mn, c i f North Atlantic ports, cargo lots, exclusive of duty: Brazilian, 46 to 48% Mn, nominal; Chilean, 47% minimum, nominal; Indian, 50 to 52%, 45¢; Caucasian, 52 to 55%, 45¢. South African, 50 to 52%, 45¢; 44 to 48%, 40¢; nominal.

Chemical grades, per ton, coarse or fine, minimum 80% MnO_2, Brazilian or Cuban, $50 in carloads, to $55 or $60 barreled. Javan or Caucasian, 85% minimum, $60 to $62. Domestic, 70 to 72 per cent, $47 to $52 in carloads, f o b mines.

Domestic manganiferous ore, 10% Mn, 35 to 40% Fe, 22¢ per unit for manganese content and 5¢ per unit of iron, per long ton, delivered Birmingham. High-grade ore, above 40% Mn, low iron content, 40¢ per unit per long ton, delivered Birmingham; under 40% Mn, 30¢ (30).

Prices on foreign ore are quoted c i f Atlantic or Pacific seaport; duty and freight from port to destination paid by consumer.

Chrome ore (31), (lump) c i f Atlantic seaboard, per gross ton (1938): South African (low grade) $16.00; Rhodesian. 45%, $22.00; Rhodesian, 48%, $25.50; Turkish, 48–49%, $25.00 to $26.00; Turkish, 45–46%, $23.50 to $24.00; Turkish, 44%, $19.00 to $19.50. Chrome concentrates (Turkish) c i f Atlantic seaboard, per gross ton: 50%, $25.50 to $26.50; 48–49%, $25.50 to $26.00.

Tin ore or concentrate. No tin ore has been mined in the U S for many years. In foreign countries, practically all tin sold is in form of concentrate. Payment is usually made for 96 to 98% of the tin content (16), depending upon the kind of ore. When all tin is paid for, the base charge is higher. Concentrate is almost always purchased

on a 60% tin basis (Spurr and Wormser). For each unit, or fraction thereof above or below 60%, the returning charge is reduced or increased at rate of 5 shillings (English currency). Net returning charge has varied from £10 to £25.

The basic price for standard tin is generally taken at £170 per long ton, and the returning charge is increased or decreased at a fixed rate per lb above or below this figure. 1% S is allowed free; above this, the penalty is 10 sh per unit (22.4 lb). 5% Fe is allowed free; above this it is penalized. Other undesirable constituents are provided for by an increased base charge.

Tungsten ores are sold in sizes from say 3 in down to fine powder. By sorting or concentration, the material usually contains about 65% tungstic acid (WO_3). It is sold at a price per unit (the unit depending on the ton used). Foreign quotations, converted into U S currency at normal exchange, per unit (20 lb) was about $8.25 in 1923 (16), as low as $3 in 1921, and as high as $40.75 in 1916. Now (Jan, 1938) for Chinese ore, duty paid, $25. Domestic scheelite, $22 to $25. No accepted practice has been evolved concerning premiums and penalties.

Bauxite is the ore of aluminum. It is used in the chemical industry and to some extent for basic refractories. For the aluminum industry, bauxite must be treated to prepare practically pure Al_2O_3. Commercial bauxite usually contains 55–58% Al_2O_3 and not over 10–15% SiO_2. It is usually sold under long-term contracts. Prices have been quoted only in the last few years. Following table gives aver sales prices, as shown in annual reports of the U S Geol Surv (16).

Price of Bauxite per Long Ton, f o b Shipping Point (1913–23)

Year	Aver price, domestic bauxite	Year	Aver price, domestic bauxite	Year	Aver price, domestic bauxite
1913	$4.75	1917	$5.48	1921	$6.38
1914	4.87	1918	5.69	1922	6.25
1915	5.10	1919	5.85	1923	6.03
1916	5.40	1920	6.23	1933	$6 to $7.50

Jan, 1938, domestic commercial, $7.50; domestic abrasive, 70–84%, $12.50 to $15 (30).

Feldspar. No basis for determining the value of feldspar, based on actual composition, has been established. Price of pulverized feldspar per short ton has varied considerably. In 1913, extra fine pulverized sold at about $13 per ton f o b on cars at mills, in car lots. In 1916 the price advanced to $15, in 1918 to $20. Jan, 1938, potash feldspar, 200-mesh, $17. Glass-spar, white, 20-mesh, $11.75 (30). Price of the lower grade is about 0.75 of first grade.

Fluorspar is usually divided into 2 grades by analysis: (*a*) metallurgical or fluxing, minimum of 85% calcium fluoride and max of 5% silica; (*b*) merchantable acid spar, 97% calcium fluoride and not over 2% silica. These are again divided into gravel, lump and ground. Gravel is used extensively in iron, steel and aluminum smelting. In finely-ground form, in the manufacture of opal-glass and in enameling iron and steel ware.

Average Annual Prices of Domestic Fluorspar per Ton of 2 000 lb, f o b at Mines (16–19)

Year	Gravel	Lump	Ground	Year	Gravel	Lump	Ground
1913	$5.87	$6.88	$12.31	1919	$23.80	$30.38	$43.02
1914	5.21	8.45	11.78	1920	23.24	29.85	43.32
1915	4.89	7.51	10.80	1921	16.11	21.39	40.00
1916	5.34	7.94	12.38	1922	16.36	18.56	36.42
1917	9.61	13.68	17.59	1923	18.61	23.32	36.86
1918	20 05	24.12	31.21				

In Jan, 1938, prices for domestic fluorspar, f o b Ky and Ill mines: washed gravel, 85% calcium fluoride and not over 5% SiO_2, $18 to $19 per ton. No 2 lump, same content, $20. Domestic No 1, ground bulk, 95–98% calcium fluoride, not over 2 1/2% SiO_2, $31.50. Foreign, 85–5 c i f Atlantic ports, duty paid, $24.50 (31).

Pyrite. Used extensively for the manufacture of H_2SO_4. Value is based on percentage of sulphur in the ore, and is paid for at a price per unit (usual basis is ton of 2 240 lb). Foreign pyrite run from 50–51% S; domestic, 35–40%. Spanish per unit of sulphur, c i f Atlantic ports, 12¢ to 12 1/2¢. Domestic graded fines, lump and furnace size, 9¢ to 11¢. The impurities, usually present in small amounts, are Zn, Pb, Sb, As, and insolubles (including SiO_2). More than 3% Zn + Pb and more than 1% As are objectionable. If the cinder resulting from the burning of the pyrite contains Cu it is recovered by leaching. The cinder, whether leached or not, is frequently desulphurized, sintered and sold as iron ore.

Sulphur. Practically all sulphur mined in U S comes from the Gulf coast. Unit of weight, 2 240 lb. From 1909 to 1927 the prices ranged from $13 to $22 per ton. In 1909 it averaged $18.52; 1914, $18.17; 1915, $13.46; 1918, $22; 1923, $16.07. During the past 10 years the price has remained constant at $18 per ton.

8. ORE CONTRACTS

Contracts for selling and purchasing should be in legal form, and supervised by engineers or metallurgists having full knowledge of the technical conditions.

Following points should be covered: 1, duration of contract, in years; 2, approx amount of ore to be delivered; 3, rate at which the ore is to be delivered (if known); 4, point at which delivery is to be made from miner to purchaser; 5, arrangement respecting freight rates; 6, method of weighing the ore; 7, determination of moisture; 8, arrangement for representation of both seller and buyer; 9, manner in which the ore is to be sampled, and sampling paid for; 10, number of final samples to be taken, and disposition of same; 11, assay and analytical methods to be used in determining the ore constituents; 12, settlement of splitting limits (Art 2); 13, mode of settlement when determinations by seller and buyer are within the splitting limits; 14, arrangement for umpire, if difference exceeds the splitting limits; 15, mode of settlement in case samples are sent to umpire; 16, terms of payment for metals; 17, the effect, if any, which fluctuations in market price will have on terms of payment; 18, the effect, if any, which the value or composition of the ore will have on these terms; 19, dates on which metals are to be paid for; 20, statement of payments to be made and penalties to be levied for certain ore constituents; 21, the base charge on which settlements will be made; 22, amount which will be advanced (if desired) by purchaser to seller, at any time after settlement has been agreed upon and before final payment is made; 23, possible conditions which shall be considered as valid reasons for failure to deliver or receive ore under the contract (Sec 29); 24, arrangements for submitting disputes to arbitrators, instead of the courts (22).

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SECTION 33

GOLD AMALGAMATION AND CYANIDATION

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GOLD AMALGAMATION

1. MILLING PRACTICE

Amalgamation of gold ores in modern plants is confined almost exclusively to recovery of the coarse free gold, as a secondary treatment, by BARREL AMALGAMATION of concentrates caught on blanket or corduroy strakes, concentration tables or jigs. PLATE AMALGAMATION, still found in older plants, is rarely installed today. Amalgamation is practically always in conjunction with or followed by CYANIDATION, FLOTATION, or a combination of both processes, and preparation of the ore is usually planned with reference to the requirements of such processes.

The advisability of making a preliminary recovery of Au by amalgamation or corduroy tables, when this is to be followed by cyanidation, must be determined for each individual ore. Where much coarse free Au occurs, often 40–80% of it can thus be recovered directly, at a lower cost than by precipitating and refining from cyanide solution. Also, by diverting the coarse Au from the cyanide plant, the time of treatment in the latter can be reduced and a satisfactory final residue obtained with coarser grinding; examples, Homestake mine, So Dakota, and West and Central parts of the Witwatersrand. Where flotation is used, the preliminary removal of as much free Au as possible is of first importance, since it has a tendency to be depressed under reagent conditions suitable for flotating sulphides. If there is no coarse Au, and fine grinding is required to release the Au from the gangue, practice tends to eliminate all forms of amalgamation or concentration; example, Far East part of the Rand. Improvements in cyanide extraction and precipitation, risk of amalgam theft, mercurial poisoning of workmen and tying up of amalgam within tube-mills, have contributed toward elimination of amalgamation at many large plants. Also, grinding in cyanide solution can not be taken advantage of where amalgamation is used.

Crushing and grinding (Sec 28). Ores must be reduced to such fineness that the Au or Ag is sufficiently freed from gangue to be attacked by mercury. Recent practice is to eliminate stamps; to crush by jaw or gyratory crusher to about 2-in size, with secondary crushing by rolls or fine gyratory-type crushers to a product at least all minus $^3/_4$ in, followed by two-stage grinding in ball- or tube-mill, or a combination of the latter. Recent installations of stamps are limited to cases where a moist, sticky ore would tend to choke a crusher, if crushed to less than about 2-in pieces.

Where size of plant does not warrant stage crushing and grinding, a gyratory crusher is used to produce a size approx 1.5 in, which is ground direct in ball- or tube-mill. For stamps, preliminary crushing is in jaw or gyratory crusher to about 2 in; tendency being toward heavy stamps and low discharge, crushing rather coarse (0.5–0.75 in), and grinding stamp-mill discharge in ball- or tube-mill.

2. PLATE AMALGAMATION

Due to adoption of coarse battery crushing, which causes rapid scouring of inside plates, inside battery amalgamation has been practically eliminated.

Amalgamation may follow the stamp battery (Sec 28) or grinding mills. When much of the coarse Au is liberated in the primary grinder, the plates should follow this unit to avoid accumulation of Au in the secondary grinder, with a second set of plates after the latter.

Apron plates following a stamp battery are wide, shallow, sloping troughs, full width of mortar and made of $^1/_8$-in annealed copper, to prevent buckling with consequent uneven flow of pulp. They are almost always silver-plated. They rest on heavy wooden tables, which, though adjustable, should be rigid and non-warping.

Amount of Ag for plating varies from 1 to 2 oz per sq ft to as high as 5 oz. Plates are resilvered after 1–3 years' use. In some mills replating is not practiced; new plates are installed when necessary, the old plates being melted down. Apron plates following a rod-, ball- or tube-mill are 1–4 ft wide, depending on requirements and constructed as for stamps.

Area of plates depends mainly on size of gold particles to be caught. Fine Au requires slower flow of a thinner layer of pulp over tables; hence, greater area. Modern practice

limits amalgamation to recovery of coarser particles of Au, and an area of 1.5–4.5 sq ft per ton milled per day is usual (Golden Chariot and Argonaut mines). Many small mills in western U S, that depend chiefly on amalgamation for recovering Au, use an area of 10–12 sq ft per ton.

Mercury traps are advisable, as apron plates may then be kept softer, with less chance of losing Hg or amalgam. Good practice avoids having much loose Hg about. An efficient trap consists of a simple transverse trough a few inches deep. The sand and Hg settling in the trap are cleaned out at intervals and returned to battery or mill.

Slope of apron plates varies from $1^1/_4$ to $2^1/_2$ in per ft; best practice, about 2 in. Flat slope requires more water. Tables for supporting plates may be adjustable, for varying the slope. This should be such as to give a pulp velocity high enough to keep the heaviest particles of ore moving, but not so high as to prevent adherence of gold to the Hg surface. Viscosity of pulp is another factor of importance, and densities over 20% of solids should be avoided, as sinking of gold through pulp layer is impaired.

Dressing and care of plates, with maintenance of maximum battery output, are the principal factors in successful amalgamation. In general, unless the gold is very coarse, too much Hg on the plates is better than too little and it should not be too thoroughly removed when dressing and scraping. Apron plates are usually dressed each day, depending on richness of ore and how bright the plates remain. In some mills, having 2 parallel plates in front of each mortar, a diverting launder throws all the feed to one plate while the other is being dressed or cleaned up. Single plates may be divided into 2 parts by a longitudinal wooden strip and one-half dressed at a time. This avoids hanging up the stamps, but the flow of battery water must meanwhile be reduced.

Details of dressing plates. Hg is first sprinkled over the plates, to soften the amalgam. This is rubbed in lightly, after which the amalgam is scraped up from lower end toward head, with a rubber or hard-wood scraper or a rag. Near the head end, scraping is done with an amalgam knife or chisel, so handled as not to cut or groove the plates. After the amalgam is removed, fresh Hg is sprinkled over the plates, more being generally used on upper end. Sprinkling is best done from a small iron bottle of gas pipe, the mouth of which is covered with muslin. When freshly wet with Hg, the plate is well rubbed until the Hg is evenly distributed. A whisk broom is generally used to distribute the mercury over the plate. A stiff broom, which will break up tough amalgam without injuring the plate, is made by stitching an extra binding across a new broom, about $^1/_3$ the way down from handle and then cutting broom to one-half length. Plates should be frequently inspected.

Amalgamators are a recently perfected kind of amalgamating trap, combining the functions of apron plate and the typical Hg trap, and in some instances have replaced the use of plates altogether. In these devices a series of amalgamated trays and deflecting baffles in troughs cause accumulations of amalgam, which increase the chance of contact between Au and Hg, while occupying far less floor space than plates. The Clark-Todd amalgamator, as used at the Homestake mine, is an example. Placed at the ball-mill discharge, the device includes a receiving box with protecting screen, a short length of transverse launder, lined with a copper box, and a longer cross launder with silvered copper plates. Scrap iron and oversize are effectively removed, while the amalgam accumulates in the boxes. Each section is covered with a locked guard screen.

Condition of mercury. The Hg fed to battery or grinding mill must be free from impurities, which SICKEN the MERCURY and form patches on the plates that will not amalgamate the gold. The sickened condition is due to fine particles of Hg becoming coated with thin films of manganese or iron sulphates, arsenides, antimonides, tellurides, clay, oil, or grease. Oil or grease must be carefully kept away from the mortar and plates. Squeezing of Hg through chamois, canvas, or muslin bags will not remove these films, and if mercury sickens it must be cleaned with weak acid, lye, or cyanide solution, or even retorted before using again. Addition of a little sodium amalgam will sometimes assist in the amalgamation of refractory gold or tellurides.

Flouring. When mercury is held too long in crushing machinery, or when too much Hg is fed, FLOURING will take place. This is very fine breaking up of the Hg, in which condition it will not amalgamate, nor will the particles reunite, as the surface tension is very great compared to the wt of the particles.

Clean-up, which is made monthly (or oftener with high-grade ore) when stamps are used, consists in thoroughly cleaning out the battery and scraping and dressing the plates, as above. With rod-, ball-, or tube-mill, a satisfactory clean-up can be made only when these mills are being relined, during which period they are thoroughly cleaned. The serious accumulation of amalgam in rod-, ball-, or tube-mills, often during a year before it can be recovered, has discouraged the adding of Hg to these units. The upper loose sands from stamps or grinding mill, containing but little amalgam, are returned to the unit after starting; the lower sands go to the clean-up room. Here the sands are treated with additional

Hg and water in a clean-up barrel or pan, and panned down over the sink to separate the amalgam, the washed sands being returned to battery or grinding mill. The amalgam thus recovered, and that scraped from the plates, is also washed with more Hg to make it liquid. Particles of iron are removed with a magnet. In large mills, the amalgam is worked over in the clean-up barrel or pan, with excess Hg; in small mills, it is ground in a Wedgwood mortar.

Cleaned amalgam is then put in a conical funnel of canvas or heavy unbleached muslin, through which the excess Hg drains off; the remaining amalgam is squeezed by hand through chamois or cloth, or in large mills by a hand or power press. Chamois is expensive to use as it wears out rapidly. Skimmings and dross from surface of the Hg strained out in cleaning amalgam are saved for separate treatment. When the ore contains much sulphide, antimonide or arsenide, the mud settling out of the washings, and the dross from cleaning amalgam, may be saved and shipped to smelters. Amalgam, squeezed quite dry and hard, contains 25 to 40% bullion. The squeezed amalgam is finally retorted for distilling off the Hg.

3. CORDUROY TABLES

Corduroy-blanket tables are generally of same size and slope as those for plate amalgamation. They are generally placed at the ball-mill discharge. Surface of table is covered with transverse strips of corduroy, placed to overlap one another a few inches; they are not fastened down, as the wet blanket clings to the table surface. But care is taken to "iron out" the air bubbles formed underneath when laying the blankets. An even distribution of the feed pulp, correct pulp velocity and density, etc, are as important as in plate amalgamation. The transverse ribs of the corduroy act as riffles, the spaces between gradually filling with a concentrate containing the free gold and consisting largely of pyrite and black sands.

An unclassified pulp, with up to 20-mesh sand at 4–5 to 1 dilution, will flow down a slope of 1 $^3/_4$ in per ft, and more dilute and finer pulp requires even less slope. Roughly, 1 sq ft is required for every 3 tons of pulp per day.

Blankets are removed for washing at intervals of about 4 hr, depending on how badly the riffles tend to pack. To change blankets, pulp is first turned off and water allowed to flow over blankets for a few minutes, to wash off surface sand; blankets are then folded up to retain the concentrates and taken to the washing box, while a clean set is put down. The washing is done by immersing the blanket, with the riffle side down, in the water and shaking it. The concentrate that falls and is washed off passes through a stout locked screen near top of the box. With 2 washing boxes, each day's concentrates can be kept separate.

Corduroy *vs* amalgam plates. There is a tendency for corduroy and other methods (see concentrating tables and jigs) to replace use of amalgam plates for the following reasons: corduroy will collect "rusty" gold and platinum-group metals, as osmiridium, which can not be caught on plates; they then require $^1/_4$ to $^1/_5$ the surface area for the same tonnage; unskilled labor can be used and less attention required; also, danger of mercury poisoning, loss of amalgam and theft are eliminated.

Prior to 1921 the Witwatersrand mills recovered 50–75% of their production by plate amalgamation, variations in percentage of recovery being due to differences in degree of grinding, perfection of equipment, and characteristics of the ore. During the following few years, however, there was an almost complete substitution of plate amalgamation by corduroy blankets, which, in one form or another, are today used almost exclusively on the Rand.

Cleaning concentrate. Corduroy concentrate, amounting to around 1% or less of the ore treated, is usually fed to a concentrating table, set rather flat and operating with a short stroke. Three products are made: clean Au, pyrite (including any iron coming from the milling machinery), and a siliceous tailing, which flows back to the mill circuit. The concentrate is collected separately in boxes having filter bottoms, and transported to the clean-up room for barrel amalgamation.

Johnson concentrator, developed on the Rand, is a continuous-discharge corduroy, consisting of a rotating cylinder slightly inclined and having a rubber liner grooved like the rifling of a gun barrel. Feed enters at the upper end, while gold and pyrite collected in the riffles are carried up out of the pulp and discharged at the highest point by sprays into a launder leading out of the cylinder.

Jigs, of the Cooley and Richards pulsating type, are finding increasing use in the ball-mill classifier circuit for removal of coarse gold as soon as it is released by grinding. The improved Denver Equipment and Pan American machines have high capacities and make clean enough concentrates for direct barrel amalgamation.

Hydraulic traps, unit flotation cells and concentrating tables, usually provided with scalping screens for removing oversize, are all in use for preliminary recovery of coarse gold, ahead of the cyanidation or flotation circuit.

Barrel amalgamation. The concentrate from tables, jigs or traps (mentioned above) is ground and amalgamated in a standard clean-up barrel, after which the amalgam is separated by panning and treated in usual manner (Art 2). These barrels are preferably cast in one piece, to avoid use of liners which tend to hold up amalgam. From 2 to 14 hr are allowed for grinding with a light ball or rod load, after which the mercury is added and another 1 1/2–2 hr allowed for amalgamation. Tailing from the barrel is returned to the main plant circuit.

Recovery of Au by corduroy tables is slightly lower on the Rand than was formerly obtained by plate amalgamation, but final residues after cyanidation have not been affected. It is reported that the Hg consumption, since introduction of corduroys, is only one-tenth of what it was before.

4. RETORTING AND MELTING

Retorts, when of large size, are horiz C-I cylinders built into a brick furnace setting, and with a pipe from the rear end to an open Hg receiver. This pipe is surrounded by a larger pipe or jacket, in which cold water is circulated for condensing the Hg. The end of the pipe is covered with wet cloth or flannel, dipping into water in the receiving pot. The level of the water in the pot must never be allowed to rise high enough to cover the end of the pipe; otherwise, if the temp of the retort falls, water may be drawn back into the hot retort, causing an explosion. Small retorts are vertical C-I pots with condenser pipe connected to the cover. Retorts are lined with chalk or wood ashes before charging, and the lumps of amalgam are often wrapped in paper, to keep the bullion from adhering to the iron. Vertical pot retorts are filled not over 2/3 full of amalgam, to avoid having particles of amalgam thrown up into the pipe in case the Hg boils due to overheating. This would clog the pipe, and might cause an explosion.

Charging retort. Amalgam is charged in horiz retorts in semi-cylindrical C-I trays (usually 4 in number), which are slid into place. The retort cover is luted with a mixture of fire clay and wood ashes, or clay with a small admixture of clean slimes. Or, gaskets can be used, cut from thin asbestos sheets, heavy oil being first applied to the joint faces. The cover is clamped on securely with a yoke and steel wedges.

Firing. The retort is gradually heated by wood or coal fire or by oil (according to design of the furnace), water running continuously through the condenser jacket, until Hg begins to pass freely into the receiver. Heat is then decreased, but should be maintained sufficiently to distil the Hg continuously. When distillation ceases, the heat is raised to a dull red and so maintained for about 0.5 hr, after which the fire is allowed to die out, and the retort to cool before it is opened. During the final heating the condenser pipe should be occasionally tapped to dislodge the last drops of Hg. Even with large retorts, distillation should be complete in 5 to 6 hr; with small pots, in say 3 hr.

Melting. The retorted mass (RETORT METAL) is melted in a graphite crucible, in a coke or coal-fired (or oil) furnace, the latter being preferable for ready control of temperature.

The crucible is gently heated before use to drive off moisture, and should be well supported in the furnace to avoid tilting. After raising it almost to red heat, a portion of flux is first charged and melted, after which the retort metal is added, a few pieces at a time, care being taken not to drop them in with a splash. The crucible may, if necessary, be charged about 0.75 full. FLUX generally used is largely borax glass, with about 0.5 the amount of sodium bicarb and a little niter. If the retort metal contains much sulphide, stirring with a heated iron rod will cause formation of a matte, but the amalgam itself usually contains enough particles of metallic iron to combine with the sulphur. Enough niter must be used to oxidize base metals, but an excess will rapidly eat away the crucible. In presence of much metallic iron, SiO_2 may be added to the flux, but not enough to form a viscous slag, which may carry shots of the bullion. Very base bullion may require 5% or more flux to produce a clean bar. After being kept molten for 20 min, the metal should be well stirred with an iron rod, to insure a homogeneous bullion, and the slag (and matte, if any) are skimmed off, or the pouring may be done with the slag.

Molds are of C I, well heated, and coated with graphite, resin smoke, or lamp-black mixed with heavy oil; oil alone, paraffine, or tallow may be used.

After cooling sufficiently the bars are plunged into cold water, or into water with a little H_2SO_4 or HNO_3. The slag (and matte, if any) are hammered off and the bar scrubbed, numbered, weighed, and drilled for assay sample. Six staggered, 3/32-in drill holes on each side (top and bottom) of the bar and about 1 in deep, give a fair sample, the surface drill-

ings being rejected. The matte is saved for separate refining, or is collected until there is enough to ship to smelter. Slags are returned to the clean-up pan or barrel, or are washed free from graphite and returned to the battery. Yield of retort metal from the amalgam should be from 30 to 40% by wt, according to amount of gold or silver in the ore. Loss in smelting is slight, if amalgam has been properly cleaned.

5. SALIVATION

Mercurial poisoning may result from absorption of Hg through the skin, from breathing Hg vapor, or swallowing soluble Hg salts.

General symptoms are: soreness of gums, tenderness of jaws, excessive secretion of saliva, swollen, inflamed gums and foul breath, later resulting in looseness of teeth, swelling of face, with fever and depression. ACUTE MERCURIAL POISONING is accompanied by tremors of arms and hands, irregular heart action and general weakness. It may result finally in loss of teeth, necrosis of the jaw bone, and in some cases in cirrhosis of the kidneys.

Salivation of the workmen in an amalgamating mill is easily prevented. Gloves should be worn when cleaning amalgam, and care taken not to breathe the Hg vapors when plates are being steamed, when Hg is being cleaned with hot water, or from leaky retorts. Men who may be exposed should maintain an active condition of bowels and kidneys, and pores of the skin should work freely. These precautions are also the principal means of eliminating Hg from the system, in cases of salivation. For acute poisoning, where mercurial salts have been swallowed, albumen or emetics are administered, or the stomach may be washed with albuminous water and magnesia solution.

CYANIDE PROCESS

Introduction. The cyanide process extracts gold and silver from ores by means of the solvent action of an alkaline cyanide solution on these metals. The first commercial application was to Au ores, in 1887–88 (McArthur-Forrest process); treatment of silver ores came later. First cyanide mills were built: 1889, at Crown mine, New Zealand; 1890, on the Rand, So Africa; 1891, Utah, U S.

Later improvements. CHEMICAL: zinc-dust precipitation of the dissolved gold and silver; aeration of leach solutions and de-aeration before precipitation; cyanide regeneration systems and, in the case of refractory ores, pre-treatment with lime, and in some cases carefully controlled roasts. MECHANICAL: replacement of stamp batteries by stage crushing and ball- or tube-mills; continuous thickening and counter-current decantation; all-slime plants; use of vacuum filters; and flotation-cyanidation processes.

The fundamental principles of the process have changed little since its inception. Improvements have been mainly mechanical, having effect of increasing extractions and lowering treatment costs. They include advances in grinding circuits and classification, resulting in more highly selective grinding on the refractory portions of the ore; the elimination of the sand-slime circuits and use of single, continuous circuits; more efficient precipitation; and, frequently, the preliminary concentration by flotation of the precious-metal values into a small bulk for cyanidation in a comparatively small plant.

Gold and silver ores containing no other valuable metal are today practically all treated by the cyanide process, either exclusively or following a primary treatment by amalgamation, flotation, or some other form of concentration.

6. ADAPTABILITY OF ORES

Nearly all gold and silver ores can be treated by cyanidation. In case of gold ores, tellurides may cause interference, but can usually be rendered soluble by very fine grinding and long agitation with excess lime.

Interfering conditions. OXIDIZED ZN AND CU ORES cause high consumption of cyanide, due to ready solubility of their oxides, carbonates, and hydrates. SULPHIDES of Cu, Ni, Co, Pb, Zn, and Fe react but slightly with cyanide solutions, though pyrrhotite tends to oxidize and acts as a cyanicide, while certain copper sulphides, as chalcocite, cause increased cyanide consumption. Some sulphide combinations of Ag with Pb and Zn are only slightly attacked by cyanide, even when finely ground. SOLUBLE SULPHIDES (especially alkaline sulphides) interfere seriously, both by combination with cyanide and by re-precipitation of the Ag and Au. ARSENICAL AND ANTIMONIAL ORES tend to be refractory and often cause high cyanide consumption, due to the formation of compounds having a strong reducing

action in the working solutions. Preliminary roasting may favorably prepare such ores for cyanide treatment. CHROMIUM compounds are some of the worst interfering substances, but can often be precipitated by adding lead nitrate. MANGANESE combined with Ag often prevents successful cyanidation. A preliminary reducing roast may increase extraction, as, for example, when conditions permit the successful application of the Caron-Clevenger process. METALLIC MERCURY (often present in tailings from amalgamation) is not readily dissolved by cyanide, and, in small quantities, may assist in precipitating soluble sulphides; present in larger quantities, Hg causes waste of Zn in precipitation boxes. A pulp containing metallic Au and Ag in COARSE PARTICLES, may require so long a cyanide treatment for complete solution of the metals as to make the process uneconomic. In such cases, preliminary removal of the metallics by amalgamation, jig or table concentration (Art 1) would be resorted to. CARBONACEOUS MATTER, if ground with the ore, tends to act as a reducing agent and re-precipitate Au and Ag before the solutions can be freed from suspended solids. The remedy is preliminary flotation to remove carbonaceous matter, or the cyanidation-flotation methods of the Edquist or Chapman processes.

Overcoming adverse conditions. The refractory nature of an ore can often be overcome by fine grinding, but as yet microscopic and chemical analysis alone can not determine the adaptability of an ore, and systematic testing is required to work out the many variables involved.

Laboratory experiments must be conducted to determine: possibility of leaching Au and Ag from coarser sands; necessary fineness of grinding; acidity of pulp and amount of lime required to neutralize it; amount of soluble sulphides in the pulp and quantity of lead acetate or oxide to be added; amount of cyanide consumed in treating the ore for the time required to effect satisfactory extraction; kind and amount of cyanicides present, and effect of pre-treatment with lime, excessive aeration, preliminary water washing or roasting of the ore. An ore not amenable to direct cyanide treatment, owing to presence of much sulphide (either mechanically including the precious metals or forming excessive quantity of cyanicides in the pulp), can often be treated by removing part of the sulphides by flotation before cyanidation.

7. CHEMISTRY OF CYANIDATION

Cyanogen (CN or Cy), a compound radical, is a colorless gas combining actively with metals. In manufactured forms it is available as KCN or NaCN. It combines with nearly all metals to form a single cyanide, and with many metals to form double cyanide groups which are only very slightly ionised in solution.

Chemical reactions occurring during solution of Au and Ag in the cyanide process have not yet been precisely determined. They vary with the nature of the ore, presence of base metals, and conditions under which solution is effected. Following fundamental reactions are generally accepted:

$$\left.\begin{array}{l} 2\,Au + 4KCN + O + H_2O = 2\,AuK(CN)_2 + 2\,KOH \\ 2\,Ag + 4KCN + O + H_2O = 2\,AgK(CN)_2 + 2\,KOH \end{array}\right\} \text{Elsner's equations.}$$

Similar equations apply when NaCN is used. They all represent the combination of CN with metallic Au and Ag, assuming the necessary presence of O. But, it is probable that an intermediate reaction takes place before formation of the double cyanide of Au or Ag, *viz*: $2\,Au + 2\,KCN + O + H_2O = 2\,AuCN + 2\,KOH$, which, with an excess of KCN, yields $AuCN + KCN = KAu(CN)_2$.

When silver sulphide is attacked, the reaction may be: $Ag_2S + 5\,KCN + O + H_2O = 2\,KAg(CN)_2 + KCNS + 2\,KOH$, or $Ag_2S + 4\,KCN = 2\,KAg(CN)_2 + K_2S$. Potassium thiocyanate (sulphocyanide) is often found in working solutions, affording a basis for the former of these two equations.

Variations in reactions are caused by presence of base metals, as sulphides or other salts, and by soluble salts of alkaline earths, always present in ores.

In practice, oxygen is absolutely necessary for the reaction, though its function is considered by some to be largely depolarizing. In certain modified processes it is supplied chemically by addition of $KMnO_4$, MnO_2, or other oxidizing salts; but this generally costs more than to bring O into contact with the pulp by mechanical aeration, or by successive leachings and drainings.

Relative solubilities of certain metals in cyanide solutions are stated by Clennell to be the same as their electrochemical activity in such solutions, in following order: Mg, Al, Zn, Cu, Au, Ag, Hg, Pb, Fe, Pt; hence any metal would replace all metals following it in the above order, and would precipitate them from a cyanide solution by what is known as a typical displacement reaction.

NaCN *vs* KCN. NaCN is generally preferred to KCN, because it is considerably cheaper and has more available CN per unit of wt, 53% against 40%; the dissolving power of NaCN is therefore 32.65% greater than that of KCN. Commercially, the strength of NaCN is sometimes quoted in percentage of equivalent KCN, though the recent tendency is to figure working solutions on basis of the NaCN actually used.

Oxidizing agents. A number of oxygen-bearing salts, when added to cyanide solutions, increase the rate of solution of precious metals, and sometimes have increased the total extraction: among these are K_2MnO_4 (potassium permanganate), Na_2O_2, and MnO_2. Their beneficial effect appears to consist in partial oxidation of interfering substances, and rapid supply of required O, but their excessive use may result in destruction of cyanide.

Nascent CN. $K_3Fe(CN)_6$ and BrCN react, without presence of O, to liberate CN in a cyanide solution, and nascent CN has greater affinity for precious metals than when in combination as KCN or NaCN. Liberation of CN by these salts is shown by:

$$K_3Fe(CN)_6 + KCN = K_4Fe(CN)_6 + CN, \text{ and } BrCN + KCN = KBr + (CN)_2$$

The latter equation represents the supposed basis of the BrCN process, in which the reaction in presence of Au is stated to be: $BrCN + 3\,KCN + 2\,Au = 2\,KAu(CN)_2 + KBr$. The reactions in this process are not thoroughly understood; probably the greater affinity of nascent CN for Au makes it possible to dissolve Au in such telluride ores as are not attacked by simple solutions of KCN or NaCN. So far as determined, no bromide of Au is formed by the use of BrCN.

When ferrocyanides are present in working solutions, $HgCl_2$ may be used to disintegrate them, by formation of double mercuric cyanides, which dissolve precious metals without additional O. The cost of $HgCl_2$ limits its use.

Strength of solution. The average Au ore requires solution strength of 1 to 2 lb NaCN per ton, and the average Ag ore 3 to 5 lb per ton. Percolation usually requires stronger solutions than agitation, probably because the necessary oxygen fails to reach the pulp as quickly and effectively.

Selective solubility. The base metal sulphides dissolve relatively slowly in cyanide solutions. Though their oxidized compounds apparently dissolve as rapidly as Au and Ag, certain of the resulting cyanides (*e g*, that of copper) are known themselves to be solvents of Au and Ag, whence the effect of such base metals is often rather to increase cyanide consumption than to lower ultimate recovery, provided free KCN or NaCN is kept up to strength.

It has been supposed that presence of certain metals retards solution of Au and Ag, but in case of such interference the cause is apt to be elsewhere. Thus, in treating certain tailings in New South Wales, in which the Au was contained in Sb_2S_3, it was found that the Sb caused no consumption of cyanide except when the solution was too alkaline; in a neutral solution, the only difficulty lay in disintegrating the Sb_2S_3 to reach the fine Au.

Heated solution frequently yields higher extractions in laboratory tests; commercially it is generally found that the increase is not appreciable, and excessive heating of solutions may destroy part of the cyanide, while also increasing its solvent action on base metals, with entailed losses of cyanide. Heating of solutions also expels oxygen, thereby necessitating increased aeration of pulp.

Chemistry of precipitation. Recovery of precious metals from cyanide solutions is based, theoretically, on the reaction: $2\,KAu(CN)_2 + Zn = K_2Zn(CN)_4 + 2\,Au$. Practically, precipitation takes place only in presence of free cyanide, thus: $KAu(CN)_2 + 2\,KCN + Zn + H_2O = K_2Zn(CN)_4 + Au + H + KOH$.

Interfering conditions. In actual operation there is always an excess of Zn, which is also dissolved by the KCN as $K_2Zn(CN)_4$, without necessarily precipitating any Au. In the precipitation boxes, other metals are generally present, forming electric couples with Zn, which electrolyze the water, liberating O and forming $Zn(OH)_2$. This combines with free cyanide, forming a soluble double cyanide (possibly an alkaline zincate). Unless excess of cyanide be present, precipitation of $Zn(OH)_2$, ZnO, or perhaps $Zn(CN)_2$, will take place, covering the Zn shavings with a white deposit, and impeding precipitation of precious metals. Free alkali, or excess of cyanide, prevents formation of this white deposit by redissolving the Zn precipitate; hence a solution to be precipitated should always contain excess cyanide or be strongly alkaline. But, the double ferrocyanide of Zn and K, which frequently deposits on the Zn shavings, is but slightly soluble in alkaline or cyanide solutions, and may form enough deposit to interfere with precipitation of the precious metals before the Zn shavings have been completely consumed, thus necessitating an earlier clean-up than otherwise required.

To a slight extent, Al_2O_3, SiO_2, CaO, and Fe may be precipitated on the Zn, retarding precipitation of Au and Ag. Cu interferes still more, as it will completely coat Zn shavings. If the shavings in the first precipitation box compartments be coated with Pb, nearly

all the Cu can be precipitated before the solution reaches the compartments in which the Au and Ag are to be collected. Zn may be thus prepared by dipping in a 10% solution of $Pb(C_2H_3O_2)_2$ (lead acetate); Pb shavings also may be mixed with the Zn in the first compartments, to furnish a Zn-Pb couple. As, Sb, and Fe are also thrown down on the Zn; but, while often causing trouble in subsequent treatment of precipitate, they do not generally interfere with the precipitation reactions.

Zn shavings and Zn dust. The use of Zn shavings has been largely replaced by Zn dust, as being more efficient, requiring a smaller installation, and showing marked economies in operation.

Aluminum dust has been successfully used instead of Zn for precipitation, especially when the ore contains considerable As. It has the advantage that Al does not replace precious metals in cyanide solution by forming a CN compound; hence the fouling of solutions with Zn and As is avoided. While not so efficient a precipitant for Au alone, in the case of solutions carrying Ag, the precipitation of both metals is complete. The presence of lime should be avoided, or soda ash added to the circuit, because any calcium aluminate formed will foul the precipitate.

The reaction is possibly: $6\ NaAg(CN)_2 + 6\ NaOH + 2\ Al = 6\ Ag + 12\ NaCN + 2\ Al(OH)_3$. The $Al(OH)_3$ is redissolved in the excess alkali to form $Na_2Al_2O_4$. Another reaction, with evolution of H, is: $2\ NaAg(CN)_2 + 4\ NaOH + 2\ Al = 2\ Ag + 4\ NaCN + Na_2Al_2O_4 + 4\ H$.

Charcoal, especially soft-wood charcoal, has been used as a precipitant. The reaction at time of precipitation is not yet understood, but it has all the characteristics of an adsorption phenomenon, and the activity of the freshly prepared charcoal has been ascribed to occluded gases, probably H or CO, which act as reducing agents. There seems to be no reaction between C and the soluble double cyanides. Gold so precipitated is not readily redissolved in KCN solution; and there is known to be a loss of KCN during precipitation.

Continued re-use of solutions ultimately results in accumulation of Zn salts, and even if proper strength of free cyanide be maintained, the solution may become foul and lose part of its dissolving power. It has been advanced that the fouling of solutions is due in part to formation of double cyanides of Zn with other alkaline earths besides Na or K, or of double cyanides of other base metals and alkalies. It has been proposed to add enough H_2SO_4 to decompose double cyanides, and then some hydrate to precipitate alkaline sulphates, with regeneration of the cyanide.

Cyanicides; loss of cyanide. A cyanicide is a substance in an ore which either combines with CN or decomposes alkaline cyanides, so that a solution loses its dissolving power. Some loss of CN may also occur, by decomposition, without presence of cyanicides. KCN and NaCN are deliquescent, and are slowly decomposed by moisture.

Loss by HYDROLYSIS is represented by the reaction $KCN + H_2O = KOH + HCN$, the CN escaping by volatilization. Loss by ABSORPTION OF O occurs thus: $2\ KCN + 2\ O + 4\ H_2O = K_2CO_3 + (NH_4)_2CO_3$. In presence of CO_2, decomposition takes place thus: $KCN + CO_2 + H_2O = HCN + KHCO_3$. In practice, the losses of CN represented by above equations are very small.

Losses due to cyanicides in the ore may be very serious. They are caused chiefly by soluble sulphides, sulphates, and arsenates, and by some salts which are insoluble in water but react with cyanide solutions. Fe, Cu and Zn salts, especially in oxidized or semi-oxidized ores, are apt to react by the formation of double cyanides of the metals with K or Na. Free acid in an ore causes liberation of HCN; obviated by maintaining sufficient alkalinity in the pulp. In the solution of sulphide ores, K_2S or Na_2S is formed; if then the cyanide solution be allowed to become weak, these alkaline sulphides may accumulate sufficiently to reverse the reaction and re-precipitate Ag as sulphide; or the alkaline sulphides may oxidize into sulphates, or may form thiocyanates, thus consuming either the available O or the CN, and decreasing the dissolving power of the solution.

Both Mg_2SO_4 and $CaSO_4$ decompose alkaline cyanide solutions; arsenates of Fe and other metals also react with CN solutions and destroy their dissolving power. While FeS_2 does not react with cyanide quickly, nor to an appreciable extent, ferrous sulphide and ferrous and ferric sulphates combine actively. Untarnished sulphides of Cu and Zn are practically unattacked by cyanide solutions, but their carbonates, oxides, and sulphates react rapidly.

8. OUTLINE OF THE PROCESS

Recovery of Au and Ag by cyanidation consists broadly of 4 operations: (*a*) preparation of the ore; (*b*) solution of precious metals; (*c*) separation of dissolved metals; (*d*) precipitation and refining of product. After preparing an ore, the pulp may be classified into coarse and fine products (SANDS AND SLIMES) which may be treated separately; or, all the

ore may be ground very fine and treated together by the ALL-SLIMING process. Variations in preliminary treatment will be described in general outline, but many ores present problems requiring special modifications.

Sands, when treated separately, are usually charged into vats or tanks having filter-bottoms, where they are given a succession of percolation leaches with cyanide solution, followed by weak solution and water washes.

Slimes refer strictly to the colloidal and semi-colloidal portion of the pulp; but in practice they are usually considered as being the portion that is composed largely of particles that will pass a 200-mesh screen. They can be treated by cyanide solution in agitation tanks, followed by separation of the metal-bearing solutions from the solids by settling and filtration.

Precipitation of precious metals from the pregnant solutions, formerly done by Zn shavings, is now almost exclusively by Zn dust. Other methods that have found restricted use include electrolytic action, sodium sulphide, aluminum, or charcoal. The precipitate is dried, melted with fluxes, refined and cast into bars.

Cyanidation of concentrates. Concentrates derived from Au and Ag ores by gravity or flotation methods consist mostly of heavy sulphide minerals, commonly FeS_2, mixed with particles of gangue.

When applied to concentrate, cyanidation must be modified. Percolation methods have not been found satisfactory, owing to the comparatively long treatment required for extraction, the difficulty of maintaining protective alkalinity during the application of successive washes, and insufficient aeration of charges.

Preparation of concentrate for cyanidation by air-agitation generally demands very fine grinding. If the concentrate must be stored to accumulate a satisfactory charge, it should be protected against exposure to air and oxidization, with formation of soluble sulphates and free acid. It has sometimes been necessary to roast concentrate; also a preliminary filtration with neutral or acid wash, or pre-liming treatment, has been used to remove oxides of base metals, and counteract the effect of flotation reagents.

Besides fine grinding, the points requiring special attention are: a sufficiently strong solution, long enough agitation to secure proper aeration, maintenance of protective alkalinity, and avoidance of settling and packing of the pulp in agitation tanks. In estimating the probable cost, the loss of more solution through fouling must be counted on than when treating ores, and also a greater destruction of cyanide in the solution. To reduce cyanide consumption, higher pulp dilutions than in direct ore treatment are desirable, and successive addition of small amounts of KCN is beneficial.

Strength of solution for continuous treatment varies from 2 to 4 lb NaCN per ton, depending on Ag content; but larger quantities may be required for the batch treatment of refractory ores. On certain Calif concentrates, as short a time as 22 hr has sufficed to extract 99%, while in New Zealand, 10 days' agitation with an 8-lb solution has been necessary to extract 96%. In some South African mills, concentrate is ground in a 4-lb solution, followed by 72 to 78-hr agitation in a 12 to 16-lb solution, the extraction being only slightly over 90%.

Advisability of cyanide treatment of concentrates depends on both metallurgical and economic factors. Where gravity methods followed by barrel amalgamation are used, the amalgamation tailing can usually be returned to the main circuit. Where flotation can be used, the cyanidation of the concentrate often results in a large enough saving over shipping to a smelter to pay for cost of plant installation. The question of all-cyanidation *vs* flotation-cyanidation is usually a matter of the association and flotability of the gold and gold-bearing minerals present. The combination-type plant is usually cheaper to build and, if the gold is not too finely disseminated in gangue material, such a plant usually yields higher net returns.

9. PREPARATION OF ORES

Crushing (Sec 28). Run-of-mine ore is crushed in jaw or gyratory crushers to approx. 2-in size. One or two stages may be used, depending on size of plant. Following the primary crushers, practice varies. Modern stamp mills tend toward heavy stamps, with comparatively coarse screens (0.25–0.75 in). If stamps are eliminated, secondary crushing is usually practiced, reducing the ore to at least all minus 0.75 in, by rolls or fine gyratory crushers, such as the Symons' short-head cone crusher and the Type B Newhouse.

Hand sorting, for removal of waste or high-grade shipping ore, may immediately follow the first crushing (Sec 28). When ore is wet and dirty, it must be screened and washed before sorting. Practice of sorting out waste is declining, due principally to improved effic of underground work in sending cleaner ore to the mill.

Screening. Best practice employs grizzlies or shaking screens ahead of primary crushers to take out undersize, and shaking or vibrating screens, closed-circuited with secondary machines, to make a uniform product for fine grinding. Slotted, or ton-cap, mesh is more generally used than square mesh, as crusher capacity is thereby increased without corresponding reduction in grinding capacity; explained by the fact that rock slivers passing such openings are broken down relatively easily.

Grinding may be in single- or multiple-stage. Crusher or stamp-mill product is reduced to the final size required by ball-, rod-, tube-mill, or a combination of these.

Under certain conditions the ore may be reduced from the primary crushers to finished product in one mill. The modern tendency is to reduce the ball-, rod-, or tube-mill feed to at least all minus 0.75 in. With multiple-stage grinding the first stage is usually carried to about 20–30 mesh in ball- or rod-mills; subsequent grinding in ball mills using a smaller ball size, or in tube-mills, using steel balls, flint pebbles, mine rock or composite loads.

To obtain max effic from any type of grinding mill, the mill should work in closed circuit with a classifier. This CLOSED-CIRCUIT GRINDING consists of operating the mill and classifier so that the mill discharge passes to the classifier, and all finished material is overflowed. Oversize material is returned to the mill. Ratio of tonnage passing in a given time through the mill to original tonnage is the CIRCULATING LOAD; within certain limits, the capacity of a ball- or tube-mill varies directly with this load. Modern practice is to carry circulating loads in these mills of 300–500%, with tendency to go higher; in rod-mills, about 175%.

Fineness required for highest economical extraction differs for each ore and can be determined only by experiment. In exceptional cases of very porous ore, satisfactory extraction can be made by percolation from sands as coarse as 0.25 in. Fineness usually must be at least 30 mesh, varying from this to all minus 200 mesh.

On the Central Rand, ore is ground to approx 75–85% through 100 mesh; on the Far East Rand, practically all the finished product is 90 mesh, about 75–85% being 200 mesh. In Ontario, Au ore is ground to about 85–90% through 200 mesh, and in some recent Canadian practice to all minus 200 mesh. In the cyanidation of concentrates it is not unusual to grind the product to all minus 325 mesh.

In general, Ag ores require finer grinding than Au ores. Mexican practice, in "all-sliming" plants on Ag ores, is to grind to approx 75–80% through 200 mesh. Nipissing mine, Cobalt, Ontario, grinds to about 98% through 200 mesh.

Grinding in cyanide solution is customary unless the ore requires preliminary washing to remove cyanicides (Art 7), or undergoes preliminary amalgamation or other treatment. The recovery by amalgamation in cyanide solution is reduced. Grinding in water causes accumulation of solution, and a part must be sent to waste. In Au ores using very low strength of cyanide (1.0 to 1.5 lb), this loss may be small, but in Ag ore, generally using a much stronger solution (3 to 5 lb), the loss of cyanide would be serious.

Preliminary treatment may consist of roasting or washing the ore before the secondary crushing, concentrating after primary or secondary grinding, floating with unit cells in the grinding circuit, or after final reduction of all the ore. If the ore contains much clay, roasting dehydrates it and renders it more porous and brittle; or, if of low value, the clay can be removed by washing. Cost of roasting is usually greater than the benefit derived. Preliminary aeration with lime may also reduce the effect of cyanicides.

Roasting also decomposes tellurides, arsenides, and antimonides. If not a dead roast, the resulting oxidized salts injure rather than aid cyanidation, and a washing step should be introduced. Roasting will not remove $CaSO_4$ or $MgSO_4$ and may not eliminate all the ferric arsenate. There is usually no appreciable loss of Au or Ag by volatilization during roasting, provided chlorides are absent and temperatures not too high.

Preliminary treatment by flotation will often remove cyanicides and interfering minerals. Addition of oil during grinding often eliminates re-precipitation of precious metals by graphitic schist. Oil apparently coats the graphite particles, rendering them inert during the process.

Tube-mills, 14–22 ft long by 4–6.5 ft diam, were formerly common for fine grinding, but are now used only where supply of pebbles is ample, as on the Rand. Current practice uses shorter mills of larger diam, with rods or balls.

Earliest mills had peripheral discharge; modern mills have a false end-screen liner and lifting scoops to assist discharge, or overflow through the discharge trunnion. Scoop feeders to assist closed-circuit classifier arrangements are desirable, though in using mine rock as a grinding medium, an open-trunnion feeder may be the only practicable device for getting the ore into the mill. Combination scoop and trunnion feeder is also used.

Speed of tube-mill depends on diam; 31 rpm is recommended for 4-ft mill, 28 rpm for 5-ft, 25 rpm for 6-ft mill. Grinding media depend on local conditions; mine rock has almost entirely replaced imported flint pebbles, but where these are not suitable, steel or white-iron balls may be used, or a composite load of balls and pebbles or mine-rock.

Lining of tube-mills may be silex blocks set in cement, steel, or white iron. Discarded rails can often be used, bolted or wedged against a false plate liner. The pebbles roll over the bars and grind on the surface of the plates. Many other types of corrugated and over-lap special steel liners are in use.

Ball-mills (Fig 1–3), cylindrical or conical, use steel or C-I balls for coarse primary grinding; length of mill, approx equal to diam; balls up to about 5 in. For finer grinding, mills are longer and, for comparatively fine original feed, balls are smaller. Initial ball charge should comprise balls of various sizes; subsequent charges, all of largest size.

Rod-mills resemble cylindrical ball-mills, but have steel rods lying horis along full length of mill. Possibly they consume less power than ball-mills, per unit of work. Initial charge of rods is of various sizes. They are of special high-carbon steel, so that when worn down they will break up instead of winding around the larger rods. Rod mills tend to produce less fines, as the grinding is more by crushing and attrition than impact. Hence, they are less suitable for very fine grinding than ball mills.

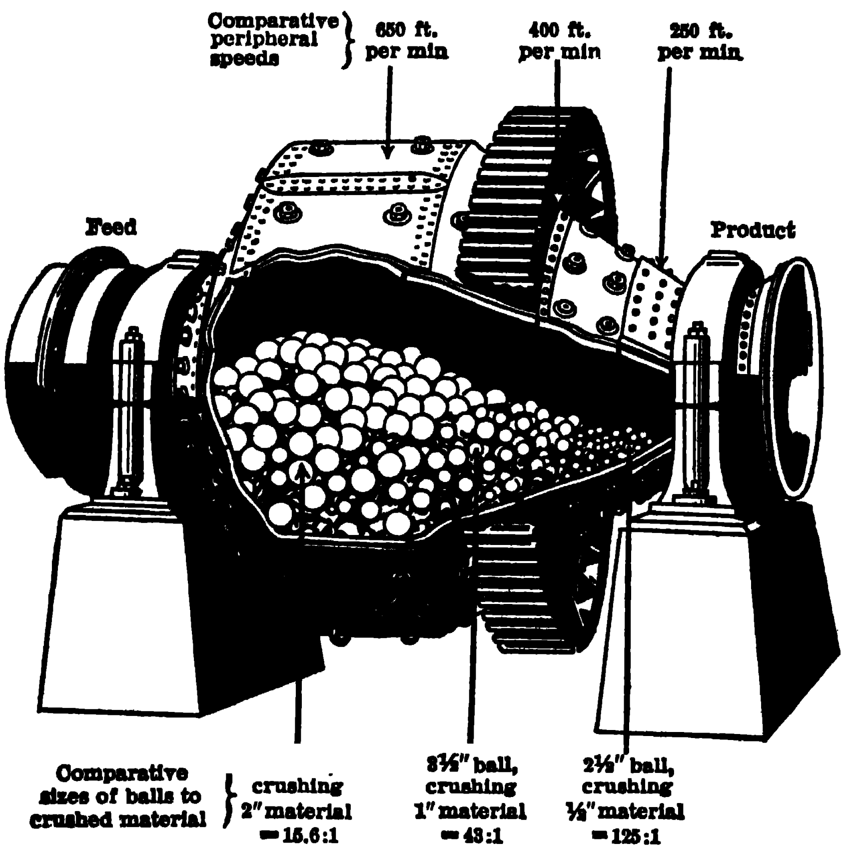


Fig 1. Conical Ball-mill (Hardinge Co, York, Pa, Bull 13-c)

Capacities and hp of mills. A tube-mill 5.5 ft diam by 22 ft long takes a pebble load of approx 14 tons, and will crush about 150 tons per 24 hr from $-{}^3/_8$ in to -90 mesh with feed rates up to 350 tons, and uses about 100 hp. Some 30–40% more power is required for starting. Diameters vary from 4 to 6.5 ft, the latter requiring 250-hp motors. A cylindrical ball mill 6 ft diam by 5 ft long, with grate discharge, will take a ball load of about 7 tons, will use 90 hp, and crush 150 tons of ${}^1/_4 - {}^3/_8$-in feed through 65 mesh. Mills 8 ft diam by 6 ft long will handle close to 500 tons per 24 hr, using about 200 hp. Corresponding conical mills for similar capacities would be about 7 ft diam by 36 in, and 8 ft diam by 60 in. The above are approximations only; actual figures in specific cases depend on the grinding characteristics of the ore and type of grinding-classification circuit employed.

Cost of tube-milling varies greatly, according to cost of power, labor, and pebbles, and is dependent also on character of ore, size of feed, and fineness of grinding. Costs range from 9 to 40¢ per ton, with 20¢ an aver figure under favorable conditions, where power represents about 30% of total cost, labor 20%, and pebbles and repairs 50%.

Classifying. All grinding in ball-, rod-, or tube-mills should be done in closed circuit, with a classifier. Change from open- to closed-circuit often increases mill capacity 25–60%. Cones were formerly always used for classification, but have given way to mechanical

classifiers, as the Akins, Dorr and Esperanza, or drag type. Disadvantages of cones are their relatively large head room, failure to handle high circulating loads, and need of constant supervision. Dorr classifier, both single-stage and bowl type, is most widely used.

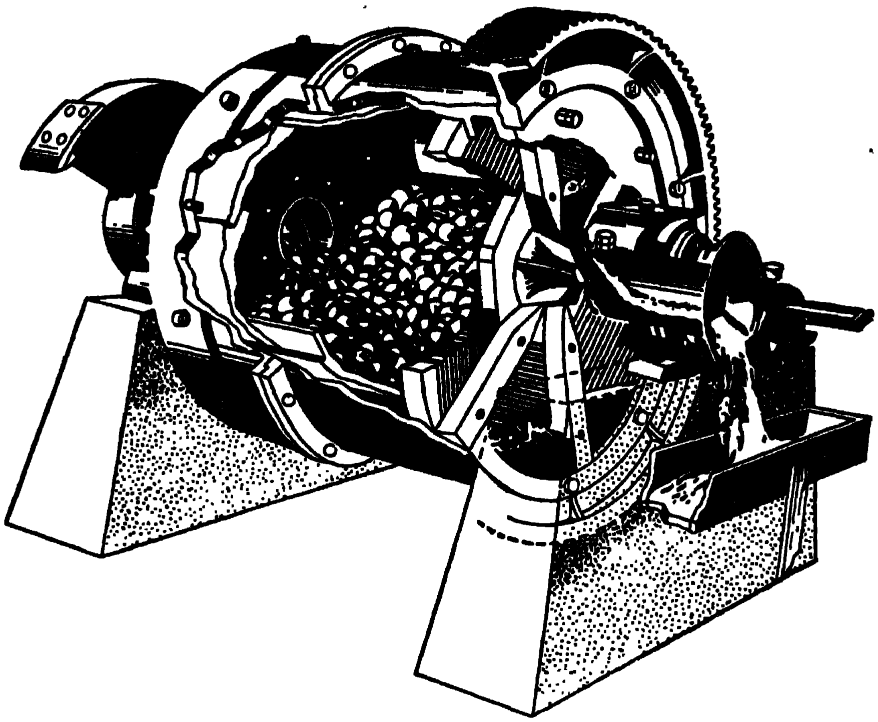


Fig 2. Marcy Cylindrical Ball-mill

Types of classifier. DORR CLASSIFIER, single-stage, consists of a settling tank in form of an inclined trough open at upper end. Feed enters near center, and the liquid and slow-

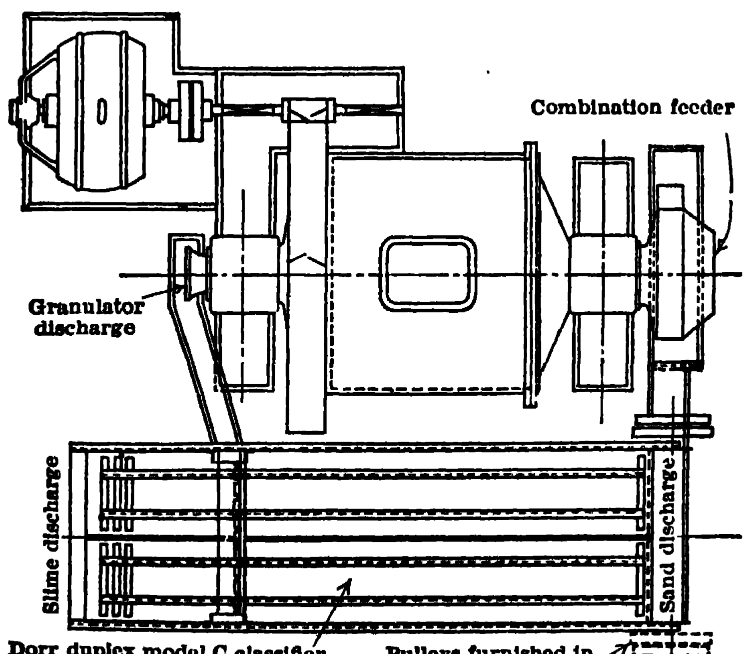


Fig 3. Allis-Chalmers Ball Granulator in Closed Circuit with Dorr Duplex Classifier

settling solids overflow at closed end; while sands or quick-settling solids are conveyed along the bottom by mechanical reciprocating rakes, and, after emerging above the liquid, are discharged at open end. All parts moving on one another are above the liquid, thus

minimizing wear. This classifier is made to handle tonnages from 4 to 1 500 per day, and at separations from 10 mesh and finer. Bowl type (Fig 4), a development of the Dorr classifier, is well suited to separations at 80 mesh and finer, especially where a clean rake

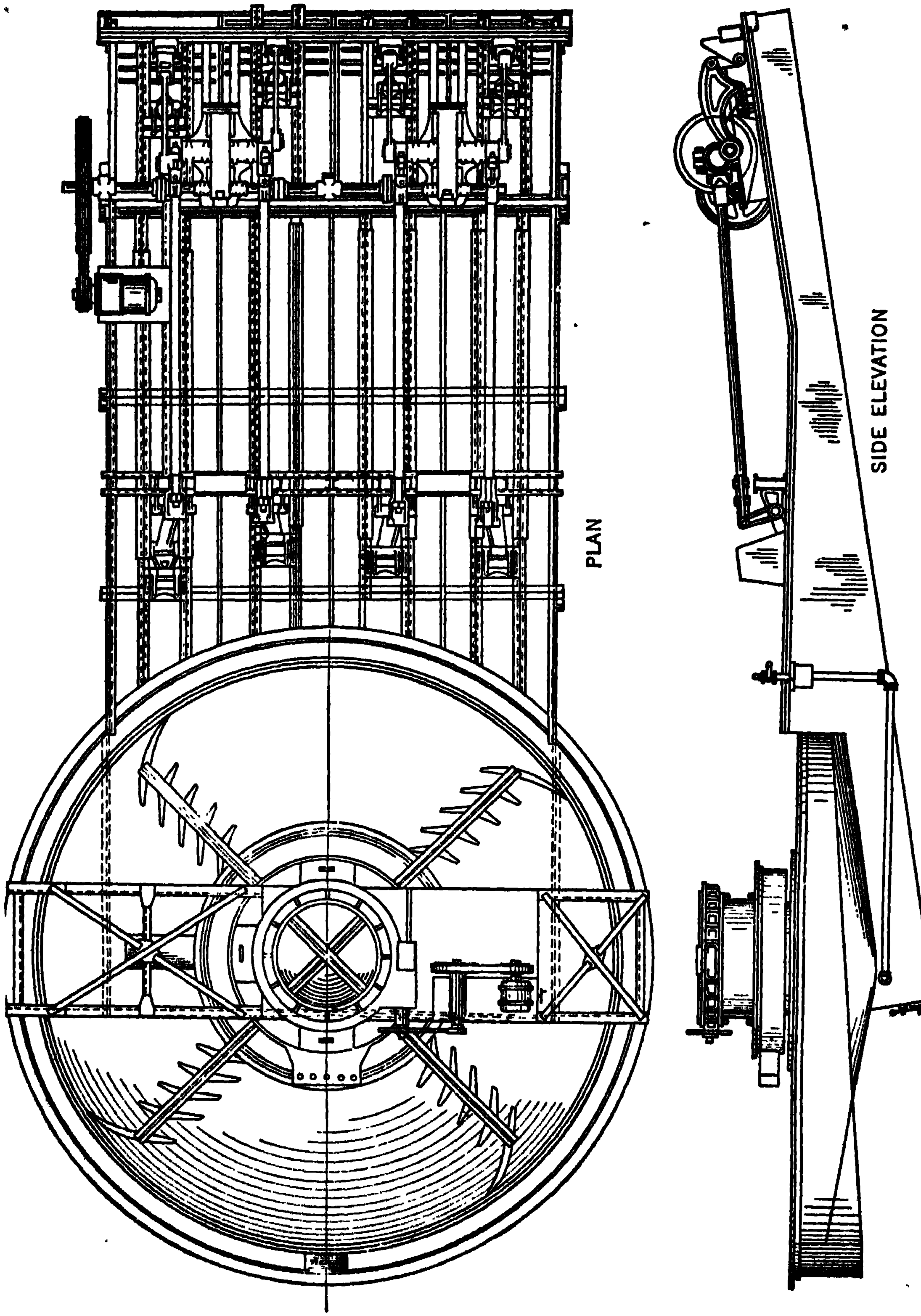


Fig 4. Dorr Turret-bowl Classifier

product is required. It is a two-stage automatic baffled return classifier, feed being introduced through a shallow well at the center. Fines overflow the periphery, and are carried off by the overflow launder. Sand or oversize is plowed to center of bowl, and discharged

into the main tank, whence it is removed by reciprocating rakes as in the single-stage machine. Wash water or solution is sprayed onto the sands just before they leave the pulp level. This type is now used almost exclusively in new plants to separate sands and slimes in a "two-product" process. Clean sands can be sent directly to treatment, eliminating costly transfer in older installations using cones or spitzkasten. AKINS CLASSIFIER (Fig 5) consists of a large-diam spiral conveyer, set in an inclined semi-circular trough. Oversize quick-settling material is conveyed by the spiral to upper discharge end. ESPERANZA CLASSIFIER is a tank placed similar to the Dorr. Oversize is removed by a continuous-flight drag line, revolving on pulleys set at both ends. This machine has found its principal use in El Oro district, Mexico.

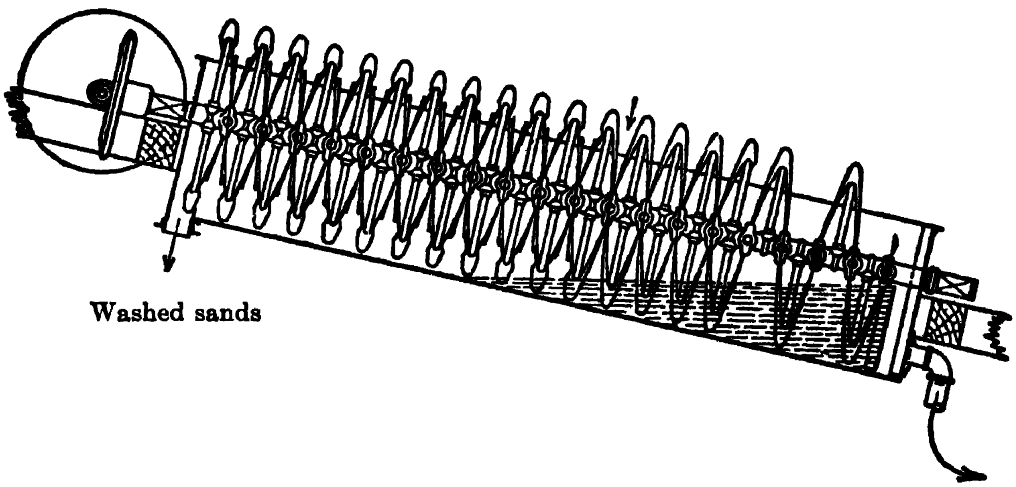


Fig 5. Akins Classifier

Dewatering and thickening. Finely ground product from classifiers, or slimes from a sand-slime separation, are dewatered or thickened in a continuous thickener.

Dorr thickener (Fig 6) is almost exclusively used for this purpose. Its mechanism may be installed in a tank of convenient size, with a circular overflow launder and a central discharge pipe. A central shaft carries radial arms with short rakes, which, by revolving slowly, move the settled slimes toward central discharge. Bottom of tank may be slightly conical, conforming to inclination of the radial arms; or, if the tank be flat, the accumulated slimes soon form a conical bottom surface. Feed enters at a central well, discharging below the overflow level to avoid surface currents. A thickener under 50-ft diam requires less than 1 hp. A diaphragm suction pump regulates the discharge and elevates it to the top of tank.

A recent development, the Torq thickener, eliminates the superstructure and employs a centerpiece carrying rakes, so supported that they ride over resistant areas in the tank. For heavy-duty work the Traction thickener has a rake drive carried on the rim instead of at the center of tank.

Balanced tray type consists of two or more superimposed thickening compartments in one tank, with radial rakes attached to a common shaft. Each compartment has a hydrostatically balanced feed and its own overflow pipe, while the thickened sludge discharges through a central opening in bottom of tank.

Combination type (Fig 6), used for counter-current decantation (Art 12), consists of a single thickening compartment superimposed on a series of tray compartments. The thickened feed passes to the agitators (not shown), and after treatment is returned to the first tray mixed with overflow wash solution from the tray immediately below. This process is repeated one or more times, and the washed pulp finally discharged from the bottom. The thickening zone is separated by an air seal, and the washing zones by an inverted cup and seal. Fig 6 shows normal design. For chemical work, with hot or caustic solutions, the tank and box are covered, and overflow is controlled by adjustable sleeves, operated from outside of box.

10. SAND TREATMENT

Tanks. Sands, after separation from slimes, are charged into tanks fitted with filter-bottoms, consisting of a grating overlaid by cocoa matting and canvas. Sometimes a layer of sand is charged on the filter proper, and over this is spread a grating to limit shoveling of the tailings when the tanks are discharged.

Bottom grating is arranged to insure free circulation of the filtered cyanide solution on the tank bottom towards the solution discharge pipe; there should be always enough orifices through the supporting frame of the bottom grating to allow all parts of the tank bottom to drain freely towards the discharge. The canvas, placed over the cocoa matting, is calked around the edge with rope; it is also calked around the discharge gate when the tank is discharged through the bottom by shoveling, sluicing, or by a mechanical excavator.

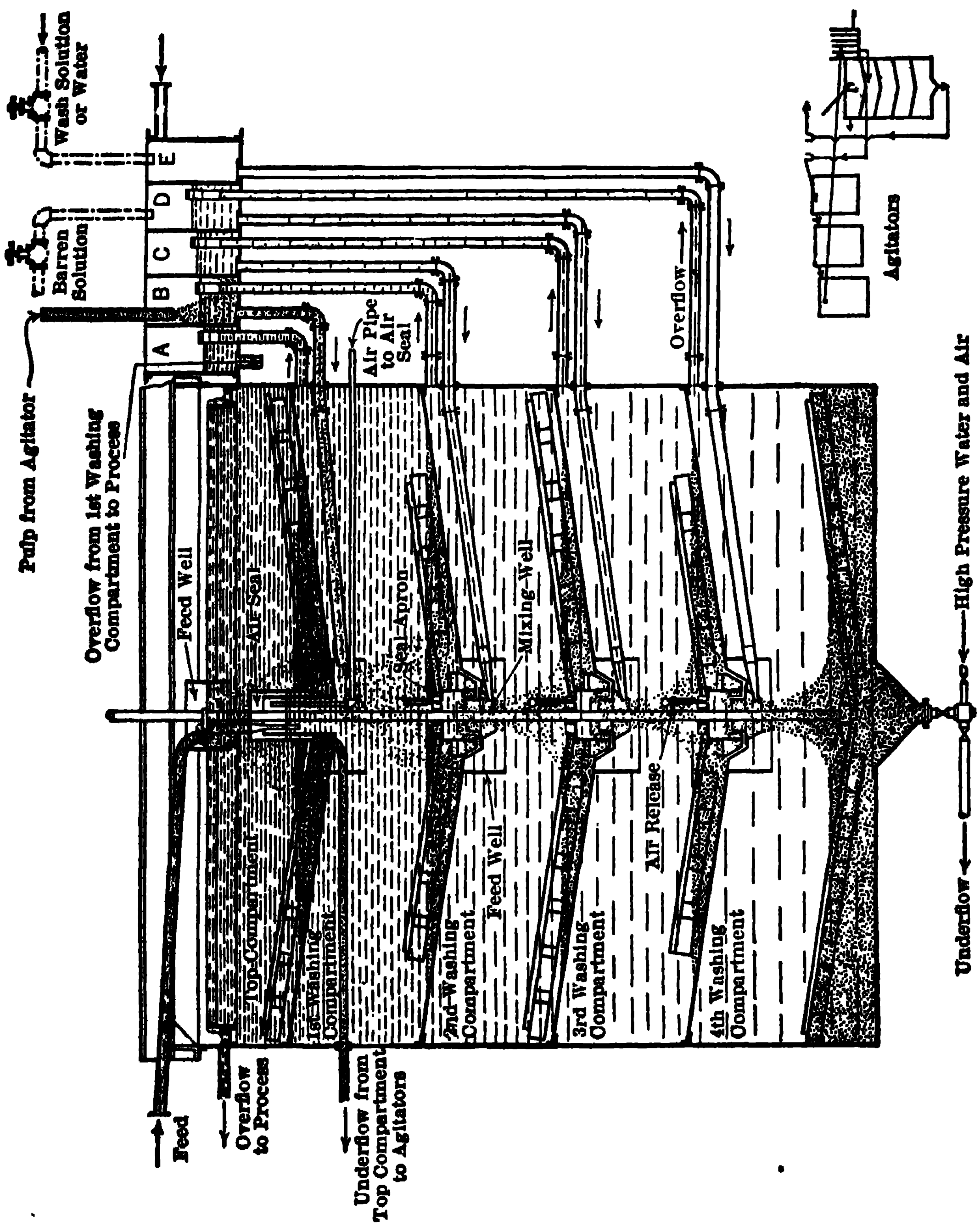


Fig 6. Dorr Washing Thickener (Combination Type)

Depth of tank depends on fineness of sand and amount of slime carried. With clean coarse sand, tanks may be deeper; in practice, depth is 6 to 10 ft. Depth is decided upon after experimenting with the ore to be treated, to determine the RATE OF PERCOLATION. This rate should be between 1 and 3 in or more per hr for satisfactory results and the pulp should be so classified that the resulting sands shall have at least this rate of percolation. The length of treatment being determined, and rate of percolation and depth of tank decided, the area of tank is made such that one or a number of tanks will hold approximately one day's mill run. Tanks have capacities from 30 to 900 tons each.

Operation. The tank is generally filled with wet pulp from the classifiers, evenly distributed by some automatic device. This usually consists of revolving pipe-arms with perforations, or modification of this device. After filling, the sands are drained, and, if crushing has been done in solution, the solution is stored for re-use until it is high enough in precious metal content to be effectively precipitated. After the first draining, the first and generally the strongest mill solution is run into the tank to cover the charge. Practice varies as to admitting the solution at top or bottom of the tank.

Experiments determine the strength and number of the successive cyanide solutions used before washing with clear water. A variation of practice is to use downward percolation; continuously drawing off the cyanide solution from the bottom, and adding fresh solution at the top until the desired extraction has taken place. But this practice does not aerate the pulp so well as intermittent flooding and draining of the charge, which generally gives a higher extraction.

Necessary precautions for percolation of sands: charge should be evenly distributed in the tanks and free from slimes which pack and prevent passage of solution. In a few cases it has been possible to treat clayey material by segregating the feed and using methods of filling tending to agglomerate the fines, and so avoid serious loss of permeability.

Variations of practice in percolation of Au ores show following range: draining off water from wet charged tank, 8 to 18 hr (aver, 13 to 14 hr); leaching, 12 to 18 hr (aver, 15 hr); washing, 16 to 22 hr (aver, 20 hr). Most mills use 1 strong and 2 weak-solution treatments, and 1 with wash water. Some mills have increased extraction by forcing air up through the filter bottom before each application of solution. Pressure used is only sufficient to aerate slowly without forming channels in the sands. Vacuum drainage is used in certain plants on the Rand.

Discharging the tanks by shoveling costs most, and is used in small plants where water is scarce. Sluicing is cheapest, and is used where water is plentiful. Mechanical excavators (radial arms, carrying circular plows that are lowered into the sands and move them to the central discharge door) are high in first cost and are adopted only for large plants. Their operating cost is generally betwen that of sluicing and shoveling.

Size of material treated, AMOUNT and STRENGTH OF SOLUTIONS used, and TIME OF TREATMENT, must be determined experimentally for each ore. An ore especially adapted to sand treatment. due to its porosity, freedom from slimes, and easy solubility of the Au, may be successfully leached by percolation, to as coarse as $^3/_8$-in size. Certain siliceous ores have given good extraction when crushed as fine as 100 mesh. A typical all-sand treatment is that of the Mountain Copper Co, Calif, where a gossan of limonite and quartz carrying \$1.85 gold is profitably leached, crushing to $^3/_8$-in size. Treatment is in ten 276-ton vats, as follows: 195 tons of 0.8-lb NaCN solution over 16 hr; 90 tons of 0.5-lb solution over 24 hr; 180 tons of 0.4 lb solution over 20 hr; and 75 tons water wash over 7.5 hr. Indicated extraction, 73% at cost of 43¢ per ton. The plant cost \$75 000.

11. SLIME TREATMENT

Agitation of thickened slimes may be done in tanks by mechanical stirrers, compressed air, or, more efficiently, by a combination of these. As oxygen is required to dissolve the Au and Ag, some aeration is always necessary.

Dorr agitator (Fig 7) is the best known device using both mechanical stirring and air. It consists of a central pipe carried by a hollow revolving vertical shaft, which has 2 arms with plows like those of the Dorr thickener. The plows draw the settled slime toward the center, where a jet of air raises it through the central shaft. From top of this, the pulp is distributed by revolving launders over the surface of the pulp in the tank. By certain modifications the Dorr agitator can be used for either continuous or batch agitation. A size 40 ft diam by 25 ft deep requires less than 3 h p.

Devereaux and Turbo agitators are mechanical types, the first employing an open propeller near bottom of tank; the other, a high-speed bladed rotor, running within a similar stator and operating on the turbine principle. Both types draw in air by a vortex action.

Brown or Pachuca tank (Fig 8). Where only compressed air is employed, this is the common type as developed in New Zealand. It is a high steel tank with conical bottom and central tube; the pulp is raised to the top, overflowing back into body of tank. A favorite size is 15 ft diam by 45 ft high. The colloidal slimes of Mexico tend to build up on the sides of the tanks, so that they require cleaning out every two weeks. As the comparatively granular slimes of the Rand are free from this tendency, the size in recent installations has been increased to 22 ft diam by 45 ft high.

A Dorr agitator, requiring less than half the power (mechanical and air) of the Brown, uses a tank of convenient size for saving in pulp elevation, and can be equipped with a

positive scraper for cleaning the tank sides. The high Brown tanks require a substantial foundation.

Dilution of pulp. To the thickened pulp charged in the agitating tanks enough stock cyanide solution is added to bring the pulp to the required consistency, at the same time maintaining the proper strength of cyanide in the charge. Dilution of the pulp depends upon character of the slime; thick clayey slime requires greater dilution for prompt extraction.

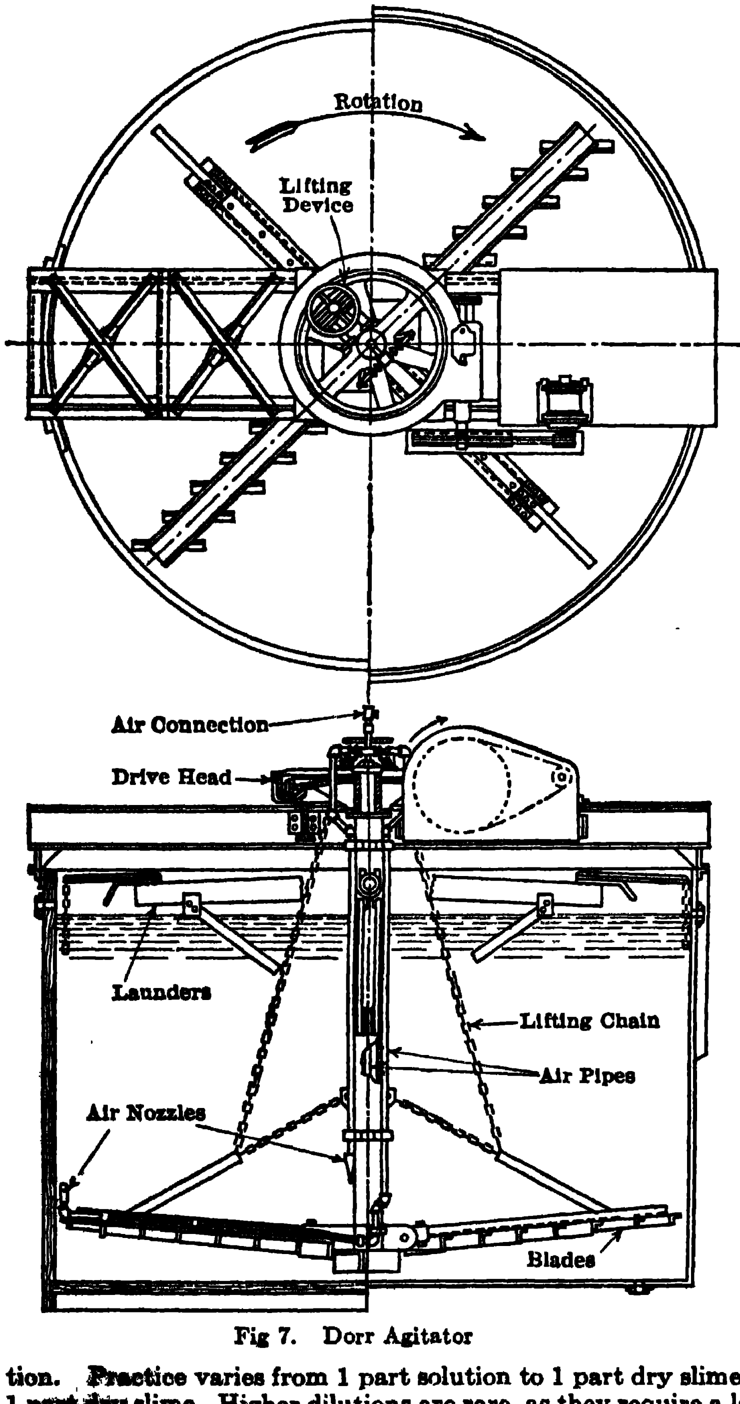


Fig 7. Dorr Agitator

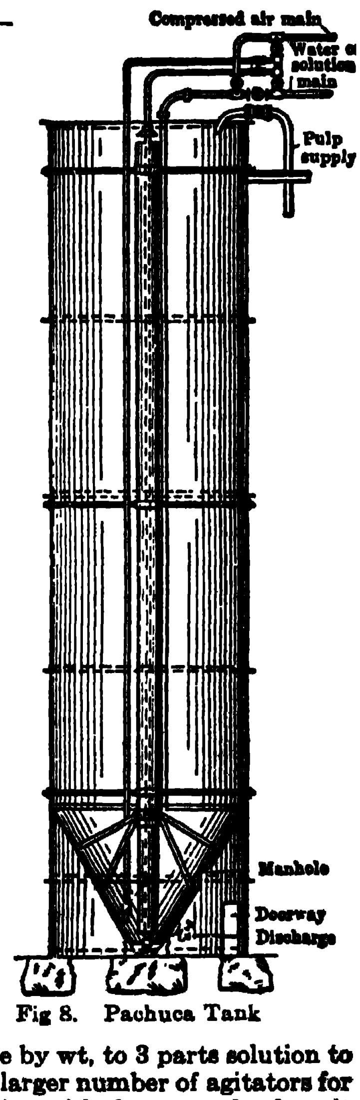


Fig 8. Pachuca Tank

Practice varies from 1 part solution to 1 part dry slime by wt, to 3 parts solution to 1 part dry slime. Higher dilutions are rare, as they require a larger number of agitators for a given treatment time. Accompanying table is useful in dealing with the sp gr of pulps; it is based on a sp gr of 2.5 for dry slime. Intermediate values in the table may be interpolated. In practice, it is usual to take samples of the thoroughly agitated pulp in a weighed flask of known capacity, and from the net wt of the pulp calculate the sp gr, the other data being obtained from sp gr tables.

Strength of solution. The amount of cyanide in the solution which forms the pulp in

the agitation tanks is determined by experiments on the ore, and varies greatly in different places. The strength may be expressed in terms of lb or per cent, but it is customary to express the strength in lb of NaCN equivalent per ton. In general, for Au ores, from 0.5 to 2.0 lb NaCN covers the range of strength of solution. For Ag ores, the range is 2.0 to 5.0 lb. Where battery crushing or tube-mill grinding has been done in solution, the strength of solution used is generally much lower than that named above, and, to bring the pulp up to the proper percentage of cyanide, a stronger solution is added in the agitation tanks to the thickened pulp; or, if the pulp reaches the agitation tanks already sufficiently dilute, cakes of cyanide are dissolved in the charged tank when agitation commences. At the same time the proper amount of lead acetate is added, which is generally done by suspending in the charge a loose burlap bag containing lead acetate corresponding in amount to that needed for the treatment of the number of tons of dry slimes in the tank.

Dry slime in pulp, %	Parts of sol to dry slime	Sp gr of pulp	Wt of 1 cu ft of pulp, lb	Cu ft pulp for 1 ton dry slime
16.67	5 : 1	1.111	69.5	172.9
20.00	4 : 1	1.136	71.1	140.8
25.00	3 : 1	1.176	73.4	108.8
28.57	2.5 : 1	1.206	75.3	93.4
33.33	2 : 1	1.246	77.8	77.7
50.00	1 : 1	1 429	89.2	44.8

The charge should also be tested for protective alkalinity; the proper amount of lime to insure this is generally crushed with the ore in the battery, or milk of lime is added to the charge in the tube-mill.

Some trouble has been experienced in air agitation, due to excessive use of lubricating oil in the compressor cylinders. Some plants use soapsuds or graphite lubricant for the air cylinders, and claim that extraction is thereby increased.

Quantity of air required. In practice a Brown tank, 15 ft diam by 45 ft high, uses 50–75 cu ft free air per min, at a press of 30 lb per sq in. A Dorr agitator, 30 ft diam by 12 ft deep, uses not over 25 cu ft free air per min at 10-lb press. From 5 to 10 hp is required for both air and mechanical operation. Violence of agitation does not necessarily increase extraction; constant agitation is needed, which will move every particle of material reaching the inlet of the air-lift. Control of aeration is important, for an excess can increase consumption of lime and cyanide, without improving extraction.

Time of agitation varies greatly with different ores; the economic limit can be determined only by laboratory tests. With Au ores, 18–36 hr generally suffices, but with Ag ores 36–60 hr is common and 90 hr are often required. A longer period often permits use of a lower-strength solution.

Agitation may be either in batch treatment or continuous. If continuous, not less than 3 agitation units should be employed to insure against short-circuiting of part of the pulp. Continuous agitation has been adopted in most districts.

12. CONTINUOUS COUNTER-CURRENT DECANTATION

A series of continuous thickeners, or a washing thickener unit, operates in such manner that the solids, with a small portion of liquid, pass successively from one thickener, or

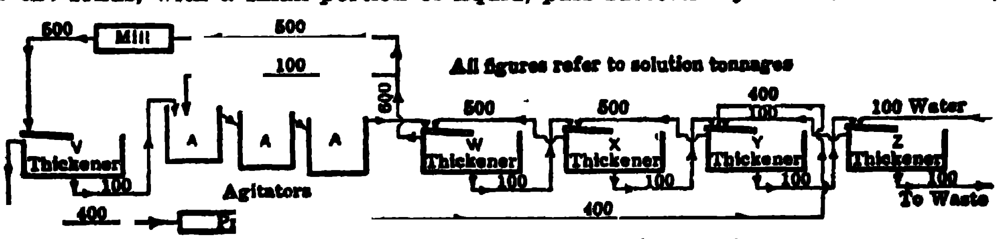


Fig 9. Diagram of Counter-current Decantation

thickening compartment, to the next, being finally discharged as tailings (Fig 9), or as a product to be further washed and dewatered on a continuous filter. The main body of liquid moves in a direction opposite to the course of the ore and is gradually enriched. By each step a large volume of low-grade solution is mixed with a very small amount of high-grade solution, so that the value of the solutions entering each successive thickener with the solids is materially reduced.

Calculations for dissolved value loss. Assume (Fig 9): 100 tons per day crushed in cyanide solution; 50% solids in thickener underflow; $10 value dissolved per ton of ore, (75% in the mill, 25% in the agitators); solution value reduced to 2¢ in the precipitation section; agitation carried out at 2 : 1 solution to solids. Then, if V, W, X, Y, Z are values in dollars per ton of solution discharged from thickeners, equations are as follows, equating in and out of each thickener:

$$
\begin{aligned}
&(1)\ 100\ V + 400\ V = 500\ W + (0.75 \times \$10 \times 100)\\
&(2)\ 100\ W + 600\ W = 500\ X + 100\,W + (0.25 \times \$10 \times 100) + 100\,V\\
&(3)\ 100\ X + 500\ X = 100\ W + 500\ Y\\
&(4)\ 100\ Y + 500\ Y = 100\ Z + 100\,X + (400 \times 0.02)\\
&(5)\ 100\ Z + 100\ Z = 100\ Y + 100 \text{ tons of water, value nil}
\end{aligned}
$$

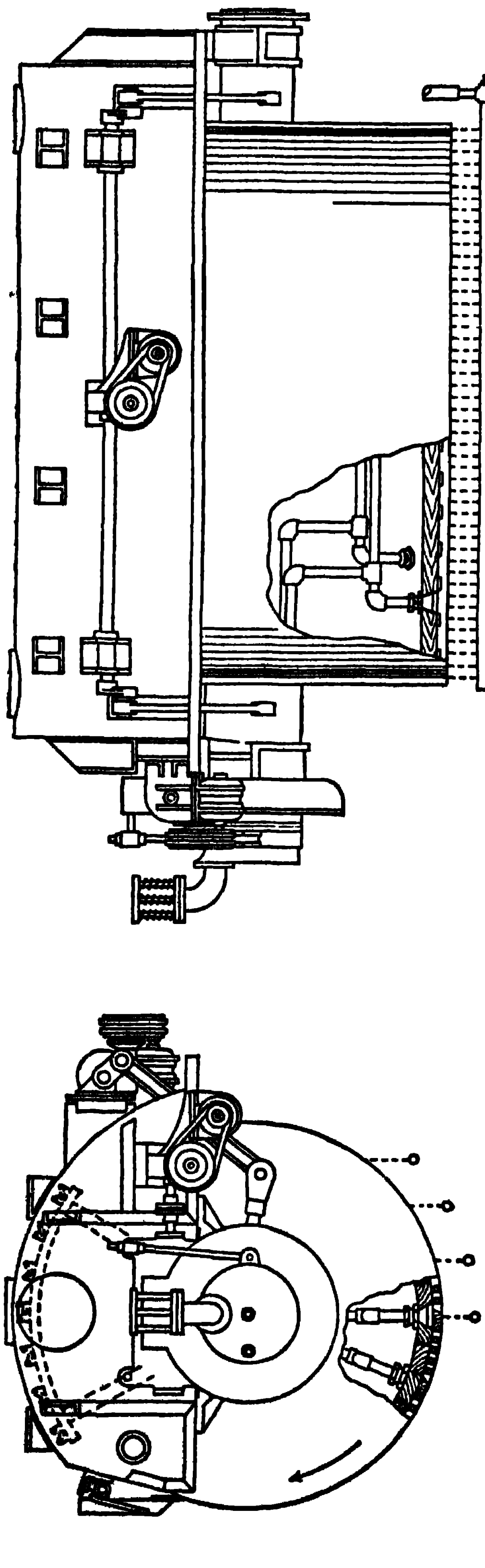


Fig 10. Oliver Continuous Cyanide Filter

Solving, $V = \$2.51332$; $W = \$1.01332$; $X = \$0.21332$; $Y = \$0.05332$; and $Z = \$0.02660$. Mechanical loss of cyanide and lime can be similarly calculated.

13. FILTRATION

Filtration by gravity (Art 10) is employed for percolation of sands. Filter presses or other appliances separate solutions from pulp in the treatment of slimes. Pressure filters, formerly used for treating granular or crystalline slimes, have practically disappeared in favor of the VACUUM FILTER for all pulps. Following are brief descriptions of the principal mechanical filters.

Oliver filter, a continuous filter of vacuum type, consists of a revolving drum partly submerged in a tank about one-third full of thickened slimes (Fig 10). During a part of the revolution of the drum, the surface of which carries the filtering medium, the solution is drawn off by a vacuum pump, connected through a rotary valve on the drum's trunnion. During passage of the drum through the slimes, a cake is formed on the drum exterior. The slime cake is sprayed through succeeding arcs with barren solution and wash water, which in turn are drawn off by the pump, and sent to weak solution storage or used in countercurrent washing in the preceding thickeners. During the final arc in the drum's revolution, the rotating valve cuts off the vacuum and connects the leaves as they pass that point with a compressed air supply. The compressed air, assisted by a scraper, discharges the slime cake for its removal to waste. ADVANTAGES: low initial cost and operating cost, continuity of operation, few complicated parts and low repair costs. Capacity of the filter varies with character of pulp, but for an aver slime is about 800–1 200 lb dry solids per sq ft of filter surface per 24 hr.

American filter is of continuous vacuum type, consisting of one or more filter disks mounted vertically about a central shaft (Fig 11). Each disk is divided into 8 or more sectors, each of which is a rigid frame holding a wire body and covered with a cloth bag clamped in place. Individual sectors are interchangeable. Drainage conduits connect each sector to a rotary valve. Cycle of operation is like that of the Oliver filter. ADVANTAGES: economy in floor space per unit of filtering area, no wiring required

for holding down the canvas, and ease with which worn or choked sectors may be replaced. **Disadvantages: considerable difficulty in obtaining effective displacement with wash water.**

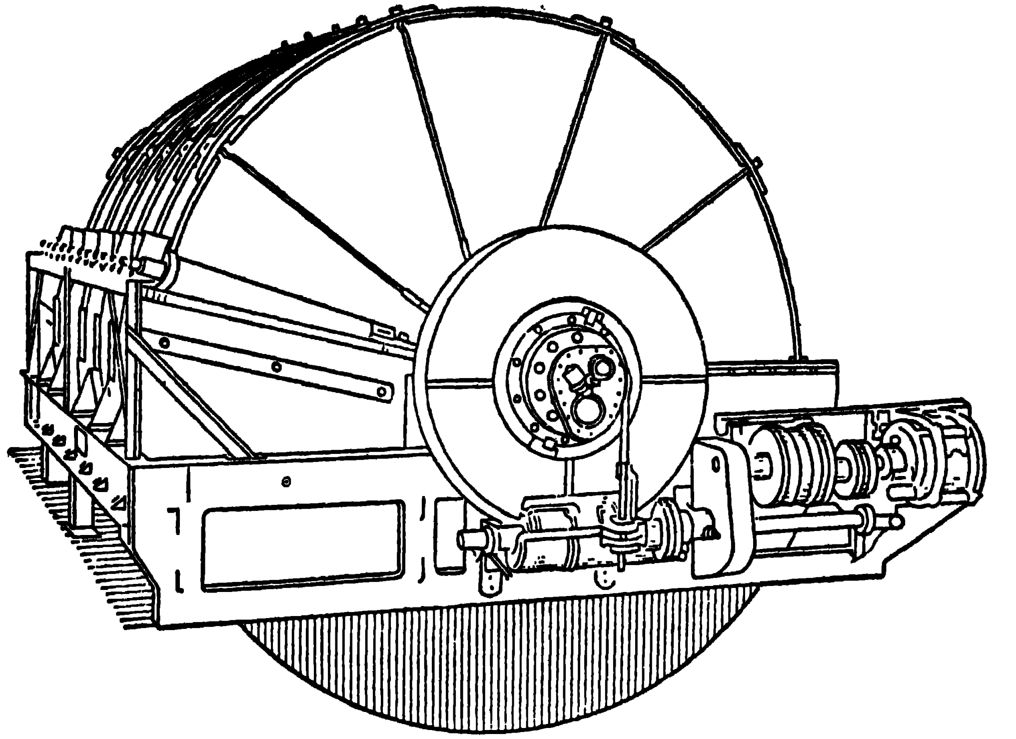


Fig 11. American Filter

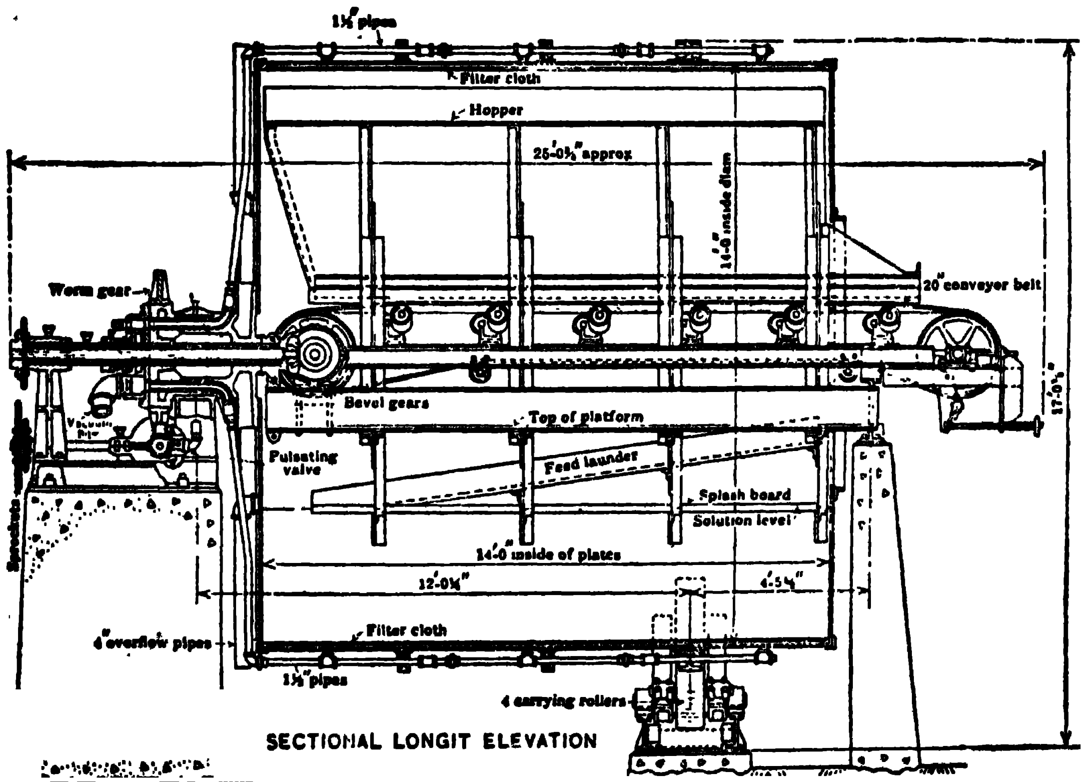


Fig 12. Dorrco Filter (Shimmin)

Dorrco filter (Shimmin) is a more recent vacuum machine, especially adapted for filtering pulps containing granular material; it consists of a rotating drum, on the inside of which is a filter medium. Fig 12 is a sectional view of the 14 by 14-ft machine, showing

the belt conveyer which removes the cake after discharge. One end of drum is open, for receiving the feed, removing the cake, and inspecting the operation. The drum itself is a container for the pulp, eliminating need for a tank and agitating mechanism to keep the pulp in suspension. Segregation of the pulp materially aids filtration, as the coarser particles fall by gravity upon the filtering medium. Hence, the action is the reverse of other vacuum filters using tanks; in the latter, segregation is a detriment, because heavy material goes to the bottom of the tank and is difficult to get onto the filter cloth. Also, the finer material is drawn onto the filter cloth first, and tends to clog it more than if the coarser particles were deposited first. The cake forming on the drum is carried around to a point at or beyond the top. At this point the filter cloth is caused to expand and contract quickly, by use of a valve which subjects the cloth alternately to pressure and vacuum, so that the cake falls from it freely, and the filter meshes are kept clean. The main vacuum for picking up the cake is applied to the drum by pipes, which terminate in the ports of a rotating valve, as for the Oliver filter (Fig 10). The cake, on being blown off the filter cloth, drops into a hopper, whence it is removed by a belt conveyer, operated by the main filter drive. The filter cloth is not wire-wound, nor is there a scraper or roller at the discharge point.

This machine can be used not only for fine pulp, but also for much coarser material than can usually be handled by vacuum filters. This is because no artificial means are required to get the coarse material onto the filter, the act of feeding the machine being sufficient.

Butters filter is a noncontinuous type and comprises a series of filter leaves, suspended vertically in a steel or wooden tank. The leaves are perforated pipe frames, supporting a filtering medium of cocoa matting and canvas cover. Each filter leaf is connected to a common pipe header. Tank is charged with slime and vacuum applied to leaves until cake is formed of sufficient thickness. Retaining a partial vacuum on leaves, excess pulp is withdrawn back to storage and tank filled with barren wash solution. Full vacuum is again applied until pregnant solution originally contained in the cake is sufficiently removed; vacuum is again lowered, excess wash solution withdrawn and cycle repeated with wash water. After being thoroughly washed, cake is dropped to bottom of tank by reversing the air pressure on the leaves and sent to waste. Time of cycle and capacity of filter vary with character of pulp and local conditions; aver, about 8 cycles per day and about 75 lb dry solids per sq ft of filtering surface (counting both sides of filter leaves) per 24 hr. ADVANTAGES claimed over continuous type of vacuum filter; better control of washing cake, enabling filter to take pulp directly from agitators, thus eliminating intermediary counter-current washing thickeners that are usually found advantageous ahead of continuous filters. DISADVANTAGES: higher initial and operating cost than continuous type; intermittent batch operation.

Acid treatment of filter surface is necessary every week or two, to remove accumulated lime from the filtering medium. The filter leaves are submerged or scrubbed in a weak solution of HCl, while under a slight vacuum.

Clarifying of solution. Pregnant solution obtained from filters or by decantation is usually cloudy. To clarify it before going to precipitation, it is filtered by gravity through beds of clean, fine sand; or, better. by a small Butters filter, using a wet vacuum pump. Several manufacturers have special pressure filters for this work.

14. PRECIPITATION AND REFINING

Precipitation on zinc shavings, still used in some of the older plants, has given way to zinc-dust precipitation in most modern plants. The clarified solution passes to a series of boxes containing the shavings. The solution overflows from one box, drops down a baffled partition and rises through the bottom of the next, and so on to the last. Each box has a false bottom, and all are contained in a longitudinal tank. The boxes, 6 or 8 in number, may be 12 to 30 in or more in width and height, with a total length of 12 to 20 ft or more, and are of wood or sheet steel.

The Zn shavings rest on a screen bottom, and are most effective when cut about $^1/_{16}$ or $^1/_8$ in wide and extremely thin, generally not exceeding $^1/_{500}$ in thick. The shavings, prepared by special automatic lathes from zinc cylinders, are packed uniformly in each compartment to insure an even flow of the solution through them. Amount of Zn is calculated at about 1 cu ft of shavings for each 1.5 ton of solution to be treated per day. The Zn is sometimes dipped in a solution of lead acetate, to form a Zn-Pb couple, making the precipitation more effective. Before cleaning the precipitate from the Zn, the strength of the cyanide solution running through is materially raised for an hr or more; the Zn is then washed or shaken in a separate receptacle and the loose precipitate is washed down into the bottom of the trough or launder containing the Zn boxes. The clean Zn is moved up from one compartment to the next, until all have been cleaned, fresh Zn being added to the lower

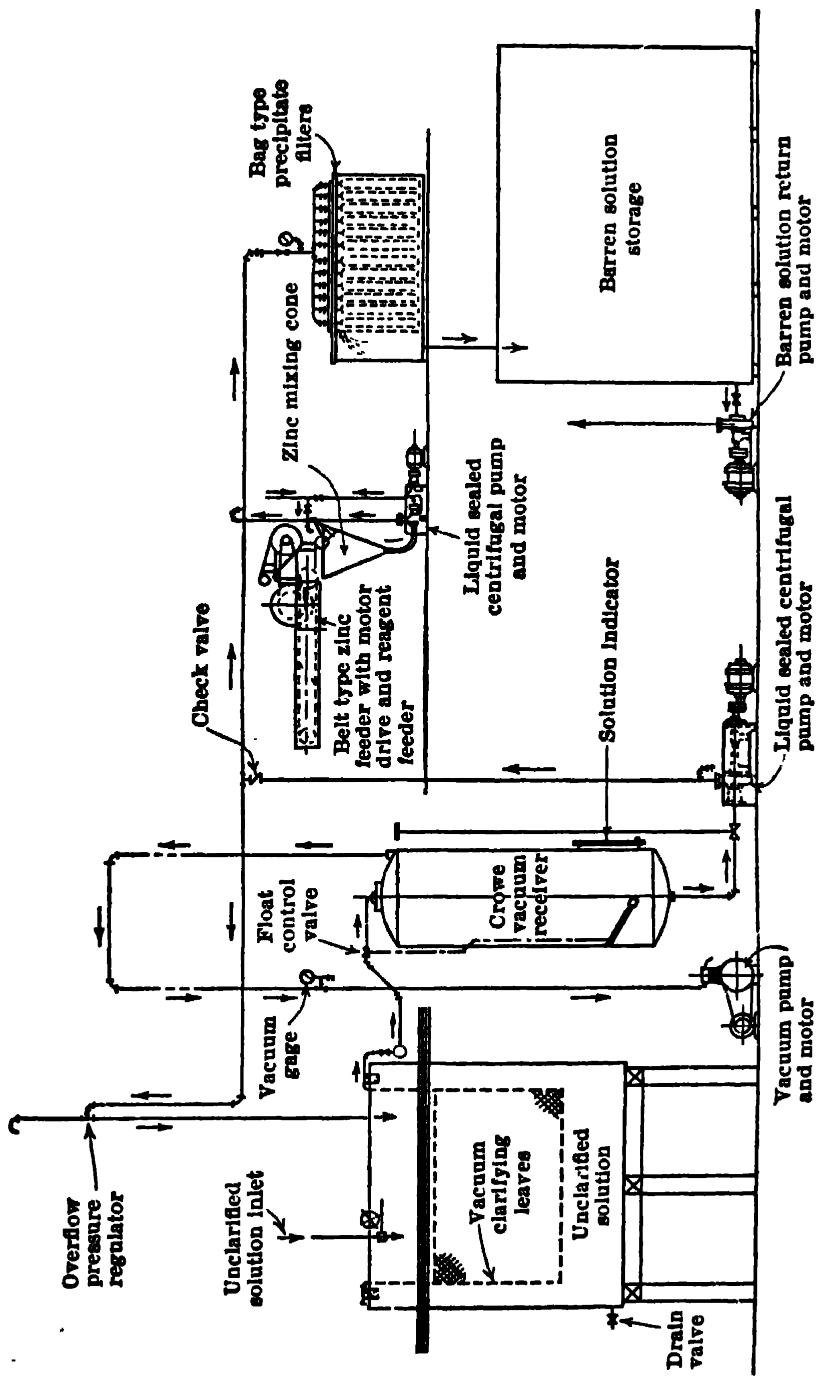


Fig 13. Merrill-Crowe Bag-type Precipitation, using Zinc Dust. Dorr's "Cyanidation and Concentration of Gold and Silver Ores," p 192

compartments. The precipitate, with such fine or "short" zinc as may have fallen with it, is then dried preparatory to further treatment.

Precipitation by zinc dust. The Merrill process of continuous zinc-dust precipitation is used in all modern plants. The dust is fed automatically into a small part of the pregnant solution, and this mixture is added to the main stream of solution going to precipitation.

The pregnant solution mixed with the Zn dust is pumped into a filter press or pressure bag filters. The bags, which are submerged, have inner and outer portions, the inner filtering medium being thoroughly washed or burned after scraping off the Au precipitate. Precipitation takes place almost immediately; the Au and Ag (and any excess of Zn dust) remaining on the filtering medium. When enough cake is formed (indicated by rapid rise of pressure), pumping is stopped and sometimes a little wash water forced through; air is then forced through the line for a few minutes, partly to dry the precipitate; the press is opened or bags disconnected, as the case may be, and the cake removed in trays for further treatment.

A modification of the Merrill process has been to use vacuum leaves in place of pressure filtering, the apparatus resembling a small Butters filter, with a wet vacuum pump. The mixture of Zn dust and pregnant solution flows by gravity to tank and is drawn through the filter leaves. An automatic control assures a submerged filter leaf, so that the Zn dust is not exposed to the air. The mixture of precipitate and solution circulates continuously over the filter leaves, assuring a uniform layer of solids on the cloth. Advantages: reduced first cost and simpler clean-up.

Crowe process. The de-aeration of pregnant solution prior to precipitation by this process has been the most striking improvement in cyanidation in recent years. It is adapted to precipitation by either Zn shavings or Zn dust, the latter being preferable (Fig 13). Few local conditions would warrant its omission from any plant. De-aeration is done by subjecting thin films of the liquid to a vacuum action within a dispersion tower or receiver. Practically all dissolved air is removed through the vacuum pump, and the solution is sent to precipitation without further opportunity to absorb air. Recent improvement in apparatus allows receiver to be placed on same level as other precipitation equipment, while vacuum clarification is made an integral part of the de-aeration system. Corresponding to importance of de-aeration prior to precipitation, the barren solution should be thoroughly aerated before its re-use in the dissolving circuit.

The Merrill-Crowe processes have reduced Zn consumption to about 0.05 lb per ton milled on the Rand, and to less than 0.5 lb per lb of bullion from Ag ores in Mexico.

Precipitation by aluminum resembles that by Zn dust. It is advantageous in some cases; but lime must be absent from the solution, as it results in the formation of insoluble calcium aluminate, which produces low-grade precipitate.

Precipitation by charcoal has been used with some success in Australia, and has possible advantages where the solutions are badly fouled with contaminating substances which interfere with Zn-dust precipitation. The combination cyanidation-charcoal precipitation-flotation processes of Edquist and Chapman have already been mentioned (Art 6).

Sodium sulphide precipitation is satisfactory where Ag only is to be considered. Advantages: low cost and regeneration of part of the cyanide. Used at Nipissing mill, Cobalt, Canada.

Regeneration of cyanide is partially accomplished by sodium sulphide, as well as sulphurous and sulphuric acid. There are (1938) as many as seven regeneration plants in operation, with an aggregate daily capacity of some 10 000 tons of solution.

Mills-Crowe process is most generally used for recovering cyanide as calcium or sodium cyanide from used mill solutions. It is claimed there is a nearly complete recovery of cyanide from the free cyanide and double cyanides of Zn and Cu. By special reagents, a partial recovery can be made from the ferrocyanide.

Process consists in acidifying the cyanide solution with sulphurous acid; the acidified solution passing through additional dispersion towers, in which the solution flows countercurrent to a relatively large volume of low-press air. The air removes practically all the cyanogen in form of HCN. By passing the air through similar towers, in which alkaline absorbent solution is circulating, the HCN is absorbed and sent out for re-use. Sulphurous acid is obtained by burning sulphur or by roasting sulphide ores (see Art 16 for costs).

The General Engineering Co uses $ZnSO_4$ to precipitate free and combined cyanide. The relatively small amount of precipitate is then acidified, the HCN re-absorbed, the $ZnSO_4$ regenerated for further use, and insoluble residues recovered by smelting.

Melting and refining. Method depends on local conditions as to quantity and quality of precipitate to be handled. Au precipitate is usually first treated with 10% solution of H_2SO_4 to dissolve out excess Zn, special precautions being taken to vent out of the building any HCN that is formed. It is then dried, fluxed and melted in graphite crucibles; Ag precipitate is generally treated in a reverberatory furnace. Silver bullion is partly cleaned

by blowing air through the molten mass, the melted bullion being poured into molds and the bars sold direct to refineries. In the Tavener process the precipitate is melted in a reverberatory furnace with litharge, the lead bullion formed being cupelled in a special furnace, producing fine bullion.

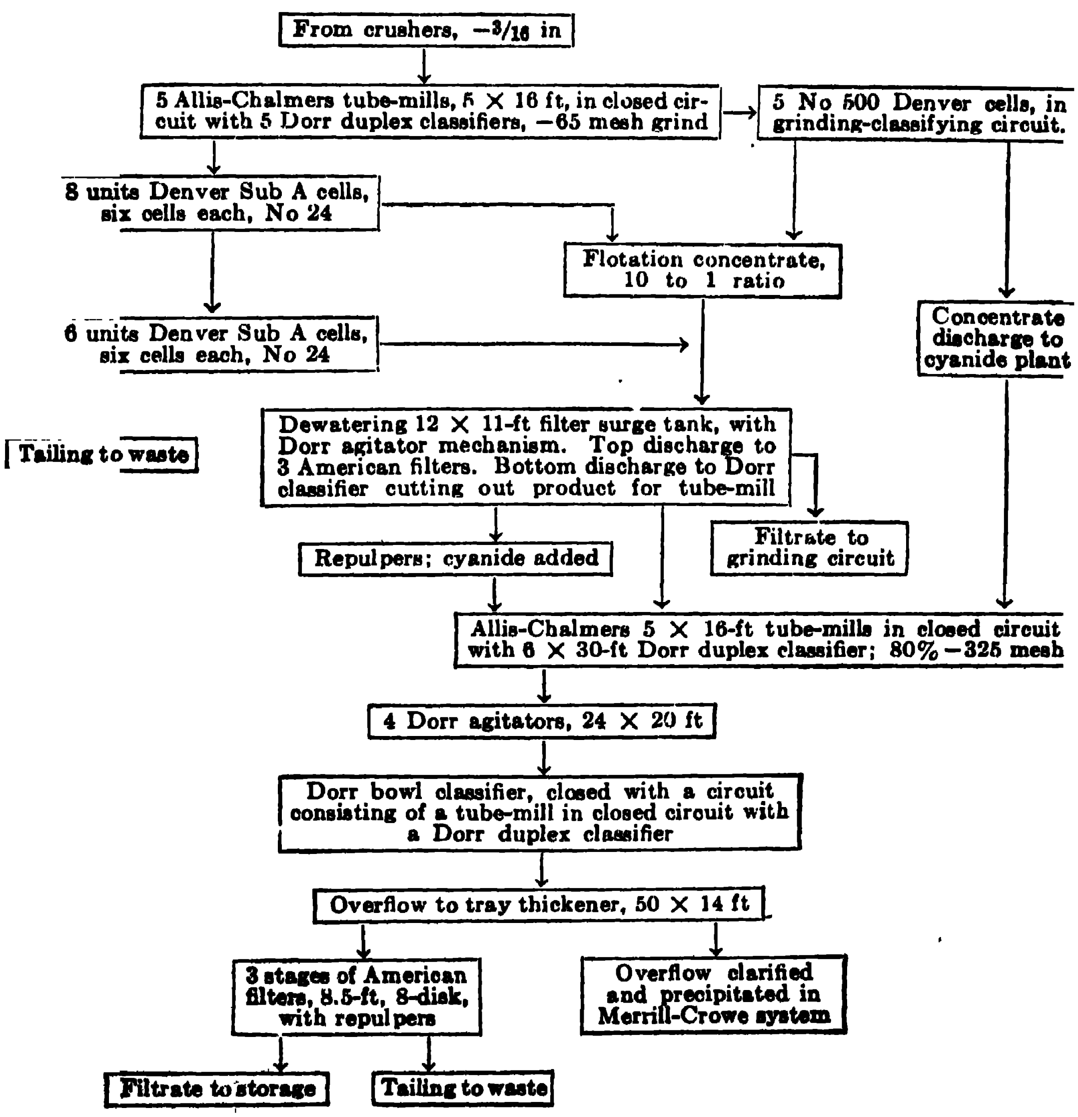
Composition of flux. For Au, aver flux is 30% Na_2CO_3, 10% clean SiO_2, and 60% borax glass. Weight of flux is 18–35% of weight of precipitate; and up to 85% if precipitate is high in Zn. A little $NaNO_3$ may be used (3–10% of wt of flux), but an excess rapidly scours the crucible. If precipitate carries much base metal, the proportion of SiO_2 may be larger. Slag and matte (if any) are saved, and re-treated or shipped when enough accumulates.

Short zinc. In using Zn shavings, short broken pieces of Zn accumulate in excess of the amount that can be returned to the boxes. They are best disposed of by treating with H_2SO_4 and adding resulting residue to final precipitate.

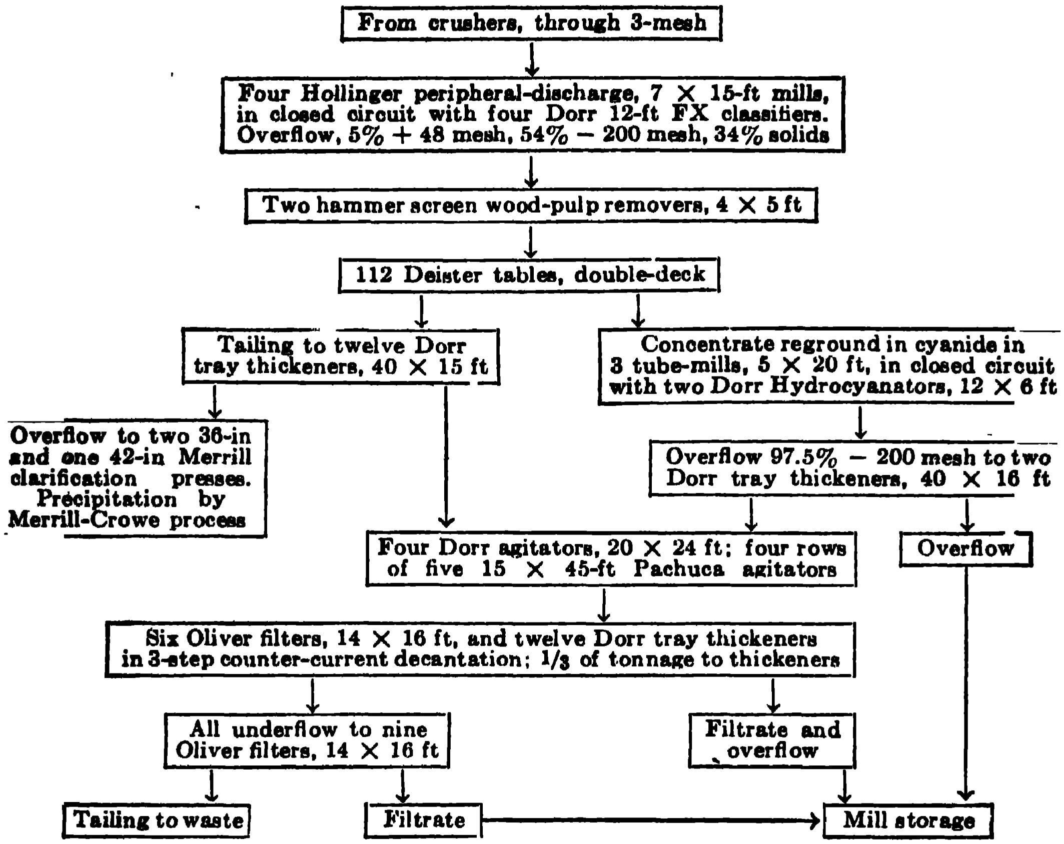
15. FLOW-SHEETS OF TYPICAL MILLS

Following are typical flow-sheets of mills operating on widely different ores.

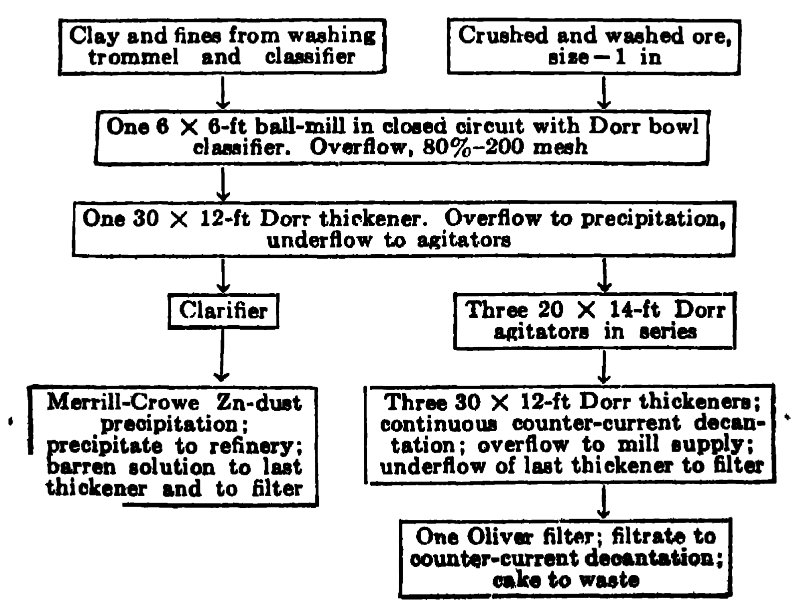
McIntyre mill, Ont, Canada. 2 400 tons daily. Gold ore, 0.218 oz per ton, in quartz, porphyry and schistose basalt, containing 8% pyrite. Grinding with unit flotation cells in mill-classifier circuit, finished tailing by main flotation circuit, regrinding and cyanidation of concentrates.



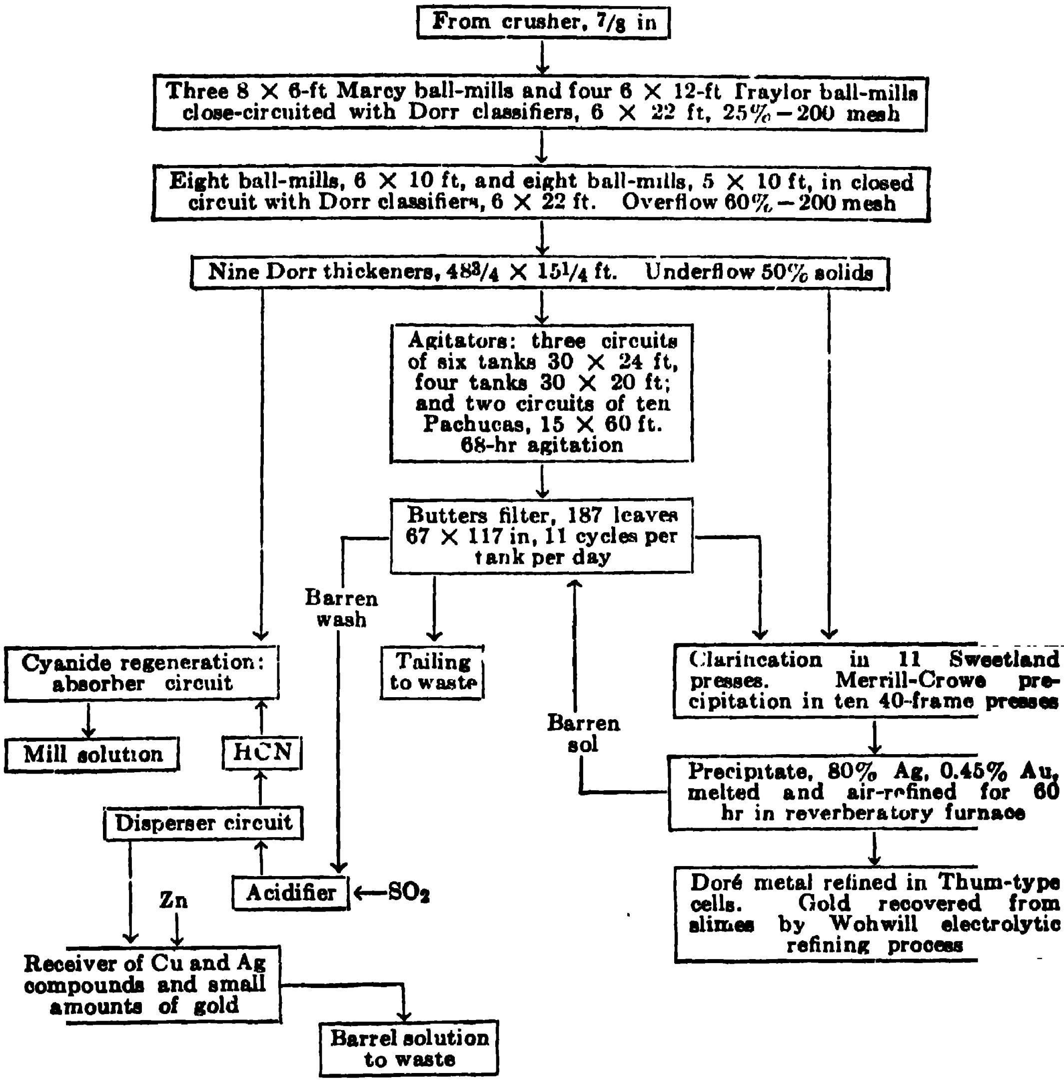
Hollinger mill, Ont, Canada. 5 000 tons daily. Gold ore, approx 0.25 oz per ton in siliceous schist containing about 4–5% pyrite, of medium hardness and 2.9 sp gr. Grinding in cyanide solution, table concentration of sulphides, concentrate reground, counter-current decantation, and two-stage filtration.



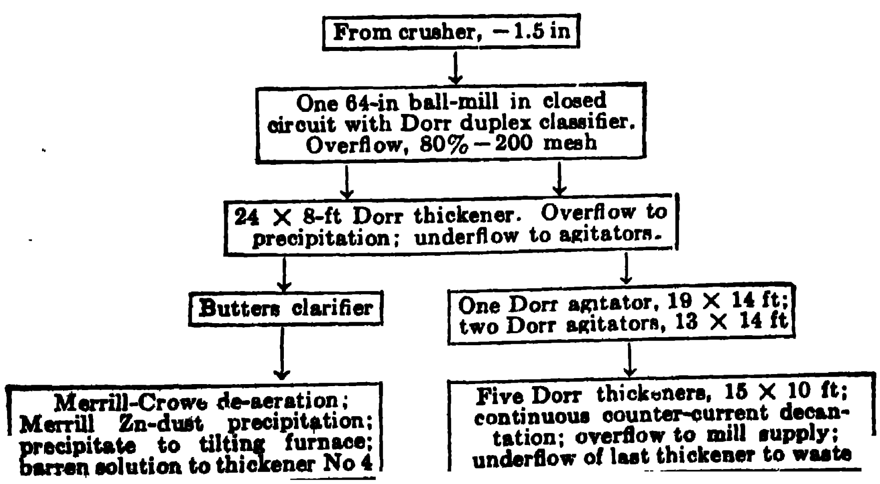
East Mindanao mill, Philippine Is. Gold ore, 100 tons per day. All-sliming and counter-current decantation, followed by a filter. Clay washed out before secondary crushing and by-passed to ball-mill. Mill heads approx 23 pesos per ton; tailings approx 1.5 pesos per ton (par value of peso, 50¢ U S cur).



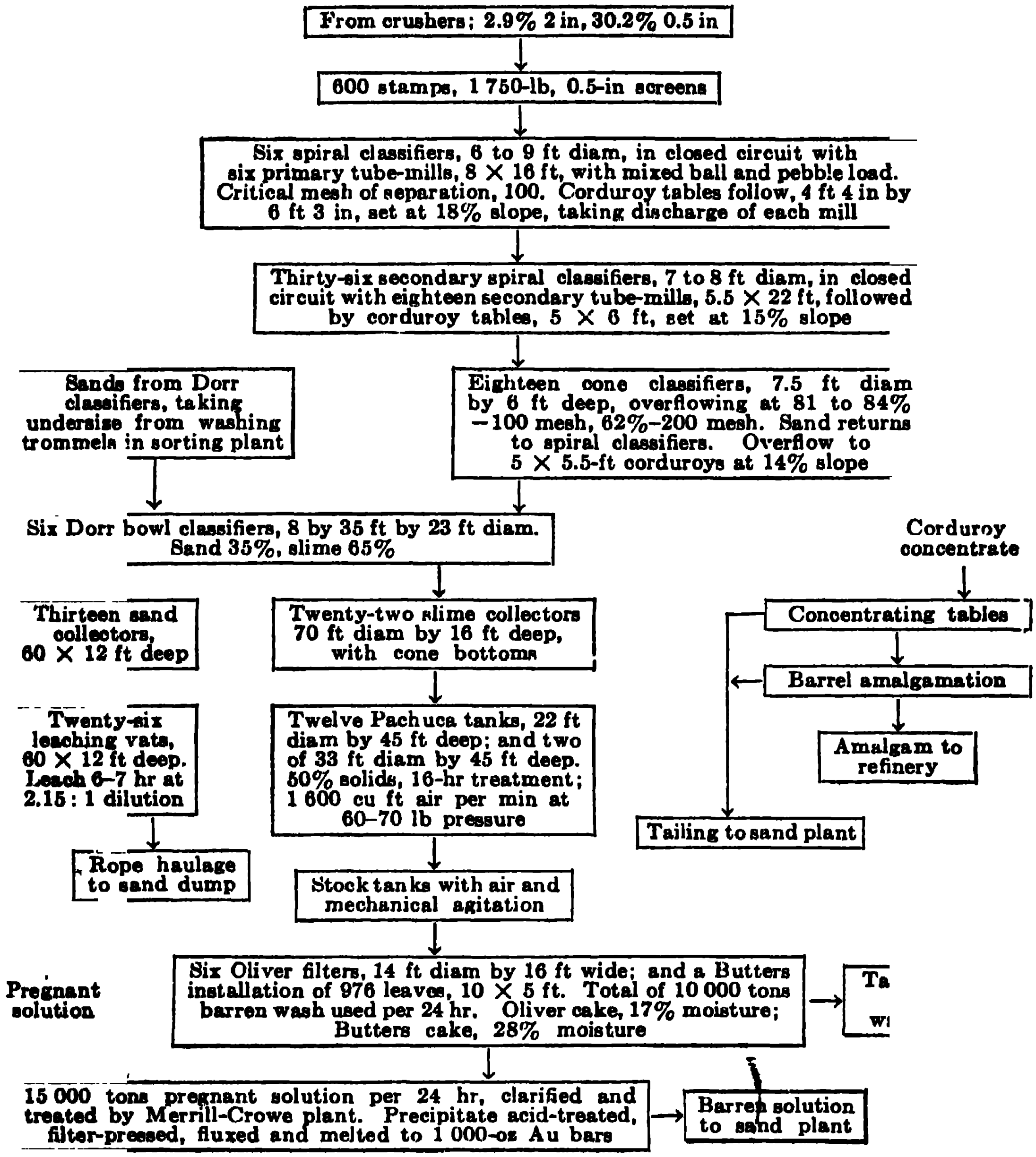
Loretó mill, Pachuca, Mex. 4 200 tons aver daily capacity; the world's largest silver cyaniding plant. Silver ore, Ag probably about 20 oz, Au 0.2 oz per ton. Two-stage grinding with ball-mills, mechanical and air agitation, Butters filtration and cyanide regeneration plant.



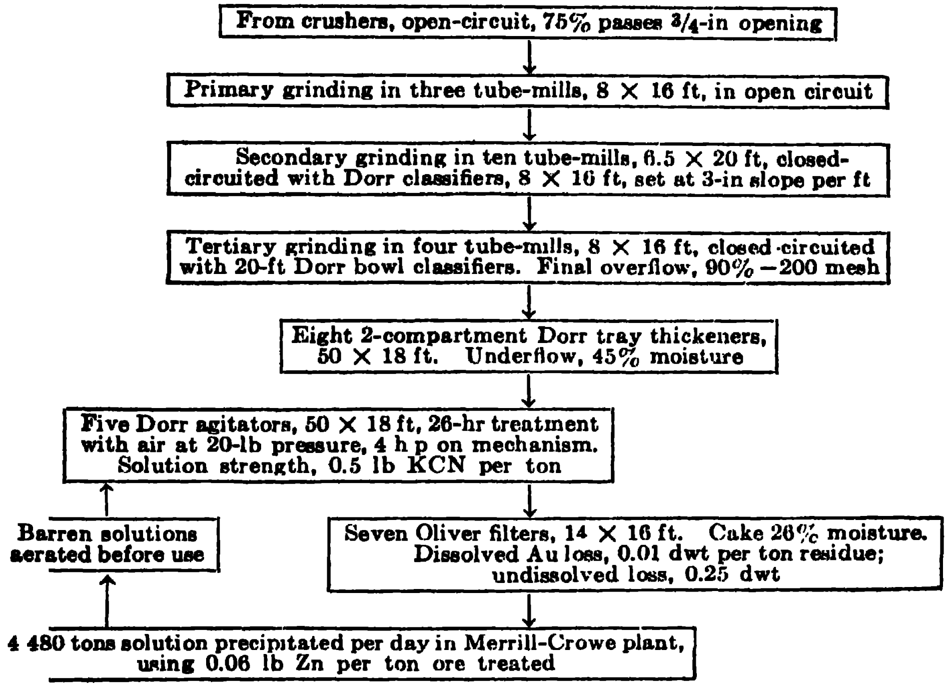
Big Jim mill, Oatman, Ariz. Gold ore, 50 tons per day. All-sliming and counter-current decantation. Hard quartz and calcite ore.



Randfontein Estates, Witwatersrand, So Africa. 12 000 tons per day; largest mill on the Rand. Gold ore, 3.76 dwt per ton. Stamp milling, corduroy concentration, tube-milling with separate sand and slime treatment. Extraction, 95.4%.



East Geduld, Witwatersrand, So Africa. 3 200 tons per day (400 tons rejected by sorting). Gold ore: 7 dwt Au, 0.7 dwt Ag, 3% pyrite. A recent all-sliming plant on East Rand. Tertiary grinding in tube-mills in water. Dorr thickeners and agitators. Extraction, 96%. Total cost of treatment, 60¢ per ton. Cyaniding, 19¢.



16. EXAMPLES OF COSTS

Homestake Mining Co, Lead, So Dak. Treating 1 400 000 tons per year by a fairly simple process of amalgamation and cyanidation. Costs per ton (1929); crushing, 3.8¢; milling, 27.6¢; cyaniding, 18.9¢; total, 50.3¢.

At the South plant (milling only); stamps, 8.9¢; rod-mills, 8.4¢; amalgamating, 1.4¢; tube-mills, 1.4¢; assaying and refining, 1.3¢; superintendence, 1.1¢; miscellaneous, 1.0¢; total 23.5¢ per ton.

Hollinger Gold Mines, Ltd, Ont, Canada. Schistose ore. Aver value, $8.70 (1935). 5 000 tons per day. Total cost, $0.647 per ton milled:

	Cents per ton		Cents per ton
General and superintendence	4.17	Filtration	4.02
Crushing	6.16	Tailings disposal	2.05
Tramming and shoveling	2.42	Precipitation	1.87
Milling (grinding)	15.28	Refining	1.36
Thickening and pumping	2.55	Cyanide and lime	8.50
Agitating and decantation	1.62	Assaying, sampling, testing	2.33
Table concentration	3.40	Heating and lighting	1.84
Sulphide treatment	3.29	Maintenance and alterations	3.88
		Total	64.74

Milling cost is subdivided into: rod-mills, 7.29¢; pebble-mills, 5.98¢; classifiers, 1.07¢; other charges, 0.94¢. Grinding was 2–3% + 48 mesh and 65% − 200 mesh.

Cyanide consumption per ton milled,	0.479 lb.	
CaO " " "	2.11 "	
Lead acetate per ton solution precipitated,	0.0085 "	
Zinc dust " " " "	0.0454 "	

McIntyre Porcupine Mines, Ont, Canada. Quartz, porphyry and schistose ore, 3–15% pyrite. Aver value in 1935, $7.61. 2 400 tons per day. Total cost, $0.698 per ton milled.

Of gold recovered, 76% is made in the unit cells in ball-mill classifying circuit. Costs per ton: crushing and conveying, 10.37¢; flotation, 27.64¢; cyanidation (concentrate treatment, see below), 26.92¢; refining, 2.24¢; assaying, 1.43¢; mill alterations, 1.20¢; total, 69.80¢. Cost of concentrate treatment, from Apl 1, 1935 to Mch 31, 1936, is shown in table.

	Cents per ton		Cents per ton
Tube-milling and classification	4.00	Precipitating	1.37
Agitation	0.94	Reagents	11.50
Thickening and pumping	2.85	Heat and light	0.37
Filtering and clarifying	3.91	Supervision	1.98
		Total	26.92

As ratio of concentration was 9 or 10 to 1, cost of treatment was approx $2.50 per ton concentrate. Consumption of chemicals per ton milled: cyanide, 0.628 lb; lime, 1.106 lb; Zn, 0.084 lb; lead acetate, 0.012 lb.

Pachuca district, Mex (1934). Costs for a 400-ton cyanide mill in this district (Bryan and Kuryla *Trans* AIME, Vol 112, 1924) are shown in table.

Prevailing rate of exchange (1934), 3.60 pesos per U S dollar.

	Pesos per ton	Distribution	Pesos per ton
Crushing and grinding	1.27	Labor	1.38
Cyanidation	1.75	Supplies	2.04
Precipitation and melting	0.30	Power	0.72
Water supply	0.14	Miscellaneous	0.24
General expense	0.92		
	4.38		4.38

Costs on the Rand in 1934 for 3 typical installations, recalculated in cents per ton (Wartenweiler, *Trans* AIME, Vol 112, p 782):

Plant No 1 treats 79 000 tons per month, rejecting 27% as waste and grinding 77% of mill feed through 100 mesh. Plant No 2 treats 208 000 tons, discarding 23%, all mine fines passing direct to tube-mills, and 92% ground through 100 mesh. Plant No 3, East Geduld, all-sliming treats about 100 000 tons per month, rejecting 12.5% as waste. Grinding to 90% – 200 mesh is done in 3 stages of tube-milling (see table for details).

	Coarse crushing and stamp milling		Fine crushing and tube-milling, East Geduld
	Plant No 1	Plant No 2	
Ore transport from shaft to plant	0.96	4 56	....
Crushing and sorting	12.22	10.20	13.0
Stamp milling	13.56	10.58	....
Tube-milling	15.48	21.94	27.0
Cyaniding	21.86	18.58	19.0
	64 08	65.86	59 0

Big Jim mill, Oatman, Ariz (U S Bur Mines, *Inf Circ* No 6824, Feb, 1935). Cost per ton milled in 1933: assaying and supervision, $0.197; crushing, $0.104; grinding, $0.686; cyaniding, $0.802; refining and marketing, $0.036; total milling cost, $1.825. Rental of mill, $0.612. Total, $2.437.

East Mindanao mill, Philippine Is (*E & M Jour*, May, 1937). Cost in pesos per ton milled, Oct, 1936: crushing and washing, 0.252; grinding and classification, 0.188; thickening, 0.044; agitation, 0.032; filtration, 0.069; clarification and precipitation, 0.069; refining, 0.107; water supply, 0.008; chemicals and reagents, 0.467; sampling and assaying, 0.166; superintendence, 0.637; miscellaneous, 0.071; total, 2.110 pesos (par value of peso, 50¢ U S cur).

17. CYANIDE POISONING

Causes and symptoms. Cyanide poisoning may result from taking cyanides internally or from inhaling HCN gas. CHRONIC form of poisoning, due to long exposure to the fumes, is manifested by disturbance of vision, pain in the forehead and in region of the heart, palpitation, difficult respiration, vertigo, coughing, and attacks of suffocation. Symptoms of ACUTE poisoning, as when taken internally, are, besides the above, constriction of throat or general contraction of muscles, and dilated pupils. Face becomes pale and bloated, nails blue, sometimes frothing at mouth. Convulsions occur, followed by coma, and death ensues from asphyxia.

Treatment. If poisoning is caused by breathing fumes, and the case is not severe, relief may be afforded by inhaling ammonia fumes and remaining in fresh air. If gas has overcome the patient, a number of hypodermic injections of 2 or 3% solution of H_2O_2 may be made, or a 10% solution of it given internally.

Artificial respiration may be resorted to; or, if available, O alone may be administered. Hypodermics of ether, strychnine or whisky are given to prevent collapse. If the cyanide has been swallowed, an efficacious antidote is $Fe(OH)_2$, which must be freshly prepared. To have this remedy ready for prompt use (as cyanide's effect is very rapid, there must be no delay), there should be kept prepared at cyanide mills the following measured quantities: a 25% solution of $FeSO_4$, a 5% solution of NaOH or KOH, and phials containing a gm of powdered MgO. These 3 ingredients are kept in well-stoppered bottles, separately. To use them, 30 cc of each of the 2 solutions are mixed with 2 gm MgO, the whole being diluted with a glassful of water and given immediately. After being retained for a few moments, the stomach should be pumped out or vomiting induced. In absence of this antidote, the stomach may be washed with a 0.3% solution of KNO_3, or a dilute solution of $Co(NO_3)_2$.

Prevention. Cyanide poisoning can be largely prevented by providing a safe supply of drinking water in the works and good ventilation.

Workmen in the melting and refining room must not expose themselves to the fumes; poisoning may be caused by HCN, and H, alone, often given off in treating precipitate with acid, will not support life. When As is present in the precipitate, AsH_3 may be produced, which is quite as poisonous as HCN. Workmen must not enter tanks that have been left closed or covered with cyanide solution, as decomposition of cyanide may give off enough gas to cause poisoning. A slight poisoning of the skin is often caused when the hands have been cut or scratched; avoided by wearing gloves in cleaning Zn boxes or handling precipitate.

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SECTION 34

PREPARATION AND STORAGE OF ANTHRACITE COAL

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PREPARATION AND STORAGE OF ANTHRACITE COAL

1. MARKET SIZES OF ANTHRACITE

Classification. Run-of-mine coal is classified, during preparation for the market, into: lump, steamboat, broken, egg, stove, chestnut (or nut), pea, buckwheat (or buck), rice, barley, buckwheat No 4 and buckwheat No 5. The large number of sizes is due to the fact that anthracite burns best when nearly of uniform size, thus permitting easy passage of air through the voids.

Use of market sizes is about as follows: lump, for locomotives; steamboat, for blast furnace and forge; broken for domestic furnace and gas; egg, stove and nut, for domestic furnace and range; pea, for domestic furnace and range, and steam boiler; buckwheat and rice for steam boiler and domestic stoker; barley, for steam boiler. Buckwheat No 4 and No 5 are used in mechanical stoker plants, to replace rice and barley and pulverized fuel.

Table 1. Market Sizes of Anthracite Coal

(Screen openings, in inches, over and through which each size is made)

Size of coal	Punched plate, round hole: Over	Through
Lump	6 1/2	
Steamboat	4 1/2	6 1/2
Broken	3 1/4	4 1/2
Egg	2 3/8	3 1/4
Stove	1 9/16	2 3/8
Chestnut	3/4	1 9/16
Pea	9/16	3/4
Buckwheat	5/16	9/16
Rice	3/16	5/16
Barley	3/32	3/16
Buckwheat No 4	3/64	3/32
Buckwheat No 5	1/32	3/64

"Prepared sizes" is the trade name for: broken, egg, stove and chestnut, which constitute about 60% of the total production. General demand for lump and steamboat has ceased, these sizes being now "broken down" into broken coal and smaller.

"Steam sizes" is the trade name for: pea, buck, rice, barley, and buckwheat No 4 and 5. These are the degradation products, resulting from mining and handling a brittle or laminated material, and are not especially desirable.

Run-of-mine is the unsized mixture of coal and impurities, as dumped into the beaker. According to method of loading cars in the mines, it may be divided into:

Class 1. Hand loaded (as in flat workings), containing lumps up to about 150 lb and relatively free from impurities in coarse lumps.

Class 2. Chute or steam-shovel loaded (as in steeply pitching seams or in strippings), containing lumps up to about 2 000 lb, and frequently wet and dirty.

Impurities in run-of-mine vary with character of seam, and with method of loading. During hand loading they may be partly removed, but in chute loading the entire seam, including all impurities and foreign matter, as clay, top and bottom rock, and broken timbers, must be loaded into the car. Impurities in Class 1 usually range from about 7 to 25%; in Class 2, 10 to 55%.

Table 1a. Run-of-mine Coal: Approximate Per Cent of Sizes from Flat and Pitch Workings, and Per Cent of Refuse in Each Size

Size	Run-of-mine, % of size		Percentage of refuse in each size			
	Flat	Pitch	Northern Field	East Middle	West Middle	Southern
Lump and steamboat	12	6	16	38	30	29
Broken	12	6	20	38	30	39
Egg	15	13	18	38	32	34
Stove	14	13	17	28	20	34
Nut	25	20	17	23	16	33
Pea	6	10	12	18	12	32
Buckwheat	7	18	12	18	12	36
Rice	5	7	....	....	....	32
Barley	3	4	....	....	....	33
Dirt	1	3	....	....	....	....

Standard of preparation designates the percentage of over or under size and of impurities allowed in each size of coal (Table 2). BONE (mixed coal and slate) is defined as containing between 40 and 65% carbon.

Mine car yield is a local term, usually indicating number of long tons of marketable coal shipped from the mines for every 100 cu ft of mine car capacity dumped. This measurement, for hand-loaded cars, is somewhat misleading. For chute loading, the car capacity is figured at its water level; to this, for hand loading, is added a volume equal to the horizontal area times 6 in. The added height of 6 in, above top of car, is called TOPPING. Should the topping exceed required 6 in, no account of excess is made in estimating mine car yield, nor is it practicable to do so. The exact method would be to figure mine car yield on the weight of loaded mine car. The present method admits a fair comparison between mines operating under similar conditions. A "yield" below a previously established record is at once detected, the trouble is investigated, and, if possible, corrected.

Table 2. Standards of Preparation

(Percentage of impurities and of over and under size allowed in each size)

Size of coal	Each size may contain			
	% of		% of next larger	% of next smaller
	Slate	Bone*		
Broken	1 1/2	2	....	15
Egg	1 1/2	2	5	15
Stove	2	3	5	15
Chestnut	3	4	5	15
Pea	5	5	5	15
Buckwheat	12 ash	....	5	15
Rice	12 ash	....	5	20
Barley	12 ash	......	5	20

* If slate is entirely removed, the percentage of bone may equal total slate and bone.

Shipments of prepared coal from a mine, per 100 cu ft of car capac (not including steam sizes), may vary from 0.75 ton, with very dirty run-of-mine, chute loaded, to 3.50 ton in clean seams, hand loaded.

Loss in prepared sizes underground depends upon: varying conditions in the coal seams; kind of explosives used; methods of loading (hand *vs* chute or steam shovel); shocks in running over poorly constructed track; bumping of cars during gravity runs or mechanical haulage; frequency of dumping. Loss in the breaker is due to: poorly constructed dumps; high drops of coal; long running chutes with abrupt turns; poor types of rolls and over-crowding of rolls; breakage in or on mechanical cleaners, scraping or belt conveyers and bucket elevators; drawing coal under pressure from deep pockets. None of these losses can be entirely eliminated, but they can be minimized by careful management and supervision over mining and transportation methods, and by painstaking design and construction of the breaker.

Breaker products. Table 3 gives average percentage of each size in run-of-mine, for Classes 1 and 2, percentage of rock or impurities, and value of pure coal in the mine

Table 3. Sizes of Coal, Per Cent of Impurities, and Value of Products from 1 000 Tons Run-of-mine Coal

Size of coal	Price per ton	Class 1				Class 2			
		% of		Coal, tons	Value, $	% of		Coal, tons	Value, $
		Coal	Rock			Coal	Rock		
Lump / Steamboat	$4.50	39.0	4.0	390	1 755.00	9.5	3.8	95	427.50
Broken	5.85	7.0	1.5	70	409.50	6.5	3.8	65	380.25
Egg	6.00	6.0	1.0	60	360.00	5.0	2.6	50	300.00
Stove	6.00	5.0	1.0	50	300 00	7.5	3.8	75	450.00
Chestnut	6.00	10.0	1.5	100	600.00	9.7	4.7	97	582.00
Pea	4.45	5.5		55	244.75	5.0	2.3	50	222.50
Buckwheat	3.50	5.0	5.5	50	175.00	5.7	3.2	57	199.50
Rice	2.75	4.0		40	110.00	6.5	3.1	65	178.75
Barley	2.00	4.0		40	80.00	4.7	2.3	47	94.00
Dirt	....	....	....	....	....	....	10.3	....	....
Totals	....	85.5	14.5	855	4 034.25	60.1	39.9	601	2 834.50

cars, with prices at mines, as of 1938, assuming 1 000 tons of material delivered to breaker.

Table 4 gives percentage of shipments of each size made by preparation of the 1 000 tons of run-of-mine in Table 3, and the average and total amounts realized, assuming each size to contain the allowable percentage of impurities, as in Table 2. Each size in Table 4 contains no under or over size, and if the maximum amounts allowed (Table 2) were put into the highest-priced sizes, the average price per ton would be increased. In practice it is impossible to screen coal without under or over size. Hence, tests are made, and the screen mesh increased or diminished, so that the greatest allowable percentages of over or under size are carried in the highest-priced sizes; or the screen mesh is adjusted to produce maximum amount of sizes required to supply the market. Usually all the barley coal, and sometimes part of the rice, is required for the colliery boiler fuel, in which case the aver price per ton does not include these sizes. Fuel burned at the colliery approximates 5 to 10% of total production. In Table 4, Class 2 shows a loss of $1 530.48,

Table 4. Percentages of Sizes and Value of Breaker Products from 1 000 Tons Run-of-mine Coal

Size of coal	Price per ton $	Class 1				Class 2			
		Per cent of shipment	Pure coal, Table 3	Coal and impurities shipped	Value $	Per cent of shipment	Pure coal, Table 3	Coal and impurities shipped	Value $
Lump, Steamboat ...	4.50	10	85.5	85.5	384.75	5	30.05	30.05	135.23
Broken.........	5.85	12	102.6	107.0	625.95	7	42.07	43.8	256.23
Egg............	6.00	15	128.25	136.7	820.20	14	84.14	89.4	536.40
Stove..........	6.00	13	111.15	120.8	724.80	14	84.14	91.4	548.40
Chestnut.......	6.00	28	239.40	266.6	1 599.60	21	126.21	140.2	841.20
Pea............	4.45	6	51.30	60.4	268.78	10	60.10	70.7	314.62
Buck..........	3.50	6	51.30	57.1	199.85	18	108.18	120.2	420.70
				834.1	4 623.93			585.75	3052.78
Aver price per ton above rice................					5.54	....	......	......	5.21
Rice...........	2.75	5	42.75	47.5	130.63	6	36.06	40.0	190.00
Barley.........	2.00	3	25.65	30.4	60.80	3	18.03	21.2	42.40
Dirt...........	....	2	17.1	.....	....	2	12.02	.....	....
Total........	....	100	855	912.0	4 815.36	100	601	646.95	3 285.18
Aver price per ton all sizes....................					5.28	....	......	......	5.08

compared with Class 1, due chiefly to difference in amount of impurities in the run-of-mine (Table 3). There would also be a further financial loss due to increased preparation cost in handling and removing the higher percentage of impurities.

Breaking down large sizes. If the lump, steamboat, broken, and egg coal, from 1 000 tons of Class 1, run-of-mine, Table 4, were broken down to stove, nut, and the steam sizes, the approximate shipments in tons and amount realized would be as shown in Table 5. The loss, $50.84, is attributable to the breakage of coal into the smaller sizes.

Table 5. Results of Breaking Down Large to Smaller Sizes of Coal

Size of coal	Price per ton, $	Total tons shipped, Table 4, Class 1	Tons made by breaking down lump, steamboat, broken and egg	Total tons after breaking	Total value, $
Lump, Steamboat	4.50	85.5	....	....	....
Broken....................	5.85	107.0	....	....	....
Egg.......................	6.00	136.7	....	....	....
Stove.....................	6.00	120.8	171.2	292.0	1 752.00
Chestnut..................	6.00	266.6	95.4	362.0	2 172.00
Pea.......................	4.45	60.4	16.2	76.6	340.87
Buckwheat.................	3.50	57.1	18.5	75.6	264.60
Rice......................	2.75	47.5	9.9	57.4	157.85
Barley....................	2.00	30.4	8.2	38.6	77.20
Dirt......................	......	....	9.8	9.8	....
Total.....................	......	912	329.2	912.0	4 764.52

The 19.3% of pea and smaller sizes is due to use of inefficient rolls. With slow-speed rolls, having sharp teeth, this percentage should reduce to approx 12%. Breaking-down losses may be partly offset by the additional percentage of impurities which the broken-down coal is allowed to carry; in Table 5 the 171.2 tons of stove may be increased by approximately 13 tons, the 95.4 tons of nut by 9.5 tons, 16 tons of pea by 2.4 tons.

Economic principle in preparation of coal is to make large shipments in prepared sizes of highest priced coal, each size containing the maximum allowable percentage of impurities. Varying percentages of the different sizes in the run-of-mine, and the impossibility of adjusting crusher and re-breaking rolls to produce, at will, a fixed proportion of any one of the prepared or steam sizes, prevent accurate prearrangement of shipments. Hence, the problem of economic preparation must be worked out in detail for each particular breaker, by frequent tests of run-of-mine and prepared coal, and the machinery must be adjusted to give results as near the ideal as possible, and consistent with market demands.

2. GENERAL COMPARISON OF METHODS OF PREPARATION

Classification: *A*, dry preparation; *B*, dry and wet preparation; *C*, wet preparation. The method to be adopted depends on the quality of coal as mined. Fig 1, 2, 3, 3*a* and 3*b* (Art 4–6) show outline flow-sheets of these three methods.

Class A is employed when the coal seams are dry, or are practically free from impurities, or when the benches of slate in them cleave free from the coal and may be removed during hand loading, the run-of-mine containing generally not over 7 or 8% of rock or slate, which may be removed by hand-picking or by dry mechanical separators. Class *A* offers the ideal breaker. Its advantages are: low cost of construction, operation, and maintenance. Shipments of dry coal are preferred by the trade, being free from risk of freezing in cars and consequent difficulties in unloading. Due to exhaustion of thick, clean veins, Class *A* is nearly obsolete.

Class B is employed when run-of-mine contains a high percentage of impurities, including rock, slate, and bone. Though this percentage may be as high as 55%, the run-of-mine must contain large pieces of pure coal, to be handled as a separate product, as in Class *A*. The remainder is sized and cleaned by wet methods to improve its appearance and to remove impurities. Class *B* possesses to some extent the advantages of dry-coal shipments, but is higher in first cost, operation, and maintenance than Class *A* or *C*. This class is now practically obsolete; it is replaced by Class *C*.

Class C is adopted when run-of-mine is high in impurities and is discolored, as is the case near an outcrop; or when entire product comes from wet, dirty seams, requiring a thorough washing to remove dirt and discoloration; or when character of the seam is such that the run-of-mine contains no large lumps. Class *C* permits no dry shipments and is higher in first cost, operation, and maintenance than Class *A*.

General design. Referring to the flow-sheet diagrams. while designers may vary the combinations of machinery, using conveyers and elevators to transfer the product from one point to another, one set of rolls to break each size of coal to the next smaller, gravity picking chutes in place of movable picking tables, different kinds of mechanical cleaners to remove impurities, and stationary bars or revolving screens in place of shaking screens, yet the general plans of preparation are identical, and the diagrams are standard for their respective classes.

Ideal breaker (Fig 1) includes the following features: dump direct from mine cars into the breaker receiving hopper; automatic feeders from hopper to breaker, and feeders to all sizing and crushing machinery; sufficient height, so that all coal may gravitate from receiving hopper to loading gates; short chutes with automatic devices to control flow of coal; minimum breakage during preparation and in loading from storage pockets; mechanical cleaners for all sizes of coal, when practicable, and shipment of dry coal; adequate outside facilities to load box and gondola RR cars quickly and with minimum breakage; sufficient empty and loaded car standing track to hold a full day's output, tracks being graded to operate by gravity or mechanical haulage.

3. GENERAL METHOD OF PREPARATION

Outline is the same for the 3 classes of breaker. Coal is dumped into receiving hopper and fed automatically, or by a hand-operated gate, on to the dump screens at the top ("head") of breaker. Lump, steamboat, and broken are sized from the run-of-mine (always the first 2 sizes, and often the broken also) and are cleaned of impurities by hand-picking. This cleaned coal is the "pure-coal" product, and is handled separately, except in a Class *C* breaker. It requires no additional attention except to be broken down, when necessary, sized and delivered to loading pockets. The prepared and steam sizes from run-of-mine (including the broken coal, when not handled with the pure coal) are

called "mud-screen product." This gravitates from dump screens to sizing mud screens, from which each size flows to its respective mechanical cleaner and then runs to storage pockets. In Class *A*, the pure and mud-screen coal are mixed and go to same pocket. In Class *B*, these sizes go to separate pockets, when dry coal is desired or practicable; otherwise they are washed together and mixed. In Class *C*, the sizes are always mixed.

Breaker refuse, of slate, bone, or rock, as removed by hand or mechanical cleaners in all classes of breakers, is conveyed by gravity or otherwise to a central point for final disposition, for which there are two general methods: (*a*) Refuse is returned into the mines to fill openings left by extraction of coal. This, called SILTING, involves crushing (in a rock breaker) the large pieces of refuse, from broken coal and larger sizes, to about egg size. This material, with all other breaker refuse, is run through a pulverizer, reducing the product to 0.5 to 0.75 in, and hydraulicking it into the mine through pipes (Sec 10). (*b*) Refuse is deposited on the surface in BANKS, by chain or belt conveyer, or by dump cars, or by hydraulicking all or part of it in open wooden troughs.

Silting should be given preference, because it adds to stability of the remaining mine pillars and helps to support the roof. When surface disposition of refuse is necessary, the hydraulic method is best, when topography permits, and there is plenty of water. When hydraulicking is not practicable, the conveying system is used, if dumping ground adjacent to breaker is available. Banks can be carried to height of 100 ft or more, and extended in length by added horiz conveying lines or use of sheet-iron chutes around top of bank. To eliminate trouble from large pieces in conveying lines, coarse refuse is crushed to about egg size. When breaker is not close to refuse bank, dump cars are advisable, as coarse refuse can then be handled without breaking.

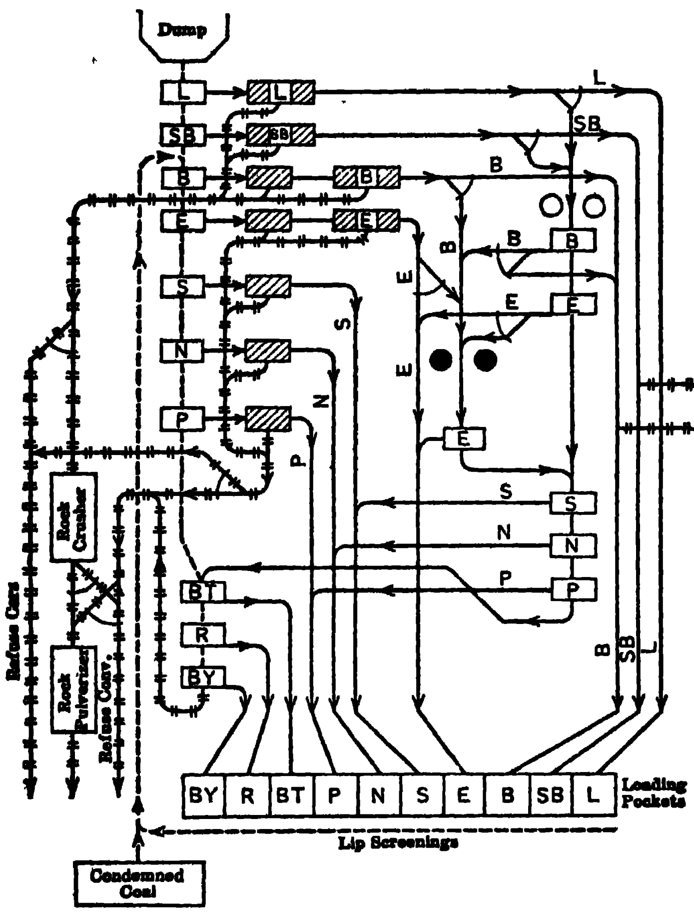
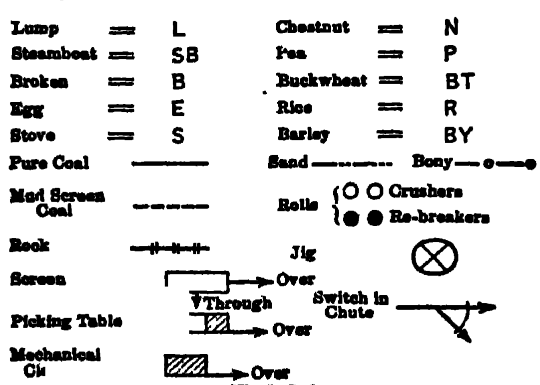


Fig 1. Flow-Sheet of Dry Preparation



Key to Fig 1, 2, 3

Fig 1. Flow-sheet of Dry Preparation

4. CLASS A BREAKER

Details of preparation (see Fig 1). Shipment of all sizes of coal is provided for, with arrangements to break down any size larger than stove into the next smaller size, or all sizes into stove, nut, and steam sizes, in order to supply varying market demands.

Run-of-mine is elevated to a receiving hopper at the head of the breaker, from which it is fed, mechanically or by hand, over a "bank" of sizing screens. Top screen removes

lump size, the remainder going through to the second screen, which removes steamboat, and the third removes broken. Material passing through the broken-coal screen is the MUD-SCREEN PRODUCT, and is sized into egg, stove, nut, and steam sizes. Sized lump, steamboat, and broken gravitate to their respective picking tables (either moving or stationary), are inspected, and the pure rock, slate, or other impurities are removed by hand to a refuse or rock chute. Pieces of rock to which coal adheres are placed on special tables, where the coal is chipped loose by hand and returned to its table. PURE-COAL PRODUCT, cleaned coal from picking tables, may go direct to its pocket; or, when there is no demand for lump, it is broken by "crusher" rolls to broken and smaller. When there is no sale for broken size, the rolls are adjusted to break it, as well as the lump, into egg and smaller. The pure-coal product from the rolls flows to a bank of 3 screens, sizing broken and egg coal. This broken coal may go direct to its pocket or to the RE-BREAKER ROLLS, which may be adjusted to make either egg or stove and smaller. The pure broken coal from crusher rolls may be mixed with the broken from picking table and gravitate with it to its pocket, or the product is divided, part going to the re-breaker rolls and the remainder to its pocket, or the entire product may go to the re-breaker rolls The pure egg coal may go direct to its pocket or to the re-breaker rolls, or may be divided in the same manner as the broken. Pure-coal product from re-breaker rolls flows to a broken-coal screen, the broken going to its pocket, while the undersize mixes with the product falling through the egg screen, set above the re-breaker rolls, and the combined material is delivered on a bank of screens arranged to size pure egg, stove, nut, and pea coal. MUD-SCREEN PRODUCTS, egg, stove, nut, and pea (the first 3 invariably and the pea usually), contain impurities in excess of that stated in Table 3, which must be partly removed, to keep within the allowable limit for marketable coal. Each size gravitates from its sizing screen to a mechanical cleaner or separator, which automatically removes impurities, the coal flowing to its pocket and the refuse to a point for disposal. Occasionally the mud-screen buckwheat requires cleaning, in which case it is handled in a similar manner; when such cleaning is unnecessary, the mud-screen buckwheat, rice, and barley are mixed with the corresponding pure-coal sizes and delivered to the buckwheat, rice, and barley screens, where each size is separated and delivered to its pocket, while the fine material through the barley screen goes with the refuse. If it is desired to make buckwheat No 4, an additional screen is installed. While loading into cars for shipment, the fine dust and screenings made during preparation are removed by passing the coal over punched steel plates or woven-wire LIP SCREENS. The lip-screen product is elevated to the sizing screens to be re-sized and returned to the pockets. After loading, the coal is inspected for size and purity, in conformity with Table 2. If it fails in this inspection, it is condemned, unloaded, and elevated into the breaker and re-treated.

Design in Fig 1 contemplates an ideal breaker. For 1 500 to 1 800 tons daily capacity, total height above loading track would not exceed 115 ft. Coal passes through breaker by gravity, eliminating conveyers and elevators. One conveyer only is required to handle lip screenings and condemned coal. Roll tests show highest percentage of prepared sizes when breaking only from one size to next smaller; that is, steamboat to broken, broken to egg, etc. The breaker would then require 4 sets of rolls, thus increasing the height, unless re-broken material is re-elevated, but the increase in prepared sizes would not warrant this installation. In practice, the steamboat and broken screens, immediately below crusher and re-breaker rolls, are omitted; because a sufficient percentage of steamboat is usually made from the picking table, in proportion to other sizes shipped and required for market, and the crusher rolls will break lump or steamboat direct into broken without oversize of steamboat, which would condemn the broken. Sufficient egg can be made above the re-breaker rolls, and any oversize will be carried by the stove. Mixing the pure and mud-screen prepared and smaller sizes is advantageous. When pure coal from the tables is broken down, each size usually carries less impurity than stated in Table 2; hence, mud-screen coal from the mechanical cleaners may contain excess of impurities without causing the resulting mixture of pure and mud-screen coal to exceed the allowable percentage.

5. CLASS B BREAKER

Details of preparation (Fig 2). This type provides for separate shipment of pure and mud-screen coal of all sizes, except buckwheat, rice, and barley. General method of preparation is practically the same as for Class *A*, except that it is not necessary to remove all impurities, but only that portion which would condemn the finished product. If run-of-mine is Class 1, one mechanical cleaner on each mud-screen size will usually suffice. If the product is Class 2 (high in impurities) a preliminary mechanical cleaner is installed for each size of mud screen coal, to remove part of the impurities. This partially cleaned

product gravitates to a second mechanical cleaner for final preparation, before going to pocket. Ordinary mechanical cleaners (except the jig) may be adjusted to remove about 50% of impurities from any one prepared size, without constant attention; and where used as a preliminary cleaner it is thus adjusted. The remainder is handled by a second cleaner under supervision of an attendant, who adjusts it to suit the varying impurities, so that the coal going to pocket will pass inspection. Broken coal is usually handled with the pure coal for Class 1 run-of-mine, and with the mud-screen coal for Class 2. If average run-of-mine contains lumps larger than 2 cu ft, this lump coal is usually broken down in a separate pair of rolls, another pair being used to break the steamboat. This prevents the

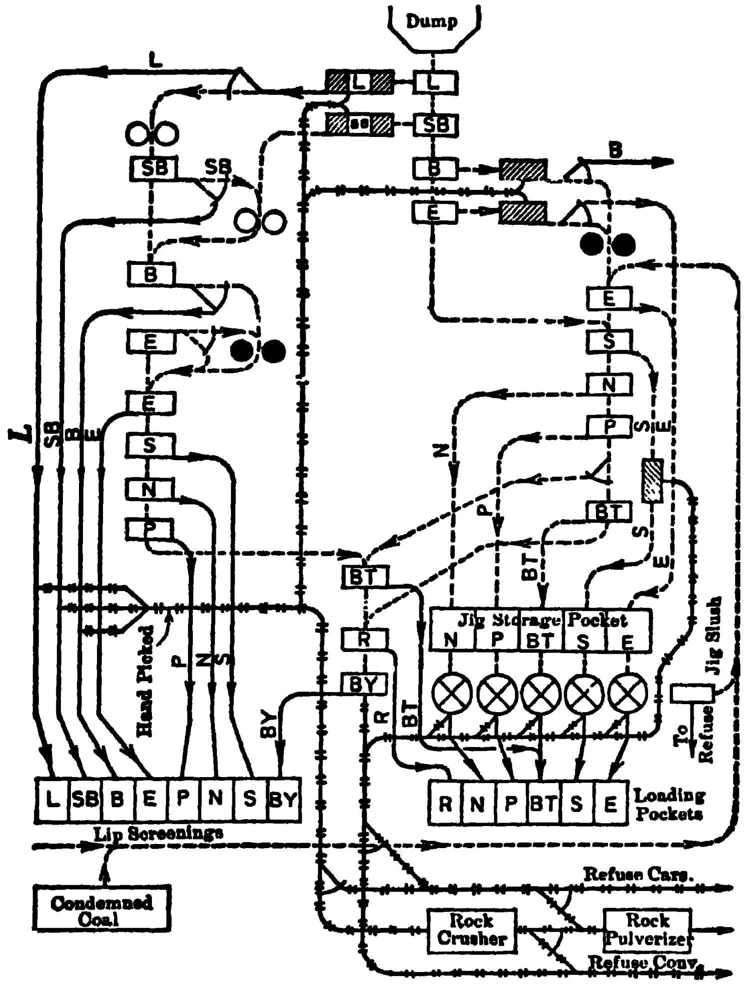


Fig 2. Flow-sheet of Dry and Wet Preparation

large degradation into steam sizes resulting from *crushing* (not breaking) large lumps direct into broken size.

When dry coal is not desired, nor practicable, the pure and mud-screen coals are mixed after cleaning. The mode of cleaning mud-screen coal is then similar to that of Class *A* breaker, respecting percentage of impurities in final mud-screen product, before mixing with pure coal and the number of mechanical cleaners for each size is entirely dependent on quality of run-of-mine.

6. CLASS C BREAKER

Details of preparation (Fig 3). This type provides for wet shipments of all sizes smaller than broken, since it is rarely practicable to produce the 3 larger sizes with arrangements to break down egg into stove and smaller. Run-of-mine is elevated to a hopper at head of breaker, from which it is fed mechanically or by hand into a bank of 4 screens, which size lump, steamboat, broken, and egg. Lump and steamboat are "skinned" on picking tables; that is, only a part of the rock is removed. After being broken down these sizes are mixed with the mud-screen coal and prepared with it. Pure lump and

steamboat are crushed to broken in rolls and flow to 2 screens, which size broken and egg. Broken coal gravitates to re-breaker rolls, and is reduced to egg or stove and smaller sizes, going thence to a central storage pocket. Egg coal gravitates direct to this pocket, or is mixed with the broken going to re-breaker rolls. Mud-screen broken and egg are usually run over preliminary mechanical cleaners, from which they are handled like the pure broken and egg, and mixed in the central storage pocket. Unsized mixture of pure and mud-screen coal is fed mechanically to a bank of screens, making all sizes, each gravitating to its respective JIG STORAGE pocket. This pocket is a reservoir for the jigs, accumulating coal during rush periods; it is necessary because jigs are operated at a fixed capacity. The jig removes enough impurity to make marketable coal, and from it each size flows

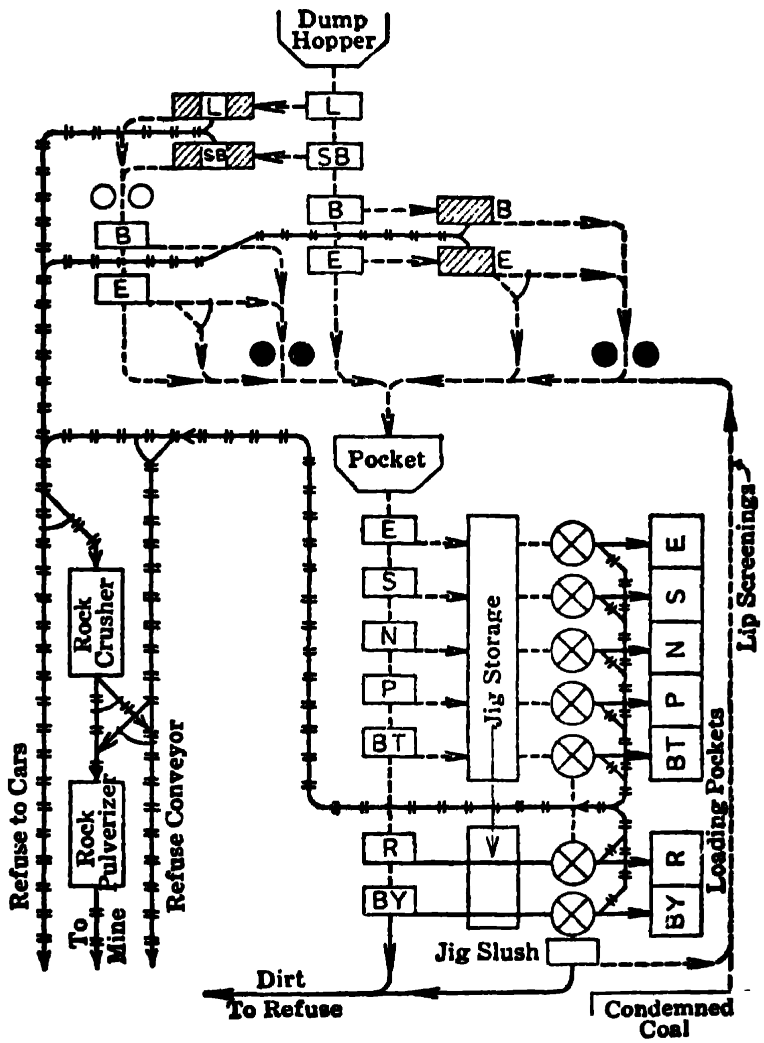


Fig 3. *Flow-sheet of Wet Preparation with Jigs*

to loading pockets. When the jig is emptied, the hutch product, containing nut coal and smaller, is washed over a JIG-SLUSH SCREEN, of a size to remove only the coal smaller than barley, oversize being re-elevated to the central storage pocket. Sized coal is loaded and inspected as in Class *A*; lip screenings and condemned coal are re-elevated with the jig-slush-screen product.

Percentage of bone coal in run-of-mine is usually small, and sized products will carry all the bone in a seam, if the rock is eliminated (Table 2). This treatment of bone is recommended above all others. When the bone percentage exceeds total allowable bone and slate, bone is handled separately, and the preparation here is identical with Class *A* or *B* breakers. A three-part separation is made of run-of-mine on picking tables: coal, bone and impurities. Re-breaker rolls reduce this material into egg or stove and smaller, the broken-down product being mixed with pure coal sizes in the proportions which each will carry. Prepared-coal mechanical cleaners also make a three-part separation; bone from egg and stove is broken into nut and smaller, or egg, stove, and nut into

pea and smaller; then screened and mixed with sized coal going to pockets. Modern methods usually disregard bone coal, and make no special provision to handle it, except to consider very poor or heavy bone as an impurity and hand pick it into refuse from the large sizes.

Flat coal, characteristic of certain seams, as compared with the more usual cubical pieces, when occurring in sufficient quantity in mud-screen prepared sizes, is removed by a flat-coal picker before going to mechanical cleaner, thus avoiding difficulty in removing impurities from a mixture of cubical and flat pieces. Flat coal flows to its own cleaner, and the clean product is mixed with cubical coal before going to pockets. The quantity of flat coal is often sufficient to spoil the appearance of prepared sizes, when loaded. If so, part of the flat coal is removed and broken smaller.

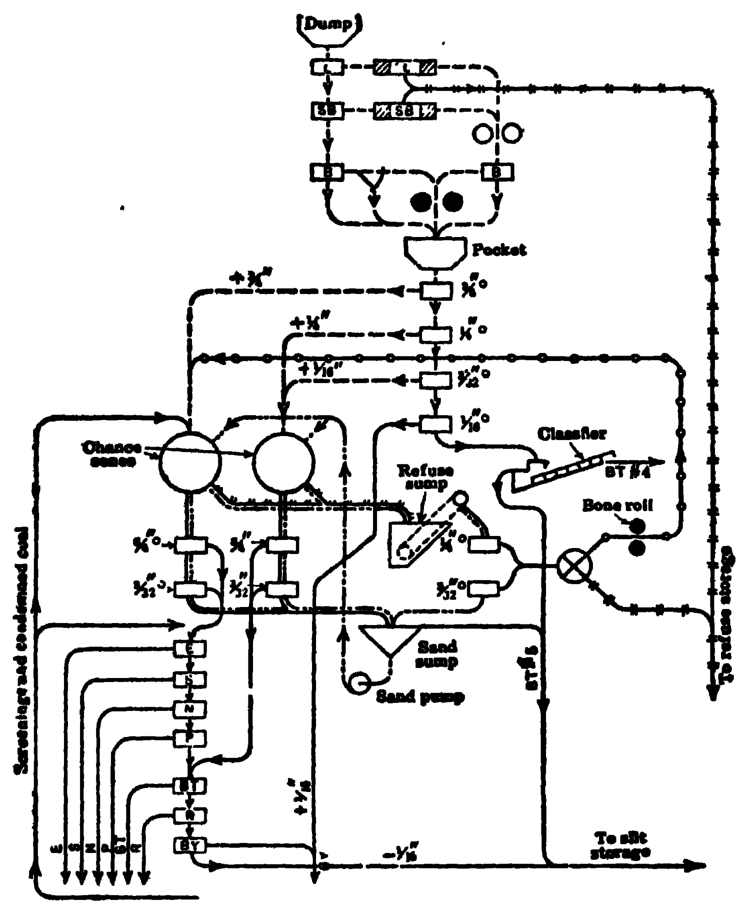


Fig 3a. Flow-sheet of Chance Preparation

Chance system (Fig 3a), for wet shipments of all sizes. Preliminary preparation above the storage pocket is the same as for Class *C* breaker. The unsized mixture is fed on a bank of 4 shakers, with $^3/_4$, $^1/_4$, $^3/_{32}$ and $^1/_{16}$-in round mesh; the particular object being to remove the minus $^3/_{32}$ in and silt, which when mixed with the sand and water in the cone would lower the gravity and cause loss of coal. Experience shows that 4 shakers are necessary for sufficient splitting of the product to permit thorough washing for removing the minus $^3/_{32}$ in.

The $^1/_{16}$-in shaker recovers the barley sized out of the mixture, and returns it to the barley passing through the cone. Two cones are required, 1 for the plus $^5/_{16}$ in and 1 for the minus $^5/_{16}$ in to plus $^3/_{32}$ in. A sp gr of about 1.75 is maintained in each; refuse of more than 1.75 sp gr sinks, while coal floats out through an overflow to a bank of 2 de-sanding shakers, of $^3/_4$ and $^3/_{32}$-in mesh respectively. The product is thoroughly washed, all flowing to the main sizing shakers, where all sizes, egg to buckwheat inclusive, are made and stored in pockets or loaded direct into cars. The minus $^3/_{32}$-in sand and silt through the de-sanding shakers flows to a sand sump, from which it is pumped back to the cones. The refuse that sinks in the cones is trapped out under hydraulic seal into a refuse sump, whence it is removed by a conveyer and delivered onto 2 de-sanding shakers with $^1/_4$ and $^3/_{32}$-in round mesh. Sand is washed out and flows to the sand sump. The plus $^1/_4$-in and larger refuse contains some coal of a sp gr greater than 1.75; also bone and half-and-half pieces. This should be treated mechanically, jigs being used at present. The minus $^3/_{32}$-in which has not passed through the cones may be partly cleaned by classifiers, as the very fines are usually high in ash and if removed produce a fairly low-ash silt (15 – 18%). Depending on character of the run-of-mine, fine-coal jigs, tables, or (preferably) Rheolaveur fine-coal plants (see below) may be used.

The sand sump is arranged as a settling tank or an upward current classifier; any fine silt in it will be carried to the overflow and run to a settling tank or basin, so that clear water may be drained off and reused.

The modern Chance breaker uses a circular cone for plus $^5/_{16}$-in material and a rectangular cone for minus $^5/_{16}$-in to $^3/_{32}$-in material. All sizes plus $^5/_{16}$-in from the first

cone are de-sanded over $^5/_{16}$-in mesh, undersize flowing to the second or minus $^5/_{16}$-in cone. A sp gr of about 1.75 is maintained in each cone, with a mixture of water and sand. The coal overflows from each cone and the minus 1.75 material is trapped out at bottom of each cone, the material de-sanded and refuse going to the refuse disposal plant, and the mixture of sand and water going to the sand sump, to be returned to No 1 cone. Sizes smaller than $^3/_{32}$-in are treated separately, either on tables or in a hydrotator. When this is done, the material is sized out before entering the cone. Some breakers of 8 000 ton per day output use a Chance system having individual cones for each size. This has the advantage that each cleaned size can be broken down to next smaller size, when required, and re-treated in the following cone, if necessary.

Rheolaveur process (Fig 3*b*), is for wet shipments of all sizes. Preliminary preparation above the storage pocket is the same as for the Chance process, Class *C*. The unsized mixture is fed into the upper end of a large-coal Rheolaveur sealed level washing trough, in which is a stream of running water, the material bedding itself according to its sp gr. Usually 2 mechanical discharge Rheo boxes are attached to this trough, the first or upper box traps refuse, which is elevated to a sealed level re-washing trough. The second Rheo box traps a middle or regulating product which is returned and mixed

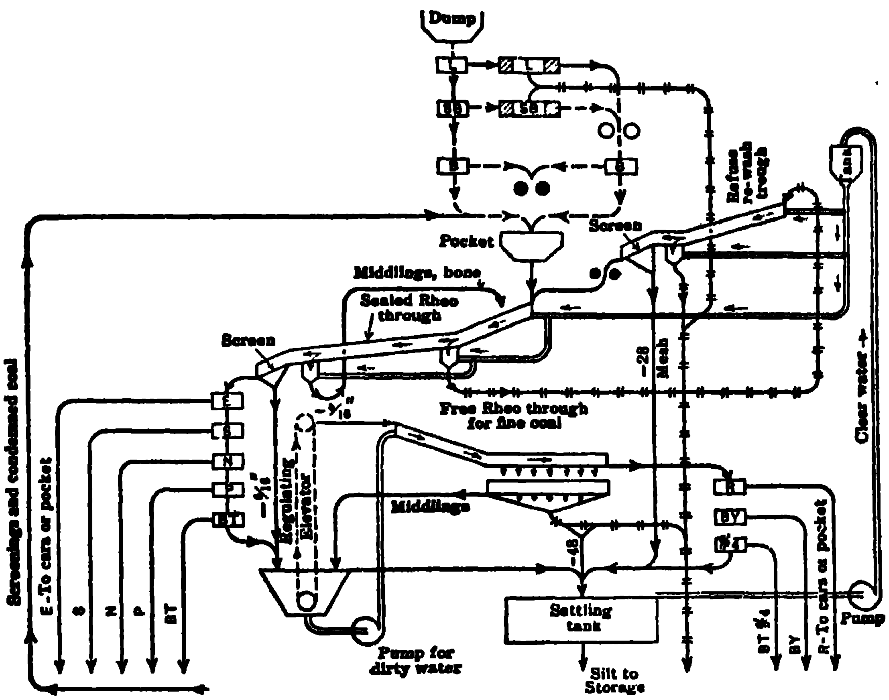


Fig 3*b*. Flow-sheet of Rheolaveur Preparation

with the feed. By varying the regulating product, the coal discharge at end of trough may be controlled.

The re-washing trough is fitted with 1 discharge Rheo box, which may be regulated to trap only pure slate, the mixture gravitating direct to refuse storage. The product discharged from the re-washing trough may be broken down and mixed with the run-of-mine feed to be retreated in the large-coal trough; or it may go to the main sizing shakers, to mix with the coal discharge from the large-coal trough, to be sized with it and be run to the pocket or cars. The large-coal trough cleans all sizes from plus $^5/_{16}$ to 3.5-in, the minus $^5/_{16}$-in being then treated in a "fine-coal free discharge plant," which cleans the fine sizes down to 50 mesh. All minus $^5/_{16}$-in material is collected in a regulating tank of about 50 tons capac, and elevated to the fine-coal plant, consisting of 1 or more troughs placed one over the other. A series of Rheo discharge boxes are placed in the bottom of each trough. A 3-part separation is made of clean coal, middlings or regulating product, and refuse. The coal is sized and delivered to pocket or cars; refuse gravitates to storage; regulating product returns to regulating tank, mixes with the feed, is elevated and re-treated. Controlling the regulating product by the valves on the Rheo boxes, quality of the cleaned coal may be varied at will. Sizes smaller than commercial fines are de-watered and stocked for shipment. (For further details, see Art 11.)

Menzies Cone Separator, Class C (Fig 3c). The principle of operation is a variable veloc of an upward current of water through a stratified mobile mass of coal and slate, within a conical separator.

Run-of-mine material, egg and smaller, enters top of cone and passes downward. In descending toward bottom of cone, the coal is washed upward over the top of cone into

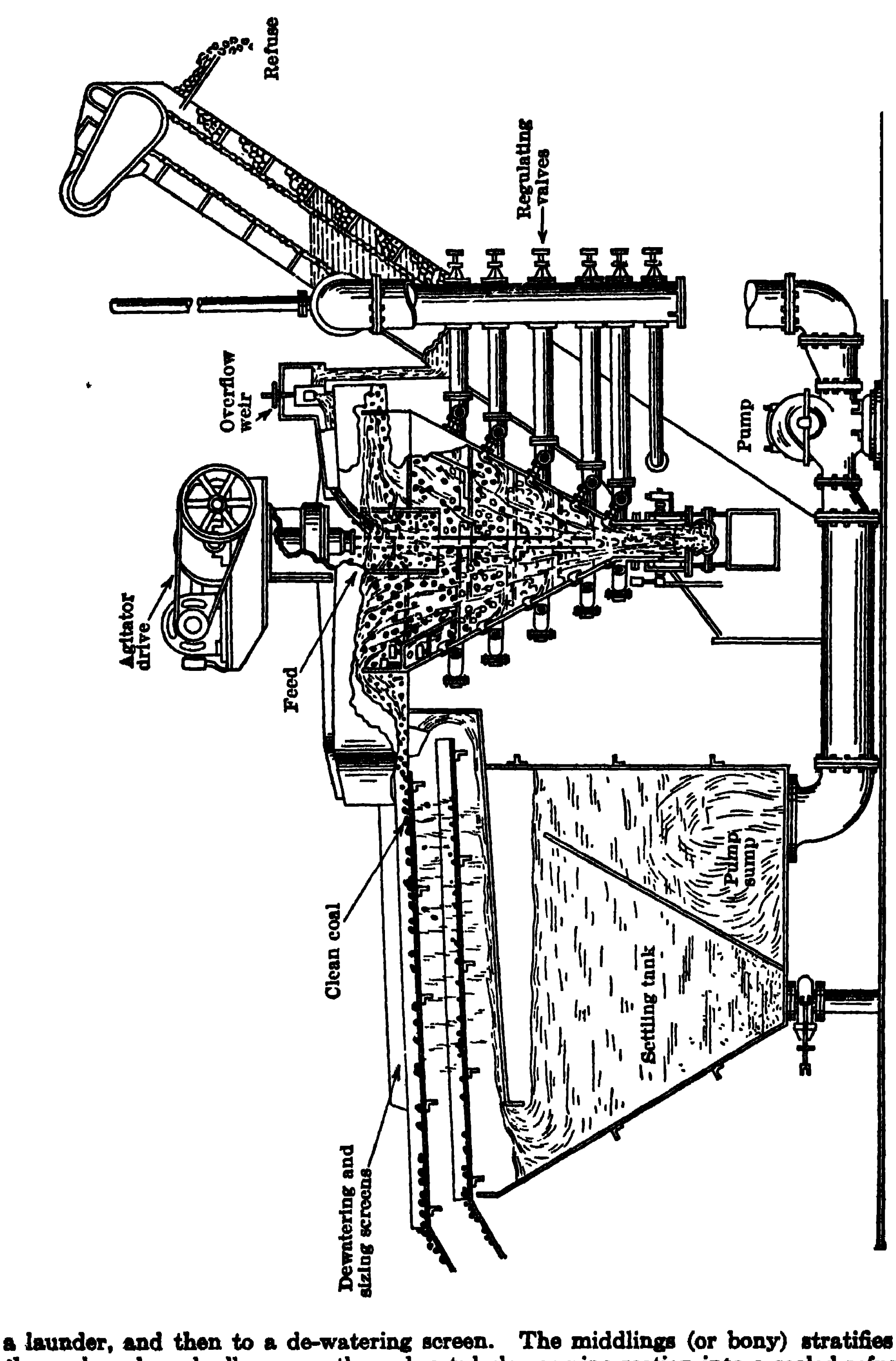


Fig 3c. Menzies Cone Separator

a launder, and then to a de-watering screen. The middlings (or bony) stratifies below the coal, and gradually passes through a tubular or pipe section into a sealed refuse conveyer, where it is discharged above the water level of the cone into refuse chute. The coal is held in suspension, or rather washed over top of cone, by a hydraulic current from a centrifugal pump, delivered to sides of the cone at fixed spaces. Each current of water

is controlled by a separate valve, to give the proper upward press. Coal leaving the cone goes over a de-watering shaker, which also acts as a sizing shaker, and then to the pocket, the water being returned by a sump to the re-circulating pump. Preliminary preparation above storage pocket is the same as for Class *C*. Two systems are in vogue in connection with the Menzies cones: (*a*) separate cones for each size to be cleaned; (*b*) mixed run-of-mine feed from egg to pea size, inclusive, in one cone, and buckwheat to barley size in the other.

Wilmot Hydrotator, Class C (Fig 3*d*). In this process, only material from chestnut and smaller are cleaned, each size going to its respective hydrotator. Preliminary preparation above the storage pocket is same as for Class *C*. Coal and refuse are fed and go into suspension in a cylindrical tank having a cone-shaped bottom. The coal rises to top and overflows onto a de-watering screen and then goes to the pocket, while the slate drops to bottom and into a sealed conveyer, being discharged above water level of the tank into the refuse chute. Water from the de-watering screen goes to the sump, and is re-circulated by a centrif pump providing the upward current. This water contains considerable undersize, consisting of coal and slate in sizes small enough to pass through the de-watering screen, this undersized material having a high spec gravity. The density thus produced causes the gravity separation. Variations in density produced by various quantities and qualities of feed, cause a fluctuating water level in the refuse conveyer, which in turn is controlled by an automatic valve admitting or shutting down the fresh water supply fed into the refuse conveyer. The re-circulating water is discharged into the hydrotator through a revolving agitator.

SIZE	A	B	C	D	E	F	G	H	I	J	J
5'0"	5'0"	2'9"	3'10"	4'9¼"	13'2½"	11'2½"	13'2"	25'10"	5'4¼"	7'4"	6"
6'0"	6'0"	2'9"	3'10"	5'0 11/16"	13'5"	11'4¼"	13'6"	26'7¾"	6'1¾"	8'4"	6"
7'0"	7'0"	2'10½"	3'10"	7'1½"	15'1"	13'0"	13'6"	27'9¼"	7'6"	9'4"	6"

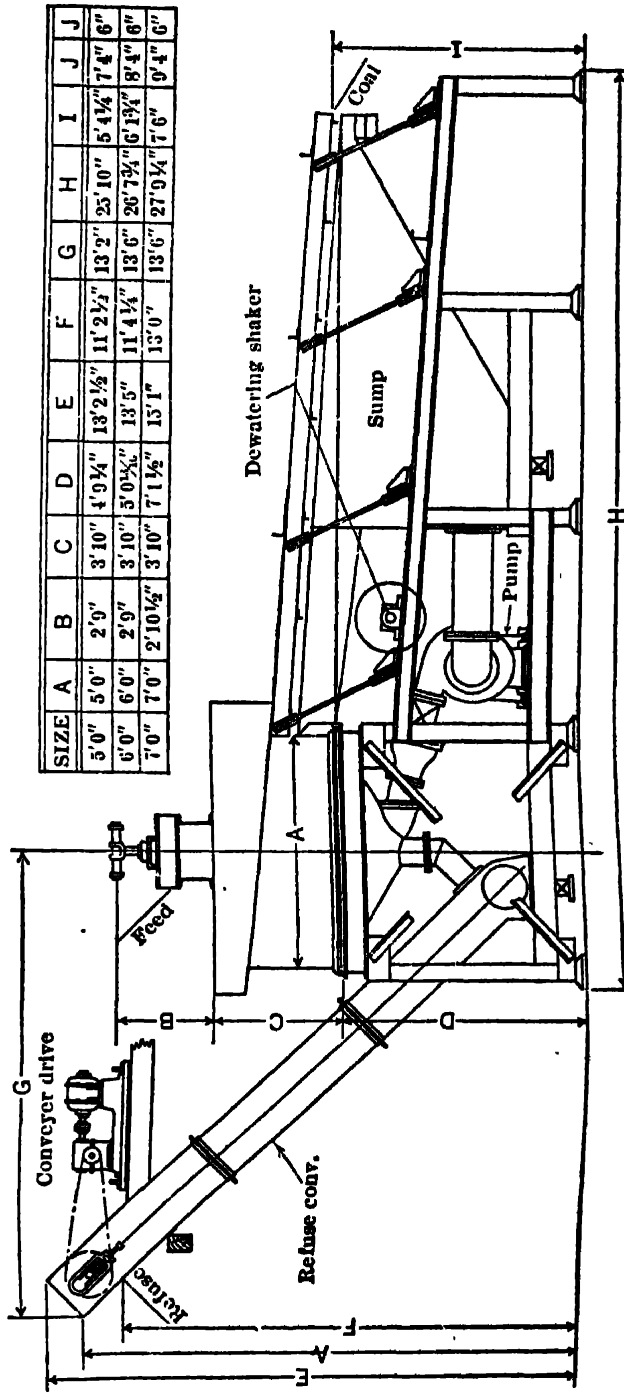


Fig 3*d*. Wilmot Hydrotator

E. I. DuPont de Nemours & Co have developed a straight gravity separation, using a "parting liquid" with gravities from 1.3 to 3.0. The process includes treatment of run-of-mine feed with "active agents," which immunise the solids against the parting liquid. The feed is treated with the active agent and enters a separator containing the parting liquid of pre-determined gravity. The "floats" are conveyed from the separator by one conveyer, while the "sinks" are re-claimed by a second conveyer. The two products are thoroughly washed to recover the parting liquid, which is collected and classified in a Dorr thickener, the excess water going to overflow, the "parting liquid" being re-claimed from the underflow and returned to the plant. This system will probably have a useful field for anthracite, but the first plant is yet to be built.

7. BREAKER STRUCTURES

Types: (*a*) all timber; (*b*) all structural steel; (*c*) all reinforced concrete; (*d*) steel base, timber superstructure; (*e*) reinforced-concrete base, timber superstructure; (*f*) reinforced-concrete base, steel superstructure. To determine best type, divide estimated cost by total estimated tonnage, and to quotient add estimated preparation cost plus yearly maintenance charges per ton of coal.

Depreciation. TIMBER STRUCTURES. For dry breakers, depreciation depends upon kind and quality of timber used, usually varying from 4% per year for yellow pine to 8% for hemlock For wet breakers, depreciation varies from 10 to 20% for yellow pine, when used for main or heavy timbers subjected to alternate wetting and drying. Hemlock should not be used in parts of structure which come in contact with water; if so used, depreciation is about 25%. White pine will outlast either yellow pine or hemlock, but is no longer available at reasonable cost. For STRUCTURAL STEEL OF REINFORCED CONCRETE, the depreciation for dry breakers is unknown, but is estimated at 1.25%. For structural-steel wet breaker, using acid mine water for washing, depreciation is estimated at $2\,^1/_2$%. Modern construction, with reasonably water-tight pockets, chutes, and $1\,^3/_4$-in tanks, should lengthen the life of such structures, depreciation being estimated at $1\,^1/_2$ to 2%. Reinforced concrete is not recommended in a wet breaker using acid mine water, because there is no certain mode of protecting concrete from its attack. In timber breakers, using wet methods, it is important that all chutes, pockets, and tanks be water-tight; such construction is more costly, but will reduce maintenance charges and lengthen the life considerably.

8. LOADING COAL FROM BREAKERS

Ideal loading pockets include following features: Chutes to lower coal into pockets with minimum breakage; sufficient storage in each pocket to hold not less than one car of coal; gates operated from loading platform; lip screens to remove undersize from lump, steamboat, and prepared sizes; loading chutes adjustable to varying heights of gondola cars, and arranged to load with minimum breakage on the center-line of the car and in the direction in which the car gravitates; special provision to load box cars, and when loading wet coal, provision to wash the sizes passing over lip screen.

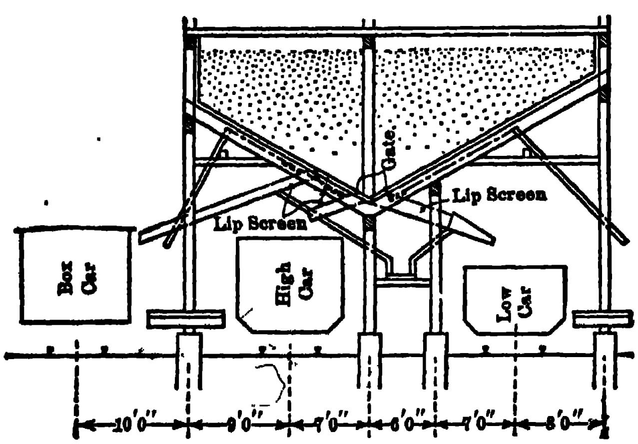


Fig 4. Direct Loading Pockets

Loading direct from storage pocket. Pockets are parallel with loading track (Fig 4). Good practice is to provide 2 tracks, for high and low cars, and a track for box cars. Such arrangement of tracks gives large loading capacity, simplifies mechanical arrangements required to handle all types of cars on one track, and permits simultaneous loading from adjacent pockets, even though less than a car-length apart.

Loading concentrated at one point. Pockets are usually placed at 90° to track (Fig 5), and coal is delivered by curved chutes to a belt conveyer parallel to pockets. Coal is discharged at tail of conveyer, over a lip screen adjustable for each size of coal. The tail of conveyer, together with lip screen, should be raised or lowered as a unit, by power, to suit different heights of car. Controls for gates, and operation and adjustment of belt, should be placed at loading point. Pockets may stand on the ground, and the conveyer be inclined to reach required height to load. Height of breaker is not much reduced by this construction, as increased pocket capac is required for storage because sizes must be loaded in sequence. Labor force is usually half that for other methods of loading, including maintenance. Rubber-covered conveying belts usually have the longest life, especially when handling wet coal; they should handle at least 2 million tons per 100 ft of conveyer. Speed of belt should vary from 250 ft per min for broken to 600 ft per min for wet rice and barley.

Loading direct from breaker chutes, with only enough chute capacity to provide storage while changing cars. General construction would be similar to that at bituminous tipples (Sec 35). Apparent advantages: no pocket breakage, no lip screens, reduced height and first cost of breaker. Disadvantages: at least 7 loading trucks are required, difficult car distribution, large loading force, and complicated arrangement to load box and gondola cars from same chute.

A recent loading plan is a combination of direct loading by chutes for egg, stove and chestnut sizes, and from storage pockets for all other sizes. This requires 4 tracks, 1

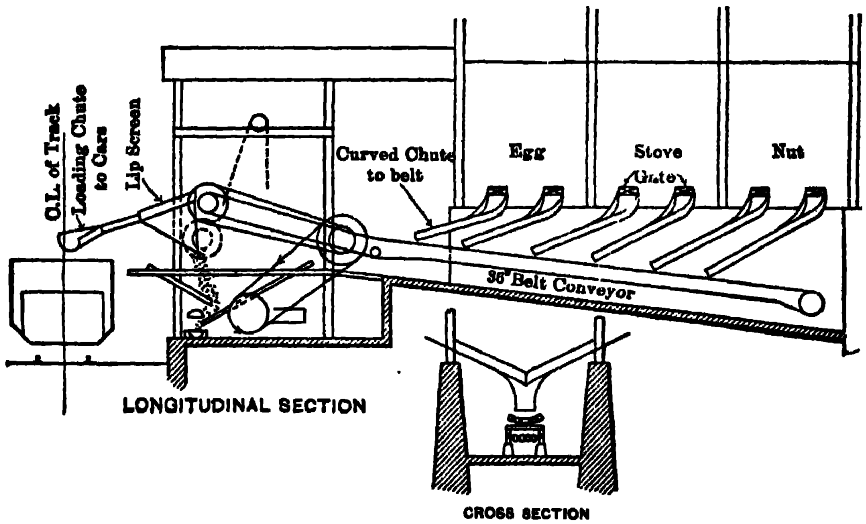


Fig 5. Concentrated Loading Pockets

each for egg, stove and chestnut, and 1 for smaller sizes. It is ideal in combination with a preparation plan in which final screening and sizing follows the cleaning machinery, so that clean and sized coal is loaded direct into cars, eliminating use of lip screens for prepared coal.

9. SIZING SCREENS (see also Sec 35)

Classification. Fixed or stationary: (*a*) adjustable bars; (*b*) finger or fixed bars; (*c*) punched plate or woven-wire segments. Movable: (*d*) cylinder or revolving screen; (*e*) "shaker" screen; (*f*) gyrating screen; (*g*) oscillating movable-bar screen; (*h*) revolving roller screen; (*i*) vibrating screen.

Stationary screens are usually built into a chute, at a pitch down which the coal will slide, while the undersize falls through the openings. They are not used when uniform sizing is required, but for preliminary separation of larger sizes from the smaller, before hand picking, and to remove dust and fine chippings made during preparation. Results are usually very unsatisfactory and they are used only for emergencies. Their only advantage is low first cost, which is more than offset by very low efficiency.

Adjustable bars are used only to size lumps out of run-of-mine, and are placed below the receiving hopper. Ratio of length of bars to width of opening varies from 8 or 10 : 1. The adjustment permits openings between bars to be increased or decreased in fixed increments, to vary percentage of material passing over them. Bar screens have low effic, considerable fines passing over with the larger pieces, so that the oversize is difficult to inspect and hand clean. Bars are T-shaped CI or round steel, 4 to 6 ft long. Their ends dovetail into fixed rests, which hold the bars parallel. Sides taper about 0.25 in in the depth, giving an inverted V-shaped opening between bars, which allows the coal to free itself. Lower end projects above the rests and passes any piece of coal which hangs between the bars. Opening between the bars is usually 5 to 6 in.

Finger or fixed bars are occasionally used to remove a portion of the undersize or dust from prepared sizes. They are made by placing in a chute equally-spaced round W I or C I bars, or angle iron with legs looking down at an angle of 45°. Opening between bars has a ratio of length to breadth ranging from 25 to 50 : 1. Specially constructed bars are used to remove pieces of flat slate or bone.

Punched-plate or woven-wire screens remove dust and chippings from prepared sizes, especially for lip screens at loading pockets; woven wire for dry, punched plate for

wet coal. When loading wet coal, a water jet is sometimes directed on the screen to remove fine material and wash the coal.

ADVANTAGES OF FIXED SCREENS: low first cost, no power required to operate, large capacity when used as lip screens. DISADVANTAGES: very poor sizing; being set on a pitch, they increase breaker height and cost; require constant attention, due to blinding; adjustable bars involve a drop at lower end, and increase breakage.

Cylinder or revolving screens, usually 6 to 8 ft diam by 12 to 24 ft long, were formerly used for all sizes of coal; they are now giving way to shaking screens. They may be single or double jacketed and the segments arranged to produce several sizes from one screen (Fig 6). Max peripheral speed should not exceed 200 ft per min. ADVANTAGES OF REVOLVING SCREENS: exact screening and sizing of coal; revolving at slow speed, they do not vibrate breaker structure; blinding of meshes is rare. DISADVANTAGES: high first cost and maintenance, as compared with shakers under similar conditions; small capacity, since coal is in contact with only about one-eighth of the screening surface at one time; require more space than shaking screens; greater breakage in the screen and in the enclosing hopper, due to high drop of coal; more difficult to inspect and repair than shakers; excessive weight as compared with shakers.

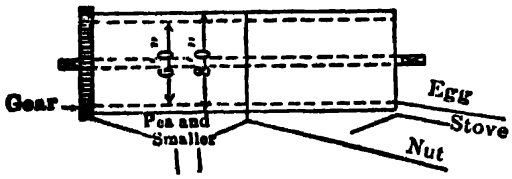


Fig 6. Revolving Screen, 6- to 8-ft diam

Shaker screen (see also Sec 35) is usually 4 to 8 ft wide by 9 to 21 ft long. Sides are of steel plate, angle iron or wood, connected by cross angles, to which the punched-plate segments are bolted. It is suspended by chains, rigid hangers with pin-connected end, or 1-in oak or hickory boards with fixed ends (Fig 6*a*).

Screen is driven by a pair of eccentrics connected to it by 3 by 6-in wooden eccentric arms with fixed ends, and reduced in section to 2 by 3 in at point of maximum bending,

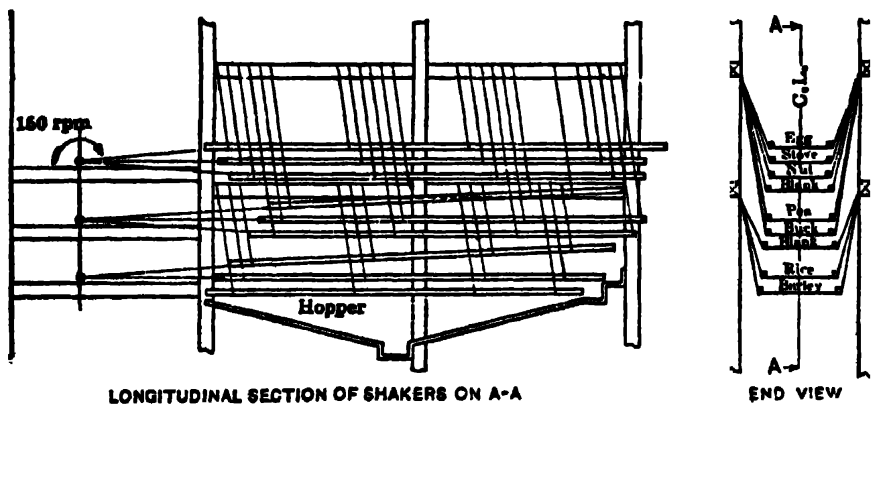


Fig 6*a*. Shaker Screen

or by forged crank arms, with metal connecting rods to a wrist pin on the screen. Usual throw is 3 in, speed 150 rev per min. Ratio of length of hanger to travel is, for rigid hangers, 3 to 6 : 1; for wood, 10 to 15 : 1. Hanger should be inclined toward back end of the screen, so that the vertical resultant acting on the coal at end of back stroke will lift the coal out of the mesh and prevent blinding; pitch of hanger varies from vertical to 3 in per ft, according to size of coal and length of hanger. Shaker screens may be built in 1, 2 or 3 decks, which should be hung in pairs, one over another, with their eccentrics on the same shaft, 180° apart, to balance vibrations. Absolute balance is impossible, due to: angularity of eccentric rods, and the fact that the screens work in different planes, with varying quantities of material on them. Resulting unbalanced load vibrates the breaker structure, which should be braced accordingly. (See Sec 35.) ADVANTAGES: exact sizing, low first cost, accessibility for repairs and inspection, simplicity of construction, reliability in operation, saving in breaker height, large capacity. DISADVANTAGE: vibration imparted to breaker structure.

Vibrating screens of various types have been experimented with on nearly all sizes of coal, but the results have not yet warranted their general adoption in place of shaker screens. Recent developments have perfected a vibrating screen particularly useful on the fine sizes, minus 5/16. The use of round mesh is specified for all sizes of anthracite, but the substitution of square wire mesh on vibrating screens in place of round punched plate requires more experimenting before its use becomes general.

Oscillating or movable-bar screens are seldom installed in new breakers; they are being superseded by shakers. Their field is limited to dump screens, for sizing lump from run-of-mine. ADVANTAGES: heavy construction, enabling them to handle large pieces of material; their action as feeders to the machinery following; saving in breaker height; slow speed; no vibration to structure. DISADVANTAGES: poor sizing; fixed space between bars, with no adjustment; high first cost; when not hand fed from a hopper, the mixed run-of-mine often rushes over onto picking table.

Table 6. Comparison of Shaker and Revolving Screens

(Sq ft of screen area required to screen one ton of coal in 10 hours. For sizes of openings, see Table 1.)

Size of coal	Shaking screen		Revolving screen	Size of coal	Shaking screen		Revolving screen
	Dry	Wet			Dry	Wet	
Steamboat	.....	1.50	......	Pea	0.20	0.69	1.75
Broken	0.60	1.20	0 75	Buckwheat	0.61	0.53	2.00
Egg	1.20	1.10	1.00	Rice	0.50	0.65	2.25
Stove	0.25	0.35	1.00	Barley	0.67	0.67	3.00
Chestnut	0 20	0.27	1.5				

Parallel revolving-roller screen consists of a number of parallel rollers equally spaced. Diam of rollers, 2 to 4 in and openings between them vary with size of coal handled, opening being varied by adjusting the roller centers. Screen is self-contained, and is usually installed in a chute at a pitch down which the coal will gravitate. Rollers revolve in same direction, and at speed of about 350 rev per min. This is not a sizing screen; it is used chiefly to remove flat coal or slate, and is often called a "slate-picker." It is occasionally used as a lip screen.

10. BREAKER ROLLS

Classification of rolls for breaking coal from a larger to a smaller size: CRUSHER or No 1 rolls (for breaking lump to steamboat or broken); MERCHANTS, or No 2 (steamboat to broken or egg); RE-BREAKERS, or No 3 (broken to egg or stove); BONEY or No 6 (egg bone to stove, stove bone to nut or pea, nut bone to pea). Rolls are also classified by their peripheral speed: high-speed (900 ft per min) and slow-speed (250 ft per min).

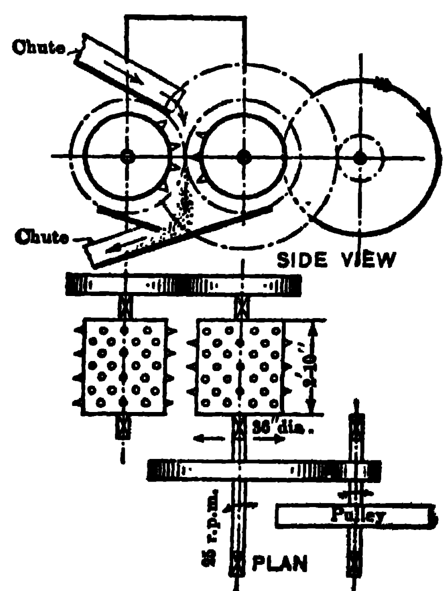


Fig 7. Toothed Slow-speed Rolls

Operation. Slow-speed rolls, with pointed teeth (Fig 7), give a higher percentage of prepared sizes than the high-speed, under same conditions. Coal should be broken by points of the teeth, and *not crushed* between roll bodies. Hence, jaw and gyratory crushers should not be used, as they crush rather than break the coal, thereby increasing degradation into steam sizes. According to design, rolls are divided into DRIVEN-TOOTH, and SEGMENT-TOOTH rolls. In the first, hardened-steel teeth are driven into drilled holes in the cast-iron roll body or drum; in the second, cast-iron or steel toothed segments are bolted to a drum or cast spiders. Pedestals supporting the driving rolls are fixed; the driven-roll pedestals are adjustable for varying the opening, which is limited by the length of gear teeth; for greater adjustment the gears are changed. Pedestals have cushion springs, which compress when hard foreign material falls into the rolls.

Table 7. Size of Rolls, Size and Spacing of Teeth, Generally Used for Breaking Coal

Roll name and number		Size of drum, in	Teeth			Size of teeth, in
			Number per row	Distance c to c	Number of rows to circumference	
Crusher	No 1	51 × 41	8	4.50	36	3 1/2× 1 3/4 sq
"	1	33 1/2× 46	8	5.50	19	"
"	1	35 × 34	6	5.25	33	4, 3 & 1.5 high
Merchant	2	30 × 36	10	3.75	26	2 3/4× 1 1/2 sq
"	2	35 × 34	8	3.50	33	2 1/2× 2 "
Re-breakers	3	30 × 36	14	2.42	39	2 × 1 1/4 "
"	3	35 × 34	13	2 3/16	55	1 3/8× 1 3/8 "
Bone	6	24 × 32	21	1.35	72	1 × 1 "
"	6	24 × 32	21	1.35	70	1 × 1 1/8 "

Table 8. Percentages of Prepared Sizes Usually Made by Breaking Down a Larger Size (See Fig 8)

Roll number	Size of drum, in	Peripheral speed, ft per min	Size of coal broken	Total percentage made of					% of prepared sizes
				Steamboat	Broken	Egg	Stove	Chestnut	
1	35×34	250	Lump	27	23.0	22.0	10.0	9.0	91.0
1	36×36	942	"	29	24.0	13.0	8.9	7.7	82.6
2	35×34	230	Steamboat	......	38.8	36.3	12.7	5.6	93.4
2	30×36	900	"	......	29.0	30.0	15.0	11.0	85.0
3	35×34	250	Broken	......	7.0	25.0	40.0	17.0	89.0
3	30×36	900	"	......	8.0	23.0	20.0	13.5	74.5
3	35×34	250	Egg	......	......	45.0	23.0	17.0	85.0
3	30×36	900	"	......	......	17.5	41.0	21.5	80.0

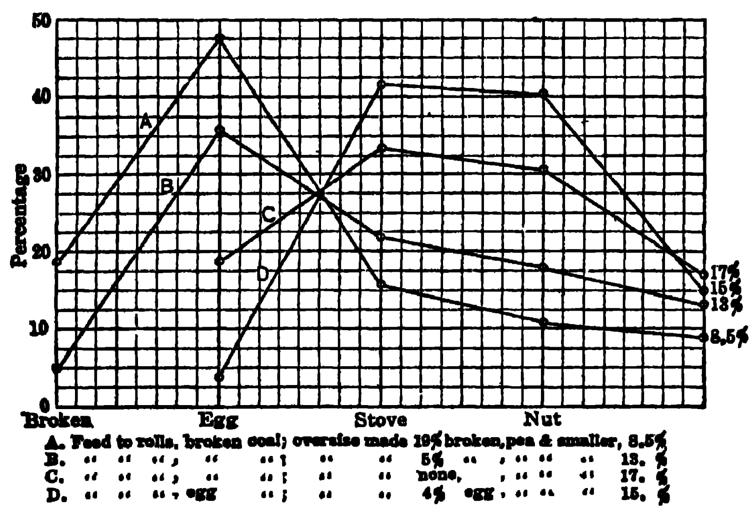


Fig 8. Percentages of Prepared Sizes made by 35- by 34-in Rolls, at 230 ft Peripheral Speed per Min

11. MECHANICAL CLEANERS

Classification. There are 3 groups, the operation of which depends on: (*a*) Difference in specific gravity; (*b*) difference in coefficient of friction; (*c*) difference in shape of fracture.

Plunger, pan, pulsator and upward-current jigs. There is no definite rule as to type of jig; personal preference seems to decide. Plunger and pan jigs give equally good results for all sizes. Upward-current and pulsator jigs are occasionally used for rice and barley coals. Combined plunger and upward-current jigs are also used. Overflow jigs cause less breakage than those with mechanical discharge. Pan jigs with mechanical discharge require less water than other types. Jigs will treat consistently a mixture high in impurities, yielding a marketable product, the refuse usually containing not over 4%

coal. Successful operation depends largely on the jig tender's skill; the coal end probably requiring some hand picking (see Table 9). So-called automatic jigs have not been entirely successful.

Broken-coal jigs for preliminary cleaning are advantageous, the jigged product being broken down to egg and smaller, and mixed with the run-of-mine for further jigging. Constant feed is essential for good results, and there should be a storage hopper back of

Table 9. Loss by Breakage in Jigs, due to Mode of Discharge, and Abrasion of Coal in Jig-tank

Size of coal	% loss into			Approx loss in value, ¢ per ton	Method of discharge
	Stove	Nut	Pea and smaller		
Broken	0.42	0.21	1.78	5.0	Overflow
Egg	0.40	0.22	1.68	5.0	"
Broken Egg	0.98	0.56	4.09	10.0	Flight conveyer
Broken Egg	0.10	0.42	3.18	8.0	Belt "
Stove		...	1.43	4.0	Flight "

each jig with 1 hr capac. Jigging of the coal refuse from the jigs improves total recovery. Jig capac varies, being dependent upon quantity of impurities in the feed. With a high percentage of impurities, the regulating gate is set low to allow more time for cleaning; conversely, with low percentage of impurities, the gate is raised.

Egg coal, containing 44% slate and 6% bone, in a 4-ft pan jig at 135 r p m, gave following results: coal discharge, 2.25% slate, 2.5% bone; slate discharge, 3.75% coal, 2.75% bone; clean coal, 6.3 ton per hr. The coal was hand picked before going to storage pockets.

Pulsator and upward-current jigs are occasionally used on rice and barley, but seldom reduce ash content below 15%. There are also special small coal jigs, but they generally produce about 15% ash coal, which seems to be the low limit practicable with jigs.

Concentrating tables are largely used. They differ from ore tables chiefly in size. The deck is approx 17 by 8.5 ft. The riffle cleats, instead of being parallel to discharge edge of table, are at an angle with it. A two-part separation is made of coal and refuse. With careful supervision, about 12% ash coal is obtainable. Constant feed and uniform water supply are important for best results. Aver.feed capac, 8–12 ton per hr, for buckwheat, rice and barley; for slush or buckwheat No 4, 4 ton. Water required is about twice the weight of coal; that is, 8-ton feed per hr requires 75 gal water per min.

Air washer or concentrator table is a specially designed table having riffle cleats mounted on a perforated metallic deck, with the usual driving mechanism; air forced through the deck agitates the feed. A 3-part separation takes place as on the wet concentrator. This table, designed for rice and barley coals, functions well on a feed containing less than 4% moisture; feed of higher moisture cannot be easily cleaned. As practically all fine coal is sized wet, the field for the air washer will be very limited, unless a cheap drying process can be devised.

Chance sand-flotation process (Fig 9). Coal is floated in a fluid mixture of sand and water, maintained at a predetermined fixed sp gr in a cone, in which slate and other refuse readily sinks. With sea or beach sand (2.6 sp gr) a fluid mixture up to 1.75 sp gr can be maintained, so that any material above that gravity will sink. Minus 30 and plus 80-mesh sand should be used. Coarse sand requires more agitation; finer sand is difficult to settle and is lost in the sand-sump overflow, unless extra large sand sumps are used.

The refuse in Table 9a carries 5.8% good coal at 1.75 gravity and 6.13% at 1.70. In run-of-mine coal there is usually merchantable coal up to about 1.90 gravity, which is not recovered directly by this process. When percentage of fine coal is high in the run-of-mine, comparatively little refuse is removed. Refuse should probably be treated by jigs to recover lost coal. Use of sand is an objection, in introducing material foreign to the coal, and, because of its excessive abrasive action on the machinery, maintenance cost is high. The cleaned coal usually looks well and is attractive to the trade. An expert attendant is necessary, who must be sure the gravity of the fluid mass is up to the desired point; otherwise, more coal will go to the refuse.

Run-of-mine coal (Fig 9) is broken to egg size and usually smaller. Silt is screened out and remaining product fed into the top of the fluid mass in the separation cone; the coal floats near the top, flows from the cone discharge with the fluid mass with which it

Table 9a. Chance Cone, Tests on Feed of 1.75 and 1.7 Sp Gr

Size of coal	Run-of-mine		Discharge			
	Feed, % coal	% slate	− 1.75 sp gr coal		+ 1.75 sp gr refuse	
			% coal	% bone	% coal	% slate
Egg	16.3	15.4	94.25	5.75	2.06	27.44
Stove	15.4	13 8	98.78	1.22	1.50	22.60
Nut	11.1	7.0	97.50	2.50	1.50	25.90
Pea	5.2	3.2	97.40	2.60	0.05	7.15
			% ash			
Buckwheat	4.5	2.1	11.2		0.21	7.29
Rice	....	....	13.0		0.20	1.40
Barley	4.9	1.2	13.0		0.28	1.62
Culm	.... .	......	13.0		....	0.8
Total	57.4	42.7			5.80	94.20
			− 1.70 sp gr		+ 1.70 sp gr	
Egg	12.4	15.2	97.63	2.37	3.30	38.3
Stove	12.1	18.7	97.75	2.25	1.50	17.0
Nut	7.9	8.9	99.00	1.00	1.00	26.1
			% ash			
Pea	5 3	2.1	11.4		0.05	3.35
Buckwheat	5.9	4.6	11.4		0.02	5.38
Rice	....	....	13.4		0.12	1.38
Barley	5.3	1.6	15.0		0.14	1.56
Culm	....	... ...	16.0		0.00	0.8
Total	48.9	51 1			6.13	93.87

is mixed, passes over a de-sanding screen and then over sizing screens. Slate and other refuse sinks and is trapped out by alternate opening and closing of the two slate valves;

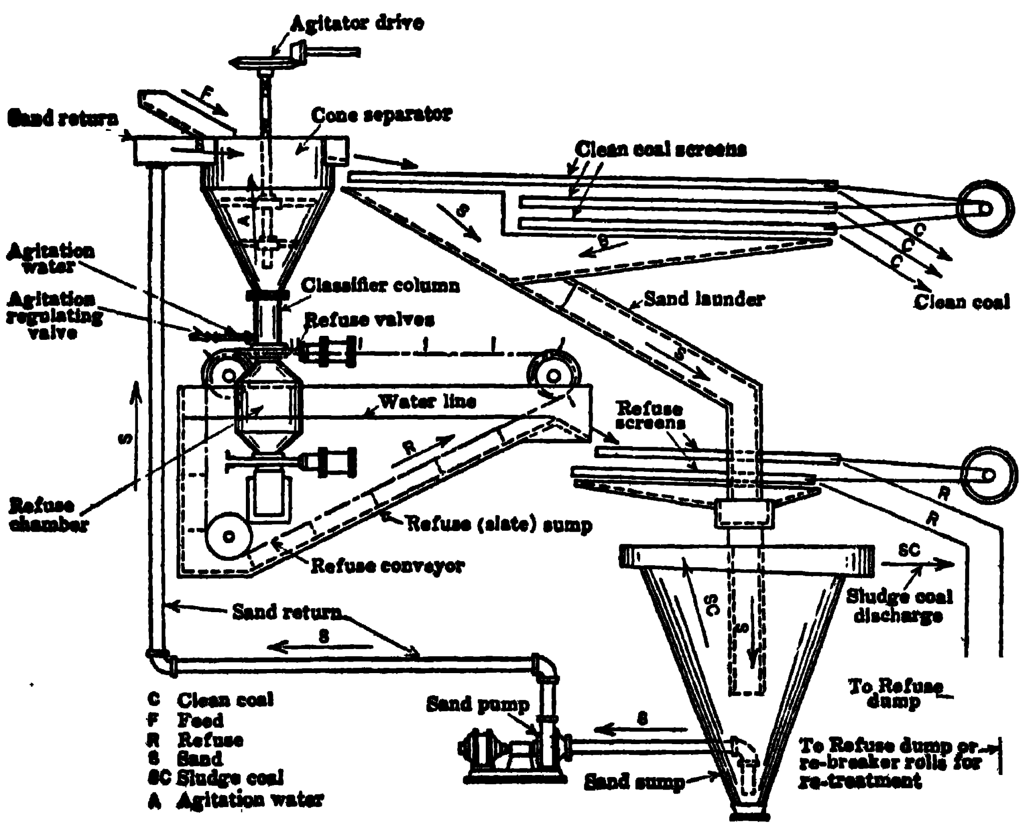


Fig 9. Chance Coal Cleaner, Sand-flotation Process

the slate falls into a sump and is elevated to a de-sanding screen. Sand and water from de-sanding screens flow to a sand sump, in which the sand settles and is pumped back

to the separator cone. Overflow, of clean water, flows to the circulating pump for reuse for agitation, shaker sprays, etc.

Rheolaveur process (Fig 10, 10*a*) is an application of the launder washer, for handling coal in size from 3.5 or 4-in to approx 48-mesh (see flow-sheet, Fig 3*a*). Invented and patented by Antoine France, Liége, Belgium; American rights owned by American Rheolaveur Corp'n.

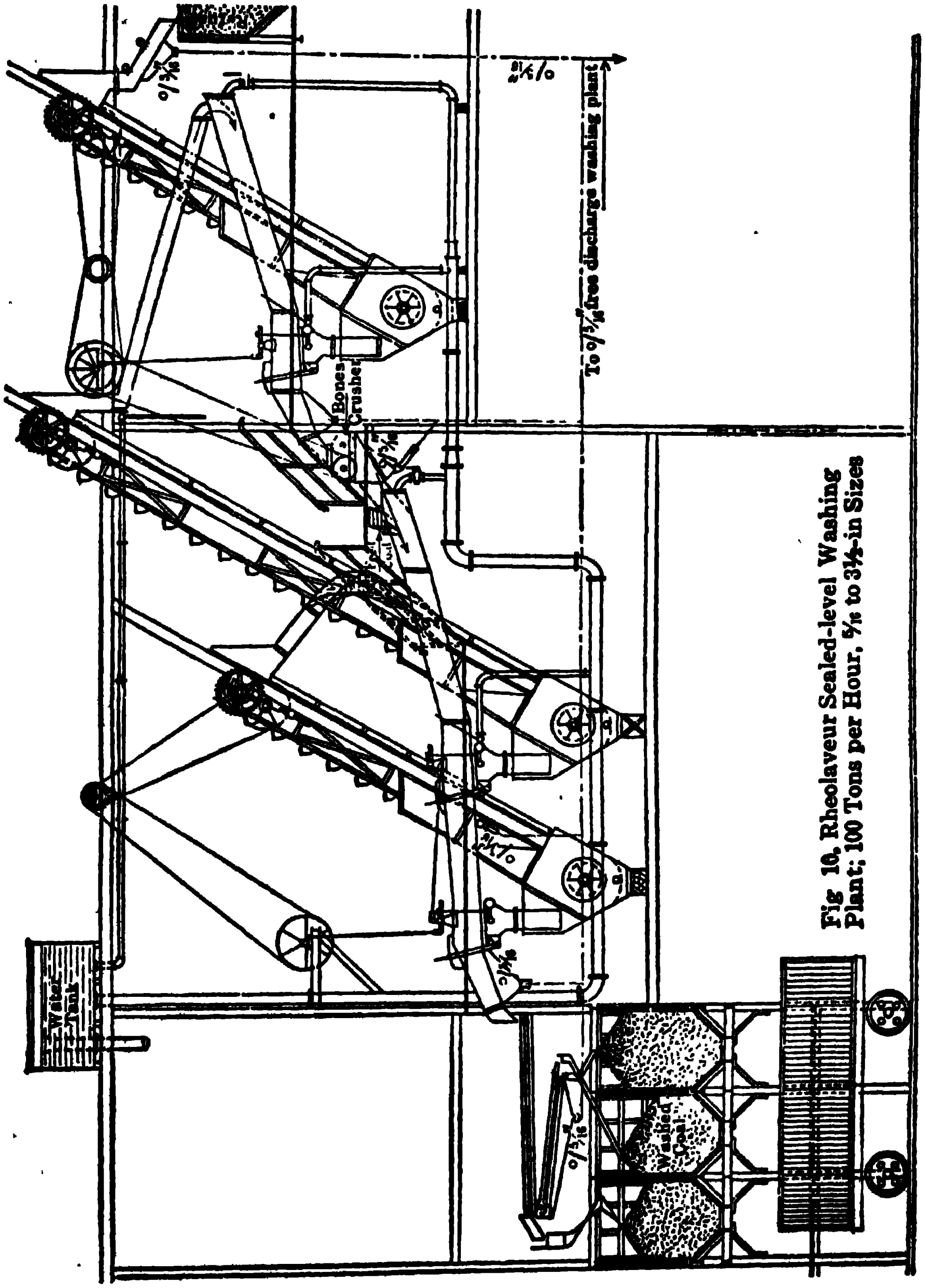


Fig 10, Rheolaveur Sealed-level Washing Plant; 100 Tons per Hour, $^5/_{16}$ to 3½-in Sizes

Classification is effected by horiz currents of water in an open launder, causing stratification of the material in proportion to its density, shape and coeff of friction. The lower layers are drawn off progressively by "Rheo" boxes; these are traps in which an ascending current prevents particles from passing to the discharge, unless their density causes them to sink. There are two types: closed or sealed (Fig 10), for the larger sizes of coal; open or free discharge (Fig 10*a*) for small sizes, usually from $^5/_{16}$-in to sludge.

Closed type. The Rheo boxes have an adjustable opening into the launder, large enough to pass the required size of coal and provided with a perforated oscillating flap to

prevent arching or clogging. A baffled water connection with a quick-acting valve admits water to create an upward current of known strength. The discharge from the boxes passes into a sealed elevator boot, maintaining the water at trough level and raising the material to a point above the water seal.

The feed enters the current at upper end of launder, which is quite steep for a distance that depends on character of feed. As the material passes rapidly down this steep grade,

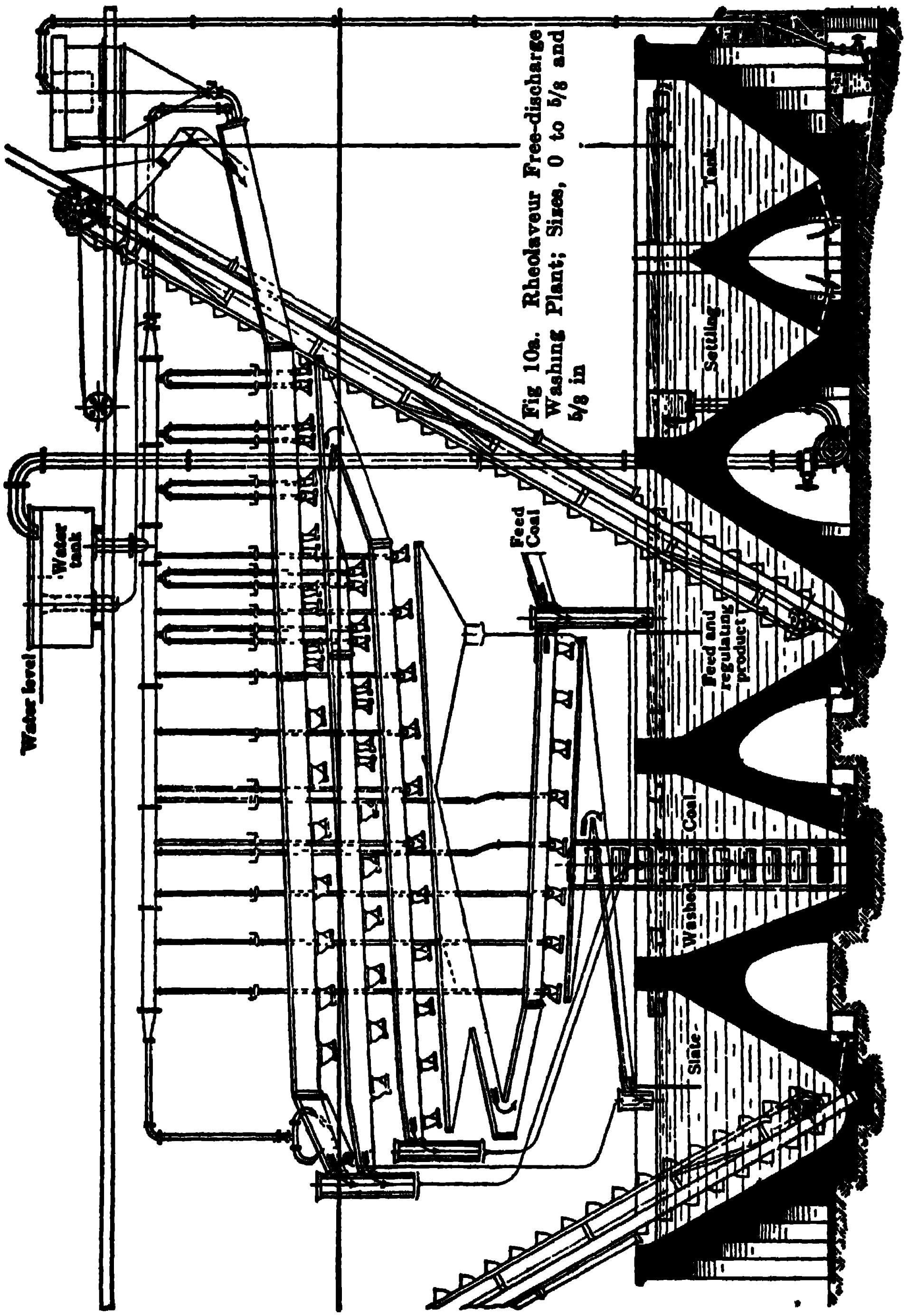


Fig 10a. Rheolaveur Free-discharge Washing Plant; Sizes, 0 to 5/8 and 5/8 in

A PRIMARY CLASSIFICATION is made, the light particles floating quickly to the discharge at end of launder. Heavier or slabby materials seek the bottom and are retarded, while those of intermediate density are classified between and move at varying speeds, according to their density and coeff of friction. From the classification end of the launder the grade flattens until it is nearly level, the heavier materials building a bed which travels very slowly along the bottom. Just beyond the beginning of the flat grade is the first Rheo box, which extracts the heaviest material. From this point the veloc of current decreases, and

stratification becomes more definite; so that, on reaching second Rheo box, particles of intermediate gravity constitute the bed on the trough bottom and are drawn off by the second box.

Materials from the first Rheo box are rewashed if they have any recoverable content, or pass directly to the refuse hopper. Those from the second box are returned to the head of the launder and mixed with the feed. Thus, the amount of intermediate material is greatly increased and builds a thick layer between the good coal and heaviest impurities. This re-circulation goes on until the intermediate material is properly classified. The results are controlled by the adjustable openings of the boxes and the upward currents.

Open or free-discharge type of launder operates as above, but the Rheo boxes are different. They are very simple, having one or more slots opening into the launder, a quick-opening regulating disk valve, and a baffled and regulated water connection for upward current when required. The launders are very long and have many boxes, which take the materials from the lowest part of the bed progressively as required for re-wash in the troughs below. There are usually 3 or 4 troughs, one above another in a "batterie cascade"; the first and second discharge the good coal, the product from the third and fourth being elevated to the head of the system for retreatment. For certain classes of coal the fourth trough may discharge bone, which, instead of going directly to the re-wash, is crushed and re-treated in a SLUDGE PLANT, for obtaining max recovery. The bottom trough evacuates the refuse. The operation is adjusted by the disk valves and upward currents of water. Fig 10, 10*a* show launders of both closed and open type, as would generally be used in a commercial plant.

The Rheolaveur process has proved very efficient in obtaining low ash and high recoveries in both anthracite and bituminous preparation. Its simplicity, low cost and upkeep, and economical operation have also made a very favorable impression abroad, where it is largely displacing other systems.

Fixed and movable cleaners (friction). FIXED type may be designed to make a 2-part (coal and slate) or 3-part (coal, bone, and slate) separation. They usually consist of a series of inclined planes, separated by adjustable cross slots. Mixed material gravitates from top to first slot, where the slow-sliding slate and bone (due to their greater coeff of fric and flatter shape) fall into the slot, the coal passing over; this is repeated until the coal is sufficiently cleaned. Slate and bone falling through the slots may be similarly treated in another cleaner: the first slot removing slate, the bone passing over. These cleaners are usually adjustable, as to slot opening and pitch of planes. They require constant attention, and hand pickers for the refuse, and are almost obsolete, as they work poorly on wet coal. SPIRAL CLEANER consists of a central column, with a series of spiral bands inclined toward center, down which coal slides. Coal maintains a helical path around the column so long as friction balances centrif force. As veloc and centrif force increase, friction is over-balanced, and coal jumps over outer edge of spiral and falls into a chute. Slate and bone adjust themselves on the spiral, according to coeff of friction, and at discharge end may be separated and run to their respective destinations. Spirals require some attention and adjustment, since a change in character or condition of run-of-mine (as from wet to dry) will vary the coeff of friction. Effic is affected also by dryness or dampness of atmosphere. MOVABLE CLEANER consists of a metallic moving band, adjusted to give a pitch in 2 directions, across the table and lengthwise, so that the band travels up the pitch. Impure mixture is fed at the high corner; coal slides obliquely across the table; slate, having greater coeff of friction, is carried up into the refuse chute.

The Anthracite Separator Co have recently developed a two-thread spiral for primary cleaning, especially for broken coal, which removes excess refuse from run-of-mine feed before final treatment of the product.

Fixed and movable cleaners (fracture). FIXED type consists of specially made bars, placed in a chute, forming long narrow slots over which cubical pieces of coal slide, while flat slate and coal fall through. Or, punched or slotted steel plates may be used, the flat pieces passing through the holes. This type is usually attached to discharge end of a shaking screen. Some forms will remove more flat slate than coal. MOVABLE type (see parallel revolving roller screen, Art 9). Slate from these cleaners usually goes by gravity to jigs or friction type of cleaner, for final preparation.

Comparison of mechanical cleaners. For high recovery, the designed capacity should not be exceeded, or a loss of coal in the refuse will result. JIGS produce clean coal, but, where the refuse in the feed exceeds 15–20%, double jigging is desirable; and if there is a high percentage of bone middlings, triple jigging may be introduced. The jig requires a trained operator, but, after it is once adjusted, neglect on his part will not seriously affect results. For high recovery, close sizing is desirable; when jigs are used, each size must have its separate machine. The larger sizes must be inspected and usually hand-picked. The CHANCE PROCESS produces clean coal; it has the advantage that, as pre-

sizing before cleaning is unnecessary, refuse is removed before sizing, with possible reduction of the screen areas. The labor force is less than that required for a jig plant. Recent installations show a preference for the Chance equipment over jigs, although more screens are required to remove the fines before entering the Chance cone, with de-sanding screens after the cone on coal and refuse.

The RHEOLAVEUR PROCESS has the advantage of handling mixed sizes, removing refuse before sizing; for high recovery, launders may be arranged in series for re-treating the refuse. CONCENTRATOR TABLES are adapted usually to fine coal, and adequate water supply, uniform feed and skilful operation are essential. The HYDROTATOR has shown good results on all sizes up to and including pea coal; but, as it will not handle mixed feeds, each size must have a separate machine.

12. CONVEYERS, CHUTES, FEEDERS, WATER SUPPLY, POWER

Conveyers are classified as: (*a*) single-strand flight; (*b*) double-strand flight (including scraping and carrying conveyers); (*c*) belt conveyers. FLIGHT conveyers with single strand may be used for all sizes of flights up to 24-in width; double strand for larger.

Sizes of flights: SINGLE-STRAND, 4 by 10 in, 6 by 12, 6 by 18, and 8 by 18; DOUBLE-STRAND: 10 by 30 in, 10 by 48 and 10 by 54. Speed 75–100 ft per min. Trough, steel or C I, the latter preferred for long life, especially in acid mine water. Breakage, 2–4%, is due to method of feeding; little breakage occurs in transit in smooth troughs. Belt conveyers are especially adapted to dry coal; when conveying mixed wet coal, they run at higher speed, to 500 ft per min, for throwing off wet fines which otherwise stick to the belt and are carried back on return belt. Breakage is probably higher than for a chain conveyer, due to drop at the discharge end at high veloc; partly overcome by using a discharge chute receiving the coal on a tangent.

Shaking chutes, replacing conveyers, have usually 3 by 8-in wood sides, with 2.5 by $^5/_{16}$-in cross angles, 3 ft long, at about 6 ft centers. The angles extend 6 in beyond the sides, and are bolted to 1 by 6-in hanger strips, 6 ft long. A pair of 3-in throw eccentrics, at 90–100 r p m, drive the chute, which is set on a pitch of about 0.5 in per ft. Width varies to suit capac, which is approx equal to area of chute × half the total travel per min, thus: area chute, 1 sq ft, 0.5-in pitch, 100 r p m, 6-in travel; 1 sq ft $\times \frac{100}{2} = 50$ cu ft per min.

Shaking chute is also used to replace inclined stationary coal and rock chutes, to reduce breakage, give uniform flow or feed, and reduce breaker height. Max safe length for a chute 24 in wide by 8 in deep is about 75 ft. For large capac, single chutes may be placed in series, the drive being in balance.

Elevators should be double-strand and of the gravity discharge type; their breakage at discharge end is less than for any other type. Loss in breakage is from 2 to 5%, according to method of feed and discharge.

Chains for elevators and conveyers working in acid water should be heavy and contain few parts, with safety factor of 6 to 10. Eye-bar links with riveted pins should not be used; rivetless chains having few parts are preferable, even if more power be required. To reduce extra stock of parts, all conveying or elevating chains at any one colliery should be of same type. (For details, see Sec 27.)

Inclined chutes for conveying coal by gravity, if badly constructed, cause a degradation loss exceeding all other losses combined. This loss is attributed to irregularities in chute bottom; striking of one piece of coal against another; drop at any point, especially at right-angle turns; and the blow which a piece of coal receives at such a turn on striking side of chute. PITCH on which coal will slide varies with quality of coal, and on chute

Table 10. Pitch and Width of Chute, for Coal Sliding on Steel Plates

Size of coal	Pitch, in per ft	Width of chute, in	Size of coal	Pitch, in per ft	Width of chute, in
Lump......	2 1/2 to 2 3/4	48	Pea..................	4 to 5	12 to 13
Steamboat..	2 1/4 to 3	36	Buckwheat..........	4 1/2 to 6	12
Broken.....	2 1/2 to 3	30	Rice................	5 1/2 to 7	12
Egg........	2 5/8 to 3 1/4	24	Barley..............	7 to 8	12
Stove......	2 3/4 to 3 1/2	18 to 24	Dirt................	8 and over	12
Chestnut...	3 to 4	18 to 24	Rock........	5 and over	18 to 60

Note.—Pitch for bottom of dump hopper, not less than 7 in per ft; for storage pockets, not less than 8 in per ft.

lining. Clean coal slides on flatter pitch than a mixture of coal and impurities. A sluggish section of inclined chute can often be remedied by replacing steel lining with bronze, if impracticable to increase pitch. Chutes for dry coal are of 2-in plank lined with sheet steel. For wet coal, use 2-in plank, lined with 2-ply smooth prepared roofing, covered with 1-in boards, and lined with sheet steel or bronze. Right-angle or 180° turns in chutes should be of laminated wood strips, $^1/_2$ by 2 in.

Vertical chutes, "telegraphs," may be designed merely for lowering coal, Fig 11, *a*, or for lowering and depositing to variable depth in a pocket, Fig 11, *b*. In the latter case,

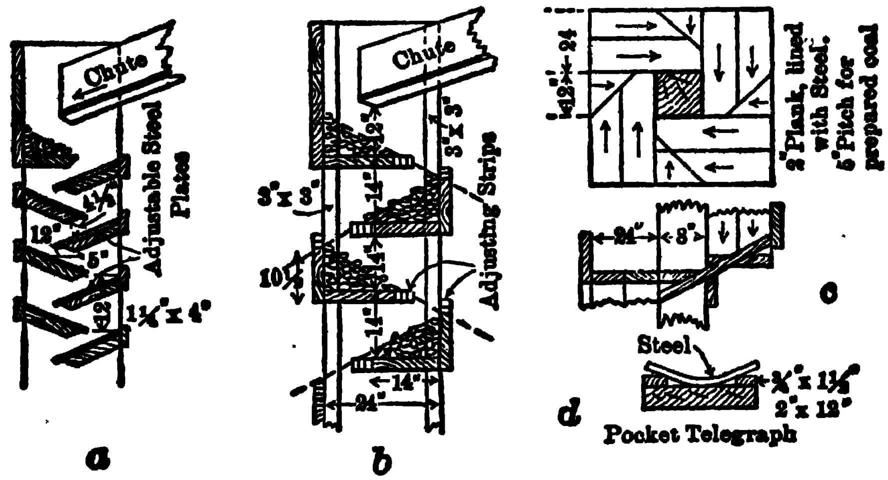


Fig 11. Vertical Chutes, or "Telegraphs"

when top of pile reaches and blocks lowermost opening, coal then issues from next higher opening and deposits on summit of pile; breakage due to subsequent avalanching down slope of pile is negligible.

Capac of a telegraph 24 in sq is 150 tons per hr; 48 in sq, 300 tons per hr; used for all sizes from egg down, and may be of any height up to 65 ft. Breakage in a well-built telegraph is only about 1%. Spiral telegraph is shown in Fig 11, *c*. Automatic control mechanism, Fig 12, is designed to keep telegraph full of coal, by discharging only as fast as fed, thus reducing breakage. Coal entering the top presses upward against pan *a*, thereby opening gate *b* at bottom.

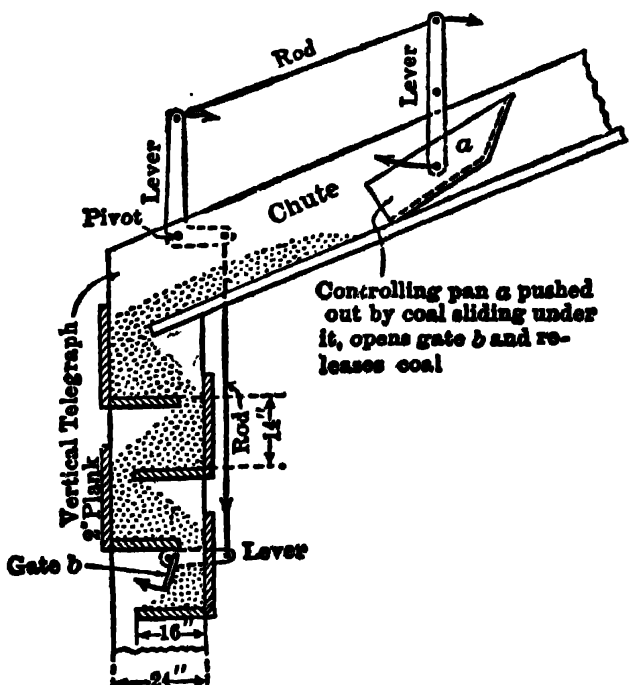


Fig 12. Automatic Control for "Telegraph"

Automatic feeders are installed between receiving hopper and lump-coal screens, and in front of rolls. Feeders may be omitted when automatically controlled chutes are used, since these serve the same purpose. Reciprocating feeder has advantage of simplicity and practically no breakage, as compared with other standard types.

Water is used in wet preparation to wash coal during sizing, on lip screens to remove fine screenings, and in jigs. The quantity required, in gal per min, is approx twice the tons shipped per day.

Gravity system gives best results, because of its constant head. It requires a storage tank in top of breaker, supplying a main feeder line, with branches to the various machines. C-I pipe is required when using acid water, and is of following sizes: shaker screens, of 60–90 sq ft, 2–2.5 in; jigs, 2 in; lip screens, of 12 to 15 sq ft, 1.5–2 in. A fan-tail spray

nozzle directs the stream onto shaker or lip screen, and against the flow of coal. Gate valves are placed at ends of all branch lines, plug valves at sprays on shaker, quick opening valves on lip screens, with spray line operated from the loading platform. All lines have blow-outs for cleaning.

Mine water is commonly used, as fresh water is seldom available. Aver acid content, 1% or less free H_2SO_4. Treatment of mine water saves largely in maintenance of pipes, chute lining and screen segments; saving soon pays for labor and material required to neutralize the water. Lime treatment is simplest; a small feeding and mixing machine is installed, and lime water fed into suction line or sump of the breaker supply pump. About 1 ton of lime per 8-hr is required for 2 000 gal per min of ordinary mine water.

Breaker drives. Both electricity and steam are used, local conditions governing choice. As between unit and combined drives, the latter is preferred. A breaker fed by conveyer should have its drive unified with the rest of the breaker machinery. Jigs should have individual drive, so they may operate during delays to other machinery or delivery of run-of-mine coal. The Chance separator should have separate unit drive, for, if stopped, the sand settles, causing delay in starting up. Belt or rope drives are used from line shaft to all power units. To avoid carrying many repair parts, all machinery should be standardized, with 48-in pulleys, 4 3/8-in shafts and 10-in belts; these sizes covering practically all conditions. Steam from a colliery boiler plant is usually cheaper than purchased electric power, but the convenience of motor drives with remote controls, as compared to steam plant, is unquestionable.

Table 11. Horse Power for Operating Breaker Machinery

	Approx h p	Speed per min
Revolving screens, 6–8 ft diam by 24 ft long	5	250 ft periph
Shaking screens, per 60 sq ft of area	2.5	150 r p m
No 1 crusher rolls, 36 by 36 in, compound	30 max	25 r p m
No 3 re-breaker rolls, 36 by 36 in, compound	20 "	25 r p m
Oscillating picking table, 5 by 24 ft	3 "	100 strokes
Plunger jig, 4 ft (conveyer discharge)	5 "	95 r p m
Simplex pan jig, 6 ft (conveyer discharge)	7.5 "	135 r p m
Double jig, 3 ft 3 in (coal overflow)	7.5 "	120 r p m
Elevators, 1 ton, 50 ft high, per min	4 "	
Chance; 15-ft cone	50	
13.5-ft cone	35	
Menzies cone, 8 ft diam	40	
Menzies cone refuse conveyer and dewatering shaker	25	
Wilmot Hydrotator, 6 ft diam	20	
Wilmot Hydrotator, 6 ft diam, with refuse conveyer	5	
Wilmot Hydrotator, 6 ft diam, with dewatering shaker	5	

Horis conveyers, 100 ft per min, 100 ft long

Size of flight and spacing, in	Weight per ft	Capac, ton per min	
6 by 12-in, at 12 in	10	0.75	For additional length, multiply by length, and divide by 100
8 " 18 " 18 "	19.7	1.5	
10 " 30 " 18 "	45	3.5	
10 " 48 " 36 "	54	5.5	

Table 12. Total Breaker Hp, with Number and Kind of Machines

Breaker capacity, tons	I hp		Machines, number and kind					
	Light	Loaded	Rolls	Shakers	Picking tables	Conveyers, no and length	Elevators, no and length	Jigs
1 500	216	280	7	33	3	2–185 ft	3–184 ft	....
2 000	342	512	4	34	....	2–135 "	4–210 "	32
2 500	354	471	4	35	....	3–225 "	4–240 "	....
1 800	274	336	3	36	1	3–220 "	1–80 "	30

13. BREAKER COSTS AND CAPACITIES. NUMBER OF EMPLOYES

While it is difficult to set a figure, the accompanying tabulation may be used for rough estimating. For structural steel frame, with metal sides and roof, and steel sash, as a basis (L.L.Y.P. construction) about 10% less.

Output, tons per day	Cost per ton per daily output
1 000	$150
2 000	137
3 000	120
4 000	120
5 000	120
6 000	115

There is a preference for high structures, so that the coal, during preparation, may flow by gravity from intake point at top of breaker, down to the storage pockets. Storage pockets, if used, practically determine ground area of breaker. Usual capac of pockets is 70–90 ton for each size, requiring 8 pockets for the 8 sizes usually made. When ample storage room is available for empty and loaded RR cars, pockets may be replaced by boom loading, with advantage of eliminating pocket degradation.

Cost of breakers is divided into about the following percentages for the classifications given: excavation, 0.37%; foundations, 6.23%; floors, 1.20%; structure, 34.60%; lumber, 8.25%; machinery, 33.40%; water piping, 8.60%; loading machy, 3.05%; misc and engineering, 4.30%; total, 100%.

Distribution of steel for a 1 500-ton Class *C* breaker. Volume of breaker, 640 000 cu ft; ground area, 8 700 sq ft; roof, 9 000 sq ft; sides and windows, 36 400 sq ft. Weight of steel, tons: columns, 125; beams, 186; bracing, 39; roof, 22; girts, 41; pocket stringers, 36; rivets and misc, 9; total, 458 tons.

Distribution of timber for 1 500-ton wooden Class *C* breaker. Volume of breaker, 870 000 cu ft. Feet, board measure: posts, 73 900; girts, 127 000; pocket bracing and supports, 140 000; roof, 41 900; siding, 75 000; machy supports, 13 100; jig tanks and supports, 67 700; bracing for chutes, walks and misc, 276 900; total, 815 500 bd ft. Erecting time, 124 days; total man-hours for construction, 67 369.

Table 13. Average Number of Employes in Breakers

	Class A. Run-of-mine, 14% refuse	Class B. Run-of-mine, 16% refuse, wet and dry	Class B. Run-of-mine, 34% refuse, all wet	Class C. Run-of-mine, 24% refuse
Boss	2	2	1	1
Ticket taker	1	1	....	1
Dumpers	2	2	1	1
Platemen and table tenders	14	9	14	7
Pickers on pure coal	8	8	4	....
Pickers on jig coal	....	2	20	2
Table refuse, pickers	....	6	....	....
Jig refuse, pickers	....	6	6	4
Mechanical cleaner, refuse picker	4	....	3	1
Mechanical cleaner, tenders	2	....	4	....
Mach'y attendants	3	7	29	6
Breaker oiler	2	2	2	1
Ropeman	1	....	2	....
Breaker cleaner	2	2	2	1
Engineer	1	2	1	1
Loaders	12	18	7	5
Total force	54	67	96	31
Tons per day	1 650	1 790	2 030	1 120

14. LOCATION AND REQUIREMENTS OF STORAGE PLANTS

Irregularities of market for all sizes of anthracite, which would interfere with continuous operation of the mines, have led to the erection of storage plants at convenient points. Due to better transport facilities from the mines, plants located on the seaboard have largely been abandoned. Local plants, in or near the anthracite region, still exist and are used to store sizes not absorbed by the market. They have the advantages of short haul from the mines; quick release of cars, which is important during a car shortage; low freight rate on coal stored; and convenience of shipping to any market. Interior plants on the Great Lakes are still in use, although the necessity for large storage is ques-

tionable, because of the loss of markets in the West and Northwest, formerly drawing their supply from this source.

Ideal storage plant should fulfil the following conditions: (*a*) production of minimum breakage in stocking and reloading; (*b*) separate storage of each size and in varying quantities; (*c*) rapid handling in storing and reloading; docks on the Great Lakes, receiving coal by steamers, should have large unloading and stocking capacity, as quick dispatch of boats is desirable; (*d*) screen house to resize all coal from storage; (*e*) separate storage for screenings and facilities for resizing them; (*f*) arrangements for reloading and screening more than one size at a time; (*g*) loading facilities for both box and gondola cars; (*h*) covered storage when in regions subject to extreme cold and heavy snows; (*i*) facilities to handle frozen coal, especially at plants receiving wet coal from the mines or at uncovered plants; (*j*) ample railroad classification yards; (*k*) ample trackage through plant, operated by a car-haulage system; (*l*) loaded and empty car scales; (*m*) minimum danger from fire; (*n*) design permitting enlargement; (*o*) low first cost per ton of capacity, and minimum operating cost.

Frozen coal is most efficiently thawed by hot water; steam jets are of little use. There should be ample provision to handle the drainage, which contains much fine dirt.

Operation of storage plants is very irregular. Night and day rushes, in unloading or reloading, are often followed by long periods of inactivity, with the plant either full or empty. Low operating cost, when the plant is working, may be offset by high fixed charges during idle periods.

15. CLASSIFICATION OF STORAGE PLANTS

Each of the two general classes, wholesale and retail, is sub-divided into non-mechanical and mechanical.

Non-mechanical	
A. Stocking on surface	Reloading by hand, yard reloaders, or steam shovel
B. Stocking on surface from trestle	Reloading by hand or yard reloaders
C. Stocking on surface from trestle	Reloading by tunnel, with cars or conveyers to a screen house, with or without yard scrapers
D. Stocking from trestle into bins	Reloading from bins into cars or wagons over lip screens
E. Stocking from trestle on hillside	Reloading by hand, dock scrapers or by hydraulicking
Mechanical	
F. Stocking by elevator and conveyers into bins	Reloading by tunnel as in *C*, or direct into cars or wagons, as in *D*
G. Stocking by cable railway and dump cars into bins or on surface	Reloading by hand, tunnel, or from bins, as in *C* and *D*
H. Dodge system. Stocking by truss trimmers in conical piles	Reloading by swing conveyers, or pivotal reloaders with inclined conveyers to screening towers
I. Stocking on hillside by traveling cantilever trimmer	Reloading by gravity and hydraulicking
J. Stocking on surface by traveling trimmer	Reloading by longitudinal and cross tunnel conveyers, fed by gravity and traveling reloaders, delivering to screen house
K. Stocking by surface cable cars, operating on traveling tramways	Reloading by tunnel conveyers to screen house
L. Stocking by grab buckets, operating on traveling or fixed tramways	Reloading by grab buckets and conveyers to screen house

16. APPLICATION OF THE DIFFERENT CLASSES OF STORAGE

A. Limited to temporary storage of steam sizes. Consists in forming a dump on a level surface, laying tracks on the accumulating stock, and raising and shifting these as the storage grows. Reloading with steam shovel or grab-bucket cranes, operated from edge of pile, or by hand. Only one size can be stocked, breakage of the prepared sizes is excessive, no screening is possible, and operating cost is almost prohibitory.

B. Commonly used for small retail yards, when first cost is the chief consideration. Consists of a trestle, the height of proposed pile, from which loaded cars are dumped. Sizes are separated by partitions. Reloading by hand or yard reloaders into wagons. Only hand screening is possible. All sizes may be stored, but breakage is excessive, due to high drops in stocking.

C. Occasionally used for large storage, where conditions do not warrant a more expensive plant. Stocking is done by trestle, as in *B*, with a reloading tunnel under the storage pile. Reloading gates in tunnel deliver into cars, or to a belt or scraping conveyer, which transfers the coal to a central loading point or screen house. Lip screens at loading gates give only a partial screening when loading cars. Screenings are collected in dust boxes and afterwards transferred by hand to a screening storage. All sizes may be stored. Approximately 60% may be reclaimed by gravity, the remainder being handled by hand or yard scrapers, with a partial screening when loading cars. Breakage is excessive, due to high drops in stocking and drawing coal under pressure in reloading. High first cost and imperfect sizing are the disadvantages of this type.

D. For large retail storage plants, for wholesale storage at seaboard, or for interior plants loading boats or steamers. Consists of bins traversed by railroad tracks, from which each size is dumped into its respective bin. Reloading is direct into cars or wagons, which pass under or alongside of bins. Loading gates in each bin feed the coal over lip screens to remove the screenings, which may be collected in dust boxes under the screens, and removed by hand or conveyers to screenings storage. ADVANTAGES: all sizes stored in varying quantities, and reloaded by gravity; can unload or reload more than one size at the same time; partial screening. DISADVANTAGES: excessive breakage, due to high drops in stocking and drawing coal under pressure when reloading; imperfect sizing.

E. For large storage, of pea coal and smaller. Consists of a hill with a side slope of 25° to 30°, at least 300 ft wide, with an abrupt change at the foot from this slope to a level surface for tracks; reasonable railroad grades to the top of the storage hill and down again. Coal is stocked from a high trestle at top of hill, and avalanches down the hillside until arrested by a retaining wall or by a level space at bottom. Reloading is through gates in the wall, but only the layer retained by the wall and lying above the angle of repose will flow by gravity to reloading gates. It is usual to reload the remainder by hand, dock scrapers, swing conveyers, or by hydraulicking, using hot water in winter. Fixed screens in front of loading gates remove screenings during reloading, which may be conveyed by cars or conveyers, or hydraulicked, to a screen house. Lip screens not used on sizes smaller than pea coal. The different sizes are separated by partitions, made movable to vary their storage capacities. Entire hillside should be covered by planking or concrete to avoid admixture of dirt.

F. For small retail storage, where location prohibits RR approach to top of bins on an easy grade. May be developed for large storage plants. Stocking is done by dumping RR cars into a receiving hopper, which feeds into an elevating and scraping or carrying conveyer, transferring coal to storage bins. Reloading is direct into cars or wagons, through gates with lip screens in the delivery chutes, the screenings collecting in a box underneath. Lowering chutes or telegraphs in each pocket reduce high drops when stocking. Stocking conveyer may encircle storage building, the lip screenings gravitating from the storage box to the lower run of the conveyer; then re-elevated and discharged into a screening bin. The space between under side of pockets and the surface may be used for storage by stocking through a gate in bin bottom. This stock coal is chuted to the lower run of the conveyer and returned to pockets.

G. For large storage plants or retail yards; especially to transfer water-shipped coal to storage or direct to screen house. Consists of a cable or gravity return car, traversing bins or a trestle over a surface storage floor, and dumping at the desired points. Reloading is by hand, or tunnel, with conveyer. Method of screening is similar to that of *C* and *F*, or, when well-sized coal is desired, it may be re-screened in a separate screen house, as with *K*. Low in first cost and operation, readily adaptable to extension and for covered storage. Breakage, into pea and smaller, may be as low as 8%, but is generally higher.

H. Dodge system, used for large storage, consists of storing on the surface in 2 conical piles by a trimmer truss supporting a flight conveyer. A unit has 2 trimmer trusses with track hopper, one central reloader, and a screening tower, with loading tracks and scales. Loaded cars are dumped into a receiving hopper and fed into the trimmer conveyer, which is built on a catenary curve from under the hopper to meet the incline of trimmer truss. Coal is discharged at apex of the growing pile, through bottom of conveyer trough. Bottom of trough is a flexible steel ribbon, which is pulled up to vary the discharge point as height of pile increases. Reloading is by a pivotal reloader (placed between the two conical piles), which carries a flight conveyer on its edge. Reloader travels on a curved track, and is drawn against the edge of the pile by wire cables winding on drums placed on the reloader. Coal is drawn to the pivotal point of reloader, where the conveyer is inclined, and is carried to screening tower. Shaking screens in this tower size the coal and deliver it direct into cars. Screenings are collected in a hopper and transferred to a screen house for sizing. Driving engines or motors are located near reloading tower. The plant is readily enlarged by adding additional piles. BREAKAGE in the prepared sizes varies from approximately 6% for egg to 2.5% for nut.

This type requires large area in proportion to its capacity. A 120 000-ton plant covers 250 000 sq ft, as compared with 105 000 sq ft for a MODIFIED DODGE PLANT, not including tracks. Practically all the coal is reclaimed by machinery. The modified plant usually has fixed trimmers in a storage building, with bulkheads between piles. Reloading is by a tunnel conveyer under each pile, delivering each size to its respective screen house, and thence to a storage pocket to be loaded into cars.

I. For large storage plants, especially for steam sizes. Consists of a side slope, as described under *E*, and single-track trestle with continuous bin chutes of sufficient height to permit feeding into a traveling cantilever trimmer, by which the coal is elevated and discharged onto storage floor. The trimmer comprises a platform mounted on trucks, traveling on a broad-gage track, parallel to dumping trestle. It carries a cantilever truss, and a scraping conveyer with a movable trough bottom, which permits discharge at any desired point. To increase capacity and prevent stored coal from flowing back on trimmer track, a bulkhead may be erected between storage floor and track. Reloading as in *E*. When surface contour will not permit an approach to the dumping trestle on easy grades, a loaded car plane can be installed. This delivers cars on the dump trestle, over which they should be operated by gravity to the apex of an empty car plane, down which the cars are lowered to the empty stand track. A steel barney, disappearing at the bottom into a pit, should be used on each plane. Plant should have ample receiving storage tracks and loaded-car stand tracks, operated by gravity, with loading scales for receiving and shipping. Hot water should be supplied for hydraulicking during winter reloading. Breakage is small when handling steam sizes.

J is a modified Dodge plant, consisting of a single-track trestle, with continuous bin chutes, on which cars are handled by a rope haulage system, spotted and dumped as desired. Hand-controlled gates feed coal from bins into an inclined trimmer conveyer, supported on a traveling truss. This traverses the length of storage floor, building up a wedge-shaped pile with rounded ends. A trimmer truss is carried on a bridge spanning the storage floor, the ends being supported on movable towers electrically operated. Reloading is by tunnel conveyer, placed under the storage floor and parallel to the trestle. A transverse conveyer receives coal from the reloading conveyer and conveys and elevates it into a screen house, where it is re-sized and loaded. Nearly 60% of stocked coal is tributary by gravity to reloading conveyers, which are fed by gates in tunnel roof; remainder is delivered to the gates by a traversing reloader. This consists of a steel truss, mounted on wheels and carrying on its edge an encircling conveyer. It is drawn against the face of storage pile by steel cables, anchored to each end of storage floor and passing around propelling sheaves mounted on reloader. Electric power, used for operating trimmer and propelling trimmer truss, is supplied by feeder rails, parallel to storage floor, and through collectors on the truss to the motors. Flexible cable supplies power to reloader from plug stations on storage floor. Tunnel conveyers and screen house are operated preferably by electricity. Plant should include extensive RR yards for receiving and shipping. Breakage is about the same as for type *H*.

K. For large storage plants and occasionally for retail yards. Stocking is by fixed or traveling tramways, spanning storage floor, traversed by cable railways. This type is readily adapted to plants receiving coal by steamers or barges, especially when storage floor is at a distance from unloading point or dock. Steeple unloading towers, at dock front, equipped with grab buckets, elevate coal to a receiving hopper. under which cable cars pass, automatically releasing a gate in the hopper which allows a fixed amount of coal to be discharged into passing car. Cars traverse a trestle to tramway and pass over it, automatically stocking the coal at any desired point, as the car door latch engages with a tripper. Reloading may be done by tunnel conveyers, transferring product to a screen house.

Quantity of coal reclaimed by gravity depends on number and location of tunnels; the remainder may be handled by dock scrapers. Cable tracks should be arranged for transferring coal direct from unloading towers to screen house. Screenings go to screenings storage, in cable cars operating in connection with an elevator. This type permits stocking of separate sizes in varying quantities by use of movable bulkheads between sizes, or by leaving valleys between them, but at a sacrifice of storage capacity. Increase in speed of stocking is accomplished by the addition of cable cars. The plant is especially adapted to irregular areas, using fixed trestles for the cable way, and for covered storage. Degradation in prepared sizes varies from 8 to 15%, due to high drops in stocking.

L. For large storage, especially at interior plants located on the Great Lakes, which receive coal by water. Grab buckets operating on traveling bridges are used for stocking. Storage floor parallels the dock front and is spanned by one or more bridges, which may be propelled along on the supporting tracks from one end of floor to the other. Bucket is suspended from a trolley, which operates across the bridge on a track hung from the

lower chord; a hinged boom, with tracks matching those on the bridge, extends over the vessel so that the bucket may unload direct from boat into storage. Reloading may be done by tunnel conveyers, or a conveyer may be placed at rear of storage floor, to receive coal from the grab bucket, either direct from vessel or picked up from stock. This conveyer transfers the coal to a screen house. For COVERED STORAGE, this type may be modified by placing fixed trolley tracks in the storage building, which match tracks on a traveling unloading tower, so that one tower may serve several tracks. Traversing trolley carrying the bucket may be operated by two methods, known as "rope" or "man-trolley." In the former, the bucket-hoisting and trolley-operating engine or motor is built into the bridge and connected to the bucket and trolley by wire cables. The operator controls the bucket and trolley. Two controlling stations are often installed, one overlooking the boat, for unloading, and the other arranged for best supervision of reloading. In the electrically-operated man-trolley, the hoisting and traversing motors are mounted on the trolley, with the controlling levers in a cab suspended from it. Usual bucket capacities are 1.5 to 3 tons for the former, and 5 to 7.5 tons for the latter. Rope trolley is cheaper in first cost, due to light construction required as compared with man-trolley; but is higher in breakage, since the man-trolley operator is always in view of his work, and can better lower the bucket onto storage pile when stocking, thus decreasing the drop and degradation. It also saves by unloading and reloading in large units. Fixed bridges may be arranged to span the screen house, thus eliminating reloading conveyers; but these are limited to small plants and require a bridge and screen house for each size stored.

This type is adapted to stocking separate sizes in varying quantities, with or without bulkheads, as under *K*, but does not lend itself to enlargement. It is usually rapid in stocking, the speed depending upon skill of operator; but speed of reloading is limited by the screen house capacity. Hoisting speed is about 500 ft per min; trolley speed, up to 1 500 ft per min. Degradation is approx 10%.

17. IDEAL SCREEN HOUSE

Essential elements. 1. Screens to remove undersize from each size reloaded.

2. Storage pockets for each size handled, to hold all the undersize made from the size being reloaded, unless sales orders can be arranged to permit loading undersize as made; otherwise, undersize must return to storage when its shipping pocket is filled.

3. Lip screens to remove pocket breakage.

4. Separate screenings storage floor, with transfer conveyers from screen house to storage, with arrangements to return them to the screens for re-sizing.

5. Arrangements to load either gondola or box cars.

6. Minimum breakage.

7. Minimum force of men, and low operating and first cost.

Design should include a receiving hopper, with feeder to shaker screens, one for each size handled, arranged one above the other and directly over the pockets. Each size of coal should gravitate directly from its shaker to its pocket, using telegraphs for reducing breakage. Loading into cars may be done as at mines, with preference for belt conveyer system (Art 8). All sizes above pea should pass over lip screens, the screenings being transferred to storage. Local and interior plants receiving box cars should be equipped with mechanical BOX CAR LOADERS. Where large tonnage is handled in reloading, the TILTING OR GRAVITY LOADER is superior to others. The first cost, approximating $50 000 complete, is warranted only where large capac is required. MOVABLE BOX-CAR LOADERS give excellent results when capac is small; cost, about $7 500 installed.

18. MISCELLANY

Mechanical car unloaders for gondola cars are generally used at shipping points at seaboard or on the Great Lakes, where large capacity is required. Car dumpers operate by elevating the car to a height from which the coal will gravitate into the vessel, at which point the car is turned over, allowing coal to run out. Box car unloaders or dumpers may be constructed like the box car tilting loader. Average rate for unloading box cars by hand is one car per hr per man.

Breakage of coal. This always occurs in handling anthracite, varying from 5 to 25%, according to method of handling. Causes are: dropping of coal, drawing coal under pressure from storage piles or pockets, poorly constructed chutes, and handling by conveyers, elevators and grab buckets. BREAKAGE FROM DROPPING varies with different classes of coal. Though it is impossible to avoid it entirely, it may be reduced by sliding

the coal in chutes or on itself; tests show that when sized coal is delivered on a pile it seeks its angle of repose by avalanching in large masses, with little breakage. DRAWING

Table 16. Breakage of Coal Due to Drop *D*, ft (R. V. Norris)

Size	Breakage into smaller prepared sizes	Breakage into small (steam) sizes	Total breakage
Broken	3% + 0.43 *D*	2% + 0.17 *D*	5% + 0.6 *D*
Egg	4% + 0.43 *D*	2% + 0.17 *D*	6% + 0.6 *D*
Stove	2% + 0.33 *D*	2% + 0.27 *D*	4% + 0.6 *D*
Nut	....	4% + 0.4 *D*	4% + 0.4 *D*
Pea	....	2% + 0.5 *D*	2% + 0.5 *D*
Buckwheat	....	1% + 0.25 *D*	1% + 0.25 *D*

COAL UNDER PRESSURE from deep bins or storage piles is a serious cause of breakage, though often neglected. When reloading from a gate under pressure, breakage is reduced by opening first those gates under the "run" or edge of the pile. CHUTES, poorly constructed, cause large breakage, and should be constructed in same manner as in breakers (Art 12). ELEVATORS and SCRAPING CONVEYERS cause breakage of from 2 to 5% for the former, and 2 to 4% for the latter. BELT CONVEYERS cause no breakage during transit, but the drop at discharge is often high unless discharge chute receives the coal at a tangent. GRAB BUCKETS cause some breakage, the percentage being in direct proportion to the length of their cutting edge per unit of coal handled; hence the greater the cubic capacity of the bucket the less the degradation.

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SECTION 35

PREPARATION AND COKING OF BITUMINOUS COAL

PREPARATION

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Note.—Numbers in parentheses in text refer to Bibliography at end of this Section.

PREPARATION AND COKING OF BITUMINOUS COAL

1. GENERAL CONSIDERATIONS

Purpose of COAL PREPARATION is to increase the value of fuel by making it more suitable for uses of the consumer (1). This is done by: (*a*) screening or sizing; (*b*) mixing or blending; (*c*) cleaning. By combining any 2 or all of these methods, coal can be prepared to standard specifications. A preparation plant should produce clean coal, and refuse free of saleable coal.

Screening or sizing was the first method of increasing the value of coal as a fuel; mixing or blending is important in preparing coal for steam, metallurgical, and gas-making purposes. For steam coals, controlling the proportions of coarse and fine increases boiler effic, control of ash and fusion, and of rate of fuel feed. For metallurgical coals, mixing and blending permit control of ash, sulphur, phosphorus, ash-fusion temperature, and volatile matter; and for gas coals are often used to control size consist and analyses. Table 1 shows distribution of $-^3/_8$-in size in raw slack and after mixing.

Table 1. Distribution of $-^3/_8$-in Size in $1\ ^1/_8$-in Slack (4)

	Raw mine slack	Prepared and mixed slack
Number of cars	113	100
Aver % of $-^3/_8$-in	50.7	47
Percent of $-^3/_8$-in	Cars, %	Cars, %
Under 35	9	5
35–40	6	11
40–45	17	17
45–50	17	38
50–55	19	18
55–60	15	9
60–70	12	2
70 and over	5	0

2. MARKET STANDARDS AND USES OF BITUMINOUS COAL (1)

Definite standards for bituminous are lacking with which to compare results; anthracite is accepted or rejected on visual inspection (Sec 34). Table 1a shows factors considered for acceptance of bituminous coal; complete list in Bib (3). Visual inspection is not always sufficiently accurate for control, due to the variety of uses, as follows (3): stationary steam generation, colloidal; pulverised coal, hydrogenation; gas and coke making; domestic; ceramic products and cement burning; briquetting; locomotive fuel, bunker; metallurgical; cargo.

Table 1a

Type of Coal	Metallurgical	Steam	Gas	Domestic
Ash	x	x	x	–
Sulphur	x	–	x	–
Phosphorus	x	–	–	–
Fusion	x	x	x	–
Moisture	x	x	–	–
Proximate analysis	–	x	–	–
Size	–	x	x	x
Visible impurities	–	–	–	x
Dustiness	–	–	–	x
Uniformity	x	–	x	–
Volatile matter	x	–	x	x
Btu	–	x	x	–

3. HAND-PICKING

Hand-picking (1) is still widely practiced on sizes above 1 in. Tests on a large scale at a commercial plant show the accompanying percentages of extraneous matter (above 1.60 sp gr) remaining in the coal. Table 2 shows amount of waste removed per picker in the same tests.

Sizes, round hole, in	Sink at 1.60 sp gr (Art 8) in cleaned product, %	
	Limits	Aver
2 to 4	0.7–4.5	2.2
1 to 2	1.2–7.1	3.2

Table 2. Tons Waste per Hour per Hand-picker

Size, round hole, in	Limits	Aver
Over 4	0.8 -2.5	1.9
Over 2	0.05–0.4	0.3
2 to 4	0.15–0.3	0.24

Compared with hand-picking, figures in Table 2a indicate the elimination obtainable on various sizes by modern preparation plants (1).

Ash in the 1.60 sp gr sink material (Art 8) in hand-picked coal usually averages between 45 and 50%; that from mechanical preparation, 35–40% ash. Loss IN REFUSE from hand-picking, which generally contains 50–60% coal, is a real loss. The value of good coal in the pickings, if recovered, is often more than the cost of picking labor. Table 3 gives data on hand-picking at 9 plants; costs adjusted to equal labor scale of 1937.

Table 2a. Results of Mechanical Cleaning

Size, round hole, in	Percent sink at 1.60 sp gr	
	Steam coal	Metallurgical coal
2 to 4	0.5	0.2
1 to 2	1.0	0.5
3/8 to 1	2.0	1.0
−3/8	2.0	1.0

Table 3. Data and Costs on Hand-picking

Size of coal picked, in	A +4	B +4	C +2	D +4	E +4	F +3	G +2	H +2.5	I 2 to 4
Percent of raw coal rejected	10.6	23.5	2.5	10.4	10.0	3.7	3.4	13.3	1.9
Number of pickers	12	4.5	12	6	6	3	16	11	8
Tons per man:									
per elapsed hr	1.9	1.6	0.4	1.1	0.8	1.0	0.05	0.86	0.24
per operating hr	1.9	2.6	0.6	1.4	1.0	...	....	1.14	....
Cost per ton of rejected material	$0.38	$0.45	$1.86	$0.67	$0.88	$0.72	$4.28	$0.83 (a)	$2.97 (b)

(a) Contained 60% of coal in addition. (b) Less than half of total impurity (4.1%) removed by picking.

Hand-picking is done on conveying or shaking tables, in chutes, or in RR cars as material is loaded. The Marcus screen is also used as a picking-table.

Conveying picking-tables are belts or apron-pan conveyers, either flat-top or with skirt boards to guard the side links and chain. Flat-top table is preferable, as pickers can work faster by sliding off large pieces instead of lifting them over the skirt boards. Belts are used as combination picking-table and loading boom. Speed of 50 ft per min on conveyer picking-tables is preferred; max, 60 ft. Width should not be over 60 in.

Shaking picking-tables have greater capac, since pieces too large to lift can be guided off the table. The material spreads evenly at one particle depth, if not retarded by perforated plates.

Tables have steel or wood frames with steel decks 3/16–3/8 in thick, supported on inclined stilts of wood or steel, pivoted or fixed at both ends; tables should not be over 60 in wide; lengths, 10–30 ft, depending on number of pickers. Top of table not over 32 in above pickers' platform; slope, 5°–8°; speed and stroke adjusted for a speed of 50 ft per min of the material passing over the table; stroke, usually 4.5 in; speed, 150–155 rpm. Drive is by crank or eccentric, with flywheel.

4. FEEDERS

Feeders give steady flow of material. Storage bins or hoppers, above the feeders have sufficient capac to provide continuous flow at a rate obviating undue delays. Feeders are adjustable, for handling dry or wet materials at a uniform rate for a given setting. Following are types of feeders.

Fan-shaped plate, sloping 10°–18°. Coal is dumped on the narrow end; the wide end usually delivering to a screen, has adjustable convex curvature for spreading the coal evenly. It is inexpensive, has no moving parts, and causes little or no degradation.

Adjustable vertical gate, for free-flowing fine materials, is placed in a bin or hopper with a bottom inclined 45°, and set for the proper opening, or raised and lowered by connecting to a power-driven eccentric.

Swing-hammer regulator. When delivering lump material from hoppers, bridging is prevented by using large openings, the regulator keeping the material in check. Heavy hammers hang in front of the opening above the feeder, limiting flow to the depth established by height of the hammers above the feeder. When a large lump appears, one or more hammers rise to let it pass.

Belt feeders are well adapted for fine, dry material under 2.5 in, at capacities to 150 tons per hr; widths are to 42 in, lengths to suit conditions; min length depends on angle of repose of material at the height of feeder opening. Belts run at 20–60 ft per min. To increase the life of a belt handling abrasive materials, steel armor strips are riveted to the carrying side, with countersunk heads on the pulley side.

Apron feeder, for large capac and large lumps, provides regularity of flow with small power consumption and little headroom. Storage bin over the feeder should be designed to relieve as much weight as possible from the feeder pan and chain.

This feeder consists of a series of overlapping steel pans, to form a continuous conveyer carried between heavy roller chains and attached rigidly to the inner links. The conveyer moves between 2 fixed skirt boards, permitting large capac. The chain runs on an angle-iron or rail track; where it undergoes undue stress, a roller-track feeder may be used; the load on feeder and wt of the chains and pans are carried, through the chain bars, on larger rollers Speeds of apron feeders vary from 5 to 50 ft per min; widths, 24–60 in; lengths, 3–10 ft between sprocket centers. Capac depends on depth of material, which, with a regulating gate, should be twice the size of lump.

Roll feeder can proportion very accurately and has a relatively large capac; headroom is greater than for some other feeders. It is not adapted for materials over 6-in diam, but for conditions within its range its simplicity and accuracy are advantages. Peripheral speed, 5–20 ft per min.

Vane feeder resembles the roll, but the rotor has 4 or more vanes. It may be used as a measuring device, giving a continuous, uniform feed, and is adjustable for predetermined deliveries. It is not adapted to wet sticky material, as the vanes soon become packed. Like the roll feeder, it may be placed at bottom of a bin or at end of a chute; the material is deposited directly or carried farther by chute. Speed, 10–20 rpm; capac, 650–3 250 cu ft per hr at 15 rpm; size, 12–24 in diam, 15–18 in wide.

Reciprocating feeder, especially useful for lump coal, is a reciprocating steel plate forming the bottom of a hopper. It is supported on rollers and driven by an eccentric or crank. On forward stroke, the material is carried away from the hopper; on the reverse, the plate slides back, thus unloading a certain amount at end of the plate. Rate of feed is varied by changing speed of the eccentric or crank, or size of opening. This feeder is simple in construction and operation, accurate in delivering specified amounts of material at a uniform rate; and requires little headroom and power. The plate is subject to considerable wear, and the rollers, unless rotating freely, develop flat spots. Feeders are 15–60 in wide and 5–12 ft long; stroke, 3–8 in; speed, 10–50 rpm; power, 0.5–15 hp. A flywheel on the drive shaft is desirable.

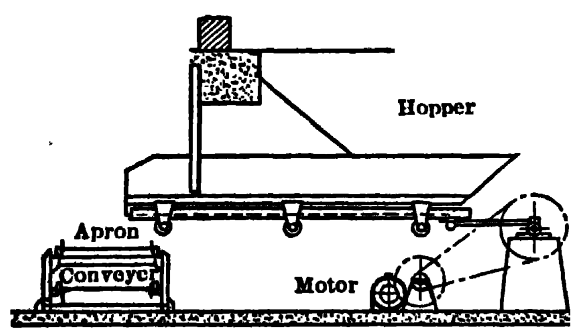


Fig 1. Reciprocating Feeder

This feeder may be supported on wheels (Fig 1), or suspended from pivoted hanger rods, or by a combination of both; feeder plate may be horiz or slope downward. It is not self-cleaning unless run at high speed. The hopper is best made with 45° sides and a vert end at least 12 in high (or a slope not less than 45°). It should take as much weight as possible from the feeder plate, which can then run faster, with less power and wear. When feeding a belt conveyer or screen with mixed sizes, the feeder ends should have extended fingers, of grizzly bars or rails, to deposit the fines on the belt as a cushion for coarse lumps.

Disk feeder. Bottom edge of the hopper is slightly elevated above the disk, to allow material to flow and assume its angle of repose within the plate's circumference. The hopper throat may be adjustable, and one or more plows set to discharge from the rotating plate. An agitator attached to the plate may extend up into the hopper to prevent clogging or bridging; this adapts the feeder to damp materials. The drive is by constant-, variable-, or multiple-speed motors, through open gear, chain drive, or gear reducers.

5. SIZING AND CRUSHING

Fixed-bar grizzly consists of screen bars set at an angle to allow the material to flow over them by gravity (2). The bars are usually of tapered or diamond-headed sections, set with the thicker edge at the top. Screens are 3–6 ft wide, 8–12 ft long; suitable for scalping large sizes when material is not sticky; used oftenest over bins or in chutes to relieve the crusher of small sizes; are the simplest and cheapest to install and operate; have practically unlimited capac, require no power and little attendance, and withstand rough work. Principal objections: (*a*) headroom required; (*b*) difficulty of changing size of product; (*c*) trouble due to changes in coeff of friction between screen and material; (*d*) low eff in exact sizing; (*e*) incomplete screening; (*f*) breakage of product.

Revolving or disk grizzly consists of a series of parallel shafts carrying disks, the spacing and diam of which leave square openings of the size desired. The shafts all revolve in same direction, but at different speeds; the first, at the receiving end of the grizzly, revolving slowest, succeeding ones slightly faster; slope, 15–20°. The openings are 1.25–4 in with 5–8-in shafts; disks, 9.5–18-in diam; screen surfaces, 24 by 68 in to 48 by 81-in; capac, 100–200 ton per hr for scalping; 75–150 ton for sizing; power, 5–7.5 hp. Space required, 73 by 56 in by 32 in high, to 84 by 78 in by 42 in high.

Revolving screen (2) is rare, due to excessive breakage; may be used as a combination breaker and screen (Bradford breaker), and a modification for breaking laminated material in the coarse refuse from a wet cleaning plant, screening out the fines and re-treating them. Max size of material, about 3 in. Screens may be 24–72 in diam, and 6–24 ft long, in 2 or more sections; they are supported on through shafts, or trunnions. Since only 1/6 of the perimeter is covered at one time, the screen has a low capac per unit area; more space is required for a given capac than other types. For damp coal, increase screen area 50% Screens are set on a slope of 5°–7.5°; peripheral speed, 225–250 ft per min; above 275 ft per min, effic decreases rapidly. Power for a screen revolving on a shaft; hp = tons per hr ÷ 10; or diam screen (ft) × length (ft) ÷ 9. Two or more screens are often set concentrically on same shaft; innermost is the coarsest, the others making additional separation. This reduces the space needed for a given plant.

Diam hole, in	Tons per hr per sq ft area
1	0.33–0.25
1.5	0.50–0.25
2	0.67–0.50

Shaking screens are advantageous in that the entire area is utilized; hence, greater capac for a given space. They require less headroom, can be used for damp coal and for dewatering, and are well adapted to sprays. The coal slides over the surface with little breakage. Disadvantage is the vibration set up in the structure. Shaking screens are widely used in tipples for sizing dry run-of-mine coal over 0.5 in and not under 0.25 in, incidentally serving as conveyers for loading on several tracks.

The frame is of steel plate and structural shapes; may have wooden sides. The screen may be used not only for sizing but also as a shaking conveying chute, by blanking off part of it with plates. Primary or run-of-mine screens are heavy, to withstand hard service. Drive and support should be well braced, to prevent flexure and shear of rivets and bolts. Lighter steel, or wood and steel, frames are suitable for secondary screening of small sizes. Light screens are run at higher speeds and with shorter strokes than for run-of-mine. Their inclination is less, permitting more exact sizing without reducing capac. Shaking screens may be up to 12-ft width; usually, 5–8 ft. Width should be figured to give a thin bed, without excessive speed of flow; aver flow, 50 ft per min; max, 75 ft; length must satisfy the tolerance of undersize in the oversize; the longer the screen, the greater the degradation.

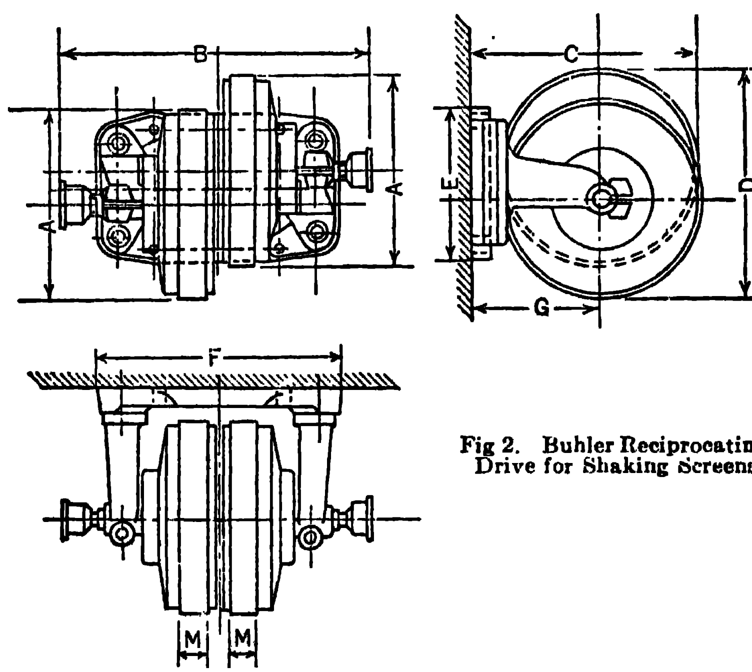


Fig 2. Buhler Reciprocating Drive for Shaking Screens

A shaking screen may have one frame, or 2 or 3 superimposed frames with one drive; or, the frames may be tandem, one drive to each 2 frames. When superimposed, the plates with larger holes are on the top deck; in tandem arrangement, the fine-hole screens are at the feed end, the others at discharge end. Superimposed frames are spaced to

allow room between decks for changing and removing the screen plates. Bottom deck may be blanked with plate, for use as a shaking conveyer. Chutes are attached to bottom or side for discharging the products at desired points. Screens are hung by rods, wire rope, wood-bound hangers, or set on wooden or steel stilts, fixed or pivoted at each end. The drive is usually an eccentric or crank. A recent application is the BUHLER RECIPROCATING DRIVE (Fig 2); its dimensions with flywheel follow:

Rpm	Dimensions, in								Weight, lb	
	A	B	C	D	E	F	G	M	Without load	Max added load
460	13 3/4	12 1/2	16 1/8	15 3/4	9 3/16	13 5/8	8 15/16	1 1/2	90	40
460	12 1/2	12 1/2	14 1/8	14 1/2	7 7/8	11 1/2	8 3/4	1 1/4	75	31
480	11	12 1/2	12 5/8	12 3/8	7 1/8	11 1/2	8 3/4	1 1/4	60	23
540	8	12 3/4	9 1/4	9 3/8	5 1/4	10	6 1/4	1 1/4	34	8

Table 4. Data on Run-of-mine Screens
(All with 4-in holes, except as noted)

	Tons per hr per sq ft of perforated area			Loaded product
	Feed	Through	Over	Moisture at 105° C
A	7.63	5.58	2.06	2.9
B	3.22	2.34	0.88	3.3
C (a)	2.33	1.30 (a)	1.03	2.5
D	3.71	3.18	0.53	2.0
E	5.16	4.36	0.80	2.0
F (b)	6.22	4.98	1.24	......
G	2.49	0.79 (c)	1.70	1.4 (d)

(a) Marcus screen; through product 2–4 in. (b) 3 3/8-in holes. (c) Through product 3 3/8–4 in. (d) Moisture at 85° C.

The 3 variables of slope, speed, and throw are interdependent, and the right combination must be ascertained for rapid and complete screening. Run-of-mine shaking screens average 100 rpm, with a throw of 6 in, there being less variation in these adjustments than in the slope, which averages 3.5 in per ft, or 16° 55′. Light fine-coal screens run at 130–160 rpm, with 3 to 5.5-in stroke. The less the slope the higher the speed and longer the stroke for a given capac. Table 4 gives capac of dry screens at 7 large installations; for other examples, see Table 9.

Parrish screen has wide applications for bituminous coal. It usually has wooden sides, is suspended on flexible wooden hangers, and driven by wooden arms, with spring pieces to provide flexibility. It is cheap and light in weight. Sides of frame are usually 3 or 4 in by 6-in oak or yellow pine, sometimes 6-in angles or a plate and angles; cross members, 2.5- or 3-in angles. Punched plate, of ordinary steel, Monel metal, bronze, or stainless steel, is flanged and bolted to sides. Wire mesh, sometimes used for fine sizes, tends to blind at coarser sizes. Screens are 4–7 ft wide and 12–70 ft long; complete deck weighs 1 000–6 000 lb. Slope, 0.5–1.5 in per ft; the steeper slope increases conveying capac at expense of effic; on a low slope, lump coal travels faster than fine; aver travel with 5-in stroke and 160 rpm is 60 ft per min.

Frames are commonly supported from above by oak, hickory, or ash boards, 1 by 8 in, rigidly fastened to screen and to the overhead structural member; flexibility of the boards allows sufficient movement of screen. Hanger boards often slope backward 1.5 in per 12 in, so that screen rises on forward stroke and accelerates movement of coal. Steel hangers or chains may be used instead of boards, and screens are sometimes supported from below on flexible legs. Single or superimposed groups of 2 or 3 screens are oscillated from cranks or eccentrics through wooden arms, rigidly bolted to each screen, with a part reduced in cross-sec to provide flexibility; steel arms with wrist-pin connection to screen frame are also used. Eccentrics are commonest on driving shafts, but cranks have advantages. If more than 1 screen is driven from same shaft, eccentrics or cranks are at angle of 180° (for 2) or 120° (for 3) to neutralize structural vibrations; power, 3–10 hp per screen. Usual adjustments: for lump coal, speed 100–150 rpm, throw 6–4.5 in; for prepared sizes, speed 125–160 rpm, throw 5–4 in; for steam sizes, speed 150–200 rpm, throw 4.5–3 in.

A new type of drive for shaker screens (Link-Belt Co) is shown in Fig 3. The motor and eccentric shaft are mounted on a counterweighted base, suspended by links from the main structure. The thrust to oscillate the screen is absorbed by the suspended base, thus eliminating vibration in the machine.

Vibrating screens. The impulses are imparted mechanically or through electromagnets; the movement may be circular, elliptical, or straight-line. The whole frame may vibrate, or the vibrating mechanism be attached to the screen cloth on the longit center at 1 or 2 points. Screens may be horiz, or, for moisture reduction, inclined 2°

upward towards the discharge end. They are 1-, 2-, or 3-deck; 2-6 ft wide and 3-20 ft long; usually supported on a fixed base, through an anti-vibrating medium of springs, rubber, cork or wood. The frame is fixed or adjustable as to inclination; suspended by wire ropes or rods, with springs and turnbuckle vert adjustment. For capacities, see Table 9.

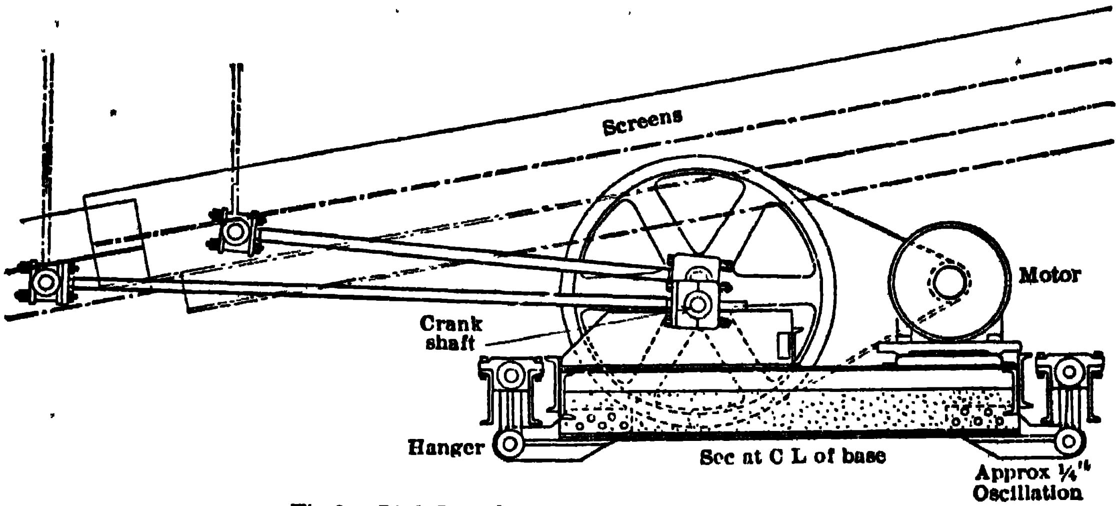


Fig 3. Link-Belt Co's Drive for Shaking Screens

Re-screening. Limited space and trackage may require loading of 2 or more coal sizes on same track; the main shaker, also, may be too short to make all the sizes. Such conditions involve re-screening of minus 2-in or 1-in slack or smaller, which may be elevated to screens set over the tracks at suitable distances from loading points, to give room for shifting RR cars. The re-screens are set over 2 or more bins, depending on number of sizes. Sized coal (except slack or fines) is deposited in the bins by spirals (see Sec 34) or lowering chutes, to avoid breakage. Vibrating screens are preferred, due to their larger capac for a given height and space.

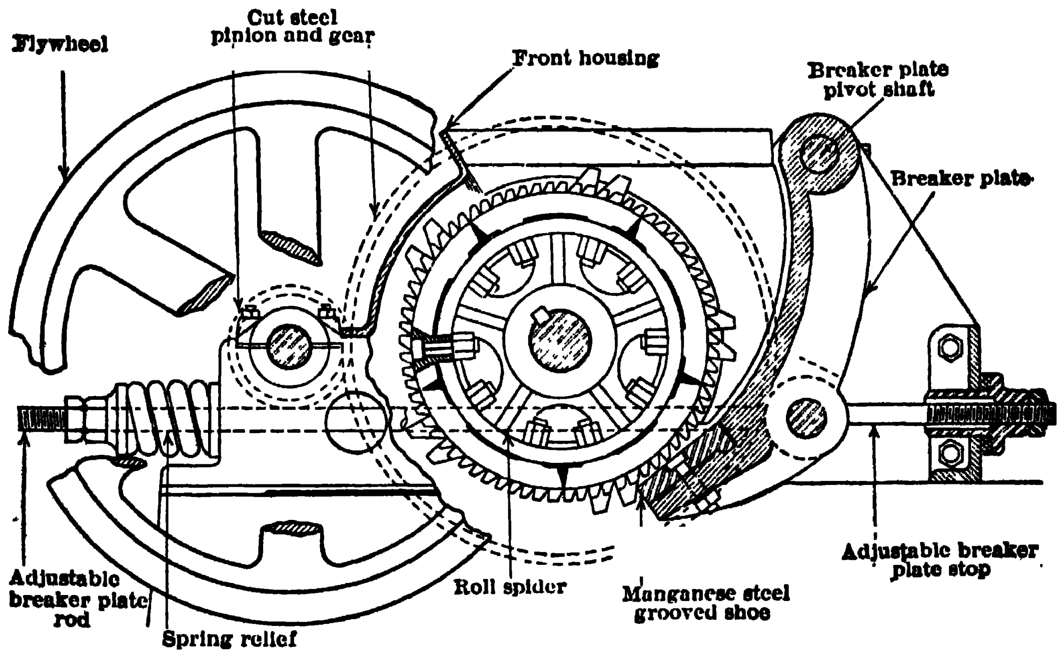


Fig 4. Single-roll Crusher

Crushers. By proper selection of equipment, the yield of coarse sizes is greater than when rough mining and handling are permitted; crushed products have more cubical pieces. Crushing may also be applied in connection with washing, to free attached particles of rock; or to raw coal prior to washing, or to refuse before re-washing. Crushers should have uniform feed, with provision for removing undersize product by by-passing. For rolls, the feed is adjusted to flow across their full face to prevent uneven wear; they should have a spring relief (Fig 4), to allow hard substances, as tramp iron, sulphur balls, or wood to pass through without breaking parts or bending a shaft.

Single-roll crusher has 2 side frames connected by cross members, on which are carried a revolving roll and a breaking plate held stationary by spring action. The drive, through gears, gives a peripheral speed of 400–600 ft per min. The roll has crushing teeth about $^7/_8$ in high and slugging teeth 1.5–4 in high, depending on size of feed. Size of product is adjusted by moving the crushing plate towards or from the revolving roll.

Double-roll crusher has smaller capac than the single roll, but is more effic, and produces larger yield of a desired size. There are several makes. One type has its 2 rolls connected by a chain drive, and has wide latitude of adjustment for different sizes of feed and product. The double-roll type requires less power, as there is less friction than against a stationary plate. Crushing is more uniform because the absence of backlash in the drive permits the teeth to mesh properly. At peripheral speed of 300–600 ft per min, rolls produce more coarse sizes than at higher speed. For large capac, oversize rolls are desirable. Rolls may be solid with inserted teeth, or in segments bolted to a C–I or steel spider; segments have integral-cast teeth, and may be of chilled C I, manganese steel, or heat-treated alloy steel, the latter being favored.

Power depends mainly upon hardness of the coal. Soft coal requires $^1/_8$ hp per ton per hr (crushing to 4-in aver cube) to $^1/_4$ hp (crushing to $^5/_8$-in). For hard coal, power per ton per hr is about doubled for corresponding sizes. Worn teeth diminish capac and increase power required.

Pick breaker pierces the coal by rows of vert picks, making stove, furnace, and egg sizes. The coal, carried on a plate conveyer, is stationary while the picks descend, and is moved forward when the picks rise. The steel frame supports a plate conveyer carried on a chain. There are 2 sets of picks, for preliminary breaking and final sizing, mounted in plates bolted to an oscillating frame.

Hammer breaker is chiefly for producing fines under 0.5-in for coking, but is applicable also to many other purposes. It is designed for products from $-^1/_8$-in to -2-in; also to break preliminary to washing, or to crush a middlings product, of laminated coal and rock, prior to re-treatment.

Bradford breaker is a slowly revolving cylindrical screen, within which the coal is raised by internal shelves, and dropped repeatedly until broken small enough to pass through the screen. Speed, 15.5–17 rpm, according to size. Maintenance cost is low; capac, 25–600 tons per hr.

Table 5 shows operating factors of several types of crushers (*Coll Eng*, Oct, 1931).

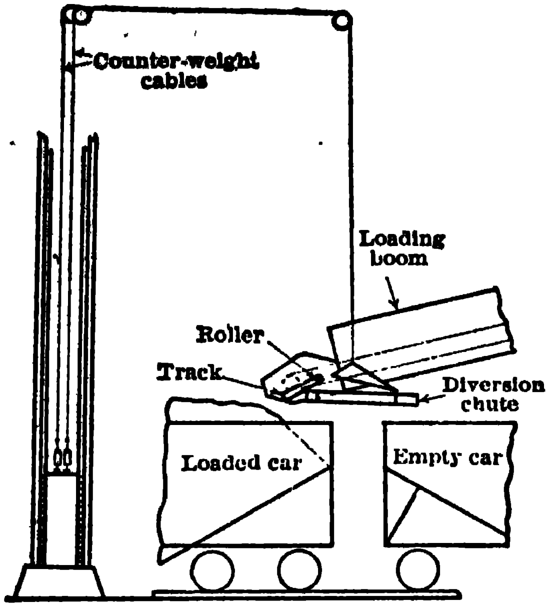
Table 5. Crusher Operating Features

Type of machine	Max feed, in	Size of product, in	Output, tons per hr	Hp per ton per hr
Vert pick..	run-of-mine	8–2	45–250	0.09–0.25
Horiz pick...	"	6–2	20–40	0.25–0.35
Single roll...	7–24	below 2	30–150	0.3 –0.75
Double roll..	7–10	2–1	10–70	0.5 –1.00
Four-roll....	6, r-o-m	2–1/2	8–40	0.6 –1.50
Flex hammer	14–24	2–1/4	12–270	0.4 –3.0
Pulverizers..	1–8	1–1/8	5–800	4.0 –10.0

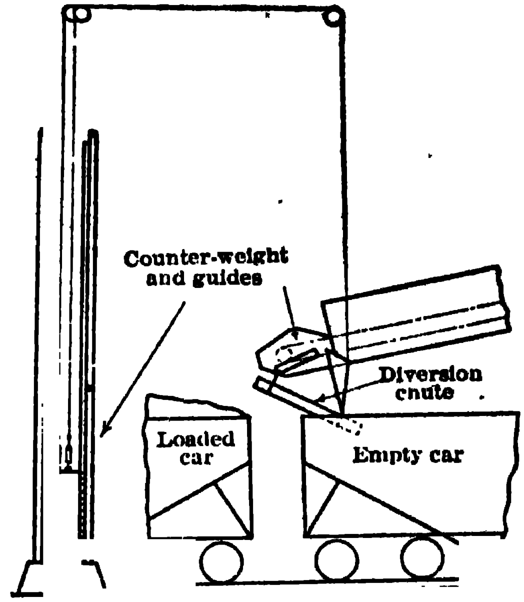
6. LOADING BOOMS AND MIXING (see also Sec 27)

These may consist of an apron or belt conveyer, shaking pan, or a low-flight, double-stranded chain conveyer with steel bottom; the conveyer is not over 60 in wide, for discharging into RR car. The boom is usually on the center line of a track and discharges down-grade, the car traveling away from it while loading; the boom may also be swiveled for discharging into a box-car loader between the tracks. For other details, see Sec 27. Speed of conveyer is from 50 ft per min on combination picking table and boom, to 100 ft per min on belt-loading booms. Capac, tons per hr = 1.55 SWT, where S = speed, ft per min; W and T = aver width and depth of material, ft. Power, depending on length of boom, capac and slope, is 5–60 hp; for boom hoists, 3–10 hp. Fig 5 shows use of an automatic loading chute at end of boom.

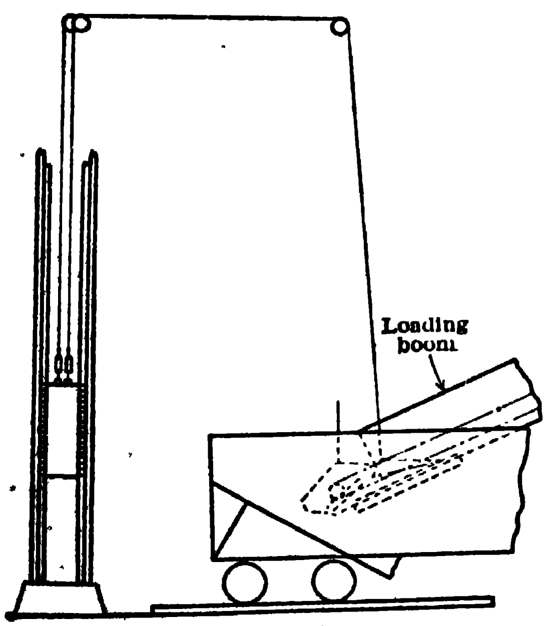
Simplest form of mixing conveyer straddles all the loading tracks and is arranged so that as many booms as desired may be raised to discharge into the conveyer on one or both of its strands, the mixed coal then being spouted to any one track. For large tonnages, the screens may discharge the different sizes onto band-conveyers running across the tracks and delivering to the horiz sections of the several booms or spouts above the tracks; each cross-conveyer has gates to discharge part or all of its load onto any boom or chute. For blending, the various sizes may be conveyed, elevated, or stored in bins having spiral lowering chutes (Sec 34) and withdrawal chutes. The coal may be drawn from bins by variable-speed feeders, and passed over degradation screens and anti-breakage spouts to mixing conveyers, loading booms, or telescopic loading chutes.



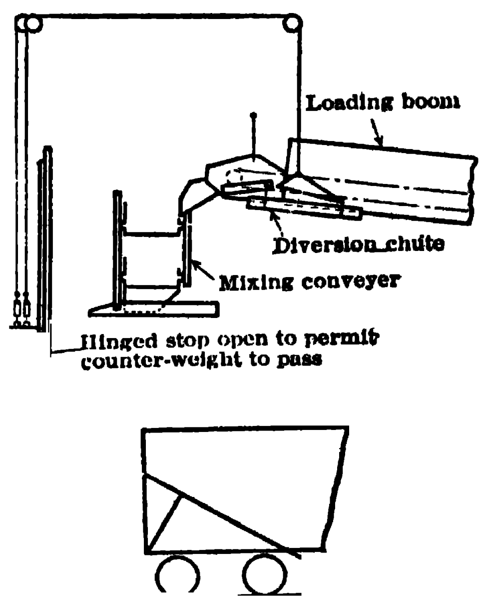
Position No. 1
Topping out car. Counter-weight at rest., to release chute upon upward movement of boom



Position No. 2
Boom raised, for chute to divert coal into oncoming car. Cables slack



Position No. 3
Empty car in loading position. Boom, dropped into car, bringing counter-weight into play and moving diversion chute under boom



Position No. 4
When mixing conveyer is used at outer end of boom, an intermediate hinged stop is required for the counter-weight. Counter-weight rests on this stop for positions 1, 2 and 3, and passes through it when boom is loading into conveyer

Fig 5. Automatic Coal-loading Chute

7. CONVEYERS. CHUTES AND LAUNDERS (see also Sec 27)

Conveyance includes: handling of raw coal; collecting for selective loading; delivery of lump and egg to rolls; re-screening operations and delivery of their products to pockets and thence to cars; delivery of coal to cleaning units, and sized products from screens to cars; disposal of refuse. Commonest types of conveyer: (*a*) inclined or horiz longit shaking; (*b*) inclined side-shaking; (*c*) flight; (*d*) belt; (*e*) pan; also, screw and bulk conveyers, chutes and launders.

Longitudinal shaking conveyers are for small tonnages of prepared sizes. They are inexpensive but noisy, and, unless designed to convey only one layer deep, cause coal breakage.

Moisture, temp, and shape of broken coal affect speed of travel; in general, to convey 100 tons per hr, a width of 30 in, slope of 6°–7°, and 150 6-in strokes per min is correct practice. Wooden hangers, with min length of 5 ft (aver 8 ft; max 12 ft), spaced 5–7 ft apart, are 8–10 in wide and 0.75–1 in thick. By inclining them forward in direction of travel, the conveyer slope may be decreased (compare shaking screens, Art 5). A conveyer with 10-ft hangers, inclined 3 in, and oscillated at 150 strokes per min, may be set horiz; speed of travel for prepared sizes, about 60 ft per min.

Side-shaking conveyers are rare. For the same service they must be steeper than the longitudinal type, and corners should be rounded to avoid breakage.

Flight conveyers (see also Sec 27), applicable to all tonnages, are especially suitable for distributing and mixing at several discharge points. They have the advantage of carrying a load on both runs, if desired.

The flights are 6–15 in deep and 12–60 in wide, of $^3/_{16}$–$^3/_8$-in plate, dished or reinforced with structural angles. Flights 16 in wide or less are carried by a single strand; wider ones by double strand of roller, strap, bar-link, or drop-forged chain. Speed for roller chain is usually 100–150 ft per min; bar-link and forged chains, 50–150 ft, 80 ft per min preferred. Roller chain is adapted to dry coal, has less frictional resistance, but weighs more per ft for a given strength than forged or bar-link; latter 2 chains are better for wet coal or to operate in water; for refuse, the bar-link is recommended. Flights 3 in high serve well for conveyers that are horiz or inclined up to the angle of repose of sized material.

Materials used in flight conveyers handling wet, corrosive, or abrasive coal or refuse: chains should be of corrosion-resisting steels; heat-treating increases strength and resistant properties; sprockets are of cast steel, for strength and to permit welding of teeth on worn sprockets; conveyer bottoms are of $^3/_{16}$–$^3/_8$-in mild steel, or of abrasion- or corrosion-resisting plates; flights of spring steel take polish and wear longer than mild steel.

Belt conveyers are for comparatively long distances and dry coal; widths, 12–60 in; speed, 200–400 ft per min; inclination, up to 18° or 20°, depending on size of coal. They are adapted to all sizes, including run-of-mine, and for capac to 1 500 tons per hr.

Pan conveyers are best for large tonnages, short distances, and inclinations to 30°. They are usually of $^3/_{16}$- or $^1/_4$-in plate, in double-headed flights, and carried on strap-roller chain at 9-, 12-, 18-, or 24-in pitch, depending on capac. Rollers travel on angle-iron or 20- to 30-lb T-rail track; speed, 75-100 ft per min.

Costs. Of the mechanical conveyers, the pan is most and belt conveyer least expensive in first cost. Operating cost and upkeep are about the same for all inclined conveyers, which are normally lower in first cost, operating, and upkeep, for same vert lift, than gravity-discharge elevators.

Chutes are conveyers only in a limited sense, their normal function being to lower coal without breakage. They have rectangular, rounded or semicircular cross-sec. Chutes that change direction should be of spiral form or with warped or triangular plates, so that coal will slide freely (Sec 34); as they wear considerably, especially in handling wet coal and refuse, bottom and sides should be made of, or lined with, abrasion- and corrosion-resisting metals. Slopes vary with moisture, temperature, shape and size of coal, and nature of chute bottom. When steeper slopes than those required for flow are unavoidable, retarding devices are placed in chutes, such as weighted chain, loops of wire rope, belting, or other baffles. Wooden chutes are lined with steel plate. To prevent blocking, width of chute is about 3 times the size of largest lump.

Storage bins, for raw coal and for sized coal intended for blending, require LOWERING CHUTES OR TELEGRAPHS (Sec 34) to reduce breakage.

Launders convey products by aid of water on flat slopes. Very little if any breakage of coal occurs in well designed launders. Capac depends on the pitch and amount of water rather than on the width. (For sand-conveying launders, see Sec 10, Art 92.) Launders may be of concrete, steel, or wood-lined; cross-sec, rectangular or semi-circular.

8. TESTING FOR METHOD

General. Design of a preparation plant should begin with careful examination of the coal to be treated. Investigation may start with samples from boreholes, to be followed, as mine development advances, by channel samples from working faces, and finally by bulk samples representing aver yield expected from the mine, and depending as much on mining method as on nature of seam. Essential factors: (*a*) distribution of sizes in run-of-mine output, ascertained by screen analysis; (*b*) distribution of impurities, all heavier than coal, among the several sizes, and their total amount in the mine output, both of which features are determined by float-and-sink analysis (see below). Correlation of screen and density analyses will then indicate: (1) whether a coal meeting the specifications of the expected market can be prepared; (2) whether a reject can be discarded (also, its amount) sufficiently free from coal to be not worth re-treating; (3) amounts of material of intermediate grade to be re-crushed and treated, if not salable as an inferior fuel; (4) at what sizes and densities separations should be made for max profit. To supply a specialized market, separations must sometimes be made at sizes or densities not favorable to the coal in question, entailing added cost for construction and operation, and increased losses in refuse.

Sampling. Methods of sampling in a mine are described in Bib (11); those for bulk or cargo lots, in Bib (12). Following data refer mainly to sampling for plant control, for examination of raw coal as to its washability, or in general, where a flowing stream of

Table 6. Increments for Plant-control Sampling; Minimum Number and Weights in Pounds (Pittsburgh Coal Co)

Purpose of sample	Testing for ash					Separation tests Gross sample			Sizing tests Gross sample			H_2O at 85° C, individual samples	
	Tests of Individual Increments		Gross sample										
Material and size	No increments	Wt per increment	No increments	Wt per increment	Total wt	No increments	Wt per increment	Total wt	No increments	Wt per increment	Total wt	No samples	Wt per sample
Raw coal, −4-in	6	20	15	20	300	25	20	(*c*)	35	20	(*c*)	3	50
″ ″ −2 or −2.5-in	6	12	15	12	180	25	12	(*c*)	35	12	(*c*)	3	30
″ ″ −3/8-in	6	3	15	3	45	25	3	75	25	3	75	3	10
Raw or cleaned coal, 2–4-in	4	10	15	10	150	25	10	250(*a*)	25	10	250	3	30
″ ″ ″ ″ 1 1/8–2-in	4	8	15	8	120	25	8	200 (*a*)	25	8	200	3	25
″ ″ ″ ″ 3/8–1 1/8-in	4	6	15	6	90	25	6	150(*a*)	25	6	150	3	20
″ ″ ″ ″ 3/8–2-in	4	8	15	8	120	25	8	(*c*)	35	8	(*c*)	3	25
Cleaned coal, −4-in	6	15	15	15	225	25	15	(*c*)	35	15	(*c*)	3	50
″ ″ −2 or −2.5-in	6	10	15	10	150	25	10	(*c*)	35	10	(*c*)	3	30
″ ″ −1 1/8-in	6	6	15	6	90	25	6	(*c*)	25	6	(*c*)	3	20
″ ″ −3/8-in	4	3	15	3	45	25	3	75(*a*)	25	3	75	3	10
Refuse (*b*), −4-in	6	30	15	30	450	25	30	(*c*)	35	30	(*c*)	3	50
″ (*b*), −2 or −2.5-in	6	15	15	15	225	25	15	(*c*)	35	15	(*c*)	3	30
″ −3/8-in	6	5	15	5	75	25	5	125	25	5	125	3	12
Filter prod, sludge, etc	.....	.....	15	2	30	.....	.....	.....	15	2	30	3	2

(*a*) Gross wt of sample is increased if any of it is to be removed, as "head" sample, for other tests. (*b*) For a sized refuse, wts to be taken are those specified for corresponding sizes of coal, plus 50%. (*c*) Total wt may not be less than sum of wts specified for individual sizes composing the gross sample; either number of increments or their individual wts may be increased to yield this total.

coal can be cut for sampling. The material taken at one cut to represent the cross sec of the flow is called an "increment"; sum of these constitutes the gross sample. The number and weight of individual increments required to yield an authentic sample depend upon: (*a*) max and min size of fragments; (*b*) tonnage represented by the sample; (*c*) nature of test to be applied to the sample, whether a screen analysis, a float-and-sink test (below) with or without previous sizing, or a chemical analysis.

Adherence to a fixed sampling schedule is desirable for comparability of results. A schedule adopted by the Pittsburgh Coal Co is given in Table 6; recommendations by 2 authorities as to min wt of samples proper for float-and-sink analyses, in Table 7. Tendency towards segregation of sizes in a flowing stream demands attention (9); wherever possible, sample should be taken at a point immediately following some mixing operation. Unaccounted losses of moisture must be avoided. Samples may be reduced in bulk by "alternate shovel" or "long pile" methods, if above 3/8-in size; smaller sizes, on mechanical riffles cutting out at least 10% of gross sample (see Sec 29). Samples for sizing tests must be handled cautiously to avoid breakage. For satisfactory size-weight ratios in samples for washability tests (rarely permitting a reduction in size before dividing the sample), see Bib (8).

Screen analysis. In modern plants, hand-picking gives place to mechanical cleaning at about 6-in size. When testing a run-of-mine sample, the sizes usually adopted are: +6-in; 6–4 in; 4–3 in; 3–2 in; 2–1.5 in; 1.5–1 in; 1–0.75 in; 0.75–3/8 in (all round-hole screens); 3/8 in–4 mesh; 4–8; 8–14; 14–28; 28–48; 48–100; 100–200; −200 mesh (Tyler std or equivalent square-hole screens) (see Sec 31). Each sized product, down to 48-mesh, may be separately tested by the float-and-sink method (below) at densities graduated systematically from 1.3 to 2.0; material finer than 48-mesh is usually considered clean coal.

Table 7. Minimum Weights (Lb) of Samples for Float-and-Sink Testing

Size, in	McMillan and Bird (14)	U S Bur Mines (7)
3 − 1.5	500	...
1.5 − 0.75	250	...
All − 1	...	500
All − 0.75	...	250
0.75 − 3/8	125	...
All − 0.5	...	125
All − 3/8	...	63
3/8 − 3/16	50	...
All − 0.25	...	32
3/16 − 20-mesh	25	...
All − 3/16	...	16
All − 20-mesh	0.5	...

Float-and-sink testing. Since all wet-cleaning processes (except flotation) depend upon differences in sp gr between coal and its impurities, determination of amount and distribution of impurities (as to size) is a valuable guide in designing a process. Run-of-mine may contain constituents ranging from pure coal (sp gr, 1.25) to clean slate (2.8) and pyrite (5.0); "bony" coal, an indefinite mixture of carbonaceous and finely divided inert matter, is very troublesome due to its intermediate density. Impurities are often in thin laminations, but separable on further crushing into light and heavy components.

Test is conducted by immersing the sample, preferably sized between 2 consecutive screens (see Screen analysis) and free from dust, in a series of heavy solutions (see below) graduated as to density, usually beginning with the less dense. Float material is removed, washed (if necessary), dried, and weighed; that which sinks is re-treated in the next heavier solution. This procedure may be reversed if the impurities disintegrate in the solution; starting with the densest solution (usually 1.70 sp gr), the heaviest (sink) components can be eliminated at the outset. For a complete test, solutions range from 1.30 to 1.70 sp gr, in steps of 0.05; for routine testing, customary densities are 1.4 and 1.6, yielding 3 products. When all sized products from the original sample (usually excluding that finer than 48-mesh) have been examined, and a composite calculated, it is possible to determine what proportion of the raw coal can be: (*a*) sold to specifications, (*b*) discarded as refuse, (*c*) improved by further crushing and repeated washing. Each item has direct bearing on design and operation of the plant.

Table 8. Composition and Density of Heavy Solutions for Float-and-sink Testing (13)

Density at ord temp	Anhydrous salt, gm per 100 cc solution		Percent by volume	
	$ZnCl_2$	$CaCl_2$	CCl_4	Toluene
1.20	..	..	44.4	55.6
1.25	26	26	51.2	48.8
1.30	31	31	58.0	42.0
1.35	35	35	64.8	35.2
1.40	39	40	71.5	28.5
1.45	42	..	78.1	21.9
1.50	46	..	85.1	14.9
1.55	49	..	91.9	8.1
1.60	52	..	....	....
1.65	55	..	....	....
1.70	58	..	....	....
1.74	60	..	....	....

Heavy solutions for coal testing are: (*a*) aqueous solutions of $ZnCl_2$ or $CaCl_2$; (*b*) organic liquids, as carbon tetrachloride (sp gr, 1.584), or bromoform (2.904), diluted with benzol or toluene. Latter group, though more expensive, has advantages of wetting dry coal freely, and evaporating from the samples at end of test, without washing. Solutions of

the metallic salts, especially at higher densities, are viscous (hence, not recommended for samples finer than $^3/_8$ in), and must be thoroughly washed from the products to avoid errors in weighing and analysis. With precautions as to washing, the chloride solutions may be used on samples down to 48-mesh. For routine and large-scale testing, $ZnCl_2$ solutions are widely used (Table 8) (13). With commercial grades of $ZnCl_2$ or $CaCl_2$, allowance must be made for their H_2O contents; sp gr of solution is tested by hydrometer. For testing at densities above 1.70, methyl iodide (2.278) and bromoform (2.904), diluted with benzol, are available. For details of float-and-sink procedure, see Bib (5, 7, 14, 15), also 2nd edn of this book, p 2009.

Float-and-sink apparatus. Small-scale tests, with lumps to 1-in size, can be made with a spoon and a beaker about $^3/_4$ full of solution at desired density. Coal sample is stirred in, a little at a time; floating material is spooned out, replaced by more sample, and operation repeated, allowing sink to accumulate. Before last removal of float, the sink is stirred to liberate entangled

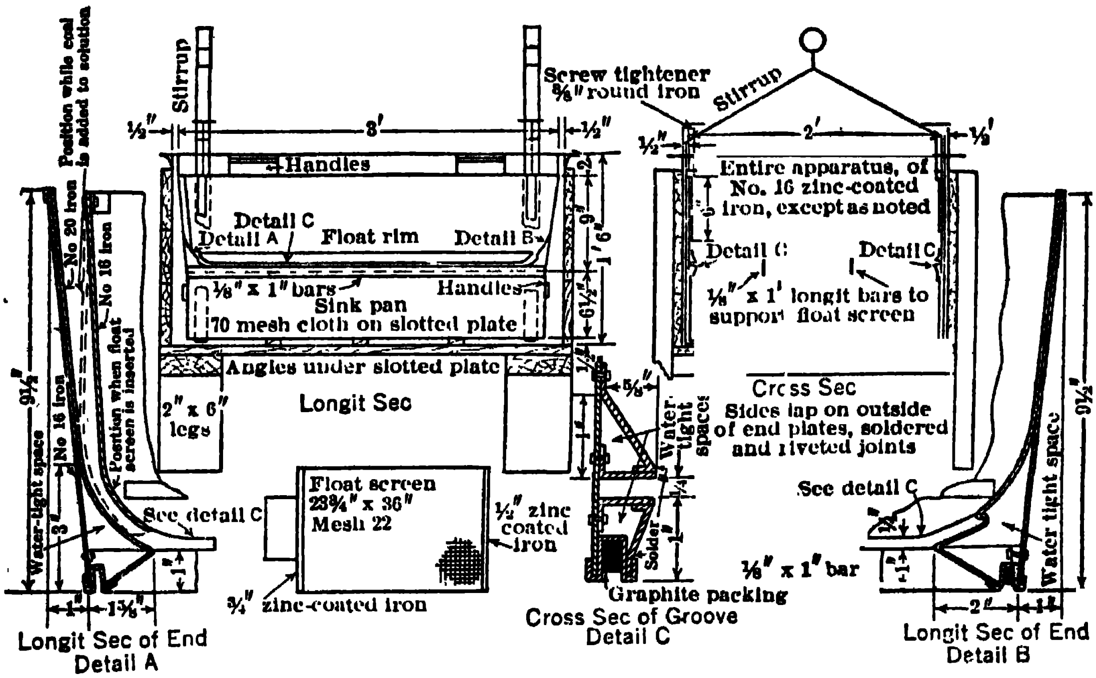


Fig 6. Details of Float-and-sink Testing Apparatus for Coarse Coal (5, 14)

light particles. For larger-scale testing, specially designed (but simple) equipment is desirable. Fig 6 shows a device used by McMillan and Bird in 1924 (5, 14). It has a flexible screen, inserted from above and outside the tank, to close the bottom of the float pan before lifting it off the sink pan and out of the tank.

9. PLANT DESIGN

Capacity of plant (1) influences design chiefly as to provision for re-treatment. In a given plant, it may not be economical to install re-treatment units; while such omission inevitably reduces effic, cost of re-treatment often exceeds the gain therefrom. In large plants, effic of cleaning is most important; inefficiency is inexcusable, except when the percentage of refuse is very low. Experience in ore concentration has shown how rarely a clean finished product and a clean refuse can be made in a single unit. Re-treatment is necessary for producing clean coal of uniform quality and a refuse reasonably free from recoverable coal; it also serves to adjust variations in quality of feed, in operating the primary units, and maintaining uniformity in both cleaned coal and refuse. **Delivery** of raw coal to the cleaning plant is a variable item in the layout, and methods show wide differences. Some cases require nothing more than a conveyer or elevator from tipple to plant. If raw coal is first stored in bins, with provision for mixing to maintain uniform feed, this may become a large first-cost item.

Screening equipment varies widely with size of raw coal and number of sizes of product; it also makes a difference whether the treated coal is slack, loaded unsized, as for coking. Sizing screens may also be used for dewatering (see sludge recovery, Art 14). Table 9 gives data on screens (see also Table 4).

Table 9. Capacities of Various Screens, Tons per Hr per Sq Ft of Perforated Area

Type of Screen	2–4-in Screening					1 1/8–2 in Screening					3/8–1 1/8-in Screening				
	Feed	Through	Over	Moist, % (a)	Moist, % (b)	Feed	Through	Over	Moist, % (a)	Moist, % (b)	Feed	Through	Over	Moist, % (a)	Moist, % (b)
Shaker (wet)	4.19	2.79	1.40	...	3.1	2.06	1.36	0.70	...	3.7	0.63	0.31	0.32	...	4.5
" "	1.74	1.67	0.07	...	...	0.83	0.70	0.13			0.33	0.15	0.18		
Shaker (wet)	1.34	0.69	0.65	...	3.9	0.44	0.09	0.35	...	4.4	0.09	0.02	0.07	...	...
Shaker (dry)	3.08	1.69	1.39 (c)	...	2.8	...	...	...	...	...	...	...	...	...	...
Vibrator (dry)	...	...	...	...	...	1.67	1.14	0.53	...	3.0	0.68	0.37	0.51	...	3.0
Shaker (wet)	2.64	2.05	0.59	...	3.1	...	...	...	...	...	1.34	0.55	0.79 (d)	...	...
" "	...	...	...	...	...	1.90	1.08	0.82	...	3.4	1.37	0.06	1.31	...	4.9
" "	...	...	...	...	...	...	...	...	...	...	0.97	0.24	0.73 (e)	...	...
" "	...	...	...	...	...	0.57	0.48	0.09 (f)	...	...	0.61	0.02	0.59 (l)	...	4.9
Shaker (wet)	2.42	1.52	0.90 (g)	2.2	...	1.19	0.96	0.23 (h)	2.9	...	0.97	0.47	0.50	5.5	...
" "	...	...	...	...	...	1.16	1.09	0.07 (i)	...	...	1.09	0.36	0.73 (j)	4.5	4.9
Vibrator (wet)	...	...	...	...	...	...	...	...	...	...	2.07	0.65	1.42 (k)	...	...
Vibrator (dry)	...	...	...	...	...	...	...	...	...	...	4.27	1.52	2.75 (m)	...	...
Shaker (wet)	3.42	2.59	0.84	2.2	...	2.39	1.37	1.02	3.2	...	0.73	0.06	0.67 (n)	7.2	...

(a) Screen oversize; moisture at 85° C, "as is"
(b) Loaded product; total moisture at 105° C
(c) +2-in screen product
(d) 3/8–2-in screen product
(e) 3/8–2-in screen product
(f) 1 1/2–2-in screen product
(g) 1 1/2–4-in screen product
(h) 1–1 1/2-in screen product
(i) 1 1/2–2-in screen product
(j) 3/8–1 1/2-in screen product
(k) 3/8–2-in screen product
(l) 3/8–1 1/2-in screen product
(m) 3/8–3 3/8-in screen product
(n) 1/4–1 1/8-in screen product

Loading and mixing depends upon mixing facilities and whether sized coal is handled on booms. Number of sizes loaded at one time determines the number of tracks, also possible necessity for storage bins for certain fine sizes. Desired mixing and blending determine the bin and conveyer system.

Refuse may be conveyed to a bin outside of the washer and loaded into RR cars; or by larry cars, aerial tram, or truck to the dump. If distance is not too great, and water abundant, refuse may be flumed to dump; when previously stored in bins, it should be partially de-watered in elevators or drainage conveyers, or, for sludge, on filters. In one installation, refuse over 3/8 in was crushed to minus 3/8 in by swing-hammer or ring-crusher, mixed with water, and pumped to dump (20). Dumps should be so situated that fumes from fires in them will not cause a public nuisance.

Costs. Analysis of OPERATING COSTS should be correlated to indicate sources of inefficiency. Table 10 compares costs at 4 types of plant; *A* is a wet plant preparing steam coal; *B*, for metallurgical coal; *C*, a dry plant for steam coal; *D*, a combination wet and dry plant for metallurgical coal. The proportionate expense for cleaning coal is shown to be approx the same, regardless of type of equipment. Relative costs of screening and picking, and of loading and mixing, vary with size of coal, numbers of pickers, and amount of sizing, blending and mixing. Table 11 itemizes CONSTRUCTION COSTS for a complete wet-process preparation plant, calculated as percentage of total cost.

Table 10. Operating Cost (Labor and Supplies Only) at Four Typical Cleaning Plants

	A	B	C	D	Aver
	%	%	%	%	%
Cleaning exclusively:					
Cleaning coal	17.5	12.3	18.0	9.5	14.4
Water and drying systems	2.4	8.5	0.4	10.5	5.4
Total	19.9	20.8	18.4	20.0	19.8
Other plant operations:					
Dumping	10.7	.....	.....	5.1	4.0
Screening and picking	7.6	8.8	23.2	12.3	12.9
Raw coal storage	0.1	7.5	1.4	2.0	2.8
Loading and mixing	8.1	16.9	15.2	17.1	14.3
Cleaning RR cars and tracks	0.4	0 3	1.8	2.1	1.1
Cleaning-up plant	2.9	4.0	8 7	4.0	4.9
Repairs to structure	3 1	1.0	0 3	1 5	1.5
Total	32.9	38.5	50.6	44.1	41.5
Incidentals:					
Sampling and testing	7.5	6.2	4.7	4.4	5.7
Waste disposal	11.3	8.0	8.0	10.6	9.5
General repairs	9.3	10.0	8.1	8.2	9.0
Heating	1.6	2.9	.....	3.0	1.9
Overhead and miscel	17.5	13.6	10.2	9.7	12.6
Total	47.2	40.7	31 0	35.9	38.7
	100 0	100 0	100 0	100 0	100.0

Drying is not included in above figures, but averages 10–12% of total cost.

Table 11. Construction Costs at a Complete Wet-process Preparation Plant

	%		%
Cleaning units	21.0	Handling raw coal	6.3
Screening units	5.2	Loading clean coal	8.3
Drying units	12.5	Handling refuse	3.8
Piping	7.3	Structure	25.0
Water-settling units	6.5	Laboratory, shops, etc	4.1
Total cleaning	52.5	Total other plant	47.5
			100 0

10. WET-CLEANING UNITS: JIGS AND LAUNDERS

In American practice, the following cleaning units are widely employed.

Baum jig. Fig 7 shows a longit and transverse sec of a 2-cell, 5-compt jig. Compressed air is the pulsating medium, thereby avoiding back suction. The jig is divided longit into 2 parts, and transversely into smaller compts, generally 3–6, according to the coal treated. The shell and dividers are of steel plates, welded or bolted to prevent weaving or "breathing."

The screens, 5–8 ft wide and supported on a steel frame, are on one side of the longit wall. The air valve is adjustable to give any desired upward veloc during any portion of the pulsation period; settling period is similarly under control. Water supply is adjustable after valves W1, W2, etc. are set. Pulsation air is admitted at valves C1 to C5, set for such quantity of air as will maintain mobility in the bed and permit rock to separate from coal by gravity. Rock settling on the screen moves out through adjustable gate *G* and rotary valve *H* to elevator. Fine refuse passing through screens reaches elevators via screw conveyers *L* and *S*. Withdrawal of refuse is continuous, as fast as it accumulates on the screen; rate is controlled automatically by a float *F* actuating an ejector drive mechanism; so long as the depth of refuse remains fixed, the ejector speed is constant. Valves *H* and *P* are controlled automatically by patented devices, both electrical and mechanical.

Table 12 gives data on a test of a Baum jig, operating as follows (17): rated capac, 80 tons feed per hr; actual feed, 100 tons; size of feed, minus 3-in round hole. First cell, 4.5 ft long, 5 5 ft wide; second cell, 10 ft long, 6.5 ft wide. Depth at overflow, 16 in. Water circulated, 1 700 gal per min, or 4.8 tons water per ton feed. Pulsations, 56 per min; amplitude of pulsations in washing compt, 12 in. Power, 0.35 hp per ton per hr.

Battelle launder is a fine-coal washer, developed by the Battelle Memorial Institute (16), and embodying features of the old trough washer. It is a single trough, with refuse

Table 12. Separation Test on Baum Jig (17)

Specific Gravity Analyses of Various Sizes of Products

Size		% Size	Float @ 1.40		1.40–1.60		Float @ 1.60		Sink @ 1.60		Ash, aver
			% Wt	Ash %	% Wt	Ash %	% Wt	Ash %	% Wt	Ash%	
3–1.5 in	Feed	15.6	79.6	11.1	10.5	27.0	90.1	13.0	9.9	67.0	18.3
	Coal	10.4	93.1	11.5	6.5	24.4	99.6	12.4	0.4	......	12.6
	Refuse	22.3	7.5	15.3	26.5	29.2	34.0	26.1	66.0	......	52.5
1.5–0.75 in	Feed	30.3	86 7	10.1	6.0	27.7	92.7	11.2	7.3	72.6	15.6
	Coal	30.4	95.2	10.1	4.3	26.4	99.5	10.8	0.5	......	11.1
	Refuse	22 8	8.2	14 5	19.0	30.3	27.2	25.5	72.8	......	57.2
0.75–3/8 in	Feed	23.5	87.4	9.0	6.4	27.6	93.8	10.2	6.4	......	13.9
	Coal	24.1	93.5	9.1	5.9	27.3	99.4	10.1	0.6	......	10.4
	Refuse	18.6	8.6	13.0	14.4	32.5	23.0	25.2	77.0	......	59.9
3/8–3/16 in	Feed	13.9	86.9	8.2	6.0	27.5	92.9	9.4	7.1	......	13.7
	Coal	16 5	93.3	8 2	5 6	27.7	98.9	9.3	1.1	......	9.8
	Refuse	14.6	8.1	11.7	10.8	31.8	18.9	23.2	81.1	......	63.5
3/16 in–20 mesh	Feed	11.7	83.1	7.2	6.9	25 7	90.0	8.6	10.0	......	14.9
	Coal	14.6	92.8	7.1	5.7	26.1	98 5	8 2	1.5	......	8.8
	Refuse	16.4	11.7	10.3	12.2	30.2	23.9	20.4	76.1	......	60.6
Through 20 mesh	Feed	5.0	71.9	6.2	11.3	25.9	83.2	8.9	16 8	......	18.7
	Coal	4.0	83.7	6.2	8.3	24.1	92 0	7.8	8.0	......	12.1
	Refuse	5.3	15.4	8.1	8.5	22.8	23.9	13.3	76.1	......	59.2
All sizes, 3 in–0	Feed	100.0	84.6	9 2	7 2	27.1	91.8	10.6	8.2	......	15.4
	Coal	100.0	93.5	9.1	5.5	26.4	99.0	10.1	1.0	......	10.5
	Refuse	100 0	9 1	12.6	16 9	30.2	26 0	24.0	74.0	......	58.3

draw pockets so closely spaced that they constitute essentially a draw along the entire bottom; space between pockets is only sufficient to house the drawing mechanism.

The pockets have wedge-shaped channels through which the refuse passes. The approach to the pocket is formed by perforated plates, the bottom plate being set at an angle of 20°–40°, depend-

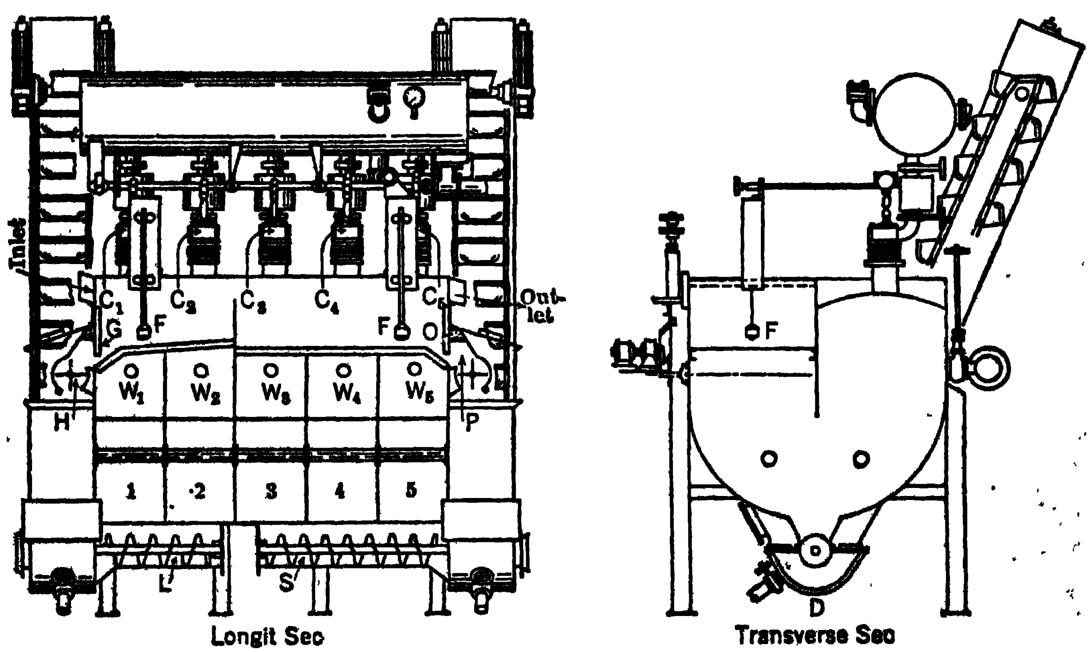


Fig 7. Baum Jig

ing upon the materials being separated. The compts below the screens have water pipes with valves and gages to regulate the water supplied to the various sections of the bed. A plant to treat [illegible] tons per hr of material under 5/16 in occupies a space 6 by 24 ft by 6 ft high; the launder being

14 in wide by 20 ft long. Refuse discharges into a sealed chamber, from which it is removed by screw conveyer or bucket elevator.

Hydro-separator (Fig 8) separates coal from heavy impurities by a rising current of water. Raw coal enters hopper *A* and passes into the separating compt *C*; here a rising current carries the coal over weir *D*, onto dead plate *E* and de-watering screen *F*, and thence into the second cell *G*, where the action is repeated. Clean coal and water pass over weir *H* into launder *J* and to a set of de-watering screens. Heavy materials in compt *C* settle in a loose mass on perforated plate *K*, and pass through gate *M* and spout *L* into conveyer *N*, which raises them above water level and discharges them as de-watered refuse. Heavy material settling in cell *G* moves similarly into bucket elevator *P*, which returns this product, if so desired, mixed with raw coal to be recirculated. Middlings may also be diverted to refuse, or saved as a saleable product. Water from the de-watering

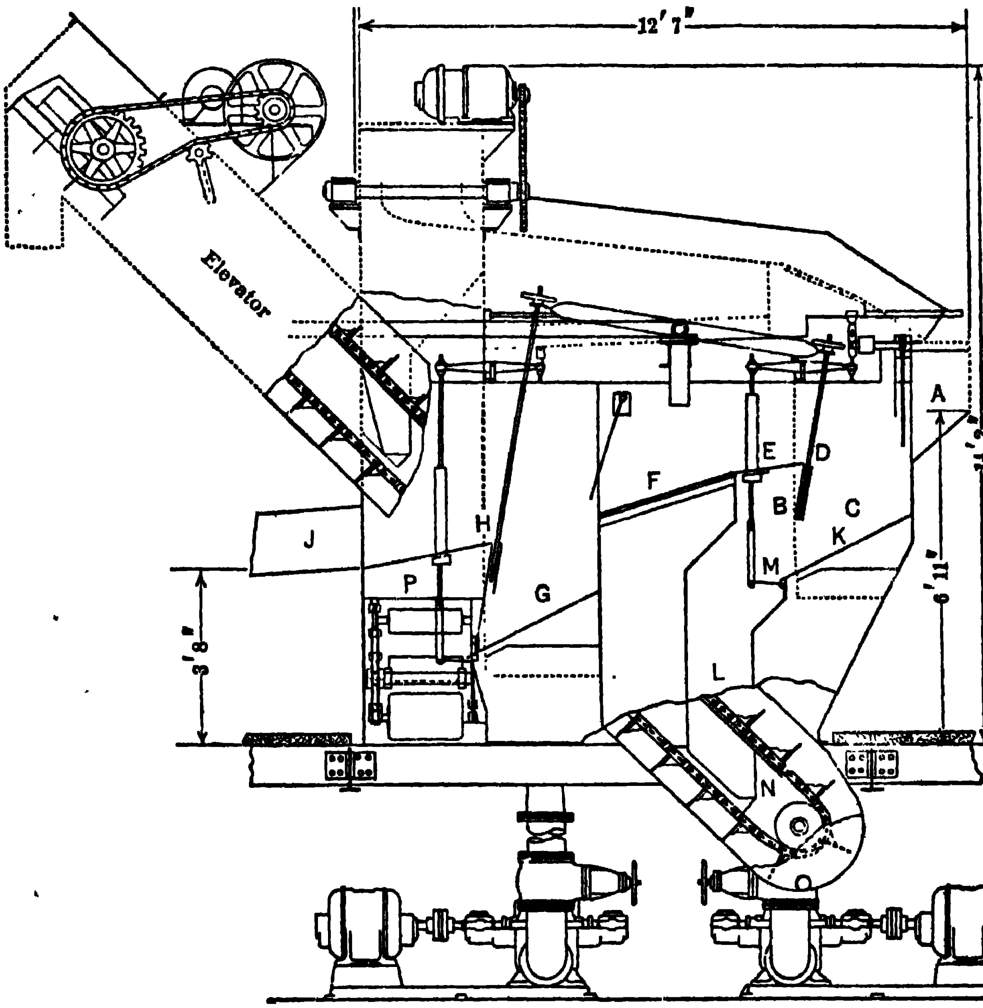


Fig 8. Hydro-separator

screens and screen *F* collects in a settling tank, from which the slurry is conveyed and the clarified water returned to the separator.

This washer is small, compact, easy to operate and control, and its initial and maintenance costs are low. It has a capac of 2–2.5 tons per in of width, and is in widths of 30–48 in, in single- and double-cell types. Feed is usually 3/8-in to max 5-in round hole. For effic operation, the size ratio in feed should not exceed 4–1. Water consumption, 6 tons per ton of feed.

Menzies coal separator. Operation is based on a variable veloc of water rising through a stratified, mobile mass of materials within a conical-shaped separator, in which the feed passes downward. Cleaned coal passes over the separator into a launder, then to a de-watering shaker screen. Material of intermediate sp gr stratifies below the coal in a loose mass, gradually increasing in density; the heavy rock descends through it into a conveyer. Quality of feed and desired product govern capac, which for aver conditions

is about 2 tons of feed per hr per sq ft of top area of separator. Feed sizes, 1.25–4 in; advisable size ratio of feed, 2–1. Water circulation, 11 tons per ton of coal. Power, 0.8–1 hp per ton of feed per hr (Table 13).

Table 13. Data on Standard Sizes of Menzies Separator

Diam cone, ft	Sq ft surface	Feed, tons per hr	Hp to drive	Hp for refuse conveyers	Hp for pump	Total hp	Hp per ton	Pump, gal per min
6	28	60 Pea	10	3	30	43	0.72	3 200
		Stove, Egg	15	3	40	58	0.97	4 300
8	50	100 Pea	15					
		Egg, Stove, Nut	30	5	40	60	0.86	4 500
10	78	160 Pea	20					
		Egg, Stove, Nut	30	5	75	100	0.91	7 500
12	113	225	40	5	125	170	1.06	12 000

Rheolaveur method (21) utilizes the classifying action of a horiz current of water in a launder of a length, shape, and slope to afford the desired stratification (for design and operation, see Sec 34, Art 6, 11). CAPAC depends on width of launder, and size and character of raw coal; widths are standardized, for combining to treat any desired tonnages; launders of coarse-coal plants are 20–56 in wide; capac, 150 to over 400 tons per hr in a main launder. Fine-coal plants have capac of 15–150 tons per hr in a single or multiple unit; individual launders of 14-, 10-, and 8-in widths are assembled to provide for this range. Table 14 gives data on water requirements of 3 plants. This washer has low initial cost per ton-hr, low power requirements, is of simple construction, has large capac, and can be varied in design to suit the type of coal.

Table 14. Water Requirements of Rheolaveur Washers (21)

	Three-box plant		Four-box plant		Five-box plant	
	Primary	Rewash	Primary	Rewash	Primary	Rewash
Water, gal per min	1 800 (*a*)	500	3 600	1 300	2 500 (*b*)	1 000
Feed, tons per hr	225	55	650	120	450	110
Gal per ton	460	550	330	650	600	550
Size of coal, in	$^3/_8$–3 $^3/_8$	$^3/_8$–3 $^3/_8$	0–4 $^1/_2$	0–4 $^1/_2$	0–4 $^1/_2$	0–2
Refuse in feed, %	3–8	17–20	8–10	40–50	8–10	40–50
Tons total water per ton feed	1.92		1.90		1.95	

(*a*) Includes the 500 gal per min overflowing the rewash. (*b*) Includes 1 600 gal per min added directly at head of primary launders, and 900 gal per min overflowing the primary rewash launder.

Belknap chloride washer uses a $CaCl_2$ solution, to which a slight rising veloc is imparted by impellers. Use of the heavier liquid increases the difference in falling veloc between coal and impurities. The slow current, rising through a slot, permits refuse to fall, while the coal moves on to its conveyer. Incoming water and chloride solution are proportioned by float-controlled valves to maintain a constant density in the tank. The washer is adapted to standard sizes of $^3/_8$–6 in; preferred size ratio is 2–1, although a range of 4–1 has been treated. CAPAC, 1 ton per hr per in of width, for coal of 1.5–3 in; 25% less for 3–6 in. The washer is made in 30-, 36-, 42-, 48-, and up to 150-in widths. POWER, 0.1 kw per ton-hr. WATER, 5 gal per ton of coal.

Sand flotation, Chance process (for design and operation, see Sec 34, Art 11). Properly operated, this separator closely approaches results of the float-and-sink test. As sp gr is the only physical property utilized, particles of any size larger than the sand can be treated. Hp for plant of 2 10-ft cones with combined capac of 300 tons per hr, sized 1–4.5 and $^3/_8$ to 1 in, is: agitators, refuse conveyer, and de-sanding screen, 75; de-sanding screens for clean coal, 30; two 6-in sand pumps, 50; 10-in circulating water pump, 50; refuse pump, 20; make-up sand and water, 15; total, 240 hp, equivalent to 0.8 hp per ton per hr of feed. WATER CIRCULATED, 4–6 tons per ton of coal depending on size and quality of feed. SAND LOSS varies with size and quality of coal; aver, 2–3 lb per ton washed coal.

Wuensch "Differential Density" process utilizes a heavy medium composed of water and the slimes (fine coal and its impurities) settled out of the circulating wash water. Middlings accumulate in a conical separating chamber and form a column, of which the density increases from top to bottom.

This "differential density" feature allows the finer refuse to settle rapidly through the less dense medium at top, while the denser medium at bottom prevents loss of good coal. The column is a sealed chamber, from which the refuse passes by gravity to an elevator discharging above the liquid level. The refuse column is kept filled with heavy medium to a level slightly above that in the cone. Most of the medium is drained from the cleaned coal on dewatering screens and returned to the separator; the remainder, washed out by sprays, is settled to a heavy density in thickeners, and then re-introduced through the refuse column at bottom of the separator.

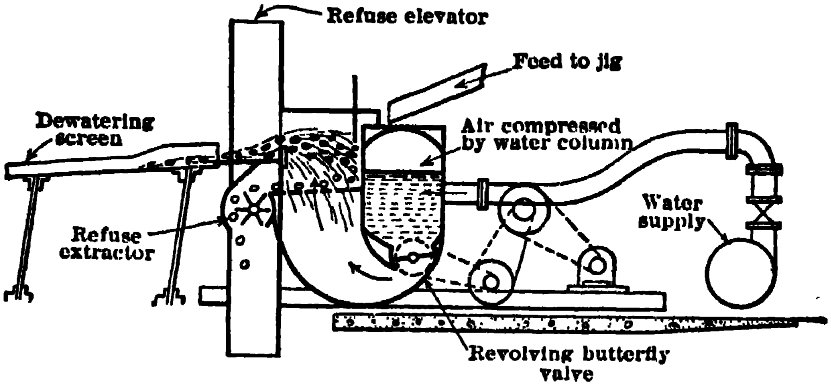


Fig 9. Vissac Pulsator Jig

Vissac pulsator jig (Fig 9) operates on same principle as the Richards ore jig; it is used on sized products (22). Refuse is removed through a gate and star valve at discharge end of the jig screen. In the nut-size jig, rotation of the valve is controlled by a float resting on the refuse bed; in the egg-size jig, by a mechanism actuated by variations in resistance offered by the bed to the upward flow of water.

Table 15. Specific Gravity Analyses of Products from Test of Vissac Jigs (18)

Size, in		% Size	Float @ 1.40 % Wt	Float @ 1.40 Ash	1.40 — 1.60 % Wt	1.40 — 1.60 Ash	Float @ 1.60 % Wt	Float @ 1.60 Ash	Sink @ 1.60 % Wt	Aver Ash
3–2	Feed	26.0	70.4	10.9	13.6	28.2	84.0	13.7	16.0	22.8
	Coal	24.3	95.6	10.5	4.3	24.1	99.9	11.1	0.1	11.1
	Refuse	35.6	16.6	17.4	36.4	27.9	53.0	24.6	47.0	45.1
2–1.25	Feed	74.0	76.1	10.3	10.9	28.1	87.0	12.6	13.0	20.2
	Coal	75.7	90.1	10.3	8.7	25.9	98.8	11.7	1.2	12.2
	Refuse	64.4	9.2	15.0	24.3	30.7	33.5	26.4	66.5	57.0
3–1.25	Feed	100.0	74.6	10.4	11.6	28.1	86.2	12.8	13.8	20.9
	Coal	100.0	91.5	10.3	7.6	25.7	99.1	11.5	0.9	11.9
	Refuse	100.0	11.8	16.2	28.6	29.4	40.4	25.6	59.6	52.8
1.25–0.5	Feed	56.7	81.8	9.9	9.2	28.9	91.0	11.8	9.0	16.9
	Coal	59.8	90.1	9.9	8.9	28.3	99.0	11.6	1.0	12.0
	Refuse	67.9	1.1	15.4	10.1	33.1	11.2	31.4	88.8	67.8
0.5–0.25	Feed	43.3	83.8	9.1	6.9	28.2	90.7	10.5	9.3	15.8
	Coal	40.2	89.1	9.0	7.1	28.4	96.2	10.4	3.8	12.3
	Refuse	32.1	2.8	10.7	3.1	32.1	5.9	22.0	94.1	74.3
1.25–0.25	Feed	100.0	82.7	9.5	8.2	28.6	90.9	11.2	9.1	16.4
	Coal	100.0	89.7	9.5	8.2	28.3	97.9	11.1	2.1	12.1
	Refuse	100.0	1.6	12.9	7.8	33.0	9.4	29.6	90.6	69.9

Table 15 gives results of a test on Vissac jigs treating egg coal (3-in round to 1.25-in sq hole) and nut coal (1.25–0.25-in sq hole). Additional data (18): air chamber, 3 by 4.5 ft; washing compt, 3.5 ft long by 4.5 ft wide; head of water, 18 ft; feed, 60 tons per hr; power, 0.1 hp per ton feed per hr; water, 1 850 gal per min, or 7.7 tons per ton coal; pulsations, 40 per min, at 4.25-in stroke for egg, and 2-in for nut coal; jig bed, 26 in deep for egg, 24.5 in for nut jig.

Koppers Llewellyn washer has a single box 6.5 ft long; or 2 boxes in tandem, comprising a sludge recovery unit, and primary and secondary washing cells. Middlings can be re-circulated, with or without crushing. The plunger, hinged at one end, produces a max activity at the feed end of the box, gradually diminishing towards discharge. Suction is eliminated by a valve in the plunger, which pumps water under the jig boxes from the overflow launder of the settling tank, thus making a circulating pump unnecessary. The refuse discharge is automatic. Cell width, 4 ft; sizes cleaned, 0.5–5 in; preferred ratio of size, 1–2; capac, 60–70 tons per hr; stroke, 4–6 in; speed, 40–30 rpm; power, 10 hp for double, 5 hp for single cell.

Prins multi-flow washer, a combined jig and launder, is small and compact. A jig with a rated capac of 135 tons raw coal per hr has a trough 18 in wide, and jig box 23 in wide. The feed moves down a central launder, where it is partially stratified, the heavier material dropping through openings into the jigging compt.

The jig basket is adjustable in length and speed of stroke; its bottom curves upwards from feed to discharge end. Water entering at different points keeps the bed fluid, floating the lighter particles to discharge end. Refuse drops through an opening in the basket into a well, from which it is conveyed for disposal. Capac, 75 tons per hr per ft of width. Re-circulation water, 10–20 gal per min per ton of coal per hr, or 2.5–5.0 tons water per ton coal treated. Power, $^1/_3$–$^1/_2$ hp per ton coal per hr.

11. COAL-WASHING TABLES

Among various types of tables, the main differences are in shape or contour of the deck and in the head motion. Decks are covered with linoleum or rubber, smooth or corrugated, with hard-wood or rubber riffles.

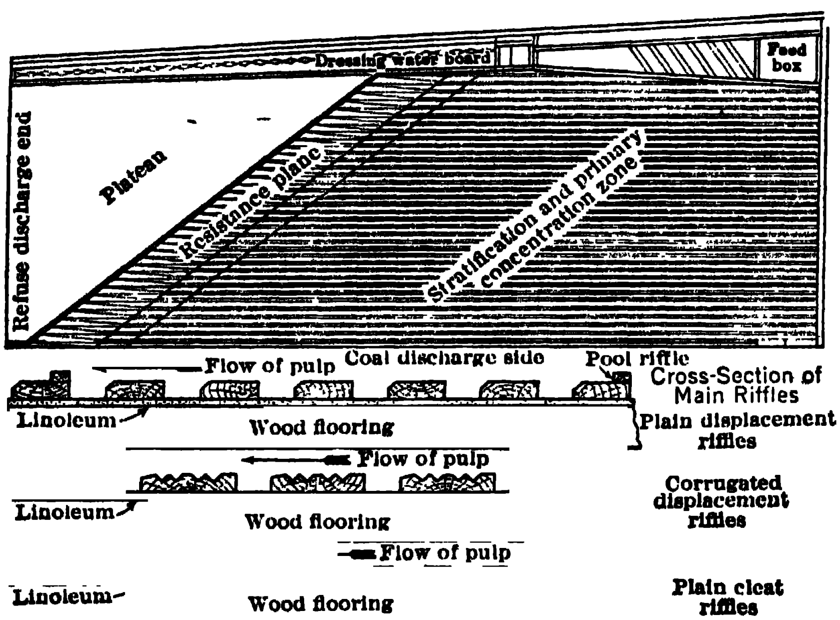


Fig 10. Plat-O Coal-washing Table

Plat-O table (Fig 10) is like that for ore, except for 55% greater deck area and heavier head motion. It cleans unsized coal, $^5/_8$-in and finer, as fed from a box attached to the table deck. The feed box itself acts as a small deck, concentrating and immediately discharging the coarsest and heaviest refuse, leaving the finer and lighter impurities to be removed on the main deck. Capac, 6–10 tons per hr. Water, 350 gal (1.5 ton) per ton of raw coal, plus a little final wash water. Running load, 0.75–1 hp; a 3-hp motor with high starting torque is usual. Speed, 255–300 rpm; stroke, $^5/_8$–1 $^5/_8$ in; usual feed contains 45–50% solids.

Deck surface of this table lies in 2 or more parallel horis planes. In the 2-plane type, universal for bituminous coal, the lower plane forms the larger part of the deck, and is the "stratification and primary concentrating" zone. The plateau or dressing zone extends back from the discharge, and joins the primary concentration zone along a diagonal line. Its height above the lower plane is about $^3/_8$ in. Between the plateau and the lower deck is a narrow, diagonal, upward-sloping "resistance plane." The main riffles end approx along the upper edge of this slope, their ends being tapered on the underside, leaving the upper edges horis. Table 16 gives data for coal sized between $^3/_{16}$ in and 48-mesh, at 6.2 tons per hr. Speed, 275 $^{15}/_{16}$-in strokes per min; transverse slope, 0.25 in per ft; longit slope (upward to discharge end) $^3/_8$ in for 12 ft; rise to upper plateau, 0.75 in.

Massco table has a linoleum-covered, riffled deck, adjustable as to transverse slope. Head motion is differential, to accelerate travel of the material with minimum agitation. The deck reciprocates endwise and horizontally, riffles being adjusted to discharge throughout its length. Capac, 4-7 tons per hr on coal passing 0.25-in round hole. Speed, about 220 rpm, requiring 1 hp.

Table 16. Coal Washing on Plat-O Table

	Wt, %	Sink at 1.60, %
Feed	100.0	6.8
Clean coal	88.9	1.1
Middlings	5.8	20.5
Refuse	5.3	91.0

Overstrom Universal table. Driving mechanism consists of an unbalanced loose pulley, on a shaft rigidly attached to the table frame; momentum of this revolving weight reciprocates the deck, which is supported by flexible hickory legs. A differential motion is produced and adjusted by a cushioned stop at forward end of stroke and coil springs on other end. Neither cams nor toggles are employed. The supporting legs slope slightly back towards the head motion, causing the table to rise in a circular arc on its forward stroke. Length of stroke varies automatically with the load. Speed, 240-280 strokes per min. Riffles curve upwards to the high side of table.

Deister-Overstrom diagonal-deck table. All riffles are parallel to direction of motion and at an angle to the discharge. This gives the advantage of deflected riffles without the whipping action, and allows the stratified refuse a free path towards the refuse edge. These tables may also have double decks, driven by same head motion, but separately asjustable as to slope and length of stroke. Speed, 250-300 strokes per min; stroke, $^5/_{16}$-1 $^3/_8$ in. Power under load, 1.5 hp. Size of feed, 2-in max. Capac, 4-8 tons per hr, varying with size and character of coal.

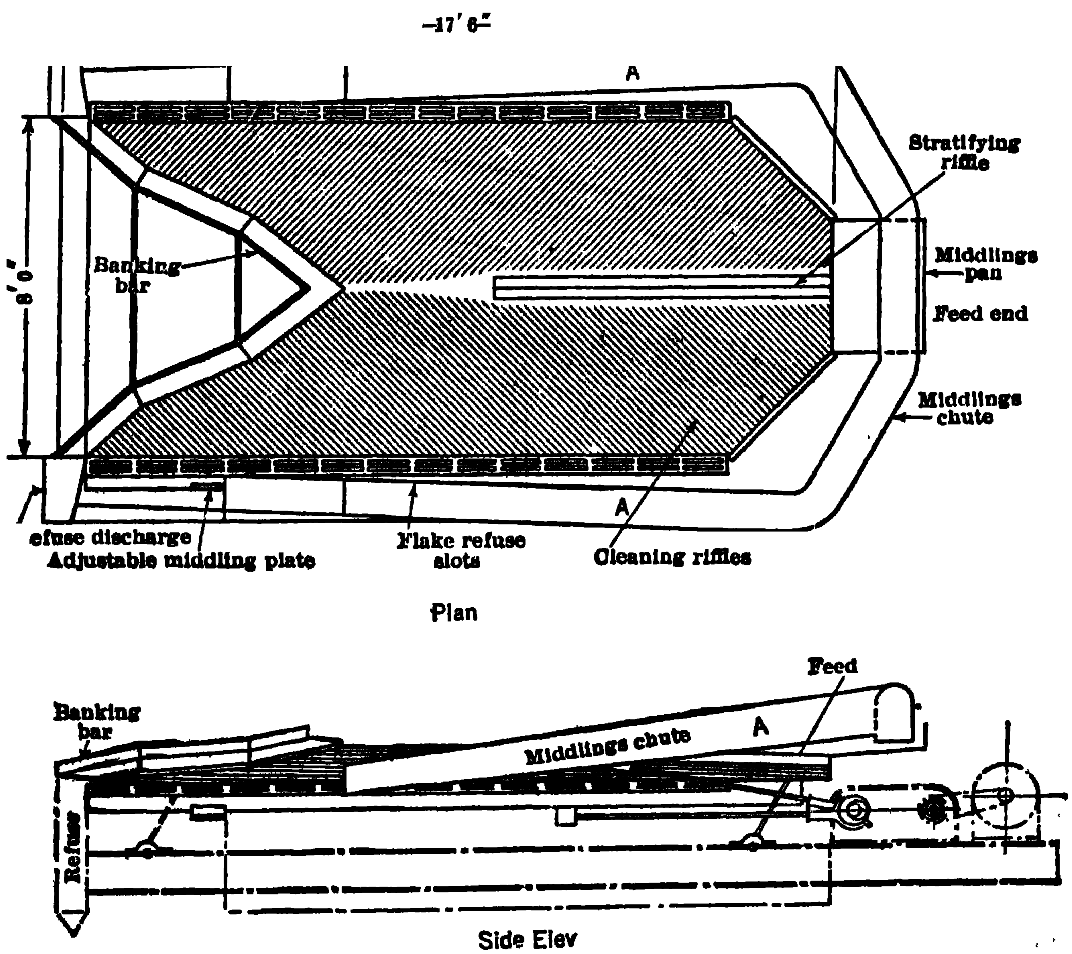


Fig 11. American Pneumatic Separator

12. DRY CLEANING

American pneumatic separator (Fig 11) utilizes its entire surface for cleaning. It ordinarily discharges 2 final products, clean coal and refuse; middlings may be made, to be

recirculated, re-treated separately or with refuse on other units, or discharged as a second-grade fuel. For re-circulation, middlings return to the feed on the deck via the anti-gravity chute A, or are conveyed back to the initial feed.

Distribution of air under different sections of the deck is adjusted by external controls: rapidity of stroke by variable-speed drive; longit slope of deck, by jack screws with lock wheels. Transverse slope of deck is fixed by construction. Proportions of the several products are varied by sliding cutter plates. The R-72-144 table requires 11 000 cu ft of air per min, at 2-in water gage, when treating minus 1/4-in feed, to 44 000 cu ft per min at 4-in gage for minus 3-in. Motor on blower fan ranges from 15 to 60 hp for minus 3-in feed; motor on separator, 5 hp.

Stump air-flow separator. The cleaning unit (Fig 12) is an air-tight metal box A, covered by a pervious deck on which rests a layer B of clay marbles of sizes depending on size of coal treated. This distributes air without requiring shutters and air ducts. Since the unit is stationary, it can be enclosed with dust-tight hoods. The layer of marbles is covered with a bronze or stainless-steel perforated deck, in which size of holes and percent of open area depends on size of coal treated. The unit is well adapted to coal of minus 3/8-in. Its initial and maintenance costs are low, and it occupies small space. Capac, 0.5–1 ton per hr per in of width, depending on size and quality of feed. Widths, 30–72 in. Air required, 250–500 cu ft per min per sq ft of deck area, at press of 2–5 in water gage under the deck, depending on size of coal.

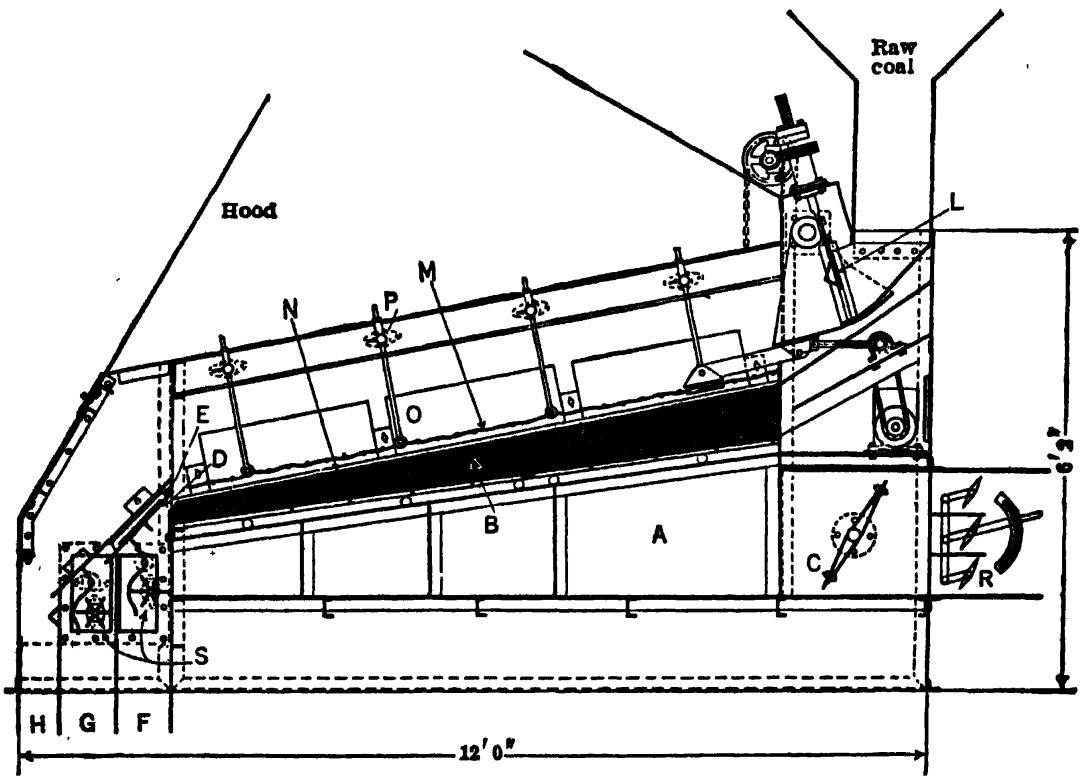


Fig 12. Stump Air-flow Separator

A, air chamber; B, bed of marbles; C, rotating valve, producing pulsations of air; D, E, adjustable plates governing proportions of refuse and middlings; F, refuse delivery chute; G, middlings

Table 17 gives results at a Stump plant treating minus 5/16-in coal at rate of 0.76 ton per hr per inch width of primary units.

Air-sand process utilizes a body of dry sand maintained in a mobile condition by an uprising stream of air. By regulation of air flow, the sand is maintained at such density that coal will float near surface, while refuse sinks through the sand.

The separator consists of 2 boxes in series, built into a trough-like unit, with dams and adjustable refuse gates to form 2 sand pools. The floor is a pervious deck. The draw-off box under each section receives air, through regulating gates, from a manifold on each side. The machine is inclined lengthwise at 5°; a continuous flow of sand passes over each compt, being fed to the upper

one at a regulated rate from a storage hopper. Washed silica sand, 20–100-mesh, is best; it is circulated at rate of 2.5 tons per ton of coal; make-up sand to replace losses, 1–2 lb per ton of coal.

Table 17. Separation Test of Size Increments, Stump Air-flow Plant

Product	Size increment	Size, % of product	Specific gravity of separation and weights of size increments, % Float @ 1.50	Sink @ 1.50 Float @ 1.60	Sink @ 1.60
Raw coal, 100%	14 mesh-5/16 in.	67.0	95 4	0.5	4.1
	48 mesh-14 mesh	20.0	95 7	0.3	4.0
	48 mesh-5/16 in	87 0	95.5	0.4	4.1
	0-48 mesh	13.0	....	...	....
Clean coal (a), 97.6% of feed	14 mesh-5/16 in	63.0	98 2	0.4	1.4
	48 mesh-14 mesh	22.5	95.5	0.5	4.0
	48 mesh-5/16 in	85 5	97.5	0.4	2.1
	0-48 mesh	14 5	....	...	....
Refuse, 2.4% of feed	14 mesh-5/16 in	96 5	14 3	1.3	84.4
	48 mesh-14 mesh	2 5	8 9	1.1	90.0
	48 mesh-5/16 in	99.0	14.2	1.3	84.5
	0-48 mesh	1 0	.	..	....

(a) Includes primary clean coal, secondary clean coal, and dust collector products

Air is supplied by a positive displacement blower at 20–30 cu ft per min per sq ft of pervious plate; pressure, 12–14 in water gage. Capac, 1.7 ton coal per hr per in of width. Sizes treated: min, 0.25 in; max, 4 in; limiting size ratio in feed, 3 to 1. Power for a 6-ft unit treating 120 tons per hr (excluding dust collection) is 0.25 hp per ton-hr, distributed as follows: separator and refuse de-sanding screens and shaking chute, 5; de-sanding screens for clean coal, 7.5; sand lift, 10; blower, 7.5; total, 30 hp.

13. DEWATERING AND DRYING

Dewatering washed coal, and reducing its free moisture to acceptable limits, are difficult problems. Fine coal may be flumed from a washing unit over an inclined FIXED SCREEN, which removes most of the water and passes the coal onward to other and more effic dewatering devices. Slope and area of this screen are adjusted so that the minimum amount of water is required to carry the coal forward without choking or spilling. "WEDGE-WIRE" screen, often used for dewatering on both stationary and shaking screens, consists of parallel wires of wedge-shaped cross-sec, supported with their wide faces uppermost on cross-bars spaced according to size of wire. The long slots thus formed have little tendency to blind; the openings are 2 to 1/16 mm, corresponding to 8- and 200-mesh (Tyler std).

Flight conveyers for drainage differ from those for dry materials (Sec 27) only in having perforated bottoms, often of wedge-wire screen. Table 18 gives data on 5 drag-flight installations, all receiving cleaned, wet coal from shaking dewatering screens.

Table 18. Data on Drainage Flight-conveyers

Coal size, in, rd-hole	Tons per hr	Size flight, in	Length dewatering section, ft	Speed, ft per min	Wedge-wire No pcs	Width and length, in	Size opening	Total area, sq ft	Free H_2O as loaded, %
3/8–4	275	7 by 45	164	47	10	48 by 36	0.5 mm	120	3.0
2–4	250	2 1/2 by 32	54	100	5	34 by 36	3/16 in	42	2.3–1.5
1–2	120	2 1/2 by 32	68	100	6	34 by 36	3/16 in	50.5	3.1
3/8–1	125	6 by 32	80	90	5	34 by 36	1/8 in	42	4.5
3/8–1	60	6 by 32	150	49	9	33 by 36	0.5 mm	74	3.8

Drainage band conveyer is inclined, with deep, troughed pans, each holding 1–2 tons, attached to a double-strand, long-pitch chain moving 1–2 ft per min. The bottom plates are perforated and slightly curved; when passing over the rollers, they squeeze the coal somewhat, and water drips into a sump. Conveyers are 60–80 ft long, holding about 40 tons in transit, and allowing 30–60 min for drainage. Discharged product, with minus 0.5-in coal, averages about 22% moisture.

Shaking dewatering screens, for coarse clean coal often have wedge-wire with 1 1/5-mm slots, usually parallel to flow of the coal. Trays with steel or wood sides are hung or supported on hickory strips, and reciprocated like dry shaking screens (Art 5); usual speed, 300 rpm, with 1 to 1.5-in throw. In recent practice, wet coal of 0.25 or 3/8 to 1-in size may pass from the shaking screen to one or more horiz vibrating screens, set on 2° up-slope towards the discharge. Table 19 gives data on several installations; all of hanger-board Parrish type except the last, which is a horiz vibrating screen.

Table 19. Data on Shaking Dewatering Screens

Coal size, in	Total screen: Length, ft	Length, in	Perf area, sq ft	Actual open area, sq ft	Tons per hr: Over screen	Per sq ft perf area	Percent moisture: In screen prod, @ 85° C	As loaded, @ 105° C
2–4	24	0	121.1	60.0	170	1 40	....	3.1
1 1/8–2	30	0	163.5	73.4	115	0.70	....	3.7
3/8–1 1/8	64	0	351.0	140.5	114	0.32	....	4.5
2–4	8	0	40.3	19.7	3	0.07	....	...
1 1/8–2	15	0	81.2	37.1	10	0.12	....	3.7
3/8–1 1/8	32	0	175.5	70.3	30	0.17	....	4.5
2–4	9	6	51.0	27.2	33	0.65	....	3.9
1 1/8–2	15	10	80.6	34.9	28	0.35	....	4.4
3/8–1 1/8	15	10	80.6	32.2	5	0.06	....	...
2–4	40	0	134.4	71.9	80	0.60	....	3.1
3/8–2	48	0	205.3	82.0	162	0.79	....	...
1 1/8–2	20	0	85.6	37.0	70	0.82	....	3.4
3/8–1 1/8	16	0	67.3	26.9	88	1.31	....	4.9
3/8–2	20	0	66.9	26.8	49	0.73	....	...
1 1/2–2	20	0	85.6	49.6	8	0.09	....	...
3/8–1 1/2	16	0	67.3	26.9	40	0.59	....	4.9
0–3/8	{ 24 24	0 0	55 } (b) 119 }	.....	114 (c)	0.66	22.0	...
1 1/2–4	48	0	161.3 (a)	85.4 (a)	121	0.75	2.2	...
1–1 1/2	48	0	205.7 (a)	102.6 (a)	39	0.19	2.9	...
3/8–1	40	0	171.2	68.5	85	0.50	5.5	...
1 1/2–2	20	0	67.3	34.3	5	0.07	2.5	...
3/8–1 1/2	20	0	67.3	26.9	49	0.73	4.5	4.9
0–3/8	{ 24 24	0 0	55 } (b) 119 }	.....	133	0.77	18.0	...
2–3 1/4	15	10	84.1 (a)	46.4 (a)	57	0.68	2.2	...
1 1/8–2	19	0	108.6 (a)	52.4 (a)	75	0.69	3.2	...
3/4–2	19	0	108.6 (a)	53.4 (a)	111	1.02	3.9	...
1/4–1 1/8	22	2	138.3	55 4	93	0.67	7.2	...
1/4–3/4	22	2	138.3	55.4	57	0.41	8.2	...
1/4–1 1/8	24	0	75.0	25.5	93	1.24	4.5	...
1/4–3/4	24	0	75.0	25.5	57	0.76	5.1	...

(a) Includes the area of Ton-Cap screens; (b) 1/5-mm wedge-wire screens; (c) includes 24 tons per hr of coal dust effluent returned.

Dewatering elevators. Fig 13 shows the commonest method of dewatering fine coal, received in a sump of wood, steel, or concrete, and cone or pyramid shaped, with an overflow around its periphery and a loading well suspended near its center. Perforated buckets allow partial drainage, the water returning to sump through the tight bottom of the elevator casing; speed, 30–50 ft per min; buckets, 16–48 in wide.

Centrifugal dryers extract moisture from minus 0.5-in coal more completely than any other mechanical type. Carpenter and Elmore dryers are used extensively in U S for coal of that size.

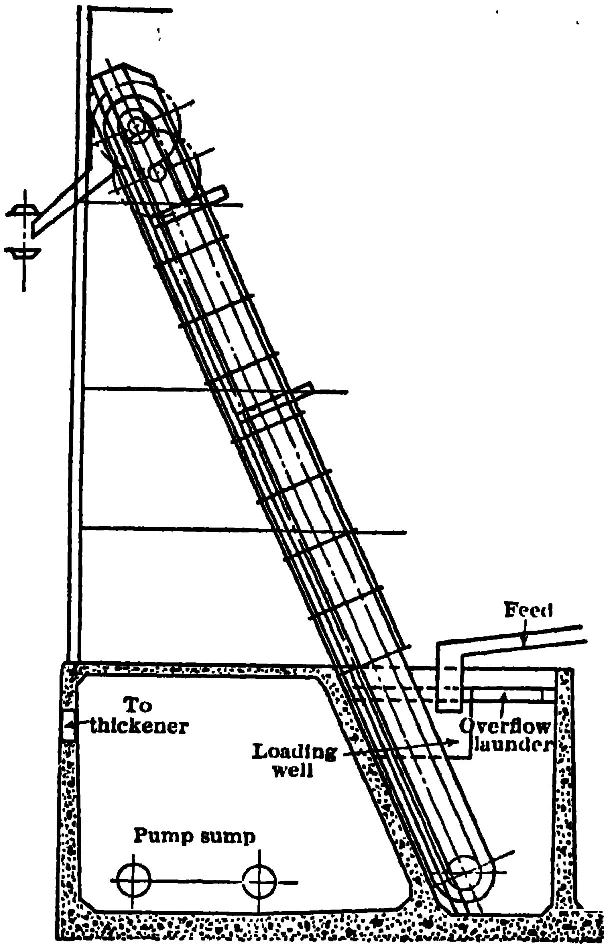


Fig 13. Dewatering Elevator

Carpenter dryer consists principally of a screen basket, in form of a stepped, truncated cone, carried on a vert shaft, with its smaller diam at top, and rotated through spiral bevel gears by V-belt from a motor mounted on a base cast integrally with the housing. Wet coal, fed into a hopper, falls on a horiz disk attached to the rotor shaft. Centrifugal force throws the coal upon the top row of screens attached to the basket; the opening between disk and screens is adjustable. The partly dried coal, on reaching the bottom of the first screens, passes over a lip and is redistributed on the second row, where extraction is continued. After passing 3 or 4 screens, the coal is gathered from the lip of the last into a hopper for final disposal. Table 20 compares 4 Carpenter dryers in one installation.

Table 20. Operating Data on Carpenter Dryers

	AR–1A	AR–1B	AR–4	AR–12
Size of feed, in	–3/8	–3/8	–5/16	–3/8
Moisture in feed, %	23.3	19.0	20.2	22.6
Moisture in product, %	5.2	9.1	7.2	6.5
Solids in effluent, %	35.7	25.1	29.6	49.6
Tons per hr (dry wt) of feed	23	21.0	30	68
" " " (dry wt) of product	19.7	20.0	27.9	52
" " " (dry wt) in effluent	3.3	1.0	2.1	16
Gal water per min in feed	28	20.0	30.8	79.4
" " " " " product	4.4	8.0	8.8	14.4
" " " " " effluent	23.5	11.8	22.0	65.0
Tons per hr + 4-mesh in feed	4.60	4.1	....	16.3
" " " + 4-mesh in product	2.85	3.4	....	8.3
" " " + 4-mesh in effluent	0.05	....	....	0.3
" " " breakage of + 4-mesh	1.70	0.7	....	7.7
% of + 4-mesh in feed broken to – 4-mesh	38.1	17.0	....	47.6
Size hole in screen plates, in	1/8 & 1/16	all 1/16	all 1/8	all 1/8

14. WATER CLARIFICATION AND SLUDGE RECOVERY

In a wet washery, regardless of size, coal characteristics, or method of processing, circulating water becomes polluted by sludge. Unless abundant water is available, it must be recovered in usable condition, and, unless the sludge is wasted, its coal must be recovered. These results are obtained by: elevator boot, rectangular sludge tank, cone, thickener, settling pond, and vacuum filter. In the first 4 methods, the sludge is flumed into the unit, passing into a baffled feed well, deflecting the stream towards the bottom, where it settles; water collects in an overflow launder around the edge, and goes to a sump. Effic of the unit depends on the ratio between the top settling area and amount of solids in the mixture, the volume, and the screen analysis of the solids. Table 21 gives data on settling at boot of elevator in 5 installations.

Table 21. Sludge Settling at Elevator Boots

Unit	Gal per min		Percent solids		% of +48 -mesh in overflow	Volume of tank, cu ft		Horiz area, sq ft at water level	Length of weir, ft	Elevator speed, ft per min	Elevator aver cap, tons, solids per hr
	Feed (*a*)	Overflow	Feed (*a*)	Overflow		Level full	Water level				
A	1 800	1 500	18.5	5.1	2.0	3 259	2 955	175	40.50	38.3	85–90
B	2 000	1 750	11.0	6.3	3.0	3 610	3 245	198	44.17	29.0	50
C	2 000	1 750	11.0	7.1	3.0	4 410	3 800	258	50.9	45.3	50
D	1 900	1 650	12.0	5.7	1.5	3 940	3 520	133	34.67	37.5	45
E	220	185	18.5	2.5	Trace	3 325	2 750	141	39.33	28.8	10

(*a*) Calculated from tons input and gal water per min overflow.

Table 22. Sludge Settling in Rectangular Tanks

	Plant *A*	Plant *B*
Gal water per min in feed	2 710	1 820
Percent solids in feed	....	5.5
Percent solids in overflow	....	4.0
Percent solids in underflow	....	73.0
Vol of tank level full, cu ft	1 455	6 135
Vol of tank at water level, cu ft	1 165	4 750
Horiz area at water level, sq ft	257	688
Length of weir, ft	45.9	39.8
Conveyer speed, ft per min	20.0	15.4
Aver elevator capac, tons solids per hr	2 0	10.0

Rectangular sludge tank, of wood, steel, or concrete, or a combination, may have sloping or vert sides. Sludge is usually flumed to one end through a feed well. A chain and flight conveyer is installed in the tank; flights are 6 in high, of wood protected by steel angles, or of angles or channels, depending on width of tank. The discharge end is extended (sometimes bottomed with wedge-wire screen) on about a 30° slope to drain the recovered solids. The chain should be barstrap or rivetless. The bottom run is carried on wearing bars or small rails; top run, on flat bars, rollers, or sprocket idlers. The conveyer, running at 10–20 ft per min, gives the material time to drain. An overflow launder along each side, starting beyond the zone of turbulence from the feed well, runs to a sump from which the clarified water is pumped for re-use. The tanks have drains for cleaning and washing. Table 22 gives data on 2 installations.

Cone settling tank (Fig 14). Washery water recovered from sump is pumped into the cone through pipe *A*, which delivers inside of a cylindrical curtain *F*. Clear water is removed through pipe *B*; *D* is an outlet to prevent water from overflowing rim of tank. Connection *E* is for cleaning by flushing the slurry pipe *C*. Valves at base of cone control discharge.

Dorr traction thickener (see Sec 33). Table 23 gives data on 4 installations for coal. Where space is available and water plentiful, SETTLING PONDS are used for sludge disposal; or the whole mixture is flumed to the pond and discarded, or allowed to settle and the residue recovered. Usually, little water is recovered from such ponds.

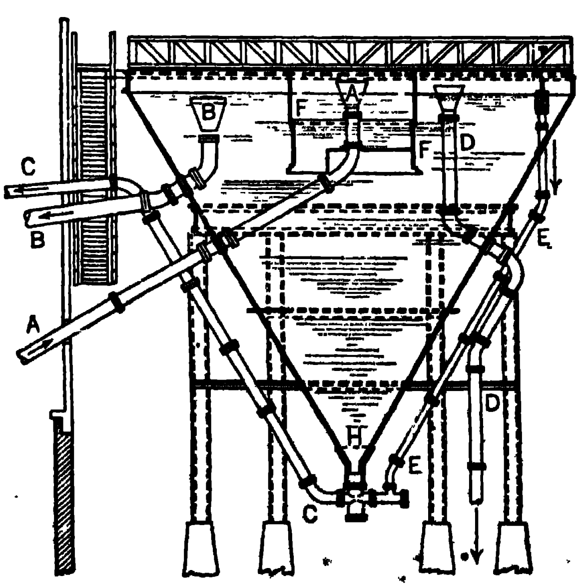


Fig 14. Cone Settler for Water Clarification

Filters. Common types are: (*a*) mechanical; (*b*) cord; (*c*) vacuum. The Bird filter is essentially a centrifuge; sludge fed into the machine passes through a high-speed impeller, which separates water and solids centrifugally. The CORD FILTER consists of endless cords mounted on 2 drums. On passing through a bath, the cords pick up the solids; water is drawn out by vacuum and the coal removed from the cords. Wright filter is an example.

In the older VACUUM type, a sectionalised cloth-covered drum rotates, with the lower part submerged in a bath. Suction collects a cake of solids on the cloth and dewaters it after rising from the bath. Just above the level at which the cake is discharged, comp air blows the cake loose and deposits it in a suitable conveyance. The DORRCO filter (Sec 33) reverses this procedure by admitting and distributing pulp to the inside of the drum, which is lined with filter-cloth. As a segment of the drum reaches the top of its travel, its load of sludge-cake drops off (assisted by press of air behind it) and falls onto the belt conveyer for disposal.

Table 23. Coal Sludge Settling in Dorr Thickeners

	Plant *A*	Plant *B*	Plant *C*	Plant *D*
Gal water per min in feed....	2 800	3 460	1 335	1 400
" " " overflow......	2 640	3 340	1 210	1 310
" " " underflow.....	160	120	125	90
Percent solids in feed.........	6.8	8.9	4.6	6.1
" " " overflow ...	4.5	8.0	0.2	3.0
" " " underflow....	36.7	40.3	43.7	45.0
Vol of tank level full, cu ft.....	41 900	22 300	31 250	31 250
" " " at water level, cu ft	37 270	19 950	28 140	28 140
Horiz area at water level, sq ft	5 215	2 640	4 065	4 065
Length of weir, ft............	256	182	225	225
Rake speed, rpm.............	0.085–0.255	0.50	0.098	0.098
Aver capac, tons solids per hr	16	13	14.5	10

15. DEDUSTING AND DUST-COLLECTION

Installation of dedusting equipment involves many factors (19). Both ash and sulphur contents of coal often increase with diminishing size. When coal containing soft powdery fusain is wet-washed, the porous fusain fines remain in the small coal or in the

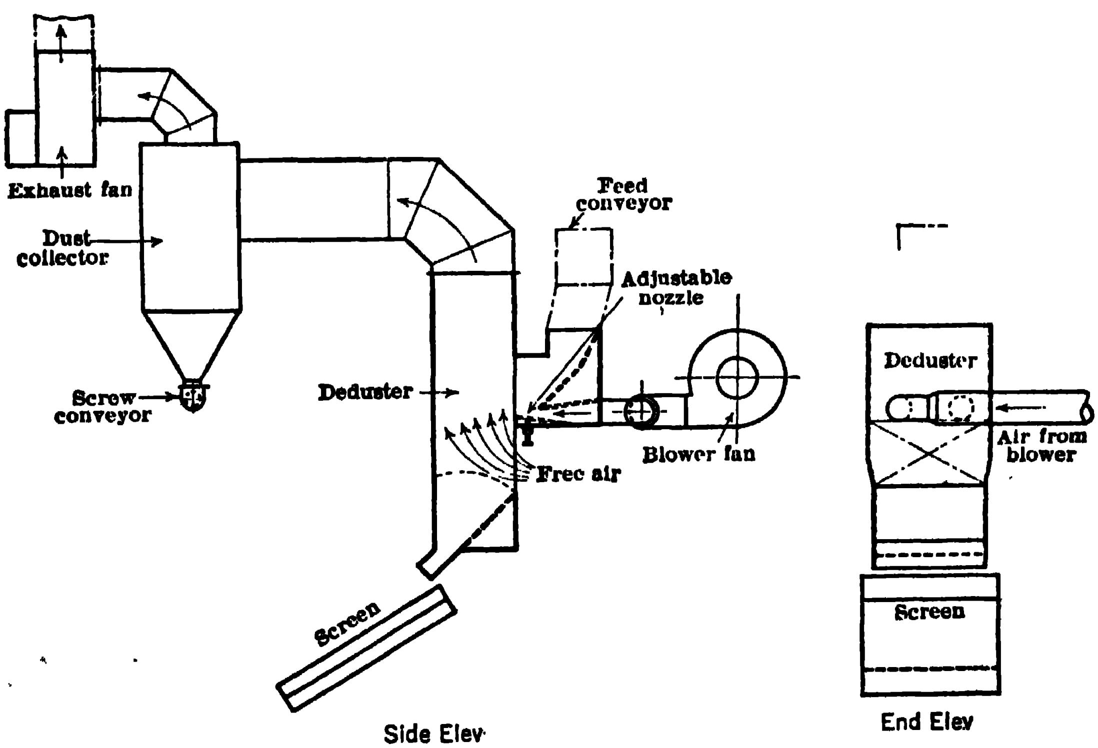


Fig 15. Koppers-Birtley Deduster

wash water, and obstruct dewatering of the coal and clarification of water. If, after investigating a specific coal, dedusting is found desirable, a problem arises as to disposal of the dust, for which a screen analysis of the coal with determination of ash contents of each size should be made. Dust is usually removed by screening or aspiration, or both.

Blaw-Knox apparatus directs 2 adjustable streams of air against falling curtains of crushed coal; dust is drawn off by 2 suction fans and delivered to collectors; air used, about 200 cu ft per min per ton-hr.

Algar deduster provides both screening and aspiration. Raw coal is fed through an air-lock onto a double-deck, circular, vibratory screen, the upper deck having the larger openings; mesh of both screens is adapted to the particular coal. Coarse material, falling in a circular curtain, is cut by rising air, which then passes through a set of conical louvres to the dust collector.

Koppers-Birtley system (Fig 15) depends on suction of a fan drawing air through an adjustable inlet, which lifts the coal off a stationary plate; a secondary inlet is provided, as shown. The modified American adaptation of the system supplies primary air by a blower and secondary air by suction fan. Coarser dust, after being lifted over a baffle, separates from the finer and is deposited in a hopper emptying automatically when enough material has accumulated to overcome a gravity-controlled sealing door. Fine dust continues through the fan to a collector. Normal air consumption, 100–200 cu ft per min per ton-hr of coal; power, about 0.18 hp per ton-hr. Table 24 shows effect of dedusting in a commercial installation.

Table 24. Screen Analysis of Dedusted Minus $^3/_8$-in Coal

Mesh	Percent by weight		
	Feed	Discharge	Dust
+4	33.0	39.0	0.0
4–14	36.5	37.0	5 0
14–48	19.0	14.5	32.5
48–100	5.0	4.0	28.5
100–200	3.0	2.5	28.5
–200	3.0	3.0	16.0
	100.0	100 0	100 0

Dust-collectors: (*a*) Cyclone, Multiclone, Polyclone, American Metallic; (*b*) Roto-Clone; (*c*) Bag collector. In group (*a*), coal particles are thrown centrifugally by an air current and collected in hoppers (19). CYCLONES are 1–4 ft diam, and consist of a cylindrical upper section above a conical bottom sloping 60°. MULTICLONE has a multiplicity of 6 or 9-in tubes, wherein high velocities are maintained; they operate under either press or suction. Units vary in size from 3 to 30 tubes; air requirements, 620 cu ft per min at 2-in water gage, to 14 640 cu ft per min at 4-in gage. Their effic is high. POLYCLONES resemble multiclones in principle, but are in units of 2 tubes in parallel. The cylindrical shells are 16–96 in diam and use 1 800–64 900 cu ft air per min at 2-in gage. The polyclone is less effic than the multiclone, but when used as a preliminary collector ahead of a multiclone, high effic is obtainable. Neither has any moving parts, nor requires much maintenance. AMERICAN METALLIC is similar in type to the Multiclone, and has been used for collecting dust from air-table cleaning. ROTO-CLONE acts as exhauster and dust separator combined. A housed impeller draws in the dust-laden air. Units require 0.75–15 hp, at capac of 400–7 810 cu ft per min at 2-in water gage, and up to 14 000 cu ft per min at 3-in gage, with 18.8 hp.

Bag collectors. The air stream passes through a fabric mounted as tubes or on frames, thus filtering out the dust; these are the most effic of all collectors. The BLAW-KNOX standard unit consists of cloth bags stretched over steel separating and beating frames. Dust-laden air enters through a duct in which it loses veloc and drops the heavier particles into a hopper, while the lighter are caught by the bags. Capacities are calculated on basis of 2.2–5 cu ft of air per min per sq ft of bag area, depending upon loading, temp and humidity of air, and moisture, size, and characteristics of the dust.

It is sometimes advantageous to operate under press rather than suction. When air is heavily loaded, it is best to install Cyclones ahead of the bag collectors. PANGBORN collector is similar to the Blaw-Knox in design, but differs in size of bags. KOPPER-WARING collector combines cyclone for coarse dust and bag filter for fine. The filter tubes are suspended above the cyclone from a frame to which they are attached by weights and springs for shaking the bags.

16. COAL DRYING

Types of apparatus: (*a*) slow-moving conveyer in a bath of warm or hot air; (*b*) screens mounted in heated and ventilated housings; (*c*) kiln-type rotary dryer.

Conveyer is usually housed by 10-gage plates. It may consist of chains and flights, flat pans and chains, or an integral-pan conveyer. Highest effic occurs when the bed of material is relatively thin. In some types, warm or hot air is blown in and exhausted by separate fans; in others, the bed is stirred by the carrier. Flue-gases are sometimes used for drying, as in the Dwight-Lloyd-Oliver dryer.

Drying screens are balanced, tandem shakers like the dewatering screen. Their tops have distributing hoods to spread hot gases over the screens. The under sides of the shakers usually have suction ducts to draw the gases through the coal. Hot furnace gases

are either blown or sucked through the screen. Some designs use a pulsating air current, synchronized with the screen motion to take advantage of the periodically loosened bed.

Rotary kiln dryer is usually a horiz drum with closed ends. The hot gases are either passed through a duct in the center, sweeping over the coal bed on its return, or pass

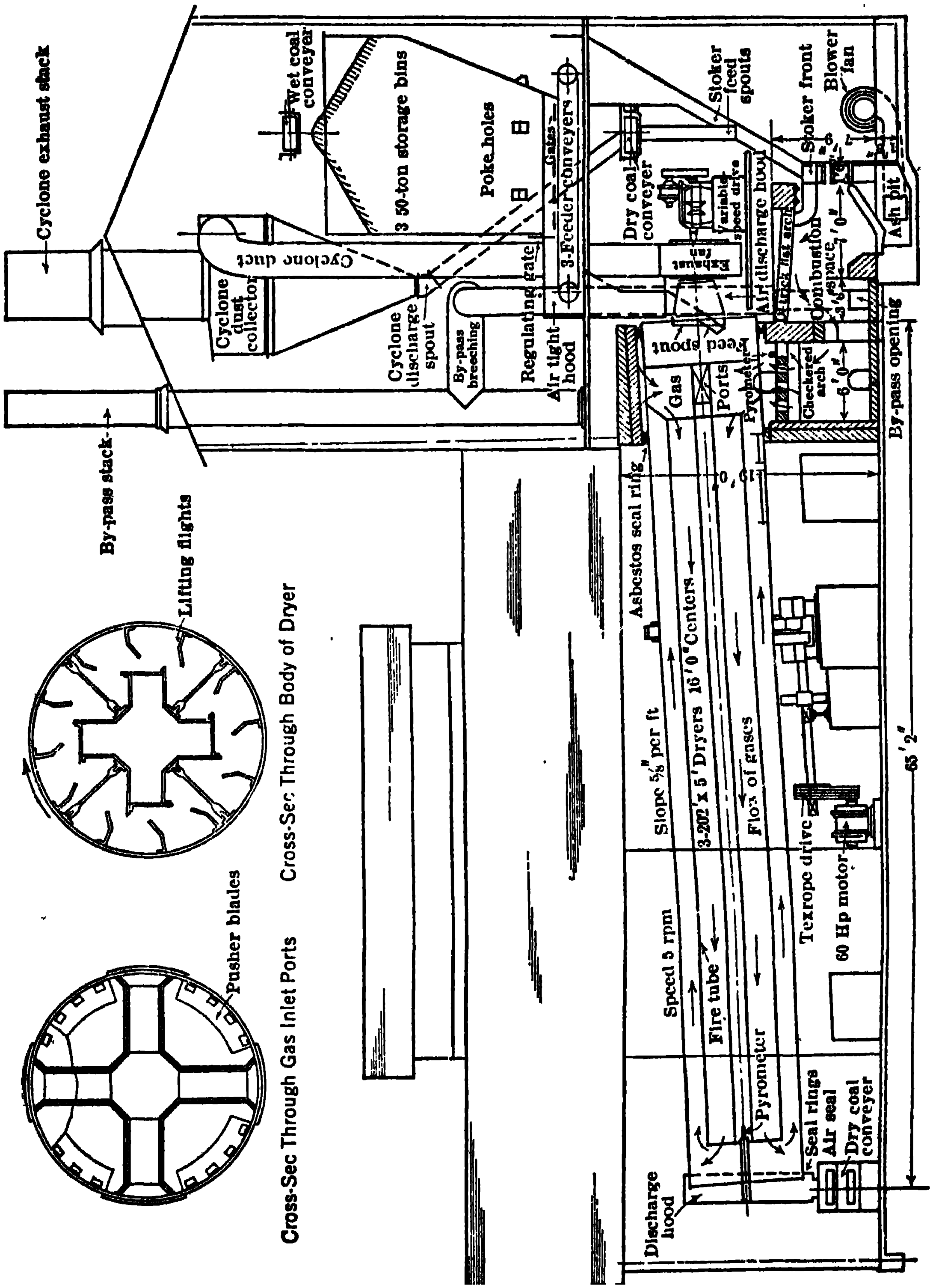


Fig 16. Christie Coal Dryer

between an inner and outer shell, coming into contact with the coal through openings. Feed end of the drum has a seal to prevent escape of coal and air. In most designs, some form of baffle stirs the coal as it rides up the sides and falls back. Dryers are set at a slope to cause the coal to travel through the drum. Fig 16 shows elev and cross-sections

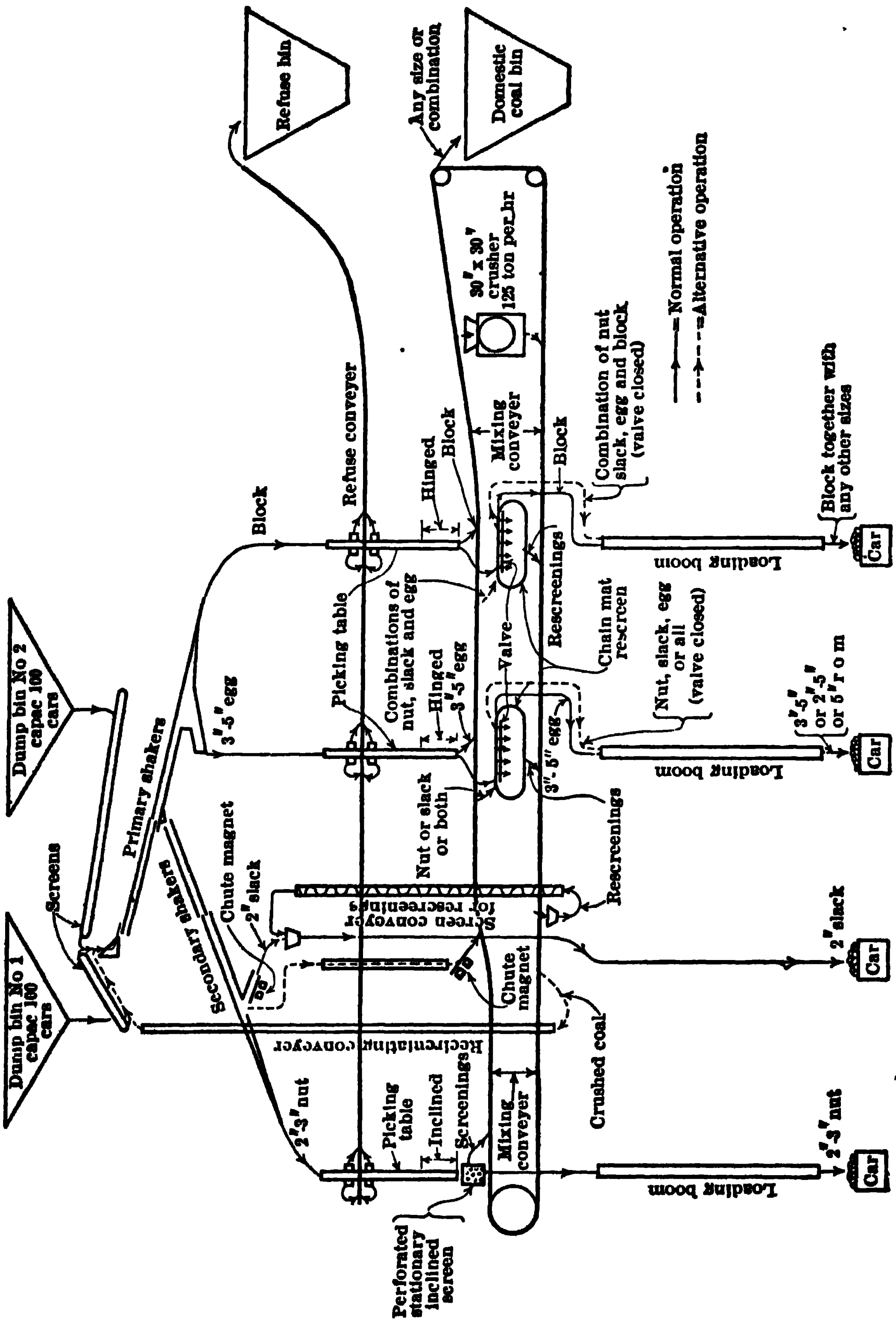


Fig 17. Tipple with Hand Picking (*Coal Age*, Jan, 1937)

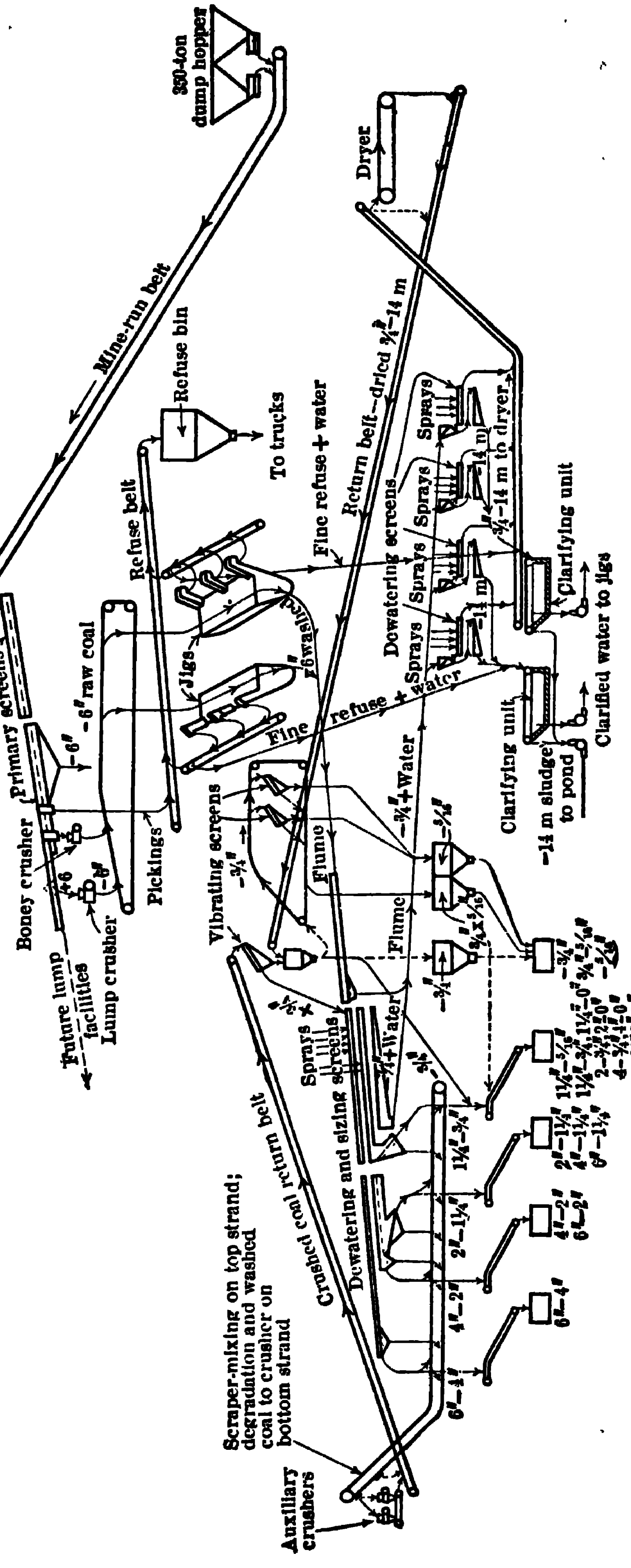


Fig 18. Baum Jig Plant

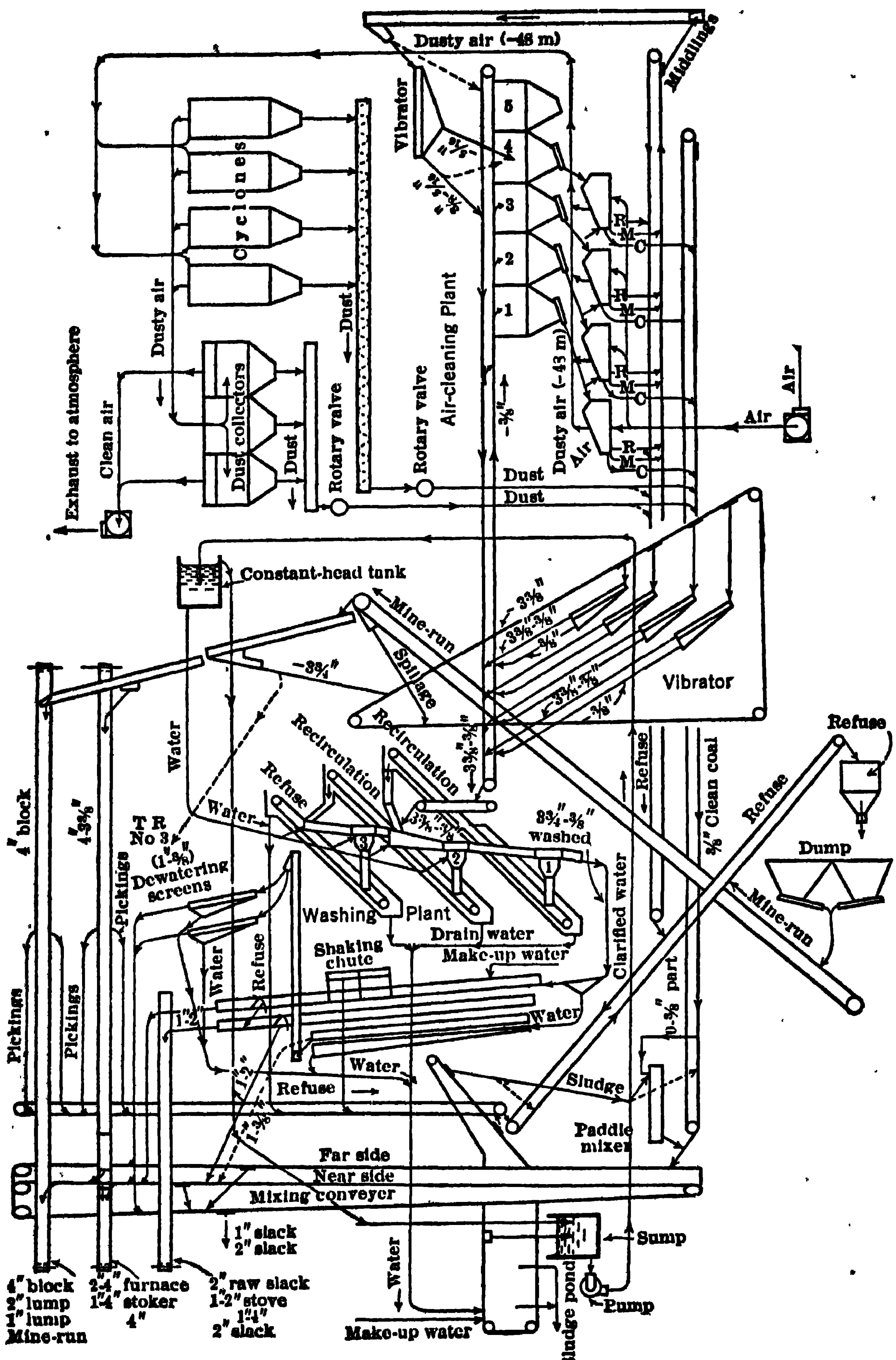


Fig 19. Combination Rheolaveur and Stump Air-flow Plant (*Coal Age*, Jan, 1938)

of the Christie dryer. Dryers usually have a variable-speed drive. Gases preferably are heated by a stoker-fed furnace, and fans used to supply the air. Cyclones sometimes recover the coal in the exhausted air, which may be added to the stoker feed.

17. FLOTATION (see also Sec 31)

Flotation, as applied to coal, separates fine particles of coal, pyrite, shale, and clay, in a pulp, by means of air bubbles produced by vigorous agitation (13). Reagents are added to cause selective adhesion of the bubbles to the coal particles. The bubbles and their burden of coal rise to the surface as a froth, which overflows and is recovered. The refuse material, to which the bubbles do not adhere, is withdrawn from below and discarded; it usually contains only a small proportion of coal.

Froth-flotation has improved the yield and quality of marketable coal in all plants where it has been installed (10). Numerous reagents are suitable. Plants located near by-product coking ovens generally use coal-tar derivatives, as creosote oil, cresylic acid, tar-oil distillates, and naphthalene or anthracene oils; others employ gas oil, petroleum, and wood-tar distillates. Reagent consumption is 1–2 lb per ton of dry raw coal. Pulp feed usually contains 20–25% solids. Coal froths contain 50–70% moisture. See Taggart's Handbook of Ore Dressing.

18. FLOWSHEETS

The preceding flowsheets show arrangement of 3 types of coal preparation plants: (*a*) Tipple equipped for hand-picking only (Fig 17); (*b*) Baum-jig plant with accessories (Fig 18); (*c*) a plant combining Rheolaveurs with Stump air-flow (Fig 19). Still more elaborate combinations are practicable, such as Baum jigs with hydro-separators and Stump air-flow equipment.

COKE

19. COMPOSITION AND PRODUCTION

Composition. Coke is the residue after driving off the volatile constituents of bituminous coal, out of contact with air; it consists essentially of carbon (the "fixed carbon" of the coal), with ash and a little water, and H, O and N gas. Color, silvery gray (from long-continued, high heat) to dull black (from quick burning, at low heat); texture, vesicular or pumice-like.

Production in United States, Net Tons (2)

1929 (Maximum year)

By-product coke,	53 411 826	tons,	from	76 758 958	tons	coal;	yield	69.6%,	without	breeze
Bee-hive	"	6 472 019	"	"	10 027 516	"	"	"	64.5%	"	"
Total,		59 883 845	"	"	86 786 474	"	"	"	69.0%	"	"

1936

By-product coke,	44 569 121	tons,	from	63 243 517	tons	coal;	yield	70.5%,	without	breeze
Bee-hive	"	1 706 063	"	"	2 698 158	"	"	"	63.2%	"	"
Total,		46 275 184	"	"	65 941 675	"	"	"	70.2%	"	"

Breeze is very fine coke, too small for furnace or domestic use, but can be burned under boilers. It is rarely recovered at bee-hive oven plants, and not entirely at by-product plants, but in 1936 an amount equal to 5.6% was actually recovered.

Aver yield of coke. Bee-hive, 1 ton coke from 1.55 tons coal, or 2.2 tons per day
By-product, 1 " " " 1.44 " " " 20.0 " " "

Methods of coking: (*a*) Mounds or heaps; time, 5 to 8 days (long obsolete). (*b*) Bee-hive and horizontal ovens; time, 48 to 72 hr (now nearly obsolete, excepting in times of large consumption). (*c*) Retorts or closed ovens, by-product; time, 14 to 22 hr.

20. BEE-HIVE OVENS

While there are still some thousands of beehive and the later rectangular ovens in existence, no more of either type are likely to be built, and those now existing are used only when the supply of coke from by-product plants is too small to meet the demand. For these reasons the reader is referred to pp 2028–2033 of the second edition of this handbook for data on bee-hive ovens and the machinery used in connection with them, for charging and drawing.

21. BY-PRODUCT OVENS (4, 5)

Coke manufacture in by-product ovens is a highly technical process, wholly different from bee-hive oven practice. The possibility of saving the by-products was known as early as 1766; satisfactory commercial results were first obtained in Europe about 1883.

The results of by-product coke making are so much superior commercially to former practice, that practically all blast-furnace plants of any size in the U S now obtain their coke from by-product ovens.

Location of plant. Bee-hive ovens, with few exceptions, have been built at the mines. By-product ovens are always at steel or furnace plants, or near large industrial centers. Such locations give the following advantages: convenience for obtaining coals from various sources; ready market for surplus gas; market for "breeze" (fines) and coke dust, always wasted at bee-hive plants; close supervision of coke making by the consumer; proximity to markets for tar and ammonium sulphate.

Kinds of oven (26–29). Although the designs of by-product ovens have been numerous, the following list of those used in the U S, Jan 1, 1937, shows that 3 types are now in most general use:

	Koppers	Semet-Solvay	Wilputte	All Others	Totals
At merchant plants......	1 953	1 080	221	421	3 675
At furnace plants........	7 833	716	505	120	9 174
Totals............	9 786	1 796	726	541	12 849

Design. Of the above types of oven, all have vert flues except the Solvay, which has horiz flues. Extensive improvements have been made in both design and capacity of the Koppers and Semet-Solvay ovens, and by-product coke making in the U S is now far ahead of foreign practice. The first Koppers oven in this country was built in 1908, and, as shown above, more than three-fourths of the total number of by-product ovens now (1938) in operation here are of this make.

Present capacity of by-product ovens is from 4.5 net tons per charge in small gas plants, to 20 net tons per charge in large steel plant installations. Coking time, 10–16 hr.

The latest designs (1929) of the Koppers (Becker type), Semet-Solvay and Otto-Wilputte by-product ovens, which are the most important types now in use, are shown in Fig 20, 21, 22.

Operation. The details of this subject being too complicated and extensive to be included in a book primarily on mining, the reader is referred for further information to Bibliography 23 to 29, at end of this section.

Gases. The coking process begins as soon as the oven has been charged, the charge leveled and the doors sealed. First gases from the ovens are rich in illuminants; usually about the first half of the gas evolved, or that given off in the first 6–10 hr, is used for illuminating purposes. The remaining gas is much poorer, and is used for heating the ovens or other fuel purposes. The rich gas has a heat value of 580 to 620 Btu per cu ft; lean gas varies from 500 to 580 Btu per cu ft. In a number of later installations, particularly those at gas plants, the ovens are heated by a low heat-value gas (such as producer or blast furnace gas) and all of the gas from the oven is utilized elsewhere.

The raw gas leaves the ovens at about 560° F, and is taken through a system of pipes, which are either sprayed or have a stream of flush water running in the bottom to keep the tar soft, this being condensed out of the gas. The condensed tar is removed by a trap, after which the gas passes to the primary coolers, of the multi-tubular, counter-flow type. Here most of the tar, the naphthalene and about 20% of the ammonia collect, the latter being dissolved in the water. Exhausters take the gas from the primary coolers to tar extractors, usually of impact type, where the remainder of the tar is removed. In most U S plants, the gas passes from the tar extractor to a reheater, where

it is heated by steam coils to 80°–130° F. It then passes to saturators, where it bubbles through a bath of H_2SO_4; the ammonia in the gas unites with the acid to make ammonium sulphate, the crystals of which fall to bottom of the tank, whence they are removed, purified and are ready for market. This process does not affect quality of the gas, and no free acid is carried away by it. The gas then passes to the final cooler, where direct contact with cooling water reduces its tempera-

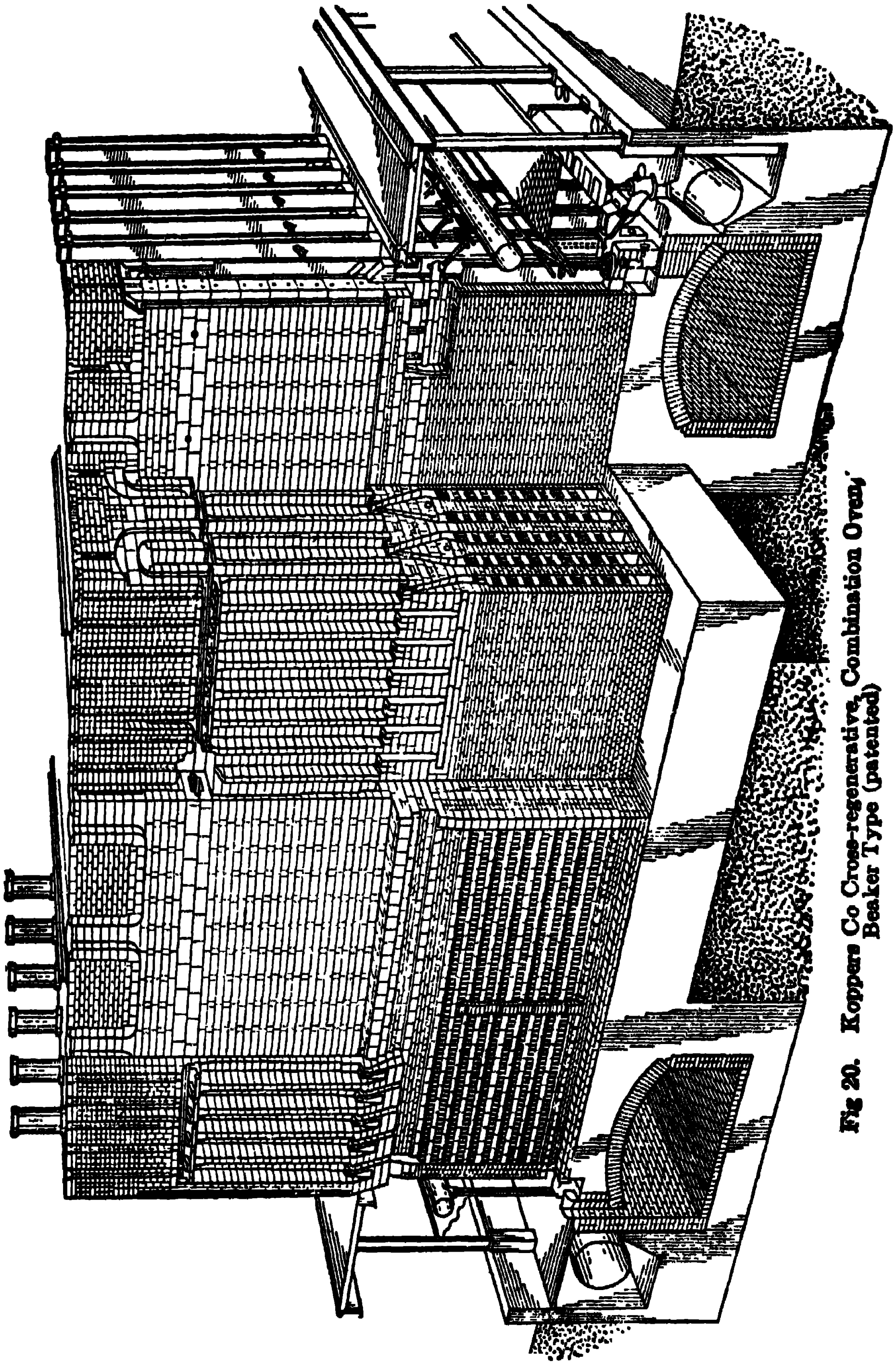


Fig 20. Koppers Co Cross-regenerative, Combination Oven, Becker Type (patented)

ture and removes the remaining naphthalene. Thence it passes to the gas holders, or (at most U S plants) to the benzol recovery plant. In the latter the gas is intimately mixed, in a counter-current flow, with "straw" oil in large washers, from which the gas goes to the holders, and the oil to stills, where the light oil is purified. This light oil is a mixture of benzol, toluol, xylol and solvent naphtha. It is distilled in large stills by steam into a number of fractions, depending upon the market for the products.

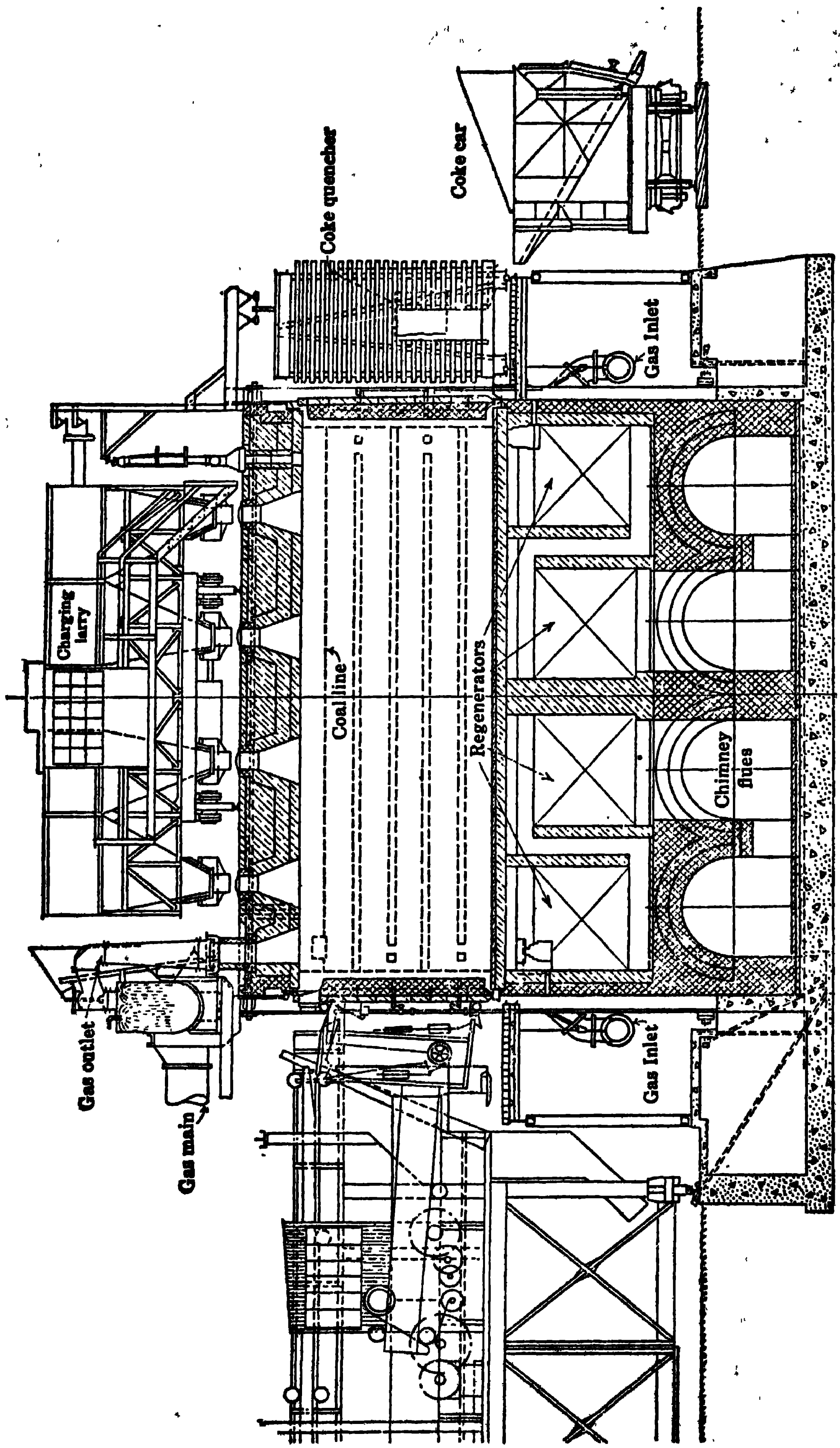


Fig 21. Semet-Solvay Standard Regenerator Gas Oven (Longit Sec)

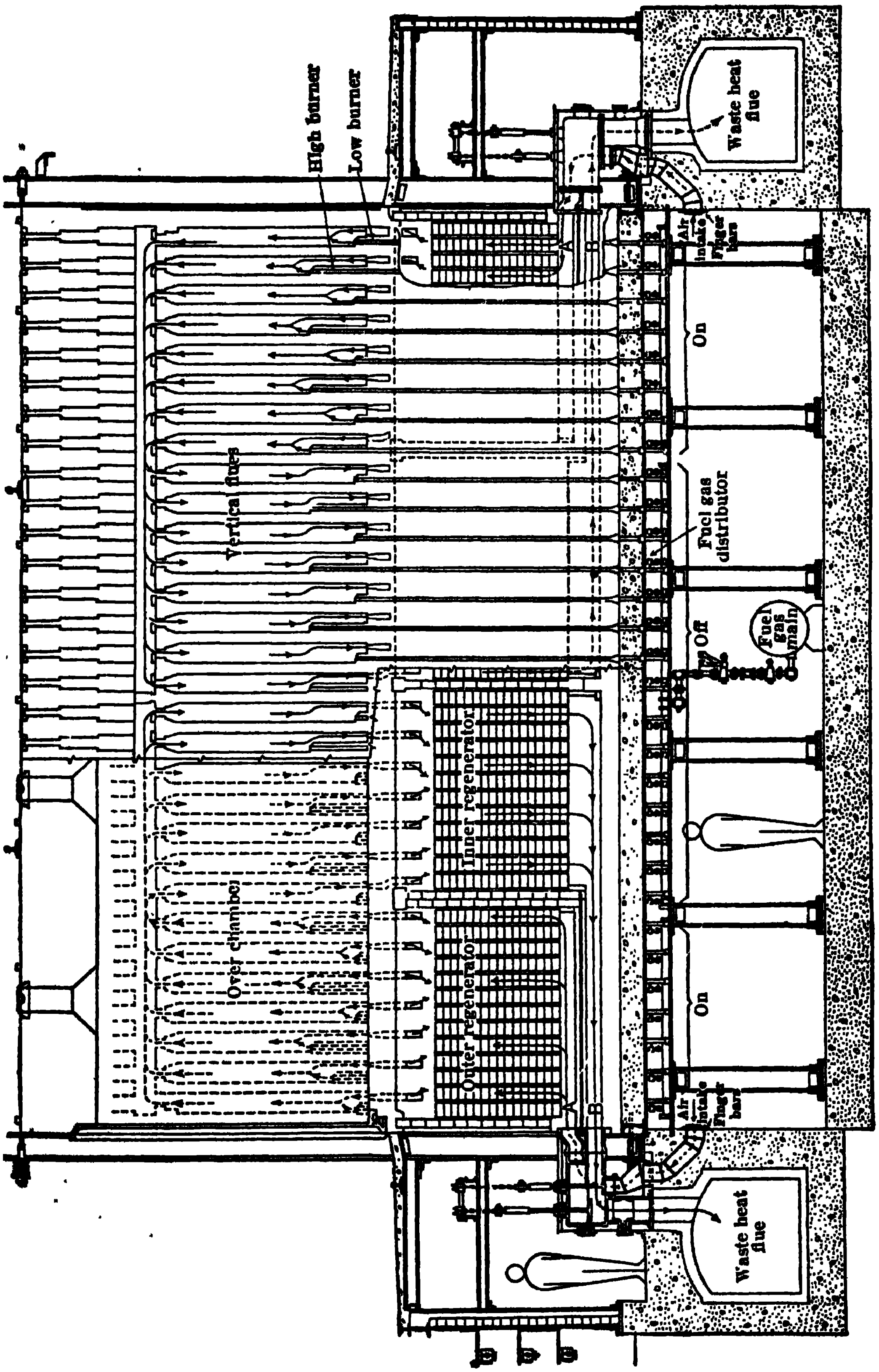


Fig 22. Otto-Wilputte Oven (Longit Sec)

Diagram Showing Derivation of By-products

- Product, metallurgical and domestic coke. Yield, 70 to 80% of coal charged
- By-products
  - Coke dust
  - Coke breeze
  - Gas.......... Surplus ranges from 4 300 to 7 500 cu ft per net ton coal charged
    - Illuminating and heating, 500 to 600 Btu per cu ft
    - Total gas yield, 10 000–12 000 cu ft per net ton of coal
    - Bensol
  - Tar.......... Yield, 6–13 gal per net ton coal
    - If heated to following temp: to 170° C
      - Ammonia liquor
      - Light oils....
        - Bensol
        - Naphtha
    - 170° to 230° C
      - Middle oils..
        - Naphthalene
        - Carbolic acid
    - 230° to 270° C
      - Creosote
      - Heavy lubricating oils
    - 270° to 360° C
      - Anthracene
      - Pitch
  - Ammonia..... Yield, 25 lb per net ton coal
    - Concentrated to 15–22% strength
    - Sulphate of ammonia, by combination with H_2SO_4; approx 20% nitrogen
    - Yield, 16–30 lb per net ton coal; aver, about 25 lb.

Uses of the By-products

Coke dust.....	In steel mill soaking pits Up to 1/4-in size, burned under boilers on automatic stoker grates
Coke breeze...	Domestic fuel As fuel in gas producers, for heating coke oven to release entire yield of oven gas
Gas..........	Domestic purposes Industrial fuel Gas engines Steel-plant heating furnaces, open-hearth furnaces, etc Cement kilns
Tar..........	For road tar, roofing tars, roofing felts Heavy-oil engines, and in open-hearth furnaces Waterproof paint, briquette binders
Bensol........	Chemical industries: for dyes; a solvent in making perfumes and medicines; in making high explosives; as motor fuel
Naphtha......	Solvent
Naphthalene..	Insecticide, moth balls, chemicals
Carbolic acid..	Disinfectant
Creosote......	Medicinal Wood preservative Sheep dip
Heavy oils....	Lubrication
Anthracene....	Chemical manufacture
Pitch.........	Paving, waterproofing, fuel briquetting Manufacture of electrodes
Concentrated ammonia	Medicinal, household and refrigeration uses In production of soda ash. Manufacture of explosives
Ammonia sulphate....	Chemicals; fertilizer, usually mixed with phosphates and potash

Recent developments have been in removing water from the gas before it enters the distribution system, in removing phenol from the waste products to prevent stream pollution, and the liquid purification of gas to completely remove sulphur.

By-products, besides coke and gas, are usually tar and ammonium sulphate. Bensol and a number of other derivatives can be made, although taking out additional products is likely to interfere with the production of the others (27–29).

Surplus gas. Total gas produced from by-product ovens is 10 000–12 000 cu ft per net ton of coal. In many of the latest plants, it has been found economical to heat the ovens with producer or blast-furnace gas and to utilize all of the coal gas produced for fuel or lighting purposes.

Design and construction of a modern by-product plant are a highly technical problem, and should be undertaken by the companies building such plants only after they have been furnished with all the necessary basic local data.

22. MISCELLANY

Basic coke. Many attempts have been made to lower the sulphur content of coke during its manufacture, some experiments being along the line of adding limestone to the coal charged, but a basic coke has not yet been commercially produced.

Coke tests. For this subject see Bibliography, 27, 28.

Low-temperature distillation of coal. For many years much effort and money have been expended, both abroad and in the U S, to develop an effic process for obtaining a more satisfactory fuel and by-products from coal at lower temperatures than those used in by-product ovens, as it is thought that the fuel will be better for domestic use, the yield of tar and oils much greater, and that a lower cost for both plant and operation will result. A number of plants of this type are in commercial use in European countries, including Great Britain; in the U S, due to differing fuel conditions, only one plant is in operation, though several others are in an experimental stage.

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SECTION 36

MATHEMATICS AND MECHANICS

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MATHEMATICS AND MECHANICS

ALGEBRA

1. ELEMENTARY OPERATIONS

Symbols of operation are the same in algebra as in arithmetic. The symbol of addition is often omitted in arithmetic. Thus, $3^2/_7$ means $3 + {}^2/_7$. If a symbol of operation is omitted in algebra, it is that of multiplication. Thus, $2\frac{x}{y}$ means $2 \times \frac{x}{y}$. $4 > x^2$ means that 4 is greater than x^2. $3 < ab$ means that 3 is less than $a \times b$.

Symbols of aggregation are the bar, | ; vinculum, ——; parenthesis, ( ); bracket, [ ]; and brace, { }. Thus, each of the expressions $\left.\begin{matrix} a \\ +c \end{matrix}\right|$, $\overline{a+c}$, $(a + c)$, $[a + c]$, $\{a + c\}$, means that $a + c$ is to be treated as a single number. If an expression within a parenthesis is preceded by a plus sign, the parenthesis may be removed. If an expression within a parenthesis is preceded by a minus sign, the parenthesis may be removed if the sign of every term within it be changed. Thus, $2 + (a - b) = 2 + a - b$; and $2 - (a - b) = 2 - a + b$. If parentheses occur within parentheses, these may be removed, in succession, by removing first the innermost parenthesis; next, the innermost of all that remain and so on:

$$a - [6 + c - \{d + 3\}] = a - [6 + c - d - 3] = a - [3 + c - d] = a - 3 - c + d$$

Addition. To add like terms, perform the indicated arithmetical operations on the numerical coefficients; this gives the coeff of the answer. Thus,

$$-7\,a + 6\,a - 2\,a + a = -2\,a$$

To add terms which are not all like, combine the like terms and write down the others, each preceded by its proper sign. Thus,

$$-2\,ax + 4\,z + 5\,ax - 3\,z - c = 3\,ax + z - c$$

Polynomials are added by adding their respective terms.

Thus, to add $m^5 - 3\,m^4n - 6\,m^3n^2$, $+\,m^3n^2 + m^2n^3 - 5\,m^4n$ and $-\,n^5 + 2\,m^5 + 7\,m^4n$, write:

$$\begin{array}{rrrrr}
+\;m^5 & -\,3\,m^4n & -\,6\,m^3n^2 & & \\
 & -\,5\,m^4n & +\;m^3n^2 & +\;m^2n^3 & \\
+2\,m^5 & +\,7\,m^4n & & & -\,n^5 \\
\hline
+3\,m^5 & -\;m^4n & -\,5\,m^3n^2 & +\,m^2n^3 & -\,n^5
\end{array}$$

Subtraction. One polynomial may be subtracted from another by changing the sign of each term of the subtrahend and adding.

Thus, $(a^3x^2 + 2\,a^2x^3 - 4\,ax^4) - (a^5 + 4\,a^3x^2 - 3\,a^2x^3 - 4\,ax^4) =$
$a^3x^2 + 2\,a^2x^3 - 4\,ax^4 - a^5 - 4\,a^3x^2 + 3\,a^2x^3 + 4\,ax^4 = -a^5 - 3\,a^3x^2 + 5\,a^2x^3$

Multiplication. To find the product of monomials, annex the literal factors to the product of the numerical factors. Thus, $4\,a \times 2\,b \times 3\,z = 24\,abz$. Like signs produce plus; unlike signs produce minus.

Thus, $4\,a \times 3\,c = +\,12\,ac$; $4\,a \times (-\,3\,c) = -\,12\,ac$; $-\,4\,a \times (-3\,c) = +12ac$; $a^2 \times a^3 = a^{2+3} = aa \times aaa = aaaaa = a^5$; $4\,a^2c \cdot ac^3 = 4\,aac \cdot accc = 4\,aaa \cdot cccc = 4\,a^3c^4$; $6\,ab^2y^3 \times 2\,b^3y^3 \times (-5\,a^2y) = -60\,a^3b^5y^7$.

To multiply two polynomials, multiply each term of one factor by each term of the other factor and add the partial products.

Thus, to multiply $(-a^3 + 2\,a^2b - b^3)$ by $(+4a^2 + 8\,ab)$, the operation may be arranged and carried out thus:

$$\begin{array}{rrrrr}
-a^3 & +\,2\,a^2b & -\,b^3 & & \\
4\,a^2 & +\,8\,ab & & & \\
\hline
-4\,a^5 & +\,8\,a^4b & -\,4\,a^2b^3 & & \\
 & -\,8\,a^4b & & +\,16\,a^3b^2 & -\,8\,ab^4 \\
\hline
-4\,a^5 & & -\,4\,a^2b^3 & +\,16\,a^3b^2 & -\,8\,ab^4
\end{array}$$

Multiplication of polynomials may be indicated by inclosing each in a parenthesis and writing them one after the other.

Thus: $(2b-3)(2b+3)(4b^2+9)$ indicates that the three binomials are to be multiplied together. The indicated operations are performed as follows:

$$\begin{array}{lll} 2b-3 & & \\ 2b+3 & & \text{Multiply the first two.} \\ \hline 4b^2-6b & & \\ \quad +6b-9 & & \\ \hline 4b^2 \quad -9 & & \\ 4b^2 \quad +9 & & \text{Multiply by the third.} \\ \hline 16b^4 \quad -36b^2 & & \\ \quad +36b^2-81 & & \\ 16b^4 \quad -81 & & \text{The final result.} \end{array}$$

Division is the operation by which, if a product and one of its factors are given, the other factor is determined. The product is called the DIVIDEND; the given factor the DIVISOR; and the required factor the QUOTIENT. The quotient is positive if dividend and divisor have like signs; it is negative if dividend and divisor have unlike signs.

$$m^5p^2x^4 \div mp^2x^2 = \frac{m^5p^2x^4}{mp^2x^2} = m^4x^2; \qquad -51\,abdy^2 \div 3\,bdy = \frac{-51\,abdy^2}{3\,bdy} = -17\,ay$$

To divide a polynomial by a polynomial, arrange both dividend and divisor according to the ascending or descending powers of some common letter, and keep this order throughout the operation. Arrange the computation thus: Dividend $\frac{\text{Divisor}}{\text{Quotient}}$. Divide the first term of the dividend by the first term of the divisor and write the result as the first term of the quotient. Multiply all terms of the divisor by the first term of the quotient. Subtract the product from the dividend. If there be a remainder, consider it as a new dividend and proceed as before.

Thus: $(22\,a^2b^2 + 15\,b^4 + 3\,a^4 - 10\,a^3b - 22\,ab^3) \div (a^2 + 3\,b^2 - 2\,ab)$

$$\begin{array}{l|l} 3a^4 - 10a^3b + 22a^2b^2 - 22ab^3 + 15b^4 & a^2 - 2ab + 3b^2 \\ \underline{3a^4 - 6a^3b + 9a^2b^2} & 3a^2 - 4ab + 5b^2 \\ -4a^3b + 13a^2b^2 - 22ab^3 + 15b^4 & \\ \underline{-4a^3b + 8a^2b^2 - 12ab^3} & \\ 5a^2b^2 - 10ab^3 + 15b^4 & \\ \underline{5a^2b^2 - 10ab^3 + 15b^4} & \end{array}$$

The quotient is $(3\,a^2 - 4\,ab + 5\,b^2)$.

Factoring is the process of finding two or more expressions the product of which is equal to a given expression. First, factor out any monomial common to each term; then treat the polynomial by one of the following TYPE FORMS:

1. $ax + ay + bx + by = a(x+y) + b(x+y) = (a+b)(x+y)$
2. $a^2 + 2\,ab + b^2 = (a+b)(a+b)$
3. $y^2 + by + c = (y+p)(y+g)$, if p and g are two numbers the sum of which is b and the product is c.
4. $ax^2 + bx + c$

Find two numbers the algebraic sum of which is b and the product is $a \cdot c$. Replace bx by two terms in x, the coefficients of which are the numbers just found, and factor by grouping terms.

Thus: $4\,x^2 - 5\,x - 6 = 4\,x^2 - 8\,x + 3\,x - 6 = 4\,x(x-2) + 3(x-2) = (4\,x+3)(x-2)$

5. $a^2 - b^2 = (a+b)(a-b)$
6. $a^4 + ka^2b^2 + b^4$. This type can sometimes be reduced to type 5 by adding and subtracting a multiple of a^2b^2. Thus:

$$\begin{aligned} 9\,x^4 + 3\,x^2y^2 + 4\,y^4 &= (9\,x^4 + 12\,x^2y^2 + 4\,y^4) - 9\,x^2y^2 \\ &= (3\,x^2 + 2\,y^2)^2 - 9\,x^2y^2 \\ &= (3\,x^2 + 2\,y^2 + 3\,xy)(3\,x^2 + 2\,y^2 - 3\,xy) \end{aligned}$$

7. $\left\{\begin{aligned} a^n + b^n &= (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 \ldots + b^{n-1}) \\ a^n - b^n &= (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 \ldots + b^{n-1}) \end{aligned}\right\}$ if n is odd.

If n is even, $(a^n - b^n)$ becomes type 5. In all other cases, if n is a multiple of 3, apply one of the special types,

$$\begin{aligned} a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\ a^3 - b^3 &= (a-b)(a^2 + ab + b^2) \end{aligned}$$

8. $a^2 + b^2 + c^2 + 2\,ab + 2\,ac + 2\,bc = (a+b+c)^2$

2. POWERS, ROOTS AND RADICALS

The relation between exponential and radical notation is expressed by the formula, $\sqrt[b]{x^a} = (\sqrt[b]{x})^a = x^{\frac{a}{b}}$, in which a and b are any numbers and b is not zero. In such a fractional exponent, the numerator indicates the power to which the number is to be raised and the denominator gives the index of the root which is to be extracted. The fundamental laws of exponents are as follows:

1. $(+a)^n = +a^n$
2. $(-a)^{2n} = -a^{2n}$
3. $(-a)^{2n+1} = -a^{2n+1}$
4. $a^m a^n = a^{m+n}$
5. $a^m \div a^n = a^{m-n}$
6. $a^m b^m = (ab)^m$
7. $a^m \div b^m = (a \div b)^m$
8. $1 \div a^m = (1 \div a)^m = a^{-m}$
9. $a^m = (1 \div a)^{-m} = 1 \div a^{-m}$
10. $(a^m)^n = a^{mn} = (a^n)^m$

11. $a^0 = 1$; $0a = 0$; $0^0 =$ indeterminate.

12. $\sqrt[2n]{+a}$ is positive or negative; $\sqrt[(2n+1)]{+a}$ is positive; $\sqrt[(2n+1)]{-a}$ is negative.

The root of $\sqrt[2n]{-a}$ is impossible or imaginary and the simplest one is $\sqrt{-1}$, which is represented by the symbol i. A complex quantity is partly real and partly imaginary:

$$i = \sqrt{-1};\quad i^2 = -1;\quad i^3 = -i;\quad i^4 = +1;\quad \frac{1}{i} = -i;$$

$$i^{4n+m} = i^m;\quad i^{4n} = +1;\quad i^{4n+1} = +i;\quad i^{4n+2} = -1;\quad i^{4n+3} = -i.$$

13. $\sqrt[m]{ab} = \sqrt[m]{a}\sqrt[m]{b} = (ab)^{\frac{1}{m}}$

14. $\sqrt[m]{a \div b} = \sqrt[m]{a} \div \sqrt[m]{b} = \left(\frac{a}{b}\right)^{\frac{1}{m}}$

15. $\sqrt[m]{\frac{1}{a}} = \frac{1}{\sqrt[m]{a}} = \frac{1}{a^{\frac{1}{m}}} = a^{-\frac{1}{m}}$

16. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = a^{\frac{1}{mn}} = \sqrt[n]{\sqrt[m]{a}}$

17. $\sqrt{a + b + 2\sqrt{ab}} = \sqrt{a} + \sqrt{b}$

The following approximations are often convenient and are sufficiently accurate if x and y are small compared with 1:

18. $(1 + x)^2 = 1 + 2x$
19. $(1 + x)^{\frac{1}{2}} = 1 + \frac{x}{2}$
20. $\frac{1}{1 + x} = 1 - x$
21. $\frac{1}{1 - x} = 1 + x$
22. $(1 + x)(1 + y) = 1 + x + y$
23. $(1 + x)(1 - y) = 1 + x - y$
24. $(1 + x)^2 (1 - 2y)^2 = 1 + 2x - 4y$

If a and b are nearly equal,

25. $\sqrt{ab} = (a + b) \div 2$

If b is small compared with a,

26. $\sqrt{a^2 \pm b} = a \pm \frac{b}{2a}$
27. $\sqrt[3]{a^3 \pm b} = a \pm \frac{b}{3a^2}$

28. $\sqrt{a^2 + b^2} = 0.960\,a + 0.398\,b$. This is within 4% of the true value if $a > b$. A closer approximation is $\sqrt{a^2 + b^2} = 0.9938\,a + 0.0703\,b + 0.3567\,\frac{b^2}{a}$.

29. $\sqrt{a^2 + b^2 + c^2} = 0.939\,a + 0.389\,b + 0\ 297\,c$. This is within 6% of the true value if $a > b > c$. For instance, for the numbers 43, 42 and 41, the error $< 5.2\%$.

3. BINOMIAL THEOREM

Binomial theorem is used to expand a binomial expression into a series as follows:

$$(a \pm b)^n = a^n \pm na^{(n-1)}\,b + \frac{n\,(n-1)}{1 \cdot 2}\,a^{(n-2)}\,b^2 + \frac{n\,(n-1)\,(n-2)}{1 \cdot 2 \cdot 3}\,a^{(n-3)}\,b^3 + \cdots$$

The series is finite if n is a positive whole number; it is infinite if n is fractional or negative; and it is convergent for $a > b$. The product $1 \cdot 2 \cdot 3 \ldots r$ is called factorial r and may be written $r!$. The coefficients of the powers are called binomial coeds, and the rth coeff is $\frac{n\,(n-1)\,(n-2)\ldots[n-(r-1)]}{1 \cdot 2 \cdot 3 \ldots r}$. The binomial $(a \pm b)^n = a^n\ (1 \pm x)^n$, in which $x = b \div a$.

It is often more convenient to expand the second form than the first. The four following special forms can be used if n is any positive integer and for any negative or fractional value of n, if x lies between 0 and 1:

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{1\cdot 2}x^2 \pm \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^3 + \frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}x^4 \pm \cdots$$

$$\frac{1}{1 \pm x} = (1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + x^4 \mp x^5 \cdots$$

$$\sqrt{1 \pm x} = (1 \pm x)^{1/2} = 1 \pm \frac{1}{2}x - \frac{1}{2\cdot 4}x^2 \pm \frac{1\cdot 3}{2\cdot 4\cdot 6}x^3 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}x^4 \pm \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8\cdot 10}x^5 \cdots$$

$$\frac{1}{\sqrt{1 \pm x}} = (1 \pm x)^{-1/2} = 1 \mp \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 \mp \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \cdots$$

Proportion $a : b :: c : d$ may be written $\frac{a}{b} = \frac{c}{d}$.

Hence, $ad = bc$, $\frac{b}{a} = \frac{d}{c}$, $\frac{a}{c} = \frac{b}{d}$, $\frac{a+b}{b} = \frac{c+d}{d}$, $\frac{a-b}{b} = \frac{c-d}{d}$, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

4. SERIES

Series is a succession of numbers which proceed according to some fixed law. Any one of the numbers is called a TERM of the series. A series continued indefinitely is an INFINITE SERIES, and one that ends with some particular term is a FINITE SERIES.

Arithmetical series or progression is a series in which the difference between any two adjacent terms is equal to the difference between any other two adjacent terms. The common difference may be plus or minus. The formula for the nth term is, $t_n = a + (n-1)d$. The sum of n terms is $Sn = \frac{n}{2}(a + t_n)$, in which a is the first term and d the common difference. From the above equations, any one of the quantities, a, d, t_n, n, or S, may be found if any three of the others are given.

Geometrical series or progression. In this, each succeeding term is obtained by multiplying the preceding term by a constant multiplier. If a is the first term, r the constant multiplier, and t_n the nth term, $t_n = ar^{(n-1)}$. GEOMETRICAL MEAN between two numbers is the number which stands between them and makes, with them, a geometrical series. If a and b denote two numbers, then $G = \sqrt{ab}$ is their geometrical mean. If it is desired to insert m means between the numbers, the constant multiplier is given by the formula $r^{(m+1)} = b \div a$. The sum of n terms is $S = \frac{a(r^n - 1)}{r-1} = \frac{rb - a}{r-1}$, b being the nth term.

Special Series:

1. $1 + 2 + 3 + 4 + \cdots + (n-1) + n = n(n+1) \div 2$
2. $p + (p+1) + (p+2) + \cdots + (q-1) + q = (q+p)(q-p+1) \div 2$
3. $2 + 4 + 6 + 8 + \cdots + (2n-2) + 2n = n(n+1)$
4. $1 + 3 + 5 + 7 + \cdots + (2n-3) + (2n-1) = n^2$
5. $1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n-1)^2 + n^2 = n(n+1)(2n+1) \div (1\cdot 2\cdot 3)$
6. $1^3 + 2^3 + 3^3 + 4^3 + \cdots + (n-1)^3 + n^3 = [n(n+1) \div 2]^2$
7. $1^4 + 2^4 + 3^4 \cdots + (n-1)^4 + n^4 = n(n+1)(2n+1)(3n^2 + 3n - 1) \div 30$
8. $\frac{1 + 2 + 3 + 4 + 5 \cdots + n}{n^2} = \frac{1}{2}$
9. $\frac{1 + 2^2 + 3^2 + 4^2 + \cdots + n^2}{n^3} = \frac{1}{3}$ as n approaches ∞
10. $\frac{1 + 2^3 + 3^3 + 4^3 + \cdots + n^3}{n^4} = \frac{1}{4}$
11. $e = \lim_{n\to\infty}\left(1 + \frac{1^n}{n}\right) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots = 2.71828$, in which e is the base of Naperian system of logarithms, and $\lim_{n\to\infty}$ means "the limit as n approaches infinity."
12. $a^x = 1 + \frac{x}{1}\log_e a + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + \cdots$
13. $e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

$$\left.\begin{array}{l}14.\ \text{Log}_e\,(1 \pm x) = \pm x - \dfrac{x^2}{2} \pm \dfrac{x^3}{3} + \dfrac{x^4}{4} \pm \dfrac{x^5}{5} \cdots \\ 15.\ \text{Log}_e\left(\dfrac{1+x}{1-x}\right) = 2\left(x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \dfrac{x^7}{7} + \cdots\right)\end{array}\right\}\text{if } -1 < x < +1$$

$$16.\ \text{Log}_e\left(\frac{x+1}{x-1}\right) = 2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \cdots\right)\text{if } -1 > x, \text{ or } x > +1$$

$$\left.\begin{array}{l}17.\ \sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \dfrac{x^7}{7!} + \cdots \\ 18.\ \cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \dfrac{x^8}{8!} \cdots \\ 19.\ \tan x = x + \dfrac{x^3}{3} + \dfrac{2x^5}{3 \cdot 5} + \dfrac{17x^7}{3^2 \cdot 5 \cdot 7} + \dfrac{62x^9}{3^2 \cdot 5 \cdot 7 \cdot 9} + \cdots\end{array}\right\}\begin{array}{l}\text{The angle } x \text{ is in RADIANS,} \\ \text{and } = \text{ angle in degrees} \\ \times \dfrac{\pi}{180}\end{array}$$

5. LOGARITHMS

A Logarithm is the power to which a given fixed number must be raised to produce another number. In $y = b^x$, b is the base, x the logarithm or power, and y the number which is produced. For common, or Briggs, logarithms the base is 10. The only other system in common use is the Naperian, natural or hyperbolic system, for which the base is 2.71828. The abbreviation *log* means the logarithm of a number to the base 10. The notation Nap log 6, or $\log_e 6$, means the logarithm of 6 to the base e; and log 6 means the logarithm of 6 to the base 10. Thus, $\log 100 = 2$, because $100 = 10^2$; $\log 10 = 1$, because $10 = 10^1$; $\log 1 = 0$, because $1 = 10^0$; $\log 0.1 = -1$, because $0.1 = 10^{-1}$. Any number > 1 has a positive logarithm and any number < 1 has a negative logarithm. Numbers which are not whole powers of 10 must result from raising 10 to some fractional or irrational power. The decimal part of such powers, or logarithms, may be obtained from a table of logarithms. Since 10 is larger than e (2.71828), it takes a higher power of e to produce a given number than it does of 10. This means that the logarithm of a number to base e, is larger than to base 10. Relation between the two systems of logarithms is:

$$\log_e n = \log_{10} n \div \log_{10} e$$

or

$$\log_e n = \log_{10} n \div 0.4343 = \log_{10} n \times 2.303$$

6. CHOICE AND CHANCE

Permutations or choice. If one thing can be done in a different ways and another thing can be done in b different ways, then both together can be done in $a \times b$ different ways. If one thing can be done in a ways, a second in b ways, a third in c ways and a fourth in d ways, the number of ways of doing all these things will be $a \times b \times c \times d$. The number of arrangements of n different elements or things, taken all at a time, is

$$n\,(n-1)\,(n-2)\,(n-3)\cdots 3 \times 2 \times 1 = n!$$

Probability or chance. If an event may happen in a ways and fail in b ways, each way being equally probable, the chance or probability that it will happen in a ways is $a \div (a+b)$; the chance that it will fail is $b \div (a+b)$. The sum of the chances is 1. Odds in favor of the event is the ratio of chance of happening to chance of failure. Odds in favor are $a \div b$; odds against, $b \div a$.

7. QUADRATIC AND CUBIC EQUATIONS

Linear equations, or those of the first degree, contain the first power only of the unknown, and have but one root. Thus, $x = -\dfrac{b}{a}$ is the root of $ax + b = 0$.

Quadratic equations, or those of the second degree, contain the second power of the unknown and have two roots. If equation is written in the form $ax^2 + bx + c = 0$, the roots are $x = (-b \pm \sqrt{b^2 - 4\,ac}) \div 2\,a$. If $(b^2 - 4\,ac) > 0$, the roots are real; if $(b^2 - 4\,ac) < 0$, the roots are imaginary; if $(b^2 - 4\,ac)$ is a perfect square, the roots are rational; if $(b^2 - 4\,ac) = 0$, the roots are equal but opposite in sign. Following is an Euclidean (graphical) construction for the roots of a quadratic. The equation should be put in the form,

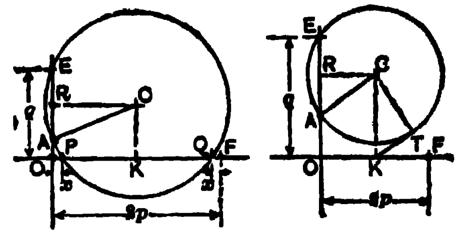
$$x^2 - 2\,px + q = 0, \text{ or } x\,(2\,p - x) = q$$

Case 1. p and q are both positive factors. On intersecting axes, ox and oy, not necessarily at right angles (Fig 1a), lay off OA = unity and $OE = q$, both upwards, and $OF = 2\,p$, to the right. From C, the intersection of the perpendicular bisectors KC and RC of lines OF and AE, as a center, with CA as a radius, describe a circle; its intercepts, OP and OQ, on OX, are the roots required.

Case 2. p negative and q positive. Lay off $2\,p = OF$ to the left, and proceed as before. Both roots will be negative, because measured to the left.

Case 3. p and q are negative factors. Lay off $OE = q$, downward, $OF = 2\,p$ to left, OA = unity upward, and proceed as before. One root will be negative.

Case 4. p positive and q negative. Lay off $OF = 2\,p$ to right, OA = unity, upwards, $OE = q$ downwards, and proceed as before. One root will be negative.



(a) (b)

Fig 1

Case 5. THE CIRCLE DOES NOT CUT OX, therefore the roots are imaginary. The real part of the imaginary root can be found by drawing a tangent to the circle from K (Fig 1b). If T is the point of tangency, KT will be the real part of the imaginary roots.

Cubic equation, or one of the third degree, contains the third power of the unknown, may contain the second, first, and zero powers; it has 3 roots, 2 of which may be imaginary. TRIGONOMETRIC SOLUTION OF THE CUBIC. If the equation is in the form $y^3 + by^2 + cy^1 + d = 0$, it is necessary to transform it to a cubic, in which the second power does not appear. This is done by making $y = x - (b \div 3)$, expanding and collecting terms; it will then be in the form $x^3 + px + q = 0$. Three cases must be distinguished:

$$\left.\begin{array}{l} 1.\ x^3 + px + q = 0 \\ 2.\ x^3 - px + q = 0, \text{ and } (p^3 \div 27) < (q^2 \div 4) \\ 3.\ x^3 - px + q = 0, \text{ and } (p^3 \div 27) > (q^2 \div 4) \end{array}\right\}$$ In these, p is necessarily positive and q is positive or negative.

Formulas for Case 1: $tan\ A = \dfrac{2\,p}{3\,q}\sqrt{\dfrac{p}{3}}$, $tan\ \dfrac{1}{2}B = \sqrt[3]{tan\ \dfrac{1}{2}A}$;

the roots are, $x_1 = -2\sqrt{\dfrac{p}{3}}\ cot\ B$, $x_2 = \sqrt{\dfrac{p}{3}}\ cot\ B + cosec\ B\sqrt{p}\sqrt{-1}$, and

$x_3 = \sqrt{\dfrac{p}{3}}\ cot\ B - cosec\ B\sqrt{p}\sqrt{-1}$

Formulas for Case 2: $sin\ A = \dfrac{2\,p}{3\,q}\sqrt{\dfrac{p}{3}}$, $tan\ \dfrac{1}{2}B = \sqrt[3]{tan\ \dfrac{1}{2}A}$;

the roots are, $x_1 = -2\sqrt{\dfrac{p}{3}}\ cosec\ B$, $x_2 = \sqrt{\dfrac{p}{3}}\ cosec\ B + cot\ B\sqrt{p}\sqrt{-1}$, and

$x_3 = \sqrt{\dfrac{p}{3}}\ cosec\ B - cot\ B\sqrt{p}\sqrt{-1}$

Formula for Case 3: $sin\ A = \dfrac{3\,q}{2\,p}\sqrt{\dfrac{3}{p}}$; the roots are, $x_1 = 2\sqrt{\dfrac{p}{3}}\ sin\ \dfrac{A}{3}$,

$x_2 = 2\sqrt{\dfrac{p}{3}}\ sin\left(60° - \dfrac{A}{3}\right)$ and $x_3 = -2\sqrt{\dfrac{p}{3}}\ sin\left(60° + \dfrac{A}{3}\right)$

8. INTEREST

Simple interest. Let principal = P, yearly rate = $r\%$, amount of \$1 for 1 year = R, number of years = n, and amount of P for n years = A; then $R = 1 + r$, the interest on P for n years = Pnr, and $A = P\,(1 + nr)$. P is sometimes called the present worth of A. If d = any number of days, simple interest for that time is given by following formulas: at 4%, interest = $Pd \div 9\,000$; 5%, $Pd \div 7\,200$; 6%, $Pd \div 6\,000$; 8%, $Pd \div 4\,500$.

As 6% is a very common rate, it is worth recalling that interest for 1 month = $(P \times 6) \div (12 \times 100) = P \div 200$; divide this by 30 to get the interest for 1 day, and multiply the last result by d to get the interest for d days. Proportional parts of these results may be taken for other percentages.

Example. The interest on \$5 284 for 30 days at 6% = 5 284 ÷ 200 = \$26.42; interest for 1 day = 26.42 ÷ 30 = \$0.8807; interest for 12 days = 12 × 0.8807 = \$10.568. By the 6% formula given above the interest for 12 days would be (\$5 284 × 12)

+ 6 000 = \$10.568. At 4%, the interest for the 12 days would be $^4/_6$ \$10.57 = \$7.04. At 7%, the interest would be $^7/_6$ \$10.57 = \$12.33.

Compound interest. If, at the end of a period, interest is added to principal, and then interest charged on the new principal, the second period's interest added to form a new principal and so on, the interest is said to be compounded. With notation as before, $A = PR^n$, or $P = A \div R^n$.

Example. In 12 years at 6%, \$400 amounts to $A = 400\ (1.06)^{12} = \$804.88$.

Present worth of \$5 000 due in 15 yr at 5% comp'd interest is $P = \dfrac{5\ 000}{(1.05)^{15}} = \$2\ 405.09$

If interest is to be added to principal g times each year, $A = P\left(1 + \dfrac{r}{g}\right)^{gn}$

Example. The amount of \$400 in 12 years with interest at 6% and compounded quarterly will be $A = 400\left(1 + \dfrac{0.06}{4}\right)^{4\times 12} = \$817.39$, instead of \$804.88 if compounded yearly, and instead of \$688.00 by simple interest.

Sinking fund is created to pay a debt that falls due, or to provide for expenditure that must be made, at some future time. If S is the sum set apart each time to be put at compound interest, and other notation is as above, then

$$A = S\ (R^n - 1) \div (R - 1) = S\ (R^n - 1) \div r, \quad \text{or} \quad S = Ar \div (R^n - 1)$$

Amortization. $d\%$ = annual dividend rate required to yield $r\%$ of simple interest, and to furnish annual instalments of P dollars each, which, if placed in a sinking fund at $s\%$ compound interest, will return original investment at end of n years:

$$d = [s + r\ (S^n - 1)] \div (S^n - 1), \quad \text{and} \quad n = \log\left[(s + d - r) \div (d - r)\right] \div \log S$$

in which S is the amount of \$1 for one year at $s\%$. (See Table 5, Sec 25.)

Buying on instalments. Interest is added to principal at time of payment and amount of instalment deducted.

Q = amount of debt, P = amount of instalment, n = number of equal instalments, $r\%$ = rate of interest, R = amount of \$1 for one instalment period, and A_n = amount of debt after payment of n instalments; then, $A_n = QR^n - [P\ (R^n - 1) \div (R - 1)]$. To cancel debt, make $A_n = 0$; then

$$P = QR^n\ (R - 1) \div (R^n - 1), \quad \text{and} \quad n = [\log P - \log (P - QR + Q)] \div \log R$$

GEOMETRY AND MENSURATION

9. CONSTRUCTIONS

Lines. To divide a given line into two equal parts (Fig 2).

Let AB be the given line. With any radius (obviously greater than 0.5 AB) describe 2 arcs with A and B as centers. The line CD, through points of intersection of the arcs, is a perpendicular bisector of the given line.

To divide a given line into any number of equal parts (Fig 3).

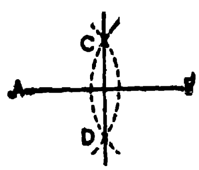


Fig 2

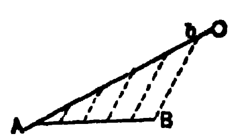


Fig 3

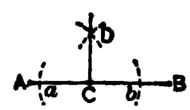


Fig 4

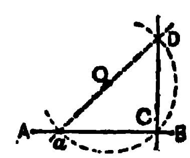


Fig 5

Let AB be the given line and let the number of equal parts be five. Draw line AC at any convenient angle with AB, and step off any 5 equal spaces from A to b. Connect b with B, and draw parallels through the other points in AC. The intersections of these parallels with AB determine the required equal parts on the given line.

To draw a perpendicular to a given line through any given point on the line.

Case 1. Point C is near the middle of the line AB (Fig 4). With C as center, describe arcs of equal radius intersecting AB at a and b. With a and b as centers, and any radius greater than Ca, describe arcs intersecting at D. CD is the required perpendicular.

Case 2. Point C is near the extremity of the line AB (Fig 5). With any point O, as center, and radius OC, describe an arc intersecting AB at a. Extend aO to intersect the arc at D. CD is the required perpendicular.

To draw a perpendicular to a given line through any given point outside of the line,

Case 1. Point C is opposite, or nearly opposite, the middle of the line AB (Fig 6). With C as center, describe an arc intersecting AB at a and b. With a and b as centers, the same radius as before, describe arcs intersecting at D. CD is the required perpendicular.

Case 2. Point C is opposite, or nearly opposite, the extremity of line AB (Fig 7). Through C, draw any line intersecting AB at a. Divide line Ca into 2 equal parts, ab and bC (method given above). With b as center, and radius bC, describe an arc intersecting AB at D. CD is required perpendicular.

Angles. To bisect a given angle.

Case 1. Vertex B is accessible (Fig 8). Let ABC be the given angle. With B as center, and a large radius, describe an arc intersecting AB and BC at a and c respectively.

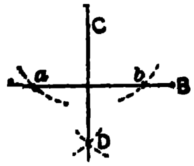


Fig 6

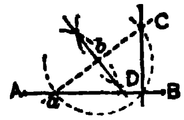


Fig 7

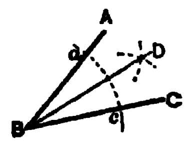


Fig 9

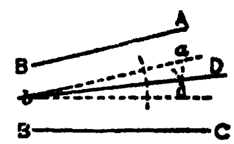


Fig 9

With a and c as centers, describe arcs of equal radius intersecting at D. DB is the required bisector.

Case 2. Vertex B is inaccessible (Fig 9). Let the given angle be the inclination between lines AB and BC. Draw lines ab and bc parallel to the given lines, and at equal distances from them, intersecting at b. Let Db bisect angle abc (method given above). Db is the required bisector.

Circles. To draw a circle through 3 given points not in same straight line (Fig 10). Let A, B and C be the given points. Bisect the imaginary chords AB and BC. Draw bisecting lines and produce them to intersect at O. Point O is center of required circle.

To draw a tangent to a circle from any given point.

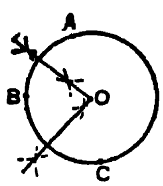


Fig 10

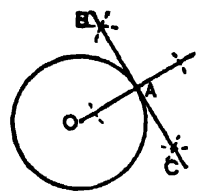


Fig 11

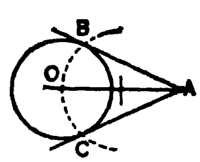


Fig 12

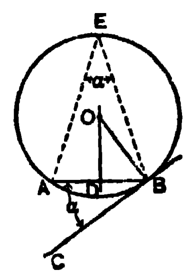


Fig 13

Case 1. Point A is on the circumference of circle O (Fig 11). Draw radius OA. Through A, perpendicular to OA, draw BAC, the required tangent.

Case 2. Point A is not on circumference of circle O (Fig 12). Two tangents may be drawn. Join O and A. With OA as diam, describe a circumference intersecting the given circle at B and C. BA and CA are the required tangents.

To construct, upon a given chord, a segment of a circle in which a given angle may be inscribed (Fig 13). Let AB be the given line, and α the given angle. Construct angle ABC equal to angle α. Bisect line AB by the perpendicular at D. Draw a perpendicular to BC from point B. With O, the point of intersection of the perpendiculars, as center, and OB as radius, describe a circumference. AEB is the required segment, and vertex E may be located anywhere on the arc AEB.

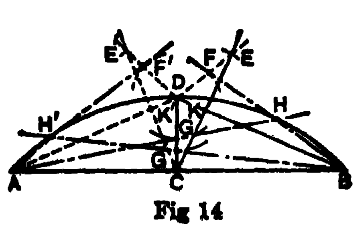


Fig 14

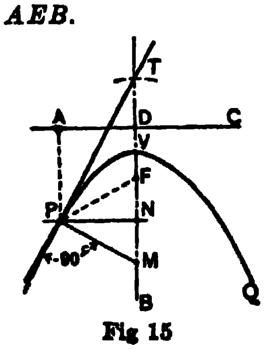


Fig 15

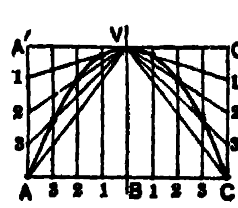


Fig 16

To construct points on a circular arc without locating the center of the circle having given the chord and the rise (Fig 14). Let ACB be the given chord and CD the given rise. Draw BD. Draw CE perpendicular to BD, and make $KE = KC$. Draw DE.

Choose any distance DF, and make $DG = DF$. Line AG, produced, meets BF at H, a point on the required arc. For other points on arc $ADHB$, use other values of DF and repeat the construction.

Parabola is a curve for which the distance of any point from a fixed line, called DIRECTRIX, is equal to the focal radius of the point; that is, $AP = PF$ (Fig 15).

To inscribe a parabola within a rectangle (Fig 16). Let $ABCC'A'$ be the given rectangle. Divide CC' and BC, also AA' and AB, into the same number of equal parts. Number the divisions on the base from the center each way. Number divisions on $C'C$ and $A'A$ from the top, downward. From the divisions on BC and AB, draw parallels to axis BV. From the divisions on CC' and AA', draw lines converging to the vertex V. The intersections of corresponding lines, such as 1 with 1 and 2 with 2, determine points on the required parabola. This construction applies equally well for inscribing a parabola within a parallelogram, in which case, axis BV is parallel to a side of the parallelogram.

To draw a tangent to a parabola from any given point.

Case 1. Point P is on the curve (Fig 15). Draw PN perpendicular to axis VB. With N as center, and radius $= 2\,NP$, describe an arc intersecting the axis at T. TP is the required tangent. NT is called the SUBTANGENT. PM being normal to the curve, NM is called the SUBNORMAL. Distance from vertex V to focus F equals 0.5 NM. Line AC, perpendicular to the axis at D, is the directrix. $DF = NM$.

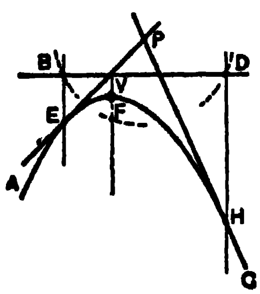


Fig 17

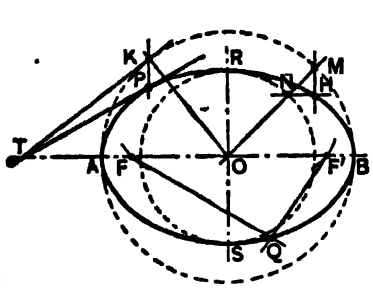


Fig 18

Case 2. Point P is not on the curve (Fig 17). Let F be the focus. With P as center, and PF as radius, draw arcs intersecting the directrix at B and D. Through B and D draw lines parallel to axis, intersecting the parabola at E and H. PE and PH are the required tangents.

Ellipse is a curve for which the sum of the focal radii to any point is a constant, that is, $FQ + QF' = AB =$ constant.

To inscribe an ellipse within a rectangle (Fig 18). Let AOB and ROS be the length and width, respectively, of the given rectangle. With O as center, and OB and OR as radii, describe circles. From O draw any radial line intersecting the circles at M and N. Through M draw a line parallel to OR, and through N a line parallel to OB. These lines intersect at H, a point on the ellipse. Repeat the construction to obtain other points. AB and RS are major and minor axes, respectively.

To locate the FOCI of an ellipse, having given the axes (Fig 18). With R as center, and radius $= AO$, describe arcs intersecting AB at F and F', the required foci.

To draw an ellipse, having given an axis and the foci (Fig 18). A cord or thread, of length equal to the major axis, should be pinned or fixed, at its ends, to the foci F and F'. With a pencil inside the loop, keeping the cord taut so as to guide the pencil point, trace the outline of the ellipse (Q represents the pencil point and length FQF' the cord).

To draw a tangent to an ellipse from any given point.

Case 1. The point P is on the curve (Fig 18). With O as center, and OB as radius, describe a circle. Through P draw a line parallel to OR, intersecting the circle at K. Through K draw a tangent to the circle, intersecting the major axis at T. PT is the required tangent.

Case 2. Point P is not on the curve (Fig 19). With P as center, and radius PF', describe an arc. With F as center, and radius AB, describe an arc intersecting the first arc at M and N. Draw FM and FN, intersecting the ellipse at E and G. PE and PG are the required tangents.

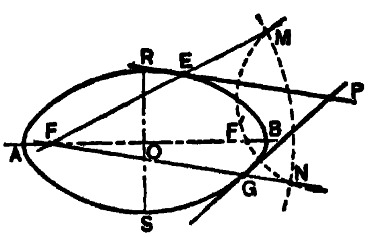


Fig 19

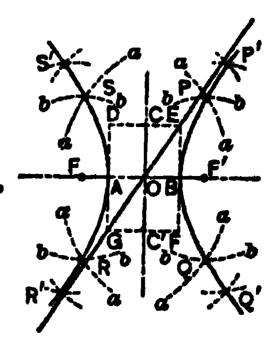


Fig 20

Hyperbola is a curve for which the difference of the focal radii to any point is a constant; that is (Fig 20), $FQ - F'Q = FQ' - F'Q' = AB =$ constant.

To construct an hyperbola by points, having given the foci and the constant difference of the focal radii. Let F and F' (Fig 20) be the foci, and transverse axis AOB the differ-

ence of the focal radii. $AF = F'B$. $AO = OB$. A and B are points on the required curve. With centers F and F', and any radius greater than FB or $F'A$, describe arcs a–a. With same centers, and radius equal to the difference between the first radius and transverse axis AOB, describe arcs b–b, intersecting arcs a–a at P, Q, R, and S, points on the required curve. Repeat the construction for additional points.

Make $BC = BC' = OF = OF'$, and construct the rectangle $DCEBFC'GAD$. The diagonals DF and EG, produced, are called ASYMPTOTES. The hyperbola is tangent to its asymptotes at infinity. If the axes CO and OB are given, the foci can be determined by making $OF = OF' = BC$.

To draw a tangent to an hyperbola from any point.

Case 1. Point P is on the curve (Fig 21). Draw lines connecting P with the foci. Bisect the angle $F'PF$. The bisecting line TP is the required tangent.

Case 2. Point P is on the convex side of the hyperbola (Fig 22). With P as center and radius PF', describe an arc. With F as center, and radius AB, describe an arc intersecting the first arc at M and N. Produce lines FM and FN to intersect the curve at E and G. PE and PG are the required tangents.

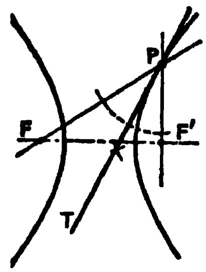


Fig 21

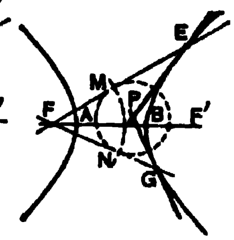


Fig 22

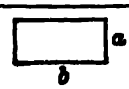
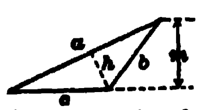
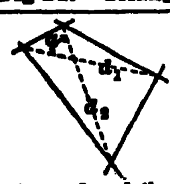
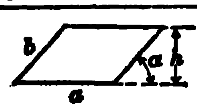
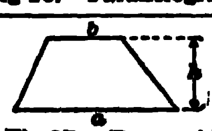
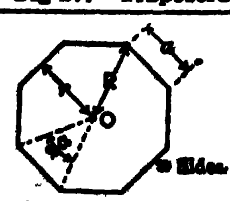
Reference units for angles. The natural unit is 4 right angles, called a PERIGON, and equals 360 degrees of angle. One degree equals 60 minutes. One minute equals 60 seconds. In circular measure the unit is the RADIAN. One radian is the measure of an angle subtended at the center of a circle by an arc equal in length to the radius.

360 degrees of angle $= 2\pi$ radians of angle.

1 radian $= 180 \div \pi$ degrees $= 57.29578$ degrees $= 57° 17' 44.8''$.

Length of arc $= r\theta$; where $r =$ radius of circular arc, $\theta =$ subtended angle, in radians.

10. LINES AND AREAS (A = Area)

 Fig 23. Rectangle	Perimeter $= 2\,(a + b)$ Diagonal $= \sqrt{a^2 + b^2}$ $A = a \times b$
 Fig 24. Triangle	Perimeter $= a + b + c$ $A = \frac{a \times h}{2} = \frac{m \times c}{2}$ If $s = \frac{(a + b + c)}{2}$, $A = \sqrt{s\,(s - a)\,(s - b)\,(s - c)}$
 Fig 25. Quadrilateral	$A = \frac{d_1 \times d_2 \times \sin \alpha}{2}$
 Fig 26. Parallelogram	$A = a \times h$ $= a \times b \times \sin \alpha$
 Fig 27. Trapezoid	$A = \frac{(a + b)\,h}{2}$
 Fig 28. Regular Polygon	Perimeter $= n \times a$ $A = \frac{nar}{2} = nr^2 \tan \alpha = \frac{nR^2}{2} \sin 2\,\alpha$

Lines and Areas (*Continued*)

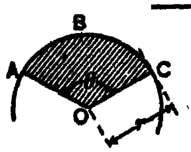


Fig 29. Circular Sector

$$A_{OABCO} = \frac{\pi r^2 \theta(\text{degrees})}{360} = \text{arc } ABC \times \frac{r}{2}$$

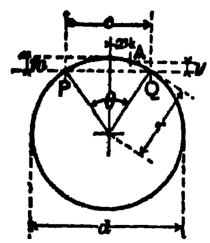


Fig 30. Circle

Circumference $= 2\pi r = \pi d$

Length of arc $PAQ = \theta$ (radians) $\times r = \theta$ (deg) $\times \frac{\pi}{180} \times r$

$= \theta$ (deg) $\times 0.017453 \times r$

Relation of radius, chord, and rise:

$$\text{Radius, } r = \frac{m^2 + \frac{1}{4}c^2}{2\,m} = \frac{\frac{1}{2}c}{\sin \frac{1}{2}\theta}$$

$$\text{Chord, } c = 2\sqrt{2\,mr - m^2} = 2\,r \sin \tfrac{1}{2}\theta$$

Rise or mid-ordinate,

$$m = r \pm \sqrt{r^2 - \frac{c^2}{4}} \quad \begin{bmatrix}\text{use } - \text{ if arc} \leqq 180^\circ \\ \text{use } + \text{ if arc} \geqq 180^\circ\end{bmatrix}$$

$$m = \tfrac{1}{2}c \times \tan \tfrac{1}{4}\theta = 2r \sin^2 \tfrac{1}{4}\theta = r + y - \sqrt{r^2 - x^2}$$

Side ordinate, $y = m - r + \sqrt{r^2 - x^2}$. $A = \pi \times r^2$

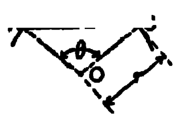


Fig 31. Circular Segment

A_{ABCA} = Area of sector A_{OABCO} minus area of triangle A_{OAC}

$$= \frac{r^2}{2}\left(\frac{\pi\theta(\text{degrees})}{180} - \sin\theta\right)$$

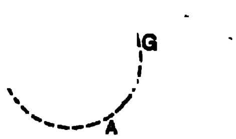


Fig 32. Circular Zone

A_{DEFGD} = Area of circle minus areas of segments $DAGD$ and $EBFE$

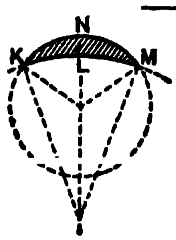


Fig 33. Circular Lune

A_{KLMNK} = Area of segment A_{KNMK} minus area of segment A_{KLMK}

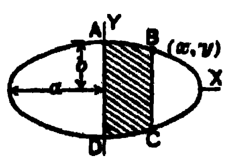


Fig 34. Ellipse

Circumference $= \pi\,(a + b) \times k$

where $k = \left(1 + \frac{c^2}{4} + \frac{c^4}{64} + \frac{c^6}{256} + \cdots\right)$

and $c = (a - b) \div (a + b)$

Circumference $= \pi\sqrt{2\,(a^2 + b^2)}$ [approx]. $A = \pi \times a \times b$

$A_{ABCD} = (x \times y) + ab \sin^{-1}(x \div a)$

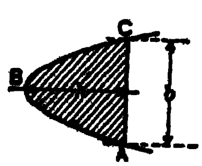


Fig 35. Parabola

If l = length of curve ABC and $n = h \div b$

$$l = b\left\{\tfrac{1}{2}(1 + 16\,n^2)^{\frac{1}{2}} + \frac{1}{8\,n} \text{ nap log} \times [4\,n + (1 + 16\,n^2)^{\frac{1}{2}}]\right\}$$

$$l = b\left(1 + \frac{8\,n^2}{3} - \frac{32\,n^4}{5} + \cdots\right) \text{ [approx]}$$

$A_{ABC} = \frac{2}{3} \times b \times h$

Lines and Areas (*Continued*)

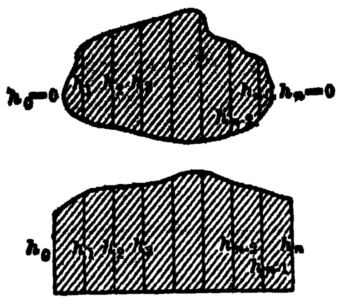


Fig 36. Irregular Figures

Determination of Area

Divide the figure into an even number of strips by equidistant ordinates, thus obtaining an odd number of ordinates. (The greater the number of strips, the greater the accuracy in the results.) The ordinates are the distances between boundary lines, represented by $h_0, h_1, h_2 \cdots h_{n-1}, h_n$.

n = the number of strips

$(n + 1)$ = the total number of ordinates, including the first and last

d = common distance between ordinates

The total area, A is approximately as follows:

Trapezoidal Rule	$A = d\left[\frac{h_0 + h_n}{2} + h_1 + h_2 + \cdots h_{n-1}\right]$
Simpson's Rule	$A = \frac{d}{3}[(h_0 + h_n) + 2(h_2 + h_4 + \cdots h_{n-2}) + 4(h_1 + h_3 + \cdots h_{n-1})]$
Durand's Rule	$A = d[0.04(h_0 + h_n) + 1.1(h_1 + h_{n-1}) + h_2 + h_3 + \cdots h_{n-2}]$
Weddle's Rule	If 6 strips (7 ordinates) are used, then $A = {}^3/_{10} \times d[5(h_1 + h_5) + 6h_3 + h_0 + h_2 + h_4 + h_6]$

11. SURFACES AND VOLUMES

S = lateral or convex surface; T = total surface; V = volume; A = area of base; A_1 = area of a right section as indicated in the illustrations.

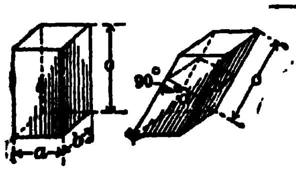


Fig 37. Parallelopiped

$S = 2c(a + b)$
$T = 2(ab + ac + bc)$
$V = a \times b \times c$

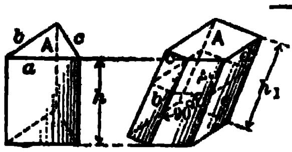


Fig 38. Prism, Right or Oblique, Regular or Irregular

$S = h(a + b + c \cdots n)$
$V = Ah = A_1h_1$

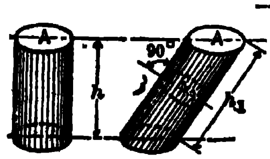


Fig 39. Cylinder (Any Cross-section)

S = perimeter of base $A \times h$
= perimeter of section $A_1 \times h_1$
$V = Ah = A_1h_1$

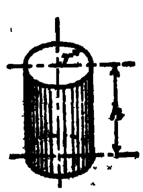


Fig 40. Right Circular Cylinder

$S = 2\pi rh$
$T = 2\pi r(r + h)$
$V = \pi r^2 h$

Surfaces and Volumes (*Continued*)

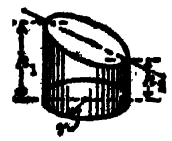


Fig 41. Frustum of Right Circular Cylinder

$$S = \pi r (h_1 + h_2)$$

$$T = \pi r \left[h_1 + h_2 + r + \sqrt{r^2 + \left(\frac{h_1 - h_2}{2}\right)^2} \right]$$

$$V = \frac{\pi r^2}{2} (h_1 + h_2)$$

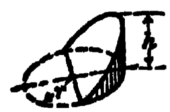


Fig 42. Ungula or Wedge of Right Circular Cylinder [Semicircular Base]

$$S = 2rh$$

$$V = {}^2/_3 (r^2 h)$$

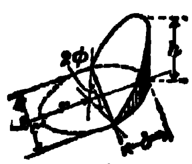


Fig 43. Ungula or Wedge of Right Circular Cylinder [Base greater or less than semicircle]

$$S = \frac{2rh}{b} \left[c + (b - r) \times \frac{\phi^\circ \times \pi}{180^\circ} \right]$$

$$V = \frac{h}{3b} \left[c(3r^2 - c^2) + 3r^2(b - r) \frac{\phi^\circ \times \pi}{180^\circ} \right]$$

$$= \frac{hr^3}{b} \left[\sin \phi - \frac{\sin^3 \phi}{3} - \frac{\phi^\circ \times \pi}{180^\circ} \cos \phi \right]$$

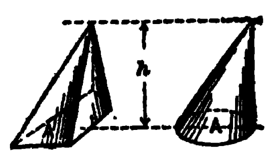


Fig 44. Pyramid or Cone, Right or Oblique, Regular or Irregular

$$V = \frac{Ah}{3}$$

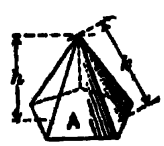


Fig 45. Right Regular Pyramid

$$S = \text{perimeter of base } A \times \frac{k}{2}$$

$$V = \frac{Ah}{3}$$

Fig 46. Right Circular Cone

$$S = \pi r \sqrt{r^2 + h^2} = \pi r k$$

$$T = \pi r (r + k)$$

$$V = \frac{\pi r^2 h}{3}$$

Fig 47. Frustum of Pyramid or Cone, Right, Regular, Parallel Ends

$$S = \frac{k}{2} \times \text{sum of perimeters of bases}$$

$$= \frac{k}{2} (P_1 + P_2)$$

$$V = \frac{h}{3} (A + A_1 + \sqrt{A \times A_1})$$

Surfaces and Volumes (*Continued*)

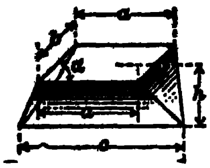


Fig 48. Wedge (Parallelogram Back)

$$V = \frac{h}{6}(2a + c) \times b \times \sin\alpha$$

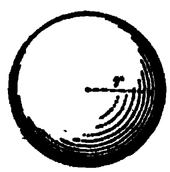


Fig 49. Sphere

$$S = T = 4\pi r^2$$
$$V = \frac{4\pi r^3}{3}$$

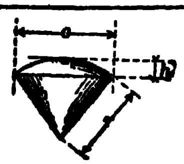


Fig 50. Spherical Sector

$$T = \frac{\pi r}{2}(4h + c)$$
$$V = \frac{2\pi r^2 h}{3}$$

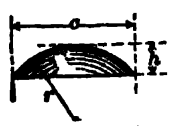


Fig 51. Spherical Segment

$$S = 2\pi rh = \frac{\pi}{4}(4h^2 + c^2)$$
$$T = \frac{\pi}{2}(2h^2 + c^2)$$
$$V = \pi h^2\left(r - \frac{h}{3}\right) = \frac{\pi h}{24}(3c^2 + 4h^2)$$

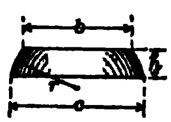


Fig 52. Spherical Zone

$$S = 2\pi rh$$
$$T = \frac{\pi}{4}(8rh + a^2 + b^2)$$
$$V = \frac{\pi h}{8}\left(b^2 + c^2 + \frac{4}{3}h^2\right)$$

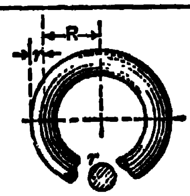


Fig 53. Circular Ring (Circular Cross-section)

$$S = 4\pi^2 Rr$$
$$V = 2\pi^2 Rr^2$$

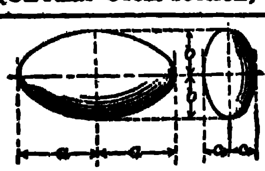


Fig 54. Ellipsoid

$V = {}^4/_3 \times \pi \times a \times b \times c$

Prolate spheroid (revolution about major axis)

$V = {}^4/_3 (\pi ab^2)$

Oblate spheroid (revolution about minor axis)

$V = {}^4/_3 (\pi ba^2)$

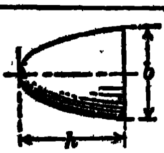


Fig 55. Paraboloid

$$S = \frac{2}{3} \times \pi \times \frac{b}{h^2}\left[\left(\frac{b^2}{16} + h^2\right)^{3/2} - \left(\frac{b}{4}\right)^3\right]$$
$$V = \frac{\pi}{8} \times b^2 \times h$$

Surfaces and Volumes (*Continued*)

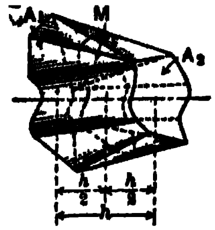


Fig 56. Prismatoid [Irregular solid with bases in parallel planes]

Prismoidal Formula.

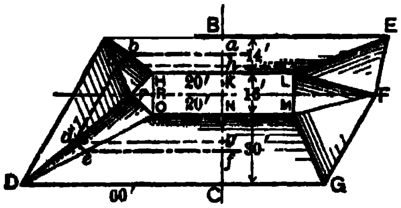


Fig 57

If surfaces connecting the bases are curved or warped they may be replaced by plane surfaces, best suited to the contour, bounded by straight lines from one of the parallel bases to the other. These planes may be parallelograms, triangles, or trapezoids.

Let A_1 = the area of one of the bases.
A_2 = the area of the other base.
M = the area of the cross-section parallel to, and midway between, the two bases.
h = the altitude of the prismatoid. (The perpendicular distance between the two parallel bases.)
V = vol of the solid = $\frac{1}{6}h[A_1 + 4M + A_2]$

[Area of cross-sec M should be computed from dimensions of the polygon formed by the intersection, with the solid, of a plane parallel to the bases through middle point of the altitude. In only exceptional cases is this cross-sec the aver of the base areas. However, the perimeter of this mid-polygon is half the sum of the basal perimeters].

Example. Find the volume of the irregular solid shown in Fig 57. $AEFGD$ is the lower base; $HLMOP$ is the upper base. To simplify the problem, let the perpendicular plane $BKNC$ divide the solid into two parts; find the volumes of the parts separately and sum them for total volume. For the left portion of the solid: $ABCD$ is the trapezoidal lower base, $HKNOP$, the pentagonal upper base. The given data are as follows: Altitude, or perpendicular distance between bases = 50 ft

AB = 40 ft BK = 14 ft KN = 16 ft BC = 60 ft
HK = ON = 20 ft DC = 60 ft NC = 30 ft HR = RO = PR = 8 ft

Dimensions necessary to calculate area of cross-section M are:
ef = 40 ft, dg = 44 ft, ch = 34 ft, ba = 30 ft, fg = 4 ft, gh = 30 ft, ha = 4 ft

Check: Perimeter of lower base = 223.24 ft; perimeter of upper base = 78.62 ft; perimeter of mid-polygon = 150.93 ft;

$$\text{half sum of basal perimeters} = \frac{223.24 + 78.62}{2} = 150.93\text{ ft}$$

Area $ABCD$ = A_1 = 3 000 sq ft; area $HKNOP$ = A_2 = 384 sq ft; area $afedcb$ = M = 1 466 sq ft; hence $V_{to\ left} = \frac{50}{6}[3\,000 + 4\,(1\,466) + 384] = \frac{50}{6}[9\,248]$ = 77 067 cu ft.

The volume to the right of plane $BKNC$ may be found in a similar manner.

PLANE TRIGONOMETRY

12. LINE VALUES OF FUNCTIONS

Let the radius of a circle (Fig 58) = AF = AB = AE = 1, the circle being called a UNIT CIRCLE, and let A = angle BAC = angle subtended by arc BE.

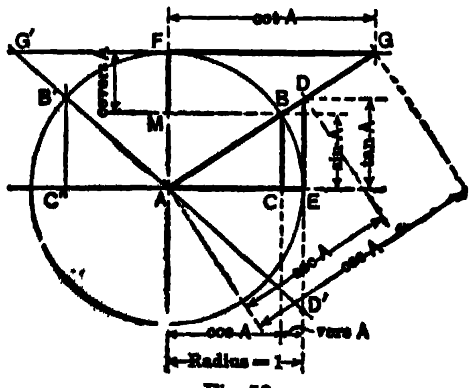


Fig 58

sine of A = $\sin A = \frac{BC}{AB} = \frac{BC}{1} = BC$

cosine of A = $\cos A = \frac{AC}{AB} = \frac{AC}{1} = AC$

tangent of A = $\tan A = \frac{DE}{AE} = \frac{DE}{1} = DE$

cotangent of A = $\cot A = \frac{FG}{AF} = \frac{FG}{1} = FG$

secant of A = $\sec A = \frac{AD}{AE} = \frac{AD}{1} = AD$

cosecant of A = $\csc A = \frac{AG}{AF} = \frac{AG}{1} = AG$

versed sine of A = $\text{vers } A = 1 - \cos A = AE - AC = CE$
coversed sine of A = $\text{covers } A = 1 - \sin A = AF - BC = FM$

If the radius AB of the circle (Fig 58) generates an angle by turning about the center A from the initial horizontal position AE, the angle will be measured by the arc described by point B, and may have any magnitude. POSITIVE ANGLES are generated by counterclockwise turning of AB; NEGATIVE ANGLES by clockwise turning. For any magnitude of angle A:

$\sin A$ = vertical projection of moving radius.
$\cos A$ = horizontal projection of moving radius.
$\tan A$ = distance from E along a tangent to intersection with moving radius produced.
$\cot A$ = distance from F along a tangent to intersection with moving radius produced.
For angle EAG': $\sin A = B'C'$; $\cos A = AC'$; $\tan A = D'E$; $\cot A = FG'$.

Algebraic signs of functions. Assume the functions of an angle in the first quadrant to be positive, then: (*a*) sines and tangents extending from the horiz diam UPWARD are positive; DOWNWARD, negative. (*b*) cosines and cotangents extending from the vert diam TOWARD THE RIGHT, are positive; TOWARD THE LEFT, negative.

13. FUNCTIONS OF RIGHT-ANGLED TRIANGLES

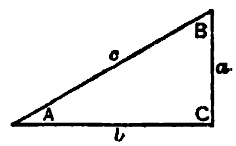


Fig 59

In any right-angled triangle ABC (Fig 59) let $AB = c$, $AC = b$, and $BC = a$.

1. $\sin A = a \div c = \cos B$
2. $\cos A = b \div c = \sin B$
3. $\tan A = a \div b = \cot B$
4. $\cot A = b \div a = \tan B$
5. $\sec A = c \div b = \csc B$
6. $\csc A = c \div a = \sec B$
7. $\operatorname{vers} A = (c - b) \div c = \operatorname{covers} B$
8. $\operatorname{covers} A = (c - a) \div c = \operatorname{vers} B$
9. $C = 90° = A + B$
10. $a = c \sin A = b \tan A$
11. $a = c \cos B = b \cot B$
12. $a = \sqrt{c^2 - b^2} = \sqrt{(c + b)(c - b)}$
13. $b = c \cos A = a \cot A$
14. $b = c \sin B = a \tan B$
15. $b = \sqrt{c^2 - a^2} = \sqrt{(c + a)(c - a)}$
16. $c = \dfrac{a}{\sin A} = \dfrac{b}{\cos A}$
17. $c = \dfrac{a}{\cos B} = \dfrac{b}{\sin B}$
18. $c = \sqrt{a^2 + b^2}$

$$19.\ \text{Area} = \frac{ab}{2} = \frac{a}{2}\sqrt{c^2 - a^2} = \frac{a^2}{2}\cot A = \frac{b^2}{2}\tan A = \frac{c^2}{2}\sin A \cos A$$

Functions of complementary angles. Each function of an acute angle is equal to the co-named function of the complementary angle, that is, $\sin A = \cos(90° - A)$; $\tan A = \cot(90° - A)$; $\sec A = \csc(90° - A)$; $\cos A = \sin(90° - A)$; $\cot A = \tan(90° - A)$; $\csc A = \sec(90° - A)$.

14. FORMULAS

$$20.\ \sin A = \frac{1}{\csc A} = \pm\sqrt{1 - \cos^2 A} = \tan A \cos A$$

$$21.\ \sin A = 2 \sin {}^1\!/_2 A \cos {}^1\!/_2 A = \operatorname{vers} A \cot {}^1\!/_2 A$$

$$22.\ \sin A = \pm\sqrt{{}^1\!/_2 (1 - \cos 2A)} = \pm\sqrt{{}^1\!/_2 \operatorname{vers} 2A}$$

$$23.\ \cos A = \frac{1}{\sec A} = \pm\sqrt{1 - \sin^2 A} = \cot A \sin A$$

$$24.\ \cos A = \cos^2 {}^1\!/_2 A - \sin^2 {}^1\!/_2 A = 2\cos^2 {}^1\!/_2 A - 1 = 1 - 2\sin^2 {}^1\!/_2 A$$

$$25.\ \cos A = \pm\sqrt{{}^1\!/_2 + {}^1\!/_2 \cos 2A} = 1 - \operatorname{vers} A$$

26. $\tan A = \dfrac{1}{\cot A} = \dfrac{\sin A}{\cos A} = \pm\sqrt{\sec^2 A - 1}$

27. $\tan A = \pm\sqrt{\dfrac{1}{\cos^2 A} - 1} = \dfrac{\pm\sqrt{1-\cos^2 A}}{\cos A} = \dfrac{2\tan {}^1/_2 A}{1 - \tan^2 {}^1/_2 A}$

28. $\tan A = \dfrac{1-\cos 2A}{\sin 2A} = \dfrac{\sin 2A}{1+\cos 2A} = \pm\sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}} = \dfrac{\operatorname{vers} 2A}{\sin 2A}$

29. $\cot A = \dfrac{1}{\tan A} = \dfrac{\cos A}{\sin A} = \pm\sqrt{\csc^2 A - 1}$

30. $\operatorname{vers} A = 1 - \cos A = \sin A \tan {}^1/_2 A = 2\sin^2 {}^1/_2 A$

31. $\operatorname{covers} A = 1 - \sin A$

32. $\sin \dfrac{1}{2} A = \pm\sqrt{\dfrac{1-\cos A}{2}} = \pm\sqrt{\dfrac{\operatorname{vers} A}{2}}$

33. $\cos \dfrac{1}{2} A = \pm\sqrt{\dfrac{1+\cos A}{2}}$

34. $\tan \dfrac{1}{2} A = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{1-\cos A}{\sin A} = \dfrac{\sin A}{1+\cos A} = \dfrac{\tan A}{1+\sec A} = \dfrac{\operatorname{vers} A}{\sin A}$

35. $\cot \dfrac{1}{2} A = \pm\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \dfrac{\sin A}{\operatorname{vers} A}$

36. $\operatorname{vers} \dfrac{1}{2} A = \dfrac{{}^1/_2 \operatorname{vers} A}{1 + \sqrt{1 - {}^1/_2 \operatorname{vers} A}} = \dfrac{1-\cos A}{2 + \sqrt{2(1+\cos A)}}$

37. $\sin 2A = 2\sin A \cos A$

38. $\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$

39. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

40. $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

41. $\operatorname{vers} 2A = 2\sin^2 A = 2\sin A \cos A \tan A = 1 - \cos 2A$

42. $\sin(A \pm B) = \sin A \cdot \cos B \pm \sin B \cdot \cos A$

43. $\cos(A \pm B) = \cos A \cdot \cos B \mp \sin A \cdot \sin B$

44. $\tan(A + B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \cdot \tan B}$

45. $\cot(A \pm B) = \dfrac{\cot A \cdot \cot B \mp 1}{\cot B \pm \cot A}$

46. $\sin A + \sin B = 2\sin {}^1/_2 (A+B) \cos {}^1/_2 (A-B)$

47. $\sin A - \sin B = 2\cos {}^1/_2 (A+B) \sin {}^1/_2 (A-B)$

48. $\cos A + \cos B = 2\cos {}^1/_2 (A+B) \cos {}^1/_2 (A-B)$

49. $\cos A - \cos B = -2\sin {}^1/_2 (A+B) \sin {}^1/_2 (A-B)$

50. $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cdot \cos B}$

51. $\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A \cdot \cos B}$

52. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$

53. $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$

15. OBLIQUE TRIANGLES

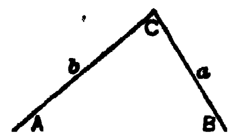


Fig 60

Given Data	To Find	Formulas
Two Angles and a Side $A, B, a,$	b, c, C Area	$b = a \cdot \dfrac{\sin B}{\sin A} \qquad c = a \cdot \dfrac{\sin (A + B)}{\sin A}$ $C = 180° - (A + B)$ $K = \dfrac{a^2}{2} \cdot \dfrac{\sin B \cdot \sin C}{\sin A}$
Two Sides and an Angle Opposite One of Them a, b, A	B, C, c Area	If $a < b$ and $A > 90°$, or if $a < b \sin A$ and $A < 90°$. No solution. If $a \geqq b$, or if $a = b \sin A$. One solution. If $a < b$, but $> b \sin A$, and $A < 90°$, there may be two solutions. If $\sin B = (b \div a) \sin A$, B may have two values, say B_1 and B_2. $C_1 = 180° - (A + B_1)$, $C_2 = 180° - (A + B_2)$ $c_1 = a \cdot \dfrac{\sin C_1}{\sin A} \qquad c_2 = a \cdot \dfrac{\sin C_2}{\sin A}$ $K = \tfrac{1}{2}\, ab \cdot \sin C$
Two Sides and the Included Angle a, b, C	A, B, c Area	$\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2} C$ $\tan \tfrac{1}{2}(A - B) = \dfrac{a - b}{a + b} \cdot \tan \tfrac{1}{2}(A + B)$ $A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$ $B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$ $c = a \cdot \dfrac{\sin C}{\sin A} = b \cdot \dfrac{\sin C}{\sin B}$ $c = (a + b)\dfrac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b)\dfrac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$ $K = \tfrac{1}{2}\, ab \cdot \sin C$
Three Sides a, b, c	A, B, C Area	Let $S = \tfrac{1}{2}(a + b + c) =$ the semi-perimeter $\sin \tfrac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{bc}}$ $\cos \tfrac{1}{2} A = \sqrt{\dfrac{s (s - a)}{bc}}$ $\tan \tfrac{1}{2} A = \sqrt{\dfrac{(s - b)(s - c)}{s (s - a)}}$ $\sin A = \dfrac{2}{bc}\sqrt{s (s - a)(s - b)(s - c)}$ $\tan \tfrac{1}{2} B = \sqrt{\dfrac{(s - a)(s - c)}{s (s - b)}}$ $C = 180° - (A + B)$ $K = \sqrt{s (s - a)(s - b)(s - c)}$

ANALYTICAL GEOMETRY

16. STRAIGHT LINE

Rectangular and polar coordinates (Fig 61). Rectangular coordinates (x, y) and polar coordinates (ρ, θ) of any point P are related thus: $x = \rho \cos \theta$; $y = \rho \sin \theta$; $x^2 + y^2 = \rho^2$; $\tan \theta = y \div x$.

Note.—The x's and y's in following formulas are rectangular coordinates.

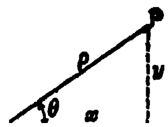


Fig 61

Fig 62

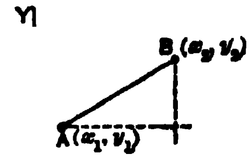


Fig 63

Slope. The slope of a line is the tangent of the angle α from the x-axis to the line. It is denoted by m, $m = \tan \alpha$. If two lines are parallel, their slopes, m and m', are equal. If the lines are perpendicular to each other, $m = -1 \div m'$. The slope of the line through A (x_1, y_1) and B (x_2, y_2), Fig 63, is

$$m = (y_2 - y_1) \div (x_2 - x_1)$$

Distance between two points, A (x_1, y_1) and B (x_2, y_2) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Line divided in given ratio. The coordinates (x, y) of the point that divides the line from A (x_1, y_1) to B (x_2, y_2) in the ratio r are

$$x = (x_1 + rx_2) \div (1 + r), \qquad y = (y_1 + ry_2) \div (1 + r)$$

Equation of straight line. Every equation of the first degree in x and y, as $Ax + By + C = 0$, represents a straight line. Its slope is $-A \div B$.

Line with slope m and y-intercept b, $y = mx + b$

Line with slope m and through point (x', y'), $y - y' = m\,(x - x')$

Line through two points (x', y') and (x'', y''), $(y - y') \div (y'' - y') = (x - x') \div (x'' - x')$

Line with x-intercept a and y-intercept b, $(x \div a) + (y \div b) = 1$

Angle θ from line with slope m to line with slope m' is found by

$$\tan \theta = (m' - m) \div (1 + mm')$$

Distance from point (x', y') **to line** $Ax + By + C = 0$ is

$$d = (Ax' + By' + C) \div \sqrt{A^2 + B^2}$$

Area of triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is

$$A = [x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)] \div 2$$

17. CIRCLE

Equation. The equation of the circle with center at the origin and radius r is $x^2 + y^2 = r^2$. If the center is (a, b) the equation is $(x - a)^2 + (y - b)^2 = r^2$. Every equation of the form

$$x^2 + y^2 + 2\,Ax + 2\,By + C = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (C)$$

represents a circle. The center is $(-A, -B)$ and the radius, $\sqrt{A^2 + B^2 - C}$.

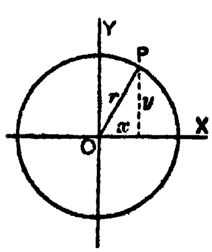


Fig 64

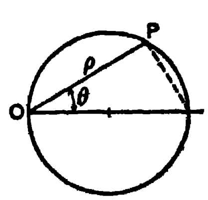


Fig 65

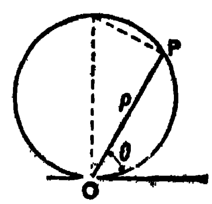


Fig 66

Tangents. Equation of tangent to the circle $x^2 + y^2 = r^2$ at the point (x', y') on the curve is $x'x + y'y = r^2$. Equation of tangent with slope m, $y = mx \pm r\sqrt{1 + m^2}$. Length of tangent from any point (x', y') outside the circle, to any other circle,

$$x^2 + y^2 + 2\,Ax + 2\,By + C = 0, \text{ is } l = \sqrt{x'^2 + y'^2 + 2\,Ax' + 2\,By' + C}$$

Circle through three points. To find equation of circle through three points, as (3, 1), (−1, 2), (5, 4), substitute these pairs of values for x and y in the equation (C) above, and find A, B, C from the resulting 3 equations.

Polar equation. Origin on circumference (Fig 65), diameter a along initial line, equation of circle is $\rho = a \cos\theta$. Diameter along line 90° from initial line (Fig 66) is $\rho = a \sin\theta$.

18. ELLIPSE

Equation of ellipse with center at origin and foci F' and F on the x-axis (Fig 67), is

$$(x^2 \div a^2) + (y^2 \div b^2) = 1 \quad \ldots\ldots\ldots\ldots (E)$$

where a is the major semiaxis and b the minor semiaxis.

Distance $OF = \sqrt{a^2 - b^2}$; distance $BF = OA = a$; focal width $QR = 2\,b^2 \div a$.

Eccentricity e of ellipse is $e = \sqrt{a^2 - b^2} \div a = OF \div OA$, and e is always less than 1. When e is nearly 0, the ellipse is nearly a circle, and the foci are near the center. When e is nearly 1, ellipse is long and narrow, and foci are near ends of the major axis. If two ellipses have the same eccentricity, the ratio of semiaxes, $a : b$, is the same for both, and conversely. The equation $(x^2 \div a^2) + (y^2 \div b^2) = k$ where a and b are fixed, represents, whenever any positive value is assigned to k, an ellipse; and all these ellipses have the same e.

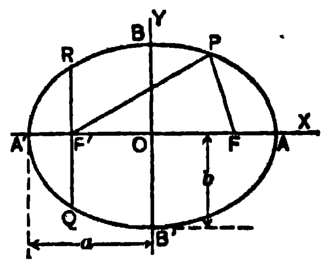


Fig 67

Confocal ellipses. If two ellipses $(x^2 \div a^2) + (y^2 \div b^2) = 1$ and $(x^2 \div a'^2) + (y^2 \div b'^2) = 1$ have the same foci, then $a^2 - b^2 = a'^2 - b'^2$. That is, $a^2 - a'^2 = b^2 - b'^2$.

Focal radii to the point P (x', y') on the ellipse are $F'P = a + ex'$ and $FP = a - ex'$. This makes $F'P + FP = 2\,a = A'A$.

Tangent to the ellipse (Eq E, above) at (x', y') on the curve is $(x'x \div a^2) + (y'y \div b^2) = 1$; with slope m, $y = mx \pm \sqrt{a^2m^2 + b^2}$.

Diameters (Fig 68). Equation of diameter AB, which bisects all chords having slope m, as CD, EF, in ellipse (E) is $y = (-b^2 \div a^2m)x$. If the slopes m and m' of two diameters of (E) are such that $mm' = -b^2 \div a^2$, each diam bisects all chords parallel to the other. They are called conjugate diameters; for example, AB and JK.

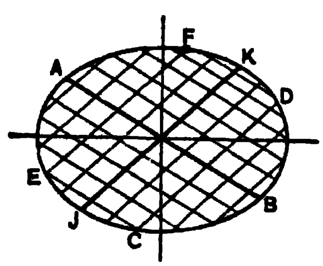


Fig 68

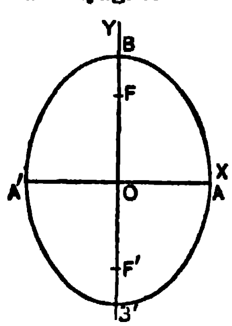


Fig 69

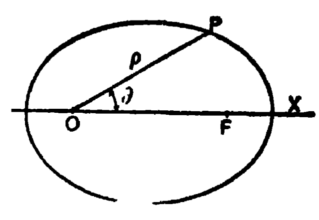


Fig 70

Foci on y-axis (Fig 69). If the foci of an ellipse are on the y-axis, its equation is (E) as above, but b is the major semiaxis, a the minor, so that $OF = \sqrt{b^2 - a^2}$, $AF = b$, and $e = \sqrt{(b^2 - a^2)} \div b$.

Polar equation (Fig 70). Initial line along major axis of ellipse, origin at left-hand focus is $\rho = b^2 \div a(1 - e \cos\theta)$. With origin at right-hand focus, $\rho = b^2 \div a(1 + e \cos\theta)$. With origin at center, $\rho^2 = b^2 \div (1 - e^2 \cos^2\theta)$, where e is the eccentricity.

19. HYPERBOLA

Equation. Equation of hyperbola having foci F' and F on the x-axis, origin at center (Fig 71):

$$(x^2 \div a^2) - (y^2 \div b^2) = 1 \quad \ldots\ldots\ldots\ldots (H)$$

where a is the principal semiaxis, b the conjugate semiaxis. Distance $OF = \sqrt{a^2 + b^2} = BA$, A being one end of principal axis, B one end of conjugate axis. Focal width $= 2\,b^2 \div a$.

Asymptotes. Equations of asymptotes to hyperbola (H): $ay = bx$ and $ay = -\,bx$. If $2\,\alpha$ is the angle between the asymptotes, $\tan\alpha = b \div a$. If P is any point on a hyperbola, the product of perpendicular distances from P to the asymptotes is constant, and is equal to $a^2b^2 \div (a^2 + b^2)$.

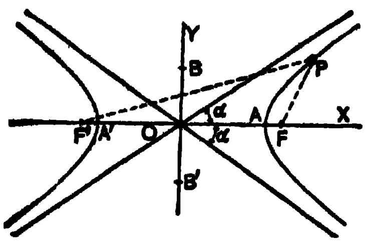


Fig 71

Eccentricity of hyperbola is greater than 1: $e = \sqrt{a^2 + b^2} \div a = \sec\alpha$, where α is half the angle between the asymptotes. When $e = \sqrt{2} = 1.4 +$, the angle $2\,\alpha$ between the asymptotes is a right angle. When $e > \sqrt{2}$, $2\,\alpha > 90°$; and when e is between $\sqrt{2}$ and 1, $2\,\alpha$ is between 90° and 0°.

If two hyperbolas, such as (H), have the same eccentricity, they have the same asymptotes, and conversely. They also have the same ratio of semiaxis, $a:b$, and conversely.

The equation $(x^2 \div a^2) - (y^2 \div b^2) = k$, where a and b are fixed, but k may take various values, represents various hyperbolas, all of which have the same ratio of semiaxes. The semiaxes are $a\sqrt{k}$ and $b\sqrt{k}$.

Focal radii to any point $P\ (x', y')$ on hyperbola (H) are

$F'P = ae + x'$ and $FP = ae - x'$ (e = eccentricity). So that $F'P - FP = 2\,a = A'A$.

Tangent to hyperbola (H) at (x', y') on the curve is $(x'x \div a^2) - (y'y \div b^2) = 1$; with slope m, $y = mx \pm \sqrt{a^2m^2 - b^2}$.

Diameter (Fig 72). Equation of diam which bisects all chords of hyperbola (H) that have slope m is $y = (b^2 \div a^2m)\,x$. If the slopes m and m' of two diameters of (H) are such that $mm' = b^2 \div a^2$, each bisects all chords parallel to the other. They are called conjugate diameters.

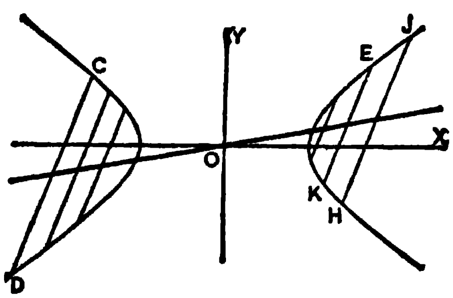


Fig 72

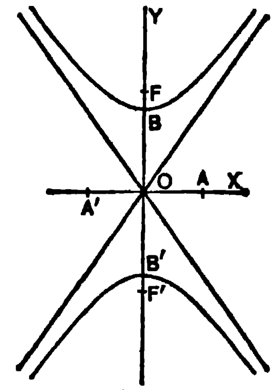


Fig 73

Foci on y-axis (Fig 73). If the foci of the hyperbola are on the y-axis, origin being at center, the equation of the hyperbola is

$$(y^2 \div b^2) - (x^2 \div a^2) = 1 \qquad (H')$$

where b is principal semiaxis, and a is conjugate semiaxis. In this case,

eccentricity $e = \sqrt{a^2 + b^2} \div b$; asymptotes are $ay = bx$ and $ay = -\,bx$.

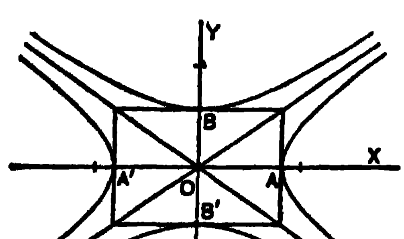


Fig 74

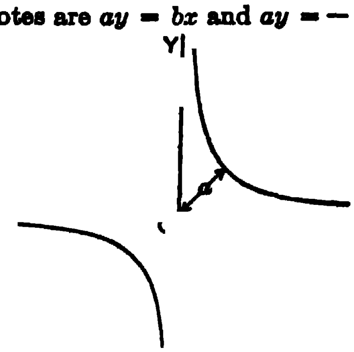


Fig 75

Conjugate hyperbolas, that is, the hyperbolas (H) and (H'), have the same asymptotes (Fig 74). The diam which bisects a set of parallel chords in one bisects the same set of

parallel chords in the other. The two segments intercepted on any straight line by two conjugate hyperbolas are equal. Eccentricities e and e' of conjugate hyperbolas are related thus: $(1 + e^2) + (1 + e'^2) = 1$.

Rectangular or equilateral hyperbola is one having its axes equal, so that $a = b$. The equation (H) is then $x^2 - y^2 = a^2$. (H'')

Asymptotes of (H'') are perpendicular to each other.

Asymptotes as axes of coordinates (Fig 75). In this case, the equation of the rectangular hyperbola is $xy = a^2 + 2$. If the hyperbola is not rectangular (Fig 76), the asymptotes are not perpendicular. But if they are taken as axes and the coordinates x and y are measured parallel to the asymptotes, the equation of the hyperbola is

$$4\,xy = a^2 + b^2$$

Polar equation is same as for ellipse, with $-b^2$ in place of b^2.

Confocal hyperbolas. If two hyperbolas, $(x^2 + a^2) - (y^2 + b^2) = 1$ and $(x^2 + a'^2)\ (y^2 + b'^2) = 1$, have the same foci, then $a^2 + b^2 = a'^2 + b'^2$.

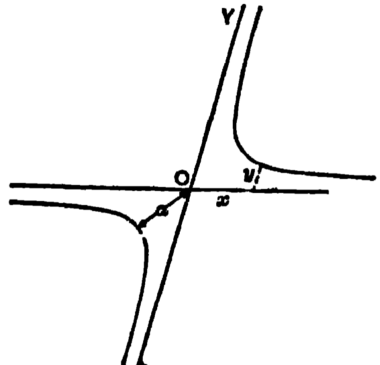


Fig 76

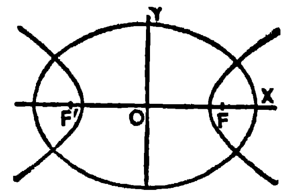


Fig 77

Confocal ellipse and hyperbola. If an ellipse and a hyperbola have same foci, they intersect at right angles (Fig 77).

20. PARABOLA

Equation. With focus on x-axis and origin at vertex, equation of parabola is $y^2 = 4\,px$, where $p = OF$ = distance from vertex to focus (Fig 78). If p is positive, parabola runs to the right; if negative, to the left.

Focal width, $QR = 4\,p$.

Focal radius to any point $P\,(x', y')$ on parabola is $FP = p + x'$.

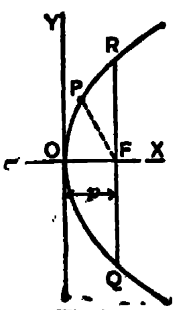


Fig 78

Tangent to parabola $y^2 = 4\,px$ at (x', y') on the curve is $y'y = 2\,p\,(x + x')$; with slope m, $y = mx + p \div m$.

Diameter. Equation of diam bisecting all chords of parabola $y^2 = 4\,px$ that have slope m is $y = 2\,p \div m$. Every diam of a parabola is parallel to the axis.

Focus on y-axis (Fig 79). Equation of parabola having focus on y-axis and origin at vertex is $x^2 = 4\,py$, where $p = OF$. This parabola runs in direction of positive end of y-axis (usually taken upward) if p is positive; in direction of negative end of y-axis if p is negative.

Polar equation of parabola, origin being at vertex, $\rho = 2\,p \div (1 - \cos\theta)$.

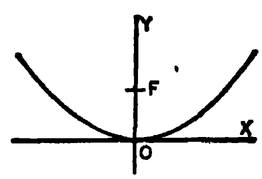


Fig 79

21. CURVES IN GENERAL

An equation of second degree in x and y,

$$ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c = 0 \qquad (G)$$

represents a conic section: an ellipse, if $h^2 < ab$; an hyperbola, if $h^2 > ab$; a parabola, if $h^2 = ab$; a circle, if $a = b$ and $h = 0$. In the special case where $abc - af^2 - bg^2 - ch^2 + 2\,fgh = 0$, the curve breaks down into a pair of straight lines, or a point.

Inclination of axis. The angle θ of inclination of the principal axis of the conic (G) to the x-axis is given by $\tan 2\,\theta = 2\,h \div (a - b)$.

Center (x', y') of ellipse or hyperbola is found by solving simultaneous equations: $ax' + hy' + g = 0$, $hx' + by' + f = 0$.

Intersections of two curves are found by solving their equations as simultaneous.

Intercepts. To find x-intercepts of a curve, set $y = 0$ in its equation, and solve for x; to find y-intercepts, set $x = 0$ in the equation, and solve for y.

Special Curves. Logarithmic and exponential curves, and the cycloid:

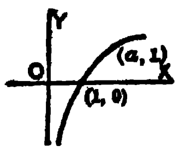


Fig 80

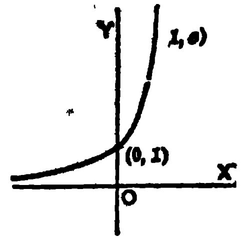


Fig 81

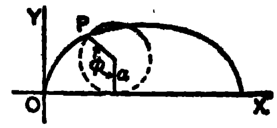


Fig 82

$y = \log_a x$ (Fig 80) $\qquad y = e^x$ (Fig 81) $\qquad \left\{\begin{array}{l} x = a\,(\phi - \sin\phi) \\ y = a\,(1 - \cos\phi) \end{array}\right\}$ (Fig 82)

Change of axes changes the equation of a curve. To find the new equation, when origin is moved to the point (h, k), axes remaining parallel to old positions, substitute $x + h$ for x, and $y + k$ for y. If axes are rotated through angle θ, with origin unchanged, substitute $x \cos\theta - y \sin\theta$ for x, and $x \sin\theta + y \cos\theta$ for y.

22. SOLID GEOMETRY

Distance from $P_1\ (x_1, y_1, z_1)$ to $P_2\ (x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

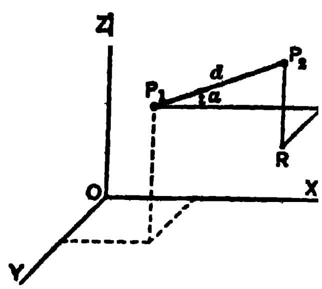


Fig 83

Also, $P_1Q = x_2 - x_1$, $QR = y_2 - y_1$, $RP_2 = z_2 - z_1$, these segments being parallel to the axes (Fig 83).

Direction cosines of P_1P_2 are

$$x_2 - x_1 \qquad \cos\beta = \frac{y_2 - y_1}{d}, \quad \cos\gamma = \frac{z_2 - z_1}{d},$$

where d is the distance from P_1 to P_2, given above. The sum of the squares of direction cosines is always 1; $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. If the direction cosines of a line are proportional to a, b, c, they are $\cos\alpha = a \div k$, $\cos\beta = b \div k$, $\cos\gamma = c \div k$, where $k = \sqrt{a^2 + b^2 + c^2}$.

Plane. Every equation of first degree in x, y, and z, as $Ax + By + Cz = D$ $\quad (P)$ represents a plane. Its intercepts on the x-, y- and z-axes are $D \div A$, $D \div B$, $D \div C$. If the plane goes through the origin, $D = 0$. The direction cosines of a line perpendicular to the plane are $\cos\alpha = A \div q$, $\cos\beta = B \div q$, $\cos\gamma = C \div q$, where $q = \sqrt{A^2 + B^2 + C^2}$.

Distance from point (x', y', z') **to plane** (P) is $r = (Ax' + By' + Cz' - D) \div q$. Distance from origin to plane is $p = D \div q$.

Angle θ between two planes, $Ax + By + Cz = D$, and $A'x + B'y + C'z = D'$, is given by $\cos\theta = (AA' + BB' + CC') \div \sqrt{A^2 + B^2 + C^2}\cdot\sqrt{A'^2 + B'^2 + C'^2}$,

or
$$\cos\theta = \cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma'$$

where $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are direction cosines of a perpendicular to one plane, and $\cos\alpha'$, $\cos\beta'$, $\cos\gamma'$ are direction cosines of a perpendicular to the other. ANGLE BETWEEN TWO LINES, the direction cosines of which are known, is given by the second of the foregoing formulas.

Perpendicular lines or planes. Two lines are perpendicular if

$$\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = 0$$

The same condition holds for planes. More simply, the two planes

$$Ax + By + Cz = D \quad \text{and} \quad A'x + B'y + C'z = D'$$

are perpendicular if $AA' + BB' + CC' = 0$.

Line of intersection of two planes, $Ax + By + Cz = D$ and $A'x + B'y + C'z = D'$, has the direction cosines: $\cos\alpha = a \div s$, $\cos\beta = b \div s$, $\cos\gamma = c \div s$, where

$$a = BC' - B'C,\ b = AC' - A'C,\ c = AB' - A'B, \text{ and } s = \sqrt{a^2 + b^2 + c^2}$$

Equations of straight line through two points (x_1, y_1, z_1) and (x_2, y_2, z_2):

$$(x - x_1) \div (x_2 - x_1) = (y - y_1) \div (y_2 - y_1) = (z - z_1) \div (z_2 - z_1)$$

Equations of line through (x_1, y_1, z_1), **with direction cosines** $\cos\alpha$, $\cos\beta$, $\cos\gamma$:

$$(x - x_1) \div \cos\alpha = (y - y_1) \div \cos\beta = (z - z_1) \div \cos\gamma$$

Plane through three points, $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, $R(x_3, y_3, z_3)$. The equation of the plane may be taken as $ax + by + cz = 1$, by dividing (P) above by D, unless $D = 0$. Substitute in this equation, for x, y, and z, the coordinates of P, then those of Q and finally those of R. This gives three equations, from which the unknowns a, b, and c can be found. This process fails if the plane through P, Q, R goes through the origin, in which case, the equation is $Ax + By + Cz = 0$, and may be divided by A, becoming $x + by + cz = 0$.

Sphere. Equation of sphere, with center (a, b, c) and radius r, is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

Cylinder. Equation of circular cylinder, with axis on z-axis and radius r, is $x^2 + y^2 = r^2$. Every equation containing only two of the variables x, y, z, represents in solid geometry a cylinder, the base of which is the curve determined in one of the coordinate planes by the given equation.

Surface of revolution. If any curve $y = f(x)$ rotates about the x-axis, the equation of the surface generated is $y^2 + z^2 = [f(x)]^2$. Thus, if the ellipse $(x^2 \div a^2) + (y^2 \div b^2) = 1$ (that is, $y = (b \div a)\sqrt{a^2 - x^2}$) rotates about the x-axis, the equation of the resulting ellipsoid of revolution is $y^2 + z^2 = (b^2 \div a^2)(a^2 - x^2)$, or $(x^2 \div a^2) + (y^2 \div b^2) + (z^2 \div c^2) = 1$.

Intercepts. The x-intercept of any surface is found by setting $y = 0$ and $z = 0$ in the equation of the surface; similarly for y and z intercepts.

Traces. The trace of any surface on the xy-plane is the curve found by putting $z = 0$ in the equation of the surface; similarly for the xz-trace and yz-trace. Thus, the xy-trace of the plane $2x - y + 3z = 11$ is the line $2x - y = 11$. The yz-trace is $-y + 3z = 11$.

Cross-section of surface. The equation of the cross-section of any surface the equation of which is known, taken parallel to the xy-plane at the distance k from the xy-plane, is found by putting $z = k$ in the equation of the surface. For cross-section parallel to yz, put $x = k$. For cross-sec parallel to xz, put $y = k$. Thus, cross-sec of sphere $x^2 + y^2 + z^2 = 36$, parallel to xy at distance $z = 2$, is $x^2 + y^2 + 4 = 36$, or $x^2 + y^2 = 32$. It is a circle with radius $\sqrt{32}$.

Dip and strike of strata. (See also Sec 10.) Suppose three points of a stratum (Fig 84), which is approx a true plane, are determined as follows:

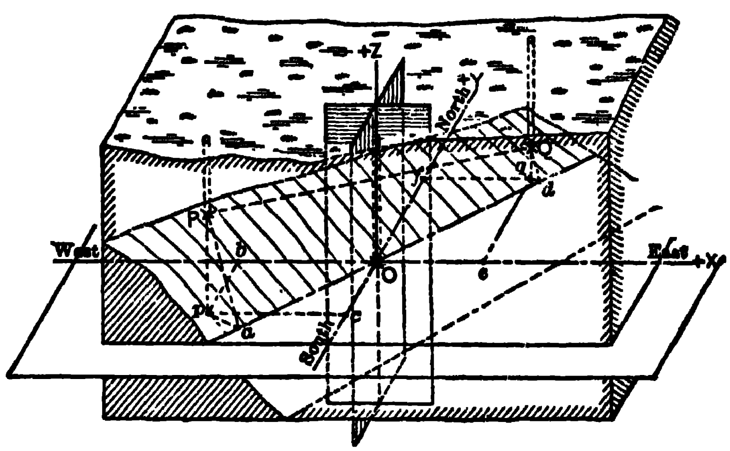


Fig 84

Point	Latitude	Departure	Elev	x	y	z
O	−312.5	305.4	517.4	0	0	0
P	−624 2	124.6	759.7	−311.7	−180.8	242.3
Q	− 48.0	558.7	621.4	264.5	−253 3	104.0

Take a horiz plane through the lowest point O as xy-plane, and the vertical through O as z-axis (elev). Then the coordinates of O, P, and Q are as above. Let the equation of the stratum plane be $x + by + cz = 0$. (See Equation of Plane Through Three Points above.) Substituting the coordinates of P, and then of Q, for x, y and z,

$$-311.7 - 180.8\,b \times 242.3\,c = 0, \text{ and } 264.5 + 253.3\,b + 104\,c = 0;$$

or $0.746\,b + c = +1.286$, and $2.435\,b + c = -2.543$. Hence $b = -2.267$, $c = 2.977$, and the equation of the stratum plane is $x - 2.267\,y + 2.977\,z = 0$ $\quad (S)$

Equation of xy or horiz plane is $z = 0$, or $0x + 0y + z = 0$ $\quad (H)$

As dip is the angle between (H) and (S) (see Angle between Two Planes),

$$\cos\theta = \frac{1 \times 0 - 2.267 \times 0 + 2.977 \times 1}{\sqrt{1^2 + (-2.267)^2 + 2.977^2} \times \sqrt{0^2 + 0^2 + 1^2}} = 0.769$$

Hence $\theta = 39° \, 40'$ approx = dip.

To find the strike, find the intersection of stratum plane (S) with horis plane (H). That is, put $z = 0$ in Eq (S). This gives the horis line

$$x - 2.267\,y = 0, \text{ or } y = 0.441\,x \qquad (L)$$

The slope of this line, $\tan\alpha$, is the tangent of the complement of the strike angle. But $\tan\alpha = 0.441$. Hence, $\alpha = 23°\,50'$, $\beta = 90° - \alpha = 66°\,10'$, and strike = N 66° 10′ E.

CALCULUS

23. DERIVATIVES

Let $f(x)$ be any function of x, and denote it by y; then $y = f(x)$. When x changes by any amount, Δx, then y changes by some amount, Δy. The derivative of the function with respect to x, denoted by $\frac{dy}{dx}$ or $f'(x)$, is

$$\operatorname*{limit}_{\Delta x \doteq 0} \frac{\Delta y}{\Delta x} = \operatorname*{limit}_{\Delta x \doteq 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{dy}{dx} = f'(x)$$

For a table of derivatives see Art 25. (Symbol $\doteq$ means "approaching.")

Slope. Value of the derivative for a particular value of x, $x = x_1$, is the slope of the curve $y = f(x)$ at the point (x_1, y_1).

Tangent and normal. Equation of the tangent to any curve $y = f(x)$ at any point (x_1, y_1) is $y - y_1 = f'(x_1)\,(x - x_1)$, where $f'(x_1)$ denotes the value of $f'(x)$ when $x = x_1$.

Equation of normal at (x_1, y_1) is $y - y_1 = (-1 \div f'(x_1))\,(x - x_1)$.

Maxima and minima. To find maximum or minimum of a given function $f(x)$, set $f'(x) = 0$ and find its roots, say $x = x_1$, $x = x_2$, ... Then $f(x)$ has a

maximum for $x = x_1$ if $f''(x_1)$ is negative,
minimum for $x = x_1$ if $f''(x_1)$ is positive,

where $f''(x_1)$ means the result of substituting x_1 for x in $f''(x)$, the second derivative of $f(x)$ with respect to x.

If, however, $f''(x_1) = 0$, compute $f'''(x_1)$, $f''''(x_1)$ and further derivatives, until one of them, say the kth, is found which is not zero when $x = x_1$. Then $f(x)$ has a maximum for $x = x_1$ if k is even and $f^k(x_1)$ is negative, a minimum if k is even and $f^k(x_1)$ is positive, but a point of inflexion on a horizontal tangent if k is odd.

Point of inflexion. To find points of inflexion of the curve $y = f(x)$, find the roots of $f''(x) = 0$. Any such root x_1 gives a point of inflexion, unless $f'''(x_1) = 0$ also. For this exceptional case, see preceding note.

Curvature. Curvature k of curve $y = f(x)$, at any point (x, y), is

$$k = f''(x) \div \sqrt{(1 + [f'(x)]^2)^3}$$

Radius R of curvature is the reciprocal of curvature, $R = 1 \div k$.

Center (a, b) of curvature,

$$a = x - f'(x)\,(1 + [f'(x)]^2) \div f''(x), \quad \text{and} \quad b = y + (1 + [f'(x)]^2) \div f''(x)$$

Angle from radius vector to tangent, in polar coordinates. If the equation of the curve is $\rho = f(\theta)$, the angle ϕ from radius vector to tangent is given by $\tan\phi = \rho(d\theta \div d)$.

Taylor's series for expanding a function $f(x)$ in powers of $x - a$, with certain restrictions, is

$$f(x) = f(a) + f'(a)\,(x - a) + [f''(a)\,(x - a)^2 \div 2!] + [f'''(a)\,(x - a)^3 \div 3!] + \cdots,$$

where $3!$ denotes $3 \cdot 2 \cdot 1$. If $a = 0$, it becomes Maclaurin's series,

$$f(x) = f(0) + f'(0)\,x + [f''(0)\,x^2 \div 2!] + [f'''(0) \div 3!] + \cdots$$

Usually this series is more and more nearly equal to $f(x)$ when more and more terms are taken, but there are many exceptions.

Derivative of arc. If s denotes length of arc of a curve $y = f(x)$, s varying as the point (x, y) moves along the curve, then $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

Function of a function. If y is a function of u and u is a function of x, $\frac{dy}{dx}$ may be found by $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

24. INTEGRATION

Integral. The integral of $f(x)$, denoted by $\int f(x)\,dx$, means the function of which the derivative is $f(x)$. For example, $\int 3x^2\,dx = x^3 + C$, C being any constant.

For a table of integrals of a few functions, see Art 26.

Definite integral. The symbol $\int_a^b f(x)\,dx$ denotes the result of finding $\int f(x)\,dx$, substituting in the resulting function b for x, then a for x and subtracting, thus:

$$\int_a^b f(x)\,dx = \phi(b) - \phi(a), \text{ where } \phi(x) = \int f(x)\,dx$$

The definite integral gives the value of the limit of the sum of terms, constructed as follows: Divide the interval from a to b into n equal parts, and denote the abscissas x of the points of division by $x_0, x_1, x_2, \ldots x_n$, x_0 being a and x_n being b. Let each part of the interval thus divided be Δx. Then

$$\int_a^b f(x)\,dx = \lim_{n \doteq \infty} [f(x_1)\,\Delta x + f(x_2)\,\Delta x + \cdots + f(x_n)\,\Delta x]$$

Area. In rectangular coordinates, the area under the curve $y = f(x)$, included between ordinates, at $x = a$ and $x = b$, is $A = \int_a^b f(x)\,dx$. If the curve crosses the x-axis between a and b, compute area to crossing, and then beyond. In polar coordinates, area between the curve $\rho = f(\theta)$ and two radii at $\theta = \theta_1$ and $\theta = \theta_2$ is given by $A = {}^1/_2 \int_{\theta_1}^{\theta_2} \rho^2\,d\theta$. For double integration the element of area dA is $dA = dy\,dx$, or $dA = \rho\,d\rho\,d\theta$.

Length of arc of curve $y = f(x)$ from $P_1\,(x_1, y_1)$ to $P_2\,(x_2, y_2)$:

$$s = \int_{x_1}^{x_2} \sqrt{1 + m^2}\,dx, \qquad \text{where } m = f'(x)$$

also,

$$s = \int_{\theta_1}^{\theta_2} \sqrt{\rho^2 + k^2}\,d\theta, \qquad \text{where } k = \frac{d\rho}{d\theta}$$

Volume and surface of revolution. If the curve $y = f(x)$ revolves about x-axis, the volume generated by area under the curve and between ordinates at $x = x_1$ and $x = x_2$, is $V = \pi \int_{x_1}^{x_2} y^2\,dx$, and surface generated by arc of curve is

$$S = 2\pi \int_{x_1}^{x_2} x\sqrt{1 + m^2}\,dx, \qquad \text{where } m = f'(x)$$

Water pressure. Pressure on vertical area bounded by the curve $y = f(x)$ and two horizontal lines at $y = y_1$ and $y = y_2$ is $P = k\int_{y_1}^{y_2} hw\,dy$, where h is the depth below water surface of a narrow horiz strip at distance y from origin, w is the width of this strip, and k is weight per unit volume. h and w must be expressed in terms of y before proceeding with the integration.

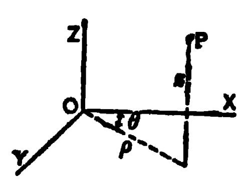


Fig 85

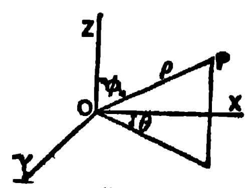


Fig 86

Element of volume, multiple integration. Element of volume under surface the equation of which is $z = f(x, y)$:

$dV = z\,dx\,dy$, for double integration ⎫ rectangular coordinates.
$dV = dz\,dx\,dy$, for triple integration ⎭
$dV = \rho\,d\rho\,d\theta\,dz$, cylindrical coordinates.
$dV = \rho^2 \sin\phi\,d\phi\,d\theta\,d\rho$, spherical coordinates.

25. TABLE OF DERIVATIVES

In this table u and v denote functions of x; a, c, and n are arbitrary constants:

$$\frac{d}{dx}c = 0 \qquad\qquad \frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \qquad\qquad \frac{d}{dx}e^u = e^u\frac{du}{dx} \quad (\text{where } e = 2.718+)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right) \div v^2 \qquad\qquad \frac{d}{dx}a^u = a^u \log_e a \cdot \frac{du}{dx}$$

Table of Derivatives (*Continued*)

$$\frac{d}{dx}u^v = vu^{v-1}\frac{du}{dx} + u^v \log_e u \frac{dv}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\log_{10} u = \frac{\log_{10} e}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\sin u = \cos u \frac{du}{dx} \quad (u \text{ in radians})$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1} u = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1} u = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$$

26. TABLE OF INTEGRALS

The base e (Naperian system) is assumed for logarithms unless otherwise noted. A constant C is understood to be added to each result

$$\int dx = x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad \text{except when } n = -1$$

$$\int x^{-1}\,dx = \int \frac{dx}{x} = \log x$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\log(ax+b)$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \quad \text{except when } n = -1$$

$$\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}}\tan^{-1}\left[x\sqrt{\frac{a}{b}}\right], \quad \text{when } a > 0 \text{ and } b > 0$$

$$\int \frac{dx}{ax^2-b} = \frac{1}{2\sqrt{ab}}\log\frac{\sqrt{a}\,x-\sqrt{b}}{\sqrt{a}\,x+\sqrt{b}}, \quad \text{when } a > 0 \text{ and } b > 0$$

$$\int \sqrt{x^2+a^2}\,dx = 1/2\left[x\sqrt{x^2+a^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right)\right]$$

$$\int \sqrt{x^2-a^2}\,dx = \text{same as above, with } -a^2 \text{ for } a^2$$

$$\int \sqrt{a^2-x^2}\,dx = 1/2\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \tan x\,dx = -\log\cos x$$

$$\int \log x\,dx = x(\log x - 1)$$

$$\int \cot x\,dx = \log\sin x$$

$$\int \log_{10} x\,dx = x\log_{10}\frac{x}{10}$$

$$\int \sec x\,dx = \log(\sec x + \tan x)$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int a^x\,dx = \frac{a^x}{\log a}$$

$$\int \csc x\,dx = \log(\csc x - \cot x) = \log\tan\frac{x}{2}$$

$$\int xe^x\,dx = e^x(x-1)$$

$$\int \csc^2 x\,dx = -\cot x$$

$$\int \sin x\,dx = -\cos x$$

$$\int x\sin x\,dx = \sin x - x\cos x$$

$$\int \cos x\,dx = \sin x$$

$$\int x^2\sin x\,dx = 2x\sin x + (2-x^2)\cos x$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\int x\cos x\,dx = \cos x + x\sin x$$

$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int x^2\cos x\,dx = 2x\cos x + (x^2-2)\sin x$$

STATICS

27. DEFINITIONS

Mechanics is that branch of science which treats of forces and motion. STATICS deals with the action of forces on bodies at rest. DYNAMICS deals with the action of forces on bodies in motion.

Force is that which changes or tends to change the state of rest or motion of a body. A force is completely specified by its magnitude, direction and point of application. The word SENSE as applied to a force refers to one of the two directions along the line of action of the force. The effect of any force applied to a rigid body at rest is the same, no matter where in its own line of action the force is applied. This is known as the principle of the TRANSMISSIBILITY OF FORCE. A force may be represented graphically in magnitude and direction by a straight line drawn parallel to its line of action, the length being proportional to the magnitude of the force; its sense is indicated by an arrowhead placed on the line. The English engineers' UNIT OF FORCE is the pound, or the earth's pull on a mass of 1 lb. A drawing which indicates the lines of action of the various forces acting on a machine or structure is called a SPACE DIAGRAM; the part in which vectors are drawn to represent the magnitudes and directions of the forces is a VECTOR DIAGRAM. A force is indicated on a space diagram by two lower-case letters placed on opposite sides of the line of action of the force; the vector, representing its magnitude and direction, by the same capital letters placed at the ends. Thus, in Fig 87, AB represents the magnitude and direction of the force W, and ab its action line. The vector being read as AB indicates a downward sense; read as BA, an upward sense.

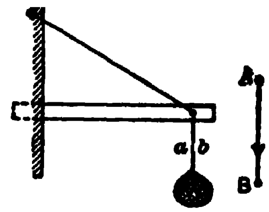


Fig 87

Forces are classified according to the arrangement of their action lines:

1. Coplanar: *a*. concurrent; *b*. nonconcurrent; *c*. parallel
2. Noncoplanar: *a*. concurrent; *b*. nonconcurrent; *c*. parallel

Any number of forces taken collectively is a system or set of forces. A system is coplanar or noncoplanar according as the lines of action of the forces do or do not lie in the same plane, and is concurrent or nonconcurrent according as they do or do not intersect in a point. Two or more forces which are the equivalent of a single force are COMPONENTS of the force. The operation of replacing a system of forces by a simpler system is called COMPOSITION of forces. RESOLUTION of forces is the operation of replacing a single force by a system of forces. RESULTANT of a force system is the simplest equivalent system. In some cases the simplest equivalent is a single force; in other cases, it is a couple; in still others it is a pair of forces which are not parallel and do not intersect (noncoplanar).

28. RESULTANTS OF CONCURRENT FORCES

Resultant of two concurrent forces. PARALLELOGRAM LAW. If two concurrent forces P and Q (Fig 88), acting at the point O of a body, are represented in magnitude and direction by the adjacent sides OB and OA of parallelogram $OACB$, their resultant is represented by diagonal OC passing through point of concurrency O. TRIANGLE LAW. If in the triangle ABC (Fig 89) AB and BC represent two concurrent forces in magnitude, direction

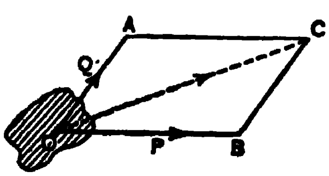


Fig 88

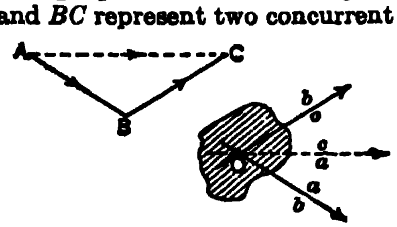


Fig 89

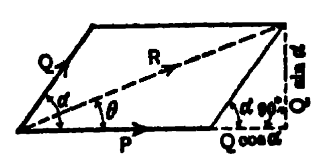


Fig 90

and sense, then AC will represent their resultant in magnitude, direction and sense; its action line will be ac, through the point of concurrency parallel to AC. The resultant may be found algebraically thus: In Fig 90, let α be the angle between the action lines of forces P and Q, and θ the angle between R and P. Then, $R^2 = P^2 + Q^2 + 2\,PQ\cos\alpha$, and $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$. If $\alpha = 90°$, $R^2 = P^2 + Q^2$, and $\tan\theta = Q \div P$.

A force may be resolved into an infinite number of pairs of components by constructing different triangles as in Fig 91. The action lines of the components must be concurrent at a point on the action line of the force. A common problem is to resolve a force into rectangular components (often called resolved parts). In Fig 92, AB and BC are the

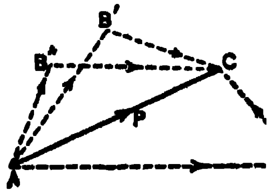


Fig 91

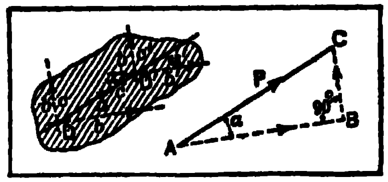


Fig 92

rectangular components of the force P. Expressed algebraically, $AB = P\cos\alpha$ and $BC = P\sin\alpha$. In general, the resolved part of a force along any line equals the magnitude of the force times the cosine of the angle between the lines of action of the force and its component. Action lines of the components are concurrent on the space diagram at some point on ac, as at D or D'.

Resultant of coplanar concurrent forces is found graphically as follows: In Fig 93, consider body G acted on by the 4 forces shown. Construct a force polygon. Plat AB parallel to ab, and scale it to represent 60 lb; from B plat BC parallel to bc, and scale it to represent 80 lb; in like manner plot CD and DE, so that the arrows lead confluently from A to E. The resultant of the system is AE in magnitude and sense = 114 lb, and its action line is ae, making an angle of $\theta = \tan^{-1} 0.146 = 8° 20'$ with the horizontal. ALGEBRAIC SOLUTION. Choose rectangular axes OX and OY. Resolve each force into its x and y components, considering components acting upward or to the right as positive, and those acting downward or to the left as negative. Arrange the results in tabular form, placing the forces in the first column, the x components in the second and the y components in the third. ΣF_x = algebraic sum of x components, and ΣF_y = algebraic sum of y components.

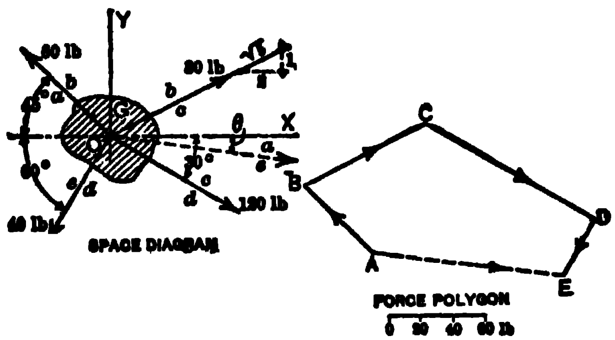


Fig 93

F, lb	F_x, lb	F_y, lb
$ab = 60$	$-60 \times 0.707 = -42.4$	$+60 \times 0.707 + 42.4$
$bc = 80$	$+80 \times 2/\sqrt{5} = +71.4$	$+80 \times 1/\sqrt{5} = +35.7$
$cd = 120$	$+120 \times 0.866 = +104$	$-120 \times 0.5 = -60$
$de = 40$	$-40 \times 0.5 = -20$	$-40 \times 0.866 = -34.6$
	$\Sigma F_x = +113$	$\Sigma F_y = -16.5$

Then $R = \sqrt{\overline{\Sigma F_x}^2 + \overline{\Sigma F_y}^2} = \sqrt{13\,041} = 114$ lb. Sense is downward and to the right (Fig 94). $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = 0.146$; $\theta = 8° 20'$. If a concurrent system has a resultant, it is a single force.

Resultant of noncoplanar concurrent forces. PARALLELOPIPEDON LAW. Consider the 3 rectangular forces, P, Q, and S (Fig 95). On these forces construct to scale a parallelopiped.

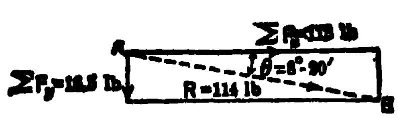


Fig 94

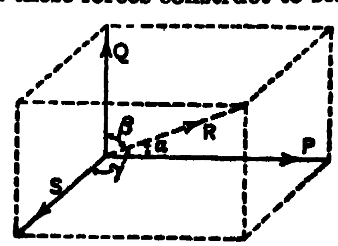


Fig 95

Resultant of the system is represented in magnitude and direction by the diagonal; its value is $R = \sqrt{P^2 + Q^2 + S^2}$. Its direction cosines with respect to the axes are: $\cos\alpha = P \div R$, $\cos\beta = Q \div R$, and $\cos\gamma = S \div R$.

Resultant of any number of concurrent forces. Let the forces be specified with respect to 3 rectangular axes passing through the point of concurrency: (*a*) Resolve each force into components along the X, Y and Z-axes; (*b*) Find the algebraic sums of the x, y and z components, and indicate them by ΣF_x, ΣF_y, and ΣF_z; (*c*) Find the resultant of these three partial resultants by the parallelopipedon law; its value is $R = \sqrt{\overline{\Sigma F_x}^2 + \overline{\Sigma F_y}^2 + \overline{\Sigma F_z}^2}$; its direction angles are $\alpha = \cos^{-1}\frac{\Sigma F_x}{R}$; $\beta = \cos^{-1}\frac{\Sigma F_y}{R}$; $\gamma = \cos^{-1}\frac{\Sigma F_z}{R}$

29. MOMENTS AND COUPLES

Moment or **torque** of a force about a point is the product of the force magnitude and the distance from the point to its action line. This perpendicular distance is called the arm of the force, and the point is the origin or center of moments. The product is the measure of the rotational tendency of the force. The name of the unit of moment is a combination of the names of force and distance units: thus: foot-pound, inch-pound or inch-ton. Moments tending to produce counterclockwise rotation of a body will be considered positive, and clockwise negative, unless contrary convention is mentioned. Thus, in Fig 96, the moment of force P about $O = P + OA$.

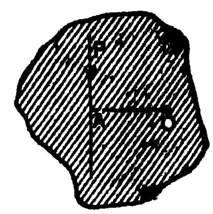


Fig 96

Moment of force about a line or axis. At any point on its action line, resolve the force into 2 rectangular components, one being parallel to the axis. The product of the perpendicular component and the perpendicular distance from the axis to the force is the moment of given force about the axis. Thus (Fig 97), $P \sin \alpha$ is the component parallel to the axis, and it has no turning effect. All the moment or turning effect is caused by the perpendicular component, and its value is $P \cos \alpha \times OE$, OE being the perpendicular distance between the axis and the parallel plane $ABCD$.

Principle of moments. Sum of the moments of any coplanar force system about any point in their plane is equal to the moment of their resultant about same point. Thus (Fig 98), R is the resultant of P and Q, and $R \times r = P \times p - Q \times q$. Sum of the moments of any force system about an axis is equal to the moment of their resultant about the axis. Thus (Fig 99), R is the resultant of the 3 rectangular forces, P, Q, and S. Moment of R about axis $X = S \times y - Q \times z$. P contributes nothing to the moment sum, as it is parallel to the axis of moments. In such a case, counterclockwise moment is called positive and clockwise negative, the observer looking toward the origin O, from the positive ends of the axes. Thus, Q has positive moment about axis Z, but negative moment about axis X. Moment of a force passing through the origin of moments is zero. Moment of a force intersecting the axis of moments is zero.

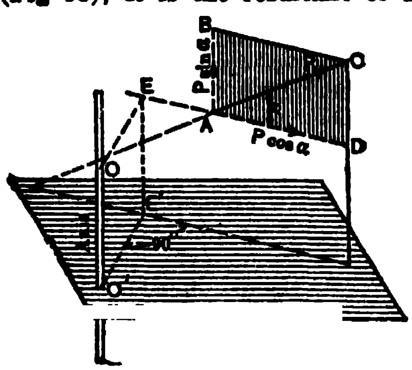


Fig 97

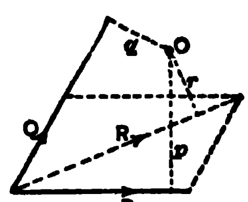


Fig 98

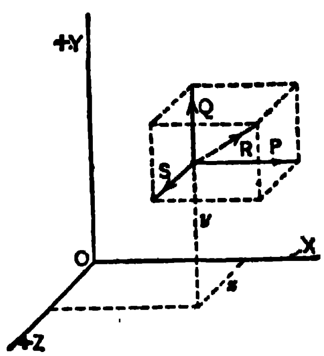


Fig 99

Couples. Two equal and parallel forces of opposite sense are called a couple. THE ARM OF THE COUPLE is the perpendicular distance between the lines of action of the forces. Moment of a couple is constant and independent of the origin of moments; it is equal to one of the forces times the arm of the couple. Its sense is positive or negative according as rotational tendency is counterclockwise or clockwise. Couples of equal moments, in the same or parallel planes, are equivalent and may be replaced one by the other. That is, a couple may be twisted or moved about in its plane, or transferred to any parallel plane without altering the resulting motion of the body on which it acts.

Resultant of any number of coplanar couples or of couples in parallel planes is a couple. Its moment and sense equal the algebraic sum of the moments of the component couples. A couple may be represented by a vector. Length of the vector to scale represents the

magnitude of the moment; it is drawn perpendicular to the plane of the couple from any origin, and an arrow is placed on it to represent the way in which the couple would cause a right-hand screw to advance. Resultant of any number of couples (in oblique or parallel planes) is a couple. The composition is effected thus: (*a*) Refer the couple vectors to three rectangular coordinate axes. (*b*) Resolve each vector into components parallel to the axes. (*c*) Take the sum of the components along the X, Y, and Z-axes. (*d*) Representing these sums by ΣC_x, ΣC_y, ΣC_z, and the resultant vector by C, its value is $C = \sqrt{\overline{\Sigma C_x}^2 + \overline{\Sigma C_y}^2 + \overline{\Sigma C_z}^2}$. Its direction angles are

$$\phi_x = \cos^{-1}\frac{\Sigma C_x}{C}\ ;\ \phi_y = \cos^{-1}\frac{\Sigma C_y}{C}\ ;\ \phi_z = \cos^{-1}\frac{\Sigma C_z}{C}$$

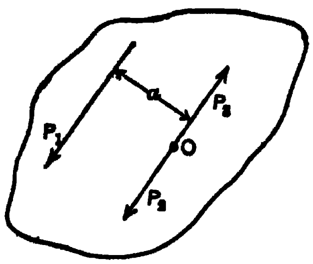


Fig 100

A force may be resolved into a force acting at a chosen point and a couple. In Fig 100, P_1 is the given force and O the chosen point. Through O apply a pair of forces, opposite in sense, equal and parallel to P_1. As P_2 and P_3 balance, no change is produced in motion of the body due to this addition. P_1 and P_3 constitute a couple of moment $= P \times a$, which is the same as moment of P_1 about O; and P_2 is a force just like P_1, but acting through the chosen point O. Conversely, a force and a couple in a plane compound into a single force parallel to given force, of same sense and magnitude, and so placed that its moment about any point in action line of given force is equal to moment of given couple.

30. RESULTANTS OF COPLANAR NONCONCURRENT FORCES

Resultant of coplanar parallel forces may be (*a*) a single force or (*b*) a couple.

Case (*a*). To find the resultant of the 4 parallel forces shown in Fig 101:

Graphical solution. (1) Plat AB to represent magnitude and sense of ab; then BC to represent magnitude and sense of bc; then CD and DE to represent the other two forces; (2) choose the pole O in any convenient position and draw the rays AO, BO, CO, DO, and EO; (3) from any point on ab, draw strings parallel to AO and OB; from intersection of ob and bc draw oc parallel to OC until it intersects cd; from that point draw od to intersect de; then oe to intersect oa at K; (4) AE, from the beginning to the end of vector diagram, represents the magnitude and sense of the resultant = 180 lb downward; its action line is ae, through K, parallel to AE, 5.3 ft to right of ab.

The polygon formed on the space diagram is called a string or funicular polygon; its sides represent the action lines of the components that replace the parallel forces. Thus

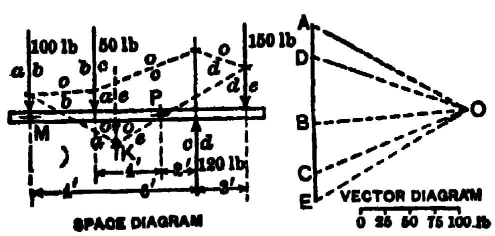


Fig 101

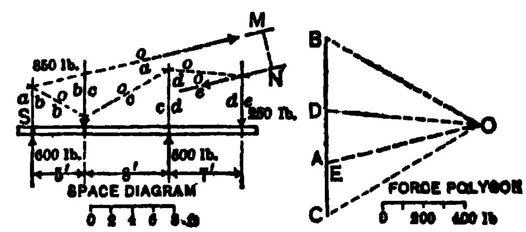


Fig 102

BC, acting in the line bo, is replaced by BO = 140 lb, and OC = 160 lb, acting in the lines bo and oc. AO and OE are components of the resultant AE. The object of the funicular polygon is to locate one point on the action line of the resultant.

Algebraic solution. The magnitude and sense of the resultant are given by $R = \Sigma F$; its position is determined by use of the principle of moments, the moment of the resultant being equal to the algebraic sum of the moments of the forces. Hence arm $= \Sigma M \div R$. It is best to call upward forces positive, and distances to right of moment origin positive. For solving example shown in Fig 101, tabulate the values as follows:

Force, lb	Arm from M	Moment about M	Arm from P	Moment about P
−100	0	0	−8	+800
−50	+4	−200	−4	+200
+120	+10	+1 200	+2	+240
−150	+13	−1 950	+5	−750
ΣF = −180 lb		ΣM_M = −950 ft-lb		ΣM_P = +490 ft-lb

R = 180 lb downward. Distance from $M = -950 \div -180 = +5.3$ ft. Distance from $P = +490 \div -180 = -2.7$ ft. Either of these distances locates the point K in Fig 101, and checks the graphical solution.

Case (*b*). $\Sigma F = 0$, and resultant is a couple.

Graphical solution. Consider the 4 parallel forces in Fig 102. The force polygon begins at A and ends at E, the same point; hence the resultant is not a single force (this is called a closed force polygon). Construct the funicular polygon as before. The first and last strings, *ao* and *oe*, being parallel, do not intersect. The forces acting in those lines, AO and OE, being equal and opposite in sense, form a couple. Hence the resultant of the system is a couple. The forces $AO = OE = 750$ lb. The arm $MN = 3.67$ ft. The moment $= 750 \times 3.67 = 2\,752$ ft-lb. The sense, by inspection of the space diagram, is clockwise. The moment of the couple is the same, no matter what pole O is selected.

Algebraic solution of the same example (see accompanying table):

Forces, lb	Arm from S	Moment about S	
+600	0	0	
−850	+ 5		−4 250
+500	+13	+6 500	
−250	+20		−5 000
$\Sigma F = 0$		$\Sigma M_s = -2\,750$ ft-lb	

Hence the resultant is a clockwise couple of moment equal to 2 750 ft-lb. Since couples of equal moment and sense in the same plane are equivalent, it is unnecessary to give more detail.

Special cases. Two parallel forces. Case (*a*). Forces alike in sense. Given the 2 forces P and Q, located as in Fig 103. The resultant lies between these forces, and is equal in magnitude to $P + Q$. The action line of the resultant is found from the equation. $a = (Q \times b) \div P$. Case (*b*). Forces opposite in sense. Given the 2 forces P and Q as in Fig 104. Resultant lies outside of the forces P and Q, is adjacent to the larger force, and is equal in magnitude to $Q - P$. The action line of the resultant is found from the equation, $a = (Q \times b) \div P$. The sense of the resultant is the same as that of larger force. In either case, the resultant divides the distance between the forces into segments inversely proportional to the adjacent forces; or, $P \div b = Q \div a = R \div c$.

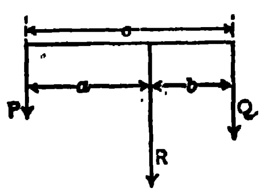


Fig 103

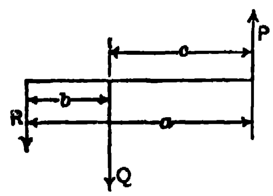


Fig 104

Resultant of coplanar nonconcurrent forces. **Case** (*a*). Resultant equals a single force.

Graphical solution. Construct a force polygon as if the forces were concurrent. The closing line gives the magnitude, sense, and angular direction of the resultant. The action line is found by drawing a funicular polygon. Thus, to find the resultant of the 4 forces in Fig 105: (1) Construct the force polygon $ABCDE$; (2) choose a convenient pole O, and draw the rays; (3) start at any point on *ab* and draw the funicular polygon; (4) determine K, the intersection of the first and last strings. This is one point on the action line of the resultant. Resultant $= AE =$ 67 lb, downward to the left, at angle $\theta = tan^{-1} \dfrac{0.784}{1.0} = 38° 6'$ to horizontal. Action line is *ae*, parallel to AE, through K.

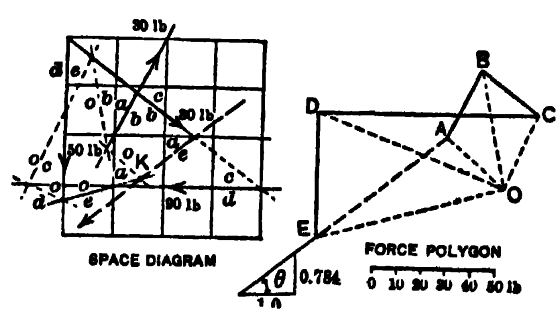


Fig 105

Second graphical method. Use the triangle law to find the resultant of two forces. Use the same law to find the resultant of this partial resultant and the third force. Repeat until the final resultant is obtained.

Algebraic solution. The resultant is found in magnitude, sense, and angular direction as if the forces were concurrent: $R = \sqrt{\overline{\Sigma F_x}^2 + \overline{\Sigma F_y}^2}$, $cos\ \theta = \dfrac{\Sigma F_x}{R}$, $sin\ \theta = \dfrac{\Sigma F_y}{R}$, in which θ is the angle between R and the X-axis. Use the principle of moments to locate the action line. Choose a convenient origin of mo-

Force, lb	F_x, lb	F_y, lb	M_o, ft-lb
50	−50	0	+150
70	−31.3	−62.6	−62.6
30	+ 7.3	−29.1	0
	$\Sigma F_x = -74$ lb	$\Sigma F_y = -91.7$ lb	$\Sigma M_o = 87.4$ ft-lb

ments and compute ΣM. The arm of the resultant $= \Sigma M \div R$, and is perpendicular to the angular direction of R found above. R is so located with respect to the origin that the sign of its moment is the same as that of ΣM. As an example, find the resultant of the 3 forces in Fig 106.

$$R = \sqrt{(74)^2 + (91.7)^2} = 118 \text{ lb downward to the left.}$$

Angle with X-axis $= \tan^{-1} \dfrac{91.7}{74} = 51° 8'$. Arm about $O = 87.4 \div 118 = 0.74$ ft.
In this case, R is to the left of O, because its moment must be $= \Sigma M_o = +87.4$ ft-lb.

Case (b). Resultant is a couple.

Graphical solution. Let forces V, P, Q, T, and S (Fig 107) represent a system of forces the force polygon of which closes, and which reduces to the 2 components AO and OF

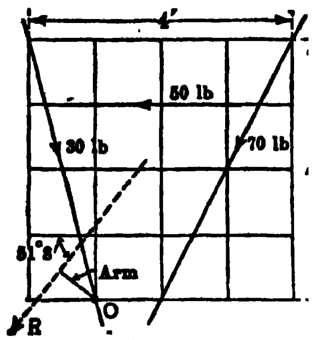


Fig 106

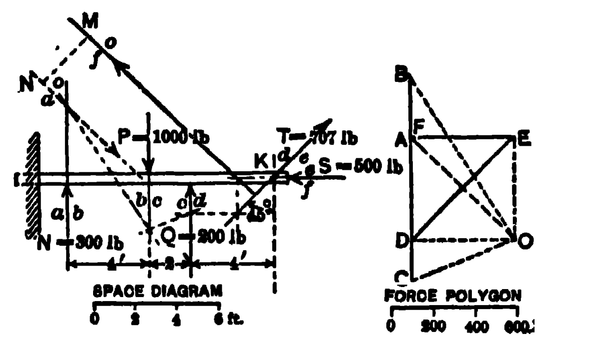


Fig 107

acting in ao and of. Since these components are equal, opposite and parallel, the resultant is a couple. The arm of the couple is the perpendicular distance MN, between the first and last strings. The moment is the product of OA (according to force scale) and the arm MN (according to scale of space diagram). The sense of the couple is counterclockwise, as found by inspection of the space diagram. The moment $= 710 \times 3.1 = 2\,200$ ft-lb.

Algebraic solution. Any couple the moment of which equals the algebraic sum of the moments of the given forces about any point may be regarded as the resultant. In Fig 107, assume forces acting as indicated. The algebraic sums of the x and y components are zero. Taking moments about point K,

$$\Sigma M_K = 1\,000 \times 6 - 300 \times 10 - 200 \times 4 = +\,2\,200 \text{ ft-lb}$$

Hence, the resultant of the system is a couple, of moment equal to 2 200 ft-lb and of counterclockwise sense.

31. RESULTANTS OF NONCOPLANAR NONCONCURRENT FORCES

Resultants of noncoplanar parallel forces may be a single force or a couple.

Choose a set of 3 rectangular axes, with the Z-axis parallel to the forces, and let the positions of the forces be specified by the x and y coordinates of their action lines. As an example, consider the 5 parallel forces shown in Fig 108 to be perpendicular to plane of the paper. Tabulate the computations as below:

Force, lb	x, ft	y, ft	M_x or $F \times y$, ft-lb	M_y or $F \times x$, ft-lb
+10	+3	+6	+ 60	+30
+30	+2	+4	+120	+60
+20	+2	−1	−20	+40
−40	−1	−1	+ 40	+40
−35	+3	+1	−35	−105
$\Sigma F = -15$ lb			$\Sigma M_x = +165$	$\Sigma M_y = +65$

$R = \Sigma F = -15$ lb, in the $-Z$ sense, or backward. $x_o = \dfrac{\Sigma M_y}{\Sigma F} = \dfrac{+65}{-15} = -4.33$ ft.

$y_o = \dfrac{\Sigma M_x}{\Sigma F} = \dfrac{+165}{-15} = -11$ ft. Hence the coordinates of the action line of the resultant are x_o, y_o, or $(-4.33$ ft, -11 ft). Position of the resultant is not indicated in Fig 108. In the above computation, the algebraic rules of signs for multiplication and division are followed throughout, so that the values of x_o and y_o may be interpreted algebraically.

This method agrees with the usual convention of signs for computing moments about the X-axis, but is the reverse of the usual convention for moments about the Y-axis.

If $\Sigma F = 0$, and if ΣM_x or ΣM_y does not equal zero, the resultant is a couple and not a force. The plane of the couple will be parallel to the Z-axis. It is necessary to deter-

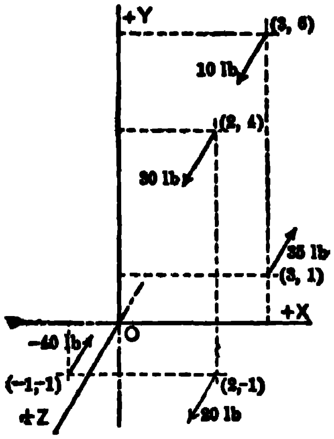


Fig 108

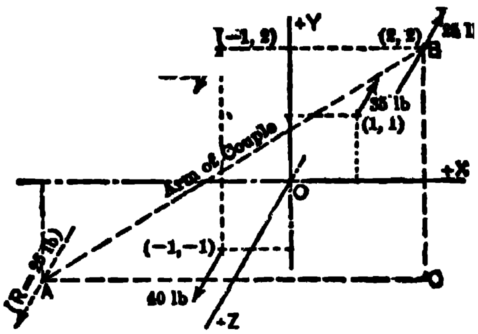


Fig 109

mine the moment of the couple, its sense, and the inclination of its plane with the XZ plane. Consider the 4 forces in Fig 109 as an example:

Force, lb	x, ft	y, ft	M_x, ft-lb	M_y, ft-lb
+20	−1	+2	+40	−20
+40	−1	−1	−40	−40
−35	+1	+1	−35	−35
−25	+2	+2	−50	−50
$\Sigma F = 0$			$\Sigma M_x = -85$	$\Sigma M_y = -145$

Since $\Sigma F = 0$, the resultant of all except one of the forces, with the omitted force, is a couple. Omitting the last force, $R = \Sigma F = +25$ lb. Its $x_o = \dfrac{\Sigma M_y}{\Sigma F} = \dfrac{-95}{+25} = -3.8$ ft, and $y_o = \dfrac{\Sigma M_x}{\Sigma F} = \dfrac{-35}{+25} = -1.4$ ft. AB is the arm of the couple, and from the triangle ABC its length is found to be 6.72 ft. The moment of the couple $= 25 \times 6.72 = 168$ ft-lb. BAC is angle its plane makes with horiz plane. Angle $BAC = \tan^{-1}\dfrac{3.4}{5.8} = 30° \, 21'$. Viewing the couple from plus end of Y-axis, its sense is observed to be counterclockwise.

Resultant of noncoplanar nonconcurrent nonparallel forces is generally a single force acting at a chosen point and a couple not coplanar with the force. The magnitude, sense, and angular direction of the single force is the same as if the forces were concurrent:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}; \quad \phi_x = \cos^{-1}\frac{\Sigma F_x}{R}; \quad \phi_y = \cos^{-1}\frac{\Sigma F_y}{R}; \quad \phi_z = \cos^{-1}\frac{\Sigma F_z}{R}$$

and R acts through the selected origin of reference. To determine the couple, compute the sums of the moments of the forces about the coordinate axes of reference. These moment sums represent 3 couples, which are axial components of the resultant couple. Consider ΣM_x as a vector along the X-axis, ΣM_y as a vector along Y, and ΣM_z along Z. The moment of the resultant couple is $C = \sqrt{(\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2}$, and the direction angles of its vector are

$$\theta_x = \cos^{-1}\frac{\Sigma M_x}{C}; \quad \theta_y = \cos^{-1}\frac{\Sigma M_y}{C}; \quad \theta_z = \cos^{-1}\frac{\Sigma M_z}{C}$$

R and C may be compounded into 2 forces which do not intersect. In a special problem R or C might be zero. Hence, the resultant might be a couple only, or a single force.

32. CONDITIONS OF EQUILIBRIUM

A body is in equilibrium with respect to adjacent bodies if it remains at rest with respect to them, or if it moves at constant velocity; that is, its state of motion does not change. A force system is in equilibrium if its resultant is zero. Such a system causes

no change in the state of motion of the body to which it is applied. If a body is in equilibrium with respect to adjacent bodies, then the external force system applied to it is in equilibrium. By this external force system is meant the pull of gravity and all the pulls or pushes exerted on the body by other bodies.

Depending on the kind of force system involved, varying numbers of tests or conditions must be applied to prove that the system is in equilibrium, or that no resultant exists. In the usual problem, a body is known to be in equilibrium; hence the system composed of all the external forces acting upon it must be in equilibrium. In such case the tests are not needed to ascertain if equilibrium exists, but they are used to set up relations involving unknown forces, distances, or angles, and the unknown elements are then computed provided they are not too numerous. Such computations are conducted either algebraically or graphically. (See following table.)

Special conditions of equilibrium. If 3 forces are in equilibrium they must be coplanar, and must be concurrent or parallel; if concurrent, each force is proportional to the sine of the angle between the other two; if parallel, each force is proportional to the distance between the other two. If a force system is in equilibrium, the resultant of any part must balance the resultant of the other part.

Necessary independent conditions of equilibrium for the various force systems

	System	Algebraical		Graphical	
		No	Conditions	No	Conditions
Coplanar	Collinear	1	$\Sigma F = 0$	1	Force polygon closes
Coplanar	Concurrent at point O	2	$\Sigma F_x = 0$, $\Sigma F_y = 0$, if the angle between x and y does not equal 180°; or, $\Sigma F_x = 0$, $\Sigma M_a = 0$, if x direction is not perpendicular to aO; or, $\Sigma M_a = 0$, $\Sigma M_b = 0$, if aOb is not a straight line	1	Force polygon closes
Coplanar	Parallel	2	$\Sigma F = 0, \Sigma M = 0$; or, $\Sigma M_a = 0$, $\Sigma M_b = 0$, if line ab is not parallel to forces	2	Force and funicular polygons close. Latter item means that first and last strings coincide
Coplanar	Nonparallel nonconcurrent	3	$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$; or, $\Sigma F_x = 0$, $\Sigma M_a = 0$, $\Sigma M_b = 0$, if x is not perpendicular to ab; or, $\Sigma M_a = 0$, $\Sigma M_b = 0$, $\Sigma M_c = 0$, if abc is not a straight line		Force and funicular polygons close
Noncoplanar	Concurrent at point O	3	$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$; or, ΣF_s in every direction and ΣM about every axis $= 0$. Combinations of moment and resolution equations can be arranged, but are not common	2	Force polygon closes. It is warped; hence plan and elevation must show closed. Not commonly used
Noncoplanar	Parallel	3	$\Sigma F_z = 0$, $\Sigma M_x = 0$, $\Sigma M_y = 0$, forces parallel to z-axis. Other combinations possible but not common		Not used
Noncoplanar	Noncon-current nonparallel	6	$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$. ΣM about every axis $= 0$, and it is often convenient to employ more than three moment equations, instead of using so many resolution equations		The projection of the system on any plane is in equilibrium, and algebraical or graphical conditions can be used to solve such projected systems

33. EQUILIBRIUM PROBLEMS

General method of solution. (*a*) Sketch or specify the free body that is in equilibrium, and indicate the complete external force system acting upon it. (*b*) Discuss the problem as to number and kinds of unknown quantities, to see if a solution is possible. If the first system tried cannot be solved, separate the original body discussed (say some machine) into simpler parts, and sketch and discuss them until one is found that permits of solution. (*c*) Apply the appropriate conditions of equilibrium and solve for the unknown forces, angles, or distances. (*d*) Make use of the acquired information to solve other systems.

If resolution equations are used, it is desirable each time to resolve perpendicular to one of the unknown forces, so as to avoid the solution of simultaneous equations and to make the answers independent of each other. If moment origins are used, it is best to select them on the action lines of some of the unknown forces. If moment axes are used, they should be selected so as to intersect some of the unknown forces. Equilibrium problems can be classified as to the kind of force system involved, and number and kind of unknown quantities. Following are a few typical problems, with details of their solutions.

Typical problem I. A SYSTEM OF COPLANAR CONCURRENT FORCES is in equilibrium and all but two are known; the action lines of these are known, required their magnitudes and senses. ALGEBRAIC SOLUTION. (*e*) Assume senses for both unknown forces. (*f*) Write a pair of equilibrium equations involving the unknown forces, and solve for the two unknowns. A plus answer indicates the sense correctly assumed. A minus answer indicates incorrect assumption. GRAPHICAL SOLUTION. (*g*) Letter the action lines of the wholly known forces, and then the action lines of the two unknown forces. (*h*) Draw the force polygon to the end of the last known vector, and close it by drawing parallels to the unknown forces through the ends of the first and last vectors, and mark the intersection of these lines as the last vertex in the polygon. (*i*) The senses must read confluently from starting point back to same point, the last two vectors representing the two unknown forces in magnitude and sense.

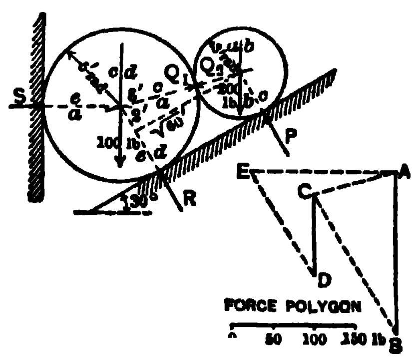


Fig 110

Example. Two smooth cylinders rest upon a 30° plane and against a vertical wall as shown in Fig 110. Determine all forces acting on each cylinder. (*a*) The forces involved are 100 lb, 200 lb, P, Q, R, and S, the last four being normal to the surfaces of contact (smooth surfaces). (*b*) Consider the two cylinders as a single free body. The external force system is 100 lb, 200 lb, P, R, and S (Q_1 and Q_2 are internal). The system is nonconcurrent, so does not come under typical problem I. Consider the large cylinder as a free body. The external force system is 100 lb, Q, R, and S. While this system is concurrent, it can not be solved because there are more than two unknown quantities. Next consider the small cylinder as a free body. The force system is 200 lb, P and Q, and this is typical problem I. ALGEBRAIC SOLUTION. Choose X and Y directions parallel and perpendicular to the plane. By (*c*) and (*e*), $\Sigma F_x = 0 = Q_1 \frac{\sqrt{60}}{8} - 200 \sin 30°$. Hence $Q_1 = \frac{800}{\sqrt{60}} = 103.3$ lb. $\Sigma F_y = 0 = P - 200 \cos 30° - 103.3 \times {}^2/_8$. Hence $P = 199$ lb. (*d*) Consider the large cylinder as a free body. $Q_1 = Q_2 = 103.3$ lb. Use the same X and Y directions. $\Sigma F_x = 0 = S \cos 30° - 103.3 \times \frac{\sqrt{60}}{8} - 100 \sin 30°$. Hence, $S = 173.2$ lb. $\Sigma F_y = 0 = R - 100 \cos 30° - 173.2 \sin 30° + 103.3 \times {}^2/_8$. Hence $R = 147.4$ lb. GRAPHICAL SOLUTION. Discussions (*a*) and (*b*) are same as above. The free body is the small cylinder. The force system is 200 lb, *ab*, *bc*, and *ca*, and the polygon is the triangle ABC (Fig 110). $BC = 199$ lb, $CA = 103.3$ lb. Next, consider the large cylinder as a free body. The force system is *ac*, *cd*, *de* and *ea*. Plat the known forces AC and CD. From D draw DE parallel to *de*, and from A, AE parallel to *ae*; these lines intersect at E. $DE = 147.4$ lb, and $EA = 173.2$ lb are the magnitudes and senses of the two unknowns. If the system is concurrent and all the forces are known except one, the two unknown elements will be one angle and one magnitude and sense. Both may be determined by writing the equilibrium equations, or by drawing the force polygon.

Typical problem II. A SYSTEM OF COPLANAR PARALLEL FORCES is in equilibrium and all are known except two; the action lines of these two are known; required their magni-

tudes and senses. ALGEBRAIC SOLUTION. (*e*) Assume senses for both unknown forces. (*f*) Write a pair of equilibrium equations involving the unknown forces and solve for the two unknowns. A plus answer indicates the sense correctly assumed. A minus answer indicates incorrect assumption. As a check on the results apply a third equilibrium equation. For problems of this type the algebraic solution is preferable to the graphical. GRAPHICAL SOLUTION. (*g*) Letter the action lines of the wholly known forces and then the action lines of the two unknown forces. (*h*) On a line of indefinite length parallel to the forces, often called the load line, form the force polygon by plotting vectors of the known forces continuously to end of last known force. The vector of the first unknown force extends from this point to some unknown point on the load line. The unknown point is to be found. (*i*) Construct a funicular polygon. Draw first string between first known force and last-lettered unknown force. Continue funicular polygon to intersection of last known string with first-lettered unknown force. Draw closing string from this point, to intersection of first string with last-lettered unknown force. Draw a ray through pole parallel to closing string; its intersection with the load line is the point sought. (*j*) The senses must read confluently from the starting point back to the same point. The last two vectors represent the two unknown forces in magnitude and sense.

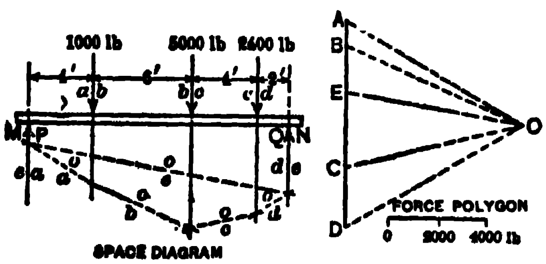


Fig 111

Example. A beam is loaded as shown in Fig 111 and supported at the points P and Q. (*a*) Determine the reactions of the supports. (*b*) Consider the beam as a free body. The external force system consists of the forces 1 000 lb, 5 000 lb, 2 400 lb, P, and Q. This is a coplanar parallel system and is typical problem II. ALGEBRAIC SOLUTION. (*c*) Assume senses for the reactions. By (*c*),

$$\Sigma M_P = 0 = -4 \times 1\,000 - 10 \times 5\,000 - 14 \times 2\,400 + 16\,Q = 0$$

Hence $Q = 5\,475$ lb; correct sense was assumed.

$$\Sigma M_Q = 0 = 2 \times 2\,400 + 6 \times 5\,000 + 12 \times 1\,000 - 16\,P = 0$$

Hence $P = 2\,925$ lb; correct sense assumed. As a check, apply condition, $\Sigma F = 0$.

$$\Sigma F = 0 = -5\,475 + 1\,000 + 5\,000 + 2\,400 - 2\,925 = 0$$

GRAPHICAL SOLUTION of the same problem involves the construction of a closed force polygon and a closed funicular polygon. Draw vectors, AB, BC, CD to represent 1 000 lb, 5 000 lb, 2 400 lb. Let DE represent Q and EA represent P. The problem is to locate point E. This is done with the aid of the funicular polygon. Draw the rays OA, OB, OC, and OD. Start on the action line of P and draw the string polygon. The closing line is oe. Draw the ray OE parallel to oe. The unknown reactions are DE and EA.

(Note.—This principle cannot be applied to beams having more than two points of support. Such problems require special treatment.)

Typical problem III. A SYSTEM OF COPLANAR NONCONCURRENT NONPARALLEL FORCES is in equilibrium, and all are known except two; the action line of one of these and a point in the action line of the other are known. Determine the magnitude and sense of the one, and the magnitude, sense, and angular direction of the other. ALGEBRAIC SOLUTION. (*a*) Assume sense for first unknown force. (*b*) Write a moment equation, center of moments being at given application point, to find first unknown force. (*c*) Assume sense and angular direction for second unknown force. (*d*) Write a pair of equilibrium equations involving the magnitude and angular direction of the second unknown force and solve for the two unknown quantities. A plus answer for the magnitude indicates the sense correctly assumed. A plus answer for the angular direction indicates it correctly assumed. A minus answer for either of the quantities indicates incorrect assumption for the one involved. Check results by means of an additional equilibrium equation. GRAPHICAL SOLUTION. (*e*) Letter the action lines of the wholly known forces, then the action line of the first unknown, and finally the force passing through the given application point. (*f*) Draw the force polygon for the known forces. Through the end of the last vector draw a line parallel to the action line of the first unknown force. Choose a pole and draw the rays. (*g*) Construct a funicular polygon, passing the first string through the application point of the second unknown force, and making the last, or closing, string pass through the same point. Draw a ray through the pole, parallel to closing string; its intersection with the vector drawn parallel to first unknown is last vertex in the force polygon. From this point, draw the closing line of the force polygon. (*h*) The senses

must read confluently from starting point back to same point. The last two vectors represent the two unknown forces.

Example. A roof truss is loaded as in Fig 112. The left end of the truss rests on a smooth horiz support. The right end is secured to a wall by means of a pin. Determine the reactions. (*a*) The external forces acting on the truss are the given loads, the left reaction P (vertical, on account of the smooth support), and the right reaction Q (inclined, through point M). (*b*) The unknown quantities are the reactions P and Q. This is typical problem III. ALGEBRAIC SOLUTION. (*c*) Assume P upward. By (*c*),

$$\Sigma M_M = 0 = 20\,000 \times 18 + 25\,000 \times 24 \cos 30° - 36\,P$$

hence, $P = 24\,430$ lb; correct sense assumed.

(*c'*) Assume Q upward to the left at angle θ with horizontal. By (*c*)

$$\Sigma F_x = 0 = 25\,000 \sin 30° - Q \cos \theta,$$
$$\Sigma F_y = 0 = 25\,000 \cos 30° + 20\,000 - 24\,430 - Q \sin \theta$$

Solving simultaneously, $Q = 21\,300$ lb, and $\theta = 54°$. Sense and direction were correctly assumed, hence Q acts upward to the left at 54° to the horiz. GRAPHICAL SOLUTION. (*d*) *ab* and *bc* are the action lines of the given loads, *cd* of the reaction P and *da* of the reaction Q. (*e*) Draw the vectors AB and BC, and a line through C, parallel to *cd*. Choose a pole and draw the rays. (*f*) Construct the funicular polygon, drawing *oa* through M, and draw closing string *od* from K to M. Draw OD through O to intersect CD at D. Draw DA. (*g*) Vectors CD and DA represent the two unknown forces, $P = 24\,430$ lb and $Q = 21\,300$ lb. The action line of Q is *da*, making angle with horiz = 54°.

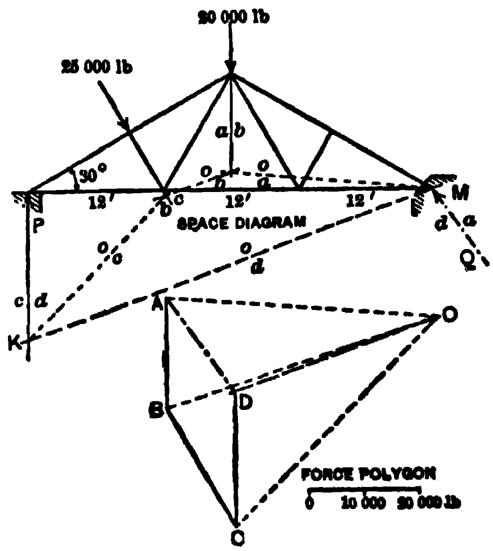


Fig 112

Typical problem IV. A SYSTEM OF COPLANAR NONCONCURRENT NONPARALLEL FORCES is in equilibrium and all except three are wholly known; the action lines of these three are known; required their magnitudes and senses. ALGEBRAIC SOLUTION. (*a*) Assume senses for the unknown forces. (*b*) To determine first unknown force, write a moment equation, center of moments at intersection of action lines of other unknown forces. (*c*) Write a pair of equilibrium equations involving the other two unknowns and solve for them. Check results by means of an additional equilibrium equation. GRAPHICAL SOLUTION. Assume any two of the unknown forces, replaced by their resultant at the point where their action lines intersect. This resultant, the third unknown force, and the known forces are in equilibrium and correspond to typical problem III. Proceed as specified in (*d*), (*e*), (*f*) and (*g*) for typical problem III, to determine the resultant and the third unknown force. (*h*) Resolve the resultant of the selected two unknown forces into components parallel to their action lines, thus determining their magnitudes and senses.

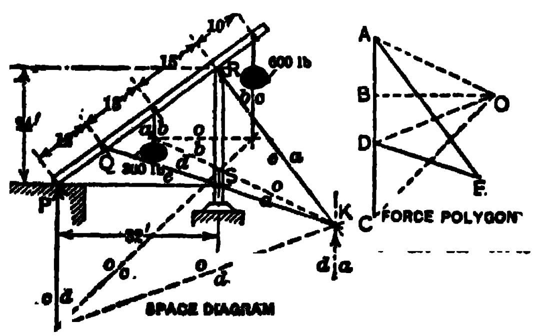


Fig 113

Example. The bar PQR (Fig 113), resting against a smooth floor at P and against a smooth post at R, is held by an inclined cord QS and carries two loads. Neglect the weight of the bar, and determine the forces acting on the bar at P and R, and the tension in the cord QS. (*a*) The external forces acting on the bar are the given loads, the reaction of the support at P (vertical on account of smooth surface), the reaction of the post at R (normal to the bar on account of smooth surface), and the pull of the cord at Q (direction is QS). (*b*) The unknown forces are the reactions P and R, and the tension Q. This is typical problem IV. ALGEBRAIC SOLUTION. (*c*) Assume P upward. By (*c*),

$$\Sigma M_K = 0 = 300 \times 35.6 + 600 \times 15.6 - P \times 55.6$$

Hence, $P = 360$ lb; correct sense assumed. Choose X and Y directions parallel and perpendicular to the bar. (*e*) Assume R upward to the left on bar, Q downward to right. By (*c*),

$$\Sigma F_x = 0 = 300 \times \frac{3}{5} + 600 \times \frac{3}{5} - 360 \times \frac{3}{5} - Q\frac{13.6}{23.5}$$

Hence $Q = 560$ lb; correct sense assumed.

$$\Sigma F_y = 0 = 300 \times \frac{4}{5} + 600 \times \frac{4}{5} + 560 \times \frac{19.2}{23.5} - 360 \times \frac{4}{5} - R$$

Hence $R = 890$ lb; correct sense assumed.

As a check, apply condition, $\Sigma M_Q = 0$: $\Sigma M_Q = 890 \times 28 - 360 \times {}^4/_5 \times 12 - 300 \times {}^4/_5 \times 13 - 600 \times {}^4/_5 \times 38 = 24\,900 - 24\,900 = 0$

GRAPHICAL SOLUTION. Assume Q and R replaced by their resultant. Its action line passes through point K. (*f*) *ab* and *bc* are the action lines of the given loads, *cd* of the reaction P, and *da* of the resultant of Q and R. (*g*), (*i*), and (*h*). Draw force and funicular polygons to determine the unknown vectors CD and DA. CD represents reaction $P = 360$ lb. (*j*) Resolve DA into components DE and EA, parallel to action lines *de* and *ea* of Q and R. (*h*) Vectors DE and EA represent the other unknown forces, $Q = 560$ lb, and $R = 890$ lb.

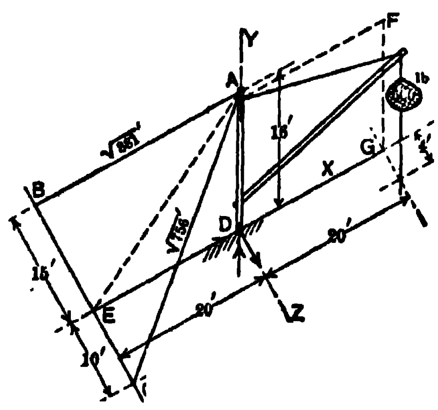


Fig 114

General noncoplanar system. Example. The crane (Fig 114), supported by a socket at the foot of the post at D, is kept from overturning by the back-stays AB and AC, and carries a load of 600 lb. (E, A, F, G, D, are in the vertical XY-plane.) Determine the axial components of the reaction on the post at D and the tensions in the back-stays. (*b*) The external forces acting on the post are the load, the reaction at D, and the tensions in the back-stays at A. The system of forces is noncoplanar, nonconcurrent, nonparallel. Moment equations are the most convenient to apply for this solution. By (*c*),

$$\Sigma M_{BC} = 0;\ D_y \quad \frac{600 \times 40}{20} \quad 1\,200 \text{ lb.}$$

$$\Sigma M_{Z_A} = 0;\ D_x \quad \frac{600 \times 20}{16} = 750 \text{ lb.}$$

$$\Sigma M_{X_A} = 0;\ D_z \quad \frac{600 \times 4}{16} = 150 \text{ lb.}$$

$$\Sigma M_{X_C} = 0 = AB \times \frac{16}{\sqrt{881}} \times 25 - 1\,200 \times 10 + 600 \times 6, \quad AB = 624 \text{ lb.}$$

$$\Sigma M_{X_B} = 0 = AC \times \frac{16}{\sqrt{756}} \times 25 + 600 \times 19 - 1\,200 \times 15; \quad AC = 454 \text{ lb.}$$

FRICTION

34. DEFINITIONS

A perfectly smooth surface is one that offers no resistance to the sliding of a body upon it. The force exerted by such a surface is normal to the surface of contact. If the surface is rough, the resultant thrust of the surface is oblique, and is called the TOTAL REACTION. The tangential component of the total reaction is called FRICTION and the normal component is called NORMAL REACTION. Friction is a passive resistance, and is developed as it is needed to prevent motion. For friction to exist there must be a tendency for a body to slip over a surface, due to the fact that the resultant of all the applied forces acting on the body (omitting total reaction of the surface) has a component parallel to the surface. If the surface is smooth, such a component causes motion of the body. If the surface is rough, it exerts a frictional force upon the body which neutralises the tendency to move. For any given conditions, there is a limit to the amount of friction, reached when slipping

impends, called LIMITING FRICTION. When the limiting friction is attained, the angle between the total reaction and the normal to the surface is the ANGLE OF FRICTION. In case there is no slipping, friction is called STATIC, and may have any magnitude from zero to the limiting friction, depending upon the tendency to move. If this tendency to move exceeds limiting friction, the body slips and KINETIC FRICTION still opposes motion; its value is less than limiting static friction.

Coefficient of friction is the ratio of the limiting friction to the corresponding normal reaction. If the angle of friction be ϕ, the normal reaction N, the limiting friction F', and the coeff of friction f, then $f = \tan\phi = F' \div N$. If a body be placed upon an inclined plane, the angle of inclination for which slipping of the body impends is the ANGLE OF REPOSE for the two rubbing surfaces. It is equal to the angle of friction.

35. FRICTION CONE

Consider a body, resting on a rough plane, to be acted upon by several forces (Fig 115). Draw the normal OP to the surface at the point where the resultant of all the applied forces, except the total reaction, cuts the surface; then draw OQ, making the angle ϕ with the normal equal to the angle of friction. The cone generated by revolving OQ about the axis OP, is the CONE OF FRICTION. The body will not slip if the resultant of all applied forces lies within this cone; it will slip if the resultant lies outside.

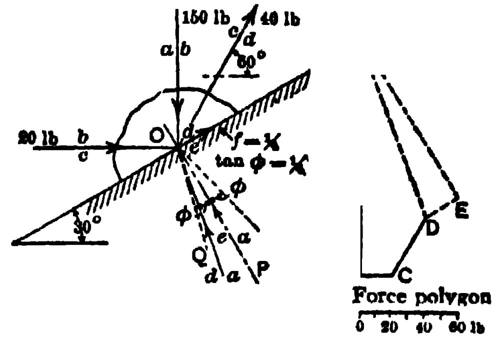


Fig 115

Assuming the data shown in Fig 115, a graphic solution is as follows: Draw a force polygon for the applied forces, $ABCD$. The resultant is AD, in magnitude and direction. Its action line ad falls inside the cone of friction, hence slipping will not occur. Total reaction of the plane is DA, opposite in sense to the resultant. Its components, parallel and normal to the surface, are DE and EA. Friction on the body is $DE = 23$ lb, upward along the surface, and the normal reaction $EA = 120$ lb, upward normal to the surface.

From the algebraic viewpoint, the normal reaction on the surface is

$$N = 150 \cos 30° + 20 \sin 30° - 40 \sin 30° = 120 \text{ lb}$$

Available or limiting friction $= fN = 120 \div 4 = 30$ lb. Sum of the force components parallel to the plane $= 150 \sin 30° - 20 \cos 30° - 40 \cos 30° = 23$ lb downward. If needed, the surface can supply 30 lb of friction; it actually does supply 23 lb, acting upwards on the body to preserve equilibrium.

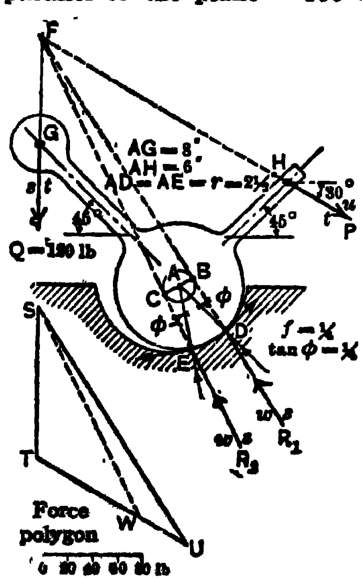


Fig 116

36. FRICTION CIRCLE

The reaction of a bearing on a journal is usually assumed to pass through the center of the journal. If the bearing is worn (as shown much exaggerated in Fig 116, for the journal A), the action line of the reaction still passes through center A if the surfaces are smooth. Otherwise, when slipping impends, the action line makes the friction angle ϕ with the normal AD to the contact surfaces on the line of contact between the journal and the bearing. If motion impends for clockwise rotation of the journal, DB is the action line; if for counterclockwise rotation, it is EC. If r be the journal radius, the perpendicular to the action lines from the journal center is AB or $AC = r \sin\phi$. For a given journal and bearing, this value is constant and may be represented by the radius of a circle with center at A. This is the FRICTION CIRCLE, and the bearing reaction is always tangent to it if slipping impends. For practical conditions, angle ϕ is so small that $r \tan\phi = r \sin\phi$ (approx). Radius of the friction circle may be taken as $r \times f$, nearly.

In a given problem, it is often impossible to observe D, the line of contact between

journal and bearing, but the action line of the bearing reaction must be tangent to the friction circle. Other known conditions may fix another point in this action line so that its position is readily found.

In Fig 116, with data as given, find what force P is needed to overcome Q (motion impending for clockwise rotation). The journal is under the action of 3 forces, P, Q, and R, concurrent at F. As R must be tangent to the friction circle, its action line is either FD or FE. The correct position is determined by resolving R into tangential and normal components, and making the choice so that friction opposes impending motion. The force polygon for the 3 forces is STU; then $R_1 = US = 226$ lb, and $P = TU = 142$ lb.

The greatest force P which Q can overcome (motion impending for counterclockwise rotation), is found from the triangle STW, corresponding to R_2; then $R_2 = WS = 188$ lb, and $P = TW = 96$ lb.

37. COEFFICIENTS OF FRICTION

Coefficients of static friction depend upon character of the surfaces, kind of material and lubricant used. Recent experiments prove that the coeff of static friction is not independent of the mormal pressure on the contact surfaces, nor of the time of contact between the surfaces.

Coefficients of Static Friction (Compiled from Rankine)

Dry masonry and brickwork.	0.6 to 0.7	Masonry on moist clay	0.33
Masonry and brickwork with damp mortar.......... .	0.74	Earth on earth	0.25 to 1.00
Timber on stone.......about	0.4	" " " dry sand, clay, and mixed earth..........	0.38 to 0.75
Iron on stone	0.3 to 0.7	Earth on earth, damp clay..	1.00
Timber on timber..........	0.2 to 0.5	" " " wet clay.....	0.31
" " metals	0.2 to 0.6	" " " shingle and gravel	0.81 to 1.11
Metals on metals	0.15 to 0.25		
Masonry on dry clay	0.51		

Coefficients of kinetic friction depend upon the velocity of the body in motion, and particularly upon the kind of lubricant. If a lubricant is used it has a greater influence upon the coeff of friction than has the kind of material in the bodies.

Coefficients of Kinetic Friction (Rough averages, compiled from Rankine)

Wood on wood, dry........	0.25 to 0.5	Leather on metals, greasy...	0.23
" " " soaped.....	0.20 to 0.04	" " " oily.....	0.15
Metals on oak, dry.........	0.50 to 0.60	Metals on metals, dry......	0.15 to 0.20
" " " wet.........	0.24 to 0.26	" " " wet......	0.30
" " " soaped......	0.20	Smooth surfaces, occasionally greased	0.07 to 0.08
" " elm, dry........	0.20 to 0.25	Smooth surfaces, continuously greased	0.05
Hemp on oak, dry...... ...	0.53	Smooth surfaces, best results	0.03 to 0.036
" " " wet.........	0.33	Bronze on lignum vitæ, constantly wet.............	0.05
Leather on oak............	0.27 to 0.38		
" " metals, dry.. ..	0.56		
" " " wet.....	0.36		

38. BELT OR COIL FRICTION

If a band is placed about a rough cylinder and a tensile force applied to each end, these forces may be very unequal without causing slipping of the band upon the cylinder, and are unequal even if slipping occurs. The band may be a belt, rope, or brake band, while the cylinder may be a pulley or sheave, post, capstan, or similar device.

Let T_1 be the larger of the forces acting upon the band, T_2 the smaller one, corresponding to the tensions in the tight and loose sides of a belt; and W the friction between band and cylinder. Then, $W = T_1 - T_2$. Let the arc covered by the band subtend an angle α at the center of the cylinder, α radians $= (\alpha \text{ degrees} \times \pi) \div 180$; e $(=2.718)$ is the base of the Naperian system of logarithms, and f the coeff of friction, care being taken to select a suitable value for either static or kinetic friction, depending on whether or not slipping occurs. Then, $T_1 = T_2 e^{f\alpha} = T_2\, 10^{\frac{\pi}{180}(0.4343) f\alpha^\circ}$. The first α is expressed in radians, the last α in degrees. For belting, the coeff of friction is often taken as 0.30.

Values of $e^{f\alpha}$ (slipping impending)

Angle of contact in decimal parts of circum = α radians ÷ 2π	Values of f (coeff of friction)								
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.1	1.06	1.1	1.13	1.17	1.21	1.25	1.29	1.33	1.37
0.2	1.13	1.21	1.29	1.37	1.46	1.55	1.65	1.76	1.87
0.3	1.21	1.32	1.45	1.60	1.76	1.93	2.13	2.34	2.57
0.4	1.29	1.46	1.65	1.87	2.12	2.41	2.73	3.10	3.51
0.425	1.31	1.49	1.70	1.95	2.23	2.55	2.91	3.33	3.80
0.45	1.33	1.53	1.76	2.03	2.34	2.69	3.10	3.57	4.11
0.475	1.35	1.56	1.82	2.11	2.45	2.84	3.30	3.83	4.45
0.5	1.37	1.60	1.87	2.19	2.57	3.00	3.51	4.11	4.81
0.525	1.39	1.64	1.93	2.28	2.69	3.17	3.74	4.41	5.20
0.55	1.41	1.68	2.00	2.37	2.82	3.35	3.98	4.74	5.63
0.6	1.46	1.76	2.13	2.57	3.10	3.74	4.52	5.45	6.59
0.7	1.52	1.93	2.41	3.00	3.74	4.66	5.81	7.24	9.02
0.8	1.65	2.13	2.73	3.51	4.52	5.81	7.47	9.60	12.35
0.9	1.76	2.34	3.10	4.11	5.45	7.24	9.60	12.74	16.90
1.0	1.87	2.57	3.51	4.81	6.59	9.02	12.35	16.90	23.14
1.5	2.57	4.11	6.59	10.55	16.90	27.08	43.38	69.49	111.32
2.0	3.51	6.59	12.35	23.14	43.38	81.31	152.40	285.68	535.49
2.5	4.81	10.55	23.14	50.75	111.32	244.15	535.49	1 174.5	2 575.90
3.0	6.59	16.90	43.38	111.32	285.68	733.14	1 881.5	4 828.5	12 391.0
3.5	9.02	27.08	81.31	244.15	733.14	2 199.90	6 610.7	19 851.0	59 608.0
4.0	12.35	43.38	152.40	535.49	1 881.5	6 610.7	23 227.0	81 610.0	286 744.0

CENTERS OF GRAVITY

39. CENTROIDS AND CENTERS OF GRAVITY

Centroid of a system of parallel forces with fixed application points is the point through which their resultant always passes, no matter how the forces may be turned (but still remaining parallel). The force of gravitation acting on the particles of a body constitutes a system of forces practically parallel; and the centroid of these forces is the CENTER OF GRAVITY of the body. Referring the application points of such a force system to a set of coordinate axes, the coordinates of the centroid, or center of gravity, are given by:

$$\bar{x} = (\Sigma F\cdot x) \div \Sigma F,\ \bar{y} = (\Sigma F\cdot y) \div \Sigma F,\ \bar{z} = (\Sigma F\cdot z) \div \Sigma F$$

In these equations F represents one force (or the weight of one particle), and x the distance of its application point from the YZ-plane.

If the body is composed of parts of known weight, and the positions of their centers of gravity are known, the coordinates of the center of gravity of the whole body are:

$$\bar{x} = \frac{w_1\bar{x}_1 + w_2\bar{x}_2 + w_3\bar{x}_3 + \cdots}{w_1 + w_2 + w_3 + \cdots}$$
$$\bar{y} = \frac{w_1\bar{y}_1 + w_2\bar{y}_2 + w_3\bar{y}_3 + \cdots}{w_1 + w_2 + w_3 + \cdots} \qquad (A)$$
$$\bar{z} = \frac{w_1\bar{z}_1 + w_2\bar{z}_2 + w_3\bar{z}_3 + \cdots}{w_1 + w_2 + w_3 + \cdots}$$

in which w_1 represents the weight of one part and $\bar{x}_1$, $\bar{y}_1$, $\bar{z}_1$ are the coordinates of its center of gravity. Any one of the above expressions may be read as, "the sum of the moments divided by the sum of the forces." If in formulas (A) the weights are replaced by volumes, areas, surfaces, or lengths of lines, they determine the coordinates of the centroids, or centers of gravity, of the volumes, areas, surfaces, or lines. These centroids are often called the centers of gravity of the corresponding magnitudes.

Center of gravity of part of a body remaining after certain parts have been taken away is determined by the rule: The moment of the remainder of a body, with respect to any plane, equals the moment of the whole body minus the moments of the parts taken away.

40. FORMULAS

Centroids of lines and areas. CIRCULAR ARC (Fig 117). Centroid is on the axis of symmetry; its distance from the center is $\bar{x} = \frac{r \times c}{s} = \frac{r \sin \alpha}{\alpha}$, α being in radians. For a semicircle, $\bar{x} = 2\,r \div \pi$; for a quadrant, $\bar{x} = 2\,r\sqrt{2} \div \pi$. TRIANGLE. Centroid of a

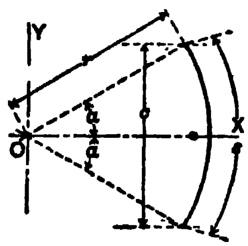


Fig 117

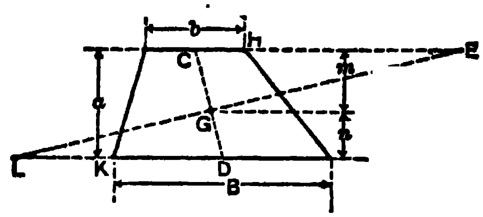


Fig 118

triangle is at the intersection of the medians; its perpendicular distance from any side equals one-third the altitude measured from that side. TRAPEZOID (Fig 118). Centroid lies on the median CD (joining the middle of the parallel sides), at a point G located by following equations:

$$m = \frac{a\,(2\,B + b)}{3\,(B + b)}\;;\; n = \frac{a\,(B + 2\,b)}{3\,(B + b)}$$

To locate the centroid graphically, make $HE = B$, $KL = b$. Draw LE, and the median CD; they intersect at G, the centroid. QUADRILATERAL (Fig 119). Divide each side of the quadrilateral $ABCD$ into thirds, and through the third points draw ef, hk, lm, and no. These lines form a parallelogram, the diagonals of which intersect at G, the centroid of the quadrilateral. SECTOR OF A CIRCLE (Fig 120). Centroid is on the axis of symmetry; its distance from the center is $\bar{x} = (2\,rc) \div (3\,s)$. For a semicircle, $\bar{x} = 4\,r \div 3\,\pi$. For a quadrant $\bar{x} = 4\,r\,\sqrt{2} \div 3\,\pi$; and the distance from each bounding radius $= 4\,r \div 3\,\pi$. SEGMENT OF A CIRCLE (Fig 121). Centroid is on the axis of symmetry; its distance from the center is $\bar{x} = (c^3 \div 12\,A)$, in which A is the area of the segment $= \frac{r^2}{2}(2\,\alpha - \sin 2\,\alpha)$, α being in radians.

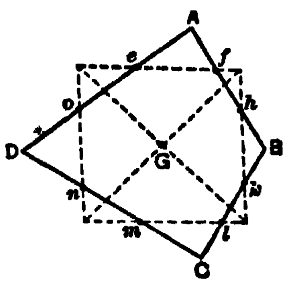


Fig 119

Centers of gravity of volumes. RIGHT CIRCULAR CYLINDER (Fig 122). Base is normal to the axis; top makes an angle α with base; mean height is h; radius of base $= r$. If

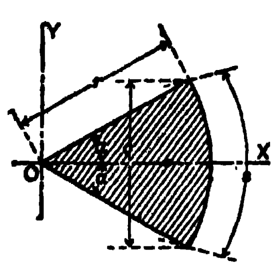


Fig 120

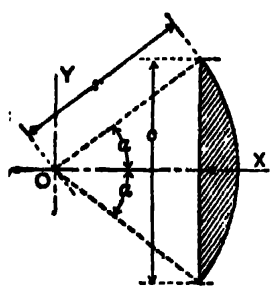


Fig 121

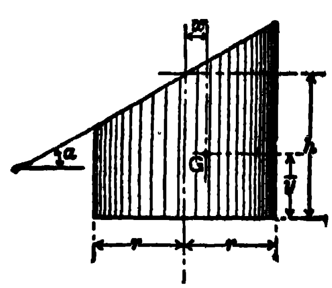


Fig 122

$\bar{y}$ is the distance of the center of gravity G from the base, and $\bar{x}$ is the distance to the right of the vertical center line,

$$\bar{x} = \frac{r^2 \tan \alpha}{4\,h}, \text{ and } \bar{y} = \frac{h}{2} + \frac{r^2 \tan^2 \alpha}{8\,h}$$

FRUSTUM OF A CIRCULAR CONE. Let R = radius of the larger base and r radius of the smaller base; a = the altitude. Distance of the centroid of the curved surface from large base is $\frac{a\,(R + 2r)}{3\,(R + r)}$ and from the smaller base $\frac{a\,(2\,R + r)}{3\,(R + r)}$. Distance from the center of gravity of the solid frustum to larger base is $\frac{a\,(R^2 + 2\,Rr + 3\,r^2)}{4\,(R^2 + Rr + r^2)}$. CONE AND PYRAMID.

Centroid of the surface (base excluded) is on a line joining the apex with the centroid of the perimeter of the base, at a distance of two-thirds its length from the apex. Center of gravity of the solid cone or pyramid is on the line joining the apex with the centroid of the base, at a distance of three-fourths its length from the apex. FRUSTUM OF A PYRAMID. If the frustum has regular bases, let R and r be lengths of the sides of the larger and smaller bases, and h the altitude. Distance of the centroid of the surface (bases excluded) from the larger base is $\dfrac{h\,(R + 2\,r)}{3\,(R + r)}$. For any bases, let A and a be the areas of large and small bases and h the altitude; the distance of center of gravity from the larger base is $\dfrac{h\,(A + 2\sqrt{Aa} + 3\,a)}{4\,(A + \sqrt{Aa} + a)}$. SPHERE. Centroid of the surface of any zone of a sphere is midway between the bases. Distance of the center of gravity of a solid segment from the base is $\dfrac{h\,(4\,r - h)}{4\,(3\,r - h)}$, where r = radius of the sphere, and h = height of the segment. For a hemisphere this distance is $3\,r \div 8$. Distance of center of gravity of a solid sector from center of the sphere $= {}^3/_8\,(1 + \cos\alpha)\,r = {}^3/_8\,(2\,r - h)$, where α = half the angle subtended by the sector. ELLIPSOID. Choose 3 coordinate axes x, y, and z; let a, b, and c denote the semi-lengths of the corresponding axes of the ellipsoid; the coordinates of the center of gravity of one octant of the solid are $\bar{x} = 3\,a \div 8$, $\bar{y} = 3\,b \div 8$, and $\bar{z} = 3\,c \div 8$.

MOMENTS OF INERTIA OF AREAS AND MASSES

41. AREAS

Moment of inertia of a plane area with respect to a given line is the sum of the products obtained by multiplying each element of area by the square of its distance from the line. The moment is called polar if the axis is perpendicular to the plane area. The common symbol for moment of inertia is I; and a subscript denotes the axis to which it refers. Let I = the moment of inertia about any axis; A the area of the figure and r the distance of any element of area from the given axis, then $I = \int r^2\,dA$. RADIUS OF GYRATION k, of any plane figure, is the square root of the moment of inertia divided by the area of the figure, thus $k = \sqrt{\dfrac{I}{A}}$, or $I = Ak^2$. PARALLEL AXIS THEOREM. $I = \bar{I} + Ad^2$, in which $\bar{I}$ is the moment of inertia with respect to an axis through the centroid and I is moment of inertia about a parallel axis at distance d.

Product of inertia J, of an area with respect to a pair of rectangular axes, is the sum of the products obtained by multiplying each element of area by its coordinates. Thus $J_{xy} = \int xy\,dA$. Product of inertia may be positive, negative, or zero, depending on the distribution of the area with respect to the axes. Moment of inertia is always positive and never zero. PARALLEL AXIS THEOREM. $J = \bar{J} + A\bar{x}\bar{y}$, in which $\bar{J}$ is product of inertia of area A with respect to a pair of centroidal axes, J is the product of inertia with respect to a set of parallel axes, and $\bar{x}$ and $\bar{y}$ are the distances from the parallel axes to the centroid of the area. For a rectangle, $J_{xy} = A^2 \div 4$; for a triangle, $J_{xy} = A^2 \div 8$ (Fig 123). J is zero for an axis of symmetry and any line perpendicular to that axis. The customary unit for both moment and product of inertia is biquadratic, inches = in^4.

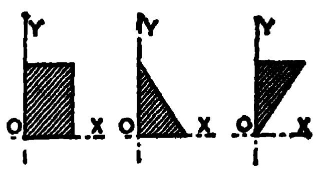


Fig 123

The principal axes of inertia at a point are those about which the moments of inertia are the greatest and the least; these axes will always be rectangular and the product of inertia for them is zero.

42. OBLIQUE AXES AND LEAST RADIUS OF GYRATION

Oblique axes. Let I_x, I_y, and J_{xy} represent moments and products of inertia for a pair of rectangular axes at a point of an area, I_u the moment of inertia for any oblique axis through that point and α its angle with the X axis; then, $I_u = I_x \cos^2\alpha + I_y \sin^2\alpha - J_{xy} \sin 2\,\alpha$. For principal axes, $\tan 2\,\alpha = 2\,J_{xy} \div (I_y - I_x)$.

The equation for I_u is interpreted by the inertia circle as follows: Suppose I_x, I_y, and J_{xy} given for the shaded area in Fig 124. To convenient scale, plot OX' and OY' to represent I_x and I_y, and $Y'A$ to represent J_{xy} (downward if negative and upward if positive). Center C is midway between X' and Y'. With CA as radius, describe inertia circle. To find I_u, draw chord AB parallel to axis OU; draw perpendicular BU'. OU' (to scale) $= I_u$, and BU' (to scale) $= J_{uv}$. OM, parallel to A 2, is axis of least I; and a parallel to A 1, through O, is axis of greatest I. O 2 (to scale) is the value of least $I = I_2$; and O 1, value of greatest $I = I_1$. Least radius of gyration is measured perpendicular to axis OM and $= \sqrt{I_2 \div \text{area}}$. If principal moments of inertia (greatest and least) at a point are equal, ($I_1 = I_2$), then I for any oblique axis through the point has same value; that is, $I_u = I_1 = I_2$.

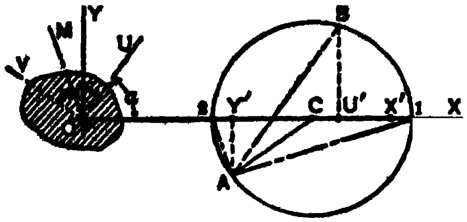


Fig 124

43. TABLE OF PLANE FIGURES. I = Moment of Inertia. k = Radius of Gyration

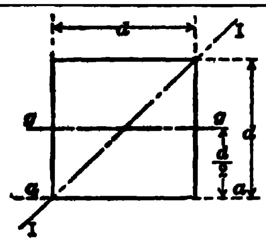


Fig 125. Square

$$I_g = \frac{d^4}{12} = I_1 \qquad I_a = \frac{d^4}{3}$$

$$k_g = \frac{d}{\sqrt{12}} = 0.289\,d = k_1 \qquad k_a = \frac{d}{\sqrt{3}} = 0.577\,d$$

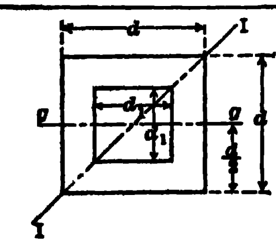


Fig 126. Hollow Square

$$I_g = \frac{d^4 - d_1^4}{12} = I_1$$

$$k_g = \sqrt{\frac{d^2 + d_1^2}{12}} = k_1$$

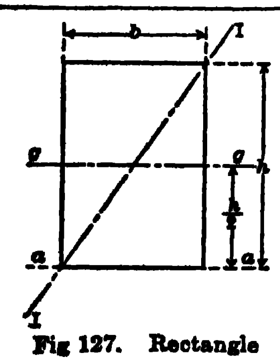


Fig 127. Rectangle

$$I_g = \frac{bh^3}{12} \qquad I_a = \frac{bh^3}{3} \qquad I_1 = \frac{b^3h^3}{6\,(b^2 + h^2)}$$

$$k_g = \frac{h}{\sqrt{12}} = 0.289\,h \qquad k_a = \frac{h}{\sqrt{3}} = 0.577\,h$$

$$k_1 = \frac{bh}{\sqrt{6\,(b^2 + h^2)}}$$

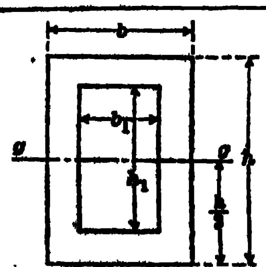


Fig 128. Hollow Rectangle

$$I_g = \frac{bh^3 - b_1h_1^3}{12}$$

$$k_g = \sqrt{\frac{bh^3 - b_1h_1^3}{12\,(bh - b_1h_1)}}$$

Table of Plane Figures *(Continued)*

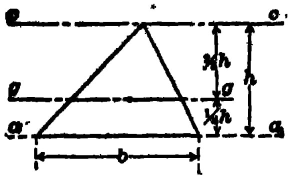


Fig 129. Triangle

$$I_g = \frac{bh^3}{36} \qquad I_a = \frac{bh^3}{12} \qquad I_c = \frac{bh^3}{4}$$

$$k_g = \frac{h}{\sqrt{18}} = 0.236\,h \qquad k_a = \frac{h}{\sqrt{6}} = 0.408\,h$$

$$k_c = \frac{h}{\sqrt{2}} = 0.707\,h$$

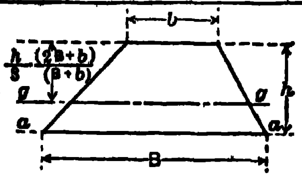


Fig 130. Trapezoid

$$I_g = \frac{h^3\,(B^2 + 4\,Bb + b^2)}{36\,(B + b)} \qquad I_a = \frac{h^3\,(B + 3\,b)}{12}$$

$$k_g = \frac{h}{6\,(B + b)}\sqrt{2\,(B^2 + 4\,Bb + b^2)}$$

$$k_a = \frac{h}{\sqrt{6}}\sqrt{\frac{(B + 3\,b)}{(B + b)}}$$

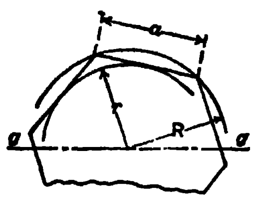


Fig 131. Any Complete Regular Polygon

A = Area of polygon

g–g is any axis through center and in plane of polygon

Axis p–p is perpendicular to plane of polygon at center

$$I_g = \frac{A}{24}\,(6\,R^2 - a^2) = \frac{A}{48}\,(12\,r^2 + a^2)$$

$$I_p = \frac{A}{12}\,(6\,R^2 - a^2) = \frac{A}{24}\,(12\,r^2 + a^2)$$

$$k_g = \sqrt{\frac{(6\,R^2 - a^2)}{24}} = \sqrt{\frac{(12\,r^2 + a^2)}{48}}$$

$$k_p = \sqrt{\frac{(6\,R^2 - a^2)}{12}} = \sqrt{\frac{(12\,r^2 + a^2)}{24}}$$

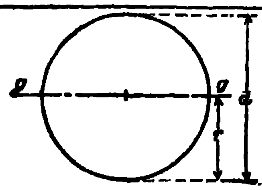


Fig 132. Circle

Axis p–p is perpendicular to circle at center

$$I_g = \frac{\pi d^4}{64} = \frac{\pi r^4}{4} \qquad I_p = \frac{\pi d^4}{32} = \frac{\pi r^4}{2}$$

$$k_g = \frac{d}{4} = \frac{r}{2} \qquad k_p = \frac{d}{\sqrt{8}} = \frac{r}{\sqrt{2}}$$

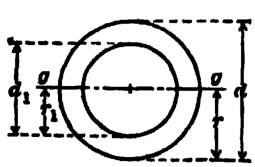


Fig 133. Hollow Circle

Axis p–p is perpendicular to hollow circle at center

$$I_g = \frac{\pi}{64}\,(d^4 - d_1^4) = \frac{\pi}{4}\,(r^4 - r_1^4)$$

$$I_p = \frac{\pi}{32}\,(d^4 - d_1^4) = \frac{\pi}{2}\,(r^4 - r_1^4)$$

$$k_g = 1/_4\sqrt{d^2 + d_1^2} = 1/_2\sqrt{r^2 + r_1^2}$$

$$k_p = \sqrt{\frac{(d^2 + d_1^2)}{8}} = \sqrt{\frac{(r^2 + r_1^2)}{2}}$$

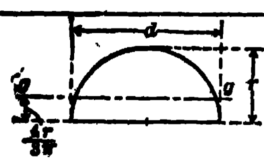


Fig 134. Semicircle

$$I_g = \frac{d^4\,(9\,\pi^2 - 64)}{1\,152\pi} = 0.00686\,d^4 = 0.110\,r^4$$

$$k_g = \frac{d\sqrt{(9\,\pi^2 - 64)}}{12\pi} = 0.132\,d = 0.264\,r$$

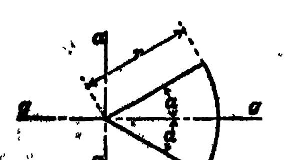


Fig 135. Circular Sector

A = Area of sector = $r^2\alpha$, α expressed in radians

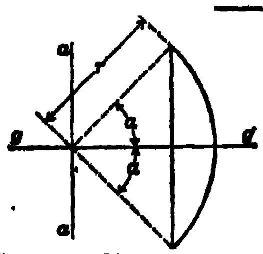
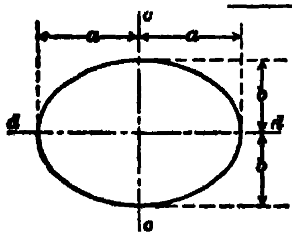
$$I_g = \frac{Ar^2}{4}\left(1 - \frac{\sin\alpha\cdot\cos\alpha}{\alpha}\right)$$

$$I_a = \frac{Ar^2}{4}\left(1 + \frac{\sin\alpha\cdot\cos\alpha}{\alpha}\right)$$

$$k_g = \frac{r}{2}\sqrt{1 - \frac{\sin\alpha\cdot\cos\alpha}{\alpha}}$$

$$k_a = \frac{r}{2}\sqrt{1 + \frac{\sin\alpha\cdot\cos\alpha}{\alpha}}$$

Table of Plane Figures *(Continued)*

 Fig 136. Circular Segment	A = Area of segment $= r^2(\alpha - \sin\alpha\cdot\cos\alpha)$, α being expressed in radians $I_g = \frac{Ar^2}{4}\left(1 - \frac{2}{3}\,\frac{\sin^3\alpha\cdot\cos\alpha}{\alpha - \sin\alpha\cdot\cos\alpha}\right)$ $I_a = \frac{Ar^2}{4}\left(1 + \frac{2\sin^3\alpha\cdot\cos\alpha}{\alpha - \sin\alpha\cdot\cos\alpha}\right)$
 Fig 137. Ellipse	$I_d = \frac{\pi}{4}ab^3$ $\quad I_c = \frac{\pi}{4}ba^3$ $k_d = \frac{b}{2}$ $\quad k_c = \frac{a}{2}$

44. MASSES

Moment of inertia of a mass, with respect to an axis, is the algebraic sum of the products of each element of mass and the square of its distance from the axis. $I = \Sigma mr^2$, in which m is any element of mass and r its distance from the axis. The common engineer's unit is slug-ft^2 (Art 53).

Radius of gyration is the distance from the axis to a point at which, if the entire mass were concentrated, the moment of inertia would not be changed. If k is the radius of gyration, and M the total mass, then $I = Mk^2$; or $k = \sqrt{I \div M}$, and is measured in ft. Value of k is always between the least and greatest values of r.

Parallel axis theorem. Let $\bar{I}$ be the moment of inertia with respect to an axis through the center of gravity, and I the moment of inertia for any parallel axis at a distance d; then $I = \bar{I} + Md^2$, and $k^2 = \bar{k}^2 + d^2$.

Principal axes and moments of inertia. The integrals $\int yz\,dm$, $\int zx\,dm$, and $\int xy\,dm$ are called products of inertia. For an oblique axis, through the origin with direction angles, α, β, and γ,

$$I = I_x\cos^2\alpha + I_y\cos^2\beta + I_z\cos^2\gamma - 2J_{xy}\cos\alpha\cos\beta - 2J_{yz}\cos\beta\cos\gamma - 2I_{zx}\cos\gamma\cos\alpha.$$

At each origin, there is one set of rectangular axes for which the three products of inertia vanish, and with respect to two of which the moments of inertia are least and greatest of all moments of inertia with respect to axes at that origin. The axes are called principal axes, and the moments of inertia with respect to them are called principal moments of inertia. If a body has three planes of symmetry, at right angles to each other, their lines of intersection are principal axes at the point of intersection. If a body has two planes of symmetry, their intersection is a principal axis for every point on it as origin, and the other two axes are in the planes of symmetry. If a body has one plane of symmetry, any perpendicular to the plane is a principal axis at the point where the axis pierces the plane. At the origin, the x-axis is a principal axis if $J_{xy} = 0$ and $J_{zx} = 0$; the y-axis is a principal axis if $J_{xy} = 0$ and $J_y = 0$; and the z-axis is a principal axis if $J_{zx} = 0$ and $J_{yz} = 0$.

45. FORMULAS FOR MASSES

Thin circular lamina, of radius a and mass m, with respect to a diameter, $I = ma^2 \div 4$. For axis perpendicular to face of lamina at its center, $I = ma^2 \div 2$.

Solid of revolution of volume V and mass M, with respect to its axis,

$$I = \left[\pi M\int y^4\,dx\right] \div 2V$$

With respect to an axis at right angles to its geometrical axis,

$$I = \left[\pi M\int\left(\frac{y^4}{4} + x^2y^2\right)dx\right] \div V$$

Thin straight rod of length l and mass M. Axis perpendicular to rod at center, $I = Ml^2 \div 12$; at end, $I = Ml^2 \div 3$. Axis inclined to rod at angle α: at center, $I = Ml^2 \sin^2 \alpha \div 12$; at end, $I = Ml^2 \sin^2 \alpha \div 3$.

Rectangular prism of dimensions a, b, c. Axis through center of gravity, parallel to edge c, $I = M\ (a^2 + b^2) \div 12$. Axis through center of base bc and parallel to edge c, $I = M\ (4\ a^2 + b^2) \div 12$.

Right circular cylinder of length h and base radius a; for its geometrical axis, $I = Ma^2 \div 2$. Axis through center of gravity, parallel to base, $I = M\ (3\ a^2 + h^2) \div 12$. Axis coinciding with diameter of base, $I = M\ (3\ a^2 + 4\ h^2) \div 12$.

Hollow cylinder, of height h and radii R and r, about axis of cylinder, $I = M\ (R^2 + r^2) \div 2$. Axis through center of gravity, perpendicular to axis of cylinder, $I = M\ (3\ R^2 + 3\ r^2 + h^2) \div 12$.

Elliptical cylinder, length h and semi-axes a and b; for its geometrical axis, $I = M\ (a^2 + b^2) \div 4$. Axis through center of gravity, parallel to minor axis $2\ b$, of elliptical section, $I = M\ (3\ a^2 + h^2) \div 12$.

Sphere of radius a, about a diameter, $I = 2\ Ma^2 \div 5$.

Right circular cone, of height h and base radius a. About its axis, $I = 3\ Ma^2 \div 10$. Axis through vertex, perpendicular to geometrical axis, $I = 3\ M\ (a^2 + 4\ h^2) \div 20$.

Ellipsoid of axes $2\ a$, $2\ b$, and $2\ c$. About the axis $2\ b$, $I = M\ (a^2 + c^2) \div 5$.

Right rectangular pyramid, of height h and sides of base a and b, about geometrical axis, $I = M\ (a^2 + b^2) \div 20$. Axis through vertex perpendicular to geometrical axis and parallel to b side of base, $I = M\ (a^2 + 12\ h^2) \div 20$. Axis through center of gravity, parallel to b edge of base, $I = M\ (4\ a^2 + 3\ h^2) \div 80$.

Circular ring of circular section, radius of section $= r$, and center of section at distance R from axis of ring. About axis of ring, $I = M\ (4\ R^2 + 3\ r^2) \div 4$. About axis through center of gravity, perpendicular to axis of ring, $I = M\ (4\ R^2 + 5\ r^2) \div 8$.

KINEMATICS

46. RECTILINEAR MOTION

A particle is in motion with respect to surrounding objects if its position with respect to them is continually changing. If A is the position of a particle at a time t_1, and B its position at a later time t_2, its displacement in the time interval $t_2 - t_1 = \Delta t$ is the vector AB, no matter whether path is straight or curved. Velocity of a particle is its time rate of displacement. Acceleration of a particle is its time rate of change of velocity.

Rectilinear motion of a particle. Let s be distance measured along the path of a particle, s_1 the distance from origin at time t_1, s_2 the distance at a later time t_2, $\Delta s = s_2 - s_1$ = displacement in time interval $\Delta t = t_2 - t_1$. Then AVERAGE VELOCITY $= \Delta s \div \Delta t$. If the position changes at a uniform rate, actual velocity at any time $= \Delta s \div \Delta t$. For every case, INSTANTANEOUS VELOCITY $= v = \dfrac{ds}{dt} = \underset{\Delta t \doteq 0}{\text{limit}} \left(\dfrac{\Delta s}{\Delta t}\right)$. UNIT OF VELOCITY is any distance unit divided by any time unit. SPEED is the name for magnitude of velocity, and does not refer to its sense or linear direction. If s is not known in terms of t, but displacements for several time intervals, beginning or ending at the instant in question, are known, an approx value of v can be obtained from a consideration of average speeds.

For example, determine the speed at time $t = 2$ sec, having given the accompanying 4 sets of observed values for Δs and Δt. Column 3 is computed from the first two. As $\Delta t \doteq 0$, the value of aver velocity in last column appears to approach a limit of 3.6 ft per sec. A value of this limit may be determined graphically by platting time intervals along the horiz (Fig 138), and corresponding values of aver velocity vertically. The curve $ABCD$, "faired" to meet the ordinate through O at E (zero time interval), determines $v = OE = 3.6$ ft per sec.

$s_2 - s_1 = \Delta s$, feet	Δt, seconds	$\Delta s \div \Delta t$, ft per sec
28.8 − 3.73 = 25.07	2 to 6 = 4	6.27
20.83 − 3.73 = 17.10	2 to 5 = 3	5.70
13.87 − 3.73 = 10.14	2 to 4 = 2	5.07
8.1 − 3.73 = 4.37	2 to 3 = 1	4.37

Let v_1 = veloc at time t_1, v_2 = veloc at a later time t_2, $\Delta v = v_2 - v_1$, change in veloc in time interval Δt; then AVER ACCELERATION $= \Delta v \div \Delta t$. If veloc changes at a uniform rate, the actual acceleration at any time is $\Delta v \div \Delta t$. For every case, INSTANTANEOUS ACCELERATION $= a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2} = \underset{\Delta t \doteq 0}{\text{limit}} \left(\dfrac{\Delta v}{\Delta t}\right)$. UNIT OF ACCELERATION is any veloc unit

divided by any time unit. The unit used herein is feet per second per second, or ft per sec². If s or v are not known in terms of t, but the displacements or veloc records are available, an approx value for a can be obtained from a consideration of average accelerations. If the displacement record only is available, values of v must first be obtained as in the accompanying table.

t, sec	v, ft per sec	Δt, sec	Δv, ft per sec	$\Delta v \div \Delta t$, ft per sec²
2	3.6	2 to 6 = 4	4.8	1.2
3	5.1	2 to 5 = 3	3.9	1.3
4	6.4	2 to 4 = 2	2.8	1.4
5	7.5	2 to 3 = 1	1.5	1.5
6	8.4			

For example, determine the acceleration at time $t = 2$ sec, having given the five sets of values for t and v as shown in first two columns of table. Columns 3 and 4 are computed from the first two. Values in last column are computed from third and fourth. As $\Delta t \doteq 0$, the value of aver acceleration appears to approach 1.6 ft per sec² as a limit. A value of this limit may be determined graphically by platting time intervals along the horizontal (Fig 139), and corresponding values of aver acceleration vertically. The curve $ABCD$, "faired" to meet the ordinate through O at E (zero time interval), determines $a = OE = 1.6$ ft per sec².

If s is given in terms of t, v and a are determined by differentiation as indicated above. If a is given in terms of t, v and s are determined by integration. The formulas are

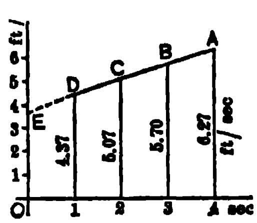


Fig 138

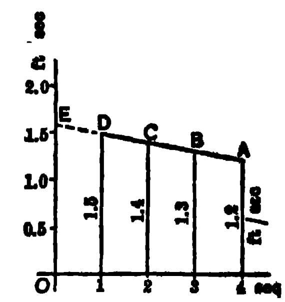


Fig 139

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt;\quad \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt;\quad \int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds;\quad \int_{t_1}^{t_2} dt = \int_{v_1}^{v_2} \frac{1}{a}\,dv = \int_{s_1}^{s_2} \frac{1}{v}\,ds$$

in which s_1, v_1, t_1, are simultaneous values. For uniform acceleration, a = constant, $v = at + v_o$, $s = {}^1/_2\,at^2 + v_o t + s_o$, v_o being initial velocity and s_o initial distance. For bodies falling from rest in vacuum, $v_o = 0$, $s_o = 0$, and $a = g$ = about 32.2 ft per sec², $s = {}^1/_2\,gt^2$, $v = gt = \sqrt{2gs}$.

47. MOTION GRAPHS

Motion graphs. DISTANCE-TIME (s-t) GRAPH (Fig 140). Any ordinate represents value of s at corresponding time. Slope of tangent represents speed $= \dfrac{ds}{dt}\cdot$ CB must be read to distance scale and AB to time scale; then speed $v = CB \div AB$. SPEED-TIME (v-t) GRAPH (Fig 141). Any ordinate represents value of v at corresponding time. Slope of tangent represents $a = \dfrac{dv}{dt}\cdot$ FE is read to speed scale and DE to time scale; then $a = FE \div DE$. Area between curve and time axis, limited by any two ordinates, represents

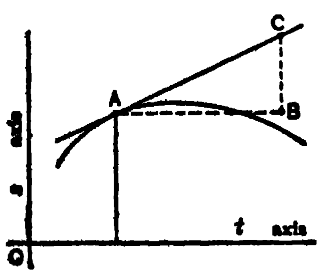


Fig 140

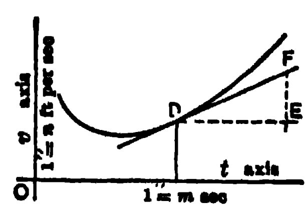


Fig 141

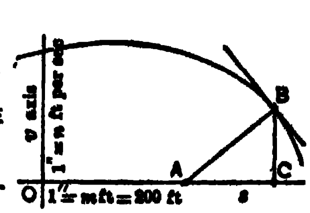


Fig 142

displacement in corresponding time interval. Δs = sq in of area × product of scale numbers = area × m × n. If area is below time axis it is considered minus. ACCELERATION-TIME (a-t) GRAPH. Any ordinate represents value of a at corresponding time. Area between curve and time axis, limited by any two ordinates, represents change in speed in corresponding time interval, Δv = sq in of area × product of scale numbers. SPEED-DISTANCE (v-s) GRAPH (Fig 142), for rectilinear or curved path. The subnormal at any point B (line AC) represents tangential component of acceleration at corresponding instant.

If m = distance scale number and n = speed scale number, then $a = AC$ (in) $\times m^2 \div n$. (In Fig 142, $m = 200$, $n = 30$.)

Simple harmonic motion. If a point P moves in a circular path of radius r at uniform speed, its projection on any diameter has simple harmonic motion. Radius r is called AMPLITUDE. PERIOD is time required for the projection to go from one end of diameter to other and back. FREQUENCY is number of periods per unit time.

When $t = 0$, let P be at P_0 (Fig 143). ϵ is called the load angle (lag, if negative). For simple harmonic motion (S H M) of V in the vertical diam, $y = r \sin (\theta + \epsilon) = r \sin (\omega t + \epsilon)$, in which $\omega = \dfrac{d\theta}{dt}$ = radians per unit time.

$$v_y = r\omega \cos (\theta + \epsilon) = r\omega \cos (\omega t + \epsilon) = \omega x$$
$$a_y = -r\omega^2 \sin (\theta + \epsilon) = -r\omega^2 \sin (\omega t + \epsilon) = -\omega^2 y$$

For S H M of H in horiz diam:

$x = r \cos (\theta + \epsilon) = r \cos (\omega t + \epsilon)$; $v_x = -r\omega \sin (\theta + \epsilon) = -r\omega \sin (\omega t + \epsilon) = -\omega y$; $a_x = -r\omega^2 \cos (\theta + \epsilon) = -r\omega^2 \cos (\omega t + \epsilon) = -\omega^2 x$

Crank and connecting-rod mechanism. Problem is to find expressions for the velocity and acceleration of any point in the crosshead, as A in Fig 144. Let $c = r \div l$, n = rev

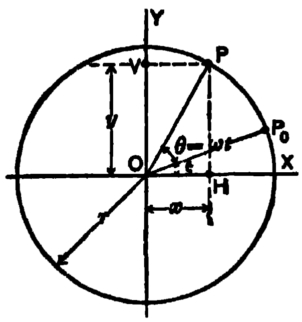


Fig 143

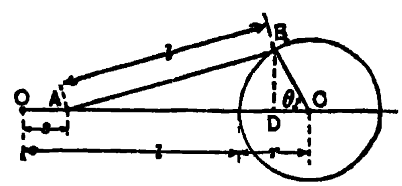


Fig 144

per sec (assumed constant), ω = radians of angle described by crank per second, and s = distance of A from its extreme left position, all distances expressed in ft. Then,

$$s = (l + r) - l\,(1 - c^2 \sin^2 \theta)^{1/2} - r \cos \theta$$

$$v = r\omega \left(\sin \theta + \frac{c \sin 2\theta}{2\,(1 - c^2 \sin^2 \theta)^{1/2}} \right); \quad a = r\omega^2 \left(\cos \theta + \frac{c \cos 2\theta + c^3 \sin^4 \theta}{(1 - c^2 \sin^2 \theta)^{3/2}} \right)$$

The above formulas are exact; close approximations are:

$s = r\,(1 - \cos \theta) + {}^1/_4\, cr\,(1 - \cos 2\theta)$; $\quad v = r\omega\,(\sin \theta + {}^1/_2\, c \sin 2\theta)$; $a = r\omega^2 \cos \theta + c \cos 2\theta)$

Exact and approximate formulas for a give same values for extreme right and left positions of point A.

48. CURVILINEAR MOTION

Curvilinear motion of a particle. The definitions of displacement and velocity, given at the beginning of article 47, apply to curvilinear motion. Velocity is a vector quantity. If s is distance measured along the curved path, the magnitude of velocity (speed) at an instant $= \dfrac{ds}{dt}$; the linear direction of the velocity is tangent to the path at the instantaneous position of the particle; and the sense of velocity corresponds to the direction of motion of the particle at the instant.

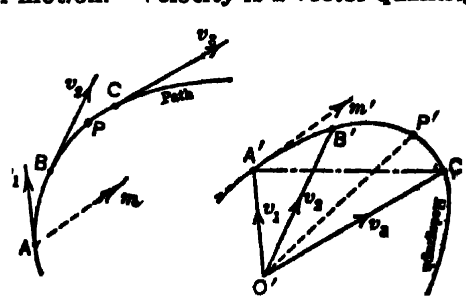


Fig 145

Acceleration for curvilinear motion is time rate of change of velocity and is a vector quantity. The velocity vector changes in magnitude and direction. In Fig 145, let A, B, C represent positions of particle P in its curved path, s, distance along the path, and v_1, v_2, v_3, velocity vectors at A, B, C. Plat velocity vectors $O'A'$, $O'B'$, $O'C'$, etc, from any origin O' to represent the velocities at A, B, C, etc. The curve $A'B'C'$, drawn through the ends of the vectors, is called a hodograph for the motion. For every position of P in

its path, there is a corresponding position P' in the hodograph; and P' describes distance s' on hodograph, while P describes distance s on path. Vector $O'P'$ represents the velocity of P. In time Δt, P moves from A to C, its velocity changes from $O'A'$ to $O'C'$, and the velocity change is $A'C'$. AVERAGE ACCELERATION for interval Δt = vector $A'C' \div \Delta t$, and it has the direction of the chord $A'C'$. INSTANTANEOUS ACCELERATION of P at $A = a$ = limit of aver acceleration as Δt approaches zero.

$$a = \lim_{\Delta t \doteq 0} \left(\frac{\text{vector } A'C'}{\Delta t}\right) = \lim_{\Delta t = 0} \left(\frac{\text{arc } A'B'C'}{\Delta t}\right) = \frac{ds'}{dt} = \text{speed of } P'$$

on hodograph. The direction of a is along the tangent $A'm'$, and as P' is moving clockwise, the sense is as indicated by arrow at m'. Hence acceleration at A is Am, parallel to $A'M'$ and $= \dfrac{ds'}{dt}$. Unit is any velocity unit divided by any time unit.

Motion graphs. Speed may be computed from a distance-time graph in the same way as for rectilinear motion. Tangential component (see Art 49) of acceleration may be computed from the speed-time graph; and the distance Δs may be obtained from the area under the curve.

49. COMPONENTS OF VELOCITY AND ACCELERATION

Components of velocity and acceleration of a particle for any curved path (not a plane curve). The position of the particle P, being defined by its coordinates x, y, z, the axial components of velocity are $v_x = \dfrac{dx}{dt}$, $v_y = \dfrac{dy}{dt}$, $v_z = \dfrac{dz}{dt}$. Resultant velocity $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$, and its direction cosines are $\cos\theta_x = \dfrac{v_x}{v}$, $\cos\theta_y = \dfrac{v_y}{v}$, $\cos\theta_z = \dfrac{v_z}{v}$. Axial components of acceleration are:

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}; \qquad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}; \qquad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}.$$

Resultant acceleration $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$; and its direction cosines are:

$$\cos\phi_x = \frac{a_x}{a}; \qquad \cos\phi_y = \frac{a_y}{a}; \qquad \cos\phi_z = \frac{a_z}{a}$$

If the path is a plane curve, $v_z = 0$, $a_z = 0$. The tangential and normal components of acceleration are $a_t = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$, and $a_n = \dfrac{v^2}{\rho}$, ρ being the radius of curvature. Resultant acceleration is $a = \sqrt{a_t^2 + a_n^2} =$

50. TRANSLATION

Translation of a rigid body is a motion such that each straight line in it remains fixed in direction. The paths of all particles of the body are exactly alike, straight or curved (not necessarily plane curves); the velocities of all particles at an instant are the same, and their accelerations at an instant are the same. For these reasons, it is customary to use the expressions, "velocity of the body" and "acceleration of the body."

51. ROTATION

Rotation of a rigid body is a motion such that one line of the body, or of its extension, remains fixed. The fixed line is the axis. The plane through the mass center perpendicular to the axis is the plane of rotation. The paths of all particles are circles with centers on the axis. Since all lines of the body parallel to plane of rotation sweep through equal angles in equal times, it is customary to describe rotation by the behavior of one radial line. In Fig 146 let θ be the angle from the x-axis to the radial line OP. $\Delta\theta = \theta_2 - \theta_1$ is the ANGULAR DISPLACEMENT of the body in the time $\Delta t = t_2 - t_1$, and is expressed in any angular unit. ANGULAR VELOCITY is the time rate of angular displacement. Average angular velocity $= \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$. The limit of this average, as Δt approaches zero, is the instantaneous angular velocity. The symbol for angular velocity is ω, and $\omega = \lim_{\Delta t \doteq 0}\left(\dfrac{\Delta\theta}{\Delta t}\right) = \dfrac{d\theta}{dt}$. The unit is any angular unit divided by any time unit, such as

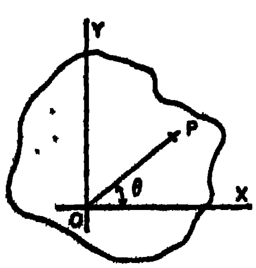


Fig 146

radians per sec, radians per min, rev per sec, or rev per min. The sign depends on the numerator of the fraction, or the way in which θ is changing. ANGULAR ACCELERATION is the time rate of angular velocity. Average angular acceleration $= \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$. The limit of this average, as Δt approaches zero, is instantaneous angular acceleration. The symbol for angular acceleration is α, and $\alpha = \lim_{\Delta t \doteq 0} \left(\frac{\Delta\omega}{\Delta t}\right) = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$. The sign of α depends on the numerator of the fraction, or on the way in which ω is changing.

Relations between v and a of any point P, of a rotating body and ω and α of the body. Let r = radius of path of P. Then $v = r\omega$, $a_t = r\alpha$, $a_n = r\omega^2$, $a = r\sqrt{\alpha^2 + \omega^4}$.

Sense of v must agree with sense of ω. Sense of a_t must agree with sense of α. Sense of a_n is always toward the axis.

52. PLANE MOTION

Plane motion of a rigid body is a motion such that each particle of the body moves in a plane at a constant distance from a fixed plane, through the center of gravity, called the plane of motion. Each line of the body parallel to plane of motion turns through the same angle θ in the same time interval. ANGULAR DISPLACEMENT of the body in time interval Δt is change of angular position of any line in the plane of motion. ANGULAR VELOCITY is time-rate of angular displacement. ANGULAR ACCELERATION is time-rate of angular velocity. The expressions, units, and rules of signs for plane motion are the same as those for rotation about a fixed axis (Art 51). Angular displacement is $\Delta\theta = \theta_2 - \theta_1$; angular velocity is $\omega = \frac{d\theta}{dt}$; angular acceleration is $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$.

A plane motion may be traced by giving the history of the movement of one point of the body (called a base point) in its own curved path, and a description of the rotation of

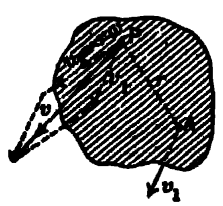


Fig 147

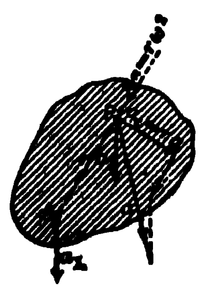


Fig 148

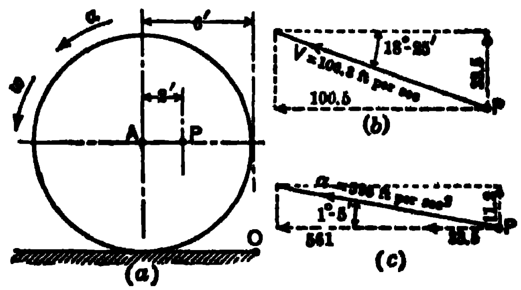


Fig 149

the body about the selected base point. The point selected as base should be one for which the motion is readily specified. In the case of a wheel rolling along a straight path, the center would be selected as a base point. VELOCITY OF ANY POINT P of the body, at an instant, with respect to a fixed point O, is the vector sum of the velocity of base point A, with respect to O, and of velocity of P with respect to A due to rotation about A. Thus (Fig 147) O is fixed point, A the moving base point, and P any other point of the body at distance r from A; v_1 is velocity of A with respect to O, and $v_2 = r\omega$ is velocity of P with respect to A. Resultant velocity of P with respect to $O = v$; or

$$v_P \text{ to } O = v_P \text{ to } A + v_A \text{ to } O$$

ACCELERATION OF ANY POINT P, with respect to a fixed point O, at an instant, has two components; one is that of the base point A with respect to O, and the other that of P with respect to base A. Acceleration of P with respect to A is rotational, and is conveniently replaced by its tangential and normal components, $a_t = r\alpha$ and $a_n = r\omega^2$. Then resultant acceleration of P, with respect to O, is the vector sum of $r\alpha$, $r\omega^2$, and acceleration of A with respect to O. Thus (Fig 148) a_1 is acceleration of A to O, and acceleration of P to A is resultant of a_t and a_n. Acceleration of P to O is a = vector sum of a_1, a_t, and a_n.

Example. A wheel of 6 ft radius rolls along a straight horiz path, and at a certain instant the point P, 2 ft from center of wheel, is in the position shown in Fig 149a. At this instant $\omega = 16.75$ radians per sec and $\alpha = 5.6$ radians per sec². Determine the velocity and acceleration of point P with respect to fixed point O at the specified instant.

SOLUTION. Select center A as base point. From relation between v and a of any point of a rotating body, and ω and α of the body (Art 51),

v_P to $A = r\omega = 2 \times 16.75 = 33.5$ ft per sec, vertically upward.

v_A to $O = r\omega = 6 \times 16.75 = 100.5$ ft per sec, horizontally toward left.

Therefore, v_P to $O = 106.3$ ft per sec, upward to left, at 18° 25′ to horizontal (Fig 149b).

a_A to $O = r\alpha = 6 \times 5.6 = 33.5$ ft per sec², horizontally toward left.

$a_t = r\alpha = 2 \times 5.6 = 11.2$ ft per sec², vertically upward.

$a_n = r\omega^2 = 2 \times (16.75)^2 = 561$ ft per sec², horizontally toward left.

Therefore, a_P to $O = 595$ ft per sec², upward to left, at 1° 5′ to horizontal (Fig 149c).

Instantaneous axis. For a body having plane motion, there is always one point in it (or in its extension), at each instant, for which the velocity with respect to A is equal and opposite to velocity of A with respect to O, that is, its velocity is zero at the instant. This point Q is called the INSTANTANEOUS (or instant) CENTER of rotation, and a line through Q, perpendicular to the plane of motion, is called the INSTANTANEOUS AXIS. Since Q is at rest for the instant, the resultant velocities of all points at the instant are purely rotational about the instant axis. The instant center is the intersection of two lines drawn from any two points, C and D, in the plane of the motion, perpendicular to their velocities. If the velocity of the point C is known, ω for the body is determined by dividing v_c by the distance of C from Q, or by r_c. The velocity of any other point E is $\omega \times r_E$, perpendicular to the radius r_E.

The position of Q in the body (or in its extension) is continually changing; its locus is a line (usually curved) fixed in the body and moving with it, called the BODY CENTRODE. The locus of the positions of Q in the fixed plane of motion is a line (usually curved) called the SPACE CENTRODE. The plane motion may be considered as produced by the rolling, without slipping, of the body centrode upon the space centrode.

KINETICS

53. FORCE AND MASS UNITS

Mass is the quantity of matter in a body; it means substance as measured by a beam scale. Standards of mass are the pound and kilogram; they are certain pieces of metal preserved in London and Paris. The mass of a body does not change with locality.

A force F acting on a particle of mass m gives it an acceleration a parallel to the force, and the relation between the quantities is given by the equation $F = Kma$, in which K is a constant. It is customary to use a KINETIC SYSTEM OF UNITS, one for which $K = 1$. For such a system, unit force gives to unit mass unit acceleration. The c g s system of units is a kinetic system; in it, force is measured in dynes, mass in gm, and acceleration in cm per sec². The system is called an absolute system, because the units are constant.

Units. The English-speaking engineers' unit of force is the pull of the earth on 1 lb of mass. This amount of pull is called ONE POUND OF FORCE. As this EARTH PULL varies with the elevation of the pound of mass, POUND FORCE is not an absolute unit. But, the range of variation is so slight, that the error involved by assuming it constant is negligible in most computations. The engineers' corresponding mass unit has no standard name, but will here be called a SLUG; it is the mass in which an acceleration of 1 ft per sec² is produced by a force of 1 lb. 1 slug of mass = about 32.2 lb of mass. In any computation that involves mass with pounds force, feet distance, and seconds time, mass should be reduced to slugs. Slugs of mass = wt of body (lb) ÷ 32.2 (approx value of acceleration due to gravity. In the following problems, the numerical value of this accel is taken as 32).

54. MOTION OF MASS CENTER AND TRANSLATION

For any particle of mass m

$$\Sigma\begin{pmatrix}\text{components in any } x \text{ direction of}\\ \text{all forces acting on the particle}\end{pmatrix} = \begin{pmatrix}\text{mass of particle times its } x \text{ component}\\ \text{of acceleration}\end{pmatrix}$$

or more briefly, if R_x represents the component of the resultant in the x direction,

$$R_x = \Sigma F_x = ma_x \qquad (A)$$

For any body, rigid or non-rigid,

$$\Sigma\left(\begin{array}{l}\text{components in any } x \text{ direction of all}\\ \text{external forces, pairs of internal}\\ \text{forces excluded}\end{array}\right) = \left(\begin{array}{l}\text{total mass of system times } x \text{ compo-}\\ \text{nent of acceleration of mass center}\end{array}\right)$$

or, more briefly, $$\Sigma F_x = M\bar{a}_x \qquad (B)$$

This is the equation of the motion of the mass center of a body or system of bodies.

Rigid body having motion of translation. Velocities of all particles are the same, and accelerations of all particles are the same. Resultant of all applied forces is a single force $R = Ma$, acting through the mass center, parallel to the acceleration,

$$\Sigma\left(\begin{array}{l}\text{components in any } x \text{ direction of}\\ \text{applied forces, including weight}\\ \text{of body}\end{array}\right) = \left(\begin{array}{l}\text{mass of body times the } x \text{ component}\\ \text{of acceleration}\end{array}\right)$$

or, more briefly, $$R_x = \Sigma F_x = Ma_x \qquad (C)$$

Also,

$$\Sigma\left(\begin{array}{l}\text{moments about any } x\text{-axis of ap-}\\ \text{plied forces, including weight of}\\ \text{body}\end{array}\right) = \left(\begin{array}{l}\text{moment of resultant about same}\\ x\text{-axis}\end{array}\right)$$

or, $$\Sigma M_x = Ma_x \times arm \qquad (D)$$

Following method of procedure is advisable in solving a problem: (*a*) Make a sketch of the body and place arrows upon it to represent all forces (including resultant at mass center), indicating unknown forces or distances by letters. Do same for each body if a system of bodies is under discussion. (*b*) Write as many resolution equations (*C*) and moment equations (*D*) as there are unknown quantities. (*c*) Solve the simultaneous equations to determine unknown quantities.

Example 1. A rectangular box (Fig 150), 2 by 2 by 8 ft high, weighing 480 lb, is placed upon a rough flat-car. If the car is running on a straight level track, what acceleration will cause the box to tip? If coeff of friction = 0.22, will the box slip or tip first? Solution. If tipping about edge O is impending, normal reaction is in position shown by N. By equation (*D*), $\Sigma M_0 = -480 \times 1 = -{}^{480}/_{32} \times a \times 4$; hence $a = 8$ ft per sec². If slipping impends, N acts through some point O' at distance x from center line. $a_y = 0$; hence $\Sigma F_y = 0 = N - 480$, shows $N = 480$ lb. Limiting friction = 0.22 N = 105.6 lb. By equation (*C*), $\Sigma F_x = 105.6 = {}^{480}/_{32} \times a$; hence, $a = 7.04$ ft per sec² to cause slipping. As this value of a is smaller than the first one, slipping will occur before tipping. x may be found from $\Sigma M_{o'} = -4 \times Ma$.

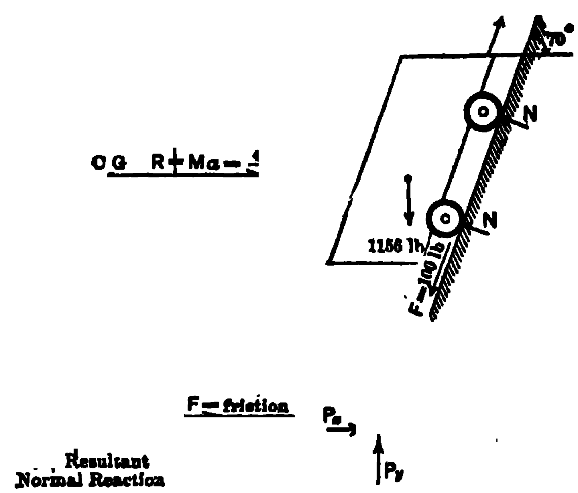


Fig 150 Fig 151

Example 2. A mine car (Fig 151), weighing 960 lb starts up a 70° slope, with an acceleration of 6 ft per sec, and is stopped at top with an acceleration of 4 ft per sec². A 196-lb man stands in the car. Determine starting and stopping tensions in the cable, if the frictional resistance amounts to 100 lb parallel to track. Also find reaction of car on man's feet for starting and stopping. Solution:

For starting, man and car: $\Sigma F_x = T - 100 - 1\,156 \sin 70° = {}^{1156}/_{32} \times 6 = 216.9$; hence, $T = 1\,402.9$ lb.

For stopping, man and car: $\Sigma F_x = T - 100 - 1\,156 \sin 70° = {}^{1156}/_{32} \times (-4) = -144.5$; hence, $T = 1\,041.5$ lb.

Man only, starting: $\Sigma F_x = P_x = {}^{196}/_{32} \times 6 \cos 70° = 12.57$ lb to right. $\Sigma F_y = P_y - 196 = {}^{196}/_{32} \times 6 \sin 70° = 34.55$ lb; hence, $P_y = 230.55$ lb up. $P = 231$ lb up to right, at angle of 3° 7′ to vertical.

Man only, stopping: $\Sigma F_x = -P_x = {}^{196}/_{32} \times (-4) \cos 70° = 8.38$ lb to left. $\Sigma F_y = +P_y - 196 = {}^{196}/_{32} \times (-4) \sin 70° = 23$ lb; hence, $P_y = 196 - 23 = 173$ lb up. $P = 173.1$ lb up to left, at angle of 2° 46′ to vertical.

55. ROTATION AND PLANE MOTION

Rigid body having motion of rotation about a fixed axis. Angular velocity $= \omega$, angular acceleration $= \alpha$, and moment of inertia of body about axis of rotation $= I$. The usual equation of motion is

$$\Sigma\begin{pmatrix}\text{moments about axis of rotation}\\ \text{of all external forces, including}\\ \text{weight of body}\end{pmatrix} = \begin{pmatrix}\text{moment of inertia about axis of rota-}\\ \text{tion times angular acceleration}\end{pmatrix}$$

or, more briefly,

$$\Sigma M = I\alpha \qquad (E)$$

Equation (E) is used to determine one unknown quantity, as a force, an arm, α, or I, according to the problem. In solving problems dealing with a connected system, in which certain bodies have motion of translation and others rotation, (C) and (E) may be used as simultaneous equations.

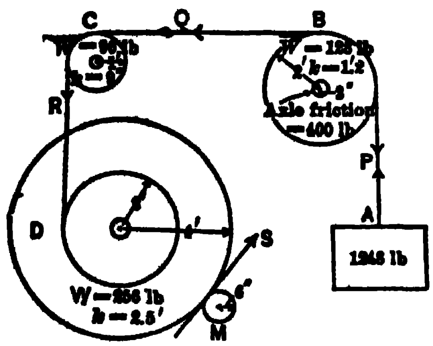


Fig 152

Example 3. Motor M (Fig 152) is geared to hoisting drum D to raise and lower weight A. Assume A descending at speed of 15 ft per sec, and determine tangential force S, exerted by motor, that will stop A in 30 ft at a uniform rate. Also, determine tensions P, Q, and R during the motion. Neglect mass of motor armature, and neglect all friction except that of axle on B (weights and sizes are not selected to conform to practice, but to illustrate the method of writing and using equations). SOLUTION. Determine acceleration of A by formulas given under Rectilinear motion (Art 46).

$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds;\quad \left[\frac{v^2}{2}\right]_{15}^{0} = a\left[s\right]_{0}^{30};\quad -\frac{(15)^2}{2} = a \times 30;\quad a = 3.75 \text{ ft per sec}^2 \text{ upward.}$$

By equations given under Angular acceleration (Art 51), $3.75 = a_A = 2\,\alpha_B = a_C = 2\,a_D$. By ($C$) and ($E$) of this Art, equations of motion for each body are:

$$P - 1\,248 = \frac{1\,248}{32} \times 3.75 = 146.25$$

$$2 \times Q - 2 \times P + 400 \times \frac{1}{4} = \frac{128}{32}(1.2)^2 \times \frac{3.75}{2} = 10.8$$

$$1 \times R - 1 \times Q = \frac{96}{32}\left(\frac{3}{4}\right)^2 \times 3.75 = 6.33$$

$$4 \times S - 2 \times R = \frac{256}{32}(2.5)^2 \times \frac{3.75}{2} = 93.75$$

Solution of these simultaneous equations determines $S = 701.4$ lb, $P = 1\,394.3$ lb, $Q = 1\,349.7$ lb and $R = 1\,356$ lb. The required motor torque is $S \times r = 701.4 \times 0.5 = 351$ ft-lb.

An axis of rotation which does not pass through the mass center must be held by forces (exerted by bearings) to keep it from shifting position. These bearing reactions depend upon the weight of the body, the manner in which the mass of body is distributed about the axis, the applied forces, the angular velocity ω, and the angular acceleration α. Generally, the resultant of the applied forces for such a body is not a single force, but a single force at a selected origin and a couple. Selecting the origin on the axis of rotation, the axial components of the single force and axial components of the couple are given by the following six equations:

$$\left.\begin{array}{ll}\Sigma F_x = M\bar{z}\alpha - M\bar{x}\omega^2 & \Sigma M_x = -\alpha\int xy\,dm - \omega^2\int yz\,dm\\ \Sigma F_y = 0 & \Sigma M_y = I_y\alpha\\ \Sigma F_z = -M\bar{x}\alpha - M\bar{z}\omega^2 & \Sigma M_z = -\alpha\int yz\,dm + \omega^2\int xy\,dm\end{array}\right\} \qquad (H)$$

In these equations, the axis of rotation is fundamentally the Y-axis; $\bar{x}$, $\bar{y}$, and $\bar{z}$ are the instantaneous coordinates of the mass center; ΣF_x, ΣF_y, ΣF_z are the sums of components of all applied forces in the axial directions; ΣM_x, ΣM_y, ΣM_z are the sums of moments of all applied forces about the axes; and the convention of signs for moments of forces, sense of ω, and measurement of θ $\left(\alpha = \frac{d^2\theta}{dt^2}\right)$ is as follows:

rotation about Z-axis from $+X$ to $+Y$ is positive,
rotation about X-axis from $+Y$ to $+Z$ is positive,
rotation about Y-axis from $+Z$ to $+X$ is positive.

Equations (H) are simultaneous at each instant. They are used more often to determine the forces exerted by the bearings on the axle, than to determine the resultant.

Special case (Fig 153). Choose the X-axis through the mass center and let XZ be a plane of symmetry of a homogeneous body. The resultant is a single force in the plane of symmetry having the Z component $-M\bar{x}\alpha$ and the X component $-M\bar{x}\omega^2$ acting at point C. $\overline{OC} = \dfrac{k^2_y}{\bar{x}}$, k_y being radius of gyration about Y-axis. If $\bar{x} = 0$, the resultant becomes a couple, in the XZ-plane, of moment $= \Sigma M_y = I_y\alpha$. If $\alpha = 0$ and $\bar{x} \neq 0$, the resultant $= -M\bar{x}\omega^2$, in the sense CO. If $\alpha = 0$ and $\bar{x} = 0$, the resultant vanishes.

Centrifugal force, body rotating about fixed Y-axis. Any particle of mass m moves in a circular path of radius r. The resultant of all forces acting on the particle has a normal component $= mr\omega^2$ and a tangential component $= mr\alpha$. The component $mr\alpha$ increases or decreases the speed of the particle; the component $mr\omega^2$ continually changes the direction of the linear velocity. The resultant of such forces for all the particles of the body is equivalent to the resultant specified by equations (H). If ω is constant and $\alpha = 0$, the resultant force acting on a particle to make it rotate in its circular path is $mr\omega^2$ toward the axis, and is called CENTRIPETAL FORCE. CENTRIFUGAL FORCE for the particle is equal and opposite to centripetal force, and is exerted by the particle upon its neighboring particles, or upon the axis of rotation. CENTRIFUGAL RESULTANT for a body is the resultant of centrifugal forces of all its particles. Generally, this resultant is not a single force; it may be computed from equations (H) by making $\alpha = 0$ and reversing senses of resultant force and couple.

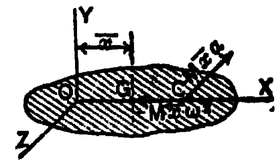


Fig 153

In the three following cases the centrifugal resultant is a single force: Case (a) Fig 154. Body has a plane of symmetry, XY, containing axis of rotation and mass center. Centrifugal resultant is a single force $= M\bar{x}\omega^2$, in XY plane, perpendicular to Y-axis, at a distance $y' = \left(\int xy\, dm\right) \div M\bar{x}$ from the XZ plane, and having sense EG. Case (b). Body has an axis of symmetry parallel to axis of rotation. Case (c). Body has plane of symmetry perpendicular to axis of rotation. In cases (b) and (c), centrifugal resultant is a single force $= M\bar{x}\omega^2$, acting through mass center outward and perpendicular to axis of rotation ($\bar{x}$ specified in Fig 154).

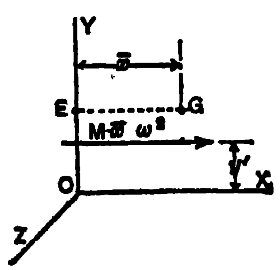


Fig 154

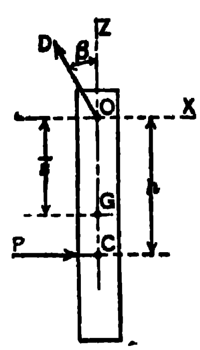


Fig 155

Center of percussion. Let Fig 155 represent a prismatic bar suspended on a horizontal Y-axis at O. G is the mass center. If a force P, parallel to X-axis, is applied to the body, the axle reaction at OD will generally be inclined to the Z-axis at some angle $\pm\beta$, the angle depending on the distance h of P from the axis of rotation. If $h = k_z^2 \div \bar{z}$, in which k_z is radius of gyration about Z-axis, P will cause no x component of axle reaction, that is β will be zero, and the point C, where the action line of P intersects OG, is the center of percussion. In impact testing machines, heavy pendulums are used to deliver blows, and proper design requires the striking point to coincide with the center of percussion, in order to avoid shock to the axle and detrimental vibration of the pendulum itself.

Body having plane motion. This motion has been described as translation of a base point in the plane of motion, and rotation about the base point (Art 52). Choose the mass center as base point and origin of coordinates, and the XY plane as plane of motion. Resultant of all applied forces may be described as a single force and a couple in the plane of motion. The couple is computed from the equation $\Sigma M_z = I_z\alpha = Mk_z^2\alpha$, in which ΣM_z represents sum of moments of applied forces about Z-axis. I_z and k_z are moment of inertia and radius of gyration about the Z-axis. The single force acts through the mass center, and its components are given by $R_x = \Sigma F_x = M\bar{a}_x$, $R_y = \Sigma F_y = M\bar{a}_y$, $R_z = \Sigma F_z = 0$. ΣF_x, ΣF_y, and ΣF_z, are sums of components of all applied forces, and $\bar{a}_x$, $\bar{a}_y$ are components of acceleration of mass center.

56. WORK, ENERGY AND POWER

Work of a constant force is the product of the force and the effective displacement of its application point. Effective displacement of application point is the component of the displacement parallel to the force. In Fig 156, the work of force F as application point describes path $AB = F \times AC$. Since $F\ (AB \cos \alpha) = (F \cos \alpha)\ AB$, the work is also equal to displacement of application point times component of force parallel to the displacement. Work of a variable force is $\int_{s_1}^{s_2} F \cos \alpha \, ds = \int_{s_1}^{s_2} F_t \, ds$, in which F is the variable force, ds is the elementary length of path, α is angle between force and element ds, and F_t is tangential component of force. The SIGN OF WORK is plus if force and effective displacement have same sense, and the sign is negative if they differ in sense.

Unit of work is unit force times unit distance 1 erg = 1 dyne cm. Work done by a body against a force is equal and opposite to work done by the force on the body.

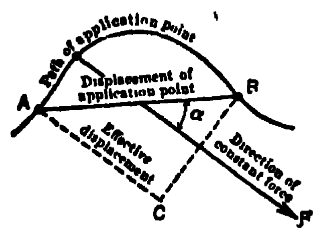


Fig 156

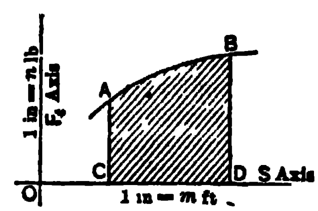


Fig 157

Work diagram (Fig 157). Plot values of F_t as ordinates, corresponding values of s as abscissas; draw curve AB through ends of ordinates. Area $ABDC$ times mn equals work, in ft-lb, done by F_t over distance $s_2 - s_1$.

Work of gravity on a body in any motion equals product of weight and change in height of the center of gravity. Work of a central force F (one always directed toward a fixed point), in any displacement of its application point, is $-\int_{r_1}^{r_2} F \, dr$, in which r_1 and r_2 are the distances of the application point from the center at the beginning and end of the displacement.

Work done by a pair of equal, opposite and colinear forces, for any motion of their application points, A and B. If force on A acts from B toward A, work of the pair $= \int_{r_1}^{r_2} F \, dr$; if force on A acts from A toward B, work of the pair $= -\int_{r_1}^{r_2} F \, dr$. In these formulas, r_1 and r_2 are the initial and final distances between A and B. Work of a torque T on a body for an angular displacement of θ radians $= T\theta$.

Power is time-rate of doing work. Let P = power and w = work; $w = \int F_t \, ds$, and $P = \frac{dw}{dt} = F_t \frac{ds}{dt} = F_t\, v$, v being speed of application point of force F. If P equals power (rate of doing work) of the steam in a locomotive, R = total resistance (grade, air, rolling, etc), M, v, and a_t being mass, speed, and acceleration of train respectively, then $P = Rv + Mva_t$; $P = 0$, for $v = 0$. Power of a torque $T = T\omega$, ω being angular velocity of rotating body. UNIT OF POWER is any work unit divided by time unit, such as ft-lb per sec, ft-lb per min, or erg per sec. (10^7 ergs per sec = 1 watt). One horsepower = 550 ft-lb per sec = 33 000 ft-lb per min.

Energy. A body possesses energy when it can do work against forces applied to it. The amount of energy possessed at any instant is the amount of work the body can do in changing to some standard state. KINETIC ENERGY (K E) of a body is energy possessed by virtue of its velocity, and the standard state is zero velocity. K E of a particle is one-half its mass times square of its velocity. K E of a body in translation $= 0.5\ Mv^2$. K E of a rotating body $= 0.5\ I\omega^2 = 0.5\ Mk^2\omega^2$, I, k, and ω being moment of inertia, radius of gyration, and angular velocity about axis of rotation. K E of a body having plane motion $= 0.5\ I\omega^2 = 0.5\ Mk^2\omega^2 = 0.5\ M\bar{v}^2 + 0.5\ \bar{I}\omega^2$, in which I and k refer to instantaneous axis, $\bar{v}$ = velocity of mass center and $\bar{I}$ is moment of inertia about axis through mass center perpendicular to plane of motion. UNIT of K E is same as unit of work, usually ft-lb. In computing K E, engineers use mass in slugs, v in ft per sec, ω in radians per sec, and k in ft.

Principle of work and kinetic energy. Total work of the applied forces acting on any body, or on any system of connected bodies, equals the change in the kinetic energy of

the body, or bodies. Work done = Δ K E. Change in energy in translation = 0.5 M $(v_2^2 - v_1^2)$, v_1 and v_2 being initial and final velocities. Change in energy in rotation = 0.5 I $(\omega_2^2 - \omega_1^2)$ = 0.5 Mk^2 $(\omega_2^2 - \omega_1^2)$, ω_1 and ω_2 being initial and final angular velocities. In plane motion, change in K E is

$$\Delta \text{ K E} = 0.5\, I\, (\omega_2^2 - \omega_1^2) = 0.5\, Mk^2\, (\omega_2^2 - \omega_1^2)$$

$$= 0.5\, M\, (\bar{v}_2^2 - \bar{v}_1^2) + 0.5\, \bar{I}\, (\omega_2^2 - \omega_1^2) = 0.5\, M\, (\bar{v}_2^2 - \bar{v}_1^2) + 0.5\, M\bar{k}^2\, (\omega_2^2 - \omega_1^2)$$

in which I and k refer to instantaneous axis, $\bar{I}$ and $\bar{k}$ to a parallel axis through mass center, and $\bar{v}$ is velocity of mass center.

Example. In example 3, Art 55 (Fig 152): (*a*) Use principle of work and K E to find constant torque M necessary to stop body A in 30 ft, from a downward veloc of 15 ft per sec; (*b*) Find tension in cord P; (*c*) Find power of motor for above value of torque, when veloc of A is 10 ft per sec. Solution:

Work of gravity is positive = 1 248 × 30 = +37 440 ft-lb
Work of axle friction is negative = −400 × $^{30}/_{2}$ × $^{1}/_{4}$ = −1 500 ft-lb
Work of motor is negative = $-M\theta = -M$ × 60 × 0.5 = −120 M ft-lb
Total work = −120 M + 35 940
Final K E = 0, and change is a loss, or negative.

For D, Δ K E = $^1/_2\, Mk^2\, (\omega_2^2 - \omega_1^2) = -^1/_2 \times {}^{256}/_{32} \times {}^{25}/_4 \times ({}^{15}/_2)^2$ = −1 408 ft-lb
For C, Δ K E = $^1/_2\, Mk^2\, (\omega_2^2 - \omega_1^2) = -^1/_2 \times {}^{96}/_{32} \times {}^{9}/_{16} \times (15)^2$ = −189.8 ft-lb
For B, Δ K E = $^1/_2\, Mk^2\, (\omega_2^2 - \omega_1^2) = -^1/_2 \times {}^{128}/_{32} \times (1.2)^2 \times ({}^{15}/_2)^2$ = −162 ft-lb
For A, Δ K E = $^1/_2\, M\, (v_2^2 - v_1^2) = -^1/_2 \times {}^{1248}/_{32} \times (15)^2$ = −4 387.5 ft-lb

Total Δ K E = −6 147.3 ft-lb

(*a*) Since W = Δ K E, −120 M + 35 940 = −6 147.3; hence, M = 351 ft-lb
(*b*) When v_A = 10 ft per sec, ω of motor = 40 radians per sec

Power = torque × ω = 351 × 40 ft-lb per sec = 351 × 40 ÷ 550 = 25.5 hp

(*c*) To find tension P, apply principle of work and K E to body A alone.

Work = (−P + 1 280) 30 = −30 P + 37 440
Δ K E = $^1/_2\, M\, (v_2^2 - v_1^2) = {}^1/_2\, {}^{1248}/_{32}\, (0 - 15^2)$ = −4 387.5 ft-lb
Then, −30 P + 37 440 = −4 387.5; hence, P = 1 394.3 lb

57. IMPULSE AND MOMENTUM

Linear impulse of a constant force F for time $t = F \times t$. If the force varies in magnitude and direction, the impulse is computed from axial components of impulse, which are found by taking the time integrals of axial components of the force. The three axial component impulses are $\int_{t_1}^{t_2} F_x dt$, $\int_{t_1}^{t_2} F_y dt$, $\int_{t_1}^{t_2} F_z dt$; the resultant of these is the impulse of the force F. Impulse is a vector quantity; hence the impulse of the force equals the square root of the sum of the squares of the components. The direction cosines of the resultant vector are determined in the usual manner (see resultant of noncoplanar concurrent forces or resultant velocity in curvilinear motion). Unit of impulse is unit force times unit time, or lb-sec in engineers' units.

Angular impulse of a force about a line for a time interval dt is the product of the moment of the force about the line and the time dt. If T represents the moment or torque of the force, the angular impulse for the time interval $t_2 - t_1 = \int_{t_1}^{t_2} T\, dt$. Unit of angular impulse is unit moment times unit time, or ft-lb-sec in engineers' units.

Sign of impulse. Impulse of a force tending to increase velocity of the body to which the force is applied is positive; that which tends to decrease velocity is negative.

Linear momentum of a particle is the product of its mass and velocity. It is a vector quantity and has the sense and direction of the velocity. Unit of momentum is unit mass times unit velocity and is equivalent to unit of impulse. Linear momentum of a body is the resultant, or vector sum, of the momentums of its particles. In any motion the linear momentum of a body is $M\bar{v}$, M being mass of the body and $\bar{v}$ the velocity of its mass center. In engineers' units this is expressed in slug-ft per sec or lb (force)-sec.

Angular momentum of a particle about an axis is the moment of its momentum about that axis. In Fig 158, let mv = momentum of particle P. Resolve the momentum into components parallel and perpendicular to the axis. DE is perpendicular distance from

axis to line AP. The momentum of $P = mv \cos \alpha \times DE$. The angular momentum of a body about an axis is the algebraic sum of the angular momentums of its particles. The angular momentum of a rotating body about the axis of rotation is $I\omega = Mk^2\omega$, I and k being moment of inertia and radius of gyration about the axis of rotation and ω the angular velocity. Engineers use I in slug ft^2, ω in radians per sec, M in slugs and k in ft; the unit of the quantity is slug ft^2 per sec or lb (force)-ft-sec.

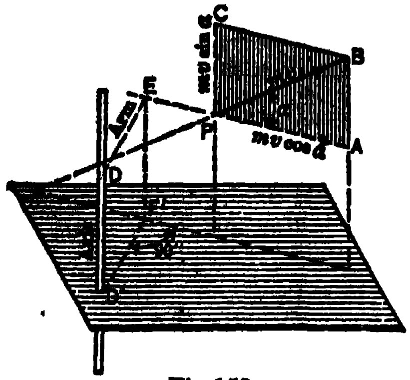


Fig 158

Principle of impulse and momentum for a body. The change in the component of linear momentum parallel to any axis x in the time $t_2 - t_1$ equals the algebraic sum of the components of the impulses of the applied forces parallel to the axis in the same time, or, more briefly, $\Delta\,(M\bar{v}_x) = \sum\int_{t_1}^{t_2} F_x\,dt$. The change in the angular momentum about any axis y in the time $t_2 - t_1$ equals the algebraic sum of the angular impulses of the applied forces about the axis in the same time, or $\Delta\,(I_y\,\omega) = \sum\int_{t_1}^{t_2} T_y\,dt$. This component of linear momentum and the angular momentum remain constant if no forces are applied.

Example. In example 3, Art 55 (Fig 152), assume the time required to stop A is 4 sec. Use principle of impulse and momentum to determine tensions P and Q. Solution:

For A,
$$\Delta\,(M\bar{v}_x) = \sum\int_{t_1=0}^{t_2=4} F_x\,dt$$
$$^{1248}/_{32}\,(0 - 15) = -585 = +1\,248 \times 4 - P \times 4; \quad \text{hence, } P = 1\,394.3 \text{ lb}$$

For B,
$$\Delta\,(I_y\omega) = \sum\int_{t_1=0}^{t_2=4} T_y\,dt$$
$$^{128}/_{32}\,(1.2)^2\,(0 - 7.5) = -43.2 = -Q \times 2 \times 4 + 1\,394.3 \times 4 - 400 \times {}^{8}/_{12} \times 4;$$
hence,
$$Q = 1\,349.7 \text{ lb}$$

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SECTION 37

CHEMICAL AND PHYSICAL NOTES AND TABLES

COMPILED BY

EDWARD K. JUDD AND ROBERT PEELE

CHEMICAL AND PHYSICAL NOTES AND TABLES

Table 1. The Chemical Elements, 1934 (a)

Element	Symbol	Atom wt (b)	Valence	Element	Symbol	Atom wt (b)	Valence
Aluminium	Al	26.97	*iii*	Mercury	Hg	200.6	*i, ii*
Antimony	Sb	121.76	*iii, v*	Molybdenum	Mo	96.0	*iv, vi*
Argon	A	39.94	*o*	Nickel	Ni	58.69	*ii, iii*
Arsenic	As	74.91	*iii, v*	Nitrogen	N	14.01	*iii, v*
Barium	Ba	137.37	*ii*	Osmium	Os	191.5	*iv, viii*
Bismuth	Bi	209.0	*iii, v*	Oxygen	O	16.0	*ii*
Boron	B	10.82	*iii*	Palladium	Pd	106.7	*ii, iv*
Bromine	Br	79.92	*i*	Phosphorus	P	31.027	*iii, v*
Cadmium	Cd	112.41	*ii*	Platinum	Pt	195.23	*ii, iv*
Calcium	Ca	40.07	*ii*	Potassium	K	39.09	*i*
Carbon	C	12.00	*iv*	Radium	Ra	225.97	*ii*
Cerium	Ce	140.13	*iii, iv*	Selenium	Se	78.9	*ii, iv, vi*
Chlorine	Cl	35.457	*i*	Silicon	Si	28.06	*iv*
Chromium	Cr	52.01	*iii, vi*	Silver	Ag	107.88	*i*
Cobalt	Co	58.94	*ii, iii*	Sodium	Na	23.0	*i*
Columbium	Cb	93.3	*iii, v*	Strontium	Sr	87.63	*ii*
Copper	Cu	63.57	*i, ii*	Sulphur	S	32.06	*ii, iv, vi*
Fluorine	F	19.0	*i*	Tantalum	Ta	181.4	*iii, v*
Glucinum	Gl	9.2	*ii*	Tellurium	Te	127.6	*ii, iv, vi*
Gold	Au	197.2	*i, iii*	Thallium	Tl	204.4	*iii*
Helium	He	4.0	*o*	Thorium	Th	232.15	*iv*
Hydrogen	H	1.008	*i*	Tin	Sn	118.7	*ii, iv*
Iodine	I	126.92	*i*	Titanium	Ti	47.9	*iii, iv*
Iridium	Ir	193.1	*iv, viii*	Tungsten	W	184.0	*ii, iv, vi*
Iron	Fe	55.84	*ii, iii*	Uranium	U	238.14	*iv, vi*
Lanthanum	La	138.92	*iii*	Vanadium	V	50.96	*iii, v*
Lead	Pb	207.2	*ii, iv*	Ytterbium	Yb	173.04	*iv*
Lithium	Li	6.94	*i*	Yttrium	Yt	88.9	*iv*
Magnesium	Mg	24.32	*ii*	Zinc	Zn	65.38	*ii*
Manganese	Mn	54.93	*ii, iv*	Zirconium	Zr	91.22	*iv*

(*a*) Omitting 33 elements of small interest to engineers. For atomic weights of all elements, see *Jour Amer Chem Soc.* The Society publishes, from time to time, such changes as may be due to further investigation. (*b*) Figures for 1934; based on oxygen = 16.

Grouping of elements based on similarity in chemical properties (same omissions as in Table 1):

1. Hydrogen
2. Chlorine, bromine, iodine, fluorine
3. Oxygen, sulphur, selenium, tellurium
4. Nitrogen, phosphorus, arsenic, antimony, bismuth
5. Carbon, silicon
6. Boron
7. Alkalies: lithium, sodium, potassium, ammonium (NH_4)
8. Alkaline earths: barium, strontium, calcium
9. Magnesium, glucinum, zinc, cadmium, mercury
10. Silver, copper, gold
11. Aluminum, yttrium, lanthanum, ytterbium, thallium
12. Iron, nickel, cobalt
13. Manganese
14. Chromium, molybdenum, tungsten, uranium
15. Tin, lead, titanium, zirconium, cerium, thorium
16. Vanadium, tantalum
17. Platinum, palladium, osmium, iridium

To compute percentage composition from a formula. Two examples:

CALCIUM PHOSPHATE

$Ca_3(PO_4)_2$

Ca....3 × 40.07	= 120.21	=	38.7%
P.....2 × 31.027	= 62.054	=	20.0
O.....2 × 4 × 16	= 128.00	=	41.3
Formula-weight	= 310.264	=	100.0

KAOLIN

$H_2Al_2(SiO_4)_2, H_2O$

H......2 × 1.008	= 2.016	=	0.8%
Al......2 × 26.97	= 53.94	=	20.9
Si......2 × 28.06	= 56.12	=	21.8
O.......2 × 4 × 16	= 128.0	=	49.5
H_2O....2.016 + 16	= 18.016	=	7.0
Formula-weight	= 258.092	=	100.0

To determine formula from analysis: divide percentage of each element by its atomic weight, and reduce quotients to least common denominator. Example:

Phosphorus.........	21.82% ÷ 31.027 = 0.703 = 1	
Hydrogen...........	0.71% ÷ 1.008 = 0.703 = 1	= Na_2HPO_4
Sodium.............	32.43% ÷ 23.00 = 1.407 = 2	
Oxygen.............	45.02% ÷ 16.00 = 2.814 = 4	
	99.98%	

Computations from analysis will not always work out with mathematical precision, but small errors can be compensated according to the fundamental law that elements always combine in simple proportions represented by whole numbers.

Determining formula of a mineral from its analysis is complicated by fact that few natural substances are "chemically pure"; certain constituents of a mineral from one locality may be found, in same mineral from a different locality, to be replaced wholly or partly by another element. First convert percentage of minor element to equivalent percentage of preponderating element of same group, equivalents being directly proportional to atomic weights; then compute ratio of elements (or molecules) in same manner as above. Example:

Analysis of a garnet:	
SiO_2.....	39.85%
Al_2O_3....	22.07
Fe_2O_3...	1.13
MgO....	0.68
CaO....	36.31
	100.04

$Fe_2O_3 : Al_2O_3$ = (2 × 55.84 + 3 × 16 = 159.68) :: (2 × 27.0 + 3 × 16 = 102.0) = 1.13 : x. x = 0.72; that is, 1.13% of Fe_2O_3 is equivalent to 0.72% of Al_2O_3. Similarly, MgO : CaO = (24.32 + 16 = 40.32) :: (40.07 + 16 = 56.07) = 0.68 : x. x = 0.94% CaO.

SiO_2............	39.85 ÷ (28.06 + 32) = 0.660 = 3
Al_2O_3...........	(22.07 + 0.72) ÷ 102.2 = 0.223 = 1
CaO............	(36.31 + 0.94) ÷ 56.07 = 0.664 = 3

Type formula: $3\,CaO \cdot Al_2O_3 \cdot 3\,SiO_2$

Table 3. Composition, etc, of More Important Industrial Compounds

(For composition of minerals, see Sec 1, Tables I–VIII. For gases, see Sec 39, Art 7–10. For additional organic and inorganic compounds, see Van Nostrand's "Chemical Annual," Merck's "Index," or U S Pharmacopœia.)

Name	Formula	Mol wt	Sp gr
Acetic acid	$HC_2H_3O_2$	60.03	1.06
Alcohol (grain)	$C_2H_5 \cdot OH$	46.05	0.785
" (wood)	$CH_3 \cdot OH$	32.03	0.791
Alumina	Al_2O_3	102.2	3.86
Alum, ammonium	$Al_2(SO_4)_3 \cdot (NH_4)_2SO_4 \cdot 24\,H_2O$	906.95	1.64
" potassium	$Al_2(SO_4)_3 \cdot K_2SO_4 \cdot 24\,H_2O$	949.06	1.76
Ammonia	NH_4OH	35.03	0.88
Am chlor (sal-ammoniac)	NH_4Cl	53.50	1.52
Ammonium nitrate	NH_4NO_3	80.05	1.73
" sulphate	$(NH_4)_2SO_4$	132.14	1.77
Arsenious oxide (white arsenic)	As_4O_6	395.84	3.74
Barium carbonate	$BaCO_3$	197.37	4.27
" sulph (blanc fixe)	$BaSO_4$	233.44	4.40
Boric acid	H_3BO_3	62.02	1.43
Calcium acetate (acet of lime)	$Ca(C_2H_3O_2)_2 \cdot H_2O$	176.13	......
" carbide	CaC_2	64.07	2.22
" carbonate	$CaCO_3$	100.07	2.85
" oxide(quicklime)	CaO	56.07	3.30
" hydroxide (slaked lime)	$Ca(OH)_2$	74.09	2.08
" phosphate (phos of lime)	$Ca_3(PO_4)_2$	310.29	3.18
"sulph(plaster paris)	$CaSO_4$	136.14	2.96
Carbon tetrachloride	CCl_4	153.84	1.58
" disulphide	CS_2	76.14	1.29
Cupric arsenite (paris green)	$CuHAsO_3$	187.54	......
" oxide	CuO	79.57	6 37
" sulphate (bluestone)	$CuSO_4 \cdot 5\,H_2O$	249.72	2.28
Ferric oxide	Fe_2O_3	159.68	5.18
Ferrous oxide	FeO	71 84	......
" sulph (copperas)	$FeSO_4 \cdot 7\,H_2O$	278.02	1.90
Hydrochloric acid	HCl	36.47	......
Hydrogen peroxide	H_2O_2	34.02	......
Lead acetate (sugar of lead)	$Pb(C_2H_3O_2)_2 \cdot 3\,H_2O$	379.20	2.50
" carbonate (white lead)	$2\,PbCO_3 \cdot Pb(OH)_2$	775.31	......
" monoxide (litharge)	PbO	223.10	9.37
" oxide (red lead)	Pb_3O_4	685.30	9.10
" sulphate	$PbSO_4$	303.17	6.23

Name	Formula	Mol wt	Sp gr
Magnesium carbonate	$MgCO_3$	84.32	3.04
" sulph (epsom salt)	$MgSO_4 \cdot 7\,H_2O$	246.50	1.68
Manganese dioxide	MnO_2	86.93	5.03
Mercuric chlor (corrosive sublimate)	$HgCl_2$	271.52	5.40
Nitric acid	HNO_3	63.02	1.53
Phosphoric acid	HPO_3	85.05	2.30
Potassium carbonate (potash)	$K_2CO_3 \cdot 2\,H_2O$	174.23	2.04
" chlorate	$KClO_3$	122.56	2.34
" chloride	KCl	74.56	1.99
" chromate	K_2CrO_4	194.20	2.73
" cyanide	KCN	65.11	1.52
" ferricyanide	$K_3Fe(CN)_6$	329.20	1.81
" ferrocyanide	$K_4Fe(CN_6) \cdot 3\,H_2O$	422.35	1.85
" hydroxide (caustic potash)	KOH	56.11	2.04
" nitrate (saltpeter)	KNO_3	101.11	2.10
" permang'n'te	$KMnO_4$	158.03	2.70
" sulphate	K_2SO_4	174.27	2.66
Silver nitrate (lunar caustic)	$AgNO_3$	169.89	4.35
Sodium borate (borax)	$Na_2B_4O_7 \cdot 10\,H_2O$	382.16	1.69
" carbonate (soda)	Na_2CO_3	106.00	2.47
" bicarbonate	$NaHCO_3$	84.01	2.20
" chloride (salt)	$NaCl$	58.46	2.17
" cyanide	$NaCN$	49.01	....
" hydroxide (caustic soda)	$NaOH$	40.01	2.13
" nitrate (Chile saltpeter)	$NaNO_3$	85.01	2.27
" silicate (water glass)	$Na_2Si_4O_9$	303.20	....
" sulphate (Glauber salts)	$Na_2SO_4 \cdot 7\,H_2O$	268.18	....
" sulphite	$Na_2SO_3 \cdot 7\,H_2O$	252.18	1.59
" thiosulphate	$Na_2S_2O_3 \cdot 5\,H_2O$	248.22	1.73
Sulphur dioxide	SO_2	64.07	....
Sulphuric acid	H_2SO_4	98.09	*
Tartaric acid	$H_2C_4H_4O_6$	150.05	1.75
Tin chloride	$SnCl_4$	260.84	2.28
Vanadium oxide	V_2O_5	182.00	3.36
Zinc chloride	$ZnCl_2$	136.29	2.91
" oxide (zinc white)	ZnO	81.37	5.78
" sulphate	$ZnSO_4 \cdot 7\,H_2O$	287.55	1.97

* Table 4. NOTE.—Wt of other materials: Sec 25, Art 11, Table 3; Sec 43, Art 17, Table 7

Table 3. Solubilities of Certain Salts in Water at 18° C (Professor Alexander Smith)

	K	Na	Li	Ag	Ba	Sr	Ca	Mg	Zn	Pb
Cl	32.95	35.86	77.79	0.0_316	37.24	51.09	73.19	55.81	203.9	1.49
	3.9	5.42	13.3	0.0410	1.7	3.0	5.4	5.1	9.2	0.05
Br	65.86	88.76	168.7	0.0_41	103.6	96.52	143.3	103.1	478.2	.598
	4.6	6.9	12.6	0.0_66	2.9	3.4	5.2	4.6	9.8	0.02
I	137.5	177.9	161.5	0.0_635	201.4	169.2	200.0	148.2	419.0	0.08
	6.0	8.1	8.5	0.0_71	3.8	3.9	4.8	4.1	6.9	0.0_22
F	92.56	4.44	0.27	195.4	0.16	0.012	0.0016	0.0076	0.005	0.07
	12.4	1.06	0.11	13.5	0.0_292	0.001	0.0_32	0.0_214	0.0_35	0.003
NO_3	30.34	83.97	71.43	213.4	8.74	66.27	121.8	74.31	117.8	51.66
	2.6	7.4	7.3	8.4	0.33	2.7	5.2	4.0	4.7	1.4
ClO_3	6.6	97.16	313 4	12.25	35.42	174.9	179.3	126 4	183.9	150.6
	0.52	6.4	15.3	0.6	1.1	4.6	5 3	4.7	5.3	3.16
BrO_3	6.38	36.67	152.5	0.59	0 8	30.0	85 17	42.86	58.43	1.3
	0.38	2.2	8.20	0.025	0.02	0.9	2 3	1.5	1.8	0.03
IO_3	7.62	8.33	80.43	0.004	0.05	0.25	0.25	6.87	0 83	0.002
	0.35	0.4	3.84	0.0_314	0.001	0.0_257	0.007	0.26	0.02	0.0_43
OH	142.9	116.4	12.04	0.01	3.7	0.77	0.17	0.001	0.0_35	0.01
	18.0	21.0	5.0	0.001	0.22	0 063	0.02	0.0_32	0.0_45	0.0_34
SO_4	11.11	16.83	35.64	0.55	0.0_323	0.011	0 20	35.43	53.12	0.0041
	0.62	1.15	2.8	0.020	0.0_410	0.0_36	0.015	2.8	3.1	0_313
CrO_4	63.1	61.21	111.6	0.0025	0.0_338	0.12	0.4	73.0	....	0.0_42
	2.7	3.30	6.5	0.0_315	0.0_415	0.006	0.03	4.3	....	0.0_65
C_2O_4	30.27	3.34	7.22	0.0035	0.0086	0.0046	0.0_356	0 03	0.0_36	0.0_315
	1.6	0.24	0.69	0.0_32	0.0_338	0.0_326	0.0_443	0.0027	0.0_44	0.0_65
CO_3	108.0	19.39	1.3	0.003	0 0023	0.0011	0.0013	0.1	0.004?	0.0_31
	5.9	1.8	0.17	$0\ 0_31$	0.0_311	0.0_47	$0\ 0_313$	0.01	0.0_33?	0.0_43

The upper figure of each pair is the number of grams of the anhydrous salt held in solution by 100 c c of water; the lower is the molar solubility, *i e*, the number of moles (molecular weight expressed as grams) contained in 1 liter of saturated solution. The numbers for small solubilities have been abbreviated; thus $0.0_64 = 0.0000004$.

Table 4. Specific Gravity of Sulphuric Acid at 15° C*

Compared with water at 4° C (Lunge)

Sp gr	Deg Baumé	Deg Twaddell	100 parts of c p acid contain: SO_3	H_2SO_4	60° Bé acid	50° Bé acid
1.00	0.0	0	0.07	0.09	0.12	0.14
1.05	6.7	10	6.02	7.37	9.44	11.79
1.10	13.0	20	11.71	14.35	18.39	22.96
1.15	18.8	30	17.07	20.91	26.79	33 46
1.20	24.0	40	22.30	27.32	35.01	43.71
1.25	28.8	50	27.29	33.43	42.84	53.49
1.30	33.3	60	31.99	39.19	50 21	62.70
1.35	37.4	70	36.58	44.82	57.43	71.71
1.40	41.2	80	40.91	50.11	64 21	80.18
1.45	44.8	90	44.92	55.03	70.52	88.05
1.50	48.1	100	48.73	59.70	76.50	95.52
1.53	50.0	106	51.04	62.53	80.13	100.00
1.55	51.2	110	52.46	64.26	82.34	102.82
1.60	54.1	120	55.93	68.51	87.79	109.62
1.65	56.9	130	59.45	72.82	93.29	116.51
1.70	59.5	140	63.00	77.17	98.89	123.47
1.71	60.0	142	63.70	78.04	100.00	124.86
1.75	61.8	150	66.58	81.56	104.52	130.49
1.80	64.2	160	70.94	86.90	111.35	139.06
1.825	65.2	165	74.29	91.00	116.61	145.60
1.830	65.5	166	75.19	92.10	118.02	147.36
1.835	65.7	167	76.27	93.43	119.72	149.49
1.840	65.9	168	78.04	95.60	122 51	152.96

* To reduce to 15° sp gravities observed at other temperatures, for each deg above or below 15° add to or subtract from the sp gr observed:

0.0006 with acids to 1.17
0.0007 " " from 1.17 to 1.45
0.0008 " " " 1.45 " 1.58
0.0009 " " " 1.58 " 1.75
0.0010 " " " 1.75 " 1.84

Table 5. Comparison of Thermometer Scales

$$\frac{C}{5} = \frac{F - 32°}{9} = \frac{R}{4}$$ (See also Sec 39, Art 6)

Cent	Fahr	Re	Cent	Fahr	Re	Cent	Fahr	Re
−273	−459.4	−218.4	+ 55	+ 131	+ 44	+1 300	+2 372	+1 040
−200	−328	−160	60	140	48	1 400	2 552	1 120
−150	−238	−120	65	149	52	1 500	2 732	1 200
−100	−148	− 80	70	158	56	1 600	2 912	1 280
− 50	− 58	− 40	75	167	60	1 700	3 092	1 360
− 40	− 40	− 32	80	176	64	1 800	3 272	1 440
− 30	− 22	− 24	85	185	68	1 900	3 452	1 520
− 20	− 4	− 16	90	194	72	2 000	3 632	1 600
− 17.8	0	− 14.2	95	203	76	2 100	3 812	1 680
− 10	+ 14	− 8	100	212	80	2 200	3 992	1 760
0	32	0	200	392	160	2 300	4 172	1 840
+ 5	41	+ 4	300	572	240	2 400	4 352	1 920
10	50	8	400	752	320	2 500	4 532	2 000
15	59	12	500	932	400	2 600	4 712	2 080
20	68	16	600	1 112	480	2 700	4 892	2 160
25	77	20	700	1 292	560	2 800	5 072	2 240
30	86	24	800	1 472	640	2 900	5 252	2 320
35	95	28	900	1 652	720	3 000	5 432	2 400
40	104	32	1 000	1 832	800	3 100	5 612	2 480
45	113	36	1 100	2 012	880	3 200	5 792	2 560
50	122	40	1 200	2 192	960	3 300	5 972	2 640

Temperatures and latent heats of fusion and evaporation, see Sec 39, Art 11.
Coefficients of expansion, see Sec 39, Art 8.
Specific gravities. Of MINERALS, see Sec 1, Art 3 and Tables I-VIII. Of STONE, EARTH, AND STRUCTURAL MATERIALS, see Sec 43, Table 7. Of GASES, see Sec 39, Table 19.

Table 6. Barometric Pressures and Altitudes

Altitude, ft	Barom press		Altitude, ft	Barom press		
	Inches mercury	Lb per sq in		Inches mercury	Lb per sq in	
0	30.00	14.75	8 000	22.11	10.87	Note.—For very accurate determinations, small corrections must be applied in second decimal place, for temperature and for height of meniscus in tube of barometer.
1 000	28.88	14.20	9 000	21.29	10.46	
2 000	27.80	13.67	10 000	20.49	10.07	
3 000	26.76	13.16	11 000	19.72	9.70	
4 000	25.76	12.67	12 000	18.98	9.34	
5 000	24.79	12.20	13 000	18.27	8.98	
6 000	23.86	11.73	14 000	17.59	8.65	
7 000	22.97	11.30	15 000	16.93	8.32	

Table 7. Boiling Points of Water at Different Barometric Pressures (Regnault)

Millimeters	Boiling point, C°	Millimeters	Boiling point, C°	Millimeters	Boiling point, C°	Millimeters	Boiling point, C°
720.15	98.5	733.21	99.0	746.50	99.5	760.00	100.0
722.75	98.6	735.85	99.1	749.18	99.6	762.73	100.1
725.35	98.7	738.50	99.2	751.87	99.7	765.46	100.2
727.96	98.8	741.16	99.3	754.57	99.8	768.20	100.3
730.58	98.9	743.83	99.4	757.28	99.9	771.95	100.4

Table 8. Weight and Volume of Water at Different Temperatures

Temp, deg F	Wt, lb per cu ft	Relative volume	Temp, deg F	Wt, lb per cu ft	Relative volume	
32.0	62.418	1.00011	100	62.02	1.00686	Note.—For sea water, multiply weights by 1.026
39.1	62.425	1.00000	120	61.74	1.01138	
50.0	62.410	1.00025	140	61.37	1.01678	
60.0	62.370	1.00092	160	60.98	1.02306	
62.0	62.355	1.00110	180	60.55	1.03023	
70.0	62.310	1.00197	200	60.07	1.03819	
80.0	62.230	1.00332	210	59.82	1.04246	
90.0	62.130	1.00496	212	59.76	1.04332	

Table 9. Tensile Strength of Metals at Ordinary Temperatures, Lb per Sq Inch

Aluminum, cast	12 500	Iron, cast	48 000
" drawn	17 000	" rolled	55 000
" rolled	19 290	Lead, sheet	1 720
" hammered	22 575	" cast	2 050
Antimony, cast	1 000	Nickel, hard-drawn	96 000
Bismuth, cast	3 000	Platinum wire, annealed	32 000
Brass	50 000	" cast	45 000
Cobalt	75 000	Silver, cast	41 000
Copper, cast	24 000	Steel, cast (ordinary)	80 000
" wire, soft drawn	35 500	" high-tensile, up to	450 000
" " hard "	60 000	Tin, cast	4 600
Gold, cast	20 000	" drawn	5 800
" wire, hard-drawn	37 000	Tungsten, hard	610 000
		Zinc	5 000

Table 10. Composition of Alloys (Liddell's Metallurgist's and Chemist's Handbook)

	Cu	Zn	Sn	Pb	Sb	Bi
Acid-resisting metal	83.0	6.0	10.8	0.1	....	....
Babbitt metal, soft	3.0	....	84.0	5.6	7.4	....
" " normal	3.0	....	90.0	....	7.0	....
" " hard	8.0	....	88.0	....	4.0	....
" " (original)	4.0	69.0	19.0	5.0	3.0	....
Bell metal	80.0	....	20.0	....	....	....
Brass, low copper	61.5	38.5	....	....	....	....
" high "	80.0	20.0	....	....	....	....
Bronze, for bearings	80.0	7.0	13.0	....	....	....
	90.0	7.0	3.0	....	....	....
" Tobin	58.2	39.5	2.3	....	....	....
Gun metal (aver of 5)	87.8	6.4	5.8	....	....	....
Magnolia metal	....	....	4.75	80.0	15.0	0.25
Monel metal	33.0	....	....	....	Ni 60.0	Fe 6.5
Munts metal	62.0	38.0	....	....	....	....
Manganese bronze	88.6	1.5	8.7	0.3	....	Fe 0.7
Pewter	1.4	....	97.0	1.6	....	....
Phosphor bronze	80.0	....	10.0	10.0	....	....
Red metal	70.0	20.0	4.0	6.0	....	....
Rose's metal	....	....	25.0	25.0	....	50.0
Solder	....	....	33.0	67.0	....	....
Type metal	...	....	3.0	82.0	15.0	....

Table 11. Chemical Analyses and Properties of Refined Copper

(Hofman, Metallurgy of Copper)

Elements	Lake, wire-bar	Lake, arsenical ingot	Electrolytic wire-bar	Best selected English
Copper	99.8900	99.4131	99.9530	99.5300
Copper and Silver	99.9000	99.4385	99.9548	99.5510
Silver	0.0096 (2.8 oz)	0.0254 (7.41 oz)	0.0018 (0.56 oz)	0.0210 (7.02 oz)
Lead	0.0031	0.0027	0.0010	0.1331
Bismuth	....	....	....	....
Arsenic	0.0062	0.3183	....	0.0071
Antimony	....	....	0.0009	0.0067
Selenium and Tellurium	0.0020	Not det'd	0.0026	0.0066
Iron	0.0028	0.0056	0.0038	0.0044
Nickel	0.0090	0.0153	0.0028	0.1112
Zinc	....	....	....	....
Sulphur	0.0016	0.0071	0.0026	0.0074
Oxygen (by diff)	0.00753	0.2143	0.0315	0.1705
Conductivity, annealed	96.49	....	100.45	....
" hard-drawn	93.84	....	97.61	....
Tensile strength, lb per sq in	67 590	....	66 300	....
Number of twists in 6 in	17	....	34	....
Elongation, %	1.03 (in 8 in)	....	1.04 (in 60 in)	....
Number of bends, annealed	11	....	14	....
Diam of wire, in	0.08	....	0.08	....

Table 12. Fuel Combustion Data; ordinary practice

(Liddell's Metallurgist's and Chemist's Handbook)

B t u in 1 lb aver coal	13 500	Lost, auxiliary pipes radiation	0.23%
Hp-hr in 1 lb coal = (13 500 × 778) ÷ (60 × 33 000)	5.3	" " exhaust	1.40"
Lost through grate	1%	" engine radiation	2.08"
" boiler radiation	5"	" " exhaust	57.30"
" chimney gases	22"	Total losses	90.56%
" main pipes radiation	1.55"	Converted to power	9.44"

Table 13. Freezing Mixtures

Parts by weight	Temp lowered to deg F
2.5 Calcium chloride crystals to 1 water	40.3
1.1 Sodium hyposulphite crystals to 1 water	40.2
1 Sodium chloride to 3 snow	36.5
45 Ammonium nitrate to 100 snow	28.3
1 Calcium chloride to 3 snow	17.8

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SECTION 38

ELEMENTS OF HYDRAULICS

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ELEMENTS OF HYDRAULICS

1. DEFINITIONS

Hydromechanics deals with the mechanics of fluids. Fluids embrace LIQUIDS (practically incompressible and the molecules of which are not fixed in position) and GASES (compressible and expand spontaneously, volume depends on press and temp, and molecules are free). Hydromechanics comprises HYDROSTATICS, dealing with fluids at rest and HYDRODYNAMICS, with fluids in motion. HYDRAULICS treats of the hydrostatics and hydrodynamics of water, but the term is often restricted to hydrodynamics only, or the mechanics of water in motion.

The principles of hydraulics are also applicable to other liquids, but hydrodynamic (not hydrostatic) phenomena are greatly influenced by internal frictional or shearing resistance as measured by viscosity and by inertia, which vitally affect motion. Viscosity, while often assumed constant for water, actually varies with the temp and is a controlling factor in flow of more viscous liquids, such as oils. The notion of velocity, basic in hydrodynamics, was clarified by Galileo, Torricelli and others in 17th century; the Bernoullis, Pitot, and Chésy pioneered in practical hydraulics in the 18th; and the determination of experimental coefficients occupied attention of Darcy, Basin, Francis and others during the 19th century. More recent work has reverted to fundamentals, to which the experiments of Reynolds have directed attention. See Art 2.

2. PHYSICAL PROPERTIES OF LIQUIDS

Compressibility. Liquids may generally be assumed incompressible; actually, water decreases about 1% in vol under press of 3 000 lb per sq in.

Unit weight. Water is assumed to have a constant wt per unit of vol, designated by w; commonly taken as 62.5 lb per cu ft (8.355 lb per U S gal). Sea water averages 64 lb, while that of salt lakes may reach 75 lb or more. Similar approx values may be used for other liquids under static conditions, although density and wt vary with temp; while such variation is unimportant with water, it may be vital in dynamic problems involving other liquids.

Mass density and specific gravity. Liquid flow is affected by inertia and viscosity. Scientifically, inertia is expressed in terms of mass. Mass density, ρ (Greek rho) in

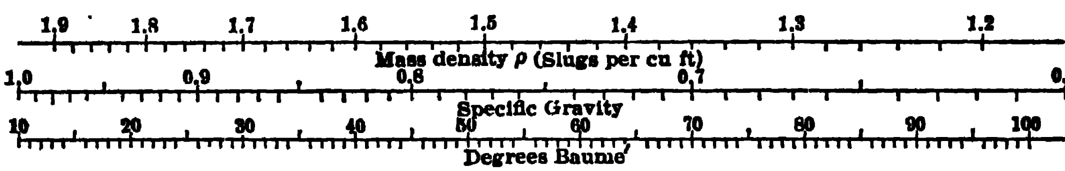


Fig 1. Density Conversion Scales

British units (SLUGS per cu ft) = w (lb per cu ft) ÷ g (ft per sec^2), where g is accel of gravity (aver value 32.16 ft per sec^2 or 980.7 dynes or cm per sec^2). Using aver values, ρ for water = 1.94, varying from 1.938 at 32° to 1.925 at 100° F.

For oils, wt is usually expressed in terms of specific gravity as measured by Baumé scale (Fig 1). With oils, sp gr is commonly determined at temp of 60° F (designated 60/60); this is not precise, though the error is usually negligible.

In flow problems, it is important to know ρ at existing temp, and charts are available for various oils. For heavy oils, with sp gr near unity, approx sp gr at any temp, t, = sp gr (60/60) − 0.00035t (°F). For lighter oils, with sp gr around 0.7, the last factor is about 0.00025t. ρ (approx) = 1.94 × (sp gr).

Viscosity. Liquids do not possess perfect fluidity; adjacent particles adhere, and a moving thread or layer of liquid tends to drag the particles with it. At low veloc, this drag constitutes a shearing resistance which is measured by the coeff of ABSOLUTE VISCOSITY, μ (Greek mu), defined as "the frictional resistance to motion, lb per sq ft, between 2 parallel

surfaces in a fluid, 1 ft apart and moving relatively to each other at rate of 1 ft per sec." In the metric system, the unit for μ is the POISE, expressed in dyne-sec per sq cm. To convert poises into lb-sec per sq ft, divide by 478.8

Viscosity is measured by viscometers of various types, and is usually expressed as time in secs required for a certain vol of liquid to pass through a small tube under a fixed head. Formulas and charts are available for converting viscosity measurements into absolute viscosity units. (See "The Viscosity of Liquids" by Emil Hatschek, London, 1928; also "The Mechanical Properties of Fluids," Van Nostrand, 1924).

Viscosity is independent of rate of change in velocity, but is affected to a small extent by press, and markedly by temp, diminishing as temp increases. For water, see table.

Temp, F°	μ (British) lb-sec per sq ft	Temp, C°	μ (Poises) dynes-sec per sq cm
32	0.000037	0	0.0179
68	0.000021	20	0.0100*
100	0.000014	56	0.0069

* Called "centipoise," one-hundredth of a poise.

Some viscosity-temp charts for oils (viscosity of which may be 50 to 300 or more times greater than that of water) have been published, but μ for the fluid in question should be known at the existing temp.

Kinematic viscosity. In flow phenomena, inertia (measured by ρ) tends to continue the existing state of motion, while viscosity (measured by μ) tends to retard such motion. Fluid flow is thus a compromise between inertia and viscosity, and the ratio $\mu \div \rho$ thus becomes a very vital characteristic of any fluid in motion. This ratio is the KINEMATIC VISCOSITY, designated by ν (Greek nu).

ν is expressed in a confusing number of units. In the British scientific system (followed here) ν is in lb-sec per sq ft $\div$ slugs per cu ft; that is, as absolute viscosity $\div$ mass density. In the metric system it is given in poises (or centipoises) $\div$ grams μ per cu cm. Since grams per cu cm = sp gr for all practical purposes, we may use poises $\div$ sp gr. Other variations are in use. To convert metric into British units, as above defined, divide former by 929. In most hydrodynamic problems, ν is simply an argument on which the friction factor depends, and no great degree of accuracy is usually required in determining its value. As viscometers actually depend upon ν for their data (rather than simply μ) viscometer readings can be directly converted into kinematic viscosity (Fig 2).

Reynolds number, *R*. Flow phenomena and coefficients vary with character of the liquid, as measured by μ and ρ, but the influence of these factors varies in turn with the scale and veloc of movement. Whereas viscous resistance varies with the first power of the veloc and of the scale of movement, the inertial forces (represented by ρ) vary with the squares of those factors. Hence, a variation must be expected in the proportionality between μlv and ρl^2v^2, where l is any linear dimension representing the scale of movement (as the diam of a pipe), and v = veloc. The formula is written:

$$\frac{\rho l^2 v^2}{\mu l v} = \frac{lv}{\nu} = R = \text{the "Reynolds number."}$$

It may be shown that R is dimensionless (containing no units of mass, length, or time) and thus it has the same value, irrespective of the units used in its calculation. *Example.* When water flowing in a pipe 1 ft diam at 4 ft per sec veloc, at 68° F, $\mu = 0.000021$, $\rho = 1.94$, and $R = 370\,000$. In metric units, at 20° C, $\mu = 0.01$ poise, $\rho = 1.0$, $d = 30.5$ cm, and $v = 122$ cm per sec, giving same value of R.

Recent studies have shown that many flow coefficients vary with R, which, as it includes general physical characteristics of the liquid, makes it possible to correlate data applying to air, oil, water, and other fluids of varying density and viscosity, and in movements on different scales.

Atmospheric pressure. At 30 in barometer (14.7 lb per sq in or 1 atmos) a water column stands 34 ft high, and water boils at 212.2° F. Boiling point is lowered about 1.8° F per in of mercury; for barom press of 20 in (= 192.4° F), water column is 22.7 ft. Roughly, the mercury barometer drops 1 in per 1 000 ft of elevation.

Water carries an amount of dissolved air varying with temp. At 60° F there are about 1 part oxygen and 4.4 parts air by wt in 100 000 of water.

Constants. Following are convenient units and constants for hydraulic computations:

1 cu ft water = 7.48 U S gal = 28.32 liter = 62.5 lb.

1 U S gal = 231 cu in = 3.78 liters = 8.355 lb.

g = acceleration due to gravity = 32.16 ft per sec; $\sqrt{2g} = 8.02$; $1 \div 2g = 0.01555$.

1 ft head = 0.434 lb per sq in; 1 lb per sq in = 2.304 ft head.

Area circle 1 ft (= 12 in) diam = 0.785 sq ft.

Area circle 3 in (=0.25 ft) diam = 0.785 ÷ 16 = 0.049 sq ft, etc.

1 acre ft = 1 ft depth over 1 acre = 43 560 cu ft = a flow of 1 cu ft per sec for 12 hr and 6 min.

1 cu ft per sec = 646 300 gal per day.

1 in of rain = 2 323 200 cu ft per sq mile.

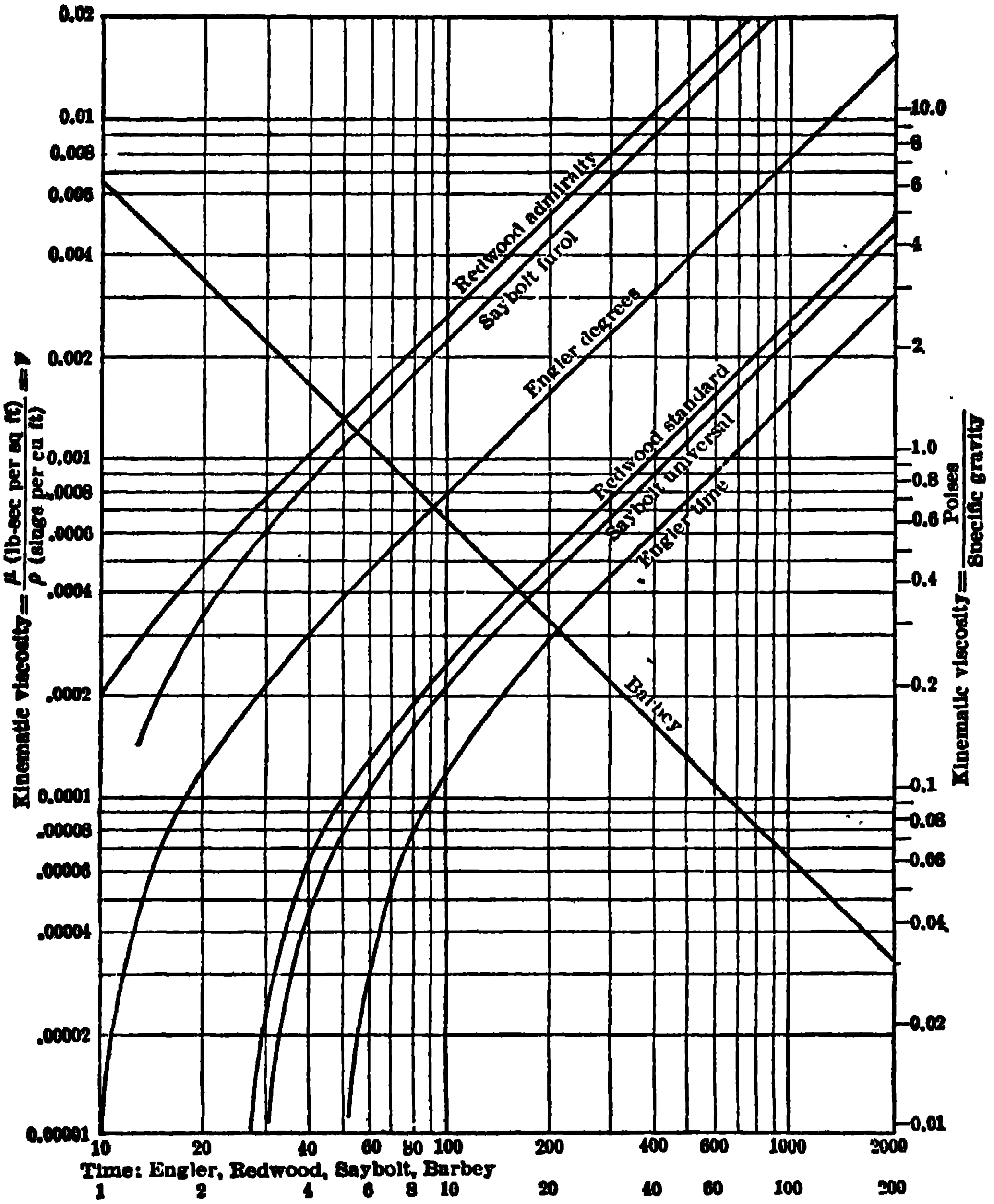


Fig 2. Viscosity Conversion Chart

HYDROSTATICS

3. TRANSMISSION OF PRESSURE

Pascal's law (1646). In a perfect fluid, neglecting effect of gravity, press exerted anywhere on the fluid is transmitted undiminished in all directions (at speed of 4 670 ft per sec = velocity of sound in water), and acts with the same force on all equal areas and at right angles to the surface.

Pascal noted that great forces could thus be produced, and that the total press increases in proportion to the surface area. A light press exerted on the water in a small tube inserted in a barrel may burst the barrel. This principle was applied by Bramah (1796) in the first successful HYDRAULIC PRESS (Fig 3), which consists of 2 cylinders, large and small, fitted with pistons and connected by a pipe through which water passes from one cylinder to other. A force P, applied to the small piston, is transmitted through the liquid and produces an equal unit press p on the large piston. Thus $W : P :: D^2 : d^2$, or $W = PD^2 \div d^2$. This is reduced by the frictional resistance of the packing, which, depending upon the type, condition and adjustment of same, may amount to as much as 10% of the applied load per cylinder.

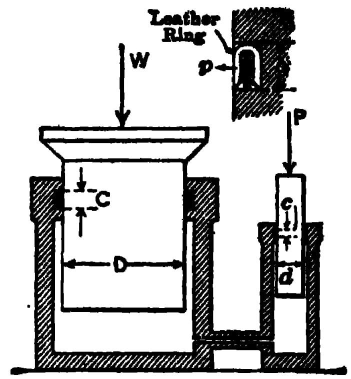


Fig 3. Hydraulic Press

4. PRESSURE PRODUCED BY GRAVITY

Considering a liquid divided into layers, the press in each layer is: (*a*) proportional to its depth, (*b*) proportional to wt of the liquid, (*c*) the same at all points in each layer, (*d*) same in all directions at any depth, (*e*) independent of size and shape of containing vessel. Also, water surface is normal to the resultant of the forces acting. Cases (*a*) and (*b*) are practically self-evident. $P = wAh$, where P = total press, lb; w = wt of 1 cu ft of water, lb; A = area, sq ft; h = head, ft. If $p = P \div A$ = unit press, then p in lb per sq ft $= wh = 62.5\ h$, or in lb per sq in $= 0.434\ h$, or h in ft $= 2.30$ times p in lb per

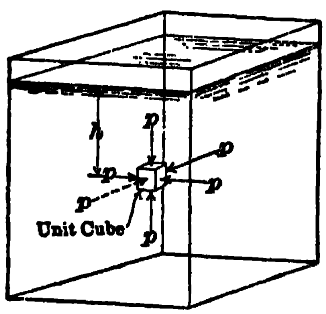


Fig 4

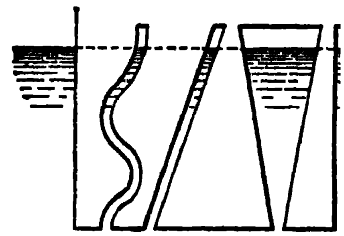


Fig 5

sq in. (*c*) and (*d*) follow from Pascal's law (see Fig 4). (*e*) is shown in Fig 5, where the bottoms of vessels are at same level, and unit pressures must be same or flow would occur.

5. TOTAL NORMAL PRESSURE AND CENTER OF PRESSURE

Total normal press, or press acting at right angles to surface $= P_n = wAh_{cg} =$ 62.5 × area of surface × distance of center of gravity of the surface below water level. This applies to any surface, plane, curved or warped. Thus the total normal press against the back face of dam shown in Fig 7 is $P_n = 62.5 \times 1 \times h\ (h \div 2) = {}^1/_2\ wh^2$ lb per lin ft.

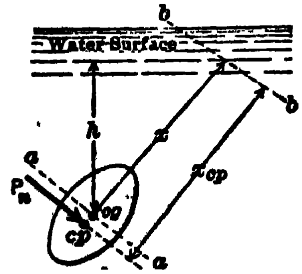


Fig 6

Component total pressure in any direction $= P' = P_n \cos\theta = wA'h_{cg}$, where θ = angle between normal and the given direction. That is, the total press in any direction = total normal press on a projection (= A') perpendicular to that direction. In Fig 7*d*, total horiz press $= P_h = wh^2 \div 2$, the same as the normal press against a vertical projection (Fig 7*a*).

Center of pressure (*cp*) of a submerged surface is the point of application of the resultant press. For a submerged rectangle, the center of press is ${}^2/_3\ h$ below the water surface, when the top of the rectangle is at the surface (Fig 7*a*). In general, the distance $x_{cp} = I \div S$, where I is the moment of inertia of the surface and S its statical moment (= area × distance of *cg* from axis) both with reference to an axis situated at intersection of the plane of the surface with the water level. x_{cp} is the perpendicular distance from this axis to the *cp*.

In applying this formula, use is made of the well known relation $I = I' + Ax^2$, where I = moment of inertia about any axis, I' = that about an axis through *cg* of surface, A = area of surface, and x = distance between axes. Thus for a circular area, Fig 6, *aa*

is axis through cg, bb is water surface axis, $A = \pi r^2$, $I' = \pi r^4 \div 4$; hence $x_{cp} = (\pi r^4 \div 4 + \pi r^2 x^2) \div \pi r^2 x = (r^2 \div 4x) + x$, or when the circle is vertical $= (r^2 \div 4h) + h$.

It is seen that the center of press is lower than the center of gravity, and that the distance between them decreases as the head increases. Practically they may be taken as coincident, when the head exceeds 3 or 4 times the vertical dimension of the surface.

6. PRESSURE AGAINST DAMS AND GATES

Fig 7 shows graphically the variation in normal, horiz, and vertical press, total press and centers of press for rectangular surfaces of unit width. The press at water surface is

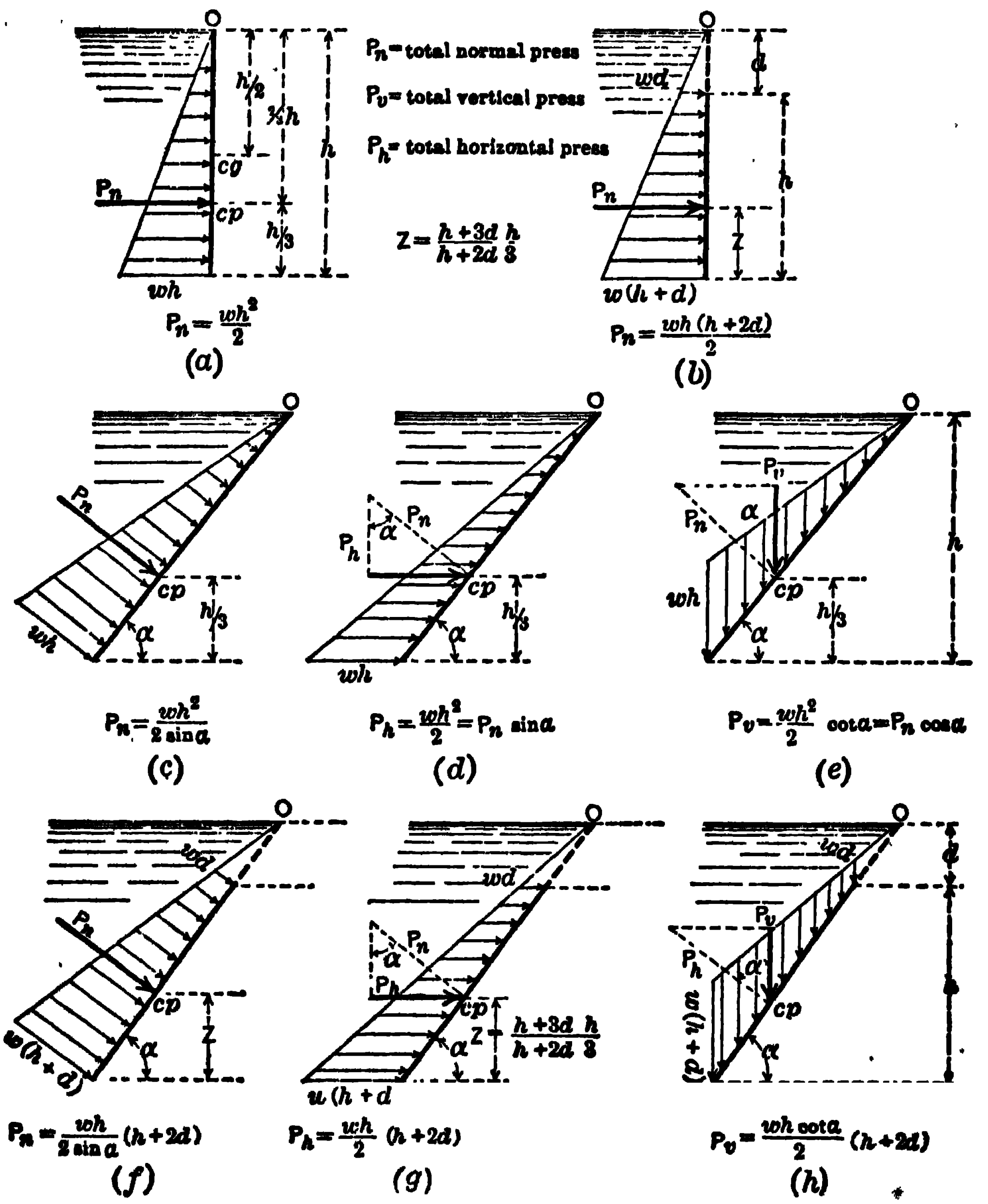


Fig 7. Water-pressure Diagrams

0, and at depth $h = wh$ in all directions; hence the variation is uniform and resultant press acts through the cg of the pressure figure.

In case of press from both sides (Fig 8), the resultant press, represented by arrows and ... by drawing the 2 press triangles, is uniform in intensity and $= wh$ for any depth at right an[gles] water surface a. It depends only on the difference in levels h. Thus a gate

anywhere between a and b would be subjected to the same resultant press, independent of its depth below a. The point of application of the total resultant press against the gate is best found by taking the difference in moments of the 2 press triangles about b.

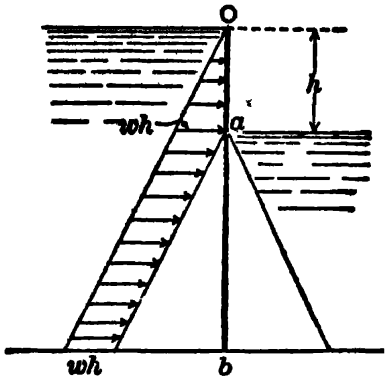


Fig 8

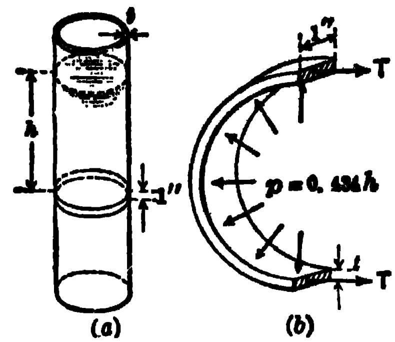


Fig 9. Pressure in Pipes and Tanks

7. PRESSURE IN PIPES AND TANKS

Hoop tension. Considering the vertical section of pipe in Fig 9a as a unit (1 in) in height, subjected to a uniform internal press of p lb per sq in, this press produces tension in the pipe $= T$ (Fig 9b), which tends to rupture the pipe longitudinally. From the principle that the total press in any direction = total normal press on a projection perpendicular to that direction, $2\ T = pd$, or T (lb per lin in of pipe) $= p$ (lb per sq in) $\times\ r$ (radius of pipe, in). If s = allowable stress in shell, lb per sq in, then required thickness t (in) $= pr \div s = 0.434\ hr \div s$, where h = head in ft.

Example. Head, 400 ft; diam, 3 ft; riveted steel pipe; joint efficiency, 70% s = 14 000 lb per sq in. $t = 0.434 \times 400 \times 18 \div (14\,000 \times 0.70) = 0.32$, or $3/8$ in. This formula holds only for thin shells. The limit is usually put at $t = 1/30\ d$. For thick shells, see *Proc* Am Soc Mech Engrs, 1912. Also the press p will be uniform only for a vert pipe, but may be so taken for a small pipe under a large head. For large pipes, besides the allowance for internal press, other factors must be taken into account (Art 14). This principle is used in designing the hoops or bands for wood-stave pipes or water tanks, or for the reinforcing steel in reinforced concrete tanks (Art 13).

8. FLOTATION AND LOSS OF WEIGHT IN WATER

(See Merriman, Treatise on Hydraulics; also Sec 1, Art 3.)

HYDRODYNAMICS

9. FLOW THROUGH ORIFICES AND NOZZLES

Torricelli's theorem (1644). The theoretical veloc of flow from an orifice is the same as that acquired by a body falling freely through a height equal to the head of water on the orifice. That is, theoretically, the veloc of flow in ft per sec from an orifice $= \sqrt{2\ gh}$, where g = accel of gravity, ft per sec per sec, and h = head on orifice, ft. This formula represents a theoretical limit which can not be exceeded by the actual veloc.

Fig 10 shows the STANDARD OR SHARP-EDGED ORIFICE. The contracted vein, or VENA CONTRACTA, of a jet issuing from a standard orifice, is due to the fact that the particles of water converge as they approach the orifice and so continue for a short distance beyond it. The contracted section occurs at a distance from the orifice of about 0.5 the diam of the orifice. Its diam is about 0.785 and its area about 0.62 that of the orifice, this latter figure being the coeff of contraction $= c'$. Experiments show that the aver veloc at the contracted section is about 0.98 ($= c_1$ = coeff of veloc) times the theoretical velocity; hence the veloc in ft per sec in the contracted section $= c_1\sqrt{2\ gh}$, the aver being $0.98\sqrt{2\ gh}$.

Since the discharge, in cu ft per sec, flowing from the orifice $= Q$ = area $\times$ veloc, the theoretical discharge $= av = a\sqrt{2\ gh}$, where a = area of orifice in sq ft. Practically $Q = c'ac_1\sqrt{2\ gh} = ca\sqrt{2\ gh}$, where $c = c'c_1$ = COEFFICIENT OF DISCHARGE, the aver

value of which is 0.605. Theoretically the shape of or head on the orifice has no effect. But experiments show that c is greater for low heads than for high, greater for rectangles than for squares, and greater for squares than for circles, and varies from 0.59 to 0.63 or more. Averages: for rectangular orifice 0.604, for circular 0.597. These averages are for water and, unless otherwise noted, coeffs given herein are for water at aver temp. For fluids in general, $c = 0.592 + 51.6 \div R_0^{0.74}$, where R_0 is a number of the Reynolds type, namely, $d\sqrt{2\,gh} \div \nu$, the $\sqrt{2\,gh}$ being substituted for v, to which it is proportional. Knowing ν for the liquid in question, this equation gives the aver coeff for a standard orifice.

The preceding figures apply also to the contracted section only. A fundamental law of hydrodynamics, the EQUATION OF CONTINUITY, is that for steady flow equal masses or weights of a fluid must pass all cross-sections in equal times. This reduces to equal volumes of liquids, since liquids are practically incompressible. Hence, the discharge in the plane of the orifice must equal that at the contracted section $= ca\sqrt{2\,gh}$, or since $v = Q \div a$, the veloc in the plane of the orifice $= v_0 = ca\sqrt{2\,gh} \div a = c\sqrt{2\,gh} = c'v$. In these formulas the HEAD ON THE ORIFICE should be taken as the head on the center of press,

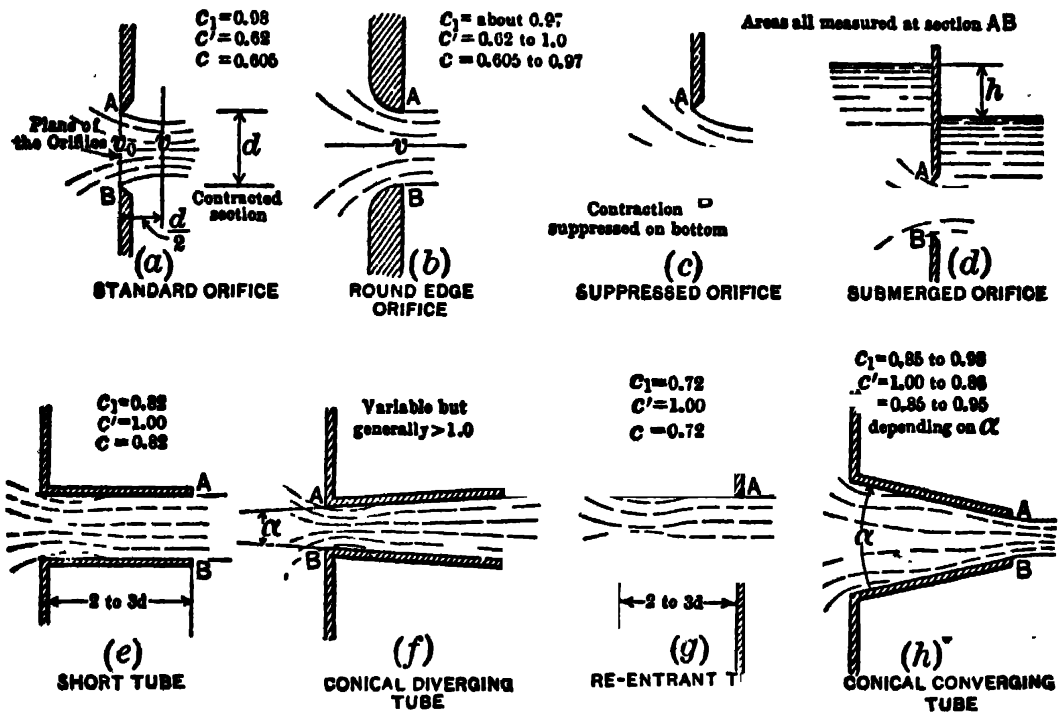


Fig 10. Orifices and Short Tubes

but if the head is greater than 2 or 3 times the vert dimension of the orifice, the head on the center of gravity of the orifice may be used. For a circular orifice the head on the center of press $= (r^2 \div 4\,h) + h$, where r = radius and h = head on cg. For rectangular orifices a convenient formula for small heads is $Q = c\,{}^2\!/_3\,b\sqrt{2\,g}\,(h_2^{3/2} - h_1^{3/2})$, where b = breadth, h_2 = depth to bottom and h_1 = depth to top of orifice. This allows for difference between cp and cg.

Loss of head in an orifice $= h' = (1 - c_1^2)\,h = (1 \div c_1^2 - 1)\,v^2 \div 2\,g = (1 \div c_1^2 - 1)v_0^2 \div c'^2\,2\,g$. For water and with a standard orifice, $c_1 = 0.98$, $c' = 0.62$ and $h' = 0.04$. Also, $h = 0.041\,v^2 \div 2\,g = 0.11\,v_0^2 \div 2\,g$.

Rounded-edge orifices (Fig 10*b*). By rounding the inner edge of an orifice, c' may have any value between 0.62 and 1.0, depending on how closely the rounding corresponds to the contour of the jet. An orifice for measuring discharge should not be round-edged, because of uncertainty in its value of c, but the increase in Q due to rounding may often be turned to advantage.

Suppression of contraction occurs when the orifice edges are nearer than 3 times the least dimension of the orifice from sides or bottom of tank. Preventing the jet from contracting increases c' and hence the discharge.

Submerged orifices. EFFECTIVE HEAD is the difference in level between the 2 water surfaces, and the action of the orifice is independent of its depth below the lower water

surface. In such case, c is slightly smaller than for free discharge into air and is usually taken as 0.60.

Velocity of approach. In above formulas h is taken as constant, but this can occur only when the inflow Q is constant. The water then approaches the orifice with an initial veloc, and the effective head is $H = h + h_v$, where $h_v = v_a^2 + 2g$ and v_a = veloc of approach. For a standard orifice,

$$v = c_1 \sqrt{\frac{2gh}{1 - c^2(a/A)^2}} \text{ and } Q = a \sqrt{\frac{2gh}{(1/c)^2 - (a/A)^2}}$$

where a = area of orifice and A = that of tank. When $a + A$ is 0.2 or less, the error due to neglecting v_a is negligible. In the case of water approaching a large orifice in a canal or conduit, the formula for a rectangular orifice under small head is $Q = c\,^2/_3\, b \sqrt{2g}\,[(h_2 + h_v)^{3/2} - (h_1 + h_v)^{3/2}]$, which is the basis for the Francis weir formulas with v_a.

Time required to empty any vessel of uniform horiz cross-sec is t (sec) $= [2A(\sqrt{H} - \sqrt{h})] + [ca\sqrt{2g}]$, where A = cross-sec area of vessel, a = orifice area, and H = initial and h = final head on orifice.

Flow under pressure may be estimated by the same formulas given above, the press being first reduced to equivalent head.

Example. Find Q when an orifice 3 in diam is first opened in a tank containing 3 ft water with 50 lb per sq in gage air press on top. Effective head $= 3 + 50 \times 144 + 62.5 = 118$ ft. $Q = 0.60 \times (0.785 + 16) \times 8.02 \times \sqrt{118} = 2.52$ cu ft per sec.

For fluids other than water the corresponding wt per cu ft is used to determine the effective head, and c must also be modified. Thus, in preceding example, if the fluid is mercury, $c = 0.62$ and wt $= 850$ lb per cu ft, or $h = 3 + 50 \times 144 + 850 = 11.5$, and $Q = 0.80$ cu ft sec. Same formula is used to find the discharge of air from an orifice, the coeff being about 0.60.

Short tubes are short sections of cylindrical or tapering pipe, used for connecting two vessels or other special purposes. Fig 10 shows some common forms, with their coeffs.

Nozzles are of the smooth or ring type. The former is the commoner. The discharge is given by $q = 29.83\, D^2 \sqrt{\dfrac{p}{\left(\frac{1}{c}\right)^2 - \left(\frac{D}{d}\right)^4}}$, where q = discharge, gal per min, p = press at the nozzle, lb per sq in, c = coeff of discharge $= c_1$ for a smooth nozzle, since there is no contraction, and $c' = 1$. (For smooth nozzle, c varies from 0.990 for a 1-in to 0.997 for a 6-in nozzle.) D = diam of nozzle, in, and d diam of pipe. If q is required in cu ft per sec, use 0.0665 instead of 29.83. This formula allows for velocity of approach, which is sometimes considerable. If p is measured by a Pitot gage held in the nozzle (Art 20), $q = 29.83\, cd^2 \sqrt{p_1}$, where p_1 = press recorded on gage in lb.

10. FLOW OVER WEIRS AND DAMS

Contracted weir (Fig 11) with end contractions is a rectangular notch in the upper edge of a vertical wall. The length of notch is less than that of the channel, and, that the end contraction may be complete, there should be a clearance of not less than 3 h on the sides, and the crest should be at least 3 h from bottom of channel. If the weir is the full width of the channel, contraction is suppressed on the ends and it is known as a SUPPRESSED WEIR, having no end contractions. There are many weir formulas. The rational formula is $Q = c\,^2/_3\, b \sqrt{2g}\, h^{3/2}$, where c is a constant or coeff of discharge, b = breadth, and h = head in ft. It is derived from the orifice formula by letting the head on the top of the orifice = 0. Francis uses $Q = 3.33\, bh^{3/2}$ for a suppressed weir (Table 1), and for a contracted weir, considering the effect of end contractions as reducing effective length of the weir by 0.1 h on each end, $Q = 3.33\,(b - 0.2\,h)\,h^{3/2}$. For Miner's Inch, see Art 20. The coeffs c for weirs do not appear to follow closely a Reynolds Number relationship, being affected also by gravity and surface-tension effects. Data on liquids other than water are lacking.

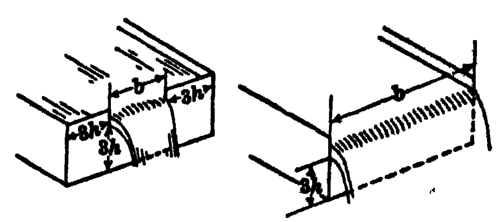


Fig 11. Weirs

Table 1. Discharge in Cu Ft per Sec per Ft of Length of Thin-edge Weirs, from $Q = 3.33\ bh^{3/2}$

Head h, ft	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000	0.003	0.009	0.017	0.027	0.037	0.049	0.062	0.075	0.090
0.1	0.105	0.121	0.138	0.156	0.174	0.193	0.213	0.233	0.254	0.276
0.2	0.298	0.320	0.344	0.367	0.391	0.416	0.441	0.467	0.493	0.520
0.3	0.547	0.575	0.603	0.631	0.660	0.689	0.719	0.749	0.780	0.811
0.4	0.842	0.874	0.906	0.939	0.972	1.005	1.039	1.073	1.107	1.142
0.5	1.177	1.213	1.248	1.285	1.321	1.358	1.395	1.433	1.471	1.509
0.6	1.548	1.586	1.626	1.665	1.705	1.745	1.785	1.826	1.867	1.909
0.7	1.950	1.992	2.034	2.077	2.120	2.163	2.206	2.250	2.294	2.338
0.8	2.383	2.428	2.473	2.518	2.564	2.610	2.656	2.702	2.749	2.796
0.9	2.843	2.891	2.938	2.986	3.035	3.083	3.132	3.181	3.231	3.280
1.0	3.330	3.380	3.430	3.481	3.532	3.583	3.634	3.686	3.737	3.789
1.1	3.842	3.894	3.947	4.000	4.053	4.107	4.160	4.214	4.268	4.323
1.2	4.377	4.432	4.487	4.543	4.598	4.654	4.710	4.766	4.822	4.879
1.3	4.936	4.993	5.050	5.108	5.165	5.223	5.281	5.340	5.398	5.457
1.4	5.516	5.575	5.635	5.694	5.754	5.814	5.874	5.935	5.996	6.056
1.5	6.118	6.179	6.240	6.302	6.364	6.426	6.488	6.551	6.613	6.676
1.6	6.739	6.803	6.866	6.930	6.994	7.058	7.122	7.186	7.251	7.316
1.7	7.381	7.446	7.512	7.577	7.643	7.709	7.775	7.842	7.908	7.975
1.8	8.042	8.109	8.176	8.244	8.311	8.379	8.447	8.515	8.584	8.652
1.9	8.721	8.790	8.859	8.928	8.998	9.068	9.137	9.207	9.278	9.348

Velocity of approach affects weir discharge, and for accurate work should be kept below 1 or 2 ft per sec. The effect of v_a can be allowed for by adding to the actual head h on the weir 1.5 h_v, which is obtained from Table 2, using the aver veloc of approach v_a as an argument.

Table 2. Head h_v Due to Velocity of Approach v_a

0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2
0.002	0.005	0.010	0.015	0.022	0.030	0.040	0.050	0.062	0.075
2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	
0.089	0.105	0.122	0.140	0.150	0.179	0.201	0.213	0.248	

Examples. (*a*) Suppressed weir, no veloc of approach, $b = 4$ ft, $h = 1.73$ ft. Find Q. From Table 1, Q per ft of length $= 7.577$ and total $= 7.577 \times 4 = 30.31$ cu ft per sec.

(*b*) Contracted weir, same as above. Q per ft of length $= 7.577$, but effective length $= b - 0.2\,h = 4 - 0.35 = 3.65$ ft, and total $Q = 27.65$ cu ft per sec.

(*c*) Contracted weir as above, with aver veloc of approach of 2 ft per sec. From Table 2, $h_v = 0.062$ and $1.73 + 1.5 \times 0.062 = 1.82$, which from Table 1 gives 8.176 per lin ft, or a total of $8.176 \times 3.65 = 29.80$ cu ft per sec.

To obtain accurate results, the weir must be constructed in a standard manner, similar to that used in obtaining the experimental coefficients: the notch must have sharp edges as for a standard orifice; crest must be level and sides vert; length should be between 4 and 8 h; veloc of approach should be small (area of cross-sec of channel of approach should be at least 6 bh); sides and crest should be 3 h from sides and bottom of channel; head should be measured accurately, 4 to 6 ft upstream, to eliminate effect of slope in the water surface toward the weir (for common work a stake may be driven in the bed of the channel of approach, Fig 12, and cut off level with the weir crest, h being obtained by measuring the water level on a graduated rod held on this stake. For accurate work a hook gage should be used); for suppressed weirs there must be free access of air to the under side of the overflowing sheet of water or NAPPE.

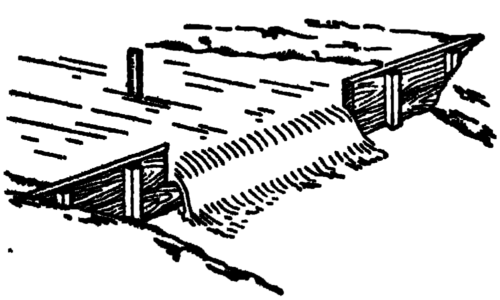


Fig 12. Measuring Flow over Weir

Example. Design a contracted weir to measure the flow of a stream 6 ft wide by 2 ft deep, the aver veloc of which, obtained roughly by a float, was 3 ft per sec. Q = approx 36 cu ft per sec. For a depth of 1 ft, Q (Table 1) $= 3.33$ cu ft per sec per ft of length, and the required length would be $36 \div 3.33$, or 11 ft, which is too long (see above). By further trial, using Table 2, it is found that a weir 8 ft long, with $h = 1.25$ ft, will give the required Q (ratio about 1 : 6.4). Crest of the weir must be about 4 ft ($3\,h = 3.75$) above bottom of stream and width of channel of approach at least 16 ft, as shown by $8 + (2 \times 3 \times 1.25) = 15.5$. If the channel is narrower when flooded, it should be widened or a suppressed weir used.

Triangular weir (Fig 13) is convenient for small quantities of water. It should have sharp inner corners, sides of equal slope, and bottom angle should be a right angle. For this weir $Q = 2.53h^{5/2}$, h being measured from bottom of notch.

Trapezoidal or Cippoletti weir (Fig 14) saves time in computations, as a table may be made or a stick graduated to read the discharge directly, as for a rectangular suppressed

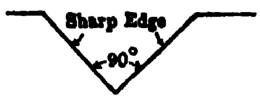


Fig 13. Triangular Weir

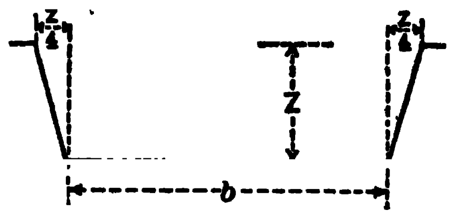
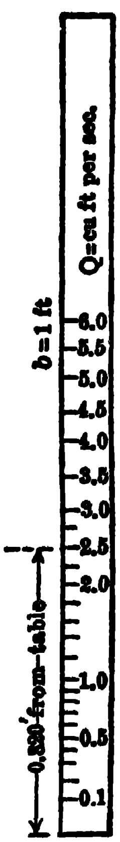


Fig 14. Trapezoidal Weir

weir. Sides are sloped to make the discharge through the triangular end portions equal to the loss due to end contractions ($3.33 \times 0.2 \times h \times h^{3/2}$). End slopes should be 1 on 4, as shown. Formula $Q = 3.367\,bh^{3/2}$ is then used, b being the bottom width. Fig 15 shows a stick graduated to read discharge directly for a trapezoidal weir 1 ft long ($b = 1$).

Spillway dams are a type of weir, but are seldom suited to flow measurement, except approximately, as the coeff in Francis formula (3.33 for standard weir) may vary from 2.5 to 4.2. A sloping upstream face on the dam generally increases the discharge, which is also modified by width and shape of crest and inclination of downstream face. In designing spillways, the flood flow Q is approx known, the breadth b is usually fixed by the flood width of channel below the dam, and the probable flood height h over the spillway may be computed from Francis formula. As it is difficult to determine flood discharge, an ample factor of safety is desirable, and the coeff used should be small enough to be on safe side (3 is a common coeff for masonry spillways).



Q, cu ft per sec	H ft.
6.0	1.470
5.5	1.387
5.0	1.302
4.5	1.214
4.0	1.122
3.5	1.026
3.0	0.926
2.75	0.874
2.5	0.820
2.25	0.764
2.0	0.707
1.8	0.659
1.6	0.609
1.4	0.557
1.2	0.502
1.0	0.445
0.9	0.415
0.8	0.384
0.7	0.351
0.6	0.317
0.5	0.280
0.4	0.242
0.3	0.200
0.2	0.152
0.1	0.096

Fig 15. Measuring Stick for Weirs

11. FLOW IN PIPES

Bernoulli (1738) stated the law of steady flow: "At any section of a tube or pipe, under steady flow without friction, the press head plus the veloc head is equal to the hydrostatic head that obtains when there is no flow." This law, together with the equation of continuity (Art 9) and a formula for loss of head in friction, is fundamental in determining flow conditions in pipes.

Loss of head by friction in pipes is inconsiderable for short tubes, and is provided for in the coeff of discharge, but for long pipes it is important and as the length increases consumes most of the available head. The proposed formulas for loss of head in friction are so numerous, and some are so complicated, that the engineer may be at a loss which to use. All reduce practically to one of the two forms given below; their chief value is not in their form, but in the fact that coefficients have been determined from experiments on certain kinds of pipes with use of a particular form of equation, so that in designing a given pipe the equation for which the best coefficients are available should be used.

Hydraulic gradient and friction head. In a pipe of uniform diam the frictional loss is at a uniform rate and hydraulic conditions may be graphically represented as in Fig 16, which shows a uniform pipe leading from a reservoir or tank. With no flow, the static head at end $A = h$. With free flow at A, h is partly converted into veloc of flow (veloc head $h_v = v^2/2g$), but is largely consumed in overcoming frictional and other resistances. A small amount is thus lost at entrance into the pipe (h_e = approx $0.5\,v^2/2g$ for flush entrance), but most of it is frictional loss. Laying these values off to scale gives the construction shown and determine the line BA, or hydraulic gradient. This is the pressure line to which liquid in pressure tubes inserted in the pipe would rise during flow and is important in design.

Entrance loss and veloc head are usually small and, in practice, it is common to neglect them and assume entire head h, available for overcoming frictional resistance. Hence data on frictional losses become vitally important. For all practical purposes, therefore, the hydraulic gradient may be drawn from free water surface to free water surface. Note also that the entire head on end of pipe can seldom be used to produce flow alone. Some terminal press is usually required in connection with use of water, as for developing power, fire service, and hydraulic mining, and h is to be taken as head available for flow only (Fig 20, 27).

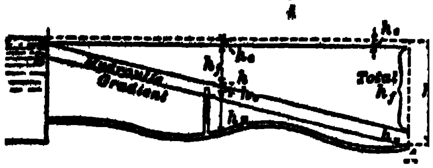


Fig 16. Hydraulic Gradient

Minor losses (usually neglected) also include: Curvature. Least for a 90° bend if the radius is about 3 d. For a 90° bend in 6-in pipe, the loss is nearly equal to that in an 8-ft length of straight pipe, and for 30-in pipe in a length of 40 ft. For intermediate sizes the loss varies roughly with diam. Loss in expansion from a small pipe with veloc v_1, to large pipe with veloc v_2, is $(v_1 - v_2)^2 \div 2g$. Loss in contraction from large to small pipe may be taken $= 0.5\,v^2 \div 2g$, where v = veloc in small pipe. Loss in gates and valves is uncertain, but for full gate is about the same as in a length of straight pipe of 6 d.

Loss of head in friction. Reynolds has shown that there are 2 major types of flow: stream-line or laminar, and turbulent. Underground flow in sands and gravels, and low-veloc flow of heavy oils in pipes, are usually stream-line, while the more rapid flow in pipes and open channels common in engineering practice is turbulent. Transition from one type to the other depends upon character of liquid and dimensions and veloc of flow. It is thus best expressed in terms of Reynolds number R (Art 2). Aver value of this number for pipes is about 2 000 or, for water at normal temps, the veloc × diam is 0.02 to 0.03.

Laminar flow. Experiments show that fluid flow is independent of press; also, it may be demonstrated that the head h_f lost (uselessly dissipated) in laminar flow in pipes $= 32\mu l v \div w d^2$, or (approx) $l^2 v \div Rd$; or the loss in press in lb per sq in per ft of pipe $= 32\mu v \div d^2$. Such flow is purely a viscous shearing phenomenon; veloc at pipe walls is zero, and loss is thus unaffected by the character (roughness or smoothness) of pipe. In porous materials, of flow V in time t in a cross-sectional area of flow A is $kAth \div l$ (Darcy's law), where k is a coeff of permeability depending upon the porosity, shape and grading of material, and the temp of flow.

Turbulent flow. Similar analysis and experiment show that h_f in turbulent flow varies: (a) directly with length of pipe, l; (b) inversely with diam d; (c) directly with the nth power of the veloc v; (d) coeff of friction f, which varies with R and with roughness of the pipe surface. As n appears to approach 2 as a limiting value, it is convenient to express this relationship in terms of the veloc head v_2, by dividing by the constant $2g$, thus: $h_f = f \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$. For smooth pipe (as glass, brass, copper) the value of f is given by the experimentally determined "Blasius-Nikuradse" curve (Fig 17). In designing oil lines (Sec 44), for which steel tubing with usual couplings is employed, the "W M S" line (Wilson, McAdams & Seltzer, *Jour* Ind Eng & Chem, Feb, 1922, p 114) gives values often used. The "Ludin" line is for a "wavy-smooth" composition pipe. Note that the scale dimension used in R for pipes is the diam d; thus $R = vd \div \nu$.

Example. Determine the press drop in lb per sq in per 1 000 ft of line in pumping 500 gal per min of an oil of 29°.1 Baumé gravity and an absolute viscosity of 0.3 poises, at a temp of 50° F, through a smooth pipe of 6-in diam. From conversion chart (Fig 1) 29°.1 Baumé = 0.88 sp gr = 1.71 slugs per cu ft. Also 0.3 poises ÷ 478.8 = 0.00063 ft-lb-sec units. Hence kinematic viscosity = 0.00037. Also 500 gal per min ÷ (7.48 × 60) = 1.11 cu ft per sec. Since d = 0.5 ft, veloc = 5.7 ft per sec. Thus $R = 5.7 \times 0.5 \div 0.00037 = 7\,700$. With this value as argument, Fig 17 (WMS line) gives $f = 0.037$. Substituting in above equation, $hf = 0.037 \times 1\,000 \times (5.7)^2 \div (0.5 \times 64.4) = 37$ ft per 1 000 ft. Or, since $p = wh$, p(lb per sq in) $= 0.434 \times$ sp gr $\times h = 0.434 \times 0.88 \times 37 = 14$ lb per sq in per 1 000 ft.

Flow of water in pipes. Except under extreme conditions of temp, ν for water is fairly constant; hence $v \times d$ may be substituted for R as an argument in determining friction factor f. Conduits used for water frequently have rough surface or, in many cases, corrode or deteriorate with use, and become rough. The smooth-pipe data used for oil lines are thus seldom applicable in the design of water conduits. Such conduits are also often of large size, beyond the range of laboratory experiments. The most valuable data have thus been secured from tests of actual lines in use; they are often fragmentary and seldom cover a wide range of v or d. Yet, empirical formulas derived therefrom have long been used, and recently have been shown to correspond with laboratory tests.

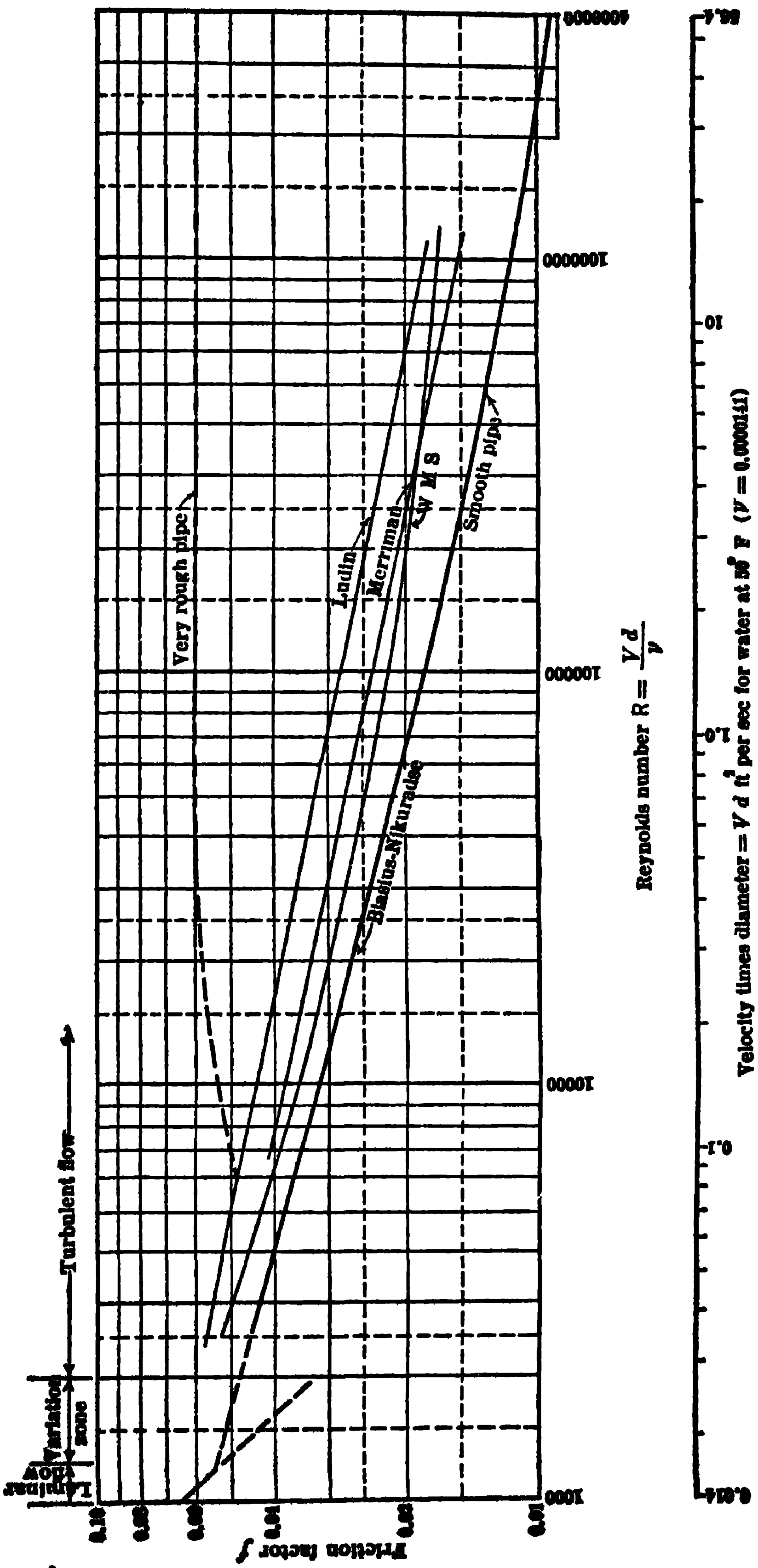


Fig 17 Design Data for "Smooth" Pipe

Chézy formula. For a uniform degree of roughness, the Chézy formula is most widely used for both pipes and open channels: $v = C\sqrt{RS}$, where v is aver veloc of flow, ft per sec. R is HYDRAULIC RADIUS, or hydraulic mean depth, a scale element of flow which, by definition = area a of flowing stream ÷ wetted perimeter p. Slope S is head available for flow, ft per ft = sine of slope of the hydraulic gradient, or practically = $h \div l$, the available head divided by the length of pipe or channel. It is sometimes expressed in ft per mile, per 1 000 ft, or in per cent. C is a coeff of smoothness, increasing with smoothness of surfaces and averaging 80–120 in value. Replacing d of the usual pipe formulas by R makes possible the application of the Chézy equation to any section of flow. For circular pipe, $R = \pi d^2/4 \div \pi d = d/4$. For rectangular flume, 6 ft wide and 3 ft deep, $a = 18$, $p = 12$, and $R = 1.5$ ft. By substituting $d \div 4$ for R, and $h \div l$ for S in the Chézy formula, this is found to be of same type as the pipe formula given above, except that $8g \div C^2$ is substituted for f. Therefore C will vary with R, as does f. This formula is applied, however, only to water, and C thus varies with the quality of the surface of channel n, and with v and d.

Simplest of the several empirical equations for C is the MANNING formula: $C = 1.486\ R^{1/6} \div n$, where n is a surface factor varying from 0.010 for smooth wood pipe, 0.012–0.015 for concrete surfaces, 0.016 for ordinary riveted steel pipes, 0.020–0.025 for good earth canals, to 0.03 for rock cut channels and 0.035–0.060 for rocky streams.

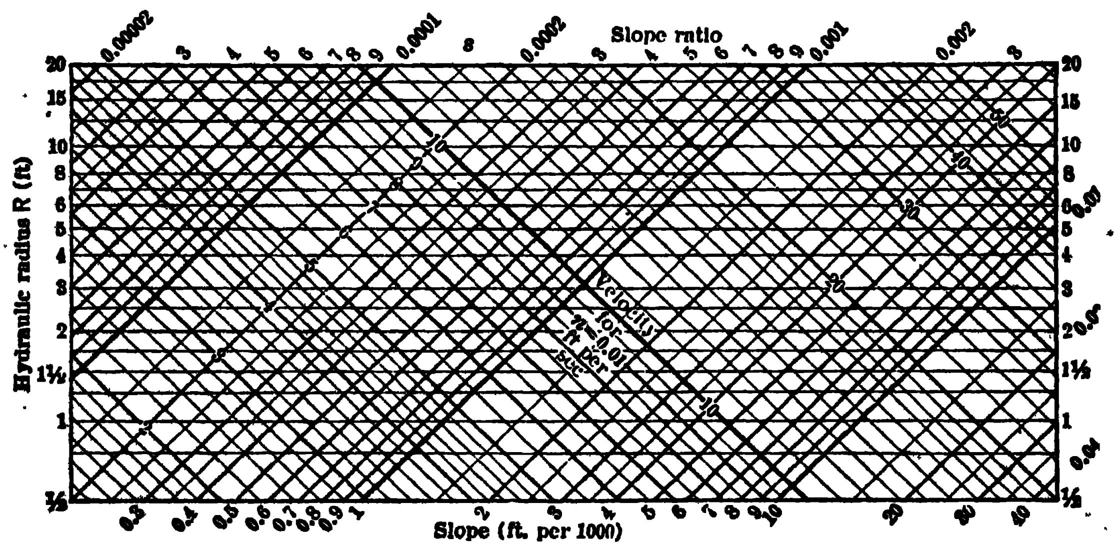


Fig 18. Graphic Solution of Chézy–Manning Formula

This equation makes C independent of v. Laboratory experiments show that for a uniform surface quality and large sizes of conduits, f and C tend to become constant. The relative effect of roughness of surface depends, however, upon the size of the channel; hence the form of the Manning equation.

Combining the Chézy and Manning formulas gives $v = 1.486\ R^{0.67}S^{0.5} \div n$, which, being exponential in form, is easily solved by a logarithmic diagram. Fig 18 shows the relationship of v, R, and S, for usual ranges in value and for $n = 0.01$. Since v varies inversely with n, the diagram may be used for any value of n by introducing a proportional factor.

Example 1. A wood-stave pipe is 6 ft diam and 2 miles long; available head, 10 ft. If $n = 0.011$, what are v and Q? $R = 6 \div 4 = 1.5$. $S = 0.95$ ft per 1 000. Enter diagram at slope 0.95 (interpolated between 0.90 and 1.0), following slope line to intersection with R value of 1.5. This point falls on v scale between 5 and 5.5 and v is estimated at 5.3 ft per sec. Since the diagram is made for $n = 0.01$, v for 0.011 would be $^{10}/_{11}$ of 5.3 or 4.9 ft per sec. Since $a = 28.2$ sq ft, $Q = a \times v = 138$ cu ft per sec. *Example 2.* A riveted steel pipe 4 ft diam and 5 000 ft long is to carry 60 cu ft per sec; $n = 0.016$. What head is required? $R = 1.0$ ft. $a = 12.6$ sq ft; hence $v = 60 \div 12.6$ or 4.8 ft per sec. Since diagram is for $n = 0.10$, whereas, here, $n = 0.016$, the v used in diagram should be $0.016 \div 0.01$ or $1.6 \times 4.8 = 7.7$ ft per sec. Enter diagram with $R = 1.0$ and run to intersection with interpolated line for $v = 7.7$. Position of this intersection on S scale gives $S = 2.7$ ft per 1 000 ft, or total h required $= 2.7 \times 5 = 13.5$ ft. For other problems, see Art 15.

Limits of application. Chézy formula is best used for velocities between 1 and 6 ft per sec, but is fairly reliable up to 10 ft. For hydraulic radii greater than 10 ft, velocities

greater than 10 ft per sec, or slopes flatter than 1 in 10 000, it should be used with caution. For R or v greater than 20, it is unreliable. Results from the formula must not be expected to be consistently closer than 5%. For values of n recommended for use in design see Art 14, 15. It can be shown that S varies as n^2 for values of R greater than 1 ft, and that v varies approximately inversely as n. An uncertainty of x% in selecting n will therefore result in an uncertainty in a computed slope of twice this amount, or 2 x%, and in a computed velocity of x%.

Exponential formulas. While the Chézy-Manning formula fits conditions for larger and rougher conduits, measurements of smaller pipes, of smoother surface, or of scattered roughness such as caused by pitting, indicate that f (or C) does not tend toward a constant value, but varies with changes in both diam and veloc.

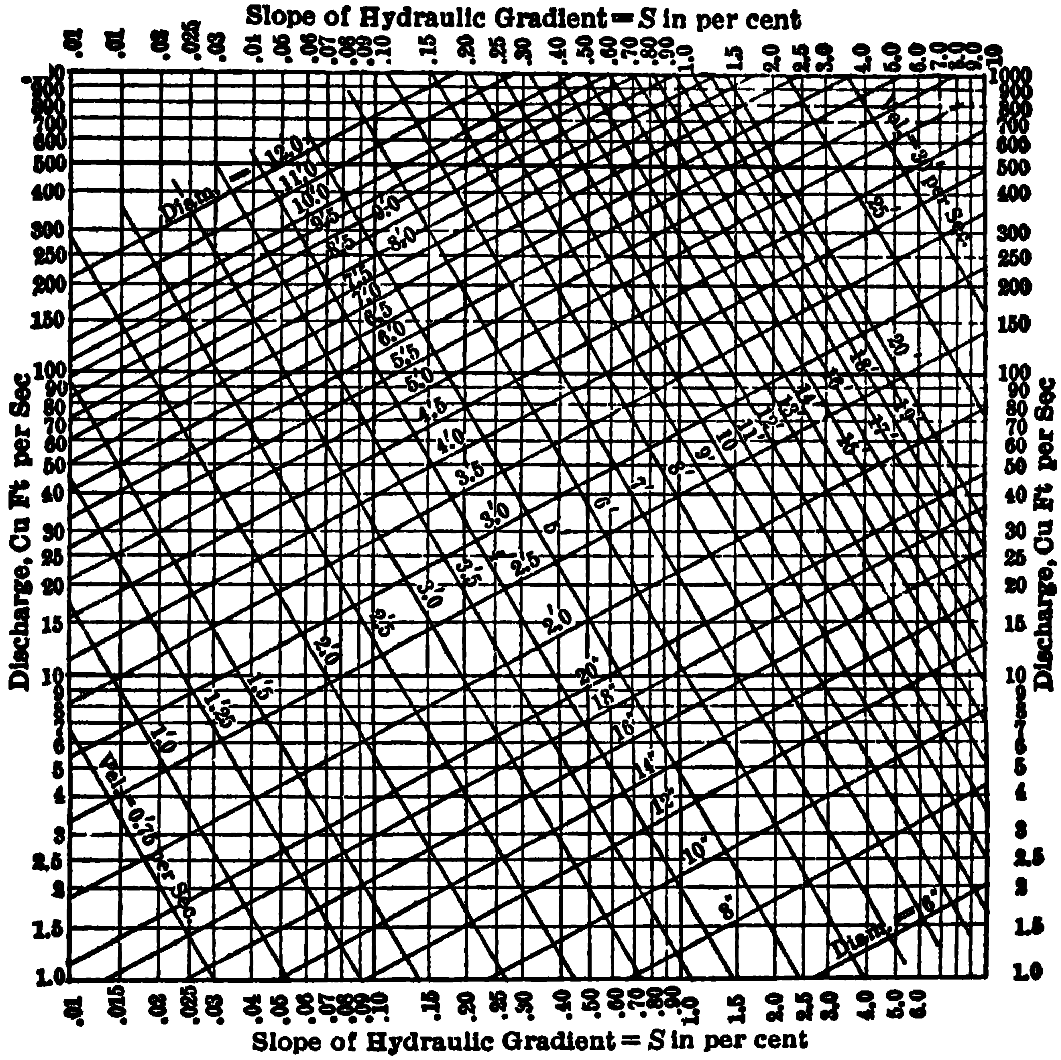


Fig 19. Diagram for Solving Hazen-Williams Formula

For example, the "Merriman" line in Fig 17 is an aver for smooth, new-coated, C-I pipe; a similar line with higher values of f and a different slope would represent older, pitted pipe. Thus, the "smooth pipe" line is the lower limit and a line such as the upper dotted "rough pipe" line of Fig 17 represents a typical upper limit in the range of f for pipes. (The Chézy-Manning equation applies to the horiz portion of this rough-pipe line and intermediate lines of similar form.) Intermediate types of pipe often appear to fall in a class giving data corresponding to the sloping "Merriman" curve, intermediate between these limits. An exact correlation of such data would require a special equation for each size of pipe, but aver data may be approximated by an exponential equation, like the Chézy-Manning, but with different constants and exponents.

Among the better known empirical equations designed to fit aver quality and range of v and d for smaller pipes is the Hazen-Williams formula, $v = 1.318cR^{0.63}S^{0.54}$ (Hydraulic Tables. Wiley, N Y, 1905.) The quantity 1.318 is introduced to make the value of c about the same as that of the Chézy constant C.

Hasen-Williams advise following values of c: for very best C-I pipe, 140; good new C-I pipe, 130; tuberculated C-I pipe, 80 to 110; designing C-I pipe, 100; new riveted steel pipe, 110; ordinary W-I pipe, 100; lead, brass, tin pipe, 140; smooth wood pipe, 120; vitrified pipe, 110; brick sewers, 100; smooth clean masonry, 140; slime-coated masonry, 130; ordinary good masonry, 120.

The diagram, Fig 19 (S. D. Bleich, *Eng Rec*, Nov 30, 1907), is convenient for solving Hasen-Williams formula. It is constructed for $c = 100$; for other values of c multiply the required discharge by $100 \div c$, and the resulting value of Q must be used to get the size of pipe and veloc. Then multiply this veloc by the ratio $c \div 100$ to get the true value.

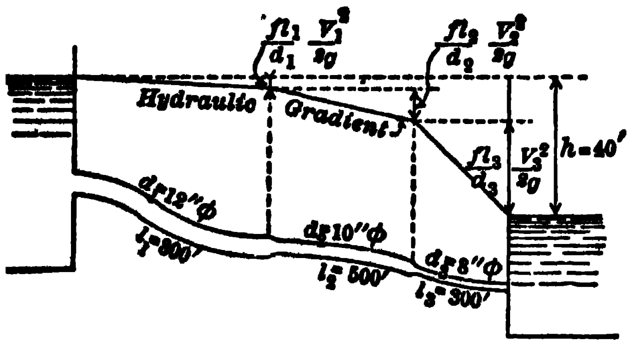


Fig 20. Hydraulic Gradient for Compound Pipe

Example 1. A C-I pipe is to carry 10 cu ft per sec, length 1 000 ft, available head 10 ft. Using $c = 100$, find required size. $S = 1$ ft per 100, and running vert from this slope in the diagram to the intersection with the horiz line representing a discharge of 10 cu ft per sec, the intersection is found to be slightly below the 18-in diam line, indicating that an 18-in pipe is needed, and the mean veloc is slightly less than 6 ft per sec. *Example 2.* A wood pipe is 4 000 ft long, and is to discharge 60 cu ft per sec with 20-ft available head. Find d and v. In this case $c = 120$. Multiply Q by $100 \div 120$, giving $Q = 50$, and enter the diagram with this value and a slope $S = 0.5$ ft per 100, as before. Hence, $d = 38$ in and $v = 6.5$ ft per sec. Actual v will be $6.5 \times 1.20 = 7.8$ ft per sec.

The Hasen-Williams formula was derived largely from a study of C-I pipe, and is best applied to design of this type. In general, however, the designer selects the nearest standard size of pipe and no great refinement in calculation is necessary. Similar formulas for other kinds of pipe have been derived by Scobey: for wood-stave, $v = 184R^{0.65}S^{0.55}$; concrete pipe, $v = 127R^{0.625}S^{0.5}$. See *Tech Bulletins* of U S Dept of Agriculture.

Compound pipes. Algebraic expressions can be derived, giving exact relations between discharge and head for compound pipes, but it is simpler to solve these problems by determining size of a single pipe that will be equivalent to the given combination; that is, such a size as will give same loss of head for a given discharge.

Example. Find the discharge from the compound pipe in Fig 20. Take $c = 100$ in Hasen-Williams formula. Assume any convenient and reasonable Q, say 3 cu ft per sec. From Hasen-Williams diagram find for the 8, 10, and 12-in pipes the required grades or loss of head in per cent, viz, 5, 1.8, and 0.7 for this Q, which, multiplied by the respective lengths, give 15, 9, and 5.6 ft, or a total of 29.6 ft necessary head. As the total length is 1 600 ft, this represents an aver loss of 1.85 ft per 100, and the uniform pipe which would give a discharge of 3 cu ft per sec with this hydraulic gradient would be (from diagram) about 9.8 in diam. Actual available head is 40 ft ($S = 2.5\%$), and Q for a 9.8-in pipe with this head is 3.5 cu ft per sec, the required value.

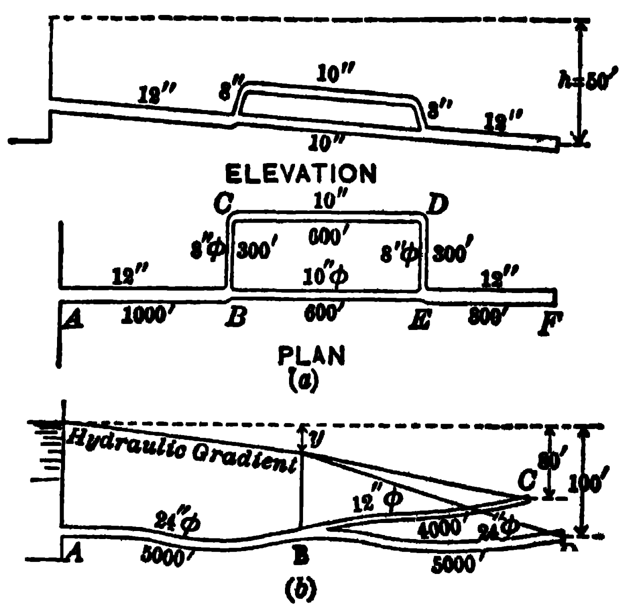


Fig 21. Branch Pipes

Branch pipes, as shown in Fig 21*a*, may be solved similarly. First find size of uniform pipe to replace *BCDE*, as above. This is found to be 8.8 in. Now the loss of head between *B* and *E* must be the same by both routes, *BE* and *BCDE*. Assume any reasonable loss of head, say 15 ft, and find discharge by each route. For the *BE* pipe (10 in, 600 ft long), $S = 2.5$ ft per 100, and from the diagram $Q = 3.7$ cu ft per sec. For 8.8-in

pipe 1 200 ft long, $S = 1.25$ and $Q = 1.8$. Total discharge = 5.5 cu ft per sec, and diam of pipe 600 ft long which will deliver 5.5 cu ft per sec under a head of 15 ft is found from the diagram to be 11.5 in. The problem is thus reduced to that in Fig 20, which when computed as above for compound pipes, assuming a Q of 6 cu ft per sec, gives uniform pipe of 11.9 in, the discharge for 2 400 ft of which under a 50-ft head is 5.2 cu ft per sec.

Fig 21b shows another problem in branch pipes. It may be solved algebraically, but more easily by trial. A certain portion y of the head up to the fork must be used up in causing a discharge through AB that will just equal the sum of the discharges through BC and BD under the remaining heads $80 - y$ and $100 - y$. Value of y is best found by trial. From Fig 21b, since lengths AB and BD are same, it is evident that y must be more than 50, and by a few trials it is found that $y = 58$ will give satisfactory results, the total discharge being 23 cu ft per sec ($BC = 2.6$ and $BD = 20.5$).

12. FLOW IN OPEN CHANNELS

In open channels, as canals or ditches, or in pipes, aqueducts or tunnels flowing part full and, hence, not under press, the hydraulic gradient coincides with water surface and slope S is slope of this surface. The Chézy formula is commonly used in design (Art 16).

Earlier practice used the Kutter rather than the simpler Manning formula for C, and in legal cases in West, this is still widely recognized in courts. (Flow of Water in Rivers and other Channels. Transl by Hering and Trautwine, Wiley & Sons.) The KUTTER FORMULA is:

$$C = \frac{\dfrac{1.811}{n} + 41.65 + \dfrac{0.00281}{S}}{1 + \dfrac{n}{\sqrt{R}}\left(41.65 + \dfrac{0.00281}{S}\right)}$$

where n, R, and S are defined as in the Chézy-Manning formula.

Usual problems in non-pressure, or open channel flow, deal with steady, established, *uniform flow*, that is, constant flow in channels of uniform slope and cross-sec. The surface and bottom slopes of the channel are the same, and S is utilized entirely for overcoming frictional resistances. Thus the "energy of flow," $E = d^2 + v^2 + 2g$ is constant, where d is depth of the channel and v the veloc of flow. This requirement may be met by two depths or stages (levels or elevations) of flow. Thus, for 50 cu ft per sec discharge, a d of 9.6 ft with an aver v of 5.2 ft per sec, or a d of 2.2 ft with $v = 22.7$ ft per sec are possible. The former is known as the TRANQUIL STAGE OF FLOW and commonly occurs in slow flowing, low-slope streams, canals and other conduits. The latter, TORRENTIAL FLOW, requires steep slopes and is found in the overflow, or spillway, sections of dams, in inclined chutes, or in the escape from partially opened sluices. The slope required to maintain such flow is generally lacking and a sudden rise in water surface, known as the HYDRAULIC JUMP, occurs when flow changes from torrential to tranquil stage. This phenomenon is important in the design of spillway dams and outlets.

Occasionally, problems in steady NON-UNIFORM FLOW are encountered, as constant flow in channels of varying cross-sec. Then, E is not constant, but energy may be added to or taken from the flowing stream. The water surface may form a BACKWATER CURVE, where veloc is retarded as depth increases, or a DROP-DOWN CURVE, where reverse conditions hold. The CRITICAL DEPTH, or depth for minimum E, is an important criterion in determining whether the flow will form a jump or a backwater curve, a drop or a drop-down curve. For these special problems of non-pressure flow, see "Hydraulics of Open Channels," B. A. Bakhmeteff, McGraw-Hill, 1932.

DESIGN

13. KINDS OF PIPE

Standard sizes. In computing the necessary diam for a pipe no great degree of refinement is ordinarily necessary, as it reduces to a question of selecting a standard size. The relative discharge capac of pipes varies as the $^5/_2$ power of the diam (about) and is shown below, using a 4-in pipe as 1. An 8-in pipe will carry 6.5 + 3, or over twice as much as a 6-in pipe; hence no very accurate computations are necessary to determine which is required.

Diam..................	4	6	8	10	11	14	16	20	24	30	36
Relative capacity......	1	3	6.5	12	20	30	43	80	130	235	390

Cast-iron pipe. Standard sizes are 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 30, 36, 42, 48, 54, and 60 in, usually made in 12-ft lengths, with bell and spigot or flanged joints (Sec 41).

The latter is used for pump connections, or wherever it may be occasionally necessary to remove sections of pipe. Bell and spigot is common for water and gas mains. The joint is made by first packing with oakum, then pouring a collar of lead around the bell, which is afterwards calked tight (Table 3). Cement joints are commonly used for gas mains. Pipe is made in standard sizes and thicknesses (Table 3); for large orders, any desired thickness is obtainable.

C-I pipe is usually in 12-ft lengths, cast with bell down, and cleaned and dipped in hot tar and linseed oil for a protective coating. In recent years, spun or centrifugally cast pipe has become the predominating type, and length has been increased, reducing number of joints. This process gives a denser, stronger metal, and thicknesses given in Table 3 can be correspondingly reduced (see catalogues of makers).

C-I pipe is subject to formation of tubercles of rust (particularly when used for impure water or water containing free CO_2), which seriously affects its carrying capac, but usually without greatly impairing its strength. Probably Hazen-Williams formula (Art 11) is the best to use in design, and while the value of c may reach 140 for new pipe, about 100 must be used if the carrying capac is to be maintained over a period of years. In laying C-I pipe, "bell holes" should be dug for the joints, so that the pipe will have a firm support over its entire length and not rest on the ends; for large sizes the back fill should be well tamped around the pipe and for a depth of 2 or 3 ft over it, to relieve the pipe from excessive external loading. C-I pipe has an aver life of 40 to 80 years or more. Cost varies considerably, averaging between $45 and $60 a ton. Special shapes, as Ys, Ts, curves and blow-off branches, aver $100 a ton. See Sec 41.

W-I pipe was formerly made in considerable quantities, but has been practically replaced by mild steel (frequently sold as wrt iron). The relative durability of the two materials has long been a matter of dispute, but since 1916 only 1 or 2 firms in U S make a real W-I pipe, so that there is seldom opportunity to choose between them.

Steel pipe is welded, ordinary riveted, spiral riveted, and lock bar. BUTT or LAP-WELDED PIPE is of mild steel, from $^1/_8$ to 15 in nominal diam (actual internal diam varies, Sec 41). It is used for steam and water connections in mechanical work, for interior piping in water supply, gas and heating, and for supply pipe lines and pumping. It is standard for most service, when the required size is 10 or 12 in or less. Quite recently the making of lap-welded pipe of larger sizes has been developed; it is now made to 72 in diam, and is used in water-power work for high heads, where the saving in metal due to its greater joint strength and perfectly smooth interior make it more economical than riveted pipe.

Riveted steel pipe is used chiefly in large sizes and for high heads. It is made of curved sheets, the longit edges of which are joined usually by a double-riveted lap joint, although in the larger sizes butt joints double or triple-riveted with single or double cover plates are used. Sizes to 30 in diam are sometimes made of single sheets in 12- or 15-ft lengths. Larger sizes are made with single-riveted lap transverse joints. Field joints between the lengths are made in several ways. For low press and pipes to 40 in diam, a slip joint may be used. A sleeve is placed inside and attached to one end of the pipe, which is covered with burlap or canvas coated with red lead or asphaltum and then driven into the end of the next length and held by wire ties. For higher press, the pipe ends are riveted to forged steel flanges, connected by bolts and gaskets. Fig 22 shows several joints. Bolted joints are also used, which not only act as expansion joints but also allow a slight deflection and permit straight pipe to be laid on easy curves. Lengths of large pipe are riveted or welded together in the field. For design of riveted joints see Sec 43.

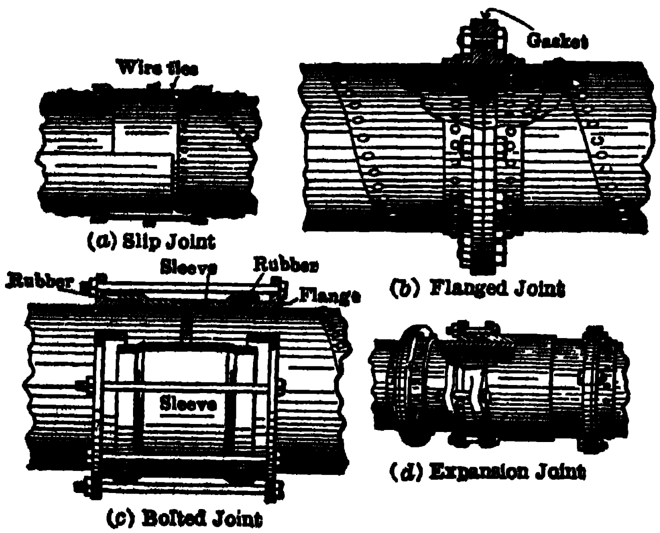


Fig 22. Steel-pipe Joints

Spiral-riveted pipe is frequently used for smaller heads and in sizes to 42 in (Sec 43). It is made in 15-ft lengths galvanized, and 30 or 40 ft, asphalt coated. Each length is of a single sheet, rolled with overlapping edges forming a single-riveted spiral seam. This gives an effic joint against bursting, and the seam is said to be the strongest part of the pipe. In large sizes it is not thicker than 0.25 in, and though used for ordinary water or

Table 8. Standard Cast-iron Bell and Spigot Pipe (U S C-I Pipe & Foundry Co)

Nominal inside diam, in	Class A 100-ft head, 43 lb press			Class B 200-ft head, 86 lb press			Class C 300-ft head, 130 lb press			Class D 400-ft head, 173 lb press			Approx lb lead per joint, 2 in thick	Approx lb hemp per joint
	Thick-ness, in	Lb per		Thick-ness, in	Lb per		Thick-ness, in	Lb per		Thick-ness, in	Lb per			
		Ft	L'th		Ft	L'th		Ft	L'th		Ft	L'th		
3	0.39	14.5	175	0.42	16.2	194	0.45	17.1	205	0.48	18.0	216	6.00	0.18
4	0.42	20.0	240	0.45	21.7	260	0.48	23.3	280	0.52	25.0	300	7.50	0.21
6	0.44	30.8	370	0.48	33.3	400	0.51	35.8	430	0.55	38.3	460	10.25	0.31
8	0.46	42.9	515	0.51	47.5	570	0.56	52.1	625	0.60	55.8	670	13.25	0.44
10	0.50	57.1	685	0.57	63.8	765	0.62	70.8	850	0.68	76.7	920	16.00	0.53
12	0.54	72.5	870	0.62	82.1	985	0.68	91.7	1 100	0.75	100.0	1 200	19.00	0.61
14	0.57	89.6	1 075	0.66	102.5	1 230	0.74	116.7	1 400	0.82	129.2	1 550	22.00	0.81
16	0.60	108.3	1 300	0.70	125.0	1 500	0.80	143.8	1 725	0.89	158.3	1 900	30.00	0.94
18	0.64	129.2	1 550	0.75	150.0	1 800	0.87	175.0	2 100	0.96	191.7	2 300	33.80	1.00
20	0.67	150.0	1 800	0.80	175.0	2 100	0.92	208.3	2 500	1.03	229.2	2 750	37.00	1.25
24	0.76	204.2	2 450	0.89	233.3	2 800	1.04	279.2	3 350	1.16	306.7	3 680	44.00	1.50
30	0.88	291.7	3 500	1.03	333.3	4 000	1.20	400.0	4 800	1.37	450.0	5 400	54.25	2.06
36	0.99	391.7	4 700	1.15	454.2	5 450	1.36	545.8	6 550	1.58	625.0	7 500	64.75	3.00
42	1.10	512.5	6 150	1.28	591.7	7 100	1.54	716.7	8 600	1.78	825.0	9 900	75.25	3.62
48	1.26	666.7	8 000	1.42	750.0	9 000	1.71	908.3	10 900	1.96	1 105.0	12 600	85.50	4.37

Nominal inside diam, in	Class E 500-ft head, 217 lb press			Class F 600-ft head, 260 lb press			Class G 700-ft head, 304 lb press			Class H 800-ft head, 347 lb press			Approx lb lead per joint	Approx lb hemp per joint
	Thick-ness, in	Lb per		Thick-ness, in	Lb per		Thick-ness, in	Lb per		Thick-ness, in	Lb per			
		Ft	L'th		Ft	L'th		Ft	L'th		Ft	L'th		
6	0.58	42.5	510	0.61	44.3	531	0.65	48.1	577	0.69	50.5	606	21.9	0.22
8	0.66	60.9	731	0.71	66.8	802	0.75	72.3	868	0.80	76.1	913	28.2	0.28
10	0.74	86.9	1 043	0.80	92.8	1 114	0.86	101.4	1 217	0.92	107.3	1 288	34.5	0.34
12	0.82	114.6	1 375	0.89	122.8	1 474	0.97	136.2	1 634	1.04	144.4	1 733	40.8	0.40
14	0.90	145.6	1 747	0.99	158.8	1 905	1.07	175.1	2 101	1.16	187.5	2 250	47.1	0.46
16	0.98	180.7	2 168	1.08	196.5	2 358	1.18	218.0	2 616	1.27	233.8	2 805	53.4	0.52
18	1.07	221.8	2 662	1.17	239.3	2 872	1.28	268.2	3 218	1.39	287.8	3 453	59.7	0.57
20	1.15	265.8	3 190	1.27	287.3	3 448	1.39	321.8	3 862	1.51	345.8	4 149	66.0	0.65
24	1.31	359.1	4 309	1.45	392.3	4 707	1.75	479.8	5 758	1.88	510.6	6 127	79.4	0.78
30	1.55	530.9	6 371	1.73	588.8	7 065	.....	.....	.....	.....	.....	.....	122.9	0.93
36	1.80	738.1	8 857	2.02	821.0	9 852	.....	.....	.....		.....	.....	146.7	1.11

low-head pipe lines, is best for construction work where its great strength as a beam saves necessity of frequent supports. Also, for its strength, it is the lightest steel pipe made.

Lock-bar pipe is a form of steel pipe in which a 100% efficient joint is obtained by bending a steel plate to the proper form. The edges, first upset, are inserted in the open grooves of a "lock bar" or small H-section, which is closed over the upset edges by a hydraulic press.

Life of steel pipe depends on so many factors that it can be given only in general terms. Unlike CI, which rusts more or less uniformly, steel often pits through in places. Thin steel pipes, $^1/_8$ in or even less in thickness, have lasted 15 or 20 years. A thickness not less than 0.25 in should give a useful life of 25 years or more.

Carrying capac of small sizes of steel pipe may be found by the Hazen-Williams formula (Art 11). For large, riveted pipe, with rivet heads and seams projecting on inside, the Chésy formula is often used with $n = 0.016$. Formula $V = 1.34\ D^{0.7}H^{0.555}$ is also used, where V = veloc, ft per sec, D = diam, ft, and H = loss of head, ft per 1 000.

Wood pipe has 2 forms: continuous-stave, built in place, and machine-banded made in lengths in factories.

Continuous-stave pipe has been built up to 14 ft diam; it is rarely made less than 2 ft diam. It is best adapted to moderate pressures, from a min of 20 ft head (considered necessary to keep the wood saturated and prevent decay) to a max of 200 to 250 ft. Advantages for large diam pipe: low cost, great carrying capac (n = 0.009 to 0.011 in Chésy's formula); easy transport of material, curves of moderate radius readily made, less liable to freeze than steel, greater strength against deformation than thin steel. Staves are milled with radial edges, to form true circular inside and outside surfaces. They have a length of 10 ft and upward, aver 16 ft, and are placed so as to break

joint by 2 ft. Joints between ends of staves are made by metal tongues inserted in saw kerfs (Fig 23). Hoops are usually of round, mild steel, in one piece for pipes to 54 in diam. They have button head on one end, and cold-rolled threads for 5 in or more on other end, which is often upset to make thread as strong as body of bar. Ends of hoop rest in a malleable iron shoe, and the band is cinched up by a hex nut. All metal work is heated and dipped in a mixture of asphalt and linseed oil, and touched up after placing. Pipe is built by using an outside form for lower half, building up both sides at same time, and an inside form for upper half resting on lower half. Sharp curves are to be avoided. A radius of 60 times the diam is the usual minimum, but this has been exceeded. MATERIAL FOR STAVES is usually Redwood, Douglas fir or Oregon yellow pine. Redwood is considered by many to be too soft. Staves are milled from standard thickness of lumber for each diam.

Diameter of pipe		Nominal thickness of staves	
10 to 14 in	54 in	1.5 in thick, 4 in wide	2.5 in thick, 8 in wide
16 to 48 in	60 to 72 in	2 in thick, 6 in wide	3 in thick, 8 in wide

Actual dimensions of staves vary with each diam. The only computation made is for size and spacing of bands. BANDS. If safe strength of band (lb) = s, then $s = s\pi r^2$, where s = safe tensile strength, usually 15 000 lb per sq in, and r = radius of band, in. Bands may cause failure by breaking in tension, or by crushing the wood fibers. The safe crushing load on wood can in this case be taken as 650 lb per sq in. Width of bearing of band on wood = r, and some specifications require that band be hammered into the wood until this bearing is secured. Hence, safe load on band would be $s = (R + t)\ 650\ r$, where R = radius and t = thickness of pipe, in. For economical design, diam of band should be such that values of s figured from either formula are equal, or $r = 0.0138\ (R + t)$. Diams of $^3/_8$, $^7/_{16}$, $^1/_2$, $^5/_8$, $^3/_4$, and $^7/_8$ in are used. SPACING OF BANDS is determined by the hoop tension due to water press, and by the tension produced by swelling of the wood. Using the minimum s (see above) the spacing of bands, in = $s \div (pR \times et)$, where p = water press, lb per sq in, and e = swelling force of the bands, which can not exceed the compressive strength of wet wood and may be taken as 100–125 lb per sq in. LIFE OF WOOD PIPE is very variable, depending on conditions under which it is used. A buried pipe line, with a cover of 2 ft or more of clean soil free from alkali or vegetable matter and maintained under a head of 20 ft or more, will have a longer life (25–50 years) than a line above ground. Under less favorable conditions, life is 6–15 years, and the pipe is best supported on cradles or sills. Connections are made by inserting steel Ts or branches in the line, to which the wood pipe is firmly secured.

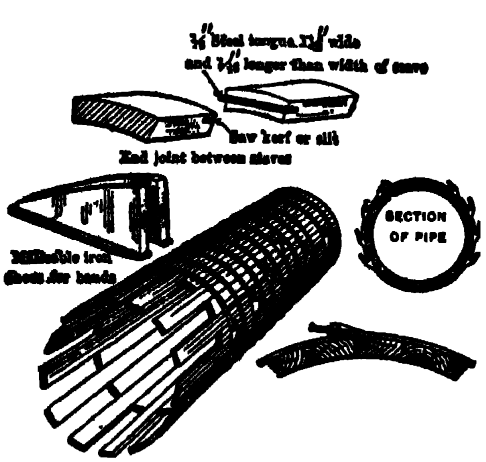


Fig 23. Continuous Wood-stave Pipe

Machine-banded pipe is built in the factory in lengths of 8–20 ft and diam ranging from 6–48 in, usually not over 24 in. Pipe is of staves with edges milled to form tongue and groove joints, banded with double galvanised steel wire, wound spirally by machine under high tension. Wire is No 8–0, size and spacing being regulated according to the press. Flat steel bands are also used. The outside of the pipe is heavily coated with hot asphalt and tar, and rolled in sawdust to make it better to handle and ship. The inserted joint (Fig 24) is for pipes to 14 in diam. A wooden collar or wood-sleeve coupling is used on larger sizes. This form has been used for water-supply pipes, instead of cast iron, also in hydraulic mining, and is useful for same service as continuous pipe, but where smaller sizes are required.

Concrete pipe has been used for drains, culverts, and for distributing water under heads of less than 15 or 20 ft. The mixture is 1 : 3 or 4 Portland cement and fine pit or bank gravel, cast in metal molds. A thin interior coating of neat cement is advisable to make the pipe less pervious. Joints are made tapering (Fig 25a). The bell end is well cleaned and filled with 1 : 2 cement mortar, and is then jammed against the taper end of previous length. Surplus mortar is removed, and a mortar band 3 in wide by 0.5 in thick is formed on outside of pipe.

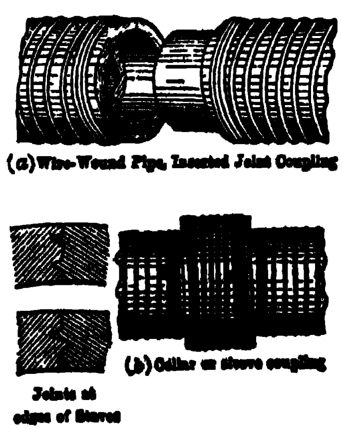


Fig 24. Machine-banded Wood Pipe

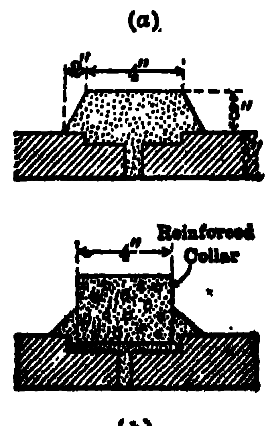


Fig 25. Concrete Pipes

Reinforced-concrete pipe for very large diam is built in place in the trench. Ordinary sizes are made in metal forms. It has been built to stand a press of 100 ft head, but the limit is generally

smaller. U S Reclamation Service has used it for upper part of wood-stave lines, where the head is 20 ft or less and wood pipe undesirable. The pipe is usually of 3 to 6 or 8-ft lengths, of 1 : 1.5 : 3 or 1 : 2 : 4 concrete, the thickness being 1 in for each ft of diam. Spiral reinforcing, held at proper pitch by spacing bars, takes the entire hoop tension. Plain wire is used, computed for a stress of 12 000 lb per sq in, and a size that gives a pitch not over 1.5 or 2 in. For joints, see Fig 25b.

Vitrified tile pipe is the commonest for sewers and drains to 36 in, but is usually 24 in and under. It is made in 24, 30, and 36-in lengths, with bell joints like C-I pipe, filled with jute packing and cement mortar. As it is never under press, only the external loading is important, and the standard thicknesses ($1/16$ to $1/12$ of the diam) are satisfactory if properly backfilled. Standard sizes, 4, 6, 8, 10, 12, 15, 18, 21, 24, 27, 30, 33, and 36 in.

14. STRESSES IN PIPES

Hoop tension is important in pressure pipes, and is the principal stress for which they are designed. Based on Art 7, the general formula is $t = \frac{(p + p')r}{s} + K$, where t = necessary thickness of shell, in, p = static press, lb per sq in (= 0.434 times head in ft), p' = allowance for water hammer, lb per sq in, r = radius of pipe, in, s = allowable stress in shell, lb per sq in, K = constant, added to the thickness determined by press considerations alone, to allow for deterioration, safety in handling, eccentricity, etc. Wood-stave pipe requires special design (Art 13).

The values of s and K depend on the material of which the pipe is made. For C I, s has been taken from 3 500–6 000 lb per sq in and K usually at 0.25 or 0.3 in. For steel pipe, s is taken at 14 000–16 000 lb or more and K at $1/16$–0.1 in. For permanent construction, thickness is usually $1/8$ in minimum for small pipes, and 0.25 in for diam over 2 or 3 ft.

Water-hammer is caused by suddenly stopping the flow in a pipe, as by quick closing of a valve at the lower end. The value of p' depends on length l (ft), veloc of flow v (ft per sec), and the time t (sec) of closing the valve. The press p_1, lb per sq in, above the normal press of the flowing water p_0, = $0.027\ lv \div t$, or = $63\ v$; the former expression to be used when t exceeds 0.000428 l, and the latter when t equals or is less than this value (Merriman). p' will then equal $p_1 + p_0 - p$. To make $p' = 0$, the time of closing is $t = 0.027\ lv \div (p - p_0)$. In water-power penstocks, relief valves, standpipes or surge tanks, are used to relieve the press caused by water-hammer. On long lines, valves are so arranged that they can not be closed quickly, and it is generally assumed that the water ram caused by closing a gate valve will not exceed 0.5 p. In designing C-I water pipes, values of p' are usually 100 to 120 lb per sq in for 4 to 10-in pipes, to 70 lb for 42 to 60-in pipes.

Backfill pressure may be important where pipe is laid in a deep trench. Steel pipe is seldom placed under deep cover. Cast iron usually has sufficient thickness, when in standard sizes or designed as above, to resist bending due to earth covering up to 20 ft or more in depth. Tile pipe is made only in standard thicknesses and must be carefully backfilled with selected material free from large stones, and well tamped around and for 2 or 3 ft over the pipe. Careful backfilling of trenches in any case is always wise. Concrete pipe is sometimes designed to withstand external loading.

Formula $t = 1/2 d\sqrt{h \div s}$ may be used in design as a rough check, t, d, and s, in, being same as for hoop tension, and h = depth of fill, ft. Wt of fill is say 100 lb per cu ft, and h is usually taken at 0.5 to $2/3$ the total depth, to allow for arch action. The amount and distribution of load is uncertain. Formula assumes the load to be uniformly distributed over the upper half of the pipe, and that it is resisted by uniform press over the lower half. This produces a moment in the shell M, in-lb per in of length = $1/16\ Wd$; where W = total load per linear in of pipe and d = diam, in. A concentrated load produces a positive moment $M = 0.159\ Wd$ at top and a negative moment = $0.091\ Wd$ at sides. Careless backfill may cause some other mode of loading (*Bull* No 22, Univ of Ill Expt Sta, 1908).

External atmospheric pressure, due to the pipe being placed above the hydraulic gradient, or to lack of or inadequate air valves, has often caused collapse of thin steel and wood-stave pipes. It is important only in lines of this kind. When the internal press becomes less than atmospheric, the shell is subjected to a uniform external press = p = difference between atmospheric (14.7 lb per sq in) and the absolute internal press; this causes circumferential compression in the pipe shell. The collapsing pressure for steel pipe, lb per sq in = p = about 50 200 000 $(t \div d)^3$. Air valves of too small a size may require a greater suction head than this amount, in order to supply air fast enough to fill the pipe when it is emptied quickly, intentionally, or due to a break. It is generally more economical to provide one or several air valves of ample area at all summits, than to increase the pipe thickness or to provide flanges to increase its collapsing strength.

Bending stresses due to wt of pipe and water are important in large steel pipes under low heads. They are usually provided for, not by increasing the thickness of shell, which

is uneconomical, but by placing a concrete saddle under the pipe, or by riveting to the pipe at short intervals curved hoops made of angles with an outstanding edge.

Maximum moment occurs at bottom of the pipe, and is due to the water load. M, ft-lb per lin ft of pipe $= CwR^3$, where C depends on angular length of the saddle (the angle at center of pipe subtended by the saddle) and has the following approx values: for $\alpha = 0°$, $C = 0.75$; $\alpha = 15°$, $C = 0.56$; $\alpha = 30°$, $C = 0.42$; $\alpha = 45°$, $C = 0.32$. W = wt of water, lb per cu ft ($= 62.5$) and R = radius of pipe. Knowing R and α, it is possible to test a pipe for bending due to this cause, the fiber stress in the shell being found by the usual formula (Sec 43), and if necessary the size and spacing required for angle stiffeners are computed.

Temperature range of a pipe depends on whether it is covered, and on its size. A short steel pipe of large diam full of water has a temp range nearly equal to that of the water (usually 32° to about 80° F); a long covered pipe line of small diam has a temp range about that of the soil, which depends on depth of cover and seldom exceeds 40° F for a 2-ft cover. If uncovered and with low flow, a small pipe may have a range almost equal to that of an empty pipe. Variations in temp produce longit stresses in a pipe that is fixed, so as neither to contract nor expand, the direction (tension or compression) and amount of which depend on the temp at which the pipe was laid. These stresses may cause buckling, and in many exposed pipes EXPANSION JOINTS are provided at intervals and the tops of saddles are made smooth to allow expansion and contraction without excessive friction. Fig 22d shows a common expansion joint. These joints are usually placed just below the anchorages on exposed pipe lines, and as anchorages occur at all bends at least one expansion joint is required on each straight portion of the pipe, and intermediate expansion joints are desirable on long stretches of straight pipe.

15. DESIGN OF PIPE LINES. VALVES AND ANCHORAGES

In general, pipe lines should be kept below the hydraulic gradient (Art 11). If they rise above it siphon action is created, which is undesirable because: (*a*) it interferes with the action of the pipe line and usually causes decreased or uncertain discharge; (*b*) the pipe at high points is under external press and liable to collapse. A profile of the proposed line should therefore be carefully studied respecting the hydraulic gradient.

Fig 26 shows profile of a pipe line. If a uniform diam is used from E to B, laid on surface of the ground (ECB), and the pipe discharges freely at B, the hydraulic gradient will run from A to B, provided the pipe is first filled and then a valve at B opened. Part of the pipe near C will be above the hydraulic gradient, and therefore subject to what is termed negative press, but which is really negative only in the sense of its being less than atmospheric, not less than zero. As long as CF is not over about 25 ft, the pipe will act as a siphon, discharging under the head h. But it is almost impossible to make the pipe tight, and air will collect at C and finally break the suction action. The pipe will then discharge at C, under the head h' (the hydraulic gradient running from A to C) and a certain quantity of water Q, less than the remaining portion CB could discharge under the head $h - h'$. The portion CB will therefore carry off this Q, flowing only partly full.

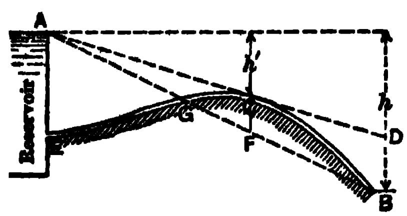


Fig 26. Profile of a Pipe Line

This action may be avoided in several ways: (*a*) Valve at B may be partly closed, causing the gradient to rise to D and acting as an orifice, the discharge of which under head DB just equals the discharge of pipe under head $h - DB$. This does away with negative press at C, but the discharge of the line is less than with siphon action. Also sudden opening of the valve at B, even if it is not opened more than the necessary amount stated above, may cause the "accelerating gradient" to fall below C until the water gets in motion and normal conditions are reached, so that air valves should be placed at C if there is danger of collapse; (*b*) if constant operation is unnecessary and negative press not liable to cause collapse, the valve at B may be closed at intervals, which puts the pipe at C under static press equal to head h'. A valve at C may then be opened and air allowed to escape. It would be advisable in this case to place a large air receiver at C, to collect the air in this tank above the pipe and not decrease the pipe section at this point; (*c*) a trench can be dug from G to B, to keep the pipe below the gradient, but this may be costly; (*d*) a compound pipe may be used, and this is usually the most satisfactory method. A large size is used from E to C, to deliver say 10% more water under the head h' than a smaller size, running from C to B, will deliver under the head $h - h'$.

Example (Fig 27). Pipe line from A to C is to deliver 2 cu ft per sec at C, with a terminal press of 25 lb per sq in ($= 25 \times 2.30 = 57.5$ ft head). If the pipe delivers this quantity when reservoir is empty, available head causing flow $= 130 - 6 - 57.5 = 66.5$ ft. For a uniform pipe, with slope 1.75 per 100, and $c = 100$, $d = 8.6$ in (Fig 19b); hence, use a 9 or 10-in pipe. With

this, the gradient will be AC', with siphon action present. Also, a uniform 14-in pipe is required to keep gradient above B, and flow at C must be throttled. Using a compound pipe for BC, the available head $= 130 - 11 - 57.5 = 61.5$ ft (slope 3.4 ft per 100) and nearest standard size (Fig 19) is 8 in. With this head an 8-in pipe will discharge 2.4 cu ft per sec. For the AB section the available slope (reservoir empty) is 0.25 ft per 100, and a 14-in pipe gives 2.5 cu ft per sec.

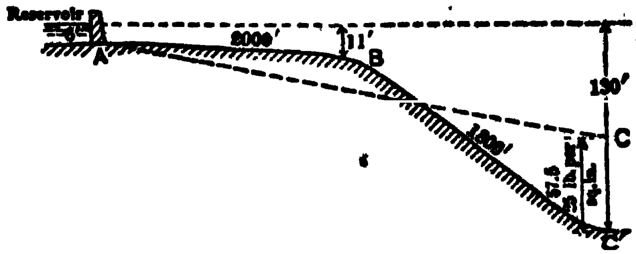


Fig 27. Profile of a Pipe Line

Fig 28 shows hydraulic gradient for conditions arising in pumping through pipes. Case *a* shows conditions in pumping from A to B against static head AC, and that the pump works under total head AD. In a pipe line for pumping it is usually economical to keep velocity (and hence friction head) low; say between 1.5 and 3 ft per sec, depending on cost of pipe and of pumping. In designing line AB, an economical velocity is computed or assumed, and area and diam of pipe found for required discharge. Friction head is then found from diagram.

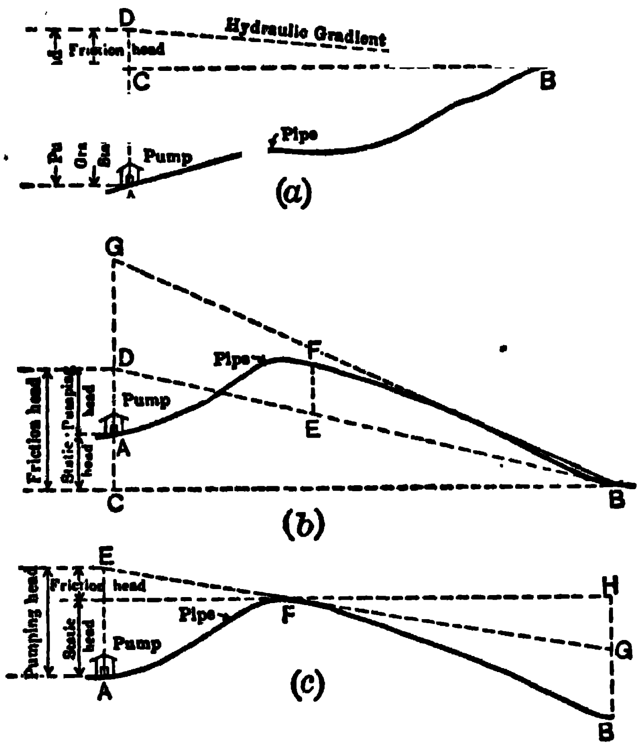


Fig 28. Examples of Hydraulic Gradient

In case *b*, if pipe AFB is once filled it would discharge under head AC, as in Fig 26, the hydraulic gradient running from A to B, provided distance EF does not exceed about 25 ft. If water is pumped through pipe AFB, the friction head is DC and the pump acts against head $DC - AC$, provided FE is as stated above. If FE be greater, the hydraulic gradient changes to that shown in case *c*, the pump then acting under head AE, and the pipe between F and B carrying off the discharge at F and flowing only partly full.

If negative press is not permissible at F, the hydraulic gradient must be kept above F. This can be done: I, by increasing the head against which the pump is working to head AG (case *b*), which involves a smaller pipe, with increase of veloc and friction head for the same discharge; II, by designing a pipe between A and F that will give an economical veloc and using a size from F to B that will just carry off this discharge under head BH; III, by placing a valve at B and throttling the flow as in case *c*. Here the gradient may be raised to line EG and kept above F. In this case air valves should be placed at F. If it is desired to take advantage of gradient DB (case *b*), steel pipe is necessary for the part above the gradient, made thick enough to withstand the difference in press (Art 14), or C-I pipe might be used. There must be a valve at B, to prime the siphon.

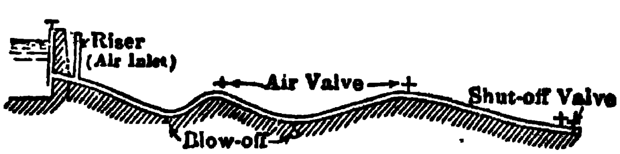


Fig 29. Profile of a Pipe Line

Valves. Fig 29 shows the profile of a pipe line, with location of valves. AIR VALVES, to permit escape of air on filling, and entrance of air on emptying, are placed at every summit and at shut-off valves. For a shallow reservoir, best type of air inlet is a simple standpipe, rising above water level in reservoir. Air valves are often automatic; float valves have been used, automatically permitting escape of air collecting at summits. Air collects at all summits, and must be removed, as by ordinary gate-valves operated by hand. The necessity of ample air valves to prevent formation of vacuum in steel pipe lines has been discussed. BLOW-OFF VALVES are placed at low points in pipe lines for emptying the pipe when necessary and for removal of sediment. These are gate-valves,

placed on a waste pipe (about 1/3 the diam of main pipe) leading to discharge in a stream or waste channel. SHUT-OFF OR STOP VALVES (ordinary gate-valves) are placed at end of pipe, and, in long pipes, at intervals of 1 to 2 miles along the line. This permits inspection and repair of any section, without emptying the entire line, and in case of breakage water may be shut off at nearest valve, thus preventing wastage and serious damage. These valves are usually placed at summits, with air valves on each side. A lock and chain are put on all valves to prevent tampering; on large lines the valves are in covered manholes with locks. For large diam lines, a small by-pass pipe with separate valve is added; it is left open to relieve the press while the large valve is being closed. Cost of gate-valves increases rapidly with the size; a saving in cost at a slight loss in head may be obtained by decreasing the diam of pipe at points where valves are placed by using suitable reducers and tapering increases. CHECK-VALVES are used mainly on pump lines, at points where breakage would permit large backward flow of water, as at foot of inclines (at A, Fig 28a, b and c), and on inlet pipe just outside tanks or standpipes. For small sizes a flap valve is used. For pipes larger than 24 in, a diaphragm or valve plate is cast in an enlarged section of the pipe, and a number of small valves attached to holes in the plate. A small by-pass is sometimes used to avoid heavy water-hammer, caused by sudden closing of valve with pulsations of pump. RELIEF (SAFETY) VALVES are occasionally used at ends of long pipe lines, or wherever water-hammer is especially to be feared. They are simple disk valves, opening outwards and held in place by springs adjusted to the press.

Freezing in pipes seldom gives trouble when there is continual flow. To prevent freezing in steel pipes, Bouche's formula is sometimes used to find necessary discharge in cu ft per sec: $q = 0.000045\ TA$, where A = exposed area of pipe and T = air temp in degrees F below 32°. Where velocities are very low at times, as in water-supply mains, pipes must be buried (3 to 5 ft, according to climate) or else protected by packing.

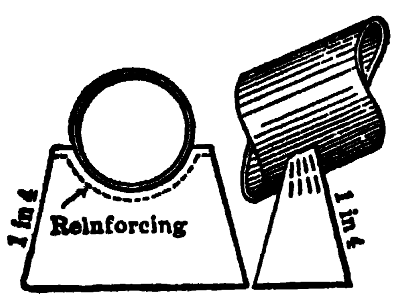


Fig 30. Concrete Pipe Support

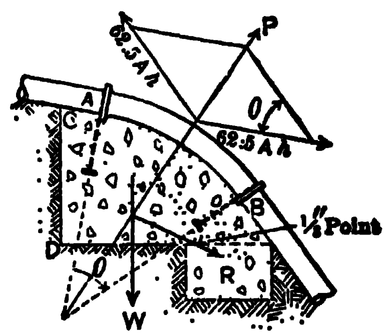


Fig 31. Concrete Anchorage

Pipe supports are required where pipe is placed above ground surface. Wood supports are common for wood pipe; for steel, concrete supports (Fig 30) are usual. Distance between supports is roughly estimated by considering the pipe as a continuous beam, loaded with its own wt plus wt of water, and using a low fiber stress.

Anchorages are necessary for exposed pipe at all bends where the resultant press may throw it out of line, at valves subjected to heavy static or water-hammer press, and on steep slopes.

On steep slopes, the pipe is usually anchored by wire or metal ties, carried to a deadman (heavy buried timber or a mass of concrete). Fig 31 shows a concrete anchorage on a bend. Anchors are at A and B. Pressures on the pipe at A and B are equal, producing an outward thrust $P = 125\ Ah\ \sin\ (\theta + 2)$, where A = area of pipe and h = static head. There is also the centrifugal force acting outward, $F = Wv^2 + 2\ gR$, where W = wt in lb of water contained in the bend between the 2 end faces A and B, v = veloc, ft per sec, g = 32.2, and R = radius of curvature of bend, ft. It may also be necessary to take into account pull produced by temp, sliding force caused by tendency of pipe to slide down hill, and earth press against the face CD. The resultant of all these forces, combined with wt of concrete acting through its center of gravity, should strike within middle third of base (Sec 43)

16. DESIGN OF DITCHES AND CANALS

(In what follows, the word "ditch" is used to include "canals.")

Hydraulic radius. When the cross-sec area of a channel and its grade are fixed, the form of cross-sec having the greatest hydraulic radius (least wetted perimeter, Art 12) will give greatest velocity and discharge. For this result, the proportions are such that the hydraulic radius = 0.5 the depth. For a rectangular section (flume or rock-cut) these considerations indicate that depth should be 0.5 the breadth, as this gives the least wetted perimeter (and hence least cost for a flume); it also gives least area for a given discharge, and hence least cost for a rock cut). For a trapezoidal section the most economical form is half of a hexagon; or, if side slopes are assumed, the best section is found by drawing

tangents to a half circle having a radius equal to the depth (Fig 32). Other practical considerations, however, result in marked variations from these sections in usual designs (see later notes).

Unlined ditches in earth. The CROSS-SECTIONAL PROPORTIONS in Fig 32 are not used, because they give sections that are too narrow and deep. Such sections are unstable; they will widen out on the sides and silt up in the center, and are seldom economical to

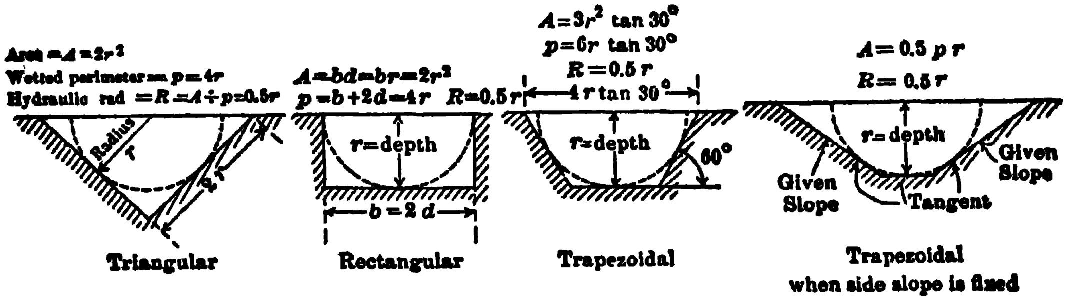


Fig 32. Cross-sections having Most Advantageous Elements

dig, as scrapers are generally used and a shallow cut is necessary. Deep narrow ditches require shoveling.

Proper proportions are largely a matter of judgment, depending on character of soil and mode of excavation. Average U S practice is to make $d = 0.5\sqrt{A}$, where d = depth, ft, and A = area, sq ft. This rule gives following depths:

Area, sq ft......	5	10	25	50	75	100	200	400
Depth, ft........	1.1	1.5	2.5	3.4	4.3	5.0	7.1	10

These values are exceeded for special conditions, as on steep hill sides where a narrow, deep section (with lining for stability if necessary) is often economical in excavation. Where water carries much silt the section may be shallower. Usual SIDE SLOPES are 1 : 1, 1.5 : 1, and 2 : 1. On steep hillsides in firm soil a 1 : 1 slope both for ditch section and outer side slope of bank is used. Any section gradually becomes rounded, the lower parts of the slopes becoming flatter, the upper parts steeper. For aver loam or gravelly loam, a slope 1.5 : 1 is used; for loose sandy loam, 2 : 1. About 1 ft freeboard is common for small, and a max of 3 ft for large ditches. Safe rule: make the freeboard $^1/_3\, d$. TOP WIDTH of ditch bank may be fixed by what is required for its use as a road (10 to 12 ft); it ranges from about 2 ft for small, to a max of 10 ft for large ditches, and is usually about equal to depth of water (Fig 33).

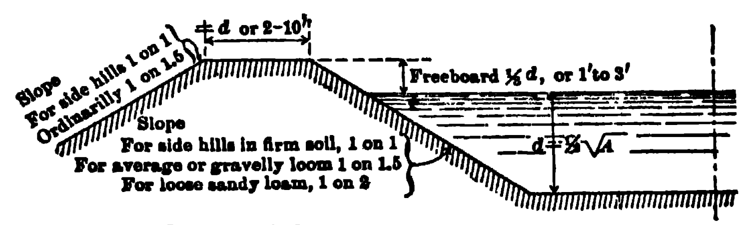


Fig 33. Side Slopes for Ditch or Canal

Velocity of water must not be high enough to erode the bottom, thus deepening the cut and making it difficult to divert water. But, there should be a certain minimum, to prevent deposit of silt and excessive growth of water plants. Experience in U S is that a veloc over 2 ft per sec will prevent deposit, and the following max mean velocities are safe against erosion. (Note.—Bottom veloc is generally about 75% of the mean.)

	Ft per sec		Ft per sec
Very light loose sand.............	1.0 to 1.5	Conglomerate, cemented gravel, soft rock......................	6.0 to 8.0
Average sandy soil...............	2.0 to 2.5	Hard rock......................	10.0 to 15.0
Average loam or alluvial soil......	2.75 to 3.0	Concrete, water carrying coarse sand	7.0 to 12.0
Stiff clay or ordinary gravel.......	4.0 to 5.0	Concrete, water carrying fine sand.	15.0 to 20.0
Coarse gravel or cobbles..........	5.0 to 6.0		

The above figures show that for ditches in earth the usual veloc is 2 to 3 ft per sec, which generally requires a slope of 4 to 7 ft per mile. A section may be designed as in the example following, and located to have the required slope; or, if the slope is fixed and small, it may be necessary to line the ditch to obtain the required discharge. On the other hand, if a large slope is available, the ditch may be excavated at the max desirable slope, and the extra head used up by constructing CHUTE DROPS, commonly designed as water-cushion

or baffle spillways (Fig 34). In some cases where no supporting ground is available, the ditch must be constructed on the given slope and lined to prevent erosion.

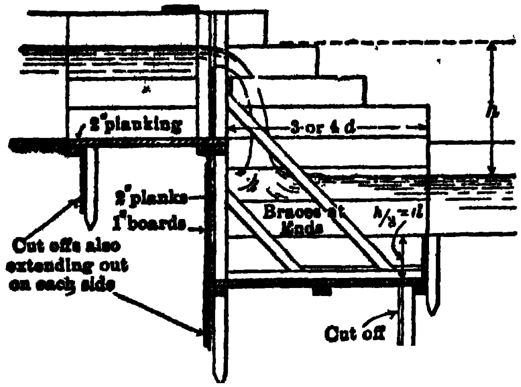


Fig 34. Vertical Drop in a Ditch

Chésy formula, with C determined from Manning (or Kutter) formula, is generally used for ditches and flumes. Most engineers take $n = 0.025$ for aver earth, n actually ranges from 0.02 for firm soil, trimmed smooth with a shovel, to 0.03 for rough gravelly surfaces, the aver for most irrigating ditches in Western U S being 0.0225.

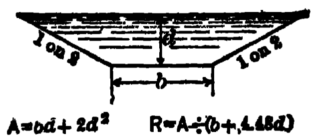


Fig 35. Cross-section of a Ditch

Example. Design a ditch in earth to carry 2 500 miner's inches of water, and compute the required slope. Assume $n = 0.025$ and safe velocity $= 2.5$ ft per sec. Discharge is $2\,500 \div 40. = 62.5$ cu ft per sec, and required area $= 62.5 \div 2.5 = 25$ sq ft. Desirable depth $= 0.5\sqrt{A} = 2.5$ ft. With 1 : 2 side slopes, area will be $bd + 2\,d^2$ (Fig 35), and $b = (25 - 12.5) \div 2.5 = 5$ ft. Hydraulic radius $R = A \div (b + 4.48\,d) = 25 \div 16.2 = 1.54$ ft. Using the diagram (Art 11, Fig 19), the required slope is found to be 1.0 ft per 1 000 ft.

The center line of the ditch should be so located that the cut will make the required fill plus the shrinkage, except where it is advisable to keep the water section entirely in cut because of danger from breaks. Breaks are especially to be feared on steep hillsides, even when the sod under the fill is furrowed with a plow to give a good bond between the fill and original ground surface.

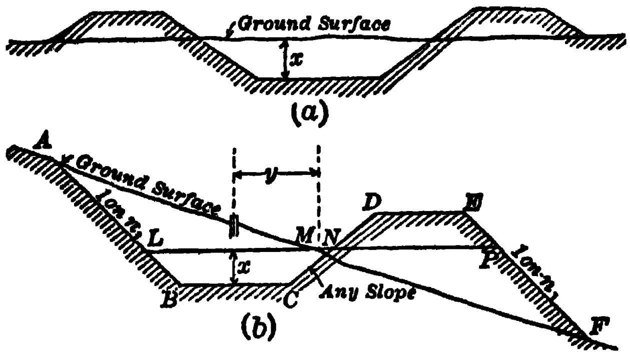


Fig 36. Balancing Cut and Fill

For level cuttings with 2 banks the economic cut x is computed to balance the section (Fig 36*a*). For side-hill work, 1 bank only is generally required, and the "pivot-point" method of locating the center line is used (Fig 36*b*). The section $ABCDEF$ being designed, a horiz line LP is drawn through the section, so that cut $LBCN$ makes the fill $NDEP$ plus shrinkage (usually 10%). Line LP is then divided into 2 parts by point M, equal if no allowance is made for shrinkage, or for 10% shrinkage such that $LM : MP :: \sqrt{100} : \sqrt{90}$. M is the pivot point, and for any slope as AF drawn through N the cut will balance the fill. M must thus be located on the ground as a point on a grade contour x ft above the bottom of the ditch, and center line will be a distance y up hill from M; x and y are independent of slope of side hill, and remain the same for any given cross-sec.

17. LOSSES IN DITCHES AND CANALS, DITCH STRUCTURES, FLUMES

Seepage losses often amount to 25% or more of the water delivered at the upper end of a ditch, and must be allowed for in computing the carrying capacity. The loss depends on the character of the material in which the ditch is excavated, whether it is or is not lined, on the relation of the water level in the ditch to the water level of the adjacent ground, and on the age of the ditch. If the ditch is below groundwater level, there will be inflow from the adjoining land and vice versa. In many cases the ditch is above groundwater influence, and the seepage loss in cu ft may be computed from the following table (Etcheverry):

Material	Loss, cu ft per 24 hr, per sq ft wetted perimeter	Material	Loss, cu ft per 24 hr, per sq ft wetted perimeter
Impervious clay loam	0.25–0.35	Sandy loam	1.00–1.50
Ordinary clay loam, silt, or lava ash loam	0.50–0.75	Loose sandy soils	1.50–1.75
Gravelly clay loam, sandy clay loam or cemented gravel	0.75–1.00	Gravelly sandy soils	2.00–2.50
		Porous gravelly soils	2.50–3.00
		Very gravelly soils	3.00–6.00

Ditch linings are to prevent excessive seepage losses, or to permit higher veloc than is permissible in earthy soils. Higher veloc is advisable in some cases for economy, the cross-sec and seepage loss being smaller; and also in cases where topography is such that supporting ground is not available for a combination of earth ditch and vertical drops or chutes to use up excessive head.

Details. About 3 gal heated heavy asphalt road oil per sq yd, applied in 2 or 3 light doses and raked into the upper 2 or 3 in of the soil, will reduce seepage loss to about 40% of value for untreated. It also prevents growth of vegetation and allows increase in veloc to about 5 ft per sec, but is not permanent, requiring renewal every 2 to 4 years. CLAY PUDDLE, consisting of a 3 to 4-in layer of well-tamped clay, is as effective stopping seepage as oiling, but does not allow any higher veloc or prevent growth of vegetation. WOOD planking, nailed to sills set in sides and bottom of ditch is effective, but due to high cost and short life is seldom as economical cement or concrete. CEMENT MORTAR lining, 1 cement to 4 sand, 1 in thick, will prevent 75% of the seepage. CONCRETE lining, 1 : 2 : 4, 2 to 4 in thick, will stop 95% of the loss. The concrete is placed with or without forms. On slopes of 1 to 1 or less, no forms are used, the lining being placed like cement sidewalks, grooved every 8 or 12 ft to reduce width of contraction joints by making them form at frequent intervals. For steeper side slopes heavier side linings are required and forms are used. Instead of trimming the bank to correct slope, it is best to use an outside form, filling in with wetted earth up to this form, and using an inner form for the concrete. A solid foundation is important, as the concrete has little strength and will crack if foundation settles or pockets are washed out under it. Proper drainage is essential to keep material compact and dry, and prevent heaving due to frost.

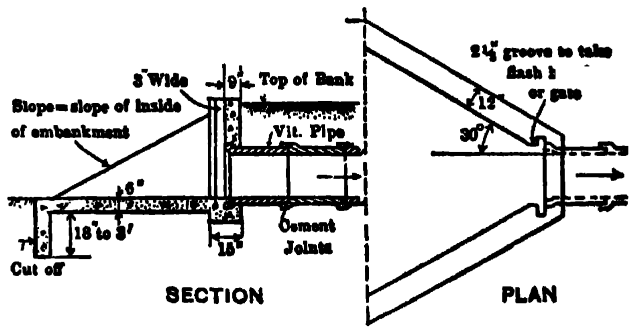


Fig 37. Turnout from Ditch

Ditch structures. TURNOUTS are to distribute water from main ditch to laterals. For small ditches, a wooden box will answer; for larger, where repairs would be costly, a permanent vitrified or cement pipe culvert is desirable (Fig 37). Connections of siphons, culverts, flumes and turnouts, to an earth ditch must be carefully made, to prevent leakage at these points, with danger of washout. CHECKS are to hold flow-back, so as to supply a turnout with proper amount of water. They are sometimes also used as small DROPS (Fig 34) to decrease the grade. CHUTES are long, lined sections on a steep grade, with a water cushion at the bottom to kill high velocity. WASTEWAYS are overflow weirs to carry off flood flows, or turnouts to a stream channel which act as blow-offs to empty the ditch. They are installed on many long lines.

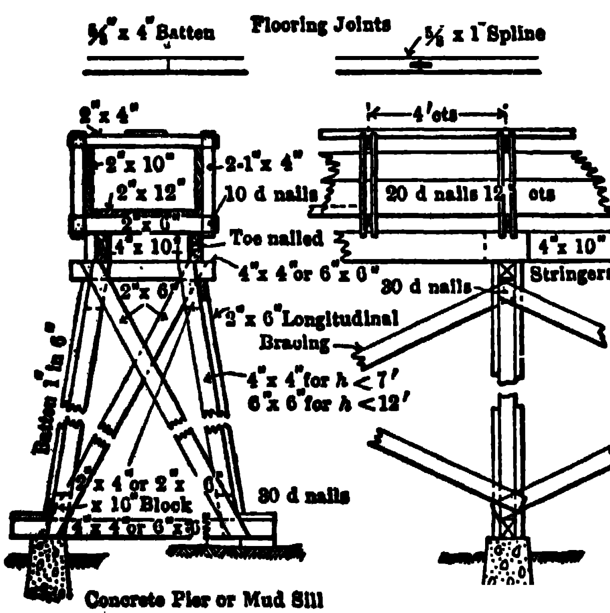


Fig 38. Details of Flume Construction

Flumes (Fig 38) are used to cross valleys, and on hillsides where the slopes are too steep or the soil is unsuited to an open ditch. The latter are called bench flumes. Breadth of flume box is generally twice the water depth (Art 16). Bench flumes are often narrower, and trestle flumes wider. Freeboard (in) may be made equal to (depth of water, ft + 12) + 2. Flume lining should be of 1.5 in, or better 2-in plank.

There are two forms of flume box: (*a*) sills are placed across the stringers, and flooring is laid parallel to direction of flow, like the sides; (*b*) flooring is transverse and nailed to stringers. The first is best and easiest to keep tight and repair. Foundations and sills are of 2 × 12 or 3 × 12-in plank, or concrete posts. Bents are spaced 12 to 18 ft centers, and stringers are designed to carry the load at a low unit stress. Semi-circular continuous wood-stave pipe has also been used for flumes; now often replaced by semi-circular steel flumes. Several patented forms, made in Denver and San Francisco, are moderate in cost, easily and quickly constructed, and watertight.

HYDRAULIC MEASUREMENTS

18. MEASUREMENT OF WATER LEVEL

Non-recording instruments. Simplest is a graduated GAGE BOARD, or piece of an old level rod, fastened to side of tank or reservoir, and read where water level cuts the rod. It can be read to

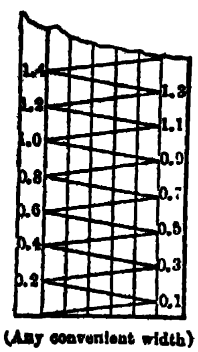


Fig 39. Gage Board

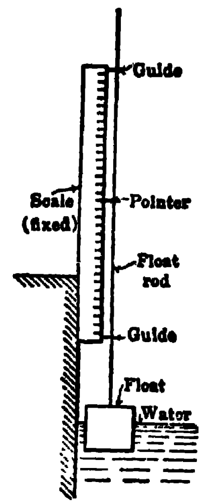


Fig 40. Float Gage

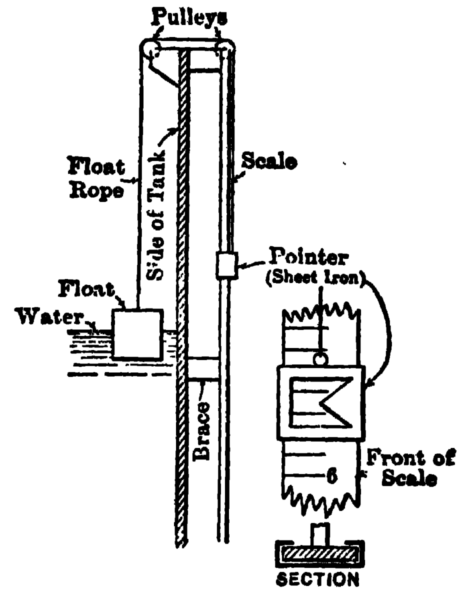


Fig 41. Float Gage for Tanks

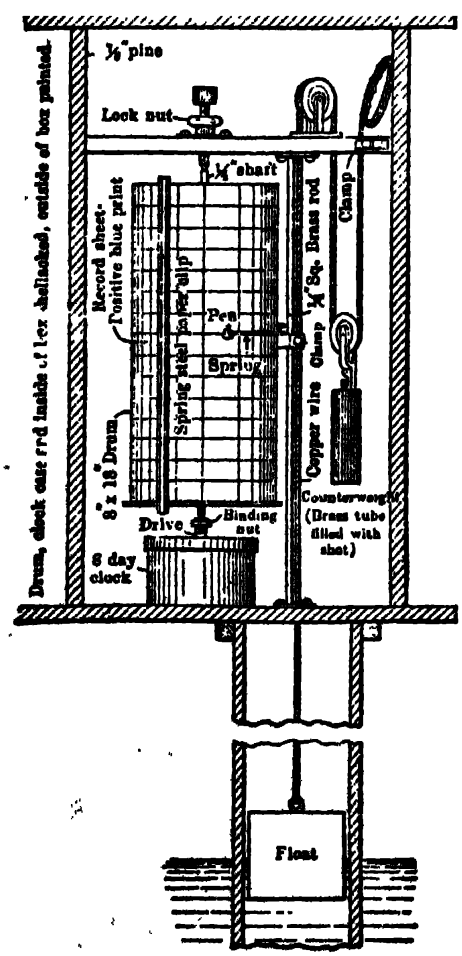


Fig 42. Recording Gage

0.005 to 0.01 ft. If the water surface is rough, a STILL WELL (a box open at the top and with a submerged opening in side) may be used and the gage placed in it. Fig 39 shows a variation from the ordinary form of gage board, the sloping lines allowing a more accurate estimate of water level. This design may be scratched with a nail on the side of a concrete tank before the concrete is fully set, or made by saw-cuts $^1/_8$ in deep on a board. Fig 40 shows a more accurate instrument, a FLOAT GAGE consisting of a fixed scale and a pointer attached to a vert rod on a float. As the reading is made on the rod, this gage is convenient where the water surface is hard to reach. If a leveling rod is used for the scale, and a target vernier attached to the float rod, readings may be made to 0.001 or 0.002 ft. Fig 41 shows a float gage for a tank. By the HOOK GAGE a skilled

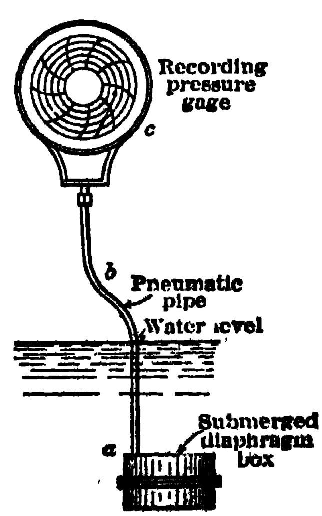


Fig 43. Bristol Recording Gage

observer can detect variations in water level of 0.0002 or 0.0003 ft. But such accuracy is seldom needed except in laboratory experimental work.

Recording gages. Fig 42 shows a simple form, which can be made by a good mechanic. The clockwork consists of an 8-day clock, with gearing to produce a revolution of the drum in any desired time. For recording weir flow, the vertical scale on the drum may be graduated to read discharge directly (Fig 15). Similar instruments, costing $75 to $150, are made by Hydro M'f'g Co, Phila, Pa; J. Fries, Baltimore, Md, and others. Fig 43 shows another recording gage. A drum *a*, containing a rubber diaphragm, is submerged in the water. Variation in water level causes variation in pressure of air in the drum, which is communicated by tube *b* to the recording gage *c*. This gage is not interfered with by floating matter in the water, nor by ice, and is said to show variations in level of 1/8 in (Bristol Co, Waterbury, Conn).

19. MEASUREMENT OF HEAD OR PRESSURE

Moderate water press in tanks or pipes is generally measured by pressure tube or PIEZOMETER; higher press, by hydraulic gage, similar to Bourdon steam gage.

For head of a few ft a WATER TUBE is convenient, consisting of a rubber tube attached to an opening in the pipe or tank, and having a glass tube at its upper end. The tube is raised or lowered until the water level shows in the glass tube, and the distance of this level above the datum (as center of pipe or bottom of tank) gives the head. The opening in the pipe should be on a straight length of pipe, at right angles to its axis, and should have a sharp inner edge. The MERCURY PIEZOMETER (Fig 44) measures heads as high as 40 to 60 ft, Hg having a sp gr of 13.6. Distance z between Hg levels in the U tube, as read on the scale, is roughly $1/13$ the actual head. True head equals ($z \times$ sp gr of Hg) $+ x$.

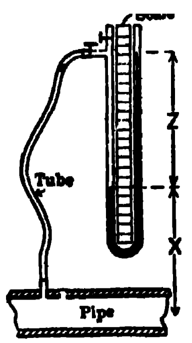


Fig 44. Mercury Piezometer

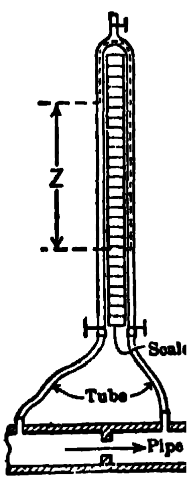


Fig 45. Oil Manometer

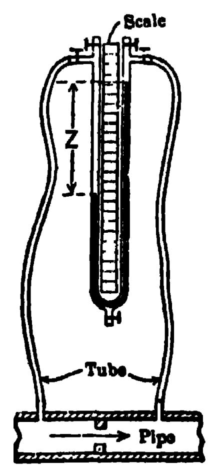


Fig 46. Mercury Manometer

Hydraulic gages measure pressures up to 1 000 lb per sq in or 2 300 ft. For measuring DIFFERENCES IN PRESSURE (if the press is very high) 2 gages are used; for very small differences, an OIL MANOMETER (Fig 45) (*Trans* A S C E, Vol 47, p 72) is used.

Kerosene oil is good. Its sp gr (which must be determined) is about 0.79. Hence the scale reading will be greater than the difference in head, which is independent of height of 0 point of the scale above datum, and equals $z \times (1 -$ sp gr of the oil$)$. There may be trouble in obtaining good readings at the junction of water and oil, due to oil adhering to sides of tube. The WATER OR AIR MANOMETER, made by tube similar to that of Fig 45, can be used for small differences in head. Air must be pumped into the tube if the press is high. The reading is the true difference in head. Two open tubes could be used if total press is small.

Mercury manometer (Fig 46) is probably the best device for work of this kind. The true difference in head $= z \times$ (sp gr of mercury $- 1$).

In all these devices the length of tubes and the quantities of liquid used must be varied to suit the difference in head to be measured.

20. MEASUREMENT OF VELOCITY AND DISCHARGE

For measuring small quantities it is best to allow the discharge for a given time to collect in a tank, where it may be measured or weighed. When this can not be done, or a permanent device is required, one of the following methods must be used. Sometimes

drop in hydraulic gradient can be measured, and discharge computed as in Art 11, but such results are liable to error due to uncertainty in value of the coeff.

Measurements in pressure pipes. There are 3 methods. For measuring water supply in 0.75 to 2-in pipes, DISK OR RING WATERMETERS are to be had in many patterns.

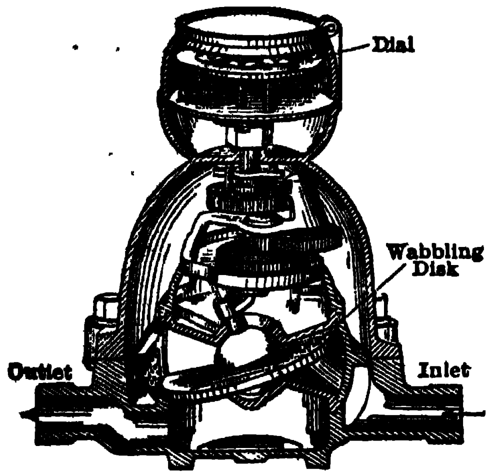


Fig 47. Disk Water Meter

The meter contains a wabbling disk or ring, so arranged that its motion is communicated to a pin which in turn works a train of clock wheels, the quantity passing being registered on a dial (Fig 47). This meter is "inferential," and must be rated to determine the discharge. Piston and other forms of meter are also used. All give reasonably accurate registration for very slow flows (within say 10% for 10 gal per hr), but their rating varies with age and condition of meter and quality of water.

Venturi meter is standard for permanent installations on pipes 3 or 4-in diam and over. They have been placed on the Catskill aqueduct for New York's water supply, pipe diam being about 14 ft. The meter consists simply of a gradual reduction in size of pipe to a throat $^1/_2$ to $^1/_3$ that of the pipe diam, and a gradual expansion back to normal size (Fig 48). There is no obstruction to the flow of the water, and no moving parts.

Formula for discharge is based on the proposition that $h_1 + v_1^2 \div 2g = h_2 + v_2^2 \div 2g$, and that $q = a_1v_1 = a_2v_2$. This gives $q = C\dfrac{a_1a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2g(h_1 - h_2)}$, where a_1 and a_2 = areas and h_1 and h_2 = pressure heads at the pipe and throat; g = acceleration of gravity and C is a coefficient to allow for friction losses (usually taken at 0.99). A manometer is generally used to measure $(h_1 - h_2)$, and for any meter the formula reduces to $q = c\sqrt{z}$, where z is the difference in manometer readings. (This meter is made by Builders Iron Foundry, Providence, R I, who also make devices for recording total quantity and rate of flow.) For rough estimates of flow in a pipe, sawdust may be introduced at upper end and time noted for passage through pipe.

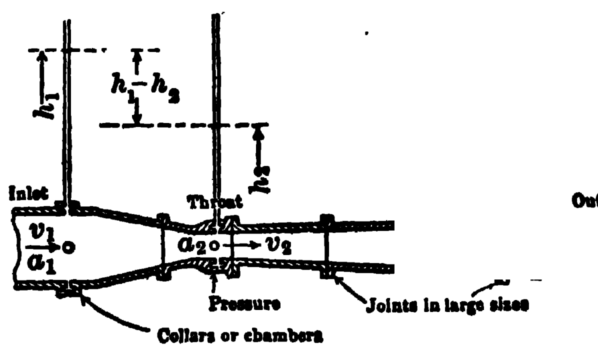


Fig 48. Venturi Meter

Pitot tube (1750) has been used in recent years. Fig 49 shows its principle. The current of water acting on the end of the curved tube causes the head in tube A to be greater than that in tube B. B measures pressure head only, and considering tube A

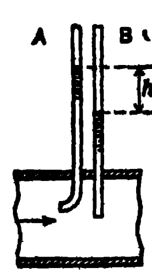


Fig 49. Pitot Tube (Diagram)

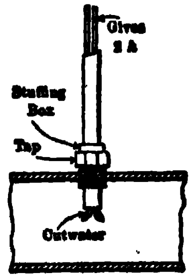


Fig 50. Pitot Tube

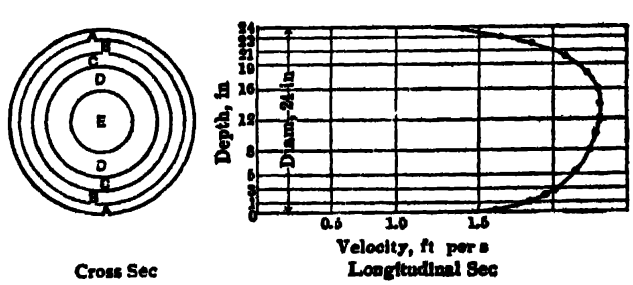


Fig 51. Diagram of Pipe Velocities

to record velocity-head only, and the difference in levels equal to h; then $v = m\sqrt{2gh}$, where m theoretically = 1. If the action of the current against the tube is considered as impact, then $m = 0.7$. It has been shown also that, when placed so as to be subject to currents of different velocities, the head is not that due to their mean, but to the mean

of their squares. For high velocities, as in nozzles, m may be taken as 1 with quite accurate results. For pipes, in which velocities are low and vary as in Fig 51, the value of m is less, and for accuracy must be determined by ratings in a pipe of same diam.

The instrument is made by the Pitometer Co, N Y, who also make a recording device. The usual form of Pitot tube (Fig 50) can be inserted in a hole tapped in a pipe, and a series of readings at different points in the diameter taken, from which the PIPE COEFF (Fig 51) is obtained. Knowing this, future measurements are made only at the center, center velocity being reduced to the mean by using the coeff. The makers advise using $m = 0.84$.

$$\text{Mean velocity} = \frac{6.202}{3.142}$$
$$= 1.97 \text{ ft per sec.}$$
$$\text{Pipe coeff} = \frac{\text{mean veloc}}{\text{center veloc}}$$
$$= \frac{1.97}{2.24} = 87.5\%.$$

Determinations of Pipe Coeff from Fig 51

Ring	Area	Aver veloc	Discharge
A	0.502	1.57	0.788
B	0.873	1.87	1.632
C	0.698	2.05	1.431
D	0.720	2.18	1.570
E	0.349	2.24	0.781
Totals	3.142		6.202

Measuring flow in open channels. The most accurate device is the weir (Art 10). Reliable data for weirs are available for a discharge up to about 200 cu ft per sec. For greater discharges, this value, which is usually higher than the limit of economy for the construction of weirs, or where weirs are not warranted or possible, floats and the current meter are used. For floats a straight, uniform section of the stream, at least 100 ft long and on an even grade, should be selected. Soundings are then taken, the stream being divided into "lanes" like those in Fig 52. If surface floats are used (small chips of wood), a large number are allowed to pass down each "lane," and the aver time noted is the surface velocity of each course.

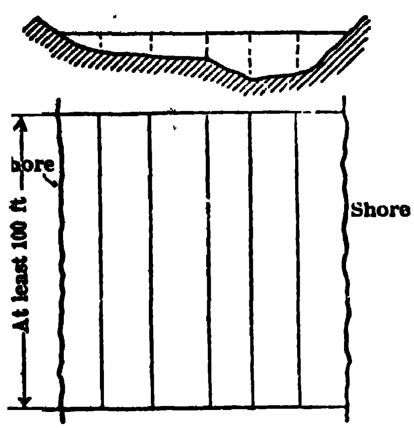


Fig 52. Soundings and "Lanes" for Measuring a Stream

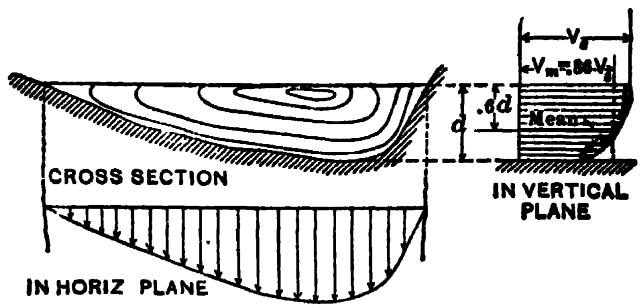


Fig 53. Velocities in Cross-section of a Stream

Fig 53 shows variation in velocity in a vertical section. To obtain the mean, the surface velocity must be multiplied by 0.79 to 0.95, varying with roughness of bed and averaging 0.86. Total discharge is found by multiplying the cross-sectional aver area of each lane by the mean veloc, and adding the results for all the lanes. Wind affects surface floats, and shallow areas of dead water on the sides must be eliminated. A rough estimate of discharge can be made, with an uncertainty usually less than 20%, by timing floats down the center of a channel, thus obtaining the max surface veloc. Mean veloc in entire cross-sec is between 0.70 and 0.85 of this; aver, 0.8.

Submerged floats (Fig 54) are used for deeper streams, and in studying tidal and other currents. By submerging the lower weighted float to a depth of about 0.6 (0.58 to 0.71) that of the vert section down which it floats, the mean veloc in the vert is obtained.

Rod floats are hollow cylinders of tin, weighted by pebbles or shot so as to stand vertically, and reaching nearly to the bottom of the channel. Francis gives $V_m = V_r(1.012 - 0.116\sqrt{d' + d})$, where V_m = mean veloc in the channel down which the rod floats with a veloc V_r, d = total depth of stream, and d' = depth below bottom of rod. d' should not exceed $0.25\,d$. In practice it is difficult to use rod floats, as they require a very uniform channel, not commonly occurring except in canals and flumes.

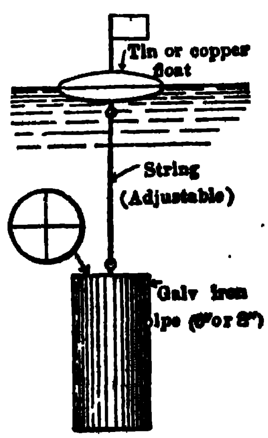


Fig 54. Submerged Float

Current meter is successfully used for measuring veloc and discharge in open channels and large pipes. It has 3 or more vanes mounted on a spindle, so arranged as to stand always normal to direction of current, the press of which causes the vanes to revolve. The number of revolutions in a given time is recorded by an electric or mechanical counter, and is approx proportional to the current veloc.

The meter is first rated by moving it through still water at known velocities and noting the revolutions. If it is then submerged, by suspending it from a rod or rope, the revolutions can be counted and the velocity obtained from the rating chart. Usually the meter is held at a depth of 0.6 that of the water, for finding mean veloc. It has been largely used by the U S Geol Surv, for obtaining curves showing the discharge of a stream at different elevations (stages) of water surface. Observers then note daily the water level, and from the curves the flow is found. Two types of meter are in common use, the Price and the Fteley: the first for deep, rapid streams, latter for shallow, slow streams (See "Use and Care of the Current Meter," *Trans* Am Soc C E, Vol 16, p 68).

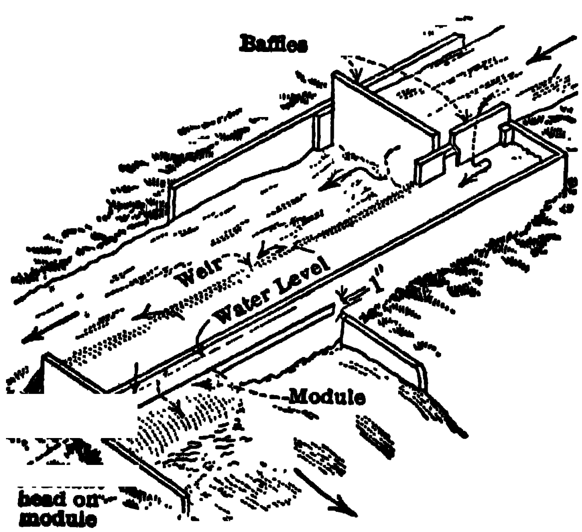


Fig 55. Foote's Weir Gage

Miner's inch, as used in western U S, is the quantity of water which will flow in 1 min from a standard vert orifice, 1 in sq, under a head at its center of 6.5 in. This would equal 1.53 cu ft, but actual value varies in different districts. In Cal and Mont, it is established by law that 40 miner's inches = 1 cu ft per sec; in Col, 38.4, and in Ariz, Idaho, Nev, and Utah, 50 is accepted by common agreement.

Fig 55 shows Foote's weir gage, one of the simpler devices for diverting a certain number of miner's inches from a canal in which flow varies. With a long weir a considerable increase in flow in canal is necessary before the head on weir is much increased. Other devices, with float and special valves, are also used to maintain constant head.

Module is an orifice used in selling water, which under constant head is to supply a certain number of miner's inches. In Fig 55, the length of opening, in, = number of miner's inches, if head is constant. As it is difficult to maintain constant head, this unit is going out of use.

WATER SUPPLY

21. ESTIMATES FOR WATER SUPPLY

Consumption is stated in gal per capita per day. While many data are available respecting requirements for small towns and cities, showing variation from 30 or 40 to 100 or more gal per capita aver daily consumption, it is only recently that the subject has received much attention in mining communities; the scanty information existing is often unsatisfactory, as such towns have been compelled to use what has been available rather than the quantity desirable. Frequently the supply for mills and power plants is the principal requirement.

Source of supply. Having estimated the quantity required, with allowance for future increase of population, a source of supply is sought and developed; either underground water or streams. The quantity available from underground water, obtained by wells or infiltration galleries, can seldom be estimated except by performance of wells driven to the same strata. To make sure that a spring or stream will give sufficient supply, measurements of its flow covering a number of years are necessary. In case of streams, which usually have large variation in flow, a reservoir may be possible, to store up the flood flows to supplement the natural flow during dry seasons (Art 22).

Main pipe lines are designed to allow for variations in consumption, unless the water goes to a small distributing reservoir, having a capac sufficient to equalize variations. The main pipe may then be designed to furnish water to the reservoir at the aver daily rate. For domestic consumption, max daily rate is about 150% of the daily aver; max hourly rate, 250% of daily aver.

Distributing pipes. The size is usually determined by fire service requirements. If the system is to be used for fire service without engines, a press of about 30 lb per sq in (70 ft head) is required at hydrants, and for an adequate stream 150–250 gal per min should be supplied. It should be possible to concentrate 2 or 3 streams at any one point without using hose over 400–600 ft long; these requirements fix the location of outlets and allowable loss of head in the pipes, and hence determine their size.

22. RAINFALL, STREAM FLOW, AND STORAGE

Water falling as rain passes off as surface flow into streams and rivers, is evaporated directly by the sun from the land surface, is taken up by plants and vegetation, or sinks into the ground forming ground storage, some of which finds its way underground to the streams and furnishes their main supply, except during and just after storms. Thus, stream flow will vary in both total amount and distribution throughout the year on two watersheds having exactly the same rainfall, but different slopes, amounts of vegetation, depth, and character of soil cover. Hence, for accurate estimates, data on flow of the stream in question are necessary; sometimes obtainable from publications of U S Geol Surv, state, city, and private reports, but often entirely lacking. It may then be possible to secure data from a similar watershed, bearing in mind what points constitute similarity, or it may be necessary to make a rough estimate from measurements of the slope and cross-sec of the stream, at its known low-water mark, applying the formulas of Art 12.

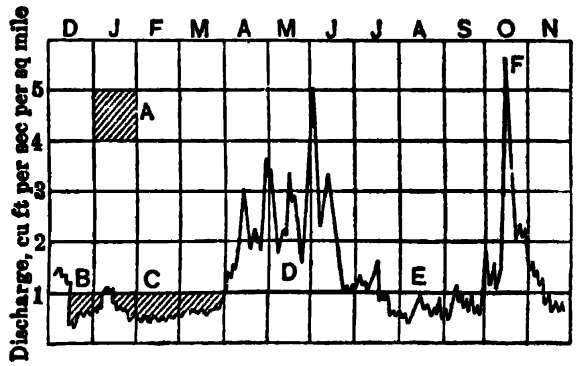


Fig 56. Hydrograph of Stream Flow

A hydrograph, like that in Fig 56, gives a complete record of stream flow, and may be used for all kinds of estimates.

For example, since a dam spillway should be designed to pass safely the flood flow shown at F, the stream records should be searched for the greatest flood, and an estimate made from high-water marks of the stream. The vert scale in Fig 56 is cu ft per sec per sq mile, which, multiplied by the area of the drainage basin gives the total flow. Area of the basin is obtained by drawing a divide line on a topographic map, enclosing the area within which all rain falling (and not lost by evaporation and seepage) would drain off to the stream in question. This area may then be taken off with a planimeter. If no storage is available, the only flow to be counted on would be the minimum shown in the hydrograph, namely, about 0.3 cu ft per sec per sq mile, which occurs in Dec. In order that the plant depending on this flow may never fail, the hydrograph used must be that for the year of lowest flow.

If storage is available, the problem is as follows. The area under the hydrograph represents total flow for the year in cu ft, or the area A of 1 square represents 2 592 000 cu ft, which, divided by 43 560 = 5.95 acre ft, or a vol of 1 cu ft per sec flowing for 1 month. For a flow of 1 cu ft per sec per sq mile, the required storage will be the shaded areas B and C, minus the small bit between them, where the hydrograph line rises above the 1-sec-ft line. This area can be taken off by planimeter and changed to cu ft, giving the required storage. This storage is filled up by the flow corresponding to D in the previous year, and will again be drawn down at E, although at E a smaller quantity is required than at B and C. If the storage is known from a contour map of the reservoir site and assumed height of dam, and it is required to find the available flow, the problem is reversed and the line corresponding to the 1-sec-ft line is found by trial so as to enclose an area equal to the given storage. The hydrograph shows that the distribution of flow throughout the year is important. Streams fed by melting snow in mountainous regions show a much steadier flow than in some southwestern rivers, which have little or no steady flow, and are subject to violent and erratic floods caused by cloudbursts. In the former case, a diversion dam may suffice, or a small storage may be needed; in the latter, a diversion dam must be built, and the flood flows diverted to a reservoir or basin having a capac sufficient to furnish the required supply for some months.

In connection with storage reservoirs note that, as evaporation losses in arid regions are large, it may be necessary to allow for 2 or 3 ft or more depth over the entire reservoir for this loss. Seepage losses also are sometimes high, especially during the first 2 or 3 years after completing a reservoir. The total losses often amount to 25% of the influent, and under best conditions are seldom less than 10%.

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SECTION 39

ENGINEERING THERMODYNAMICS

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Note.—Numbers in parentheses in text (except numbers of equations) refer to Bibliography at end of this section.

ENGINEERING THERMODYNAMICS

Introduction. The term thermodynamics has here been extended to include not only part of the theory of the subject, but also formulas and numerical data on air compression, steam, air and internal-combustion engines, fluid flow, combustion and related reactions. This section, therefore, is one of engineering, rather than of pure thermodynamics. In it reference is frequently made to Prof C. E. Lucke's work on Engineering Thermodynamics (1). Methods of computing engine power, steam and fuel consumption, power to compress air and similar problems are given in Art 3, 4, 5, leaving to Sec 40 the subjects of mechanical construction, sizes, costs and actual test performance data of engines. Tables of physical and chemical constants frequently used in engineering computations are included in their respective articles.

1. WORK AND POWER

Work in terms of pressure and volume. Work is expressed as the product of force and distance; for the units of work see Sec 36, on Mechanics. Since pressure is force per unit area, following equations show that work is also the product of press and vol:

$$PA = F, \quad \text{hence } W = PAL; \quad \text{but } AL = V, \quad \text{whence } W = PV,$$

where P = press per unit area, A = area on which press acts, F = force acting, W = work, L = distance, ft, and V = volume swept through while pressure is acting. The result is in ft-lb, if press is in lb per sq ft abs and vol is in cu ft.

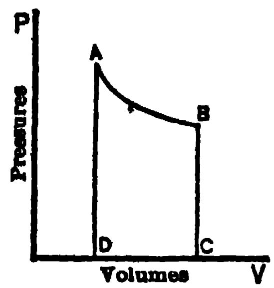

Fig 1. Graphical Representation of Work

Graphical representation of work. In Fig 1, the coordinates are press in lb per sq ft abs and vol in cu ft. Area denoting work done during any process is that under the curve, as AB, down to line of zero press; thus the work done during the volume change A to B is the area $ABCD$.

Work of expansion or compression alone. When a gas changes in volume, work is done irrespective of a change in press; without change in volume, no work is done.

$$\text{For constant press, } W = P(V_1 - V_2) \tag{1}$$

$$\text{variable press, } W = \int P dV \tag{2}$$

The general expansion law for all gases is $PV^s = K$ (constant). When integrated, equation (2) assumes 2 forms, one when value of s in the general law is unity, the other when s has a value other than unity. Results of integration are given below in several convenient forms, in which P_1 is the greatest press and P_2 the least, V_1 being the least vol and V_2 the greatest.

For special cases s has definite values for every gas. The most important case is isothermal expansion or compression, in which the temp is constant during expansion or compression, and $s = 1$ for all gases. Another important case is adiabatic compression or expansion, in which no heat is given to or removed from the gas during the process, and s equals specific heat at constant press divided by the specific heat at constant vol. Table 1 gives some adiabatic values of s, lying between 1 and 1.5.

$s = 1$	s not equal to 1	
$W = P_1V_1 \text{ Nap log} \frac{P_1}{P_2}$	$W = \frac{P_1V_1}{s-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right]$	(3)
$= P_1V_1 \text{ Nap log} \frac{V_2}{V_1}$	$= \frac{P_1V_1}{s-1}\left[1 - \left(\frac{V_1}{V_2}\right)^{s-1}\right]$	(4)
$= P_2V_2 \text{ Nap log} \frac{P_1}{P_2}$	$= \frac{P_2V_2}{s-1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} - 1\right]$	(5)
$= P_2V_2 \text{ Nap log} \frac{V_2}{V_1}$	$= \frac{P_2V_2}{s-1}\left[\left(\frac{V_2}{V_1}\right)^{s-1} - 1\right]$	(6)

Vapors may be assumed to behave as gases only when considerably superheated; when near their saturation point, s is variable, and work determination by this method is inexact. When steam expands so that it is just dry or follows the saturation law, $s = 1.0646$. For ordinary expansion of s in a cylinder the average value of s is 1, but the expansion is not isothermal.

Table 1. Adiabatic Values of "s"

Substance	s	Authority	Substance	s	Authority
Air	1.4066	Smithsonian Tables	Nitrogen	1.41	Casin
			Nitrous oxide	1.291	Wullner
Ammonia, wet	1.1	Average practice	Pintsch gas	1.24	Pintsch Co
Ammonia, superheated	1.3	Average practice	Sulph dioxide	1.26	Casin
Carbon dioxide	1.3	Röntgen	Steam, superheated	1.3	Smithsonian Tables
Carbon monoxide	1.403	Wullner			
Carbon disulphide	1.2	Beyne	Steam, wet	1.111	Rankine
Hydrogen	1.41	Casin	Steam, wet	1+0.014 × % moist	Perry
" sulphide	1.276	Müller	Steam, wet	1.035+0.01 × % moist	Gray
Methane	1.316	Müller			

For adiabatic expansion or compression the work done is given by Eq 7, as well as by Eq 3 to 6.

$$W = JC_v(T_2 - T_1)w \tag{7}$$

where J = Joule's equivalent = 778 approx; C_v = sp heat at constant vol, T_1 and T_2 = abs temp before and after compression, and w = weight of gas. The heat added to or removed from a gas during expansion or compression is given by Eq 8.

$$Q = \frac{W}{J} \times \left(\frac{C_p}{C_v} - s\right) \div \left(\frac{C_p}{C_v} - 1\right) \tag{8}$$

where Q = heat, C_p and C_v = specific heats at constant press and constant vol, J = Joule's equivalent, W = work, and s = exponent of V in $PV^s = K$. For the isothermal case, $Q = W \div J$, heat and work being equal when expressed in same units, since $s = 1$. For the adiabatic case, $Q = 0$, since $s = C_p \div C_v$.

Example 1. 5 cu ft of air expanded to vol of 25 cu ft. If expansion occurs (a) at constant press, and (b) so that $s = 1.4$, what will be the work done, the original press being 100 lb per sq in abs?

$$(a)\quad W = 144 \times 100(25 - 5) = 288\,000 \text{ ft-lb} \qquad (b)\quad W = \frac{144 \times 100 \times 5}{1.4 - 1}\left[1 - \left(\frac{5}{25}\right)^{1.4-1}\right] = 85\,500 \text{ ft-lb}$$

Example 2. How much heat is added during the expansion of Example 1, and how much if $s = 1$ and 1.2?

For constant press, $s = 0$, since if $s = 0$, $PV^s = K$ becomes $P = K$; hence for first case from Eq 8,

$$Q = \frac{288\,000}{778} \times \frac{\dfrac{0.24}{0.17}}{\dfrac{0.24}{0.17} - 1} = 1\,270 \text{ B t u}$$

$$\text{For } s = 1.4,\quad Q = 0 \qquad \text{For } s = 1,\quad Q = \frac{144 \times 100 \times 5 \text{ Nap log } 5}{778} = 149$$

$$\text{For } s = 1.2,\quad Q = \frac{0.5\,W}{778}, \text{ and from Eq 4, } W = \frac{14\,400 \times 5}{0.2}\left[1 - \left(\frac{1}{5}\right)^{0.2}\right]$$

or

$$W = 100\,800 \quad\text{and}\quad Q = 64.3$$

Work of complete cycles. Expansion or compression of gases rarely occurs alone, but in connection with other processes, as admission of gas to a cylinder or its delivery from one. A single process, as expansion, is a phase; several phases forming a complete operation constitute a cycle, net work of a cycle being the algebraic sum of the work of each phase. The sign of the work of a particular phase is determined by whether the work is done on the gas or by it; it is immaterial which is considered positive or negative, provided the designation is consistent throughout the cycle.

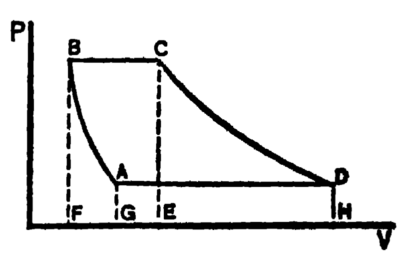

Fig 2. Complete Work Cycle, to P–V Coordinates

Thus, consider the cycle $ABCD$ (Fig 2) composed of: adiabatic compression AB, constant press vol increase BC, adiabatic expansion CD and constant press vol decrease DA. Such a cycle, considered in the order named, represents an engine case or, in reverse order, a compressor case. Net work will be $W_{bc} + W_{cd} - W_{da} - W_{ab}$, or areas $BCEF + CDHE - DHGA - ABFG$. Value for each area may be found from Eq 1 to 6, their algebraic sum giving area $ABCD$, representing net work of the cycle. Numerical value of the work is equal in either case, but the sign for the engine cycle will be opposite to that of the compressor, since the work is done by the gas in one case and on it in the other.

Example 2. Consider the pressures in Fig 2 to be 10 and 100 lb per sq in abs, and the vol at A and C to be 5 and 8 cu ft respectively. Find net work done, if $s = 1.43$.

$$W_{ab} \text{ by Eq 5} = \frac{144 \times 10 \times 5}{0.43}[10^{0.3} - 1] = 16\,800 \text{ ft-lb}$$

$$P_a V_a^{1.43} = P_b V_b^{1.43} \quad \text{or} \quad V_b = V_a \left(\frac{1}{10}\right)^{0.7} \quad \text{or} \quad V_b = 1 \text{ cu ft}$$

$$W_{bc} \text{ by Eq 1} = 144 \times 100 \times (8 - 1) = 100\,800 \text{ ft-lb}$$

$$P_c V_c^{1.43} = P_d V_d^{1.43} \quad \text{or} \quad V_d = V_c(10)^{0.7} \quad \text{or} \quad V_d = 40 \text{ cu ft}$$

$$W_{cd} \text{ by Eq 5} = \frac{144 \times 10 \times 40}{0.43}[10^{0.3} - 1] = 134\,000 \text{ ft-lb}$$

$$W_{da} \text{ by Eq 1} = 144 \times 10 \times (40 - 5) = 50\,400$$

hence,

$$\text{net work} = -16\,800 + 100\,800 + 134\,000 - 50\,400 = 167\,600 \text{ ft-lb}$$

Mean effective pressure (m e p) is that press which, if acting for one stroke, would give the same work as actually accomplished during the entire cycle irrespective of number of strokes of the cycle. If the area under curve BCD of Fig 2 be divided by the length of diagram, the result will be the mean height or press for that portion of the cycle, and the mean press for the remaining portion may be similarly obtained. The former quantity is termed the mean forward press, as it occurs during the forward stroke of the machine, the latter the mean back press. The difference between them is the mean effective press (m e p), which may be obtained algebraically in a similar manner to the work, or experimentally by the indicator.

Indicated horsepower (I H P) is the power developed in the cylinder of a machine, and is so named since it is usually obtained by the indicator. Numerically it is equal to (m e p) Lan + 33 000, where (m e p) is the mean effective press in lb per sq in, L is length

Table 2. Engine Horsepower Constants

Diam of cyl, in	Speed of piston, ft per min								
	100	200	300	400	500	600	700	800	900
8	0.1523	0.3046	0.4570	0.6093	0.7616	0.9139	1.0662	1.2186	1.3709
8 1/2	0.1720	0.3439	0.5159	0.6878	0.8598	1.0317	1 2037	1.3756	1.5476
9	0.1928	0.3856	0.5783	0.7711	0.9639	1.1567	1.3495	1.5422	1.7350
9 1/2	0.2148	0.4296	0.6444	0.8592	1.0740	1.2888	1.5036	1.7184	1.9532
10	0.2380	0.4760	0.7140	0.9520	1.1900	1.4280	1 6660	1.9040	2.1420
11	0.2880	0.5760	0.8639	1.1519	1.4399	1.7279	2.0159	2.3038	2.5818
12	0.3427	0.6854	1.0282	1.3709	1.7136	2.0563	2.3990	2.7418	3.0845
13	0.4022	0.8044	1.2067	1.6089	2.0111	2.4133	2.8155	3.2178	3.6200
14	0.46o5	0.9330	1.3994	1.8659	2.3324	2 7989	3.2654	3.7318	4.1983
15	0.5355	1.0710	1.6065	2.1420	2.6775	3.2130	3.7485	4.2840	4.8195
16	0.6093	1.2186	1.8278	2.4371	3.0464	3.6557	4.2650	4.8742	5.4835
17	0.6878	1.2756	1.9635	2.6513	3.3391	4.0269	4.6147	5.4026	6.1904
18	0.7711	1.5422	2.3134	3.0845	3.8556	4.6267	5.3978	6.1690	6.9401
19	0.8592	1.7184	2.5775	3.4367	4.2959	5.1551	6.0143	6.8734	7.7326
20	0.9520	1.9040	2.8560	3.8080	4.7600	5.7120	6 6440	7.6160	8.5680
21	1.0496	2.0992	3.1488	4.1983	5.2479	6.2975	7.3471	8.3966	9.4462
22	1.1519	2.3038	3.4558	4.6077	5.7596	6.9115	8.0634	9.2154	10.367
23	1.2590	2.5180	3.7771	5.0361	6.2951	7.5541	8.8131	10.072	11.331
24	1.3709	2.7418	4.1126	5.4835	6.8544	8.2253	9.5962	10.967	12.338
25	1.4875	2.9750	4.4625	5.9500	7.4375	8.9250	10.413	11.900	13.388
26	1.6089	3.2178	4.8266	6.4355	8.0444	9.6534	11.262	12.871	14.480
27	1.7350	3.4700	5.2051	o.9401	8.6751	10.410	12.145	13.880	15.615
28	1.8659	3.7318	5.5978	7.4637	9.3296	11.196	13.061	14.927	16.793
29	2.0016	4.0032	o.0047	8.0063	10 008	12.009	14.011	16.013	18.014
30	2.1420	4.2840	6.4260	8.5680	10 710	12 852	14.994	17.136	19.278

of stroke, ft, a is the net area of piston, sq in, and n the number of cycles per min. The term nL is the PISTON SPEED, and Lan + 33 000, the ENGINE CONSTANT. Horsepower (h p) results from multiplying the constant by (m e p). Values for the constant are given in Table 2 for different size cylinders and piston speeds.

Piston speed is best obtained as the product of number of working chambers, length of stroke and rev per min, divided by number of revolutions to complete one cycle. In 2-stroke cycle machines, which include all steam and air engines, compressors, pumps and many oil engines, the no of cycles per working chamber = no of rev. In 4-stroke cycle, mainly gas and gasolene engines, no of cycles = half the revolutions.

Example 4. What is the I H P of a 9 by 12-in single-cylinder, double-acting engine, running at 300 rev per min, when the m e p is 50 lb per sq in? Diam of cylinder is always the first dimension given when expressing size of an engine. In this case there are 2 working chambers, and a cycle is completed in 1 revolution; hence piston speed is 600 and constant is 1.16, giving 58 I H P.

Example 5. What is the h p of a 6-cylinder, single-acting, 4-stroke cycle gas engine, 6 by 6 in, running at 1 000 rev per min, if the m e p is 80 lb per sq in?

Piston speed = (6 × 0.5 × 1 000) ÷ 2 = 1 500; engine constant = 1.29; I H P = 103.2.

Brake or shaft h p, so called because found by a brake, is that actually available at the shaft or belt pulley, and equals the indicated minus friction h p. Ratio of brake to indicated is the mech effic of the engine. In a combined machine, as a direct-connected air compressor or pump, the mech effic is usually taken as the ratio of I H P of the air or water end to that of the steam end. But in a pump, the output is sometimes considered as the h p equivalent of lb of water pumped × head.

Thermal efficiency is the ratio of work done in the engine cylinder to the work equivalent of the heat required to do it.

Example 6. A pump delivers 1 000 gal of water per min against a net head of 150 lb per sq in. The I H P of water end is 100 and of steam end 120, and it uses 75 lb steam per min, each lb containing 1 200 heat units. What are the mech and thermal effics? Considering the mech effic to be the ratio of work in the cylinders, there results 100 ÷ 120 = 0.833 or 83.3%. Considering the output to be the net work of the pump, which is (1 000 × 8.3 × 150 × 2.31) ÷ 33 000 = 87, there results 87 ÷ 120 = 0.725 or 72.5%. Work done per min in the steam cylinder is 33 000 × 120, and the work equivalent in the steam is 778 × 75 × 1 200; hence thermal effic is 0.056 or 5.6%.

2. FLOW OF GASES AND VAPORS

Flow through nozzles and orifices, with large pressure drop. When gases or vapors expand in nozzles the expansion occurs so rapidly that there is no time for escape or addition of heat (although actually a portion of the energy of the jet is turned back into heat by friction against the nozzle walls), hence the process is assumed to be adiabatic, and the preceding equations apply as well in this case, called free expansion, as when the expansion occurs in a cylinder, called balanced expansion. As the fluid must be brought to the nozzle and that which has expanded must be removed, the complete cycle is as in Fig 3, the area $ABFE$ representing the work done by the fluid approaching the nozzle, area $BCGF$ that of expansion, and $CDEG$ that required to remove the expanded fluid and hence negative. Algebraic sum of these areas is

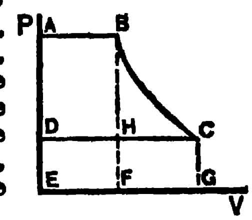

Fig 3. P–V Diagram Representing Nozzle Flow

$$W = \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} \right] \tag{9}$$

If V_b represents the volume of 1 lb under the initial conditions, W is the work done by 1 lb, and if this work is used in accelerating the same 1 lb of fluid, then,

$$W = (u_2^2 - u_1^2) \div 2g \tag{10}$$

In nozzle work, u_1, the original velocity, is generally small as compared to u_2, the final velocity, so that u_1^2 may be neglected; in which case, from Eq 9 and 10,

$$u_2 = 8.02 \sqrt{\frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} \right]} \tag{11}$$

Weight of fluid flowing through the nozzle follows directly, since it equals the product of velocity, cross-section and wt per cu ft. Let A = area at point of max veloc, then,

$$w = uA\delta_c = 8.02\, A\delta_c \left\{ \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} \right] \right\}^{0.5}$$

where w = lb per sec, A = area in sq ft, and δ_c = wt of 1 cu ft at the low press. It is generally more convenient to express δ_c in terms of original conditions and since

$$\delta_c = \frac{1}{V_c} = \frac{1}{V_b} \left(\frac{P_c}{P_b} \right)^{\frac{1}{s}}$$

$$w = \frac{8.02\, A}{V_b} \left(\frac{P_c}{P_b} \right)^{\frac{1}{s}} \left\{ \frac{s}{s-1} P_b V_b \left[1 - \left(\frac{P_c}{P_b} \right)^{\frac{s-1}{s}} \right] \right\}^{0.5} \tag{12}$$

w becomes a maximum when first differential $= 0$, which occurs when $\left(\frac{P_c}{P_b}\right)^{\frac{1-s}{s}} = \frac{s+1}{2}$, or for the maximum flow,

$$P_c = P_b \left(\frac{2}{s+1}\right)^{\frac{s}{s-1}} \tag{13}$$

Reducing the lower press will not increase wt, but final velocity only. Throat area of a nozzle is designed to pass the desired wt for the above press ratio, and the mouth area to permit of the complete expansion. A relation between throat and mouth area (Moyer) is

$$\left.\begin{aligned} \frac{\text{Mouth area}}{\text{Throat area}} &= 0.172\,\frac{P_b}{P_c} + 0.7, && \text{if } \frac{P_b}{P_c} < 25 \\ &= 0.175\left(\frac{P_b}{P_c}\right)^{0.94} + 0.7, && \text{if } \frac{P_b}{P_c} > 25 \end{aligned}\right\} \tag{14}$$

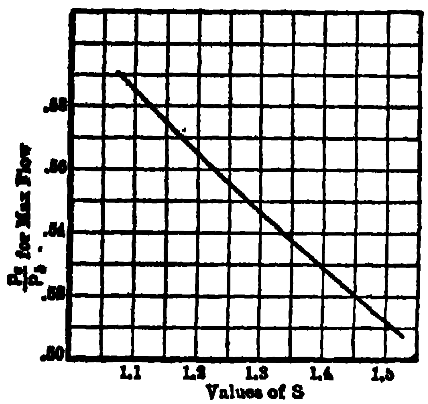

Fig 4. Pressure Ratio for Max Wt Discharged by Nozzle for any Value of "s"

The line joining mouth and throat varies greatly, but a common form is almost a straight line and a 20° angle.

The curve in Fig 4 shows the pressure ratio $P_c \div P_b$ for max flow for different values of s. For all vapors, unless considerably superheated, s varies in value throughout the expansion, and hence results for vapors by above method are not strictly accurate. For an exact method see Art 10.

The above equations are for perfect nozzles or orifices, so that the actual wt discharged, or the area to pass a given wt, may differ somewhat from that calculated, due to stream contraction. For orifices with sharp corners the effective area may be as low as 60% of the actual, while for nozzles it may be as high as 95%.

For flow occurring when the low press is below the critical value, the empiric equations in Table 3 apply.

Table 3. Formulas for Flow of Steam and Air from Orifices

Lb dry steam per hr per sq in of orifice	$60\,p^{0.97}$	Grashof
	$51.43\,p$	Napier
	$49.6\,p$	Harter
	$3.6\,p\,[16.36 - 0.96 \log p]$	Rateau
Lb superheated steam per hr per sq in of orifice	$60\,p^{0.97} + [1 + 0.00065 \times \text{degrees superheat}]$	Moyer
	$49.6\,p$ to $45\,p$ for superheat 0° to 185° at 160 lb	Harter
Lb of air per hr per sq in of orifice	$1\,900\,p_1 + \sqrt{T_1}$ for flows from 2 atmos or more to 1 atmos	Fliegener
Lb of wet steam per hr per sq in of orifice	$60\,p^{0.97} + \sqrt{\text{dryness fraction}}$	Grashof

Example 7. Calculate loss of air per hr from a tank in which the press is maintained at 90 lb per sq in gage and temp is 100° F, if there be a leak amounting to 1/8 in of effective opening. Quantity of air escaping can be no greater than if the press without the tank were 53% of that within, so this value will be used for P_c in Eq 12. For conditions given, $V_b = 2$ cu ft; hence, by Eq 12, wt per hr =

$$3\,600 \times \frac{8.02 \times 0.125}{144 \times 2} \times (0.53)^{0.71}\{3.5 \times 144 \times 105 \times 2[-(0.53)^{0.29}]\}^{0.5} = 1\,070 \text{ lb}$$

By Fliegener's empiric equation, wt per hr $= (1\,900 \times 105) + (\sqrt{560} \times 8) = 1\,030$ lb. Should the press within the tank be only 10 lb gage, the empiric formula would no longer apply, since the low press is above the critical value, and the result must be found by Eq 12. For above press, $V_b = 8.4$ cu ft, hence the wt per hr will be

$$3\,600 \times \frac{8.02 \times 0.125}{144 \times 8.4} \times (0.6)^{0.71}\{3.5 \times 144 \times 25 \times 8.4[1 - (0.6)^{0.29}]\}^{0.5} = 253 \text{ lb.}$$

Flow through orifices with small pressure drops occurs in all engine and compressor valves, and in discharge from pipes. Use is made of this drop in press for measurement purposes. For water and non-expansive fluids the velocity of flow is merely $\sqrt{2\,gh}$; but with expansive fluids the work done in expanding must also be accounted for. In Fig 3,

for a non-expansive fluid the work causing increase in velocity is represented by area *ABHD*; for expansive fluids the area *ABCD*. Naturally, for small press differences the net work of expansion *BCH* may be neglected. Hence, there are 2 sets of expressions for flow for small drops: (*a*) the exact, for expansive; (*b*) the exact, for non-expansive and approximate for expansive.

$$\left.\begin{array}{r}\text{Lb per sec per sq ft}\\ \text{of effective orifice}\end{array}\right\} = \sqrt{2g\frac{s}{s-1}\frac{P_1}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{2}{s}} - \left(\frac{P_2}{P_1}\right)^{\frac{s+1}{s}}} \quad (a)$$
$$= \sqrt{2g\frac{P_1 - P_2}{V}} \quad (b) \qquad (15)$$

where P_1 and P_2 are initial and final press, in lb per sq ft; V_1, the vol per lb at initial press; and V, the vol per lb at aver press, $(P_1 + P_2) \div 2$. The effective opening is always less than the actual; Table 4 gives values for the ratio of effective to actual.

For air Eq (16) is useful, and Table 5 gives values for C found by Durley (6). These values apply exactly for air flowing into a press of 30 in of mercury, between 40° and 100° F, when the orifice is less than 1/20th the area of the approaching pipe or chamber, has sharp edges, and a thickness of 0.057 in. For a distance, they apply fairly well beyond the above conditions, the character of the orifice edge being the variable having the most influence; values of C as high as 0.99 having been found for orifices with rounded approach.

Table 4. Values of Ratio of Effective to Actual Openings in Orifices (Weisbach)

$P_1 \div P_2$	1.05	1.09	1.43	1.65	1.89	2.15
C for orifice, diam = 0.39 in	0.555	0.589	0.692	0.724	0.754	0.788
$P_1 \div P_2$	1.05	1.09	1.36	1.67	2.01	.....
C for orifice, diam = 0.84 in	0.558	0.573	0.634	0.678	0.723	.....
$P_1 \div P_2$	1.05	1.1	1.3	.....	.....	.....
C for short tube, d = 0.39, l = 1.18	0.730	0.771	0.830	.....	.....	.....
$P_1 \div P_2$	1.41	1.69	.....	.....	.....	.....
C for short tube, d = 0.56, l = 1.67	0.813	0.822	.....	.....	.....	.....

$$\text{Lb air per sec to atmosphere} = 0.6283\, Cd^2 \sqrt{h_w \div T}, \text{ through a circle } d \text{ in in diam} \qquad (16)$$

where h_w = press drop, in of water, and T = abs temp.

Table 5. Discharge Coeff C for Different Heads and Diam of Orifice (Durley)

Diam of orifice, in	Drop in press, in of water				
	1	2	3	4	5
5/16	0.603	0.606	0.610	0.613	0.616
1/2	0.602	0.605	0.608	0.610	0.613
1	0.601	0.603	0.605	0.606	0.607
1 1/2	0.601	0.601	0.602	0.603	0.603
2	0.600	0.600	0.600	0.600	0.600
2 1/2	0.599	0.599	0.599	0.598	0.598
3	0.599	0.598	0.597	0.596	0.596
3 1/2	0.599	0.597	0.596	0.595	0.594
4	0.598	0.597	0.596	0.594	0.593
4 1/2	0.598	0.596	0.596	0.593	0.592

In general, the constants for different types of orifice may be taken between the limits shown in Table 6, no rule for specific cases being possible.

If the gas being measured is other than air the wt of flow is found from Eq 17:

$$w_G = w_A \sqrt{V_A \div V_G} \qquad (17)$$

where w_G and w_A are the wt of gas and air respectively, and V_G and V_A their vol per lb.

Table 6. General Limits of Constants (Weisbach)

Type of orifice	Constant
Rounded entrance	0.97–0.99
Sharp edge thin plate	0.55–0.80
Short straight tubes	0.80–0.85
" rounded entrance tube	0.92–0.93
Converging orifice	0.90–0.99

Venturi tubes are used commercially for measuring gases and vapors, and here again 2 expressions result:

$$\left.\begin{aligned}\text{Lb per sec} &= A_2\left(\frac{P_2}{P_1}\right)^{\frac{1}{s}}\sqrt{2\,g\,\frac{s}{s-1}\,P_1\delta_1\,\frac{1-\left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}}{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{P_2}{P_1}\right)^{\frac{2}{s}}}} \qquad (a)\\ &= \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2\,g\,(P_1-P_2)\,\delta} \qquad (b)\end{aligned}\right\} \qquad (18)$$

where A = area in sq ft, P = press in lb per sq ft, δ = wt per cu ft of gas, and s = ratio of specific heats, $C_p \div C_v$, of the gas. Subscript 1 refers to upstream, and 2 to throat conditions, no subscript referring to the mean value. For steam the exact expression is simplified, since the factor

$$\frac{s}{s-1}\,\frac{P_1}{\delta_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{s-1}{s}}\right] = 778 \times \text{work of Rankine cycle in B t u}$$

between pressures P_2 and P_1, and may be found by the Mollier diagram, Fig 27, hence,

$$\text{Lb steam per sec} = A_2\left(\frac{P_2}{P_1}\right)^{\frac{1}{s}}\delta_1\sqrt{\frac{2\,g \times 778\ (\text{work of Rankine cycle})}{1-\left(\frac{A_2}{A_1}\right)^2\left(\frac{P_2}{P_1}\right)^{\frac{2}{s}}}} \qquad (19)$$

Pitot tube is also used for measuring gases. As it is primarily a velocity meter, 2 sets of values are possible for its results, one neglecting the work of expansion, the other accounting for it. But, to secure accurate readings the velocities must be high, so that the use of the exact expressions is more important than in the orifice or Venturi meter. As the readings are often inches of water or mercury, as well as in lb, the equations are made to apply to these units also.

$$\left.\begin{aligned}\text{Lb per sec} &= A\sqrt{Z}\sqrt{2\,g\,\delta_1\left[1-\frac{K}{2s}\,\frac{1+s}{6\,s^2}K^2\right]-\left[\frac{(1+s)(1+2\,s)}{24\,s^3}\right]K^3} \qquad (a)\\ &= A\sqrt{Z}\sqrt{2\,g\,\delta_1}\\ \text{where}\quad Z &= P_2-P_1 = 5.2(hw_2-hw_1) = 70.8\,(hm_2-hm_1) \qquad (b)\end{aligned}\right\} \qquad (20)$$

$K = \left(\frac{P_2}{P_1}-1\right)$, s = ratio of specific heats, $C_p \div C_v$, δ = lb per cu ft of fluid, $(P_2 - P_1)$ = diff in press in lb per sq ft, $(hw_2 - hw_1)$ inches of water, and $(hm_2 - hm_1)$ inches of mercury, subscript 1 referring to the tangential orifice and 2 to the impact orifice.

Flow in pipes. For flow of gases and vapors through pipes there is little more reliable information than empiric formulas, some of the commoner being given below (Gebhardt). For theory of pipe flow, see (1); for pipe sizes used in practice, see (5).

$$\text{Veloc, ft per min} = A\sqrt{\frac{(p_1-p_2)\,d}{\delta L}}, \quad \text{where } A = \begin{cases}9\,240 & \text{Geipel \& Kilgour}\\ 9\,976 & \text{Hawksley}\\ 10\,350 & \text{Martin}\\ 10\,360 & \text{Hurst}\end{cases} \qquad (21)$$

$$\text{Flow, lb per min} = B\sqrt{\frac{(p_1-p_2)\,\delta\,d^5}{L}}, \quad \text{where } B = \begin{cases}50.2 & \text{Geipel \& Kilgour}\\ 54.4 & \text{Hawksley}\\ 56.5 & \text{Martin}\\ 56.5 & \text{Hurst}\end{cases} \qquad (22)$$

$$\left.\begin{array}{l}\text{Pressure drop,}\\ \text{lb per sq in}\end{array}\right\} = \frac{C\,w^2\,L}{\delta\,d^5}, \quad \text{where } C = \begin{cases}0.000396 & \text{Geipel \& Kilgour}\\ 0.000337 & \text{Hawksley}\\ 0.000313 & \text{Martin}\\ 0.000313 & \text{Hurst}\end{cases} \qquad (23)$$

$$\text{Pipe diam, in} = D\sqrt[5]{\frac{w^2\,L}{(p_1-p_2)\,\delta}}, \quad \text{where } D = \begin{cases}0.2087 & \text{Geipel \& Kilgour}\\ 0.2010 & \text{Hawksley}\\ 0.1990 & \text{Martin}\\ 0.1990 & \text{Hurst}\end{cases} \qquad (24)$$

In above equations, d = diam, in; $(p_1 - p_2)$ = press drop, lb per sq in; w = lb flow per min; L = length of pipe, ft, and δ = lb per cu ft of fluid.

Following values by Hunt show loss in press due to 90° elbows, expressed in equivalent ft of straight pipe:

Pipe, diam	Resistance	Pipe, diam	Resistance	Pipe, diam	Resistance	Pipe, diam	Resistance
1	1.5	5	20	9	44.4	16	90.1
2	4.9	6	25.9	10	50.7	18	104
3	9.4	7	32	12	63.7	20	117
4	14.5	8	38	14	76.7	22	130
....	....	....	....	......	....	24	144

Draft due to convection flow, as found in chimneys and in some ventilating systems, depends on temp and height of stack or duct. For STATIC conditions the difference in press of draft in in of water (the common unit) is $H\left(\frac{7.64}{T_C} - \frac{7.95}{T_H}\right)$ for chimney gases, or $7.64\,H\left(\frac{1}{T_C} - \frac{1}{T_H}\right)$ for air. Due to flow conditions, wall resistances, bends, changes of cross-sec in duct work, many of which can not be evaluated, the actual draft will be less. For chimneys the same is true, besides which there are fuel-bed, boiler-tube and economizer resistances, rendering any but empiric formulas of little practical value. Stirling suggests a corrected equation for actual draft:

$$\text{Actual draft, in of water} = 7.64\,H\left(\frac{1}{T_C} - \frac{1}{T_H}\right) - \frac{fw^2 CH}{A^3} \qquad (25)$$

where H = height, ft, T_C = abs temp of air, T_H = mean abs temp of hot gases, w = wt of gases per sec, C = perimeter of stack, A = area of stack, and f = a constant varying from 0.001 to 0.002, 3 values for which are: 0.001 for steel stack (abs temp = 800), 0.0015 for steel stack (abs temp = 1 060) and 0.002 for brick stack (abs temp = 760).

Tendency of heated liquids and gases to rise is often utilized in heating or cooling systems to produce flow, the thermosyphon consisting merely of 2 pipes or flues, connected at top and bottom; through one pipe the warmer, lighter fluid rises, through the other the cooler and heavier descends.

Capacity of chimneys does not increase indefinitely with stack temp, but is maximum when the temp is approx twice the abs outside temp. Formulas are numerous and are all empiric. Chimney capacities are generally stated in boiler horsepower (B H P). Common formulas are:

$$\left.\begin{aligned} \text{B H P} &= 3.25\,A\sqrt{H} \text{ (Christie)} \\ \text{B H P} &= 3.33\,E\sqrt{H} \text{ (Kent)} \\ D &= 4.92\,(\text{B H P})^{2/5} \text{ (Stirling)} \end{aligned}\right\} \qquad (26)$$

The last formula can also be stated as B H P $= 12.6\,A^{5/4}$.

In these formulas, A = area of top of chimney, sq ft; H = ht of chimney, ft; E = effective area, after deducting 4 in from diam or width, to allow for chimney friction; D = diam of chimney, in. The first 2 formulas include a function of H, and therefore require previous determination of an H that will produce the draft required to overcome the combined resistance of the fuel-bed, boiler, and flue. It is claimed that the Stirling formula gives the stack area that costs least to construct, and that this area does not change with H, determined independently.

As the coal burned and flue gas produced per B H P are indefinite, the formulas are re-stated below, giving capacity in lb of flue gas per hr; using in first formula 4 lb coal per B H P (Christie); in second, 5 lb coal per B H P (Kent); in both formulas, 24 lb flue gas per lb coal, and in the last, the Stirling value of 120 lb flue gas per B H P.

$$\left.\begin{aligned} \text{Lb flue gas per hr} &= 312\,A\sqrt{H} \text{ (Christie)} \\ \text{" " " " "} &= 400\,E\sqrt{H} \text{ (Kent)} \\ \text{" " " " "} &= 1\,508\,A^{5/4} \text{ (Stirling)} \end{aligned}\right\} \qquad (27)$$

3. WORK AND CAPACITY OF AIR COMPRESSORS (See also Sec 15)

Isothermal compression requires the least work, but is impossible to attain in practice even with the most efficient cylinder jacketing.

Actual compressors operate with the compression line somewhere between isothermal and adiabatic. Leakage of air from the cylinder during compression causes displacement of compression curve, and gives a lower value to exponent s (Art 1). Some compressor

diagrams show apparently isothermal compression, whereas result is really due to leakage. Fig 5 shows an ideal diagram for the 2 cases in a no-clearance, single-stage machine, area $ABCD$ representing the work for isothermal compression, $ABC'D$ that for adiabatic, and CBC' the saving due to the former.

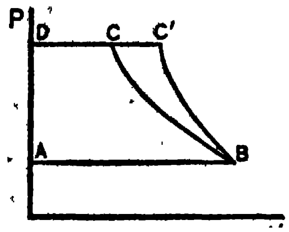

Fig 5. P–V Diagram for Ideal Single-stage Compression

Stage compression. Isothermal compression is more nearly approached if the gas be compressed in one cylinder to some intermediate press, delivered to an intercooler, and the compression completed in a second cylinder. This is termed staging, the number of stages being 2, 3, or 4, most commonly 2. Fig 6 shows the case for an ideal, no-clearance, two-stage machine. Area $ABCC'DE$ represents the process as given, area $ABCD'E$ that for single-stage adiabatic compression for same amount of air, and area $ABC'D'E$ for single-stage isothermal compression. Saving due to staging is area $CC'DD'$.

The air is generally cooled to about the original temp in an intercooler, and in the following equations this assumption is made. The press to which the air is compressed, termed the RECEIVER PRESS, affects the saving due to staging. Maximum saving occurs in 2 stages when

$$\text{receiver press} = (\text{low press} \times \text{high press})^{0.5} \tag{28}$$

and in 3 stages when

$$\left.\begin{aligned} \text{first receiver press} &= (\text{low press})^{0.67}\,(\text{high press})^{0.33} \\ \text{second receiver press} & (\text{low press})^{0.33}\,(\text{high press})^{0.67} \end{aligned}\right\} \tag{29}$$

As real compressors have a CLEARANCE vol, not all the air in cylinder can be expelled on each stroke, and the vol remaining must reëxpand to the low press before any fresh air

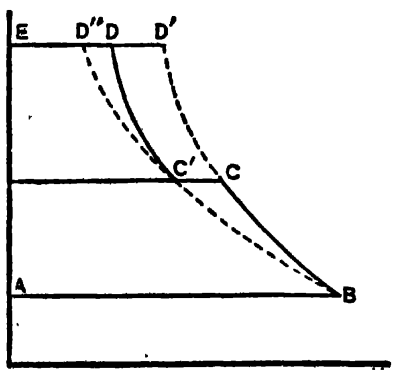

Fig 6. P–V Diagram for Ideal Two-stage Compression

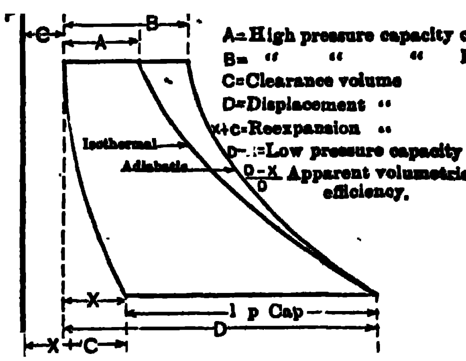

Fig 7. P–V Diagram Showing Effects of Clearance in Compressors

can be drawn in. Hence, the quantity of air compressed is always less than the piston displacement (Fig 7). The ratio of the low-press capacity to displacement is the APPARENT VOL EFFIC. Since the entering air comes in contact with the warm cylinder walls and pistons, it expands, and hence the quantity of air drawn in, when measured under external conditions of press and temp, is even less than would appear from the value of apparent vol effic. The ratio of the actual vol drawn in to the cylinder displacement is the TRUE VOL EFFIC. Eq 30 by Lucke is an attempt to formulate this quantity.

$$\text{True vol effic} = \frac{\text{apparent vol effic}}{1 + (0.3 \text{ to } 0.5)\left(\dfrac{\text{abs temp air delivered}}{\text{abs temp air supplied}} - 1\right)} \tag{30}$$

Air delivered in other than isothermal compression is at a higher temp than that supplied, and when cooled to original temp decreases in vol. Vol actually delivered is termed the high-press capacity hot; when cooled, the high-press capacity cold. These volumes for isothermal compression are equal.

Expressions for work and capacity of single and 2-stage compressors are given in Tables 7 and 8, in which following symbols are used:

lp = low press, lb per sq in absolute
hp = high press " " " "
rp = receiver press, lb per sq in absolute
c = clearance, as a fraction of displacement
cap = capacity, cu ft
d = diam of cylinder, in
l = stroke, in
n = cycles per min
D = displacement, cu ft per min
s = val of exponent of volume in equation for the compression curve $PV^s = K$, = 1.406 for adiabatic compression
T_1 = temp of entering air, abs
T_2 = temp of delivery air, abs
δ_1 = wt per cu ft of entering air
R_p = ratio of high press over low press
E_v = apparent vol effic
C_v = specific heat at constant volume
w = wt of air

Subscripts 1 and 2 refer to low and high-press cyl respectively.

Work required to compress a given quantity of air is independent of the clearance, but the size of the cylinder required varies with the clearance. A comparison of the expressions for work for the entire single-stage compressor cycle and for compression alone (Eq 5) shows that the work for the entire cycle is s times that for compression alone, that is, in Fig 8, area $ABCD$ is s times area

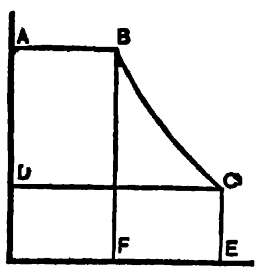

Fig 8. Ideal Compressor Card

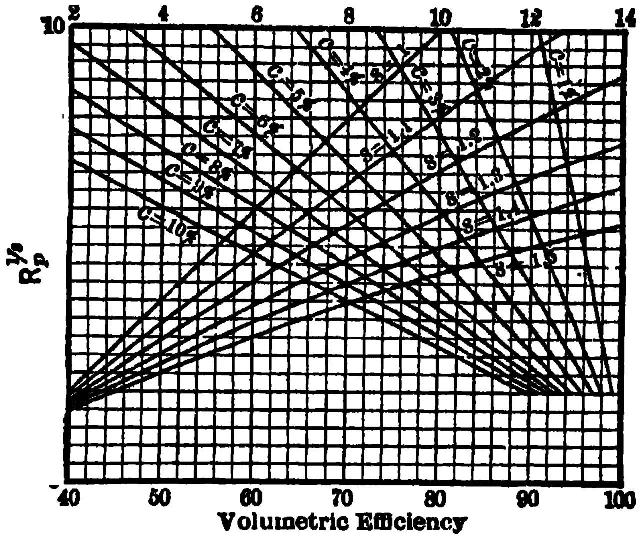

Fig 9. Chart to Find Apparent Volumetric Effic

$BCEF$. Hence, Eq 7, if multiplied by s, gives the complete work for adiabatic compression.

$$\text{Work of complete cycle for adiabatic compression} = JsC_v\,w\,(T_2 - T_1) \qquad (31)$$

Those quantities of Tables 7 and 8 most frequently required may be found graphically from Fig 9 to 11. Fig 9 is for finding the apparent vol effic for any clearance, press ratio, and s. Project from upper horiz scale to the proper curve for s, then horiz to the proper clearance curve, and then down

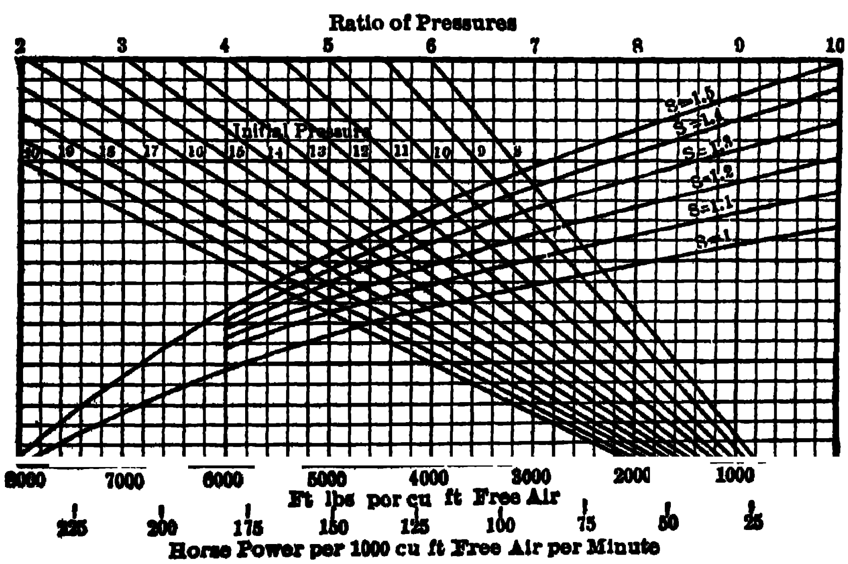

Fig 10. Horsepower for Single-stage Air Compressor

to lower horiz scale, where answer is read. By projecting from the s curve to left-hand scale the value of Rp^{1+s} is found, which gives the high-press capac hot, when the low-press capac or displacement is known, since the high-press capac hot = low-press capac + Rp^{1+s}. Fig 10 is to find the

work required to compress 1 cu ft, or the hp required to compress 1 000 cu ft of free air per min in one stage, between any pressures and for any value of s. The upper of the 2 lower scales, when multiplied by apparent vol effic, gives the m e p in lb per sq ft. To use, project downward from the value of R_p to the proper s curve, then horiz to the low-press curve, and finally downward to read the

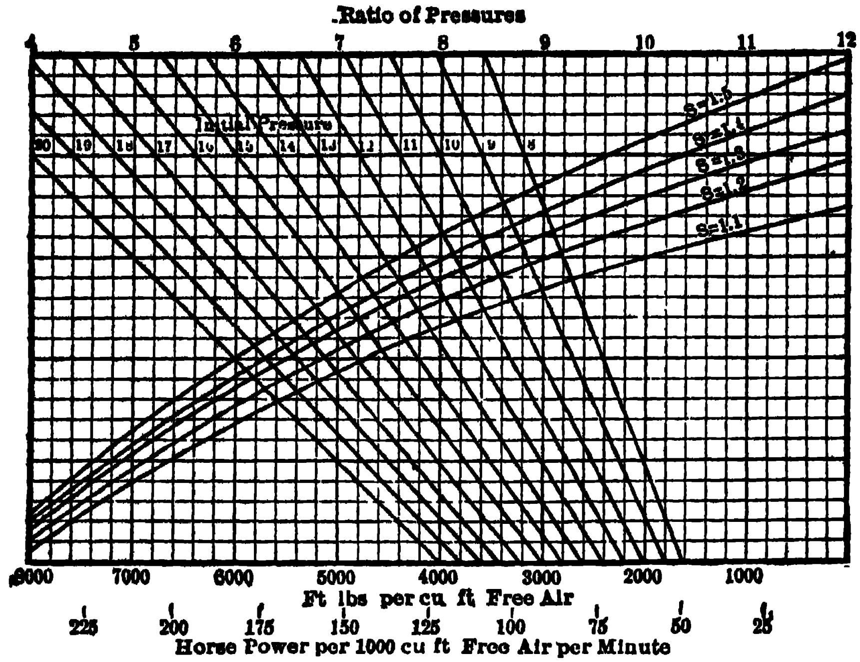

Fig 11. Horsepower for Two-stage Compression of Air, with Best Receiver Press and Perfect Intercooling

answer. Fig 11 is a diagram for 2-stage compression with best receiver press and perfect intercooling. Results obtained by the expressions of Tables 7 and 8, or from the curves, are for perfect compressors. The amount of saving by compressing in 2 stages over single-stage adiabatic is seen in Fig 12. Here the horiz scale is the ratio of high press to low, the vertical scale the ratio of the work required to compress a given amount of air in two stages to that required to compress an equal amount to the same pressure in one stage. As the value of s effects the result, curves for various values of this are given on the figure. Knowing the press ratio, project from the value to the proper s curve and then horizontally; the number so found when multiplied by the work required in single stage will give that required for 2-stage..

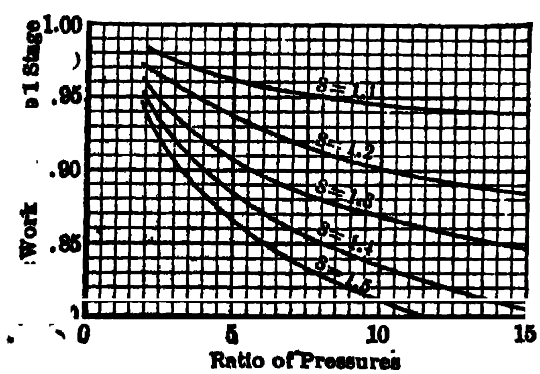

Fig 12. Diagram Showing Saving in Two-stage Air Compression

Blowers are used for compressing large volumes of air or other gas to press usually below 1 lb. As the work to compress a given quantity of gas from one press to another is independent of the type of machine, all the above work formulas apply equally to blowers. In case of fans the press to which the air is compressed is so little above that of the atmosphere that the work of compression alone becomes negligible, so that the work may be represented by the volume multiplied by the increase of press. But, increase of press must include not only that actually observed, but also that required to produce the velocity with which the air leaves the fan or blower, or,

$$W = 144(p_s + p_v)Q \tag{32}$$

where Q = cu ft of air per min, p_s = increase of press, and p_v = velocity head, both in lb per sq in. Since

$$v = \frac{Q}{60A} \qquad p_v = \frac{s}{144 \times 2g}\left(\frac{Q}{60A}\right)^2,$$

$$W = 144\,Q\left[p_s + \frac{s}{144 \times 2g}\left(\frac{Q}{60A}\right)^2\right] \tag{33}$$

in which s = wt per cu ft of gas, and A = area of discharge opening, sq ft.

The above equations and curves are for the ideal or perfect machine. Actual machines require greater hp, to allow for frictional and other losses. Relation between the ideal and actual hp, termed the DIAGRAM FACTOR, varies from 0.5 to 0.85; that is, the power

Table 7. Expression for Work and Capacity of Single-stage Compressors

	Isothermal compression	Non-isothermal compression
Work per min	$144\,(l\ p)\,(l\text{-}p\ cap)\ \text{Nap}\log R_p$	$144\,\frac{s}{s-1}\,(l\ p)\,(l\text{-}p\ cap)\left[(R_p)^{\frac{s-1}{s}}-1\right] = sc_v w\,(T_2-T_1)$*
Work per cu ft low press gas	$144\,(l\ p)\ \text{Nap}\log R_p$	$144\,\frac{s}{s-1}\,(l\ p)\left[(R_p)^{\frac{s-1}{s}}-1\right] = sc_v\delta_1\,(T_2-T_1)$*
Work per cu ft high-press gas hot	....	$144\,\frac{s}{s-1}\,(l\ p)\left[(R_p)^{\frac{s-1}{s}}-1\right]R_p^{\frac{1}{s}} = sc_v\delta_1\,(T_2-T_1)\,R_p^{\frac{1}{s}}$*
Work per cu ft high-press gas cold	$144\,(h\ p)\ \text{Nap}\log R_p$	$144\,\frac{s}{s-1}\,(h\ p)\left[(R_p)^{\frac{s-1}{s}}-1\right] = sc_v\delta_1\,(T_2-T_1)\,R_p$*
M e p, lb per sq in	$(l\ p)\,E_v\ \text{Nap}\log R_p = (l\ p)\,[1+c-cR_p]\ \text{Nap}\log R_p$	$\frac{s}{s-1}\,(l\ p)E_v\left[(R_p)^{\frac{s-1}{s}}-1\right] = \frac{s}{s-1}\,(l\ p)\left[1+c-cR_p^{\frac{1}{s}}\right]\left[(R_p)^{\frac{s-1}{s}}-1\right]$
Displacement per min, cu ft	$\frac{d^2ln}{2\ 200}$	$\frac{d^2ln}{2\ 200}$
Low-press capacity per min	$DE_v = D\,[1+c-cR_p]$	$DE_v = D\left[1+c-cR_p^{\frac{1}{s}}\right]$
High-press capacity hot per min	....	$\frac{DE_v}{R_p^{\frac{1}{s}}} = \frac{D\left[1+c-cR_p^{\frac{1}{s}}\right]}{R_p^{\frac{1}{s}}} = \frac{l\text{-}p\ cap}{R_p^{\frac{1}{s}}}$
High-press capacity cold per min	$DE_v \div R_p = D\,[1+c-cR_p] \div R_p = (l\text{-}p\ cap) \div R_p$	$\frac{DE_v}{R_p} = \frac{D\left[1+c-cR_p^{\frac{1}{s}}\right]}{R_p} = \frac{l\text{-}p\ cap}{R_p}$
Apparent vol effic	$1+c-cR_p = \frac{l\text{-}p\ cap}{D}$	$1+c-cR_p^{\frac{1}{s}} = \frac{l\text{-}p\ cap}{D}$

* For adiabatic compression only

Table 8. Expression for Work and Capacity of Two-stage Compressors with Perfect Intercooling

	Any receiver pressure	Best receiver pressure
Work per min	$144\frac{s}{s-1}(l\,p)\,(l\text{-}p\;cap)\left[\left(\frac{r\,p}{l\,p}\right)^{\frac{s-1}{s}}+\left(\frac{h\,p}{r\,p}\right)^{\frac{s-1}{s}}-2\right]$	$288\frac{s}{s-1}(l\,p)\,(l\text{-}p\;cap)\left[(R_p)^{\frac{s-1}{2s}}-1\right]$
Work per cu ft low-press gas	$144\frac{s}{s-1}(l\,p)\left[\left(\frac{r\,p}{l\,p}\right)^{\frac{s-1}{s}}+\left(\frac{h\,p}{r\,p}\right)^{\frac{s-1}{s}}-2\right]$	$288\frac{s}{s-1}(l\,p)\left[(R_p)^{\frac{s-1}{2s}}-1\right]$
Work per cu ft high-press gas hot	$144\frac{s}{s-1}(l\,p)\left(\frac{h\,p}{r\,p}\right)^{\frac{1}{s}}\left(\frac{r\,p}{l\,p}\right)\left[\left(\frac{r\,p}{l\,p}\right)^{\frac{s-1}{s}}+\left(\frac{h\,p}{r\,p}\right)^{\frac{s-1}{s}}-2\right]$	$288\frac{s}{s-1}(l\,p)\,(R_p)^{\frac{s+1}{2}}\left[(R_p)^{\frac{s-1}{2s}}-1\right]$
Work per cu ft high-press gas cold	$144\frac{s}{s-1}(h\,p)\left[\left(\frac{r\,p}{l\,p}\right)^{\frac{s-1}{s}}+\left(\frac{h\,p}{r\,p}\right)^{\frac{s-1}{s}}-2\right]$	$288\frac{s}{s-1}(h\,p)\left[(R_p)^{\frac{s-1}{2s}}-1\right]$
Displacement per min, cu ft	$\frac{d_1{}^2l_1n}{2\,200}$ for *l-p* cylinder	$\frac{d_1{}^2l_1n}{2\,200}$ for *l-p* cylinder
Low-press capacity per min	$D_1\left[1+c_1-c_1\left(\frac{r\,p}{l\,p}\right)^{\frac{1}{s}}\right]$ for *l-p* cylinder	$D_1\left[1+c_1-c_1R_p{}^{\frac{1}{2s}}\right]$ for *l-p* cylinder
High-press capacity hot per min	$D_1\left[1+c_1-c_1\left(\frac{r\,p}{l\,p}\right)^{\frac{1}{s}}\right]\left(\frac{l\,p}{r\,p}\right)\left(\frac{r\,p}{h\,p}\right)^{\frac{1}{s}}=(l\text{-}p\;cap)\left(\frac{l\,p}{r\,p}\right)\left(\frac{r\,p}{h\,p}\right)^{\frac{1}{s}}$	$D_1\left[1+c_1-c_1R_p{}^{\frac{1}{2s}}\right]\div R_p{}^{\frac{s+1}{2s}}=(l\text{-}p\;cap)\div(R_p)^{\frac{s+1}{2s}}$
High-press capacity cold per min	$D_1\left[1+c_1-c_1\left(\frac{r\,p}{l\,p}\right)^{\frac{1}{s}}\right]\div R_p=(l\text{-}p\;cap)\div R_p$	$D_1\left[1+c_1-c_1R_p{}^{\frac{1}{2s}}\right]\div R_p=(l\text{-}p\;cap)\div R_p$
M e p, referred to 1st stage	$\frac{s}{s-1}(l\,p)\,E_{v1}\left[\left(\frac{D_1E_{v1}}{D_2E_{v2}}\right)^{\frac{s-1}{s}}+R_p\left(\frac{D_2E_{v2}}{D_1E_{v1}}\right)^{\frac{s-1}{s}}-2\right]$	$\frac{2s}{s-1}(l\,p)\,E_{v1}\left[(R_p)^{\frac{s-1}{2s}}-1\right]$
Apparent vol effic, 1st stage	$1+c_1-c_1\left(\frac{r\,p}{l\,p}\right)^{\frac{1}{s}}=\frac{l\text{-}p\;cap_1}{D_1}$	$1+c_1-c_1R_p{}^{\frac{1}{2s}}=\frac{l\text{-}p\;cap_1}{D_1}$
Apparent vol effic, 2nd stage	$1+c_2-c_2\left(\frac{h\,p}{r\,p}\right)^{\frac{1}{s}}=\frac{l\text{-}p\;cap_2}{D_2}$	$1+c_2-c_2R_p{}^{\frac{1}{2s}}=\frac{l\text{-}p\;cap_2}{D_2}$

required in aver practice will be 2 to 1.2 that calculated for isothermal compression. For further data on the compression of air, see Sec 15.

Example 8. What cylinder displacement per min and hp are required to compress adiabatically 1 200 cu ft of air from atmos press to 60 lb gage, in single-stage compressors, cylinder having 3% clearance? From Table 7 the apparent vol effic is $1 + 0.03 - (0.03 \times 5) = 94\%$. Assuming the inlet air to be at 60° F, the real effic (Eq 30) is

$$\frac{0.94}{1 + 0.4[1.58 - 1]} = 76\%.$$

The displacement per min will be 1 580 cu ft, and the work

$$\frac{144 \times 15 \times 3.5 \times 1\,200}{33\,000}[5^{0.29} - 1] = 162 \text{ h p.}$$

Similar results are obtainable from Fig 9, 10.

Example 9. What are the high and low-press capacities of a 2-stage compressor, 10 and 17 by 12-in cyl, speed 120 rev per min, with 3% clearance in low-press cyl, operating at best receiver press and perfect intercooling? Compression is to 9 atmos abs. What power is required and how does it compare with that for a single-stage machine?

From Eq 28, receiver press $= (15 \times 135)^{1/2} = 45$ lb; hence, low-press apparent vol effic is $1 + 0.03 - (0.03 \times 3) = 96.5\%$. Assuming an initial temp of 60° F, the real effic (Eq 30) is $0.965 \div [1 + 0.4\,(1.36 - 1)] = 85\%$. Low-press capacity will be 85% of the displacement, or 323 cu ft. High-press capacity hot (Table 8) is $323 \times (1/9)^{0.86}$, or 49 cu ft, the high-press capacity cold being $323 \div 9$, or 36 cu ft. Power required for an ideal compressor is (Table 8 or Fig 11) 53.6, and from Fig 12 is seen to be about 84% of that for compression in one stage.

4. POWER, FUEL CONSUMPTION AND THERMAL EFFICIENCY OF PISTON STEAM AND AIR ENGINES

Simple engines are those in which the fluid does work on the piston, and is then exhausted to the atmos or to a condenser. The fluid may be admitted for the entire stroke, or may be cut off at some point of the stroke and then expanded. Similarly, the exhaust may be for an entire stroke, or for only a part, followed by compression. Fig 13 shows the series of operations forming a cycle, for which 2 strokes are required. When the fluid is admitted to one end of cylinder only, engine is single-acting; if to both ends, double-acting. Referring to Fig 13: A = admission, inlet valve opens; B = cutoff, inlet valve closes; C = release exhaust valve opens; D = compression, exhaust valve closes; A–A' = admission at constant vol; A'–B = admission at constant press; B–C = expansion; C–C' = exhaust at constant vol; C'–D = exhaust at constant press.

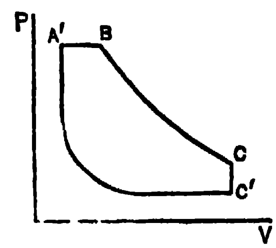

Fig 13 P-V Diagram for Simple Engine Cycle

Expansion curve BC may be of any form, but in steam practice it is a logarithmic curve, so that $PV = K$; for air, it is usually assumed to be adiabatic, so that $PV^{1.41} = K$. Two sets of expressions are given in Table 9, one for $PV = K$, the other a general one for $PV^s = K$, where s may have any value other than 1. Four degrees of expansion and compression may exist (Fig 14, 15, 16, 17). In complete expansion the fluid expands in the cyl to the exact press of the exhaust, in incomplete expansion it fails to reach exhaust press, and in over expansion it falls below it. Similarly, for complete compression the steam trapped in the cyl at the end of exhaust is compressed to the initial press.

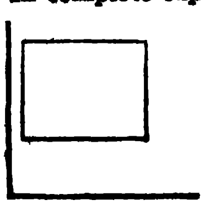

Fig 14. Zero Expansion and Compression

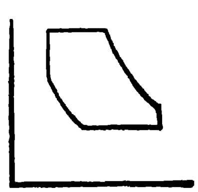

Fig 15. Incomplete Expansion and Compression

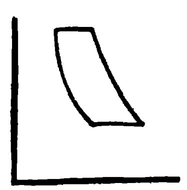

Fig 16. Complete Expansion and Compression

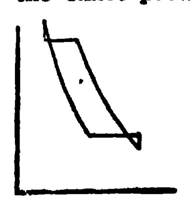

Fig 17 Over Expansion and Compression

Indicated horsepower (i h p) of an engine is given by the expression $pLan \div 33\,000$, where p is the mean effective press (m e p), lb per sq in, L the length of stroke, ft, a the net area of piston (area of piston minus area of rod, if there be one), sq in, n number of cycles per min. The quantity $Lan \div 33\,000$ is the ENGINE CONSTANT (see Table 2 and its explanation).

Consumption of engines is expressed in cu ft, or in lb of high-press fluid per hp hr, the latter value being termed the WATER RATE. In the equations, the letters have following designations: Z = cutoff = fraction of stroke completed when inlet valve closes; X = compression = fraction of stroke remaining when exhaust valve closes; c = clearance expressed as a fraction of the stroke; δ_1 = wt of a cu ft of fluid at the high press, as found from steam tables; (in pr), (bk pr), and (rel pr) = initial, exhaust, and release press, in lb per sq in abs (Fig 18).

Table 9. Expressions for M e p and Consumption of Steam or Air Engines

	For $PV = K$	For $PV^s = K$
m e p	$(\text{in pr})\left[Z + (Z + c)\text{ Nap log}\dfrac{1+c}{Z+c}\right] - (\text{bk pr})\left[\begin{array}{c}(1 - X) + (X + c)\\ \times \text{ Nap log}\dfrac{X+c}{c}\end{array}\right]$	$(\text{in pr})\left\{Z + \dfrac{Z+c}{s-1}\left[1 - \left(\dfrac{Z+c}{1+c}\right)^{s-1}\right]\right\} - (\text{bk pr}) \times \left\{(1 - X) + \dfrac{X+c}{s-1}\left[\left(\dfrac{X+c}{c}\right)^{s-1} - 1\right]\right\}$
Cu ft of fluid per hr, per i h p	$\dfrac{13\,750}{\text{m e p}}\left[(Z + c) - (X + c)\dfrac{\text{bk pr}}{\text{in pr}}\right]$	$\dfrac{13\,750}{\text{m e p}}\left[(Z + c) - (X + c)\left(\dfrac{\text{bk pr}}{\text{in pr}}\right)^{\frac{1}{s}}\right]$
Lb of fluid per hr, per i h p	$\dfrac{13\,750}{\text{m e p}}\left[(Z + c) - (X + c)\dfrac{\text{bk pr}}{\text{in pr}}\right]\delta_1$	$\dfrac{13\,750}{\text{m e p}}\left[(Z + c) - (X + c)\left(\dfrac{\text{bk pr}}{\text{in pr}}\right)^{\frac{1}{s}}\right]\delta_1$
Release press	$\left(\dfrac{Z+c}{1+c}\right)(\text{in pr})$	$\left(\dfrac{Z+c}{1+c}\right)^s(\text{in pr})$

For complete expansion and compression Z and X have special values as follows:

For $PV = K$ For $PV^s = K$

$$Z = (1 + c)\frac{\text{bk pr}}{\text{in pr}} - c \qquad Z = (1 + c)\left(\frac{\text{bk pr}}{\text{in pr}}\right)^{\frac{1}{s}} - c \tag{34}$$

$$X = c\left(\frac{\text{in pr}}{\text{bk pr}}\right) - c \qquad X = c\left(\frac{\text{in pr}}{\text{bk pr}}\right)^{\frac{1}{s}} - c \tag{35}$$

Diagram factor of an engine is the ratio of the m e p actually realized in the cyl to that calculated as above. Actual m e p is always less, due to valve interference with steam flow and other causes. Values of diagram factors for different types of engines are given under description of engines, averaging about 0.8 to 0.7. SHAFT OR BRAKE HP of an engine is always less than the i h p, due to friction losses. Ratio of brake hp to i h p is the MECHANICAL EFFICIENCY.

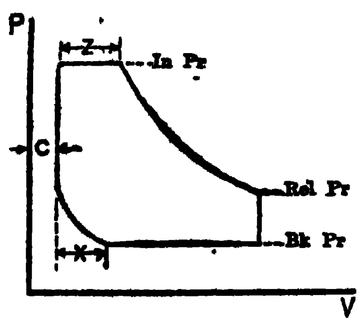

Fig 18. Diagram Representing Cutoff, Compression, and Pressures in Steam Engine Cycle

Actual consumption of engines is greater than the above values, due to leakage past valves and from one side of piston to other, and to condensation of steam in the cyl. During exhaust stroke the cyl tends to cool down, since the temp of exhaust steam is relatively low. On admission of high-press steam, some is condensed on striking the cooler cyl walls, allowing more to enter.

Missing water is the difference between actual and computed consumption. It is difficult to predict, as the condition of the engine (frequently unknown) affects it greatly. Following empiric formulas (Perry) are in common use:

$$\text{Non-condensing engines, } R = m\frac{1 + \frac{1}{Z}}{d\sqrt{N}}. \quad \text{Condensing engines, } R = 120\frac{1 + \frac{1}{Z}}{d\sqrt{np}} \tag{36 (37)}$$

in which R is missing water divided by computed consumption, Z the cutoff, d the diam of cyl, in N the rev per min, n the cycles per min, p the initial press, and m a constant varying from 5, in engines in very good condition and with jackets, to 30 or more in engines without jackets.

Multiple-expansion engines are those in which the steam, after doing work in one cyl, is exhausted to one or more cyls and does more work by further expansion. Amount of work which 1 cu ft of steam can do in expanding between initial and back press is independent of the number of cyls in which expansion takes place, but a greater range of expansion can be had in a multiple than in a single-cyl engine. This, together with a decrease in condensation and leakage losses, is the reason for employing compound and triple-expansion engines. Should expansion occur in 2 stages, the engine is termed a compound, a triple if in 3 stages, and quadruple if in 4. But, 2 cyls may be used instead of 1 large one for the lower press cyl, so that a compound is not necessarily a 2-cyl, or a triple a 3-cyl engine. Exhaust from 1 cyl may be led directly to the next, or to an intermediate receiver. Should the fluid be heated between cyls, the receiver is a reheating receiver. It is considered good practice to reheat air, but not steam. Since the size of receiver, arrangement of cranks, type of expansion curve, clearance, cutoff, and compression, in each cyl all enter into the expressions for m e p for multiple-expansion engines, these expressions become too complicated for presentation here (1). (See textbooks.)

Expressions for cu ft or lb of fluid consumed, as given for simple engines, may be used for multiple-expansion engines as follows: Z, X, c, and the pressures may be taken for either cyl and the m e p taken as referred to that cyl for which the values of Z, etc, are taken. The m e p referred to 1 cycle is a hypothetical m e p, which, if existing in a single cyl of the same size as that to which it is referred, gives the same hp as the whole engine.

$$\text{For compound engines:}\quad \begin{cases} (\text{m e p})\text{ referring to high press} = (\text{m e p})_H + (\text{m e p})_L \dfrac{D_L}{D_H} \\ (\text{m e p})\text{ referring to low press} = (\text{m e p})_H \dfrac{D_H}{D_L} + (\text{m e p})_L \end{cases} \tag{38}$$

where D_H and D_L are the displacements of the high and low-press cyls.

Thermal effic of steam engines is the ratio of output to input. Reducing both to 1 hp, this becomes thermal effic = 2 545 ÷ (water rate × H_1), in which H_1 is the heat content of 1 lb of steam above the feed-water temp, for values of which see Art 12.

Thermal effic of air engines is generally considered as the work obtained from the engine for a given quantity of air, divided by the work to compress the same quantity. Should there be preheating and reheating, the work-equivalent of the heat so supplied must be added to the work of compression. For the work of engines, see above; for work of compressors, see Sec 15. Work-equivalent of the heat of preheat or reheat will be $778\, wC_p\,(T_2 - T_1)$ where w = wt of air, C_p = specific heat at constant press = 0.24 for air, and $(T_2 - T_1)$ the number of degrees through which the air is heated.

Example 10. Find the hp which a 15 by 18-in double-acting engine, with 5% clearance, running at 200 rev per min, could probably furnish at the belt pulley for following conditions. Initial press, 125 lb gage; back press, atmosphere; cutoff, 1/2; compression, 20%. What is the probable steam consumption? Taking s as 1 (customary for steam work), from Table 9 the ideal m e p would be

$$140\left[0.05 + 0.55 \text{ Nap log} \frac{1.05}{0.55}\right] - 15\left[0.8 + 0.25 \text{ Nap log} \frac{0.25}{0.05}\right] = 102.$$

Assume a diagram factor of 80%, and a mech effic of 85%, as fair values. Then probably m e p = 0.8 × 102 = 82. The piston speed = 2 × 200 × 1.5 = 600; hence from Table 2, h p constant is 3.21. The i h p will then be 3.21 × 82 = 263 and the b h p, 0.85 × 263 = 223. From Table 9 the indicated steam per i h p per hr is $\frac{13\,750}{82}\left[0.55 - 0.25 \times \frac{15}{140}\right] \times 0.31 = 27.5$ lb. From Eq 37, $R = \frac{30 \times (1 + 2)}{15\sqrt{200}} = 0.42$. The total steam per hr will then be 27.5 × 1.42 × 263 = 10 270 lb.

Example 11. If the engine in above problem be run for best economy, complete expansion and compression must be had; hence (Eq 34, 35), $Z = 1.05 \times \frac{15}{140} - 0.05 = 0.062$ and $X = 0.05 \times \frac{140}{15} - 0.05 = 0.42$. From Table 9, the ideal m e p is 21. Probable b h p is then 46. The indicated consumption (Table 9) is $\frac{13\,750}{17}\left[0.112 - 0.47 \times \frac{15}{140}\right] \times 0.31 = 15.5$, and R (Eq 37) is 2.4. Total steam per hr is 15.5 × 3.4 × 54 = 2 846 lb.

5. POWER, FUEL CONSUMPTION AND THERMAL EFFICIENCY OF INTERNAL COMBUSTION ENGINES

In these engines the heat is developed directly in the working cyl by burning the fuel therein. Such engines may be divided into 2 classes; (a) those burning the fuel at con-

stant vol; (b) those at constant press. Fig 19 and 20 show the cycles on which they operate. Fig 19 shows the Otto cycle, where

AB = adiabatic compression	BC = heating at constant volume
CD = " expansion	DA = cooling " " "

Fig 20 shows the Diesel cycle, where

AB = adiabatic compression	BC = heating at constant pressure
CD = " expansion	DA = cooling " " volume

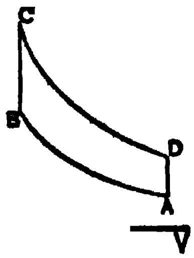

Fig 19. P–V Diagram of Otto Cycle

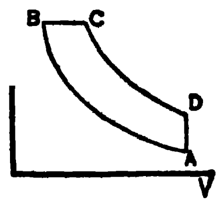

Fig 20 P–V Diagram of Diesel Cycle

As actually operated, both types draw in a new charge in each cycle, and expel the products. These transfer phases may each take 1 stroke, or occur at the dead-center periods. In first case, the engine is a 4-cycle, in second, a 2-cycle, the 4 or 2 indicating the number of strokes per cycle. Expressions for m e p, thermal effic and fuel consumption, for Otto and Diesel cycles are given in Table 10.

Table 10. M e p, Efficiency, and Fuel Consumption for Otto and Diesel Cycles

	Otto cycle	Diesel cycle
M e p...........	$\dfrac{JQ_1\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}\right]}{144\,V_a\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}\right]}$ or $5.4\dfrac{H_1}{a+1}\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}\right]$	$\dfrac{J\left\{Q_1-C_vT_a\left[\left(1+\frac{Q_1}{C_pT_b}\right)^s-1\right]\right\}}{144\,V_a\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}\right]}$
Thermal efficiency	$1-\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}$ or $1-\left(\frac{V_b}{V_a}\right)^{s-1}$ or $1-\frac{T_a}{T_b}$	$1-\dfrac{c_vT_a\left[\left(1+\frac{Q_1}{C_pT_b}\right)^s-1\right]}{Q_1}$
Fuel consumption per hr per i h p	$\dfrac{2\,545}{EH}$	$\dfrac{2\,545}{EH}$

J = Joule's equiv = 778; Q_1 = heat added per lb of working gases in cyl; P_a = press at beginning of compression; P_b = press at end; V_a = vol per lb of air-gas mixture at beginning and V_b at end of compression; C_p and C_v = specific heats of gases at constant press and constant vol; H_1 = B t u per cu ft of gas or vapor; H = B t u per cu ft of gas or per lb of oil; E = thermal effic, a = the air required to burn 1 cu ft gas or vapor; T_a = abs temp at beginning of compression; T_b = abs temp at end of compression.

Efficiency of Otto cycle depends only upon the degree of compression and is independent of amount of heat added. It may be expressed in terms of temp, vol or press, before and after compression, the latter being commonest. Table 11 gives values for efficiency for different compression ratios for the usual value of $s = 1.4$. Efficiency of Diesel cycle varies with the degree of compression, and with amount of heat added. In practice, efficiency is further decreased at heavy loads due to incomplete combustion when large amounts of fuel are added. The m e p in each case is work divided by change in volume. In above expressions, work is the product JQ_1E, and vol change is expressed as a function of original vol. Heat liberated per cycle Q_1, divided by vol of mixture admitted, or vol change, is the heat content per cu ft of mixture; hence, the m e p expression for Otto cycle may have the simpler form given in Table 10. Table 12 gives the quantity $\frac{H_1}{a+1}$ or B t u per cu ft of air-fuel mixture for the common fuels, at 32° F and 14.7 lb. For other conditions of press and temp, the values vary directly as the abs press and inversely as abs temp. Hence, solutions of problems on the Otto cycle are chiefly a matter of finding tabular quantities. For solution of m e p by the other expressions, the heat liberated per lb of stuff in cyl must be known. Considering V_a to be the vol of 1 lb of mixture external to the cyl,

$$V_a\left[1-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}\right]\frac{H_1}{a+1}=Q_1.$$

Table 11. Values of Otto Cycle Efficiency

$\frac{P_b}{P_a}$	$1-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}$	$1-\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}=E$	$\frac{P_b}{P_a}$	$1-\left(\frac{P_a}{P_b}\right)^{\frac{1}{s}}$	$1-\left(\frac{P_a}{P_b}\right)^{\frac{s-1}{s}}=E$
3	0.544	0.269	5.8	0.715	0.395
3.2	0.564	0.283	6	0.722	0.401
3.4	0.583	0.295	7	0.751	0.427
3.6	0.599	0.307	8	0.774	0.448
3.8	0.615	0.317	9	0.792	0.466
4	0.628	0.327	10	0.807	0.482
4.2	0.641	0.336	15	0.855	0.538
4.4	0.653	0.345	20	0.882	0.575
4.6	0.664	0.353	25	0.900	0.601
4.8	0.673	0.361	30	0.912	0.622
5	0.683	0.369	35	0.921	0.638
5.2	0.692	0.376	40	0.928	0.651
5.4	0.700	0.382	45	0.934	0.663
5.6	0.708	0.389	....	....	....

Table 12. Values of B t u per Cu Ft of Best Air and Gas Mixture

Fuel	$\frac{H_1}{a+1}$ at 32° F and 14.7 lb	Fuel	$\frac{H_1}{a+1}$ at 32° F and 14.7 lb
Carbon monoxide...............	100	Average coal gas.............	90
Hydrogen.....................	88	" oil gas..............	97
Methane......................	91	" coke-oven gas........	90
Benzene......................	103	" blast-furnace gas.....	60
Alcohol......................	101	" water gas............	88
Gasolene.....................	107	" anthracite producer gas	67
Crude oil....................	110	" lignite producer gas...	69
Kerosene.....................	107	....	....

Note.—Values at any other press and temp = values at 32° and 14.7 lb × $33.45\frac{p}{T}$, where p is press in lb per sq in and T is abs temp in deg F. (Constant 33.45 is 492 + 14.7.)

Table 11 gives values for $1-(P_a \div P_b)^{\frac{1}{s}}$ and as the air-fuel mixture is largely air the vol per lb may be considered the same as air, values for which are given in Table 20. As the fuel mixture external to engine is generally at atmos press and temp, an aver value of 13 is not greatly in error.

For liquid fuels the value for $\frac{H_1}{a+1}$ averages about 105, for city and natural gases 90, and producer gases 70. Values given for m e p and effic are all for perfect engines. The diagram factor varies far more than for steam engines, but a rough value is 45 to 55%. Values for different types of engines are given in Sec 40.

Example 12. Find full-load hp, fuel consumption, and thermal effic of a single-acting Otto 4-cycle 3-cyl 10 by 12-in engine, running at 500 rev per min, with a compression press of 90 lb gage; fuel, alcohol. Compression ratio = (90 + 15) ÷ 15 = 7. Effic (Table 10) = $1-\left(\frac{1}{7}\right)^{\frac{s-1}{s}}=$ 0.427, as also shown in Table 11. Assuming the diagram factor = 0.5, real effic = 0.21. Fuel per hp hr (Table 10), and value of 12 100 B t u per lb alcohol (Table 26) = 2 545 ÷ (0.21 × 12 100) = 1. This value is the probable consumption, since the probable effic of 0.21 was used. Same result could be found by using the hypothetical value 0.427, and dividing by diagram factor 0.5. B t u per cu ft mixture, at say 60° F and atmos press (Table 12) = 95.5; hence m e p = 5.4 × 95.5 × 0.427 = 220 lb. Applying same diagram factor, real m e p = 110. Piston speed = (3 × 500) ÷ 2 = 750. Hp constant from Table 2 = 1.785; hence, hp = 196.

Example 13. Find the m e p, fuel consumption, and thermal effic, of a single-cyl, 2-cycle Diesel engine, operating on crude oil at full load. Compression ratio, 40. At theoretical full load in a Diesel there would be heat liberated due to best mixture, but actually there is always excess air. Assuming best mixture, Q_1 = 13 × 104 × 0.93 = 1 250, for temp of 60° F external to cyl. From the expressions following Eq 42, the temp ratio equals the press ratio raised to $(s-1) \div s$ power, hence T_b = 2.87 × T_a = 1 500 abs;

$$\text{hence } E = 1 - \frac{0.17 \times 520\left(1+\frac{1\,250}{0.24 \times 1\,500}\right)^{1.4}-1}{1\,250} = 0.515$$

Assuming diagram factor of 0.5, real thermal effic becomes 26%. Fuel consumption, assuming 19 000 B t u per lb oil, is 2 545 ÷ (0.26 × 19 000) = 0.51 lb.

$$\text{M e p} = (778 \times 1\,250 \times 0.515) \div (144 \times 0.13 \times 0.93) = 290 \text{ lb.}$$

Applying same diagram factor, real m e p = 145 lb.

6. HEAT AND TEMPERATURE UNITS

Thermometer scales in common use are Cent and Fahr, usually the latter in American engineering practice. The fixed points on each are the freezing and boiling points of water, at standard press of 760 mm of mercury. Freezing point is denoted by 0° and 32°, boiling point, by 100° and 212°, on the Cent and Fahr scales respectively. 1° F = $^5/_9$° C, or 1° C = $^9/_5$° F. Temp F = $^9/_5$ temp C + 32, or temp C = $^5/_9$ (temp F − 32). (See also Sec 37.)

Absolute temperature. Scales given above are purely arbitrary, and can not be used in thermodynamic calculations for which abs temps are required. The more permanent gases, as air, hydrogen, oxygen, etc, or those much above their condensation temp, when cooled at constant press, lose a definite amount in vol per degree of cooling. If cooled at constant vol, there is a definite decrease in press of $^1/_{273}$ per deg C, which is the same as the decrease in vol for constant-press cooling. Considering a unit press or vol at 0° C, cooled through 273°, there would be zero press if the cooling were at constant vol, or zero vol if cooling were at constant press. Temp of −273° C is therefore called the abs zero Cent, and 0° on the same scale becomes 273° C abs. Similarly, 0° on Fahr scale is 460° abs. The above values are in the nearest round numbers. More exact values are:

0° C scale = 272.85° abs (Regnault), and 0° F scale = 459.64° abs (Marks & Davis).

Units of heat. There are 2 units of heat, one corresponding to each of the above thermometer scales. That corresponding to the Cent scale, or metric unit, is the calorie, or heat required to raise the temp of 1 kg of water from 15° C to 16° C. This unit is sometimes called the large calorie, to distinguish it from the small calorie, which is the heat required to raise the temp of 1 gm water an equal amount. The unit used in English-speaking countries is the British Thermal Unit (B t u), and is the heat required to raise the temp of 1 lb water from 62° F to 63° F (Peabody) or $^1/_{180}$ of the heat required to raise 1 lb of water from 32° F to 212° F (2). These values would be identical were the specific heat of water constant. The variation is slight, and for practical purposes the error arising from considering it constant and equal to unity may be neglected. For detailed discussion of different values of specific heat of water, see (17, 21). 1 B t u = 0.252 calorie; 1 calorie = 3.968 B t u.

Mechanical equivalent of heat varies in different latitudes; a common value in engineering work is 1 B t u = 778 ft lb (Table 13).

Table 13. Power Conversion Factors

Ft lb	B t u	Calorie	Hp sec	Hp min	Hp hr
1	1.26×10^{-3}	0.324×10^{-3}	1.818×10^{-3}	0.303×10^{-4}	$5\ 05 \times 10^{-7}$
778	1	0.252	1.414	$2\ 356 \times 10^{-2}$	$3\ 93 \times 10^{-4}$
3 086	3.968	1	5.61	$9\ 35 \times 10^{-2}$	1.56×10^{-3}
550	0.707	0 178	1	1.67×10^{-2}	2.78×10^{-4}
3.3×10^4	43.44	10 70	60	1	$1\ 67 \times 10^{-2}$
1.98×10^6	2 545	641	3 600	60	1

7. SPECIFIC HEATS

When substances are so heated that no change of state occurs, as from solid to liquid or liquid to vapor, and no heat is conducted away, the temp rises. Quantity of heat required to cause temp of equal weights of different substances to rise the same number of degrees varies. The standard is the heat required to raise 1 lb water 1° F. Ratio of the heat required to raise 1 lb of a substance 1° F to that required to raise 1 lb water 1° F is its SPECIFIC HEAT.

Heat may be applied to a substance at constant press or at constant volume. In first case the vol changes, and hence work is done; in second case no work results. As work is done at the expense of heat, more is required to cause a given temp rise in the case of constant-press heating. Hence, there are 2 specific heat values, C_p for constant press, and C_v for constant vol conditions. But, in case of solids and liquids, the change in volume is so small as to be negligible in engineering work, hence for these substances only one

value is reported. Specific heats of substances are not constant over the entire range of temps, but have the form of equation, $C = a + bt + ct^2$, where a, b, and c are constants. As the values of b and c are not definitely determined, and expressions involving specific heats become very complicated when the variable values are used, it is customary to use the constant values and apply correction factors where necessary at end of the calculation. See Tables 14, 15, 16.

Table 14. Specific Heats of Solids
(Compiled from Smithsonian and Landolt-Bornstein Physical Tables)

Substance	Sp heat	At deg F	Substance	Sp heat	At deg F
Aluminum	0.2089	32	Iron wrt	0.115	60–212
Asbestos	0.195	70–210	" "	0.199	1 800–2 200
Brass	0.092	70–210	Lava, Ætna	0.26	90–1 400
Brickwork	0.21	....	Lead	0.031	70–212
Bronze	0.104	70–210	Limestone	0.216	60–212
Calcspar ($CaCO_3$)	0.2	32–212	Magnetite (Fe_3O_4)	0.156	70–115
Carbon, diamond	0.113	52	Manganese	0.124	70–212
" graphite	0.160	52	Marble	0.21	32–212
Cement	0.271	82	Mica	0.208	70–210
Clay	0.197	....	Nickel	0.13	932
Coal	0.2–0.25	......	Pyrites (FeS)	0.13	60–210
Copper	0.094	32–212	Quartz, (SiO_2)	0.174	32
Galena (PbS)	0.0466	32–212	"	0.305	750–2 200
Glass	0.16–0.18	......	Rock salt	0.219	55–110
Gold	0.032	32–212	Sandstone	0.22	....
Granite	0.192	50–212	Silver	0.056	22–212
Hematite (Fe_2O_3)	0.1645	60–212	Steel	0.118	70–212
Hornblende	0.1952	70–210	Sulphur	0.18	32–120
Ice	0.5	....	Tin	0.055	70–212
Iron, cast	0.119	70–212	Zinc	0.0935	32–212

Table 15. Specific Heat of Liquids (Compiled from Smithsonian Physical Tables)

Substance	Sp heat	At deg F	Substance	Sp heat	At deg F
Alcohol, ethyl	0.648	100	Lead (melted)	0.0356	620–590
" methyl	0.6	70	Mercury	0.033	212
Ammonia	1.07	32	Petroleum	0.511	70–135
" solution	1	.....	Sea water	0 9 to 1.0	62

Table 16. Specific Heat of Gases

Substance	Specific heat C_p	Specific heat C_v	At deg F	Authority for C_p
Air	0.2389	0.1703	70–212	1
Ammonia	0.5202	0.4011	70–212	1
Benzole	0.2990	0.2131	70–240	1
Carbon dioxide	0.2025	0.1558	60–212	2
" monoxide	0.2425	0.1734	75–210	1
Ethylene	0.4040	0.3404	50–400	2
Hydrogen	3.4100	2.4219	70–212	1
Methane	0.5929	0.1734	65–405	2
Nitrogen	0.2419	0.1715	70–825	3
Oxygen	0.2240	0.1603	70–825	3

1. Wiedeman. 2. Regnault. 3. Holborn-Austin. All values for C_v from (1).

For a mixture of substances the specific heat may be found by Eq 39:

$$\text{Specific heat of mixture} = \frac{c_1w_1 + c_2w_2 + c_3w_3}{w_1 + w_2 + w_3} \tag{39}$$

where c_1, c_2, etc, are the specific heats and w_1, w_2, etc, the weights of the components.

Equation 40 (Dulong and Petit) is useful where no experimental values are available:

$$\text{Specific heat of solids} = 6.4 \div \text{atomic weight} \tag{40}$$

Quantity of heat absorbed by or removed from a body on heating or cooling is given by $H = Cw(t_2 - t_1)$, where H = heat, w = wt, C = specific heat, and t_2 and t_1 temp.

8. COEFFICIENTS OF EXPANSION

Substances may be heated at constant vol or at constant press, but the change in vol of liquids and solids when heated at constant press is very small and is usually expressed in the form of a coefficient or change per deg. Solids may undergo both linear and volumetric change, the latter being substantially 3 times the former. Liquids are subject to volumetric expansion only. All gases have substantially the same coef over a wide range, which is $^1/_{460}$ of their vol at 32° F and atmos press. A gas following this law exactly is termed a perfect gas. Most of the common gases follow the law closely enough for practical purposes, but vapors, being gases only when at temperatures considerably above their condensation points, follow it less closely, so that for them results are only approx. Tables 17 and 18 give the coef of expansion for common substances.

Table 17. Coefficient of Linear Expansion of Solids
(Compiled from Smithsonian Tables)

	Coef × 10^4 per deg F	At deg F	Substance	Coef × 10^4 per deg F	At deg F
Aluminum	0.129–0.175	100–1 100	Marble	0.065	60–100
Brasses and bronzes	0 095–0.117	32–1 700	Masonry	0 025–0.05	....
Concrete	0 0795	....	Nickel	0.071	100
Copper	0.0927	32–212	Porcelain	0.023	70–100
Glass	0.032–0.05	32–212	Silver	0 1065	100
Ice	0 283	4–30	Steel	0 0735	100
Iron, cast	0.059	100	Tin	0.1241	100
Iron, wrt	0 0635	0–212	Wood, with grain	0 014–0 053	35–90
Lead	0 1625	100	Wood, cross grain	0 13–0 3	35–90

Table 18. Coefficient of Cubical Expansion of Liquids and Solids
(Compiled from Smithsonian Tables)

Substance	Coef × 10^4 per deg F	At deg F	Substance	Coef × 10^4 per deg F	At deg F
Alcohol	7.96	−35–160	Mercury	0.1	75–575
Bromine	6.49	20–140	Petroleum	0.058	75–250
Fluorspar	0.35	50–115	Quartz	0.196	122–140
Iceland spar	0.08	122–140	Rock salt	0.67	122–140
Magnetite	0.16	32–212	Zincite	0.015	100

9. PRESSURE, VOLUME, AND TEMPERATURE RELATIONS FOR GASES

$$\text{For a perfect gas, } PV = wRT \tag{41}$$

where P = abs press, lb per sq ft; V = vol, cu ft; w = wt, lb; R = the gas constant = $778\,(C_p - C_v)$; T = abs temp, deg F.

$$\text{When a gas expands, } P_1V_1^s = P_2V_2^s \tag{42}$$

and a comparison with Eq 41 shows the following relations:

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^s = \left(\frac{T_1}{T_2}\right)^{\frac{s}{s-1}} \qquad \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{s}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{s-1}} \qquad \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{s-1}{s}} = \left(\frac{V_2}{V_1}\right)^{s-1}$$

These relations are shown graphically in Fig 21, the vertical scales representing press ratios, horiz scale at top vol ratios, and horiz scale at bottom, temp ratios. The curves denote values of s. Left-hand group of curves give relations between press and vol, right-hand, between press and temp Thus, for air expanding from 7 atmos to 1 atmos, when expansion is such that s = 1.4, the press ratio = 7, final vol will be 4 times original, and final abs temp 1 ÷ 1.74 times original abs temp.

Volume per lb of a gas may be experimentally determined for a given press and temp; Table 19 gives values for common gases. Specific vol may be found from the molecular wt of the gases by Avogadro's law, that "equal volumes of gases at the same press and temp contain the same number of molecules"; hence $\delta_1 \div \delta_2 = m_1 \div m_2$, where δ = density of the gas, lb per cu ft, and m = molecular wt. Density of hydrogen is 0.00559 lb per cu ft, at 32° F and atmos press; its molecular wt is 2. Considering this as the most reliable gas on which to base the others, the wt of any gas in lb per cu ft = 0.00279 × molecular wt, and its vol in cu ft per lb = 358 ÷ molecular wt. These equations

Table 19. Specific Volumes and Densities at 32° F and 14.7 Lb per Sq In and Gas Constants of Common Gases

Gas	Lb per cu ft	Cu ft per lb	Gas constant	Authority
Air	0.0807	12.39	53.33	Rayleigh
Ammonia	0.0476	21.02	90.47	Leduc
Carbon dioxide	0.1227	8.15	35.08	Rayleigh
" monoxide	0.0781	12.81	55.14	Leduc
Ethylene	0.0795	12.58	54.15	Saussure
Hydrogen	0.0056	177.91	765.89	Rayleigh
Methane	0.0447	22.35	96.20	Thomson
Nitrogen	0.0783	12.77	55.98	Rayleigh
Oxygen	0.0892	11.21	48.24	

give the values at 32° F and atmos press of 14.7 lb per sq in. To convert this value into that for any other press and temp use the following relation:

$$\text{Vol at any temp and press} = \text{vol at } 32° \text{ F and } 14.7 \text{ lb} \times \frac{T}{492} \times \frac{14.7}{p} \tag{43}$$

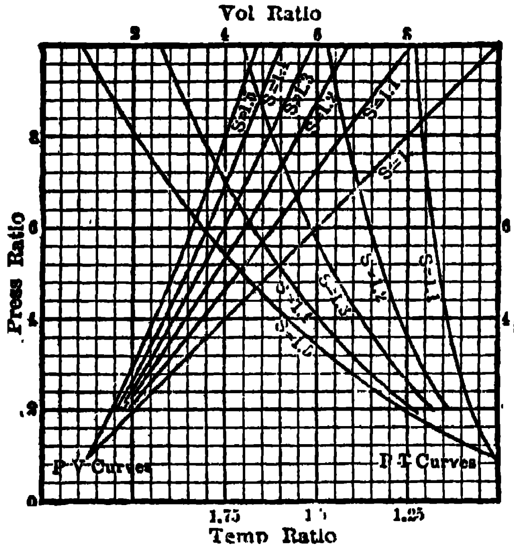

Fig 21. P–V–T Change Relations for Gases

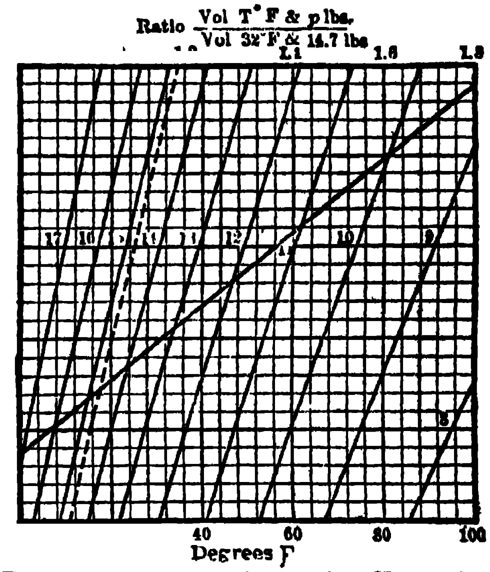

Fig 22. Diagram to Determine Change in Specific Vol, with Press and Temp

where T is abs temp F, and p abs press, lb per sq in. Fig 22 gives the ratio of the vol under the desired conditions to that under standard conditions. Lower scale is in deg F; the upper is the ratio, and the diagonal lines are different pressures. Should the vol of a certain gas be desired at 100° F and 10 lb press, when the vol at standard conditions is known, simply multiply the standard vol by the factor found by projecting upward from 100 on lower scale to the heavy diagonal, then horiz to the 10-lb line, and then again up to the top scale, where 1.67 is read; hence actual vol = 1.67 times that under standard conditions. Density, or lb per cu ft under the actual conditions, is of course the reciprocal of the vol per lb. The vol per lb dry air at different press and temp being often required is given in Table 20. Table 21, with the following simple equations, takes the place of tables giving the wt of dry air and of water vapor contained per cu ft of moist air, and lb of moisture per lb of air for all conditions of saturation, temp and barom press, which would be too bulky for insertion here.

t = temp, deg F; P_B = barom press, in of Hg; P_V = partial press of vapor (Art 10) at $t°$, lb per sq in abs; P'_V = partial press of vapor at $t°$, in of Hg; P_A = partial press of saturated air, lb per sq in abs; P'_A = partial press of saturated air, in of Hg; Z = cu ft per lb dry air; T = abs temp = $460 + t°$; δ = density of vapor = wt per cu ft; $P_A = P_B - P_V$; $Z = 0.37\dfrac{T}{P_A} = 0.7534\dfrac{T}{P'_A}$; Lb vapor per lb air = $Z\delta$. Total wt per cu ft of mixture = $(1 + Z) + \delta$

For non-saturated conditions, results sufficiently accurate for most purposes can be obtained by finding dew point from tables or curves (Sec 23, Fig 1), computed for standard atmos conditions.

Example 14. Find wt of 1 cu ft of air, saturated with moisture at 100° F; wt of the air alone for the same conditions; and wt of water per lb of air, for barom reading of 27 in of Hg. For 100° the vapor press is 0.946 lb per sq in, the total press at 27 in of Hg is 13.25 lb per sq in; hence, the partial press of air is 13.25 − 0.95 = 12.30. Wt of vapor per cu ft (Table 21) is 0.0028 lb. Wt of air per cu ft is $1 \div Z$, or $1 \div \left(0.37 \times \dfrac{510}{12.3}\right) = 0.0653$, and total wt = 0.0653 + 0.0028 = 0.0681 lb. Water per lb air = 0.0028 × 15.3 = 0.0429 lb.

Table 20. Volume of 1 Lb Dry Air at Different Pressures and Temperatures

Abs press, lb per sq in	Temperature, Deg F											
	10	20	30	40	50	60	70	80	90	100	110	120
8	21.75	22.21	22.68	23.14	23.60	24.06	24.52	25.00	25.45	25.91	26.38	26.83
9	19.11	19.75	20.18	20.58	21.00	21.40	21.80	22.21	22.60	23.05	23.45	23.90
10	17.40	17.77	18.14	18.51	18.88	19.25	19.62	20.00	20.36	20.73	21.18	21.47
11	15.81	16.16	16.50	16.82	17.17	17.51	17.85	18.19	18.50	18.85	19.15	19.50
12	14.50	14.80	15.12	15.42	15.71	16.04	16.35	16.65	16.95	17.29	17.59	17.90
13	13.38	13.67	13.96	14.23	14.52	14.80	15.10	15.38	15.62	15.92	16.22	16.51
14	12.42	12.70	12.96	13.22	13.49	13.74	14.03	14.29	14.52	14.81	15.08	15.35
15	11.60	11.84	12.10	12.33	12.58	12.83	13.10	13.33	13.56	13.82	14.08	14.35
16	10.87	11.10	11.34	11.57	11.80	12.03	12.22	12.50	12.23	12.95	13.19	13.41
17	10.23	10.45	10.67	10.89	11.10	11.32	11.53	11.77	11.96	12.20	12.41	12.63
18	9.56	9.85	10.10	10.30	10.50	10.70	10.90	11.10	11.30	11.50	11.70	11.95
19	9.16	9.31	9.53	9.74	9.92	10.12	10 32	10.52	10.70	10.90	11.11	11.31
20	8.70	8 89	9.07	9.25	9.44	9.62	9.81	10.00	10.18	10.37	10.55	10.74
25	6.96	7.12	7.24	7.40	7.56	7.72	7.84	8.00	8.16	8.28	8.44	8.60
30	5.80	5.92	6.05	6.17	6.30	6.42	6.54	6.67	6.79	6.91	7.03	7.16
35	4.98	5.08	5.18	5.28	5.38	5.50	5.62	5.72	5.80	5.92	6.02	6.14
40	4.34	4.44	4.53	4.62	4.72	4.81	4.91	5.00	5.09	5.18	5.27	5.37
45	3.87	3.94	4.04	4.12	4.18	4.28	4.36	4.44	4.51	4.60	4.68	4.78
50	3.48	3.56	3.62	3.70	3.78	3.86	3.92	4.00	4.08	4.14	4.22	4.30
55	3.17	3.24	3.30	3.38	3.44	3.50	3.56	3.64	3.70	3.76	3.84	3.90
60	2.90	2.96	3.02	3.10	3.16	3.22	3.28	3.34	3.40	3.46	3.52	3.58
65	2.68	2.73	2.79	2.85	2.90	2.96	3.02	3.08	3.12	3.18	3.22	3.30
70	2.49	2.54	2.59	2.64	2.69	2.75	2.81	2.86	2.90	2.96	3.01	3.07
75	2.32	2.37	2.41	2.47	2.51	2.57	2.61	2.67	2.72	2.76	2.81	2.87
80	2.17	2.22	2.26	2.31	2.36	2.41	2.45	2.50	2.54	2.59	2.63	2.68
85	2.05	2.09	2.13	2.18	2.22	2.26	2.31	2.35	2.39	2.44	2.48	2.53
90	1.93	1.97	2.02	2.06	2.10	2.14	2.18	2.22	2.26	2.30	2.34	2.39
95	1.83	1.86	1.91	1.55	1.98	2.02	2.06	2.10	2.14	2.18	2.22	2.26
100	1.74	1.78	1.81	1.85	1.89	1.93	1.96	2.00	2.04	2.07	2.11	2.15
105	1.66	1.70	1.72	1.76	1.80	1.84	1.87	1.90	1.94	1.97	2.01	2.05
110	1.58	1.62	1.65	1.69	1.72	1.75	1.78	1.82	1.85	1.88	1.92	1.95
115	1.51	1.55	1.58	1.61	1.64	1.68	1.71	1.74	1.76	1.80	1.84	1.87
120	1.45	1.48	1.51	1.55	1.58	1.61	1.64	1.67	1.70	1.73	1.76	1.79
125	1.39	1 42	1.45	1 48	1.51	1.54	1.57	1.60	1.63	1.66	1.69	1.72

Table 21. Properties of Saturated Air

$t°$ F	Vapor press		Wt per cu ft = δ
	Lb per sq in	In of Hg	
20	0.0492	0.1026	0.000176
25	0.0622	0.1298	0.000222
30	0.0807	0.1640	0.000276
35	0.1000	0.2034	0.000340
40	0.1217	0.2477	0.000410
45	0.1475	0.3002	0.000492
50	0.1780	0.3625	0.000587
55	0.2140	0.4357	0.000700
60	0.2562	0.522	0.000828
65	0.3054	0.622	0.000977
70	0.3626	0.739	0.001148
75	0.4288	0.873	0.001346
80	0.505	1.029	0.001570
85	0.594	1.209	0.001832
90	0.696	1.417	0.002131
95	0.813	1.655	0.002467
100	0.946	1.926	0.002851
105	1.098	2.236	0.003282
110	1.271	2.589	0.003766

Example 15. Considering Example 14 as for air 54% saturated. The dew point (Sec 23, Art 1) is 80° F. Vapor press is then 0.505 lb per sq in; hence, partial press of the air is 13.25 − 0.505 = 12.75 lb; wt of vapor = 0.0016, wt of air = 0.0676, total wt = 0.0016 + 0.0676 = 0.0692 lb. Water per lb of air = 0.0016 × 14.8 = 0.0215 lb, by same method as in Ex 14.

10. VAPORS

A vapor is a gas near its condensation temp, or the gaseous form of a substance normally a liquid or solid. Any liquid will evaporate or give off vapor at any temp, but only while the press exerted by the vapor is less than a certain value, different for each temp and each substance. On reaching this press evaporation ceases, but if the temp be increased evaporation again takes place until a new pressure value is attained. Thus, for a liquid and vapor together, fixing the temp fixes the vapor press, and the curve showing this relation to $P - T$ coordinates is called the vapor press-temp curve, or the vapor-tension curve, of the substance. Relation between press and temp may be expressed by an equation.

The press referred to in above paragraph is press of the vapor alone, and is independent of the presence of other gases. Thus, if water be placed in a vessel in which there is an absolute vacuum, and the temp maintained at 170° F, the press will rise to 6 lb per sq in. If, instead of having a vacuum

in the vessel, it had been filled with dry air at 170° F and 15 lb, and the water then added, the press would have risen to 21 lb = 15 + 6, the air pressure not influencing the water. Here, the values 6 and 15 are termed the partial pressures of the water vapor and air. Vapor in presence of its liquid can not have a temp other than that fixed by the press; under these conditions it is SATURATED, but if removed from the liquid it may be heated to a temp greater than the saturated value. Vapor under these conditions is SUPERHEATED, and the amount of superheat is expressed in deg above saturation temp.

Dalton's Law. When a space is filled with 2 or more gases, the press exerted equals the sum of all the partial pressures, and the mixture behaves as a single gas. Ratio of any partial press or vol to that of the whole may be expressed as a function of the weights and gas constants as follows:

$$\frac{P_1}{P_m} = \frac{w_1 R_1}{w_m R_m} \text{ and } \frac{V_1}{V_m} = \frac{w_1 R_1}{w_m R_m} \tag{44}$$

where P = press, V = vol, w = wt, and R = gas constant, the subscript 1 referring to a single constituent and m to the mixture. If one constituent is a vapor, its partial press is limited by the temp, and, when the press is that corresponding to the temp, the gas is said to be saturated, since the maximum amount of vapor is present. Actually, it is the space that is saturated, as the presence of the gas does not affect the vapor, but it is customary though quite incorrect to say that the gas is saturated. For this case, the following relation holds true:

$$W_v + W_g = P_v M_v + P_g M_g \tag{45}$$

where W = wt, P = press, M = molecular wt, subscript v = vapor and g = gas. If the gas contains less wt of vapor than corresponds to the saturated state, the ratio of actual wt to saturation is termed the relative humidity (Sec 23, Art 1). If a gas-vapor mixture, for which relative humidity is less than 100%, is cooled at constant press, the temp at which moisture begins to condense (that is, when saturation occurs), is called the dew point.

Two humidity problems are important in dealing with moisture in the atmos, and with preparation of liquid fuels for internal-combustion engines. The humidity of the air is found by the psychrometer (Sec 23, Art 1) (4). The problem of humidifying air with liquid fuels can not be treated briefly enough to be inserted here (10, 15). See "Conditioning of Air," Art 17; also Sec 14.

Sublimation is the process by which solids pass directly to the vapor state without the intervening state of liquid.

11. FUSION AND EVAPORATION

In Art 7 it was stated that if substances were heated without change of state the heat was manifested by a change in temp. But, under continued heating, solids and liquids

Table 22. Temperature and Latent Heat of Fusion (From Smithsonian Tables)

Substance	Melting point, deg F	Latent heat of fusion, B t u	Substance	Melting point, deg F	Latent heat of fusion, B t u	Substance	Melting point, deg F	Latent heat of fusion, B t u
Aluminum	1 217	139	Iron	2 770	40–90	Potass sulphate	372	....
Antimony	1 166	......	Lead	620	10	Silicon	2 583	....
Barium	1 563	......	chloride	940	.....	Silver	1 760	38
nitrate	1 100	......	Magnesium	1 205	.....	chloride	850	....
Bismuth	518	24	chloride	1 306	.....	nitrate	417	....
Borax	1 040	......	sulphate	129	.....	sulphate	1 230	....
Boric acid	365	......	Manganese	2 237	.....	Sodium	207	57
Cadmium	610	66	chloride	189	....	chloride	1 463	....
Calcium	1 481	......	sulphate	129	....	hydrate	140	....
chloride	1 404	73	Mercury	−38	5	nitrate	600	117
Carbon	7 630	......	Mercuric chloride	554	.....	chlorate	527	....
dioxide	−70	......	Nickel	2 642	8.3	carbonate	1 565	....
disulphide	−170	....	nitrate	135	.....	sulphate	1 585	....
Chromium	2 740	......	sulphate	210	.....	Sulphur	235–245	17
Chrome alum	192	......	Phosphorus	111	9	Stannic chloride	−27	....
Cobalt	2 714	......	Platinum	3 190	49	Stannous "	482	....
Copper	1 980	77	Potassium	144	29	Tin	450	25
Cupric chloride	930	......	carbonate	1 544	.....	Zinc	788	58
Cuprous "	815	......	chlorate	680	.....	chloride	504	....
Gold	1 945	......	chloride	1 436	.....	nitrate	97	....
Iodine	237	21	nitrate	644	88	sulphate	122	....

finally reach temp at which fusion, or change from solid to liquid, and evaporation, or change from liquid to vapor, will occur. During either process, the temp is constant if the press be fixed, and the heat added is directly proportional to the wt of substance changed. Quantity of heat required to fuse or evaporate 1 lb of a substance is the latent heat of fusion or vaporization, and is exactly equal to the quantity which must be removed to cause the condensation or solidification of 1 lb of the substance. LATENT HEAT OF VAPORIZATION varies considerably with change of press, but the heat of fusion is practically constant. Any change in the latter is of little importance in engineering, as fusion or solidification in most cases takes place at atmos press. For equations expressing the heats of fusion and vaporization in terms of other quantities and their derivation, see Bib (8). The temp and latent heats of fusion, and temp and latent heats of evaporation, at atmos press, for common substances are given in Tables 22 and 23.

Table 23. Temperature and Latent Heats of Evaporation at Atmos Press
(Compiled from Smithsonian and Landolt-Bornstein Physical Tables)

Substance	Temp of evap, deg F	Latent heat of evap, B t u per lb	Substance	Temp of evap, deg F	Latent heat of evap, B t u per lb	Substance	Temp of evap, deg F	Latent heat of evap, B t u per lb
Acetic acid	245	153	Hydrobr'c acid	90	......	Phosphorus	558	......
Acetylene	−121	......	Hydrochloric "	118	......	trichloride	167	......
Air	−314	80	Iodine	> 200	43	trioxide	343	......
Alcohol, ethyl	172	372	Iron	4 442	....	trisulphide	914	......
methyl	151	482	nitrate	257	......	Potassium	1 372	......
Aluminum	3 272	....	Lead	2 777	......	Silicon chloride	136	......
Antimony	2 622	.....	Magnesium	2 047	......	Silver	3 552	......
Bensole	212	238	nitrate	289	.....	Sodium	1 382	......
Bismuth	2 607	....	Manganese	3 452	......	Sulphur	837	......
Bromine	142	82	Mang'se chloride	223	.....	dioxide	12.6	......
Cadmium	1 440	......	nitrate	265	.....	trioxide	114	......
Carbon	6 510	......	Mercury	674	......	Sulphuric acid	640	......
disulphide	115	151	chloride (ic)	579	......	Tin	4 118	......
Chlorine	−29	.....	nitrate	278	......	chloride(ic)	237	......
Chromium	4 000	......	Nitric acid	187	......	" (ous)	1 143	......
nitrate	258	.....	oxide	−243	......	Zinc	1 686	......
Copper	4 192	......	Nitrogen	−412	90	chloride	1 347	......
nitrate	338	.....	Nitrous oxide	192	......	nitrate	268	......
Cupric chloride	1 820	..	Oxygen	−297	110			

12. PROPERTIES OF STEAM

Properties of dry saturated and superheated steam are given in Tables 24 and 25. For discussion of sources of data, see the original tables (8, 9).

Dry saturated steam is steam having the temp due to its press, but containing no moisture; it is rarely found in practice, as commercially "dry" steam usually contains from 1 to 4% of water, or is superheated. Quality of steam is designated by the amount of dry steam present in 1 lb of actual steam stuff, or by the degrees of superheat; for example, 95% dry steam means steam-stuff containing 95% of steam and 5% of water, and 100° superheat means steam having a temp 100° above that due to the press. As the values in the saturated table are for 100% dry steam, made from water at 32° F, corrections must generally be applied, first for quality and second for the original temp. Two methods are available, first to follow the procedure actually occurring in the boiler, and second, to correct the table values for the heat not added to the steam.

Example 16. Find the heat required to make 1 lb of steam at 100 lb abs press, 95% dry, from feed water at 100° F. By first method there must first be added enough heat to raise water from 100° F to the boiling point at 100 lb. From the tables the heat of the liquid at 100° F is 68, and at 100 lb is 298, or, a difference of 230. If the specific heat of water were unity throughout the entire range, this should be the same as the difference between the temp at 100 lb and 100° F, or 228. But, the resulting error is negligible for practical work, and hence the heat to raise the water to the boiling point from feed-water temp may be considered as equal to the temp difference. When water has been raised to boiling point, 95% is evaporated; therefore, to the heat necessary to heat water is added 0.95 of latent heat of evaporation, or in this case 0.95 × 888, hence total heat = 230 + 0.95 × 888 = 1 074. By the second method the total heat from tables is, for dry steam, 1 186. But, there was

Table 24. Properties of Saturated Steam

(Condensed from Marks and Davis's Steam Tables and Diagrams (4), 1909, by permission of the publishers, Longmans, Green & Co). See also Goodenough's Tables (19)

Vacuum, in of mercury	Abs press, lb per sq in	Temp, °F	Total heat above 32° F. In the water h heat-units	Total heat above 32° F. In the steam H heat-units	Latent heat, $L = H - h$ heat-units	Volume, cu ft in 1 lb of steam	Wt of 1 cu ft steam, lb	Entropy of the water	Entropy of evaporation
29.74	0.0886	32	0.00	1073.4	1073.4	3294	0.000304	0.0000	2.1832
29.67	0.1217	40	8.05	1076.9	1068.9	2438	0.000410	0.0162	2.1394
29.56	0.1780	50	18.08	1081.4	1063.3	1702	0.000587	0.0361	2.0865
29.40	0.2562	60	28.08	1085.9	1057.8	1208	0.000828	0.0555	2.0358
29.18	0.3626	70	38.06	1090.3	1052.3	871	0.001148	0.0745	1.9868
29.89	0.505	80	48.03	1094.8	1046.7	636.8	0.001570	0.0932	1.9398
28.50	0.696	90	58.00	1099.2	1041.2	469.3	0.002131	0.1114	1.8944
28.00	0.946	100	67.97	1103.6	1035.6	350.8	0.002851	0.1295	1.8505
27.88	1	101.83	69.8	1104.4	1034.6	333.0	0.00300	0.1327	1.8427
25.85	2	126.15	94.0	1115.0	1021.0	173.5	0.00576	0.1749	1.7431
23.81	3	141.52	109.4	1121.6	1012.3	118.5	0.00845	0.2008	1.6840
21.78	4	153.01	120.9	1126.5	1005.7	90.5	0.01107	0.2198	1.6416
19.74	5	162.28	130.1	1130.5	1000.3	73.33	0.01364	0.2348	1.6084
17.70	6	170.06	137.9	1133.7	995.8	61.89	0.01616	0.2471	1.5814
15.67	7	176.85	144.7	1136.5	991.8	53.56	0.01867	0.2579	1.5582
13.63	8	182.86	150.8	1139.0	988.2	47.27	0.02115	0.2673	1.5380
11.60	9	188.27	156.2	1141.1	985.0	42.36	0.02361	0.2756	1.5202
9.56	10	193.22	161.1	1143.1	982.0	38.38	0.02606	0.2832	1.5042
7.52	11	197.75	165.7	1144.9	979.2	35.10	0.02849	0.2902	1.4895
5.49	12	201.96	169.9	1146.5	976.6	32.36	0.03090	0.2967	1.4760
3.45	13	205.87	173.8	1148.0	974.2	30.03	0.03330	0.3025	1.4639
1.42	14	209.55	177.5	1149.4	971.9	28.02	0.03569	0.3081	1.4523
lb gage	14.70	212	180.0	1150.4	970.4	26.79	0.03732	0.3118	1.4447
1.3	16	216.3	184.4	1152.0	967.6	24.79	0.04042	0.3183	1.4311
3.3	18	222.4	190.5	1154.2	963.7	22.16	0.04512	0.3273	1.4127
5.3	20	228.0	196.1	1156.2	960.0	20.08	0.04980	0.3355	1.3965
7.3	22	233.1	201.3	1158.0	956.7	18.37	0.05445	0.3430	1.3811
9.3	24	237.8	206.1	1159.6	953.5	16.93	0.05907	0.3499	1.3670
11.3	26	242.2	210.6	1161.2	950.6	15.72	0.0636	0.3564	1.3542
13.3	28	246.4	214.8	1162.6	947.8	14.67	0.0682	0.3623	1.3425
15.3	30	250.3	218.8	1163.9	945.1	13.74	0.0728	0.3680	1.3311
17.3	32	254.1	222.6	1165.1	942.5	12.93	0.0773	0.3733	1.3205
19.3	34	257.6	226.2	1166.3	940.1	12.22	0.0818	0.3784	1.3107
21.3	36	261.0	229.6	1167.3	937.7	11.58	0.0863	0.3832	1.3019
23.3	38	264.2	232.9	1168.4	935.5	11.01	0.0908	0.3877	1.2925
25.3	40	267.3	236.1	1169.4	933.3	10.49	0.0953	0.3920	1.2841
27.3	42	270.2	239.1	1170.3	931.2	10.02	0.0998	0.3962	1.2759
29.3	44	273.1	242.0	1171.2	929.2	9.59	0.1043	0.4002	1.2681
31.3	46	275.8	244.8	1172.0	927.2	9.20	0.1087	0.4040	1.2607
33.3	48	278.5	247.5	1172.8	925.3	8.84	0.1131	0.4077	1.2536
35.3	50	281.0	250.1	1173.6	923.5	8.51	0.1175	0.4113	1.2468
37.3	52	283.5	252.6	1174.3	921.7	8.20	0.1219	0.4147	1.2402
39.3	54	285.9	255.1	1175.0	919.9	7.91	0.1263	0.4180	1.2339
41.3	56	288.2	257.5	1175.7	918.2	7.65	0.1307	0.4212	1.2278
43.3	58	290.5	259.8	1176.4	916.5	7.40	0.1350	0.4242	1.2218
45.3	60	292.7	262.1	1177.0	914.9	7.17	0.1394	0.4272	1.2160
47.3	62	294.9	264.3	1177.6	913.3	6.95	0.1438	0.4302	1.2104
49.3	64	297.0	266.4	1178.2	911.8	6.75	0.1482	0.4330	1.2050
51.3	66	299.0	268.5	1178.8	910.2	6.56	0.1525	0.4358	1.1998
53.3	68	301.0	270.6	1179.3	908.7	6.38	0.1569	0.4385	1.1946
55.3	70	302.9	272.6	1179.8	907.2	6.20	0.1612	0.4411	1.1896
57.3	72	304.8	274.5	1180.4	905.8	6.04	0.1656	0.4437	1.1848
59.3	74	306.7	276.5	1180.9	904.4	5.89	0.1699	0.4462	1.1801
61.3	76	308.5	278.3	1181.4	903.0	5.74	0.1743	0.4487	1.1755
63.3	78	310.3	280.2	1181.8	901.7	5.60	0.1786	0.4511	1.1712
65.3	80	312.0	282.0	1182.3	900.3	5.47	0.1829	0.4535	1.1665
67.3	82	313.8	283.8	1182.8	899.0	5.34	0.1873	0.4557	1.1623
69.3	84	315.4	285.5	1183.2	897.7	5.22	0.1915	0.4579	1.1581
71.3	86	317.1	287.2	1183.6	896.4	5.10	0.1959	0.4601	1.1540

Table 24. Properties of Saturated Steam (*Continued*)

Gage press, lb per sq in	Abs press, lb per sq in	Temp, °F	Total heat above 32° F: In the water h heat-units	Total heat above 32° F: In the steam H heat-units	Latent heat, $L = H - h$ heat-units	Volume, cu ft in 1 lb of steam	Wt of 1 cu ft steam, lb	Entropy of the water	Entropy of evaporation
73.3	88	318.7	288.9	1184.0	895.2	5.00	0.2001	0.4623	1.1500
75.3	90	320.3	290.5	1184.4	893.9	4.89	0.2044	0.4644	1.1461
77.3	92	321.8	292.1	1148.8	892.7	4.79	0.2087	0.4664	1.1423
79.3	94	323.4	293.7	1185.2	891.3	4.69	0.2130	0.4684	1.1385
81.3	96	324.9	295.3	1185.6	890.3	4.60	0.2172	0.4704	1.1348
83.3	98	326.4	296 8	1186.0	889.2	4.51	0.2215	0.4724	1.1312
85.3	100	327.8	298.3	1186.3	888.0	4.429	0.2258	0.4743	1.1277
87.3	102	329.3	299.8	1186.7	886.9	4.347	0.2300	0.4762	1.1242
89.3	104	330.7	301.3	1187.0	885.8	4.268	0.2343	0.4780	1.1208
91.3	106	332.0	302.7	1187.4	884.7	4.192	0.2336	0.4798	1.1174
93.3	108	333.4	304.1	1187.7	883.6	4.118	0.2429	0.4816	1.1141
95.3	110	334.8	305.5	1188.0	882.5	4.047	0.2472	0.4834	1.1108
97.3	112	336.1	306.9	1188 4	881.4	3.978	0.2514	0.4852	1.1076
99.3	114	337.4	308.3	1188.7	880.4	3.912	0.2556	0.4869	1.1045
101.3	116	338.7	309.6	1189.0	879.3	3.848	0.2599	0.4886	1.1014
103.3	118	340.0	311.0	1189.3	878.3	3.786	0.2641	0.4903	1.0984
105.3	120	341.3	312.3	1189.6	877.2	3.726	0.2683	0.4919	1.0954
107.3	122	342.5	313.6	1189 8	876.2	3.668	0.2726	0.4935	1.0924
109.3	124	343.8	314.9	1190.1	875.2	3.611	0.2769	0.4951	1.0895
111.3	126	345 0	316.2	1190 4	874.2	3.556	0.2812	0.4967	1.0865
113.3	128	346.2	317.4	1190.7	873.3	3.504	0.2854	0.4982	1.0837
115.3	130	347.4	318.6	1191.0	872.3	3.452	0.2897	0.4998	1.0809
117.3	132	348.5	319.9	1191.2	871 3	3.402	0.2939	0.5013	1.0782
119.3	134	349.7	321.1	1191.5	870.4	3.354	0.2981	0.5028	1.0755
121.3	136	350.8	322.3	1191 7	869.4	3.308	0.3023	0.5043	1.0728
123.3	138	352.0	323.4	1192 0	868.5	3.263	0.3065	0.5057	1.0702
125.3	140	353.1	324.6	1192.2	867.6	3.219	0.3107	0.5072	1.0675
127.3	142	354.2	325.8	1192.5	866 7	3.175	0.3150	0.5086	1.0649
129.3	144	355.3	326.9	1192.7	865.8	3.133	0.3192	0.5100	1.0624
131.3	146	356.3	328.0	1192.9	864 9	3.092	0.3234	0.5114	1.0599
133.3	148	357.4	329.1	1193.2	864.0	3.052	0.3276	0.5128	1.0574
135.3	150	358.5	330.2	1193.4	863 2	3.012	0.3320	0.5142	1.0550
140.3	155	361.0	332.9	1194.0	861.0	2.920	0.3425	0.5175	1.0489
145.3	160	363.6	335.6	1194.5	858.8	2.834	0.3529	0.5208	1.0431
150.3	165	366.0	338.2	1195.0	856.8	2.753	0.3633	0.5239	1.0376
155.3	170	368.5	340.7	1195.4	854.7	2.675	0.3738	0.5269	1.0321
160.3	175	370.8	343.2	1195.9	852.7	2.602	0.3843	0.5299	1.0268
165.3	180	373.1	345.6	1196 4	850.8	2.533	0.3948	0.5328	1.0215
170.3	185	375.4	348.0	1196.8	848.8	2.468	0.4052	0.5356	1.0164
175.3	190	377.6	350.4	1197.3	846.9	2 406	0.4157	0 5384	1.0114
180.3	195	379.8	352.7	1197.7	845.0	2.346	0.4262	0.5410	1.0066
185.3	200	381.9	354.9	1198.1	843.2	2.290	0.437	0.5437	1.0019
195.3	210	386.0	359.2	1198.8	839.6	2.187	0.457	0.5488	0.9928
205.3	220	389.9	363.4	1199 6	836.2	2.091	0.478	0.5538	0.9841
215.3	230	393.8	367.5	1200.2	832.8	2.004	0.499	0.5586	0.9758
225.3	240	397.4	371.4	1200.9	829.5	1.924	0.520	0.5633	0.9676
235.3	250	401.1	375 2	1201 5	826.3	1.850	0.541	0.5676	0.9600
245.3	260	404.5	378.9	1202.1	823.1	1.782	0.561	0.5719	0.9525
255.3	270	407.9	382.5	1202.6	720.1	1.718	0.582	0.5760	0.9454
265.3	280	411.2	386.0	1203.1	817.1	1.658	0.603	0.5800	0.9385
275.3	290	414.4	389.4	1203.6	814.2	1.602	0.624	0.5840	0.9316
285.3	300	417.5	392.7	1204.1	811.3	1.551	0.645	0.5878	0.9251
335.3	350	431.9	408.2	1206.1	797.8	1.334	0.750	0.6053	0.8949
385.3	400	444.8	422	1208	786	1.17	0.86	0.621	0.868
435.3	450	456.5	435	1209	774	1.04	0.96	0.635	0.844
485.3	500	467.3	448	1210	762	0.93	1.08	0.648	0.822
535.3	550	477.3	459	1210	751	0.83	1.20	0.659	0.801
585.3	600	486.6	469	1210	741	0 76	1.32	0.670	0.783

already in feed water the heat necessary to raise it to 100° F from 32° F, or 68 units, also, 5% of latent heat was not utilised, because only 95% of water was evaporated; hence, actual total heat = 1 1[illegible] − 68 − (0.05 × 888) = 1 074, as before. For the superheated condition, the correction for h[illegible] the feed water only is required.

Table 25. Properties of Superheated Steam

v = spec vol in cu ft per lb, h = total heat, from water at 32° F, in B t u per lb, n = entropy, from water at 32°. (Condensed from Marks and Davis's Steam Tables and Diagrams) (4)

Abs press, lb per sq in	Temp sat steam		Degrees of superheat									
			0	20	50	100	150	200	250	300	400	500
20	228.0	v	20.08	20.73	21.69	23.25	24.80	26.33	27.85	29.37	32.39	35.40
		h	1156.2	1165.7	1179.9	1203.5	1227.1	1250.6	1274.1	1297.6	1344.8	1392.2
		n	1.7320	1.7456	1.7652	1.7961	1.8251	1.8524	1.8781	1.9026	1.9479	1.9893
40	267.3	v	10.49	10.83	11.33	12.13	12.93	13.70	14.48	15.25	16.78	18.30
		h	1169.4	1179.3	1194.0	1218.4	1242.4	1266.4	1290.3	1314.1	1361.6	1409.3
		n	1.6761	1.6895	1.7089	1.7392	1.7674	1.7940	1.8189	1.8427	1.8867	1.9271
60	292.7	v	7.17	7.40	7.75	8.30	8.84	9.36	9.89	10.41	11.43	12.45
		h	1177.0	1187.3	1202.6	1227.6	1252.1	1276.4	1300.4	1324.3	1372.2	1420.0
		n	1.6432	1.6568	1.6761	1.7062	1.7342	1.7603	1.7849	1.8081	1.8511	1.8908
80	312.0	v	5.47	5.65	5.92	6.34	6.75	7.17	7.56	7.95	8.72	9.49
		h	1182.3	1193.0	1208.8	1234.3	1259.0	1283.6	1307.8	1331.9	1379.8	1427.9
		n	1.6200	1.6338	1.6532	1.6833	1.7110	1.7368	1.7612	1.7840	1.8265	1.8658
100	327.8	v	4.43	4.58	4.79	5.14	5.47	5.80	6.12	6.44	7.07	7.69
		h	1186.3	1197.5	1213.8	1239.7	1264.7	1289.4	1313.6	1337.8	1385.9	1434.1
		n	1.6020	1.6160	1.6358	1.6658	1.6933	1.7188	1.7428	1.7656	1.8079	1.8468
120	341.3	v	3.73	3.85	4.04	4.33	4.62	4.89	5.17	5.44	5.96	6.48
		h	1189.6	1201.1	1217.9	1244.1	1269.3	1294.1	1318.4	1342.7	1391.0	1439.4
		n	1.5873	1.6016	1.6216	1.6517	1.6789	1.7041	1.7280	1.7505	1.7924	1.8311
140	353.1	v	3.22	3.32	3.49	3.75	4.00	4.24	4.48	4.71	5.16	5.61
		h	1192.2	1204.3	1221.4	1248.0	1273.3	1298.2	1322.6	1346.9	1395.4	1443.8
		n	1.5747	1.5894	1.6096	1.6395	1.6666	1.6916	1.7152	1.7376	1.7792	1.8177
160	363.6	v	2.83	2.93	3.07	3.30	3.53	3.74	3.95	4.15	4.56	4.95
		h	1194.5	1207.0	1224.5	1251.3	1276.8	1301.7	1326.2	1350.6	1399.3	1447.9
		n	1.5639	1.5789	1.5993	1.6292	1.6561	1.6810	1.7043	1.7266	1.7680	1.8063
180	373.1	v	2.53	2.62	2.75	2.96	3.16	3.35	3.54	3.72	4.09	4.44
		h	1196.4	1209.4	1227.2	1254.3	1279.9	1304.8	1329.5	1353.9	1402.7	1451.4
		n	1.5543	1.5697	1.5904	1.6201	1.6468	1.6716	1.6948	1.7169	1.7581	1.7962
200	381.9	v	2.29	2.37	2.49	2.68	2.86	3.04	3.21	3.38	3.71	4.03
		h	1198.1	1211.6	1229.8	1257.1	1282.6	1307.7	1332.4	1357.0	1405.9	1454.7
		n	1.5456	1.5614	1.5823	1.6120	1.6385	1.6632	1.6862	1.7082	1.7493	1.7872
220	389.9	v	2.09	2.16	2.28	2.45	2.62	2.78	2.94	3.10	3.40	3.69
		h	1199.6	1213.6	1232.2	1259.6	1285.2	1310.3	1335.1	1359.8	1408.8	1457.7
		n	1.5379	1.5541	1.5753	1.6049	1.6312	1.6558	1.6787	1.7005	1.7415	1.7792
240	397.4	v	1.92	1.99	2.09	2.26	2.42	2.57	2.71	2.85	3.13	3.40
		h	1200.9	1215.4	1234.3	1261.9	1287.6	1312.8	1337.6	1362.3	1411.5	1460.5
		n	1.5309	1.5476	1.5690	1.5985	1.6246	1.6492	1.6720	1.6937	1.7344	1.7721
260	404.5	v	1.78	1.84	1.94	2.10	2.24	2.39	2.52	2.65	2.91	3.16
		h	1202.1	1217.1	1236.4	1264.1	1289.9	1315.1	1340.0	1364.7	1414.0	1463.2
		n	1.5244	1.5416	1.5631	1.5926	1.6186	1.6430	1.6658	1.6874	1.7280	1.7655
280	411.2	v	1.66	1.72	1.81	1.95	2.09	2.22	2.35	2.48	2.72	2.95
		h	1203.1	1218.7	1238.4	1266.2	1291.9	1317.2	1342.2	1367.0	1416.4	1465.7
		n	1.5185	1.5362	1.5580	1.5873	1.6133	1.6375	1.6603	1.6818	1.7223	1.7597
330	417.5	v	1.55	1.60	1.69	1.83	1.96	2.09	2.21	2.33	2.55	2.77
		h	1204.1	1220.2	1240.3	1268.2	1294.0	1319.3	1344.3	1369.2	1418.6	1468.0
		n	1.5129	1.5310	1.5530	1.5824	1.6082	1.6323	1.6550	1.6765	1.7168	1.7541
350	431.9	v	1.33	1.38	1.46	1.58	1.70	1.81	1.92	2.02	2.22	2.41
		h	1206.1	1223.9	1244.6	1272.7	1298.7	1324.1	1349.3	1374.3	1424.0	1473.7
		n	1.5002	1.5199	1.5423	1.5715	1.5971	1.6210	1.6436	1.6650	1.7052	1.7422
400	444.8	v	1.17	1.21	1.28	1.40	1.50	1.60	1.70	1.79	1.97	2.14
		h	1207.7	1227.2	1248.6	1276.9	1303.0	1328.6	1353.9	1379.1	1429.0	1478.9
		n	1.4894	1.5107	1.5336	1.5625	1.5880	1.6117	1.6342	1.6554	1.6955	1.7323
450	456.5	v	1.04	1.08	1.14	1.25	1.35	1.44	1.53	1.61	1.77	1.93
		h	1209	1231	1252	1281	1307	1333	1358	1383	1434	1484
		n	1.479	1.502	1.526	1.554	1.580	1.603	1.626	1.647	1.687	1.723
500	467.3	v	0.93	0.97	1.03	1.13	1.22	1.31	1.39	1.47	1.62	1.76
		h	1210	1233	1256	1285	1311	1337	1362	1388	1438	1489
		n	1.470	1.496	1.519	1.548	1.573	1.597	1.619	1.640	1.679	1.715

13. COMBUSTION AND ITS EFFECTS

The process of combustion is exothermic or heat liberating, and consists in the rapid oxidation of practically only 2 chemical elements, C and H, either free or in combination with each other. Few natural fuels consist of these elements in the free state; they are

generally combined in a complex manner. The formation of the compound took place with the liberation or absorption of heat, so that when the fuel is burned there is liberated, not the quantity of heat which would result from the burning of an equivalent quantity of free C and H, but a greater or less quantity depending upon whether the breaking up of the compound develops or absorbs heat. The HEAT OF COMBUSTION is then the heat of formation of the products less the heat of formation of the compound. In engineering calculations, it is generally sufficiently exact to consider heat of combustion as equal to the heat of formation of products, inasmuch as in exact or test work the heat of combustion of a fuel should be determined by calorimeter.

When H burns, the product is H_2O in the gaseous state, equal to 9 times the wt of H originally present. Usually the products pass off at a temp greater than 212° F, but at lower temp the vapor will condense to liquid, giving up 970 B t u per lb, or 9 × 970 B t u per lb of H. Hence, fuels containing H may have 2 heating values, the HIGH VALUE, when the products cool to less than 212°, and the LOW VALUE, when they do not No definite rule exists as to which should be used, nor is it material, provided there be a clear understanding between the parties interested. Generally, the high values are used for boiler and other open fires, and the low values for explosive combustion. The values found by calorimeter are high values.

Table 26 gives heating values per lb and per cu ft at 32° F and 14.7 lb, for some simple substances, and for common liquid and gaseous fuels. The latter are merely averages, but give a general idea of what may be expected from each class.

Table 26. Heats of Combustion. (Selected from Lucke's Engineering Thermodynamics)

Symbol	Substance	High value B t u		Low value B t u	
		per lb	per cu ft	per lb	per cu ft
C_2H_2	Acetylene	21 400	1 529	20 727	1 578
C_2H_5OH	Alcohol ethyl	13 246	1 720	12 100	1 570
CH_3OH	" methyl	10 203	917	9 113	819
	Anthracite producer gas, av	2 043	142	1 935	135
	Benzene, av	17 976	3 942	17 305	3 795
	Bituminous producer gas, av	2 280	165	2 160	155
	Blast furnace gas, av	1 343	109	1 323	107
C	Carbon to CO	4 351	......	......	......
C	" " CO_2	14 544	......	......	......
CO	Carbon monoxide	4 325	338	......	......
	Coal gas, av	18 970	605	17 023	543
	Coke-oven gas, av	15 930	642	14 384	580
C_2H_6	Ethane	22 027	1 845	20 280	1 700
C_2H_4	Ethylene	21 200	1 700	20 053	1 595
	Gasolene	21 300	6 050	2 520	5 732
C_6H_{14}	Hexane	20 610	4 970	19 195	4 630
H_2	Hydrogen	60 626	341	51 892	292
	Kerosene	20 100	9 550	19 000	9 050
	Lignite producer gas	2 420	172	2 277	160
CH_4	Methane	23 646	1 066	21 463	960
	Natural gas (Kansas)	23 500	1 050	21 300	950
	Oil gas	23 096	996	18 650	804
S	Sulphur	4 000	......	......	......
	Water gas, av	7 825	357	7 228	330
	" " carburetted, av	13 800	719	12 888	670

Heating value of a fuel. To compute it, the chemical analysis must be known. The fractional quantity of each constituent is then multiplied by its heating value, and the sum of these products is the heating value of the fuel. Analyses of solid and liquid fuels are reported in percentages by wt, gaseous fuels, by volume. Coals are reported in 2 ways: proximate and ultimate analysis, the former ascertaining moisture, volatile matter, fixed carbon and ash, the latter the chemical constituents, C, H, S, O, and ash. As the character of the volatile matter can not be determined, attempts to compute the calorific value of coals from proximate analysis are not very successful, the ultimate values being generally used. Following formula, by Lucke, applies with fair accuracy over a rather wide range of coals:

$$Q = 14\,544\,C + 27\,000\,V\left[1 - \frac{1}{\frac{C}{V} + 0.5}\right] \qquad (46)$$

in which Q = B t u per lb, C = fractional wt of fixed carbon, V = fractional wt of volatile matter. When ultimate analysis shows both O and H, it is assumed that the O,

and as much of the H as can chemically combine with it, exist as water, and hence only the remaining H is available as a heat producer. Following formula, proposed by Am Soc Mech Engs, is based on this assumption:

$$Q = 14\,600\,C + 62\,000\left(H - \frac{O}{8}\right) + 4\,000\,S \tag{47}$$

in which Q = B t u per lb, and C, H, O, and S are the fractional weights of carbon, hydrogen, oxygen, and sulphur per lb of coal.

Example 17. The calorific value of a coal was 13 900 by calorimeter. Proximate analysis: moisture, 1.25; vol matter, 39.03; fixed C, 53.72; ash, 6.00. Ultimate analysis: H_2, 2.47; C, 87.57; N_2, 1.09; O_2, 2.87; S, 0; ash, 6.00. Calculated values would be:

(a) by Eq 46, $14\,544 \times 0.537 + 27\,000 \times 0.39\left[1 - \dfrac{1}{1.38 + 0.5}\right] = 12\,750$ B t u

(b) by Eq 47, $14\,600 \times 0.876 + 62\,000\left(0.0247 - \dfrac{0.0287}{8}\right) = 14\,085$ "

(c) by calculation from ultimate analysis,

$$14\,544 \times 0.876 + 60\,626\left(0.0247 - \frac{0.0287}{8}\right) = 14\,003 \text{ ''}$$

Sherman and Kropff found that the heating value of oils is a function of density, and deduced following expression, which is within 2 to 3% of the calorimetric values:

$$Q = 18\,650 + 40\,(B\acute{e} - 10) \tag{48}$$

$B\acute{e}$ = deg on Beaumé hydrometric scale, the usual mode of reporting oils. Stanton and Strong revised the constant in Eq 48 for gasolene, and Allen and Strong for kerosene:

$$\left.\begin{aligned} Q &= 18\,320 + 40\,(B\acute{e} - 10), \text{ for gasolene} \\ Q &= 18\,440 + 40\,(B\acute{e} - 10), \text{ for kerosene} \end{aligned}\right\} \tag{49}$$

with an error of less than 1% in all tests.

Example 18. Find heating value of the following gas high and low values:

CO	H_2	CH_4	C_2H_4	N_2	O_2	CO_2
7	38	40	3	8	2	4

Multiply quantity of each combustible constituent by its heating value (see table).

Results found are for 32° and 14.7 lb, since the heating values for the constituents are for this press and temp. For any other press and temp the value will be the above $\times$ 33.5 $p \div T$, where p = press, lb per sq in abs, and T = abs temp F. At 70° F, the values are 576 and 527.

High value	Low value
7 × 338 = 24	7 × 338 = 24
38 × 341 = 122	38 × 292 = 111
40 × 1 060 = 424	40 × 960 = 384
3 × 1 700 = 51	3 × 1 595 = 48
621	567

Air needed for combustion is most readily computed from the chemical equations representing the reactions. The quantity required to satisfy the equations is that which is chemically necessary. But, in burning fuels, with the possible exception in the case of internal-combustion engines, considerably more air must be supplied completely to burn the fuel than is apparently required. The difference between these amounts is termed the excess air.

Chemical symbols indicate the amounts as well as the substances entering into a reaction. Thus the equation, $CH_4 + 2\,O_2 = CO_2 + 2\,H_2O$ may be stated in words in 3 ways: (a) CH_4 (methane) combining with O forms CO_2 and water vapor; (b) 16 lb CH_4 combined with 64 lb O forms 44 lb CO_2 and 36 lb water vapor; (c) 1 cu ft CH_4, combined with 2 cu ft O forms 1 cu ft CO_2 and 2 cu ft water vapor. Thus, not only does the symbol name the substance, but also gives the wt and vol reacting, as each symbol represents 1 unit vol, or the molecular wt of the substance in any units. In engineering practice the wt is taken in lb and the vol in cu ft. The molecular wt of any substance is the sum of the atomic wt of the elements forming it, taken as many times as they occur; CH_4, for example, being composed of 1 C (atomic wt, 12) and 4 H atoms (atomic wt, 1), has a molecular wt of $12 + (4 \times 1) = 16$, and the symbol CH_4 indicates 16 lb of methane. It should be noted that the molecule of simple gas is composed of 2 atoms, hence 1 cu ft of H, O, or N is represented by H_2, O_2, or N_2, and not by H, O, or N. For atomic weights of the elements, see Sec 37.

As air is approx 21% O by vol, or 23% by wt, the air required to burn a substance is found by dividing the vol of O required by 0.21, or the wt by 0.23. Table 27 gives the air required per lb or cu ft of different substances.

To find the air required by a mixture of substances, it is merely necessary to find the sum of the quantities of air needed for each constituent. In general it is simpler to find the wt of air required for solids and liquids, and convert to cu ft if needed, and to find the cu ft needed for gases and convert to lb.

Table 27. Air Chemically Required for Complete Combustion

Symbol	Substance	1 lb requires		1 cu ft requires	
		cu ft	lb	cu ft	lb
C_2H_2	Acetylene	165.1	13.3	11.95	0.96
C_2H_5OH	Alcohol, ethyl	112.0	9.0	14.34	1.17
CH_3OH	" methyl	80.5	6.5	7.17	0.58
	Anthracite producer gas, av	14.4	1.16	1.00	0.08
C_6H_6	Benzene	165.1	13.3	35.84	2.89
	Bituminous producer gas, av	14.4	1.16	1.00	0.08
	Blast-furnace gas, av	0.9	0.73	0.73	0.05
C	Carbon to CO	71.6	5.8	......	....
	" " CO_2	143.1	11.6	......	....
CO	" monoxide	30.6	2.5	2.39	0.19
	Coal gas, av	157.5	12.7	5.01	0.40
	Coke-oven gas, av	135.2	10.9	5.43	0.44
C_2H_6	Ethane	200.2	16.2	16.73	1.35
C_2H_4	Ethylene	184.9	14.9	14.34	1.16
	Gasolene	188.6	15.2	52.74	4.26
C_6H_{14}	Hexane	188.6	15.2	45.45	3.66
H_2	Hydrogen	429.2	34.6	2.39	0.19
	Kerosene	187.3	15.1	88.71	7.15
	Lignite producer gas, av	18.6	1.5	1.31	0.11
CH_4	Methane	214.6	17.3	9.59	0.77
	Natural gas (Kansas)	211.0	17.0	9.40	0.76
	Oil gas, av	185.0	14.9	11.40	0.92
S	Sulphur	53.5	4.3	......	....
	Water gas, av	48.4	3.9	2.15	0.17
	" " carburetted, av	110.5	8.9	5.98	0.48

Example 19. Find the air required to combine chemically with 1 lb coal, the ultimate analysis of which is given under Example 17:

(a) By chemical equations:

$$H_2 + O = H_2O \qquad C + O_2 = CO_2$$
$$2 + 16 = 18 \qquad 12 + 32 = 44$$

hence 1 lb H_2 requires 8 lb O_2, and 1 lb C requires 32 ÷ 12 lb O_2; therefore, the O required by coal per lb is

$$\left(0.0247 - \frac{0.0287}{8}\right) \times 8 + 0.8757 \times (32 \div 12) = 2.51 \text{ lb}$$

and the air required is 2.51 ÷ 0.23 = 10.9 lb. Should the vol be required, the wt is multiplied by the cu ft per lb at the press and temp desired.

(b) By Table 27,

$$\left(0.0247 - \frac{0.0287}{8}\right) \times 34.6 + (0.8757 \times 11.6) = 10.9 \text{ lb}$$

Example 20. Find the air required for chemical combination with a gas, the analysis of which is given under Example 18:

(a) By chemical equations:

$$CO + O = CO_2 \quad H_2 + O = H_2O \quad CH_4 + 2\,O_2 = CO_2 + 2\,H_2O \quad C_2H_4 + 3\,O_2 = 2\,CO_2 + 2\,H_2O$$
$$1 + 0.5 = 1 \qquad 1 + 0.5 = 1 \qquad 1 + 2 = 1 + 2 \qquad 1 + 3 = 2 + 2$$

hence 1 cu ft H_2 or CO requires 0.5 cu ft O_2, 1 cu ft CH_4 requires 2 cu ft O_2, and 1 cu ft C_2H_4 requires 3 cu ft O_2; therefore, the gas requires

$$(0.07 \times 0.5) + (0.38 \times 0.5) + (0.4 \times 2) + (0.03 \times 3) - (0.02 \times 1) = 1.095 \text{ cu ft } O_2,$$

and the air needed = 1.095 ÷ 0.21 = 5.22 cu ft. To convert to lb, multiply by the density at the existing press and temp.

(b) By Table 27,

$$\text{Air} = (0.07 \times 2.39) + (0.38 \times 2.39) + (0.4 \times 9.59) + (0.03 \times 14.34) - (0.02 \div 4.8) = 5.22 \text{ cu ft.}$$

$$\text{Wt air per lb fuel} = 11.5\,C + 34.6\,H \text{ (approx)} \tag{50}$$

The above expressions and methods are useful in finding the air chemically required. The actual quantity of air for combustion is found by analysis of the flue gas. Then, the air required per lb of C is determined by Eq 51, for fuels containing but little H; when there is much H, the relations are too complex to be of much use.

$$\text{Lb of air per lb } C = (3\,N_2 \div 0.165\,O_2) \div (CO + CO_2), \tag{51}$$

where N_2, O_2, CO, and CO_2 are percentages by vol by flue-gas analysis.

Products formed by combustion are chiefly CO_2, water vapor, and the N from the air. When excess air is used this is also found in the products. When insufficient air is sup-

plied, it is generally impossible to predict the analysis of the products, because the quantity of the available air received by each combustible can not be foretold.

Example 21. Compute the products of combustion for the fuels of Example 19 for the case of the necessary and of twice the necessary air. From the equations, 1 lb C makes 44 ÷ 12 lb CO_2, and 1 lb H_2 makes 9 lb H_2O; hence, 9 × 0.0247 = 0.222 lb H_2O + (44 ÷ 12) × 0.8757 = 3.21 lb CO_2 + 0.0109 lb N_2 from fuel, and 0.77 × 10.9 = 8.4 lb N_2 from air. With 100% excess air, the water vapor and N from the coal will be the same, and the N from the air twice as great, and there will be 10.9 × 0.23 lb of O from the excess air.

Loss due to incomplete combustion of C to CO instead of CO_2 is found from flue gas analysis by Eq 52:

$$\text{Loss (B t u) due to unburned CO per lb C} = 10\,193\left(\frac{CO}{CO + CO_2}\right) \qquad (52)$$

Producer reactions. Gas producers produce a combustible gas from solid fuel, as coke, anthracite coal, and to a less extent from bituminous coal, lignite, etc. The nearer pure carbon the fuel, the better the result. If the producer blast be air alone, the reactions are: $C + O = CO$, $C + O_2 = CO_2$, and $CO_2 + C = 2\,CO$; if with steam alone, not a continuous process, the reaction is $C + H_2O = CO + H_2$; or steam and air may be used together, in which case the composition of the gas formed is too complex to discuss here (1).

Final temp and press due to combustion. Combustion may take place at either constant press or constant vol, explosively or nonexplosively, a rise in press being characteristic of explosive combustion, but not necessarily defining it. For explosive combustion a mixture of fuel and O, with or without neutral gases, as N, must be present and intimately mixed, so that a flame once started proceeds rapidly throughout the mass.

A rise in temp always occurs, but which can only be estimated, because of uncertainty of specific heats and other phenomena. Mode of procedure is to apply correction factors found by experience to the hypothetical results computed. Temp rise due to combustion is

$$\left.\begin{aligned} t_2 - t_1 &= C\frac{Q}{0.17w}, \text{ for constant vol} \qquad (a)\\ t_2 - t_1 &= C\frac{Q}{0.24w}, \text{ for constant press} \qquad (b)\end{aligned}\right\} \qquad (53)$$

where t_1 and t_2 are the initial and final temp, Q the heat liberated, w the wt of products formed, and C a constant. Value of C lies between 0.45 and 0.55 for both furnace combustion and gas-engine work, when there is about the chemically correct amount of air, but it has higher values for excess air. The vol and press changes due to combustion are

$$V_2 - V_1 = \frac{V_1}{T_1}\left(\frac{CQ}{0.24w}\right) \text{ and } P_2 - P_1 = \frac{P_1}{T_1}\left(\frac{CQ}{0.17w}\right), \qquad (54)$$

where V_1, P_1, T_1 are the original vol in cu ft, press in lb per sq ft, and temp in deg abs; V_2, P_2, T_2 the final values, Q the heat liberated, C the constant as above, and w the wt of products. The press rise is more conveniently expressed by

$$\begin{matrix}\text{Press rise,}\\ \text{lb per sq in}\end{matrix} = 72.3\,C\left(\frac{\text{initial press, lb per sq in abs}}{\text{initial temp, °F abs}}\right) \times \left(\frac{\text{B t u per cu ft of mixture}}{\text{at 32° F \& 14.7 lb abs}}\right) \qquad (55)$$

Temp of ignition is the temp to which a substance must be heated before combustion takes place. Experiments by Falk and others indicate that this temp is not constant.

Falk proved that for gaseous mixtures, it varies with the proportion of gas to air. He gives following values, as true over a 100% range of proportions:

Ignition temp of gasolene mixtures with air .... 986° F
" " " alcohol " " " .... 1 292° F

Beyond certain limits of the air-gas ratio, mixtures will not ignite. Table 28 gives this limit for several gases, expressed in terms of the percentage of detonating mixture which must be present for ignition to occur. By DETONATING MIXTURE is meant the combustible portion of fuel plus the O chemically required for combustion.

Table 28. Percentage of Detonating Mixture in Total Air-gas Mixture at Explosive Limits of Proportion (Bunte)

Gas	For excess air	For excess gas	Gas	For excess air	For excess gas
Carbon monoxide	24.8	15.8	Ethylene	16.4	23.9
Hydrogen	14.2	21.2	Alcohol	15.0	24.2
Water gas	18.2	21.0	Methane	18.3	27.5
Acetylene	11.7	14.0	Benzene	19.9	22.3
Coal gas	17.4	31.2			

Rate of combustion of coal is controlled by the draft. Following relation (1) holds for small-sized anthracite and bituminous coal:

$$\text{Lb coal per sq ft grate surface per hr} = C\sqrt{\text{draft, in of } H_2O} \qquad (56)$$

where C has the following values: anthracite rice, 17.3; anthracite No 1 buckwheat, 24.5; anthracite pea, 31.5; semi-bituminous, 50.2; bituminous, run-of-mine, 81.6; bituminous, slack, 59.3.

Efficiency of combustion in furnaces is the ratio of heat developed to that contained in the coal; sometimes termed furnace effic, and ranges from 80 to 95%. In producers, the effic is a matter of gasification, and is the ratio of the heat in the gas from 1 lb of coal to the heat originally in the coal (Sec 37, Table 12).

14. HEAT TRANSFER

Except in the burning of explosive mixtures, heat is practically never developed in the substance heated. In all other cases, heat must be conducted or transmitted to the desired point, generally through separating walls. Flow of heat may be grouped in 3 classes, internal conduction, radiation, and convection.

Table 29. B t u Transmitted per Hr per Sq Ft per Deg F Difference in Temp per In Thickness

Substance	K	Substance	K
Air	0.1405	Limestone, marble, calcite, dolomite, etc	13.6–16.2
Aluminum	966 (1 + 0.0003 (t − 32))	Magnesia	0.45–1.3
Antimony	128 (1 − 0.00058 (t − 32))	Magnesium	1 090
Asbestos paper	1.245	Mercury	0.0148–0.0189
Bismuth	51 (1 − 0.00041 (t − 32))	Micaceous flagstone	
Blotting paper	0.435	along cleavage	18.3
Brass yellow	592 (1 − 0.00136 (t − 32))	across "	12.8
red	713 (1 − 0.00089 (t − 32))	Nickel	412
Cadmium	638 (1 − 0.00039 (t − 32))	Paraffine	0.66 at 32° to 4.88 at 212°
Carbon	1.32	Pasteboard	1.3
monoxide	0.145	Petroleum	1.03
dioxide	0.0891	Plaster of Paris	2.03
Carborundum	1.45	Platinum	483
Castor oil	1.23	Portland cement	2.06
Chalk	0.58	Quartz	1.04
Constantin	1 565–1 855	Sand	2.7
Copper	2 080 (1 − 0.00003 (t − 32))	Sandstone	15.8–16.4
Cotton wool	0.125	Sawdust	0.348
pressed	0.0957	Silver	3 180
Cork	2.08	Slate	
Diatomic earth	0.377	along cleavage	16–19
Ebonite	1.1	across "	9.2–10.4
Felt	0.252	Snow, packed	1.48
Firebrick	0.81	Soil	0.96 dry, 4.6 wet
German silver	203 (1 − 0.00148 (t − 32))	Steel	180–320
Glass	3.2–6.7	Strawboard	0.871
Granite	15.6	Tin	443 (1 − 0.00038 (t − 32))
Haircloth	0.122	Water	3.5–6.5
Ice	6.47–16.48	Wood	
Iron	507 (1 − 0.00083 (t − 32))	cross grain	0.261
wrt	600	with grain	0.871
Lead	242 (1 − 0.00048 (t − 32))	Zinc	443
Leather (cowhide)	1.22		
Lime	0.84		

Internal conduction is the passage of heat along a bar of any substance, from a hot to a cooler zone. In this case, the characteristics of different materials are fairly well known.

Values for common substances are given in Table 29, in which K is the B t u transmitted per hr through a plate 1 ft square and 1 in thick per deg difference in temp. K is not constant, but varies with the temp in a linear relation; that is, $K_t = K_{32}[1 + a(t - 32)]$. Where value of a is known, it is given in Table 29; when not given, the single value found must be used regardless of temp. For great temp range, the mean value of the coeff must be used, the mean value of K between 2 points being

$$K_{32}\left(1 + \frac{a}{2}[(t_2 - 32) + (t_1 - 32)]\right),$$

hence the heat flow through a body by internal conduction is

$$Q = \frac{A}{l} K_{32} \left(1 + \frac{a}{2}[(t_2 - 32) + (t_1 - 32)]\right)(t_2 - t_1), \tag{57}$$

where Q = B t u per hr, A = cross-sec of path in sq ft, l = length of path, in, K_{32} = coeff at 32° F, t_2 = high temp, t_1 = low temp. Temp t_2 and t_1 must be taken in the body itself; not in the fluid surrounding it.

Radiation of heat by a body is the transfer of heat from a hotter to a cooler body, without warming the intermediate space. Most common example is the sun's heat, which warms the earth, but not the intervening medium. The permeability of different materials to heat waves varies as with light waves, but a substance easily passed by one is not necessarily passed by the other. The most accepted law for heat radiation is that of Stefan and Bolzman; the heat radiated from an incandescent body, black when cold, varies as the difference of the fourth powers of the temp, thus:

$$\text{B t u radiated per hr per sq ft} = 16 \times 10^{-10}\,(T_2^4 - T_1^4), \tag{58}$$

where T_2 = abs temp F of the radiating, and T_1 = that of the receiving body.

Practically the only black body (that is, one which is 100% efficient in absorbing heat rays and reflecting or transmitting none) is carbon, the crystalline form excepted. All other bodies reflect some heat and fail to radiate to the same extent. As radiation increases with roughness and darkness, surfaces from which radiation is not desired are frequently polished. Table 30 gives relative radiating and reflecting power of different substances, compared to carbon as unity for radiation and zero for reflecting power. Of course heat rays striking at an angle are partly reflected, and for small angles nearly all the heat may be reflected.

Table 30. Radiation and Reflection Coefficients (Kent)

Substance	Radiating or absorbing power	Reflecting power	Substance	Radiating or absorbing power	Reflecting power
Brass cast	0.11	0.89	Ivory, Jet, Marble	0.93–0.98	0.07–0.02
Brass polished	0.07	0.93			
Carbon, porous	1.00	0.00			
Carbonate of lead	1.00	0.00	Mercury	0.23	0.77
Cast iron	0.25	0.75	Platinum, polished	0.24	0.76
Copper, hammered	0.07	0.93	" sheet	0.17	0.83
" varnished	0.14	0.86	Silver polished	0.03	0.97
Glass	0.90	0.10	Steel "	0.17	0.83
Gold, plated	0.05	0.95	Tin	0.15	0.85
Ice	0.85	0.15	Wrought iron	0.23	0.77
			Zinc	0.19	0.81

Convection is the actual conveyance of heat by the movement of particles of a subtance. The most important example is the heating of water in vessels, where the heat applied to the bottom is carried up through the liquid by the particles of water, which, on becoming warmer, grow less dense and tend to float or rise through the mass being heated. Convection can occur only in liquids or gases, substances in which the particles are free to move, and as radiation and conduction generally take place at the same time it is difficult to isolate the convection effect.

Heat transfer between separated fluids. When heat flow passes from one body to another, a high resistance to flow is encountered, even a carefully-made joint being equivalent to several inches of the bar itself. Hence, boiler seams are not allowed in contact with the fire, because the high resistance of the joint would prevent the carrying away of heat fast enough to prevent the fire side from overheating. Between a plate and a fluid a film adheres to the plate, causing high resistance; by reducing this, agitation of the fluid or high velocity increases the rate of heat transfer.

Generally, in heat transfer apparatus, there is high resistance to flow on one side or the other of the separating wall, and but little improvement is realized by agitation or similar effort on the low-resistance side. In boilers, for example, the high resistance lies between the gases and tubes, and effort spent on the water side avails little. The total resistance to heat transfer is therefore made up of the resistance of the materials through which the heat must pass, plus the joint resistances, plus the film resistances, of which the values of very few can be determined. Considering a boiler tube, the path is made up of a layer of soot and ash of unknown thickness and conductivity, a film resistance between the gases and this, a joint resistance between the soot or ash and tube, the tube itself and a film resistance between tube and water. The practical solution of such a problem employs a coeff of heat transmission experimentally determined for various frequently-occurring conditions, and including all the separate resistances and radiation and convection effects. Such coeff, generally

designated by U, expresses the B t u transmitted per hr per sq ft of surface per deg difference in temp, measured in the separated fluids, one of which is receiving heat from the other. Values of U in aver practice are in Table 31. The heat transmitted per hr is

$$Q = AUt_m, \tag{59}$$

where Q = B t u per hr, A = area in sq ft, U = coeff, t_m = mean temp difference between the fluids.

Table 31. Coefficients of Heat Transfer U. Averages Used in Practice

Giving heat	Receiving heat		
	Liquid warming	Liquid boiling	Gas warming
Liquid cooling	50–75	{25–50, 100*}	2–6
Gas cooling	2–5	2–5	2–5
Vapor condensing	{150–350, 1 000*}	400–600	2–4

* High liquid velocity or agitation.

Mean temp difference is merely the difference between the hot and cold fluids, if throughout each the temp be constant. But, in most cases the temp of one or both is changing in accordance with which one of the 4 possible cases shown below is being considered:

	Substance losing heat may be	Substance receiving heat may be
(a)	at constant temp	at constant temp
(b)	" " "	rising in temp
(c)	falling in temp	
(d)	" " "	at constant temp

Case (c) may occur for either parallel or counter-current flow; in the former, both fluids enter at same end of the system, the final temps approaching the same value; in the latter, the fluids enter at opposite ends, final temp of one approaching initial of other. For cases where either or both fluids are changing in temp,

$$\text{Mean temp difference} = \frac{\text{Initial temp difference} - \text{final temp difference}}{\text{Nap log}\left(\dfrac{\text{Initial temp difference}}{\text{Final temp difference}}\right)} \tag{60}$$

Example 22. A solution the specific heat of which is 1.2 is to be heated to 170° from 60° by exhaust steam at atmos press and 90% dry. Find the surface required to heat 50 000 lb per hr, also the steam needed, neglecting any heating by condensed steam.

By Eq 60, $t_m = \dfrac{(212 - 60) - (212 - 170)}{\text{Nap log}\left(\dfrac{212 - 60}{212 - 170}\right)} = 86.$ $Q = 50\,000 \times 1.2 \times 110 = 6\,600\,000$ B t u

$$\text{Heat per lb steam} = 970 \times 0.9 = 873, \text{ and lb steam} = \frac{6\,600\,000}{873} = 7\,550$$

$$\text{From Table 31, } U = 150 \text{ to } 350, \text{ say } 300, \text{ and from Eq 59, } A = \frac{6\,600\,000}{300 \times 86} = 256 \text{ sq ft}$$

Example 23. The same quantity of the same liquid as in Example 22 is to be used as it cools to heat boiler-feed water, by passing the liquids in a counter-current direction through concentric pipes. 10 000 lb of water per hr are to be heated from 50° to 150°. Find required heating surface; also the required surface should it be desired to raise the final temperature of the feed water to 160°. The heat required to be transmitted will be 10 000 × 100 = 1 000 000 B t u, hence fall in temp of solution will be 1 000 000 ÷ (50 000 × 1.2) = 16.7°, and final temp will be 153.3°.

$$\text{By Eq 60, } t_m = \frac{(153.3 - 50) - (170 - 150)}{\text{Nap log}\left(\dfrac{153.3 - 50}{170 - 150}\right)} = 50.5$$

From Table 31, assume U = 60, whence from Eq 59, A = 1 000 000 ÷ (60 × 50.5) = 332 sq ft. For second case, Q = 1 100 000; final temp of sol = 151.6; t_m = 39.3; A = 464; showing the large increase in area needed for increased final temp of the cold liquid as it approaches the initial temp of the hot liquid.

Capacity of boilers and other heat-transfer apparatus is usually expressed in BOILER HORSE POWER (b h p). A b h p was originally assumed to be such that a boiler of a certain hp would supply enough steam to run an engine of the same hp. The term became too

indefinite, since engines differed widely in consumption of steam for 1 hp, and the following definitions were adopted by the Am Soc of Mech Engs:

1 b h p = 34.5 lb steam made at standard atmos press, or temp of 212° F per hr from feed water at 212° F, or 33 305 B t u absorbed per hr.

Originally these terms were equivalent, but later investigations show that if the first definition is true the second should be 33 479 B t u. The first is generally used. For ordinary feed-water temp and steam press, 34.5 lb from and at 212°, as it is termed, do not differ greatly from 30 lb under common conditions, and this being an even number is generally used as the h p unit for rating boilers, feed-water heaters, condensers, etc. That is, a feed-water heater of 100 b h p capacity will heat 30 × 100 = 3 000 lb water per hr. Boilers require from 10 to 12 sq ft of heating surface per hp, depending on type, and are frequently rated by dividing the sq ft of heating surface by these numbers.

Efficiency of a boiler is the ratio of the heat absorbed by the water or steam to that in the coal and may be separated into 2 parts:

$$\left.\begin{array}{lll}(a) \text{ Furnace effic} & = \text{heat liberated} \div \text{heat in coal} \\ (b) \text{ Surface effic} & = \text{heat absorbed} \div \text{heat liberated} \\ (c) \text{ Total boiler effic} & = a \times b = \text{heat absorbed} \div \text{heat in coal}\end{array}\right\} \quad (61)$$

Boiler effic defined above is sometimes designated as boiler and furnace effic. Among the losses tending to reduce furnace effic are coal lost to ash pit, carbon burning to CO instead of CO_2, H, and hydrocarbons passing out of stack unburned; that tending to reduce surface effic is the sensible heat of the stack gases above the room temp. For description of a complete heat balance, defining each loss and a manner of determination, see (18).

15. ENTROPY AND ENTROPY DIAGRAMS

Definitions. Just as work may be expressed as the product of press and vol change, when these quantities are expressed in proper units, so may heat be expressed as the product of temp and the change in a quantity termed entropy. One of the best definitions of entropy is that it is "heat per degree of temperature." In engineering calculations it is the CHANGE IN ENTROPY during a process, rather than the ABSOLUTE ENTROPY at any one time, that is important; hence any arbitrary zero of entropy may be chosen.

For steam work the zero generally taken is that of 1 lb of water at 32° F, and in steam tables (Art 12) the values given are above that value as a base. For ordinary work on gases the point in the process having the least entropy is selected as the zero. Entropy is usually designated by the Greek letter ϕ, and a diagram drawn to temperature-entropy coordinates is the $T\phi$ or heat diagram, precisely as one drawn to P-V coordinates is the P-V or work diagram. The area representing the amount of heat change during a process is the area under the curve down to line of zero temp, this quantity being expressed to the absolute scale. Thus, in Fig 23, the heat change from A to B is represented by area $ABCD$.

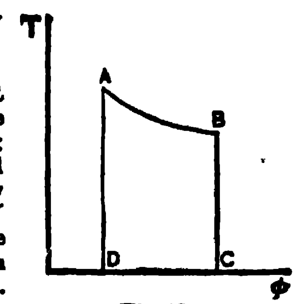

Fig 23.

Change in entropy, being the change in heat divided by the temp at which it occurs, is simply expressed if the change takes place at constant temp. If the temp is also changing, a differential form is required, for which a relation between heat and temp is necessary. This is found in the quantity known as specific heat, and the change in entropy is found by integrating. The results are given in Eq 62 for constant specific heats, the expressions resulting from varying specific heats being too complex for general use:

$$\left.\begin{array}{l}\phi_2 - \phi_1 = w\,Q \div T, \text{ for constant temp} \\ \phi_2 - \phi_1 = w \text{ Nap log } (T_2 \div T_1), \text{ for water (sp ht} = 1) \\ \phi_2 - \phi_1 = wC_v \text{ Nap log } (T_2 \div T_1), \text{ for gases at constant vol} \\ \phi_2 - \phi_1 = wC_p \text{ Nap log } (T_2 \div T_1) \text{ '' '' '' '' press}\end{array}\right\} \quad (62)$$

For superheated vapors the values for gases may be used with fair accuracy, especially for high degrees of superheat. The four commonest processes, isothermal and adiabatic expansion constant press-vol change, and constant vol-press change for gases, are shown in Fig 24 to P–V and T–ϕ coordinates.

Temperature-entropy diagram. For vapors 2 entropy diagrams are used, the temp-entropy and the B t u-entropy or Mollier diagram. Fig 25 shows on a small scale the temp-entropy diagram for steam.

The liquid line ABB''' represents the heating of water from 32° F, the area below it denoting the heat required, or the heat of the liquid at any point above 32° F. Lines BC, $B'C'$, etc, represent

steam-making at constant temp, the area under any horiz line denoting the latent heat at that temp. The third group CD, $C'D'$, etc, represents superheating at constant press. Lines of constant vol, constant quality or constant heat may also be added (omitted from this diagram for clearness). Adiabatic expansion or compression is represented by a vertical line; one of the chief uses of the diagram is to determine the quality after an adiabatic change. The length of any of the horizontals BC is equal to the latent heat divided by the temp. Should the steam be wet, all of the latent heat will not be used; hence the line will be shorter and C will move to the left to some indefinite position X.

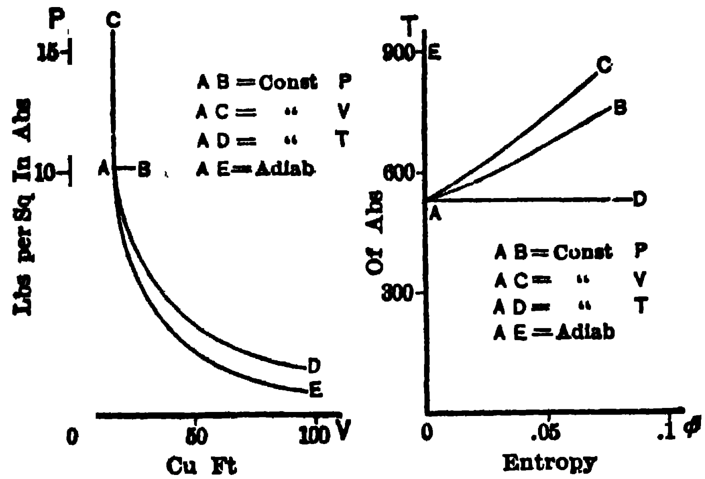

Fig 24. Constant Press, Vol, Temp, and Adiabatic Lines, for Air to P–V and T–ϕ Coordinates

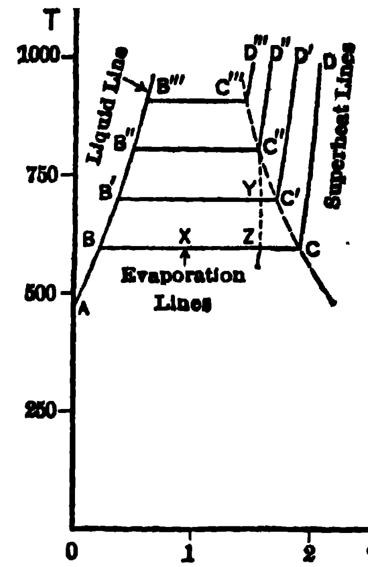

Fig 25. T–ϕ Diagram for Steam

Then as BX is proportional to the quantity of latent heat used, and BC to all the latent heat for the same press, $BX \div BC$ is the percentage of steam present, or the dryness fraction, and $XC \div BC$ the percentage of moisture present. The quality after any adiabatic change can be quickly determined by projecting from the original point up or down on the diagram. For example, if adiabatic expansion occurs from point C'' (representing dry steam at some press), when the press drops to Y the quality will be $B'Y \div B'C'$, and when pressure Z is reached, the quality will be $BZ \div BC$. The diagram shows that adiabatic expansion of dry steam causes wetness, expansion of superheated steam causes loss of superheat, and expansion of very wet steam causes drying. Compression under the above conditions causes the reverse action. For more complete description see (1).

Mollier diagram for steam, drawn to as large a scale as possible, is shown in Fig 26. This, named from its originator, Prof Mollier, is of much wider use than the temp-entropy diagram. On it, horizontals are lines of total heat; verticals, lines of constant entropy; lines nearly horiz but sloping toward the lower right-hand corner represent constant quality, and lines making an angle of more than 90° with them are lines of constant press. By projecting to the side from the intersection of a quality and press line the total heat may be read directly, or the entropy by projecting downward. Adiabatic expansion or compression is represented by a vertical line, constant total heat by a horiz.

The greatest use of the diagram is in solving Rankine-cycle problems (Art 16) for effic and for velocity of steam jets. To find the work done in a Rankine cycle it is merely necessary to read the total heat corresponding to any given press and quality, project from the original point downward to the line representing final press, read the total heat at this point and subtract the two. The final quality needed for determining m e p is found from the nearest quality line intersecting the final press line.

As explained in Art 2, the velocity of a jet is substantially equal to $\sqrt{2\,g \times \text{work}}$ and as the work is readily found by the Mollier diagram it is customary to add an extra scale giving velocity directly for any total heat difference.

Example 24. Find the quantities required in Example 26 by means of the Mollier diagram.

Heat originally in steam, per lb	= 1 248
" finally " " " "	= 1 045
Work per lb................	203 B t u
Final quality................	90%
Jet velocity................	3 190 ft per sec

Remaining quantities are found as in Ex 26. For discussion of the Rankine cycle and method of solution without the Mollier diagram, see Art 16.

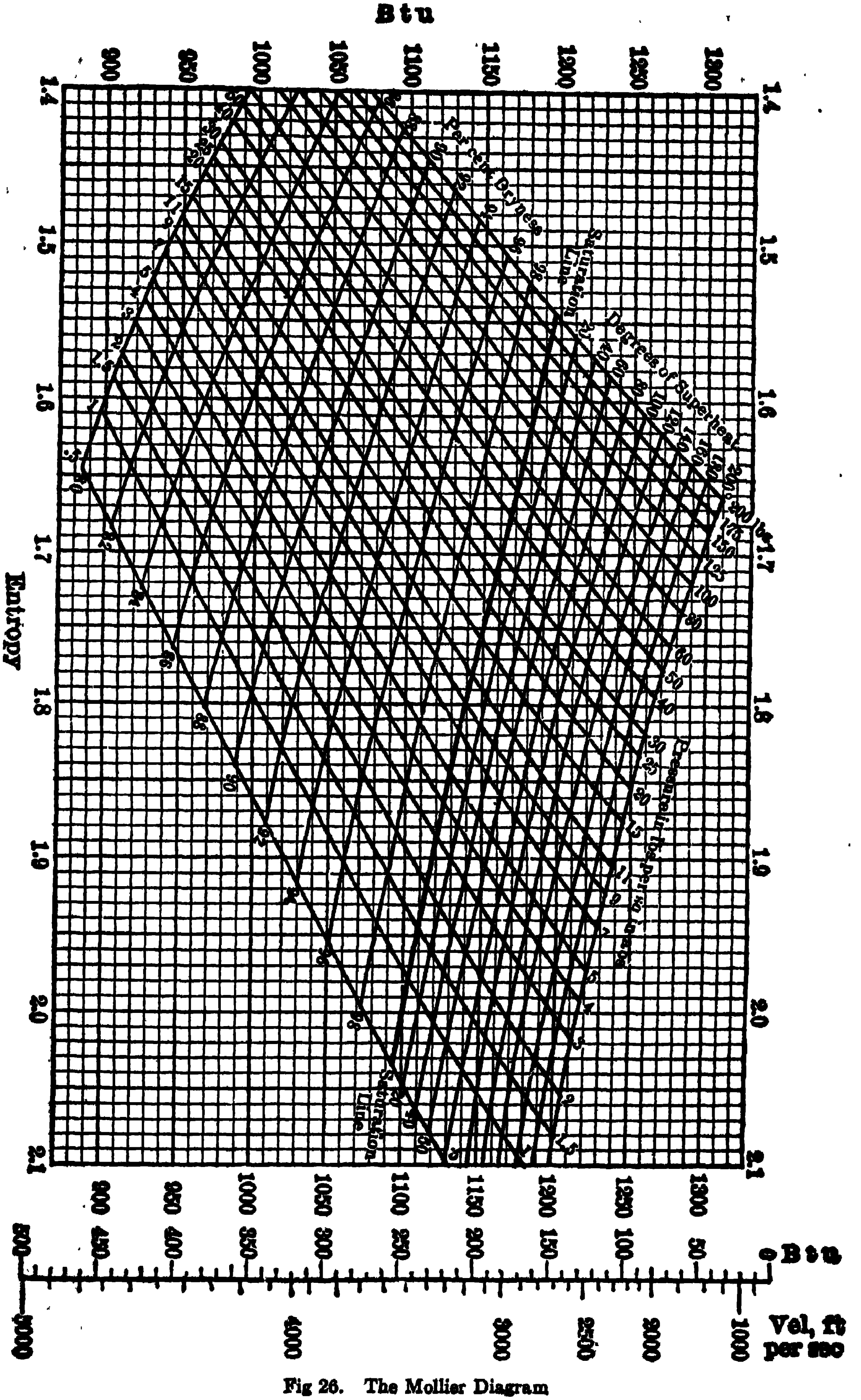

Fig 26. The Mollier Diagram

The Mollier diagram is useful also for finding the quality of steam by the throttling calorimeter. The temp and press of steam are read on the low-press (generally atmosphere) side of an orifice through which the steam has expanded, and subsequently brought to rest, being the equivalent of constant total-heat expansion. Knowing the temp and press of the steam at the low press, the amount of superheat for this condition is readily found; if there be no superheat at this point this method of obtaining the steam quality is not applicable. When the degree of superheat and low press are known, a point may be found on the diagram and, projecting horizontally (at constant total heat) to the high press, the quality line passing through the intersection of the horiz and high-press line gives the quality at the high press.

Example 25. In a throttling-calorimeter chamber, attached to a steam line in which the press was 110 lb gage; temp after expansion was 222° F, and the pressure, atmospheric; situation, sea level. Determine the quality in the line. Press at sea level, 14.7 lb abs, and saturation temp = 212° F, hence 10° superheat. Projecting from 14.7 and 10° on Mollier diagram horizontally to intersect with (110 + 14.7 lb) shows quality to be 96%.

16. HEAT CYCLES

Definitions. Work is always done at the expense of heat. During the process some heat may or may not be added; if none is added, the work must be at the expense of the heat already contained in the substance, and hence the decrease in this heat content is exactly equal to the work done. The energy stored up or contained in a body is internal or intrinsic energy, and may be reckoned above any arbitrarily-chosen zero. As with entropy, however, it is the change in intrinsic energy during a process, rather than the absolute quantity at any one time, which is of interest. If, while work is being done, heat is added, the difference between the added heat and work done is the change in intrinsic energy, and this relation is one way of expressing the first law of thermodynamics. It applies, of course, only to reversible processes.

For a complete cycle the intrinsic energy is the same at the end of the process as at the beginning, or the change is zero; hence, all the work must have come from the heat added during the different phases. As heat may have been abstracted as such at some part of the cycle, or work added as such, the term added heat must be the net heat added, or the algebraic sum of the heats added; similarly for the work done; hence,

$$\text{Work done in any complete cycle} = \text{Heat added} - \text{heat abstracted,} \qquad (63)$$

and as the effic of any thermal cycle is the ratio of work done to heat supplied,

$$\text{Effic of any complete cycle} = 1 - (\text{Heat abstracted} \div \text{heat added}) \qquad (64)$$

Steam-engine cycles. In Art 3, the action of steam in a cylinder was described on the basis of press-volume analysis. It can also be dealt with on the basis of heat-cycle analysis, and expressions for effic and m e p derived in this way. For the phases of constant press, admission and exhaust of the P–V diagram are substituted for the equivalent constant press-vol change processes of steam making, with possible superheating and condensation, on the heat diagram.

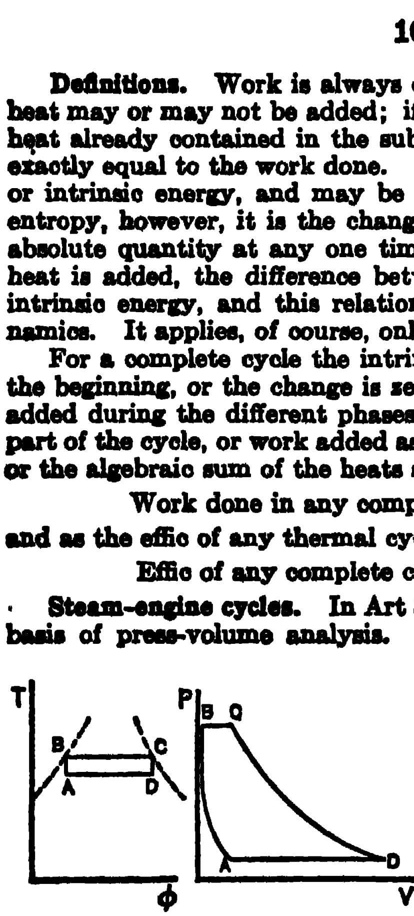

Fig 27. The Carnot Cycle

Two standard cycles represent the ideal steam engine process, the Carnot and the Rankine. Both are shown to *P–V* and *T–φ* coordinates; the former for dry steam only in Fig 27, the latter for wet and dry steam, and steam with different degrees of superheat, in Fig 28, as follows:

ABCD, initially wet, finally more wet; *ABC'D'*, initially dry, finally wet; *ABC''D''*, initially superheated, finally wet; *ABC'''D'''*, initially superheated, finally dry; *ADC*^{IV}*D*^{IV}, initially superheated, finally superheated.

Carnot cycle, which assumes complete adiabatic expansion and compression, was formerly universally accepted as standard, chiefly because it was the most efficient cycle between any 2 temperatures, and hence for steam pressure limits. Its effic depends only on the temp, or,

$$\text{Carnot-cycle effic} = (T_2 - T_1) \div T_2 \qquad (65)$$

where T_2 and T_1 are the high and low abs temp, respectively.

Rankine cycle assumes complete adiabatic expansion, but no compression, and is now widely used as a standard of comparison, chiefly because it represents an effic more nearly attainable, is equally applicable to superheated and to dry steam, and serves as well for a

basis for turbine work as for engine work, features not possessed by the Carnot. Engines are spoken of as attaining a certain fraction of the Rankine cycle possibility.

In Fig 28 the constant-vol, press-rise line of the P–V diagram is represented on the T–ϕ diagram by a liquid heating line, which is substantially a constant-vol process. Heat is added from A to B and then from B to C, C', etc, as the case may be, adiabatic expansion follows to D, D', etc, and finally heat is removed from D, D', etc, to A. Considering one case only, that of initially dry steam, for example (as the rest are similar), the heat added is $H_{C'} - H_A$, that removed $H_{D'} - H_A$, and the work done is $(H_{C'} - H_A) - (H_{D'} - H_A)$ or $H_{C'} - H_{D'}$. Thus, the work of the Rankine cycle is merely the difference in the total heats at beginning and end of expansion. That at beginning is readily found from the steam tables (Art 12), when the initial press and quality are known, or from the Mollier Diagram (Fig 26). The final quality is not known, but can be computed or more readily obtained directly from Fig 28. For methods of computation, see Example 26. Effic will be $(H_{C'} - H_{D'}) \div (H_{C'} - H_A)$, H_A being the heat in the liquid at beginning of the cycle, or at the temp corresponding to the back press, and may be found from the steam tables. The m e p is the work done divided by the vol change, and since originally there is only liquid present, the vol of which may be neglected as it is very small compared to the final vol, the vol change may be taken as equal to the final vol. This quantity is the tabular value (from steam table) times the quality, if wet, and times the actual temp divided by the saturation temp, both absolute, if superheated. Rankine cycle applies equally to turbines, and all of the above except the expression for m e p applies equally to both piston engines and turbines. But, there is one more quantity applicable to turbines only, viz, the velocity of the jet. Considering 1 lb of steam and a zero initial velocity, the final velocity will be $\sqrt{2\,gW}$ or $8.02\sqrt{H_{C'} - H_{D'}}$. These expressions are grouped in Table 32.

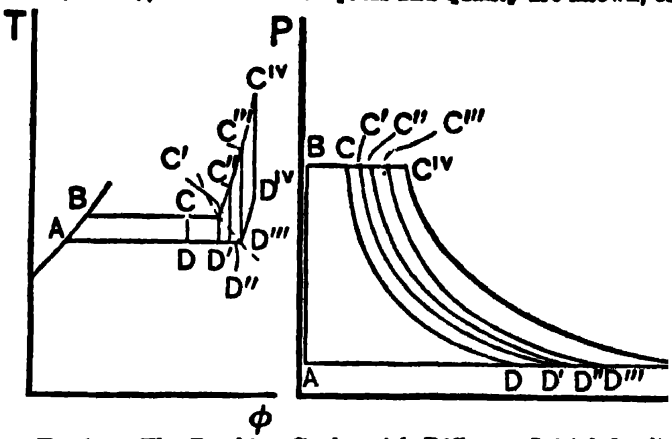

Fig 28. The Rankine Cycle, with Different Initial Qualities

Table 32. Rankine Cycle

Work	$H_2 - H_3$
Efficiency	$(H_2 - H_3) \div (H_2 - H_1)$
M e p (lb per sq in)	$778\,(H_2 - H_3) \div 144\,V_3$
Heat consumption per hr per h p	$2\,545\,(H_2 - H_1) \div (H_2 - H_3)$
Steam consumption per hr per h p	$2\,545 \div (H_2 - H_3)$
Jet velocity	$\sqrt{(H_2 - H_3) \times 778 \times 64.4}$

H_1 is the heat in the liquid at the low press; H_2, the heat in the steam prior to expansion; H_3, the heat in the steam after expansion.

The thermal effects of change of initial press, back press, superheating, jacketing, and reheating, in a steam engine are best seen from examination of a T-ϕ diagram of a Rankine cycle. In Fig 29 (*a*), *ABCD* represents a cycle between the pressures of 100 lb and 20 lb, the area being proportional to the work of 1 lb of initially dry steam. *AB'C'D* shows the case for an initial press of 160 lb, all other conditions being the same, the gain in work being *BB'C'C*. *A'BCD'* is the case of the original initial press, but with back press lowered to 1 lb, the gain in work being *A'ADD'*. Thus, the gain in the latter case is considerably greater than in the former, although the change in press is less. Efficiencies in the 3 cases are: *ABCD* ÷ *EABCF*, *AB'C'D* ÷ *EAB'C'F*, and *A'BCD'* ÷ *E'A'BCF*. Increase of initial or decrease of exhaust pressure both tend to increase the efficiency. Complete expansion is assumed in all cases of the Rankine cycle, which is not possible to realise in real engines with high ratios of expansion, that is, with either very high initial or very low-back press, so that the gains in cyclic performance may not be fully attained in practice or may possibly be losses in the case of piston engines. Expansion always being complete in turbines, any cyclic gains offered by changes in press or superheat may be realised to a great extent in this type of machine. Hence, higher initial press and degree of superheat are now in more common use than formerly believed possible. For the same reason, exhaust pressures have been lowered to within a fraction of a pound of abs vacuum.

In Fig 29 (*b*), *ABCD* represents the work of a cycle for saturated steam between certain press limits, and *ABCC'D'* for steam with 300° of superheat and the same press limits, the area *DCC'D'* denoting the gain in work. Efficiencies for the 2 cases are *ABCD* ÷ *EABCF* and *ABCC'D'* ÷ *EABCC'F'*. It is obvious that superheat causes a gain in work, but thermally with but little increase in efficiency. In practice, however, the smaller leakage and heat transmission losses generally produce a greater improvement than would be expected from the cyclic effect.

In Fig 29 (c), cycle *ABCD* is for saturated steam with adiabatic expansion, and *ABCD''* represents a case with jackets imparting heat enough to the steam to cause it to remain saturated during expansion. Other cases, in which insufficient or more than sufficient heat is added during expansion to produce continued saturation, are possible, but ideally all show a loss in thermal efficiency and in practice either a loss or such a small gain as generally to render the use of jackets inadvisable. Cycle *ABCxyzD'* represents the case of reheating after partial expansion, a process, if used at all, occurring between the cylinders of a multiple-expansion engine, or between an engine and a low-press turbine. Point *X*, to which expansion is allowed to proceed, is arbitrary, as is also the reheating temp. A common practice is to make the receiver pressure *X* such that equal work is done in each expansion. Reheating can not well be carried above the original temp. As with jackets, the gain due to reheating is either negative or too small to warrant its use on engines. Efficiencies for the 3 cycles in Fig 29 (c) are *ABCD* ÷ *EABCDF* for the original, *ABCD''* ÷ *EABCD''F''* for jackets, and *ABCxyzD'* ÷ *EABCxyzF'* for reheating.

Example 26. Find the cyclic effic, water, and steam consumption of an engine operating on initial press of 140 lb abs, with 100° superheat and back press of 10 lb abs. Also find the jet velocity for same press drop, and the m e p.

From the steam tables (Art 12) the heat at beginning of expansion is 1 248 B t u and that originally in the liquid at 10 lb abs (= 193° F) is 161 B t u. The quality after expansion is found as follows.

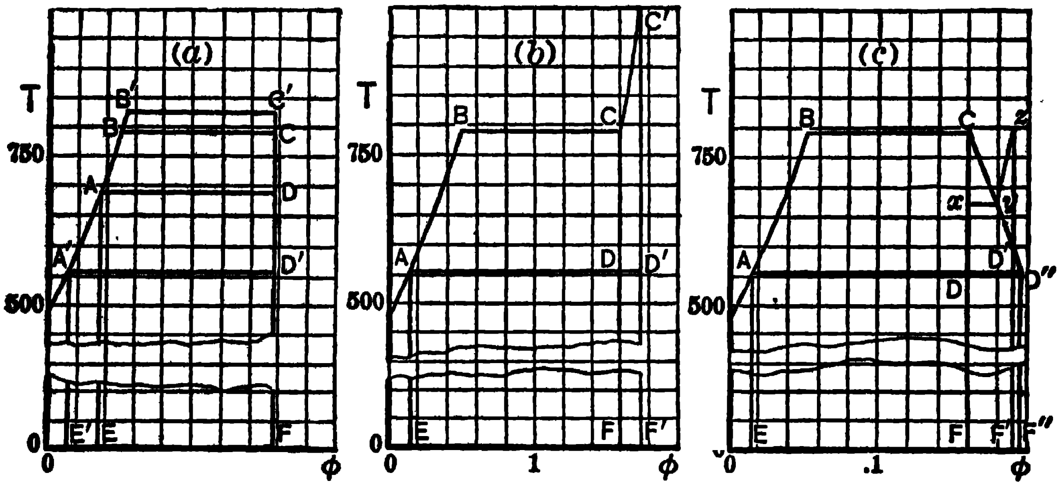

Fig 29. Rankine Cycles Showing Effect of Initial and Back-Press Changes, Superheat, Reheat, and Jacketing

The entropy originally (Table 25) is 1.6395, and since expansion is adiabatic the final entropy must have same value. The total entropy of steam is made up of 2 parts if wet, or 3 if superheated, precisely as is the total heat. If for any press the entropy is less than the dry saturated value for that press, the steam is wet and the missing entropy must be part of that of evaporation; the percentage of moisture is the amount of missing entropy divided by the entropy of evaporation for the given press. Entropy for dry steam at 10 lb is 1.7874, hence the amount missing is 1.7874 − 1.6395 = 0.1479. As the entropy of evaporation for this press is 1.5042, the percentage of moisture is 9.8, or dryness is approx 90; hence, the total heat is 1 045 B t u. The final quality could be read directly from a temp-entropy or a Mollier diagram (Art 15).

By Table 32 the effic will be (1 418 − 1 045) ÷ (1 240 − 161) = 18.7%. Steam consumption will be 2 545 ÷ (1 248 − 1 045) = 12.5 lb, and jet velocity

$$= \sqrt{(1\,248 - 1\,045) \times 778 \times 64.4} = 3\,190.$$

Final quality being 90%, the final vol will be 90% of the tabular, or 0.9 × 38.4 = 34.6 cu ft; hence, m e p = 778 (1 248 − 1 045) ÷ 144 × 34.6 = 37 lb. Same results could be obtained with practically no calculation by the Mollier diagram, and this example is worked by that method in Art 15.

17. CONDITIONING OF AIR

Working conditions in deep mines, where temp is high, may be improved by the cooling action of the air supplied (Sec 14). The heat that can be absorbed by air is a function of change in its temp and moisture content; that is, its relative humidity (Sec 23, Art 1). In computing these changes, the customary unit is a mixture containing 1 lb of dry air. Heat and moisture content for relative humidity at any temp may be computed from humidity tables (19); or, for temp above 32° F, from steam tables (Art 12). Fig 30 shows a graphical solution. For saturated conditions, the moisture content is given in the tables; for incomplete saturation, it is the amount for saturated air at the temp for which the vapor pressure, divided by vapor pressure at given temp, gives a value equal to the relative humidity. That is, moisture content for air 50% saturated at

64° F is the same as for saturated air at 45° F; since 0.3 in of mercury, the vapor pressure at 45° F, is 50% of 0.6 in mercury, the vapor pressure at 64° F.

The heat content of mixed air and moisture is considered to be the heat necessary to warm the air from some base point, usually taken as 0° F, plus heat required to evaporate the moisture present, evaporation being assumed to occur at saturation temp. The heat to warm the water and superheat the vapor are generally neglected, since it is very small compared with the other quantities. Heat and moisture absorbed or given up, when air changes from one temp and relative humidity to another, are merely the differences between these values at original and final conditions.

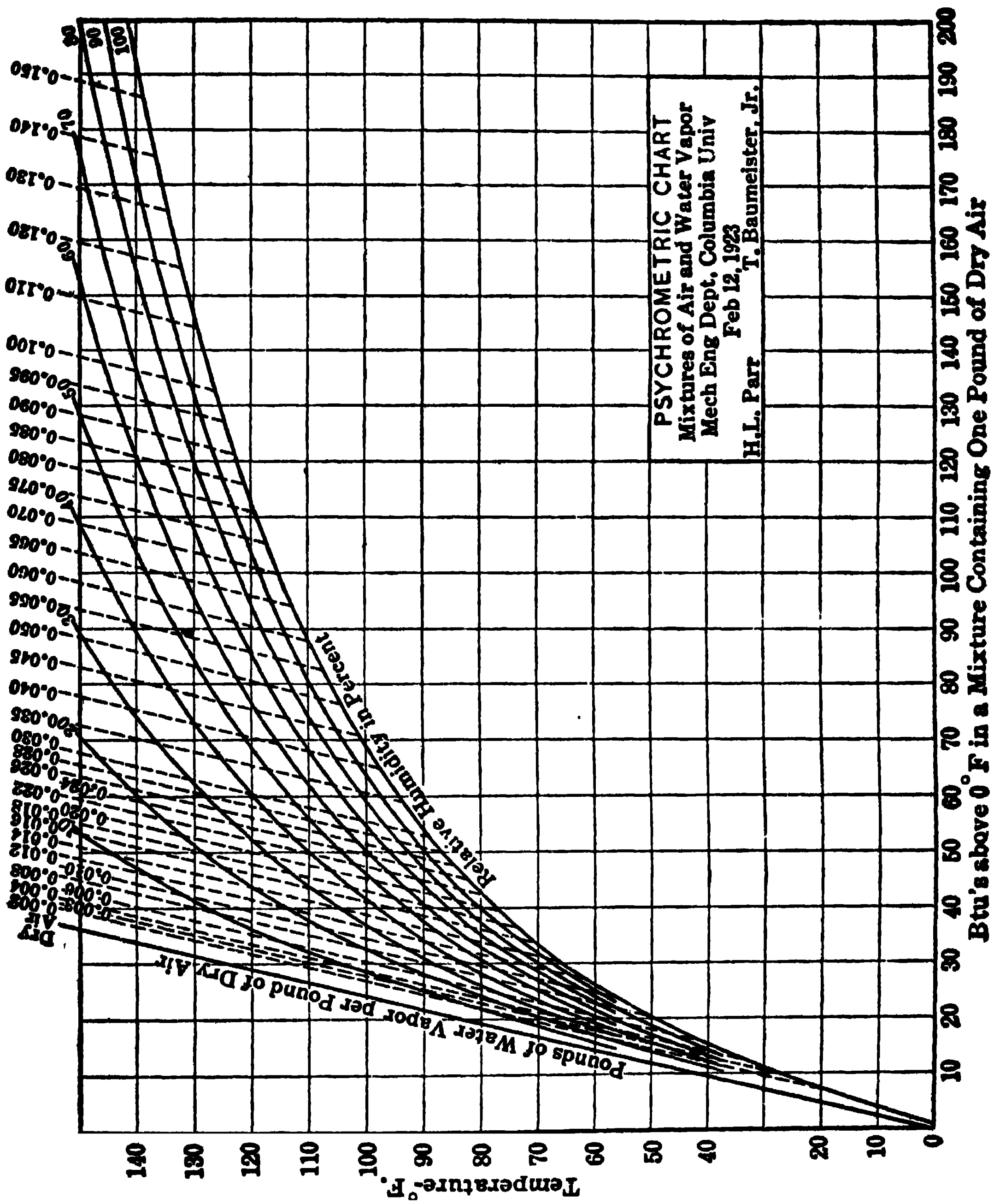

Fig 30. Psychrometric Chart

Example 27. How much heat and moisture must be removed from saturated air at 90° F, to bring it to 40° with a relative humidity of 30%?

Moisture and heat content for original condition are read directly in the tables (19), as 0.0311 lb and 54 B t u, respectively, for a mixture containing 1 lb dry air. Vapor press at 40° F is 0.25 in mercury; hence press at saturation is 0.25 × 0.3 = 0.075 in mercury, and the corresponding temp, 13.5° F. At this temp the moisture content is 0.0015 lb. Heat content of final air is 0.24 × 40 + 0.0015 × 1 260 = 11.5 B t u. Hence, 0.0311 − 0.0015 = 0.0296 lb of moisture, and 54 − 11.5 = 42.5 B t u, which must be removed per lb dry air.

Example 28. How much heat and moisture could be absorbed if the air warms to 70° F, with 60% humidity? Vapor press at 70° F is 0.74 in mercury; hence, saturation press is 0.6 × 0.74 = 0.444 in mercury, for which the temp is 55.5° F. Moisture content for final condition is thus 0.0093 lb, and heat content 0.24 × 70 + 0.0093 × 1 075 = 26.8. Hence, there will be absorbed 0.0093 − 0.0015 = 0.0078 lb moisture, and 26.8 − 11.5 = 15.3 B t u per lb dry air.

Values for moisture and heat content in above examples may all be read directly from Fig 30, in which the vert scale is temp and the horiz, B t u. Solid curves represent % relative humidity; dotted curves, lb moisture in a mixture containing 1 lb dry air. By projecting over from any temp on the vert scale to a given relative humidity curve, the heat content may be read on the horiz scale and the moisture content on the dotted line passing through point of intersection. For the final condition in Ex 28 (70° F and 60%), the heat content is 27 B t u; moisture content, 0.0095 lb (approx) as computed. The method of computing heat and moisture content from data in ordinary steam tables is too complicated to be given here.

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SECTION 40

POWER AND POWER MACHINERY

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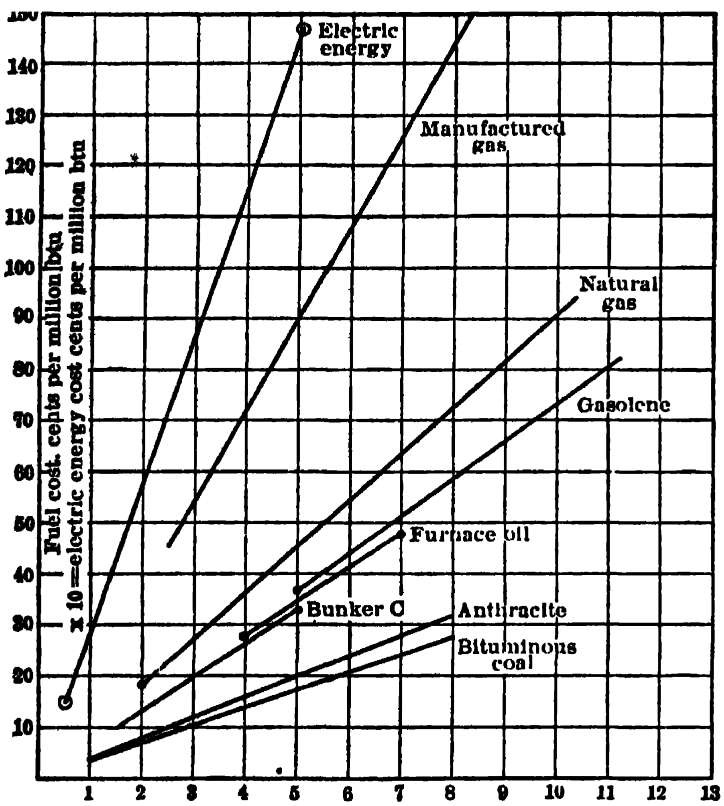
POWER AND POWER MACHINERY

1. POWER SYSTEMS

Extent of requirements of the mining industry for adequate and economic sources of power is indicated in Table 1. Power service can be: (1) purchased from a central station; or (2) generated in a self-contained plant. Basic sources of energy are: (*a*) an elevated water supply; or (*b*) heat-energy of fuel. Plants include: (1) water power; (2) steam, utilizing any type of fuel; (3) internal-combustion engines, requiring oil or gaseous fuels; (4) waste heat. Stationary plants are chiefly represented by steam and hydro; portable plants are mostly internal combustion (Table 2a).



Fuel prices. Dollars per short ton for all coals
Cents per gal, gasolene, furnace oil, bunker C
Cents x 10 per MCF, natural and illuminating gas
Cents per kw-hr, elec energy

Fig 1. Fuel Costs

Fuel-burning systems are of major importance. Continuity and reliability of fuel or water supply and fuel cost control selection. Condensing steam plants typically require 500 to 1 000 lb circulating water per lb coal burned. Steam plant site and cycle selection are dictated by circulating water adequacy. Internal-combustion plants do not have this limitation. With any fuel-burning installation, fuel cost is the largest single operating item. Comparisons between fuels are best made on basis of cost in cents per million btu. Fig 1 aids conversion to this basis. Typical competitive fuel prices are shown in Table 3, with bunkerage data. Prices given are for heat in fuel, and must be modified by various efficiencies to determine their worth at point of utilization.

Thermal performance of fuel-burning plants. By the laws of thermodynamics, heat can be converted into work only by sacrifice of availability. Mechanism inefficiency, coupled with cyclic limitations, result in low overall thermal effic (Table 4). Heat rate is more directly usable than thermal effic, and is defined as:

$$\text{Heat rate, btu per kw-hr} \quad \frac{\text{Heat input in fuel, btu}}{\text{Energy output, kw-hr}} = \frac{3\,412.75}{\text{Thermal effic}} \times 100$$

Minimum heat rates in steam plants are obtained only with the largest equipment sizes and max press and temperature. Internal-combustion engines give good heat rates, even in smallest sizes. Fuel price, cents per million btu, can be multiplied directly by heat rate, to give fuel cost per kw-hr generated.

Table 1. Power Requirements of Typical Mining and Metallurgical Operations

Operation	Kw-hr per ton	Notes to Bib at end of Section
Aluminum smelting and refining	24,000	Fed Power Comm
Anthracite mining	15–25	T. D. Lewis
Bituminous coal mining	4–6 average 3–16 extremes	Jackson and Grow, *Coal Age*, Jan, 1937
Cement mills	75–100	
Copper smelting and refining	370	Fed Power Comm
Copper mining	2–5	
" milling	5–15	
Gold mining	20–30	
" milling and refining	15–25	
Lead and zinc mining	4–77	Bur of Mines, *Bull* 381, 1935
Limestone mining	4–5	S. M. Shallcross
Lime plant	50	S. M. Shallcross, per ton of lime
Ore mining:		
Open stopes	1–35	Bur of Mines, *Inf Circ* 6785
Shrinkage stoping	1.6–38	" " " " "
Cut and fill stoping	9–78	" " " " "
Square-set stoping	9–86	" " " " "
Undercut block caving	2–3	" " " " "
Sublevel caving	6–16	" " " " "
Top slicing	2–16	" " " " "
Sulphur	10	E. C. Meagher, per ton of sulphur, elec operations only

Table 2. Total Installed Horse Power of Prime Movers in U S, 1936

	H p		H p
Electric central stations	44 670 000	Automotive plants	965 000 000
Industrial power plants	20 130 000	Airplane plants	3 500 000
Electric RR plants	2 500 000	Locomotive plants	88 000 000
Isolated non-industrial plants	1 500 000	Marine plants	30 000 000
Mine and quarry plants	2 750 000		
Agricultural prime movers	72 760 000	Total	1 230 810 000

Distribution of Prime Movers in U S (1935)

Service	Steam	Internal combustion	Hydro
Transportation	8%	92%	....
Stationary	76%	2%	22%

Table 2a. Distribution of Annual Energy Generation Among Plant Types in U S (1935)

Service	Annual generation, kw-hr
Transportation	150×10^9
Stationary, total	142×10^9
Steam	100×10^9
Hydro and internal combustion	42×10^9
Total, U S	292×10^9

Table 3. Typical Competitive Fuel Prices for a Specific Market

Fuel	High heating value, btu	Unit price	Cost, cents per million btu	Wt per million btu, lb	Space per million btu, ft³
Bituminous coal	14 500 per lb	$3 per ton	10	69	1.25
Anthracite	12 500 "	$4 "	12	80	1.33
Furnace oil	19 500 "	5¢ per gal	35	51	0.92
Bunker C fuel oil	18 500 "	3¢ "	20	54	0.89
Gasolene	20 000 "	10¢ "	73	50	0.95
Natural gas	1 000 per ft³	50¢ per M cu ft	46	36	910 (NTP)
Manufactured gas	550 "	50¢ " "	90	72	1 820 (NTP)
Elec energy	3 413 per kw-hr	2¢ per kw-hr	585	..	.....

Loads and load curves. Power plants can not be properly designed or operated without considering the load and its hourly, daily, seasonal, and annual variations. Fig 2, 2*a*, 2*b* give typical daily load curves for a coal mining plant; ANNUAL LOAD DURATION CURVE for same data is in Fig 3, the latter curve showing: (1) demand of 50 kw exists for 8 760 hr; (2) peak is 920 kw; (3) annual energy generation is the area under the curve, 2 650 000 kw-hr; (4) area divided by the product of peak and the 8 760 hr of the period is the annual load factor, 33%. LOAD FACTOR is defined by the relation: Load factor = average load ÷ peak load, and covers a definite operating period, as a day or year. Both loads are net delivered outputs. Peaks are not instantaneous, but generally taken over a 30-min interval. CAPAC FACTOR is defined by the relation: Capac factor = gross generation ÷ plant capac × hr in period, and is usually expressed as an annual amount. It shows the extent to which installed capac is used, bearing no relationship to load factor. Equipment reliability is measured by the

Table 4. Typical Thermal Performance of Fuel-burning Power Plants

Type of plant	Heat rate, btu per kw-hr	Thermal effic, %
Public utility aver, steam, 1935	19 000	18
Industrial aver, steam, 1935	39 000	9
All stationary steam, aver, 1935	27 000	13
Best record, large utility steam plant, 1937	10 800	31
Condensing steam plant, overall	20 000	17
Non-condensing steam plant, overall	35 000	10
Diesel plant, overall	11 500	30
Natural gas-engine plant, overall	13 500	25
Gasolene engine plant, overall	16 000	21
Producer gas engine plant, overall	18 000	19

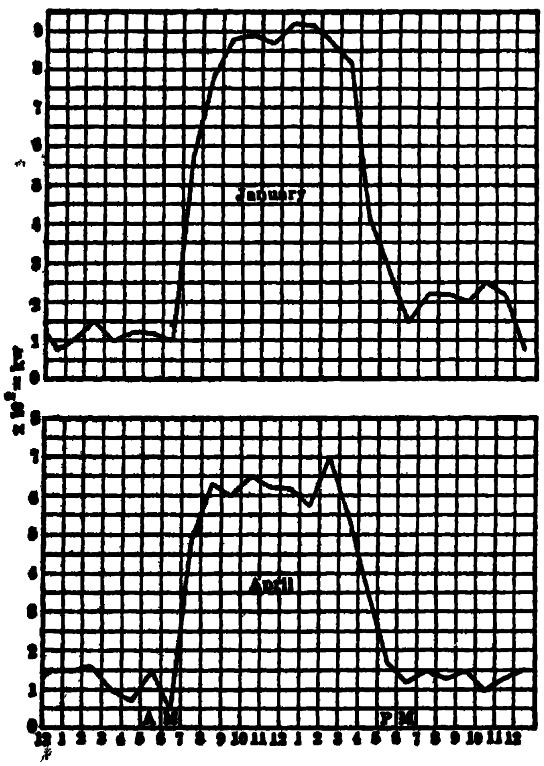


Fig 2. Typical Daily Load Curves, Coal Mine

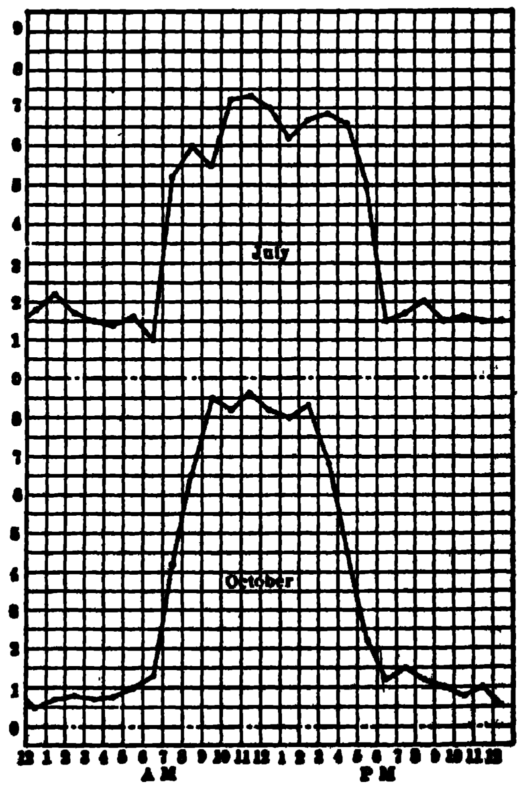


Fig 2*a*. Daily Load Curves, Coal Mine

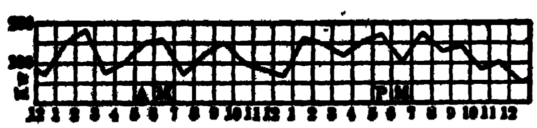


Fig 2*b*. Typical Load Curve, Coal Mine, Saturdays, Sundays and Holidays

AVAILABILITY FACTOR, defined by the expression: Availability factor = Annual hr operated or operable ÷ 8 760. It is best applicable to specific equipment within a plant. For modern power machinery it exceeds 85%, with good maintenance and operating technique. Equipment reliability reflects the need for spare capac to provide continuity of service. While reserve factors may be of some worth, it is more rational to install spare capac at least equal to

largest unit in the plant. For large plants this often offsets the merits of lower unit first cost and operating economy. Cold, hot, and spinning reserve are all affected.

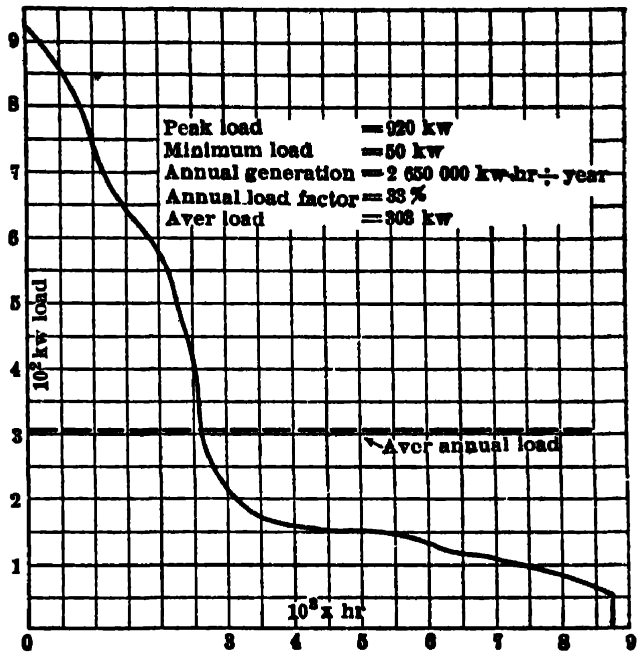


Fig 3. Typical Annual Load Duration Curve, Coal Mine

2. COST OF POWER

Total cost is the sum of: (1) fixed charges; (2) operating charges or production expense. FIXED CHARGES include: (1) cost of money; (2) insurance and taxes; (3) depreciation; and are usually computed as an annual percentage of the investment. COST OF MONEY includes interest, discounts, and financing charges, and is not under control of the designer. It ranges from 5 to 10% per annum. INSURANCE varies with the nature of the risk and embraces fire, boiler explosion, machinery breakdown, public liability, and workmen's compensation. TAXES cover real estate, franchise, incorporation, receipts, income, and social security. Annual charges for insurance and taxes are from 1 to 3%. DEPREC charges reflect loss in value of equipment due to use and passage of time, and result from physical deterioration, obsolescence, change in use, and inadequacy of equipment. While physical deterioration can be largely offset by proper maintenance, functional deprec is beyond control. Replacement of equipment will ultimately be required. Deprec reserves should therefore be accumulated on some convenient basis, as straight line, sinking fund, or observed deprec. Table 5, 6, 7 give depreciable life data; annual charges are 1–10%. The usual aggregate annual allowance for all FIXED CHARGES on power plant equipment is 10–20%; commonly 12.5% for steam plants. In case of short-lived enterprises, industrial undertakings, or possible depletion of ore reserves in mining, fixed costs are often computed on other bases.

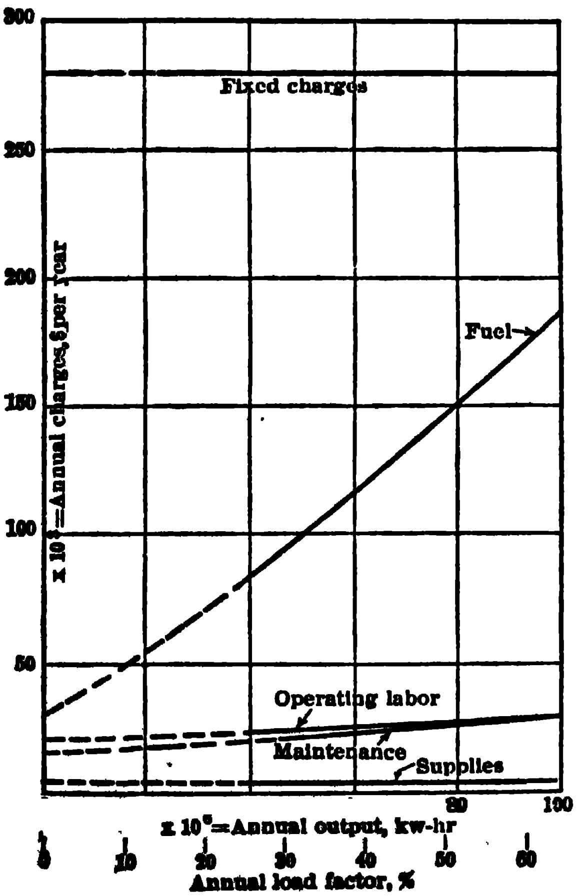


Fig 4. Annual Cost of Generation; Small Steam Plant

Operating charges or production expense (see also Sec 21) vary with output and load factor and are classified as: (1) fuel; (2) labor and supervision; (3) maintenance, materials and labor; (4) misc supplies and expense. FUEL COST is the largest item, and varies with heat rate, fuel price, load factor, and operating practice. There is one best effic point for any plant, which is generally approximated in design to occur at "average" load (Fig 4). OPERATING LABOR is largely constant, irrespective of plant load. A straight line, with an intercept equal to 50–100% of the full load cost, is representative. MAINTENANCE follows a characteristic similar to labor, but varies so widely in

different plants as to preclude statistical comparison. For preliminary estimates, annual maintenance can be taken as 1–3% of investment cost. MISC SUPPLIES can be treated as a constant charge of 5–15% of the maintenance or labor cost.

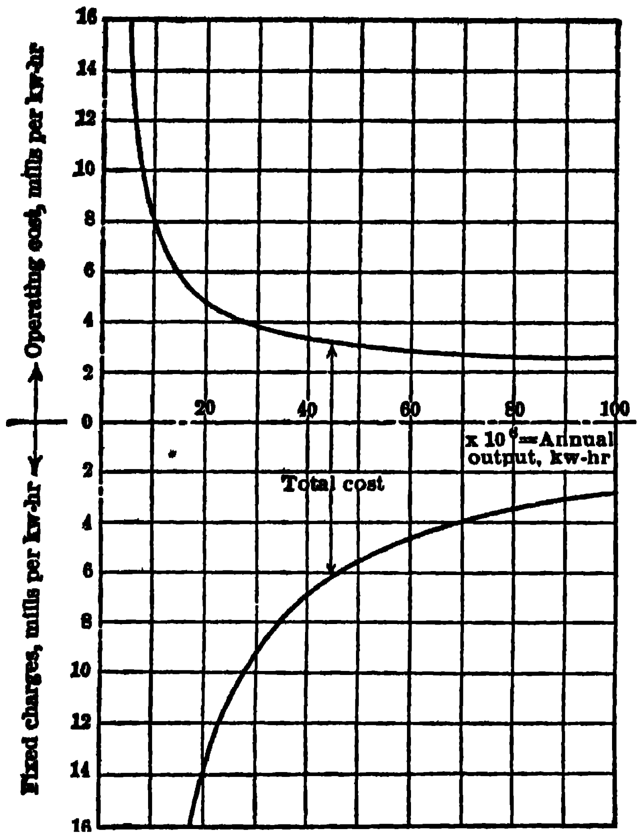


Fig 5. Unit Cost of Generation, Small Steam Plant

Total cost of power. The annual costs of generation are plotted for a small steam plant in Fig 4, and the results transferred to a two-quadrant plot for unit costs in Fig 5. Total cost of generation (Fig 6) reflects a constant portion or intercept, regardless of output. The cost of service can be approximated by the relation: Annual cost of service = $K_1 + K_2$ (kw-hr output per kw capac) $= K_1 + K_2$ (8 760) (capac factor). More accurate equations in 3 or 4 parts, can be written to reflect: (1) capac charge; (2) peak load; (3) hrs of service; (4) energy charges. Such relations show the small significance of direct comparison of unit costs for power. Table 8 gives production costs for several power plants; their values are illustrative, and should be used cautiously for comparison with data from other sources.

Cost of power at point of use. Distinction must be made between cost of generation at the power plant and cost of service at point of use. Fuel transport, elec transmission lines, comp air or water lines, all add to cost of energy. All forms of power plant are not equally suitable from this viewpoint. Waterpower plants must be located at the hydro site; steam plants, if condensing, must have adequate circulating water; internal-combustion engines require little water and may be portable.

Purchased *vs* generated power. Purchased power from a central station is usually the best basis for measuring the worth of any proposed plant. Reliability of service and rate schedules should be studied to assure continuity and economy of service. Purchased power generally minimises capital investment on the part of the industrial plant and results in a short term commitment. Rate schedules are usually based on cost equations, as given above, to reflect a demand plus an energy charge. Discounts are often made for prompt payment of bills; high power-factor loads; off peak power; and secondary service. Special contracts and rates can also be obtained under favorable conditions.

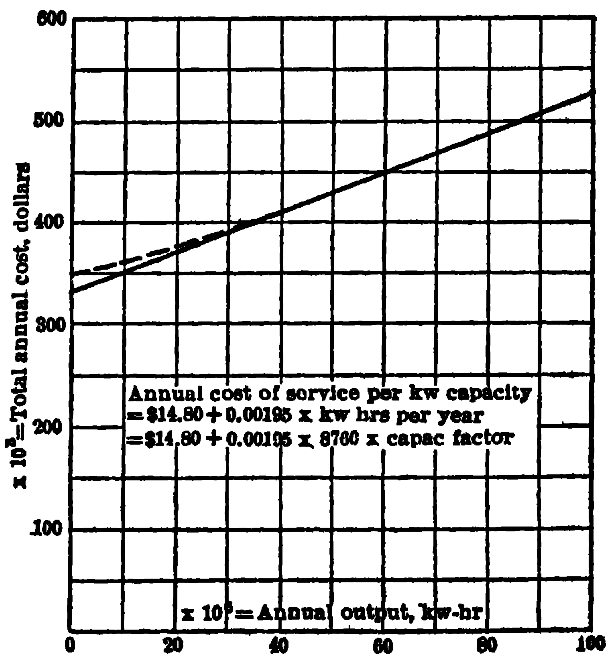


Fig 6. Total Cost of Generation, Small Steam Plant

Table 5. Probable Life of Structures and Apparatus for Steam Power Plants
(Justin and Mervine, Power Supply Economics. Pub, Wiley)

Description	Years	Description	Years
Accumulators	15	Foundations	Same as life of equipment supported
Boilers, water-tube	20	Fuel oil handling equipment	20
Boiler accessories	20	Furniture and fixtures	15
Breechings, steel	10–30	Feed-water heaters	20
Buildings:		Pipe and pipe covering	15–25
Brick	30	Pumps, reciprocating	15–20
Wood or wood frame	20	Pumps, centrifugal	20
Cables and feeders	15–25	Stacks, brick or concrete	30
Coal and ash machinery	20	Steam turbines	20
Compressors, air	20	Steel	12–15
Condensers	20	Stokers and other fuel burning equipment	20
Cranes	30	Superheaters	20
Economizers and air preheaters	15	Switchboards and their equipment	20
Electric generators	20	Tools and shop machinery	15
Electric motors	20	Transformers	15
Engines, small steam	15		
Fences	12		

Table 6. Probable Life of Structures and Apparatus for Hydro-electric Plants

	Years		Years
Concrete structures	*	Miscellaneous power house equipment	25
Cranes and hoists	60	Pipe lines, concrete	40
Dams, concrete	*	" " steel	20
" earth	*	" " wood	20
" timber	30	Power house, substructure	*
Flumes, concrete	20	" " superstructure	50
" steel	40	Screens and rakes	50
" wood	10	Steel structures, exposed	40
Gates	50	Transformers	15–25
Generators	50	Water wheels	50

* These items kept in perpetual usefulness by proper maintenance.

Table 7. Probable Life of Apparatus for Internal-combustion Plants

Item	Years	Item	Years
Automobiles	5	Diesel engines:	
Batteries:		Slow speed	15–20
Storage, stationary	10	High speed	5–10
Storage, automotive	1–2	Fuel oil systems	20
Blowers and fans	20	Gas engines	20
Cooling towers	15	Gas producers	15
Compressors:		Gas supply systems	20
Stationary	20	Gasolene engines, high speed	5
Portable	5	Tanks, steel	25

Table 8. Selected Power Plant Production Costs

Plant	A	B	C	D	E	F	G	H
Plant type	Steam central station	Steam central station	Steam industrial	Steam industrial	Hydro	Hydro	Diesel	Diesel
Plant capacity, kw	300 000	100 000	5 000	1 000	100 000	15 000	1 000	3 000
Load factor, %	53	75	60	40	75	50	45	40
Capacity factor, %	43	70	50	30	65	40	40	30
Fuel cost, ¢ per million btu	15	6	20	20	..	..	35	35
Production cost:								
Fuel, mills per kw-hr	2.1	0.72	5.0	8.0	....	...	4.0	4.5
Operating labor, mills per kw-hr	0.3	0.2	0.4	3.0	0.2	0.5	2.0	2.4
Maintenance, mills per kw-hr	0.4	0.3	0.8	2.0	0.15	0.2	1.8	0.5
Supplies, mills per kw-hr	0.2	0.1	0.3	1.5	0.1	0.4	2.1	0.7
Total, mills per kw-hr	3.0	1.32	6.5	14.5	0.45	1.1	9.9	8.1
Heat rate, btu per kw-hr	14 000	12 000	25 000	40 000	....	...	11 500	13 000

3. STEAM POWER CYCLES

Heat balance diagrams (Fig 7) give data on fuel, steam, water, and energy for each item of equipment. Preparation of such diagrams involves the operating performance, characteristics, costs, and limitations of all the machinery. Fig 7 and Table 9 give data

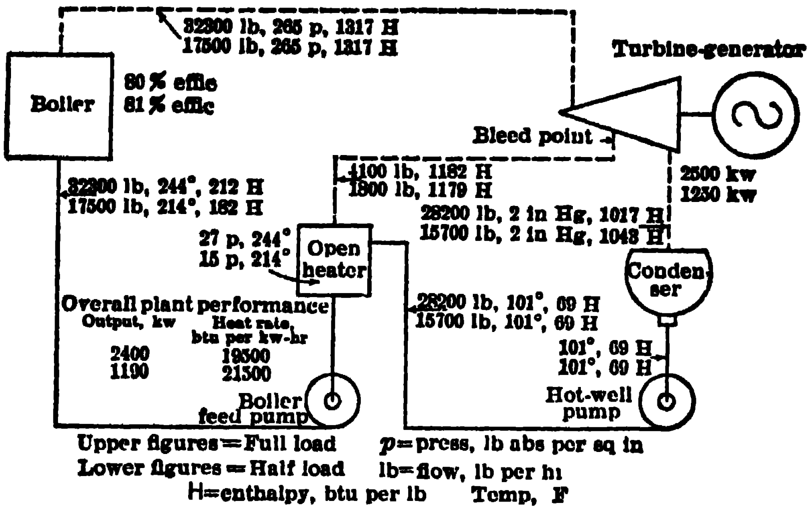


Fig 7. Heat Balance Diagram, 2 500-kw Unit

for an elementary plant containing one 2 500-kw turbine-generator, using steam at 265 lb and 600° F, exhausting at 2 in abs, and with extraction feed heating. Calculations depend on the turbine state lines of Fig 8, using 75% Rankine effic ratio, and plotted on a

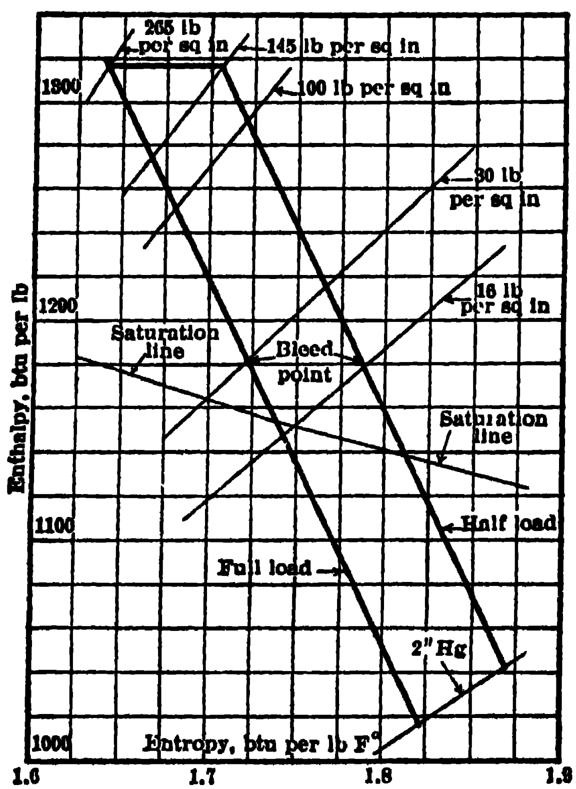


Fig 8. Turbine State Lines, 2 500-kw Unit

Mollier chart (Sec 39, Fig 26). Overall thermal performance includes allowances for generator effic = 95%; boiler effic = 80%; elec-driven auxiliaries = 4%; plant realization ratio = 96%. While elec drive gives favorable cost and operating economy for auxiliaries

spare equipment, as steam-driven feed pumps, should be provided. The diagram in Fig 7 may be extended to include other items, as stage heating, air ejectors, gland condensers and generator air coolers.

Table 9. Heat Balance Calculations, 2 500-kw Unit (see also Fig 7)

Load	Full	Half
Condenser:		
Flow, lb	28 200	15 700
Pressure, inches Hg	2	2
Hot-well temp, Fah	101	101
Open heater:		
Turbine shell press, p	30	16
Press drop through line, p	3	1
Heater press, p_0	27	15
Heater temp, Fah	244	214
Enthalpy, water in, H	69	69
Enthalpy, water out, H	212	182
Heat added, H	143	113
Water flow to heater, lb	28 200	15 700
Heat added, btu per hr $\times 10^6$	4.05	1.77
Enthalpy bleed, H	1 182	1 179
Heat available, H	970	997
Extraction required, lb	4 100	1 800
Boiler feed, lb	32 300	17 500
Main turbine:		
Enthalpy at throttle, H	1 317	1 317
Enthalpy at bleed point, H	1 182	1 179
Heat drop to bleed point, H	135	138
Flow to bleed point, lb	32 300	17 500
Generation to bleed point, kw	1 270	700
Enthalpy at exhaust, H	1 017	1 043
Heat drop to exhaust, H	165	136
Flow to exhaust, lb	28 200	15 700

Load	Full	Half
Main turbine (*continued*):		
Generation from bleed point to exhaust, kw	1 360	630
Gross turbine generation, kw	2 630	1 330
Generator efficiency, %	95	94
Output at gener terminals, kw	2 500	1 250
Throttle water rate, lb per kw-hr	12.9	14.0
Condenser water rate, lb per kw-hr	11.3	12.5
Boiler:		
Enthalpy at superheater outlet, H	1 317	1 317
Enthalpy at feed, H	212	182
Heat added, H	1 105	1 135
Steam made, lb	32 300	17 500
Heat added, btu per hr $\times 10^6$	35.8	19.9
Boiler efficiency, %	80	81
Heat required in fuel, btu per hr $\times 10^6$	44.8	24.5
Gross plant heat rate, btu per kw-hr	17 900	19 600
Auxiliary power, %	4	5
Net plant sendout, kw	2 400	1 190
Plant utilization ratio, %	96	96
Net plant heat rate, btu per kw-hr	19 500	21 500

4. BOILERS AND THEIR APPURTENANCES

Boiler room must be carefully designed, to reduce power cost. No other part of the plant can accumulate losses so rapidly from poor effic or maintenance. This concerns both the combustion chamber (furnace) and the boiler proper, which are constructed under rigorous laws and codes, such as that of Am Soc of Mech Engrs, regulations of boiler insurance companies, and local laws.

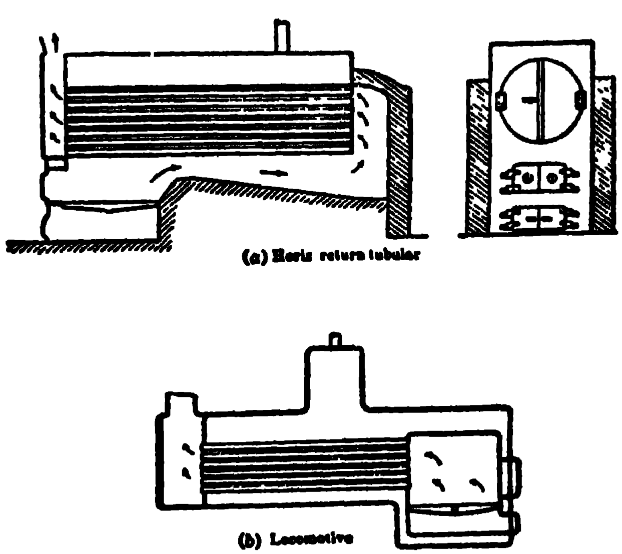
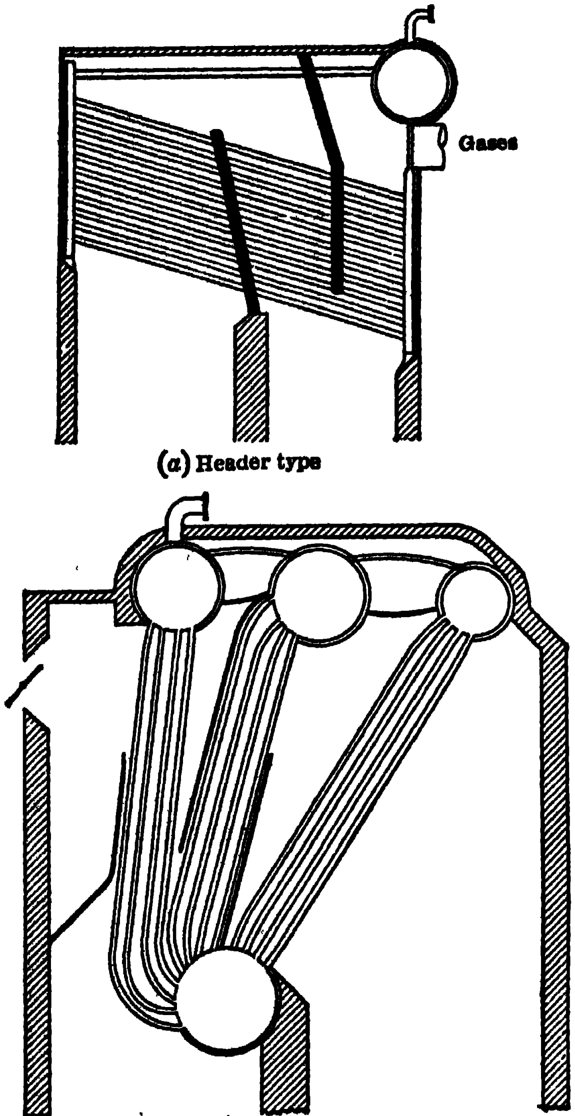


Fig 9. Fire-tube Boilers

Chief objectives: high steaming capac per sq ft, low first cost, and high effic; but high capac and high effic are directly opposed, because the former requires high mean temp differences; the latter, low mean temp differences. Max steaming capac and effic can not therefore be expected in the same boiler; in heat transfer there is a complicated thermal path through gas film, soot, metal, scale, and water film. As the relative thermal resistances of metal, water and gas are as 1 : 100 : 1000, the controlling resistance, with clean surfaces, is on the furnace gas side. Vigorous circulation, scrubbing, mixing, and high mean hydraulic depth are needed on the gas side to promote heat transmission. On the water side, the metal surface must be kept wet, to prevent a shift in the controlling resistance and consequent overheating

of the metal (steel loses its structural strength rapidly with increase in temp above 800° F). Thermal action due to the difference in density of water and steam provides necessary circulation. Surfaces must be kept clean of scale further to prevent overheating, and clean of soot to maintain effic.

Boilers are either fire-tube, in which the gases pass through the tubes, or water tube, in which the water flows through the tubes. FIRE-TUBE BOILERS comprise a nest of straight 3–4 in tubes, between the heads of the shell; they are externally fired as in Fig 9*a* (horiz return tubular), or internally fired as in Fig 9*b* (locomotive); the former require brick setting, the latter are self contained and portable; both are limited to low press and low capac services (less than 250 lb per sq in and 250 boiler hp); are low in first cost, but can not be operated at high ratings with sustained effic. WATER-TUBE BOILERS (Fig 10) are header or drum types. The headers, into which the tubes are rolled, may be of box or sectional design; the former are suitable for press below 400 lb per sq in and require stays for bracing; the latter have only one vert row of tubes, which, when made sinuous, allow close spacing and good gas mixing. Hand holes give access to tubes for cleaning. Tubes are limited to 20–24 ft length. Larger capac is obtained by increasing the number of vert sections. Drums are horiz or slightly inclined. Cross drums (Fig 10*a*) permit of high capac with one drum; for the longit arrangement, 2 or 3 drums are needed. Drum size is determined by disengaging surface requirements. Hot gases circulate over the tubes in a 1, 2, or 3-pass construction (Fig 10*a*), but longit baffling may be used instead of cross baffling. DRUM BOILERS, having no headers, cost less and the curved tubes permit a variety of designs. Tubes are spaced farther apart than in header boilers, to allow tube replacement. There are no hand-hole covers. Some water-tube boilers produce steam at rate of 1 000 000 lb per hr in a single unit and at press of 1 400 lb.



(*a*) Header type

(*b*) Drum type

Fig 10. Water-tube Boilers

Superheaters may be of the convection type in the gas pass, or the radiant type in the furnace wall. With the superheater at top of the first pass (Fig 10*a*) the final superheat temp increases with boiler output; in the radiant position, steam temp decreases with increase in load. Internal boiler dampers, interdeck positions, and convection-radiant series arrangements allow of superheat control. Steel is suitable for steam temp below 750° F, but alloy steels are needed for sections exposed to higher temp (950° F max).

Fuels are mixtures of C and/ or H, either as elements or in combination. A fuel burner and its furnace must bring the necessary O molecules from the air supply into contact with the combustible elements. Table 10 gives analyses of power-plant fuels, their heats of combustion and chemically required air. The air is estimated for coals and fuel oils by the relation:

$$\text{Lb air chemically required per lb of fuel} = \frac{\text{High heating value, btu per lb}}{1\,300}$$

No furnace can give perfect combustion; excess air for fuels containing negligible amounts of nitrogen is calculated from analysis of the combustion products by the equation: Excess air,

$$\% = \frac{3.78\left(O_2 - \frac{CO}{2}\right)}{N_2 - 3.78\left(O_2 - \frac{CO}{2}\right)} \times 100,$$

where O_2, CO, and N_2 are vol percentages in the dry flue gas. Fig 11 shows relationship between CO_2 and excess air for several fuels. The wt of products is determined by adding to the wt of fuel actually converted into the gaseous state, the wt of chemically-correct air and excess air. Table 11 gives excess air performance figures, together with firing rates, for different furnaces. For chemistry of combustion, see Sec 39.

Gaseous fuels and furnaces. Gas is the ideal fuel. Flame length is controlled by the proportions of primary and secondary air. Ash

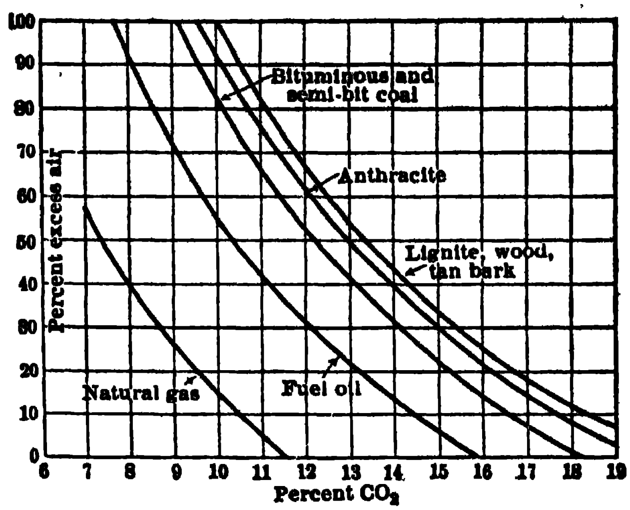


Fig 11. CO_2 and Excess Air in Products of Combustion of Typical Fuels

Table 10. Analyses of Some Typical Fuels

Fuel	Coals								
	VM	FC	Moist	Ash	H	C	O+N+S	High heating value, btu per lb	Lb air per lb fuel
Lignite	17	35	39	9	2	36	14	6 500	4.8
Sub-bituminous	31	38	22	9	3	54	12	8 800	7.2
Low rank bituminous	37	42	11	10	4	65	10	11 600	8.8
Middle rank bituminous	37	50	5	8	4	75	8	12 800	10.0
High rank bituminous	29	60	3	7	5	77	8	14 000	10.6
Low rank semi-bit	22	69	3	6	5	79	7	14 600	10.8
High rank semi-bit	13	76	5	6	4	79	5	14 500	10.5
Semi-anthracite	9	77	6	8	3	79	3	13 700	10.2
Anthracite	2	85	3	10	2	82	3	13 000	10.2
Coke	..	..	..	13	..	86	1	12 500	10.0

Fuel	Liquids								
	H	C	S	N	O	H_2O	High heating value, btu per lb	Sp gr, deg API	Lb air per lb fuel
Bunker C	11.5	85	1.5	0.5	1.0	0.5	18 500	14–18	13.7
Diesel fuel	13–13.5	85	1.5–2.0			....	19 000–19 500	22–28	14.3
Furnace oil	13.5	85	1.5				19 500	30–35	14.4
Kerosene	14	85	0–1.0			0.5	19 800	45	14.6
Gasolene	15	85	0–1.0				20 000	55	14.9

Fuel	Gases							
	H	CO	CH_4	HH	O_2, N_2, CO_2	High heating value, btu per cu ft NTP	Sp gr (air = 1)	Cu ft air per cu ft gas
Natural gas	.....	.....	96	2	3	1 100	0.6	10
Coke-oven gas	50	6	32	5	7	600	0.4	5.2
Producer gas	15	26	2	1	56	150	0.85	1.2
Blast-furnace gas	5	23	..	..	72	90	1.0	0.67

problems are absent, unless the gas is impure. Refractory furnace-wall construction is relatively simple.

Oil fuels and furnaces. Fuel oil is generally a refinery residue, as Bunker C or No 6, which has 100–300 seconds Saybolt Furol viscosity, 10°–16° API density; and contains impurities of sand, S, and water. External strainers, and heaters to raise the temp above 150° F, condition the fuel for spraying into the furnace. Steam, comp air, or mechanical atomizing nozzles are used; fine sprays reduce flame length, but some coarseness is desirable for penetration. Air enters in the secondary form, through louvres surrounding the burner tip. Checkerbrick and "bouncing walls" are sometimes used to increase furnace turbulence. The combustion chamber must effectively mix the fuel and air. Straight refractory walls prevail, due to absence of ash and slag. Firing rates are controllable by burner multiplicity, together with regulation of individual burners.

Table 11. Typical Average Furnace Firing Rates

Fuel	Heat liberation rate, btu per hr per cu ft	Excess air, %
Natural gas	20 000–40 000	10– 30
Fuel oil	20 000–40 000	15– 30
Pulverized coal	20 000–40 000	15– 30
Stokers, large	20 000–40 000	20– 40
Stokers, small	15 000–25 000	30– 60
Hand fired, industrial	5 000–10 000	50–150
Hand fired, domestic	less than 5 000	more than 100

Coal and coal-burning furnaces. While coal is the commonest fuel, ash and volatiles cause difficult firing problems. Volatile is driven off rapidly, and must be intimately mixed with enough air while at high temp. Chilling by contact with cool surfaces lowers combustion, and forms smoke and soot. Ash composition and improper ash fusion temp may destroy furnace linings by fluxing with the refractory. Water-cooled or steam-cooled walls, with varying amounts of firebrick facing, may be required. With stationary grates for HAND FIRING the ash must be pulled out through the firing door; it is difficult to keep air spaces clear of ash and clinker. Rocking grates are dumped, or worked by levers at the front of boiler.

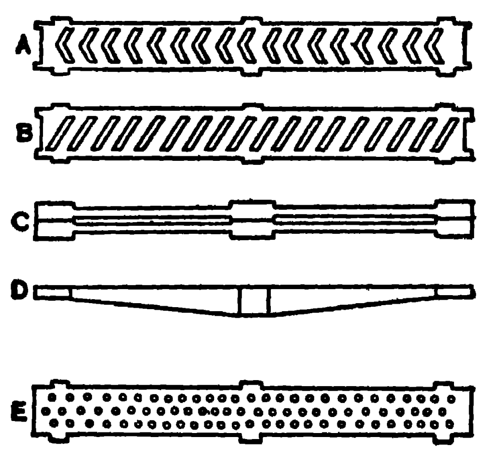


Fig 12. Grate Bars

Grates are divided into 2 or more sections, so that coal may be piled on one part while cleaning another. Fig 12 shows ordinary grate bars: A, herringbone; B, Tupper; C, D, common straight bar; E, pinhole. The bars are about 3 ft long, and arranged in sections to form a rocking grate. Air openings must be suited to size and kind of coal. Table 12 shows sizing standards; uniformity of sizing aids materially in firing. Hand-fired grates are limited in length to the 10–12 ft that a fireman can toss coal, as with anthracite; and 6–7 ft in slicing, as with bituminous. Hand-firing is limited to furnaces burning less than 0.5 ton per hr, and where high effic is not sought. Savings by experienced firemen repay the extra labor cost. Bituminous coal should be spread evenly, in beds 8–10 in thick, and the fire kept without holes, and not be allowed to become dirty, as evidenced by a dark ash pit. If a heavy layer is added, the volatile is driven off without burning; smoke and inefficiency result. Instead of uniform spreading, fresh coal may be piled on a dead plate in the front, where the volatile is distilled, and the resulting coke then pushed over the fire. The coking method is preferable for low fires. The smokeless burning of anthracite is simpler, because of low volatile. Steam sizes (Table 12) form a dense bed, not over 3–4 in thick, to reduce draft losses. Care is needed to maintain uniformity on thin beds, prevent formation of holes, and attain effic firing.

Table 12. Round-hole Openings in Testing Screens for Anthracite (A S T M)

Size	Passing through	Retained on
Broken	4 3/8″	3 1/4
Egg	3 1/4	2 7/16
Stove	2 7/16	1 5/8
Chestnut	1 5/8	13/16
Pea	13/16	9/16
No 1 Buckwheat	9/16	5/16
No 2 Buckwheat (Rice)	5/16	3/16
No 3 Buckwheat (Barley)	3/16	3/32

Tolerances of ±3% on aver openings and 10% in max openings are permissible.

Mechanical stokers give high capacities at sustained effic. Feed is continuous, at controllable rates, with minimum labor, and air is supplied in controlled proportions to

each bed section, so as best to distill, gasify, reduce, or oxidise the fuel. The fire is kept even with reasonable excess air, the furnace being maintained at a high even temp. Ash is automatically removed on completion of combustion. The best stokers accomplish all these purposes; cheaper ones are available, which only partly fulfill the requirements.

While stokers may be designed to give good performance, constant, intelligent supervision in the fireroom is essential. From a hopper in front of the furnace, OVERFEED STOKERS feed raw coal above, UNDERFEED STOKERS, below the incandescent bed. In some small overfeed types the coal is mechanically or pneumatically distributed over horiz or inclined grates; in others, it is deposited in a coking zone or on a dead plate at the hopper end. The coked fuel is then controllably moved down the stepped and inclined grate by oscillation of the grate bars, or is conveyed by a chain or traveling grate toward rear of furnace. In underfeed, coal is fed from the hopper by rams or screws, operating in troughs or retorts, beneath the air admission zone. The rams gradually feed from the cold, through the volatile distillation zone, to the incandescent zone on top. Air enters through tuyeres on the top edges of the retorts, which are horiz or inclined; in the former, there is a single retort with ash removal laterally; in the latter, a multiplicity of retorts, and ash is discharged to the rear. The agitating action of the rams makes the underfeed stoker suitable for caking coals of high ash fusion temp; traveling grates are best for non-caking, non-clinkering bituminous and anthracite. Underfeed stokers carry thick (2-ft) beds with combustion rates controlled by ram speed and draft. Chain grates carry thinner fires (6 in), with control effected by depth of bed, grate speed, and draft. Best results are by forced draft, with combustion rates of 30–40 lb per hr per sq ft of grate area.

Pulverized coal firing requires a grinding plant to reduce the coal to proper size. Standard sieve sizes are given in Table 13; it is usual to specify 60–75% through 200 mesh; uniformity of particle size is as important as fineness.

Pulverizers may be of the hammer, impact, ball, tube, or attrition type. Mill size, power requirements, and maintenance are dependent on coal grindability, moisture content, and desired fineness. Energy requirements for mill operation are usually 10–20 kw-hr per ton. Mills formerly were arranged in a bin-and-feeder system, but have been largely displaced by the unit or direct system, in which the fuel is burned as soon as it is ground, dryers controlling the moisture content. The classified coal dust is handled by fans and fed through burners in which say 15% of the air is primary and 85% secondary. Burner and furnace arrangements mix the fuel and air, contribute turbulence, shorten the flame, and increase volumetric heat-release rates. Ash problems are often severe, and fusion, slagging and fluxing require proper furnace-wall construction, first-row tube spacing, and ash removal system. Low-fusion ash (2 000° F) can be removed in a molten state from a tap or drip type furnace; high-fusion ash (2 500° F+) is usually dealt with in a powder form from a hopper. Correct ash fusion temp must be specified, and the furnace designed to handle ash over the entire operating load range. Pulverized coal and stokers are in close competition in the steam making range of 50 000–150 000 lb per hr, with stokers prevailing below the lower limit and powdered coal above the upper.

Table 13. Tests for Fineness of Powdered Coal, ASTM Standards, 1936 (Abbreviated)

U S St'd sieve	Sieve opening			Wire diam	
Series No	Microns	Millimeters	Inches	Millimeters	Inches
16	1 190	1.19	0.0469	0.54	0.0213
30	590	0.59	0.0232	.33	.0130
50	297	.297	0.0117	.188	.0074
100	149	.149	0.0059	.102	.0040
200	74	.074	0.0029	.053	.0021

Square mesh screens for bituminous coal, crushed to less than 1.5 in.

Sieve analysis of crushed bituminous coal reported in percentage to nearest 0.1%

Retained on 1.050 percent
Through 1.050 and retained on 0.742 "
0.742 " " " 0.525 "
0.525 " " " 0.371
0.371 " " " 0.263
0.263 " " " 0.185
0.185 " " " 0.131
0.131

Furnace walls. Refractory walls are suitable for gas and oil firing, but with coal, especially under high firing rates, the furnace may grow too hot and lead to ash and slag problems. WATER-COOLED screens and arches prevent washing away of furnace refractories. Heat transfer rates of 100 000 btu per hr per sq ft are attained on these surfaces, and strong water circulation is essential to prevent overheating. The walls may be bare tubes, or faced with blocks, fins, studs and varying thickness of refractory to control furnace temp. AIR-COOLED walls are no longer used. Radiant superheaters may be employed for part of the walls.

Economizers are placed in the boiler exit gas stream to preheat feed water and lower the stack temp. C-I tubes were formerly used, but steel tubes, plain or protected, prevail with use of oxygen-free feed water. AIR-HEATERS preheat the air for combustion to

improve boiler effic by lowering the stack temp and shortening the flame length. Degree of preheat is fixed by furnace conditions and firing methods; 500° F is the usual max. Preheaters are either recuperative (plate or tubular) or regenerative. Heat transfer rates of air heaters and economizers are low (2–6 btu per hr per sq ft per deg); hence they require large surfaces. Boilers may use economizers or preheaters, or combinations of both; or they may be omitted entirely. Stack temp should never be below the dew point, because of flue gas corrosion; 250° F is the usual minimum.

Draft systems provide the necessary air, and remove products of combustion. The combustion chamber should be maintained at a press just below the atmos (−0.1 in water) to minimize air leakage into the furnace; the press should not be positive or flame will pass through cracks in the setting. FORCED DRAFT provides air at positive press to the grates or burners; INDUCED DRAFT removes the stack gases by maintaining negative press at the flue. Balanced draft results when forced and induced draft maintain a press of −0.1 in water. Chimneys, fans, blowers, or steam jets are used. Chimneys provide only induced draft; the other devices are for both forced and induced draft. Draft varies with type, design and condition of equipment and the load carried. CHIMNEYS provide natural draft by difference in density between the vert column of hot gas inside and the similar column of cold air outside the stack. Theoretic draft (Sec 39) is a function of the gas temp, chimney height, and physical properties of the gases. Effective draft is obtained by deducting friction and veloc head losses from the theoretic draft. Gas velocities of 30–40 ft per sec at full load are common. Stack temp is 400–700° F with natural draft.

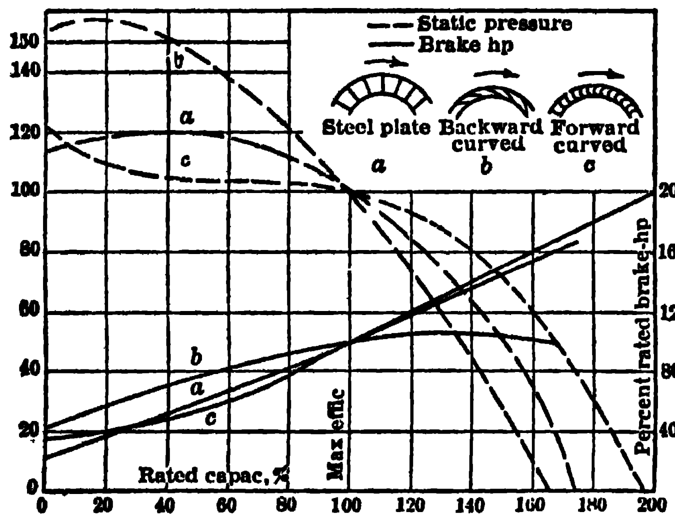


Fig 13. Fan Performance Characteristics, Percentage Basis

Chimneys are of brick, concrete, or steel, lined or unlined. Their effic is only 0.1–0.2% thermodynamically. For high rates of firing they are not practical, and must be supplemented by mechanical draft (fans and jet blowers). FANS are generally centrifugal; direct, geared, or belt-connected to elec motors, steam turbines, or engines. High-speed drives reduce space and wt of the units. Design details fix fan performance (Fig 13). The curves are definitive, because a fan must operate on its characteristic. If the resistance demands do not coincide with the fan characteristic, the supply must be throttled, or the speed varied. Flat or steep head characteristics, self-limiting hp, inherent high speed, or inherent high capac can be built into a fan. Forced-draft fans handle only air and hence are smaller and run faster than induced-draft fans, which must handle hot, dust-laden flue gas. Propeller or disk fans, set in the ash pit wall and direct-connected to turbines or motors, are favored for smaller plants. STEAM JET-BLOWERS, entraining gas by a steam jet in the throat of a Venturi tube, may be installed in the ash pit wall or at base of the stack; they are uneconomical and may consume 5–20% of the steam generated, but are low in first cost, foolproof, suitable for small plants, or where natural draft must be supplemented to carry peak loads.

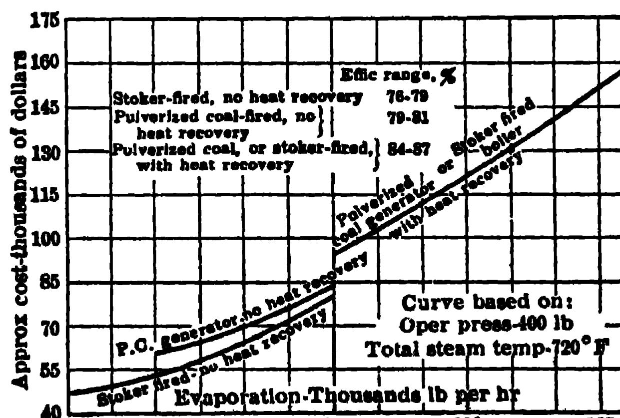


Fig 14. Variation of Steam Generating Unit First Cost, with Size and Type

Boiler settings and casings. Boilers which are not self-contained require supporting steel, and a casing of brick or brick and sheet steel, with doors for inspection and repairs. Settings must be gas tight, resist strain due to heating and cooling, and minimize radiation losses.

Accessories. BOTTOM BLOW-OFF VALVES remove accumulated solids in the boiler; SURFACE BLOW-OFF VALVES remove floating matter or scum. GAGE GLASSES indicate water level in the drum and are supplemented by TRY COCKS for operation when the gage glass is out of service. PRESSURE GAGES are also needed. SAFETY VALVES, required by law, are of the spring-loaded pop type, with adjustments for relieving press and the degree of blowdown. With superheated steam, the first safety valve to relieve should be on the superheated outlet, to prevent burning of the elements. FEED-WATER REGULATORS may be manual or automatic. Draft and damper regulators, draft gages, and CO_2 recorders, aid good operation. The entire steam producing function can be made automatic by COMBUSTION-CONTROL EQUIPMENT. It is expensive, but assures high sustained effic. SOOT BLOWERS, served by steam or comp air, keep gas side surfaces clean.

Boiler performance. Capac was formerly measured in boiler hp. A developed boiler hp was defined as the heat added, in steam, to evaporate 34.5 lb of water per hr, from and at 212° F; or 33 479 btu per hr. Makers rated hp was defined as the heating surface ÷ 10; the % rating is the ratio of developed to rated hp. In current practice, as a boiler is seldom on the line below 100% rating, the concept of hp has been abandoned in favor of lb of steam per hr, or heat added per hr. BOILER EFFIC is the ratio of heat absorbed by the steam to heat supplied in fuel. Heat-balance calculations are made according to methods specified in the Boiler Test Codes, to allocate inefficient items. The largest loss in boiler operation is generally represented in the sensible heat of stack gases. Standby, banking losses, and variable loads always make the actual effic lower than test figures; 80% is readily obtained by well designed and operated modern plants; poor design, maintenance, or operation may give effic below 50%. Actual evaporation, in lb steam per lb of fuel, is a convenient means of measuring effic. The evaporation factor is the ratio of actual heat added per lb of steam to the latent heat at 212° F (970.4 btu per lb). Equivalent evaporation is actual evaporation × evaporation factor, or lb of steam made from and at 212° F, per lb of fuel.

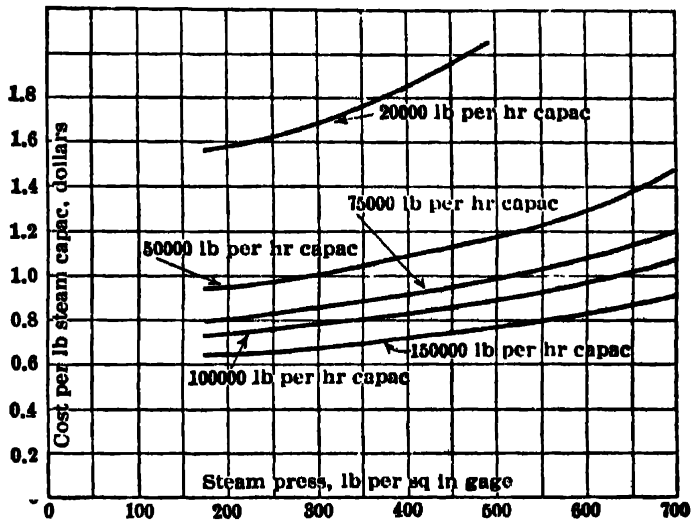


Fig 15. Variation of Steam Generating Unit First Cost, with Size and Pressure (Kolflat, Powerfax, Winter, 1938)

Costs of steam generating units, as influenced by size, type and press, are shown in Fig 14, 15.

5. STEAM TURBINES

Steam power prime movers are turbines or reciprocating engines. Turbines operate by steady flow, with free expansion of steam to utilize the energy in its kinetic form; in engines, with intermittent flow and balanced expansion, the press acts on a piston.

Turbines comprise a group of nozzles, in which steam expands and is accelerated in accordance with the relation, $(H_1 - H_2)\ 778$, where u = veloc, ft per sec; H = enthalpy, btu per lb; subscripts 1 and 2 denote initial and final conditions. If initial veloc u is zero, the veloc of the issuing steam jet is $u_2 = \sqrt{2g(H_1 - H_2)\ 778} = 223.5\sqrt{\text{enthalpy drop}}$. Enthalpy drop is most conveniently determined by a Mollier chart (Sec 39). The nozzles of IMPULSE TURBINES are fixed in the casing, the issuing jet impinging on a row of curved vanes on the wheel periphery. By Newton's second law of motion, the momentum of the fluid is altered by change in direction and magnitude by friction and impact, to give the turning effort. In REACTION TURBINES the nozzles are on the moving wheel. Accel of the steam turns the wheel by the reactive force opposite to direction of the issuing jet, and in accordance with Newton's third law. Commercial turbines employ both principles. Reaction turbines are distinguished by a press drop across the moving buckets. Vectorally, it can be demonstrated that for best effic the tip speed should be 40–45% of jet speed on impulse turbines, and 85–90% on reaction turbines. Expansion from supply to exhaust press must be in stages, to keep the bucket speed within practical limits (600 ft per sec). Impulse turbines may be of the RE-ENTRY

TYPE, with staging on a single wheel. Steam passes several times through the single row of moving blades by means of fixed reversing buckets. These turbines are suited to small sizes (100 kw); are inexpensive, and seldom exceed 50% Rankine effic ratio (50% of isentropic enthalpy drop). With multiple-wheel construction, the staging may be veloc (Curtis) or press (Rateau). With VELOCITY STAGING, all expansion occurs in the row of fixed nozzles, and steam flows axially through alternate rows of moving and reversing buckets. Curtis staging is practically limited to 2 or 3 stages, due to friction losses. In PRESSURE STAGING, the construction is cellular, with a single row of moving buckets in each cell and nozzles in the separating diaphragms. With the reaction principle, multiple rows of fixed and moving nozzles give the Parsons design.

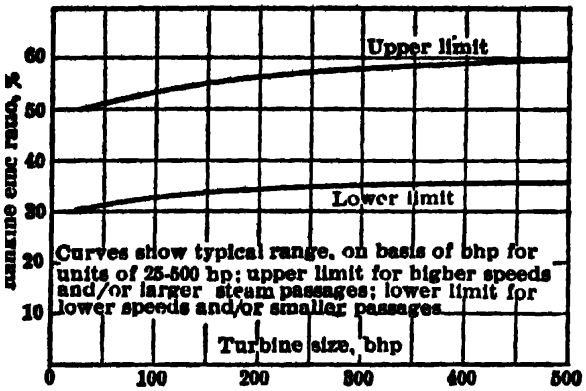


Fig 16. Rankine Effic Ratios of Small Steam Turbines

Commercial turbines are combinations of these types, with drive either direct or geared.

Turbine sizes as small as 5 kw, and as large as 60 000 kw at 3 600 rpm available; when less than 250 kw they have advantages of high speed, low first cost, and small floor space, but are less effic than reciprocating engines. Blade heights are limited to 0.5 in minimum, due to friction losses, and 40 in max, because of structural problems. Blade speeds of 500 ft per sec are common, but special designs may run at 1 200 ft. Turbines can utilize the highest vacua. Steam temp is fixed by the allowable moisture in the exhaust (not more than 10–12%); more moisture causes excessive vane erosion and increased water rate; reduction of 1% moisture in any stage causes approx 1% improvement in effic in that stage. Proper selection of material, as stainless steels and irons, reduces erosion. Labyrinth packings and glands decrease interstage leakage. Fig 16, 17 show typical Rankine effic ratios; effic is not limited to these data, but it is difficult to justify higher values.

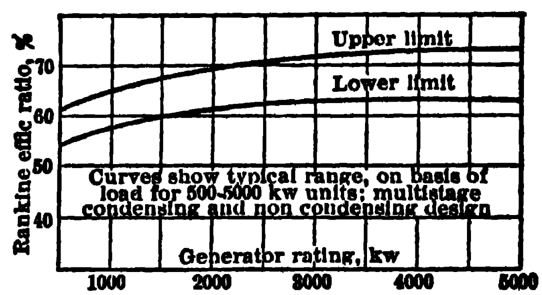


Fig 17. Rankine Effic Ratios of Steam Turbine Generator Sets

Table 14. First Cost of Turbine-generator Sets, 750° Steam Temp

A. Condensing units			B Non-condensing units			
Steam press, lb per sq in	Size, kw	Cost	Steam press, lb per sq in	Back press, lb per sq in	Size, kw	Cost
200–450	500–5 000	$17 000 + 18 6 × kw	450	0–215	500–5 000	$13 000 + 17 × kw
200–450	5 000–7 500	5 000 + 21 × kw	450	0–215	5 000–7 500	3 000 + 21 × kw
451–650	500–5 000	17 000 + 20 × kw	451– 650	0–265	500–5 000	16 000 + 20 × kw
451–650	5 000–7 500	5 000 + 22 5 × kw	451– 650	0–265	5 000–7 500	6 000 + 22 × kw
651–850	5 000–7 500	15 000 + 22.2 × kw	651– 850	0–265	2 000–5 000	24 000 + 20 × kw
			651– 850	0–265	5 000–7 500	11 000 + 22.5×kw
			851–1 450	200+	5 000–7 500	21 000 + 23 × kw

Table 15. Effect of Size of Turbo-generators on Price (Clarke. Industrial Power, *Trans* ASME, 1939)

Rating, kw	Cost per kw, %	Rating, kw	Cost per kw, %
1 000	Base	7 500	0.625
1 500	0.850	10 000	0.687
2 000	0.775	15 000	0.652
2 500	0.730	25 000	0.623
5 000	0.635	50 000	0.462

Turbines are governed by throttling the steam supply, or cutting out nozzles on the first stage; the former is less expensive but less effic at light loads. Water rates at part loads, can be estimated if a Willans' line (Sec 39) is used with an intercept (no load) steam flow of 10–25% of full load flow. An internal governor may be built into the turbine to maintain constant press at a selected extraction point. For costs of turbine generator sets, see Table 14, 15.

6. STEAM ENGINES

Reciprocating engines compete with steam turbines in many services. Advantages of engines: low speed, high torque, and ready reversibility. Turbine advantages: high speed, minimum wt and space, and utilization of highest vacua. Max size for engines, 5 000 kw; for turbines, 200 000 kw. In sizes less than 250 kw, engines have better water rates than turbines. In large sizes turbines give max steam economy. Engines are suited to direct drive of air and refrigeration compressors, and piston pumps; turbines, to direct drive of elec generators, centrifugal pumps and fans, and turbo blowers.

Table 16. Steam Engine Diagram Factors* at Rated Load

Overall range, extreme conditions..	0.50–0.95
Usual range, normal conditions....	0.75–0.95
Simple, slide valve, small size.....	0.50–0.80
Simple, single valve, automatic, well maintained................	0.80–0.90
Uniflow..........................	0.85–0.95
Simple, four valve, or Corliss......	0.85–0.95
Compound, four valve............	0.75–0.85

* Diagram factor = Actual mep ÷ hypothetical mep

Engines are vert, horiz, angle, or inclined; single or multi-cylinder; simple or compound; condensing or non-condensing; uniflow or counterflow. Single-valve engines (D-slide, balanced, or piston valve) prevail on sizes below 100 kw, one valve controlling admission and release at both ends of the cylinder. Multiple-valves divide these functions among mechanisms such as oscillating plugs, plugs with trip gear (Corliss), or poppet valves. In uniflow designs, exhaust is through a central ring of ports, as in 2-cycle internal-combustion engines.

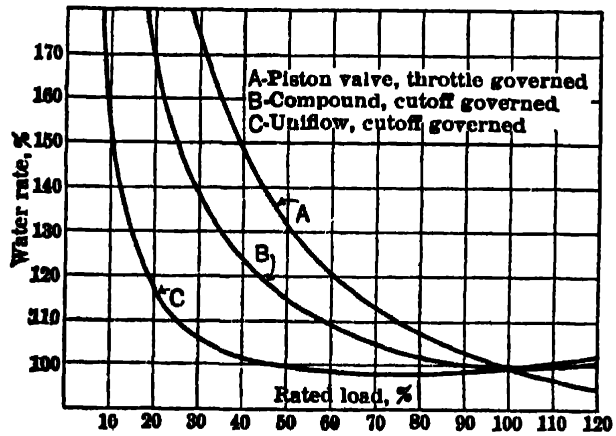


Fig 18. Typical Steam Engine Water-rate Curves

Mean effective pressure. Hypothetical values are calculated by thermodynamic methods (Sec 39). Actual values are obtained by using diagram factors (Table 16) as multipliers. Indicated hp (ihp) is then obtained by the equation ihp $= \dfrac{\text{mep} \times L \times a \times n}{33\,000}$, where mep = mean effective press, lb per sq in; L = stroke, ft; a = piston area, sq in; n = cycles completed per min. Mep is 50–100 lb per sq in for the usual steam press of 100–200 lb, and cutoff at 10–30% of stroke. The bore-stroke ratio ranges from stroke = 1 × bore, to stroke = 2 × bore, with 1.25 rather common. Commercial sizes are 4 to 36 in stroke. Speed is primarily a matter of machine design. Slide or poppet valves prevail in the 300-rpm group; plug and Corliss valves in the 100-rpm group. Piston speeds are 500–750 ft per min with the lower values on smaller engines.

Cylinder condensation and internal leakage cause higher actual water rate than computed from indicator cards; the increase may reach 50 or 75%; good maintenance reduces leakage. Use of superheated steam, and compound or uniflow designs reduce cylinder condensation; 10° superheat improves the water rate about 1%. The economy of a simple uniflow engine is nearly the same as that of a compound engine. The Rankine effic ratio of 500 hp condensing (5 in abs back press) engines is 60–70%; for non-condensing engines, the ratio is 5–10% higher. Smaller engines, especially slide-valve designs of less than 100 hp, show Rankine effic ratios of 30–50%, when operated non-condensing. The friction mean press is 8–10 lb per sq in, and results in mechanical effic of 85–95% at normal loads.

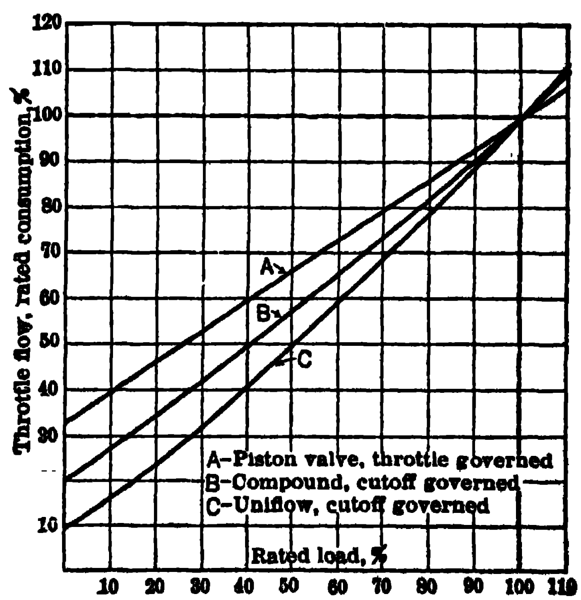


Fig 19. Some Typical Willans' Lines for Steam Engines

Governing and regulation, either for constant or variable speed, is obtained by throttling the steam, or varying the cutoff. Fig 18, 19 show comparative water rates, and Willans' lines for methods of governing different engines. Flyball and flywheel governors are not equally suitable to all valves and valve gear. "Automatic" engines have slide valves, and use flywheel governors to change the cutoff. Engines are made reversible by linkage mechanisms, which alter the valve eccentric position or its equivalent. Some first cost data on reciprocating engines are given in Table 17.

Table 17. First Costs of Reciprocating Steam Engines

Type	Size range, hp	Cost, dollars
Simple engines		
Single valve, automatic.	40– 320	930 + 11.7 × hp
Four-valve..........	150– 800	2 560 + 11.3 × hp
Corliss, slow speed....	107– 575	3 410 + 6 8 × hp
Uniflow.........	350–1 000	3 900 + 14.6 × hp
Four-valve, cross-compound.............	480–1 250	5 420 + 12.7 × hp
Corliss slow-speed, cross-compound.. .	400– 650	7 110 + 7.6 × hp

7. CONDENSING PLANT

The expansion of a lb of steam from boiler press to high vacuum produces approx twice the work that would be done by expansion to atmos press only. The condenser maintains the requisite vacuum. Engines can effectively utilize 25–26 in vacuum, while turbines realize the full benefit of 28 or 29 in (referred to 29.92 in, standard sea-level barometer). Condensing plants are not expensive, and are customary unless exhaust steam can be used for process heating purposes, or unless weight or bulk of equipment is prohibitive.

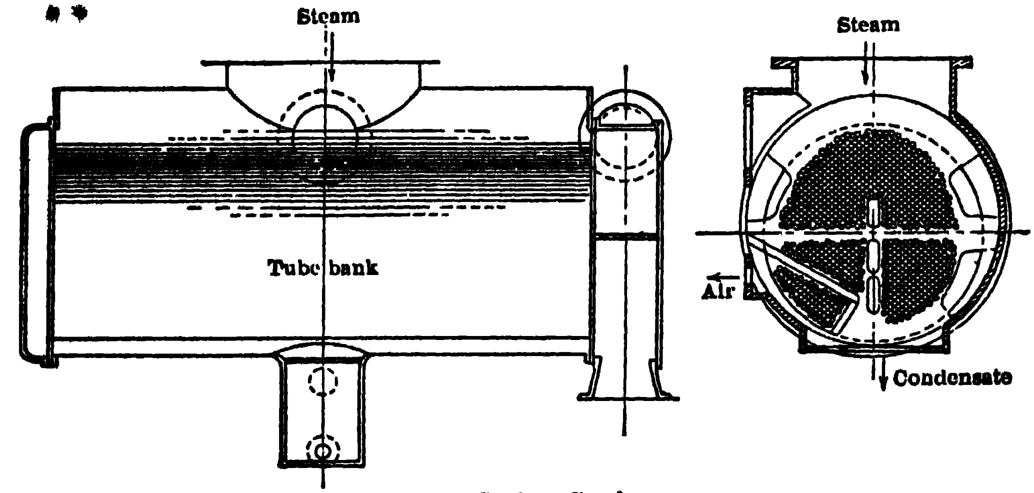


Fig 20. Surface Condenser

Condensers are of the mixture, contacting type, or surface, non-contacting (Fig 20, 21). Jets and sprays mix water and steam in the former; in the latter, a bank of tubes, less than 1 in diam, separate steam from water. In both, circulating pumps bring the cooling water to the condenser; hot-well pumps remove the condensate (or mixed condensate and circulating water), and air removal pumps abstract non-condensible gases which otherwise would accumulate and destroy the vacuum. A barometric leg may be substituted for the hot-well pump on a jet condenser to remove water by gravity. JET CONDENSERS will not maintain the highest vacua, nor preserve distilled condensate; hence, used only in small plants, with low steam press, and reciprocating engines. In SURFACE CONDENSERS, water generally flows through copper alloy tubes, steam surrounding the tubes, which are within a steel or C–I shell.

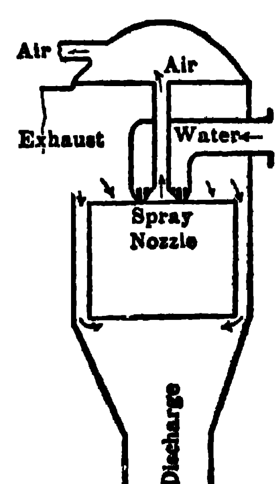


Fig 21 Jet Condenser

Vacuum is obtained in accord with the vapor tension curve for steam; lowest available temp is the atmos; latent heat of exhaust steam must be absorbed at this level. Water is preferable to air for cooling, due to its higher heat capac. Allowable temp rise of circulating water is limited to maintain

lowest practical steam temp. Fig 22 gives hypothetical water requirements for abstracting latent heat of dry steam at various vacua; actual condenser specifications often use 950 btu per lb latent heat with turbines, and 1 000 btu per lb with reciprocating engines. Allowable water temp rise is usually 10–20° F.

Heat transfer rates vary with water veloc, surface cleanliness, water and steam distribution, tube size and spacing, surface drainage, and air concentration. Heat transfer relation is $U = C\sqrt{V}$, where U = heat transfer rate, btu per hr per sq ft per deg F; V = water veloc, ft per sec; C (constant) = 251 for 1-in, and 270 for 0.75-in tubes. Heat transfer rates are 400–800 btu per hr per sq ft per deg F, with the veloc of 3–8 ft per sec. The surface required is usually 0.5–2 sq ft per kw of capac. The above formula (given by "Standards of Heat Exchange Inst," Condenser Section, p 7, edn 1939) is used by leading condenser makers.

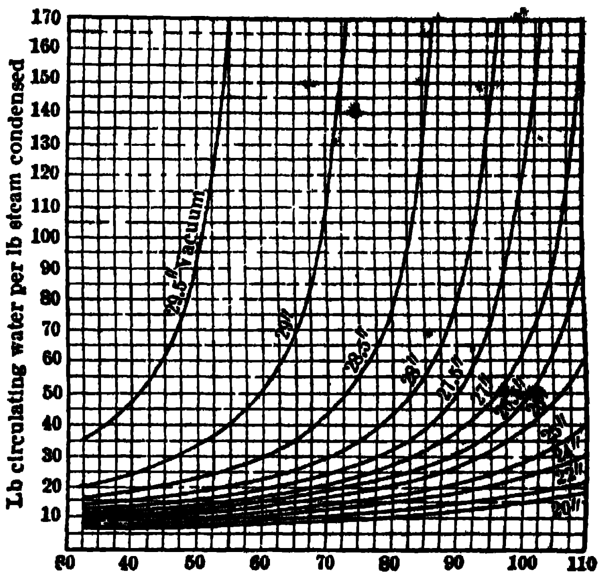


Fig 22. Hypothetical Condenser Circulating Water Requirements (29.92 in Barometer)

Condenser tubes are mounted in the tube sheets: (1) with ferrules and packings at both ends; (2) rolled at one end and packed at the other; (3) rolled at both ends with floating heads or bowed tubes. Tubes are kept clean by chlorination of circulating water, or by mechanical cleaners, as wire brushes or rubber slugs. ADVANTAGES OF JET CONDENSERS are cheapness, lower maintenance, less space, and fewer complications; surface condensers preserve pure feed, and give better vacuum.

Circulating pumps are of high capac (0.5–2.0 gal per min per kw) and low head (15–30 ft with surface condensers). The circulatory system is a syphon loop, the pump having only to provide for friction and veloc heads. Centrifugal and propeller types are best for this service, but direct-acting pumps can be used on small installations. Surface condenser, HOT-WELL PUMPS, have 0 01–0 04 the capac of circulating pumps; heads usually 100–200 ft. Centrifugal pumps prevail in larger installations. AIR PUMPS may be wet vacuum, removing both air and condensate, or dry vacuum, handling only air. The former are used only for small capac, and may be reciprocating or rotary; dry vacuum pumps may be reciprocating, rotary, hydraulic jet, or steam jet. The latter prevails because of small bulk and low cost, and is made in 1, 2 or 3 stages, with heat recovery condensers after each stage. Air pump capacities for 25 000 lb of steam condensed per hr are 0.25 lb dry air per min for turbines and 0.5 lb for engines. Atmospheric relief valves protect condensers from positive pressures due to interruption of the circulating water supply. Vacuum breakers prevent flooding the prime mover if the water level runs too high on a jet condenser.

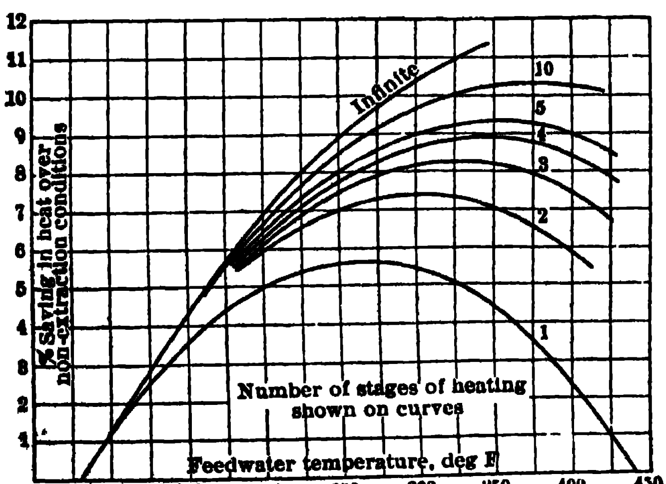


Fig 23. Feed-water Heat Saving by Extraction; Steam Conditions, 400 p, 700° F to 1 in Abs Back Pressure

Circulating water supply may be from natural or reclaimed source; it should be as cold as possible. The discharge should not mix with the cooler supply. Intake systems are protected by screens and racks to remove rubbish. If the natural supply is inadequate, reclaimed water can be used with an evaporation loss of 1–2%. Spray ponds and cooling towers increase power requirements, because of the 5–10 lb per sq in nozzle press and elevated head.

Cost of condensing systems, is $3–$6 per sq ft of condensing surface.

Feed water heaters are most important heat-saving devices. The use of exhaust or extracted steam for heating gives an approx 1% fuel saving for each 25 deg F; nearly all plants justify heating to 200° or 220° F. The greatest economy results when max work

is obtained from the steam before using it for heating. Main unit-extraction-stage heating gives max effectiveness (see Fig 23). OPEN HEATERS mix steam and feed by use of sprays and trays, with heat exchange rates of 200 000 btu per hr per cu ft of contact space. CLOSED HEATERS have a tube bank with water inside and steam outside the tubes. Tubes are less than 1 in diam and may be straight, plain, corrugated, U-shaped, spiral, or helical, with provisions for expansion and cleaning. Heat transfer rates are 250–750 btu per hr per sq ft per deg F, with terminal temp differences of 2°–10° F. Some advantages of open heaters are: (1) higher feed temp with same steam press; (2) convenient precipitation and removal of salts and scale; (3) degasification of feed, if properly vented. Closed heater advantages: are adaptability to any operating press; feed pumps can be placed before or after the heater; and contamination of otherwise pure feed by oily or dirty exhaust steam is prevented. Closed heaters vary widely in price, according to pressure; \$5–\$10 per sq ft being typical.

8. FEED-WATER HEATING AND PURIFICATION

Injurious substances occur in practically all water. They are detrimental either to the boilers or to their proper operation. Troubles from impure feed water are: formation of scale, corrosion, and priming or foaming. Solid matter may be precipitated as sediment, or may form scale on the heating surface. SEDIMENT causes little trouble; it is regularly drawn off at the blow-off. SCALE seriously retards heat transfer, and if allowed to accumulate the heat may weaken the metal sufficiently to cause rupture. CORROSION causes general weakening of the boiler, finally resulting in a breakdown at some point. PRIMING causes steam to carry with it more or less water, which, if it enters an engine cyl in sufficient quantity, may blow-out the cyl head. Substances causing these troubles may be natural constituents of the water, or be due to waste from a plant or factory at a higher point on the stream from which the water is drawn. Where the condensed steam from surface condensers is pumped back to the boilers the oil should be effectively separated. Condensed steam from turbines does not give trouble, as oil is used only in the bearings.

Water treatment. Scale-forming substances may be precipitated by heating or by adding chemicals to the water supply, and eliminated by filtration; or chemicals may be added to the feed water to cause the solid matter to form as a sediment instead of incrusting the heating surface. Anti-scale chemicals are known as BOILER COMPOUNDS. Substances causing corrosion or priming are treated in a similar manner. Electrolytic action, sometimes causing pitting and corrosion, may be neutralised by hanging zinc plates inside the boiler.

Table 18. Feed-water Purification

Trouble	Cause	Remedy
Incrustation	Sediment, mud	Filtration
	Readily soluble salts; Bicarb of magnesia; Lime, iron	Blowing off. Heating feed water and precipitating caustic soda, lime, and magnesia
	Organic matter	See Sewage, below
	Sulphate of lime	Sodium carbonate; Barium chloride
Corrosion	Organic matter	Precipitate with alum or ferric chloride and filter
	Grease, oil	Add slaked lime or sodium carb and filter
	Chloride or sulphate of magnesium; Sugars, acid	Sodium carbonate
	Dissolved CO_2 and O	Slaked lime, caustic soda; heating
	Electrolytic action	Zinc plates
Priming	Sewage	Precipitate with alum or ferric chloride, and filter
	Alkalies	Heat feed water and precipitate
	Sodium carbonate in large quantities	Barium chloride

Impure feed water should be analysed, to determine what it contains and the remedy, and also the wt of solids in a given wt of water and hence the proper wt of chemicals to be added for neutralising. Boiler compounds, if used in excessive quantity, may themselves cause boiler troubles. Avoid all secret preparations. They are either worthless or contain ordinary chemicals for which a higher price is charged under the name of a special preparation. In the latter case it is only a [illegible] chance that the chemicals are suited to the water. A number of concerns specialise in [illegible] impure feed water.

The importance of proper treatment is shown by the extensive purification systems installed at many large plants. EVAPORATORS. The most effective treatment of feed water is by using a closed system and adding only pure distilled water as make-up. On straight power cycles the makeup is only 1–3%. When make-up is too high, evaporators may be precluded. When used they are single or multiple in effect, and of submerged tube or circulating types. Cleanability of their surfaces is of prime importance. Dissolved oxygen in feed water corrodes steel parts of boilers and economizers; it is removed by DE-AERATORS, of either the chemical, or more generally the mechanical type. They resemble an open heater, but must have a scrubbing action, and not less than 30° temp rise to sweep the entrained O from the water. They operate at either positive or negative press. Good designs of condenser hotwells will give O concentrations as low as 0.03 cc per liter, but for greater purification a separate deaerator is necessary.

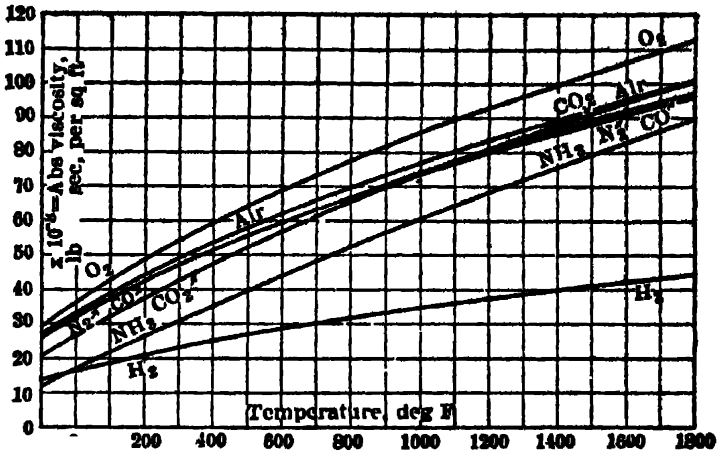


Fig 24. Viscosity of Gases, Calculated by Sutherland's Equation

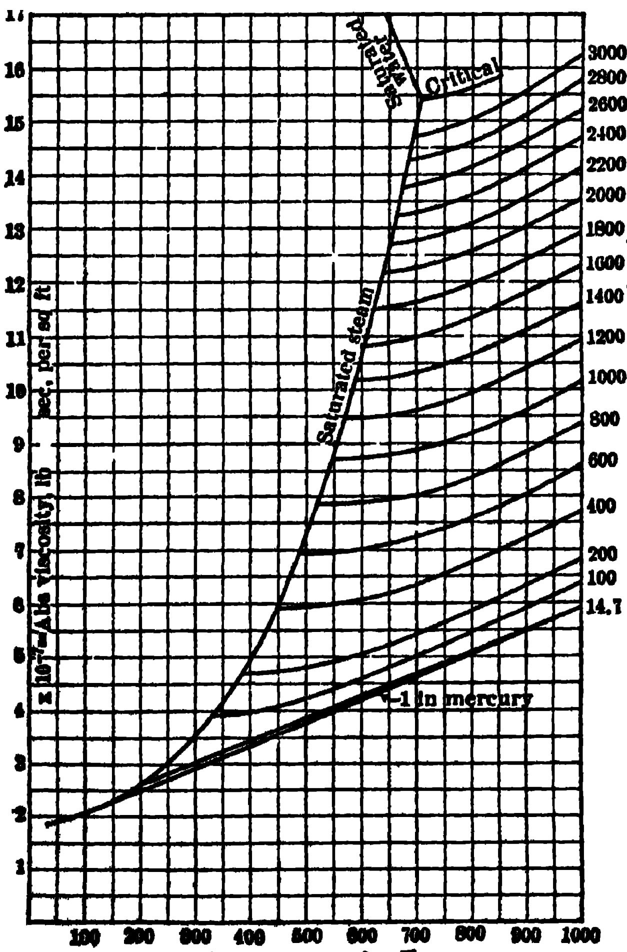


Fig 25. Viscosity of Steam

9. PIPING AND DUCT SYSTEMS

Selection of piping and duct sizes is a matter of balancing the operating cost, due to loss of energy by friction, against investment charges, to give the lowest total cost. Friction drops in straight pipe are determined by the relation: $\Delta H = C_D \frac{2Lv^2}{gD} = C_D \frac{Lv^2}{2gm}$, where ΔH = head loss due to friction, ft of fluid; L = length, ft; v = aver veloc, ft per sec; g = gravitational accel, 32.2 ft per sec²; D = pipe diam, ft; m = mean hydraulic depth, ft = $D \div 4$ for round pipe; C_D is the friction factor defining the flow pattern, and is a function of the "Reynolds' Number" (see Sec 39). The Reynolds' Number can be calculated from Fig 24, 25, 26. Data on C_D are in Fig 27. When an approx solution is sought, C_D can be taken as 0.005 or 0.006 for most duct and pipe installations. For the usual allowable piping veloc, see Table 19. The friction drop caused by fittings is generally expressed as $\Delta H = F \frac{v^2}{2g}$, where F = resistance coeff, for which typical values are given in Table 20, for approx estimates.

Expansion joints are necessary in pipe lines subject to temp changes. Piping must be so supported that no damaging thrusts will occur on connected equipment. They must be firmly anchored at intervals; supported by hangers, floor stands, or wall brackets; and have long-radius loops, U bends, sleeve, slip, or bellows joints for expansion and contraction. Hangers permit movement by swinging; brackets and stands have rollers.

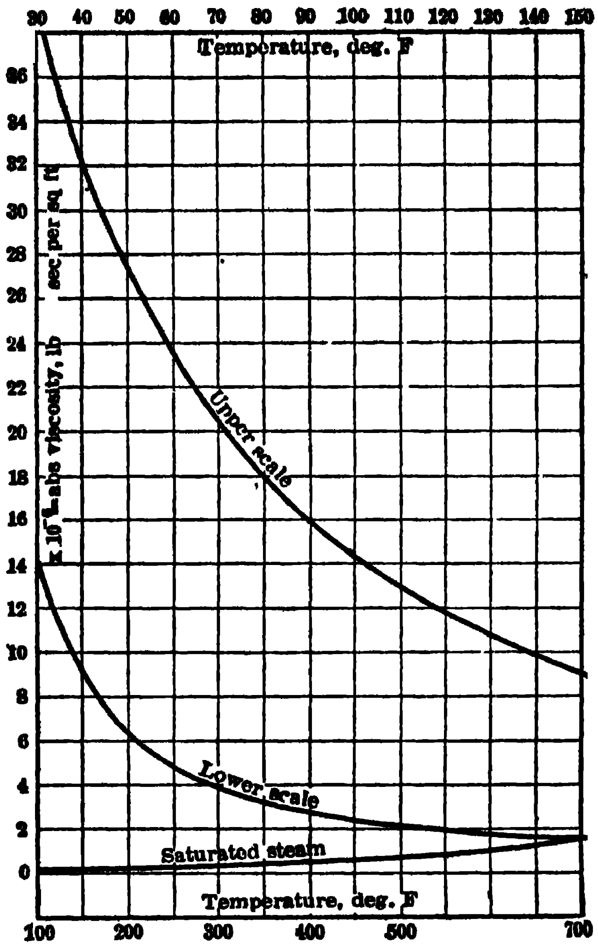


Fig 26. Viscosity of Water

Pipe insulating coverings. Cork, hair and felt are used for refrigeration and water pipes; waterproofed against external moisture condensation. Air-cell covering is satisfactory for steam temp below 250° F, while mixtures of magnesia and asbestos, in powdered or molded form with a cloth cover, are used for higher steam temp. Piping identification can be provided by color schemes, but instructional stencils on their surface are more effective.

Table 19. Allowable Velocities in Pipes and Ducts

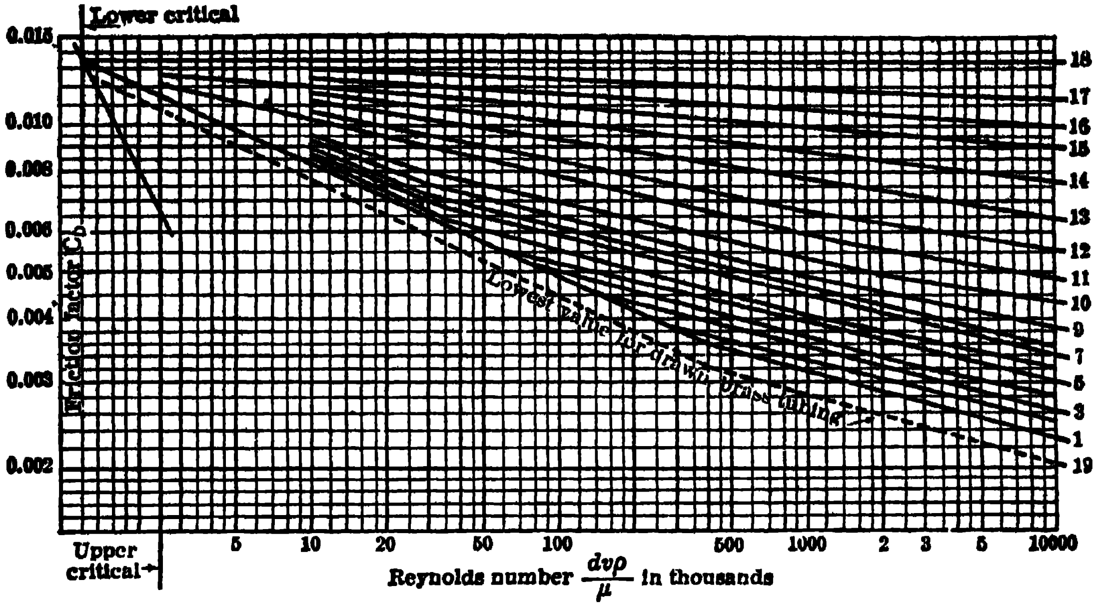
Service	Veloc, ft per min
Air ducts	2 000– 3 000
Comp air pipe	1 500– 2 500
High-press steam	10 000–15 000
Low-press steam	10 000–20 000
Water pipe	300– 600

Table 20. Friction Factors for Pipe Fittings

Fitting	Factor
Elbow, standard, 90°	0.75–1.00
Elbow, long sweep, 90°	0.20
Elbow, blade	0.25
Elbow, 45°	0.25
Gate valve	0.20
Globe valve	2.00
Orifice, sharp edge	0.04
Orifice, bell mouth	0.04
Re-entrant nozzle, L > 3 diam	1.00
Tee, straight run	0.25
Tee, turn	1.00–2.00

Drainage of steam lines prevents dangerous accumulation of condensate. Simple drainage chambers or drop legs may be used, with or without supplementary separators. SEPARATOR types: (1) reverse current; (2) centrifugal; (3) baffle plate; (4) mesh; (5) capillary.

Drainage chambers should have sufficient tankage, water gages, steam traps, and by-passes. TRAPS automatically remove drips, without allowing steam to escape; they comprise: float, bucket, gravity, differential expansion, thermostatic, and diaphragm. A manually operated by-pass is used when the trap is out of order. Drips are collected in a tank, hot-well, or open heater.



d = diam of duct, ft

v = mean velocity, ft per sec

ρ = fluid density = $\frac{\text{lb per cu ft}}{32.2}$

μ = viscosity, lb sec per sq ft

C_D = friction factor (above diagram)

L = length of duct, ft

Press drop, lb per sq ft = $\frac{2 C_D \rho v^2 L}{d}$

Diam of duct, inches (for drawn tubing, actual inside diam)

Curve	Roughness, percent	Clean steel, wrought iron	Clean, galvanized steel	Best cast iron, cement, light-riveted steel	Aver cast iron, rough-formed concrete	First class brick, heavy- and spiral-riveted steel	Drawn tubing, tin, brass, lead, glass
1	0.2	72	....	....	....	....	0.35 up
2	0.45	48–66					
3	0.81	14–42	30	48–96	96	220	
4	1.35	6–12	10–24	20–48	42–96	84–204	
5	2.1	4–5	6–8	12–16	24–36	48–72	
6	3.0	2–3	3–5	5–10	10–20	20–42	
7	3.8	1 1/2	2 1/2	3–4	6–8	16–18	
8	4.8	1–1 1/4	1 1/2–2	2–2 1/2	4–5	10–14	
9	6.0	3/4	1 1/4	1 1/2	3	8	
10	7.2	1/2	1	1 1/4	....	5	
11	10.5	3/8	3/4	1	....	4	
12	14.5	1/4	1/2	....	....	3	
13	19.0	1/8					
14	24.0	....	3/8	....	....	....	1/8
15	28.0						
16	31.5	....	1/4				
17	34.0						
18	37.5	....	1/8	....	....	....	0 06

Fig 27. Flow of Fluids in Closed Conduits (Pigott)

10. WATER WHEELS

Water wheels function as demonstrated by Bernoulli's principle, relating the potential, press, and kinetic energy, at entrance to and exit from the wheel. The obsolete over-shot wheels directly utilize the wt of water as potential energy. Modern water-wheels, like steam turbines, function on the conversion of static into veloc energy. IMPULSE WHEELS (Fig 28). The veloc is generated in fixed nozzles, the emergent stream impinging on buckets mounted on the wheel periphery. With REACTION WHEELS (Fig 29) the nozzles are in the runner. There must be some accel in the speed ring, to bring the water to the wheel with proper entrance veloc; varying degrees of accel are therefore required in the

casings of different types of wheel. The vector diagrams of Fig 30 are illustrative. Fig 30(a) shows the impulse wheel, where the entire accel is in fixed nozzles; $v_1 = C_v\sqrt{2gH}$, where v_1 = jet speed, ft per sec; H = head, ft; and C_v = 0.98–0.99, the coeff of veloc. If the nozzle angle β were zero, and 180° reversal were attainable in the buckets ($\alpha_2 = 0$),

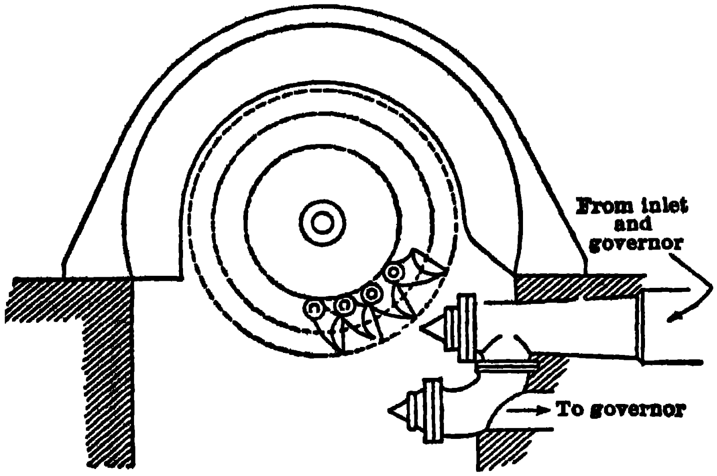


Fig 28. Impulse Water Wheel

then, for best energy utilization, bucket speed u would be half the jet speed v_1. As complete reversal is impossible in practice, impulse wheels operate with bucket speed = 0.42–0.45 × jet speed. If the conversion of head into veloc is only partial, in the fixed nozzles, and expansion completed in nozzles on the wheel, then vector diagrams, Fig 30(b), 30(c) obtain. With less expansion in the casing nozzles, the jet veloc becomes progressively smaller and the entrance vane angle, α is increased for max effic. The ratio of bucket to jet speed ($u_1 \div v_1$) is therefore about 0.9 in Fig 30(b), and may range to 1.5 to 2.0, as in Fig 30(c). Fig 30(a) is typical of Pelton wheels, Fig 30(b) of mixed press Francis runners, and Fig 30(c) of propeller wheels. The exit veloc triangles should always give minimum length for the final water veloc, v_2; attained when perpendicular to u_2, and only for one condition of flow. Effic curves must therefore show a peak value. In commercial designs the vector diagrams are more complicated, due to three-dimensional flow considerations.

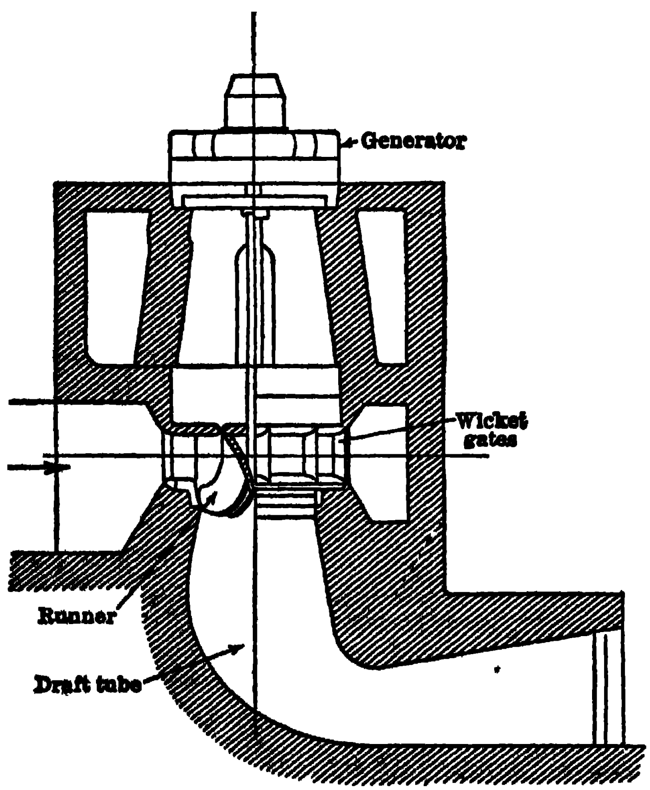
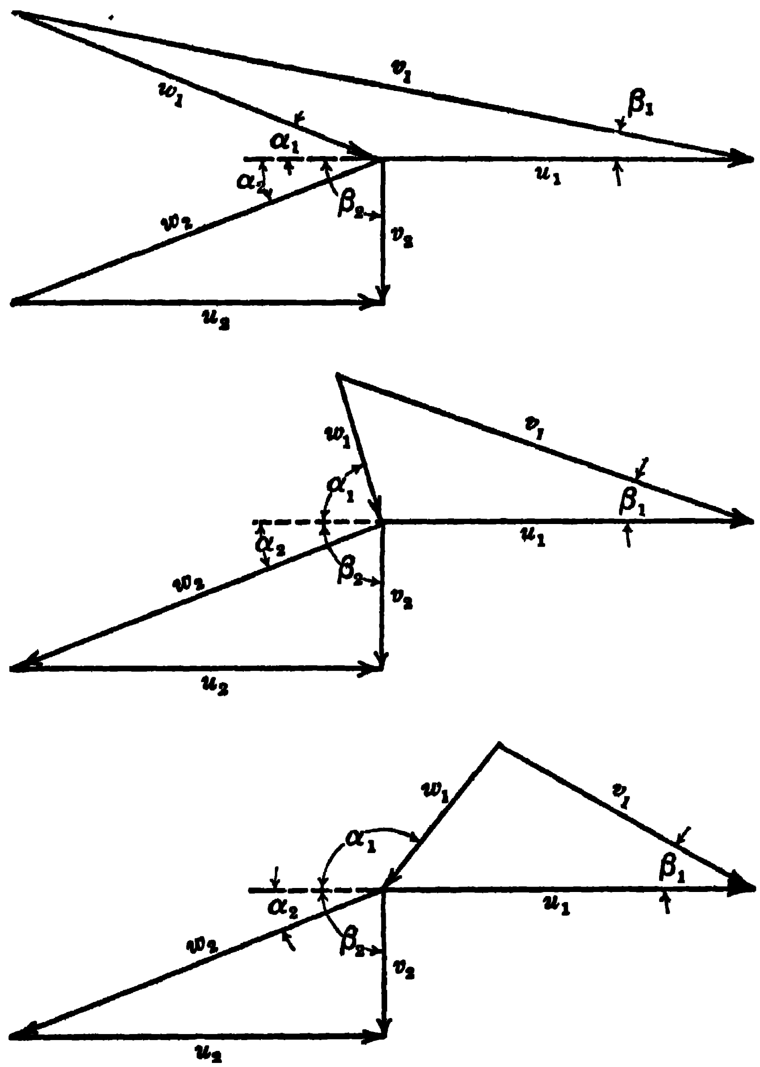


Fig 29. Francis Reaction Water Wheel and Setting

Pelton wheel (Fig 28) is the only impulse wheel of importance. The water jet is created in a stream-lined needle nozzle; only 1 or 2 jets are used per wheel. Governing and regulation is by use of a deflecting hinged nozzle, or a double nozzle, the lower one being for waste and opens only during load changes to avoid water hammer.

Reaction wheels operate full of water, as required by the criterion of a press drop through the runner. Fig 29 shows how the water is brought through a concrete, C-I, or steel-plate scroll to the gate ring and runner. The water is accelerated partly in accordance with vector requirements, and is directed by the speed ring to the runner passages.

The inward radial, and subsequent axial, flow complete the expansion. Water is discharged to a draft tube; thence to the tail race. Control and regulation are through the gate settings. Gates may be wicket, register, or cylindrical. FRANCIS RUNNER, with inward radial flow, predominates for heads of 50–500 ft. At less than 500 ft, the flow becomes more axial (Fig 31), ultimately reaching the propeller type. This results in high, sustained effic; see Fig 32, which includes other curves for comparison. Propeller wheels



Subscript 1, applies to inlet conditions; 2, to exit
v = abs water veloc;
u = abs bucket veloc;
w = relative veloc, water to bucket:
α = vane angle;
β = nozzle angle.

Fig 30. Water-wheel Vector Diagrams

are generally for heads under 100 ft. KAPLAN RUNNER vanes are adjustable and under governor control. SPECIFIC SPEED, N_s, defined by $N_s = \dfrac{\text{rpm} \times (\text{bhp})^{0.5}}{(\text{head})^{1.25}}$, where rpm = rev per min of the runner; bhp = brake hp; head = head of water, ft. Specific speed, a criterion that exactly classifies a runner (Fig 31), is the speed at which an homologous runner of proper diam would run in order to develop 1 hp under 1 ft head. It is usually given for the point of max effic, or at rating. High spec speed means a high-speed runner. With low heads, the practical problem is to make the rotative speed high enough. Hence, high spec speeds are for low-head service, and vice versa.

Fig 32 contains curves from numerous installations and authorities, to show the relations between spec speed and head. The spec speed of Pelton wheels is fixed by the ratio of wheel and

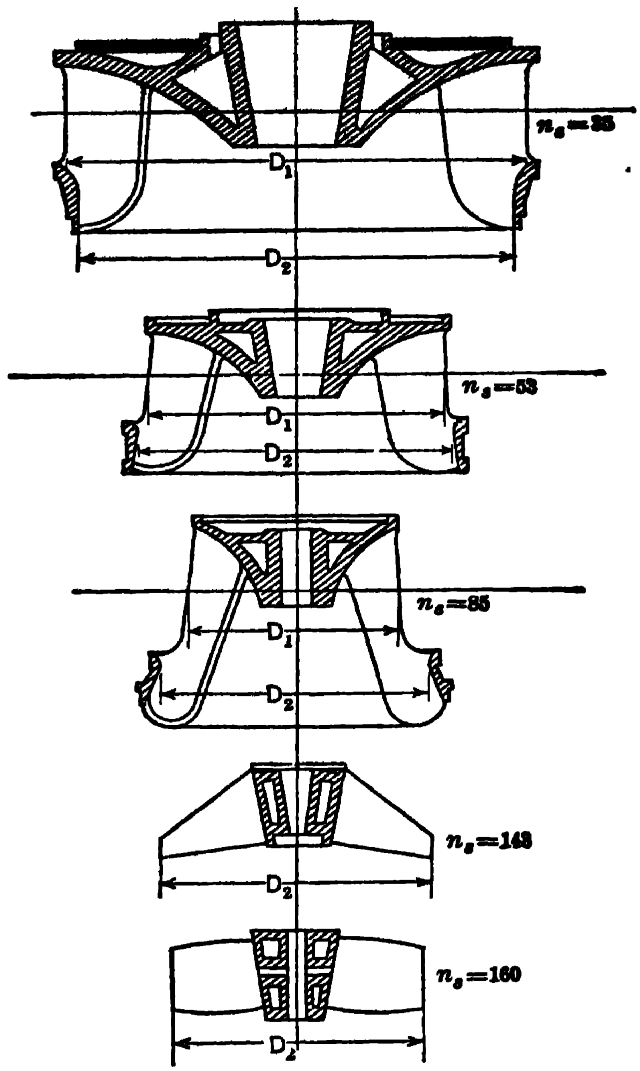


Fig 31. Comparison of Runners of Equal Power, but Different Specific Speeds (n_s)

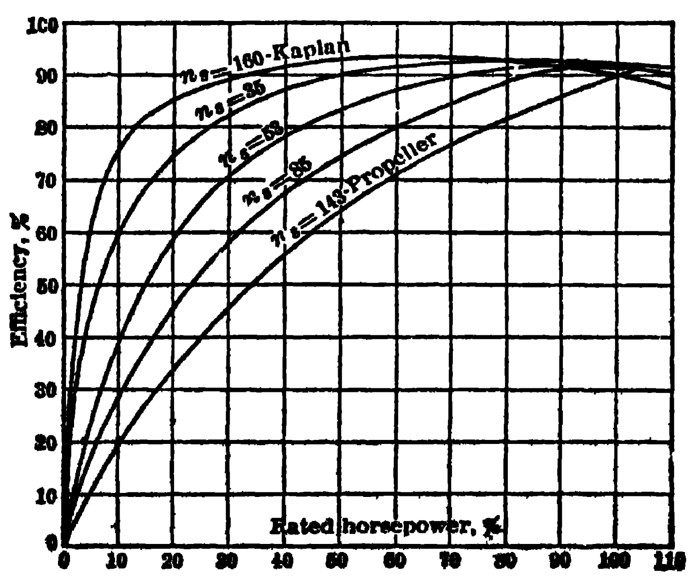


Fig 32. Water-wheel Efficiency Characteristics

jet diam, and is increased by the square root of the number of jets. Commercial spec speeds are: 3–8 on Pelton wheels; 15–50 on Francis; 50–100 on mixed-press wheels; and 100–200 on propeller and Kaplan wheels. All commercial runners except the Pelton operate full of water and hence require a continuous water column from head to tail race. The runner discharge connection is not a simple pipe, but a DRAFT TUBE, for the supplementary purposes of placing the machinery safely above tail water without loss of available head, and reducing to a negligible amount the loss incident to the discharge veloc from the runner. A plain cylindrical tube satisfies the first requirement, but a conical tube is needed for the second. While the theoretic height is the barometric (33.8 ft at sea level), the practical height is generally about 20 ft, because of veloc conversion, vapor press,

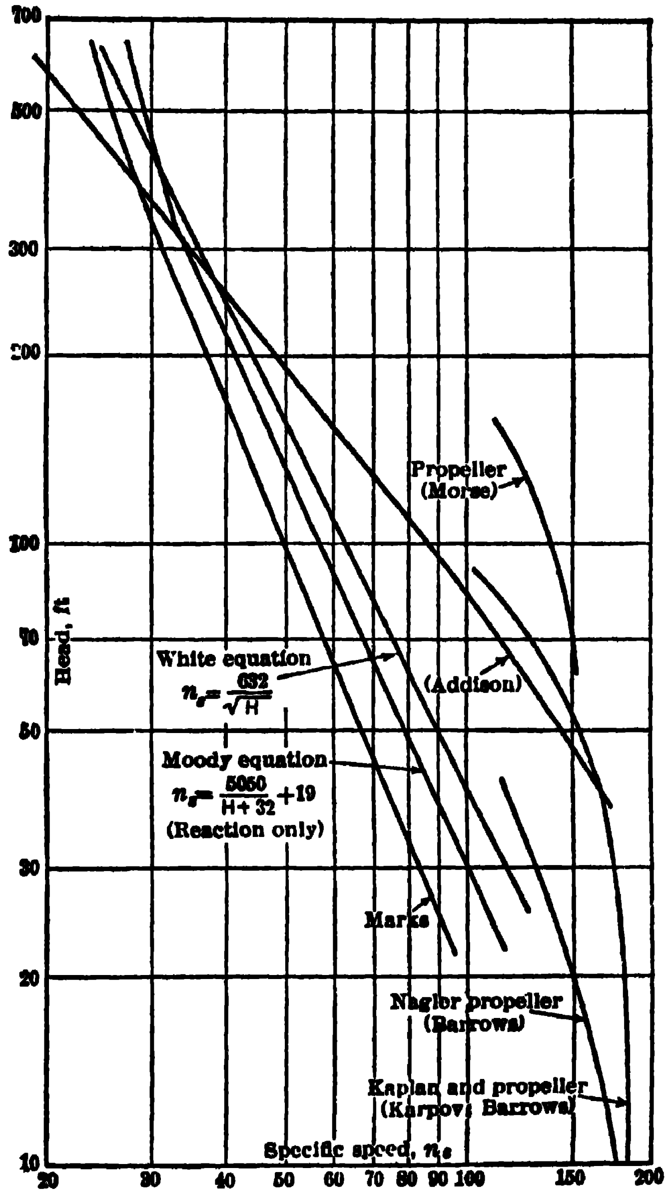


Fig 33. Water Wheel Experience Data; Specific Speed vs Head

air in solution, cavitation, and vibration. Draft tubes are usually of concrete, the design depending on site conditions. Draft-tube effic is the ratio of regain of press energy in the tube to veloc energy at its entrance; values run to 90% on the best designs.

Cost of hydraulic turbines varies widely, because most installations are of special design. Wheel and generator cost \$20–\$50 per kw; the lower prices for large size and high head. Turbine and generator usually represent only a small part of total cost of plant, which may be \$100–\$400 per kw of capac. Most of the cost is for structures, dams, spillways, head and tail works, land, and water rights.

11. PUMPS

Head. The TOTAL DYNAMIC HEAD of fluid (*TDH*) in ft delivered by a pump is given by: $TDH = h_s + h_d + \frac{V_d^2}{2g} - \frac{V_s^2}{2g} + d$, where h_s = suction head, ft, measured by a gage on the suction side; h_d = discharge head, ft, measured by a gage at foot of column pipe; V_s = veloc, ft per sec, in suction pipe at gage point; V_d = veloc in discharge pipe at gage point; d = vert distance, ft, between gage centers. SUCTION HEAD h_s = pipe entrance head loss + $\frac{V_s^2}{2g}$ + suction-pipe friction loss + suction lift. DISCHARGE HEAD h_d = static head at pump + discharge pipe friction loss to gage point.

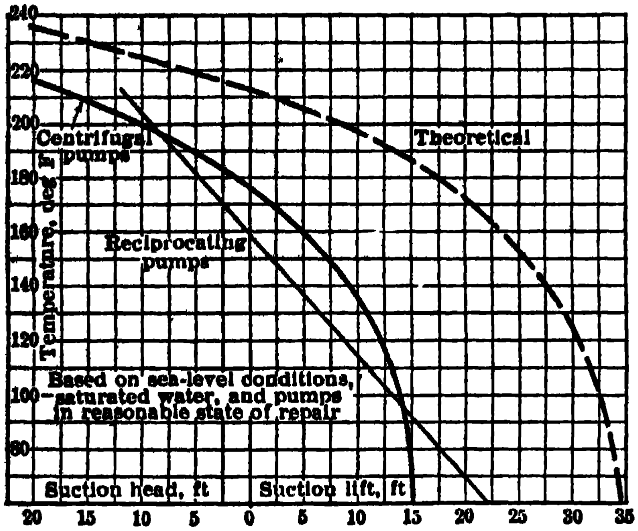


Fig 34. Theoretical and Practical Suction Lifts for Water at Various Temps

Water may be brought to the pump under negative press (suction lift) or positive press (suction head). With SUCTION LIFT, the fluid enters the pump by atmos press on the free liquid surface in the well or sump. Theoretical lift equals barom press in ft of fluid (33.8 ft at sea level). Actual allowable suction lift, h_s, is equal to this barom head h_a, minus the following: (1) h_{vp} = vapor press of the liquid; (2) h_v = veloc head in the suction pipe; (3) h_g = press of gas in solution; (4) h_f = friction loss in suction pipe. Conservative practice requires inclusion of a safety factor. In Fig 34 are given practical data. With liquids at or near the flash point, the fluid must be fed to the pump under positive head, to prevent separation in the column and cavitation. Thick liquids must flow to pumps under positive press.

Capacity is the net useful vol delivered per unit of time, expressed as gal per min (gpm). With positive pumps, piston displacement is sometimes listed as capac.

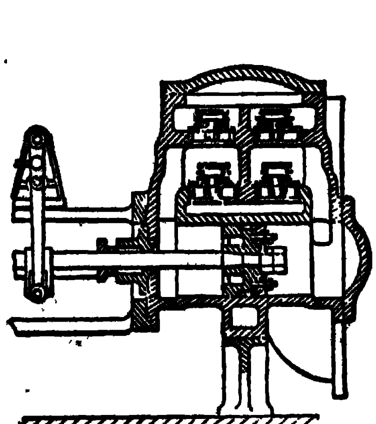


Fig 35. Water End of Piston Pump

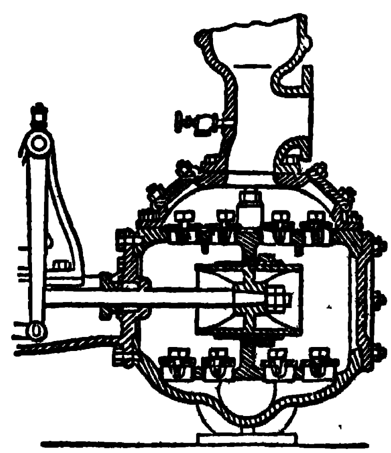


Fig 36. Piston-and-ring Pump

Hp and effic. Theoretical water hp is given by $whp = \frac{gpm \times TDH \times \text{sp grav}}{3\,960}$. Mechanical effic, $n_m = \frac{whp}{bhp} \times 100$, where bhp is brake hp. Standard density used for water is 62.318 lb per cu ft, at 68° F. DUTY of a pump is the ft lb of energy delivered to the water per million btu (the term is gradually being abandoned). SLIP is loss in displacement due to internal leakage and short stroking expressed as a percentage of displacement; usually less than 5%. SIZE designation. An 8 by 5 by 12-in direct-acting pump

has 8-in diam steam cyl, 5-in water cyl, and 12-in stroke. A 6-in pump, reciprocating or centrifugal, means 6-in nominal pipe size for discharge. CLASSIFICATION: (*a*) Positive displacement, including reciprocating, direct-acting and power pumps, and rotary pumps; (*b*) Steady flow, jet and vane pumps, including centrifugal and axial-flow propeller pumps.

Reciprocating pumps. Water-end classification comprises the simplex pump, having one water cyl; duplex, with 2 water cylinders in parallel; and triplex, with 3 water cylinders. A pump is single-acting when it takes water on one side of the piston only, and double-acting when it takes water on both sides.

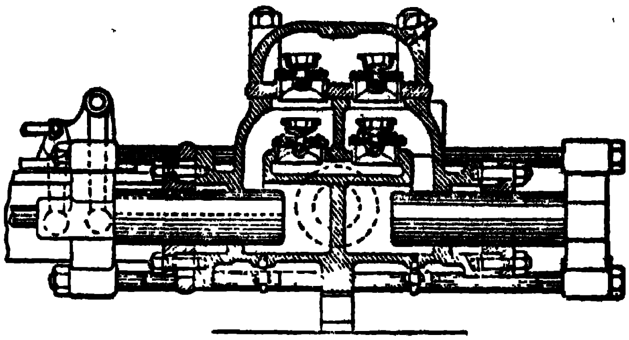


Fig 37. Outside, End-packed Plunger Pump

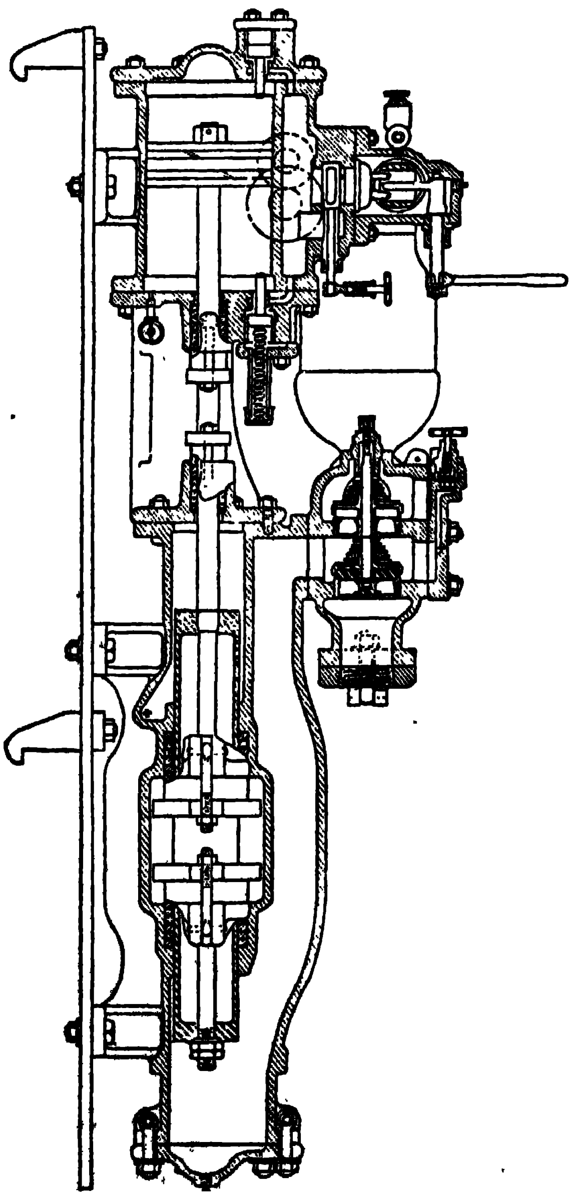


Fig 38. Vert Plunger Sinking Pump (Ingersoll-Rand)

The water cyl has a piston or plunger. A piston, making a loose fit in the cyl (Fig 35), is packed with hemp or metallic packing ring. Piston-and-ring pump (Fig 36) is a cheaper construction, but is apt to score with gritty water. Both types permit undetected leakage past the piston. They may be used for heads up to 150 lb per sq in. For pressures of 150–300 lb, outside-packed plunger pumps (Fig 37, 38) are recommended; for pressures exceeding 300 lb they are necessary. With outside packing leakage is readily detected, and stopped while the pump is running. Scoring (cutting) of the plunger is also plainly visible. Plunger pumps cost more than piston.

When both suction and discharge valves are above the barrel (Fig 35, 37), the pump is of the submerged type; when the suction valves are below (Fig 36) it is of the straightway type. Submerged type is used for hot liquids and condenser service, the pistons being surrounded by cool and de-aerated water, which protects the packing and improves suction action. But, as flow of water in the cyl is reversed at every stroke, it is not recommended for strokes over 14 in. Straightway type costs more, but the water has a more direct flow through the pump, and is therefore used for high water veloc.

Direct-acting pump has the water piston or plunger directly connected to steam end, through the piston rod. Steam-end classification: pumps are simple, compound, and triple expansion, according as each water cyl is served by a single steam cyl, a high and low-press cyl, or a high, intermediate, and low-press cyl. A crank shaft, connecting rod and flywheel are used in some steam designs. If motor-driven, the pump is called a POWER PUMP; Fig 39 shows a triplex pump of this type. Cylinders of all designs may be single, duplex or triplex. Water flow through a reciprocating pump is intermittent. Use of several cylinders and AIR CHAMBERS aids uniformity of flow and reduces shocks. Air chambers are generally mounted on the delivery side, and vacuum chambers are sometimes used on the suction side. Air chamber volume should be 6–8 times piston displacement on simplex pumps;

3-4 times, on duplex pumps; the chambers should have a gage glass and means for air charging.

Reciprocating pumps (Table 21-25) have 2 to 36 in stroke, the water cyl bore being 0.3-0.7 times the stroke. Actual stroke of steam pumps is 0.7-0.9 of rated stroke; power and crank-and-flywheel pumps do not short-stroke. Piston speeds and rpm are low, to reduce shock and cavitation. Fig 40 contains recommendations of Hydraulic Inst for standard pumps handling water, or liquids with viscosity less than 200, Saybolt Universal. Higher speeds are obtainable with positive suction feed.

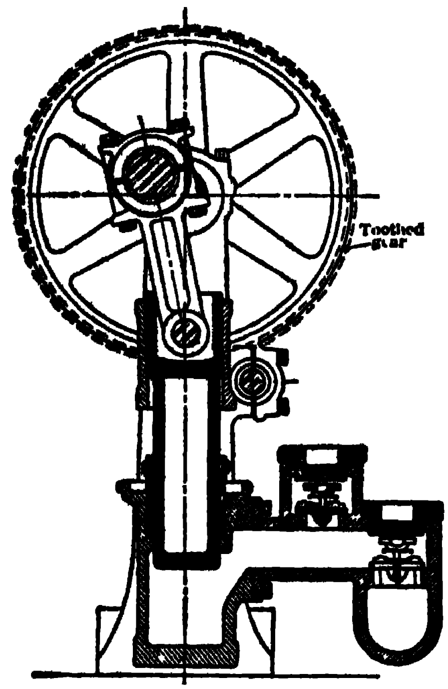


Fig 39. Triplex Power Pump

Commercial classification: REGULARLY-FITTED pump has a C-I plunger or piston working in a bronze ring, liner, or sleeve; piston rod is of steel, and valve seats, springs, and all inside bolts and nuts, are of bronze; valve disks, of rubber or bronze. BRASS-FITTED. Besides the bronze of regularly-fitted pumps, the rods are of "tobin bronze," piston is solid bronze or bronze-lined, and stuffing-box throats and glands are bronze-lined. In ALL-IRON FITTED pumps, for ammonia or tar, no bronze comes into contact with the fluid pumped. Valve seats are of malleable iron, springs of steel, the C-I piston works in the solid-bore of the cyl, and plunger nuts are iron. ALL-BRONZE pumps, for acids that attack iron, are entirely of bronze, which in some cases must contain no zinc. LEAD AND WOOD-LINED pumps. Large pumps, too costly if of bronze, may be made of iron, with the interior lined with an acid-resisting material, as lead, wood, or cement. By LARGE OPENINGS is meant that the valve seats are not obstructed by ribs. Ball or clack valves are of this type (see below).

"Special-fitted" pumps for different liquids:

"How fitted"	Liquid pumped	"How fitted	Liquid pumped
All-Iron................	Ammonia	Regularly-fitted	Calcium brine
"	Bichloride of mercury	" "	Calcium chlorate
"	Calcium chloride	" "	Citric acid
"	Caustic soda	" "	Cyanide of potassium
Brass-fitted............	Brine	Lead-lined, no zinc.	Hydrochloric acid
" "	Copper liquors	" " " ..	Hydrosulphite acid
" "	Glycerine; oils	All-bronze, or lead or wood-lined C-I, no zinc	Sulphuric acid
" " (large openings)	Sewage		

Table 21. Simplex Piston Pumps for General Service

Diam steam cyl, in	Diam water cyl, in	Stroke, in	Gal per stroke	Capacity at ordinary speed, gal per min	Steam pipe, in	Discharge pipe, in	Floor space, in	Wt, lb	Price, iron water cyl, steel piston rod
4	2	6	0 081	12	3/8	1	40×10	210	$240
5	2.5	6	0.12	18	1/2	1 25	40×11	260	280
6	4	7	0.21	28	3/4	1.5	47×13	418	330
7	4	7	0 39	50	3/4	2	51×16	457	470
8	4	12	0 65	65	1	2.5	58×18	864	650
7	5	13	1.43	130	1	3	63×20	1 160	700
10	5	13	1.10	100	1 1/4	3	64×21	1 345	750
10	7	13	2.18	206	1 1/4	3.5	64×21	1 411	800
12	7	13	2.18	206	1 1/2	4	66×24	1 928	940
14	9	18	4.96	330	2	5	81×30	3 126	.....
16	10	18	6.13	408	2 1/2	6	90×37	4 920	.....
18	12	20	8.5	510	3	8	103×41	6 080	.....

For boiler feed service, reduce these capacities by 50%.

Valves are almost always automatic, operating by unbalanced press on the two sides of a moving element. They include: (a) disk, for general service; (b) wing-guided, for high press; (c) ball,

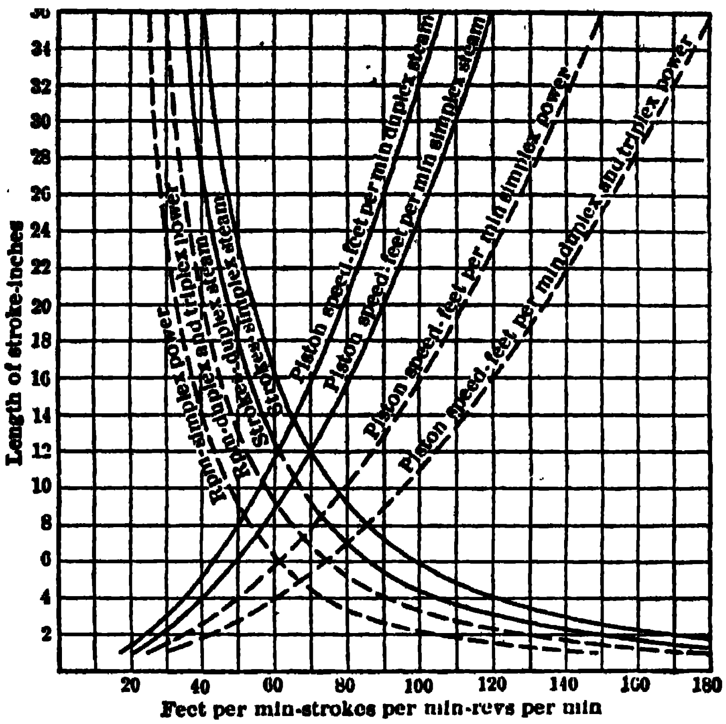


Fig 40. Basic Speeds, Simplex and Duplex Direct-acting Steam Pumps, and Simplex, Duplex and Triplex Power Pumps (Hydraulic Institute)

for viscous liquids; (d) clack, for lightest services. Disk valves, the commonest, seat on a bronze grid, and are spring-loaded on the downstream side. The disks are of medium or soft rubber, for press below 150 lb per sq in; hard rubber or composition, for hot water or press of 300 lb; metal for high press; standard diam, 3 to 4 in. Total valve area, 50–100% of piston area; highest for viscous liquids. Lift, 10–20% of valve diam; spring loading, 0.25–1.0 lb per sq in of valve area on suction valves, to 5 lb on discharge valves.

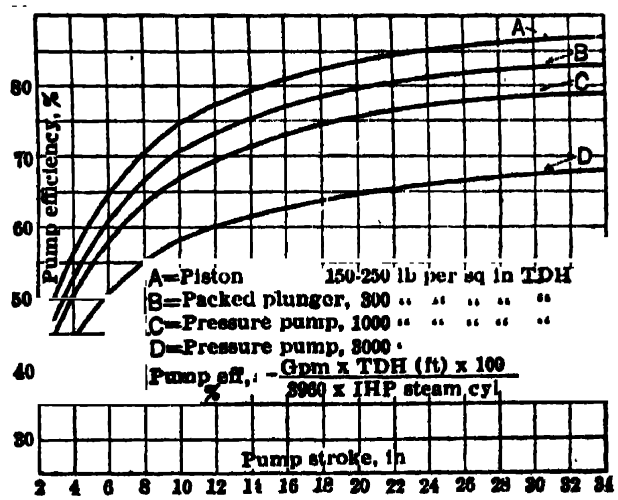


Fig 41. Efficiency of Direct-acting Steam Pumps

Efficiencies of direct-acting pumps are given in Fig 41. Performance is measured on basis of mean press, rather than mechanical effic. Friction losses, solid and fluid, are best expressed as equivalent mean press; which is practically constant, irrespective of operating head, and has decreasing effect as the head increases. STEAM CONSUMPTION is estimated by: lb per hr = $whp \times 90$; air consumption, by cu ft of free air per min $= 1.4\dfrac{asp}{p_a}$, where a = area of water piston, sq in; s = piston speed, ft per min; p = air press at pump required for water press, lb per sq in abs; p_a = atmos press, lb per sq in abs. See Tables 21–25.

Rotary pumps. Rotary motion may be used for an element within a fixed casing, to give positive displacement (Fig 42). The rotary element may consist of gears, lobes, screws, cams, vanes, or combinations of these. Rotary pumps are suited to direct connection with elec motors, and for oily liquids. Sizes are from the smallest to several thousand gal per min; heads, to 1 000 lb per sq in.

Table 22. Compound Duplex Pumps (Outside end-packed, double-plunger, pot-valve pattern)

Diam h-p cyl, in	Diam l-p cyl, in	Diam water cyl, in	Stroke, in	Gal per stroke, 1 plunger	Strokes per min, 1 plunger	Gal per min displacement of 4 plungers	Steam pipe, in	Discharge pipe, in	Length, ft and in*	Width, ft and in
10	16	8 1/2	18	4.42	40 to 80	354 to 708	2	6	18 11	4 0
12	18 1/2	8 1/2	18	4.42	40 to 80	354 to 708	2 1/2	6	18 11	4 0
14	20	8 1/2	18	4.42	40 to 80	354 to 708	3	6	18 11	4 6
16	24	8 1/2	18	4.42	40 to 80	354 to 708	3 1/2	6	19 5	6 0
10	16	9	18	4.96	40 to 80	397 to 794	2	6	18 11	4 5
12	18 1/2	9	18	4.96	40 to 80	397 to 794	2 1/2	6	18 11	4 5
14	20	9	18	4.96	40 to 80	397 to 794	3	6	18 11	4 6
16	24	9	18	4.96	40 to 80	397 to 794	3 1/2	6	19 5	6 10
20	30	9	19	4.96	40 to 80	397 to 794	5	6	19 5	6 10
12	18 1/2	10	18	6.12	40 to 80	489 to 978	2 1/2	8	20 0	6 10
14	20	10	18	6.12	40 to 80	489 to 978	3	8	20 0	6 10
16	24	10	18	6.12	40 to 80	489 to 978	3 1/2	8	20 6	6 10
20	30	10	18	6.12	40 to 80	489 to 978	5	8	20 6	6 10
16	24	12	18	8.81	40 to 80	705 to 1410	3 1/2	10	21 6	6 10
20	30	12	18	8.81	40 to 80	705 to 1410	5	10	21 6	6 10

* Dimensions are with plungers in extreme outward position. Pumps will stand 350 lb press.

Table 23. Simplex Horiz Boiler-Feed or Pressure Piston Pumps (For 250 lb max working steam and water press)

Size			Gal per stroke	Max per min			Boiler hp pump will feed at slow speed	Steam pipe, in	Discharge pipe, in	Floor space, in
Diam steam cyl, in	Diam water cyl, in	Stroke, in		No single strokes	Piston speed, ft	Gal				
3 1/2	2 1/4	4	0.068	150	50	10	45	3/8	3/4	29×8
4 1/2	2 3/4	6	0.15	130	65	20	70	1/2	1	39×11
5 1/2	3 1/4	7	0.25	124	72	31	100	1/2	1 1/4	44×11
6 1/2	4 1/8	8	0.46	118	78	54	160	3/4	2	50×13
7 1/2	4 1/2	10	0.69	108	90	75	300	1	2 1/2	59×15
8	5	12	1.02	100	100	102	500	1	3	67×15
10	6	12	1.47	100	100	147	750	1 1/4	3 1/2	70×15
12	7	12	2.00	100	100	200	1 000	1 1/2	4	70×15
14	8	12	2.61	100	100	261	1 300	2	4	75×18
16	10	18	6.12	67	100	410	3 500	2 1/2	6	96×22

For continuous work, from 25 to 50% less than max listed speed is recommended.

Table 24. Vertical Plunger Sinking Pumps

Diam steam cyls, in	Diam plunger, in	Stroke, in	Capac per stroke, gal	Capac at ordinary speed, gal per min	Steam pipe, in	Discharge, pipe, in	Space occupied in shaft, in	Weight, lb	Price, iron plunger, steel rod
8	4	12	0.65	65	1	2 1/2	25×25	1 435	$700
10	5	13	1.10	100	1 1/4	3	31×30	2 285	860
12	5	13	1.10	100	1 1/2	3	32×33	2 620	1 100
12	7	13	2.16	200	1 1/2	4	34×33	3 400	1 250
14	7	13	2.16	200	2	4	40×35	3 850	1 350
18	9	16	3.28	247	3	4	42×45	5 000	1 700
14	7	13	2.24	206	2	5	40×38	4 150	1 450
16	10 1/2	16	5.97	447	2 1/2	5	42×45	5 220	1 900
18	10 1/2	16	5 97	447	3	5	42×45	5 575	2 050

Centrifugal pump is essentially an impeller, with backward-curved vanes, rotating in a fixed casing. It may be volute or turbine (Fig 43); the former has a simple casing; the latter, a diffuser ring, with or without guide vanes, and also a concentric or volute casing. Impellers may be single or double suction (Fig 44, 45, 46).

Table 25. Duplex, Pot-valve, Solid-end Pumps (Outside end-packed plungers)

Diam steam cyl, in	Diam plungers, in	Stroke, in	Gal per rev	Max rev per min	Total capac, gal per min	Steam pipe, in	Discharge pipe, in	Approx length, ft and in	Approx width, ft and in
10	4	12	2.63	40	105	2 1/2	3	11 0	2 10
12	4	12	2.63	40	105	2 1/2	3	11 0	2 10
12	5	12	4.14	40	165	2 1/2	4	11 2	3 10
14	5	12	4.14	40	165	3	4	11 2	3 10
14	6	12	5.9	40	235	3	5	11 4	4 4
16	6	12	5.9	40	235	3	5	11 4	4 4
18	6	12	5.9	40	235	3	5	11 4	4 4
16	6	18	8.8	33.3	295	3	5	15 0	4 4
18	6	18	8.8	33.3	295	3	5	15 0	4 4
18	7	18	12.0	33.3	400	3	6	15 6	4 6
20	7	18	12.0	33.3	400	4	6	15 6	4 6
20	8	18	15.6	33.3	525	4	6	15 6	4 8
22	8	18	15.6	33.3	525	4	6	15 6	4 10
24	8	18	15 6	33.3	525	4	6	16 0	4 10

These pumps are good for lifts up to 900 ft.

The pumps are vert or horiz; right or left hand, when viewed from the coupling end. Casings may be: (a) split, in a plane parallel to shaft; (b) drum or barrel, where impellers and diffusers are assembled in series from one end of a cylindrical case; or (c) ring, where successive stages are mounted from one end with through bolts. Shafts are supported in external bearings. End thrust, especially with single-inlet impellers, requires hydraulic balancing drums and thrust bearings. Stuffing boxes, with hard or soft packings, are suitable for heads of several hundred ft; high press requires supplementary labyrinth glands. Materials specification should be with the maker's advice. The impeller should be removable with minimum disturbance of pipe connections.

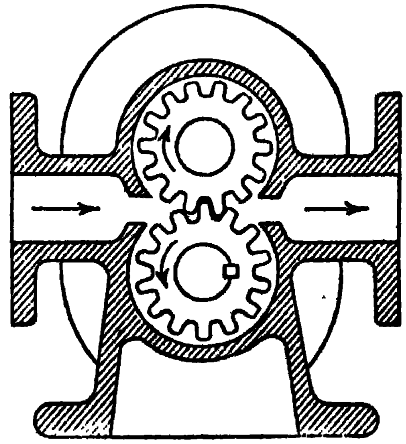
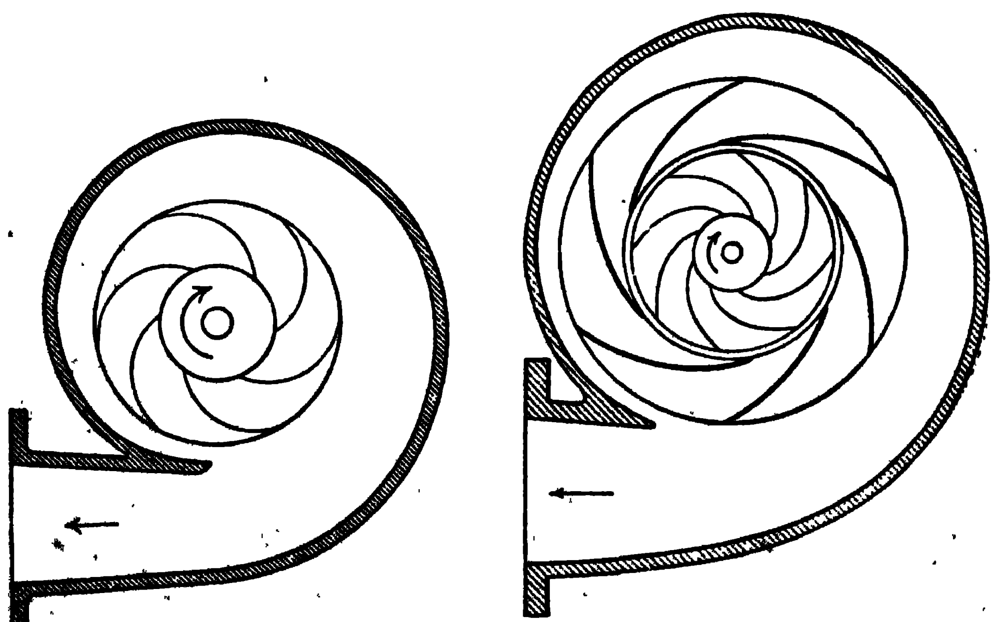


Fig 42. Gear Type Rotary Pump

Design and performance. Centrifugal pumps function as a combination of forced and free vortices. The static head theoretically $= U_2^2 \div 2g$, and total head $= U_2^2 \div g$: wherein U_2 = tip speed, ft per sec; g = gravitational accel, ft per sec^2. In practice, pumps give heads differing materially from the theoretic values, being influenced by blade



Volute pump

Turbine or Diffuser Pump

Fig 43. Centrifugal Pumps

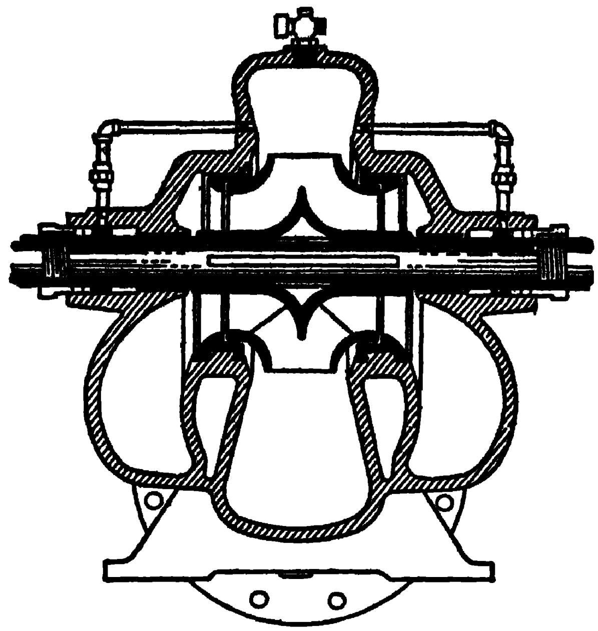


Fig 44. Single-stage, Double-suction, Volute Pump (Worthington Pump & Mach Co)

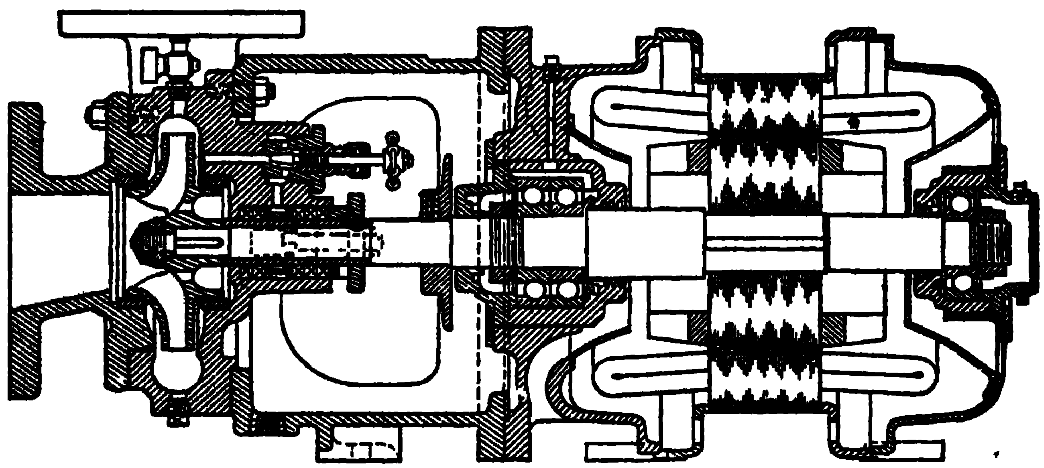


Fig 45. Single-stage, General Service, Centrifugal Pump (Ingersoll-Rand)

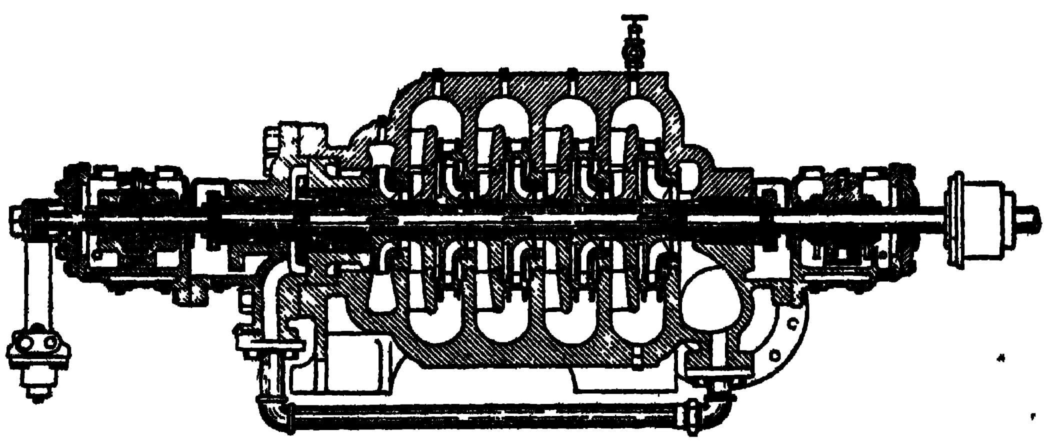


Fig 46. Five-stage Centrifugal Pump (Ingersoll-Rand)

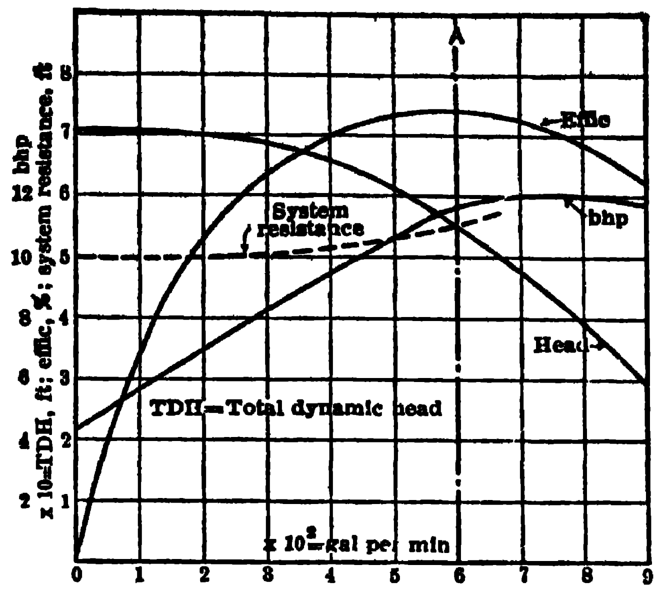


Fig 47. Characteristic Curves, Single-stage Centrifugal Pump running at 1 750 rpm

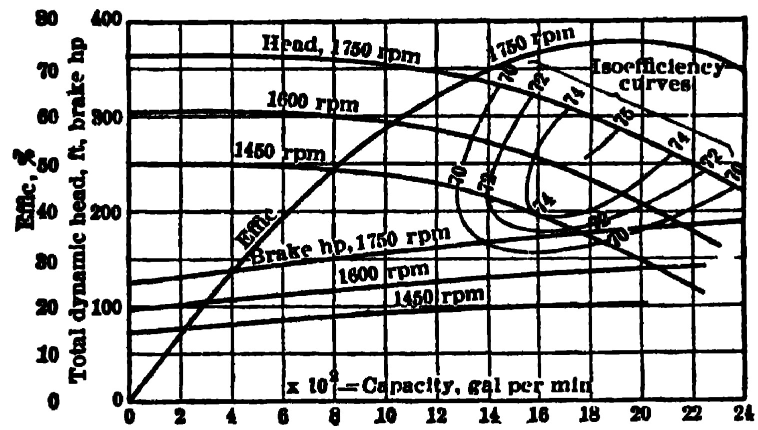


Fig 48. Characteristic Curves, Two-stage Centrifugal Pumps at Various Speeds

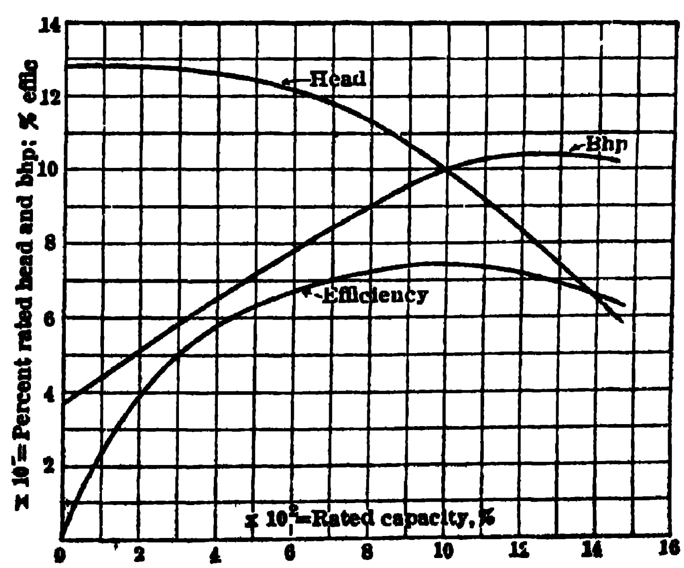


Fig 49. Characteristic Curves, Single-stage Centrifugal Pump; Percentage Rating Basis

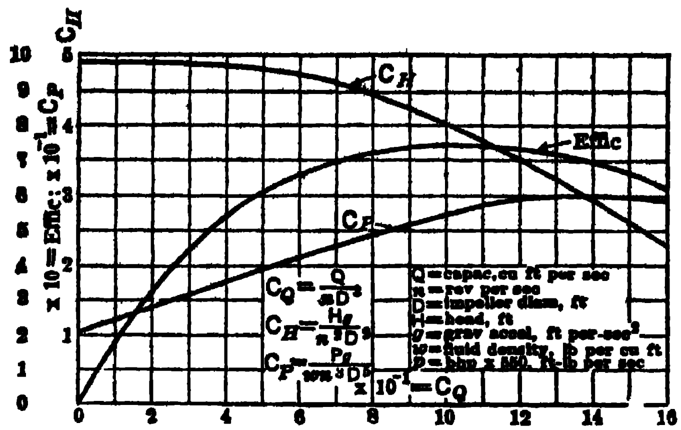


Fig 50. Single-stage Centrifugal Pump, Characteristic Curves, Dimensionless Coeff Basis

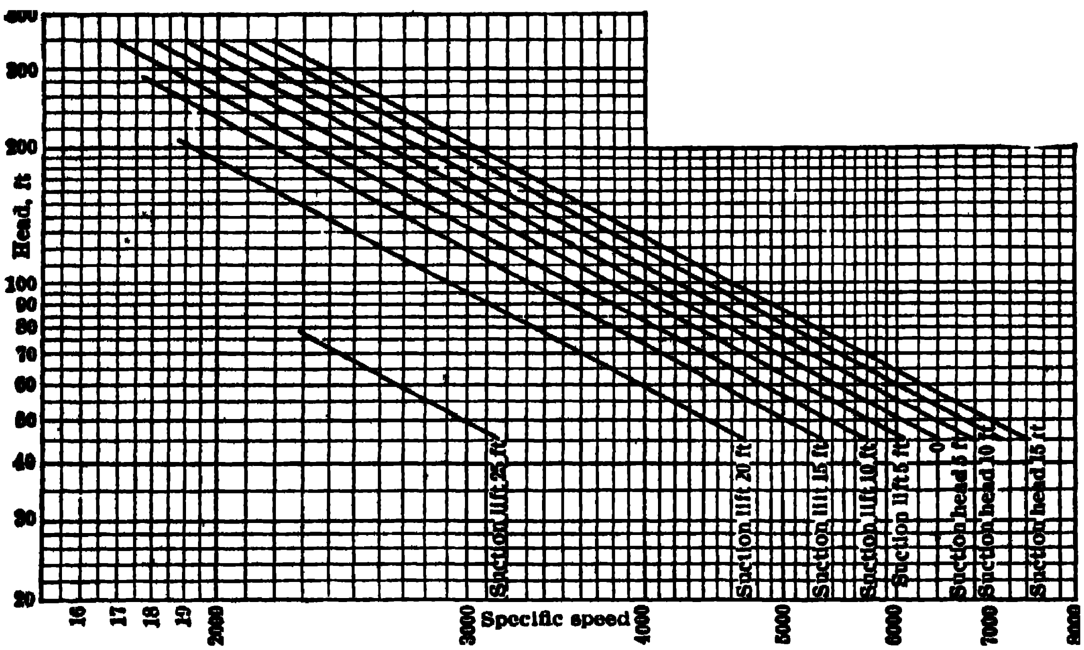


Fig 51. Upper Limits of Specific Speeds for Double-suction, Single-stage, Centrifugal Pumps (Hydraulic Institute). Normal water temp, 80° F, at sea-level

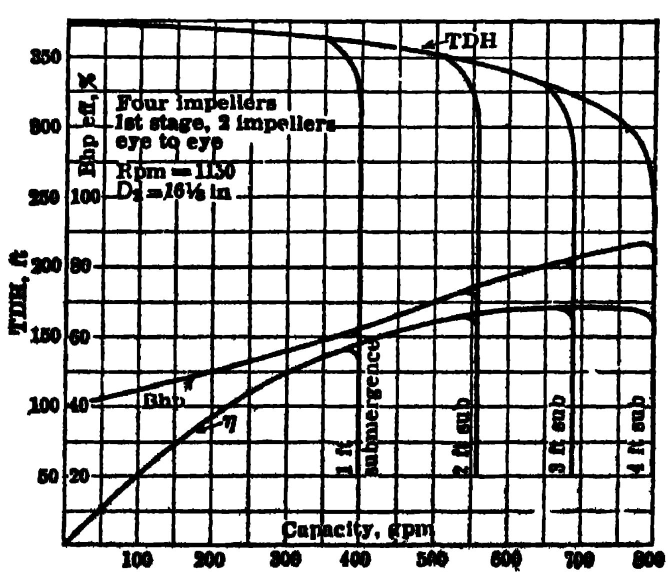


Fig 52. Hot-well Pump, 3-stage

curvature at heel and tip, axial blade length, radial depth, number of blades, sidewall details, casing proportions, inlet form and proportions, fluid properties and friction. Variations in design are reflected in the CHARACTERISTIC CURVES (Fig 47), which are definitive. A pump can operate only on its characteristic. If excess head is developed (capac less than A in Fig 47), the difference must be throttled, or speed altered; the latter procedure economizes power consumption, but the carrying charges for speed-control equipment are not always justified. APPROX PERFORMANCE can be estimated at different speeds by the relations: (1) capac is directly proportional to speed; (2) head, directly proportional to speed squared; (3) hp directly proportional to cube of speed. These relations apply to the same point on the effic curve and give characteristics for different speeds. The relations are not exact, because they do not reflect scale effect or Reynolds' Number influence. The relations are sufficiently accurate for practical purposes. Fig 48 shows a set of curves from test. The characteristics may be plotted on the percentage basis (Fig 49), or on the dimensionless coeff basis (Fig 50).

Specific speed (Fig 51), $N_s = rpm \times (gpm)^{0.5} \div \text{head}^{0.75}$, where head is in ft per stage. It is rigorously fixed by the pump design, and is the speed (rpm) at which a geometrically similar impeller would revolve to deliver one 1 gal per min under 1 ft head. High-speed impellers have high values of N_s. Spec speed is useful in pump selection, because it relates head capac and pump speed for best effic and absence of possible suction cavitation. Fig 51 contains recommendations for double-suction, single-stage pumps. Usual spec speeds are: (1) less than 4 000 for single-inlet impellers; (2) less than 6 000 for double-inlet; (3) 4 000–9 000 for mixed-flow impellers; (4) greater than 9 000 for axial-flow pumps. Impellers for high heads have low spec speeds; those for low heads, high spec speeds.

Fig 51 shows that centrifugal pumps are sensitive to suction conditions. With liquids at or near the flash point, positive suction head is necessary to prevent cavitation in the impeller eye; as demonstrated by the hot-well pump characteristics of Fig 52. For very viscous liquids, pump makers should be consulted. Centrifugal pumps are generally limited to 200–400 ft head per stage, and a tip speed less than 150 ft per sec. Multistaging (Fig 46) is needed for higher heads. Actual head per stage is from 0.9 to 1.3 $\times \left(\frac{U_2^2}{2g}\right)$. Effic is 50–90%, the higher values for large pumps of few stages. Head characteristic curves may be flat, steep, or rising. Hp characteristics may be straight inclined lines; or curved, with concavity upwards or downwards; the last has advantage of a self-limiting feature and avoids over-motoring. Steep head characteristics are best for parallel operation, as they give equitable load division.

Axial-flow propeller pump (Fig 53) has high speed, low head and large capac. Spec speeds exceed 9 000. Its characteristic curves (Fig 54) differ from those for centrifugal pumps; showing decreasing head and hp for increasing capac; max power is required at shut off. They are especially suited to such low-head service as condenser circulating water and irrigation plant. For higher heads they are multistage, with guide vanes between stages. Axial-flow and centrifugal types often merge in a single mixed-flow design, where the impeller eye approx equals the diam of the outer periphery; this permits combining in a single unit the best features of both.

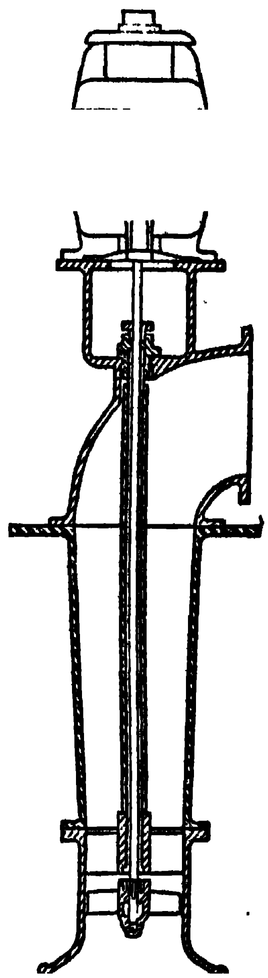


Fig 53. Axial-flow Pump

Cost of centrifugal pumps is affected not only by size or capac, but also by head pumped against, speed, number of stages, quality of construction, and commercial conditions (Table 26).

Cost of 4-in, single-stage pump. Single-suction: iron, \$200; brass impeller and brass-sheathed shaft, \$320; bronze ditto, \$425. Double-suction: iron, \$320; brass impeller and brass-sheathed shaft, \$575; bronze ditto, \$700.

Cost of 2-stage pumps is about 3 times that of single-stage, for same discharge and speed, but double the head. As number of stages increases, the cost approaches the number of stages × cost of a single-stage pump. For similar head, speed, and construction, cost varies directly as capac. Water-supply pumps cost about \$2 000 per million gal daily capac (DeLaval Steam Turbine Co).

Table 26. Average Cost of Iron Single-stage Centrifugal Pumps

Capacity, gal per min	A	B	C	D	Capacity, gal per min	A	B	C	D
100	$110	$160	$300	$400	1 500	$580	$840	$1 200	$1 400
200	180	270	400	500	2 000	675	1 000	1 425	1 575
500	300	430	660	800	5 000	1 325	1 800	2 375	2 600
1 000	440	650	1 025	1 150	10 000	2 600	3 800	......	......

Note.—*A* = low-head pumps, with iron impellers, single suction, belt-drive, and no performance guarantee. *B* = double-suction pumps, belt or direct-drive, but otherwise similar to *A*. *C* = pumps of better construction and heads up to 150 ft. *D* = pumps of fairly good construction, suitable for direct motor-drive.

For pumps suitable for high heads (as for mine drainage), the design is special and cost varies widely. Examples: 2 500 gal per min, 600-ft head, $10 000; 5 560 gal per min, 280-ft head, $50 000; 5 500 gal per min, 300-ft head, $55 000.

Cost per lb wt is nearly constant, decreasing slightly as size of pump increases; for small iron pumps, about 35¢ per lb; higher for brass and bronze-fitted.

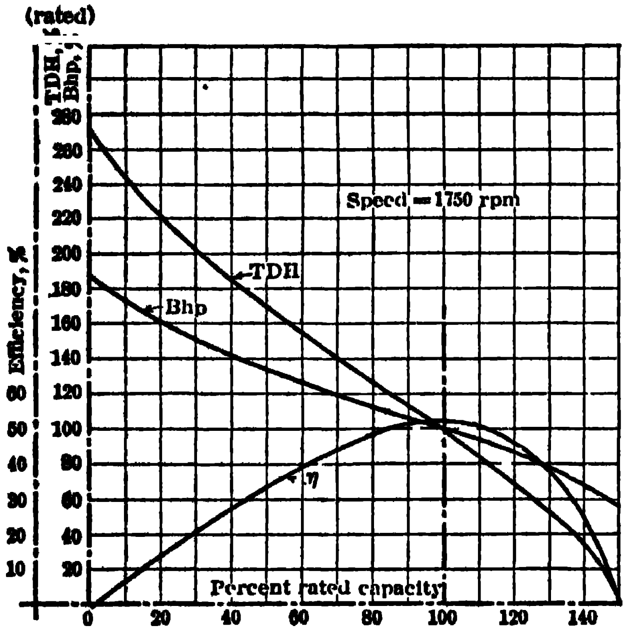


Fig 54. Characteristic Curves, Axial Flow, Propeller-type Pump, Percentage Rating Basis

12. INSTALLATION AND OPERATION OF PUMPS

Location. Pumps should be accessible, set on rigid foundations, with base slightly above the floor. Care must be taken in setting, not to exceed proper suction height (including losses in suction pipe), see Art 8. If bolted to foundations instead of being grouted, the base must be in perfect alinement when tightening the bolts. The pump shaft is alined with that of the driver by putting a steel scale on the coupling flanges. If alinement is accurate, the scale will rest on both flanges all around the surface. See that the clearance between the 2 half couplings is equal at all points of circumference. If bolts instead of grouting are used, make sure after the bolts are tight, that the pump and driver are still in line.

Piping. Piping should have as few bends as possible, and these should be of long radius. Introduce no unnecessary bends or other sources of friction in the suction line. Suction line must be free from air leaks, especially where it is long, or the pump is at some height above water level. Avoid air pockets in suction line; if any exist, provide means to get rid of the air.

Valves. In multiple-stage and single-stage high-press pumps, a CHECK VALVE is placed in the discharge line between gate valve and pump. Where there is a foot valve and possibility of water hammer, the discharge valve should be closed before shutting off power for stopping the pump. FOOT VALVE. Where the suction lift is not high, a foot valve is often advisable; it simplifies priming the pump. There must be a strainer to keep the foot valve from becoming choked. A foot valve is not advisable for a pump working against a high static head; on shutting off the power, the pump would stop suddenly, and the water rushing back might close the foot valve before the check valve could act, thus producing heavy water hammer. The foot valve should be of flap type, and of ample size to minimize friction.

Bearings must be cleaned before starting the pump for the first time, as foreign substances may get in during shipment or erection. They should then be filled with pure, clean mineral oil, which must be changed when it becomes dirty and the bearings thoroughly cleaned at same time.

Stuffing boxes and packing. Before starting the pump, the stuffing boxes should be carefully cleaned, and packed with enough packing back of the water-seal ring so that the inlet water for sealing is brought in at the ring and not at the packing. Piping supplying the water seal should fit tightly, so that no air can leak in; a little air entering here might cause the pump to lose its suction. If the water is acid or gritty, the sealing water must be obtained from some clear source.

of supply. Valves for the water seal must be open before starting up. If packing is too tight, it may cause burning of the packing and cutting the shaft. Packing-ring joints should be at angle of 45°, and the joints of the several rings staggered. On starting the pump, it is better to have the packing slightly loose (without causing an air leak); instead of putting too much press on it, place heavy oil in the gland until the pump works properly and then gradually tighten the gland. Since packing boxes are water sealed, see that the valves are open before starting; a slight leakage of sealing water through the gland is desirable.

Air in water may be liberated and collect in the pump passages. Hence, the air cocks on top of the casing should be opened occasionally; or, if much air, they may be kept partly open during operation, connecting them to drain pipes. If the delivery falls off without apparent reason, open the pump and examine its interior and the impeller passages, as foreign bodies may have been drawn into the pump.

Priming. Unless the water supply flows to the pump with sufficient head to fill the casing, the pump must be primed. PRIMING BY EXHAUSTER or ejector is convenient where steam or comp air is available. The discharge pipe must have a tight valve close to the pump, to allow air to be exhausted from casing and suction line. A foot valve is desirable on a steam exhauster, for priming the first time, or when the suction line has been emptied. PRIMING BY FOOT VALVE can be done where there is a supply of water available under pressure, by merely allowing the suction pipe and pump casing to fill. All pet cocks should be opened during the filling to allow escape of air. PRIMING BY VACUUM PUMP. Where steam is not available and it is impracticable to fill the suction line with water, a hand or power air pump may be used. A valve must then be placed on the discharge line. In priming multiple-stage pumps, the air must be exhausted thoroughly from each stage; if primed by exhauster, it should be connected to each stage.

Starting. The pump casing must be entirely filled before starting; if run empty, the clearance rings and shaft sleeves, which have very small clearances, will bind, heat, and cut. When first starting the motor, be sure that its direction of rotation agrees with that of the pump; pumps must run in the direction for which they are designed; this is usually stamped on the casing. After priming a centrifugal pump, the shaft should be turned over 1 or 2 rev, to allow all air to free from the impeller vanes.

13. INTERNAL-COMBUSTION ENGINES (see Sec 39)

Advantages for stationary power service: (1) when water supply is small or poor; (2) when coal is not available, or is high in price; (3) for small power requirements at high thermal effic, and with good reliability and reasonable first cost; (4) for poor load factors, because of low stand-by losses and quick starting.

Classification: (1) Otto, Diesel, or mixed cycle; (2) mixture or injection engines; (3) gaseous or liquid fuel; (4) volatile or non-volatile liquid fuel; (5) spark or compression ignition; (6) 2 cycle or 4 cycle; (7) air or solid (airless) injection Diesel engines; (8) high or low speed.

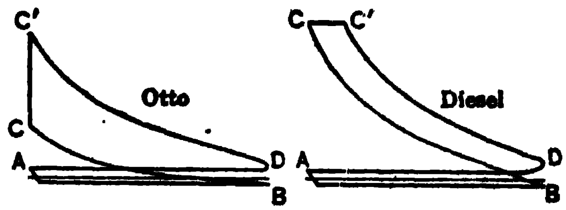


Fig 55. Otto and Diesel 4-cycle Indicator Cards

Four-cycle engine indicator cards for Otto and Diesel cycle (Fig 55). In the OTTO, a new charge is drawn into the cyl on suction stroke AB, and compressed into the clearance space on compression stroke BC, at or about the end of which it is ignited by an elec spark, and in burning causes the press rise CC', at approx dead center. The high-press gases formed by combustion expand during the third or expansion stroke C'D, driving the piston forward. On this stroke only is power delivered to the engine, enough being stored in the flywheel to run the engine until next power stroke. On exhaust stroke DA, the burned gases are forced out. In the DIESEL, a charge of pure air only is drawn in, AB, and compressed as in the Otto, but to a higher press, sufficient to raise the temp above ignition temp of the fuel, which is injected during the first part, CC', of next stroke, burning as it enters. Rate of injection determines slope of line CC', which should be nearly horiz. During remainder of stroke expansion occurs, followed by exhaust as in the Otto. The 4-cycle engine has 2 valves, inlet and exhaust, both mechanically operated.

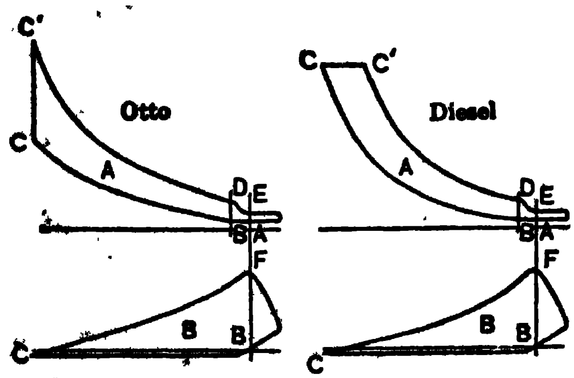


Fig 56. Otto and Diesel 2-cycle Indicator Cards

Two-cycle cards (Fig 56). A power stroke occurs once in each rev, the burned gases being removed, followed by a new charge during latter part of expansion and early part of compression strokes. Combustion occurs, CC', followed by expansion C'D, as in 4-cycle. At D an exhaust port is uncovered by the piston, allowing much of the gases to escape. A fraction of the stroke later, at E, a transfer port is uncovered, allowing a

new charge, previously compressed to 6–10 lb above atmos, to flow into the cyl, and thereby force out the products of previous cycle. On its return stroke, the piston closes transfer port at A and exhaust port at B, permitting compression to occur during remainder of stroke BC. During compression in the power cyl, suction BC occurs in the compressing cyl, followed by compression CF, while expansion occurs in the engine. Prior to admission to power cyl, the charge may be compressed in various ways: in small engines, by front end or step-piston compression; in large engines, by a separate compressor. Due to leakage at exhaust port during charging, 2-cycle type is generally limited to injection engines, in which fuel is not added until after compression is over, thereby eliminating fuel wastage through exhaust port. A 2-cycle engine, having twice the cycles for the same rpm, gives greater hp for same cyl size; but not twice as great, because the mep and effic are less than for 4-cycle. Chief advantages are: less piston area per hp, lighter flywheel due to greater frequency of impulses, and reduction of valves and valve gear; all resulting in smaller weight and cost.

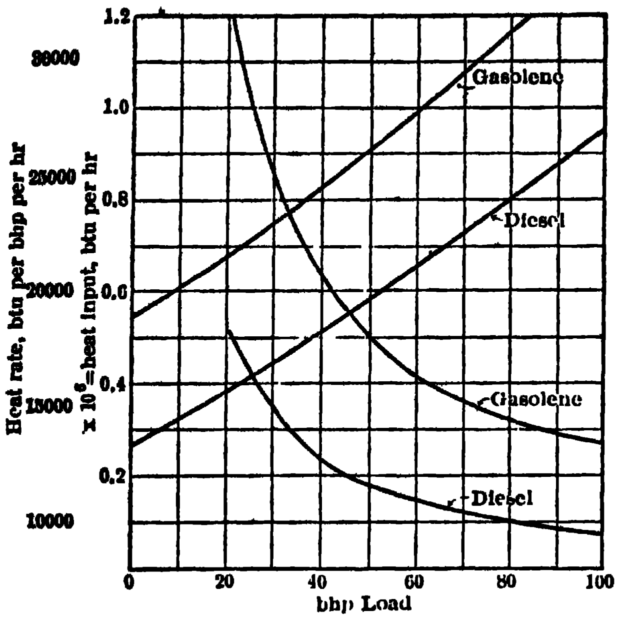


Fig 57. High-speed Gasolene and Diesel Engines. Comparative Performances (based on gasolene of 125 000 btu per gal; Diesel fuel of 142 000 btu per gal)

Externally-made-mixture engines operate on: (*a*) gaseous fuels, as natural, producer, or blast-furnace gas; (*b*) volatile liquid fuels, as gasolene, alcohol, benzol, or blends. The high speed, multicylinder, automotive engine, using gasolene, is the most important. Low and medium speed gas or gasolene engines are used in some stationary services.

Injection engines utilize non-volatile liquid fuel, wherein air only is compressed in the cyl and fuel injected near end of compression stroke; ignition is by the heat of compression. Low-compression, hot-wall engines were formerly used. Diesels now prevail; they may be solid injection (fuel directly injected by a pump), or, rarely, air injection (fuel injected by comp air at 1 000 lb per sq in). Air injection has been practically abandoned in favor of solid injection, due to trouble with high-press air, and improvements in solid injection fuel systems. The trend with all of these is away from low speeds and large cylinders, except where long life and minimum operating charges are sought. Multi-cylinder, high-speed design reduces wt and first cost. High-speed engines make little or no effort to follow Diesel or Otto cycle, but generally operate as a mixed cycle.

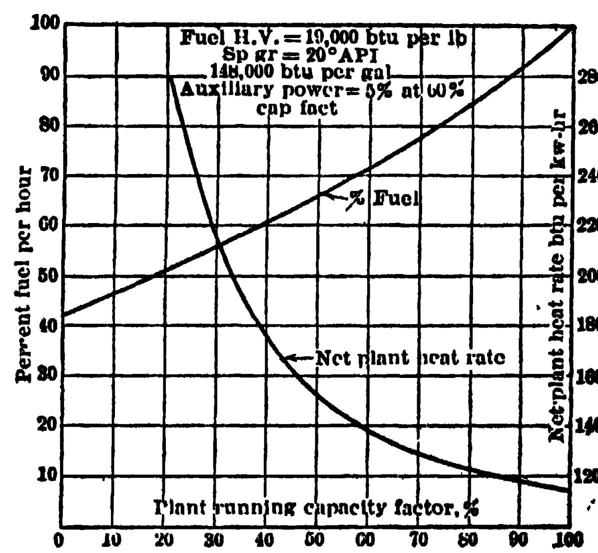


Fig 58. Diesel Plant Heat Rate

Power and efficiency. Analyses and heating values of fuels are in Table 10, Art 4; abbreviated ASTM Diesel fuel specifications, in Table 27. Performance data for different classes of engines are given in Table 28, and Fig 57, 58 show performance curves for stationary Diesel and gasolene engines. Lubricating oil consumption of Diesel plants is reflected by the relation: Gross kw-hr generated per gal lubricating oil = 24 × (plant running at capac factor, %).

Costs vary widely, due to differences in design and to competition. Prices of gasolene and kerosene stationary engine are: 1–6 hp, \$50–\$60 per hp; 6–10 hp, \$35–\$50; 10–25 hp, \$25–\$40 per hp. Vert, multicylinder medium speed, natural gas engines, \$35–\$40 per hp.

High-speed Diesel engine generator sets (10–100 kw). \$50–\$90 per kw. Low and medium speed Diesels, 4 cycle, \$50–\$75 per hp; 2 cycle, \$40–\$60 per hp.

Table 27. Diesel Fuel-oil Classification (ASTM abbreviated)

	Grade of Fuel				
	1-D	3-D	4-D	5-D	6-D
Saybolt Universal Viscosity:					
at 100° F minimum	35	35	....	....	....
at 100° F maximum	50	70	250	....	....
Saybolt Furol Viscosity, at 122° F max.	....	....	....	100	300 max
Pour point, °F max	35	35	35	....	....
Cetane No.	45	35	30	....	....
Diesel index	45	30	20	....	....
Flash point, °F min	115	150	150	150	150
Carbon residue, % by wt, max	0.2	0.5	3.0	6.0	....
Ash, % by wt, max	0.02	0.02	0.04	0.08	....
Sulphur, % by wt, max	1.5	1.5	2.0	2.0	no limit
Water and sediment, % by vol	0.05	0.1	0.6	1.0	2.0

Grade of fuel	Type of engine
1-D	Solid injection engines, running at more than 1 000 rpm
3-D	Solid injection engines, running at 360–1 000 rpm
4-D	Solid injection engines, cylinders more than 16 in diam, running at less than 240 rpm Air-injection engines, running at less than 400 rpm
5-D	Air-injection engines, running at less than 240 rpm
6-D	Used only in isolated cases

Table 28. Performance of Typical Internal-combustion Engines
(F. H. Dutcher, Columbia University)

Type			Fuel	Bhp	Compression ratio	Brake, mep	Piston speed, ft per min	Wt, lb per cu in piston, displacement	Wt, lb per bhp	Bhp hr per gal fuel
Mixture engines	Automotive engines		Gasolene	10–200	4.5–6.5	50–90	800–1 600	3–6	10–50	12±
Mixture engines	Automotive engines		Kerosene	10–200	3.5–4.5	40–75	800–1 600	3–6	15–55	12±
Mixture engines	Stationary gas engines		Natural gas	150–800	4–7	50–70	600–1 200	4–8	50–140	10 000 btu per bhp hr
	Solid injection, spark ignition		Diesel fuel	25–100	5–7	50–80	800–1 200	3±	12–15	14±
Injection engines	Air injection Diesel		Diesel fuel	300–5 000	12–15	50–75	600–1 000	4–8	25–200	18±
Injection engines	Solid injection, compression ignition	High speed	Diesel fuel	20–1 200	12–17	50–110	900–1 500	3.5–8	15–100	18±
Injection engines	Solid injection, compression ignition	Medium speed	Diesel fuel	50–750	12–15	40–75	800–1 500	3–6	20–100	18±
Injection engines	Solid injection, compression ignition	Low speed	Diesel fuel	100–5 000	13–14	40–75	600–1 000	4–8	25–100	18±

14. OPERATION OF INTERNAL-COMBUSTION ENGINES

Governing. External-mixture engines are practically all controlled by throttling the mixture on its way to the cyl, manually or by governor; injection engines, by varying amount of fuel injected.

Ignition for external-mixture engines is always by elec spark, the commonest system being the jump-spark, in which the secondary circuit of an induction coil is led into the cyl through the insulated terminal of a spark plug. At the proper moment the primary circuit is closed and a series of sparks jumps a gap in the secondary circuit at end of plug. Injection engines require no

ignition system, except in the hot-wall type, where an uncooled portion of the cyl heats the fuel to ignition point.

Starting devices. Large engines are started by admitting comp air to one or more cyls at proper time, until a mixture has been drawn into and fired in one of them, whereupon the air is shut off. Small engines are started by turning them over by hand until an explosion occurs, or by elec motors. Automobile engines have elec starters, which become generators and supply current for ignition, storage battery, and lighting.

Cooling is by water jacketing the cyl, except in very small sizes where, by ribs on the cyl, enough radiating surface is obtained for cooling. In larger sizes, the exhaust valves, and in double-acting engines, the pistons and rods, are also water-cooled. Cooling water must not deposit much salt on heating; dirty or hard water fills the jackets with mud or scale, causing overheating. Water may be allowed to go to waste, or artificially cooled; 3–5 gal per hr per hp are generally sufficient.

Carbureters prepare explosive mixtures from gasolene or alcohol. Strictly speaking, the term carbureter is limited to apparatus where no external heat is applied, those requiring heating being called vaporizers. But, with the use of heavier grades of fuel, many so-called carbureters now have means for applying heat, from engine-jacket water or exhaust. Carbureters proportion the fuel and air, vaporizing the fuel to some extent and mixing the two, by allowing fuel to flow from a reservoir, in which the level is kept constant, into the air-inlet pipe. As the press in this pipe drops with increase of air flow, more fuel will flow. If air and fuel followed the same law of flow with press drop, a carbureter adjusted for any condition of load or speed would be correct for all conditions. But, flow of fuel increases faster than that of air, tending to cause too rich a mixture at heavy loads or high speeds. Most carbureters use a manual or automatic device, or a combination of these, for introducing extra air when conditions normally cause a rich mixture. The fuel reservoir is called the float chamber, when the liquid level in it is regulated by a float. This device is limited to engines in which the supply is under a gravity or press head. For stationary engines insurance regulations usually forbid its use, in which case a pump forces fuel to the reservoir in excess of the engine demand, surplus returning to the tank. The fuel flows from the reservoir through a spray nozzle, controlled by a needle valve.

For light fuels the heat in the atmos suffices for vaporization; for heavier grades, heat is applied by drawing the air partially or wholly from a jacket on the exhaust pipe, or by allowing the engine-jacket water to pass through a jacket on the carbureter. Excessive heating is bad, as engine power is proportional to density of the charge. Incomplete vaporization, especially in multi-cyl engines, is apt to cause a non-homogeneous mixture. For mixing action, carbureters depend chiefly on the throttle and valve obstruction, and on bends in the supply pipe.

15. GAS PRODUCERS

Construction. A gas producer is a metal shell, lined with fire brick and having provision for supporting the fuel bed and blasting it with air or a mixture of air and steam. Fuel bed is 4 ft or more thick, comprising a zone of ash at bottom, next a high temp zone of burning fuel and finally a layer of fresh coal. The bed may be supported on a grate, fixed or revolving; or on a concrete pier rising from a pool of water, into which the sides of the producer extend, thus forming a water seal through which ash is raked out. Fuel is charged through a hopper, to prevent outrush of gas or inrush of air.

Operation consists in roasting off volatile matter, and then oxidizing the carbon to CO or CO_2 by the O in the air and steam; H of the steam is set free, and the ratio of CO to CO_2 depends on temp and thickness of the bed. Steam reduces the temp, the exothermic reaction of the oxidation of the C being partly neutralized by the endothermic reaction of breaking down the steam. Without steam the hemp is usually high enough to fuse the ash, forming clinker and preventing flow of air through the bed.

Fuels. Though any fuel rich in carbon may be used, those containing much volatile are disadvantageous, due to variation in gas quality and difficulty in removing tar and soot formed by reduction of the heavier hydrocarbons. Coking coal is unsatisfactory, as it tends to block the flow of gas like clinker; some producers have mechanical pokers to break up the cake. Fuels used in producers: anthracite or bituminous coal (non-caking), coke, lignite, charcoal, oil, wood, peat, and waste products.

Blast may be forced through the bed by building up press before the bed is reached, as in PRESSURE PRODUCERS, or by creating a deficiency of press beyond the bed, as in SUCTION PRODUCERS. In press producers (commoner in the larger sizes), the blast is caused by a steam jet or positive blower (usually the former, as steam is required in the blast and the blower serves as a good mixing device). In the suction type, the engine piston draws the blast through the producer (a small auxiliary blower being used for starting). Though this type requires no gas holder, it is disadvantageous to employ the engine for producing blast, and a rotary blower or fan is generally used to draw gas from the producer and deliver it to the engine. The suction producer is limited to the better grades of fuel, as anthracite and coke, but is safer, because, as leaks are inward only, the burning of some gas in the producer is all that can result; whereas leak in a press producer allows escape of CO_2, or may form an explosive mixture in the room or building. The draft may pass up through the bed, UP-DRAFT PRODUCER; downward, DOWN-DRAFT PRODUCER; or both ways, COMBINED-DRAFT PRODUCER. In the two latter cases, the volatile matter passes through the combustion zone and is reduced to staple forms or is completely burned.

Producer auxiliaries. The term PRODUCER SET covers producer, auxiliaries, and engine. The most important is the cleaning and purifying device, usually comprising a scrubber and filter or tar extractor. The SCRUBBER is a chamber filled with coke or similar material, through which the gas rises while water passes down; sometimes it is a pipe, up which the gas travels, passing water sprays at short intervals. FILTERS, of saw dust, excelsior, or mineral wool, extract tar and other liquids, but are only partially successful. TAR EXTRACTORS, other than filters, are centrifugal fans or beaters for churning the gas and water, causing tar particles to collect in drops, which float away. A recent filter depends on static elec action in causing minute particles of tar to coalesce into drops large enough to fall by gravity in an ordinary separator; tar so recovered is water-free, and has value as a by-product. Suction producers must also have apparatus to supply steam for humidifying the blast. As no press is required, the steam may be made in the producer jackets, or in an evaporator heated by the gas on its way to the scrubber. Proportions of steam and air must be controlled; best done by saturated air, but of different temp. For a press producer, a separately fired boiler generates steam for the blower.

Mond producers recover ammonia from the coal. An excess of steam is supplied to the producer, reducing the temp of the reaction and hence increasing the CO_2 content, but preventing breaking down of the ammonia. After purification the gas passes to a scrubber supplied with dilute H_2SO_4 instead of water, which absorbs the ammonia as sulphate, and upon evaporation solid sulphate is obtained.

Rating of producers is more uncertain and difficult than that of boilers; the term horsepower of a producer is incorrect, as no mechanical work is done. However, a producer is usually designated as being of a certain hp, meaning that it will supply gas for an engine of that hp. On the basis of 1–1.5 lb coal per hr per engine hp, and a rate of combustion of 7–10 lb per sq ft of producer grate surface (figures realized in aver units of both types), a producer will supply about 7 hp per sq ft of grate.

Table 29. Producer Dimensions (Smith)

Hp rating	Coal per hr, lb	Space required, ft			Hp rating	Coal per hr, lb	Space required, ft		
		Length	Width	Height			Length	Width	Height
50	65	12	7	20	175	220	15.5	9.5	22
75	95	12.5	7.5	20	200	250	16	10	22
100	125	13	8	20	250	315	17	11	22
125	160	13.5	8.5	20	300	375	18	12	22
150	190	14	9	22	.....	......	. .	..... .	......

Producer costs. Following figures are from actual installations before 1917:

Suction producers, up to 300 hp,	$252 + (14 \times hp)$	**Note.**—For approx 1938 costs, double the figures obtained from above formulas.
Pressure " " " 300 "	$860 + (15 \times hp)$	
Suction " and auxiliaries, 0–200 hp,	$570 + (46 \times hp)$	
Suction producers and auxiliaries,	$150 + (11 \times hp)$	
Pressure " " " to 200 hp,	$1\,000 + (16 \times hp)$	
" " " " over 200 hp,	$2\,000 + (15 \times hp)$	

16. APPARATUS FOR TESTING POWER PLANTS

Codes governing the testing of power-plant machinery are published by Amer Soc Mech Engs. Following suggestions, as to methods and procedure, may assist in laying out and running tests.

Preparation. Define the object of test, and decide upon the limit percentage of error allowable in the results. Determine quantities to be measured, by analysing the expressions denoting the final results (generally compound units) into quantities of a single dimension, since there are very few methods or instruments for measuring quantities of more than one dimension. Thus, the determination of water rate calls for the wt of dry steam per hr per ihp, and the measurements necessary are: wt of steam used, dryness factor, length of run, mep in the cyl, length of stroke, area of piston, and strokes per min.

Choice of instruments and methods of measurement depend upon the limit percentage of error allowable. If an aggregate error of 20% is permissible, apparatus of extreme accuracy is unnecessary, but, if results will be worthless unless accurate within a very few per cent, it is useless to proceed except with accurate methods.

After deciding upon the method, a DIAGRAMMATIC SKETCH is made of the machine to be tested, with all instruments shown in their relative positions. This is a guide in setting up the apparatus. A log of all of the readings is then drawn up, with a column for each variable and space for each constant. The log should also contain spaces for description of apparatus, date of test, names of observers, and remarks concerning unusual occurrences. Completeness of the log and sketch

should be proved by seeing that there is a place for every instrument reading, and that the formulas used for calculating final results can be fully satisfied for the log readings.

Pressure measurements. Ordinary press units are: lb per sq in, in of mercury, in and ft of water. 1 lb per sq in = 2.0359 in mercury at 32° F = 27.71 in or 2.309 ft water at 62°. Lb per sq in is used for all ranges of press, except vacuum and press to 10–15 lb above atmos, which are expressed in inches of mercury. Furnace draft, press in ventilating ducts, and other small pressures, are expressed in inches of water. For hydraulic work, ft of water are used almost exclusively.

Pressure gages of the Bourdon tube type, with scales ranging from 30 in vacuum (15 lb press) to several thousand lb, are of indicating or recording form. Gages should be CALIBRATED before and after use. The most reliable gage tester is the "dead-weight" tester. Measurements by mercury manometer, of either U-tube or cistern form, are standard if the mercury be pure. Density of mercury is altered by dissolving tin in it; evidence of such adulteration is found by subjecting mercury and water columns to the same press, and comparing the ratio with that for the pure liquids. Pure water should be used in water manometers, but ordinary impurities in fresh water will not materially affect readings. For measuring very small press at temp below 32°, other liquids may be used. Such readings should be transposed into inches of water, by direct comparison with a water manometer, or by measuring the sp gr of the liquid by a hydrometer. Very small press may be measured by a water manometer with tube inclined at a considerable angle to the perpendicular, thus causing large longit travel of the surface of the liquid for small change in elev.

Temperature measurements are made by the Fah scale by engineers, though chemists use the Cent scale. 1° F = $^5/_9$° C. Temp in °F = temp in °C × $^9/_5$ + 32. (For conversion table, see Sec 37.) The mercury thermometer, generally used for measuring temp, has a range of scale from −40° to 1 000° F. Indicating and recording instruments are also made, in which the actuating portion is a bulb filled with liquid or gas, connected by a flexible tube to a Bourdon tube which moves the recording pen. Elec pyrometers, indicating or recording, are of the thermo-couple or resistance type, for all ranges of temp to 3 000° F. Optical and radiation PYROMETERS are for measuring temp from 1 000° F up. High temp is sometimes gaged by the fusing of materials having well-defined melting points. The temp of a gas flowing through a flue may vary considerably in different parts of the cross-sec, and the true aver is hard to determine. Also, the instrument must not be exposed to radiation from some object at a temp higher than that of the gas. Failure to shield the instrument from such radiation may cause very large errors. As gases are transparent they will not radiate heat, and, if optical or radiation pyrometers are used, a solid object must be placed in the gases at the point where temp is to be measured and upon which the instrument is sighted. Such an object should be dark colored; all optical or radiation pyrometers are calibrated for a black object.

17. MEASUREMENT OF WEIGHT, DIMENSIONS, SPEED AND POWER

Measurement of wt is made by lever or spring balance. Spring balances are often more convenient, but their length of scale is limited and percentage error relatively high. After long use their accuracy is more questionable than that of lever balances. Wt may be determined indirectly by measuring the vol of an object when its sp gr is known.

Length, area, and volume. Small irregular areas on drawings may be measured directly by a planimeter. Irregular volumes may be determined by weighing in air and immersed in water; from which volume is computed. Differences in altitude may be determined by measuring the difference in press in a water pipe running from one point to the other. The water must not be flowing when measurement is made.

Speed. Many types of stroke or revolution counters are used. Some are attached to the machine; others for hand use. They are the most reliable for general service, when the speed varies. TACHOMETERS indicate or record directly rev per min. They operate by the centrifugal force of solids or liquids, voltage of an elec generator, or synchronous vibration. They should be calibrated before use in important work, and their readings checked by a revolution counter. They are unsatisfactory for variable speeds, because averages must be estimated. Chronographs make a graphical record of each rev, with simultaneous time record.

Power measurements. Standard unit in the U S is the hp = 33 000 ft lb per min = 0.7457 kw. Power developed in cyls of engines, pumps, or compressors is indicated horsepower (ihp). It is measured by an engine INDICATOR, a special form of press gage which records on a piece of paper wrapped on a drum, caused to follow the motion of the piston. In nearly all indicators the press-recording mechanism is driven by a spring-opposed piston, the cyl of which is connected with the engine cyl. Springs have a wide range of strength, and the scale marked on the spring indicates that the recording pencil will rise 1 in for that unit press. Mean effective cyl press (mep) is the aver height of the recorded diagram, inches, multiplied by scale of spring used. The indicator drum, usually 1.5–2 in diam, is driven from the engine crosshead, with the necessary reducing motion. The indicator is connected to the cyl through a special cock, which has a standard 0.5-in pipe thread on one end, the other end being special for each make of indicator. Indicators are often used for recording rapidly fluctuating press in pipes, the drum being designed so that a continuous roll of paper is passed over it.

Power output of machines is measured by absorption or transmission dynamometers. The absorption dynamometer or PRONY BRAKE is easily made in any shop, the details of its design being governed by the work to be done. Fig 59 shows a Prony brake made of wood blocks held together by steel bands. The arms are of pipe or bar iron. Friction,

is adjusted by the handwheel. Bhp = $2\pi L \times$ rev per min $\times W + 33\,000$, where W = lb wt registered on the scale, less the unbalanced wt of the brake and its support, and L = length of brake arm, ft.

Prony brakes are seldom used to absorb more than 50 or 75 hp, the limitation lying in the difficulty in removing the heat generated; this may be met by water-cooling the brake, but best by using a brake pulley with an internally-flanged rim, which when running will hold a body of water in contact with the pulley rim. Another absorption dynamometer is the ALDEN BRAKE, made with a rotating steel disk surrounded by a casing. Within the casing are copper plates, pressed against the disk by water press. Power is determined by measuring the torque of the casing, the heat generated being carried off by the water.

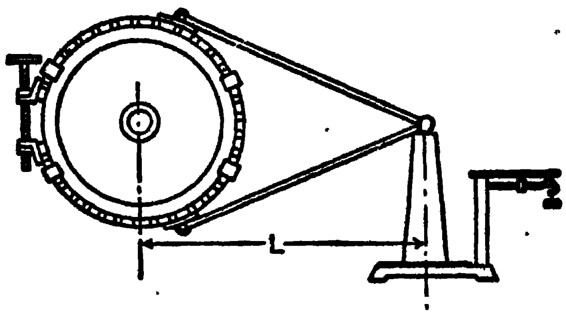


Fig 59. Diagram of Prony Brake

A number of HYDRAULIC BRAKES are on the principle of a centrifugal pump, with vanes on both casing and motor. Power is absorbed by the eddy-current friction, and measured by the torque of the casing. These brakes have been made to absorb over 5 000 hp. At low speeds their capac is low, but increases about as the cube of the speed.

An elec generator may be used for measuring power, if its effic is known. ELECTRIC BRAKES are on this principle, with the generator stator hung on ball bearings, so that the torque can be measured. By running them as motors they will accurately measure power input to a machine.

18. MEASUREMENT OF FLOW OF WATER AND STEAM

Measurement of water or other liquids. The most accurate mode of weighing liquids is in tanks on scales. When measured volumetrically, by change of level in a calibrated tank, the area of the water surface should be as small as possible at the points where depth measurements are taken, so that a small error in depth will not make a large error in vol. Flowing water is measured by weirs, Venturi meters, pitot tubes, orifices, and nozzles. Piston and disk meters, also used for small and intermittent flows, should be set up for easy calibration. Disk meters will not handle hot water.

Measurement of steam flow should be made in the form of water, either before it is fed to the boiler or after it leaves a condenser. STEAM METERS operate on the pitot tube, Venturi meter, orifice, or float principle. Chief source of error is in the recording or indicating device, which, for accuracy, should correct for press and moisture content or superheat. A steam meter is calibrated by placing it in a steam line from a boiler, the feed water being weighed, or in the line to an engine exhausting to a condenser.

Measurement of gas flow is more difficult than that of steam, as the meters are not so easily calibrated. Small quantities of gas can be measured in gasometers or gas holders, and wet or dry gas meters give fair accuracy for small flow. But, large quantities of gas must be measured by a RATE-FLOW METER. Pitot tubes, Venturi meters, orifices of various forms, and anemometers, are used. The standard meter for measuring gas is the Thomas elec gas meter, in which the gas is heated by an elec resistance coil and amount of current measured. The wt of gas flowing can then be computed from the rise in gas temp and the specific heat of the gas. The apparatus is expensive, and is generally used for calibrating the simpler and less costly forms of meters.

Steam calorimeters. For most work a throttling calorimeter is best. It consists of a thoroughly lagged metal chamber (often made of pipe fittings), with a thermometer cup at the top and an opening to the atmos at bottom. It is connected close to the steam line and steam flows into it through a small orifice. Total heat in the steam is computed from amount of superheat indicated by the calorimeter thermometer. As the principle of operation depends upon the difference in total heat, the range of this instrument is limited, about 4% moisture being the max at 100 lb steam press and 5.5% at 200 lb press. For very wet steam a separating, or combination separating and throttling calorimeter should be used. The chief source of error in steam calorimetry lies in the difficulty of obtaining a truly representative sample of steam; hence care should be taken in placing the sampling nipple.

Fuel calorimeters comprise those which operate continuously and those operating on single charges. Continuous calorimeter is used for gas and light fuel oils. The common form resembles a small fire-tube boiler, set vertically, with an internal fire box. The fuel burner is placed within the fire box, air for combustion being drawn in at the bottom. Water circulates around the tubes, and the heating surface is sufficient to allow gases of combustion to be cooled to atmos temp. The relative quantities of water and fuel are so adjusted that the rise of water temp is 20–30° F. The btu per lb or cu ft of fuel are computed from measurements of amount of fuel burned, wt of water heated, and the rise in water temp. The discontinuous type of calorimeter (bomb calorimeter),

is for solid fuels and heavy oils. A weighed amount of fuel is placed in a heavy chamber or bomb, together with O gas under press, or a chemical containing O to support combustion. The bomb is placed in a fixed wt of water, in an insulated chamber, and the charge fired by an elec spark. The rise of water temp measures the heat units set free.

19. CONTRACTS

When contracts for power machinery contain a clause guaranteeing the performance, the wording should be such that its meaning can not be mistaken, and that the operating conditions are clearly specified under which the performance is to be made. Unless otherwise agreed upon, the acceptance test is run by the engineers of the purchaser. The contractor is allowed to have a representative present, who shall have access to all readings, though without right to interfere in the running of the test; but he may see that the machine is properly adjusted before the test is begun, and in case of boiler tests he is generally permitted to direct the firing, but not to furnish the firemen. The engineer in charge is responsible for maintaining the operating conditions for which the guarantee is made. When there may be difficulty in maintaining such conditions, some agreement should be reached by the contracting parties as to allowances made in fulfillment of the guarantee on account of such variations of conditions. When it is necessary to make measurements by other than generally accepted methods, a mutual agreement on the matter should be reached before the test begins. All readings should be entered upon the official log sheet, which should be retained in its original form, because, in case a lawsuit develops upon the results of the test, that sheet alone has a legal standing in court as evidence.

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SECTION 41

MECHANICAL ENGINEERING MISCELLANY

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MECHANICAL ENGINEERING MISCELLANY

1. GEARING

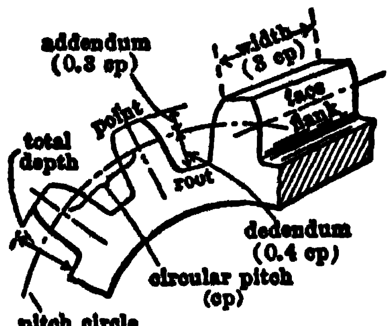


Fig 1. Involute Gear Teeth

Characteristics. The term as here used means tooth gearing; friction gearing and other special forms will not be considered. According to the shape of tooth, gears are divided into involute and epicycloidal; former has many advantages, and the latter is little used. PITCH CIRCLE (Fig 1) is the base to which all gear calculations are referred. The pitch circle diams of 2 intermeshing gears are equal to the diams of 2 equivalent cylinders, rolling in contact without slipping. CIRCULAR PITCH is the distance measured on pitch circle from center to center of teeth; hence cir pitch $= \frac{\text{pitch circumference}}{\text{no of teeth}} = \frac{\pi d}{N}$, where d = diam of pitch circle and N = number of teeth in the gear. Circular pitch is not convenient for determining number of teeth or pitch diam; hence, the DIAMETRAL PITCH is used, which is the number of teeth per in of diam, and is either a whole number or a simple fraction. Then, no of teeth = pitch diam × diametral pitch, and cir pitch $= \frac{\pi d}{N} = \frac{\pi d}{d \times \text{diametral pitch}} = \frac{\pi}{\text{diametral pitch}}$ In this way gear proportions are readily calculated. Thus, for a 2 diametral pitch, if the wheel has 20 teeth the pitch diam is 10 in; or, if a wheel has 60 teeth and the pitch diam is 20 in, the diametral pitch is 3, and the circular pitch is $3.1416 \div 3 = 1.05$.

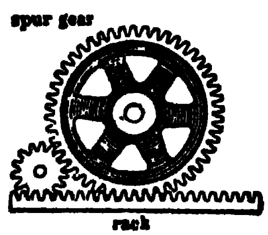


Fig 2. Spur Gears and Rack

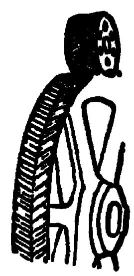


Fig 3. Herringbone Gears

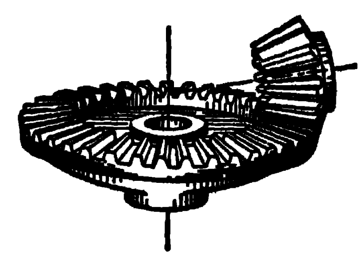


Fig 4. Bevel Gears

Spur gears are gears in one plane, with their axes parallel. If the radius of pitch circle is infinite, the circle becomes a straight line, resulting in the RACK. Spur gears having the same pitch are interchangeable (Fig 2). HERRINGBONE GEARS are used when end thrust on shafting is to be avoided. Their teeth are cut at an angle with the axis (Fig 3).

Bevel gears are those of which the axes are in one plane and intersect; they are called miter gears if both are of the same diam. Bevel gears are not interchangeable for the same pitch, but must be made in pairs (Fig 4).

Worm and worm-wheel gears are gears with axes at right angles and in parallel planes (Fig 5). They must be made in sets.

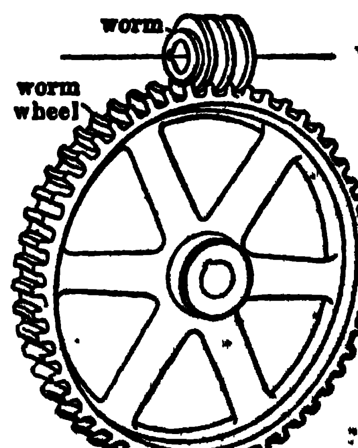


Fig 5. Worm and Worm-wheel

Construction. Metal gears are usually of C I or steel; brass, white metal, raw-hide, paper, and cloth, are used for special cases. Teeth cut in the solid rim give the most perfect form of tooth,

and are known as CUT GEARS. If cast in a sand mould the tooth outline is less accurate than in the cut gear; in operation they are noisy, noise increasing with speed. Small gears are cast in steel moulds under pressure, known as diecasting. They are accurate in form, but can be made only of soft metal and commercially must be duplicated in large quantities because of the expensive dies. Gearing is also made of the inserted tooth type. The teeth, of maple wood, are inserted in a C-I rim; one wheel of a pair has wooden teeth, the other, ordinary metal teeth (Fig 6). Advantages: they run quietly and in case of accident the wooden teeth, which are the ones to be damaged, are readily replaced. Wooden teeth are made in coarse pitches only. They are wider on the pitch circle than iron teeth, in the proportion of 0.6 circular pitch for the wooden to 0.4 circular pitch for the iron, making strength and wear more uniform. Gears of large sizes are made in halves or sections, bolted together, and can therefore be placed anywhere on a shaft, not necessarily at the end.

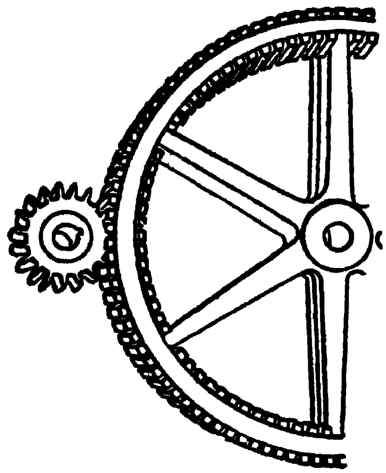


Fig 6. Wooden-tooth Gear

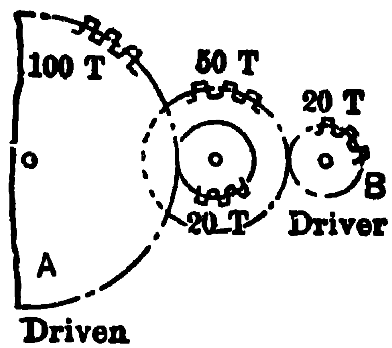


Fig 7. Compound Gear Train

Spur-gear ratios for single reduction of speed seldom exceed 6 : 1; in some special cases as high as 10 : 1. When greater reduction is required, gear trains may be used; in that shown in Fig 7 the driver has 20 teeth and meshes with a 50-tooth wheel, giving a 2.5 : 1 reduction; attached to the same shaft as the 50-tooth wheel is a 20-tooth wheel, forming a COMPOUND GEAR. The latter wheel meshes with a 100-tooth wheel, giving a 5 : 1 reduction, or a total of $2.5 \times 5 = 12.5 : 1$.

A handy rule for calculating gear ratios in compound trains is $\frac{\text{Product of the drivers}}{\text{Product of the driven}} =$ gear ratio; in which diameters, numbers of teeth, or rev per min may be used. Where single reductions of 3 or 4 : 1 are made, the teeth on small gears wear much faster than those on the large. To equalize wear, the smaller or pinion gear is made of harder metal, as steel, while the larger is of C I. When the number of teeth are exact multiples, any tooth on the pinion will always mesh with the same tooth on the large wheel, and, in case there were opposed hard and soft spots in the teeth, wear would be rapid. But, where exact gear ratios are unnecessary, as for triplex pumps, the larger gear has an extra tooth called a HUNTING TOOTH; thus, if the pinion has 20 teeth and the large wheel 101 teeth, every tooth on the pinion will come in contact with every tooth on the large gear, thus distributing the wear more evenly over both wheels.

Fig 8

Strength of gear teeth. Weakest part of a tooth is at the root in the smallest wheel (a, Fig 8), the root being bounded by the radial lines, as shown. Formulas for strength of teeth are constructed upon this basis, the smallest number of teeth being 12. Stub tooth gears have overcome this objection and even 5-tooth pinions can be cut. The root width a and the strength increase with the number of teeth. Width of face for good contact is generally assumed to be 3 times the circular pitch (sometimes exceeded). Spur and bevel gears are obtainable of almost any size, stock sizes having pitches from 0.5 to 46 diametral pitch. Selection of proper pitch depends upon size of gear and power transmitted; in general, the larger the gear the greater the power transmitted and the coarser the pitch. WORM-GEAR REDUCTIONS are much greater than for other types of gearing. If the worm is a single thread, for each revolution of worm the worm-wheel moves 1 tooth; for a double, triple, or quadruple thread, the wheel advances 2, 3, or 4 teeth. The wheel should have at least 25 teeth, and as the diams of worm and worm-wheel have no influence on the gear ratio, the correct proportions must be carefully worked out for efficient operation. In makers'

catalogues, ratios of 15 : 1 for quadruple threads, or 129 : 1 for single thread, are quoted. In all gearing the ratios are positive; there is no slipping as with belting.

2. BELTING

Principles of pulley and belt transmission. Since slippage always occurs, exact speed ratios can not be obtained. Resistance to slippage depends upon the coeff of friction, area of surface in contact, and pressure on pulley face due to belt pull. The effective pull is equal to the difference between the tensions on the tight and slack sides of the belt. The angle over which contact takes place is the angle of wrap. Coeff of friction depends upon the materials in contact and the condition of the two surfaces. For ordinary iron or wooden pulleys and leather belting, the coeff is 0.3 to 0.4 aver for dry surfaces, and say 0.15 to 0.2 for wet or oily surfaces.

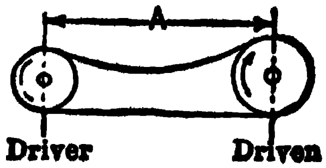


Fig 9. Favorable Belt Drive

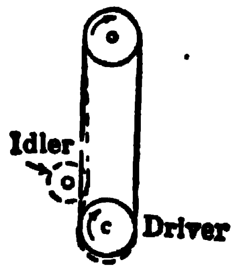


Fig 10. Unfavorable Belt Drive

For efficient operation the friction coeff must be as large as possible. It is evident that the best drive is that shown in Fig 9, that is, with the tight side below, since any stretch in the belt increases the angle of wrap, one of the power factors. The worst condition is shown in Fig 10. The lower pulley being the driver, any stretch in the belt causes slipping and excessive wear. In practice this form of drive requires an idler pulley to maintain proper tension.

Power transmitted by a belt depends first upon the difference in tension between tight and slack sides, and second upon the belt speed. With surfaces in good condition, and 180° angle of wrap, the ratio of tension on tight side (T_n) to tension on slack side (T_0) averages about 2.5, that is, $T_n \div T_0 = 2.5$. This ratio varies from 2 in small sizes to 3 in large, the pulley diams being equal or nearly so. To solve this formula either T_n or T_0 must be known. T_n is obtained by finding the breaking strength of a piece of belting, from which is determined the safe working stress. Since belting varies in thickness, the tension per in of width is generally given, rather than tension per sq in of cross sec.

Belt speed. Increase in speed, other conditions unchanged, increases power delivered, up to a certain limit. But, as the belt travels over the pulley, centrifugal force throws it away from the pulley face and partly neutralizes the driving force. The limit is reached at a speed of about 5 000 ft per min; any further increase of speed decreases the power transmitted. Hp = [pull of belt (lb) × veloc (ft per min)] ÷ 33 000. In practice, 3 000 to 4 500 ft per min are aver speeds. Velocity must sometimes be sacrificed, due to the large diam of pulleys required for given rev per min of shafting, and the belt width is increased to transmit the required hp. Belting is usually of leather, or several plies of canvas covered with rubber and pressed together. Woven belts, or thin steel bands, are also used.

Leather belts are of single, double, or triple thickness. Makers' catalogues give tables of the hp transmitted by different kinds of belting at given speeds, the figures being based upon an arc of contact of 180°, and an aver working tension per in of width found satisfactory in practice, viz: 40 lb for single, 70 lb for double, and 100 lb for triple thickness. Thus, if a belt running at 3 000 ft per min is required to deliver 60 hp, the total pull would be $P = 33\,000 \times 60 \div 3\,000 = 660$ lb. For a double belt a pull of 70 lb per in of width is allowable, giving $660 \div 70 = 9.5$, or say a 10-in belt. It is good practice to design a belt drive capable of transmitting 25% excess power for small sizes and 5% excess for large drives. One maker recommends the following rule, which includes reserve power: for single-thickness leather belt, the hp per in of width = belt speed in ft per min ÷ 800; for double thickness, divide the speed by 500.

Leather belting should be cleaned and dressed with a reliable compound to keep it soft and pliable, and be systematically inspected. Belts exposed to dampness or acid fumes require special treatment in manufacture. If a belt is stretched to its limit to transmit a given hp, its life is short. Since tension produces a heavy pull on shafting and bearings, causing friction and wear, it should be a minimum for the work to be done. Low tension gives long life and increased first cost; high tension, a short life and lower first cost, but with more stoppages for belt repairs. Shafts can be driven at almost any angle, if provided with idler or guide pulleys. DISTANCE BETWEEN SHAFT CENTERS: if ample, the weight of belt on slack side produces the necessary tension, and stretch increases

angle of contact; if short, the belt must be stretched for necessary tension. Slippage, which will occur after a short time, due to increased stretch, can be overcome by an adjustable idler pulley on slack side, close to the driving pulley, so as to increase arc of contact. Short-center drives, capable of heavy power transmission, are now built with special type of idlers. If the distance between pulley centers is excessive, the tight side will sag, decreasing the angle of contact, and the belt is liable to flap, causing the faces to rub on each other. To get proper tension when putting on new belting some makers recommend cutting out 2 in of belt for every 10 ft of tape line measurement. For running shafts in opposite directions, crossed belts (Fig 11) are used, but should be avoided if possible, because the belt rubs at point of crossing. For other types of belt drive, the makers should be consulted. Leather belting can be had from 0.5 in to 72 in wide; widths from 0.5 to 1 in vary by 1/8 in, from 1 to 4 in by 0.25 in, 4 to 7 in by 0.5 in, 7 to 26 in by 1 in, 26 to 40 in by 2 in, and above 26 in by 4 in.

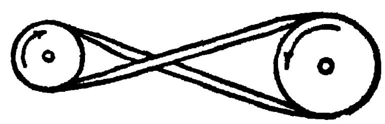


Fig 11. Crossed Belt

The figures in Table 1 are based on first-class short-lap belting, made from backs of pure oak-bark tanned leather, run under ordinary conditions and on pulleys of medium and equal size. When the driven pulley is smaller, the power transmitted will be as much less than the tabulated values as that part of the surface of the driven pulley which is covered by the belt is less than half its surface.

Rubber belts consist of several plies of canvas covered with rubber. They can be used in damp or wet places, or in drives exposed to the atmosphere or to acid fumes.

Table 1. Hp Transmitted by Leather Belts, American Leather Belting Association (1939)

Horse power per inch of width

Belt speed, ft per min	Single ply		Double ply			Triple ply	
	11/64″	13/64″	18/64″	20/64″	23/64″	30/64″	34/64″
	Medium	Heavy	Light	Medium	Heavy	Medium	Heavy
600	1.1	1.2	1.5	1.8	2.2	2.5	2.8
800	1.4	1.7	2.0	2.4	2.9	3.3	3.6
1 000	1.8	2.1	2.6	3.1	3.6	4.1	4.5
1 200	2.1	2.5	3.1	3.7	4.3	4.9	5.4
1 400	2.5	2.9	3.5	4.3	4.9	5.7	6.3
1 600	2.8	3.3	4.0	4.9	5.6	6.5	7.1
1 800	3.2	3.7	4.5	5.4	6.2	7.3	8.0
2 000	3.5	4.1	4.9	6.0	6.9	8.1	8.9
2 200	3.9	4.5	5.4	6.6	7.6	8.8	9.7
2 400	4.2	4.9	5.9	7.1	8.2	9.5	10.4
2 600	4.5	5.3	6.3	7.7	8.9	10.3	11.0
2 800	4.9	5.6	6.8	8.2	9.5	11.0	12.1
3 000	5.2	5.9	7.2	8.7	10.0	11.6	12.8
3 200	5.4	6.3	7.6	9.2	10.6	12.3	13.5
3 400	5.7	6.6	7.9	9.7	11.2	12.9	14.2
3 600	5.9	6.9	8.3	10.1	11.7	13.4	14.8
3 800	6.2	7.1	8.7	10.5	12.2	14.0	15.4
4 000	6.4	7.4	9.0	10.9	12.6	14.5	16.0
4 200	6.7	7.7	9.3	11.3	13.0	15.0	16.5
4 400	6.9	7.9	9.6	11.7	13.4	15.4	16.9
4 600	7.1	8.1	9.8	12.0	13.8	15.8	17.4
4 800	7.2	8.3	10.1	12.3	14.1	16.2	17.8
5 000	7.4	8.4	10.3	12.5	14.3	16.5	18.2
5 200	7.5	8.6	10.5	12.8	14.6	16.8	18.5
5 400	7.6	8.7	10.6	12.9	14.8	17.1	18.8
5 600	7.7	8.8	10.8	13.1	15.0	17.3	19.0
5 800	7.7	8.9	10.9	13.2	15.1	17.5	19.2
6 000	7.8	8.9	10.9	13.2	15.2	17 6	19.3
Minimum pulley diam. — Belts under 8″ wide	3″	5″	6″	8″	12″	20″	24″
Minimum pulley diam. — Belts 8″ and over wide	5″	7″	8″	10″	14″	24″	30″
	These are the minimum allowable pulleys for the above thickness belts						

For belt speeds over 6 000 ft per min consult a leather belting manufacturer

Quality of rubber determines grade of the belt, new rubber being better than reclaimed product. Strength depends upon the wt of canvas used, which is given in oz per sq yd of canvas duck. Rubber belting is cheaper than leather. Makers advocate a difference of tension of 60 lb per in of width for 4-ply belts, made of new rubber, with minimum pulley diam of 24 in. Smaller pulleys can be used, but the life of belt will be shortened, due to bending over the pulleys, the outside layers being stretched, while the inner are compressed, thus causing working of the layers on one another. On this basis makers give the following figures, 3 or 4-ply rubber belts being rated as the equivalent of single-thickness leather, and 5 or 6-ply the equivalent of a double leather belt transmitting about 50% more power than a single. The arc of contact is taken as 180°.

4-ply	rubber	belt,	60-lb	tension per	in of width,	min diam	of pulley,	24 in
5 "	"	"	80 "	"	"	"	"	30 "
6 "	"	"	100 "	"	"	"	"	36 "
7 "	"	"	120 "	"	"	"	"	42 "
8 "	"	"	140 "	"	"	"	"	48 "

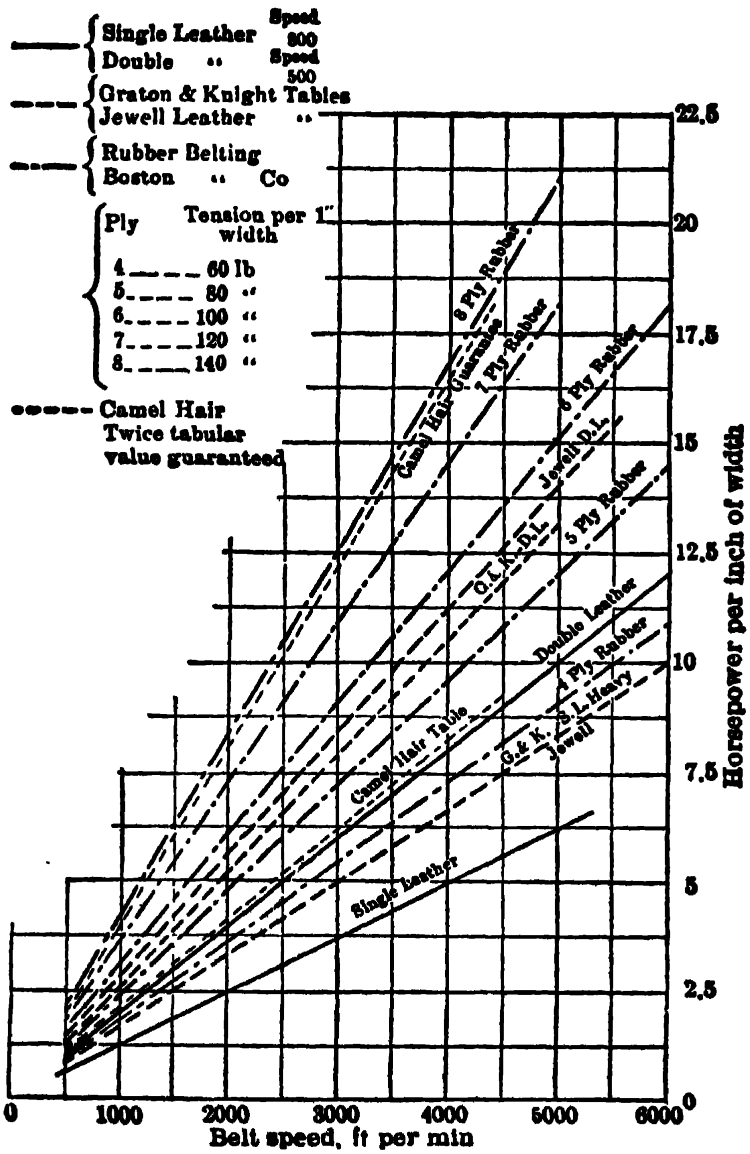


Fig 12. Belt Speeds and Horsepower

Some engineers estimate the aver value for tension at 20 lb per in of width per ply of canvas, which is higher than the values given above. In laying out belt drives, ample clearance must be given between machine frame and the walls of wooden buildings; the belt rubbing against a timber may produce enough heat to cause a fire, besides destroying the belt. If the outside rubber coating is damaged, moisture or fumes will penetrate the

canvas and destroy the belt. Application of resinous substances to pulley faces to increase adhesion also destroys belting by tearing the rubber. Power transmitted (Fig 12), based upon previous formula for 180° arc of contact, is $hp = P \times V \div 33\,000$; if the arc is 240°, allow 150%; if 120°, allow 60%; if 90°, allow 30%. An allowance of 1/8 to 0.25 in per ft of tape line measurement for stretch in rubber belts is recommended by makers. CANVAS BELTS are made by sewing plies of canvas together; woven belts by a weaving process as for cloth, animal fiber being used in some cases. In using this class of belting the tension must be the same over the entire width, otherwise the belt will not run true. CAMELS-HAIR BELTING is guaranteed to transmit twice the power of a leather belt, based upon the increased coeff of friction and strength of belt.

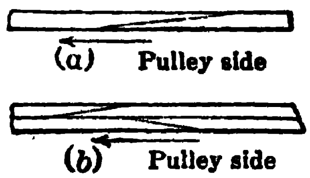


Fig 13. Scarf Joints, Leather Belt

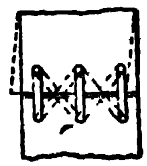


Fig 14. Belt Lacing

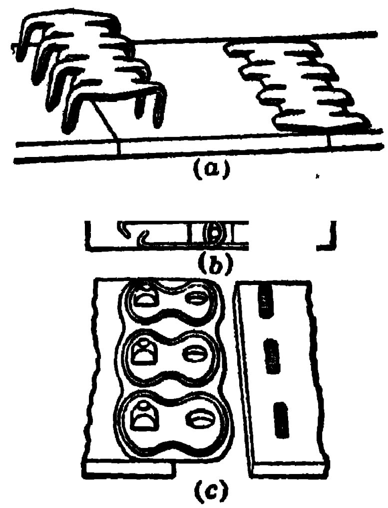


Fig 15. Metal Belt Fasteners

Belt fastenings. For leather belting, joints are made by scarfing and gluing the ends together, making an endless belt. Some machines are designed to be shifted on their frames to take up slack, but when the limit is reached the belt joint must be made over. For single leather belts the joint should be as in Fig 13 *a*; for double belts, as in Fig 13 *b*. Rubber belting can be similarly spliced. After the surfaces are covered with a rubber cement and allowed to dry they are stitched together with narrow rawhide lace. LACING should have uniform tension; it is straight on pulley side but crosses on the back; for narrow belts, the holes are in one line; for wider, they may be staggered. In lacing a belt the 2 ends must meet square and in line; if offset, as shown by dotted lines, Fig 14, the edges will be damaged. Metal fasteners are good and are made in many designs. For canvas and woven belts, the maker generally recommends some special type, best adapted to his own belting. In the fastener shown in Fig 15 *b* the wire is inserted by a special machine, the ends being held together by a rawhide pin. It is a hinge joint and can be readily taken apart and replaced.

3. PULLEYS

Pulleys for belt drives are of C I, wood, fiber, or pressed steel. C-I pulleys are solid, or in halves bolted together. A solid pulley must be installed by slipping it over the end of the shaft; split pulleys can be set anywhere on the shaft by bolting them in place.

Principal dimensions for ordering pulleys are bore, diam, width of face, and size of keyway (Fig 16). The face may be crowned or flat (Fig 17). With a CROWNED PULLEY, if the tension is not equal on both sides of belt, or the shafting is slightly out of line, the belt will seek the highest veloc, which is at the middle of the face, and is thus made to run true. Crowning is generally about 1/8 in per ft of width; if excessive, it damages the belt by overstretching it in the center. Flanges on the pulley face to keep the belt from running off should be avoided, as they wear the edges of the belt. If the belt becomes slack it may ride up on the flange and tear. Untrue running of belts is caused by unequal tension or poor alinement of shafting. Machinery that must be started and stopped frequently is sometimes fitted with crowned TIGHT AND LOOSE PULLEYS, the belt being shifted from one to the other to start or stop the machine. The driving pulley has a flat face, and a width equal to the combined widths of the 2 driven pulleys. BELT SHIFTER should be placed on the side where the belt leads on to the pulley. Very wide C-I pulleys may have double spokes or arms (Fig 18). C-I pulleys are heavier than other types. They vary from 3-in diam and 2-in face to 120-in diam by 60-in face. (For details, see

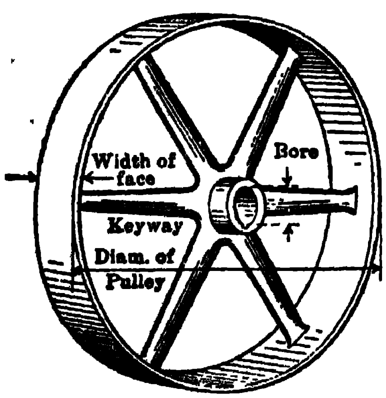


Fig 16. Cast-iron Pulley

Fig 17. Flat and Crowned Pulleys

catalogues.) Solid C-I pulleys for single-thickness belt are the cheapest. Split C-I pulleys cost about 50% more for small sizes, to about 20% more on medium sizes. C I (with the foundry scale attached) withstands action of acid fumes or dampness better than steel or wood. PRESSED STEEL PULLEYS (Fig 19) are made in halves, with double sets of arms when necessary. Sizes vary from 6-in diam by 2-in face to 120-in diam by 36-in

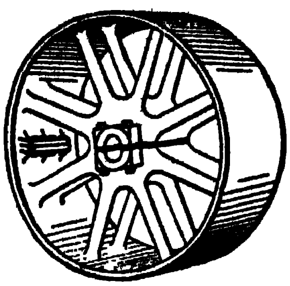


Fig 18. Double-arm Pulley

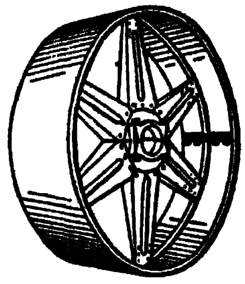


Fig 19. Pressed-steel Pulley

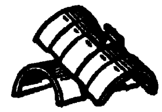


Fig 20. Pulley Bushings

face, prices averaging about the same as C-I split pulleys for single belts. Fig 20 shows a set of bushings which adapts this type of pulley to several sizes of shaft. WOODEN PULLEYS are made by gluing together strips of wood so arranged as to avoid warping. They should not be used in damp places. They are light in wt, and have the same advantages as the pressed-steel pulley. Sizes are from 3-in diam by 3-in face to 120-in diam by

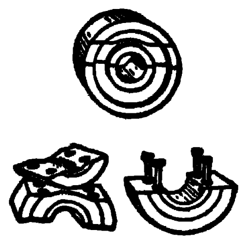


Fig 21. Small Wooden Pulley

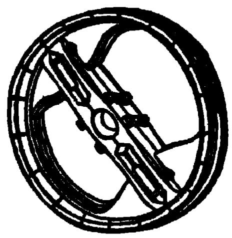


Fig 22. Large Wooden Pulley

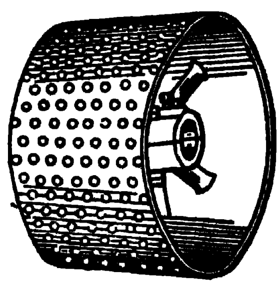


Fig 23. Pulley with Inserts for High Coeff of Friction

24-in face (Fig 21, 22). Prices of wooden pulleys for double belts aver slightly less than solid C-I pulleys. Catalogues give information as to special types and sizes. If the coeff of friction can be increased, the tension on tight side can also be increased, and more power transmitted; hence, the pulley face is sometimes covered with leather, canvas, or cork inserts (Fig 23), increasing the power transmitted for the same drive.

4. SHAFTING, HANGERS, AND BEARINGS

Shafting is of steel, turned and ground, or cold-rolled. The latter has a hard surface, which, if damaged by the cutting of a keyway, may warp the shaft. Shafting is carried in stock in sizes from $^3/_{16}$ to 1.5-in diam, in lengths from 1 to 24 ft, and from $1\,^9/_{16}$ to 7 in diam, in lengths from 5 to 24 ft. Diam always increases by $^1/_{16}$ in. Where longer shafts are required, stock lengths are joined by couplings.

Hangers and bearings. Shafting is carried in bearings mounted in adjustable hangers attached to the ceiling, walls, floor, or posts of a building. Special fittings are required for steel-frame buildings. Hangers are of C I or pressed steel. Fig 24 is a ceiling hanger, which can be reversed and made a floor stand. In Fig 25 the bolts *c* pass through slotted holes to permit adjustment; the bearing can be raised or lowered by screws *d*, which support it. The bearing in Fig 26 has a spherical seat

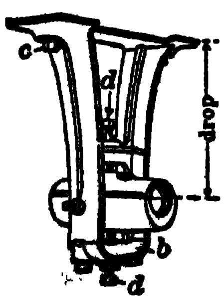


Fig 24. Ceiling Hanger

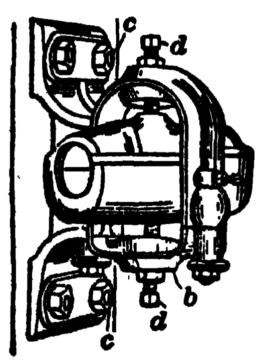


Fig 25. Adjustable Post Hanger

at *a*, for alining in any direction about its center. The lower cap *b* (Fig 25) can be removed, the box taken out, or shafting removed, without interfering with the hanger; shafting can be assembled on the floor, hoisted into position in the hangers, and bearings and caps readily put on; convenient for heavy work. Boxes should be ring-oiled and dustproof, to minimize attendance. The points for ordering hangers are: style of hanger, drop or distance from center of shaft to the base (Fig 24), and diam of shaft. When in places difficult of access, shafting and bearings are apt to be neglected. When hangers are attached to ceiling beams, the spacing of these determines the distance between bearings; 8 to 10 ft is considered good spacing; if greater, heavier shafting should be used to prevent bending. Length of bearings is given in terms of diam of shaft; for diam less than 2 $^3/_8$ in bearings are 5 diam long; for heavier shafting, 4 diam long. Shafting is subjected to torsion, due to the power transmitted, and to bending, due to wt of the pulleys, its own wt, and the wt and tension of the belts. All pulleys should be as close to the hangers as possible, and heavy drives placed near the point where the power is received. For good service, shafting should be carefully alined, with a small end play (say $^1/_8$ in), and with collars to prevent any excess end motion. Where the power is received, collars should be placed on both sides of one hanger, leaving the ends of the shaft free to expand and contract with temp changes. Motors driving shafting in dusty places can be enclosed, and a slight increase of air press above atmos maintained on the inside, to prevent entrance of dust. Bearing pressures are determined per sq in of projected area (diam × length of bearing). The aver allowable press, varying with load conditions, speed, lubricant, and method of lubrication, is about 60 lb per sq in of projected area.

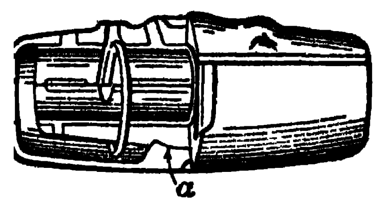


Fig 26. Hanger with Spherical Seat

5. ROPE DRIVES

Hemp or cotton-rope drives are of two kinds: continuous or American, and multiple or British. American system. The rope is wound around the pulleys as many times as the power to be transmitted requires; it then passes over a tension carriage (see below), which keeps the rope at constant tension and returns the last wrap to the first groove of driving sheave. This drive has only one splice, which is the weakest part of the rope. Breakage of the rope causes a complete shut down, usually avoidable by careful inspection. This system is best for vertical or angular drives, and can be used for heavy powers with short center distance. British system has separate ropes for each loop, and hence has as many splices as ropes. Maintenance of constant tension in all the ropes is difficult but important. If 1 or 2 should fail the others will transmit the power until repairs are made. Low tension lengthens life of the drive, which should average 8 to 10 years.

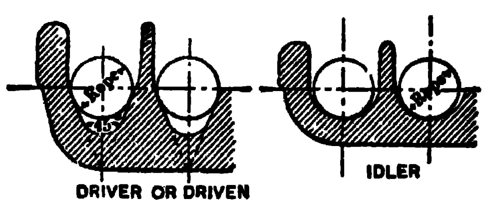


Fig. 27. Sheave Grooves, British Rope Drive

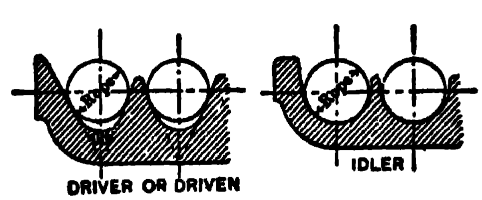


Fig 28. Sheave Grooves, American Rope Drive

Sheaves and rope. Fig 27 shows shape of grooves for main sheaves and idler, of the English system; Fig 28, for the American system. Grooves must be accurately machined, with surfaces free from imperfections; sand holes cause rapid wear on ropes. Being necessarily heavy, the sheaves act as fly-wheels for variable loads, and should be well balanced. Minimum sheave diam is 36 times rope diam; 40 times is better. Rope must not touch bottom of grooves of driver or driven sheaves, otherwise no wedging action occurs. Some makers design the groove to cause the rope to rotate on its axis, and so produce uniform wear.

Cotton ropes are softer than hemp, cost more, and are more used in Great Britain than in U S. Hemp transmission ropes are laid with internal lubricant to reduce wear. Manila rope has 3, 4, or 6 strands (Fig 29). Diams from 0.75 to 2 in increase by $^1/_8$ in; from 2 to 2.5 in by 0.25 in. (For strengths and wt of hemp rope, see Sec 12.) To lie properly in sheave grooves, splices must not increase the rope diam. Strength of a good splice is about 85% that of the rope.

In the American drive (Fig 30), constant tension is maintained by a TENSION CARRIAGE, to compensate variation in length, from stretch or shrinkage due to moisture. A tension carriage will take care of 3 000 to 3 500 ft of rope.

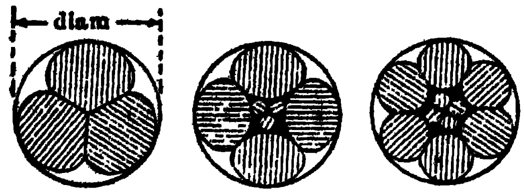


Fig 29. Drive Ropes

Power transmitted by rope drive depends upon the same elements as for belting. The friction between the ropes and sides of grooves can be increased by an increase of tension, and the difference between the 2 tensions and centrifugal force is the pull in lb. Rope speed is generally the same as for belting; but, since rope weighs less than belt, the tension can be increased at high speeds to offset effect of centrifugal force; 80 ft per sec is an aver speed.

Driving force is $P = T_n - T_0$, where T_n = tension on tight side, and T_0 is the resultant of the tension on slack side (taken as $0.5\ P$) and centrifugal force C. Hence, the power may be written, $P = T_n - (C + 0.5\ P)$, reducing to $P = {}^2/_3(T_n - C)$, and the hp $= {}^2/_3(T_n - C) \times V \div 33\ 000$, where V = rope veloc, ft per min. The expression for centrifugal force is $C = Wv^2 \div 32.16$, W being the wt of rope per ft and v the veloc, ft per sec. With fixed rope speed and min diam of sheave it is more difficult to design rope than belt drives. If the desired rev per min can not be obtained with a certain rope diam, the rope diam may have to be reduced, so as to use a smaller sheave. To transmit the same power, this requires more ropes, increasing wear and maintenance. It is good practice to use few ropes of large diam; 1.75-in rope is a good commercial size.

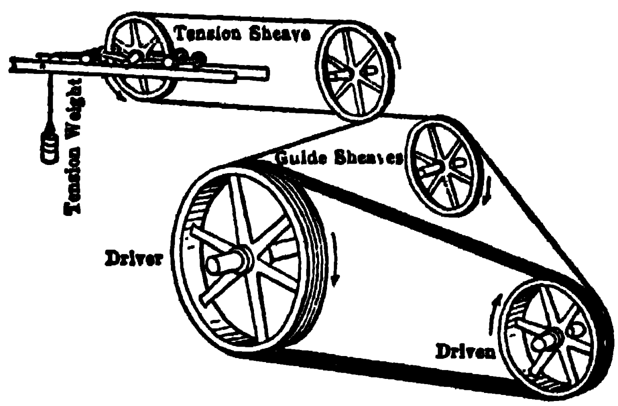


Fig 30. American Rope Drive

Table 2.

Width of face for one rope

Rope	¾"	⅞"	1"	1⅛"	1¼"	1⅜"	1½"	1¾"	2"
Eng	1½"	1⅝"	2"	2¼"	2½"	2⅝"	3½"	3⅝"	4"
Amer	1⅜"	1¾"	2"	2¼"	2½"	2¾"	3"	3¼"	4"
For each add'l extra rope add									
Eng	1⅟₁₆"	1⅜"	1½"	1⅝"	1¾"	2"	2⅛"	2½"	2¾"
Amer	1⁵⁄₁₆"	1¼"	1⅜"	1⁹⁄₁₆"	1¹¹⁄₁₆"	1⅞"	2"	2⅛"	2⁵⁄₁₆"

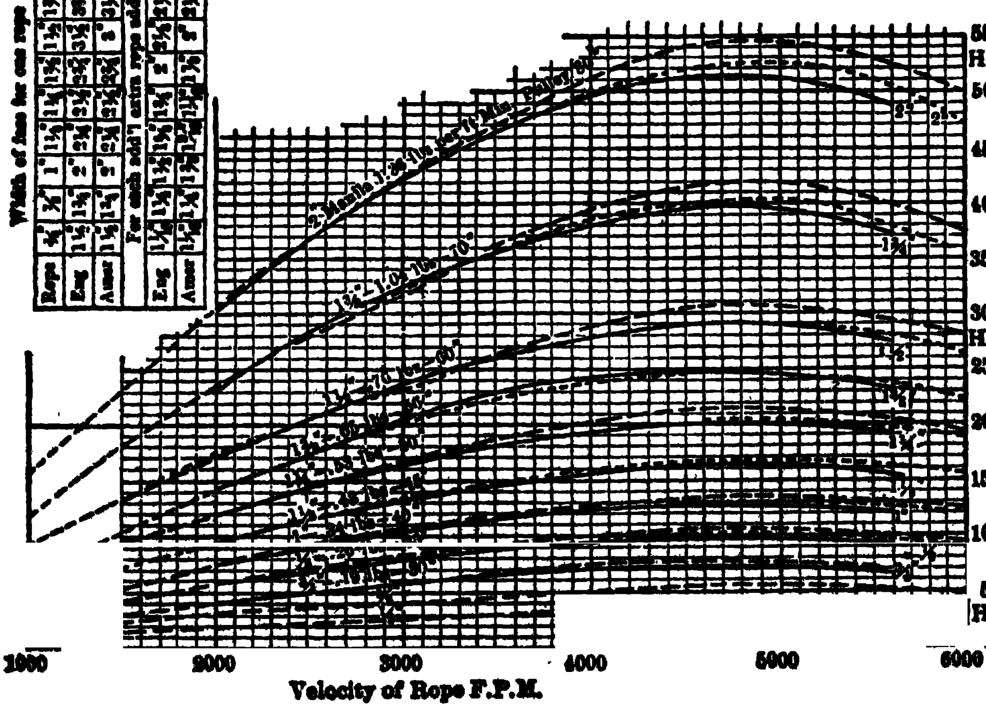


Fig 31. English (Multiple) Hemp Rope Drives (C. W. Hunt Co, broken lines; G. V. Cresson Co, dotted lines; Rice & Sergeant, full lines). L. DeG. Moss

Comparison of power drives. Gearing is used for minimum center distances; it gives a positive drive, but is noisy. Chain drives are positive, and special forms can be run at high speeds; center distances are greater than for gears, less than for belts. The elements of chain drive are given in Sec 27. Belts can be shifted from one pulley to another, can be run at high speeds or at any angle, require good alinement of shafting, are not noisy, but produce electricity, which is sometimes objectionable. Rope drives, used for max center distances, are quiet in operation, produce no electricity, do not require accurate alinement of shafting, can be run at any angle, occupy less space than belts, and are cheaper than belting for heavy power and long center distances.

Wire-rope drive (Fig 33), consisting of a single loop of wire rope around a pair of pulleys, can be used for distances of 70 to 400 ft between sheave centers. The sheave grooves are "filled," and the rope should not touch the sides of the grooves, thus requiring good alinement.

Driving power depends upon coeff of friction between rope and sheave filling (see below). (For wire ropes, see Sec 12.) 7-wire rope will stand more wear, but requires larger pulleys. Steel rope is stronger and has a longer life, but the size can not be reduced, due to the wt required; a small rope must have more tension and damages the pulley filling. Bending stresses (Sec 12) generally exceed those due to direct pull. Speed limit is 5 000 ft per min. If the rope lashes at high speed, the direct tension may be momentarily doubled, and, if the sheaves are small, the bending stresses plus the tension may strain the rope beyond the elastic limit.

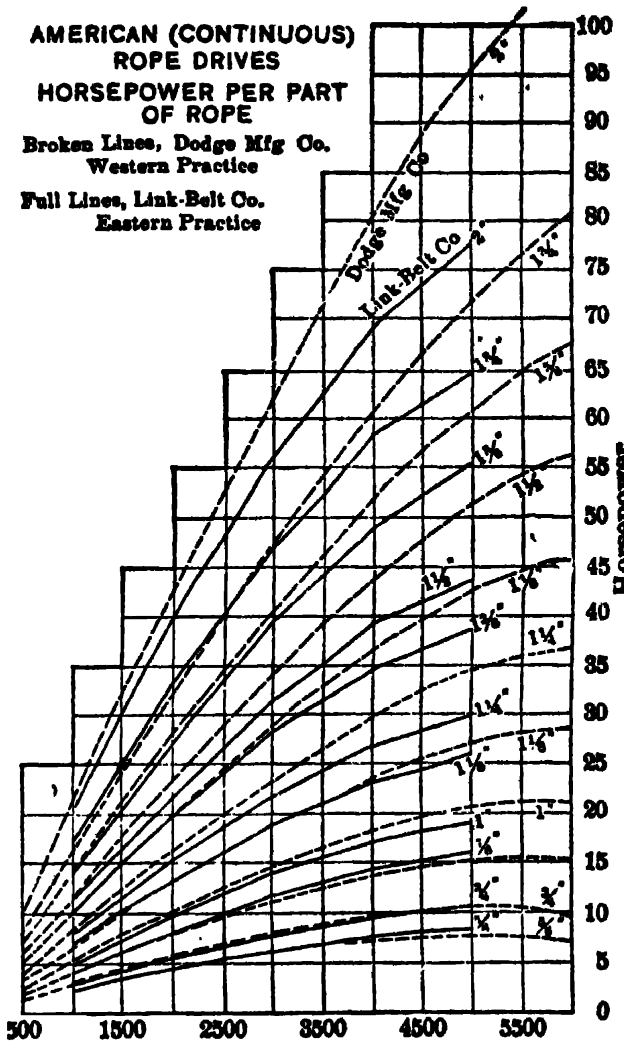


Fig 32. (L. DeG. Moss)

Sheaves should be as large as practicable, to give high rope speed and hence permit use of min diam of rope. Fig 34 shows sheave groove and filling. Filling consists of alternate blocks of leather and rubber; rubber has high coeff of friction, while the leather has resisting qualities. Hard-wood blocks are sometimes used. 5 000 ft per min rim speed is about the safe limit for C-I sheaves or for the filling. Sheaves must be accurately

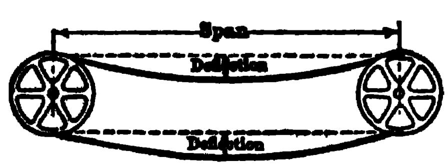


Fig 33. Wire-rope Drive

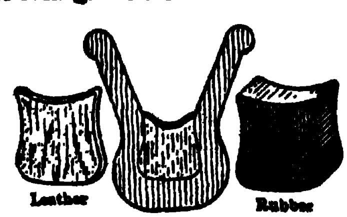


Fig 34. Filling for Wire-rope Drive Sheaves

balanced, to prevent lashing of the rope. At less than 70 ft center distance, tension becomes excessive, increasing wear on rope and filling. For spans greater than 400 ft, multiple loops may be employed. Where there are sudden variations in load, a flywheel can be attached to the sheave shaft. For changing direction of the drive, bevel or friction gears are used. The heavier the rope, the greater the tension and farther apart may the

sheaves be placed, with smaller sag or deflection. Makers advise a sag of $^1/_{36}$ the span in both ropes when idle; in operation, the sag on tight side is about 0.02 the span and on the slack side 0.04. This makes the tension ratio about 2 : 1; or difference in tensions may be computed by taking the wt of 1 rope between sheaves and multiplying by 3. The hp can then be found: hp = $T \times V \div 33\,000$, T being difference in tensions and V veloc, ft per min. No deductions are made for centrifugal force, or bending stresses, as they are negligible under usual conditions. Idler sheaves may be used to prevent the rope from striking obstructions. When on the following side, they are 0.5 the diam of the driver or driven wheel; if in the middle of a span, the tension and power transmitted will be halved. Tightener sheaves may be placed on short spans, but are not recommended. Sheaves are sometimes mounted on movable bearings, for taking up slack without resplicing. Catalogues of makers give tables of hp transmitted, center distances, diam of sheaves, and rev per min for different size ropes.

6. LUBRICANTS

Requirements. Lubricant must have sufficient BODY at the probable temp to which it will be subjected, to prevent it from being squeezed out by press between bearing surfaces; it must not decompose nor evaporate at ordinary working temp; it must contain no acid, which may injure bearing surfaces; its coeff of friction must be low; it must be as fluid as is consistent with the required body; and it must not freeze at any temp to which the bearing may be exposed.

Ordinary lubricants: animal fats, vegetable and mineral oils, flake graphite, soapstone, powdered mica.

Animal fats or oils are often used for bearings subjected to low temp, but are generally mixed with mineral oils. They decompose at moderate temp and set free acid. Vegetable oils are used where the bearing press is light and temp low. Mineral oils are obtainable to suit practically all conditions of lubrication; they do not easily decompose and contain no acid. Greases are mixtures of mineral oil and animal or vegetable soap; useful for slow-moving bearings under heavy press.

Solid lubricants, used alone, fill the pores of the bearing metal and form a film on it. They are commonly mixed with oil or grease, and generally improve lubrication. They must not be used for ball bearings, as they build up a surface on the ball race which often splits the bearing.

Testing lubricants. The value of ordinary tests is relative rather than absolute. Two oils, showing approx the same characteristics, will probably serve equally well in the same bearing. Oils are tested for the following characteristics. Special machines are used for coeff of friction tests. Other tests are for adulteration and presence of acids. VISCOSITY, or degree of fluidity, is a measure of the body of an oil. It is determined by viscosimeters. Nearly all operate on the principle of finding the time required for a given volume of oil to flow through an orifice. Their design varies greatly, and 2 different instruments may give widely different results for the same oil. Viscosity is generally expressed as specific or as relative viscosity (Table 3), which denote the relation between the times required for the same volume of oil and water to flow through the orifice. Tests are made at different temps. FLASH POINT is the temp at which an oil will give off enough vapor to ignite and flash, when a flame is held over the surface. The heating should be done slowly. Types of apparatus are the open cup and closed cup. The closed cup has an opening in the cover, closed by a slide. As the oil is heated the slide is periodically removed, and a flame inserted until the flash point is reached. During tests a thermometer is hung in the body of the oil. BURNING POINT is the temp at which an oil will give off enough vapor to ignite and continue to burn. CHILL POINT is the temp at which an oil freezes. Best determined by placing the oil in a test tube with a thermometer. The oil is then frozen by an ice and salt mixture. After freezing, the test tube is removed from the ice and the temp observed at which the oil melts. FRICTION TESTS are made by special machines, designed to reproduce working conditions, and to measure the temp of the oil and coeff of friction. The speed and press between bearing surfaces may be fixed at will. The simpler machines give only comparative results, but valuable data have been obtained from specially designed apparatus.

Table 3. Characteristics of Lubricating Oils

Kind of oil	Specific viscosity, 70° F	Specific grav, water = 1	Flash point, deg F	Burning point, deg F
Heavy engine and machine	185	0.875	400	445
Ordinary engine and machine	180	0.873	410	460
Light machine	100–150	0.870	400	440
High-grade cylinder	175–200	0.903	600	650
Ordinary cylinder	185	0.899	570	615
Gas engine	300	0.893	320	350
Automobile cylinder	195	0.877	430	485

Detection of adulterants. High-grade oil is sometimes adulterated with low-flash mineral oil, which is detected in flash-test apparatus. Presence of vegetable or animal oil in mineral oils is determined by mixing a sample with a solution of sodium or potassium hydroxide and heating to about 200° F. Animal or vegetable oils will saponify and separate from the mineral oil. Presence of mineral oils in animal or vegetable oils is determined by the same process, as the mineral oil will not saponify. Acids are detected by litmus paper.

Most of the above oils have a chill point slightly below freezing point of water. Oils may be obtained the chill point of which is considerably lower.

Selection of lubricants. For engine cylinders using high-press steam, high-grade cyl oil; for engine cylinders using steam below 100 lb press, second-grade cyl oil; for stationary gas engines, a heavy oil with high flash and burning points (it should preferably have a liquid base); for air compressors and high-speed internal combustion engines, a liquid-base oil with a relatively low viscosity and a high flash point. Engine and machine oils are sold in many grades to suit all conditions. For slow-moving bearings under heavy press, use grease. If the press is exceptionally heavy, add graphite to the grease.

7. PIPING

List sizes of welded W-I or steel pipe indicate the nominal inside diam. Actual diam varies considerably from the nominal, especially in standard wt pipe of small sizes. Outside diam of all weights is the same, so that the threads may be uniform for same nominal size. Added thickness of metal reduces the internal diam. Ordinary welded pipe is of mild steel. If iron pipe be desired, it should be specified. Tubes are listed according to their actual outside diam. Pipe is regularly supplied in random lengths, from 18 to 22 ft, threaded on both ends, and 1 coupling is furnished with each length. An extra charge is made for pipe in equal lengths. Prices vary with the metal market, and the place of sale.

List prices rarely change, and those in following tables hold good for 1938. List prices are subject to large and varying discounts and prevailing rates should be obtained from manufacturers before cost estimates are made.

In the following tables a fairly complete list of pipes and fittings is given, but valves and cocks are omitted, as they are made in too great variety to be listed here. Dealers' catalogues should be consulted regarding them.

Table 4. Line Pipe, for Oil, Gas, and High-press Mains

Nominal inside diam, in	Actual outside diam, in	Wt per ft, lb	No of threads per in of screw	Nominal inside diam, in	Actual outside diam, in	Wt per ft, lb	No of threads per in of screw
2	2.37	3.61	11 1/2	6	6.62	18.76	8
2 1/2	2.87	5.74	8	7	7.62	23.27	8
3	3.50	7.54	8	8	8.62	28.18	8
3 1/2	4.00	9.00	8	9	9.68	33.70	8
4	4.50	10.66	8	10	10.75	40.06	8
4 1/2	5.00	12.34	8	12	12.75	49.00	8
5	5.56	14.50	8				

Table 5. Boiler Tubes

Actual outside diam, in	Thickness		Wt per ft, lb	Actual outside diam, in	Thickness		Wt per ft, lb
	B.W.G.	In			B.W.G.	In	
1	13	0.095	1.037	3	12	0.109	3.838
1 1/4	13	0.095	1.328	3 1/4	11	0.120	4.555
1 1/2	13	0.095	1.619	3 1/2	11	0.120	4.921
1 3/4	13	0.095	1.910	4	10	0.134	6.286
2	13	0.095	2.201	4 1/2	10	0.134	7.103
2 1/4	13	0.095	2.492	5	9	0.148	8.720
2 1/2	12	0.109	3.171	5 1/2	9	0.148	9.622
2 3/4	12	0.109	3.504	6	7	0.180	12.750

Tubes may be obtained in other thicknesses and larger sizes than those given above.

Table 6. Root's Spiral-riveted Pipe

Black and asphalted pipe is in lengths of 25 ft or less, galvanized in lengths of 20 ft. Wt is approx for black pipe only; asphalted and galvanized weight 10 to 20% heavier. Prices, $ per linear ft, with plain, crimped, or sleeved ends. Sleeves when furnished are included in linear measurement.

Diam, in	Prices: Black	Prices: Asphalted	Prices: Galvanized	Wt per 100 ft, lb	Approx bursting press per sq in, lb	Diam, in	Prices: Black	Prices: Asphalted	Prices: Galvanized	Wt per 100 ft, lb	Approx bursting press per sq in, lb
		No 24, B W G						**No 20, B W G**			
3	0.20	0.23	0.30	100	....	3	0.27	0.30	0.38	150	900
4	0.25	0.29	0.38	130	....	4	0.35	0.39	0.48	200	700
5	0.30	0.35	0.45	160	....	5	0.40	0.45	0.60	250	550
6	0.33	0.39	0.50	185	....	6	0.46	0.52	0.68	300	450
7	0.37	0.44	0.60	210	....	7	0.51	0.58	0.75	325	400
8	0.42	0.50	0.65	240	....	8	0.58	0.66	0.85	360	350
9	0.48	0.57	0.75	280	....	9	0.66	0.75	0.97	410	325
10	0.54	0.64	0.85	300	....	10	0.72	0.82	1.05	500	275
11	0.60	0.71	0.90	330	....	11	0.78	0.89	1.20	550	250
12	0.68	0.80	1.05	400	....	12	0.90	1.02	1.35	600	225
		No 18, B W G						**No 16, B W G**			
3	0.34	0.37	0.46	185	1 300	4	0.50	0.54	0.70	320	1 250
4	0.42	0.46	0.58	245	1 000	5	0.60	0.65	0.85	415	1 000
5	0.50	0.55	0.70	300	800	6	0.70	0.76	1.00	500	800
6	0.57	0.63	0.85	360	700	7	0.80	0.87	1.10	550	700
7	0.63	0.70	0.90	400	600	8	0.93	1.01	1.28	650	600
8	0.73	0.81	1.05	460	500	9	1.08	1.17	1.47	750	550
9	0.82	0.91	1.18	525	450	10	1.15	1.25	1.55	800	500
10	0.90	1.00	1.30	575	400	11	1.20	1.31	1.70	850	450
11	0.95	1.06	1.40	625	360	12	1.45	1.57	2.05	1 025	400
		No 14, B W G						**No 12, B W G**			
6	0.89	0.95	1.15	610	1 100	6	1.25	1.31	1.90	800	1 330
7	1.02	1.09	1.35	700	950	7	1.40	1.47	2.10	910	1 140
8	1.15	1.23	1.50	825	825	8	1.55	1.63	2.30	1 040	1 000
9	1.32	1.41	1.70	925	750	9	1.70	1.79	2.50	1 180	880
10	1.40	1.50	1.80	1 025	650	10	1.90	2.00	2.75	1 300	800
11	1.50	1.61	1.95	1 125	600	11	2.25	2.36	3.00	1 425	725
12	1.80	1.92	2.35	1 325	550	12	2.50	2.62	3.25	1 700	660

Note.—Straight-seam riveted pipe is obtainable in form of punched and formed sheets, at list prices from 7 to 8.5¢ per lb black and 10 to 13¢ per lb galvanized, depending on thickness of metal.

Table 7. Spiral-riveted Pipe (Approx bursting strength, lb per sq in)

Nominal inside diam, in	Thickness, U S Standard Gage: 18	16	14	12	10	8	6	Nominal inside diam, in	Thickness, U S Standard Gage: 18	16	14	12	10	8	6
3	1 666	....	....	....	....	....	....	22	....	....	355	497	639	781	....
4	....	1 563	....	....	....	....	....	24	....	....	325	455	586	716	....
5	....	1 250	1 562	....	....	....	....	26	....	....	....	420	541	661	781
6	....	1 047	1 300	1 825	....	....	....	28	....	....	....	390	502	614	725
8	....	780	975	1 366	....	....	....	30	....	....	....	364	469	573	677
10	....	625	781	1 093	....	....	....	32	....	....	....	....	439	537	635
12	....	521	651	911	1 172	....	....	34	....	....	....	....	413	505	597
14	....	448	558	781	1 004	1 228	....	36	....	....	....	....	390	477	564
16	....	391	488	683	879	1 074	....	38	....	....	....	....	370	452	534
18	....	....	434	607	781	955	....	40	....	....	....	....	351	430	508
20	....	....	390	546	703	859	....	42	....	....	....	....	335	409	483

Pipes are tested at approx one third of the bursting pressure.

Table 8. C-I Water Pipe, Standard Weight

Size............in	3	4	6	8	10	12	14	16	18	20	24	30	36
Thickness........in	1/2	1/2	1/2	1/2	9/16	9/16	3/4	3/4	7/8	7/8	1	1 1/8	1 3/8
Weight per ft.....lb	17	22	33	42	60	75	117	125	167	197	250	350	475

Table 9. Wrought-iron or Steel Welded Steam, Gas, and Water Pipe

1 in and below, proved to 300 lb per sq in, hydraulic press; 1 1/4 in and above, proved to 500 lb

Nominal inside diam, in	Actual inside diam, in	Actual outside diam, in	Thickness, in	Length per sq ft inside surf, ft	Length per sq ft outside surf, ft	Inside sec area, sq in	Outside sec area, sq in	Length containing 1 cu ft, ft	Nominal wt per ft, lb	No of threads per in of screw	Contents of 1 ft length, gal	List price per linear ft, $
1/8	0.27	0.40	0.07	14.15	9.44	0.06	0.12	2 500	0.24	27	0.002	0.05 1/2
1/4	0.36	0.54	0.08	10.50	7.07	0.10	0.22	1 385	0.42	18	0.002	0.06
3/8	0.49	0.67	0.09	7.67	5.65	0.19	0.35	751.5	0.56	18	0.005	0.06
1/2	0.62	0.84	0.10	6.13	4.50	0.30	0.55	472.4	0.84	14	0.010	0.085
3/4	0.82	1.05	0.11	4.63	3.63	0.53	0.86	270.0	1.12	14	0.023	0.115
1	1.04	1.31	0.13	3.67	2.90	0.86	1.35	166.9	1.67	11 1/2	0.040	0.17
1 1/4	1.38	1.66	0.14	2.76	2.30	1.49	3.16	96.25	2.24	11 1/2	0.063	0.23
1 1/2	1.61	1.9	0.14	2.37	2.01	2.03	2.83	70.65	2.68	11 1/2	0.091	0.27 1/2
2	2.06	2.37	0.15	1.84	1.61	3.35	4.43	42.36	3.61	11 1/2	0.163	0.37
2 1/2	2.46	2.87	0.20	1.54	1.32	4.78	6.49	30.11	5.74	8	0.255	0.58 1/2
3	3.06	3.5	0.21	1.24	1.09	7.38	9.62	19.49	7.54	8	0.367	0.765
3 1/2	3.56	4.0	0.22	1.07	0.95	9.83	12.56	14.56	9.00	8	0.500	0.92
4	4.02	4.5	0.23	0.94	0.84	12.73	15.90	11.31	10.66	8	0.652	1.09
4 1/2	4.50	5.0	0.24	0.84	0.76	15.93	19.63	9.03	12.34	8	0.826	1.27
5	5.04	5.56	0.25	0.75	0.62	19.99	24.29	7.20	14.50	8	1.02	1.48
6	6.06	6.62	0.28	0.63	0.57	28.88	34.47	4.98	18.76	8	1.46	1.92
7	7.02	7.62	0.30	0.54	0.50	38.73	45.66	3.72	23.27	8	2.00	2.38
8	7.98	8.62	0.32	0.47	0.44	50.03	58.42	2.88	28.18	8	2.61	2.88
9	9.00	9.68	0.34	0.42	0.30	63.63	73.71	2.26	33.70	8	3.30	3.55
10	10.01	10.75	0.36	0.38	0.35	78.83	90.79	1.80	40.06	8	4.08	4.12
11	11	11.75	0.37	0.34	0.32	95.03	108.43	1.50	45.0	8	4.93	4.65
12	12	12.75	0.37	0.32	0.30	113.09	127.67	1.27	49.0	8	5.87	5.07
13	13.25	14.0	0.37	0.29	0.27	137.88	153.94	1.04	54.0	8	6.89	....
14	14.25	15.0	0.37	0.27	0.25	159.48	176.71	0.90	58.0	8	8.00	....

Table 10. Extra Strong and Double Extra Strong W-I or Steel Welded Pipe

Extra Strong Steam, Gas, and Water Pipe						Double Extra Strong Steam, Gas, and Water Pipe					
Size, in	Actual outside diam, in	Nominal inside diam, in	Thickness, in	Nominal wt per ft, lb	Price per ft, $	Size, in	Actual outside diam, in	Nominal inside diam, in	Thickness, in	Nominal wt per ft, lb	Price per ft, $
1/8	0.405	0.205	0.100	0.29	0.11	1/2	0.84	0.244	0.298	1.70	0.32
1/4	0.540	0.294	0.123	0.54	0.11	3/4	1.05	0.422	0.314	2.44	0.35
3/8	0.675	0.421	0.127	0.74	0.11	1	1.315	0.587	0.364	3.65	0.37
1/2	0.840	0.542	0.149	1.09	0.12	1 1/4	1.66	0.885	0.388	5.20	0.52
3/4	1.05	0.736	0.157	1.39	0.15	1 1/2	1.90	1.088	0.406	6.40	0.65
1	1.315	0.951	0.182	2.17	0.22	2	2.375	1.491	0.442	9.02	0.91
1 1/4	1.66	1.272	0.194	3.00	0.30	2 1/2	2.875	1.775	0.560	13.68	1.37
1 1/2	1.90	1.494	0.203	3.63	0.36	3	3.50	2.284	0.608	18.56	1.86
2	2.375	1.933	9.221	5.02	0.50	3 1/2	4.00	2.716	0.642	22.75	2.30
2 1/2	2.875	2.315	0.280	7.67	0.81	4	4.50	3.136	0.682	27.48	2.76
3	3.500	2.892	0.304	10.25	1.05	4 1/2	5.00	3.564	0.718	32.53	3.26
3 1/2	4.000	3.358	0.321	12.47	1.33	5	5.563	4.063	0.750	38.12	3.86
4	4.500	3.818	0.341	14.97	1.50	6	6.625	4.875	0.875	53.11	5.32
4 1/2	5.000	4.280	0.360	18.22	1.95	7	7.625	5.875	0.875	62.38	6.35
5	5.563	4.813	0.375	20.54	2.16	8	8.625	6.875	0.875	71.62	7.25
6	6.625	5.750	0.437	28.58	2.90	.....	.....	.....	.....	.....	.....
7	7.625	6.625	0.500	37.67	3.80	.....	.....	.....	.....	.....	.....
8	8.625	7.625	0.500	43.00	4.30	.....	.....	.....	.....	.....	.....

Extra strong and double extra strong will be shipped in random lengths and plain ends unless otherwise ordered. For pipe fitted with threads and couplings, extra charge will be made above regular. For cut lengths, extra charge will be made above random. For galvanized or asphalted, extra charge will be made above black.

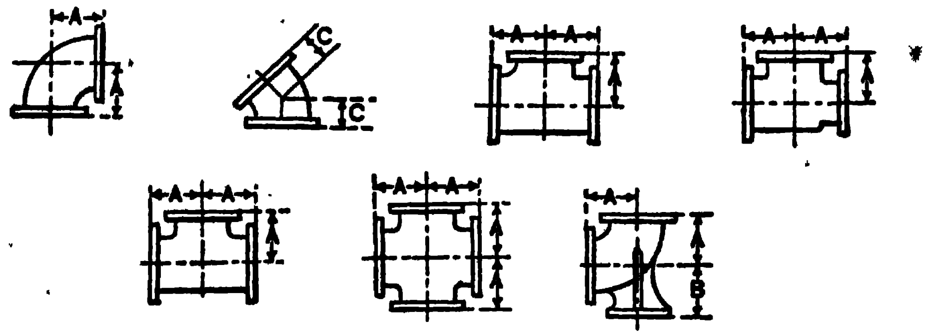
Weights given are standard, but can be changed when pipe is made to order. Coated inside and out, and tested 300 lb per sq in, hydrostatic press. Made in 12-ft lengths, 440 lengths per mile.

Use of welded joints, instead of screwed or flanged joints, is rapidly gaining favor, especially for high pressure and high temperature piping. Particulars concerning welded

joints and accepted codes governing use of piping of various types and materials will be found in Kent's Mechanical Engineer's Handbook, Vol II, Power, 1936.

8. PIPE FITTINGS AND COVERINGS

Table 11. C-I Flanged Fittings (American Standard). For 125 lb working press



Size of run......in	In	2	2 1/2	3	3 1/2	4	4 1/2	5	6	7		
Outlets......	In	All reducing fittings, 2 in to 9 in, inclusive, have same dimensions as straight fittings, center to face										
Face to face......	AA	9	10	11	12	13	14	15	16	17	18	20
Center to face......	A	4 1/2	5	5 1/2	6	6 1/2	7	7 1/2	8	8 1/2	9	10
Center to face of 45° ells..	C	2 1/2	3	3	3 1/2	4	4	4 1/2	5	5 1/2	5 1/2	6
Center to base of base ells	B	5	5 1/2	5 3/4	6 1/4	6 1/2	6 3/4	7	7 1/2	8 1/4	8 3/4	9 1/2
Diam flanges......	.....	6	7	7 1/2	8 1/2	9	9 1/4	10	11	12 1/2	13 1/2	15
Thickness flanges......	.....	5/8	11/16	3/4	13/16	15/16	15/16	15/16	1	1 1/16	1 1/8	1 1/8

Size of run......	In	10	12	14	15	16
Outlets......	In	7 and larger	9 and larger	10 and larger	10 and larger	12 and larger
Face to face......	AA	22	24	28	29	30
Center to face......	A	11	12	14	14 1/2	15
Center to face of 45° ells..	C	6 1/2	7 1/2	7 1/2	8	8
Center to base of base ells	B	10	10 1/2	13 1/2	14	14 3/4
Diam flanges......	.....	16	19	21	22 1/4	23 1/2
Thickness flanges......	.....	1 3/16	1 1/4	1 3/8	1 3/8	1 7/16

Tees having outlets larger than the run have same length center to face all openings as a tee with all openings of same size as outlet.

Table 12. Prices of Standard C-I Fittings

Size......in		1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4
Elbows, R H......	each	0.05	0.05	0.06	0.08	0.10 1/2	0.16	0.20	0.28	0.50	0.75	1.05	1.20
" R and L...	"	0.06	0.06	0.07	0.09	0.12	0.18	0.23	0.32	0.60	0.85	1.20	1.50
" L H......	"	0.06	0.06	0.07	0.09	0.12	0.18	0.23	0.32	0.60	0.85	.....	.....
" reducing..	"	.....	0.06	0.07	0.09	0.12	0.18	0.23	0.32	0.60	0.85	1.20	1.40
" pitched...	"	.....	.....	.....	0.10	0.13	0.20	0.25	0.35	0.65	1.00	1.30	1.50
" with side outlet....	"	.....	.....	0.18	0.24	0.30	0.48	0.60	0.84	1.50	2.25	3.15	3.60
45° elbows......	"	0.06	0.06	0.07	0.10	0.12	0.19	0.24	0.34	0.60	0.90	1.25	1.45

Size......in		4 1/2	5	6	7	8	9	10	12	14	15
Elbows, R H......	each	1.75	2.00	2.75	4.70	6.75	9.00	13.50	20.00	57.00	70.00
" R and L......	"	2.25	2.40	3.40	.....	.....	.....	.....	.....	.....	.....
" reducing......	"	2.00	2.30	3.15	5.40	7.75	10.50	15.50	23.00	63.00	77.00
" with side outlet......	"	5.25	6.00	8.25	14.00	20.00	26.00	40.00	60.00	.....	.....
45° elbows......	"	2.20	2.50	3.45	5.90	8.50	11.25	17.00	25.00	55.00	68.00

Table 13. Prices of 250-lb C-I Fittings

Size . . . in	3/4	1	1 1/4	1 1/2	2	2 1/2	3
Elbows	.30	.35	.45	.60	.75	1.25	2.00
" reducing	.40	.45	.55	.75	.95	1.55	2.50
" 45°	.40	.44	.55	.70	.90	1.50	2.50
Tees	.45	.55	.70	.90	1.15	1.80	3.00
" reducing	.60	.70	.90	1.15	1.40	2.25	3.75
	3 1/2	4	5	6	8	10	12
Elbows	2.75	3.50	5.50	8.00	17.00	28.00	40.00
" reducing	3.40	4.40	6.80	10.00	21.00	35.00	50.00
" 45°	3.50	4.50	6.75	9.75	21.00	34.00	48.00
Tees	4.25	5.50	8.25	12.00	25.00	42.00	60.00
" reducing	5.30	6.85	10.25	15 00	31 00	52.00	75.00
	3/4	1	1 1/4	1 1/2	2	2 1/2	3
Crosses	.65	.70	.90	1.20	1.50	2.50	4.00
Flanged unions, screwed	.70	.80	1.00	1.15	1.50	1.90	2.25
Plugs	.03	.04	.05	.07	.10	.18	.25
	3 1/2	4	5	6	8	10	12
Crosses	5.50	7.00	11.00	16.00	34.00	....	....
Flanged unions, screwed	2.70	3.15	4.75	6.00	10.50	....	....
Plugs, square head	.38	.42	.88	1.20	2.75	3.75	5.00

Table 14. C-I Fittings. Standard and low-press flanged valves, flanged fittings and flanges, suitable for 125 lb working press

Size, in	Diam of flanges, in	Bolt circle, in	Number bolts	Size, bolts, in	Length bolts for standard, in	Size, in	Diam of flanges, in	Bolt circle, in	Number bolts	Size bolts, in	Length bolts for standard, in
1	4	3	4	7/16	1 1/2	10	16	14 1/4	12	7/8	3 3/4
1 1/4	4 1/2	3 3/8	4	7/16	1 1/2	12	19	17	12	7/8	3 3/4
1 1/2	5	3 7/8	4	1/2	1 3/4	14	21	18 3/4	12	1	4 1/4
2	6	4 3/4	4	5/8	2	15	22 1/4	20	16	1	4 1/4
2 1/2	7	5 1/2	4	5/8	2 1/4	16	23 1/2	21 1/4	16	1	4 1/4
3	7 1/2	6	4	5/8	2 1/2	18	25	22 3/4	16	1 1/8	4 3/4
3 1/2	8 1/2	7	4	5/8	2 1/2	20	27 1/2	25	20	1 1/8	5
4	9	7 1/2	4	3/4	2 3/4	22	29 1/2	27 1/4	20	1 1/4	5 1/2
4 1/2	9 1/4	7 3/4	8	3/4	3	24	32	29 1/2	20	1 1/4	5 1/2
5	10	8 1/2	8	3/4	3	26	34 1/4	31 3/4	24	1 1/4	5 3/4
6	11	9 1/2	8	3/4	3	28	36 1/2	34	28	1 1/4	6
7	12 1/2	10 3/4	8	3/4	3 1/4	30	38 3/4	36	28	1 3/8	6 1/4
8	13 1/2	11 3/4	8	3/4	3 1/2	36	45 3/4	42 3/4	32	1 3/8	6 1/2
9	15	13 1/4	12	3/4	3 1/2						

Table 15. Prices of Standard Flanged Fittings

	Elbows				Tees				
Size, in	Center to face, in	Diam of flanges, in	With flanges faced	With flanges faced and drilled	Center to face, in	Face to face, in	Diam of flanges, in	With flanges faced	With flanges faced and drilled
2	4 1/2	6	$3.00	$3.60	4 1/2	9	6	$4.35	$5.25
2 1/2	5	7	3.15	3.75	5	10	7	4.55	5.45
3	5 1/2	7 1/2	3.45	4.15	5 1/2	11	7 1/2	5.00	6.00
3 1/2	6	8 1/2	4.05	4.90	6	12	8 1/2	5.85	7.10
4	6 1/2	9	4.50	5.50	6 1/2	13	9	6.50	8.00
5	7 1/2	10	6.25	7.25	7 1/2	15	10	9.10	10.60
6	8	11	7.60	8.90	8	16	11	11.10	12.95
8	9	13 1/2	12.00	13.60	9	18	13 1/2	17.40	19.80
10	11	16	19.00	21.70	11	22	16	27.50	31.50
12	12	19	28.00	31.00	12	24	19	40.50	45.00

Mineral wool prices (net) per ton in 1939:

	Car-load lots	Less than car-load lots
Loose.....	$47.00	$53.00
Granulated	63.00	69.00

Temperature limit, 1000° F. Mineral wool is also made in blanket form, felted between metal fabric, asbestos paper or other materials. Mixed with various binders, it is in form of blocks, or special shapes to suit requirements.

Table 16. Pipe Covering

Inside diam of pipe, in	Price per linear foot, canvas jacketed	Size of fittings, in	Elbows	Tees	Crosses	Globe valves
1/2	$0.22	1/2	$0.30	$0.36	$0.48	$0.54
3/4	0.24	3/4	0.30	0.36	0.48	0.54
1	0.27	1	0.30	0.36	0.48	0.54
1 1/4	0.30	1 1/4	0.30	0.36	0.48	0.54
1 1/2	0.33	1 1/2	0.30	0.36	0.48	0.54
2	0.36	2	0.36	0.42	0.54	0.60
2 1/2	0.40	2 1/2	0.42	0.48	0.60	0.78
3	0.45	3	0.48	0.54	0.70	0.96
3 1/2	0.50	3 1/2	0.54	0.60	0.80	1.20
4	0.60	4	0.60	0.75	0.95	1.50
4 1/2	0.65	4 1/2	0.72	0.90	1.10	1.85
5	0.70	5	0.90	1.20	1.50	2.25
6	0.80	6	1.30	1.60	2.00	2.80
7	1.00	7	1.80	2.20	2.80	3.60
8	1.10	8	2.40	3.00	3.60	4.40
9	1.20	9	3.00	3.80	4.40	5.30
10	1.30	10	3.60	4.60	5.20	6.20
12	1.85	....	....	....	....	....
14	2.10	....	....	....	....	....

Block covering for boilers, drums, and tanks, made with wire netting, smooth cement finish: 1.5 in thick, 0.90¢ per sq ft; 2 in thick, 1.20¢ per sq ft.

Hair felt. Per sq ft: 0.5 in, 4.75¢; 0.75 in, 5.75¢; 1 in, 6.5¢; 1.5 in, 10¢; 2 in, 13¢.

Table 17. Block Covering

Thickness, in	Price, sq ft
1/2	0.54
3/4	0.54
7/8	0.54
1	0.60
1 1/8	0.68
1 1/4	0.76
1 3/8	0.84
1 1/2	0.90
1 5/8	0.98
1 3/4	1.06
1 7/8	1.14
2	1.20
2 1/8	1.28
2 1/4	1.36
2 3/8	1.44
2 1/2	1.50
2 5/8	1.58
2 3/4	1.66
2 7/8	1.74
3	1.80
3 1/4	1.96
3 1/2	2.10
3 3/4	2.26
4	2.40

Any thickness made to order.

Table 18. Fittings for Light Weight Pipe, with standard flanges attached. No 10 gage thickness (0.1406 in)

Center to face dimensions are same as 125-lb American Standard C-I fittings

Nominal inside diam, in	90° elbows			90° long rod elbows			45° elbows			Tees		
	Black	Galvanized	Approx wt, lb	Black	Galvanized	Approx wt, lb	Black	Galvanized	Approx wt, lb	Black	Galvanized	Approx wt, lb
3	$6.00	$6.80	8	....	....	.....	$6.00	$6.80	7	$8.70	$10.90	13
4	8.00	9.00	12	....	....	.....	8.00	9.00	10	10.10	13.00	19
5	9.00	10.60	18	....	....	.....	9.00	10.60	14	11.80	14.60	27
6	10.00	11.90	25	$13.50	$15.90	30	10.00	10.90	20	13.00	16.00	37
8	13.00	15.80	33	18.00	21.50	43	13.00	15.80	25	17.00	20.00	46
10	20.00	24.00	53	26.00	31.00	67	20.00	24.00	41	25.00	30.00	79
12	24.00	29.00	65	33.00	39.20	87	24.00	29.00	51	29.00	35.00	97
14	32.00	39.00	90	43.00	52.00	119	32.00	39.00	67	41.00	50.00	135
16	44.00	54.00	124	55.00	70.00	160	44.00	54.00	95	71.00	86.00	185
18	79.00	97.00	148	90.00	110.00	194	61.00	75.00	112	97.00	118.00	222
20	90.00	108.00	168	100.00	125.00	224	70.00	85.00	125	108.00	130.00	252
22	100.00	125.00	215	115.00	140.00	280	80.00	98.00	158	125.00	150.00	322
24	120.00	145.00	263	135.00	160.00	337	90.00	110.00	195	142.00	175.00	395

9. WIRE GAGES, BOLTS, RIVETS, SPIKES

Table 19. Wire Gage Standards Used in United States

No of gage	American or Brown & Sharpe	Birmingham or Stubbs	Washburn & Moen Mfg Co	Imperial Wire gage	Stubbs steel wire gage	U S Stand for sheets and plate	Approx thickness, fractions of in	Wt per sq ft, lb
0000000	....	....	....	....	......	0.50	1/2	20.00
000000	....	....	....	0.464	......	0.46875	15/32	18.75
00000	....	....	....	0.432	......	0.4375	7/16	17.50
0000	0.46	0.454	0.3938	0.400	......	0.40625	13/32	16.25
000	0.40964	0.425	0.3625	0.372	......	0.375	3/8	15.0
00	0.3648	0.38	0.3310	0.348	......	0.34375	11/32	13.75
0	0.32486	0.34	0.3065	0.324	......	0.3125	5/16	12.50
1	0.2893	0.3	0.2830	0.300	0.227	0.28125	9/32	11.25
2	0.25763	0.284	0.2625	0.276	0.219	0.265625	17/16	10.625
3	0.22942	0.259	0.2437	0.252	0.212	0.25	1/4	10.0
4	0.20431	0.238	0.2253	0.232	0.207	0.234375	15/64	9.375
5	0.18194	0.22	0.2070	0.212	0.204	0.21875	7/32	8.75
6	0.16202	0.203	0.1920	0.192	0.201	0.203125	13/64	8.125
7	0.14428	0.18	0.1770	0.176	0.199	0.1875	3/16	7.5
8	0.12849	0.165	0.1620	0.160	0.197	0.171875	11/64	6.875
9	0.11443	0.148	0.1483	0.144	0.194	0.15625	5/32	6.25
10	0.10189	0.134	0.1350	0.128	0.191	0.140625	9/64	5.625
11	0.090742	0.12	0.1205	0.116	0.188	0.125	1/8	5.0
12	0.080808	0.109	0.1055	0.104	0.185	0.109375	7/64	4.375
13	0.071961	0.095	0.0915	0.092	0.182	0.09375	3/32	3.75
14	0.064084	0.083	0.0800	0.080	0.180	0.078125	5/64	3.125
15	0.057068	0.072	0.0720	0.072	0.178	0.0703125	9/128	2.8125
16	0.05082	0.065	0.0625	0.064	0.175	0.0625	1/16	2.5
17	0.045257	0.058	0.0540	0.056	0.172	0.05625	9/160	2.25
18	0.040303	0.049	0.0475	0.048	0.168	0.05	1/20	2.0
19	0.03589	0.042	0.0410	0.040	0.164	0.04375	7/160	1.75
20	0.031961	0.035	0.0348	0.036	0.161	0.0375	3/80	1.50
21	0.028462	0.032	0.03175	0.032	0.157	0.034375	11/320	1.375
22	0.025347	0.028	0.0286	0.028	0.155	0.03125	1/32	1.25
23	0.022571	0.025	0.0258	0.024	0.153	0.028125	9/320	1.125
24	0.0201	0.022	0.0230	0.022	0.151	0.025	1/40	1.0
25	0.0179	0.02	0.0204	0.020	0.148	0.021875	7/320	0.875
26	0.01594	0.018	0.0181	0.018	0.146	0.01875	3/160	0.75
27	0.014195	0.016	0.0173	0.0164	0.143	0.0171875	11/640	0.6875
28	0.012641	0.014	0.0162	0.0149	0.139	0.015625	1/64	0.625
29	0.011257	0.013	0.0150	0.0136	0.134	0.0140625	9/640	0.5625
30	0.010025	0.012	0.0140	0.0124	0.127	0.0125	1/80	0.5
31	0.008928	0.01	0.0132	0.0116	0.120	0.0109375	7/640	0.4375
32	0.00795	0.009	0.0128	0.0108	0.115	0.01015625	13/1280	0.40625
33	0.00708	0.008	0.0118	0.0100	0.112	0.009375	3/320	0.375
34	0.006304	0.007	0.0104	0.0092	0.110	0.00859375	11/1280	0.34375
35	0.005614	0.005	0.0095	0.0084	0.108	0.0078125	5/640	0.3125
36	0.005	0.004	0.0090	0.0076	0.106	0.00703125	9/1280	0.28125
37	0.004453	....	....	0.0068	0.103	0.006640625	17/2560	0.265625
38	0.003965	....	....	0.0060	0.101	0.00625	1/160	0.25

Table 20. Track Bolts (U S Standard Hexagon Nuts)

Rails, lb per yd	Bolts	No in keg, 200 lb (with nuts)	Kegs per mile	Rails, lb per yd	Bolts	No in keg, 200 lb (with nuts)	Kegs per mile
45 to 85	3/4 × 4 1/4	230	6.3	30 to 40	5/8 × 3 1/2	375	4.0
	3/4 × 4	240	6.0		5/8 × 3	410	3.7
	3/4 × 3 3/4	254	5.7		5/8 × 2 3/4	435	3.3
	3/4 × 3 1/2	260	5.5		5/8 × 2 1/2	465	3.1
	3/4 × 3 1/4	266	5.4	20 to 30	1/2 × 3	715	2
	3/4 × 3	283	5.1		1/2 × 2 1/2	760	2
					1/2 × 2 1/4	800	2
					1/2 × 2	820	2

Note.—For tables of R R spikes, and cut and wire nails and spikes, see Sec. 11, Table 3, and Sec 43, Table 14.

Table 21. Corrugated Iron or Steel Plates

Wt, lb per 100 sq ft (Amer Sheet and Tin Plate Co)

Corrugations	5/8 in		1 1/4 × 3/8 in		2 × 1/2 in		2 1/2 × 1/2 in		3 × 3/4 in		5 × 7/8 in	
U S Standard sheet metal gage	Painted	Galvanized	Painted	Galvanized	Painted	Galvanized	Painted	Galvanized	Painted	Galvanized	Painted	Galvanized
28	72	87	72	87	68	85	68	85	68	85	68	85
27	79	94	79	94	76	91	76	91	76	91	76	91
26	86	101	86	101	83	98	83	98	83	98	83	98
25	100	115	100	115	96	111	96	111	96	111	96	111
24	114	129	114	129	110	125	110	124	110	124	110	124
23	.....	.....	128	143	123	138	123	138	123	138	123	138
22	.....	.....	142	157	136	151	136	151	136	151	136	151
21	.....	.....	156	171	150	165	150	165	150	165	150	165
20	.....	.....	170	185	163	178	163	178	163	178	163	178
18	.....	.....	.....	.....	217	232	217	232	217	232	217	232
16	.....	.....	.....	.....	271	286	271	286	271	286	271	286

Table 22. Square-head Bolts, Wt in Lb per 100 (Hoopes & Townsend)

Diam, in	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2
Length														
1 1/2 in	9.7	14.7	20.4	26.0	37.0	58.0	.....	.....	.....	.....	.....	.....	.....	.....
2 "	11.3	16.5	22.4	29.0	39.9	63.2	97.7	145	.....	.....	.....	.....	.....	.....
2 1/2 "	12.9	18.5	25.0	32.2	44.1	69.0	105.6	153	.....	.....	.....	.....	.....	.....
3 "	14.5	20.5	27.8	35.4	48.3	75.2	113.8	163	240	309	350	480	.....	.....
3 1/2 "	16.1	22.6	30.6	38.7	52.5	81.4	122.0	174	253	325	370	500	.....	.....
4 "	17.7	24.7	33.4	42.0	56.7	87.6	130.2	185	267	342	390	520	800	.....
4 1/2 "	19.2	26.8	36.2	45.3	60.9	93.8	138.4	196	281	359	410	545	833	.....
5 "	20.7	28.9	39.0	48.6	65.1	100.0	146.6	207	295	376	430	570	866	1 370
5 1/2 "	22.2	31.0	41.8	51.9	69.2	106.1	154.9	218	309	394	450	595	900	1 414
6 "	23.7	33.1	44.6	55.2	73.4	112.2	163.2	229	323	412	470	620	934	1 458
6 1/2 "	25.2	35.2	47.4	58.5	77.6	118.3	171.5	240	337	430	490	645	968	1 502
7 "	26.7	37.3	50.2	61.8	81.8	124.4	179.8	251	351	448	510	670	1 002	1 546
7 1/2 "	28.2	39.4	53.1	65.1	86.0	130.5	187.1	262	365	466	530	695	1 036	1 590
8 "	29.7	41.5	56.0	68.5	90.0	136.6	195.4	273	379	484	550	725	1 070	1 634
9 "	33.1	45.7	61.5	75.2	98.0	148.8	212.0	295	407	518	590	775	1 138	1 722
10 "	36.5	49.9	67.0	81.9	106.3	161.0	229.0	317	435	552	630	825	1 206	1 810
11 "	40.0	54.1	72.5	88.7	114.6	173.2	246.0	339	463	586	670	875	1 274	1 898
12 "	43.5	58.3	78.0	95.5	122.9	184.4	263.0	361	491	620	710	925	1 342	1 986
13 "	....	....	83.5	102.3	131.2	196.6	280.0	383	519	655	751	975	1 410	2 074
14 "	....	....	89.0	109.1	139.5	208.8	297.0	405	547	690	793	1 025	1 478	2 162
15 "	....	....	94.5	116.0	148.0	221.0	314.0	427	575	725	835	1 075	1 548	2 250
16 "	....	....	100.0	123.0	156.5	233.2	331.0	449	603	760	877	1 125	1 616	2 338
17 "	....	....	105.5	130.0	165.0	245.4	348.0	471	631	795	919	1 175	1 684	2 426
18 "	....	....	111.0	137.0	173.5	257.6	365.0	493	659	830	961	1 225	1 752	2 514
19 "	....	....	116.5	144.0	182.0	269.8	382.0	515	687	865	1 003	1 275	1 820	2 602
20 "	....	....	122.0	151.0	190.5	282.0	399.0	537	715	900	1 045	1 325	1 888	2 690

Table 23. Lag Screws, Weights and Dimensions

Length, in	Diam, in (lb per 100)					Length, in	Diam, in (lb per 100)				
	3/8	7/16	1/2	5/8	3/4		3/8	7/16	1/2	5/8	3/4
1 1/2	6.88	....	....	....	....	4 1/2	14.82	19.50	28.25	42.62	67.88
1 3/4	7.50	11.75	16.88	....	....	5	16.50	21.25	30.37	47.75	71.37
2	8.25	12.62	17.18	....	....	5 1/2	17.37	23.56	33.88	51.62	79.37
2 1/4	9.25	12.88	18.07	....	....	6	18.82	25.31	35.37	55.12	86.62
2 1/2	9.62	13.28	19.18	....	....	7	....	....	38.94	61.88	92.75
3	10.82	16.62	22.00	34.07	....	8	....	....	44.37	68.75	97.50
3 1/2	11.50	18.18	24.00	35.88	....	9	....	....	....	77.00	108.75
4	13.31	18.88	26.82	39.25	64.00	10	....	....	....	90.00	124.75

Table 24. Weight of Rivets and Round-head Bolts per 100, without Nuts

Length under head, in	Diam, in							
	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
1.25	5.5	12.8	22.0	29.3	43.9	66.6	93.3	127
1.5	6.3	14.2	24.1	32.4	48.2	72.1	100	136
1.75	7.0	15.5	26.3	35.5	52.5	77.7	107	145
2	7.9	16.9	28.5	38.7	56.7	83.3	114	153
2.25	8.7	18.3	30.7	41.8	61.0	88.8	121	162
2.5	9.4	19.7	32.8	44.9	65.2	94.4	128	171
2.75	10.2	21.1	35.0	48.0	69.5	100	136	179
3	11.0	22.5	37.2	51.1	73.7	105	143	188
3.25	11.7	23.9	39.3	54.3	78.0	111	150	197
3.5	12.6	25.3	41.5	57.4	82.3	116	157	205
3.75	13.4	26.7	43.7	60.5	86.5	122	164	214
4	14.1	28.1	45.9	63.6	90.8	128	170	223
4.25	14.9	29.4	48.0	66.7	95.0	134	177	231
4.5	15.7	30.8	50.2	69.9	99.3	139	185	240
4.75	16.5	32.2	52.4	73.0	104	145	192	249
5	17.2	33.6	54.5	76.1	108	150	199	258
5.25	18.1	35.0	56.7	79.2	112	156	206	266
5.5	18.8	36.4	58.9	82.3	116	161	213	275
5.75	19.6	37.8	61.1	85.5	120	166	220	284
6	20.4	39.2	63.2	88 6	124	172	227	292

10. SPRINGS

Formulas relating to springs are based on torsion formulas, modified according to form of spring and cross-section of the wire or rod used. Following brief notes relate especially to springs of mine hoisting cages.

Helical spring, of round wire, is shown in Fig 35; Fig 36 illustrates its action. A load, P lb, is attached by a cord to the rim of a pulley, of a diam D = mean diam of the spring coils, and produces torsion in the rod. A line AB, originally straight, becomes a helix AC, length of rod being l. Angle between AB and AC is the angle of shear ϕ, and angle $BOC = \theta$, is the angle of twist. For equilibrium the external and internal moments must be equal, hence (Fig 36) $P\dfrac{D}{2} = f_s Z_p = f_s \dfrac{\pi d^3}{16}$, where Z_p equals section modulus (Sec 43, Art 4) derived from the polar moment of inertia, and d equals diam of wire or rod. From the equation, the load P which the spring will carry can be determined for any given fiber stress f_s by transposition, $P = \dfrac{2\pi d^3}{16D} f_s = \dfrac{\pi d^3}{16R} f_s$, all dimensions being in inches. The length of a coil of the spring is generally taken as πD; accurately, it is $\sqrt{(\pi D)^2 + p^2}$. On applying the load, the rod is twisted through an angle θ; or through a length of arc measured on the circumference of $D = \delta$, which is the moment of the weight P. If the distance from B to C measured on the rod circumference $= x$, then

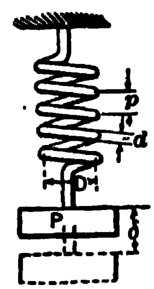


Fig 35. Helical Spring

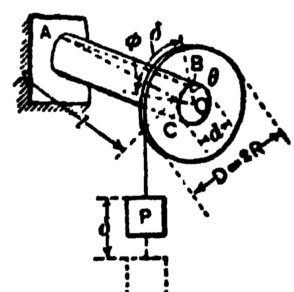


Fig 36

$$\frac{x}{l} = \frac{\text{Shear stress}}{\text{Modulus of rigidity}} = \frac{f_s}{G}.$$

Angle $\phi = x \div l$ and angle $\theta = x \div r$ or $2x \div d$; hence, $\theta = 2f_s \div Gd$, and $\delta = R\theta l = 2f_s l \div Gd$. Since $f_s = 16PR \div d^3$, $\delta = 32PR^2 l \div Gd^4$, which gives the deflection of the spring, where $l = 2\pi Rn$, if n = the number of complete coils. The resilience = 0.5 PS, providing the spring is not too open.

For rectangular wire or rod, the max stress occurs at centers of the longer sides. If b = thickness and h = width of section of wire,

$$I_p = \frac{b^3h^3}{3(b^2 + h^2)}, \quad \text{and} \quad Z_p = \frac{b^2h^2}{3\sqrt{h^2 + b^2}}.$$

Following the same analysis, $P = \dfrac{f_s}{3R}\dfrac{b^2h^2}{\sqrt{b^2+h^2}}$, and $\delta = \dfrac{3PR^2l}{G} \times \dfrac{b^2+h^2}{b^3h^3}$.

For wire of square section, $b = h$, $I_p = b^4 \div 6$, and $Z_p = b^3 \div 3\sqrt{2}$.

Flat spiral spring (Fig 37). In this the center is fastened to an arbor, the other end being attached to the framework. The formulas are correct if the spring winds uniformly and the center remains the center of gravity. If the load is applied at B, the greatest bending stress may occur at a point diametrically opposite A, or at the spindle if the curvature is great. For the first condition the external moment $= P \times 2R$ and $f = 2Pr \div Z = 12PR \div bh^2$; in the second case, $f = 6PR \div bh^2$. The load is $P = fbh^2 \div 12R$ (first case), or $fbh^2 \div 6R$ (second case).

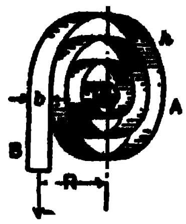


Fig 37. Flat Spiral Spring

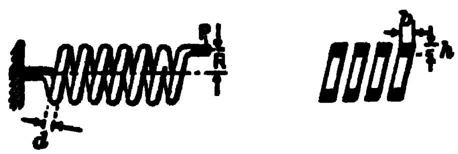


Fig 38. Round and Flat Helical Springs

From the theory of the elastic curve, the angle of deflection $\beta = PRl \div EI$, and $\delta = \beta R = PR^2l \div EI = 12PR^2l \div Ebh^3$, and resilience $= 0.5PR^2l \div EI$, and $I = bh^3 \div 12$.

Round and flat helical springs (Fig 38). For a spring subjected to a force P, tending to twist the spring at a radius R, the load $PR = fZ = f\pi d^3 \div 32$ for the circular section, and $PR = fbh^2 \div 6$ for the square section; also $\delta = 64PR^2l \div Ed^4$ for circular section, and $\delta = 12PR^2l \div Ebh^3$ for square section; the resilience being determined as above.

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SECTION 42

ELECTRICAL ENGINEERING

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ELECTRICAL ENGINEERING

1. DEFINITIONS, UNITS, AND STANDARDS

Potential (E or V), or DIFFERENCE IN POTENTIAL between two points, is theoretically the work required to move a unit quantity of electricity from one point to the other. It is analogous to difference in pressure, or head, of a current of water. The practical unit is the (International) VOLT, or electrical pressure which, when steadily applied to a conductor, the resistance of which is 1 ohm, will produce a current of 1 ampere. Difference of potential is measured in practice by a VOLTMETER (Art 4). For comparison and calibration the Weston Normal Cell is used, the electromotive force (e m f) of which is 1.0183 volt at 20° C. These cells are calibrated by Bureau of Standards, Washington. A millivolt is 0.001 volt.

Current (I) expresses the quantity of electricity flowing past a given point in 1 sec; it is analogous to gal per sec of a current of water, or to the miner's inch. The practical unit is the (International) AMPERE, or the unvarying electric current which, when passed through a solution of $AgNO_3$ in water, in accordance with standardized specifications, deposits silver at the rate of 0.001118 gm per sec. Current is measured by an AMMETER (Art 4). A milliampere is 0.001 ampere. DIRECT CURRENT (d c) is officially one which always has the same direction, though it may pulsate in value; but the term is popularly applied to a unidirectional current of constant value, officially called a CONTINUOUS CURRENT (c c). ALTERNATING CURRENT (a c) is one which changes its direction and amount in regularly recurring periods.

Resistance (R) in an electric circuit is similar to friction in mechanics. It is the property by which a circuit limits the amount of current caused by a steady voltage. The (International) OHM is the practical unit of resistance, and is the resistance offered to an unvarying electric current by a column of mercury, at temp of melting ice, which is 14.4521 gm in mass, of constant cross-sec and a length of 106.3 cm. Resistance is usually determined by computing the ratio between volts and amperes in a circuit according to Ohm's law (Art 2).

Power unit is the watt or kilowatt (kw). A watt equals 10 million units of power in the centimeter-gram-second system, and in practice equals the product of 1 volt by 1 ampere, or is the rate of expenditure of energy by 1 ampere in a circuit of 1 ohm. A KILOWATT is 1 000 watts; a horsepower (U S) = 746 watts.

Energy. The theoretical unit is the JOULE, which is 10 million ergs; in practice, equal to 1 watt times 1 sec. Commercial unit is the kilowatt-hour (kw-hr), equal to 3 600 000 joules. A circuit taking 10 amperes at 100 volts receives 1 kw of power, or 1 kw-hr of energy per hr; hence 1 kw-hr = 2 655 000 ft lb.

Coulombe is the unit of quantity of electricity (corresponding to a gallon of water), or the quantity transferred in 1 sec by a current of 1 ampere.

Capacity is the ability of an electric circuit to store energy in the space between conductors, measured in coulombes or quantity of electricity. A condenser is a device to provide capacity. FARAD (C) is the unit of capacity; or the capacity of a condenser which will be charged to a potential of 1 volt by 1 coulombe of electricity. A microfarad is one-millionth of a farad.

Maxwell (ϕ) is the unit quantity of magnetic flux, and is equal to 1 line of force.

Gauss (B) is the unit of magnetic flux density, and is equal to 1 line or 1 maxwell per sq cm.

Gilbert (F) is the unit of magneto-motive force (m m f), and is that m m f which will produce 1 line or 1 maxwell in a path in air having a cross-sec of 1 sq cm and a length of 1 cm. One gilbert = 1.257 ampere-turns.

Oersted is the unit of magnetizing force or magneto-motive force per unit length of path, expressed in gilberts per centimeter, or $0.4\pi NI \div 1$.

Reluctance. The practical unit is defined as the reluctance of a magnetic circuit in which a magneto-motive force of one ampere-turn produces a flux of one weber.

Weber is the practical unit of quantity of flux and is equal to 10^8 maxwells.

Henry (L), the unit of inductance, is the inductance of a circuit in which a variation in current of 1 ampere in 1 sec will induce an e m f of 1 volt.

Hysteresis. When iron or steel is magnetized by an electric current, the value of magnetic flux does not vary directly with the current. An increasing current does not produce the same flux for a given current as a decreasing current. This lagging of the

flux behind the magnetizing force is called hysteresis, and may be considered as the effect of friction between the molecules of iron which, in alining themselves with the flux like little magnets, rub against each other. Hysteresis causes loss of energy and heating of the iron, when the direction or magnitude of flux is changed. If the magnetizing force varies through complete cycles, positive and negative, the power lost in hysteresis is:

$$\text{Watts} = k_1 f V (B_m + 1\,000)^{1.6} \times 10^{-6}$$

where $k_1 = 6$ to 18, depending upon quality of iron; f = frequency in cycles per sec; V = volume, cu in; B_m = maximum magnetic density in lines per sq in.

Eddy currents are local currents induced in any conducting material located within the influence of any varying magnetic field. The term is especially applied to currents set up in the iron core of a machine, or in a conductor carrying an a c. These currents cause heating and loss of power and energy.

Effective resistance of a circuit carrying an a c is found by dividing the total loss in watts due to the combination of true resistance, eddy currents and hysteresis, by the square of the effective value of the current.

Series. Two circuits are in series when the same current passes through both successively.

Shunt. A circuit is in SHUNT, in MULTIPLE or in PARALLEL, with another when they form a divided circuit, so that a part of the total current passes through each, while the potential across them is common.

Constant potential circuit is one across which the potential is kept as nearly as possible constant, irrespective of the current flowing. It implies a supply from a circuit of low resistance. Most house lighting (incandescent), motor supply, and transmission systems are of constant potential type.

Constant current. Certain systems of lighting by arc lamps require constant value of current in all the lamps, which are therefore connected in series and the voltage across the whole circuit is varied to suit the number of lamps in operation. A special type of generator or transformer is required. In Europe the Thury system of long-distance transmission employs a constant d c, the voltage generated being varied with the load.

Conductance (G) of a circuit is that quantity which, if multiplied by the voltage, will give total current if voltage is unvarying, and which will give the component of current in phase with voltage ($I \cos \phi$) if voltage is alternating. For d c, conductance is the reciprocal of resistance. For a-c conductance, see Art 7. Conductance is used in analyzing multiple circuits. 1 volt applied to a circuit of 1 MHO conductance will cause 1 ampere to flow.

Rating of an electrical machine is the load in kw which it will give for a specified period of time without injury to itself. There are several limitations to the output, but the one most frequently met and inferred is injury to insulation by the heat developed. The limit is the temp of the hottest part of the machine, and the allowable temp depends upon character of the INSULATING MATERIALS. These are divided into 3 classes: (*a*) cotton, silk, and other fibrous materials, not so treated as to increase their heat-resisting qualities; (*b*) similar materials, but treated and impregnated, including enameled wire; (*c*) mica, asbestos, and other materials capable of resisting high temp. Temp of machines is usually determined by thermometer or by resistance measurements. Since neither method gives the temp of the hottest spot, it has been agreed to assume the temp of the hottest part as 15° C higher than indicated by a thermometer on the surface, and 10° C hotter than indicated by resistance measurement. Temp of surrounding air (ambient temp) influences directly the temp of the machine. Provision must be made that, when the air is at its highest possible temp (assumed at 40° C), the machine will not become excessively hot. This is done by specifying that the rise in temp above surrounding air shall be such that, if operating in a room of 40° C, its hottest spot shall not exceed certain established values. CONTINUOUS RATING is the load in kw which a machine will carry continuously without exceeding the above temp rise. INTERMITTENT RATING, or short time rating, is the load in kw a machine will carry for a short specified period (up to 2 hr) without exceeding these limitations. Unless otherwise specified, continuous rating is understood (Art 6).

Table 1. Temperature Conventions (Standard Rules of Am Inst Elec Engs)

Class of insulation	Highest permissible temp	Limiting temp by thermometer	Limiting temp rise	Limiting temp by resistance	Limiting rise by resistance
a	95° C	80° C	40° C	85° C	45° C
b	105	90	50	95	55
c	125	110	70	115	75

2. PRINCIPLES

Electric circuit. OHM'S LAW: $E = IR$, where I = current, amperes; E = e m f or difference of potential, volts; R = resistance, ohms. Used for solution of all simple d-c circuits and for a-c circuits having no reactance. For an a-c circuit having reactance, $E = IZ$, where I and E are effective values and Z is impedance in ohms (Art 7).

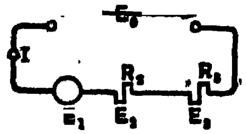


Fig 1. Series Circuit

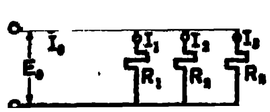


Fig 2. Multiple Circuit

Kirchhoff's laws are for solving any combination of series, multiple, or series-multiple circuits. FIRST LAW, for series d-c circuits: $E_0 = E_1 + E_2 + E_3$, where E_0 = generated e m f, E_1 = counter e m f (motor), and $E_2 = IR_2$, etc, are the drops in voltage in different resistances (Fig 1). Special case: $R_0 = R_1 + R_2 + R_3$. SECOND LAW (for multiple d-c circuits): the algebraic sum of all currents meeting at a point = 0 (Fig 2). $I_0 = I_1 + I_2 + I_3$, where I_1 may be found by $I_1 = E \div R_1$, whence

$$I_0 = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}; \text{ or, in general, } \frac{1}{R_0} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Magnetic circuit is calculated similarly to electric circuit: $\phi = \dfrac{\text{m m f}}{\text{reluctance}}$ where ϕ = flux in maxwells, m m f is in gilberts and reluctance $= l \div \mu A$, where l = length of path, cm; A = cross-sec of path, sq cm; μ is permeability of the material, varying from 1 for air to 3 000 in some steels.

Permeability. Since μ varies greatly with different materials, and with different densities in same material, it is more practical to calculate a magnetic circuit by means of magnetisation curves, which show relation between m m f per unit length of path and flux density in maxwells or lines per sq cm. Such a curve must be made for each different material used. In U S it is customary to plot the magnetisation curve between the ampere turns per in length of path and maxwells or lines per sq in (Fig 3). To determine the current to produce a given flux, divide number of lines of flux by cross-section of circuit in sq in, and find required magnetic density in lines per sq in. Referring to magnetisation curve (Fig 3) of the particular material to be used, the number of ampere turns per in of path is obtained for this particular density. Multiplying this number by length of path (in) gives total ampere turns required, which may be divided into any number of amperes or any number of turns, the product of which will give the required number. For a path in air, or in any non-magnetic material, the ampere turns per in = 0.313 times the density in lines per sq in.

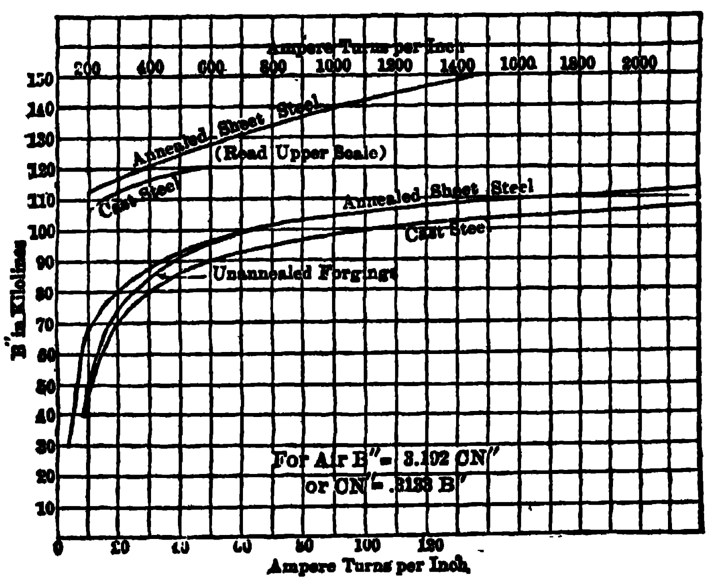


Fig 3. Typical Magnetisation Curves

Force on a conductor. Experiment shows that if a conductor carrying a current I and having a length l is placed in the magnetic field of density B gausses, at right angles to the direction of lines of force, a mechanical force will be exerted on the conductor, at right angles to the conductor and to the flux, the value of which will be $f = BlI$, where f = dynes, if the other quantities are expressed in c-g-s units. Practical form of this equation is $F = 0.89\ BlI\ 10^{-7}$, where F = lb, B = lines per sq in, l = in, I = amperes. This relation is basis for calculation of torque of any motor. In this case l must be the length of conductor in the active field only. The pull of any magnet is given by equation:

$$P = B^2A \div 72\,134\,000$$

where P = lb, B = lines per sq in, A = cross-sec of core, sq in. This is the case when 2 magnetic elements are in contact; when separated, the average gap density must be used.

Generation of an electromotive force (e m f). When a conductor is moved in a magnetic field so that it cuts lines of the field, an e m f is generated, numerically proportional to rate of cutting the flux. When conductor cuts 10^8 lines per sec, one volt is generated. If Z conductors are connected in series, as is usual in practice, $E = Z\phi \div 10^8 T$, where E = average volts generated and ϕ = flux cut in T sec. In the armature of any machine the relations are:

$$\text{Average } E \text{ of a d-c machine} = (4PNS\phi) \div (120 \times 10^8)$$

$$\text{Effective } E \text{ in an a-c machine} = (4.44PNS\phi) \div (120 \times 10^8)$$

where P = number of poles, N = rev per min, S = turns in series between terminals, ϕ = flux per pole, maxwells.

3. CONDUCTORS

Copper is generally used, iron and aluminum being next in importance. Though all metals are conductors, only these 3 are of commercial value, since their cost is reasonable and loss of energy is low. SILVER is the best conductor, but is too expensive for practical use. COPPER is nearly as good as silver, and is best economically. ALUMINUM is next; in the form of solid wire it is as cheap as copper, but lacks uniformity in mechanical strength. It is better when in form of stranded cables, but is then more expensive than copper of equal conductivity. IRON and STEEL approach copper in economic value, and for small-diam conductors are preferable due to their mechanical strength. An INSULATOR is a material used to prevent flow of current, but the division between insulators and conductors is not sharp; all insulators conduct some current. Common insulators are air, glass, porcelain, rubber, cotton, paper, and dry wood.

Table 2. Relative Characteristics of Conductors of Equal Resistance and Length

	Cross-sec for equal resistance	Wt for equal resistance
Copper	1	1
Aluminum	1.6	0.48
Iron, steel	8	7.1

Table 3. Resistances of Conductors

Material	Ohms resistance per mil-foot at 20° C, ρ	Temp coef, per cent increase per deg C, from 20° C
Silver, annealed	9.5	0.377
Copper, annealed (100%)	10.3	0.388
" , hard drawn	10.5	....
" , cast	12.0 to 97.0	....
Gold, annealed	13.5	0.365
Aluminum, commercial	16.8	0.388
Zinc	38.0	0.379
Platinum	57.0	0.24
Iron, pure	61.5	0.56
" , cast	34.0 to 68.0	....
Steel, soft *	104.0	0.388
" , rails *	121.0	....
" , special *	84.0	....
Nickel	84.0	0.56
Tin, pure	86.0	0.365
Lead	130.5	0.387
Mercury	570.0	0.072

* Steel being an alloy has varying spec resist. Steel used for track rails has approximately the value given in table, while for the "third" or conductor rail a special composition is used with lower spec resist.

Resistance of any conductor is given by: $R = \rho L \div A$, where R = resistance, ohms; L = length of conductor, ft; A = cross-sec of conductor, circular mils; ρ = specific resistance, or resistance of a conductor of unit cross-sec and length. Though the cm and sq cm are the more scientific units, it is convenient to use the circular mil for unit of cross-sec and the foot for unit of length, as adopted for commercial wires. A circular mil is the area of a circle having a diam of one mil (0.001 in). A circle 1 in diam has an area of 1 000 000 circ mils, and 1 sq in = 1 270 000 circ mils. Hence, in above equation, if length be 1 ft and A = cross-sec in circ mils, then ρ has values given in Table 3

Specific resistance ρ varies directly with the temp. Table 3 gives a constant for determining the percentage of increased resistance per deg C in excess of 20° C. General law of increase in resistance of copper with rise in temp is $R_t = R_0\,(1 + 0.0042\,t)$, where R_t = resistance at any temp, t° C; R_0 = resistance at 0° C; t = temp above 0° C.

Wire gages. Standard classification of wires in the U S is the Brown and Sharpe or American wire gage (Table 4).

B & S gage sizes have definite relations, readily remembered: A wire 3 sizes larger than another has nearly half the resistance, twice the area, and twice the wt per unit length. Thus, No 7 has

Table 4. Brown & Sharpe Wire Table

B & S or A W G size	Bare diam, in	Double cotton covered, diam, in	Area, circ mils	Wt, lb per 1 000 ft	Resistance: Ohms per 1 000 ft, 75° F	Resistance: Ohms per mile, 75° F	Current capacity, rubber insulation, amperes
......	1.152	......	1 000 000	3 050	0.01051	....	650
......	1.035	......	800 000	2 440	0.01313	....	550
......	0.891	......	600 000	1 830	0.01751	....	450
......	0.819	......	500 000	1 525	0.02101	....	390
......	0.728	......	400 000	1 220	0.02627	....	325
......	0.590	......	250 000	762	0.04203	....	250
0000	0.460	......	211 600	640	0.04906	0.2589	225
000	0.4090	......	167 805	507	0.06186	0.3265	175
00	0.3648	......	133 079	402	0.07801	0.4117	150
0	0.3248	......	105 538	318.8	0.09838	0.5189	125
1	0.2893	0.303	83 694	252.8	0.12404	0.6546	100
2	0.2576	0.272	66 373	200.5	0.15640	0 8254	90
3	0.2294	0.243	52 634	159	0.19723	1.0409	80
4	0.2043	0.216	41 742	126.1	0.24869	1.3125	70
5	0.1819	0.194	33 102	100	0.31361	1.6550	54
6	0.1620	0.174	26 250	79.3	0.39546	2.0871	50
7	0.1442	0.156	20 816	62.9	0.49871	2.6318	38
8	0.1284	0.140	16 509	49.88	0.62881	3.3184	35
9	0.1144	0.126	13 094	39.56	0.79281	4.184	28
10	0.1018	0.112	10 381	31.37	1	5.277	25
11	0.0907	0.101	8 234	24.88	1.2607	6.654	20
12	0.0808	0.091	6 529	19.73	1.5898	8.390	17
13	0.0719	0.082	5 178	15.65	2.0047	10.580	14
14	0.0640	0.075	4 106	12.44	2.5278	13.341	13
15	0.0570	0.067	3 256	9.84	3.1150	16.822	....
16	0.0508	0.059	2 582	7.81	4.0190	21.213	6
17	0.0452	0.053	2 048	6.19	5.0683	26.749	....
18	0.0403	0.048	1 624	4.19	6.3911	33.729	3

half the resistance of No 10. A wire 10 sizes larger has 0.1 the resistance and 10 times the area and wt. No 10 is approx 0.10 in diam, has area of 10 000 circ mils, resistance of 1 ohm per 1 000 ft at 20° C, and weighs 32 lb per 1 000 ft.

Resistance and wt of standard sizes of wire of other materials are found by using following ratios: ALUMINUM, multiply resistance given for copper by 1.61 and the wt by 0.301. IRON telegraph wire, resistance is 5.96 and wt 0.877 that of copper. STEEL wire, resistance is 8.82 and wt 0.883 that of copper.

Birmingham wire gage (B W G), also known as Stubbs, is largely used in Great Britain and somewhat in this country. Metric system is used in France and Germany. Roebling table is used in the U S for steel wires and cables (see Sec 41, Art 9).

Table 5. High-resistance Wires

	Composition	Resistance relative to copper	Temp coef, % increase per deg C	Max allowable temp, deg C
German silver, 18%	Ni-Cu	19.0	0.0310	....
" " 30%	"	28.0	0.0310	....
Iala (Boker)	"	27.3	0.0005	300
Advance	"	28.6	0.0018	480
Manganin *	Cu-Fe-Mn-Ni	24.0 to 43.0	0.0011	....
Superior	Ni-Steel	49.0	0.0720	....
Climax	"	51.00	0.0540	540
Nichrome	....	56.0	0.0430	1 200

* Manganin is used in electric meters because of its low temp coef.

4. MEASURING INSTRUMENTS

Solenoid. Simplest form of measuring instrument is the PLUNGER TYPE of solenoid. It consists of a coil of thick wire carrying current to be measured, and a soft steel plunger which is pulled into axis of coil by effect of the current acting against weight of plunger,

or against a spring. Operates with both d c and a c, but is not accurate with either. MAGNETIC VANE is a special form of solenoid instrument. Inside the coil carrying the current is a thin steel vane, pivoted in center of coil, and a fixed vane of triangular cross-section. When current flows in the coil, both vanes are magnetized alike and repel each other; hence, pivoted vane moves away from fixed vane, moving force being proportional to square of the current. Used both for d c and a c, but its accuracy varies with frequency and wave shape. INCLINED COIL solenoid instrument is very generally used to measure a c. A cylindrical coil of wire carrying current to be measured is set inclined to a spindle or axis carrying a steel vane. When no current flows, the vane is held in plane of coil by a spring. When current flows, the magnetic field is perpendicular to plane of coil and the pivoted vane, in moving to set its longer axis in the line of flux, rotates the spindle to which pointer is attached. Force is proportional to square of the current. Used for d c and a c, but is affected by wave shape and frequency. D'ARSONVAL PRINCIPLE, used in almost all instruments for d c, employs a coil of fine wire placed in the field of a permanent magnet, suspended so that it can rotate. Current to be measured, or a definite part of it, passes through the movable coil, causing a torque which is resisted by a spring. The force is directly proportional to the current. Only operative with d c, and is affected by stray magnetic fields.

Hot-wire instruments may be used to measure both d c and a c. The current is passed through a wire, which it heats and lengthens. Increase in length is taken up by a spring or weight, the movement of which is proportional to square of current.

Dynamometer consists of 2 coils, fixed and movable, so arranged that their magnetic fields act upon each other. The current, passed through the two in series, develops a torque (as in a series motor) proportional to square of the current. Movement of the coil is resisted by a spring and indicated by a pointer. As no iron is present, this instrument is accurate for d c and a c of all frequencies. The principle is used in the Siemens dynamometer, in some a-c voltmeters and ammeters and in all wattmeters. In wattmeters one coil is in series with the line and the other coil across the load, receiving current through a high resistance proportional to the potential.

Induction-type instruments utilize the fact that, if a coil carrying an a c has within it a piece of copper or aluminum, eddy currents will be induced in the metal, and these will react on the main current, giving a force or torque. The torque is proportional to square of current. Instrument operates only on a c, and is affected by varying frequency.

Ammeter operates on any one of above-mentioned principles. Its electrical circuit must have very low resistance, and usually the measuring coil carries only a small though definite part of the current. In parallel with working circuit, there is usually a metallic "shunt," which has such resistance that the maximum current to be measured causes a drop of about 50 millivolts (Art 1, par 1) in the shunt. Some meters have interchangeable external shunts, while others have shunts placed inside the case and permanently connected in the circuit.

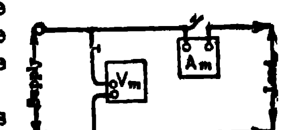


Fig 4. Connections of Voltmeter, Ammeter, and Protecting Switches

When the instrument is used with an external shunt it is sometimes called a MILLIVOLTMETER, its scale being marked in millivolts. An ammeter must never be left connected in the circuit without its shunt. Most ammeters for measuring d c operate on the d'Arsonval principle, as it gives the most accurate results. For a c, the inclined coil is in most general use. Most a-c meters will indicate as correctly on d c as on a c, but not as accurately as an instrument of the d'Arsonval type (Fig 4).

Voltmeter resembles an ammeter and in reality measures current, but its electrical circuit has a high resistance connected in series with it, to limit the current to a reasonable value. Since, in accordance with Ohm's law, with a constant resistance the current is proportional to the potential across the circuit, the scale deflection will indicate volts. The resistance must be non-inductive, and must not vary with temperature. When the resistance coil is separate from voltmeter the coil is called a multiplier.

The dynamometer type voltmeter is to be preferred for a c, and a potential transformer may be used instead of a multiplier if required to measure a voltage higher than the range of the voltmeter proper. ELECTROSTATIC VOLTMETER, a special instrument for measuring very high voltages, utilizes the force exerted between two charged bodies. A moving vane is connected to one side of the circuit, a fixed vane to the other. As the high potential charges these vanes, a force is exerted between them causing the movable vane to rotate. This instrument is used for either a c or d c.

Wattmeter is used on both a-c and d-c circuits for measuring the true or active power in watts; EI in d-c circuits, $EI \cos \phi$ in a-c circuits. In d-c circuits a voltmeter and an ammeter may be used to measure power, the product of the volts and amperes giving true

power, but in an a-c circuit this is not true. The wattmeter has a potential coil connected in parallel with the load (like a voltmeter), and a current coil connected in series with the load (like an ammeter). If the needle shows negative deflection, the terminals of one coil or other must be reversed. Range of the wattmeter must be selected with reference to the volt and current capacity of its coils. A "multiplier" may be used in the potential coil and a current transformer in the circuit of the current coil.

Watt-hour meter measures energy by recording watt-seconds. Coils are arranged to form a motor, which causes a disk to revolve at a velocity proportional to the power. A counter registers number of revolutions of the disk, and this number represents energy. Watt-hour meters for a-c circuits are constructed differently from those for d-c. (Art 11.)

Power-factor meter determines the phase angle (ϕ) between voltage and current of an a-c circuit. Since in practice the power factor is the more important, the scale is graduated in $\cos\phi$. Instrument is constructed like a wattmeter having 2 coils, one for potential and one for current.

Frequency meter indicates the frequency in electrical cycles per second of the circuit voltage. The two common types are the vibrating reed and the induction. They are connected to the line like voltmeters.

Recording meters are often used in power stations to provide a continuous record of voltage, current or power. A pen, attached to the indicating element, rests on a strip or disk of paper, and draws a line as the paper moves by clock work.

5. DIRECT-CURRENT GENERATORS

Application. Direct-current (which is officially named continuous current) apparatus is used for short transmission distances. For long distances the a-c system (Art 7) is preferable, because the transformer (Art 13) permits transmission at very high voltages. However, since the d-c motor is more convenient and more easily controlled than the a-c motor, the d-c system is used whenever possible. It is often found advisable to generate and transmit by means of a-c, which is then converted to d c at the place where the motors are located. Thus, most generating stations produce a c, though many of the motors supplied by these stations use d c provided by synchronous converters or rectifiers.

Fundamental principle of a dynamo is the production of e m f (Art 2) in one or more conductors by the motion of these conductors in a magnetic field. In most machines the e m f is alternating; first positive, as conductor passes a north pole, then negative, as it passes a south pole. For use as a d-c machine a commutator must be provided to rectify voltage and current. The commutator is the distinguishing feature between d-c and a-c machines. Small machines are usually of the bipolar type, with a more or less enclosed frame of cylindrical shape. Large machines are multipolar; that is, having a large number of inwardly projecting radial pole pieces. Commutating pole or INTERPOLE machines have small auxiliary poles placed alternately with respect to the main poles and excited by a few turns in series with the load. Effect of these poles is to improve the commutation of the machine. Machines are divided generally into belt-driven and direct-connected, the former having the higher speed.

Shunt machine, either generator or motor, is one in which the entire field excitation is derived from a circuit of many turns and high resistance, connected in shunt or multiple with the armature circuit (Art 6, Fig 8). The characteristic of a shunt generator is poor regulation; that is, voltage decreases as load increases, and hand regulation of a rheostat in the field is necessary to provide steady voltage.

Compound-wound machine has on each field pole, in addition to its shunt winding, a few turns of thick wire which carry the load current and are known as series winding (Fig 5, 6). This causes excitation to increase as load increases, and tends to keep the terminal voltage constant or even to increase it. If the field windings are proportioned to cause a higher voltage at full load than at no load, the machine is over-compounded. A flat-compounded machine has the same voltage at full load as at no load; an under-compounded machine has a lower voltage at full than at no load. A compound-wound machine may be connected either in short shunt (Fig 5), which is usual practice, or in long shunt (Fig 6), in which the field winding is connected from outside of the series field to outside of the armature.

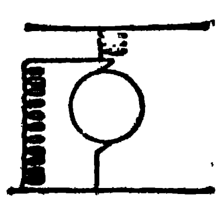


Fig 5. Short Shunt Connections

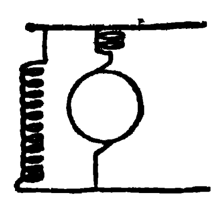


Fig 6. Long Shunt Connections

Constant potential vs constant current. In constant-potential type an endeavor is made to maintain a constant potential across the load. For particular purposes machines sometimes have their field connected in series with the armature, and with a special regulating device for giving constant current irrespective of load. These machines were formerly much used in arc lighting, but are not now built.

Installation. In installing and erecting a d-c machine, certain features must receive careful attention, so that the machine shall operate properly and not deteriorate too rapidly. The most important considerations are: (*a*) For large machines the base is bolted to the foundation in accordance with the drawings. (*b*) Bearings are lined up, cleaned, and oiled. (*c*) Field coils are tested for open circuit and wrong connections. Test is made with a compass for correct polarity. For a self-excited generator there is one particular connection of the field to the armature for each direction of rotation. (*d*) Armature must be properly centered so that the air gap is correct at all points. Gap is measured by taper wedges. (*e*) Magnet frame is bolted to base. (*f*) Commutator is smooth and polished; use sand paper, never emery paper, to polish commutator. (*g*) Brushes are properly and accurately spaced around the commutator, sandpapered and fitted to its curvature, and their pressure adjusted to the correct value, usually 1.5 to 2 lb per sq in of contact surface. (*h*) Field connections are adjusted for correct direction of rotation and substantial connections made. (*i*) Machine must be protected from moisture during shipment; if it be damp it must be dried out by heat before starting.

Operation. In starting a single generator it may be necessary to CHARGE THE FIELD by exciting the shunt field separately for a moment to set up residual magnetism. To cause the machine to PICK UP, or generate voltage by self-excitation, the rheostat connected in series with the shunt field must be cut out, or short-circuited. If total resistance of shunt-field circuit exceeds a certain critical value the machine will not "pick up," however much time is allowed.

Parallel operation. For economical operation of a power station there must be a number of machines, whose aggregate capacity equals the maximum demand on the station. As the demand varies the number of machines in operation is adjusted so that those running are operating at a load near their rating, and therefore at good efficiency. To operate SHUNT GENERATORS in parallel (that is, feeding the same bus-bars); it is only necessary to adjust all to the same polarity and voltage, connect them to the bus-bars, and adjust division of load by strengthening the field of the underloaded machine if voltage of the bus-bars is low, or weakening the field of the overloaded machine if voltage is high. To operate COMPOUND GENERATORS in parallel there must be an equalizer connection, making a common connection on all the machines at the point between the armature and the series field (Fig 7). The equalizer divides the load current in proper proportion between the series fields of the different generators, preventing the machines from acting as series generators or differential motors, which would cause short-circuits. For successful operation of compound machines in multiple all those connected to one set of bus-bars must have same amount of compounding, and same voltage at no load. The compounding curves of machines must be investigated before they are operated in parallel. With unlike curves, one machine may be overloaded while another is underloaded, unless the field current of one of them is adjusted.

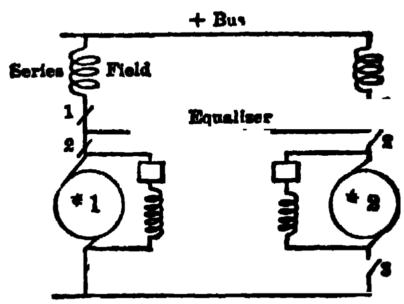


Fig 7. Parallel Operation of Compound-wound Generators

If a machine is to be connected in parallel with those already in operation it must have proper polarity and voltage, the equalizer circuit must be made, and the switches are closed in the order 1, 2, 3, as in Fig. 7. With any other order the effect is the same as having no equalizer. In shutting down one machine, switch No 3 must be opened first; then No 2 and No 1. For MANAGEMENT, see Art 6.

Testing. D-c machines are tested for regulation, commutation, and heating, by subjecting them to a load, but their efficiency is best determined by the separate loss method, for which the machine need not be run under load. Customary commercial tests are:

(*a*) **Resistance measurements** are made on armature winding, shunt field, and series field, at known temp, before the machine is run and again after the heat run (*d*). Resistance of a lap or multiple winding is found by measuring between two diametrically opposite points on commutator. If this resistance be R', then the actual armature resistance is $R_a = 4R' + p^2$, where p = number of poles on machine. Resistance of a series-wound armature is found by measuring between two commutator segments, separated by a distance equal to periphery of commutator divided by number of poles. Resistance of the brush contact can not be accurately measured by any simple means, as its value is affected by value of the current, speed of commutator and pressure of brushes. An accepted method is to assume a total loss of 2 volts in all brushes.

(*b*) **Core loss and friction (stray power).** Machine is run as motor without load, field current being held at proper value and the voltage applied to armature adjusted to give desired speed. Input to armature is measured; subtracting from this the calculated I^2R of armature circuit, the net value of input = core loss + friction.

(*c*) **Load runs.** Machine is run and the excitation adjusted to give proper voltage at no load. Load is then added and terminal voltage noted. If E_0 = voltage at no load and E = voltage at full load, the regulation is $(E_0 - E) \div E$. Brushes must be set in proper position for commutation, which is judged by observing action of brushes at no load, full load, and 150% load.

(*d*) **Heat runs** are made by operating at full load until the temps of the various parts that can be noted during operation become constant. The greater the machine's capacity, the longer is the testing time required. Heat runs are sometimes made at 1.25 or 1.50 times rated load for 2 hr. Heat runs may be made either by the DEAD-LOAD METHOD, in which a resistance load such as a water rheostat is connected to the terminals and full rated load power is required to drive the machine; or by the Hopkinson or PUMPING-BACK METHOD, in which 2 machines of similar characteristics are run in multiple, one acting as a generator to supply electrical power to the other as motor, which in turn drives the first by a belt or equivalent mechanical connection. For this test a LOSS SUPPLY is required, which may consist of a source of either electrical or mechanical power. The power required is from 10 to 20% of the rating of one machine. During a heat run, thermometer readings are taken at stated periods, showing temp of frame, field-coils, bearings, and surrounding air. After the run thermometers are placed on various parts of the machine, the bulbs being protected from radiation by small cotton pads, and the temps are noted of armature core surface, ventilating ducts and winding, commutator surface, pole-tips, field-coils, bearings, frame, and the room. Resistance of the elec circuits should be measured, and average temp of the copper calculated from: $t_1 = (R_1 \div R_0)(234 + t_0) - 234$, where R_1 and R_0 are the hot and cold resistances, and t_1 and t_0 are the hot and cold temps.

(*e*) **Compounding test.** To adjust the current in series field so that a compound-wound generator will give specified voltages at no load and full load, the machine is first operated at no load and current in shunt field adjusted to give desired no-load voltage. Load is then put on, and it will usually be found that the terminal voltage is too great. Strips of German silver or other resistance metal are then connected across the series field terminals, until by shunting current from the series field the machine voltage is reduced to desired value. This "series-field shunt" is then insulated, and made up into permanent form. Before making the no-load adjustment, it is advisable to over-excite the shunt field for a moment to overcome hysteresis.

(*f*) **Insulation tests.** After the heat run the insulation is tested, first by measuring its resistance and then by applying a high-potential test. INSULATION RESISTANCE is found by connecting one terminal of a 500-volt d-c circuit to the windings through a 500 voltmeter, and connecting the other terminal of the 500-volt circuit to the machine frame. If voltmeter resistance is R_v and the deflection during above connection is x, then insulation resistance in ohms is: $R = R_v\,(500 - x) \div x$. The value of R should be approximately one megohm (1 000 000 ohms) for each 1 000 volts of rated potential of the machine. HIGH-POTENTIAL test is made by applying an alternating high potential between each winding and the frame of the machine for 1 min, in accordance with specifications adopted by Am Inst Elec Engs. This test is usually made only at the factory and should be under supervision of an experienced person.

6. DIRECT-CURRENT MOTORS AND MOTOR-GENERATOR SETS

Shunt motor (Fig 8) has only one exciting winding, which is connected across armature terminals, and is in parallel or shunt with the armature. Field winding consists of a large number of turns of fine wire on each pole, and all poles are usually connected in series in one circuit. Field current depends upon line voltage and resistance of field winding. Resistance of field winding is made high, so that the field current will be between 1% (large motors) and 5% (small motors) of the full-load current of motor. The characteristic of the shunt motor is a fairly constant speed for all reasonable load values.

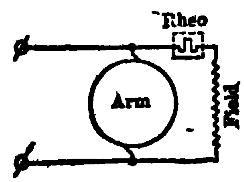


Fig 8. Shunt-motor Connection

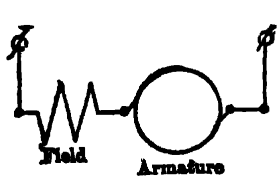


Fig 9. Series-motor Connection

Series motor (Fig 9) has only one exciting winding, which is connected in series with the armature, and all the current flows through both field and armature. Field winding consists of a few turns of thick wire on each pole, the windings on all poles being connected in series. Field current depends upon the load; large with heavy load and small with light. Resistance of field winding is made low, so that loss of voltage and power in that circuit will be small. Characteristics of a series motor are: a speed varying with every change in load, high speed at light load, and low speed at heavy load. Efficiency is high throughout a wide range of speed. Speed will be dangerously high at no load; thus a series motor must always be connected rigidly to its load. Since torque is high at low speeds, this motor is especially adapted to work requiring frequent starting.

Compound motor. Usual form is the cumulative motor, each pole of which has both series and shunt winding, wound and connected for mutual assistance in producing the magnetic field. It is a combination of shunt and series motor, designed to give the good starting qualities of the series motor and to avoid danger of excessive speed at light loads.

Differential motor has opposed shunt and series winding; not much used and not recommended.

Inclosed vs open type referring to the mechanical housing of the motor. Open type has all parts freely exposed to the air and is therefore well ventilated. Intended to be used indoors or in protected places. Inclosed type is for use in exposed locations, where there is liability of dampness and dirt. Special means must be provided to circulate air inside the machine, but even then an inclosed motor is larger and more expensive than an open motor of same capacity. Relative capacities in output of open, semi-inclosed and totally inclosed motors, are shown by Table 6. Totally inclosed motor weighs about 15% more than an open motor of same capacity, notwithstanding that it is allowed by commercial convention to operate at 15° C higher temp.

Table 6. Data of Typical Commercial Motors

Type	Output, h p	Temp rise, deg C	Weight, lb	Speed, r p m
Open............	10.0	40	970	700
Semi-inclosed....	8.0	40	970	700
Totally inclosed.	5.75	55	970	775

Rating. Motors are rated on basis of their continuous or their intermittent capacity. Continuous rating is the output in h p (or kw) which a motor will give continuously with a maximum temp rise (measured by thermometer above the surrounding air) not exceeding 50 or 70° C in field or armature, depending upon character of insulating material (Art 1). Intermittent rating is the output which the motor will give for 1 hr (starting at room temp), with a maximum rise in temp above surrounding air not exceeding the above values (Standardization Rules of ASA C-50).

Voltage and current. Usual values of voltage for d-c motors are: 110–125 for small units on lighting circuits; 220–250 for motors in factories, shops, and mines, on power mains or on outside mains of a 3-wire system; 500–600 for general railway work; 1 200–2 400 for heavy railway work. Current in amperes required for any motor is: $I =$ (output in h p $\times$ 746) $\div$ (effic $\times$ voltage). Usual efficiencies of motors of different sizes are given in Table 7.

Applications of motors. Chief applications of d-c motors: shunt motor, driving shafting, machine tools, blowers, reciprocating pumps, motor-generators; series motor, R R, and all transportation work, hoists, cranes; compound motor, elevators, hoists, or machinery that is stopped and started frequently.

Speed characteristics. Shunt motor is always used when constant speed is desired. It may be obtained with characteristics such that the drop in speed from no load to full load is 5 to 10%. At full load the speed of any well-designed shunt motor may be increased to equal the no-load speed, by putting resistance into the field circuit. Variable-speed motors are of the series type; they decrease in speed as load increases, every change in load causing change in speed. Multispeed motors are of shunt type, and usually have commutating poles so that they may operate with different strengths of field without suffering from bad commutation. They may be controlled by varying the field resistance, or the voltage impressed upon the armature. By increasing resistance in series with the field of a shunt motor speed is increased, due to weakening of the field. If the motor has commutating poles, to assure good commutation, speed may be varied in ratio of 1 to 2, or even 1 to 4 in small sizes. Motor speed is stable with this mode of control and efficiency is good.

Potential control. By means of several generators and several wires, various definite voltages are available. Thus, with one main generator giving 240 volts and 3 smaller ones giving 40, 80, and 120 volts, combinations may be made giving 40, 80, 120, 160, 200, and 240 volts. By connecting the armature terminals to these voltages, the motor will run at speeds proportional to the voltages. The shunt field is left continuously connected to one circuit, usually the maximum voltage circuit. A shunt motor with normal excitation will be stable at each speed; that is, will operate at fairly constant speed irrespective of load. Efficiency will be good at all speeds. A number of wires are needed to make the different voltages available at the places where the motors are located.

Variable-speed motors. By connecting a rheostat in series with a shunt motor armature, the voltage impressed upon the armature will be reduced by an amount proportional to the current, and speed will thereby be reduced. The speed is unstable, changing with every change of load, and efficiency is poor.

Starting box (rheostat) is always employed in starting d-c motors, to reduce the voltage impressed on the armature when not running at a speed high enough to generate the

proper counter e m f. Fig 10 shows usual connections of a starting box to the line and motor.

Starting box contains: (*a*) means of opening and closing the circuit supplying current to the motor, including the field current; (*b*) set of resistance steps in series with motor armature, and a means of short-circuiting this resistance step by step; (*c*) a magnet coil connected across the motor terminals to open the circuit if impressed voltage fails (low-voltage release); (*d*) a magnet coil carrying main current to actuate a spring and open main circuit, if current exceeds a specified value (overload release).

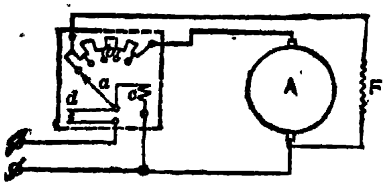


Fig 10. Starting Box Connections for Shunt Motor

Specifications. In calling for bids for d-c motors it is customary to specify: kind of service and load, method of drive, voltage, horse-power, speed, and whether open, semi-inclosed or inclosed, series, shunt, or compound-wound; if shunt-wound, whether shunt field rheostat is to be supplied; if compound-wound, whether cumulative or differential; dimensions of pulley and shaft extension, whether rails or base are required; whether starting box is to be supplied and if so, the style; temp rise for continuous full load; temp rise for 25% overload; efficiency at 25, 50, 75, 100, 125, and 150% load; starting torque required; necessity for withstanding moisture; regulation; percentage of variation in speed between no load and full load, with field resistance constant; variation in speed at full load obtainable by variation of field rheostat.

Installation. (See Art 5.)

Costs. (See Art 19.)

Operation. All motors should be frequently inspected and following points noted: (*a*) Bearings filled with proper amount of oil; (*b*) brushes securely held in proper position; (*c*) brushes fit properly; (*d*) commutator smooth: look for "high mica," or projection of the insulation above the bars; (*e*) air gap true; (*f*) commutator not worn in grooves.

Troubles. Following troubles that may occur in operating d-c motors, and their causes, are stated by Crocker and Wheeler: (*a*) Sparking at commutator. Causes: armature carries too much load, brushes improperly spaced or not in proper position, rough commutator, poor brush contact, internal short or open circuit, field too weak, unequal strength of poles, vibration. (*b*) Heating of commutator and brushes. Causes: sparking, bearing trouble, bad connections, brush friction too great. (*c*) Heating of armature. Causes: overload, internal short circuit, moisture or ground, reversed coil. (*d*) Heating of field. Cause: internal short circuit. (*e*) Heating of bearings. Causes: bearings dry or dirty, shaft out of true, bearings out of line, thrust due to belt, unbalanced magnetic pull. (*f*) Noise. Causes: armature not balanced, brushes dry or not set at proper angle, armature strikes. (*g*) Speed too low. Causes: wrong voltage, overload, armature strikes, bearing too tight. (*h*) Speed too high. Causes: wrong voltage, field too weak. (*i*) Motor stops, or fails to start. Causes: overload, open circuit, wrong connection.

Tests. A motor is usually tested for resistance of insulation and of windings, heat run, stray power, regulation and commutation. Regulation test consists in measuring speed at no load and at full load. Other tests are the same as made on d-c generators (Art 5).

Constants. Efficiency of any particular size of electrical machine may vary widely according to its design. For weight and cost, see Art 19.

Table 7. Average or Usual Constants for D-C Machines

Kw rating	1	5	10	20	50	100	200	500
Efficiency at full load	0.80	0.84	0.86	0.88	0.90	0.91	0.92	0.93
Rev per min	2 000	1 100	850	680	650	600	450	350

Motor-generator set is a combination of motor and generator having separate fields and armatures, but mounted on same shaft with common base and bearings. The two machines may both be for d c, both for a c, or one for each, in accordance with the use to which the plant is to be put. Principal applications are:

Balancers. Two similar d-c machines mounted on the shaft, and connected in series across the mains of a 250-volt circuit. From the common connection a third wire is brought out, thus giving a 3-wire system for 125-volt lamps on each leg, and for 250-volt motors on the outside main. With compound machines the regulation on the 125-volt circuits is good.

Boosters. To raise the voltage on a particular feeder a series booster is used, consisting of a shunt motor driving a series generator, the latter being connected in series with supply and feeder. The set runs at constant speed, and the voltage increases with the current to the load. Voltage at

full load is from 10 to 20% of the rated voltage of the circuit. The combination may be designed to neutralize loss in voltage due to drop in the feeder.

Multiple-voltage systems. Motor-generator sets are used to give the fractional voltages required for these systems. Each set consists of 2 or 3 machines operating as a balancer; that is, each may be a motor at one moment and a generator at the next, depending upon which circuit is supplying the load.

Ward-Leonard system. For regulating the speed of a motor a special generator may be provided, which, being driven by its own motor, forms a motor-generator set. By varying the field strength of the generator any voltage may be generated, from 0 to full rated voltage. If this generator supplies only the armature of the motor, the speed of which is to be regulated, its speed may be controlled from 0 to full speed by merely varying the small current flowing in the field of the special generator. Main supply circuit furnishes current for the motor of the set, and fields of the generator and the controlled motor.

Ilgner sets. Where a motor must start and accelerate a heavy load frequently, as in mine hoists and rolling mills, the motor is advantageously supplied from a special set, consisting of a motor (shunt or induction) driving a special generator, with a heavy flywheel forming part of the set. For such service the set motor must have poor speed regulation. As the accelerating motor starts, it draws a heavy current from the generator, causing speed of set to drop and the flywheel to give up considerable stored energy. This energy supplies the peak demands, and when the peak load period has passed the set speeds up gradually and stores more energy in the flywheel. This combination is expensive, but is used where conditions limit the maximum demand which may be made on the electrical supply system. (See Sec 16.)

7. ALTERNATING-CURRENT CIRCUITS

Sine wave. Every generator for d c as well as for a c has induced in its own windings an alternating voltage. The commutator of the d-c machine rectifies this, giving direct or continuous voltage. Since an a-c machine has no commutator, this voltage is brought to the load in its alternating form by slip-rings.

In designing machines every endeavor is made to obtain an e m f which follows a true sine law: $e = E_m \sin \theta$, in which e is the instantaneous value, E_m the maximum value, and θ gives the phase time, or abscissa at which the instantaneous value is specified. All calculations of a-c circuits are based on assumption that voltage and current do follow a sine law.

Frequency. The series of values from a to c (Fig 11) constitute a cycle or period. Time in seconds required for one cycle is the periodic time T, and $1 \div T$ (number of cycles per sec) is the frequency f. In commercial practice frequencies of 25, 50, and 60 cycles per sec are used, the first for general power, the latter for lighting purposes. If θ is expressed in radians, then $\theta \div t = 2\pi f$, and $\theta = 2\pi f \times t$. Thus phase θ is a conventional means of expressing a given instant of time t. A cycle may be divided into 360°, and θ may be the abscissa expressed in degrees.

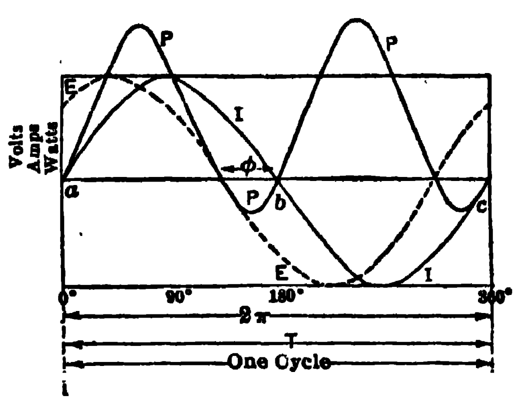


Fig 11. Alternating-current Voltage and Power, Rectangular Coordinates

Effective, virtual, or root mean square value of an a c is equal to that value of a d c which would produce the same heating effect or average power loss in a resistance RI^2. If current follows a sine law, the effective value is equal to maximum value divided by $\sqrt{2}$. Effective value of e m f is that which, when multiplied by the effective value of current in a simple resistance, will give the true power in watts. It is also equal to maximum value divided by $\sqrt{2}$. Effective values of current and voltage are the values shown by all instruments, and are the only ones used in practical engineering. (See Sec 16.)

Inductive reactance, inductive circuit. Any circuit in which the presence of a current produces a magnetic flux is an inductive circuit, the quantity of inductance depending upon the number of turns and the quantity of flux produced by one ampere. When an a c flows in an inductive circuit the current lags behind the impressed e m f, and the relation between voltage and current in such a circuit is $E = 2\pi fLI$, where E = effective value of e m f, f = frequency in cycles per sec, L = inductance in henries, I = effective value of current. $X_L = 2\pi fL$ is the inductive reactance in ohms. $E = X_L I$ in a circuit having inductance only, and the current will lag behind the voltage by 0.25 of a period.

Capacity reactance, anti-inductive circuit. If an alternating e m f is impressed upon a circuit having capacity only, the current which flows precedes or leads the e m f by 90°, and its value is $I = 2\pi fCE$, where C is the capacity in farads and the other quantities are as in preceding paragraph. $X_c = 1 \div 2\pi fC$, called capacity reactance, is expressed in ohms and $E = IX_c$.

Power and power factor. In an a-c circuit containing inductance or capacity, as well as resistance, the product of effective volts and effective amperes (EI) does not give true power, because the current will be out of phase with the voltage; that is, the maximum value of current will occur either later than or before the maximum value of voltage. Therefore EI is the volt-amperes or "apparent power." Ratio of true power P to apparent power is the power factor. It may be proved that the power factor is equal to the cosine of the angle of phase difference between the voltage and current, as shown by equation: Power factor $= \cos\phi = P \div EI$, where ϕ is the time difference in phase between voltage and current, expressed in angular measure.

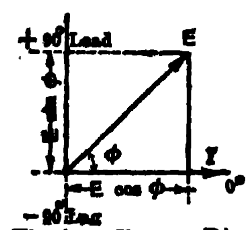


Fig 12. Vector Diagram of Alternating-current Voltage

Fig 11 shows an alternating voltage E and an a c I differing in phase by an angle ϕ, which means that their maximum or their zero values occur at a difference in time of $(\phi \div 360)\ T$ sec, where T is the periodic time of one cycle. Fig 12 is a vector representation of these quantities, and conventionally I is said to lag behind E by an angle ϕ. $E \cos\phi$ is the component of the voltage in phase with the current, and is called the "active component." The useful or heat power in watts is $P = EI\cos\phi$. $E \sin\phi$ is the component of the voltage out of phase with the current, and is called the "reactive component." $Q = EI\sin\phi$ is the reactive power, as it reacts on the impressed forces without producing useful work or heat. Total volt-amperes or apparent power is $EI = \sqrt{P^2 + Q^2}$.

Kilovolt-ampere (*kva*) equals 1 000 volt-amperes.

Calculations. A corollary of the law of conservation of energy is that the sum of all the true (heat) power consumed in a circuit is equal to the input of true power, $P_0 = P_1 + P_2 + P_3$. The sum of all the reactive power consumed in a circuit is equal to the input of reactive power, $Q_0 = Q_1 + Q_2 + \text{etc}$, and $E_0 I_0 = \sqrt{P_0^2 + Q_0^2}$.

Kirchhoff's laws for a-c circuits. First law (for series circuits) (Fig 13 and 14): The sum of all active components of voltage ($E\cos\phi = IR$) in a closed circuit is equal to zero, and the sum of all reactive components ($E\sin\phi = IX$) is equal to zero. Hence,

$$E_0 \cos\phi_0 = E_1 \cos\phi_1 + E_2 \cos\phi_2 + \text{etc}$$

$$E_0 \sin\phi_0 = E_1 \sin\phi_1 \pm E_2 \sin\phi_2 + \text{etc}$$

$$E_0^2 = (E_0 \cos\phi_0)^2 + (E_0 \sin\phi_0)^2$$

Reactive component of voltage in an inductance is positive; that in a capacity is negative. Second law (for multiple circuits) (Fig 15 and 16): The sum of all active components of current ($I\cos\phi = ER \div Z^2$) meeting at a point is equal to zero, and the sum of all the reactive components ($I\sin\phi = EX \div Z^2$) is equal to zero.

$$I_0 \cos\phi_0 = I_1 \cos\phi_1 + I_2 \cos\phi_2$$

$$I_0 \sin\phi_0 = I_1 \sin\phi_1 + I_2 \sin\phi_2 - I_3 \sin\phi_3$$

$$I_0^2 = (I_0 \cos\phi_0)^2 + (I_0 \sin\phi_0)^2$$

Reactive component of current in an inductance is negative; in a capacity, positive.

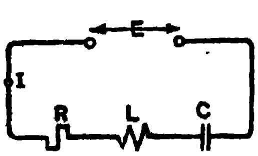


Fig 13. Series A-c Circuit, with Resistance Inductance and Capacity

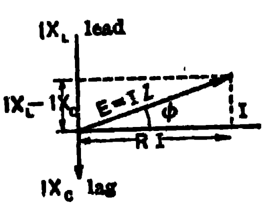


Fig 14. Vector Diagram of Series Circuit

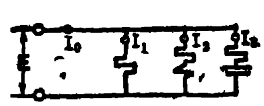


Fig 15. Multiple A-c Circuit with Resistance Inductance and Capacity

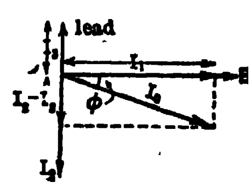


Fig 16. Vector Diagram of Multiple Circuit

Ohm's law for series a-c circuits (Fig 13). In a circuit having resistance R, inductive reactance X_L, and capacity reactance X_c in series, the impedance in ohms is: $Z = \sqrt{R^2 + (X_L - X_c)^2}$, the current $I = E \div Z$, and the power factor $\cos\phi = R \div Z$. Voltage across the resistance RI (Fig 14) is in phase with the current. Voltage across the inductance IX_L is 90° ahead of the current, and the voltage across the capacity IX_c is 90° behind the current. These combine geometrically to give $E = IZ = \sqrt{(RI)^2 + (X_L I - X_c I)^2}$, which differs in phase from I by the angle ϕ, the $\cos$ of which $= R \div Z$, and $\sin = (X_L - X_c) \div Z$.

Multiple circuits (Fig 15, 16). In multiple circuits having resistance and reactance in various branches:

Conductance $G_0 = R_1 \div Z_1^2 + R_2 \div Z_2^2 + R_3 \div Z_3^2$
Susceptance $B_0 = X_L \div Z_2^2 + X_c \div Z_3^2$
Admittance $Y_0 = 1 \div Z_0 = \sqrt{G_0^2 + B_0^2}$
Current $I_0 = E_0 Y_0$ and $\cos\phi_0 = G_0 \div Y_0$

The negative sign is used to denote a lagging current in an inductance in multiple circuits.

Resonance. If an inductance and capacity are connected in series, the voltages across them are opposite in sign; if equal in magnitude their resultant or difference is zero. This is resonance in series circuits. If an inductance and capacity are connected in parallel, the two currents will oppose each other; if the currents are equal a large current may flow in each, but the resultant current in the supply circuit is zero or very small. This is resonance in multiple circuits. The phenomenon is utilized to regulate long transmission lines. The line has inductance, while near the load a synchronous motor is installed, which can act like a large condenser. By adjusting the excitation of the latter its capacity effect is varied until it neutralizes the line inductance, and it is possible to obtain a higher voltage at the load than is given by the generating station.

Polyphase circuits. The single-phase a-c system, while simpler to calculate and best for lighting purposes, is not satisfactory for general transmission and power purposes. This is because: (*a*) a polyphase machine, whether generator or motor, weighs and costs less than a single-phase machine: (*b*) single-phase motors will not start from rest without complicated auxiliary devices; (*c*) in transmission work the 3-phase system requires only 75% as much copper for given power and voltage as the single or 2-phase systems. Therefore some commercial installations use 2-phase, while the great majority use 3-phase.

Two-phase system (sometimes called quarter-phase) consists of 2 circuits, which are usually independent and come from independent circuits in the generator. Voltage of each circuit is the same, and it is endeavored to maintain same current in each. This is called balanced conditions. Generally 4 wires are required, though sometimes 3 wires are used, but the latter arrangement is undesirable as it causes poor regulation. The wiring connections of 2-phase circuit are simple, as each phase is treated as an independent single-phase circuit.

Three-phase system saves 25% of the copper required for transmission line, as compared with any other commercial system, and is in general use. Theoretically, it consists of 3 independent circuits, in which the 3 e m f's differ in phase by 1/3 of a period, or 120 electrical degrees. Since the currents in the 3 circuits combine to neutralize each other, the 3 return wires are not necessary, and the outgoing current in each wire returns by the other two without interference with regulation. It is, therefore, common in 3-phase machinery to connect the 3 windings inside the machine so that only 3 conductors are brought out from the machine. There are two methods of making this connection. Fig 17 shows a Y-connected generator feeding a 3-phase line, at the other end of which is a delta (Δ) connected motor. The different voltages in this system have definite ratios as follows: Let E be the voltage between any 2 wires of the transmission line, and I the current in each line: Generator voltage per phase $= E \div \sqrt{3}$. Line voltage (neutral) $= E \div \sqrt{3}$. Motor voltage per phase $= E$. Generator current per phase $= I$. Motor current per phase $= I \div \sqrt{3}$.

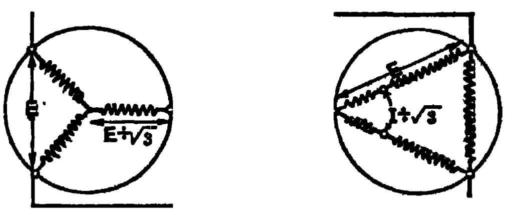


Fig 17. Three-phase Y and Δ (delta) Connections

Choice of Y or delta-connections is a matter of convenience, except for some exceptional pieces of apparatus. To the operator it makes no difference which connection is used, and in fact it is impossible to tell without having the design drawings.

Polyphase transformations. There may be systems of 5, 6, or any number of phases, but the 1, 2, and 3-phase systems are the only ones used for transmission. Some machines have 6-phase windings, but these are always supplied by 3-phase transmission lines. For methods of converting from one polyphase system to another see Art 13.

8. ALTERNATING-CURRENT GENERATORS

Synchronous generators. Field structure consists of a number of poles, excited by coils carrying a d c. The relative motion of armature conductors and field structure generates the voltage. Frequency of the voltage depends directly upon number of field poles and rev per min of revolving part. Revolving-field generators make more effective use of material than those with revolving armatures, and are more easily insulated for high voltages. Most generators of greater capacity than 300 kw are of the revolving-field

type. Advantages: good regulation, high efficiency, and satisfactory operation in parallel. Most alternators built at present are separately excited, because voltage regulators are now available which are simpler and more reliable than the self-excited machines formerly in use. INDUCTION GENERATORS are a special form of synchronous machine, having a revolving-field structure consisting of a spider with bare projecting poles, and one coil of d c which excites all poles.

Induction generators are constructed like induction motors. The magnetic field is of the rotating polyphase type, produced by a c in same windings as the load current. An induction generator must always be connected to a synchronous generator, from which it must receive its excitation. Only a minority of the units of a station may be of the induction type. Advantage: less violent effects in case of short circuits.

Connections (single, two, and three phase). Single-phase generator is usually about 30% heavier and more costly than a polyphase of same rating. Two and three-phase generators of same capacity and voltage are practically of same dimensions, weight, and cost. By changing the internal armature connections any polyphase machine may be reconnected as a 3, 2 or single-phase machine. As transmission by 3-phase currents is more economical of copper than by 2-phase or single-phase currents, all transmission lines are 3-phase; hence, the 3-phase generator is preferable.

Voltage. Alternators are now built for voltages up to 16 000 and 22 000 between lines, either single or polyphase. For higher voltage, owing to extra cost of insulation and danger of damage, it is cheaper to install transformers with a lower voltage alternator.

Frequency depends upon speed of rotation and number of poles. If the rotative speed of revolving part is given in rev per min, the frequency is $f =$ (r p m $\times$ poles) $\div$ 120. Formerly it was found more economical to run alternators at high speeds. Hence frequencies as high as 133 and 125 cycles per sec were customary; but, as systems increased in size and complexity, high frequencies caused electrical difficulties. Usual frequencies are now 60, 50, 40, and 25 cycles per sec, of which 60 is standard in U S. In Europe 50 cycles are used instead of 60.

Single phase. Voltage per phase is the same as between lines, and current per phase same as current per line. The product of voltage and current = volt-ampere rating.

Two-phase or quarter-phase. Each phase supplies half the rating; thus the voltage and current per phase in the machine are the same as the voltage and current per phase of the line. The current is

$$I = \text{Power output in watts} \div 2\,Ep$$

where I = amperes in each line; E = volts between lines; p = load power-factor.

Three-phase machines may be either Y or delta-connected. In both cases the relation between output, current in each line, and voltage between lines is the same:

$$I = \text{Power output in watts} \div \sqrt{3}\,Ep$$

Tests. Principal commercial tests on a-c generators are: resistance, cold and hot; friction; saturation and core loss; synchronous impedance and load loss; heat run and insulation. Friction, core loss, and saturation tests are made during one run; synchronous impedance and load loss during another. For testing, the generator is driven by a small motor of about 1/10 the capacity of the generator. Synchronous impedance is used to calculate regulation of the generator; results of the other tests give the efficiency. Losses are friction, core loss, excitation, armature I^2R or copper loss, and load loss. Efficiency is the output divided by the output plus all losses at that output. Armature I^2R and load loss vary as the square of the load; other losses are approximately constant. Heat runs may be made by operating the generator under normal load conditions, or by a compromise method in which the generator supplies current to a synchronous motor, which runs free and is under-excited, the current being wattless or reactive. Temp of windings after the heat run is determined by comparing their resistance at that time with their resistance when machine was at a known (room) temp.

Usual efficiencies at full load for different sizes of generator are given in Table 8. These values are merely indications and vary with frequency, voltage, speed, power-factor, etc. A 60-cycle low-voltage machine will probably have a better efficiency than a machine of same rating for 25 cycles or high voltage.

Table 8. Usual Efficiencies

Rating, *kva*	Efficiency, %
100	91
500	94
1 000	95
2 000	96
3 000	96.5
5 000	97
10 000	97.2

Regulation of a generator is defined as the ratio of the difference in terminal voltage at no load and at full load to the voltage at full load, the field excitation being kept constant at full load value and the speed being constant. Expressed as a percentage, the regulation is 100 $(V_0 - V) \div V$, where V is terminal voltage at full load and V_0 is terminal voltage at no load, with excitation for full load. Usual value, 5 to 25%.

Operation. In the operation of an a-c generator several factors are considered: EXCITATION. A-c generators of the synchronous type (that in most general use), require d c to excite their field windings; usually obtained from an exciter, which is driven by a separate engine, or by an electric motor, or from the generator itself by belt or other mechanical connection. Usual potential for field circuit is 125 volts, though some machines are designed for excitation at 250 volts. Since most machines have revolving fields, the d c is led into the field coils by collector rings on the main shaft. In series with exciter and field circuit a rheostat is connected, to vary the current in field coils, and thereby the excitation and voltage of the generator. Usual power for excitation is from 0.5 to 2.5% of rating. Before STARTING A GENERATOR its bearings are inspected, cleaned, and oiled. Machine is then brought up to speed, and bearings again inspected to see that oil rings are running properly. Exciter or excitation circuit is then put in readiness, and the rheostat in alternator-field circuit adjusted for maximum resistance. Before exciting the field the armature insulation must be thoroughly dry; otherwise, the armature is short-circuited through an ammeter, and run for several hours at a partial excitation to give about rated current in the short-circuited armature. When insulation is thoroughly dry the short circuit is removed and the excitation adjusted to give rated voltage at armature terminals at correct speed. To stop the machine, load is first removed by opening the circuit breaker; then the field rheostat is turned to maximum resistance, as is also the rheostat in the exciter field if there is an individual exciter. Field circuit is then opened.

Paralleling of generators. Before connecting a generator to the bus bars (to which one or more other generators are connected), following conditions must be satisfied; (*a*) frequency of generator must be the same as that of bus bars; (*b*) frequency of generator, and therefore its speed, must be constant for an appreciable interval of time; (*c*) voltage of generator must be the same as that of bus bars; (*d*) generator and bus-bar voltage must be in phase.

If two machines have not the same frequency, or if frequency is not constant, a condition will occur intermittently in which the voltages are 180° apart, or the machines are in series on a short circuit, and a dangerous current will flow. If voltages are not equal, a large wattless current may flow, and if they are not in phase, a large power current will flow, causing a mechanical shock. Two methods are used for determining when these conditions are favorable for connecting the machines together without disturbance; that is, for indicating when the machines are in SYNCHRONISM. For synchronising with lamps, the connections as in Fig 18a are such that the lamps remain dark when above conditions are satisfied; with connections as in Fig 18b, they will remain bright. If the frequencies are wrong, the lamps will flicker (the slower the flicker the nearer the frequencies). If voltages are wrong, lamps will glow slightly but steadily. SYNCHRONOSCOPE is an instrument which indicates the relative frequencies and phases of 2 alternators; usually installed on switchboard of a station, if a number of generators are used.

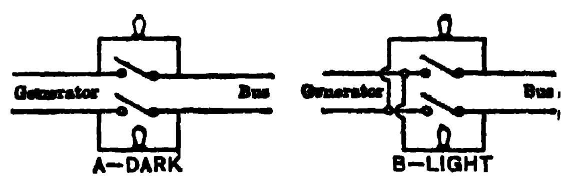


Fig 18. Connection of Lamps for Synchronising

Cost and weight. Generators vary widely in their specific weights and costs, that is, weight and cost per *kva* rating; but they may be divided into classes in which these characteristics are fairly definite. Conditions affecting specific weight are: method of rating, speed, frequency, voltage, and size. Method of rating is fundamental. For comparison, rating is taken as the output in *kva* which each machine will give continuously with a temp rise not exceeding 50° C. On this basis, specific weight decreases as the speed, frequency or capacity increases, and it increases with increase of voltage. A-c generators are divided into 3 classes according to their speeds and purposes: high speed, as turbine-driven generators; medium speed, as belt-driven and water-wheel driven; slow speed, or engine-driven. See Table 17, Art 19.

9. SYNCHRONOUS MOTORS

Operation. Any alternator will operate as a motor. If two synchronous alternators are connected in parallel to bus bars supplying a load, and the driving power is removed from one prime mover, the alternator connected to this prime mover will continue to run at same speed, taking power from the other alternator and driving its own prime mover or other apparatus coupled to it; this alternator thus acts as a motor. Speed of such a motor depends solely upon the speed of the generator or generators supplying electric energy to it; it is therefore said to run in "synchronism" with source of supply and is called a synchronous motor. The speed of a synchronous motor having p poles, and supplied with current of a frequency of f cycles per sec, is: r p m $= 120f \div p$. If load on the motor increases, speed will not decrease, unless load reaches a value so excessive that

the maximum output or "pull-out torque" is reached; then the motor will drop out of step and come to rest, while the current taken will increase to short-circuit value and torque will decrease to a negligible value. Difference in construction between an alternator and a synchronous motor is that the latter has, in the face of the field poles, a squirrel-cage winding, intended to give good starting torque and prevent "hunting" while running. HUNTING defines the occasional undesirable action of synchronous machines, of varying in speed, current, and voltage, at a frequency observable in meters. If excessive, a short circuit results (see Specifications below). A standard 2 300-volt generator will operate satisfactorily as a motor at 2 080 volts, and as these are the natural values of the generated and delivered voltages, this characteristic of the synchronous motor accords well with customary distribution practice. A standard generator may have a squirrel-cage winding added to its poles and become a good synchronous motor.

Number of phases. Synchronous motors may be single, 2, or 3-phase. The single-phase motor is not self-starting, and has a considerably lower efficiency than the polyphase. It is also more liable to hunt and be unstable, and is therefore far less desirable than a polyphase motor. The 2 and 3-phase motors are very similar in all their characteristics.

Terminal voltage. Since synchronous motors are usually built with a revolving field and a stationary armature, the armature winding can be insulated for voltages as high as 13 000, thus often obviating need of transformers.

Relations of voltage and current. Relations between line voltage and phase voltage are the same as in a-c generators (Art 8). Current in each line of a 3-phase motor is: $I = \dfrac{746\,P}{\sqrt{3}\,eE\cos\phi}$, where P = output, h p; E = voltage between lines; e = efficiency at assumed load; $\cos\phi$ = power factor (may be unity). Usual values for efficiency are about the same as for a-c generators.

Advantages of synchronous as contrasted with induction motors: higher efficiency, higher power factor, controllability of power factor with constant speed, high voltage, lower cost. DISADVANTAGES: need of an exciter, will not start as great a load.

Applications. To transform from a c to d c, or from one kind of a c to another differing in frequency, potential, or phase relation, motor-generator sets, consisting of a synchronous motor direct connected to one or more generators, are often employed (Art 6). The potential of the secondary or distribution circuit is thus made independent of the variation in potential of the primary circuit supplying power to the motor. In certain cases it is desired to take power from a 25-cycle circuit and supply power at 60 cycles for lighting purposes. Here a synchronous motor-generator set would be used; often called a "frequency changer." In some applications of electric drive by induction motor one synchronous motor is installed to make it take leading current, in order to neutralize the lagging current taken by the induction motor. This effect is produced by over-exciting the fields of the synchronous motor. The motor may be used to drive any machinery not requiring much starting torque. Such a motor is a "rotary phase modifier" or "rotary condenser" (see below).

Tests of synchronous motors. The first four tests of an a-c generator (Art 8) apply also to synchronous motors.

Phase characteristics, or V-curves at no load, full load, and any other specified load. Machine is operated as motor with specified load kept constant throughout the run. Voltage and frequency impressed upon the motor are also kept constant. Current in the field is varied from minimum at which motor will operate to the maximum (from 0.25 to 1.5 normal), and the variation in current input to armature noted. Readings are taken of load, volts armature, amperes armature, and amperes field. A curve is plotted with amperes armature as ordinates, and amperes field as abscissas. This gives the characteristic V-curves. At point of minimum current input for each load the power factor is unity. At lesser values of field current the armature current lags and power factor is poorer; at greater values the armature current leads.

Starting. All synchronous motors have a squirrel-cage winding in the faces of the poles, which enables them to start as induction motors with a torque of 40% to 80% of rated torque. The induction motor action brings the speed up to about 95% of synchronous speed. Then the field switch is closed and a strong field supplied. This causes the rotor to pull suddenly into exact synchronism. Frequently there is a "Field Breakup Switch," which is kept open during starting and closed just before pulling into synchronism. This protects the field windings from breaking down, due to the high voltage induced in them at low speeds. If there is considerable flywheel mass in the load it is difficult for the motor to pull into synchronism. Special synchronous motors are available for unusual starting requirements. After the motor has reached synchronous speed, the full load may be applied and the field rheostat adjusted to give minimum armature current, or a leading power factor if the motor is designed for that particular operation.

Installation and operation. Precautions to be taken in installation are the same as for a-c generators (Art 8). Direct current must be provided for excitation. If a synchronous motor is operated on a polyphase system having unbalanced voltages, it will take unequal currents in the different lines and tend to balance the voltages. But these unequal currents increase the heating for a given load.

Specifications. Synchronous motors are rated in same manner as synchronous generators, and same heating limits and specifications apply (Art 8). It is customary to specify the value of current taken by the motor in starting with no load other than the friction of its own bearings; or its own friction plus that of the machine to which it is connected, in case it is part of a motor-generator set. It is sometimes stated in specifications that the motor will not hunt, provided the total resistance drop between generator and motor is less than some specified value (10 or 15%).

Dimensions, weight, and costs are given in Table 19, Art 19.

10. INDUCTION MOTORS

Principles. Induction motor is a machine having distributed windings, like those of the armature of a d-c machine, on both stationary and revolving members. One winding, the primary, receives polyphase currents from the supply circuit and thereby sets up a rotating magnetic field. This field cuts the conductors of the secondary winding, induces currents in it, and thereby drags it along with the field. The action is reversible; either member may be the primary, provided it is connected to the line. That member which remains stationary, whether it be primary or secondary, is called the stator; the other, the rotor. In U S the stator is usually primary.

Synchronous speed. Speed of rotation of the magnetic flux is called the synchronous speed. At light loads the rotor speed is nearly equal to the synchronous speed. If f is the frequency of the current, and p the number of poles of motor windings, the synchronous l is: rev per min = $120 f \div p$.

Slip. At any appreciable load the rotor speed is less than the synchronous speed. Difference between the actual rotor speed N_1, and synchronous speed N is "slip." It may be expressed as a percentage or a fraction: $S = (N - N_1) \div N$.

Methods of rating. Amer Inst of Elec Engs recommends that the rating of an induction motor shall be the h p which it will deliver continuously at the shaft with a maximum rise in temp not injurious to its insulation (Art 1).

Starting and break-down torque. In addition to the ability to carry its rated load without excessive heating and with reasonable values for efficiency, power factor, and slip, the motor should be able to start such loads as must be brought up to speed, because good starting ability in an induction motor involves certain complications and expense. It is important also that the motor be able to carry momentary overloads without "breaking down," that is, gradually coming to a standstill. To prevent this the max motor output must be known and should be at least 50% greater than rated output.

Frequency and speed. Induction motors can be built for any frequency. The higher frequencies are satisfactory where the load never exceeds the normal. Frequencies as low as 25 are preferable where overloads are common, or large starting torques are required. Rotor speed of an induction motor at normal loads approaches within 5 to 10% of the synchronous speed, which is fixed by the frequency of the system and its number of poles (see above). Hence, for a given frequency of supply circuit, only certain speeds are available. Thus, for 60 cycles, the rev per min are: 3 600 for 2 poles; 1 800 for 4 poles; 1 200 for 6 poles, etc.

Phase connections. Two-phase or quarter-phase motors are usually wound with independent phase windings. Three-phase motors are connected in Y or delta, depending upon convenience of designing engineer. In single-phase and 2-phase motors the voltage and current per phase are the same as voltage between lines and current in line; in a Y-connected 3-phase motor, the current per phase is equal to line current, and voltage per phase is equal to line voltage divided by $\sqrt{3}$; in a delta-connected 3-phase motor the current per phase is equal to line current divided by $\sqrt{3}$, and voltage per phase is equal to line voltage.

Table 9. Power Factors and Efficiencies at Full Rated Load for Polyphase Motors

H p	25 cycles		60 cycles	
	Effic	Power factor	Effic	Power factor
1	0.79	0.78	0.78	0.78
5	0.85	0.88	0.82	0.88
20	0.88	0.91	0.84	0.91
50	0.90	0.92	0.87	0.92
100	0.905	0.925	0.89	0.92
200	0.91	0.925	0.905	0.92

Currents taken by motors. Let: P_0 = output, h p; I = current in each line; e = efficiency as a decimal fraction; $p = \cos\phi$ =

power factor as a decimal fraction; E = voltage between lines (between one outside wire and the middle wire for 3-wire, 2-phase line). Then, for 2-phase: $I = 373\,P_0 + peE$; for 3-phase: $I = 431\,P_0 + peE$.

Testing. Induction motors may be given an "input-output" test (Art 5) at working load, from which efficiency, power-factor, slip, and maximum output may be determined; or, a no-load excitation and a no-load short-circuit test, from which all characteristics may be calculated. The former is similar to a stray-power test (Art 5) on a d-c motor, the latter to a resistance measurement. No-load tests require very little power, and are preferable for large motors, where it would be expensive to supply power and inconvenient to dissipate the energy. They are equally accurate.

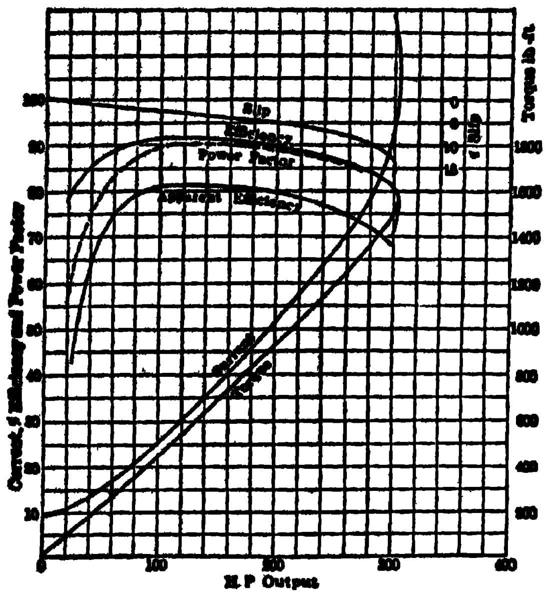


Fig 19. Characteristic Curves of a Three-phase Induction Motor

Characteristic curves. Fig 19 shows examples of the usual characteristic curves of an induction motor. Typical break down of all the curves at maximum output is shown at 300 h p. Power-factor curve shows relation between the true power input and the apparent power, or volt-amperes. Low power factor involves no greater registration of the watt-hour meter, or cost of energy to operate the motor, but does involve poor voltage regulation of the system as a whole and a larger capacity of wiring, transformers, etc. Apparent efficiency is the product of the power factor and true efficiency, and is equal to output in watts divided by input in volt-amperes. Its value determines the actual capacity of the line and transformer supplying the motor.

Starting. To start an induction motor without injurious heating, a resistance must be connected in the secondary circuit, or the voltage impressed upon the primary must be reduced. These methods are known as RHEOSTATIC CONTROL and POTENTIAL CONTROL. They are also available to control the motor speed.

Potential control consists in reducing and regulating the voltage impressed on the primary, usually by a starting compensator or auto-transformer, which provides 1 or 2 fractional voltages. For this the secondary must be of higher resistance than with other methods of starting, but no changes of connection of the secondary are necessary. Hence, a squirrel-cage rotor winding is used, made of one bar per slot, all bars being connected at both ends to solid copper rings. To start the motor the primary is connected to taps on the compensator, which give a voltage of from $^1/_2$ to $^2/_3$ the rated voltage if it is a small motor, and $^1/_3$ to $^1/_2$ if it is large. A small motor may be brought up to speed with this voltage, but a larger one may require an intermediate step. When motor is up to speed (indicated by the low, steady value of the current) it is connected to full line potential.

Rheostatic control. Better apparent torque efficiency, that is, more torque for a given current, is obtained by inserting in the secondary circuit a much greater resistance than can be permanently used. To do this a special starting resistance is connected in series with each of the 3 armature windings, and a switch provided for short-circuiting the resistance, either step by step or as a whole, as the motor speeds up. There are two commercial methods. The first is for use only when the torque required at starting is not great, in which case the starting resistance may be small and placed within the armature spider. The switch lever is so arranged that the resistance can be short-circuited in steps while armature is revolving, thus obviating the need of collector rings and external connections. Second method, for large starting torque, consists of bringing the 3 terminals of the secondary winding to collector rings. From brushes on these rings conductors lead to external resistances, with steps or taps, so that the resistance can be short-circuited gradually by a controller.

Speed control. Speed of an induction motor may be controlled in 5 ways: (*a*) By varying the potential applied to the primary of a motor having suitable permanent resistance in the secondary. (*b*) By varying the resistance in external circuit of the secondary. (*c*) By so changing the internal connections of primary winding as to change the number of poles. (*d*) By connecting two motors in "concatenation" (similar to series connection). That is, the stator of first motor is connected to the supply, rotor of first motor is connected directly to rotor of the second, and stator of second motor is treated as its secondary. (*e*) By changing frequency of the applied voltage. Method (*b*) (rheostatic) is the most general and practical where more than 2 speeds are required; any speed may be obtained. Method (*c*) is best where only 1 motor is available and only 2 speeds are required. Method (*d*) requires 2 motors and gives 2 speeds; not as desirable as (*c*). Method (*e*) is of theoretical interest only.

Installation. Induction motors, even in large sizes, are usually self-contained, the bearings being a part of the end frame of the motor. They are either direct-connected or

belted. Belt drive is the commoner, since an induction motor can be built only for a definite speed, corresponding to a certain number of poles. Small motors need no foundation, and are often attached to the wall or ceiling, the bearings and end shield being so made that they may be turned through 90° or 180°, for proper operation of the oil rings. Most motors must have reasonable ventilation, free from dust and dirt. For severe service, as in cement mills or mines, motors are built totally inclosed (" iron-clad ") and may then be even submerged in water. This increases both size and cost.

Operation. Small motors are designed to start merely by closing the main switch. With larger motors, if starting switch is in proper position, potential may be applied to the motor, and the starting resistance gradually cut out by the switch. Induction motors are very sensitive to variations in the impressed voltage. A decrease from 100 to 80 volts will cause the maximum output and starting torque to decrease from 100 to 64, with a roughly proportional increase in heating for a given load. UNEQUAL VOLTAGES in the different phases cause decrease in maximum output, and increase in heating for a given output. An unbalancing of 25% in voltage doubles the heating effect at full load; giving the same heating as 50% overload. CARE IN STARTING. Before starting for the first time, see that the starting device is in operating condition and in proper position, to avoid injurious heating. Wiring must be proportioned to carry the starting current without an excessive drop in voltage (see above).

Faults. Some of the common faults in induction motors, with their indications and remedies, are: SECONDARY OPEN-CIRCUITED. Motor will not start, and will not take a current greater than exciting current; probably due to the starting resistance not being connected in. ONE PHASE OF SECONDARY OPEN-CIRCUITED. Motor tends to remain at half synchronous speed, although the current is apparently normal. If the armature is blocked, the currents in primary will be unbalanced. ONE PHASE OF PRIMARY OPEN. Motor will not start, and current will be unbalanced. ONE PHASE OF PRIMARY REVERSED. Currents in primary will be very much unbalanced when motor is running, and starting torque will be very slight. SHORT-CIRCUITED COIL IN PRIMARY. There will be humming when potential is applied to the motor and excessive local heating around the short-circuited coil. VIBRATION due to mechanical unbalancing is chiefly noticeable at high speeds, particularly in high-speed machines. If vibration is due to magnetic unbalancing, it is probably caused by inequality in the air gap at different portions of circumference, and at different positions of armature. It may be detected by measuring the air gap with taper wedges at different points around the circumference, with the armature in several positions.

Specifications. Following memoranda (from Amer Handbook for Elec Engs) will assist in framing specifications: PRINCIPAL CHARACTERISTICS AND CONDITIONS OF SERVICE. Use to which motor is to be put; kind of load and method of drive; voltage and number of phases; rating, horse-power; frequency and speed. STYLE AND DESCRIPTION, CONSTRUCTION DETAILS. Whether to be open, semi-inclosed or inclosed; requirements regarding pulley and length of shaft; whether rails are required; method of starting; compensator, external resistance or internal resistance; whether motor is to be run at speeds other than full speed; whether starting devices are to be supplied. PERFORMANCE AND TESTS. Final temp rise at full-rated load; temp rise in 2 hr at 25% overload applied immediately after full-load run; efficiency at 25, 50, 75, 100, 125, and 150% load; starting torque with full-load current, ft-lb; high-potential tests of insulation; requirements as to effect of moisture upon insulation.

Weight and cost of induction motors vary with type of armature winding and character of mechanical frame and housing; also with speed, frequency, and voltage. Table 18, Art 19, gives weights and costs for standard 60-cycle motors.

Single-phase motors may be of either induction or commutator type. INDUCTION TYPE is the commoner, as its constant-speed characteristic is more generally appropriate than the variable speed of the usual commutator a-c motor. A 2- or 3-phase induction motor may be operated as a single-phase machine after it is once up to speed; but it has lower efficiency and power factor, and smaller maximum output than it would have as a polyphase motor. Slip, for a given output, is less in a single than in a polyphase motor. Because of the poorer operating characteristics, and especially the smaller maximum output or maximum torque, the motor must be rated at lower capacity when run single phase. A single-phase motor having same frame and wt of iron and copper as a polyphase motor will rate at from 60 to 70% of the capacity. Single-phase induction motors usually consist of standard 3-phase motors with their rating changed; thus a 10-h p, 90-volt 3-phase motor would make a typical 7.5-h p, 120-volt single-phase motor. STARTING. A single-phase induction motor has no torque at standstill, and must be started by some such means as a phase-splitting device. A common method is to connect a Capacitor in series with one phase winding of a special wound polyphase motor. This gives a good starting torque and improves the power factor under running conditions. Or, a commutator on the rotor may be used, making it possible to operate as a repulsion motor (see

below) having good starting qualities. After reaching considerable speed the brushes are removed and the secondary windings are short-circuited.

Commutator type. The several types of a-c commutator motors designed to operate on single-phase circuits differ chiefly in their electrical connections. They may be divided into series motors and repulsion motors. Both have an armature wound like that of a d-c motor and a commutator. SERIES A-C MOTOR is connected like a d-c series motor, and has the same general characteristics. Principal difference is that the a-c motor has an extra winding placed in the face of the poles and connected in series with armature and field. This winding improves the power factor. Limiting feature of most a-c commutator motors is sparking at the brushes, which is much worse than in d-c motors. REPULSION motor has a stationary structure or field (primary) with a completely distributed winding like that of an induction motor. The winding may be for any voltage. The armature or secondary is like that of a d-c commutator motor. The commutator brushes are short-circuited upon themselves, and are placed at a small angle away from the neutral. This motor acts like a combination of transformer and series motor, having the variable speed, high torque, and runaway characteristics of the series motor. It is started by applying a reduced potential to the primary, and is reversed by moving the brushes or by reversing the current in one particular portion of the primary winding known as the exciting turns. REPULSION-INDUCTION TYPE of motor is made in small sizes to drive machinery requiring good starting torque. It is a repulsion motor with extra brushes on the commutator, the brushes being connected across a portion of the primary. This gives a constant speed characteristic like a shunt motor, and a good power factor.

11. SYNCHRONOUS CONVERTERS AND RECTIFIERS

Since it is more economical to transmit electrical energy in the form of a c and more convenient to utilize it in the form of d c, some means of converting it from one form to the other is desirable. For this purpose synchronous converters, motor-generator sets (Art 6) and rectifiers are available.

Synchronous converter, also called "rotary converter," is similar to a d-c generator, in which certain commutator segments, or the conductors leading from them, are connected to 2, 3, 4, or 6 collector rings. When the movable member rotates, the voltage between any 2 rings is alternating. Such a machine may be operated as an a-c generator, or as a "double-current" generator giving a c from its collector rings and d c from its commutator. If the rings are connected to an a-c source, the machine will run as a synchronous motor, and d c may be obtained from the commutator brushes; that is, the machine with but one set of windings acts simultaneously as an a-c motor and a d-c generator. It therefore has the friction, core loss, and excitation loss of one machine instead of two: and since the motor and generator currents flow in the same winding, and opposite directions, they more or less balance each other, and the armature RI^2 loss is much less than in either a motor or generator alone.

Converter *versus* motor-generator. A converter is much more efficient, weighs and costs less than a motor-generator set of same capacity, and occupies less space. But, since only one winding is used, there is a definite relation between the voltages of the a-c and d-c terminals. Maximum value of the alternating wave bears a definite relation to the direct e m f (see below). The converter must therefore be supplied with a voltage of the same order as the direct voltage, which requires transformers if a high-voltage transmission line is used to supply the converter. EFFICIENCY of a converter approximates 93%, and of the transformers, 97%; the efficiency of the combination is therefore about 90%. Efficiency of a synchronous motor is about 93% and of a d-c generator, 92%; hence the combination motor-generator set has an efficiency of 85.5%. If the supply voltage is greater than 13 000 volts, transformers will also be needed for the motor-generator set, the net efficiency being then 83%.

Application of converters is commonest in electric railway work. Nearly all motors for electric traction are of d-c series type, operating at 500 to 600 volts. For this service the energy is transmitted over long distances, requiring a high-voltage a-c transmission line and converters to link the d-c distribution with the a-c transmission.

Phases and rings. SINGLE-PHASE CONVERTER has 2 collector rings, each connected to the windings by as many equally spaced taps as there are pairs of poles. The taps for the 2 rings alternate

at equal spaces. A single-phase converter is therefore a 2-ring converter. THREE-PHASE CONVERTER has 3 rings and 3 equally spaced taps (one for each ring) for every pair of poles. A 4-phase or quarter-phase converter has 4 rings and 4 taps, and a SIX-PHASE CONVERTER has 6 rings and 6 taps, per pair of poles.

Shunt and compound-wound converters. A converter may be shunt or compound wound, depending upon the service. Series winding is intended to make the converter take leading current when the load increases, and thus increases the voltage at the a-c terminals, but the ratio of the a-c terminal voltage to the d-c voltage remains unaltered.

Inverted converter converts from d c to a c. It works satisfactorily, but its speed depends upon nature of the a-c load. An inductive load in the a-c circuit causes the armature to demagnetize the fields, with a resultant increase in speed. It is therefore dangerous to operate an inverted converter on an inductive load, unless it is provided with a speed-limit device. This does not occur when the machine operates as an a-c motor, since its speed is fixed by the frequency of the supply circuit.

Connections and voltage ratios. The ratio of voltage on the a-c side to that on the d-c side depends upon the number of rings and type of connection (Table 10). In practice the current on input side must be greater than that given in the table, in order to supply the converter losses, and the a c will also vary inversely as the power factor, which is taken as unity in the table. There are other losses besides RI^2; hence practical figures for output differ slightly from those in the table.

Table 10. Voltage, Current, and Capacity Relations of Converters

	D-C generator	Converters					
		2 ring	3 ring	4 ring	6 ring diametrical	6 ring double delta	12 ring
D C, volts	100	100	100	100	100	100	100
A C, volts between lines	...	71	61.2	71	71	61.2	71
A C, volts between rings	...	71	61.2	50	35	35	18
D C, amperes	100	100	100	100	100	100	100
A C, amperes in line	...	141	94	71	47	47	24
A C, amperes in winding	...	71	55	50	47	47	45
Relative RI^2 loss	100	137	55	37	26	26	20
Relative output, unity power factor	100	85	134	165	197	197	224
Relative output, 87% power factor	...	. .	99	115	129	129	135

Testing. Converter tests are practically the same as those on a synchronous motor (Art 9), with the addition of a test on ratio of a-c to d-c voltage, taken at no load and at full load.

Regulation. A converter, like a synchronous motor, may be used to cause a rise in voltage in the transmission line supplying it. It is done by over-exciting the fields. The voltage on the d-c side will rise correspondingly, and thus the machine may be compounded. For this to take place there must be considerable inductance between generating station and converter.

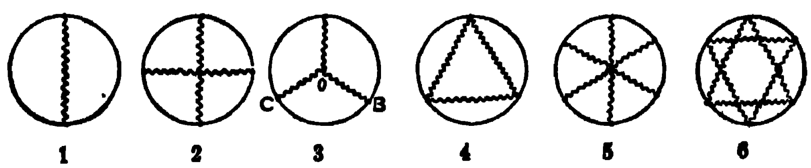


Fig 20. Transformer Connections and Vector Relations of Synchronous Converters

Operation. As a converter exerts no appreciable mechanical torque, its structure and foundations are light. Fig 20 shows usual modes of connecting transformers to supply converters. The transformer secondaries must give a voltage which is a definite fraction of the direct voltage desired (Table 10).

Starting. A converter may be started by either of the methods described for synchronous motors, or it may be speeded up as a shunt motor, and then synchronized with the a-c supply circuit like an alternator (Art 8). All converters are supplied with a "field break-up switch," which opens the field circuit in several places to avoid the strain of the high potential induced in field windings during starting, and to reverse direction of field current after the machine is up to speed, in order to reverse the polarity if necessary. This is usually a double-throw switch with several poles. For normal operation it is thrown down to "running" position. To change the polarity it is thrown up to "reverse" position and left closed only for a moment; then it is returned to "running" position. SPEED-LIMITING DEVICE. If a converter be disconnected from the main a-c supply circuit, and still remain connected to the d-c circuit, it will tend to operate as a

d-c motor, and its speed might become dangerously high. To avoid this a centrifugal governor placed on the shaft opens the main d-c switches of the converter.

Rectifiers. The operation of a mercury-arc rectifier depends on the facts that: (1) a tube containing mercury and mercury vapor under low pressure (less than 0.01 mm), having one electrode in the mercury (the cathode) and the other of some conductor, usually graphite (the anode), offers very high resistance to a current tending to flow from the mercury to the anode, but (2) offers very little resistance to a current flowing from anode to cathode, provided the current is once started by forming an arc inside the tube. An alternating e m f impressed on this tube will cause a unidirectional current to flow in separate pulses. With two anodes and one cathode, with suitable connections, the pulses are largely smoothed out, and with some inductance in the circuit a unidirectional current with but little variation is obtainable. In practice a 3-phase alternating current is usual, with 3, 6 or 12 anodes in the tube, producing a direct current of almost no pulsation.

Losses. In such a rectifier there is a loss in voltage of 20 to 30 volts, no matter what the working voltage or current, and the product of this voltage and the current used gives the rectifier loss in watts. There is practically no other loss in the rectifier, but there are other losses in the auxiliary apparatus, transformers, vacuum pumps and cooling water circulator. An auxiliary "starting anode" is necessary. This draws an arc giving the initial ionisation to the mercury vapor; after which the working current maintains the ionised conditions.

The anode tank is of steel, insulated from the ground, with elec connections to the several electrodes carried through the tank walls by special "insulating seals," of porcelain or a special glass, in which the different materials have the same heat expansion coefficient. This tank is surrounded by another tank and between them circulates water for cooling. The inner tank is maintained at a vacuum of 1 to 5 microns (the upper limit of successful operation being 10 microns, or 0.010 mm) by two pumps in series, a mercury condensation pump operating continuously for fine adjustment and a rotary pump for coarse work. The latter operates only occasionally.

Applications. Mercury-arc rectifiers are chiefly used to supply direct current to railways at voltages of 600 and greater. They are at their best at high voltages, because their efficiency is higher, but operate satisfactorily at 200 volts with a lower efficiency and greater bulk and cost per kw.

Hot-cathode mercury rectifier, known to the trade as "phanotron," has a hot cathode of a suitable metal which emits electrons, and contains a small amount of mercury in form of vapor. This operates as a rectifier with much smaller values of current than the pool type, but with about half as much loss in voltage and therefore with a higher efficiency at low voltages, and is suitable for circuits of 125–250 volts, though not yet in very general commercial use.

Tungar rectifier is similar to the preceding, except that it uses argon gas in a glass tube and is suitable for lower voltages, 60 volts or less, and small currents, 15 amp or less. It is useful for charging storage batteries and is cheaper.

Copper-oxide rectifier is a very simple device, consisting of a pile of copper plates whose surfaces have been treated in a particular manner. It can rectify alternating voltages of the order of a few volts, but has current capacity up to hundreds of amperes.

12. ELECTRIC POWER PLANTS (See also Sec 16)

Location of an electrical power plant is governed by: form of power; distribution of load; cost of ground; possibility of good foundations; room for extension; convenience of coal, and of water supply for evaporation and condensing; limitation of voltage used.

Form of power, whether steam or water, is usually definitely determined by local conditions. Choice between steam, gas, or oil depends chiefly on relative cost of fuel and of maintenance of an oil or gas engine. If steam is to be used, the choice between turbines and reciprocating engines is governed largely by size of units; in large sizes the steam turbine costs less per h p and is cheaper to operate, while in small sizes the reciprocating engine is more economical. In designing a power plant and selecting the number and size of units, it is important that the total capacity be sufficient to carry the maximum load with margin, and that it be divided into units of a size to carry average load economically. Each unit is usually capable of carrying 25 or 50% overload for 2 hr. There should be at least 4 units in a large plant, and nothing is gained by having more than 8 units. A sufficient number of exciters are necessary to carry the peak load, with one exciter to spare. Transformers are commonly arranged in groups, one group for each generator. In hydro-electric plants it is often advisable to install 1 or 2 steam-driven units and boilers, to serve if water supply should fail (see also Sec 16).

Alternating vs direct current. Choice depends upon length of transmission line and value of energy. For utilising all the energy within a short distance of the generating

station d c is preferable, because d-c motors (particularly in small sizes) are more convenient, easier to control, and are available in greater variety of form and characteristics than a-c motors. But the voltage obtainable from a d-c generator is limited by the commutator to about 600; and, for transmission to a considerable distance by d c, the quantity of copper becomes prohibitive; hence a c is used, and with the a-c transformer the transmission line potential is independent of the voltage at either generators or motors. The dividing line between the best fields for d-c and for a-c plant is not clear cut, and can be determined only by careful calculation and comparison of costs. Some installations use a-c generators and a c for transmission, but these are converted into d c in sub-stations near the point of utilization, and at a distance from the generating station. When the sub-station cost is less than the difference between the cost of the copper requisite for d c and for a c, the a-c system with sub-station is preferable. If the a-c system be chosen, 3-phase generators are advisable, as 3-phase transmission requires 25% less copper than 2 phase or single phase, and there is no advantage in the 2 phase over the 3 phase. If the power plant is for a varied service, including lights as well as motors, the frequency should be 60 cycles, because incandescent lights operated at 60 cycles are satisfactory, while at 25 cycles the flickering is harmful to the eyes. Transformers for 60 cycles are considerably cheaper than those for 25 cycles.

Voltage. The most convenient and widely used voltage for a local power plant is the 220–240 volt 3-wire system, in which two 110-volt generators are connected in series, or a 220-volt generator and a balancer set provide the 3 voltages. In this case the electric lights and small motors are connected to the 110 volts from the neutral to either of the outside wires, while all the large motors receive 220 volts from the 2 outside wires, thereby saving in copper. If 110 volts were used, either an excessive amount of copper would be required, or the voltage on the lamps would vary enough to cause dissatisfaction. For a number of large motors, 500 volts is generally used, but the lighting problem then becomes difficult, as the lamps must be connected in groups of 4 in series, and if any one of the 4 burns out, all in that group go dark. 500 volts is also undesirable in damp places, as in a mine, because there is danger of an unpleasant shock, particularly to horses or mules. The 220-volt, 3-wire system is therefore best, if d c be used.

For the 3-phase a-c system the distribution circuits are best run with 4 wires, with 208 volts between the outside wires and 125 volts between each phase and the neutral. Lights are connected between each phase and the neutral, balancing the load on the 3 phases as nearly as possible, and 3-phase 208-volt motors used on the 3 phases.

Lowest standard voltage for transmission is 2 200, which will take care of reasonable loads within a radius of 1 or 2 miles from the power station, stepdown transformers being used to supply the different sections of the area.

Arrangement. It is common practice to divide the power station, if steam or gas driven, into two parts, separated by a fireproof, dustproof wall. On one side are the boilers or gas producers; on the other, the prime movers, generators, and all electrical apparatus. In the simplest form the boilers are set in a single row along one side of this wall, and the generating units along the other side, the piping being so arranged that any engine may be supplied from any boiler. If transformers are used, they are generally placed in the room with the generators, but on opposite side and on the lowest level, the switchboard being in a gallery above. Cables from generators to switchboard are laid underneath the floor and running up the side of the wall.

Switchboards are of 2 kinds: DIRECT CONTROL, in which case the current is carried by bus bars immediately behind the switchboard panel, the opening and closing of switches in this circuit being effected directly by the operator; and REMOTE CONTROL, in which the current is carried by conductors placed in brick compartments in the basement, and the connections are made by electrically operated switches controlled by a small relay circuit coming from control switches on a benchboard above. If potential is 2 200 or less, and power is less than 5 000 kw, direct control is satisfactory and feasible, but for greater voltage or power the remote control system is preferable.

Switchboard is divided into a number of units, each known as a panel. There are generator panels, exciter panels, and feeder panels, and at one end of the switchboard is a voltmeter; also, in an a-c station, a synchroscope is placed on a swinging bracket, which may be distinctly seen from any part of the switchboard. A circuit-breaker is a switch which opens the circuit when current is excessive; operated by hand or automatically by the current itself. While the make-up and arrangement of the different panels may vary greatly according to individual ideas, the essentials are common to all, as follows. D-C GENERATOR PANEL, 2-wire system: 1 circuit-breaker at top of board, 1 ammeter, 1 hand-wheel for rheostat, 1 field rheostat back of board, 1 single-pole field switch, 1 triple-pole main switch (or 1 double and 1 single-pole) for the equalizer, 1 4-point voltmeter receptacle. D-C FEEDER PANEL 2-wire system: 1 single-pole circuit breaker, 1 ammeter, 1 double or 2 single-pole main switches, 1 4-point voltmeter receptacle, 1 watt-hr meter. A-C GENERATOR PANEL, 3 phase: 3 ammeters, 1 3-phase wattmeter, 1 field ammeter, 1 double-pole

field switch, 1 handwheel for rheostat, 1 synchronizing receptacle, 1 potential receptacle, 1 3-phase oil switch, 3 disconnecting switches, 2 current transformers, 1 potential transformer, 1 voltmeter for group, 1 synchronizer for group. A-c FEEDER OR OUTGOING LINE PANEL, 3-phase: 3 ammeters, 1 3-phase oil switch, 3 disconnecting switches, 1 polyphase watt-hr meter, 3 current transformers. A VOLTAGE REGULATOR may be used, requiring an independent panel.

Costs. See Art 19.

13. ELECTRIC TRANSMISSION

Types. Transmission may be in form of single-phase, 2-phase or 3-phase a c, or as d c. Single-phase and d-c systems require 2 wires; 3-phase, 3 wires; 2-phase, 4 wires. For given voltage between wires the relative weights of copper are: single-phase and 2-phase, 100; 3-phase, 75; d c, 50. Because of the limited voltage at which d c is obtainable, it is not applicable to long distances. Single-phase system is used where the object is lighting only, on account of its simple connections. For important transmission problems the 3-phase system would be chosen (Sec 16).

Line drop is the difference between voltage at generating station and at the load. It is usually expressed as a percentage of the delivered voltage, and most transmission lines are designed for 10% drop when transmitting full or rated load power. In general, the weight of copper required varies inversely as the line drop allowed. This is not strictly true in a-c systems, but is exact in d-c systems, and the relation may be written: $W = 125\ PD^2 + KE^2$, where W = wt of copper, lb; P = power delivered, watts; D = distance one way, in thousands of ft; K = ratio of power lost in line to power delivered; E = voltage delivered.

In an a-c system the inductive reactance must be considered, as well as the resistance. Resistance for a certain length of given size of wire is obtained from a wire table, and is prorated for the given distance. Inductance in henries per mile for one wire of a transmission system is given by: $L = 74 \times 10^{-5} \times \log_{10} (2\ D + d)$, where D = distance between wires, and d = diam of wire, both in inches. Reactance in ohms is $x = 2\ \pi fL$, where f is the frequency. In a single-phase transmission the total resistance and reactance are twice the above values. In a 2-phase circuit, the resistance and reactance per phase are twice the above values. In a 3-phase circuit, the resistance and reactance per line or per Y-phase are used, and these are the values given by the above formulas.

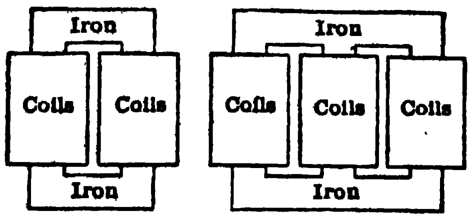
Line regulation depends upon: line resistance and reactance, the current, and power factor of the load. It may be calculated by: $E_0^2 = (pE_1 + IR)^2 + (IX \pm qE_1)^2$, where E_0 = voltage at generating end; I = load current per line; $p = \cos\phi$ and $q = \sin\phi$ at the load; qE_1 is negative for anti-inductive load; E_1 = voltage at receiving end; X = reactance of line; R = resistance of line. For single-phase, E_1 is the voltage at the load, and R and X are taken as the sum of outgoing and return circuits. For 2-phase, E_1 is the voltage per phase at the load, and I is the current per phase in a 2-phase system (Art 7); R and X being taken for the complete circuit of one phase, outgoing and return. For 3-phase, E_1 is the voltage to neutral at the load = 0.58 × voltage between lines; I is the current per line, or Y current; R and X are taken for one line, one way. This gives E_0, the voltage to neutral at generating station. Voltage between lines is then 1.73 E_0.

Size of wire for an a-c transmission. Select first a wire which gives an allowable resistance drop (IR = 5 to 10%), and then find loss in voltage [E_0 (generator) − E_1 (load)] due to combined effect of R and X. If loss be too great, select a larger wire and try again. To obtain a solution in one calculation is too complicated a problem except for experts. Remember that the impedance drop in the line (IZ) is usually much greater than the difference between generator and load voltage due to vector relations. Usual voltages for transmission work are: 2 200, 6 600, 13 000, 19 000, and 33 000 volts between lines, whether single, 2 or 3-phase. A rough rule for choice of voltage is to allow 1 000 volts for each mile of distance between generating station and load. Usual sizes of wire are from No 4 to No 0000 B & S gage in solid conductors and larger sizes in stranded conductors (Table 4). Smaller wire than No 4 has insufficient mechanical strength. Wires larger than No 0000, if solid, would be too stiff, and would break from the repeated bending to which they are subjected.

Stranded conductor. Gage number or rating of a stranded conductor is that of a solid conductor having same cross-section of metal. Resistance and weight of a stranded conductor are greater for a given length than of a equivalent solid conductor, due to the twisted path (and hence greater length) of the individual wires. Actual length of path of the current is increased by about 7%. In spacing the 2 wires of a line, it is usual to allow about 1 ft for every 10 000 volts.

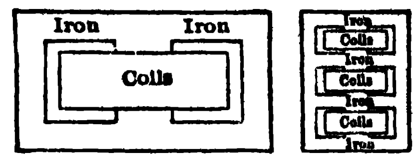
Transformer, commonly called static transformer, is used to transform a c of a given voltage to a higher or lower voltage. Single-phase transformer consists of 2 electrical circuits, usually of a large number of turns, interlinked with a common magnetic circuit of iron. Since the power is approx the same in both windings, the currents are inversely value

proportional to the voltages. Polyphase transformer is essentially two or more single-phase transformers, made into a single piece of apparatus, and so designed that at least part of the magnetic circuit is common to all the phases. A 3-phase transformer has 3 high-tension and 3 low-tension windings, arranged on an iron core (Fig 21). The winding by which energy enters the transformer is the PRIMARY; that by which it leaves, the SECONDARY. Since either winding may be connected to the source of energy, these terms are not definite unless the manner of connection is also stated. HIGH-TENSION WINDING and LOW-TENSION WINDING are used to distinguish the two windings, the high-tension being the one with the greater number of turns. When the high-tension winding is connected to the source of supply, it is a STEP-DOWN TRANSFORMER; when the low-tension winding is so connected, it is a STEP-UP TRANSFORMER.



Single Phase   Three Phase

Fig 21. Core-type Transformers



Single Phase   Three Phase

Fig 22. Shell-type Transformers

Classification of transformers. (A) according to operating characteristics. CONSTANT-POTENTIAL transformers are intended to give an approx constant potential on the secondary side; CONSTANT CURRENT, to give an approx constant current on the secondary, for arc lamps. Both are designed to operate on a constant-potential supply circuit. SERIES TRANSFORMERS are connected in series with the main circuit and receive the line current in the primary. The secondary circuit includes only a meter and the secondary current will be inversely proportional to the transformer ratio. They are used to step down very heavy currents for purposes of measurement by low-reading ammeters and wattmeters. AUTO-TRANSFORMERS or COMPENSATORS, sometimes called single-circuit transformers, consist of one electric circuit interlinked with a magnetic circuit and a tap brought off from some part of the winding. As the voltage between this tap and either terminal of the circuit is a fraction of the total, a fractional voltage may be secured. Windings on each side of the tap are usually proportioned to the current to be carried. Auto-transformers are desirable where the ratio of voltages approximates unity, as they then require much less copper than regular transformers. POTENTIAL REGULATORS are transformers in which the voltage of one member may be varied from zero to a fixed maximum, either by changing the direction of the magnetic flux or by changing the phase of the e m f of the secondary with respect to that of the primary.

(B) according to construction. There are two methods of arranging the electric and magnetic circuits of transformers, the corresponding construction being called "core-type" and "shell-type." CORE-TYPE transformer (Fig 21) has a single magnetic circuit interlinked with 2 electric circuits, each electric circuit containing primary and secondary coils. It is best adapted to small sizes or high voltages and to oil cooling, and is the commoner. The 3-phase core-type is a combination of 3 single-phase transformers into one, to save material and space. SHELL-TYPE (Fig 22) has 2 magnetic circuits in parallel, interlinked with 1 electric circuit containing primary and secondary. Best for air cooling and large currents. Three-phase shell-type is a consolidation of 3 transformers into one.

(C) according to method of cooling. As a transformer is a very compact piece of apparatus, the problem of carrying away the heat is important. NATURALLY-COOLED type has no special means of cooling, but relies upon ordinary circulation of air. Only used in very small sizes, as for meters. In the OIL-COOLED, the core and windings are submerged completely in a tank of oil, and are subdivided by ducts so that oil may circulate and abstract heat from internal parts. The heat is carried to surface of tank containing the transformer, and then dissipates into the surrounding air. Tank must be specially designed, with deep corrugations or projecting vanes or tubes, to provide large air-cooling surfaces. In the AIR-BLAST type there are passages through which air is forced by a blower. WATER-COOLED transformer resembles the oil-cooled type, but a coil of pipe carrying running water is submerged in the oil. Transformers artificially cooled by oil are used when the size is too great for the self-cooling oil type and no water is available. Oil circulates through external coils or tanks, to afford greater cooling surface.

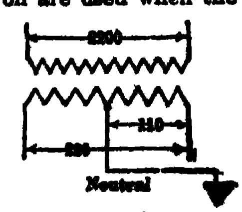


Fig 23. Single-phase, Three-wire

Transformer connections commonly used in lighting and power service are: SINGLE-PHASE SYSTEM with 3-wire secondary (Fig 23), standard in household lighting with a-c system, the neutral wire being grounded on the low-tension side, the primary side not being grounded. Lamps or motors operating at 110 volts are connected between the neutral and either side. Maximum potential between any secondary and ground is 110 volts, but if either outside wire becomes grounded a short-circuit occurs on that half of the transformer. TWO-PHASE, or quarter-phase 4-wire system. The standard is essentially 2 independent single-phase systems, which are

usually electrically independent throughout. TWO-PHASE 3-wire system is occasionally used for distribution of power in small systems. Return wire is common to both circuits. There may be a slight saving in copper, but the chances of unbalanced voltage and bad regulation, especially with an inductive load, render it objectionable. THREE-PHASE Y AND DELTA CONNECTIONS (Fig 24, 25). Transformation in a 3-phase system, with 3 single-phase transformers or one 3-phase transformer, having 3 primary coils and 3 secondary coils, is accomplished by connecting the primary either in Y or in delta, and the secondary either in Y or in delta. Following relations exist between voltage per transformer winding and voltage between lines, current in transformer windings, and current in lines. Power in the 3 transformers in any case is $3 \times 0.58 \times EI = 1.73\,EI$. THREE-PHASE OPEN DELTA, or V-connection, saves expense by omitting one transformer from the delta connection. Is recommended only for low voltages, as 2 300. Regulation and

Connection	Volts between lines	Volts per winding	Current per line	Current per winding
Y	E	$0.58\,E$	I	I
Delta (Δ)	E	E	I	$0.58\,I$

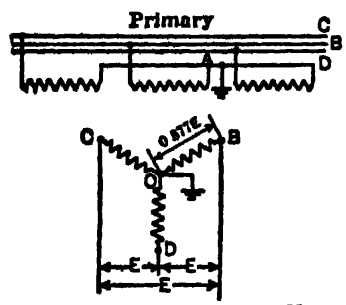


Fig 24. Three-phase Y

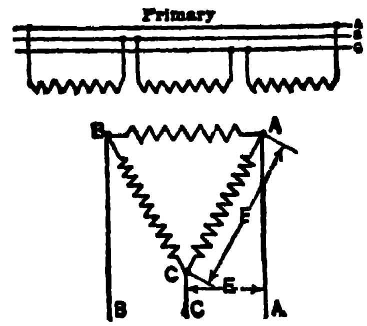


Fig 25. Three-phase Delta

efficiency are poor, as one phase of the load receives power from 2 transformers in series. Aggregate capacity of the transformers should be 15% greater than the load. TWO-PHASE TO 3-PHASE (Fig 26). Scott, or T-connection, consists of 2 transformers which, on the 2-phase side, are connected in normal 2-phase manner. On the 3-phase side one transformer has a tap at the middle point, the other a tap giving 87% of full-transformer voltage. Fig 26 shows method of connecting, and the currents in primary and secondary with balanced loads. Total transformer capacity must be 15% greater than the load.

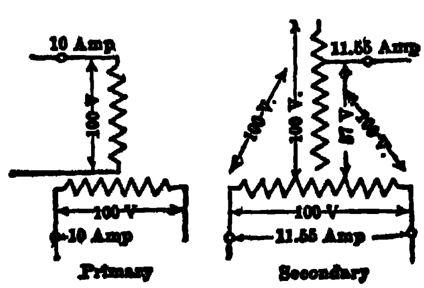


Fig 26. Two-phase to Three-phase

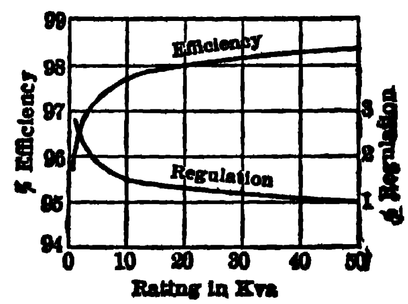


Fig 27. Efficiency and Regulation of a Line of Transformers

Efficiency is equal to the power output divided by output plus core loss plus total copper loss, and is very high. "All-day efficiency" is equal to the energy output for a day divided by the input; with short periods of load it is low, because core loss goes on when there is no load. If P is the load in watts for h hr per day, A the core loss, and B the copper loss, the all-day efficiency is $100\,hP \div (hP + 24A + hB)$ in percentage.

Regulation is $(E_0 - E) \div E$, where E_0 = secondary voltage at no load and E = secondary voltage at full load. It should be from 1 to 4% for load of 100% power factor, and is poorer for inductive loads. Fig 27 shows efficiency and regulation at full load of a line of 60-cycle lighting transformers for 2 300 volts.

Installation. Transformers require no special foundations, but merely provision to carry their dead weight. They must be in a dry place, unless of the totally enclosed "subway" type. They are often placed on poles. They must be thoroughly cooled.

Cost and weight. (See Art 19.)

Substation is a building containing an equipment for transforming or converting the energy of transmission line into a convenient form for use in small units. Since the energy may be used as a c or d c, there are 2 general types of substations: a c to a c and a c to d c. In either case the energy must be distributed to the various consuming devices at a much lower voltage than that of transmission, sometimes at a different frequency, and sometimes in an entirely different form. The substation is therefore the dividing point between the transmission and distribution systems. A-c SUBSTATION in simplest form comprises step-down transformers for lowering the voltage, and incidental regulating and protective devices. There may be a group of 3 single-phase transformers, or one 3-phase. Single-phase transformers are preferable for moderate sizes, as they require a smaller reserve. Since the transmission system is always 3-phase, these groups provide low-voltage 3-phase currents for different groups of motors, or the individual phases may be wired separately for lighting purposes. If transmission system is 25 cycles it may be necessary to install motor-generator sets (Art 6) (frequency changers) to give 60-cycle current for lighting. These usually consist of a synchronous motor driving an alternator. If the distribution system voltage must be kept constant, some form of voltage regulator is installed, which will keep the voltage constant on each individual circuit, or on the 3-phase circuit as a whole. A switchboard with the necessary switches and measuring instruments is requisite. D-c SUBSTATION consists of stepdown transformers, converters, motor-generator sets, or rectifiers with incidental regulating and protective devices. The converter (Art 11) is cheaper and more efficient than the motor-generator set, and is advisable in absence of any local reason for using motor-generators. Converters may be arranged to give constant voltage irrespective of load, or may be over-compounded. The substation then resembles a d-c generating station.

Lightning arresters are to protect the apparatus of a station from natural lightning discharges, and from "internal lightning" caused by sudden changes in current or voltage conditions in the system itself. A lightning arrester permits a current to flow from conductor to ground, whenever the potential exceeds the normal, and stops this current when the excess voltage ceases to exist. This is usually done by letting the excess voltage break down a spark gap, by means of an arc which stops as soon as the voltage drops below a critical value depending upon the width of gap.

Types of arresters. HORN-GAP ARRESTER is simplest and is commonly used for high voltages, but is not sensitive enough for moderate and low-voltage systems. It consists of 2 copper rods, bent like horns, separated by a fraction of an inch at the nearest point and diverging from there. The arc starts at the small end of gap, lengthens as it rises, and finally breaks. The "Autovalve" and the "Thynite" lightning arresters are two new commercial forms of the valve type; that is, they can discharge a very large current for a short time with a very small increase in line potential and can shut off this current as soon as the line potential returns approximately to normal. Each arrester consists of three functional parts, all in series in the circuit from line to ground: (*a*) an air gap, so set as to spacing of terminals that it will break down and discharge a very large current when the line potential rises to a point 20–50% above normal. This initiates the discharge; (*b*) a set of porous blocks or disks, made up of finely divided porcelain and carborundum, stacked and electrically in series. These blocks contain an almost infinite number of very small gaps and resistances in series and multiple, and regulate the flow of current during discharge. Each block is good for about 3 000 volts and enough are put in series to take care of line potential; (*c*) a quench-gap, whose duty is to open the circuit and shut off all current as soon as the discharge has diminished to a reasonable value and the dangerous voltage has been eliminated.

Functional part (*b*) is always used, but in medium-voltage applications (*c*) may not be used and for 600 volts neither (*a*) nor (*c*) is necessary.

14. ELECTRIC DISTRIBUTION

Principles. Three fundamental considerations determine the cross-section and weight of copper conductors for a particular purpose (see also *Art 13*): (*a*) Voltage lost in the line, or IR drop. This depends upon the line current and the resistance of outgoing and returning lines. It is important in affecting the saleability of the energy and desirability of the apparatus served. It causes considerable variation in the light given by lamps, and a variation in speed of motors. (*b*) Energy lost in the line due to I^2R, which represents an actual money loss. (*c*) Current density in conductors. This determines their temp rise, which, if excessive, causes danger of fire.

Usual loss in voltage allowed in distribution systems is less than in transmission; it varies from 2 to 5% of the delivered voltage. In a d-c system the proper cross-section of conductor in circular mils (Table 4) is given by: cm = $(21.6\,DI) \div e$, where D = distance one way, ft; I = current, amperes; e = specified loss, volts. In an a-c system the drop due to resistance of wire is usually the determining factor; reactance drop is probably negligible, as the wires are strung close to each other. Choice between a c and d c for distribution work depends upon character of the load apparatus and the available current. D-c motors are generally preferable, and therefore d-c distribution systems are used, unless the cost of the converting substation is too great.

Examples

Series-arc system (Fig 28). Problem: 80 arc lights, requiring 10 amperes at 50 volts each in a 5-mile series circuit. Allow 3% loss. Voltage consumed = $50 \times 80 = 4\,000$. Let R = resistance per 1 000 ft of wire. Length of wire in thousands of feet = $5 \times 5.28 = 26.4$. Voltage loss $= 4\,000 \times 0.03 = 120 = 10 \times 26.4 \times R$; hence $R = 0.455$. From Table 4, the nearest size of wire is No 7 B & S.

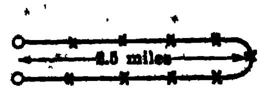


Fig 28. Series-arc Distribution

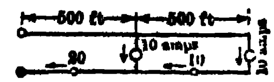


Fig 29. Multiple Distribution

Multiple system (Fig 29). Problem: two loads at 500 and 1 000 feet respectively from distribution center. Each load, 10 amperes. Allowable maximum drop, 5 volts. Find size of wire. Let R = resistance per 1 000 ft of wire. Then, $5 = 20 \times 2 \times 0.5\, R + 10 \times 2 \times 0.5\, R$; hence $R = 0.166$. Choose No 2 B & S wire, of which $R = 0.156$.

Anti-parallel system. Problem: find size of wire to give a maximum drop (on middle group) of 5 volts on loads shown in Fig 30. Let R = resistance per 1 000 ft of wire. Then, $5 = 30 \times 0.2\, R + 20 \times 0.2\, R + 20 \times 0.2\, R + 30 \times 0.6\, R$; hence $R = 0.156$. Choose No 2 B & S wire.

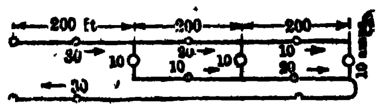


Fig 30. Anti-parallel Distribution

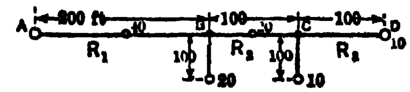


Fig 31. Tapered-conductor Distribution

Tapered conductor. Problem: given 3 loads as in Fig 31, and maximum drop of 5 volts, to find sizes of wires. Assume drop proportional to distance: $AB = 2.5$, $BC = 1.25$, $CD = 1.25$. Then,

$2.5 = 40 \times 2 \times 0.2\, R_1$; hence $R_1 = 0.156$ per 1 000 ft; choose No 2 wire.
$1.25 = 20 \times 2 \times 0.1\, R_2$; " $R_2 = 0.312$ " 1 000 " " " 5 "
$1.25 = 10 \times 2 \times 0.1\, R_3$; " $R_3 = 0.625$ " 1 000 " " " 8 "

Feeder and main. Problem: given loads as in Fig 32, and max drop of 10 volts. Find size of wire. Assume drop to junction point = 8 volts (0.75 of total).

Feeder, $8 = 15 \times 2\, R_1$; hence $R_1 = 0.266$; choose No 4 wire.
Main, $2 = 5 \times 2 \times 0.2\, R_2$; " $R_2 = 1.0$ " " 10 "

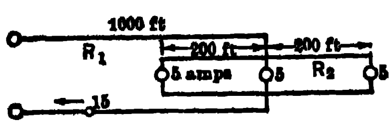


Fig 32. Feeder and Main Distribution

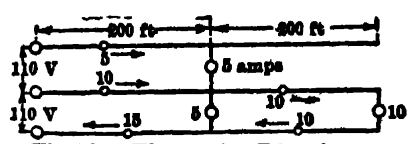


Fig 33. Three-wire Distribution

Three-wire system. Principle: at any given distance from generator the sum of outgoing currents equals sum of returning currents (Fig 33). Problem: Given 3 loads, 200 and 400 ft from generator, as shown. Neutral or middle wire to be same size as outside wires. Max drop, 5 volts. Let R be resistance per 1 000 ft. $5 = 10 \times 0.4\, R + 10 \times 0.2\, R + 15 \times 0.2\, R$; hence $R = 0.555$. Choose No 7 wire.

Construction. Distribution systems may be overhead or underground. Underground system costs about 5 times as much to install as the overhead, but less to maintain. Steel poles are used for high voltage or regular transmission systems, but wooden poles are satisfactory for ordinary distribution. Cedar, chestnut, pine, and cypress are good, costing \$10 to \$20 per pole, according to location. They are usually 35 to 40 ft long, have a taper of 1 in in 5 to 8 ft, are 7 to 8 in diam at top and set 5 to 8 ft in ground. Spans, from 75 to 125 ft. Poles are guyed to anchors on curves, at ends, and at every 20th pole, to take care of breaking of wires. Glass insulators on cross arms are used for moderate voltages, porcelain insulators for higher. In stringing wires, sag must be adjusted with reference to the temp; a large sag in summer, small in winter.

In cities most of the distribution is laid underground in elaborate conduits, which are expensive and unnecessary for mining work. Good results are obtained by laying a lead-covered double conductor cable in a shallow trench and covering it with earth.

Interior wiring of buildings. Definite methods, practices, and regulations covering installation of distributing circuits are based on following considerations: (*a*) avoidance of danger from fire or risk of shock; (*b*) limitation of loss of power and energy; (*c*) maintenance of reasonably constant voltage at the load; (*d*) assurance of reasonable mechanical strength; (*e*) compliance with local specific regulations. STANDARD VOLTAGES: for lighting and incidental power, 100 to 125 volts; for power and lighting on 3-wire system, 220 to 250 volts; for power alone, 550 volts. Usual allowance for drop in building wiring, 2%. LOCAL REGULATIONS to be observed: National Electric Code, for

obtaining insurance on buildings; Municipal Inspection in cities; regulations of local Light and Power Co, if energy is purchased. These regulations are so elaborate that any one expecting to be responsible for an installation of wiring must be acquainted with the National Electric Code, on which most of the other regulations are based. ALLOWABLE CURRENT-CARRYING CAPACITY OF WIRE. Values given in Art 3 are for rubber-covered wire installed indoors. For other insulation, and for exterior work, somewhat greater current is allowed. In laying out a distribution system, a CUT-OUT OR FUSE must be inserted at every junction where size of wire changes, and there must not be more than 660 watts in lamps on any one fuse. Larger motor loads are cared for by a CIRCUIT-BREAKER.

Methods of indoor wiring. OPEN WIRING ON CLEATS. Wire must be insulated with material resisting fire, and dampness and corrosive vapors, if present. Wires must be separated from each other and from any nearby surface by specified distances and the cleats may be spaced not more than 4.5 ft apart (local regulations). No 14 is the smallest allowable wire. MOLDING, either wood or metal, may be used to hold and protect wires, if difference in potential does not exceed 300 volts and the power transmitted is within specified limits. KNOBS AND TUBES. If not prohibited by special regulations, wires may be attached to knobs and run through walls in porcelain tubes; a cheap form of wiring.

Conduit. Best but most expensive method is to run the wires in conduits, built into the walls, floors, or framework of building. Conduit may be lined or not, which has a bearing on amount of insulation required on the wires. Conduit may be rigid or flexible; the former is better and more expensive, and is often required in large cities. ARMORED CABLE, commonly known as "BX," may be used in buildings not provided with conduits, and where very reliable installation is required. Lead-armored cable is used where dampness is expected.

Metering. Charges for electric energy are based upon the kw-hr, which is the total integrated amount of energy delivered in 1 hr to a circuit in which the average power is 1 kw. Cost per kw-hr varies from 1¢ in large quantities, to 10 and 15¢ for small quantities. The ampere-hr is not an accurate measure of energy, unless the circuit voltage remains absolutely constant. See also Sec 16.

Instrument used for measuring electric energy is the watt-hr meter, sometimes called integrating wattmeter. Electric connections are similar to those for wattmeters used in measuring power. MEASUREMENT OF TOTAL POWER OR ENERGY: (*a*) in d-c or single phase a-c circuit, by one meter having its current coil in series with the load and its potential coil in parallel with the load; (*b*) in a 3-wire d-c or in a 2-phase circuit, balanced or unbalanced, by 2 meters connected as if in 2 independent single-phase circuits (total power being the sum of the 2 readings); (*c*) in a balanced 3-phase system, by 1 meter connected in 1 of the 3 receiving circuits (its reading being multiplied by 3) or by 1 meter connected with its current coil in 1 of the line wires and its potential coil connected from 1 line wire to the neutral (total power being obtained by multiplying the reading by 3); (*d*) in a 3-phase system, balanced or unbalanced, by 2 meters, connected as in Fig. 34. Total power is the sum of the 2 readings, if the power factor be greater than 0.5, or the difference of the readings if it be less than 0.5, in which case the potential leads of 1 meter must be reversed, to bring the pointer on the scale.

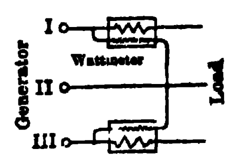


Fig 34. Power and Energy in Three-wire Distribution

Watt-hour meter comprises 3 parts: (*a*) a small electric motor, of commutator, mercury and disk, or induction type, which runs at a speed proportional to the power; (*b*) a brake or generator system, usually consisting of an aluminum disk revolving in the field of permanent magnets to give a drag by eddy currents proportional to the speed; (*c*) a train of gears driving pointers for recording number of revolutions. While the type of motor varies with the make and purpose of the meter, the brake and recording devices are common to all. COMMUTATOR TYPE consists of a small d-c motor, the field being excited by the load current in current coil and the armature connected in potential coil carrying a current proportional to potential. It is operative on both d-c and a-c circuits, but is used only on d-c, the induction type being cheaper and better for a-c. MERCURY-MOTOR watt-hr meter contains a motor consisting of a disk of copper or aluminum floating in a well of liquid mercury. Main current is led to the mercury, but passes through the disk because of its lower resistance. This forms an elementary armature. Potential coils are on an iron core; they form the motor field and produce a torque in the armature which revolves it. Operative on both a-c and d-c circuits, but the induction type is preferable for a c. INDUCTION TYPE watt-hr meter is similar to a 2-phase induction motor, the current coils forming 1 phase, the potential coils the other. This gives a rotating magnetic field which drives a metal disk rotor. Operates only on a c. Cheaper than either of the other types, and more reliable and accurate. POLYPHASE WATT-HR meter contains 2 current coils and 2 potential coils, connected as 2 wattmeters would be connected; but both elements act on same shaft and drive same set of gears, thus mechanically taking care of any positive and negative readings in either element, and giving correct total energy in one reading. Sometimes a single-phase watt-hr meter is used on a balanced 3-phase circuit (such as one containing motors only). Its current coil is connected in one line, and its potential coil is connected between one line and a neutral obtained by connecting 3 equal high resistances in Y. Total energy is 3 times the reading.

Meter capacity. In dwellings it should be 50% of the total capacity of the connected load; in offices, 75%; for elevators, hoists, etc, 150% of the load.

Installation. Watt-hr meters must be installed on a rigid support free from vibration, and must be level. They should be at least 15 in between centers, and must not be near iron girders nor steam pipes, nor conductors carrying large currents. They must be protected from mechanical shock, weather, heat, dirt, vermin, and dampness.

Sources of error. Adjustments are provided to take care of normal friction loss. Variation in friction of brushes or bearings causes error. Mechanical shock, dirt, or dampness may alter the friction. External magnetic fields cause variation in adjustment. In induction meters variations in frequency, voltage, or load power factor may cause a slight error. Meters should be inspected and calibrated at least once a year. To CALIBRATE a watt-hr meter, connect it to a constant known load and count revolutions of the disk per min. Note the constant K painted on the disk, and substitute in formula: watts = 60 r p m × K.

15. ELECTRIC LIGHTING

Light distribution follows the law that the intensity is inversely as the square of the distance, providing the greatest dimension of the light source is less than 0.1 of the distance between source and object.

Definitions. INTENSITY of light is the relative amount of luminous energy given by any source and is measured in candles or candle power. CANDLE POWER (c p) is a measure of light intensity, determined by comparison with a well-seasoned incandescent electric lamp, that has been accurately checked with a standard lamp at the National Bureau of Standards, Washington. ILLUMINATION is measured in ft-candles. One ft-candle is intensity of illumination on a surface 1 ft distant from a source of one c p. At 2 ft distance the illumination would be 0.25 ft-candle. Light is the means; illumination, the end. INTRINSIC BRILLIANCY of a source is measured in c p per sq in. When excessive it is harmful to the eye. Intrinsic brilliancy of tungsten lamp is 1 000 c p per sq in. A light source generally gives different intensities in different directions. Hence, c p means nothing unless direction is specified. MEAN HORIZONTAL CANDLE POWER (m h c p) is the average c p of a lamp in all directions in a horizontal plane passing through center of the source, and is usually obtained by rotating the lamp about a vertical axis. MEAN SPHERICAL CANDLE POWER (m s c p) is the average c p of a lamp in all directions, or the c p of a uniform source giving the same total flux of light. It is directly proportional to the total light given by the lamp, and is measured by taking intensity readings in all directions, or by placing the lamp in a hollow dull white sphere and measuring average intensity. Flux of light is measured in LUMENS and is the quantity emanating from a source of unit intensity (one c p) and contained in a unit-soled angle. There are 4π or 12.56 lumens emanating from a source of unit intensity and 12.56 N lumens from a source of N spherical c p. Luminous effic is expressed in lumens per watt consumed and is from 10 to 20 in modern incandescent lamps, the higher values for the larger lamps (see mfrs guarantees).

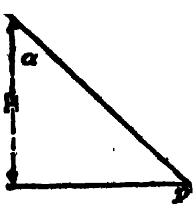


Fig 35. Law for Intensity of Illumination

In applying the laws of photometry to the calculation of a problem in illumination certain qualifications are necessary: source of light must be small compared to the distance of the object illuminated; luminous surface must be at right angles to the line of direction from the object to be illuminated; reflection and refraction must be negligible. The most useful and general application of the law is expressed by: $I_h = (\text{c p} \div H^2) \cos^3 \alpha$; where I_h = intensity of illumination in ft-candles at a given point p (Fig 35) on any horiz plane; c p = intensity of light source in given direction, expressed in candles; H = height in ft of lamp above horiz plane containing point p; α = angle from vertical.

General illumination of reasonable intensity (0.5 to 1.0 ft-candles) facilitates a general view and perspective of the surroundings, while a greater intensity (1 to 5 ft-c) facilitates reading or work. In DIFFUSED ILLUMINATION light comes from all directions, casting no deep shadows; extreme diffusion gives flat lighting and may be monotonous. To protect the eye some diffusion is usually desirable, for it tends to remove glaring reflection from objects in view; thus, when reading from glazed paper or working on polished metals light must not be strongly directional. In DIRECT ILLUMINATION, shadows are dark, contrasts great, and forms are clearly outlined. Details are best brought out by a light rich in the color corresponding to that of the object viewed. DIRECT LIGHTING unit sends most of its light at once to the object to be lighted. SEMI-INDIRECT unit consists of a diffusing medium between the lamp and the object, which directs most of the light to the walls and ceiling, from which it is reflected. TOTALLY INDIRECT unit directs all light from the lamp to the walls or ceiling, from which it is reflected. While more light is lost with the indirect system it is easier on the eyes, glare is avoided, the eyes are more sensitive, and vision is improved.

Arc lights are generally in large units (1 000 c p), are among the most efficient sources, have high first cost and high maintenance cost, compared to incandescent lamps.

They operate on either d-c or a-c circuits (but are not interchangeable), and may be of series or multiple type. The d-c lamp is usually a little more efficient and more reliable than the a-c. SERIES-TYPE ARCS are in series on one circuit, supplied by a generator or transformer designed to give a constant current irrespective of the lamps in operation. MULTIPLE-TYPE ARCS operate either singly or in pairs in series, on the usual constant-potential circuits with motors and incandescent lamps. Arc lamps other than the mercury and sodium-vapor type are becoming obsolete.

Mercury-vapor lamp is used on d c multiple circuits; 110 volts with individual auxiliary starting and regulating device gives a blue green light, an effic of 15 to 20 lumens per watt and comes in units of 300 to 500 watts. Sodium-vapor lamp is a new development, particularly adapted to highway lighting. It requires considerable auxiliary apparatus, gives a yellow light and has very high effic. Operates in multiple on 125-volt a c circuits.

Incandescent lamps are made in many forms and sizes and for many voltages and currents, for operation in multiple or series. The multiple type for 115, 120 or 125 volts, either a c or d c, is the most common. They consist of a coiled filament of fine tungsten wire, enclosed in an inside frosted bulb containing an inert gas, nitrogen or argon.

Rating of incandescent lamps. All multiple lamps are now rated in watts input, and in lumens rather than candle power; thus the 25-watt lamp gives 250 lumens; the 40-watt 420; 60-watt, 750; 100-watt, 1 500. The useful life is from 1 000 to 1 200 hr; that is, at end of this time the output of light is reduced to 80% of the original value. All incandescent lamps are very sensitive to variations in impressed voltage, respecting life, as well as effic and candle power. The smaller sizes give 10–15 lumens per watt when new.

Fluorescent lamp consists of a mercury-vapor arc in a tube with fluorescent or phosphorescent coating, which gives the light a wide range of agreeable colors; it operates on a c or d c, more efficiently on a c. It is more efficient than the incandescent lamp.

Fixtures, known as luminaires, are very important. The direct type sends all the light downward, the indirect type sends the light upward and depends upon a good, reflecting ceiling. There are many intermediate types. Utilization factor of a fixture for a room is the ratio of the useful lumens of light delivered to the working plane to the total lumens output of the lamps. It varies from 0.15 to 0.72; mean value, 0.5. The reflecting qualities of the surroundings have an important bearing. To determine the number and rating of lamps required: $F = AE \div u$, where F = total lumens required; A = horiz area to be illuminated, sq ft; E = desired illumination in ft-c (Table 11); u = utilization factor for the particular surroundings. Choosing the number of units desired, based upon the uniformity of illumination required, gives the lumens per lamp.

Cost per hr of supplying a given illumination comprises cost of lamp and of energy: $C = \frac{Pn}{L} + \frac{n\,w\,K}{1000}$; where C = total cost of light required, cts per hr; P = price of lamps, cts per lamp; n = number of lamps required obtained as above; L = aver life of lamps, hr; w = watts per lamp; K = price of energy, cts per kw-hr.

Interior Illumination. Interiors are generally illuminated by incandescent lamps on multiple circuits, and to supply general illumination of even intensity a number of medium size units (60 to 250 watts) are usually preferable to one large unit. Units should be placed or shaded so as to be out of line of sight of eye. Best method of distribution is to divide the room into equal squares, having sides of 10 to 20 ft, depending upon degree of illumination desired and height of ceiling, and placing a source of light at ceiling in center of each square (not at corners). The side of each square should not exceed twice the height of the lamp above the working plane. Any local concentrated illumination may be provided by special local fixtures. Diffusing globes should be used on all lamps in line of vision, and frosted lamps in all fixtures unless completely shaded. Customary criterion for good interior lighting of factories by Masda lamps is to supply 1 to 2 watts per sq ft of floor space; the higher value when work is exacting or walls are of dark color. This is based on an illumination of from 3.5 to 7.0-ft candles on a plane 30 in above floor, using tungsten lamps.

Table 11. Usual Intensities of Illumination (Ft-candles) in Plants, as Mine Plant

Service	Ft-candles	Service	Ft-candles
Desk work	8–30	Steel works: Unloading yards	2–5
Factory, general	3–5	Open-hearth floors	4–10
Local bench, for fine work	25–100	Blast furnace	2–5
Local bench, for coarse work	6–20	Rolling mills	4–10
Machine tools	10–20	Wire drawing	3–5
Pattern shops	10–20	Threading, pipe mills	2–5
Power house	3–5	Warehouse, Wharf	2–5

Mine lighting. Tunnels, shaft stations, and all places of traffic, are usually lighted by 40- or 60-watt Masda lamps, with weatherproof sockets and steel porcelain enamel reflectors. In gaseous

mines lamps are enclosed in heavy vapor-tight glass globes protected by iron guards. 200 to 300-volt circuits are used in mines, for which voltage lamps are available. Working places may be lighted by portable storage battery lamps, carried on the cap with separate battery or in the hand with battery combined (Sec 23, Art 10). To be approved by the Bureau of Mines these lamps must be free from possibility of igniting gas, must be of light weight (not over 5 lb) and must not spill or leak electrolyte. They must illuminate a 7-ft circle, give about 0.7 c p, and are usually for 2 volts (Bureau of Mines Schedule 6).

Reflectors should be used with all lamps. Reflector for arc lamps for general lighting is an integral part of lamp, and designed to throw the light downward and outward. Reflectors for incandescent lamps may be extensive, intensive, or focussing; extensive throws the light downward, within a solid angle of about 90°, intensive in an angle of 60°, and focussing in an angle of 60 to 40°. Industrial reflectors may be made of white enameled steel, aluminum, or painted metal; for interior use, plain glass or prismatic glass. Latter uses the principle of refraction to send downward the rays that would normally pass through the glass. Reflectors and lamps must be cleaned regularly, to prevent loss of efficiency. Tungsten lamp should be cleaned while the current is on, as there is less liability of breaking the filament.

16. APPLICATIONS OF ELECTRIC TRANSMISSION TO MINE SERVICE (See Sec 16 for details)

17. ELECTROCHEMISTRY

Equivalent weight or chemical equivalent of any substance is its molecular weight divided by its highest valency: thus, equivalent weight of copper in $CuSO_4$ is 31.78, being the atomic weight of copper 63.57 divided by its valency 2. Equivalent weight of the SO_4 radical is $[32.07 + (4 \times 16)] \div 2 = 48.03$. Equivalent weight gives the relative weights of various substances deposited in electrolytic work by a given current.

Gram molecule or mol is the number of grams of a substance which are equal to its molecular weight; thus, 1 gram molecule of $CuSO_4$ is 159.64 gm. GRAM EQUIVALENT is the number of grams of a substance which are equal to its equivalent weight; or to its mol divided by its valency. The gram equivalent of $CuSO_4$ is $159.64 \div 2 = 79.82$.

Electrolyte. When an electric current passes through certain substances, chemical action takes place at the points where the current enters and leaves the substance. Such substances are called electrolytes and the chemical action produced is electrolysis.

Electrodes are the conductors by which the current enters and leaves the electrolyte. ANODE is the electrode by which the current enters; CATHODE, that by which it leaves. The anode is connected to the positive terminal of the generator. Hydrogen and metal atoms of salts flow to cathode and are called CATHIONS; oxygen, and acid and basic radicals flow towards anode and are called ANIONS. All anions and cathions are called "ions."

Electrochemical equivalent OF AN ION is the mass in gm of the ion which would be deposited by one coulomb (Art 1) of electricity. ELECTROCHEMICAL CONSTANT OR FARADAY (F) is the number of coulombs required to liberate one gram equivalent of any ion; value is 96 450 coulombs for all ions.

Faraday's Laws. Quantity of an electrolyte decomposed by an electric current is directly proportional to total quantity of electricity which passes. Rate of decomposition is directly proportional to current. Quantity decomposed is theoretically independent of voltage, current density, size of electrodes, and concentration of electrolyte. In practice local reactions cause deviation from this law. A given quantity of electricity always decomposes equivalent weights of different electrolytes in definite proportions for different elements. Total weights of different substances deposited by a given current in a given time are proportional to the equivalent weights of those substances. Some elements, as copper, may have different valencies in different combinations. Weight of such substances deposited is inversely proportional to the valency.

Electrochemical equivalent OF A SUBSTANCE is the number of gm of the substance deposited by 1 ampere flowing for 1 sec. The ampere is the unit of electric current, which will deposit 0.001118 gm silver in 1 sec. For calculating weights deposited by different currents in commercial practice it is more convenient to use the value of grams deposited by 1 ampere in 1 hr (Table 12).

The electrochemical equivalent of any ion may be calculated by dividing the gram equivalent of the ion by the Faraday. Thus, grams deposited in 1 sec by 1 ampere equals the gram equivalent of the ion $\div$ 96 450. Weight of any substance in gm deposited in 1 hr is obtained by multiplying the values given in Table 12 by the current in amperes.

Decomposition or critical voltage is the minimum potential which must be impressed to cause decomposition of a substance. With lesser potential no action takes place; with greater potentials action is proportional to current. This value depends upon chemical composition of the electrolyte, and is a function of the chemical energy of the process

Table 12. Electrochemical Equivalents of Common Elements Deposited per Ampere-hr

	Grams		Grams
Aluminum	0.337	Lead	3.865
Bromine	2.981	Magnesium	0.454
Calcium	0.7476	Nickel (val = 2)	1.093
Chlorine	1.322	Oxygen	0.2984
Copper (val = 1)	2.371	Platinum (val = 2)	3.640
" (val = 2)	1.186	Silver	4.024
Gold (val = 1)	7.36	Sodium	0.8576
" (val = 3)	2.453	Tin	1.107
Hydrogen	0.0375	Zinc	1.219
Iron (val = 2)	1.042		
" (val = 3)	0.6944		

Table 13. Decomposition Voltages of Solutions used in Metallurgical Industries

	Volts		Volts
Acid, hydrochloric	1.31	Hydrate, potassium	1.67
" nitric	1.69	" sodium	1.69
" oxalic	0.95	Nitrate, lead	1.52
" perchloric	1.65	" potassium	2.17
" phosphoric	1.70	" silver	0.70
" sulphuric	1.67	" sodium	2.15
Chloride, nickel	1.85	Sulphate, cadmium	2.03
Chloride, sodium	1.98	" nickel	2.09
		" zinc	2.35

Values given in Tables 12 and 13 are useful in checking efficiency of any electrolytic action; for, with these constants, the theoretical value of grams separated per kw-hr may be calculated and compared with the results obtained in practice. Thus, if A is the value of grams deposited per ampere-hr (Table 12), and B is the critical voltage for that substance (Table 13), then the theoretical grams per kw-hr = $(A \div B) \times 1\,000$. Example. Copper is deposited from a cupric solution at the rate of 1.186 gm per ampere-hr (A), and requires 1.59 volts to maintain the action (B). Therefore the maximum possible recovery is $(1.186 \times 1\,000) \div 1.59 = 748$ gm per kw-hr. Similarly, zinc is decomposed at rate of 1.219 gm per ampere-hr (A), and requires a critical voltage of 2.35 (B). Hence, a primary battery consumes a minimum of $(1.219 \times 1\,000) \div 235 = 520$ gm per kw-hr. The kw-hrs required to deposit 1 kg of material are given by the formula: kw-hr = $B \div A$

18. BATTERIES, STORAGE AND PRIMARY

Lead storage battery consists of 2 lead electrodes in dilute H_2SO_4. The positive plate has lead peroxide supported by a grid of pure lead; the negative is lead sponge similarly supported. The chemical reaction is:

$$\underbrace{\underset{+\text{ plate}}{PbO_2} + 2\,H_2SO_4 + \underset{-\text{ plate}}{Pb}}_{\text{(charged)}} = \underbrace{\underset{+\text{ plate}}{PbSO_4} + 2\,H_2O + \underset{-\text{ plate}}{PbSO_4}}_{\text{(discharged)}}$$

Reading from left to right gives the action of discharging; from right to left gives charging. There are 2 types, the Planté and the Faure. PLANTÉ TYPE consists of plates of pure lead with surface area increased by roughening by webs, ribs or grooves, and formed by charging electrolytically. There are 3 forms of construction of Planté plates: central web, cast lead, and pellet. In general, Planté cells are heavier and more bulky than Faure cells, but last longer. FAURE TYPE (paste type) has active material consisting of a paste containing litharge (PbO) or red lead (Pb_3O_4) applied to all grids and dried. Plates are then formed by being charged and discharged. Faure cells are lighter and therefore preferable for vehicles. Positive plates are those from which the current flows to the load during discharge. They have a dark chocolate color when charged, with hard smooth surface. Negative plates are gray, with softer surface. The ELECTROLYTE H_2SO_4, diluted with distilled or pure water, should have sp gr of about 1.15 when new, but different values are used for different services. It must be free from chlorine, nitrates, copper, iron, mercury, arsenic, acetic acid, and platinum, and should be made for the particular purpose. When cell is fully charged, the sp gr is 1.21 to 1.3; when discharged, about 1.15. Open-circuit voltage is 2.

Capacity of a lead battery is stated in ampere-hr. Normal discharge current is that which the cell will give for 8 hr. Ampere-hr rating is this current multiplied by 8.

Special ratings may be stated for a 1-hr or 4-hr discharge. Rate of charge and discharge varies from 6 to 10 amperes per sq ft of plate area, not including area obtained by corrugating. Any current may be obtained from a cell by putting enough plates in multiple. Watt-hr efficiency is the ratio of output to the input necessary to bring the battery back to its original condition. It is 90 to 92% for a battery floating on the line, and 75 to 90% for a battery completely discharged before recharging. A cell is discharged until its terminal voltage while discharging has decreased to 1.8 volts, and is charged until it requires 2.5 volts to force rated current through it. If a cell is discharged so that its voltage at normal current is less than 1.75, $PbSO_4$ may form, making it difficult to charge again.

Care of lead storage batteries. All terminals and electric connections of a lead cell must be of lead; joints are formed by burning, or are lead covered. Instructions for care of batteries are given by makers, and should be followed carefully according to conditions and service. Electrolyte must be free from the injurious substances mentioned above, must be of definite sp gr for each condition, and must cover all plates. Cells must be cleaned before any deposit of sediment on bottom is high enough to touch the plates. Exterior of the cells must be cleaned and kept dry to prevent grounding and deterioration of terminals. Any leak must be repaired at once. Charge as soon as possible after cell has been discharged, at a current slightly in excess of the 8-hr discharge current, continuing until the negative plates give off gas steadily; then stop. In discharging, stop when terminal voltage drops to 1.8 for normal 8-hr current and 1.6 volts for the 1-hr rate. Keep temp of the battery below 105° F. Cells to be put out of commission for some time should first be fully charged, electrolyte removed and replaced by pure water. Then discharge through a very low resistance, until practically discharged (avoiding heating of cells). Electrolyte is then removed, plates washed, dried, and put away.

Edison storage battery contains positive plates consisting of perforated steel tubes heavily nickeled and filled with alternate layers of nickel hydroxide and thin flakes of pure metallic nickel. Tubes are supported by a grid of cold-rolled nickel steel. Negative plates consist of a nickel-steel grid, holding rectangular pockets filled with powdered iron oxide. All plates are insulated from each other and from steel container. Electrolyte consists of a 20% solution of potash in distilled water. Chemical reaction is:

$$\underset{+\text{ plate}}{2\,NiO_2} + 2\,KOH + \underset{-\text{ plate}}{Fe} = \underset{+\text{ plate}}{Ni_2O_3} + 2\,KOH + \underset{-\text{ plate}}{FeO}$$

Read from left to right for discharge; from right to left for charge.

Rating. Batteries are rated at the current which they will give for 5 consecutive hr. Ampere-hr rating is this current × 5 hr. Charging current is same as discharging, but time is about 7 hr instead of 5. Ampere-hr capacity is practically independent of rate of discharge, as battery is not injured by reasonable overload and may be discharged to 0 volts. Watt-hr efficiency is 60 to 65%. Average voltage per cell at normal rate of discharge (5 hr) is 1.2, varying from 1.45 to 1.0 volt. During charging at normal rate voltage is 1.55 to 1.85.

Operation. Steel containers must be insulated from each other, kept clean and dry, and plates kept covered with electrolyte. Charge at 75 to 85° F; discharge at 120 to 125° F. Sp gr of electrolyte should be constant at 1.2. Batteries are shipped with plates discharged, and after setting up should be charged at their normal rate for 12 hr. This overcharge should be repeated after the first 30 and 60 days, and thereafter whenever electrolyte is renewed. Battery may stand idle indefinitely if level of electrolyte is above the plates. It should be ventilated, as the hydrogen gas given off is explosive. As in case of all storage batteries the maker's instructions must be carefully followed.

Weight and cost. Lead storage batteries rated on 8-hr basis cost about $50 per kw-hr of capacity, or the cost may be $500 to $600 per kw capacity on 8-hr basis. Weight: 120 lb per kw-hr for portable style, 260 lb in glass jars, 370 lb in lead-lined tanks. Nickel storage batteries rated on 5-hr basis cost about $75, and weigh about 90 lb per kw-hr capacity. On this basis the cost per kw rating is about $400. The nickel-iron storage cell gives about 12 watt-hr per lb of gross weight per charge; portable type of lead cell, about 8 watt-hr per lb.

Primary batteries consume zinc or other metal by H_2SO_4, KOH, or similar electrolyte, and produce electrical energy. Zinc is usual for negative pole, and carbon or copper for positive. Inside a primary battery current flows from zinc to carbon. Polarisation results from new chemical substances (usually hydrogen) at the poles, causing a back e m f. Depolarisers are chemical reagents used to combine with these substances and prevent or reduce back e m f. There are 3 general types: wet, dry, and standard. Wet cells are being superseded by dry cells on account of their greater convenience. Typical wet cells are: Daniell, giving 1.07 to 1.14 volt; Gravity, 1 volt;

Bunsen, 1.9 volt; Edison-Lalande, 0.75 volt; LeClanche, 1.5 volt. The Daniell and Gravity are similar, and are good for continuous small currents. LeClanche cell is good for intermittent service, as its depolarizer takes time to operate. It is the commonest wet battery. DRY CELLS are of LeClanche type, consisting of a zinc container which is also the negative pole. A carbon rod forms the positive pole, the electrolyte being sal-ammoniac and zinc chloride held in blotting paper, saw dust, or the like. Manganese peroxide is used as a depolarizer. Voltage is 1.5 to 1.6. Internal resistance is 0.05 ohm new and 0.5 ohm after 10 months. The short-circuit current through an ammeter of 0.01-ohm resistance should be from 18 to 30 amperes. High temp increases the current. Potential decreases with age, due to drying out. Cells are not good after 10 to 12 months, even if not used. Their ampere-hr capacity is 24 to 30; higher on intermittent service, but capacity and life are very poor on continuous service. Life of cell depends greatly upon relative external and internal resistance, a matter requiring careful consideration. By putting 2 cells in parallel their combined life may be much more than twice the life of one alone on the circuit. Usual size of cell is 6 in high by 2.5 in diam. Cost is 10 to 40¢ each, depending upon quantity and quality purchased. (Standard Cell, see Art 1.)

19. COSTS (as of 1938)

The war has altered all price schedules, but the 1938 data indicate relative costs and show weights of various sizes.

D-c generators are usually of the compound-wound type, and comprise 3 classes: high speed, to be belted to prime mover; moderate speed and low speed, to be direct-driven by an engine. The last class is least important. Table 14 gives costs and weights of the first 2 classes, for 125 or 250 volts. Machines to operate at lower speeds cost more. Add about 10% for installation.

Table 14. Weight, Cost and Speed of D-C Generators

Kw	High speed			Moderate speed		
	Speed, r p m	Wt, lb	Cost	Speed, r p m	Wt, lb	Cost
25	1 450	1 090	$ 790	850	1 825	$ 930
50	1 150	2 035	1 250	700	3 550	1 800
75	850	3 550	1 750	575	4 900	2 550
100	700	4 900	2 250	575	6 160	3 000
150	575	7 500	3 100	500	7 620	4 300

D-c motors. Shunt motors, usually rated at the output they will give continuously without injury to themselves, are divided into classes according to speed (small high- and moderate-speed units, Table 15). For installation, excluding freight, add 5 to 10%.

Table 15. Weight, Cost, and Usual Speed of Shunt Motors for 115 or 230 Volts

H p	High speed			Moderate speed		
	Speed, r p m	Wt, lb	Cost	Speed, r p m	Wt, lb	Cost
0.5	1 750	55	$ 60	1 150	70	$ 74
1	1 750	105	92	1 150	130	115
2	1 750	160	115	1 150	200	141
5	1 750	285	225	1 150	350	280
10	1 150	485	370	850	570	430
25	1 150	1 000	600	700	1 300	770
50	850	1 500	1 100	690	1 800	1 300
100	850	2 450	1 600	690	3 200	1 950

Table 16. Weight, Cost and Usual Speed of Series Motors for 230 Volts

H p, 1 hr	Speed, r p m	Wt, lb	Cost
10	725	790	$ 696
25	575	1 670	1 042
50	500	2 970	1 531
75	475	3 900	2 052
100	460	5 000	2 600

Table 17. Cost, Weight and Usual Speed of A-C Generators

Kva	R p m	Wt, lb	Cost
100	450	4 400	$2 300
200	450	6 500	3 400
400	400	10 000	5 000
800	400	13 000	7 000

Series motors are usually of the "Mill Motor," totally enclosed type, rated at the output they will give for 1 hr without injury.

Table 17 gives data on a line of slow-speed, 60-cycle, three-phase generators, for 440 to 2 200 volts. For installation and cost of exciter add about 15%.

Three-phase induction motors for 60 cycles are standard. In Table 18 costs and weights are given for a standard line having the good starting characteristics obtained by a definite wound secondary. Squirrel-cage motors are slightly less expensive. Single-phase motors cost 30% more for given output and speed. Installation is simple for most a c motors and its cost can be taken at 4% to 8% of first cost.

Table 18. Weight, Cost and Usual Speed of 60-cycle, Wound Rotor, Induction Motors

H p	Speed, r p m	Wt, lb	Cost
1	1 800	137	$ 95
2	1 800	175	135
5	1 800	280	210
10	1 800	410	300
25	1 200	825	580
50	1 200	1 400	800
100	1 200	2 100	1 150

Table 19. Weight, Cost and Usual Speed of Synchronous Motors for 60 Cycles

H p	Speed, r p m	Wt, lb	Cost
20	450	1 465	$ 750
50	450	1 625	900
100	360	2 200	1 200
200	300	3 000	1 700

Costs of synchronous motors for 60 cycles, three-phase, are given in Table 19. Cost of installation and an exciter must be added, aggregating 10%.

Motor-generator sets, with 2 d-c machines, or 1 a-c and 1 d-c, cost about $23 per kw rated output for a 200-kw set running at 750 r p m; more for smaller sets. Flywheel sets and specially controlled sets cost more because of special features.

Table 20. Cost of Oil-cooled Transformers for 60 Cycles

Kva	Voltage	Wt, lb	Cost
3	2 400	170	$116
5	2 400	225	144
10	2 400	330	205
25	2 400	535	360
50	2 400	1 080	580
100	2 400	1 550	800

Transformers. Weights and costs of oil-cooled transformers for 60 cycles are given in Table 20; 25-cycle transformers cost 20 to 30% more. Cost of installation is very small, as there are no moving parts.

Transmission lines for 3-phase, 33 000 volts, with No 2 B & S wire, wooden poles and porcelain pin insulators, may be erected for $3 600 to $4 000 per mile, if topography of district is not difficult.

Power plants. Costs of building and equipping are: STEAM PLANTS, with reciprocating engines, vary according to size and character of plant from $90 to $140 per rated kw. STEAM-TURBINE PLANTS, from $60 to $150 per kw. HYDROELECTRIC PLANTS, from $100 to $300 per kw, including dam and other equipment. GAS ENGINE PLANTS, with gas producers, from $90 to $125 per kw. In each case the lower figure applies to very large installations.

Cost of operation of power plants varies so widely with local conditions that great detail is necessary for accurate estimates. A value of 0.5¢ per kw-hr (excluding fixed charges) is low, and is attainable only in large plants. 1¢ per kw-hr delivered to bus bars is common where load factor is reasonable, say 0.3 to 0.4, and is a figure commonly used for preliminary estimates. LOAD FACTOR is the ratio of average load for some period (as 24 hr) to maximum load during that time. Duration of PEAK LOAD must be specified (as 1 min or 15 min); it is usually a short period during which the machinery may carry the overload without injury. Total cost of a kw-hr, including fixed charges, is high when the load factor is low and vice versa.

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SECTION 43

ELEMENTS OF STRUCTURAL DESIGN

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Note.—Numbers in parentheses, except numbers of formulas, refer to Bibliography at end of this section.

ELEMENTS OF STRUCTURAL DESIGN

1. GENERAL PRINCIPLES (1)

In structural design, one or all of 3 conditions may govern: safety, economy, and esthetic effect. Safety involves knowledge of the stresses produced by the external forces acting, mechanical properties of materials of construction, and details of design best suited to the purpose.

To compare two designs from standpoint of economy, the elements of cost are reduced to a common basis, using either capitalized or annual cost:

$$P = C + \frac{M}{r} + \frac{E}{(1+r)^m - 1} + \frac{R}{(1+r)^n - 1}$$

where P = capitalized cost, or sum which would build and perpetually maintain the structure; C = first cost; M = annual maintenance and operation; E = cost of extraordinary repairs every mth year; R = cost of renewal of structure in nth year; r = rate of interest. Annual cost = Pr.

Mine structures have a useful life not greater than life of the mine, and a sinking fund should be included = $S \div [(1+r)^x - 1]$, where x = useful life of structure in years. Esthetic principles are seldom considered except in important structures like bridges or architectural work. Following case is also common. An addition to a plant will be required n years hence and will cost A dollars. The expenditure that can be afforded now to avoid this future outlay equals the present worth of A dollars due in n years, or $p = A \div (1+r)^n$. See Tables on these subjects in Sec 45.

MECHANICS OF MATERIALS

2. STRESS, STRAIN, AND ELASTICITY (2, 3)

Strain or deformation is the change in dimension produced on any material by an external force; usually expressed as a ratio, inches per inch. Poisson's ratio of lateral to direct strain is about 0.25–0.3 for metals. STRESS is the resistance offered to deformation. For equilibrium it must equal the external force which it resists. It is expressed in units of force per unit area, lb per sq in or per sq ft. There are 3 kinds of strain and stress: TENSION, COMPRESSION, and SHEAR. Bending and torsion are combinations of these, though sometimes classed separately.

Modulus of elasticity or Young's Modulus = E = unit stress $\div$ unit strain = a constant (within elastic limit); or " stress is proportional to strain " (Hooke's law).

The modulus is generally the same in tension and compression for all materials in common use; a fundamental assumption in the theory of beams and columns. In shear or torsion, modulus = G = about 0.45 E. Hooke's law holds only for "elastic" materials (not plastic), and imperfectly for C I, concrete, and some other materials. Thus, a steel bar 1 in diam by 12 in long, under tensile load of 23 000 lb (= 29 300 lb per sq in), taking E = 30 000 000, will stretch 29 300 $\div$ 30 000 000 = 0.001 in per in, total 0.012 in. If this law held until strain = unity, the length of bar would be doubled; hence E is sometimes defined as force which would stretch a rod of unit cross-sec to double its length.

Elastic limit is the limiting stress, usually in lb per sq in, up to which Hooke's law holds. Most easily determined in testing by "drop of the beam" of the testing machine, which occurs at the YIELD POINT, or commercial elastic limit; higher than true elastic limit (see Fig 1). Yield point is not well defined in some materials; for C I, average is about 0.5 of ultimate strength.

Ultimate strength is the greatest stress, lb per sq in, which a material develops before failure; sometimes greater than BREAKING LOAD. Some materials in compression fail to give a true ultimate, but continue to compress and increase in cross-sec.

Resilience is a measure of DUCTILITY of material, and hence its ability to withstand shock or suddenly applied loads. It is the work required to produce a given stress.

Direct or static stresses are those acting always in one direction. REPEATED STRESSES vary in intensity in one direction, but never pass through zero. ALTERNATING STRESSES pass through zero, as from tension to compression. Rupture may be caused not only by a force exceeding the ultimate resistance of the material, but also by repeated action of forces of these two types, acting between certain limits which are less than the ultimate; known as the FATIGUE.

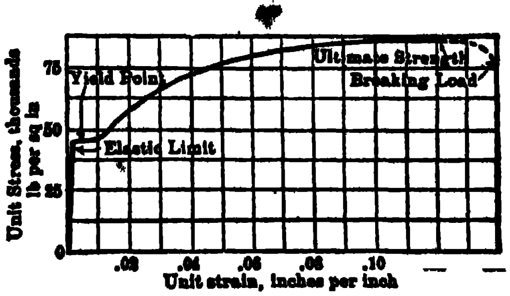


Fig 1. Stress and Strain for High-carbon Steel

Suddenly-applied loads, shock, and impact. A load is considered suddenly applied when its full amount acts instantly upon the material loaded. It produces strains and stresses double those produced by a gradually-applied load. Shock is produced when a load moves freely before acting upon the material. The effect is that of a suddenly-applied load plus the effect of the motion. In practice suddenly-applied loads seldom occur; shock is avoided if possible, but, in structures like RR bridges, an allowance for effect of impact (a combination of sudden application of load and of shock due to irregularities in roadbed, etc) is always made, as in designing hoisting ropes, and links and pins for couplings.

Working stress is the max considered safe by the engineer and used by him in designing. FACTOR OF SAFETY = ult strength divided by working stress; it depends on: (*a*) uniformity, reliability, and durability of material; (*b*) kind of stress and character of loading, and certainty with which amount of load is known; (*c*) character and life of structure. For uniform materials like steel, factor of 4 may be used; for variable materials like wood, 8–10. Factor of safety is greater for alternating than for direct stress; less for temporary structures than for permanent. In many cases (as for steel columns) safety factor should be computed on basis of elastic limit rather than ultimate strength.

3. BENDING MOMENTS AND SHEARS IN BEAMS (2, 3)

Beams: (*a*) simple beams or horiz members, with end supports; (*b*) cantilevers, having but 1 support, or projecting from a support; (*c*) constrained beams, having 2 supports and being rigidly fixed at one or both, or projecting over one or both, so that moments occur at the support; (*d*) continuous beams, with more than 2 points of support. Beams are subject to bending and shear; struts or columns to bending and compression. External bending moments and shears in (*a*) and (*b*) are easily determined as below. The derivation of formula for (*c*) is based on equation of the elastic or deflection curve, while (*d*) involves special treatment, based on Theory of Three Moments. Table 1 gives formulas for (*a*), (*b*), and (*c*); for (*d*) see (2, 3).

End reactions must equal total load, from law $\Sigma V = 0$. Also $\Sigma M = 0$; hence they may be found by taking moments about either support.

Example, Fig 2, M about $A = (2\,000 \times 3) + (4\,000 \times 4) = R_B \times 10$, or $R_B = 2\,200$ lb, and $R_A = 2\,000 + 4\,000 - 2\,200 = 3\,800$ lb.

Bending moment at any section is the algebraic sum of moments of all forces to left of the section, calling forces tending to cause clockwise rotation positive and others negative.

In Fig 2, bending moment at section $xy = (3\,800 \times 7) - (2\,000 \times 4) - (4\,000 \times 3) = +6\,600$ ft lb, and also equals $2\,200 \times 3$, or the moments to the right, if counter-clockwise be called plus.

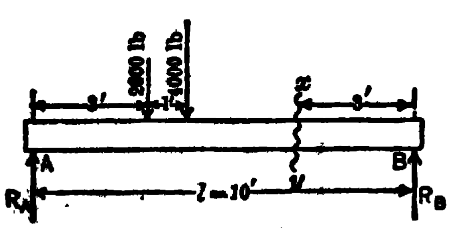
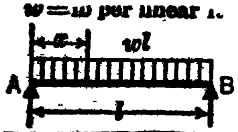
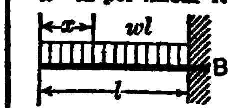
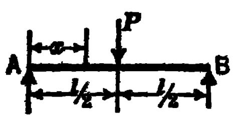
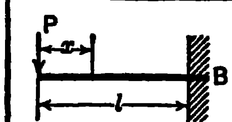
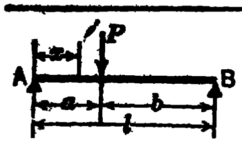
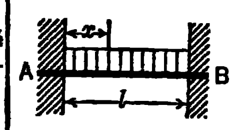
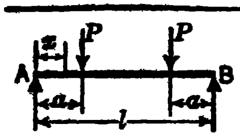
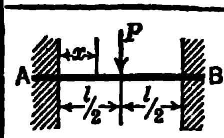
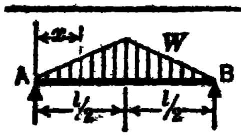
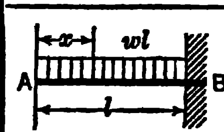
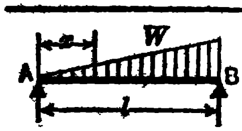
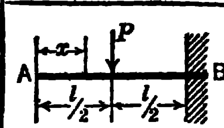


Fig 2. Loaded Beam

Vertical shear. At any section in a beam 2 equal and opposite forces may be acting, one on right and one on left, tending to shear the beam at that point. Vert shear at any section is equal to algebraic sum of forces to the left of the section, upward forces being taken as positive, downward forces, negative. Forces to right of section may also be used, changing the sign.

In Fig 2, vertical shear at $xy = 3\,800 - 2\,000 - 4\,000 = -2\,200$ lb. Fig 3 shows moments and shears at different sections of a beam.

Table 1. Reactions, Bending Moments, and Deflections of Beams

Simple Beams. Reactions, Bending Moments and Max Deflection (For Uniform Cross-sec)				Cantilever and Restrained Beams. Reactions, Bending Moments and Max Deflection (For Uniform Cross-sec)			
Beams and loading	**Reactions**	**Bending moments**	**Max deflection**	**Beams and loading**	**Reactions**	**Bending moments**	**Max deflection**
	$R_A = R_B$ $= 1/2\, wl$	$M = 1/2\, wx(l - x)$ $M_{max} = \frac{wl^2}{8}$	$\frac{5wl^4}{384EI}$		$R_B = wl$	$M = -\frac{wx^2}{2}$ $M_{max} = -\frac{wl^2}{2}$	$\frac{wl^4}{8EI}$
	$R_A = R_B$ $= 1/2\, P$	$x < 1/2\, l$ $M = \frac{P}{2}x$ $M_{max} = 1/4\, Pl$	$\frac{Pl^3}{48EI}$		$R_B = P$	$M = -Px$ $M_{max} = -Pl$	$\frac{Pl^3}{3EI}$
	$R_A = \frac{b}{l}P$ $R_B = \frac{a}{l}P$	$x < a$ $M = \frac{Pb}{l}x$ $M_{max} = \frac{Pba}{l}$	$\frac{Pb(6la - 3a^2)^{3/2}}{81EIl}$		$R_A = R_B$ $= 1/2\, wl$	$M = \frac{w}{12}(6lx - l^2 - 6x^2)$ $M_{max} = -\frac{wl^2}{12}$	$\frac{wl^4}{384EI}$
	$R_A = R_B$ $= P$	$x < a$ $M = Px$ $x > a < (l - a)$ $M = M_{max} = Pa$	$\frac{Pa(3l^2 - 4a^2)}{24EI}$		$R_A = R_B$ $= 1/2\, P$	$x < l \div 2$ $M = \frac{P}{8}(4x - l)$ $M_{max} = \pm\frac{Pl}{8}$	$\frac{Pl^3}{192EI}$
	$R_A = R_B$ $= 1/2\, W$	$x < l \div 2$ $M = Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$ $M_{max} = \frac{Wl}{6}$	$\frac{Wl^3}{60EI}$		$R_A = 3/8\, wl$ $R_B = 5/8\, wl$	$M = \frac{wx}{8}(3l - 4x)$ $M_{max} = -\frac{wl^2}{8}$	$\frac{wl^4}{185EI}$
	$R_A = \frac{W}{3}$ $R_B = \frac{2W}{3}$	$M = \frac{Wx}{3l^2}(l^2 - x^2)$ $M_{max} = \frac{2}{9\sqrt{3}}Wl$	$.01304\frac{Wl^3}{EI}$		$R_A = 5/16\, P$ $R_B = 11/16\, P$	$x < l \div 2$ $M = -5/16\, Px$ $x > l \div 2$ $M = P \div 16\,(8l - 11x)$ $M_{max} = -(3Pl \div 16)$	$\frac{1}{\sqrt{5}}\frac{Pl^3}{48EI}$

Maximum moments and shears determine the cross-sec of a beam. It is generally economical to use same cross-sec throughout, hence only max moments and shears need be computed. Point of max moment is found from principle shown in Fig 3; it occurs where shear passes through zero. Max shear occurs at supports. In designing structures for moving loads, a series of concentrated loads is used, similar in spacing and amount to the cars or moving load to be used. These loads may occupy different positions, and the max moment is determined as in Art 19. Table 1 gives formulas for reactions, max bending moments, moments at any section and max deflection (for uniform cross-section), for common forms of beams and loading.

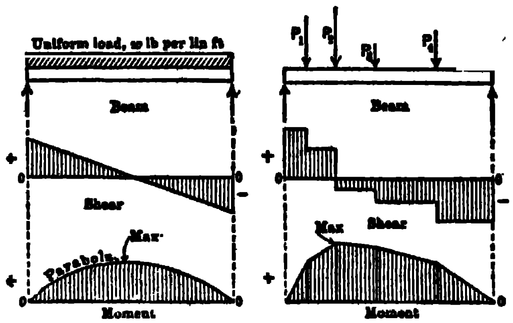


Fig 3. Graphic Representation of Moments and Shears in a Beam

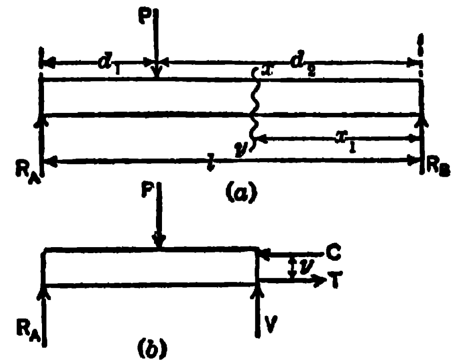


Fig 4. Bending Moment in a Simple Beam

4. FLEXURE OF BEAMS (2, 3)

All beams, when subjected to loading, are stressed in tension, compression and shear. Vert loading produces vert shear, causing equal horiz shearing forces, which give to the horiz fibers of the beam their tension and compression stresses. Investigation of beams is based on the 3 laws of equilibrium (Sec 36, Art 32), the external forces being usually coplanar and at right angles to beam.

Thus, if beam in Fig 4*a* be cut at any section xy, the bending moment at $xy = P\,d_1x_1 \div l$, and shear at $xy = P\,d_1 \div l$ (Art 3). This moment must be resisted by the inner forces of beam acting on the right of section xy, or 2 forces C and T (both equal since ΣH must $= 0$) and acting with lever arm y. Assuming E equal in tension and compression, that Hooke's law holds, and that plane sections of a beam before bending remain plane after bending (Navier's hypothesis), then C and T are resultants of uniformly varying stresses acting over the upper and lower sections of beam, having 0 intensity at a horiz plane passing through the center of gravity of section (known as the NEUTRAL AXIS), and max intensity at the extreme fibers of beam, directly proportional to distance of these fibers from neutral axis. The internal, or RESISTING MOMENT, is then $= M = kI \div d = kS$, where M is in inch-lb, k = extreme fiber stress, lb per sq in, I = moment of inertia of the section, in^4, d = distance from neutral axis to extreme fiber, in, and $S = I \div d$ = SECTION MODULUS.

Example. Assuming a T-section (Fig 5), subjected to external bending moment of 9 000 inch-lb, I is found $= 2.34$, the center of gravity being 2.09 in from bottom, or 0.91 in from top, giving extreme fiber stress in compression, $k = dM \div I = (0.91 \times 9\,000) \div 2.34 = 3\,500$ lb per sq in, while for tension side, $d = 2.09$ and $k = 8\,050$. For a symmetrical section, k is equal in tension and compression. In design, the greatest d should be used. For moments of inertia of various sections see Art 26 and Sec 36, Art 41–45.

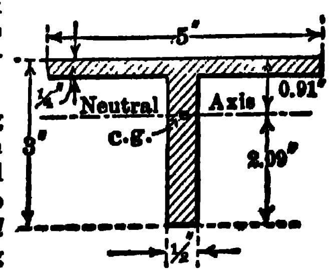


Fig 5. T-Section Beam

Working value of k used in design is found by reducing by a proper factor of safety the k found by testing a beam to failure. Value at failure is the MODULUS OF RUPTURE, and is computed on assumption that above equation holds to failure (actually true only to elastic limit), since M, I and d are known. The work of design is facilitated by tables giving values of S for different beam sections. Instead of S, the COEFF OF STRENGTH is often given. Thus, assuming M in ft-lb, let C = coeff of strength $= 8\,M$ ($= Wl$ for uniform loading or $2\,Wl$ for concentrated, where W = total load, lb, and l = span, ft), then $C = {}^2/_3\,kS$; and, knowing value of C, the safe load is computed, or vice versa. C is in ft-lb.

Shear. In Fig 4*b*, a force V, equal to the vert shear, must be applied at section xy for equilibrium. $V \div$ the area of cross-sec of beam $= v =$ the aver intensity of shearing stress, and for rectangular beams the max intensity $= 3\,v \div 2$ or $3\,V \div 2\,bh$, and occurs

at the neutral axis, where b = breadth and h = depth of beam. Usually, the limiting factors in design are axial tension and compression, it being unnecessary to increase size of member because of shear. In short or deep beams, shear may become the limiting factor, and in short timber beams especially, due to laminated structure, horiz shear often controls. Diagonal stresses are only important in reinforced concrete beams (Art 14).

5. COLUMNS AND STRUTS (2, 3)

Compression members are of 3 classes, depending upon mode of failure, which in turn depends on ratio of length of column to its least RADIUS OF GYRATION (Sec 36, Art 42), or in solid sections to its least diam, expressed as $l \div r$, where l = length, in, and r = least radius of gyration, in (Art 28).

Short columns, in which $l \div r$ is less than 10 or 15, generally fail by compression only, and are sometimes termed short blocks. The safe working stress in compression is used in their design.

Short blocks are tested for determining strength of materials in compression; most concrete and many timber columns fall in this class. They fail by compression or by shearing in a plane making an angle of slightly over 45° with the axis of block. Shear failure occurs when resistance to shear is less than 0.5 resistance to direct stress, as compression induces shearing stress on all oblique planes and max intensity (= 0.5 that of the direct stress) on a 45° plane. Hence length of test specimens should always be stated, and generally will be 3 times breadth; many tests fail to agree, due to use of variable lengths.

Ordinary columns, in which $l \div r$ ranges from 40 to 120, fail by combined compression and bending. There are 2 kinds of formulas used: (*a*) those derived from theory, in which constants are inserted from tests; (*b*) empirical formulas, from tests only. For column design, see Art 14, 21, 28. For eccentric loading, see Art 6. The most widely used of the first kind is Gordon's or Rankine's formula (demonstrated first by Tredgold): $P = cA \div \left(1 + a\frac{l^2}{r^2}\right)$, where P = ult strength, lb; A = area, sq in; l and r as above, and c and a are constants depending upon kind of material, and l and a depending on condition of fixity of ends of column. This formula may be used for design by reducing P by a proper factor of safety.

Theoretically, the strongest column has flat ends, a column with round or pin ends which is free to bend and not restrained at the ends being only 0.25, and a column with one end round and one end free only about 0.5 as strong. Practically, pin-end columns appear to be as strong as flat-end, probably due to difficulty in getting an even distribution of loading and the resulting eccentric stress with the latter. Hence a should be determined by tests of both flat and pin-end columns.

The principal formula of type (*b*) is the STRAIGHT-LINE FORMULA, so called because its graph is a straight line. This line is obtained by plotting, for a series of tests of columns of one type, the ultimate strength and $l \div r$ ratio, and drawing an aver line through these points. The general form is $p = A - B\,(l \div r)$, where $p = P \div A$ = intensity of stress, and A and B come from the tests. This formula is widely used, often displacing the Gordon. As before, p is properly reduced for design. LONG COLUMNS, in which $l \div r$ exceeds say 200, are barred from structural practice and will not be discussed (2).

6. TORSION AND COMBINED STRESS

Torsion, a type of shearing stress, is fundamental in the design of shafts and other members subjected to a twisting moment. For circular section the formula is $d = 1.72 \sqrt[3]{M \div s}$, where d = diam, in, of a shaft capable of resisting a twisting moment M, in-lb, with a max torsive shear in the extreme fibers of s in-lb per sq in.

Example. Assume a working torsive shearing stress of 8 000 lb per sq in, in designing a circular shaft to carry a twisting moment produced by a force of 2 000 lb acting with a lever arm of 48 in. $M = 2\,000 \times 48 = 96\,000$ in-lb, and from the formula $d = 4$ in (about).

Combined stress is the condition of stress occurring when a member is simultaneously subjected to 2 of the 3 elemental forms of stress. A common case, compression and bending, occurs in the eccentric loading of columns and joints, and foundations of masonry structures.

Eccentric loading of columns. Whenever possible in column design the load is applied to coincide with the axis of the column. Through use of brackets, some columns are loaded at a distance e

from the axis, as in Fig 6. This increases compression on left side and decreases it on the right, and must be provided for by increasing the section until the max stress does not exceed the allowable. The unit compression due to direct stress $= p_0 = P \div A$. The unit bending stress caused by the moment $M = Px$ will be $p' = Md \div 2I$, where I = moment of inertia of the section. Max unit stress $= p = p_0 + p'$, and must not exceed the safe value. Minimum stress $= p_0 - p'$. These formulas, though in common use, are only approx, as increase in stress due to deflection is not considered. (See Art 8 for foundations.) Combined stress also occurs in inclined bridge members, as eye bars and chords, which are subjected to bending due to their own wt as well as the direct stress of tension or compression. The common method of procedure is like to that given above. Shafts also frequently act under the combined stress of torsion and bending (as in drum shafts of hoists).

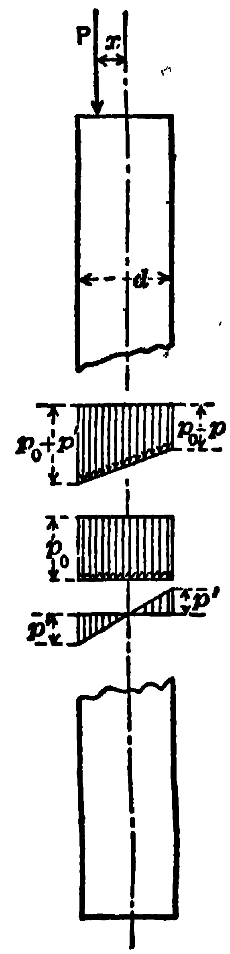


Fig 6. Eccentric Loading of Columns

FOUNDATIONS

7. BEARING POWER OF SOILS (5, 6)

Failure of foundations is generally caused by use of empirical rules, neglect of fundamental principles and poor judgment of designer.

Requisites of foundations: (*a*) They should be leveled off approx at right angles to the direction of loading. This insures that no component of the press will be tangential to the foundation surface, hence no tendency to sliding. Natural inequalities in rock surfaces usually hold the footing in place and make expensive stepping unnecessary. (*b*) When built on earth, design should be such that center of press and center of base coincide. In retaining walls, it is too costly to adhere to this rule; low foundation pressures are required and settlement is common. On rock, little or no settlement can take place. (*c*) Each foundation for separate parts of a structure supported at several points, as walls and interior columns in a building, should be so designed, when on a compressible material, as to be subjected to pressure intensities that will cause uniform settlement.

Safe bearing values of foundation materials depend on character, condition, depth and amount of moisture present, of the foundation material, general sub-surface conditions at the site; also the type of structure and character of loading. Borings, or test pits and soil tests, should be made for important structures. Permanent foundations should be well below the frost level. Some soils become more compact as depth increases and loads may be increased. Also the surcharge effect, or lateral confinement, permits higher loads where foundations are below general level of excavation. Danger of disturbance by adjacent excavations or operations must be considered. A foundation fails when settlement, especially unequal settlement, partially destroys the use of the structure placed upon it.

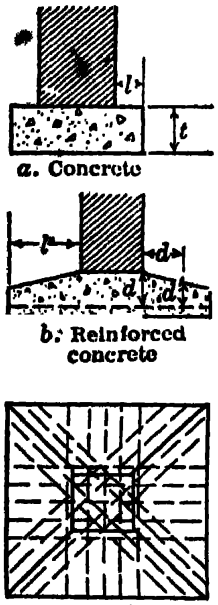
Table 2. Allowable Pressures of Foundation Bearing Materials (Boston Build'g Code)

Class	Material	Allowable bearing value (tons per sq ft)
1.	Massive bedrock, as granitic rock; also gneiss, trap rock, felsite and well cemented conglomerates, in sound condition (sound condition allows some cracks)	100
2.	Laminated, such as slate, schist, in sound condition (some cracks allowed)	35
3.	Shale in sound condition (some cracks allowed)	10
4.	Residual deposits of shattered or broken bed rock of any kind except shale	10
5.	Hardpan	10
6.	Gravel and sand-gravel mixtures, compact	5
7.	Gravel and sand-gravel mixtures, loose; sand, coarse, compact	4
8.	Sand, coarse, loose; sand, fine, compact	3
9.	Sand, fine, loose	1
10.	Hard clay	6
11.	Medium clay	4
12.	Soft clay	1
13.	Silt, shattered shale, or any deposit of unusual character not provided for herein	Value determined by tests

The wide variation in bearing values in tons per sq ft of various materials is illustrated by Table 2 (Boston Bl'd'g Code), which is a guide in design but must be used with judgment and caution. In recent years the science of soil mechanics has been rapidly developed, and design, although still largely a matter of experience, is based more and more on scientific data and knowledge of the physical characters of soils.

8. SPREAD FOUNDATIONS (5)

The commonest foundation for walls and buildings is a spread footing, which distributes load over a larger area at the base to reduce the press per sq ft to that considered safe from a study of site and soil. TOTAL LOAD comprises dead plus live load. The DEAD LOAD, or wt of structure, is computed as in Art 17. The LIVE LOAD is often uncertain in amount, and for buildings on compressible soil, the footings of interior columns should be designed with allowance for the fact that usually only a small part of total live load acts at one time. Otherwise unequal settlement will occur, as the footings carry chiefly fixed dead load.



a. Concrete

b. Reinforced concrete

c. Column footing of type *b* (plan)

Fig 7. Spread Footings

Depth of foundation below lowest level of structure will depend on: (*a*) necessity of going below frost line, to prevent heaving due to freezing; (*b*) depth required to reach suitable foundation material. Frame buildings are quite flexible, loads usually light, and foundation wall is seldom carried deeper than 1.5–2 ft, or much below cellar level. Masonry buildings require deeper and heavier foundations to prevent excessive settlement and cracking. In larger structures design is more involved.

For moderate loads, wall and column footings of concrete may be used, the projection of the footing beyond face of wall or pier not being large enough to demand special reinforcement. In such cases the thickness of the footing t should be proportioned to the offset l (Fig 7*a*) and should generally be in the ratio of about 2 : 1. Where wider spread is required, modern practice uses a reinforced footing or grillage of steel I-beams encased in concrete. For column footings the spread is in 4 directions and reinforcement must be in 2 or more layers with bars as in Fig 7*c*.

The footing in Fig 7*b* may be designed as in Art 14. Experiments indicate that the shear should not exceed the allowable at a distance d from face of wall; hence d is usually greater than required for bending, being fixed by d' which depends on shear. When the required offset l must be large to keep unit press low, large masses of masonry and deep excavations are avoided by using a grillage consisting of 2 or more layers of timber or steel beams (17), or a reinforced footing with rods in both directions. For more complicated designs, see Bib 5.

9. PILE FOUNDATIONS (5)

Pile foundations are intermediate in type between the spread footing and costly caisson foundations carried to great depths. They are used: (*a*) where a soft, compressible soil of poor bearing capac overlies a firm layer of good quality. The pile then becomes, in effect, a column, partly supported by the surrounding material, but transmitting its load largely by point bearing to the good material below; (*b*) where the foundation material is of gradually increasing capac and the pile carries its load both by point bearing and by skin friction; (*c*) in loose or silty soils of low bearing capac, which can be compacted by pile driving to permit increased bearing loads and where skin friction is most important. In case (*a*) the total load carried will be the safe load per pile, x, and the no of piles, provided the point reaches rock or a very resistant soil. In general, clusters of piles act with a "group effect" and may carry far less than this value.

Design of pile footings is not a simple matter and no one rule is always applicable. For piles in sandy material (*b*, *c* above) some indication of safe load P, lb, may be obtained from the empirical equation, $P = 12\ Whe \div (s + c)F$, where W is wt of the driving hammer, lb; h, the fall or drop, ft (or equivalent ht for a steam hammer); e an effic allowance, 75% for drop and about 90% for steam hammers; s, aver penetration of the pile under the last 10 blows; c, a constant, 1.0 for drop and 0.1 for steam hammers; F, factor of safety 3–6. Soft materials may develop some "structure" (interlocking of particles) and excessive pile driving may destroy this natural quality and cause lower bearing capac. Minimum spacing of piles is about 3 diams. Overdriving wood piles causes splitting.

Wood piles are used for both temporary and permanent foundations, and when below water level will last indefinitely. In temporary work, as trestles, the pile serves the purpose of both foundation and column. Oak, pine and spruce are in common use, depending upon difficulty in driving, length required, and purpose, oak being good for hard driving or where pile is subjected to abrasion. For very hard soil, piles are pointed with iron shoes and the butt is chamfered off and banded with iron 0.5–1 in thick and 2–3 in wide. Special driving caps, with a tapered recess, are also effective. Wood piles should generally be at least 6 in diam at point (preferably 8 in) and about 14 in at butt (not over 18 in).

Concrete piles are cast-and-driven or cast-in-place. They are for permanent construction where excavation to water level, required for wood piles, would be too costly. Recently piles of type (a) have been sunk by driving steel pipe with hammer, with or without use of water jet, blowing out interior with compressed air, and filling with concrete. Such piles are essentially small caissons; used to sizes 2 or 3 ft or more in diam.

Capping of piles to provide a foundation footing is usually done by excavating 12 in below tops of piles and encasing with concrete carried to 12 in or more in thickness over top of piles.

MASONRY AND CONCRETE STRUCTURES

10. LIME, CEMENT AND MORTAR (4)

Common lime is calcined limestone, which, when fresh, should be in lumps with little dust. It should be stored in a dry place. When slaked with water, lime paste may be kept indefinitely under water (protected from air). If exposed to air in a thin layer lime may be slaked to a dry powder, sold as HYDRATED LIME.

Cement is of 3 kinds, Natural, Portland, and Pozzuolana. NATURAL CEMENT is calcined cement rock, containing proper proportions of lime and clay, ground to a fine powder. It is not now used for structures requiring great strength, having been superseded by Portland cement, which costs but little more and is stronger. It was early made at Rosendale, N Y, hence sometimes called Rosendale cement. PORTLAND CEMENT is made by grinding to a fine powder the clinker from calcining to incipient fusion a powdered mixture of properly proportioned lime and clay materials. Limestone mixed with clay may be used, also cement rock and clay or limestone, marl and clay, limestone and slag.

Production of Portland cement in U S has increased rapidly, to over 100 million bbl annually. Cement is measured in bbl and usually sold in paper bags or in bulk, the aver price at the mills being \$2–\$3 per bbl. Wt of Portland cement averages 100 lb per cu ft. A bbl weighs 376 lb; a bag, 94 lb. A bbl contains about 3.8 cu ft. Natural cement weighs 265–300 lb per bbl and 90–100 per bag. POZZUOLANA CEMENT proper is made by grinding hydrated lime and pozzuolana, a volcanic material occurring near Naples, and used by the ancient Romans. So-called SLAG CEMENT is of this class, although slag is also used for a true Portland cement which differs in no essential from other Portland.

Testing cement. Portland cement has now become, in name, if not fully in fact, a standardised product and for all except largest works is usually bought without specific testing. It is well to specify A S T M standards (Am Soc for Testing Materials), and it is common to require that specimens of concrete be tested both as a check on the cement and as a measure of the quality of the aggregate and mixing.

Cement grout is a mixture of 1 vol of cement with 1, 2 or more vols of sand. It is used in a semi-liquid form for grouting column bases, bearing plates and wherever space requires a fluid mix.

Cement mortar, usually in proportion of 1 vol cement to 2–3 sand, is used for masonry. Pure cement mortar is "short" and difficult to handle with a trowel. Addition of 5–10% hydrated lime to the cement makes the mortar "slick" and easier to work. A cu yd of 1 : 2 mortar requires about 2.6 bbl cement and 0.9 cu yd sand; a 1 : 3 mortar, about 1.9 and 1.0, for aver sands.

11. STONE MASONRY.

Cut-stone masonry, formerly widely used, has, due to rising costs of labor, been superseded in practically all work except where architectural effect justifies cost. But when labor is cheap and cement costly, rubble masonry of unsquared or roughly squared stone, can sometimes be used to advantage. In such work stones should be laid in irregular courses in natural bedding plane, with joints overlapped so as to interlock; with frequent headers extending into wall, to tie in or bond stretchers (see brickwork for terms) and with all voids filled with small stones and mortar. The hammering or sliding of stones on work under construction should be prohibited and porous stone should be wetted before use. Aver quantity of mortar per cu yd of rubble varies with shape and amount

of dressing of stones, from 0.2 to 0.4 cu yd. Rubble masonry will carry 10–15 ton per sq ft; best quality cut-stone masonry may safely carry 30 or more.

12. BRICK AND BRICKWORK

COMMON BRICK is used for ordinary construction. Best brick, when broken, shows a fine, uniform texture, deep salmon color, contains no fissures, air bubbles, pebbles nor lumps of lime. It should also have plane faces and parallel edges (no warping), and give a clear ringing sound when struck (thorough burning). Usual size is 8.25 × 4 × 2.25 in. Overburned brick is dark in color and liable to be warped and brittle. Light or yellow color indicates soft or underburned brick. PRESSED BRICK is brick molded under heavy press. It is uniform in shape and of greater density, absorbing only 10–20% of its wt of water, while common brick may absorb 30–35%. It is used for exterior faces of walls, or for neat finish. ENAMELED BRICK is made from clay containing a large amount of fire clay, on which a surface finish of enamel is placed before or after burning. GLAZED BRICK is produced by applying a slip to the unburned brick, which adheres to clay and holds a glaze composed of readily fusible materials giving a vitreous surface. FIREBRICK is of fireclay burned at a high temp. Good fireclay is free from pyrites, lime, and other ingredients which would cause swelling or vitrification. Fireclay brick should be of uniform color, white or white speckled with brown throughout; should be laid in thin paste of fireclay, with joints as thin as possible. SAND-LIME BRICK is generally pure white. Made by molding 4 to 6% of finely powdered lime with silica sand. CEMENT BRICK is of sand, with Portland cement as the binding material.

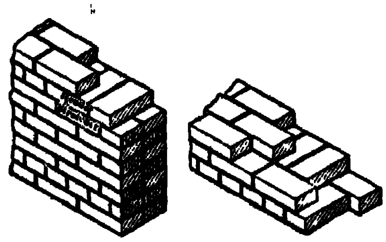


Fig 8. Bonds for Brickwork

Bond results from laying brick so as to tie the wall together longitudinally and transversely. Fig 8 shows the common bond, consisting of 4 to 7 courses of STRETCHERS to 1 course of HEADERS, the latter tying the wall together transversely. In laying brick, vert joints in header courses must be of such thickness that they will not come over a joint in the stretcher course. Brick should be thoroughly wet before laying; joints $^1/_4$–$^3/_8$ in thick, and for best work brick should be placed on a full mortar bed and shoved into place ("shove joints"). All vert joints should be thoroughly "slushed" with semi-liquid mortar, so the brick will be well bedded. Face of the wall may be finished with either flush, struck or weather joints.

Number of brick required depends on size. For common masonry 1 cu ft generally requires 20 brick; 1 cu yd, 500 to 600 brick.

Quantity of mortar. For aver joints ($^1/_4$–$^3/_8$-in) 1 cu yd common brickwork requires $^1/_4$–$^1/_3$ cu yd mortar, or about 0.5 cu yd per 1 000 brick.

Working stresses for compression are: for common brick in Portland cement mortar, 175 lb per sq in. Brick piers may be designed by the formula, $p = P - 6H + D$, where p = allowable unit stress, lb per sq in; P = allowable direct compressive stress, lb per sq in; H = height, in; D = least thickness of pier, in.

Common brick are sold by the thousand (M); aver price, $12–$20 per M. One bricklayer will lay 1 200–1 600 brick per 8 hr, in straight thick walls, but only 250–500 for work involving many corners or angles.

13. CONCRETE (7)

Composition. Concrete is a mixture of broken stone, gravel, cinders, or slag, called the COARSE AGGREGATE, and sand or stone screenings, known as the FINE AGGREGATE, with a cementing material, as cement or asphalt. Portland cement is almost invariably used in general construction and for reinforced concrete.

Proportions of ingredients are usually expressed by vol as, 1 : 2 : 4 (a relatively "rich" mix), or 1 : 3 : 6 (a lean mix), indicating parts by vol of cement, fine and coarse aggregates.

Strength of Portland cement concrete (hereafter referred to as concrete) depends upon quality of the 3 basic ingredients; also (*a*) quantity of cement in proportion to total aggregates, (*b*) ratio between coarse and fine aggregates, (*c*) amount of water used in mixing (cement-water ratio), (*d*) adequacy of mixing, (*e*) method of placing, (*f*) method of curing (setting). All other factors being the same, the more cement used the stronger the mix. Ratio of sand to stone by vol is usually about 1 : 2. Great caution should be exercised in using "run of bank" gravel, on assumption that 1 part cement to 6–9 parts total aggregate will be satisfactory. Sand, present usually far in excess of coarse material, is defined as material passing $^1/_4$-in screen and bank gravel requires that part of sand be

screened out; otherwise, the mortar binding the coarse aggregate may be as weak as 1 : 4 or more and concrete is of poor quality. For high density, strength and impermeability, the minimum of water required for setting should be used. As such a mix can not be successfully placed, amount of water is determined by "workability" and will vary with difficulty of placing. Excess water leaves "water voids" in mix, tends to produce "segregation" (settling of coarse aggregates), and excess water flushing to surface washes out cement and leads to later surface disintegration or "map cracking." Workability is measured by slump test. Fill conical shell 4 in diam at top, 8 at bottom and 12 in high, with proposed mix rodded in place and "struck off" at top. When shell is lifted vertically drop of mass, in = slump. For mass work or highway construction water to give slump of 1 to 3 in is used. For building and similar construction requiring placing around reinforcing bars, 3 to 6 in. This usually requires 5 to 8 gal water per cu yd of mix. Varies with character of aggregate.

Hand mixing is now seldom used, mixing machines being available in sizes from a few cu ft to 10 yd or more. Mixing should start with dry materials, cement added to sand, then coarse aggregate and, finally, water.

Care must be exercised in placing, so as to fill forms and avoid pockets or "honeycombing" (areas of coarse aggregate without enough mortar to fill voids). Spading face with tamping bars, hand tamping and vibrating mechanical or elec tampers are used. Avoid excessive tamping, as it causes segregation, as for excess water noted above. Concrete may be placed under water by means of metal pipes, the concrete being fed in at the upper end as fast as it goes out at the lower end, thus preventing it from falling through the water, which would wash out the cement. In joining old and new surfaces, care should be taken to remove all dirt and laitance (scum which forms on surface) and thoroughly to wet the surface of the old work. In long walls, expansion joints should be provided every 30 to 50 ft. Fig 9 shows common forms of joints. Concrete is not injuriously affected by oils and acids, unless there is enough seriously to affect other materials. Salt or alkali water causes disintegration at the water line, due to unknown action, and sand containing alkali should not be used.

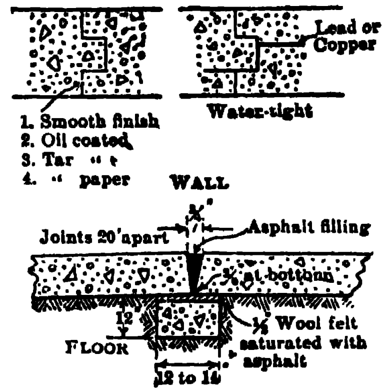


Fig 9. Concrete Floor Joints

Proper "curing" has the object of providing adequate moisture until concrete has set and is vitally important. Fresh concrete should be protected from direct rays of sun by covering with burlap or straw kept constantly damp. An asphalt emulsion coating has recently been used for highway work. In cold weather concrete must be kept from freezing until set, and protected from alternate freezing and thawing until fully hardened.

Coarse aggregate may consist of gravel or broken stone, its quality being at least equal to that of hardened cement mortar, preferably graded in size (although much " single-size " stone is used), clean (broken stone or washed gravel best), durable, and contain no soft, flat, elongated or friable particles. Max size depends on dimensions of construction. For massive work " cobble concrete " or a max size stone of 3 in or more can be used. For thinner walls 1.5 in, for coping and thin walls 1 in, while for reinforced work 0.75-in is usually required. For cinder concrete, hard " steam " cinders should be used, free from soot or excessive fine particles.

Specifications. For small work so-called standard mixes are used as follows: RICH MIXTURE, 1 : 1.5 : 3; for columns or other parts subjected to high stresses or requiring to be completely watertight. STANDARD MIXTURE, 1 : 2 : 4; for reinforced floors, beams and columns, arches, reinforced engine or machine foundations subject to vibrations, tanks, sewers, conduits or other watertight work. MEDIUM MIXTURE, 1 : 3.5 : 5; for ordinary machine foundations, retaining walls, abutments, piers, thin foundation walls, building walls, ordinary floors, sidewalks and sewers with heavy walls. LEAN MIXTURE, 1 : 3 : 6; for unimportant work in masses, for heavy walls, for large foundations supporting a stationary load, and for backing for stone masonry. It is always desirable on important work to experiment and test various mixtures to secure best yield and satisfactory quality. Tendency is to specify required strength for various parts of work rather than set proportions.

Strength of stone concrete in compression, when tested in cylinders (usually 8 in diam by 16 in high) is 1 000 to 4 000 lb per sq in. Strength increases with age, the allowable stresses in design being given in ratios of the ultimate compressive strength at 28 days ($= f'_c$, Art 14). The tensile strength of concrete is even more markedly affected than its compressive strength by factors noted above, hence is usually not counted on in design. For a 1 : 2 : 4 mix, it varies from 200 to 300 lb per sq in; for 1 : 3 : 6, from 100 to 175. Shearing strength is difficult to determine, but it is probably about 0.5 the compressive strength. Modulus of elasticity is also variable, the material not usually showing true

elastic behavior; for working loads, it is 2 500 000–3 500 000 lb per sq in, depending on mixture and age.

Quantities of materials required for 1 cu yd concrete with various aggregates depends so much on character of aggregates, voids, etc., that it is best determined by trial. As a rough approximation for aver broken stone concrete, divide 11 by sum of parts = bbl cement per cu yd. Multiply this by the number of parts of sand and by the cu ft in 1 bbl cement (3.8 aver) and divide by 27. This equals the cu yd of sand required. Similarly for the broken stone.

Example. 1 : 3 : 5 cement concrete. 11 + 9 = 1.22 bbl cement per cu yd. 1.22 × 3.8 × 3 + 27 = 0.51 cu yd sand and 1.22 × 3.8 × 5 + 27 = 0.84 cu yd broken stone.

Rates of work and costs. One man will mix, wheel 50 ft, level and ram, about 1.5 cu yd of concrete per 8-hr day. A gang of 15 men will handle as above an aver of about 20–25 cu yd, and 1 man will level and ram in 6-in layers about 8 cu yd per 8-hr day. Cost varies from $15 to $30 or more per yard, depending on labor and material costs, as well as the kind of work.

Forms for concrete are made of partly seasoned pine or spruce, planed on one side (to give smooth surface and even thickness) and for best work tongued and grooved. The lagging is generally 1 in thick, but for heavy work and where the forms are to be used several times 2 in is better. Studding is commonly of 2 by 4 in to 4 by 6 in, spaced 1.5 to 2 ft centers for 1-in lagging, and 4 to 5 ft for 2-in. Pressure due to concrete is about that of a liquid weighing 80 to 140 lb per cu ft, depending upon the amount of water used in mixing and rate of placing. Since forms cost 0.25 to 0.5 of the entire cost of the work, a concrete structure should be designed with a view to simplicity in the form work, a little extra concrete often being economical. Forms should be carefully designed and constructed, so as to be easily removed without injury to the structure or the forms themselves; this is done by making them in units, joined by cleats, the pressure from the concrete being taken up by transverse tie wires, which are left in the concrete, or by external braces and struts.

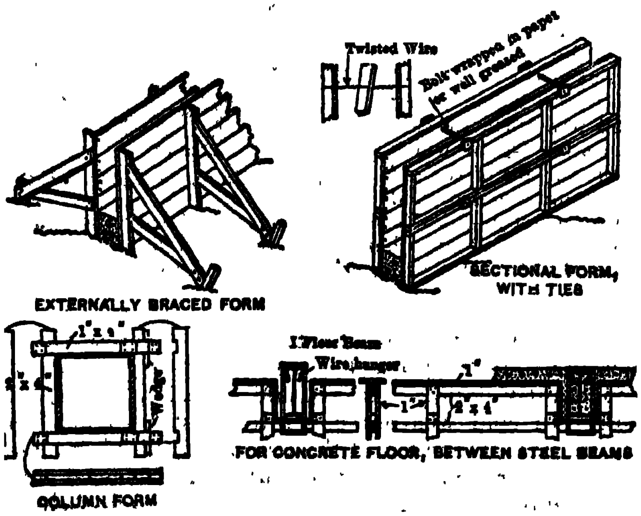


Fig 10. Forms for Concrete

Adhesion of concrete to the forms can be prevented by applying crude oil, soap or grease. Fig 10 shows typical forms. The time which must elapse before forms can safely be removed depends on weather conditions and how soon the structure will receive its load. Time under ordinary conditions: walls in mass work, 1 to 3 days or until concrete will bear press of thumb without indenting; thin walls and columns, summer 2 days, cold weather 5 days, provided they carry no load; slabs, up to 7 ft span, beams and girders, summer 6 days, cold weather 2 weeks; long-span slabs and girders, summer 10 days, cold weather 3 weeks to 1 month; arches, small 1 week, large 1 month.

14. REINFORCED CONCRETE (7, 8)

Concrete, reinforced by steel bars, is one of the most useful and important structural materials, and, for permanent structures, has to some extent replaced older materials. Its economy lies in its greater durability, requiring less maintenance, fire-resistance, and the ease with which it can be molded into shapes. Structures can be built of it economically wherever Portland cement and suitable aggregate are available, and for the most part by unskilled labor. To get the desired results, details of design should be simple, completely worked out and shown on the plans, and construction supervised and inspected by competent engineers. Failures can usually be traced to neglect of these principles.

Reinforcing bars are usually 0.25–1.5 in. The 0.75-in bar is known as BASE, there being no extra charge for this size or larger, but smaller bars cost more. Round bars are used more than squares, especially in sizes less than 1 in. There are 7 standard round

bars and 4 squares (Table 3). Except under unusual circumstances, designs should be limited to these bars. DEFORMED BARS are used to provide a bond between steel and concrete, independent of adhesion. They are obtainable in the 11 standard sizes in Table 3. Wire mesh and various forms of expanded metal are much used in slabs, walls and other thin sections.

Table 3. Data on Standard Bars

Size, in	Shape	Area, sq in	Wt, lb per ft	Perimeter, in
1/4	Round	0.049	0.167	0.785
3/8	Round	0.110	0.376	1.178
1/2	Round	0.196	0.668	1.571
1/2	Square	0.250	0.850	2.000
5/8	Round	0.307	1.043	1.963
3/4	Round	0.442	1.502	2.356
7/8	Round	0.601	2.044	2.749
1	Round	0.785	2.670	3.152
1	Square	1.000	3.400	4.000
1 1/8	Square	1.266	4.303	4.500
1 1/4	Square	1.562	5.313	5.000

Beams. Working formulas, based on Navier's hypothesis and other assumptions listed below, are for design purposes only; that is, within normal working conditions. Since the limit of extensibility of concrete is only about 0.0002 in per in, tension in the concrete is neglected, the steel being supplied to take all the tension. The first time the beam is loaded to its full design load, fine cracks will develop on the tension side of the beam and the above assumption will be very nearly satisfied. In addition, these formulas are predicated on a perfect bond between steel and concrete, and a constant modulus of elasticity in the concrete. The span of a reinforced concrete beam is taken as the clear span plus the depth of the beam, but need not exceed the c-c distance of supports.

Standard notation. For convenience the complete notation is given herewith (lb, in, lb per sq in).

Rectangular Beams and Slabs

- f_s unit fiber stress in steel
- f_c unit fiber stress at its extreme fibers
- f'_c ultimate compression strength at 28 days, when cast in standard 6 by 12-in cyl
- E_s modulus of elasticity of steel
- E_c modulus of elasticity of concrete
- A_s area of cross-sec of steel
- e_s unit strain of steel due to f_s
- e_c unit strain of concrete due to f_c
- n ratio of E_s to E_c
- T total tension in cross-sec of steel
- C total compression in cross-sec of concrete
- M external moment acting on beam due to loads
- M_s resisting moment of steel in tension
- M_c resisting moment of concrete in compression
- b width of beam
- d effective depth of beam = distance from compression face to center of gravity of the steel
- k ratio of depth of neutral axis to effective depth
- j ratio of arm of resisting couple to effective depth
- p ratio of A_s to bd
- r ratio of f_s to f_c
- e thickness of concrete from outside of bottom row of steel to tension face (protective covering)
- i diam of bar
- s side of sq bar

T-Beams

- b width of flange
- b' width of stem
- t thickness of flange

Shear and Bond

- V total external shear
- V_c portion of total shear carried by concrete
- V' portion of total shear carried by web reinforcement
- v, v_c, v' unit shear stresses (see above)
- A_v total area of stirrup, two legs
- f_v unit tensile stress in stirrup
- u bond stress, per unit of surface area of bar
- Σo sum of perimeters of horiz bars

Rectangular beam and slab formulas (Fig 11). From Navier's hypothesis and Hooke's Law (Art 2):

$$r = \frac{f_s}{f_c} = n\,\frac{(1-k)}{k} \tag{1}$$

and the resisting moment of the beam based on working strength of steel is:

$$M_s = Tjd = A_s f_s jd = pf_s jbd^2 = C_s bd^2 \tag{2}$$

where

$$C_s = pf_s j$$

also for the concrete

$$M_c = Cjd = b\cdot\frac{kd}{2}\cdot f_c jd = C_c bd^2 \tag{3}$$

where

$$C_c = \tfrac{1}{2}\,jkf_c$$

If the resisting moment from Eq (2) is greater than that from Eq (3), the beam, if tested to failure, will fail by crushing the concrete and is said to be OVER-REINFORCED. If the reverse condition is true, the beam fails by reaching the yield point of the steel,

and the beam is UNDER-REINFORCED. If a beam has equal strength in each, it is a BALANCED BEAM. For latter with n and r known, from Eq (1) k is fixed and

$$k = \sqrt{np(np+2)} - np \qquad (4)$$

Hence with k fixed, required p is fixed and C_s equals C_c as required. M_s is then equal to M_c and both must equal the external moment, M. Hence, from Eq (2) or (3),

$$d = \sqrt{\frac{M}{Cb}} \qquad (5)$$

where $C = C_s = C_c$

The value of d (Eq 5) is the balanced depth, as it is the only depth that will simultaneously insure full working values of both steel and concrete with n and k fixed. Thus solving Eq (1) for k,

$$k = \frac{n}{n+r} \qquad (6)$$

and Eq (4) may be solved for p and combined with Eq (6):

$$p = \frac{k^2}{2n(1-k)} = \frac{n}{2r(n+r)} \qquad (7)$$

also

$$j = 1 - \frac{k}{3} \qquad (8)$$

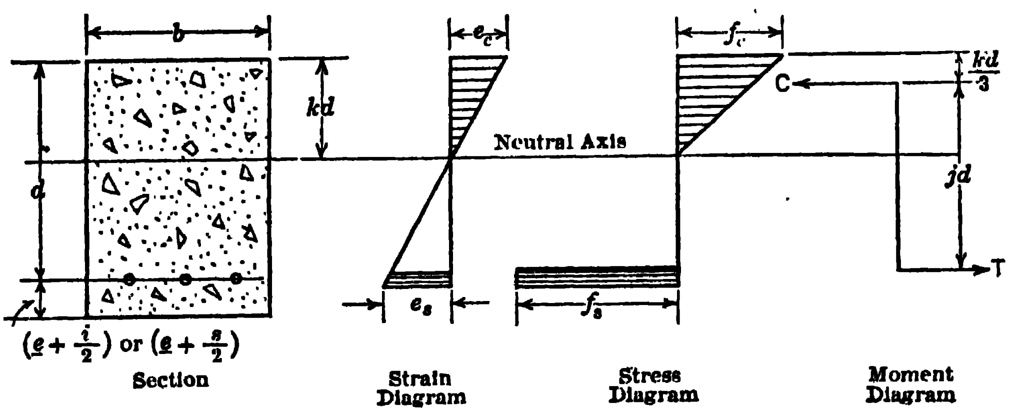


Fig 11. Stresses in Concrete Beams

Thus, with n and r fixed, k is obtained from Eq (6), p from Eq (7), j from Eq (8), and C from Eq (2) or (3). Table 4 gives values of these constants for common values of f_s and f_c, and n. For economy, b should be small, since for equal strength, a deep narrow beam has a smaller area than a wide shallow one. For proper design, this can not be done, as b should be between $0.5d$ and $0.75d$. In fulfilling this specification, most designers proportion all beams for widths of even integral inches, so that a plank of standard width may be used for the bottom form. Using proper proportions for b and d, a balanced depth is readily designed, and closely approximates the most economical. Often the difference from the theoretically most economical design is more than counterbalanced by the saving in form costs.

Table 4. Constants for Balanced Beams

f_s = 18 000 lb per sq in						f_s = 20 000 lb per sq in					
k = 0.400		j = 0.867		S = 1 300 ft-lb C		k = 0.375		j = 0.875		S = 1 458 ft-lb C	
f'_c	f_c	n	p	ft-lb	in-lb	f'_c	f_c	n	p	ft-lb	in-lb
2 000	800	15	0.0089	11.57	138.8	2 000	800	15	0.0075	10.94	131.3
2 500	1 000	12	.0111	14.43	173.2	2 500	1 000	12	.0094	13.70	164.5
3 000	1 200	10	.0133	17.29	207.5	3 000	1 200	10	.0113	16.48	196.8
3 750	1 500	8	.0167	21.71	260.5	3 750	1 500	8	.0141	20.56	246.8

In determining A_s, two formulas are used. By definition

$$A_s = pbd \qquad (9)$$

also from Eq (2)

$$A_s = \frac{M}{f_s jd} = \frac{M}{Sd} \qquad (10)$$

Eq (9) applies only for balanced depth, whereas Eq (10) gives results close enough for other depths, and is used instead of Eq (9). Table 4 gives values of S ($= f_s j$) in ft-lb, which may be used with Eq (10), when M is in ft-lb.

Three general problems may arise: (*a*) to design a balanced beam; (*b*) to design a beam whose depth is fixed by clearance requirements; (*c*) to find the safe load on an existing beam.

Example 1. Using $f'_c = 3\,000$ (working stress $f_c = 1\,200$) and $f_s = 18\,000$; design a balanced beam for a load of 1 000 lb per lin ft on a span of 10 ft (load includes wt of beam itself). Ordinarily a dead wt must be assumed for the beam, and a tentative design made which is modified if the resulting wt does not check the assumed wt. For this loading $M = wl^2 + 8 = 12\,500$ ft-lb. From Table 4, $C = 17.29$ ft-lb units and by Eq (5),

$$d = \sqrt{\frac{12\,500}{17.29b}}$$

A value of b is now assumed, which will give satisfactory proportions. By trial is obtained a 6-in width and a depth of 11 in. From Table 4, p is .0133, and from Eq (9), A_s is 0.88 sq in, or from Eq (10) and using $S = 1\,300$ from Table 4,

$$A_s = \frac{12\,500}{1\,300 \times 11} = 0.88 \text{ sq in}$$

Table 3 shows that several combinations of bars may be used to give this area. Thus two 0.75-in round bars (area = 0.884) or three 0.625-in rounds (0.921) are suitable.

In choosing the number and size of bars, observe the following points. A minimum number is desirable (2 for this beam), and if 2 are used the area is less than for 3, giving a saving in cost; and there is no "size extra" for the 0.75-in size, while there is for the $^5/_8$. But if web reinforcement is necessary (see below), more than 2 bars are advisable. Also, 3 bars will have a greater surface area than 2 bars for the same cross sec area, which will lessen the unit bond stress. Finally, there must be a certain minimum clear distance between bars (1.5 diameters for rounds and 2 × side for squares), and there must be a certain thickness of concrete protection over the steel. Concrete protection for reinforcement shall be at least 1 diam for rounds and 1.5 × side for squares. For surfaces not directly exposed to ground or weather, the protective covering should be 0.75 in for slabs and walls; and 1.5-in for beams, girders and columns. Table 5 gives the least width of beams for different numbers of bars of diam i, or side s.

In Example 1, if 2 bars are used, a beam $5.5 \times 0.75 = 4.12$ in is necessary; or for 3 bars, $8 \times {^5/_8} = 5$ in. Hence from this standpoint either combination is satisfactory. Fig 12 shows detail of beam using 2 bars. Since d is the effective depth to the center of steel, to get the total depth an amount e must be added for protection. Hence $e = 1 \times 0.75 = 0.75$ in; but this is less than the 1.5 in specified as minimum for beams. Hence total depth $= (11.0 + 1.5 + 0.5) \times 0.75$, or 13 in to the nearest 0.5 in.

Table 5. Spacing of Reinforcing Bars

No of rows of bars	Least width, b	
	Square, s	Round, i
1	4	3
2	7	5.5
3	10	8
4	13	10.5
5	16	13
6	19	15.5
7	22	18
8	25	20.5
9	28	23
10	31	25 5

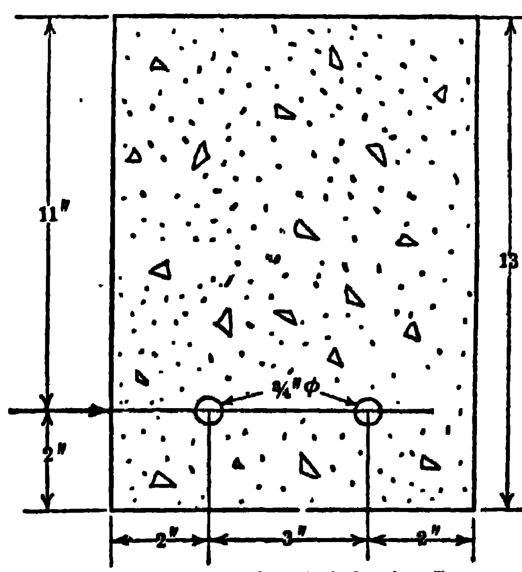


Fig 12. Beam with 2 Reinforcing Bars

Example 2. Suppose on account of clearance, the total depth is limited to 11.5 in, and the effective depth is 9.5 in, or less than the balanced depth of *Example 1.* The concrete will control and be stressed to full working strength, while the steel will be understressed. To keep f_c below 1 200, the beam must be overreinforced. Thus solving Eq (3) and (8) with $d = 9.5$ in; $k = .570$. From Eq (7), $p = .0378$ (against .0133 for balanced depth), and $A_s = .0378 \times 6 \times 9.5 = 2.16$ sq in. Also f_c will be 1 200 and f_s [from Eq (1)] will be only 9 050. A double-reinforced (both tension and compression bars) beam may be more economical in this design.

Example 3. Suppose a total depth of 14.5 in ($d = 12.5$ in) had been used. There will be less steel, stressed to full value, while the concrete will be understressed. The beam will be underreinforced. Using $S = 1\,300$, as for a balanced beam, Eq (10) gives a sufficiently close approx for

A_s, or 0.770 sq in. Hence $p = .0103$ and k, from Eq (4) is .363. From Eq (8), j is .879 and from Eq (10), $f_s = 17\,730$ p s c. From Eq (1), $f_c = 1\,010$ p s i.

Example 4. Slabs may be similarly designed. Required a reinforced concrete floor slab of balanced depth to carry 500 lb per sq ft on a 10-ft span. Assume $f_c = 800$, $f_s = 20\,000$ *psi*; M is 6 250 ft lb, for a strip 12 in wide as a simple beam. From Table 4, C is 10.94, and since b is 12 in, from Eq (5), d is 7 in. Also from Eq (9), A_s is 0.612 sq in per ft width of slab. Necessary spacing in inches of bars of any size to obtain this area = 12 times area of 1 bar divided by required area (A_s) per ft of width. Thus $^5/_8$-in rounds, 6 in c-c, or 0.75-rounds 8 in c-c, could be used. Bar spacings are given in even inches if possible. Minimum protection is 1 diam or 0.75 in. Thus for $^5/_8$-bars, total slab depth would be $7 + .75 + {}^5/_8$ or say 8-in and 8.5-in for the 0.75-in rounds. Most economical slab, $^5/_8$-in bars with 8 in depth.

Example 5. In using these formulas to compute SAFE LOAD for an existing beam, first find p from Eq (9), then k from Eq (4) and M_s and M_c from Eq (2) and (3). Equate the smaller of these to the external moment, and solve for the load. Find the safe load for a rectangular beam simply supported on a 10-ft span. Beam is 8 in wide, and 12 in deep to center of steel, and has 0.96 sq in steel. Take $f_c = 1\,000$ and $f_s = 20\,000$ lb per sq in, hence from Eq (9) p is .01; k from Eq (4) is .383; j from Eq (8) is .872; M_s is 16 750 ft lb and M_c 16 000 ft lb. This beam is over reinforced and would fail by crushing the concrete. It could carry 16 750 ft lb moment, and the steel stress would not exceed 20 000 lb per sq in, but the concrete stress would exceed 1 000. Hence M must not exceed 16 000 ft lb, at which time $f_s = 19\,100$ from Eq (2). Also $M = wl^2 \div 8 = 16\,000$ and $w = 1\,280$ lb per lin ft, including wt of the beam, or about 1 180 lb of live load.

Diagonal tension, shear and bond. Besides failing by crushing of the concrete or yielding of the steel, the beam may fail by diagonal tension or by bond failure. Horiz and vert shear, modified by the fiber stress due to bending, produce max tensile stresses having direction shown by curved lines in Fig 13; hence, best position of reinforcing bars would be curved to follow these lines.

If horiz bars only are used, since concrete has little tensile strength, a beam may fail due to inclined or diagonal tensile stress at ends, causing cracks A, A (Fig 13). As it is impracticable to compute diagonal tension, which is dependent on the shear, shear is used as a measure of the diagonal tension in experiment and design. To prevent failure in this way 3 methods are used:

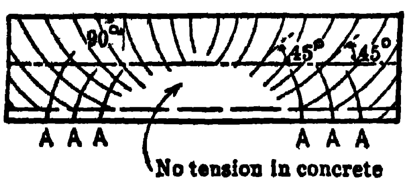


Fig 13. Diagonal Tensile Stress

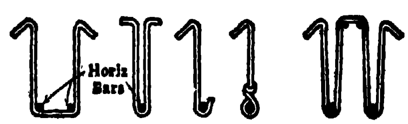


Fig 14. Common Forms of Stirrups

1. Shear is kept below safe value. The max intensity of shear is $v = V \div bjd$ (11)

With no web reinforcement, this should not exceed v_c, which is limited by specification to $v_c = 0.02f'_c$. In slabs where it is impractical to place web reinforcement, depth d is increased over that required for bending, if necessary, to comply with the above condition.

2. Bent-up longit bars, with inclination of 15° or more to axis of reinforcement. If this is done, v may not exceed $.06\,f'_c$; that is v' cannot exceed $.04\,f'_c$ and v_c equal to $.02\,f'_c$. Compute the distance from support to where v equals v_c and beyond which no web reinforcement is necessary. Bend up one or more bars (preferably symmetrically) at angle θ. Compute aver total vert shear V' over a length of beam l', in which a portion of the shear is to be taken by bent-up bars. Stress in bar is

$$f_v = \frac{V'l'}{A_v jd(\sin\theta + \cos\theta)} \tag{12}$$

which should not exceed safe stress for the bar. Suitable adjustment of distance l' will give a solution which meets all conditions. Note that bars can not be bent up until they are no longer required in tension reinforcement for bending. The distance from support x, where bars of area t may be bent up is given by

$$x = l\left(1 - \sqrt{\frac{t}{m}}\right) \tag{13}$$

where l is span and m is total area of bars. This formula is derived on basis of a uniformly distributed load and can only be used for such. Also the bond stress in the remaining horiz bars should not exceed a safe value of $.04\,f_c$ for plain bars, or $.05\,f'_c$ for deformed bars. This stress u will be greatest at support and equals

$$u = \frac{V}{\Sigma_o jd} \tag{15}$$

Bars must extend into the support at least 10 diam and if the above bond stress values are exceeded, special anchorages (hooks) must be provided.

3. Stirrups are used (Fig 14). If these are computed to take the excess v', over what the concrete takes, v_c, the shear stress v should not exceed $.06\,f'_c$, unless the longit steel is specially anchored and in no case should v exceed $.12\,f'_c$.

Procedure for stirrup design (Fig 15): (1) find length to be reinforced, $X = \frac{L}{2}(1 - v_c \div v)$; (2) total area of stirrups, $A_v = \frac{v'bX}{2f_v}$ (required in length X); (3) approx number of stirrups, $N = \frac{3X}{d}$; (4) area of 1 leg of 1 stirrup, $a = 1/2\,\frac{A_v}{N}$; (5) select nearest size to give area a,

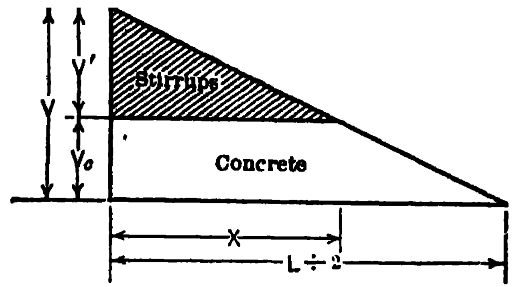


Fig 15.* Shear Diagram

(1/4", 3/8", or 1/2"); (6) adjust number of stirrups for size selected, $N = 1/2\,\frac{A_v}{a}$; (7) space according to Table 6, without exceeding .50d. Adjust spacing to nearest 1/2 in and add stirrups where allowable spacing is exceeded.

Table 6. Coefficients for Spacing Stirrups

Total number of stirrups

2	3	4	5	6	7	8	9	10	11	12	n
.13	.09	.07	.05	.04	.04	.03	.03	.03	.02	.02	1
.53	.29	.21	.16	.13	.11	.10	.09	.08	.07	.07	2
	.62	.39	.29	.23	.20	.17	.15	.13	.12	.11	3
		.67	.45	.36	.29	.25	.22	.19	.18	.16	4
			.70	.50	.40	.34	.29	.26	.23	.21	5
				.73	.54	.44	.38	.33	.29	.27	6
					.75	.57	.47	.41	.36	.32	7
						.76	.59	.50	.44	.39	8
							.78	.61	.52	.46	9
								.79	.63	.55	10
									.80	.65	11
										.81	12

n = number of the stirrup from the support.
To find distance from support to any stirrup, multiply the factor from table by X, in.

T-beams. The area of concrete below neutral axis in a rectangular beam is of no value against bending. If part of it is removed the resulting T-beam (Fig 16) will be stronger, as it is relieved of some dead load. Hence, as long as t exceeds kd, the beam is essentially rectangular, and the formulas for rectangular beams are used. If t is less than kd, use Eq 16–20. As shown in Fig 16 for rectangular beam, Eq 1, 6, 10 hold equally well for T-beams. The compression carried by the small portion of the stem above neutral axis is usually neglected. Practically

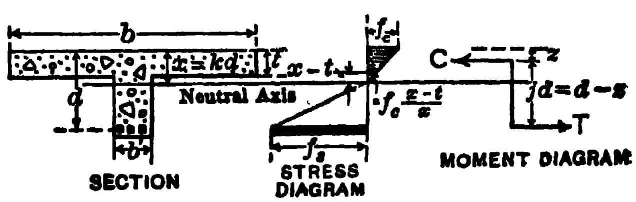


Fig 16. Design of T-beam

$$f = \left(\frac{3kd - 2t}{2kd - t}\right)\frac{t}{3} \qquad (16)$$

and can never exceed $t \div 2$; hence, approx,

$$M_s = f_s A_s\left[d - \frac{t}{2}\right] \qquad (17)$$

Actually

$$M_s = f_s A_s jd \qquad (9)$$

where

$$j = d - f$$

Also

$$M_c = \frac{(2kd - t)btf_c jd}{2kd} \qquad (18)$$

In designing T-beams, a balanced depth is seldom used, but the proper proportions of stem are found from other factors, as area required for shear and headroom. In general, proper proportions for stiffness require that b' shall be between $^1/_3$ and $^1/_2\, d$. Also, where the stem forms part of a floor, the slab of thickness t being the flange, for economy the relation of b' and d should be such that

$$d = \sqrt{rM + f_s b'} + t + 2 \tag{20}$$

where r = ratio of unit cost of steel to concrete, or 50–70. The value of t, where beam forms part of a floor slab, is fixed, and b should not be taken greater than $16t + b'$, the distance c–c of T-beams, or over 0.25 the span. For independent T-beams, b should not exceed $4b'$, and t should not be less than $0.5d$. Three problems may arise.

Example 1. T-beam is part of a floor system; t = 4.5 in. Design stem for moment of 92 700 ft lb. Max reaction = 16 800 lb; span 22 ft. Use f'_c = 2 000 lb per sq in; f_s = 20 000. Max allowable shear = $.06 f'_c$ = 120 lb per sq in. Min area of stem $b'd$ for shear = 16 800 ÷ $^7/_8$ × 120, Eq (1) = 160 sq in, or say 9 by 19, as b' would be between $0.3d$ and $0.5d$ for proper proportions. But from Eq (20) when b' = 9, it is economical to make d = 23.0 in. From Eq (17), A_s = 2.68 sq in, which can be made up by several different combinations, as six .95-in rounds, placed in 2 rows of 3 each. Since d = depth to center of gravity of steel, the total depth is determined as in Fig 17, where e = 1.5 in. Checked by the exact formula, $b = (16t + b')$ = 81 in, or b = 0.25 × 22 × 12 = 66 in. Use 66 in for b, as it is the smaller. From Eq (19), as kd = 4.70 in, this is a true T-beam. From Eq (16), t = 1.56 in; from Eq (10); f = 19 600, lb per sq in, and from Eq (1), f_c = 334, which is satisfactory. Stirrups must be provided.

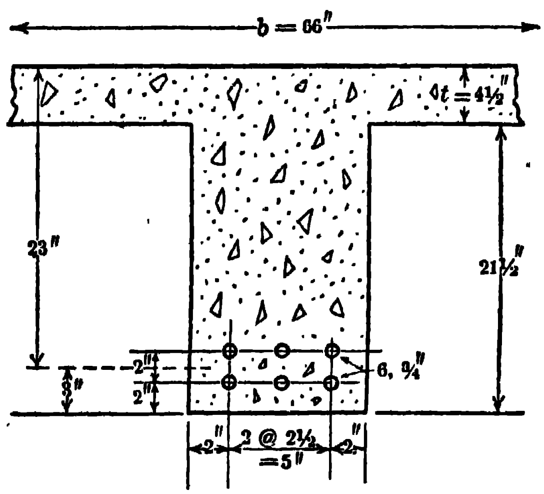


Fig 17. Design of T-beam

Example 2. Design an independent T-beam for 40-ft span, to carry 3 000 lb live load, and assumed dead load of 1 400 lb per lin ft. $M = wl^2 \div 8$ = 10 560 000 in-lb. Max V = 88 000 lb; hence min area for shear is 838 sq in, using f_s and f'_c as in Example 1. Selecting b' = 18, and d = 47 in as good proportions, this may be designed as a balanced T-beam. Thus from Eq (6), k = .375 and kd is 17.6 in. Selecting t as 16 in, since it must be at least $^1/_3 d$, this is a true T-beam. From Eq (16), f = 5.78 in and jd = 41.22 in. From Eq (9), A_s = 12.80 sq in, and solving Eq (18), b = 36.7 in, which is less than $3b'$ as required and gives a good design. To get the steel area required, 9 1 $^1/_8$-in and 1 1.25-in square bars could be used. To place this steel properly in 2 rows, the width of beam might be 20 in, instead of 18 in. End bearing required is at least

$$\frac{88\,000 \times 2}{18 \times 450} = 21.8 \text{ in}$$

This assumes an allowable compression of 450 lb per sq in, and increases the load by 100% to allow for unequal distribution over bearing surface.

Example 3. Given a T-beam forming part of floor; t, b', d and A_s being known, find safe load; b from data above, and kd from Eq 19. If kd is less than t, use rectangular beam formulas; if greater, it is a true T-beam and z is found from Eq 16, giving jd. M_s is computed from Eq 10, and M_c from Eq 18, and the smaller used. Also, test strength in shear and bond.

Reinforced concrete columns. General formulas apply to "short columns," where unsupported length does not exceed 10 × least lateral dimension. Principal columns in buildings shall have a min diam or thickness of 10 in and min gross area of 120 sq in. Noncontinuous posts between stories must have min diam of 6 in. Unsupported length of columns is taken as: (1) clear distance between floor slabs (or to bottom of capital in flat slabs); (2) clear distance between floor and underside of deeper beams framing into column at next higher floor level; (3) clear distance between lateral struts providing adequate lateral support.

Spirally reinforced columns (Fig 18). Permissible axial load P, on columns with closely spaced spirals enclosing a circular concrete core, reinforced with longitudinal bars; $P = A_g(0.22f'_c + f_s p_g)$, where A_g = gross area of column (including protective concrete covering, f'_c = compressive test strength of concrete; f_s = nominal working stress in vert column reinforcement = 40% of yield point (16 000 for intermediate grade, or 20 000 lb per sq in for rail or hard grade steel, max not to exceed 30 000), p_g = ratio of effective cross-sec area of vert reinforcement to gross area, A_g.

Limitations for vertical reinforcement. p_g is not less than 0.01 or more than 0.08. Min number of bars = 6, min diam = $^5/_8$-in round. Also, c-c spacing of longit bars within periphery of core is not

less than 2.5*d* for round bars, or 3 × side for sq bars, or a clear spacing not less than 1.5 in, or 1.5 × max size of aggregate used in the concrete mix. Protective covering, poured monolithically with core, not less than 1.5 in or 1.5 × max size aggregate (not less than special requirements for fire protection or weathering).

Tied columns. Permissible load, when reinforced with longit bars and separate lateral ties, shall be 70% of that given for spirally reinforced columns: p_g not less than .01, or more than .04; longit reinforcement not less than 4 bars, min dia = $^5/_8$ in. Lateral ties at least 0.25 in diam, spaced not over 16 bar diameters, 48 tie diameters, or least dimension of column. Each bar must be held with lateral support equal to that provided by a 90° corner tie.

For composite columns (encased structural steel or C-I and pipe columns and long columns) see Am Concrete Inst Building Code.

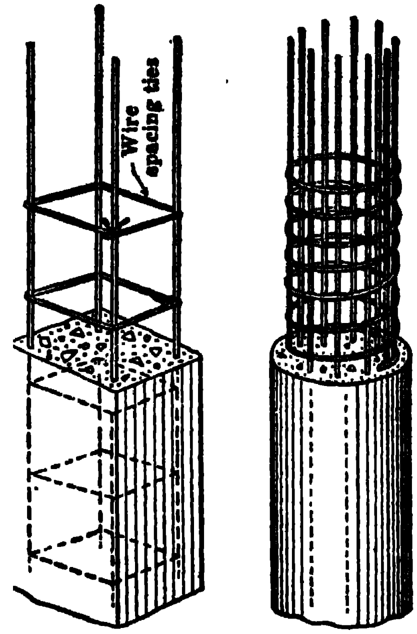


Fig 18. Reinforced-concrete Columns

15. RETAINING WALLS (7)

General. Retaining walls are of masonry, sometimes laid dry for unimportant walls, concrete or rubble concrete, or reinforced concrete. Principal forces acting are lateral earth thrust and wt of the wall itself. In gravity walls these are the only forces to be considered, but in reinforced walls the stability of the various parts must be investigated.

Gravity walls, when small, are built without special computation and of more or less standard section, that is, a top width of 1.5–2 ft, bottom width 0.4–0.6 of total height, front face battered 1 : 12. Foundations are carried below frost line (3–5 ft) and back sloped or preferably stepped as required by these dimensions. Drainage of back fill is most important in reducing press, and is cheaper than to use a heavier wall. Selected back fill of sand, cinders or gravel that drains readily is desirable where natural fill is of bad quality. The design of larger walls involves checking stability against failure by tilting forward due to excessive settlement, or sliding forward on base (Fig 19). Considering 1 linear ft of wall, the earth thrust E may be combined with wt of wall W, producing resultant R, and if R passes outside toe of wall A the wall will overturn. R may be resolved into a vertical component V, which produces a varying base press, greater at the toe of the wall A than at the back of the base (hence usual unequal settlement), and horiz component H, which tends to produce sliding.

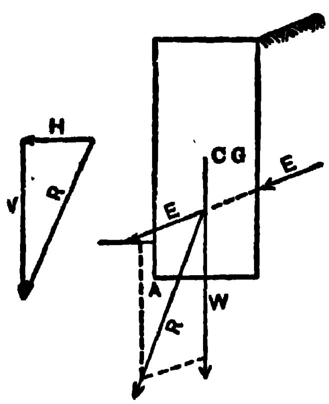
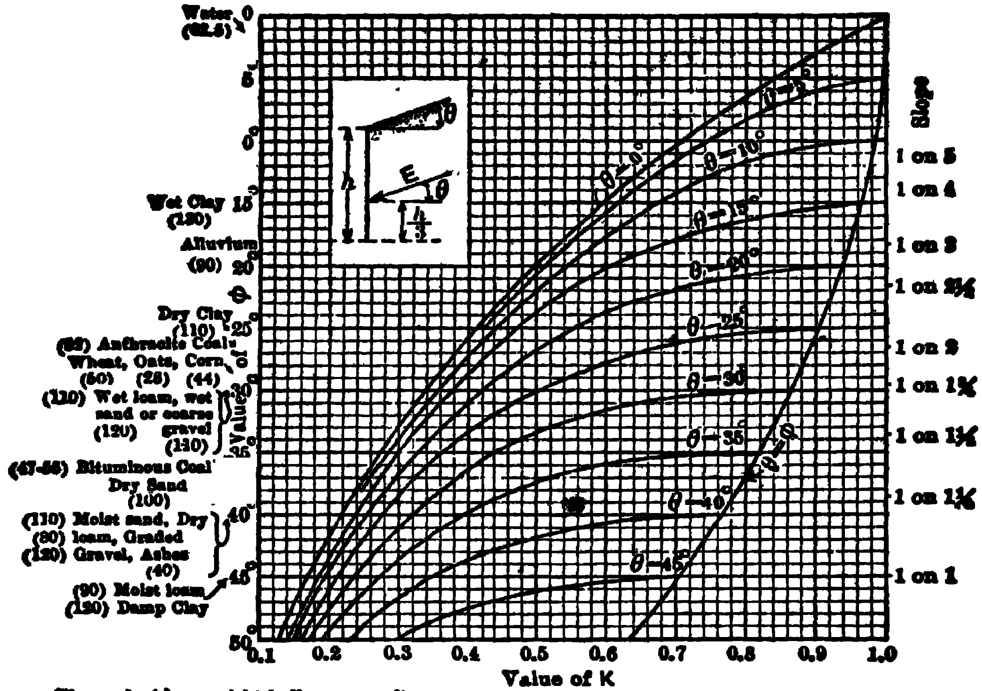


Fig 19. Gravity Retaining Wall

Earth pressure. Modern studies and observations indicate: (1) E (representing about a min value of the equivalent hydrostatic press) = $Kwh^2 \div 2$, where w is wt of earth, lb per cu ft, h is total height in ft, and K is a coeff usually 0.3–0.6; (2) K depends not only upon character of the earth backing, amount of moisture present and time after backing has been placed, but also on settlement of wall which causes its tilting forward; (3) Rankin's formula $K = \cos\theta \dfrac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}$ gives theoretical value of K for a vert wall with fill sloping upward from top at an angle θ, the fill having a coeff of friction represented by angle ϕ. For dry, granular material and horiz fill ($\theta = 0$), as in design of ore bins, this value can be used. Fig 20 gives aver values of K for a number of materials, but must be used with caution and judgment. For important earth retaining walls certain possible modifications must be considered, namely, that theoretically E acts at $^1/_3$ height above base; actually, depending on amount of wall yield, it may act as high as 0.5 and in design it is best to assume 0.4; theoretically, also, E acts parallel to slope of back fill, thus having, for surcharged walls, a vert component (= E sin θ) which aids stability; actually E may act horiz, depending on presence or lack of friction on back of wall and on its tilting; (4) factors noted in (3) may increase effect of E to values twice those indi-



Figures in ( ) = weight in lbs. per cu. ft

Fig 20. Pressure Factor K for Granular Materials

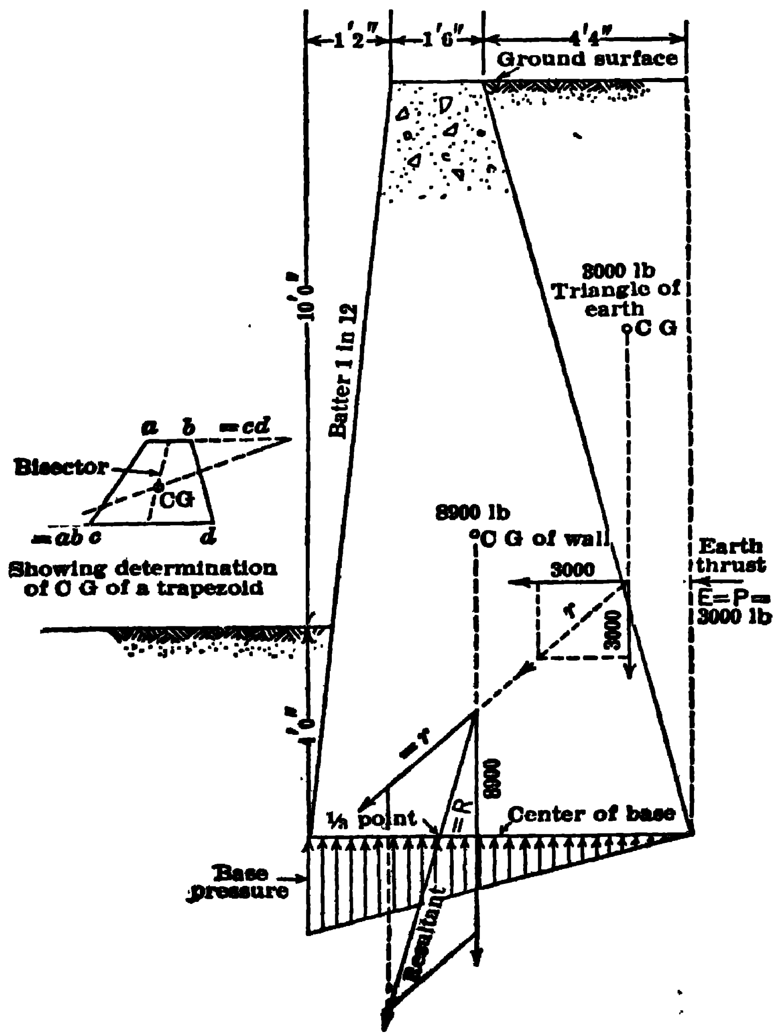


Fig 21. Graphical Design of Retaining Wall

cated by theory, hence conservative design requires ample margin of safety; (5) most failures occur (*a*) directly after heavy rain, due to building up of hydrostatic press back of wall, so that provision for carrying off surface water from back fill and for draining fill by tile drains is most important, or (*b*) by gradual tilting forward of wall due to unequal settlement of base. This is liable to occur on soft, wet foundations and emphasizes importance of drainage.

Example. Check design of a wall for a clear height of 10 ft with horiz fill ($\theta = 0$). Foundation, 4 ft deep. Assume w for fill, 100; wt of wall, 150 lb per cu ft; material, well drained gravel, $\phi = 33°$; suitable top width, 18 in; trial width of base, 0.5 total height, or 7 ft. From Fig 20, $K = 0.3$, E against a vert (Fig 21) = 3 000 lb per ft of wall. Combining this graphically with wt of triangle of earth back of wall, 3 000 lb, gives resultant press on wall, r. This combined in turn with wt of wall, 8 900 lb, gives resultant R which cuts base slightly inside the third point Assuming a uniformly varying distribution of base press, this would vary about 3 400 lb per sq ft at the front (toe) to approx 0 at the rear (heel) of the base. This design is to be regarded as a minimum; E being taken as acting at 1/3 point of height (see under 3 above). For poorly drained soil K would exceed 0.3, so that a wider base would be required. Computed base press is moderate, but, while V would not change (11 900 lb), the distribution of this press depends on moment of E, and thus the max value would be affected by changes in b and E.

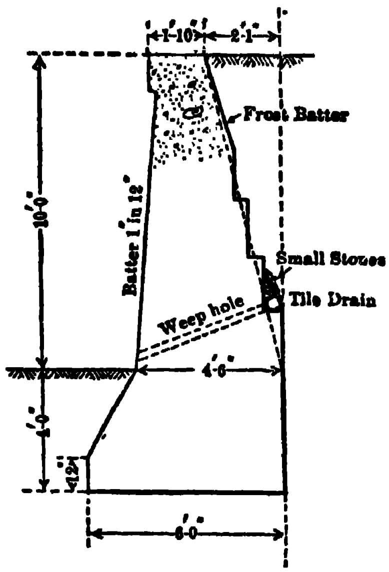


Fig 22. Stepped Retaining Wall

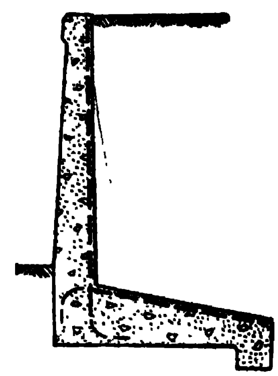


Fig 23. L-type Cantilever Retaining Wall

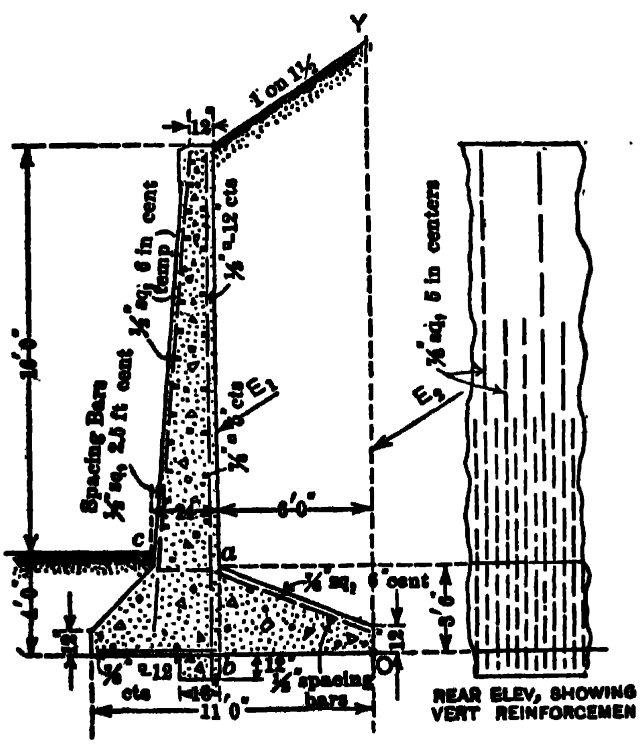


Fig 24. T-type Cantilever Retailing Wall

In the final design, the back of the wall would preferably be stepped, the sloping line in Fig 21 being an aver section. This insures vert action on wall of triangle of earth (3 000 lb), which can act only on the sloping back if friction (uncertain) exists between wall and fill. The upper 3 ft are often left with slope forming a frost batter to relieve possible thrust from freezing (note also drainage). Expansion joints must be provided at intervals of 30–60 ft (Art 13). The variation from the trapezoidal form in Fig 22 results in a slight saving of material and smaller base press.

Reinforced-concrete retaining walls are of 3 types: cantilever, counterfort and cellular. Forms for the counterfort wall cost more than for cantilever, but the counterfort is more economical for heights over 20 ft, while the cantilever is cheaper than the gravity wall, except for very low walls. The cellular type has been used where foundations are poor, and piles can not be used, and where wall must be built close to property line.

Cantilever wall of L-type (Fig 23) is required when the front face is on the property line, and the foundation can not extend beyond it. The base press in such walls is liable to be high, requiring the use of piles. Whenever possible, the T-type (Fig 24) is used, as it not only requires a smaller base and is therefore economical, but it gives smaller base press. The vert stem should be placed at about the third point of the base.

Design of a cantilever wall (Fig 24). (*a*) Estimate is made of thickness of footing *ab*, which determines approx height of the vert wall. Top width is assumed as 12 in, and base *ac* is found by Eq 5 (Art 14), considering the wall as a vertical cantilever slab subject to the earth thrust E_1, and taking moments about *a*. If necessary, this thickness is increased to keep shear below 40 lb per sq in, as web reinforcement is not desirable. This also applies to *ab*. (*b*) Dimensions of base are assumed, earth thrust E_2 on plane *OY* is computed and combined with wt of earth and wall to find position of resultant on base. Front toe is then designed to keep resultant inside third and p_{max} as desired. (*c*) Rear footing. Take moments about *a*. Downward load is wt of earth in front of *OY*, and the vert component of E_2, which is taken as being uniformly distributed over the footing. Foundation press acts upward. Design *ab* for bending and shear as in (*a*), taking steel required from Eq 10, Art 14. (*d*) Design front footing as in (*c*). Only force considered is the upward foundation press. (*e*) Redesign vertical wall, using correct value of *ab* from (*c*). All steel need not go to top. For A_s use Eq 10, as in (*c*).

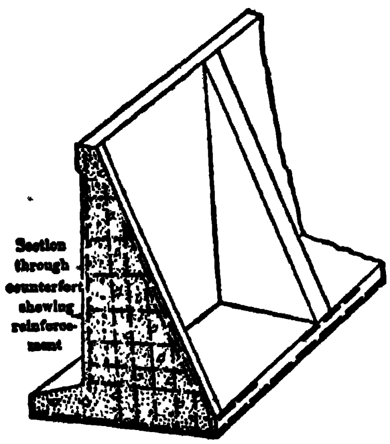


Fig 25. Counterfort Retaining Wall

Bottom projection is put on base to prevent sliding; also gives sufficient grip (Eq 15, Art 14) for vert bars below *a*. Spacing bars are used to preserve the required spacing of main reinforcing (by wiring bars to them), and to bind entire structure together; usually 0.5-in bars, 18 in to 3 ft centers. Temperature reinforcement, consisting of small longit bars (0.5-in here) having a total cross-sectional area of 0.3 to 0.6 of the area of exposed wall, and placed near exposed surface, localizes contraction cracks at frequent intervals (not visible) and no expansion joints are necessary. As in the gravity wall problem above, this section must be regarded as a minimum; *E* may be greater, act higher and also be essentially horiz, thus eliminating the vert component and requiring modifications in design.

Counterfort walls are as in Fig 25. The vert wall is designed as a continuous slab, loaded with the earth thrust and supported by the counterforts. Footing is also designed as a slab, loaded with the downward wt of the earth, and subject to the upward reaction of the soil and spanning between counterforts. The counterfort is subjected to a varying load on 2 faces, enough horiz and vert rods being usually put in to take the entire slab and footing reactions.

16. DAMS AND CULVERTS (7, 9)

A dam comprises 4 parts: the dam proper, spillway, outlet and gates. Sometimes the entire dam is a spillway and in some diversion works it is made up of a series of gates or outlets placed between piers. Dams are built of earth, timber, masonry, concrete and reinforced concrete. The design of dams can not be fully treated here; it requires extensive knowledge and experience, and is under state supervision in several states (9). Earth dams are built either with or without a masonry core. Their design, while formerly based largely on empirical rules and still depending on judgment and experience, has recently been improved through increased knowledge of soil physics and mechanics. The basic problem is the design of an earth fill of a form and on a foundation which will not settle excessively or shear under vert wt, even when much of fill is saturated, and will not permit dangerous seepage. The former requires coarse, stable material; the latter, the presence of fine, well compacted material to fill voids. If suitable materials are available for a mix of sand and gravel for stability, with enough fines and clay for low permeability, no core wall is used. Such dams are constructed: (*a*) by placing the fill in layers, with compacting by wetting and rolling with sheep's-foot rollers (study of soil characteristics and control of water used determines amount of rolling required); (*b*) by full or semi-hydraulic methods. Full hydraulic method is probably the cheapest, but is difficult to control and can be used only where suitable material is available at a sufficient height above the dam. Best material is a mixture of sand and gravel with 25–35% clay. This is sluiced to the dam site with hydraulic giants (Sec 10) and delivered to the dam at the up and down-stream faces from a trestle, either erected in the dam and built in as work progresses, or supported on scows floating in the pond of water which is constantly maintained over the entire section of the dam. The heavy material in the water is deposited at the edges, which are being built up by stones raked from the deposit, and by brush and stop planks. The finer material is carried in suspension in the water, finally settling in the central part of the structure and forming a dense embankment.

Essentials of design (9) are: (*a*) suitable foundation and bond between dam and foundation; if foundation is earth, its quality both to support dam and as to permeability must be satisfactory and all loose soil, boulders and stumps should be removed and the exposed surface roughened by plowing to give bond with fill; if rock, low concrete cut-off or core wall extending into fill may be needed to prevent seepage; (*b*) no earth fill can be

fully impervious to water, but line of saturation through the dam must be below its downstream face; (*c*) moderate slopes for embankment are thus essential; slopes may be from 1 on 2 to 1 on 4 or more; (*d*) elev of crest above highest possible water level must be sufficient to preclude possibility of overflowing. On the water side the slope is usually protected from wave action by dry rubble paving or riprap. The downstream side may be sodded or seeded to prevent wash. Top width is usually 5–30 ft, or about 0.2 the height +5 ft. Elev of crest above spillway is found by adding to the max depth of flood over the spillway the probable height for waves, and to this an allowance of 2–5 ft for frost. This insures that the material exposed to water action will be below frost action. Allowance for waves is: for a "fetch," or exposed water surface, of 0.5 mile, 2.5 ft; 1 mile, 3 ft; 2 miles, 3.5 ft. When suitable material is not available a CORE-WALL is necessary. A puddle core-wall is made by filling a trench left in center of dam with clay placed in 4 to 6-in layers, made plastic with water, and carefully worked by spading and ramming so as to make it homogeneous (Fig 26). Core-walls must go down to an impervious foundation and be securely bonded into it, and should reach to highest water level; they are placed near center or on upstream side of the center of the dam.

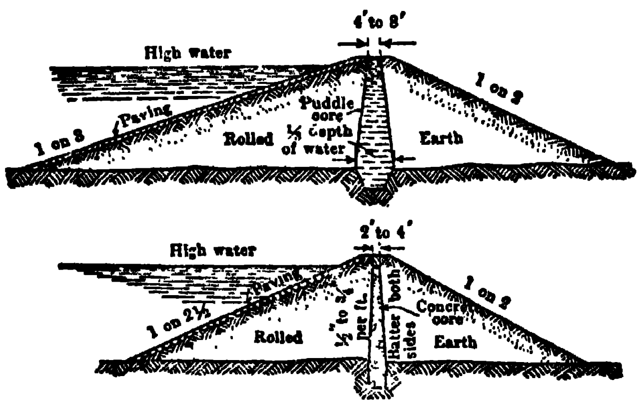


Fig 26. Puddle and Concrete Core-walls for Earth Dams

Rock-fill dams (Fig 27) are a Western type, used where no suitable earth is available, transport difficult and rock abundant. The bulk of the structure is loose rock, dumped into position or placed by derricks, the dam being made impervious by use of earth, lumber, concrete or steel. Where lumber is cheap 2 layers of 2-in planking are used, sometimes with tar paper between, the planks being spiked to stringers bolted to hori[illegible]s in the rock fill. For more permanent construction a steel face may be used, coated with cement or asphalt concrete. Concrete decking consisting of reinforced slabs with vert and horiz joints resting on concrete sills have been used in large dams.

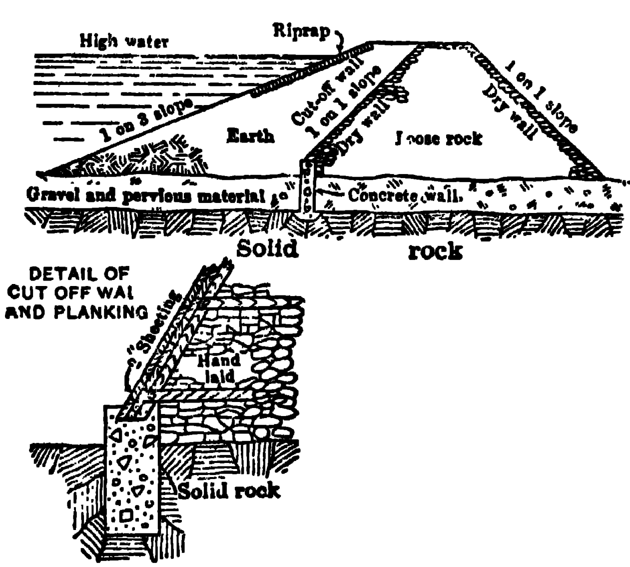


Fig 27. Rock-fill Dam

Masonry dams are now made of Portland cement concrete, this being generally less expensive than the rubble masonry formerly used or than cyclopean concrete (concrete in which large stones or "plums" are embedded). Modern practice emphasizes the importance of a dense concrete, to reduce percolation and insure greater permanency, and thus relatively dry mixtures with careful tamping to avoid segregation are advised.

Design. The basic form is a triangular section with upstream face vert, or nearly so, and downstream slope determined by base width. Principal forces acting are water press (Sec 38). For vert back, $P = wh^2 \div 2$ and wt of structure. Assuming sp gr of masonry $= \rho$, wt of sec per ft is $\rho wbh \div 2$. It is usually required that resultant R (Fig 28) pass inside $1/3$ point of base as in retaining walls. Moments about $1/3$ point give $Ph \div 3 = Vb \div 3$, from which $b = h \div \sqrt{\rho}$. For $w = 62.5$ and $\rho w = 140$ lb per cu ft, $\rho = 2.24$ and the required b is thus about 0.7 h. For extremely high dams, or dams on poor foundations, b is determined by allowable press rather than the above condition. Top of dam is made 2 or 3 ft above high water level, top width about 0.1 h by adding triangle shown in Fig 28. Other forces, which must frequently be allowed for (requiring greater b), are uplift press in foundation or joint between base and foundation and, more rarely, ice thrust. Uplift may vary from $C'wh$

lb per sq ft acting vertically upward at heel O, to zero at toe A, where C is 0.3–0.6 or more, ; on character of foundation.

Foundation. Small dams, to 20 ft or so in height, have been built on hardpan, compact sand or (with cut-off sheet piling) on gravel. In general and for higher dams a rock foundation is required. Design must always be adapted to foundation conditions and thus requires knowledge of site and experience.

Curved or arched dams of a sec similar to gravity are often used, especially for the higher structures. Curving a gravity dam does not add to stability or lead to economy unless arch action is provided for in the design and construction, that is, an arched-gravity sec is actually designed (see Bib).

Timber dams (Fig 29) are built of inclined timber cribs well fastened, filled with stone to give wt and decked with plank. Rough computations can be made to check stability and sliding. Brush dams, brush tied in bundles with butts downstream and weighted with boulders, often suffice for temporary construction. Buttress dams, of reinforced concrete with inclined reinforced concrete or arched deck supported by triangular buttresses at intervals of 15 to 30 ft, may be economical in special locations.

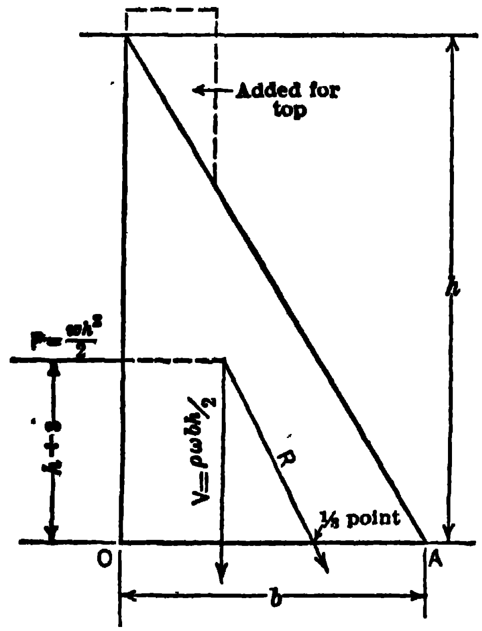


Fig 28. Gravity Dam

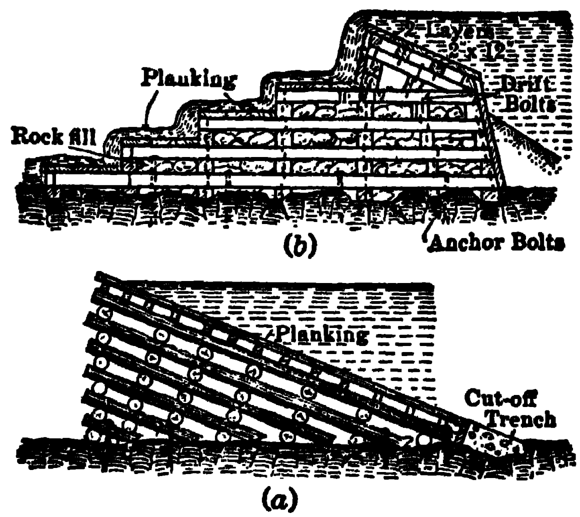


Fig 29. Timber Dams

Spillways, or waste weirs, must be able to pass safely flood flows, and are therefore designed with a liberal safety factor. When possible the spillway should be apart from main dam. Its length is usually fixed by flood width of the stream channel below, and probable depth of flow H can be found as in Sec 38, Art 10.

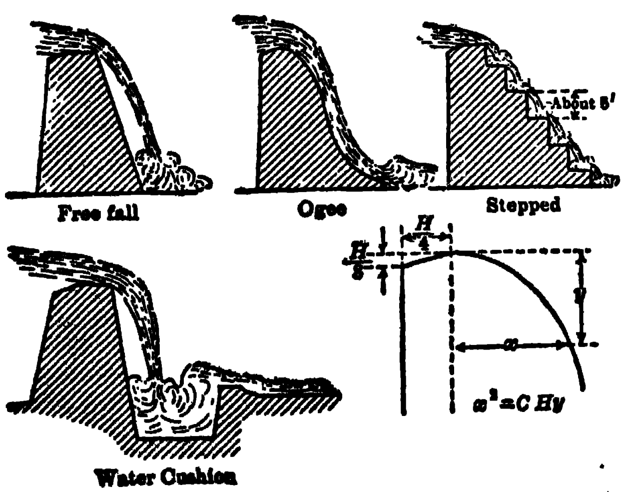


Fig 30. Spillways for Dams

Fig 30 shows types of spillways. When the stream bed is solid rock, the fall can be free, if not over about 10 ft high, although a height of 20 ft has been used. In the ogee, or reverse-curve section (an American type) the water leaves foot of dam with a direction parallel to stream bed, its velocity being destroyed by the backwater from below. Where no backwater exists, and stream bed is of soft or loose material, an apron of concrete or paving is necessary to prevent scour and undermining. Upper part of spillway curve is full enough to cause the overflowing water to adhere to the face of the section; otherwise a partial vacuum might occur, also hurtful vibration. Fig 30 shows the curve used, C being 1.6 for high, to 2.3 for low, dams. For gravity spillways, the section must be heavy enough for stability. The lower curve is often made the reverse of the upper. The water cushion, often used by English engineers, is nature's method in many natural waterfalls and is also used with ogee and sloping sections. The depth is generally made about ⅓ the fall; the width, 3 or 4 times the depth.

Outlets and gates should be kept away from the dam if possible, so an accident may not endanger the main structure. For low earth dams the arrangement shown in Fig 31 can be used; for higher dams an inlet tower near toe of inner slope is preferable. This allows water to be drawn from different levels, which is desirable for water-supply works. Screens are provided to keep out large rubbish and fish. Area of screen openings should be 2 or 3 times the area of the pipe, and veloc of flow through the screen should be low. Finer screens should be put on portable frames, for ready removal and cleaning.

Gates are generally of the sluice-gate type, obtainable in circular or rectangular form up to 6 ft diam, or 8 by 12 ft, and even larger. They move in vertical guides set in the masonry, and are operated by hand wheels, cranks, or electric motors. A blow-off outlet for draining the reservoir is essential.

Seepage around the outlet pipes of earth dams may cause failure; all parts should be accessible for inspection and repair. Pipes are bedded in concrete, with wide collars to prevent direct flow of water (Fig 31). In more costly construction the pipe is in a special concrete or masonry tunnel.

Headworks for a dam for diverting water into a canal are shown in Fig 32. A scouring sluice is essential, when much sediment is carried, which is deposited behind the dam due to decrease in velocity of stream at this point. Sediment can be kept from the canal by opening the scouring sluice during flood flows.

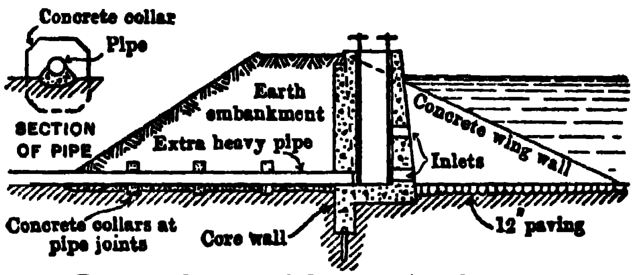


Fig 31. Outlet and Gates for Earth Dam

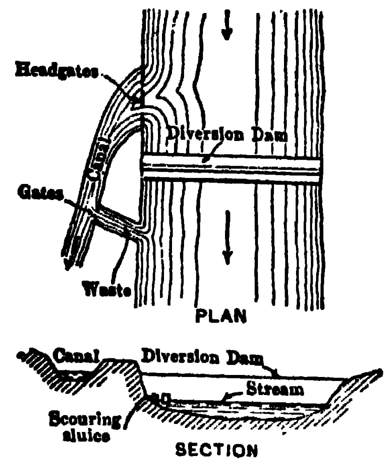


Fig 32. Headworks for Diversion Dam

Culverts are small openings to carry stream flow or drainage, under an embankment or roadway. Formerly C–I pipe or timber, masonry or concrete box sections were used. Today corrugated iron pipe is common, especially for smaller sizes, and concrete pipe for larger, but the iron type has been used up to a diam of 6 ft or more. End walls of concrete or masonry prevent scour and entrance of wash and rubbish. Most important point is selecting size of culvert. Diam less than 1 ft should seldom be used, as smaller sizes are apt to clog. Ample area is desirable and may be estimated from cross sec of stream in time of flood, as shown by high water marks. Culverts of larger size become small bridges and are designed as such (Art 14, 23, 29).

ANALYSIS OF FRAMED STRUCTURES

17. SIMPLE FRAMEWORKS AND LOADS (10, 11, 13)

Truss is a framework designed to carry loads over larger spans than are economical for beams. Fig 33 shows types for bridges; Fig 34, for roofs.

The loads from the floor of a bridge or covering of a roof truss are transferred to the panel points (where members join) by the floor beams or purlins. Hence, truss members do not carry loads directly; they are only subjected to longit stresses due to live load. But, since their own dead wt produces cross-bending, the truss members are subjected to combined bending and direct stress. As bending is unimportant except in large structures, it is customary to consider the dead wt of the truss as concentrated at panel points, and the truss members are designed simply for tension or compression, a − sign indicating compression and + tension.

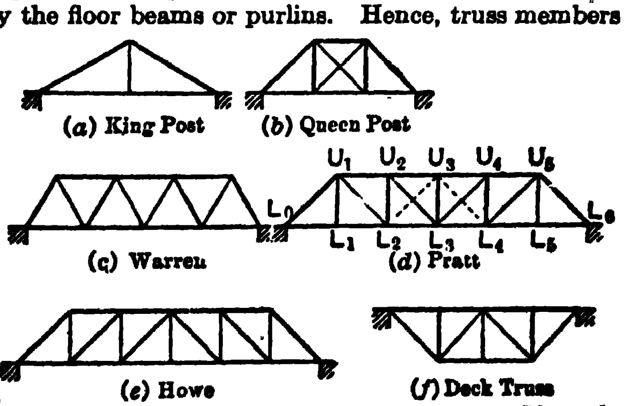


Fig 33. Typical Bridge Trusses (*a, b, c, d, e* are Through Trusses)

In bridge work, if the floor system is supported at the panel points of the lower chord it is called a THROUGH TRUSS, if by upper chord a DECK TRUSS. The principle of division into triangles is also used for the bracing of towers and other structures.

In the following analysis only simple trusses will be considered. Structures containing REDUNDANT MEMBERS and other special forms can not be investigated by ordinary methods, as the stresses depend upon the relative elongation of the different members and are computed by the Theory of Least Work. Thus, the braced bent in Fig 39 would be STATICALLY INDETERMINATE, if both diagonals were counterbraced members capable of taking both tension and compression, that is, were considered as acting simultaneously. Loading carried is of 2 kinds, dead load and live load.

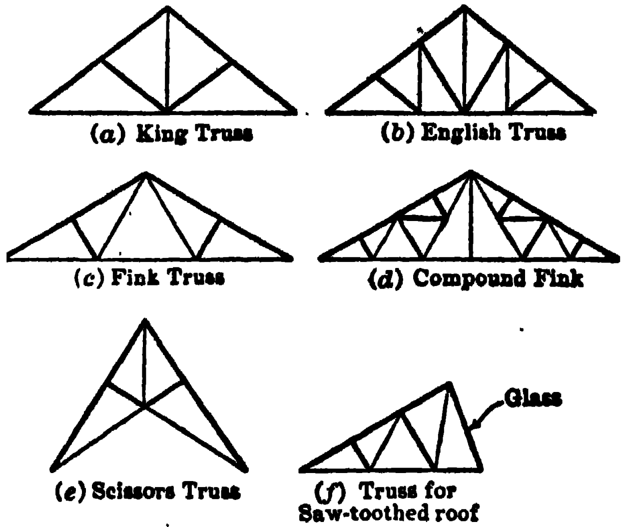


Fig 34. Typical Roof Trusses

Dead load is the wt of the structure itself, and can be closely determined. Table 7 is useful for computing dead loads. In designing, the dead wt must generally be estimated, and computations revised if the estimate subsequently proves to be in error.

For ROOF TRUSSES the approx wts are found as follows. For wooden trusses, $W = 0.5\ al\ (1 + 0.15\ l)$, where W = total wt of 1 truss, lb, a = distance c-c of trusses, ft, and l = span of truss, ft. For steel roof trusses, $W = 16\,\frac{wal^2}{s}$, where w is the load in lb per sq ft of horiz projection which the truss supports, including the wt of truss and bracing, and s is the working stress in tension, lb per sq in.

Table 7. Weights of Materials, in Pounds

Masonry	lb per
Brickwork	115 cu ft
Cement (Portland)	100 " "
" "	376 bbl
" natural	282 "
" barrel	20 lb
Concrete, cinder	110 cu ft
" stone or gravel	145 " "
" cyclopean	155 " "
" reinforced stone	150 " "
Granite, ashlar masonry	165
Plaster, 2 coat	6 sq ft
T C floor arches 6″	26 " "
8″	31
10″	37
12″	42
partitions 3″	16
4″	17
6″	25
Mortar, Portland cement	135 cu ft
Stone	
Gneiss	170 cu ft
Granite	170 " "
Limestone	165 " "
Sandstone	150 " "
Marble	165 " "
Loose rock	100 " "
Gravel, clean	100 " "
Earth	
Loam, loose	76 cu ft
Dry, rammed	100 " "
Mud	110 " "
Sand, dry	90 " "
Clay, loose	65 " "
Cinders	45 " "

Metals	lb per
Cast iron	450 cu ft
Wrought iron	480 " "
Steel	490 " "
Brass, cast	505 " "
Lead	711 " "
Zinc	438 " "
Timber	
Chestnut	41 cu ft
Hemlock	25 " "
Oak	50 " "
Pine, short leaf	35 " "
" long "	40 " "
Spruce	25 " "
Pine, white	25 " "
Roofs	
Corrugated steel	2–3 sq ft
Shingles	4–6 " "
Tiles	6–8 " "
Slate	6–8 " "
Gravel and tar	8–10 " "
Tin, with sheathing	6 " "
Paving and track	
Stone block paving	160 cu ft
Brick paving	160 " "
Asphalt and concrete paving	130 " "
Wood paving	70 " "
Crossties and guard rails	200 lin ft
Rails and fastenings	200 " "
Miscellaneous	
Coal, broken	53 cu ft
Grain	48 " "
Ice	58 " "
Snow, fresh	8 " "
Water	62.5 " "

For bridges many formulas are used, depending on type of bridge and loading. Data on the wt of girders and trusses may be found in "Bridge Engineering," by J. A. L. Waddell. For RR girder bridges, the formula $W = kl^2$ is often used, where k is 12–16

for aver loads and working stresses. Hudson's formula for RR trusses is $W = \frac{100}{3} Al$, where A = net area of the largest tension chord. In analyzing the stresses due to DEAD LOADS, the loads obtained from these formulas are divided by the number of panels, and considered as being applied at the panel points. LIVE LOADS are uncertain in amount, and are generally fixed by specification. Max possible loading must always be used.

Floor loads in buildings are usually designed for a uniformly distributed live load of 75–250 lb per sq ft, depending on type of building and on the local building code. For

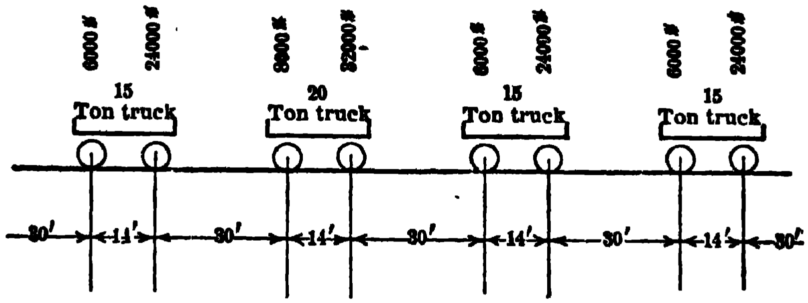


Fig 35. H–20 Loading for Highway Bridge

highway bridges, the so-called H-TRUCK-TRAIN LOADINGS are commonly used. Fig 35 shows arrangement of the H-20, from which the others can be prorated; 3 classes are used, H-20, H-15 and H-10, the particular truck train selected depending on importance of the highway. For RR bridges, a concentrated load followed by a uniform load are used, representing 2 locomotives followed by a train, and are known as E-50, E-60, etc, indicating the engine axle load. Fig 36 shows E-50 loading (for single track bridge half of each axle load is carried by each truss or girder). E-60 is used for second-class roads; E-72 is

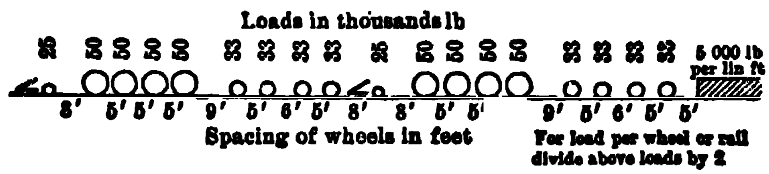


Fig 36. E–50 Loading for R R Bridge

recommended for first-class roads. To obtain E-72 from the diagram, multiply the axle loads given by 72 ÷ 50, the axle spacings remaining the same.

Wind load. For exposed structures, specifications range from 15 to 30 lb per ft of area exposed to the wind, the area taken being the vert projection. For sloping roofs, wind and snow effects are often combined in a single vert uniform load.

Snow loads must be considered in designing roofs in cold climates; in some structures temperature stresses also. Table 8 gives values in lb per sq ft of roof surface, of equivalent vert uniform load to replace combined wind and snow loads for calculating max stresses in roof trusses.

Table 8. Equivalent Combined Snow and Wind Loads (Vert)

Locality	Flat	20°	30°	45°	60°
Northwestern and New England States	40	35	24	26	28
Western and Central States	35	30	24	26	28
Southern and Pacific States	30	20	24	26	28

Impact stresses must be allowed for in structures subject to moving loads. This is usually done by multiplying the live load stresses by an IMPACT FACTOR. For highway bridges this factor I is often given as $I = \frac{50}{L + 125}$, where L is length in ft of the portion of span which is loaded to produce the max stress in the member considered. For RR bridges the impact factor is $I = (1 - 0.006L) + 0.2$. This expression takes care of both the lurching and direct vert effect, and is good for bridges less than 100 ft long.

18. METHODS OF TRUSS ANALYSIS (10, 11)

Algebraic method. Stresses in the members of simple trusses are found by applying the 3 laws of equilibrium for coplanar forces. $\Sigma V = 0$, $\Sigma H = 0$, and $\Sigma M = 0$ (Sec 36, Art 30–33). Two procedures are common, method of joints and method of moments.

Fig 37 shows half of a Fink truss. The loadings P, P, etc, are determined by adding to the roof load, brought to the joints 1, 2, 3, etc, by the purlins, 1/9 the total wt of the truss, which is supposed to be equally divided among the upper panel points. Vert load 3.5 P, acting downward at A, can be neglected, as it is carried directly by the support. By the method of joints (Fig 37a) a section xy is passed, cutting members A 1 and A 5. Since ΣV and $\Sigma H = 0$, by the resolution of forces, and since $R = 3.5\,P$, the stress in A 1 $= 3.5\,P \div \sin\theta$, and stress in A 5 $= 3.5\,P \tan\theta$. By method of moments, a section is passed generally cutting 3 members, the stress in one of which is desired, and the formula $\Sigma M = 0$ is applied, taking moments about the point of intersection of the other 2 forces.

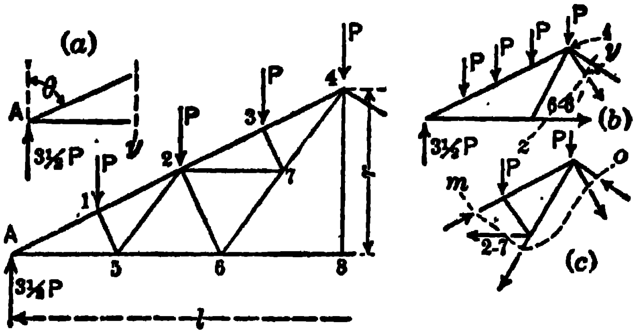


Fig 37. Truss Analysis by Method of Joints

Thus (Fig 37b), passing the section yz and taking moments about 4, $3.5\,P\frac{l}{2} - \frac{6}{8}Pl -$ (stress in 6–6′) $r = 0$, or stress in 6–6 $= Pl \div r$. The section passed may be either straight or curved. Thus (Fig 37c), passing the section mo and taking moments about 4, $(Pl \div 8)$ + (stress in 2–7) $r \div 2 = 0$, or stress in 2–7 $= Pl \div 4\,r$. For the common rise of 1 in 4 ($= r \div l$), the stresses in the various members found by the continued application of the above methods are shown in Fig 38 in terms of P. The member 4–8 serves simply to support member 6–6′ at center and is not an essential part of the truss and figures as carrying 0 stress. It actually carries 0.5 the wt of 6–6′.

In applying the method of moments to the braced bent in Fig 39, members AC and FD must be prolonged to intersection at O, outside the truss, and moments taken about O.

Thus, for the wind load shown, reactions are equal and may be found by moments about A or F. (Diagonal members are supposed to be in tension, and the dashed members are not taking stress for the loading shown.) To get stress in AE, take a sec through AB, AE, EF (Fig 39a). Taking moments about O, $P_1b + P_2(a + b) - AEx' = 0$, giving stress in AE. To get stress in DE, take sec through DE, BE and BA (Fig 39b), and moments about B; thus $P_1a - DEy = 0$, giving DE. To get BE, take same sec (Fig 39c) and taking moments about O, $BE\,(a + b) - P_1b - P_2(a + b) = 0$, giving BE.

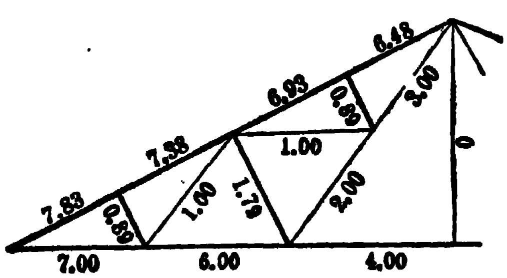


Fig 38. Truss Analysis by Method of Moments

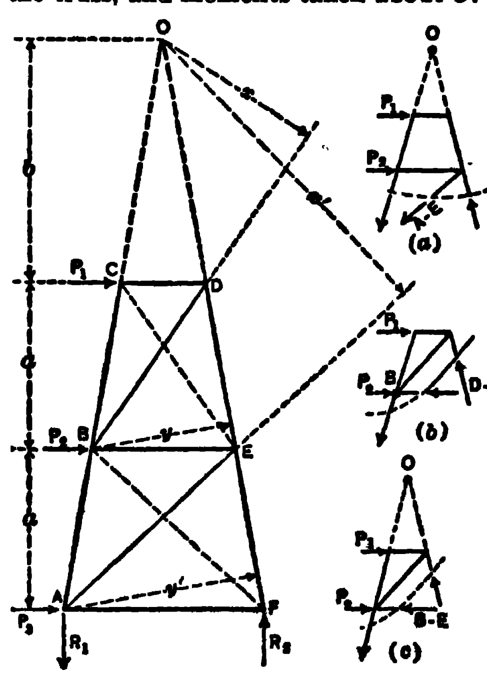


Fig 39. Analysis of a Braced Bent

Graphical method. This is often easier and shorter than the algebraic method, due chiefly to the Bow system of notation. There are 2 diagrams; one a TRUSS DIAGRAM, showing lines of action of the forces in connection with the structure itself, and the other simply a FORCE POLYGON. Reactions in many cases are determined graphically, but it may be simpler to obtain them algebraically by method of moments (see above).

Thus, take a simple kingpost truss (Fig 40a), carrying a concentrated load at its center. Following Bow's notation, the spaces between the lines of action of any 2 adjacent forces on the diagram,

or the space within any triangle the sides of which are lines of action of any forces, are marked by capital letters. Employing a right-hand system of reading, indicated by the curved arrow, the reaction at 1 is called AB, the concentrated load CA, the member 1–2 is BD, etc. Referring to the force diagram (Fig 40*b*), the reactions are evidently equal to 0.5 the center concentration. Starting at any point, the line *ab* is laid off upward from *a*, representing to scale the left reaction in direction and magnitude. Line *bd* is then drawn parallel to BD, beginning at *b*, and line *da* parallel to DA, beginning at *a*; their intersection fixes point *d*. In a similar manner the diagram is completed, as shown.

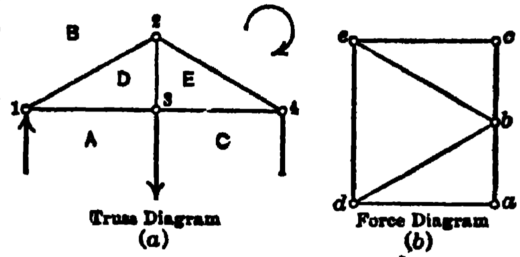


Fig 40. Graphical Method of Truss Analysis

The lines in the force diagram are now scaled off, giving the stresses in different members. The kind of stress may also be determined as follows. Considering any panel point, say 2, in the truss diagram, the member 1–2 is read DB. Noting *db* in the force diagram, it is seen that the direction from *d* to *b* is toward panel point 2, and DB is therefore in compression. Or, at panel point 1, member 1–3 is read DA, and, as direction from *d* to *a* is toward right or away from 1, this member is in tension. Analysis of stresses in hoisting headframes by the graphical method is shown in Sec 12.

19. ANALYSIS OF MOVING LOADS (10, 11)

For a fixed system of concentrated loads the max reaction is easily found, giving max shear, and the max moment is then computed by finding the section at which the shear passes through zero (this usually occurs under one of the concentrations known as the critical wheel), the moment at that point being a max. It is usual in small beams to design simply for max moments and shears, and to use the same section throughout. A moving load must therefore be so placed, with reference to the span, as to produce the max shear or moment.

Max reaction or shear occurs at the support when the loads are as near as possible to one end of the span. Thus, in Fig 41*a*, the 2 loads must be moved so that wheel 1 will be a very small distance from the support, and reaction equals $P_2y + P_1$. For a locomotive (as in Fig 36) the first driver must be over the support (Fig 41*b*) and be considered as acting just to the right of the support.

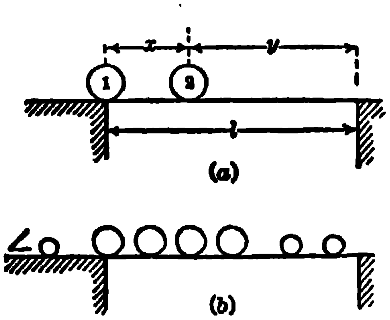


Fig 41. Analysis of Moving Loads

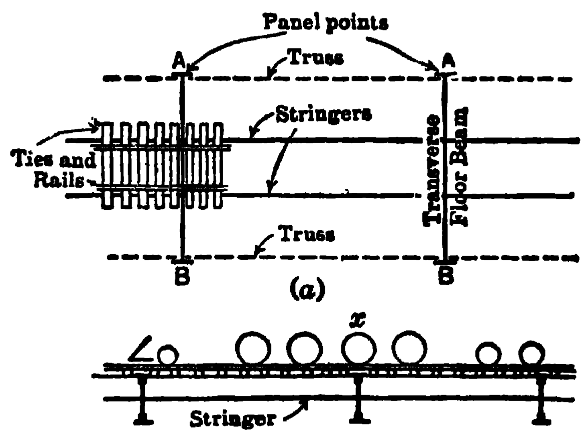


Fig 42. Live Load on Truss Bridge

Live load on truss bridges is transferred from the longit stringers to transverse floor beams AB which carry it to the panel points (Fig 42*a*). Beams AB must therefore be designed to carry 2 concentrations, at x and y, equal to the max reaction from the longit members framing in at these points. Sum of reactions from the 2 equal spans will be max when total load is max, and evenly divided between the spans. For a locomotive concentration this generally requires one of the middle drivers at x.

Max moment at any section in a beam occurs when total load is divided in proportion to distances of this section from each end of beam. Thus, max moment at sec xy (Fig 43) will occur when sum of loads to left ($= W'$) is to sum of all loads ($= W$) as l' is to l. It is sufficiently accurate to determine M by this CRITERION for center of beam, and call it the max moment. The max moment will not be at the center, but at a point such that the distance between this point and the center of gravity of entire load is bisected by the center of span (Fig 44*c*). EXAMPLE. Take 2 equal loads of 1 000 lb, 4 ft apart, span 10 ft. Max M at center is when 1 load is on either side of center, and equals 3 000 ft-lb (Fig 44*a*). Max possible moment is 3 200 ft-lb at point *e*, when load is placed as in Fig 44*b*. General formula for equal loads (Fig 44*c*): max M occurs where shear $= 0$, when

$x = (l + 2) - (a + 4)$, and will equal $(P + 2\,l)\,[l - (a + 2)]^2$. This holds to $a = 0.586\,l$, after which max M is for 1 load at center of span.

Stresses in truss members due to concentrated moving loads (Bib 10, 11, 12). Referring to Fig 33*d*, for example, the dead load stresses, or those due to a fixed live load, may be found by the methods of Art 18. But, if a moving live load is considered, it must be moved to positions producing max stresses in each member. Thus, for max stress in end post L_0U_1, the reaction of moving load at L_0 must be a max, and max stress then equals this reaction + *cos* of angle $L_0U_1L_1$. U_1L_1 is simply a hanger, carrying the max floor beam reaction at L_1.

For max live-load tensile stress in diagonal U_2L_3, the load is advanced from right so as to produce greatest shear in panel L_2L_3. The CRITERION for this (see above) is that the total load on bridge must equal n times the load on the panel in question. Stress in U_2L_3 will then = shear + *cos* of angle $L_2U_2L_3$. Max shear in U_2L_3 = reaction at L_0, minus the floor-beam reaction at L_2. This condition of loading also gives the max stress in post U_3L_3, which will equal the vertical component of U_2L_3, or the shear. For a load advancing from left, stress in U_2L_3 will be compression. This member can be made as a counterbraced member, capable of taking tension or compression, or to take tension only, and another tension member U_3L_2, called a "counter," can be put in, as shown by dotted lines.

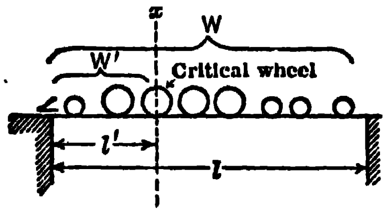


Fig 43. Max Moment in Truss (Approx)

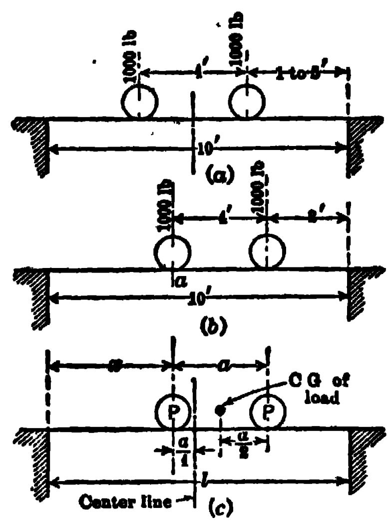


Fig 44. Max Moment in a Truss (Exact)

For max stress in a chord member, the load must be placed so as to produce the max moment in the chord. Criterion for this is the same as for max moment at any section of beam, or $W' : W$ as $n' : n$, where n = total panels in truss, n' = the number to left of section, W = total load and W' = load to left of section. For U_2U_3, stress = M + height of truss, where M = reaction at L_0 times its lever arm minus floor beam reactions at L_1 and L_2, each multiplied by their lever arms, the center of moments being taken at L_3, or any point in the panel L_2L_3.

TIMBER STRUCTURES

20. TIMBER (12)

Trade classification of woods: evergreens or soft woods (pines, spruce, hemlock, cedar, cypress, fir); hard woods (oak, chestnut, hickory, ash, walnut, maple, birch, whitewood). Some " hard woods " are softer than the so-called soft woods.

Pine. Most useful of all woods. WHITE or northern white, from northern U S and Canada: light, soft, not very strong, but easily worked; for interior finish, best grades for pattern making. WESTERN PINE covers timber sold as white pine, coming from Arizona to Washington. IDAHO WHITE, from northern Idaho, western Montana and eastern Washington; not as strong as white pine, but resembles it and used for same purposes. SOUTHERN YELLOW is of 2 principal kinds: Long-leaf is from southern coast, Virginia to Texas, heavy, hard, strong, close-grained, tough; used for heavy framing and floors; can not be used in contact with ground, or built in solidly in masonry, as it decays rapidly unless dry, and dry-rots unless open to air. Short-leaf or Carolina is used as substitute for long-leaf; variable in quality, the short-leaf being inferior. RED or NORWAY PINE, used for all construction purposes, is light, hard, coarse-grained. DOUGLAS FIR or OREGON PINE from north Pacific coast is the most valuable lumber of the Pacific region; used for structural purposes instead of eastern hard pines; two kinds, red more valuable than yellow; imported to east when large size or length is required.

Spruce. BLACK or eastern, from northern U S and Canada; light, soft, close-grained, tough in fiber; desirable for studs, joists and framing in buildings, also for piles, submerged cribs, etc, as it lasts well under water and resists destructive action of crustacea. WHITE, similar to black but not so common. WESTERN or SITKA, north Pacific coast. Giant spruce, soft, satiny, but inclined to be cross-grained; used for interior finish, fences, boats and cooperage.

Hemlock. EASTERN: brittle, splits easily, liable to be "shaky"; used in cheap, rough framing, as substitute for spruce, which it closely resembles in appearance. WESTERN: said to be harder and superior in quality, but liable to shrink and warp.

Redwood. Giant tree and most valuable timber of California; used for same purposes in West as pine in East; soft and splinters easily; durable for fence posts, telegraph poles and R R ties (tie plates are used on ties to prevent wear).

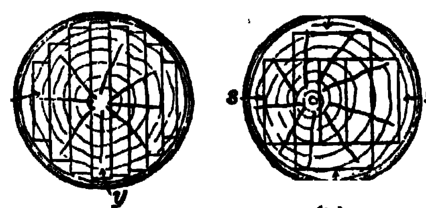
Cedar. WHITE or arborvitæ: soft, fine-grained, durable, but lacks strength and toughness; used for tanks, shingles, boats, fencing, cooperage. RED: small size, very durable but brittle, strong, pungent odor; used for fence posts, shingles, etc.

Cypress, similar to cedar, used in northern U S for shingles; in south as widely as pine in north.

Oak. WHITE, from eastern U S; heavy, strong, hard, tough and close-grained; liable to check if not well seasoned; used where great strength is required in framing, boat building, etc. RED oak is similar, except in color.

Miscellaneous. HICKORY is hardest, toughest and strongest of all American woods; for tool handles. ASH, for interior work; is unfit for outside, as it decays rapidly. ELM, tough and very close-grained; used in wagons, cars, boats, barrel-staves.

Structure and sawing. The mature, compact inner layers are called HEARTWOOD; the outer 0.25 to 3 or 4 in being SAPWOOD, soft, open, and full of sap which hastens decay. Bark should always be removed. The fibers of the tree are in circular layers separated by ANNUAL RINGS, which are interrupted by radial cells, communicating with the pith at center and the sapwood, and known as medullary rays.



(b)

Modes of Sawing Logs

Fig 45 shows modes of sawing common lumber from a log: (*a*) for plank of uniform thickness; (*b*) for joists, etc, of same thickness and width. The part of log section from which lumber comes is of great importance. Thus, in (*a*) piece *x* is liable to warp or curl, due to unequal length of fibers on 2 sides, and if used in flooring is liable to splinter off. *y*, the best cut, is said to be RIFT-SAWED or comb-grained; makes best and most durable flooring; *sss* are slabs. QUARTER SAWING, in plane of medullary rays, gives handsome finish.

Commercial sizes. FLOORING embraces 4, 5 and 6 quarter-in thicknesses, by 3 to 6 in wide, excluding 1.5 by 6. BOARDS, thicknesses under 1.5 in by over 6 in wide. PLANK, sizes from 1.5 to under 6 in thick, by 6 in and over wide. SCANTLING, sizes above 1.5 and under 6 in thick, and 2 to 6 in wide. DIMENSION TIMBER, sizes over 6 by 6 in. STEPPING, 1 to 2.5 in thick by 7 in and over in width. ROUGH-EDGE, or flitch, all sizes, 1 in thick and up by 8 in and up wide; sawed 2 sides only.

In U S, lengths increase by 2 ft; in Canada, by 1 ft. In dimension timber, sizes above stated may be obtained, but stock sizes vary in both dimensions by even inches; thus, 6 by 6, 6 by 8, 8 by 8, etc. All sizes less than 1 in thick are counted as 1 in, in computing quantity. The unit is the BOARD FOOT, 12 by 12 by 1 in.

Finish. Rough timber and lumber, sawed to standard size, shall be not over 0.25 in scant. Thus, 2 by 6 may be 1.75 by 5.75 in. In small sizes, this greatly reduces strength. Not more than 0.25 in is allowed for dressing each surface. The term *S* 1 *S* 1 *E* indicates that 1 side and 1 edge are surfaced; *S* 4 *S* means surfaced on 4 sides, etc. A 2 by 6 *S* 1 *S* 1 *E* is usually worked to 1 $^5/_8$ by 5 $^5/_8$ in.

Shrinkage. Shrinkage is greatest tangentially to the fibers, that is, circumferential shrinkage; radial shrinkage is less. Flat-sawed lumber (Fig 45*a*, *x*) may crack and shrink 3–10% of width, thus opening spaces between boards or flooring, if not thoroughly seasoned. Longit shrinkage is small, generally less than 0.1%, which must be considered in designing structures where settling is important; if possible, all timbers with horiz fibers should be eliminated from vert supports. Effect of moisture is the reverse, and should also be considered.

Seasoning. Natural seasoning, used for lumber for construction purposes, requires 1 to 3 years, depending on kind of lumber. Object is to remove moisture and dry up natural gums, thus guarding against shrinkage and decay. Lumber should be piled under cover, high and dry, thorough air circulation being allowed by placing strips between layers. WATER SEASONING consists in immersing timber in water about 2 weeks, to dissolve out soluble substances, and then air drying. Liable to make heartwood brittle or sapwood brashy. KILN-DRYING consists in placing lumber, usually only small sizes, in a kiln and exposing to a current of air at 100 to 200° F for 2 to 6 or 8 days. It generally produces inferior product, due to excessive drying of exterior and imperfect drying of interior.

Defects and inspection (2). Two things must be considered: quality and dimensions. Strongest and most durable timber comes from trees of slow growth, indicated by closeness

of annular rings. Best timber comes from heartwood; should be straight in fiber, free from large or dead knots, flaws, shakes and blemishes. Dull, chalky appearance, or disagreeable odor, indicates decay and bad timber. Good timber is sonorous when struck.

Table 9. Timber Sizes for Use in Design

Nominal size, in	American standard dressed size, in	Area of section, sq in	Wt per ft, lb	Moment of inertia, in^4	Section modulus, in^3
2×4	1 5/8×3 5/8	5.89	1.64	6.45	3.56
6	5 5/8	9.14	2.54	24.1	8.57
8	7 1/2	12.2	3.39	57.1	15.3
10	9 1/2	15.4	4.29	116	24.4
12	11 1/2	18.7	5.19	206	35.8
14	13 1/2	21 9	6.09	333	49.4
16	15 1/2	25.2	6.99	504	65.1
18	17 1/2	28.4	7.90	726	82.9
3×4	2 5/8×3 5/8	9.52	2.64	10 4	5.75
6	5 5/8	14.8	4.10	38.9	13.8
8	7 1/2	19.7	5.47	92.3	24.6
10	9 1/2	24.9	6.93	188	39.5
12	11 1/2	30.2	8.39	333	57.9
14	13 1/2	35.4	9.84	538	79.7
16	15 1/2	40.7	11.3	815	105
18	17 1/2	45.9	12.8	1 172	134
4×4	3 5/8×3 5/8	13 1	3.65	14.4	7.94
6	5 5/8	20.4	5.66	53.8	19.1
8	7 1/2	27.2	7.55	127	34.0
10	9 1/2	34.4	9.57	259	54.5
12	11 1/2	41.7	11.6	459	79.9
14	13 1/2	48.9	13.6	743	110
16	15 1/2	56.2	15.6	1 125	145
18	17 1/2	63.4	17.6	1 619	185
6×6	5 1/2×5 1/2	30.3	8.40	76.3	27.7
8	7 1/2	41 3	11.4	193	51.6
10	9 1/2	52.3	14.5	330	82.7
12	11 1/2	63.3	17 5	697	121
14	13 1/2	74.3	20.6	1 128	167
16	15 1/2	85.3	23.6	1 707	220
18	17 1/2	96.3	26.7	2 456	281
20	19 1/2	107.3	29.8	3 398	349
8×8	7 1/2×7 1/2	56.3	15.6	264	70.3
10	9 1/2	71.3	19.8	536	113
12	11 1/2	86.3	23.9	951	165
14	13 1/2	101.3	28.0	1 538	228
16	15 1/2	116.3	32.0	2 327	300
18	17 1/2	131.3	36.4	3 350	383
20	19 1/2	146.3	40.6	4 634	475
22	21 1/2	161.3	44.8	6 211	578

Nominal size, in	American standard dressed size, in	Area of section, sq in	Wt per ft, lb	Moment of inertia, in^4	Section modulus, in^3
10×10	9 1/2× 9 1/2	90.3	25.0	679	143
12	11 1/2	109	30.3	1 204	209
14	13 1/2	128	35.6	1 948	289
16	15 1/2	147	40.9	2 948	380
18	17 1/2	166	46.1	4 243	485
20	19 1/2	185	51.4	5 870	602
22	21 1/2	204	56.7	7 868	732
24	23 1/2	223	62.0	10 274	874
12×12	11 1/2×11 1/2	132	36.7	1 458	253
14	13 1/2	155	43.1	2 358	349
16	15 1/2	178	49.5	3 569	460
18	17 1/2	201	55.9	5 136	587
20	19 1/2	224	62.3	7 106	729
22	21 1/2	247	68.7	9 524	886
24	23 1/2	270	75.0	12 437	1 058
14×14	13 1/2×13 1/2	182	50.6	2 768	410
16	15 1/2	209	58.1	4 189	541
18	17 1/2	236	65.6	6 029	689
20	19 1/2	263	73.1	8 342	856
22	21 1/2	290	80.6	11 181	1 040
24	23 1/2	317	88.1	14 600	1 243
16×16	15 1/2×15 1/2	240	66.7	4 810	621
18	17 1/2	271	75.3	6 923	791
20	19 1/2	302	83.9	9 578	982
22	21 1/2	333	92.5	12 837	1 194
24	23 1/2	364	101	16 763	1 427
18×18	17 1/2×17 1/2	306	85.0	7 816	893
20	19 1/2	341	94.8	10 813	1 109
22	21 1/2	376	105	14 493	1 348
24	23 1/2	411	114	18 926	1 611
26	25 1/2	446	124	24 181	1 897
20×20	19 1/2×19 1/2	380	106	12 049	1 236
22	21 1/2	419	116	16 150	1 502
24	23 1/2	458	127	21 089	1 795
26	25 1/2	497	138	26 945	2 113
28	27 1/2	536	149	33 795	2 458
24×24	23 1/2×23 1/2	552	153	25 415	2 163
26	25 1/2	599	166	32 472	2 547
28	27 1/2	646	180	40 727	2 962
30	29 1/2	693	193	50 275	3 408

All properties and weights given are for dressed size only.
The weights given above are based on assumed aver wt of 40 lb per cu ft.

In inspecting, timber is graded according to number, position and kind of defects visible in any piece. Thus, lumber is graded as first, second and third clear, or as No 1 or No 2, depending upon STANDARD DEFECTS shown, as pin and standard knots, sap stains, pitch pockets or streaks, checks, wanes or shakes, sap. The number of defects allowed depends upon kind of inspection: square-edge, merchantable, prime and clear inspection (2).

Decay. Life of timber depends on manner and time of felling, seasoning and working, and is always subject to attack by insect and vegetable life. Best time to cut timber is in winter and drier summer months, when cells are dormant and amount of sap small. Alternate wetting and drying, heat and confined air, fungi, insects, and worms are the chief agencies producing decay. Well-seasoned wood, in well-ventilated and uniform condition as to moisture, should never decay. Constant immersion weakens and softens, but does not cause decay.

Dry rot is due to excretion of ferments by a fungus which attacks cell walls, and breaks them down in the presence of moderate warmth, dampness, or lack of ventilation. Occurs in wall pockets at ends of timbers and in cores of timber columns. WET ROT is caused by presence of moisture and warmth, which dissolves out cell walls in sapwood. COMMON ROT is due to improper seasoning without thorough ventilation, and is shown by yellow color at points of contact or yellow dust in checks and cracks. The most active worm is the TEREDO, or shipworm. It attacks timber in clear, unpolluted, sea water, from mean tide to ground level, and is especially prevalent on calcareous shores.

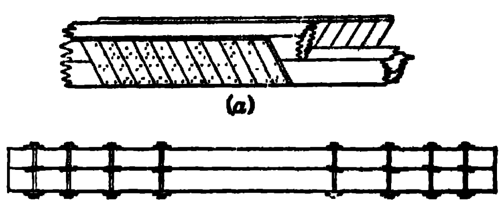
Preservation. CREOSOTING is the best preservative, and is also effective against the teredo. It consists in impregnating with dead oil of coal tar or distillates from the wood tars (creosote). Timber is first placed in cylinders and sap vaporized by steam. After 0.5 to several hr, steam and sap are drawn off by a pump and the cylinder filled with oil, at about 160° F. When gage stands constant, showing no more oil is being absorbed, the surplus is drawn off. Entire treatment takes about 24 hr. Amount of oil varies, with kind of wood and seasoning, from 5 to 12 or 18 lb per cu ft for green and hard timber. BURNETTIZING consists in impregnation with $ZnCl_2$, in a similar manner as for creosoting. About 0.24 lb of $ZnCl_2$ is used per cu ft. Not suitable for timber exposed to moisture, as the Zn will leach out. ZINC-TANNIN or Wellhouse process is same as Burnettizing, except that a small quantity of glue is added to the Zn, and after completing the Zn treatment a solution of tannin is run in under pressure, which forms, with the glue, a leathery coating, closing the pores and preventing leaching out of the $ZnCl_2$. Several other methods and variations in the above methods are in use. KYANIZING, or impregnating with Hg_2Cl_2 and $CuSO_4$, is also used. Timber may be made incombustible by injecting Fe_2SO_4 and a solution of $CaSO_4$ or Na_2SO_4.

21. COLUMNS AND BEAMS

Tests and strength. Timber shows wide variation in strength depending on kind and age of tree, moisture content, size of test specimen, imperfections and knots. Due to low resistance of timber to shear parallel to the fiber, relatively short, deep planking or beams must be checked for both bending and horiz shear.

Since $M = KS$ (Art 4), Tables 10–12 may be used directly in design. *Example.* Required safe uniform load for 6 by 8 in spruce beam spanning 4 ft. From Table 9, $S = 51.6$, from Table 10, aver K = say 1 000 lb per sq in; hence safe M = 51 600 in-lb; or, since $M = Wl \div 8$, with l = 48 in, total load is 8 600 lb; or, divided by 4, a safe load of 2 150 lb per ft. For horiz shear, using max from Table 10 of aver 100 lb per sq in, since V max = $3/2\ V \div bd$, the max permissible shear = 0.66 × 100 × 6 × 8 = 3 200 lb, and safe load per ft = 1 600 lb, which controls. Note. When ratio of depth of beam to span (1 : 8 in this problem) exceeds ratio of allowable shear to direct stress (1 : 10 as above), horiz shear rather than bending controls safe load. This is shown in Table 11 for plank floors (based on K = 1 000 and horiz shear = 100 lb per sq in), where values below the heavy line depend on bending; those above being computed for horiz shear.

Timber columns are usually of sq sec. *Example.* Design redwood strut 10 ft long to carry 50 000 lb. For a selected stick, if safe load (Table 10) is 1 000 lb per sq in, about 50 sq in or 7 in sq sec is required. With this sec, $l \div d$ = about 17 and safe stress would be 900, giving sec of 7.5 or, nearest commercial size, 8 by 8. For common timber a larger sec, say 8 by 10, would be required.


(a)

(b)

Fig. 46. Compound or Keyed Beams

Deepened, built-up, compound or keyed beams. When no timber of required size is available, beams of this kind are used for temporary work and sometimes for permanent construction, though trussed beams are much lighter. If 2 beams are superposed, they have only twice the strength of one, neglecting friction between them. But, if they are bolted together (Fig 46) they act as one beam of twice the depth, and are 4 times as strong as the single stick.

Compound beam in Fig 46a is fastened by sheeting of planks at least 1/8 the width of timbers, with nails extending at least 0.5 their length into the main members; it has a strength of about 70% that of a solid beam of same dimensions. Beams with hardwood or metal keys (Fig 46b), preferably of a wedged type, are 75 to 80% as strong as solid beams. The keys transfer the horiz shear from one beam to the other, and are of such size that the safe end compression is not exceeded. Spacing must be determined for the safe horiz shearing stress. Keys should be about 2.5 times as wide as thick, and beam should be about 0.5.

Design. *a.* Design cross-sec of beam for bending = 1.25 to 1.33 times that computed (to allow for effic of 75–80%). *b.* Compute external vert shear = V, at governing points. *c.* Compute intensity of horiz shear at neutral axis = $(3 \div 2\ V) \div bd$. *d.* Compute total horiz shear between

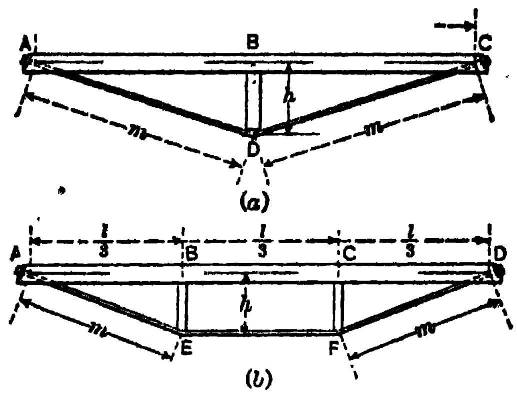


Fig 47. Trussed Beams

end and point of zero shear (total horiz shear = aver intensity between any 2 points × b × distance between points). *e.* Select number of keys (usually 5) and divide (*d*) by this number, designing keys for bearing accordingly. *f.* Draw vert shear diagram (horiz shear is proportional) and divide into number of equal areas (= number of keys) by vert lines. Place keys in back of these lines. *g.* Check design for shear between keys. *h.* Design bolts as for tabled fish-plate joint.

Trussed beams are economical for spans over 20 ft, where headroom permits, or for shorter spans where saving in wt is an object. King and queen-post trusses (Fig 47) are used. Usually the beam is a single unspliced piece, extending from end to end, with iron

Table 10. Structural Timber Allowable Unit Stresses, lb per sq in (Dept of Agriculture)

Species	American standard grade	Bending stress: In extreme fibers	Bending stress: Horizontal shear	Compression stress: Parallel to grain	Compression stress: Perpendicular to grain	Modulus of elasticity
Cedar, Western Red	Select	900	80	700	200	1 000 000
	Common	720	64	560		
Cedar, Northern and Southern White	Select	750	70	550	175	800 000
	Common	600	56	440		
Cedar, Port Orford	Select	1 100	90	900	250	1 200 000
	Common	880	72	720		
Cedar, Alaska	Select	1 100	90	800	250	1 200 000
	Common	880	72	640		
Cypress, Southern	Select	1 300	100	1 100	350	1 200 000
	Common	1 040	80	880		
Douglas Fir (Western Wash and Ore)	Dense Select	1 750	105	1 285	380	1 600 000
	Select	1 600	90	1 175	345	
	Common	1 200	72	880	325	
Douglas Fir, Rocky Mountains	Select	1 100	85	800	275	1 200 000
	Common	880	68	640		
Fir, Balsam	Select	900	70	700	150	1 000 000
	Common	720	56	560		
Fir, Golden, Noble, Silver, White (Commercial White)	Select	1 100	70	700	300	1 100 000
	Common	880	56	560		
Hemlock, West Coast	Select	1 300	75	900	300	1 400 000
	Common	1 040	60	720		
Hemlock, Eastern	Select	1 100	70	700	300	1 100 000
	Common	880	56	560		
Larch, Western	Select	1 200	100	1 100	325	1 300 000
	Common	960	80	880		
Oak, Commercial White and Red	Select	1 400	125	1 000	500	1 500 000
	Common	1 120	100	800		
Pine, Southern Yellow	Dense Select	1 750	128	1 285	380	1 600 000
	Select	1 600	110	1 175	345	
	Common	1 200	88	880	325	
Pine, Calif, Idaho and Northern White, Lodgepole, Pondosa, Sugar, Western Yellow	Select	900	85	750	250	1 000 000
	Common	720	68	600		
Pine, Norway	Select	1 100	85	800	300	1 200 000
	Common	880	68	640		
Redwood	Select	1 200	70	1 000	250	1 200 000
	Common	960	56	800		
Spruce, Red, White, Sitka	Select	1 100	85	800	250	1 200 000
	Common	880	68	640		
Spruce, Englemann	Select	750	70	600	175	800 000
	Common	600	56	480		
Tamarack, Eastern	Select	1 200	95	1 000	300	1 300 000
	Common	960	76	800		

Strength of wood depends considerably on moisture content. For wet locations, working stresses should be reduced according to conditions and timber used.

tie rods passing through oblique holes bored in the ends. If 2 pieces are used, they are separated with tie rods between.

Table 11. Allowable Load on Plank, lb per sq ft.

Span, in	Thickness of plank, in							
	7/8	1	1 1/8	1 1/4	2	3	4	6
12	1 020	1 333	1 688	2 000	3 200	4 800	6 400	9 600
14	750	979	1 240	1 713	2 740	4 120	5 480	8 220
16	575	750	950	1 172	2 400	3 600	4 800	7 200
20	368	480	608	750	1 920	2 880	3 840	5 760
24	255	333	422	521	1 333	2 400	3 200	4 800
30	163	214	270	333	852	1 920	2 560	3 840
36	113	148	188	233	593	1 333	2 133	3 200
42	83	109	138	170	435	980	1 740	2 740
48	......	83	105	130	333	750	1 333	2 400
54	......	......	83	103	263	593	1 055	2 130
60	......	......	......	83	213	480	853	1 920
66	......	......	......	......	176	397	705	1 587
72	......	......	......	......	148	333	593	1 333

Table 12. Structural Timber Columns, Allowable Unit Stresses, lb per sq in (Dept of Agriculture)

Species of timber	American standard grade	Ratio of length to least dimension ($l \div d$)										
		10 and less	12	14	16	18	20	25	30	35	40	50
Ash, Commercial White..	Select	1 100	1 076	1 055	1 023	978	913	658	457	336	257	164
	Common	880	868	857	840	818	784	647				
Cedar, Western Red; Fir, Balsam..............	Select	700	686	674	656	629	592	438	304	224	171	110
	Common	560	553	547	538	524	505	425				
Cedar, Northern and Southern White.......	Select	550	540	530	516	496	468	351	244	179	137	88
	Common	440	435	430	423	412	398	338				
Chestnut; Pine, Northern White, Idaho White, Sugar Calif White, and Pondosa..............	Select	750	733	718	695	663	617	438	304	224	171	110
	Common	600	591	583	572	556	532	434				
Cypress, Southern; Larch, Western..............	Select	1 100	1 063	1 030	981	909	810	526	365	268	206	132
	Common	880	861	843	818	781	729					
Douglas Fir (Coast Region); Pine, Southern Yellow; Beech; Birch, Yellow and Sweet; Maple, Sugar.........	Dense Select	1 285	1 251	1 222	1 176	1 112	1 022	702	487	358	274	175
	Select	1 175	1 149	1 127	1 093	1 045	975	702				
	Common	880	870	861	847	826	796	675				
Douglas Fir (Rocky Mtn Region); Spruce, Red, White, Sitka; Norway Pine; Alaska Cedar; Elm, Slippery and White; Sycamore; Gum, Red and Black; Tupelo	Select	800	786	774	753	726	688	526	365	268	206	132
	Common	640	632	627	617	602	582	500				
Hemlock, West Coast....	Select	900	885	872	852	823	783	614	426	313	240	153
	Common	720	712	706	696	680	660	573				
Hemlock, Eastern; Fir, Commercial White.....	Select	700	689	678	664	641	611	482	335	246	188	121
	Common	560	554	549	542	530	515	449				
Oak, White and Red.....	Select	1 000	982	967	943	908	860	658	457	336	257	164
	Common	800	790	783	771	753	728	625				
Redwood...............	Select	1 000	972	947	910	856	781	526	365	268	206	132
	Common	800	786	773	754	726	688					
Spruce, Englemann......	Select	600	586	574	556	530	494	351	244	179	137	88
	Common	480	473	466	457	444	426	347				
Tamarack..............	Select	1 000	976	955	923	877	817	570	896	291	223	142
	Common	800	788	777	761	737	706	566				

No column shall be used in which the unsupported length is more than 50 times the least diam; l and d must be in same unit of measurement.

Strength of wood depends considerably on moisture content. For wet locations working stresses should be reduced according to conditions and timber used.

When the load is concentrated at the vert struts, that is, for panel point loading, the stresses are found as in Art 17 and members designed accordingly. The only bending stress is due to wt of members themselves, which can be ignored except for long spans and small live loads. A uniformly-distributed load is carried partly by the beam and partly by truss action. The stresses can not be computed by ordinary means, but may be found as follows. In Fig 47a, king-post truss, W = total load, lb, on beam uniformly distributed. Compression in $BD = 5\ W + 8$; compression in $AB = 5\ Wl + 32\ h$; tension in $AD = 5\ Wm + 16\ h$. Design AD and BD for these stresses. Divide compression in AB by area of beam $= bd$, giving unit direct compression in AB; add to this the extreme fiber stress due to load acting on AB as a simple beam $= 2.25\ Wl + bd^2$, and design AB so this total will not exceed allowable. All lengths are in ft, except b and d = breadth and depth of AB, in. When the truss is above the beam, AD will be in compression, BD and AB in tension, and the latter must be designed for max tension due to direct and bending stress, using same formulas. (Fig 47b) queen-post truss. Approximately, compression stress in BE and $CF = W + 3$; compression in AD = tension in $EF = Wl + 9\ h$; tension in AE and $FD = Wm + 3\ h$. Extreme fiber stress in AB, BC, or CD, acting as a beam, $= Wl + bd^2$.

22. FASTENINGS AND JOINTS

Nails are classified as cut and wire. For lumber liable to split, cut nails are better than wire. They should be driven so that the tapering sides bear against the bent ends of the wood fiber, thus preventing splitting and acting like a wedge. Wire nails may be clinched or bent down; cut nails can not be bent. Table 13 gives common forms, designated in size by the "penny" system (d), probably originally the number of lb to 100 nails. Number per lb varies with different makers. Cost about 2–5¢ per lb. Nails should not be depended on for important joints, even in temporary structures. Large cut or wire nails are known as SPIKES. Boat or ship spikes are also made, forged from square bars with square head and chisel point. R R track spikes have a special head.

Table 13. Nails and Spikes

Size	Length, in	Cut		Wire		Boat spikes				
		No of nails per lb	No of spikes per lb	No of nails per lb	No of spikes per lb	Length, in	Size, in			
							3/8	7/16	1/2	5/8
2d	1	740	....	900	....	3	1 320	...	...	...
3d	1.25	460	....	615	....	4	1 140	...	...	...
4d	1.5	280	....	322	....	5	940	...	...	...
5d	1.75	210	....	250	....	6	800	600	450	...
6d	2	160	....	200	....	7	650	590	380	...
7d	2.25	120	....	154	....	8	600	510	340	260
8d	2.5	88	....	106	...	9	530	400	300	240
9d	2.75	73	....	85	...	10	480	360	280	220
10d	3	60	....	74	37	11	...	320	260	210
12d	3.25	46	....	57	32	12	...	230	240	190
16d	3.5	33	17	46	29	14	...	...	...	180
20d	4	23	14	29	23	16	.	.	.	160
30d	4.5	16	10	23	18					
40d	5	12	9	17	13					
50d	5.5	10	8	14	10					
60d	6	8	7	10	9					
...	6.5	...	6	...	8					
...	7	...	5	...	7					
...	8	...	....	...	6					
...	9	...	...	...	5					
...	10	...	...	...	4					
...	12				3					

A keg of nails or spikes weighs 100 lb. Table of boat spikes gives approx number in a keg of 200 lb.

Drift bolts are long, steel pins, generally round (square are used), with or without heads. 3/4 and 7/8-in diam are common sizes, but 1-in diam and up to 30 in long have been used. A hole should be bored 1/16 to 1/8 in less diam than bolt.

Treenails (wooden pins) are hard-wood pins, sometimes used as drift bolts, but commonly in mortise and tenon joints, etc. DOWELS are wood or iron pins extending into, but not through, 2 members, to connect them; they have neither point nor head. In heavy work, they are usually 0.75 to 2 in diam and 5 to 12 in long.

Screws comprise wood and lag screws. Wood screws are made to 6 in long, and some lengths in as many as 18 different diams. Thus, 3-in screws are from 8 to 26 gage (diam): 3.5-in, 8 to 26; 4-in, 8 to 30; and 4.5, 5, and 6-in, 12 to 30. Gage refers to special screw-gage, which is approx as follows: 6 = 0.14 in, 10 = 0.19, 15 = 0.26, 20 = 0.32, 25 = 0.39, and 30 = 0.45 in. For all large screws a hole should be bored less than diam of shank and about 0.5 its depth. For lag screws bore a hole equal in diam and length to shank, and then a smaller hole for about 0.5 length of thread. Lag screws have square or hexagonal heads, and are from 5/16 to 7/8-in diam for 3 in length;

5/16 to 1-in for 3 1/2 to 6-in length; 7/16 to 1 for 6.5 to 9-in; 0.5 to 1 for 10 to 12-in. Lengths vary by 0.5 in up to 8 in.

Bolts have round, square or hexagon heads, and are either rough or finished, rough being used for woodwork. Where used to prevent sliding of one member over another, the hole should be bored with a diam of 1/8 in less than the bolt, to give a close driving fit. Bolts have diam differing by 1/16 or 1/8-in. Where subject to vibration, lock nuts are used, or a nail is driven alongside the nut, or the thread may be "burred" to prevent nut working loose.

Washers (Fig 49) are important in timber work, to provide sufficient bearing area so that compression on surface of the timber may not exceed a safe value. They are of C I, steel or W I, and malleable iron. Common C-I washer is the Ogee (marked O G) form. Special washers can be cast when the number required warrants it. Steel and W-I washers are cut from plate, and are often too small and thin properly to distribute the bearing load. They should be about 4 times the diam of the bolt and 0.5 as thick.

Fig 48. Timber Fastenings

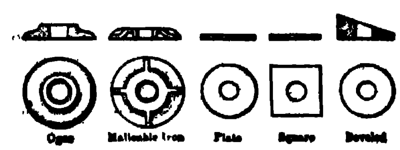


Fig 49. Washers

Table 14. C-I Ogee Washers (dimensions, in)

Diam of bolt	Diam of washer	Thickness	Wt of 100	Bearing area	Diam of bolt	Diam of washer	Thickness	Wt of 100	Bearing area
3/8	1 1/2	5/16	8.5	1.57	1	4	7/8	180	11.57
1/2	2	3/8	22	2.83	1	4	1	194	11.57
1/2	2 3/8	1/2	37.5	4.12	1 1/8	4 1/2	1	215	14.68
5/8	2 1/2	1/2	45	4.47	1 1/8	4 1/2	1 1/8	295	14.68
5/8	3	5/8	75	6.63	1 1/4	5	1 1/8	320	18.15
3/4	3	5/8	82	6.97	1 1/4	5 3/8	1 1/4	469	21.21
3/4	3 1/4	3/4	100	7.69	1 3/8	5 1/2	1 1/4	485	21.99
7/8	3 1/2	3/4	115	8.84	1 1/2	6	1 3/8	525	26.20
7/8	3 5/8	7/8	150	9 54	1 1/2	6	1 1/2	688	26.20

Anchor bolts have a thread cut at upper end to take a nut, lower end being embedded in the foundation. Neat cement grout has been shown to be best material for setting the bolt; if there is no vibration, plain rods may be used, proportioned for 15 000 lb per net section of bolt, or 100 to 150 lb per sq in of embedded surface of bolt. Embedment of about 20 diam is needed to develop full strength of bolt. For machine foundations, it is best to use a bolt with nut and large washer at lower end and build it into the concrete.

Resistance to pulling of nails and screws varies with character of wood, depth of penetration, and for screws with size of hole used. Cut nails have higher holding power than wire, and the safe load may be taken at 4d in yellow pine and 7d in oak, where d = penny number of nails driven across grain, to depth of 0.4 length; for wire nails, 2.5 and 4d. If subject to shock, the resistance is very low. For wood screws the safe resistance of a No 20 gage screw is twice that for a nail of same length. Lag screws show an ultimate resistance varying from 400 for white pine to 1 400 or more lb per sq in for oak; area being length

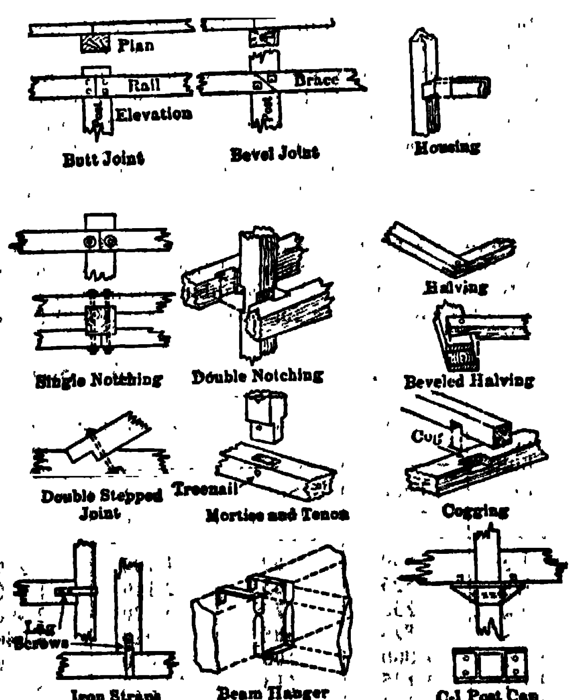


Fig 50. Timber Joints

(exclusive of point) times circumference of screw. Safe lateral resistance of nails is roughly twice the resistance to pulling.

Timber joints. (Fig 50, 51.) Butt joint is effective for compression members, the ends being secured by other connecting members or by fish plates. Notching is widely used, especially for cribs; sometimes called GAINING or "dapping"; or "sizing" when used in heavy framing to allow for inequalities in dimensions of timbers. HALVING is common in frame building construction, to join sills and wall plates. MORTISE AND TENON is a favorite joint for strength and rigidity. For 10 to 12-in timbers treenails should be 1 to 1.5 in diam, with a taper of $^1/_8$ to $^1/_4$ in. Hole in tenon should be about $^1/_3$ the length of tenon from shoulder, and slightly nearer the shoulder than that in mortise, so the pin will draw the timbers together. STEPPING is common in all kinds of framing.

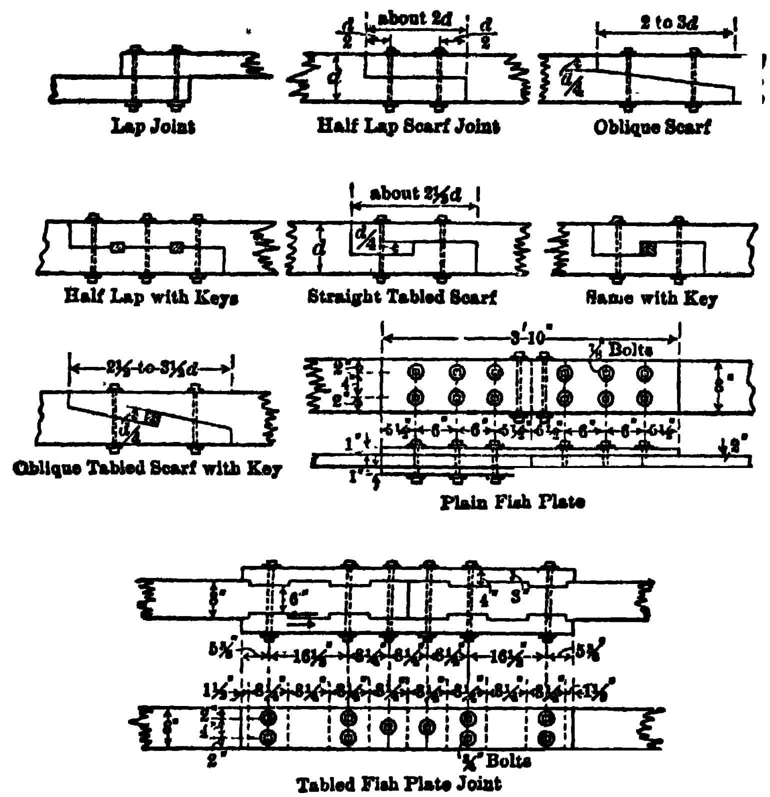


Fig 51. Lap and Scarf Joints

Metal straps are sometimes used in framing structures subject to shock. Metal plates are used in heavy construction, and for joints in structures like derricks, which are often taken down and set up again. Beam hangers and column caps are used in heavy floor construction. Lap joint is simple, but only used for connections in minor members, due to its eccentricity and consequent bending of pins or bolts. Half-lap scarf is used for joining members carrying small stresses, as guard rails on bridges, sills in buildings. Oblique scarf has more than double the strength of the half lap. Both joints are often used with fish plates in tension and compression members subjected to bending. Tabled scarf with key will take some tension, and when the amount is known the joint is easily designed for shear. Fish-plate joints, with wood or steel plates, are effective for tension. Bolts should be brought to solid bearing, to prevent timbers from buckling and to exclude moisture. Safe bearing loads per lin in are for safe bearing of 1 000 and 100 lb per sq in respectively. For other unit stresses, safe loads are obtained by proportion. For bending, the bolt is computed as a restrained beam with fiber stress of 22 500 lb per sq in (Table 15).

Design of a plain fish-plate joint (Fig 51). A tension timber to be spliced is 2 by 8-in longleaf yellow pine. Allowable stress, 1 200 lb per sq in net section. 2 fish plates of same material will have to be 1 in thick each. Number and size of bolts are found from Table 15. Thus, try 0.75-in bolts. Safe end bearing for the timber is 1 400 lb per sq in. Hence, safe bearing load on 0.75-in

Table 15. Strength of Bolts

Diam of bolt, in	Safe bearing, lb per lin in		Total safe uniform bending load on bolt, lb. a = thickness of member, b = thickness of fish plates					
	End grain	Side grain	$a = 2$ $b = 1$	$a = 3$ $b = 1.5$	$a = 4$ $b = 2$	$a = 6$ $b = 3$	$a = 8$ $b = 4$	$a = 10$ $b = 5$
1/2	500	50	1 100	740	550	370	280	220
5/8	625	63	2 160	1 440	1 080	720	540	430
3/4	750	75	3 720	2 490	1 860	1 250	930	750
7/8	875	88	5 920	3 940	2 960	1 970	1 470	1 180
1 1/8	1 000	100	8 820	5 880	4 410	2 940	2 210	1 760
1 1/4	1 125	113	12 560	8 390	6 290	4 190	3 140	2 510
1	1 250	125	17 250	11 500	8 640	5 750	4 300	3 450

bolt = 750 × 2 (= length of bearing) × 1 400 ÷ 1 000 = 2 100 lb. For bending, safe load = 3 720 lb; hence, bearing controls. Area of net section of member = 2 × (8 − 2 × $^7/_8$) = 12.5 sq in, assuming that bolts are placed in pairs and holes are $^1/_8$ in larger than bolt. Total tensile load = 12.5 × 1 200 = 15 000 lb, and number of 0.75-in bolts required = 15 000 ÷ 2 100 = 7. As an even number of bolts makes a better design, 6 $^7/_8$-in bolts will be used. Length of plate is determined by spacing of bolts, which tend to shear out a portion of the plate between planes tangent to top and bottom of bolt. Round bolts also tend to split the plates, so only 80% of safe shear with grain is used, or 80% of 150 lb per sq in = 120. Hence, the distance between bolt holes should be not less than 14 400 (= total load on net section for $^7/_8$-in bolts) ÷ [6(= number of bolts) × 2 (= thickness of 2 fish plates) × 2 (double shear) × 120 (shearing stress)] = 5 in, or, adding 1 in for bolt hole, = 6 in c to c and 5.5 for ends. The TABLED FISH-PLATE JOINT in Fig 51 was designed for a load of 32 tons in longleaf yellow pine. The design is as follows, using for tension 1 800, for compression on end of grain 2 000, and shear along grain, 250 lb per sq in. *a*. Find areas required for tension, compression and shear, or 64 000 ÷ 1 800 = 35.6, 64 000 ÷ 2 000 = 32, and 64 000 ÷ 250 = 256 sq in, respectively. 4 tables are usually effective, and the shearing area of each would be 256 ÷ 4 = 64 sq in. In order that the length of table shall be not less than $^2/_3$ its width (representing good proportions), the width of timber must be about 8 in, and the table a little over 8 in long to allow for bolt holes. Depth of tables must be 32 ÷ (4 × 8) = 1 in, to furnish necessary bearing area on their ends. Assuming $^3/_4$-in bolts and $^7/_8$-in holes, the net depth of main member for tension must be 35.6 ÷ (8 − 2 × $^7/_8$) = 5.75 in, making total depth of timber 8 in, or an 8 × 8 is needed. The fish plates, each taking 0.5 the total load and being of same material as main member, should be 0.5 as thick. Size of bolts may be roughly checked. Unlike the plain fish-plate joint, they transmit stress by tension. Thus, the pull on fish plate is approx through its center (see arrows in cut), and this is transmitted to the main member by end bearing on the tables, approx at their centers. This causes a moment of (6 400 ÷ 4) × (1.5 + 0.5) = 32 000 in lb per table, tending to cause it to rotate about its edge. This is resisted by tension in the bolt and compression on face of table. The lever arm of this couple may be taken as 0.5 the length of table, or 4 in, giving a pull on the bolt of 4 000 lb, assuming 2 bolts per table. A 0.75-in bolt is ample for this stress, using 15 000 lb per sq in, net sec. The washers should have sufficient area for bearing, using 550 lb per sq in. A standard 3.25 C-I washer has an area of 7.69 sq in, and will transmit a total stress of 4 230 lb, which is OK. Bolts should be pulled up to a tight bearing.

Special joint fastenings have been developed in recent years which greatly increase effectiveness of timber joints. These consist of various types of bushings in the form of washers, which are countersunk in the timber and increase the bearing diam of the bolts. When available or where size of construction warrants special purchase, these patented fastenings are advantageous.

23. TRESTLES, TRUSSES AND BRIDGES (12)

Trestles consist of posts framed together to form bents, supporting a floor system of stringers, ties, and guard rails. They are used for semi-permanent work, to avoid expenditure for permanent bridge or fill construction in the early days of a new R R, for making large fills, and in connection with mine dumps.

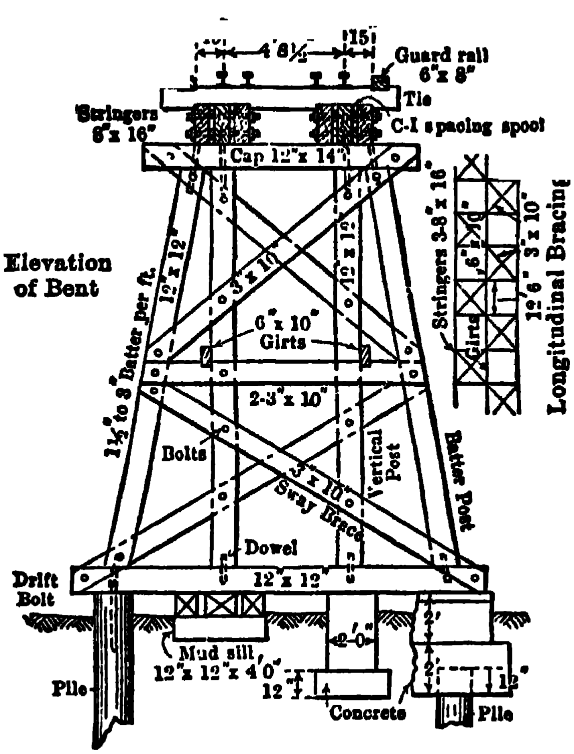


Fig 52. R R Trestle Bent

Fig 52 shows a trestle bent, with different types of foundations used indicated. Where piles are used they may be long enough to form the posts (pile-bent), being sway-braced as for a framed bent. Bents are built in stories of 15 or more ft high. Fig 52 is a 2-story bent. Posts may be designed as columns, but are made larger than computations require to allow for decay. Stringers should be continuous over 2 spans, and are designed for bending stresses. Bents are spaced 12–16 ft apart, depending upon wt of loading. For light mine cars, the construction would be much lighter than that shown, which is for R R work.

Roof truss of heavy design is shown in detail by Fig 53. Load from the rafters is carried to the panel points by the purlins; the stresses may be found (Art 17) for any assumed loading, and the members and details designed accordingly.

Small highway bridge, consisting of a queen truss with plank floor, is shown in Fig 54.

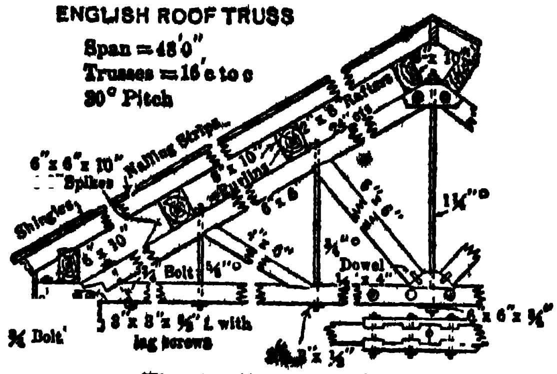


Fig 53. Heavy Roof Truss

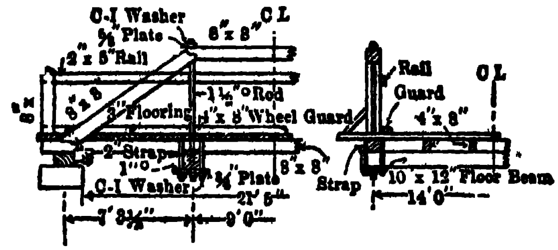


Fig 54. Small Highway Bridge

24. FRAME BUILDINGS (13)

Framing is of 3 kinds: (*a*) Mortise and tenon frame, in which the members are mortised and pinned together. This requires a large amount of labor and for light buildings has been practically abandoned; (*b*) Balloon frame, composed of lighter members, nailed together, and more quickly erected at lower cost; (*c*) Combination frame, in which main members are mortised as in (*a*) and the studding and bracing is nailed, used in the best modern construction. Fig 55 shows typical balloon frame for a 2-story house. Computations are rarely necessary. Sill is usually 4 by 6 in, halved at corners, on foundation walls or piers. Floor joists are of 2-in plank, 12 or 16 in c to c; for joists over 12 in deep, 3-in thickness should be used. Joists are braced laterally by cross bridging, 1 row for 8 to 16-ft spans, and 2 for 18 to 24-ft spans, with 1 by 3-in bridging, or 2 by 3 for 14-in beams, nailed with 2 10d nails. Corner posts, usually 4 by 4 or 4 by 6-in, are set up and temporarily secured by "stay lath." Studs are held by a temporary board nailed against inside. Studs are 2 by 4 in, usually 16-in centers; they are spiked to the sills at lower ends, and when practicable studs and joists are placed against and spiked to each other. Studs extend from the sill up to the plate, the second floor joists being supported on 1 by 7-in ledger boards, notched into the studs. The plate is usually of two 2 by 4-in pieces. Joints in studs are made by nailing a fish plate on each side. Studding is doubled at window and door openings. 1 by 6-in boards, notched into sill, post and studs, are used for corner braces. Rafters, 2 by 6 in, for lengths of 12 ft, 2 by 8 to 18, 2 by 10 over 18 ft, are spaced 16 to 24 in centers. Combination frame posts are usually 4 by 8 in, and the ledger board is replaced by a 4 by 8-in girt, framed into the posts, the studding being separate for each story.

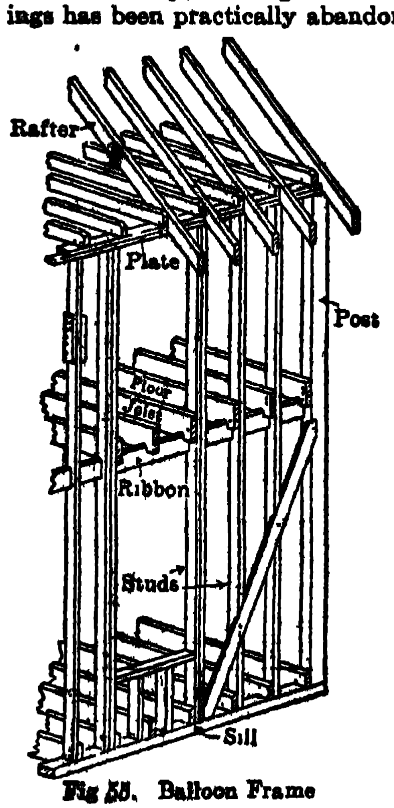


Fig 55. Balloon Frame

Partitions are usually of 2 by 4-in studding, spaced 12 to 16-in centers. In framing, it should be remembered that timber shrinks but little endwise of the grain, and sometimes as much as 0.5 in per ft crosswise. Where settlement is to be avoided, all horiz members should be eliminated from vertical supports of the buildings. Partitions should rest on the cap of the partition below when possible (not on the floor), or on a sole piece.

Covering. For cheap construction ship-lap is often placed vertically, with battens over the cracks. Beveled siding is commonly 6 in wide and laid 4.5 in to the weather. Molded siding (Novelty siding), is of 7/8-in boards, with 5/8-in lap. In best work, the frame is first covered with 1-in diagonal sheathing, which in turn is covered with building paper before the siding is placed. Shingles or corrugated sheets are also used for siding.

Shingle roof. Shingles vary from 16 to 24 in long, laid with 4.5 to 5.5 in (a little less than 1/3 the length) to weather. 4 bundles usually contain a "thousand" shingles, that is, the equiva-

lent of 1 M shingles 4 in wide, the widths varying from 3 to 14 in. Table 17 gives number of standard 4-in shingles for 100 sq ft of roof. Cost laid, \$8.00 to \$10 per square (= 100 sq ft).

Tin roofing. There are 2 common sizes of tin, 14 by 20 in and 20 by 28 in, latter being most used. The usual thicknesses are IC (No 29 gage) and IX (No 27). Value of tin roofing depends on amount of tin used in coating the iron. If well painted every 2 or 3 years it will last 25 or 30 yr. Laid with flat seam for flat roofs. Using 0.5-in locks, 1 box (112 sheets) of 14 by 20-in tin will cover 192 sq ft; 20 by 28-in, 399 sq ft. For steep roofs, a standing seam is used. With $^3/_8$-in lock, and 1-in standing seam, a 14 by 20 box will cover 168 sq ft, and the larger size 365 ft. Aver cost in place, including material and painting, \$15 to \$20 per square.

Table 16. Shingle Roofing

Length	Weather	No of shingles	No of shingle nails	Lb of shingle nails
14 in	4 in	900	1 800	4.5
16 "	5 "	720	1 440	3.6
18 "	5 1/2 "	655	1 310	3.3
20 "	6 "	600	1 200	3.0
22 "	6 1/2 "	554	1 110	2.8

Corrugated iron sheets, for roofs and sides of buildings, are made in lengths of 5, 6, 7, 8, 9 and 10 ft, and a covering width of 24 in. For siding, a side lap of 1 corrugation and 4-in end lap is usual; for 1 to 4 pitch roofs, 1.5 corrugations are used on edges and 6 in on ends. Usual corrugation is 2.5 in wide by 0.5 in deep, and for roofs No 20 and 22 gage are common. Sheets are made painted or galvanized, in No 12, 14, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28 U S Standard Gage thicknesses. Purlins should be less than 6 ft centers. Sheets, held by nails, or by steel straps encircling the purlins and placed 12-in centers, should be well secured at eaves and gable ends of roof to prevent their being loosened by wind. Safe uniform distributed load in lb per sq ft = 25 000 $td \div L^2$, where t = thickness of sheet, in, d = depth of corrugation, in, and L = unsupported length of sheet, ft.

Table 17. Corrugated Sheets (See also Sec 41, Table 21)

Description of sheets				Width, in		Sq ft of sheets to cover 100 sq ft			
Corrugations									
Width, in		Approx depth, in	No per sheet	Full sheet	Covers approx	End lap, in	Side lap corrugations		
Nominal	Actual						1	1.5	2
5	4 2/3	7/8	6	28	24	1	110	116	123
3	2 8/9	5/8	9	26	24	2	111	117	124
2 1/2	2 3/5	1/2	10	26	24	3	112	118	125
2	2 4/11	1/2	11	26	24	4	113	119	126
1 1/4	1 1/4	3/8	20	25	24	5	114	120	127
5/8	25/26	3/16	26	25	24	6	115	121	128

Standard lengths, 5, 6, 7, 8, 9 and 10 ft; max length, 12 ft for 1.25 to 5-in corrugations.

Tar and gravel roofing is often used for flat roofs. When laid on wood a layer of building felt (unsaturated) is first placed. This is covered with tarred felt, nailed down. Final covering, of 1 to 3 layers of tarred felt, is placed on the roof, each layer being thoroughly mopped down with hot pitch. Entire outer surface is covered with pitch, in which $^1/_4$ to $^5/_8$ in dry gravel or slag is embedded while hot (400 lb per square). 4 or 5-ply roofing costs \$15 per square. TAR PAPER, nailed on with large tin washers, costs from \$4 to \$6 per square. Tar paper shingles, surfaced with crushed slate, are common. Several patented forms are also used, costing \$2–\$4, up; for more permanent construction, slate, tile, and asbestos shingles.

Partitions and ceilings. When plaster is used, 2 coats (rough or scratch, and finish coats) are applied to lath. Lath is of wood (0.25 by 1.5 in by 4 ft) nailed to studding, or perforated sheet metal or mesh, or patented composition board. The rough coat is lime and sand mortar; finish coat, usually of special plaster. Patented wall boards (fiber and plaster board) have largely replaced plaster in cheaper work and cost 6–10¢ sq ft in place. Made in sheets usually 4 ft wide and 8 or 12 ft long. Metal ceiling (stamped steel sheets) is also available.

Flooring. Single flooring is usual, of 1-in tongued and grooved pine or Douglas fir, from 12 in down to 3 or 4 in wide. Where subjected to hard wear, as in halls, and not carpeted, edge-grain or rift-sawed flooring is best (Art 20). Flooring comes in many grades, depending on grain, knots, etc, and in lengths of 8 to 20 ft. Double flooring, for higher class work. In figuring the amount of flooring, generally add about 25% to area to be covered, to allow for face width being less than actual width, and for cutting to bring joints over floor joists.

Stairs. Usual rise, 7 to 7.5 in; max, 8 in. Tread = run + overhang (1.5 in), and run should be such that rise + run = 17 or 17.5 in. With no nosing (overhang), tread should be 10 to 12 in wide.

Door and window frames are placed in the openings left in the studding for the purpose; they are smaller in size than these openings so they can be set plumb and true. In cheapest construction, no frames are used, the windows and doors being set in the openings between studs.

Doors. Stock doors are 1 $^1/_8$, 1 $^3/_8$ and 1 $^3/_4$ in thick; 1 $^3/_8$ in is usual for inside doors less than 3 by 7 ft. Outside doors are 1.75 in. Common sizes are 2–3 ft wide and 6.5 to 7 ft high, varying by even in.

Windows. Double-hung windows have 2, 4, 8 and 12 lights and frames are 1 3/8 and 1 3/4 in thick, usually with 3/8-in parting strips. Two-light windows are for glass from 16 by 24 to 30 by 40 in; 4-light windows, 10 by 20 to 15 by 32; 8-light, 9 by 12–14 by 24; 12-light, 8 by 10–12 by 24. Windows are generally 4 in wider and 6 in higher than the width and combined height of glass. Single-thickness glass is about 1/12 in thick; double-thickness, scant 1/8 in, is usual for widths over 24 in.

Cost. Simple frame buildings, as outlined above, vary in total cost from 15 to 30¢ per cu ft, depending on finish.

Slow-burning or mill construction is often used for more important buildings, as for manufacturing and storage. Its chief characteristic is absence of small timbers. Floors are generally supported by heavy timber columns. Beam hangers and column caps are widely used (Art 25) and flooring is heavier. Such construction is carefully computed to carry required loads.

STEEL STRUCTURES

25. STRENGTH OF IRON AND STEEL (4)

Table 18 gives aver physical and mechanical properties of metals for structural work.

Cast iron is, in general, somewhat brittle, is subject to hidden defects, such as blow holes, cold-shuts, and shrinkage cracks, has relatively low tensile strength, no true elastic behavior and is used only for thick, heavy parts, as short columns, engine frames and bed plates and pipes. Good castings should have clear, sharp corners and should be soft enough to dent under a hammer. Better grades of malleable castings are used for small machine parts and pipe fittings.

Wrought iron is tough, ductile, malleable and easily welded, but can not be tempered. Formerly widely used, it has been superseded by low-carbon steel. WI pipe is said to resist corrosion better than steel; purer iron under various trade names, used for galvanized culverts, is best of all. Etching for 20 min with 3 parts H_2SO_4, 1 part HCL in 9 parts water, produces a characteristic fibrous structure in WI.

Table 18. Physical Properties of Iron and Steel

Material	Wt, lb per cu ft	Modulus elasticity*		Yield point†		Ult strength†			Working stress†		
		T & C	S (t)	T & C	S (t)	T	C	S	T	C	S
Iron: Gray, cast	450	15	6	..	..	25	100	25	4	16	4
Malleable, cast	475	22	8.8	..	..	45	110	45	8	20	8
Wrought	480	37	11	25	25	50	70	40	12	17	10
Steel: 0.1–0.2% carbon	490	30	12	35	20	60	90	48	18	18	12
0.3–0.4% "	490	30	12	40	24	80	45	64	20	20	16
0.7–0.8% "	490	30	12	60	36	125	70	85	30	30	21
Nickel, H T	490	30	12	85	55	112	95	80	25	25	20
Brass, rolled	520	15.5	6.2	25	17	73	30	47	18	15	11
Bronze, "	535	15	5.6	35	30	65	25	43	16	12	10
Aluminum, structural alloy	173	10	3.7	35	35	58	58	35	14.5	14.5	8.8

Values in thousands of lb per sq in, except as noted. T = Tension. C = Compression. S = Shear. t = Torsion. * Millions of lb per sq in. † Thousand lb per sq in.

Structural steel, first made in quantity by Bessemer process, is now largely manufactured in open-hearth furnaces. Low-carbon steel is used for work where ductility is required, medium grades in structural work, but high carbon is being more widely used in reinforcement bars and other work. Various special alloy steels have been used in large bridge construction, as nickel and silicon, but are not available for smaller structural work. In tool and machine design alloy steels (chrome and tungsten) are used; also special heat treatment and other refinements to seecure metal best adapted to needs and loads are characteristic of modern practice. A structural aluminum alloy (Table 18) is available, but has not yet been widely employed.

Specifications. Standard specifications of Am Soc for Testing Materials are generally followed. For bridge work, these limit material to open hearth make, with a phosphorus content not over 0.04–0.06% and sulphur not over 0.05%. A tensile strength of 55 000–65 000 lb per sq in, a yield point of 0.5 of this, or not less than 30 000, and an elongation in per cent in an 8-in length of 150 000 ÷ tensile strength, is specified. Rivet steel is of somewhat softer grade, with tensile strength of 46 000–56 000 lb per sq in. Certain limits are also placed on variations in thickness of plates and wt per ft.

Table 19. American Standard I-Beams

Nominal size, in	Wt per ft, lb	Area of section, sq in	Depth of section, in	Width of flange, in	Web thickness, in	Axis 1-1			Axis 2-2		
						I in⁴	S in³	r in	I in⁴	S in³	r in
24 × 7 7/8	120.0	35.13	24.00	8.048	0.798	3 010.8	250.9	9.26	84.9	21.1	1.56
	115.0	33.67	24.00	7.987	.737	2 940.5	245.0	9.35	82.8	20.7	1.57
	110.0	32.18	24.00	7.925	.675	2 869.1	239.1	9.44	80.6	20.3	1.58
	105.9	30.98	24.00	7.875	.625	2 811.5	234.3	9.53	78.9	20.0	1.60
24 × 7	100.0	29.25	24.00	7.247	.747	2 371.8	197.6	9.05	48.4	13.4	1.29
	95.0	27.79	24.00	7.186	.686	2 301.5	191.8	9.08	47.0	13.0	1.30
	90.0	26.30	24.00	7.124	.624	2 230.1	185.8	9.21	45.5	12.8	1.32
	85.0	24.84	24.00	7.063	.563	2 159.8	180.0	9.33	44.2	12.5	1.33
	79.9	23.33	24.00	7.000	.500	2 087.2	173.9	9.46	42.9	12.2	1.36
20 × 7	100.0	29.20	20.00	7.273	.873	1 648.3	164.8	7.51	52.4	14.4	1.34
	95.0	27.74	20.00	7.200	.800	1 599.7	160.0	7.59	50.5	14.0	1.35
	90.0	26.26	20.00	7.126	.726	1 550.3	155.0	7.68	48.7	13.7	1.36
	85.0	24.80	20.00	7.053	.653	1 501.7	150.2	7.78	47.0	13.3	1.38
	81.4	23.74	20.00	7.000	.600	1 466.3	146.6	7.86	45.8	13.1	1.39
20 × 6 1/4	75.0	21.90	20.00	6.391	.641	1 263.5	126.3	7.60	30.1	9.4	1.17
	70.0	20.42	20.00	6.317	.567	1 214.2	121.4	7.71	28.9	9.2	1.19
	65.4	19.08	20.00	6.250	.500	1 169.5	116.9	7.83	27.9	8.9	1.21
18 × 6	70.0	20.46	18.00	6.251	.711	917.5	101.9	6.70	24.5	7.8	1.09
	65.0	18.98	18.00	6.169	.629	877.7	97.5	6.80	23.4	7.6	1.11
	60.0	17.50	18.00	6.087	.547	837.8	93.1	6.92	22.3	7.3	1.13
	54.7	15.94	18.00	6.000	.460	795.5	88.4	7.07	21.2	7.1	1.15
15 × 6	75.0	21.85	15.00	6.278	.868	687.2	91.6	5.61	30.6	9.8	1.18
	70.0	20.38	15.00	6.180	.770	659.6	87.9	5.69	28.8	9.3	1.19
	65.0	18.91	15.00	6.082	.672	632.1	84.3	5.78	27.2	8.9	1.20
	60.8	17.68	15.00	6.000	.590	609.0	81.2	5.87	26.0	8.7	1.21
15 × 5 1/2	55.0	16.06	15.00	5.738	.648	508.7	67.8	5.63	17.0	5.9	1.03
	50.0	14.59	15.00	5.640	.550	481.1	64.2	5.74	16.0	5.7	1.05
	45.0	13.12	15.00	5.542	.452	453.6	60.5	5.88	15.0	5.4	1.07
	42.9	12.49	15.00	5.500	.410	441.8	58.9	5.95	14.6	5.3	1.08
12 × 5 1/4	55.0	16.04	12.00	5.600	.810	319.3	53.2	4.46	17.3	6.2	1.04
	50.0	14.57	12.00	5.477	.687	301.6	50.3	4.55	16.0	5.8	1.05
	45.0	13.10	12.00	5.355	.565	284.1	47.3	4.66	14.8	5.5	1.06
	40.8	11.84	12.00	5.250	.460	268.9	44.8	4.77	13.8	5.3	1.08
12 × 5	35.0	10.20	12.00	5.078	.428	227.0	37.8	4.72	10.0	3.9	.99
	31.8	9.26	12.00	5.000	.350	215.8	36.0	4.83	9.5	3.8	1.01
10 × 4 3/4	40.0	11.69	10.00	5.091	.741	158.0	31.6	3.68	9.4	3.7	.90
	35.0	10.22	10.00	4.944	.594	145.8	29.2	3.78	8.5	3.4	.91
	30.0	8.75	10.00	4.797	.447	133.5	26.7	3.91	7.6	3.2	.93
	25.4	7.38	10.00	4.660	.310	122.1	24.4	4.07	6.9	3.0	.97
8 × 4	25.5	7.43	8.00	4.262	.532	68.1	17.0	3.03	4.7	2.2	.80
	23.0	6.71	8.00	4.171	.441	64.2	16.0	3.09	4.4	2.1	.81
	20.5	5.97	8.00	4.079	.349	60.2	15.1	3.18	4.0	2.0	.82
	18.4	5.34	8.00	4.000	.270	56.9	14.2	3.26	3.8	1.9	.84
7 × 3 3/4	20.0	5.83	7.00	3.860	.450	41.9	12.0	2.68	3.1	1.6	.74
	17.5	5.09	7.00	3.755	.345	38.9	11.1	2.77	2.9	1.6	.76
	15.3	4.43	7.00	3.660	.250	36.2	10.4	2.86	2.7	1.5	.78
6 × 3 3/8	17.25	5.02	6.00	3.565	.465	26.0	8.7	2.28	2.3	1.3	.68
	14.75	4.29	6.00	3.443	.343	23.8	7.9	2.36	2.1	1.2	.69
	12.5	3.61	6.00	3.330	.230	21.8	7.3	2.46	1.8	1.1	.72
5 × 3	14.75	4.29	5.00	3.284	.494	15.0	6.0	1.87	1.7	1.0	.63
	12.25	3.56	5.00	3.137	.347	13.5	5.4	1.95	1.4	.91	.63
	10.0	2.87	5.00	3.000	.210	12.1	4.8	2.05	1.2	.82	.65
4 × 2 3/4	10.5	3.05	4.00	2.870	.400	7.1	3.5	1.52	1.0	.70	.57
	9.5	2.76	4.00	2.796	.326	6.7	3.3	1.56	.91	.65	.58
	8.5	2.46	4.00	2.723	.253	6.3	3.2	1.60	.83	.61	.58
	7.7	2.21	4.00	2.660	.190	6.0	3.0	1.64	.77	.58	.59
3 × 2 3/8	7.5	2.17	3.00	2.509	.349	2.9	1.9	1.15	.59	.47	.52
	6.5	1.88	3.00	2.411	.251	2.7	1.8	1.19	.51	.43	.52
	5.7	1.64	3.00	2.330	.170	2.5	1.7	1.23	.46	.40	.53

26. TABLES AND SPECIFICATIONS

Standard rolled shapes (Fig 56). Structural parts are composed of, or built up of, regular standard shapes, comprising I-beams, channels, angles, and tees. These are

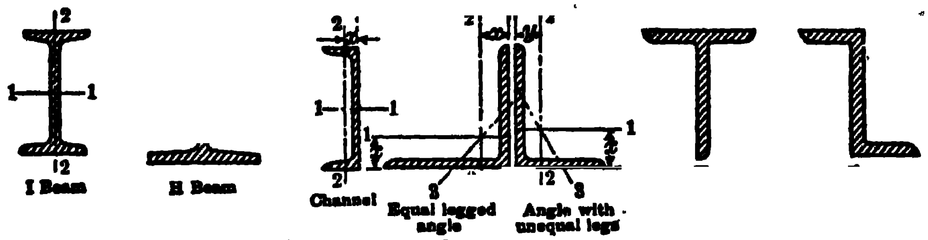


Fig 56. Standard Rolled Shapes

Table 20. American Standard Channels

Nominal size, in	Wt per ft, lb	Area of section, sq in	Depth of section, in	Width of flange, in	Web thickness, in	Axis 1–1 I in⁴	Axis 1–1 S in³	Axis 1–1 r in	Axis 2–2 I in⁴	Axis 2–2 S in³	Axis 2–2 r in	Axis 2–2 x in
18 × 4	58.0	16.98	18.00	4.200	0.700	670.7	74.5	6.29	18.5	5.6	1.04	0.88
	51.9	15.18	18.00	4.100	.600	622.1	69.1	6.40	17.1	5.3	1.06	.87
	45.8	13.38	18.00	4.000	.500	573.5	63.7	6.55	15.8	5.1	1.09	.89
	42.7	12.48	18.00	3.950	.450	549.2	61.0	6.64	15.0	4.9	1.10	.90
15 × 3 1/2	55.0	16.11	15.00	3.814	.814	429.0	57.2	5.16	12.1	4.1	.87	.82
	50.0	14.64	15.00	3.716	.716	401.4	53.6	5.24	11.2	3.8	.87	.80
	45.0	13.17	15.00	3.618	.618	373.9	49.8	5.33	10.3	3.6	.88	.79
	40.0	11.70	15.00	3.520	.520	346.3	46.2	5.44	9.3	3.4	.89	.78
	35.0	10.23	15.00	3.422	.422	318.7	42.5	5.58	8.4	3.2	.91	.79
	33.9	9.90	15.00	3.400	.400	312.6	41.7	5.62	8.2	3.2	.91	.79
12 × 3	40.0	11.73	12.00	3.415	.755	196.5	32.8	4.09	6.6	2.5	.75	.72
	35.0	10.26	12.00	3.292	.632	178.8	29.8	4.18	5.9	2.3	.76	.69
	30.0	8.79	12.00	3.170	.510	161.2	26.9	4.28	5.2	2.1	.77	.63
	25.0	7.32	12.00	3.047	.387	143.5	23.9	4.43	4.5	1.9	.79	.68
	20.7	6.03	12.00	2.940	.280	128.1	21.4	4.61	3.9	1.7	.81	.70
10 × 2 5/8	35.0	10.27	10.00	3.180	.820	115.2	23.0	3.34	4.6	1.9	.67	.69
	30.0	8.80	10.00	3.033	.673	103.0	20.6	3.42	4.0	1.7	.67	.65
	25.0	7.33	10.00	2.886	.526	90.7	18.1	3.52	3.4	1.5	.68	.62
	20.0	5.86	10.00	2.739	.379	78.5	15.7	3.66	2.8	1.3	.70	.61
	15.3	4.47	10.00	2.600	.240	66.9	13.4	3.87	2.3	1.2	.72	.64
9 × 2 1/2	25.0	7.33	9.00	2.812	.612	70.5	15.7	3.10	3.0	1.4	.64	.61
	20.0	5.86	9.00	2.648	.448	60.6	13.5	3.22	2.4	1.2	.65	.59
	15.0	4.39	9.00	2.485	.285	50.7	11.3	3.40	1.9	1.0	.67	.59
	13.4	3.89	9.00	2.430	.230	47.3	10.5	3.49	1.8	.97	.67	.61
8 × 2 1/4	21.25	6.23	8.00	2.619	.579	47.6	11.9	2.77	2.2	1.1	.60	.59
	18.75	5.49	8.00	2.527	.487	43.7	10.9	2.82	2.0	1.0	.60	.57
	16.25	4.76	8.00	2.435	.395	39.8	9.9	2.89	1.8	.94	.61	.56
	13.75	4.02	8.00	2.343	.303	35.8	9.0	2.99	1.5	.86	.62	.56
	11.5	3.36	8.00	2.260	.220	32.3	8.1	3.10	1.3	.79	.63	.58
7 × 2 1/8	19.75	5.79	7.00	2.509	.629	33.1	9.4	2.39	1.8	.96	.56	.58
	17.25	5.05	7.00	2.404	.524	30.1	8.6	2.44	1.6	.86	.56	.55
	14.75	4.32	7.00	2.299	.419	27.1	7.7	2.51	1.4	.79	.57	.53
	12.25	3.58	7.00	2.194	.314	24.1	6.9	2.59	1.2	.71	.58	.53
	9.8	2.85	7.00	2.090	.210	21.1	6.0	2.72	.98	.63	.59	.55
6 × 2	15.5	4.54	6.00	2.279	.559	19.5	6.5	2.07	1.3	.73	.53	.55
	13.0	3.81	6.00	2.157	.437	17.3	5.8	2.13	1.1	.65	.53	.52
	10.5	3.07	6.00	2.034	.314	15.1	5.0	2.22	.87	.57	.53	.50
	8.2	2.39	6.00	1.920	.200	13.0	4.3	2.34	.70	.50	.54	.52
5 × 1 3/4	11.5	3.36	5.00	2.032	.472	10.4	4.1	1.76	.82	.54	.49	.51
	9.0	2.63	5.00	1.885	.325	8.8	3.5	1.83	.64	.45	.49	.48
	6.7	1.95	5.00	1.750	.190	7.4	3.0	1.95	.48	.38	.50	.49
4 × 1 5/8	7.25	2.12	4.00	1.720	.320	4.5	2.3	1.47	.44	.35	.46	.46
	6.25	1.82	4.00	1.647	.247	4.1	2.1	1.50	.38	.32	.45	.46
	5.4	1.56	4.00	1.580	.180	3.8	1.9	1.56	.32	.29	.45	.46
3 × 1 1/2	6.0	1.75	3.00	1.596	.356	2.1	1.4	1.08	.31	.27	.42	.46
	5.0	1.46	3.00	1.498	.258	1.8	1.2	1.12	.25	.24	.41	.44
	4.1	1.19	3.00	1.410	.170	1.6	1.1	1.17	.20	.21	.41	.44

usually designated by nominal outside dimensions and wt per ft as 15 I 42.9, indicating a 15 in I-beam weighing 42.9 lb per lin ft. For complete data on wide flange, regular, standard and special shapes, see *Steel Construction*, published by Am Inst Steel Const. Tables 19–22 give the regular shapes, made by all mills rolling these products; other shapes, wide flange, column sections, T's and Z's, are only occasionally rolled, and may not be in stock, or are rolled only on order. In the tables, I = moment of inertia, S = section modulus, and r = radius of gyration (Art 4, 5).

Table 21. Angles, Equal Legs

Size, in	Thickness, in	Wt per ft, lb	Area of section, sq in	Axis 1–1 and axis 2–2				Axis 3–3
				I in⁴	S in³	r in	x in	r in
8 × 8	1 1/8	56.9	16.73	98.0	17.5	2.42	2.41	1.56
	1 1/16	54.0	15.87	93.5	16.7	2.43	2.39	1.56
	1	51.0	15.00	89.0	15.8	2.44	2.37	1.56
	15/16	48.1	14.12	84.3	14.9	2.44	2.34	1.56
	7/8	45.0	13.23	79.6	14.0	2.45	2.32	1.57
	13/16	42.0	12.34	74.7	13.1	2.46	2.30	1.57
	3/4	38.9	11.44	69.7	12.2	2.47	2.28	1.57
	11/16	35.8	10.53	64.6	11.3	2.48	2.25	1.58
	5/8	32.7	9.61	59.4	10.3	2.49	2.23	1.58
	9/16	29.6	8.68	54.1	9.3	2.50	2.21	1.58
	1/2	26.4	7.75	48.6	8.4	2.50	2.19	1.59
6 × 6	1	37.4	11.00	35.5	8.6	1.80	1.86	1.17
	15/16	35.3	10.37	33.7	8.1	1.80	1.84	1.17
	7/8	33.1	9.73	31.9	7.6	1.81	1.82	1.17
	13/16	31.0	9.09	30.1	7.2	1.82	1.80	1.17
	3/4	28.7	8.44	28.2	6.7	1.83	1.78	1.17
	11/16	26.5	7.78	26.2	6.2	1.83	1.75	1.17
	5/8	24.2	7.11	24.2	5.7	1.84	1.73	1.18
	9/16	21.9	6.43	22.1	5.1	1.85	1.71	1.18
	1/2	19.6	5.75	19.9	4.6	1.86	1.68	1.18
	7/16	17.2	5.06	17.7	4.1	1.87	1.66	1.19
	3/8	14.9	4.36	15.4	3.5	1.88	1.64	1.19
5 × 5	1	30.6	9.00	19.6	5.8	1.48	1.61	.97
	15/16	28.9	8.50	18.7	5.5	1.48	1.59	.97
	7/8	27.2	7.98	17.8	5.2	1.49	1.57	.97
	13/16	25.4	7.46	16.8	4.9	1.50	1.55	.97
	3/4	23.6	6.94	15.7	4.5	1.51	1.52	.97
	11/16	21.8	6.40	14.7	4.2	1.51	1.50	.98
	5/8	20.0	5.86	13.6	3.9	1.52	1.48	.98
	9/16	18.1	5.31	12.4	3.5	1.53	1.46	.98
	1/2	16.2	4.75	11.3	3.2	1.54	1.43	.98
	7/16	14.3	4.18	10.0	2.8	1.55	1.41	.98
	3/8	12.3	3.61	8.7	2.4	1.56	1.39	.99
4 × 4	3/4	18.5	5.44	7.7	2.8	1.19	1.27	.78
	11/16	17.1	5.03	7.2	2.6	1.19	1.25	.78
	5/8	15.7	4.61	6.7	2.4	1.20	1.23	.78
	9/16	14.3	4.18	6.1	2.2	1.21	1.21	.78
	1/2	12.8	3.75	5.6	2.0	1.22	1.18	.78
	7/16	11.3	3.31	5.0	1.8	1.23	1.16	.78
	3/8	9.8	2.86	4.4	1.5	1.23	1.14	.79
	5/16	8.2	2.40	3.7	1.3	1.24	1.12	.79
	1/4	6.6	1.94	3.0	1.1	1.25	1.09	.80

Examples. Steel makers' handbooks give tables for safe load for shapes used as beams and columns, but these estimates are easily secured from the data given. (*a*) Required size of I-beam to carry central load of 2 tons on span of 15 ft. From Table 1, $M = Pl \div 4 = 180\,000$ inch-lb. Assuming approx wt of beam as 15 lb per ft, there should be added for dead load about 5 000, making total $M = 185\,000$ inch-lb. From Art 4, $M = kS$, or required section modulus S, using a fiber stress $k = 18\,000$, is 10.3. From Table 19, this will require a 7-in I, of 15.3 lb per ft. (*b*) Find the safe load for an 8 by 6, 0.5-in L, 10 ft high acting as a column. From Table 22, least radius of gyration $r = 1.30$, hence $l \div r = 92$ and from Art 5 and 29, $p = 17\,000 - 0.485\,(l^2 \div r^2) = 12\,900$ lb per sq in. Also from Table 22 area of sec $= 6.75$ sq in, hence safe load $= 6.75 \times 12\,900 = 87\,000$ lb.

Table 22. Angles, Unequal Legs

Size, in	Thickness, in	Wt per ft, lb	Area of section, sq in	Axis 1-1				Axis 2-2				Axis 3-3
				I in⁴	S in³	r in	x in	I in⁴	S in³	r in	y in	r in
8×6	1 1/8	49.3	14.48	88.9	16.8	2.48	2.70	42.5	9.9	1.71	1.70	1.28
	1 1/16	46.8	13.75	84.9	15.9	2.48	2.68	40.7	9.4	1.72	1.68	1.28
	1	44.2	13.00	80.8	15.1	2.49	2.65	38.8	8.9	1.73	1.65	1.28
	15/16	41.7	12.25	76.6	14.3	2.50	2.63	36.9	8.4	1.73	1.63	1.28
	7/8	39.1	11.48	72.3	13.4	2.51	2.61	34.9	7.9	1.74	1.61	1.28
	13/16	36.5	10.72	67.9	12.6	2.52	2.59	32.8	7.4	1.75	1.59	1.28
	3/4	33.8	9.94	63.4	11.7	2.53	2.56	30.7	6.9	1.76	1.56	1.29
	11/16	31.2	9.15	58.8	10.8	2.54	2.54	28.6	6.4	1.77	1.54	1.29
	5/8	28.5	8.36	54.1	9.9	2.54	2.52	26.3	5.9	1.77	1.52	1.29
	9/16	25.7	7.56	49.3	9.0	2.55	2.50	24.0	5.3	1.78	1.50	1.30
	1/2	23.0	6.75	44.3	8.0	2.56	2.47	21.7	4.8	1.79	1.47	1.30
	7/16	20.2	5.93	39.2	7.1	2.57	2.45	19.3	4.2	1.80	1.45	1.31
8×4	1	37.4	11.00	69.6	14.1	2.52	3.05	11.6	3.9	1.03	1.05	.85
	15/16	35.3	10.37	66.1	13.3	2.52	3.02	11.1	3.7	1.03	1.02	.85
	7/8	33.1	9.73	62.5	12.5	2.53	3.00	10.5	3.5	1.04	1.00	.85
	13/16	31.0	9.09	58.7	11.7	2.54	2.98	10.0	3.3	1.05	.98	.85
	3/4	28.7	8.44	54.9	10.9	2.55	2.95	9.4	3.1	1.05	.95	.85
	11/16	26.5	7.78	51.0	10.1	2.56	2.93	8.7	2.9	1.06	.93	.85
	5/8	24.2	7.11	46.9	9.2	2.57	2.91	8.1	2.6	1.07	.91	.86
	9/16	21.9	6.43	42.8	8.4	2.58	2.88	7.4	2.4	1.07	.88	.86
	1/2	19.6	5.75	38.5	7.5	2.59	2.86	6.7	2.2	1.08	.86	.86
	7/16	17.2	5.06	34.1	6.6	2.60	2.83	6.0	1.9	1.09	.83	.87
7×4	1	34.0	10.00	47.7	10.9	2.18	2.60	11.2	3.9	1.06	1.10	.85
	15/16	32.1	9.43	45.4	10.3	2.19	2.58	10.7	3.7	1.07	1.08	.86
	7/8	30.2	8.86	42.9	9.7	2.20	2.55	10.2	3.5	1.07	1.05	.86
	13/16	28.2	8.28	40.4	9.0	2.21	2.53	9.6	3.2	1.08	1.03	.86
	3/4	26.2	7.69	37.8	8.4	2.22	2.51	9.1	3.0	1.09	1.01	.86
	11/16	24.2	7.09	35.1	7.8	2.23	2.49	8.5	2.8	1.09	.99	.86
	5/8	22.1	6.48	32.4	7.1	2.24	2.46	7.8	2.6	1.10	.96	.86
	9/16	20.0	5.87	29.6	6.5	2.24	2.44	7.2	2.4	1.11	.94	.87
	1/2	17.9	5.25	26.7	5.8	2.25	2.42	6.5	2.1	1.11	.92	.87
	7/16	15.8	4.62	23.7	5.1	2.26	2.39	5.8	1.9	1.12	.89	.88
	3/8	13.6	3.98	20.6	4.4	2.27	2.37	5.1	1.6	1.13	.87	.88
6×4	1	30.6	9.00	30.8	8.0	1.85	2.17	10.8	3.8	1.09	1.17	.86
	15/16	28.9	8.50	29.3	7.6	1.86	2.14	10.3	3.6	1.10	1.14	.86
	7/8	27.2	7.98	27.7	7.2	1.86	2.12	9.8	3.4	1.11	1.12	.86
	13/16	25.4	7.47	26.2	6.7	1.87	2.10	9.2	3.2	1.11	1.10	.86
	3/4	23.6	6.94	24.5	6.3	1.88	2.08	8.7	3.0	1.12	1.08	.86
	11/16	21.8	6.40	22.8	5.8	1.89	2.06	8.1	2.8	1.13	1.06	.86
	5/8	20.0	5.86	21.1	5.3	1.90	2.03	7.5	2.5	1.13	1.03	.86
	9/16	18.1	5.31	19.3	4.8	1.90	2.01	6.9	2.3	1.14	1.01	.87
	1/2	16.2	4.75	17.4	4.3	1.91	1.99	6.3	2.1	1.15	.99	.87
	7/16	14.3	4.18	15.5	3.8	1.92	1.96	5.6	1.9	1.16	.96	.87
	3/8	12.3	3.61	13.5	3.3	1.93	1.94	4.9	1.6	1.17	.94	.88
6×3 1/2	3/4	22.4	6.56	23.3	6.1	1.89	2.18	5.8	2.3	.94	.93	.75
	11/16	20.6	6.06	21.7	5.6	1.89	2.15	5.5	2.1	.95	.90	.75
	5/8	18.9	5.55	20.1	5.2	1.90	2.13	5.1	1.9	.96	.88	.75
	9/16	17.1	5.03	18.4	4.7	1.91	2.11	4.7	1.8	.96	.86	.75
	1/2	15.3	4.50	16.6	4.2	1.92	2.08	4.3	1.6	.97	.83	.76
	7/16	13.5	3.97	14.8	3.7	1.93	2.06	3.8	1.4	.98	.81	.76
	3/8	11.7	3.42	12.9	3.3	1.94	2.04	3.3	1.2	.99	.79	.77
5×3 1/2	3/4	19.8	5.81	13.9	4.3	1.55	1.75	5.6	2.2	.98	1.00	.75
	11/16	18.3	5.37	13.0	4.0	1.56	1.72	5.2	2.1	.98	.97	.75
	5/8	16.8	4.92	12.0	3.7	1.56	1.70	4.8	1.9	.99	.95	.75
	9/16	15.2	4.47	11.0	3.3	1.57	1.68	4.5	1.7	1.00	.93	.75
	1/2	13.6	4.00	10.0	3.0	1.58	1.66	4.1	1.6	1.01	.91	.75
	7/16	12.0	3.53	8.9	2.6	1.59	1.63	3.6	1.4	1.01	.88	.76
	3/8	10.4	3.05	7.8	2.3	1.60	1.61	3.2	1.2	1.02	.86	.76
	5/16	8.7	2.56	6.6	1.9	1.61	1.59	2.7	1.0	1.03	.84	.77
5×3	3/4	18.5	5.44	13.2	4.2	1.55	1.84	3.5	1.6	.80	.84	.64
	11/16	17.1	5.03	12.3	3.9	1.56	1.82	3.3	1.5	.81	.82	.64
	5/8	15.7	4.61	11.4	3.5	1.57	1.80	3.1	1.4	.81	.80	.64
	9/16	14.3	4.18	10.4	3.2	1.58	1.77	2.8	1.3	.82	.77	.65
	1/2	12.8	3.75	9.5	2.9	1.59	1.75	2.6	1.1	.83	.75	.65
	7/16	11.3	3.31	8.4	2.6	1.60	1.73	2.3	1.0	.84	.73	.65
	3/8	9.8	2.86	7.4	2.2	1.61	1.70	2.0	.89	.84	.70	.65
	5/16	8.2	2.40	6.3	1.9	1.61	1.68	1.8	.75	.85	.68	.66

27. RIVETED CONNECTIONS (13, 14, 18)

Rivets are of low-carbon round steel rod, with one rounded head. Holes are punched or drilled, or subpunched and reamed, through the parts to be riveted together, and 1/16 in larger than nominal diam of the rivet shank. The rivet is inserted after heating to light yellow color (not over 1 950° F), is held in place by a tool fitting the rounded head, and the other head is formed by hand hammering, by a pneumatic hammer with cup-shaped head, or, in shops, by riveting machines.

Rivets are used: (a) to fasten together various shapes and plates thus building up composite sections, as girders and columns, having a larger area, and hence greater strength, than the standard shapes; (b) to join shapes or composite sections together to form framed construction, as trusses and floor systems; (c) in joining curved or other plates to form boilers, pressure vessels and pipes.

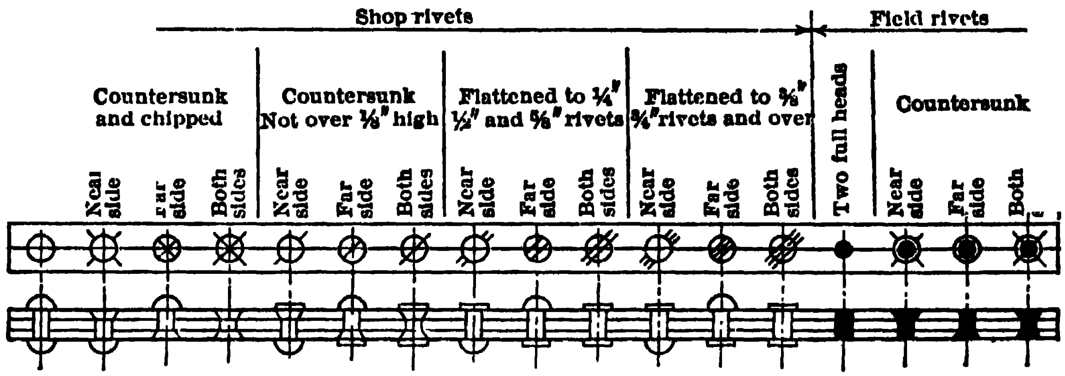


Fig 57. Conventional Signs for Riveting

Bolts are used in holding parts together temporarily, or for minor connections, but rivets are preferred for important work. The nominal diam of rivets varies from 0.5 to 1.5 in by eighths, the common size in structural work being 0.75 in. Fig 57 shows the types of rivets and conventional signs used to designate them.

Spacing and clearances. Rivets are placed in rows, the center lines of each row being known as the gage line. Pitch is the distance c–c of rivets in a row. Standard gage lines are used for angles (Table 23) and in standard beam connections. To leave space to

Table 23. Standard Gage Lines

Usual Gages for Angles, in															Crimps
Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1	$b = t + 1\frac{1}{2}''$ Min = 2"
g	4½	4	3½	3	2½	2	1¾	1⅝	1⅛	1	⅞	⅞	¾	⅝	
g_1	3	2½	2¼	2											
g_2	3	3	2½	1¾											

form the rivet head, a certain minimum pitch is required and minimum clearances between the gage line and outstanding flanges and crimps. The minimum dist between centers should be at least 3 × diam of rivet, but is preferably 0.5 in greater. This applies to rivets in 1 row, or to diagonal dist between staggered rivets in 2 or more rows (chain riveting refers to identical spacing in 2 or more rows).

Stresses. Riveting is so designed as not to subject a rivet to tension, but to cause it to act in shear. Strength of a rivet is thus measured by its shearing resistance ($R_s = \pi d^2 f_s \div 4$), or its resistance in double shear ($R_{2s} = 2R_s$), or by the capac of the metal pierced by the rivet to carry the press produced by the bearing of the rivet shank against it; $R_B = dtf_b$, d being the rivet diam, t length of bearing or thickness of plate pierced by the rivet, and, (A I S C specifications) f_s = 15 000 and f_b 32 000 lb per sq in for machine-driven rivets.

Tension joints. In these the main sec of the plate is reduced by the rivet holes and the ultimate strength of a joint designed with enough rivets is thus determined by the net sec of the plate.

Example 1 (Fig 58a). A lap joint with two 7/8-in rivets $b = 3$ and $t = 0.5$ in. Net sec is $b - (d + 1/8)tf_t$, where f_t is the allowable tensile stress, taken as 20 000 lb per sq in, and an allowance of 1/8 in for the rivet hole. This gives (3 − 1) (1 ÷ 2) 20 000, or 20 000 lb, which, for a fully riveted joint would be the max load. But, as there are only 2 rivets, the safe load in single shear (area of 7/8-in shank = 0.601 sq in) = 2 × 0.601 × 15 000 = 18 030 and in bearing = 2 × 7/8 × 1/2 × 32 000 = 28 000 lb. Hence R_s controls and, since strength of the full plate = 3 × 1/2 × 20 000 = 30 000 lb, the effic of the joint = 18 030 ÷ 30 000 = 60%.

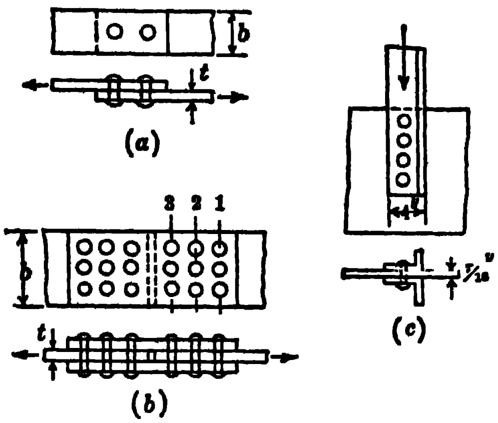


Fig 58. Riveted Connections

Example 2 (Fig 58b). A double-strap butt joint, with 9 7/8-in rivets, $b = 12$, t of main plate = 0.75 and of straps = 0.5 in. Assuming that the stress is uniformly distributed to the rivets, R_s = 2 × 9 × 0.601 × 15 000 = 162 400 lb, as the rivets are in double shear. Also R_B on side plates = 9 × 2 × 7/8 × 1/2 × 32 000 = 252 000 lb. As f_b may be increased 25% (AISC) for the center plate, which has support from both side plates, R_B for center plate = 9 × 7/8 × 3/4 × 40 000 = 23 620 lb. Finally the tension in the net sec of the main plate is (12 − 3 × 1) (0.75) × 20 000 = 135 000, and in straps = (12 − 3) (2 × 0.5 × 20 000 = 180 000 lb. Hence tension in the net sec of main plate limits the working load on this joint. Since the gross sec of main plate will carry 180 000 lb, the effic of the joint is 135 000 ÷ 180 000 = 75%.

A fuller analysis of this design will show that the stress is not carried uniformly, as assumed, but, because of the varying elongation of the plate between lines of rivets, the rivets in row 1 (Fig 58b) carry most of the load and those in row 2 the least. The design can be increased in effic to approx 90% by using tapered straps and a different arrangement of rivets.

Compression joints. There is no essential difference in design, except net section is unimportant and shear or bearing usually control.

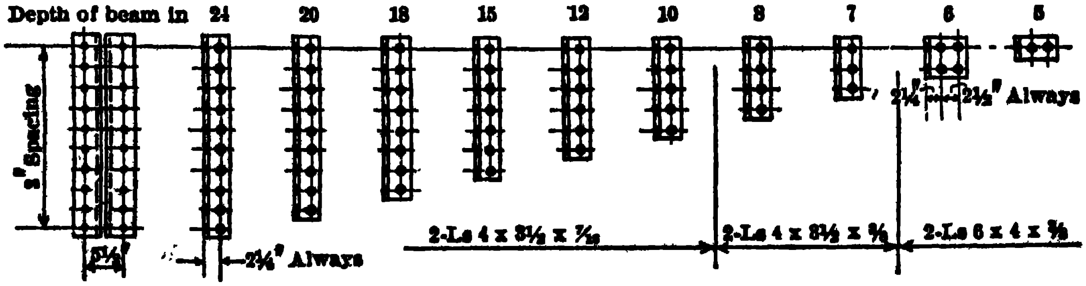
Example 3 (Fig 59c). Two 4 × 3 × 3/8-in angles acting as a compression member and carrying 50 000-lb load are connected to a 7/16-in plate (gusset plate), as is common in designing joints in framed structures.

Find the number of 0.75-in rivets required. R_{2s} is found to be 13 250 lb, and R_B on the 7/16-in plate is 13 100 (allowing 25% increase as above), and for the 2 angle legs is 18 000 lb. Hence R_B on the 7/16-in plate controls and the number of rivets required is 50 000 ÷ 13 100 = 4. For more detailed analysis, particularly of eccentrically loaded joints, see (13).

Standard connections have been adopted for standard I-beams (Table 19), as shown in Table 24. There is also given the minimum span for uniformly distributed loads, below which the strength of the connection (assuming 0.75-in rivets) determines safe load on the beam rather than its cross-sec in bending.

Table 24. Standard Two-angle Connections

Approx min l ft	15-21	9-12	11-13	8-11.5	7.5-9	6-8	3.5-4.5	5-6	4-4.5	2.5-3
Depth of beam in	24	20	18	15	12	10	8	7	6	5



28. WELDED CONNECTIONS (19).

Use of gas or arc welding for connections in steel construction is now common. Joints are formed by fusing additional metal along the edges of parts to be joined, or, less often and in smaller parts, by spot welds. Fig 59 shows typical joints, those requiring special

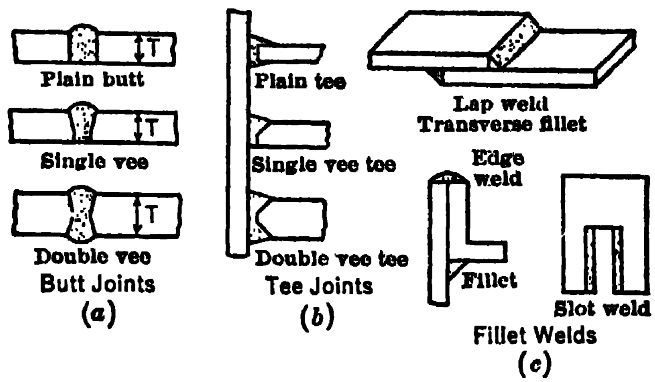


Fig 59. Types of Welded Connections

preparation of the edges V types being used for the heavier thicknesses, double V for over 0.5 in.

In forming the weld (Fig 60) the tool is moved continually forward, fusing the *base metal* to a penetration of 1/32–1/8 in, while *welding metal* is added to build up the *bead*. If properly timed there will be no overlap (due to surplus welding metal), or undercut (due to lack of metal), and a bead is formed of uniform contour with a surface free from oxide coating and marked with a series of ripples with a *crater* at the end, where the operation stopped. In gas welding, the limit of thickness

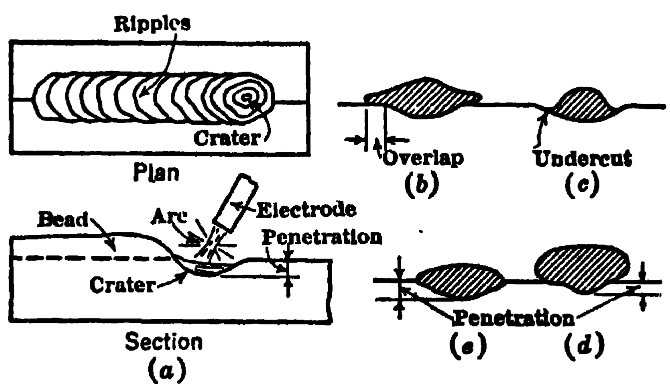


Fig 60. Welding Nomenclature

is determined by size of the molten pool of metal which can be maintained. In arc welding, single layers seldom exceed 1/8 in, but any total thickness can be built up in multi-layers. Fillet welds of 0.25 in are made in one pass.

The operation requires intense heat and thus extreme local expansion, followed by cooling and shrinkage. Annealing to relieve residual stresses is desirable, but rarely possible. To minimise

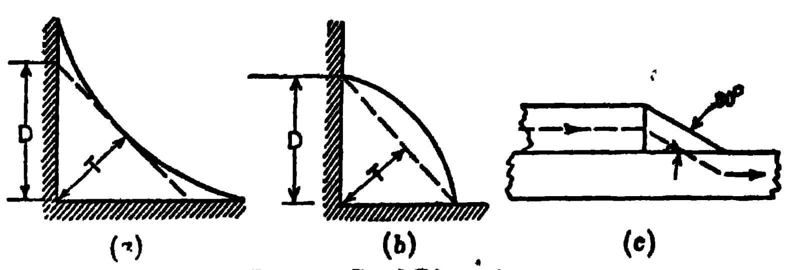


Fig 61. Bead Dimensions

these stresses, the parts should be free to move with the contracting metal, and the design of the joint and sequence of the operation (reducing the length of weld metal deposited in one operation, welding opposite sides in short lengths or simultaneously) must be planned to allow for heat effects. Inspection involves overlap, undercut, burned metal, quality of surface and depth of bead, but skill is

essential. For further details, see "Procedure Handbook of Arc Welding Design and Practice," of Lincoln Elec Co; also (19).

Design. The computation of welded joints is far simpler than for riveted connections. In butt welds (Fig 59 *a* and *b*) there is a surplus of metal, termed *reinforcement*, the weld sec being thicker than the plate but dimensions of latter (Tee, Fig 59*a*) control design.

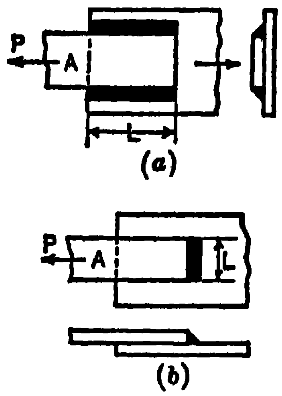


Fig 62. Design of Fillet Welds

Example. In fillet welds dimensions are determined by minimum included triangle, as in Fig 61 *a* or *b*, where D = depth of weld and T = throat or minimum sec. Assuming that longit shear is uniform, if f_s = shear value of weld, lb per sq in, strength of weld in lb $P = Tf_sL$, or $= 0.7Df_sL$, where L = length of weld. Using allowable shear on throat (Am Welding Soc), f_s = 11 300 lb per sq in, a 0.25-in weld will carry 2 000; a 3/8, 3 000, and a 0.5, 4 000 lb per lin in. Thus for the joint in Fig 62*a*, in which a 0.5-in plate 6 in wide is connected by 0.5-in side fillet welds to give a tensile stress of 20 000 lb per sq in, the total load = 0.5 × 6 × 20 000 = 60 000 lb, and the total length of weld $2L$ = 60 000 ÷ 4 000 = 15 in, to which 0.5 in is added on each side to allow for crater, making L = 8 in. Another type of welded connection for this joint is the transverse weld (Fig 62*b*), where a transverse fillet is used. A better stress distribution is secured by using a 30° weld (Fig 59*c*), rather than the usual 45° sec. Although the stress on the throat area is combined tension (allowable tension in welds (A W S), 13 000 lb per sq in) and shear, it is generally assumed that shear controls as in longit welds.

29. COLUMNS AND GIRDERS (13)

Although much larger steel sections are now being rolled than in the past and, whenever available, solid rolled sections are used, it is necessary when larger areas are required, or for special members in frame construction, to build up composite sections of plates and angles. Special wide-flange beam sections up to 36 × 16.5 in are rolled and wide-flange H-column sections are now used.

Girders are built-up of a web plate with flange angles and cover plates (Fig 63), giving, in effect, a large I section. The basic computations are the same as those for smaller

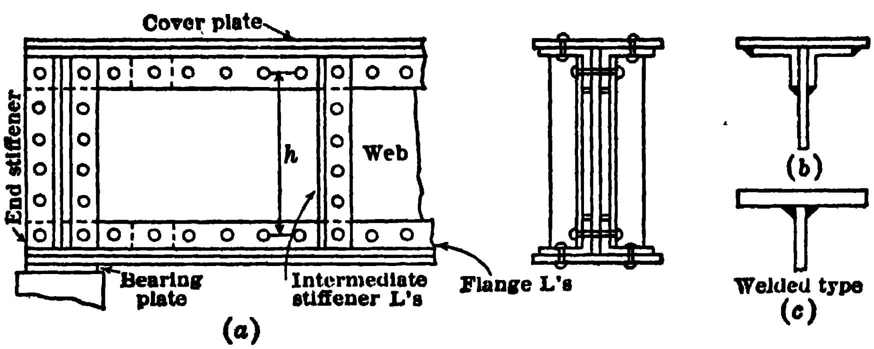


Fig 63. Girders

beams except: (*a*) while the web plate and flange angles are usually of uniform thickness, the cover plates may be reduced in thickness (number) toward the supports, in accordance with the decreasing moment; (*b*) the web will require intermediate stiffener angles to brace it against buckling due to shear under heavy loads. For details, see (13). Effective depth d, in $M = KI \div d$ (Art 4), is measured between centers of gravity of flange sections. Where a welded girder section is used, the most economical form is that of Fig 63*c*, rather than a section like the riveted form in Fig 63*b*.

Struts or compression sections in lighter framed construction are often composed of two angles back to back, as in steel roof truss (Fig 64).

Washers, the same thickness as gusset plates, are used to permit riveting of angles at such intervals that $l \div r$ (Art 5), or 2 angles considered separately, shall not exceed that of composite section (Fig 65). For axially loaded columns with $l \div r$ not over 120 (required for main members) the safe load (AISC) per sq in $= p = 17\,000 - 0.485\,(l^2 \div r^2)$, while for bracing and less important members, where $l \div r$ may reach 200, $p = 18\,000 \div (1 + l^2 \div 18\,000r^2)$. For details of design of heavier bridge and building columns see (13).

Tension members are also frequently made of standard sections (as angles in Fig 64) instead of round rods, due to ease of making riveted connection and increased stiffness given to entire

frame. Tie rods, with or without turnbuckles, and with threaded or loop ends, are also used, while in heavier construction with pin joints eye bars are often employed.

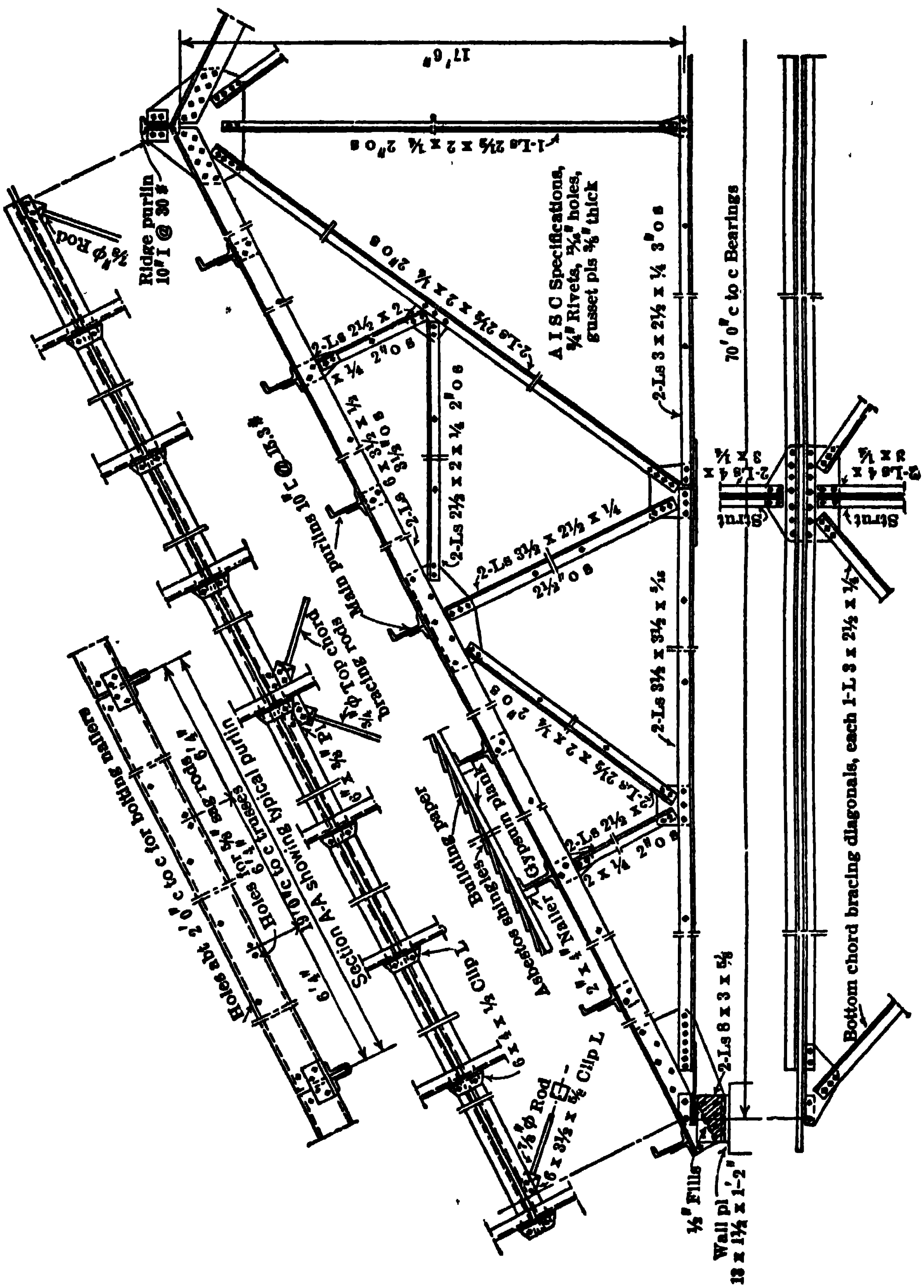


Fig 61. Steel Roof Truss

30. BRIDGES AND BUILDINGS

Bridges. For relatively light loads, the possibility of using 2 or more standard I-beams should always be investigated. Built-up girders are used for R R loadings in spans to 120 ft or more, the depth, usually about 0.1 the span, being limited by shipping conditions

(bridge and tunnel clearances). When possible, ties or floor should rest on top of beams or girder (*deck construction*), but clearances may require more costly *through* construction. Trusses are used only for larger spans. In former practice spans to 150 ft were generally built with riveted joints; longer spans pin connected, for easier erection. Solidly riveted connections are now usually employed for greater rigidity. For details of steel bridge design see (13).

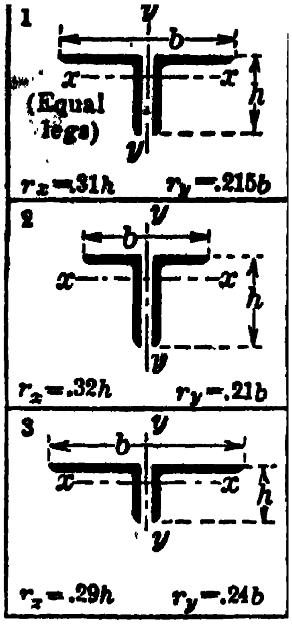


Fig 65. Compression in Angles

Buildings. Steel mill buildings are of 3 types: (*a*) steel truss and braced columns, covered with corrugated iron (Art 24), or thin concrete walls, supported by the steel frame; (*b*) same as above, except columns are braced by thin self-supporting masonry walls; (*c*) steel roof trusses, supported on masonry walls.

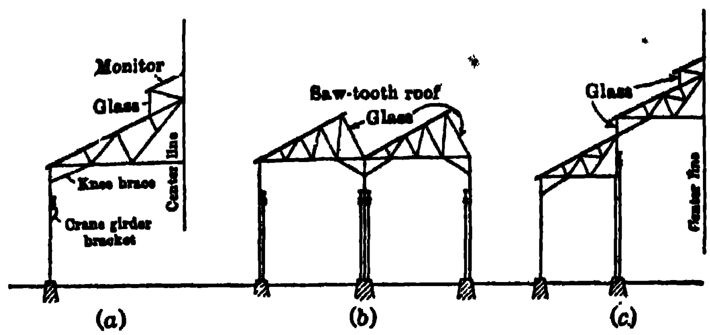


Fig 66. Forms of Steel Buildings

Fig 66 shows 3 common forms. Saw-tooth roof is usually so designed that the glass has a northern exposure. Fig 67 shows cross-sec of a building of type (*a*), with wind force acting against it. Dead and snow-load stresses in the roof truss are computed as in Art 17, but column, brace, and wind stresses depend largely on wind load and condition of fixity of columns. Fig 67 shows conditions when columns are assumed as usual as fixed at upper and hinged at lower end. Wind load is taken as acting at panel points, the individual loads being W_1, W_2, W_3, W_4, and W_5. The resultant W, acting at height h, is found by taking moments about the column base, the forces required for equilibrium being shown. In column P_1 max moment producing bending is just below the knee brace, and $= Wb \div 2$; the column must be designed (Art 6) to allow for this, in addition to direct

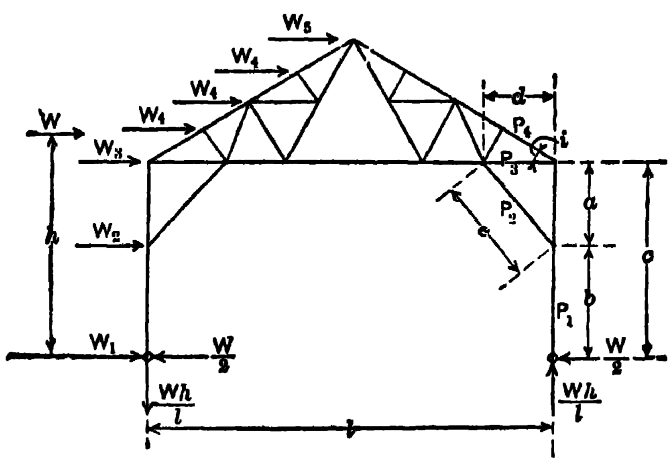


Fig 67. Distribution of Wind Pressures on Roof Truss

load from truss. The shear above brace $= Wb \div 2a$, and below brace $= W \div 2$. Direct stress due to wind below the brace is $Wh \div l$; above brace $= (Wc \div 2d) - (Wh \div l)$; in brace, $P_2 = Wce \div 2ad$. Member $P_4 = (Wc \div 2d) - (Wh \div l) \operatorname{cosec} i$. Chord $P_3 = P_4 \cos i - Wb \div 2a$. Stresses in other members are found by applying forces P_2, P_3, and P_4 and solving by methods of Art 18.

In many mill and factory buildings the column design is complicated by need of providing an overhead crane, to be carried by crane track girders supported by brackets attached to inside of columns. Crane loads thus produce excentric loading on columns; also a side thrust of 5–10% or more of lifting capac of the crane should be provided for, together with some impact allowance (17).

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SECTION 44

PETROLEUM PRODUCTION METHODS

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Note.—Numbers in parentheses in text refer to Bibliography at end of this section.

PETROLEUM PRODUCTION METHODS

1. PETROLEUM DEPOSITS

Geological occurrence. Oil deposits are found in sand strata, and in beds of porous limestone where structural conditions permit accumulation and retention of oil and gas. These conditions are: (*a*) sufficient structural relief, (*b*) a porous body (sand, limestone, or in a few cases porous serpentine), of sufficient thickness and porosity to form the necessary reservoir; (*c*) source beds, whence the oil has migrated to the reservoir (1).

Structures favorable to oil accumulation are anticlines, domes, monoclines, terraces, lenses, faulted areas, part dome and part fault, and unconformities. Fig 1 shows a classic

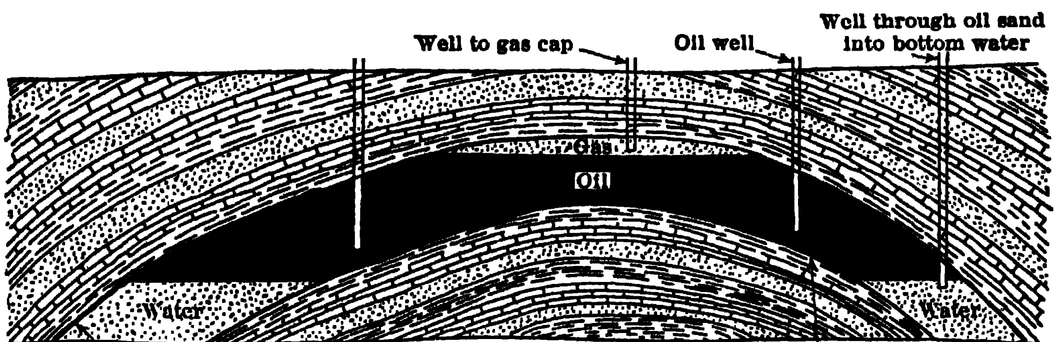

Fig 1. Ideal Vert Sec through Dome containing Oil, Water and Gas

example of an oil reservoir, in this case a sand stratum with cap-rock preventing the upward escape of oil and gas. Free gas occupies the upper part of the dome, underlain by the horiz oil deposit, below which is the "edgewater," preventing oil and gas (now under considerable press) from escaping down the slopes of the dome. This is either water which has backed into the reservoir, or (in part) water displaced by oil and gas (2).

Oil and gas in underground reservoirs are usually under pressure, roughly equal to press in a water column of same depth below surface, though there are many cases of press greater or less than required by this rule. Examples are given in Table 1. Pressures observed in early years were probably "casing-head pressures," not actual "bottom-hole pressures," which are more accurately recorded today with instruments lowered to the well bottom. The last 7 examples in Table 1 (except Oklahoma City) were taken with bottom-hole press bombs, and their close approach to water-column press of 0.434 lb per sq in per ft of depth is noteworthy.

Table 1. Pressures of Gas in Various Reservoirs

Field	Depth, ft	Lb per sq in	Lb per sq in per ft depth
Kokomo, Indiana	650	328	0.503
Findlay, Ohio	950	400–450	0.421–0.473
Butler Co, Penna	1 452	600	0.413
Smackover, Ark	2 200	900	0.407
Cotton Valley, La	2 556	900	0.353
Barbour Co, W Va	2 980	1 420	0.476
East Texas, Texas	3 600	1 650	0.458
Keokuk Falls, Okla	4 000	1 820	0.455
Talco, Texas	4 325	1 800	0.416
Plymouth, Texas	5 600	2 420	0.432
Rodessa, Texas	6 100	2 670	0.437
Oklahoma City, Okla	6 500	2 600	0.400
Anahuac, Texas	7 100	3 260	0.458

In general, oil under press contains gas in solution, the quantity varying with the press and the character of oil. High-gravity oils usually contain more gas than heavier oils. "Saturated" oil contains all the gas that can be dissolved at the reservoir press, and any excess must then exist free, in a gas-cap at top of the geologic structure. "Undersaturated" oil has less than the max gas content at the existing press. Fig 2 shows the

quantities of gas dissolved in oils of various fields; for example, that dissolved in one bbl of oil in the Seminole field under 1 000-lb press is 290 cu ft.

When a reservoir is tapped, oil rises in the well to the height corresponding to reservoir press, provided the gas above the oil column can escape from the casing head. If enough gas is associated with the oil, the oil will flow naturally from the well, until reservoir press has declined to the point where the gas content can no longer lift the oil to the surface, whereupon the well will flow in heads and finally cease altogether. In deep deposits, the gas content seems to increase sufficiently with depth to maintain more rapid flow than in shallow wells. After natural flow ceases, artificial means must be provided to lift the oil.

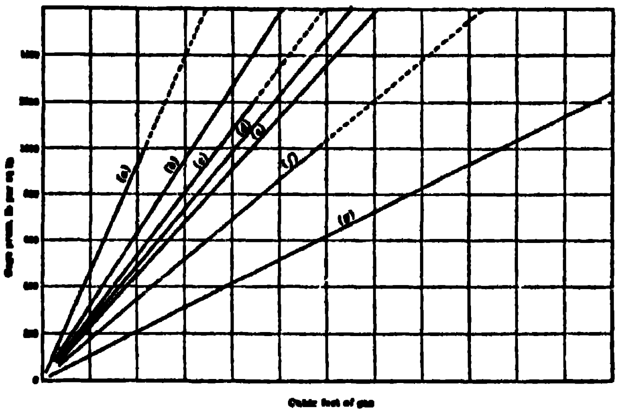

Fig 2. Cubic Feet of Gas Dissolved in One Barrel of Crude Oil. (*a*) Raccoon Bend, Tex, 20.3° API. (*b*) Sugarland, Tex. (*c*) Raccoon Bend Crude, 32.1° API. (*d*) Oklahoma Crude, 31° API. (*e*) Wyoming Crude, 36° API. (*f*) Seminole Crude. (*g*) Hobbs Crude

Methods of extracting petroleum described herein are classed as primary or secondary, depending chiefly upon age of the well and degree to which oil has been extracted. These classes were formerly more sharply distinguished, but are now beginning to overlap. Primary methods include natural flow and direct-lifting methods, beginning with completion of the well and ending when these methods can no longer extract oil profitably, but must be supplemented by secondary methods of repressuring (Art 12, 13).

PRIMARY METHODS OF EXTRACTION

2. NATURAL FLOW

In most fields, a completed well will flow naturally for a period dependent on the quantity of gas associated with the oil, depth of well, reservoir press and diam of casing. If the quantity of gas is large enough and can automatically adjust itself to the quantity of oil entering the well bottom, natural flow may continue throughout the life of the well, and such cases have been known (3). Usually, the quantity of gas declines until the well stops flowing or "dies," whereupon artificial means of lifting the oil are necessary.

Depth of well is an important factor in the duration of natural flow, since a definite amount of work must be done for each foot through which the oil is lifted, whence (other conditions being constant) the deeper the well the greater the quantity of gas required to lift a bbl of oil (4). For a given flowing press, assuming that the casing is delivering oil at its most effic rate, the greater the diam of casing the less gas is required to lift a bbl of oil. Table 2 shows ft-lb of work developed by a perfect gas, when expanding isothermally from a given gage press to atmosphere. Gas associated with oil is not a perfect gas, nor does it expand isothermally, but in each case its characteristics bear a definite relation to isothermal expansion; whence, for any one well the observed characteristics of flow, and the flowing effic, based on isothermal theory, provide a measure of lifting effic accurate enough throughout the period of natural flow. Between wells of the same pool there is little difference in the factors for deviation of compressibility under flow conditions.

Flowing press at the sand face is another important factor. If this press declines until the associated gas has too little energy to lift the oil to the surface, the well ceases continuous flow. It may then flow intermittently, as gas accumulates from time to time at the well bottom in sufficient quantity to lift the accumulated oil, but this flow is uncertain and subject to almost no control; whence, at this point some artificial means of lifting is usually necessary (5). But flowing press may be partly or fully maintained by the gas associated with the oil, or by water (if under press head) at the edges of the field, tending to drive oil to the well. In general, when a well is opened and begins natural flow, some of the oil and gas near the well bottom is drawn off, lowering press at the sand face and allowing reservoir press to drive or push more oil to the well; this process continuing until, at some rate of flow, reservoir press is balanced by the press required to drive oil through the sand to the well and through the casing and discharge piping at the surface (6).

Desired production of oil governs the manner of "producing" the well. For max production the well is opened wide at surface, and back press at that point is held to a minimum. If there is enough associated gas to maintain a high rate of flow, the well is

Table 2. Ft-lb of Work to Compress 1 Cu Ft of Gas Isothermally to a Given Press

Lb per sq in	Ft-lb of work	Lb per sq in	Ft-lb of work	Lb per sq in	Ft-lb of work	Lb per sq in	Ft-lb of work	Lb per sq in	Ft-lb of work
....	....	....	.....	260	6 190	510	7 568	760	8 392
1	139	20	1 819	270	6 274	520	7 608	770	8 420
2	270	30	2 354	280	6 347	530	7 647	780	8 446
3	393	40	2 782	290	6 417	540	7 685	790	8 473
4	510	50	3 137	300	6 486	550	7 723	800	8 499
5	620	60	3 441	310	6 552	560	7 760	810	8 525
6	725	70	3 707	320	6 616	570	7 798	820	8 550
7	824	80	3 943	330	6 678	580	7 833	830	8 576
8	920	90	4 156	340	6 739	590	7 868	840	8 601
9	1 011	100	4 349	350	6 798	600	7 902	850	8 625
10	1 107	110	4 526	360	6 855	610	7 937	860	8 649
11	1 183	120	4 689	370	6 911	620	7 971	870	8 674
12	1 263	130	4 841	380	6 965	630	8 003	880	8 697
13	1 341	140	4 982	390	7 018	640	8 036	890	8 721
14	1 416	150	5 115	400	7 069	650	8 068	900	8 744
15	1 489	160	5 240	410	7 120	660	8 100	910	8 767
16	1 559	170	5 358	420	7 169	670	8 131	920	8 790
17	1 627	180	5 469	430	7 218	680	8 162	930	8 812
18	1 692	190	5 575	440	7 264	690	8 192	940	8 835
19	1 757	200	5 676	450	7 311	700	8 222	950	8 857
....	....	210	5 772	460	7 356	710	8 251	960	8 879
....	....	220	5 865	470	7 400	720	8 280	970	8 900
....	....	230	5 953	480	7 443	730	8 309	980	8 922
....	....	240	6 038	490	7 485	740	8 337	990	8 943
....	....	250	6 119	500	7 527	750	8 365	1 000	8 964

flowed through the casing (7). Sometimes the sand at the well bottom is loose and enough may enter the well eventually to choke it (as in many wells on Gulf coast and in Calif), in which case it may be necessary to choke back the flow, by partly closing the discharge valve at surface, or by inserting a "choke" or "bean" or "flow-nipple" in the discharge line. Choking may also be effected by using a diam of well tubing suited to desired flow.

When there is an oversupply of oil, it may be necessary at times to choke or restrict the production of a well, usually effected by running tubing inside the casing, of a diam likely to be used later when producing the well artificially. Closer control of the rate of flow is obtained by inserting in the flow-line a choke or flow-nipple, of diam suited to the rate desired. During the past ten years the large oversupply of oil has made it necessary to restrict the production of large wells, and various methods of prorating the restricted production equitably between operators have been applied. From data obtained during these years of proration, some engineers hold that restriction increases the ultimate recovery of oil, especially where there is a strong water drive; on the theory that the right degree of restriction would so adjust the advance of the water as to maintain reservoir press, while sweeping the oil gradually to the wells, thus maintaining natural flow throughout their life (8). Some engineers also believe that under restriction the gas-oil ratios are held to a minimum, thus tending to maintain higher reservoir press than when the wells are produced at max capacity; but, it has been observed in many fields that flowing at max capacity has not increased, but rather reduced, the gas-oil ratio. Whether ultimate recovery is increased or reduced by restricting production is a question which continued observation during the next 10 or 15 years may tend to decide. Meanwhile, as long as a large oversupply of oil exists, production must be restricted regardless of ultimate recovery, and the restriction will be applied to large flowing wells rather than to small wells operated by artificial lifting methods. "Kick-off" valves (Art 4) are used in flowing wells when gas press and volume have declined until the wells will not start flowing after they have been shut in for a time, or else flow in a surging condition.

3. PRESSURE MAINTENANCE

Maintenance of reservoir pressure and indefinite natural flow without any resulting disadvantage would be the ideal mode of producing oil (9). If no oil or gas were drawn off, the reservoir press would be maintained indefinitely; and a lowered rate of production means a slower decline in press, though this may serve only to spread production over a longer period.

Reservoir press depends upon quantity and press of gas associated with the oil, activity of the water-drive behind the oil, degree to which the reservoir sand or rock becomes compacted as oil and gas are removed, and effectiveness of returning some fluid (gas, water, or oil) to the reservoir. It has been held that, with an active water-drive, some restricted rate of production can be found at which the water will follow up the oil without loss of press, thus maintaining this press on the oil and gas in the reservoir (10). To the extent

that this balance is achieved, restriction is of benefit whenever the longest possible maintenance of natural flow is desired, such flow being usually the cheapest known method of production. Restriction where there is a strong water-drive is found in the East Texas, Hobbs, Rabbs Ridge, Anahuac and other fields.

Reservoir pressure can be maintained to some degree by injecting into the reservoir a volume of fluid (oil, water or gas) equivalent to the volume removed (11). The usual method is to inject gas into the highest wells on the structure, thereby forcing the oil down to the lower wells. To do this, the gas produced with and separated from the oil is used, being recompressed and returned to the input wells (12). Also, oil or water can be injected into wells lower on the structure, to push oil upward to those higher.

In Persia, heavy crude oil (left after tapping the gasolene) has been injected into the reservoir, tending to maintain press to the extent that it replaces oil and gas removed. In a few fields, a large part of the gas produced has been returned for the same purpose (as in Colombia, Perú and Sumatra, operated by subsidiaries of Standard Oil Co of N J). Much oil in the Talang Akar-Pendopo field, Sumatra, has been produced by the press maintenance method, with apparently favorable results. The method has also been applied to the Tepetate field, Louisiana (13) and the Sugarland (14) and Raccoon Bend fields in Texas, of which the last named has not responded very well; but results in the Sugarland field seem encouraging enough to suggest continuing until exhaustion of the pool, or until definite conclusions can be reached. Data of these last two fields are in Table 3. The South Burbank (15) and Keokuk Falls (16) fields, Okla, are operated by returning a large part of the gas produced with the oil, though perhaps the latter field is not suited to the method. Results in the South Burbank field are not yet definitely established.

Table 3. Pressure Maintenance Operations, Sugarland and Raccoon Bend Fields, Tex

	Sugarland Field		Raccoon Bend Field	
	Nov, 1934	Since Apl, 1930	Nov, 1934	Since July, 1930
Oil produced, bbl	179 995	15 147 793	104 367	8 661 567
Gas produced, M cu ft (2-lb base)	46 385	4 378 349	151 199	13 128 861
Gas returned, M cu ft (2-lb base)	41 082	3 869 605	91 959	6 773 874
Gas returned, %	88.7	88.4	58.5	51.5
Gas-oil ratios, cu ft per bbl:				
Total	258	289	1 449	1 516
Returned	228	255	881	782
Net	29	34	568	734

4. GAS-LIFT, CONTINUOUS FLOW

Air- or gas-lift was tried and used on a small scale soon after the first commercial wells were produced in Penna (1865), and in early days on the Gulf Coast, at Humble, Evangeline, Goose Creek, Orange, West Columbia; in various Calif fields, as Huntington Beach, Kern River, Long Beach, Santa Fe Springs and Seal Beach; in Okla, at Blackwell, Burbank, Oklahoma City, Seminole, Wewoka, etc; abroad in Argentina, Canada, Colombia, Perú, Rumania, Russia, Sumatra, Trinidad and Venezuela.

Gas-lift follows natural flow in logical sequence; used to good advantage where natural flow has ceased, or has fallen off for lack of gas with the oil. Its principle is identical with that of natural flow itself, and of the air-lift for raising water; for theory and details, see Sec 15, and Bib (17, 18). Tubing is installed inside the well casing, and air or gas injected, either through the tubing (the oil flowing between tubing and casing), or vice versa.

Casing sizes for gas-lift are 5 1/2 to 9 5/8 in; tubing sizes, 4 in down to 1 in or less; in general, casing size should increase with well depth (19). In the Seminole field (20), usual sizes were 5 1/2 and 6 5/8-in casing with 2-in tubing, 7-in casing with 2 1/2-in tubing, and 8 5/8-in casing with 3- and 4-in tubing; the oil usually flowed between tubing and casing, for lifting as much oil as possible, but in periods of restricted production it was lifted through 3- and 4-in tubing. In Oklahoma City, sizes are 6 5/8-in casing with 2-in tubing; 7-in casing with 2- and 2 1/2-in tubing; and 8 5/8, 9- and 9 5/8-in casing with 3-in tubing. In early years in this field, when production was greatly restricted and reservoir press still high, oil was flowed through the tubing in some wells, through the casing in others, but when press had declined from 2 600 lb to 700 lb or less, the oil was usually raised through the annular space, to obtain larger production with less vol of gas per bbl than by raising through the tubing (21). General type of well hook-up for gas-lift used in the Tonkawa, Seminole and Oklahoma City fields is shown in Fig 3. Variations were made to suit local conditions.

Quantity of gas required to raise oil by continuous gas-lift depends on well depth, flowing press at bottom of tubing, size of casing and density of oil. Table 4 shows the quantity required to lift a bbl of oil at max flow.

Table 4. Approx Quantity of Output Gas for Lifting One Bbl of Oil, When Producing at Max Capacity

Flowing pressure, lb per sq in	Cu ft gas per bbl of oil							
	Burbank 2 900 ft	E Texas 3 600 ft	Seminole 4 000 ft	Corpus Christi 4 100 ft	Lucien 5 200 ft	Okla City 6 500 ft	Turner Valley 7 000 ft	Buckeye 7 900 ft
2 080	.....	.....	.....	.....	.....	.....	40	110
1 500	.....	.....	.....	.....	55	110	144	240
1 000	17	40	70	95	160	290	327	480
800	53	70	123	157	255	430	487	680
600	100	140	230	268	425	690	744	1 080
500	145	200	330	374	572	950	1 000	1 480
400	225	300	480	563	805	1 230	1 300	2 020
300	388	490	750	870	1 310	1 700	2 000	2 660
200	692	930	1 410	1 570	2 370	2 580	3 182	3 850
100	1 810	2 650	3 240	3 580	4 670	6 000	7 900	8 570
50	4 350	5 330	6 190	6 720	9 500	13 000	15 300	18 300

The above quantities are total, including gas associated with the oil and flowing with it into the well; the latter should be deducted to find the quantity to be injected for lifting purposes. Thus, a Seminole well producing at 200 lb flowing press, with 800 cu ft of gas accompanying the oil, requires an input of 1 410 minus 800, or 610 cu ft per bbl (Table 4).

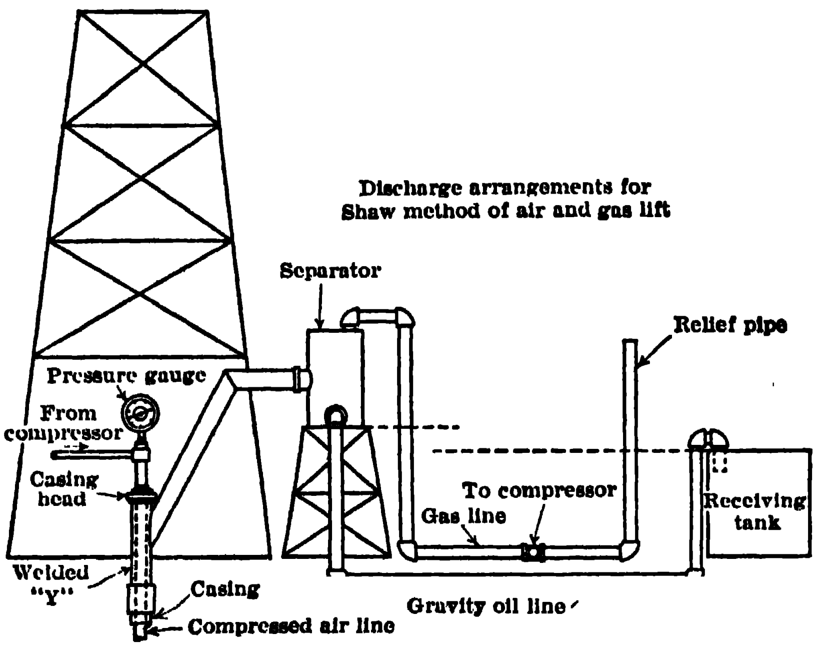

Fig 3. General Layout for Gas-lift

For gas-lift, the capacity of tubing and casing in two fields follows the formula $P = CB^n$, wherein P = flowing press at bottom of tubing, in lb per sq in, B = bbl of oil lifted per 24 hr, and C and n for various sizes of tubing or casing are as follows:

Table 5. Constants in Formula for Capacity of Tubing and Casing

Casing	Tubing	Seminole Field		Okla City Field	
		C	n	C	n
Oil flowed through tubing..........	3-in	10.22	0.465	14.21	0.4722
" " " "	4-in	3.965	0.507	8.018	0.510
7-in O D..........................	2 1/2-in upset	2.546	0.554	6.222	0.5128
8 5/8-in..........................	3-in upset	1.021	0.617	......	......
8 5/8-in..........................	4-in upset	1.166	0.62	......	......
9-in O D..........................	3-in	......	......	1.479	1.6267
9 5/8-in O D......................	3-in	......	......	1.139	0.625
9 5/8-in O D......................	4-in	......	......	1.525	0.620

Gas for lifting may be supplied from large centrally located plants, from small plants on the individual lease, or from gas wells; the last named furnishes the most satisfactory supply if press and volume are sufficient (22, 23, 24).

"Kick-off" valves are used to start wells on gas-lift, when the press is higher than can conveniently be handled by the gas-line press. They are inserted in couplings in the tubing at intervals of about 300 ft, with the uppermost approx at the static fluid level in the well. When press in the fluid column exceeds that in the gas column by a predetermined amount, the valve opens and allows gas to enter the liquid column, starting it on gas-lift and thus lowering its press at the valve, which then closes. Gas is admitted to the next lower valve, and the liquid blown off. This process continues until the bottom of the tubing is reached, when all valves are closed and the well flows by means of gas entering lower end of the tubing.

Ultimately in most fields, gas will have to be compressed in a plant installed for the purpose. For a number of wells within a 1/2-mile or even 1-mile radius, a central plant effects savings; which may be partly offset, however, by friction loss in pumping gas to the wells, unless large-diam pipe is used (25 to 29).

Table 6. Cost of Compressor Plants in Various Oil Fields

	Total cost	Displacement per day, cu ft
Seminole Field		
Small compressors, electric driven	$ 26 000	2 000 000
" " " "	46 000	4 000 000
" " " "	55 000	6 000 000
90-hp direct-driven gas-engine compressors	80 000	3 600 000
165-hp " " " " "	25 000	1 700 000
165-hp " " " " "	200 000	10 000 000
Oklahoma City Field		
Small compressor, electric driven (used)	12 500	2 000 000
90-hp direct-driven gas-engine compressors (new)	41 000	2 700 000
90-hp " " " " " (used)	23 000	2 700 000
190-hp " " " " " (new)	32 500	2 000 000
165-hp " " " " " (used)	18 000	2 550 000
230-hp " " " " " (new)	83 000	6 700 000
East Texas Field		
Small compressors driven by 50-hp electric motors (used)	2 500	400 000
90-hp direct-driven gas-engine compressors (new)	8 500	600 000
90-hp " " " " " (used)	4 000	500 000

In Seminole field the required compressor capac was about 1 500 000 to 2 000 000 cu ft displacement per day per well, when producing 1 000 bbl or more per day (30). In Okla City field, the displacement per well is as high as 6 000 000 cu ft per day, when producing 12 000 bbl or more, and about 3 000 000 cu ft per day, when pressures have so declined that only 200 bbl can be lifted. To the present time, the allowable 20 bbl per day per well in the East Texas field has made unnecessary the use of large plants for lifting the oil. Compressors often serve only to start the wells flowing, after which they continue on natural flow. In the East Texas field, a gas-lift compressor of 500 000 cu ft per day displacement will serve up to 20 wells, if near the plant, thus making the aver installed cost about $450 per well (31).

Operating costs, not including deprec, of compressor plants in Okla City field are as in accompanying table. Lifting cost of the gas is equal to quantity of gas per bbl of oil, multiplied by cost for 1 000 cu ft. Thus, at Seminole, to lift oil by 100-lb bottom-hole press required a total output consumption of 3 240 cu ft per bbl (Table 4), of which 800 was the aver quantity of formation gas accompanying a bbl of oil, leaving the required input gas at 2 400 cu ft per bbl. At 1¢ per 1 000 cu ft, the lifting cost of gas would be 2.4¢ per bbl.

Plant	Total M cu ft per year	Total operating cost	Cost per 1 000 cu ft gas, cts
A	422 662	$ 8 059	1.91
B	3 150 737	29 025	0.92
C	4 262 661	31 429	0.93
D	3 250 438	33 390	1.02
E	2 039 155	20 300	1.00

Gas-lift without modification is termed "continuous" or "straight" gas-lift; when in connection with auxiliary centrifugal or plunger pumps, a "combination" gas-lift (Art 5); and when gas is admitted to the well at intervals, it is an "intermittent" gas-lift (Art 6).

5. COMBINATION GAS-LIFT METHODS

The gas-lift Reda pump combination method (see Art 7) has thus far been the most successful (32). An electric-driven centrifugal pump, suspended from a string of tubing, is lowered into the well. A "packer" (seal between tubing and casing) is set at 100 to 1 000 ft above bottom of casing, and perforations in the tubing above the packer allow oil to pass through the pump into the casing. Gas pumped down the tubing issues with the oil through the perforations and lifts the oil from packer to surface. Below the packer, the action is strictly that of a centrifugal pump; above, it is that of continuous gas-lift.

Much oil has been raised in Okla City field by this method, even after reservoir pressures declined to 100 lb per sq in or lower, and it operates with reservoir press as low as 25 lb, provided an excessive quantity of sand or gas is not drawn into the pump (33). In a well with reservoir press of 50 lb, completed with 9 5/8-in O D casing and 3-in tubing, a production of 4 500 bbl per day was maintained for some time. Consumption of gas and power for various capac rates in the Okla City field is given in Table 7. The combination Reda pump has been successful in Beebe, Hendricks, Powell, Seminole, and various pools in Kansas, especially where much liquid is handled.

Table 7. Consumption of Gas and Power in Combination Gas-lift Reda Pump

Well	Bbl per month	Input gas				Electrical current			
		Cu ft per month	Cu ft per bbl	Cost per M cu ft, cts	Cost per bbl, cts	Kw-hr per month	Kw-hr per bbl	Cost per kw-hr, cts	Cost per bbl, cts
A	98 390	139 586	1 490	1.35	2.01	59 730	0.61	1.955	1.19
B	50 680	91 967	1 810	0.69	1.25	35 520	0.70	1.75	1.22
C	24 850	55 956	2 250	1.05	2.36	22 520	0.91	1.55	1.41
D	15 030	36 025	2 330	1.02	2.38	13 830	0.92	1.75	1.61

Gas-lift hydraulic-pump combination method has been used in the Okla City field to a limited extent, but enough to indicate its possibilities (34). A Kobe hydraulic pump is attached to end of a section of 4 or 5-in tubing, suspended in the well from 3-in tubing, which is sealed to the casing by a packer usually placed within 500 ft of lower end of casing string. The 3-in tubing (perforated just above the packer) encloses 1 1/4-in tubing, through which "power-oil" is passed to operate the pump, lifting oil from bottom into the casing above the packer; whence gas injected between the 3-in and 1 1/4-in tubing lifts it to surface. In a 6 500-ft well, with reservoir press of about 50 lb, and 9-in casing, this method produced 1 500 bbl or more per day.

Sucker-rod pump in connection with gas-lift. The pump is suspended from 2 1/2 or 3-in tubing, and a packer seals off the well between tubing and casing. Perforations in the tubing above the packer allow injected gas to mix with the oil, thus lifting it to surface. The lowering of press enables the pump to speed up and handle a greater load. This method, employed a short time in the Seminole field, was abandoned because slits were cut in the tubing by sliding of the rods against the tubing. Apparently, gasified oil was less effic as a lubricant than dead oil.

Norod plunger pump. The plunger is driven by a compressed-gas piston pump, lowered on a string of tubing, with packer set between tubing and casing, and the compressed gas passing through the pump joins the oil above the packer, thus raising the oil through the casing.

6. GAS-LIFT, INTERMITTENT FLOW

This type of gas-lift has been employed in several forms, of which 5 are described below (35, 36).

1. Gas entering the tubing lifts oil through the casing without chamber, packer, or valve at bottom of tubing. The gas is admitted intermittently, by manually operated or automatic time-controlled valves at surface (Table 8).

2. A tubing string is run into the well, with a chamber at lower end and a valve at bottom of chamber. A packer is set between casing and tubing, close to lower end of casing. Gas is admitted to the tubing intermittently, thus closing the valve, displacing the oil from the chamber into the casing above the packer, and then lifting it to surface. This is the Clark "bottom-hole intermitter" (Table 9) (37).

3. Gas is admitted to the annular space between tubing and casing, thus forcing the oil through the tubing to surface. No chamber or packer is used (Table 10).

Table 8. Intermittent-flow Gas-lift, Without Auxiliary Devices

Well	Depth	Casing diam, in	Tubing diam, in	Bbl per day	Cu ft input gas per bbl
A	4 208	7 O D	2 1/2	350	685
B	4 349	7 O D	2 1/2	260	950
C	4 125	7 O D	2 1/2	783	5 380
D	4 164	7 O D	2 1/2	104	6 000
E	4 158	7 O D	2 1/2	70	9 000

Table 9. Intermittent-flow Gas-lift, with Packer, and Flowing Through Casing

Well	Depth	Casing diam, in	Tubing diam, in	Bbl per day	Cu ft input gas per bbl
A	6 550	7 O D	2 1/2	888	507
B	6 485	6 5/8 O D	2 1/2	786	1 085
C	6 485	6 5/8 O D	2 1/2	534	2 727
D	6 555	6 5/8 O D	2 1/2	280	4 940
E	6 580	6 5/8 O D	2 1/2	200	9 850
F	6 564	6 5/8 O D	2 1/2	51	11 250

Table 10. Intermittent-flow Gas-lift, Through Tubing Without Chamber

Well	Depth	Casing diam, in	Tubing diam, in	Bbl per day	Cu ft input gas per bbl
A	4 190	7 O D	2 1/2, 3, 4	30.5	2 650
B	4 117	7 O D	2 1/2, 3, 4	12.4	3 644
C	4 125	7 O D	2 1/2, 3, 4	11.1	4 878
D	4 245	7 O D	2 1/2, 3, 4	4.8	2 500

Table 11. Intermittent-flow Gas-lift, Through Tubing with Chamber

Well	Depth	Outer tubing, in	Inner tubing, in	Bbl per day	Cu ft input gas per bbl
A	2 820	2	1	12	3 300
B	2 806	2	1	6	5 000
C	2 863	2	1	4 1/2	4 500
D	3 390	2	1	22	4 200
E	4 349	4	2 1/2	225	4 500
F	4 308	4	2 1/2	125	7 800
G	4 344	3	1 1/2	45	5 500

4. Two strings of tubing are run concentrically. A chamber, with valve at its lower end, is suspended from the outer string. Smaller tubing extends close to bottom of chamber. Gas is admitted to the annular space between, thus closing the valve at bottom of chamber, driving the oil in the chamber into the inner tubing and lifting it to surface (Table 11).

5. Hughes plunger-lift (Fig 4). A tubing string, reamed to smooth bore, is run into the well. A valved plunger, fitting closely to the tubing, drops to the bottom through the column of oil, and strikes a bumper which closes the plunger valve. The gas admitted to the casing then lifts the plunger to surface through the tubing, with its accumulated load of oil, and drops down for another load (39) (Table 12).

Table 12. Intermittent-flow Operation of Hughes Plunger-lift

Well	Depth	Tubing diam, in	Bbl per day	Cu ft input gas per bbl	Casing press, lb
A	1 829	4	400	465	...
B	3 977	2 1/2	168	....	128
C	3 848	2 1/2	160	323	109
D	3 939	2 1/2	52	280	83
E	4 193	4	84	1 560	32
F	5 159	2 1/2	114	....	71
G	6 528	4	309	1 360	...
H	6 528	4	120	1 962	147
I	6 500	3	113	2 440	167
J	6 540	2 1/2	56	6 000	80
K	6 231	2 1/2	30	7 150	35
L	6 736	3	140	2 700	108
M	6 800	3	105	1 140	68
N	7 803	4	468	1 645	260
O	8 200	4	250	3 340	300

Formula for capacity of Hughes plunger-lift is $P = CB^n$, where P is the press, lb per sq in at bottom of tubing, and B the bbl of oil lifted per day. Values of constants C and n are given in accompanying table.

Constants in Formula for Capacity of Plunger-lift

Ft depth	Tubing, diam, in	C	n
3 900	2 1/2	2.906	.7408
6 500	2 1/2	4.292	.7738
6 500	3	3.381	.7497
6 500	4	2.085	.7417

Various devices are employed to admit gas intermittently to the well, the usual ones being controlled by a clock mechanism opening and closing a pilot valve, which opens a diaphragm-operated main valve. Another type consists of a cylinder, in which water is displaced by compressed gas from a compartment at one end to another at opposite end, thus causing the cylinder to tip down and open the pilot valve, which in turn operates a diaphragm-valve in the pipe carrying the gas supply. The number of tips per hr is regulated by a valve placed between the two compartments, thus controlling the quantity of water or oil passing in a given time.

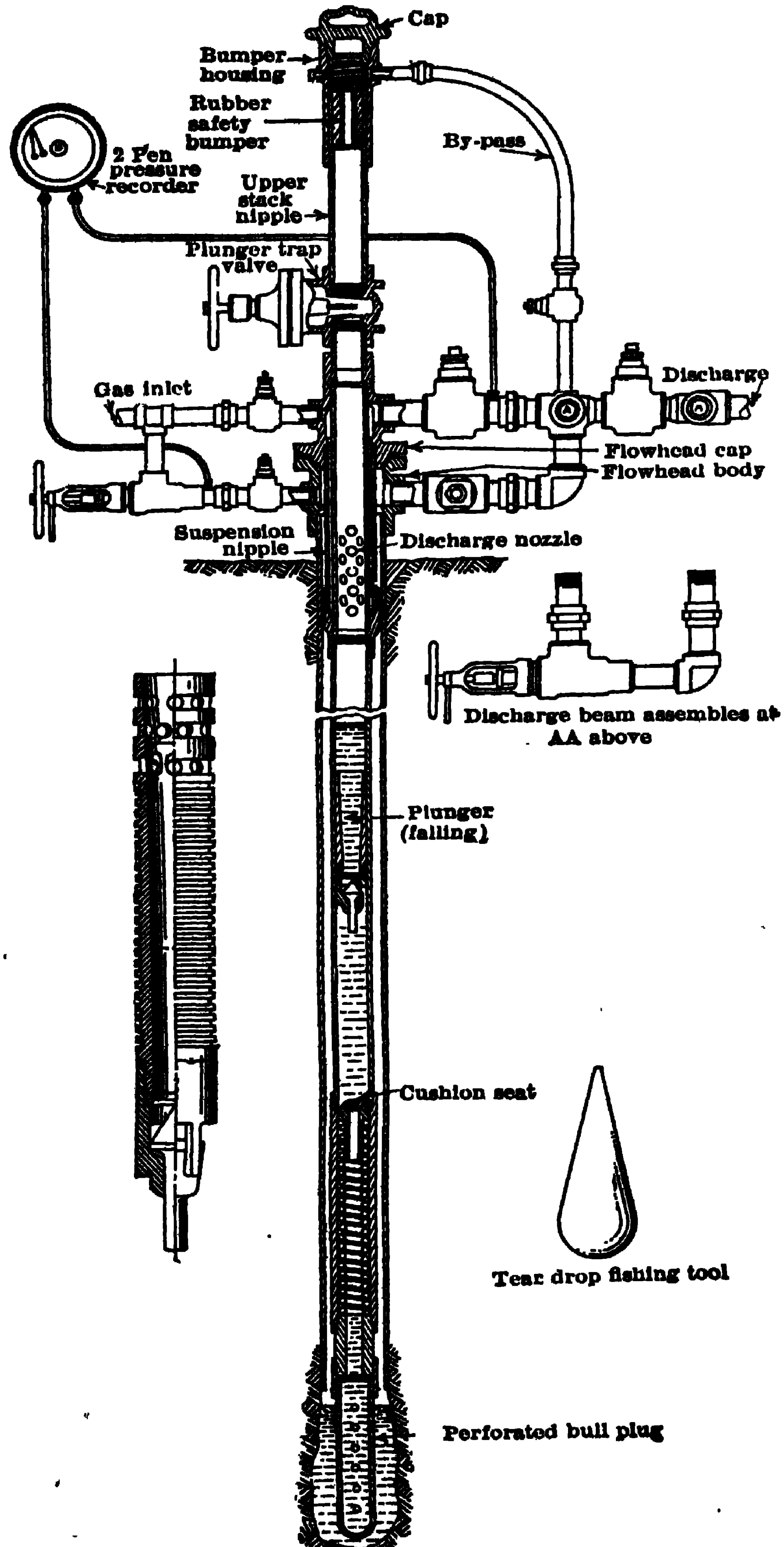

Fig 4. Hughes Plunger-lift

7. CENTRIFUGAL PUMPS

Submerged electric-driven centrifugal pump is used in several fields (Fig 5). In connection with gas-lift it is termed the gas-lift Reda combination method (Art 5); when without gas-lift it is called "straight" Reda pump. The unit consists of a multi-stage centrifugal pump, with the shaft connected directly to an elec motor through a protector section. As the entire assembly is of small outside diam, it can be run to bottom of a well having standard sizes of casing. In operating position the assembly is suspended by standard tubing, submerged in the fluid of the well, with a cable extending to surface, for supplying electricity to motor. Operation of the motor is controlled by a switchboard at surface, having several automatic-control features. In the unit's usual form, the Reda motor is below the pump, but in one type of assembly the pump intake is at the bottom, thus permitting max production from wells of very low fluid level.

The motor is of the squirrel-cage, induction type, operating at 3 600 rpm, and filled with oil for lubrication and cooling. The protector, between motor and pump, contains a spring-backed piston, grease chamber and oil chamber; it equalizes internal and external press and excludes well-fluid from the motor. The pump is a vert assembly of centrifugal impellers and diffusers, enclosed in a steel housing; their number, type and size being determined by the vol of fluid to be raised and the required discharge head. The electric cable is oil proof, unaffected by hydrostatic pressure, steel armored, and flexible enough to permit easy spooling.

The Reda pump is installed by lowering the motor-protector-pump assembly into the well on the tubing like a working barrel, the cable being supported by clamping it to the tubing at intervals. Surface end of cable being attached to switchboard, the pump is ready for work. Reda units now in operation are of 7.5 to 97.5 hp. Largest volume handled is 14 000 bbl per day; the smallest, 18 bbl. The "straight" Reda pump has been largely used in the Okla City field, handling 400 or 500 to 1 200 bbl per day, with an elec consumption of 2 1/2 to 6 1/4 kw-hr per bbl lifted, if no water is mixed with the oil (40, 41) (Table 13).

Table 13. Operation of a "Straight" Reda Centrifugal Pump

Well	Bbl per mo	Total kw-hr	Kw-hr per bbl	Cost of elec current, cts	
				Per kw-hr	Per bbl oil
A	19 300	50 791	2.62	1.72	4.51
B	14 600	48 900	3.34	1.45	4.84
C	7 000	31 850	4.55	1.55	7.05
D	4 620	28 620	6.17	1.75	10.8

8. HYDRAULIC PUMPS

Hydraulic plunger-pumps are coming into use for lifting oil under certain favorable conditions. The most successful thus far is the Kobe (42), a triplex pump mounted on a filter tank and operated by electric motor or multiple-cylinder gas engine (Fig 6).

Table 14. Data on Wells Equipped with Kobe Hydraulic Pump, Oklahoma City

Well	Pump size	Diam tubing		Bbl per day		Pump, speed, r p m	Pump vol effic, %	Kw-hr per bbl fluid	Power cost, ¢ per bbl fluid*
		Inner	Outer	Fluid	Oil				
A	2 1/2	1 1/4	3	89	89	26	70	2.62	4.45
B	2 1/2	1 1/4	2 1/2	161	161	40	82	1.52	2.58
C	3	1 1/4	3	94	94	35 1/2	28	2.82	4.80
D	3	1 1/4	3	183	183	21 1/2	90	1.36	2.31
E	3	1 1/4	3	252	252	29	92	1.45	2.47
F	3	1 1/4	3	359	344	40	95	1.39	2.36
G	4	2	4	475	475	24 1/2	92	...	....

* Based on 1.7¢ per kw-hr.

Table 15. Capacity Rating of the Kobe Hydraulic Pump

	Pump size			
Outside diam, in	2	2 1/2	3	4
Inside diam, in	1 7/8	2 5/16	2 7/8	3 13/16
Length, in	78 1/2	105 1/2	136 1/2	173
Stroke length, in	12	18	24	30
Strokes per min at rated capac	47.7	40.7	42.4	38.1
Capac, bbl per day	100	200	400	800
Displacement per stroke, bbl	0.00146	0.00342	0.00656	0.0146

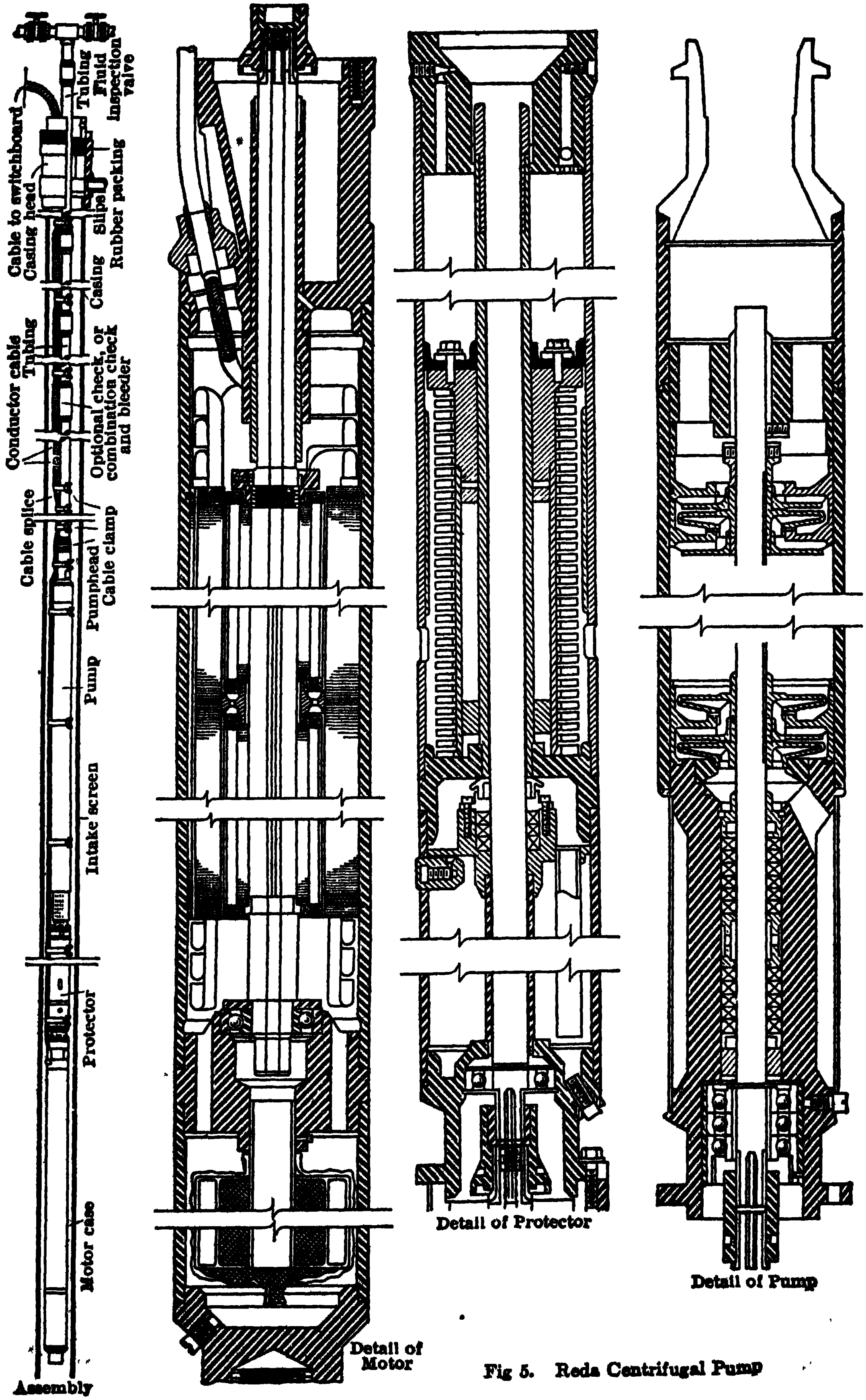

Fig 5. Reda Centrifugal Pump

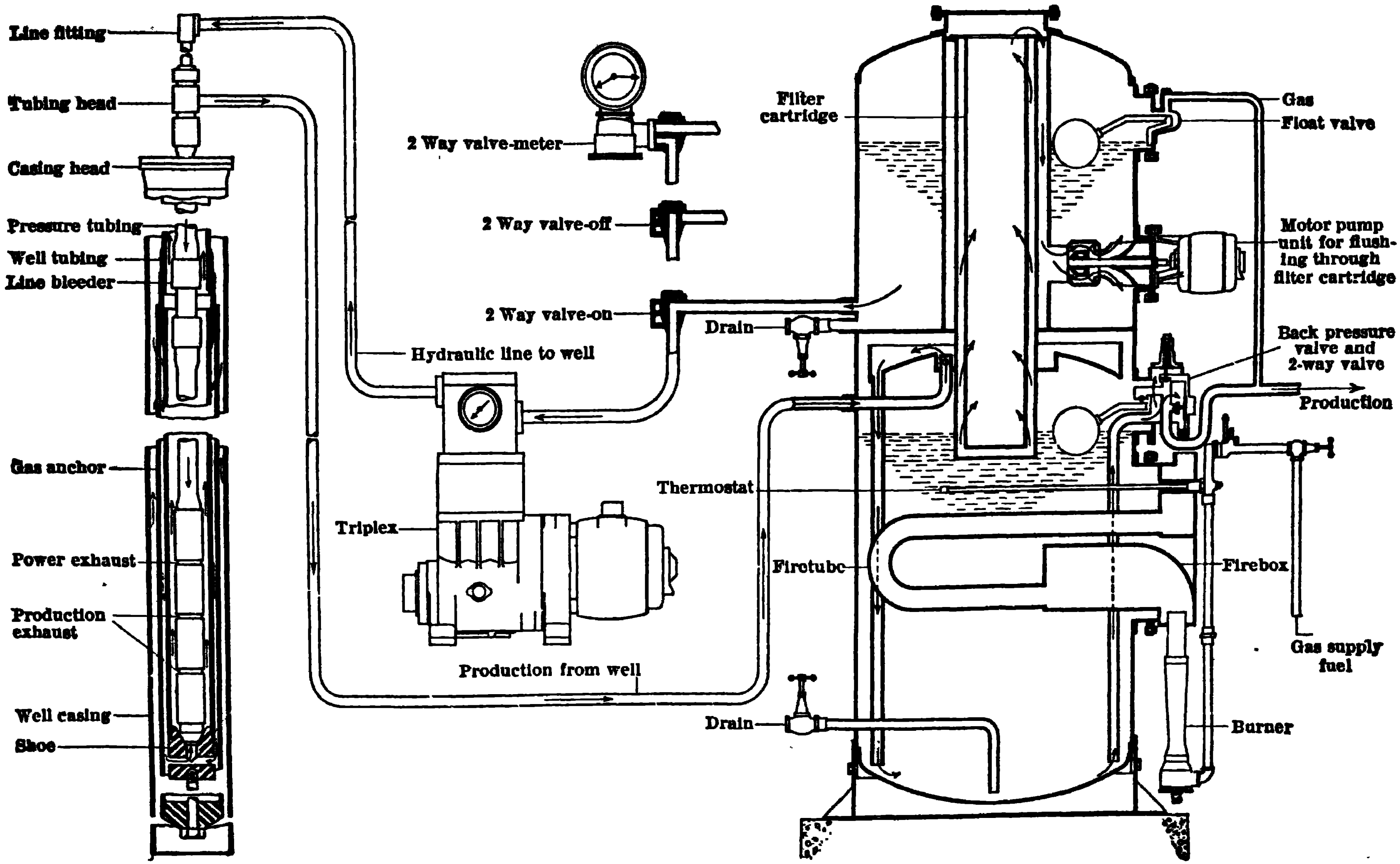

Fig 6. Kobe Hydraulic Pump Layout

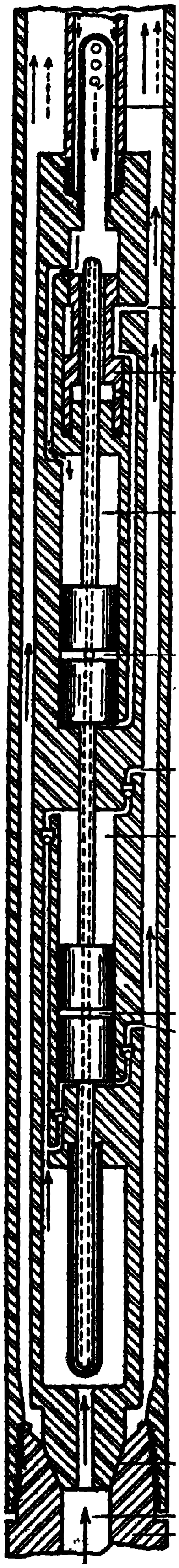

Fig 7. Details of Kobe Hydraulic Pump (see Fig 6)

The bottom-hole Kobe production unit is a double-acting plunger pump connected directly to a fluid engine (Fig 7). The only mechanical connection between the pump and the surface is a string of small-diam tubing, to conduct the power fluid to engine. The surface power unit, supplying fluid under the desired press, consists of a filter-separator system (when required) and a combined prime motor and triplex pump. In equipping a well for this method, a full string of tubing (the "production string") is installed with a gas-anchor and seating device at its lower end. The bottom-hole engine and pump form a single unit, inserted within the production string and run to the bottom on a smaller string (the "pressure string"). The power fluid, conducted down through the pressure tubing, is discharged to surface through the annular space between the two strings, being mixed with the fluid pumped from reservoir.

In the Oklahoma City field the fluid from the well passes through a heater and then to a filter, whence enough filtered oil is returned to the well bottom to actuate the fluid engine. Usually the quantity of power oil is approx the same as that of the oil produced from the well. In this field, when using 3-in tubing with 1 1/4-in inner tubing, the pump has a max capac of 300–400 bbl per day (43). With 4-in tubing and a 2-in inner tubing, the max capac is 700–800 bbl per day (Table 14).

9. SWABBING AND BAILING

Swabbing is often used for lifting oil in the interval between cessation of natural flow and completion of permanent equipment with some artificial lifting method. A close-fitting rubber swab, enclosed in a wire cage on the end of a wire line, is run down into the fluid as far as will provide a safe load for the hoisting equipment to raise. The oil passes through a check valve into the swab, as it is lowered. The valve closes, and the rubber flattens out against the casing or tubing walls, making a fairly tight seal when the swab is lifted. Upper end of swab has a pin-joint connection with the drilling tools, to provide enough weight to carry the swab rapidly down the well and then to sink it into the oil.

This method has been widely used in the Boryslaw field, Poland, for depths to 5 000 ft. There are usually two independent hoists, so that no time is lost if one is disabled. Some wells have 3 engines, two driven by steam, one by elec motor. These installations are rather elaborate, as the lifting speed is high. Sand can be lifted with the swab, but wear is rapid and the rubber must frequently be replaced. The load of oil, especially when mixed with sand, must not be too great to be lifted properly; the casing has sometimes had to be pulled from the well to remove the swab, when frozen in the pipe by the load of sand.

Bailing is sometimes used to lift oil, when accompanied by much sand, as in the Baicoi field, Rumania. The bailer consists of 10 to 40 ft of pipe, of a diam permitting free passage through the casing, and allowing gas to pass freely between casing and bailer. At the lower end of bailer there is a dart-valve,

through which the oil passes when bailer is lowered, and which is seated tightly when being hoisted. Bailers range from 4-in diam to sizes that will pass through casings as large as 14 in.

In the Baicoi field, many holes were very crooked, partly due to movement of sand in the steeply dipping formation, and bailers with flexible joints were used. Oil and sand are dumped from bailer into mine cars, from which the sand is ladled by hand and trammed away. When production at Baicoi declined, thus causing less movement of sand, bailers were replaced by pumping jacks actuated by central power.

Bailing is rarely practicable, due to cost of wire ropes, which must often be replaced, and also to their tendency to wear slits in the casing of a crooked hole. The bailer rope winds on the hoist drum, which is 3 to 5 ft or more in diam. For high speed in deep wells, considerable engine power is required.

10. SUCKER-ROD PUMPS WITH INDIVIDUAL DRIVE

These have been common since the first discovery of oil in the U S. Probably 75% of pumping wells in this country use sucker-rod pumps, and perhaps 60% of the world's oil production is raised by this method. There has been little change in the principle, but great improvements in design and effic of equipment. Stresses on the sucker-rods increase at some geometrical ratio with the depth (21). However, as in the Oklahoma City field where depths of as much as 6 500 ft are reached, sucker-rod pumps often lift 300 bbl per day; sometimes 500 bbl (44).

The sucker-rod pump has a working barrel with hollow plunger, and a standing valve at the lower end of a string of tubing. It is actuated by the sucker rod and contains a traveling valve like the standing valve. The valves are steel balls on metal seats, inclosed in a cage screwed to the seats. The plunger is operated by a rod-line, $^1/_2$–1 in diam, suspended from a walking beam or other device at surface. On up stroke the standing valve opens, allowing fluid to follow the rising piston into the barrel. The traveling valve is shut, causing all fluid above the plunger to be raised. On down stroke, the standing valve shuts and the traveling valve opens, allowing oil in the barrel to pass through the plunger. The pump is actuated by an individual engine, or by a "pumping-jack" driven from a central power plant serving several wells. A few wells have air or steam piston heads, or hydraulic lifts. In many cases, a rod-line may be hitched to an individual walking beam unit, and connected to another well having a pumping jack. As many as 3 or 4 wells have thus been operated.

These pumps are now common. Notwithstanding the heavy press against which they work, and the constant tendency for sand to enter the pump, together with the difficulties due to frequent presence of corrosive liquids and gases, these pumps operate efficiently at the end of a sucker-rod string a mile or more in length (45). INSERT-PUMPS are of such diam that they can be run inside the tubing, so that the tubing need not be pulled if the pump only is to be removed (46). The pump is set on a tight-fitting seat at bottom of the tubing. FLUID-PACKED PUMPS, without a plunger, have two concentric cylinders, one sliding up and down over another close-fitting cyl. They are effective for large volumes of oil and water. CASING PUMPS are used for lifting large volumes against a moderate head. A packer (Art 5) is set on the tubing within the casing near bottom of well, the oil being thus lifted into the casing. Above the packer, the sucker-rod string travels through the casing without use of tubing.

Sucker-rods are of best grade steel or wrought iron; diam from $^1/_2$ or $^3/_4$-in for wells to say 1 000 ft deep, to 1 in for wells of 6 500 or 7 500 ft, like those at Okla City, and some even deeper in Calif. Rods are in lengths of 30, 25, and 20 ft, with box-and-pin joints, and tapered threads to make a tight fit quickly. For very deep wells, 1-in rods are often used to about 2 000 ft; then $^7/_8$- or $^3/_4$-in to the bottom. Too wide a range of tapering is not good practice. Care must be taken to prevent bends or kinks in the rods, and avoid striking them by a heavy blow, which might cause breakage in rods carrying a heavy load (47). Crooked holes are troublesome for sucker-rods. Box-and-pin joints become badly worn and subject to frequent breakage. This can be partly overcome by using sucker-rod protectors, but most operators depend on case-hardened joints to effect a minimum of galling.

"Polish rod" is the connecting link between the pumping jack, or walking beam, and the string of sucker-rods. It passes through a stuffing box in the top of the tubing, to its connection with the sucker-rod string.

Tubing is usually 2 or 2 $^1/_2$ in diam, though some wells use tubing of 3 in or more for large-scale pumping. For small production in shallow wells the tubing is 1 $^1/_2$ or 2 in; 2 $^1/_2$ in tubing has been used for greatest depths thus far reached. In shallow wells standard couplings can be used; for depths of 3 000 to 6 000 ft or more, the ends of the tubing lengths are upset to strengthen the

threaded coupling and lessen danger of breakage, especially when the string is being pulled from a hole in which sand or debris has caved around the tubing (48).

Gas anchors are attached to lower end of tubing in gassy wells, to allow gas to dissociate from the oil, so far as possible, before it enters the working barrel. The anchor surrounds the barrel, the oil passing through perforations in upper part of the anchor, then downward to the pump intake. Gas thus separated passes out of the anchor into the casing and then escapes through the casing head. Gas collecting in the working barrel, especially in a pump having considerable clearance between the standing and traveling valves, tends alternately to expand and compress, thus reducing the pump's capacity.

Gas anchors should be designed with care; they are commonly more harmful than beneficial, due to faulty design or application. Mere length beyond 5 or 6 ft has no advantage, unless space for a sand trap is required. Constrictions in line of flow through the anchor should be avoided, as consequent press drop due to frictional resistance tends to bring gas out of solution. The anchor must not have protuberances nor a large diam, which might cause difficulty in pulling out of the well. In low-fluid-level wells, it is desirable to place the anchor around the working barrel, rather than below it, to obtain the effic resulting from max submergence.

Tubing-catchers usually accompany long strings of tubing, to prevent the string from dropping to the bottom if accidentally released while being run down, or if the tubing parts. Upset tubing is now common for deep wells, to reduce danger of parting, but it is best practice to use tubing catchers on all strings in deep pumping wells, as the full weight of the column of oil and that of the tubing must be sustained by the string.

Tubing anchors are sometimes used in deep wells, especially if crooked, when there is a tendency for the tubing string to creep up and down with the sucker-rod strokes, or if sand lodges between the plunger and working barrel, thus freezing them together. This creeping action wears the tubing collars and casing walls. Anchors are set about 1 000 to 1 500 ft above lower end of tubing, to prevent the tubing from creeping lower. Though not always effective, many operators favor them.

Prime movers for pumping are of many makes and designs, comprising steam, gas, gasolene and oil engines, and elec motors (49). Steam engines were formerly common, the usual fuel being oil or gas obtained from the property. At present, steam is generally limited to cases where the drilling engine is retained only until the well is thoroughly tested; then if it proves profitable, more effic equipment is substituted. Gas engines are the commonest, the fuel being nearly always available from the well at no cost. They comprise slow-motion horiz, 2-cycle engines of 10 to 90 hp; vert 2- or 3-cylinder, 2-cycle engines up to 60 hp, working at 300 to 600 rpm; and multiple-cylinder 4-cycle engines to 80 hp at speeds of 600 to 1 800 rpm (50). Slow speed 2-cycle horiz engines, as the Bessemer, Fairbanks-Morse, Frick-Reid, Superior, Titusville, and Weber, have had wide use. Vert 2-cycle engines at 300 to 600 rpm, which came in during 1936, include the Cooper-Bessemer, Clark Bros, Fairbanks-Morse, Superior, Weber, and others. Multiple-cyl 4-cycle engines have been used for several years, the best-known being the Buda, Case, Caterpillar, Climax, Hercules, International, and Waukesha. OIL ENGINES are seldom used except where gas is very scarce (51). ELEC MOTORS are widely and increasingly used, despite the fact that elec power must generally be purchased. Facility in starting and stopping motor-driven pumps has much in their favor, and lost time caused by engine trouble is minimized. First cost is usually larger than with other types of drive.

As the engine speed is never that at which the walking-beam travels, there must be a reduction in speed, for which the band-wheel is commonly used. Between band-wheel and engine pulley there is a counter shaft and pulleys. If the engine serves other wells also, the counter shaft has a reversible clutch. At one end of the band-wheel shaft is a crank with several holes, so that the pitman can provide the desired length of stroke to the walking beam. Also, at one side of the band-wheel, there is a grooved pulley over which a manila rope runs to the bull-wheel. (For details see Sec 9.) Reduction-gear units were recently introduced in place of the band-wheel (52). Some have a grooved pulley and rope for driving the bull-wheel; or the grooved pulley and bull-wheel are replaced by a portable pulling unit. Reduction units are made by the Continental Supply Co, Foote Co, Lufkin Foundry, National Supply Co, Oil Center Supply Co, Oil Well Supply Co, and others. They are generally used with multiple-cyl engine, or electric motor (53). Flat belts have been used, but the present tendency toward the V-belt drive, between motors or multiple cylinder engines and the reduction unit, is preferable.

Walking beams are usually of steel (Sec 9, Art 5).

Counter-balances of various types are used to balance the weight of sucker-rods and column of oil. The "grasshopper" type is common; also pieces of cast-iron, bolted to the reduction unit crank, or to the opposite side of crank arm on band-wheel shaft. Proper balancing is important for uniform loading on the engine, and the most effic filling

of the working barrel with oil (54). Several types of hangers are used to obtain a vert pull on the "polish rod," from which the sucker-rods are suspended, the movements of the beam being varied for same purpose. The Parkersburg long-stroke unit imparts a vert motion by toggles attached to the beam.

Miscellaneous parts. Cushions are used to dampen the strain on the polish rods at end of stroke; sucker-rod rotators, for turning polish-rod and sucker-rods a certain number of degrees at each stroke, to cause uniform wear on the rods at boxes, and prevent the plunger from wearing along one side; wobbler stuffing-boxes, sometimes used where the polish rod enters the tubing, to take up irregularities in the motion and thus prevent side movement of the casing head. GARBUTT RODS, inserted in the plunger, make contact with the standing valve-cage when the rods, plunger and standing valve are pulled, but are not practical where the oil is grassy or must be pumped at high speed, owing to the danger of overtravel which might cause the rods to strike the standing valve-cage. Double-ball traveling valves are used to reduce wear and gas-locking, and to distribute better the load pressure.

Portable servicing units are used when rods or tubing must be pulled, or other work done. They comprise hoisting facilities for greater speed than is obtainable with an ordinary pumping outfit. But, if frequent pulling jobs are necessary in a deep well, it may be better to install a special hoist than to risk loss of considerable production while awaiting arrival of a portable outfit.

Costs. In the Okla City field, most wells employ portable units, built in sizes costing up to $15 000 or more. A walking-beam outfit, including a self-servicing hoist, costs $1 000 for a shallow well, to $15 000 for a 6 500-ft well. For a 6 500-ft well, pumping units driven by small, vert, medium-speed gas engines, cost installed about $10 000.

Cost of a steam-engine, walking-beam pump in the Coalinga field, Calif (55) is: engine, 23-hp complete, $296.09; boiler, 40 hp, $473; boiler connections $116.15; engine house, concrete foundations, lumber and labor, $66.80; total $952.04.

Cost of gas-engine outfit for a pumping well in the Midway-Sunset field is: engine, with pipe and fittings, $2 025; circulating tank, 50 bbl, $170; cement $45; labor, including foundations, hauling and setting engine, $155; miscell, 5%, $119.75; total $2 514.75.

Aver cost for equipping 367 wells in Mid-Continent field: gas engine, 35 hp, $1 950; foundations and floor, 16 cu ft at $35, $560; connecting and erecting $200; water tank, setting up and grading, $165; gas and water pipe, 800 ft at 35¢ per ft, $280; total $3 155.

Aver cost for equipping 241 wells with elec motor: motor and control $1 355; foundations and floor, 8 cu yd, at $35, $280; transformers, switches, etc, $275; electric lines $120; connecting and erecting $100; turbo gear, ratio 5 to 1, $775; flexible couplings $175; shaft, pulley and bearings $100; total $3 175.

Cost of installing a walking-beam pump unit in Okla City field in 6 500-ft well: pumping unit $4 400; vert, 2-cyl, 2-cycle gas engine, $1 700; house $280; foundations $150; installation, labor, and hauling $500; pipe $100; fittings $200; belting $95; gas regulator $20; pulling winch $90; guards $75; platform $25; stuffing box $10; polish rod $10; polish-rod grips $20; pump $300; gas anchor $10; gas cleaner $10; sucker rods $1 300; water tank $125; total $9 410.

A self-servicing pumping outfit, comprising a 75-hp horiz 2-cycle gas engine, with clutch, geared pumping unit, steel walking beam, bull-wheel, pump, and rods, installed in a 6 500-ft well, costs approx $15 000.

Capacity of sucker-rod pumps depends on diam of working barrel, length and speed of stroke, clearance in working barrel, and other smaller factors (56). Speed of stroke can be increased in shallow wells, as there is not enough friction between rods and tubing to prevent the sucker-rod string from dropping back rapidly on down stroke. Large-diam pumps can be used in shallow wells, capable of a large production. Speed in shallow wells is 10 to 30 strokes per min; length of stroke, 6 to 48 in. Number of strokes in deep wells: 10 to 25 per min; length of stroke, 36 to 132 in.

Tubing in shallow wells is usually of smaller diam than in deep wells, as there is less friction loss in a short length of pipe. After natural flow ceases, shallow wells usually require a pump of small capac. In 2 000-ft wells, a 4-in pump suspended on 5 1/2-in casing-string lifted about 2 000 bbl of fluid per day, but breakages of moving parts were excessive. Capacities of shallow wells range from almost zero to 2 000 bbl per day. In deep wells, as in Okla City field, the capac varies from say 10 bbl per day (the lowest limit economical to pump) to 500 bbl; aver, between 100 and 200 bbl per day (57).

Space between standing and traveling valves should be as small as possible, to minimize accumulation of gas between the valves. The valves should be spaced after the well pumps up, due to tendency toward excessive overtravel at that time; if their proper position has been previously determined, the well should be pumped slowly to prevent pounding down. GAS-LOCKING, which causes low effic (58), is reduced by close spacing;

by proper design of pump to give the least possible space between the standing and plunger valves; by minimizing all constrictions, bends, and small diameters in the stream ahead of the traveling valve; by obtaining the max ratio between stroke of pump and pump clearance; by using oversize valves or multiple valves; and by obtaining the max submergence. OVER-STROKING is caused by inertia of the sucker-rod string and is affected by speed and length of stroke, and weight of the rods. Overstroking may be beneficial, if the barrel is long enough to prevent pounding down. It is increased when there is no oil in the tubing, or by using heavy rods, or by reducing the friction loss with large-diam tubing, or by faster stroking. For a given "polish-rod" displacement, a long, slow stroke gives less overstroke than a fast short stroke. SUBMERGENCE of a pump should be as large as possible; thereby greatly increasing its capac (59). The amount of submergence can often be ascertained by closing in the casing to different back pressures, and observing the production made under these conditions. DYNAMOMETERS are used to determine the load on the sucker-rod string throughout the stroke. By observing the chart of a dynamometer test, many pump troubles can be discovered and corrected (60).

STEELS of various alloy are used for rods, pump, etc, depending on the prevailing conditions. For ordinary strength and minimum corrosion, manganese steel generally suffices. When extra strength is required, chrome and nickel steels are used; for very corrosive conditions, wrought iron, chrome, and galvanized steel.

11. SUCKER-ROD PUMPS DRIVEN FROM CENTRAL POWER PLANT

In groups of wells, connected to a central power plant, the pump is operated by an engine or elec motor (see Art 10). The engine is usually belt-connected to a band-wheel, carrying one or more eccentrics from which transmission lines, or "pull-," or "shackle-rod," run to the different wells (61). A counter-shaft, with friction clutch, is usually interposed between engine and band-wheel belt. From one to 40 shallow wells can thus be operated. The pull-lines are usually $^1/_2$ to $^3/_4$-in steel rods, which are preferable to wire rope, as they stretch less; discarded sucker-rods are also often used. The pull-lines

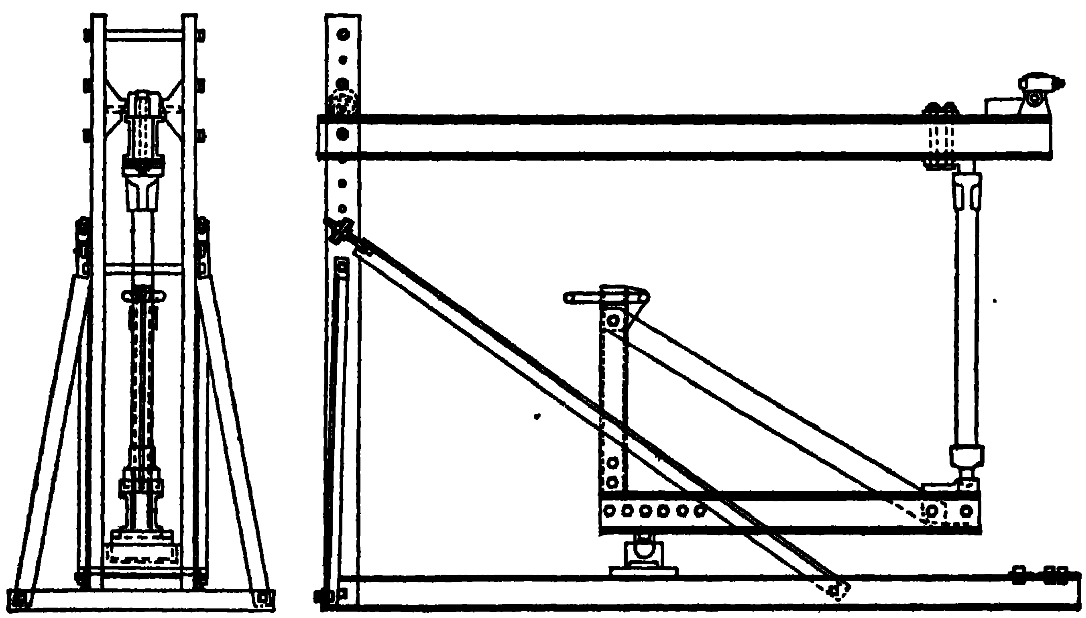

Fig 8. Oklahoma Pumping Jack. (Permission of National Supply Co)

are best run on rollers or guides. Various types of angle turns, rockers, or swings are used for changing direction of a line, to avoid objects that can not readily be removed; such turns may also afford a better balance for the band-wheel.

Pumping jacks above the well operate the sucker-rod string through the pull-rod line (62). The jacks in common use are the Oklahoma (Fig 8) and the Jones and Hammond (Fig 9). Speed of stroke, 10 to 20 per min; length of stroke, which is 12 to 24 in, can be adjusted by changing the hook-up of the rod-line in the pumping-jack frame. In these two types of jack a vert pull is rare, because of the short stroke; but is sometimes made by a "mule-head," where the polish-rod is attached to the jack. Special forms of

jacks, made by Jensen Bros, National Supply Co, and others, are substantially built, and cause a vert pull on the Polish rod.

Sullivan pumping head is a special form of sucker-rod pump, having a single-acting cylinder, suspended in the derrick by pull-rods, the piston-rod being attached to the sucker-rods. The piston, actuated by compressed air or gas, imparts a straight-line reciprocating motion to the rods. Intake air or gas lifts the rods and fluid, forcing the piston upward. Then the valve mechanism reverses and sufficient discharge press is maintained to support the load during the down-stroke.

The unit comprises the compressor, heater, and pump-head. For installations having an individual or central compressor plant, the discharge from the pump-head is piped back to the compressor intake, giving a low-compression ratio and resultant low-compression hp (63). Throughout

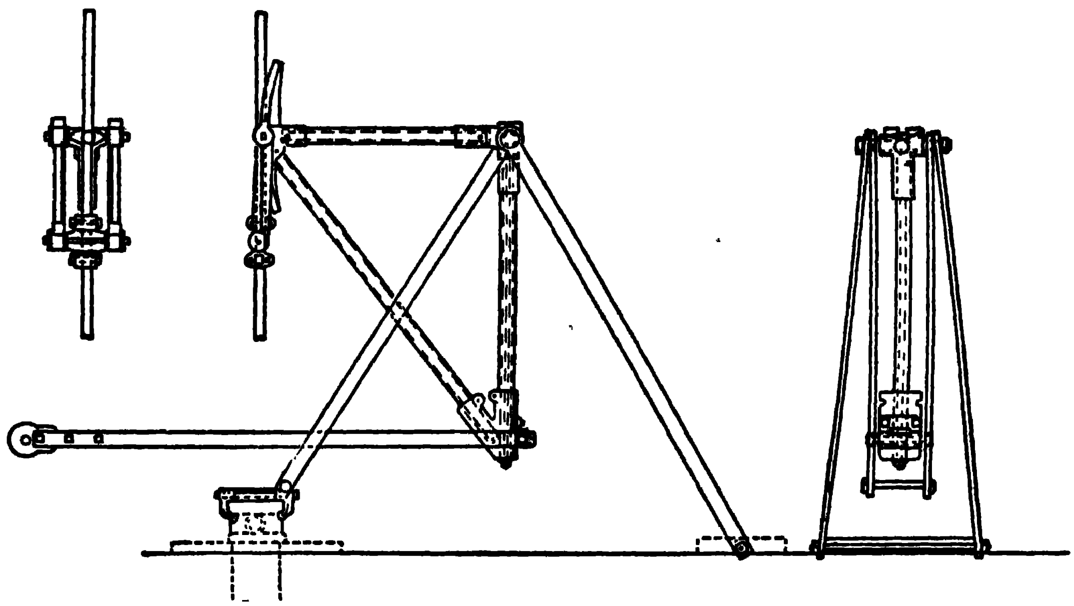

Fig 9. Jones & Hammond Pumping Jack (Permission of National Supply Co)

the entire cycle, the sucker-rod is protected by a cushion of compressed air or gas. The Sullivan head, used in Ill, Mich, Okla, Tex, and elsewhere, gives good service (64). Several in the Okla City field have pumped at depth of 6 500 ft for the past 3 years, the equipment cost, not including the compressor, being about $3 900.

Simple steam-heads are sometimes used, but waste power unless the steam lines are well insulated and freezing at the exhaust prevented in winter. The head may be driven by compressed air, if it can be supplied at low cost. Tendency to freeze is lessened, but not entirely eliminated, due to moisture in the compressor intake air. These heads have been used for years in the Bradford, Penn, field.

12. REPRESSURING WITH AIR OR GAS

Advantage of repressuring a depleted oil field with compressed air was discovered by Dunn and Smith in 1911, when compressed air was injected into a well in Ohio. On reopening the well after a time, considerable oil was raised by the escaping air. This experiment was followed by injecting gas into an input well, and allowing it to find its way through the sand to a producing well; resulting in increased production. The method then spread through Ill (65), Ohio, Penn (66, 67) and W Va, but was not applied in the Mid-Continent and other producing areas until about 1924 (68). It was successful also in certain shallow fields in Kansas and Okla (69) between 1926 and 1930, and has since spread to most oil-producing districts of the world.

Factors entering into repressuring. No definite procedure for all cases has yet been developed. Some operators inject the gas down the dip; others at the upper part of the structure; still others arrange the input wells in patterns distributed through the area to be repressured. Data as to whether a given area has been depleted, or enough oil remains to warrant repressuring, are obtainable by drilling new wells between the old ones, coring the sand, and finding the degree of oil saturation in the areas between wells. Porosity of the sand is important for determining whether the gas will permeate readily

or not. Uniform porosity and sand thickness are generally favorable for repressuring, but presence of much water is unfavorable (70). In fields where repressuring has given good results, there has been a definite increase in both daily production and ultimate recovery (Fig 10).

Old wells used as intake wells present no difficulties in some fields, while in others it is better to drill new wells. To use an old well for intake, the casing should be tight, to prevent escape of gas into sands other than those being repressured. As the deeper sands have usually been under

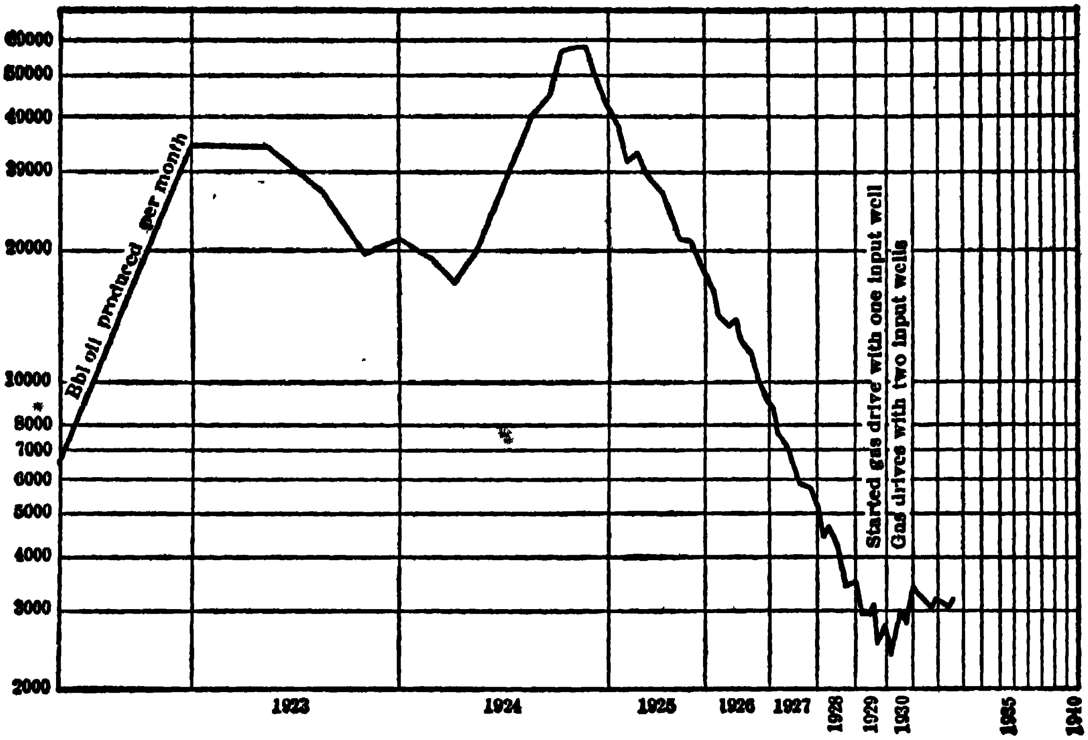

Fig 10. Effect of Gas Drive (Repressuring) in the Burbank Pool, Okla

higher initial press, larger wells are generally found at these greater depths, and the recovery percentage by primary methods is greater than in shallow sands. Hence, it is reasonable to expect less favorable results from repressuring deep wells than shallow ones. If the recycled gas contains much gasolene, its extraction may go far toward paying for repressuring.

Compressed air or gas is used for repressuring. Compressed gas makes less emulsion, if water has encroached, and insures a fuel supply for the engines, uncontaminated by air and less liable to contain explosive mixtures. But, as air dissolves in oil less readily than in gas, it may have a better driving effect.

Pressures employed in different fields range from 50 to 500 lb per sq in; perhaps 200 lb is a fair aver. But in maintenance operations, the press may reach 1 500 lb, as at Sugarland, Texas, or 3 000 lb or more, as in the Tepetate field, La (13), though these do not present the type of repressuring considered here.

Table 16. Cu Ft of Gas per Bbl of Increased Recovery

State	Range per bbl	Aver per bbl
Illinois.....	....	7 400
Kansas.....	....	2 750
Kentucky...	....	4 800
Oklahoma..	650 to 9 000	4 500
Penna......	3 000 to 15 000	8 300
Texas......	450 to 8 670	4 000
West Va....	2 550 to 103 000	8 000

Table 17. Pressures for Repressuring, Lb per Sq In

State	Range	Aver
Calif.............	160 to 1 500	500
Kansas...........	70 to 170	125
Ohio.............	35 to 300	150
Oklahoma.........	18 to 400	175
Penna............	14* to 475	75
Texas............	10 to 1 300	300
West Va..........	26 to 350	25

* Indicates vacuum, in of mercury.

Volume of gas for repressuring depends upon several factors, that can seldom be determined quantitatively until actual operations have been under way for some time.

If the oil sand is thick, and much gas has been removed during the flush stage of production, the reservoir must often be flooded with a large vol of gas before any results are obtained. If the sand has low permeability, the vol required may be less, but with a higher injection press (Table 16).

For pressures to 50 or 75 lb per sq in, single-stage compression can be used, if intake press is not less than atmospheric. For press to 300 lb, 2-stage compression is usual; above 300 lb, and if the plant is to operate several years, 3-stage compression gives lower costs. With 3-stage at 300 lb, operating effic increases about 7 1/2% over that of 2-stage, and effic increases with the discharge press.

Cost question for repressuring is important. To handle gas at high press, equipment and operating costs are both greater, thus increasing cost per bbl of oil. Hence, pipe lines on surface and in the well must be designed to minimize friction losses. Compressing gas for repressuring costs 1/2¢ to 10¢ per 1 000 cu ft, depending upon size of plant, operating press, and kind of power for the compressors. To compress 25 000 000 cu ft of gas per day to 300 lb per sq in, operating costs are about 1/2¢ per 1 000 cu ft, assuming that gas engines are employed and fuel gas is available on the property at no cost, as is often the case in oil fields. For a plant handling 1 000 000 cu ft per day at 300 lb and with fuel gas at no cost, operating costs will be about 2¢ per 1 000 cu ft (72). If electric power is purchased, the first cost for machinery will be considerably less than for internal-combustion engines, and plant upkeep will be less than for gas or oil engines; but cost of electric power will range from 3¢ to 5¢ per 1 000 cu ft of gas compressed to 300 lb (Table 18).

Table 18. Data on Repressuring Operations

State	Field	Bbl per day	Pressures, lb per sq in	Input gas, thous of cu ft per day	Cu ft gas per bbl of increased production
Calif.	Brea Canyon	......	590– 830	2 100	....
	Buena Vista	......	160– 200	125–250	....
	Domingues	......	440– 775	17 450	....
	Elk Hills	......	360– 500	135	....
	Seal Beach	......	1 400–1 500	1 600	....
	Shiells Canyon	......	230	200	....
Kansas	Eldorado	1 500	....	....	2 667
	Eldorado	54	70– 150	1 300	2 850
	Miami Co	......	160– 170	....	....
La.	Haynesville	450	....	....	3 800
Ohio	Byers	9	35– 120	....	18 800
	Carrol Co	......	150– 300	....	....
	Graham	......	275– 300	....	....
	Macksburg	......	45	....	....
	Trail Run	......	46	....	....
Okla.	Alluwe	......	75– 155	....	....
	Alluwe	......	35	20	....
	Avant	......	65	....	....
	Burbank	345	55– 175	900	4 600
	Burbank	......	20– 288	5 085	....
	Cromwell	300	18– 240	315	3 000
	Delaware Ext	......	225– 250	....	....
	Healdton	147	....	....	650
	Lenapah	165	180	1 048	9 000
	Nowata	60	....	....	3 900
	Ponca City	100	225	230	5 500
Penna.	Bingham	......	18	800	....
	Bradford	50	475	617	15 000
	Clarendon	27	....	....	11 900
	Fagundus	80	....	....	3 000
	Hamilton	30	....	....	13 400
	Harmony	9	25	25	3 000
	McKee	20	....	....	6 500
	Knox Plant	42	....	....	3 500
	Poverty Hill	10 1/2	25	80	11 400
	Sheffield	9	....	....	4 700
	Sherard	11.7	....	....	16 800
	Tiedoute	......	35	200	....
Texas	Harmel	900	10– 45	150	520
	Hatchett	......	85– 100	52–70	....
	Iowa Park	65	175	350	7 800
	Oldham	475	45– 60	90	450
	Olney	85	199	975	11 400
	Petrolia	400	85	1 760	587
	Red River	1 600	85	1 600	4 000
	Saratoga	......	75– 185	17	....
	Turbeville	1 093	15	....	700
West Va.	Belmont	100	15	700	11 670
	Boggs	103	....	....	5 180
	Henderson	120	30	450	7 500
	Holliday Cove	......	18	35	....
	Mannington	25	....	....	2 550
	Prunty	61	350	236	4 630
	Richardson	23	....	....	103 000
	St. Mary's	4	15	50	21 700

13. WATER FLOODING OF OIL SANDS

This is being employed for secondary recovery in a few fields of the U S, where profitable production by other methods has almost ceased. So far as known, this method was first used at Bradford, Penna, where it has had its greatest success, and where its usefulness is supposed to have begun accidentally, due to water breaking into nearly exhausted areas, thus unexpectedly increasing the output. The method consists in forcing water

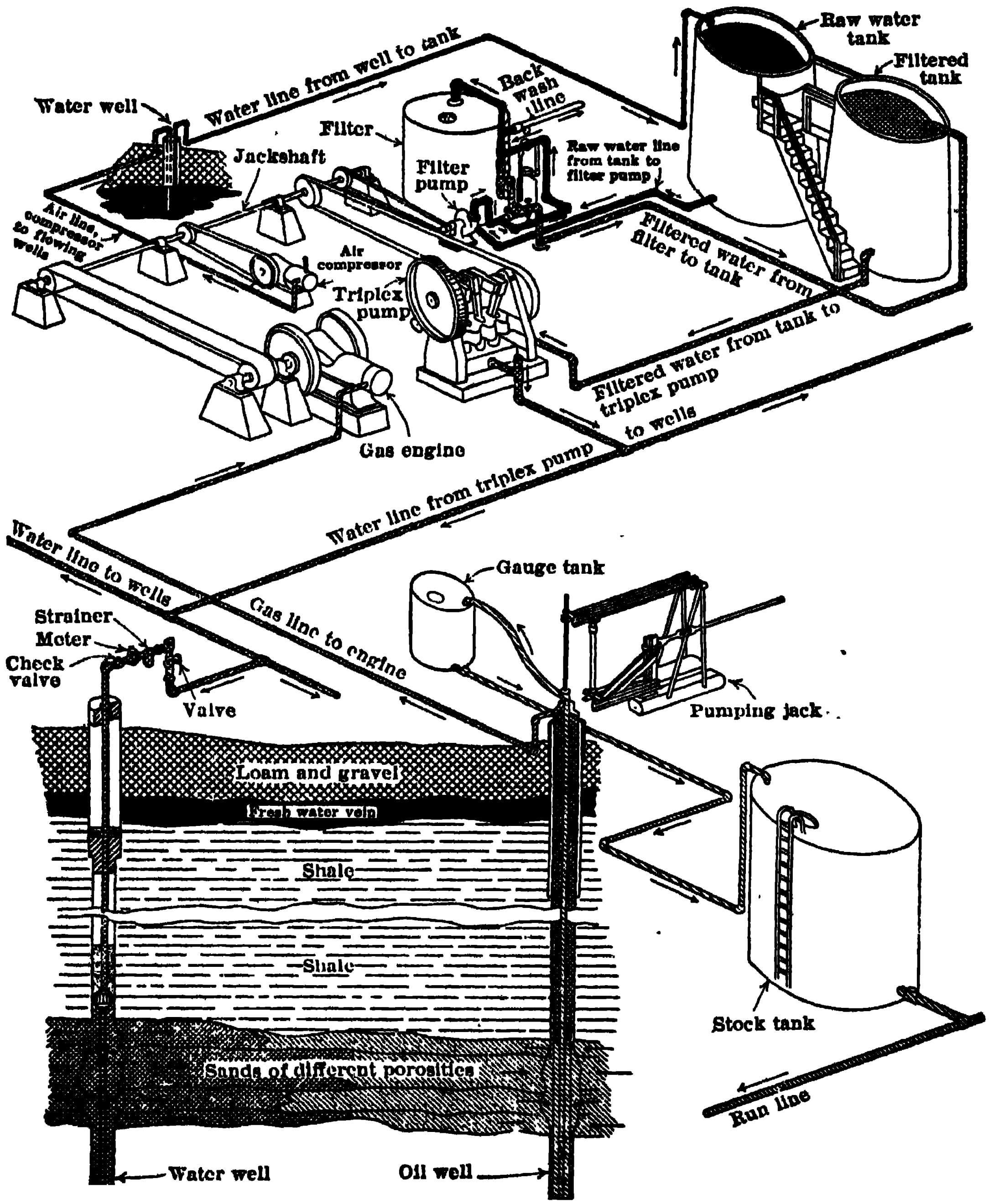

Fig 11. Water-flood Layout in Pennsylvania (Permission of *The Oil Weekly*)

into the oil sand, to drive the oil to a producing well, where it is lifted to surface (Fig 11). Flooding was introduced into the Nowata area, Okla, in 1931, by Bert-Collins and the Carter Oil Co, by whom tests were made near Chelsea. Since 1935 tests elsewhere in that area have produced good results.

Modes of flooding oil sands in the Bradford field are: (1) circle-flood; (2) line-flood; (3) 4-spot; (4) 5-spot; (5) 7-spot (73). The circle-flood might be termed a hit-and-miss

method, as it consists in injecting water into a well selected almost at random, which is surrounded by oil-producing wells, without any systematic procedure; but it can be developed into one of the other methods by spacing the water wells at regular intervals. After finding that production could be thus increased, a study of the factors involved indicated that a systematic arrangement would better serve the purpose. The next step was the line-flood, in which water wells were drilled in a line below the oil wells. The water, introduced in measured quantities, moves up the slope through the sand, pushing the oil into the oil wells, from which it is pumped. When the flood reaches the oil wells, another line of water wells is drilled and process continued until the field is depleted.

Probably the most common method is the 5-spot, as it conforms best to property lines. When a series of 5-spot wells are laid out, they develop into line-floods. As a rule, the sands are quite compact, and permeable in only a low degree; hence, the flood advances slowly, sometimes not more than 50 to 100 ft in a year, unless large volumes of water under high press are pumped into the wells.

In adopting water-flooding a careful investigation of the conditions in the oil sands must be made, to determine the chances of success. Conditions tending to success (74) are where: (1) the oil sand is a continuous, uniform body; (2) porosity is not less than 7% (at Bradford, it is about 11 1/2%); (3) permeability is such that the water will advance through the sand uniformly, and the oil move ahead of the waterflood and not be impeded by too great irregularities in sand conditions; (4) flood can begin at a point low on the structure and move up the dip; (5) flood can move from thin to thick sand sections, if there be such variations; (6) there is little water in the reservoir to be flooded; (7) there is no gas stratum depleted of its gas content, and into which the water would enter. In such case the gas sand must be packed off before the flood is started; (8) the initial gas-oil ratio has not been unusually high, which might indicate that relatively little water would be required to fill the spaces once occupied by gas. If much gas is associated with the oil, there is a fair chance that the oil has been largely extracted by movement of gas toward the producing wells; (9) oils of low viscosity move ahead of the water-flood more freely than those of high viscosity, which have greater resistance to displacement; (10) oils not forming permanent emulsions act better in water-flooding than others. The cost of breaking down permanent emulsions may be serious; (11) an ample water supply is necessary for flooding, obtained from a source containing no dirt or salts that would clog the sands. Dirty water must be settled, or filtered, and salts removed, especially if they precipitate and thus impede the flow.

Water-intake wells are prepared by drilling and shooting the sand, to reduce movement of water from the well into the sand. In the Bradford field, 2-in tubing is usual, packed off and the packer cemented in place. The water injected is measured, and a continuous, permanent record kept of the quantity injected through each well. It is usually distributed to the wells by vert triplex pumps; sometimes by centrifugals.

Delayed drilling is sometimes more satisfactory than repressuring. The water wells are drilled in the pattern deemed advisable, and water is pumped into them until the oil is forced into the area of the oil wells to be drilled. Wells may flow naturally for a time, so that enough recoverable oil is extracted by flowing to make unnecessary the use of lifting equipment. This method has shown a recovery as much as 20% greater than with ordinary water-flooding (75).

Back-pressuring has been employed in water-flooding to prevent so high a rate of production as to lower the press to the point requiring lifting equipment. With back-pressuring, oil is extracted more slowly, the output peak being delayed.

For profitable operation in the Bradford field, Penna, the minimum quantity of oil that must be recovered, besides cost of leasehold, is 3 000 to 3 500 bbl per acre. In the Mid-Continent, 6 000 to 7 000 bbl per acre should be recovered to insure profit, unless the wells are very shallow, thus giving low development cost. Natural flow and pumping in the Bradford field are said to have recovered about 35% of the contents of the reservoir, and water-drive accounts for an added 25 to 35%.

In Cody's Bluff field, Nowata Co, Okla, oil wells are spaced at one per 2 1/2 acres, with water wells on 5-spot patterns, which means another set of wells on basis of one well to 2 1/2 acres. This is close spacing as compared with practice in deeper sands, but seems necessary here for satisfactory recovery (76). Delayed drilling of 45 to 75 days is being done at Cody's Bluff in the Bartlesville sand, at depth of about 500 ft, and sand thickness of 40 to 50 ft. The water is treated and filtered before use (77).

Cost of preparation for water-flooding in Penna is about $3 000 per acre (78). Pressures for pumping the water reach 1 600 lb per sq in. The water is treated before pumping into the sand. One lease of 160 acres, making 40 bbl per day, was increased to 3 000 bbl per day by applying flooding.

14. PETROLEUM MINING

Ordinary mining methods were applied to production of petroleum long before the present-day practice of drilling wells. Shafts were sunk or tunnels driven into the oil-bearing strata, or along outcrops and surface seepages. The oil was collected in pits, or skimmed from underground pools of water. This procedure has been followed for many years in Rumania, Russia, Burma, Japan and elsewhere.

In ancient times, oil mining accounted for most of the total production; now, only a very small percentage is so obtained. Underground mining methods are still employed in the Pechelbron field, Alsace (79, 80), at Wietse, Germany (81), and in Canada and Texas.

According to recent data, the total production at Pechelbron is about 520 000 bbl per year, of which 230 000 bbl are recovered by mining. In 3 mines of the district there are 6 shafts, 495 to 775 ft deep, with some 90 miles of underground workings. Other data show that about 17% of the Pechelbron oil comes from borings from the surface, 43% from drainage of oil into mine drifts, and much of the remaining 40% is recoverable when the sands are finally hoisted out and washed. Similar practice is followed at Wietse.

At Bustenari, Rumania, 3-ft diam shafts have been sunk by hand to oil-bearing sands, at depths of 200 to 800 ft. On reaching sand, small holes are drilled ahead from the shaft to locate bodies of oil, gas or water, thus averting danger to the miners from sudden inrushes.

In the Nacodgoches field, Tex, oil is mined through shafts, 6 by 7 ft section. Good showings were found in one shaft at 60 to 70 ft depth. Another shaft has recently been sunk to 225 ft to an expected oil sand (82).

Ranney process (83). A shaft is sunk to rock, either above or below the oil sand, and cross-cuts are driven at regular intervals. In holes drilled from the cross-cuts to the sand, pipes are inserted to convey the oil to a central point, whence it is pumped to surface. Through other holes compressed air or gas is injected into the sand, to force the oil into the oil pipes.

15. TREATMENT OF OIL

Following is a very brief summary. Oil containing no water or gritty matter can usually go direct to the purchasing pipe-line company; hence, called "clean pipe-line oil." Sand accompanying the oil, especially if fine, must be removed to the degree where oil is acceptable to the pipe-line company. In certain fields of Rumania, the oil contains so much sand that it is discharged at the well into mine cars for settling, the sand being conveyed by other cars to the dump. Often the oil is run to a series of connecting pits, the sand settling out as it flows through them. Thick viscous oil tends to retain sand or rock particles, and requires considerable time for settling. In the U S it is settled in large pits; or, if very heavy, it must be heated, or diluted, to remove sand.

Emulsification of the oil usually follows entrance of water into the well. Emulsions may form while flowing through the sand; by leakage of water and oil through valves; or by flowing through restricted passages, as flow nipples and sharp turns. Emulsions due to low water percentages (10% or less) present more serious treatment problems than high water percentages; also, low-gravity oils are usually more difficult to treat than high-gravity.

Emulsified oil is treated by heating, chemical or mechanical means, or electrical dehydration (84). Heat is a factor in most methods. Unless the liquid from the well is already at 100° to 150° F (as in some Gulf Coast fields), it is heated: (*a*) while flowed through a boiler; (*b*) by mixture with hot water in a tank under thermostatic control; or (*c*) heated in a flow-treater, where separation takes place. To the hot liquid, flowed to a "gun-barrel" or treatment tank, a small quantity of treatment compound is added, such as "Breaxit," "De-Hydro," "Tret-O-Lite" or "Vez," causing separation. Clean oil overflows to the stock tank and thence goes to pipe-line; water is siphoned (at a rate to maintain fairly constant tank level) from the bottom of the gun-barrel tank to a settling pond, where any oil or basic sediment contained is caught and pumped back, the clear water being discarded or (if used for heating) returned so far as needed to circulation (86).

Cost of a treatment plant in the East Texas field, using a 750-bbl gun-barrel tank, 24 ft high, with capac of 40 to 80 bbl per hr, is as follows (87): 750-bbl steel tank gun-barrel $1 083.65; heating boiler $500; centrifugal circulation pump, electric-driven, capac 80 bbl per hr against 40 ft head, $138; connections and fittings $409; labor $219; teaming and trucking $78.80; gas regulator and thermostat $76.70; total $2 505.15.

A recent development in treating oil is the use of special "flow treaters," through which the emulsified oil passes. In these, constant heat is maintained by gas-fired burners set within the treater shell, and under thermostatic control. Discharge of water, oil, and gas from the flow-treater is controlled by pilot-operated diaphragm valves, and fluid-level

control-valves. Exceptionally clean separation is thus obtained in the Okla City, Seminole and other Okla fields.

Mechanical treatment. One method consists in passing the emulsified oil through a tank filled with excelsior, on its way to the tank battery. At Sulphur Bluff and Talco (Texas) both heat and chemical treatments are applied before the oil goes to the excelsior-filled tanks (87). In some fields, where oil containing much water is discharged under press, free water is removed before the oil goes to treatment tanks, by a water "knock-out" with a choke at its end. This permits the water to be blown out periodically, the knock-out being operated manually, or automatically by the water level in the vessel.

Electrical de-hydration is sometimes employed, the oil being heated before entering the apparatus, as in the Luling field, Government Wells, Cayuga, and other Texas fields. Cost of an elec de-hydration unit is $1 500 to $3 000 (55).

Treatment costs range from a fraction of a cent to 20¢ per bbl of oil. Heating in open tanks or pits costs 15¢ to 20¢ per bbl. Cost of electrical treatment, 1 1/2¢ to 3 1/2¢; of chemical treatment, from a fraction of a cent to perhaps 5¢ per bbl.

16. TRANSPORTATION OF OIL

Petroleum is moved from the oil field to refineries by pipe lines, RR tank cars, automobile trucks, barges or tank ships. Most oil goes through pipe lines, of which there is a network in the different oil-producing states, with connections to trunk lines for interstate transport to refineries in the central and eastern states. From individual leases the oil is collected by gathering lines, through which it is pumped, or flows by gravity, to a field-central pumping station. Thence it goes to the main pumping system, unless the field-central station is a unit in that system (88, 89, 90, 91, 92).

Booster stations are maintained at points along main lines, to avoid necessity for excessive initial press. In level country these stations are 10 to 30 miles apart, depending on diam of the pipe line and quantity of oil handled.

Reciprocating or centrifugal pumps are used, the latter being now preferred. Pumps are driven by gas or oil engines, or elec motors, depending on kind and cost of power available. Storage tanks at each booster pumping plant take care of temporary accumulation exceeding capac of the plant, or for storing oil during a shut-down for repairs. Heaters are provided at booster stations, when the oil is very viscous, as in a few Calif fields.

Pipe lines range from 4 to 12-in or more in diam, and pressures up to 800 lb per sq in are sometimes used in pumping stations. Temperatures of the oil at pumping stations are 120° to 180° F, dropping to from 60° to 120° at end of a pipe-line station, before the oil is taken up by the next booster pumps. Pipe-line capacities range from 15 000 to 25 000 bbl per day, when pumping through an 8-in line, and operating under normal conditions. For further details relative to influence of viscosity in flow through pipes, see Sec 38 Art 2.

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SECTION 45

ENGINEERS' TABLES

COMPILED BY

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1. Logarithms of Numbers from 1 to 100

N	Log	N	Log	N	Log	N	Log	N	Log
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1 986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

Following is a complete table of six-place logarithms from 100 to 1000.

2. Common Logarithms of Numbers

N	0	1	2	3	4	5	6	7	8	9	Diff.
100	**000000**	**000434**	**000868**	**001301**	**001734**	**002166**	**002598**	**003029**	**003461**	**003891**	432
1	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174	428
2	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415	424
3	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616	420
4	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775	416
5	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896	412
6	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978	408
7	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021	404
8	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028	400
9	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998	397
110	**041393**	**041787**	**042182**	**042576**	**042969**	**043362**	**043755**	**044148**	**044540**	**044932**	393
1	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830	390
2	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694	386
3	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524	383
4	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320	379
5	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083	376
6	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815	373
7	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514	370
8	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182	366
9	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819	363
120	**079181**	**079543**	**079904**	**080266**	**080626**	**080987**	**081347**	**081707**	**082067**	**082426**	360
1	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004	357
2	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552	355
3	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
4	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562	349
5	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026	346
6	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462	343
7	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871	341
8	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253	338
9	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609	335

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
434	43.4	86.8	130.2	173.6	217.0	260.4	303.8	347.2	390.6
432	43.2	86.4	129.6	172.8	216.0	259.2	302.4	345.6	388.8
430	43.0	86.0	129.0	172.0	215.0	258.0	301.0	344.0	387.0
428	42.8	85.6	128.4	171.2	214.0	256.8	299.6	342.4	385.2
426	42.6	85.2	127.8	170.4	213.0	255.6	298.2	340.8	383.4
424	42.4	84.8	127.2	169.6	212.0	254.4	296.8	339.2	381.6
422	42.2	84.4	126.6	168.8	211.0	253.2	295.4	337.6	379.8
420	42.0	84.0	126.0	168.0	210.0	252.0	294.0	336.0	378.0
418	41.8	83.6	125.4	167.2	209.0	250.8	292.6	334.4	376.2
416	41.6	83.2	124.8	166.4	208.0	249.6	291.2	332.8	374.4
414	41.4	82.8	124.2	165.6	207.0	248.4	289.8	331.2	372.6
412	41.2	82.4	123.6	164.8	206.0	247.2	288.4	329.6	370.8
410	41.0	82.0	123.0	164.0	205.0	246.0	287.0	328.0	369.0
408	40.8	81.6	122.4	163.2	204.0	244.8	285.6	326.4	367.2
406	40.6	81.2	121.8	162.4	203.0	243.6	284.2	324.8	365.4
404	40.4	80.8	121.2	161.6	202.0	242.4	282.8	323.2	363.6
402	40.2	80.4	120.6	160.8	201.0	241.2	281.4	321.6	361.8
400	40.0	80.0	120.0	160.0	200.0	240.0	280.0	320.0	360.0
398	39.8	79.6	119.4	159.2	199.0	238.8	278.6	318.4	358.2
396	39.6	79.2	118.8	158.4	198.0	237.6	277.2	316.8	356.4
394	39.4	78.8	118.2	157.6	197.0	236.4	275.8	315.2	354.6
392	39.2	78.4	117.6	156.8	196.0	235.2	274.4	313.6	352.8
390	39.0	78.0	117.0	156.0	195.0	234.0	273.0	312.0	351.0
388	38.8	77.6	116.4	155.2	194.0	232.8	271.6	310.4	349.2
386	38.6	77.2	115.8	154.4	193.0	231.6	270.2	308.8	347.4
384	38.4	76.8	115.2	153.6	192.0	230.4	268.8	307.2	345.6
382	38.2	76.4	114.6	152.8	191.0	229.2	267.4	305.6	343.8
380	38.0	76.0	114.0	152.0	190.0	228.0	266.0	304.0	342.0
378	37.8	75.6	113.4	151.2	189.0	226.8	264.6	302.4	340.2
376	37.6	75.2	112.8	150.4	188.0	225.6	263.2	300.8	338.4
374	37.4	74.8	112.2	149.6	187.0	224.4	261.8	299.2	336.6
372	37.2	74.4	111.6	148.8	186.0	223.2	260.4	297.6	334.8
370	37.0	74.0	111.0	148.0	185.0	222.0	259.0	296.0	333.0
368	36.8	73.6	110.4	147.2	184.0	220.8	257.6	294.4	331.2
366	36.6	73.2	109.8	146.4	183.0	219.6	256.2	292.8	329.4
364	36.4	72.8	109.2	145.6	182.0	218.4	254.8	291.2	327.6
362	36.2	72.4	108.6	144.8	181.0	217.2	253.4	289.6	325.8
360	36.0	72.0	108.0	144.0	180.0	216.0	252.0	288.0	324.0

N	0	1	2	3	4	5	6	7	8	9	Diff.
130	**113943**	**114277**	**114611**	**114944**	**115278**	**115611**	**115943**	**116276**	**116608**	**116940**	333
1	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245	330
2	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525	328
3	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781	325
4	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012	323
5	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219	321
6	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403	318
7	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564	316
8	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
9	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818	311
140	**146128**	**146438**	**146748**	**147058**	**147367**	**147676**	**147985**	**148294**	**148603**	**148911**	309
1	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982	307
2	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032	305
3	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061	303
4	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068	301
5	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055	299
6	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022	297
7	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968	295
8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
9	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802	291
150	**176091**	**176381**	**176670**	**176959**	**177248**	**177536**	**177825**	**178113**	**178401**	**178689**	289
1	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558	287
2	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407	285
3	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239	283
4	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051	281
5	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846	279
6	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623	278
7	195900	196176	196453	196729	197005	197281	197556	197832	198107	198382	276
8	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124	274
9	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848	272

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
358	35.8	71.6	107.4	143.2	179.0	214.8	250.6	286.4	322.2
356	35.6	71.2	106.8	142.4	178.0	213.6	249.2	284.8	320.4
354	35.4	70.8	106.2	141.6	177.0	212.4	247.8	283.2	318.6
352	35.2	70.4	105.6	140.8	176.0	211.2	246.4	281.6	316.8
350	35.0	70.0	105.0	140.0	175.0	210.0	245.0	280.0	315.0
348	34.8	69.6	104.4	139.2	174.0	208.8	243.6	278.4	313.2
346	34.6	69.2	103.8	138.4	173.0	207.6	242.2	276.8	311.4
344	34.4	68.8	103.2	137.6	172.0	206.4	240.8	275.2	309.6
342	34.2	68.4	102.6	136.8	171.0	205.2	239.4	273.6	307.8
340	34.0	68.0	102.0	136.0	170.0	204.0	238.0	272.0	306.0
338	33.8	67.6	101.4	135.2	169.0	202.8	236.6	270.4	304.2
336	33.6	67.2	100.8	134.4	168.0	201.6	235.2	268.8	302.4
334	33.4	66.8	100.2	133.6	167.0	200.4	233.8	267.2	300.6
332	33.2	66.4	99.6	132.8	166.0	199.2	232.4	265.6	298.8
330	33.0	66.0	99.0	132.0	165.0	198.0	231.0	264.0	297.0
328	32.8	65.6	98.4	131.2	164.0	196.8	229.6	262.4	295.2
326	32.6	65.2	97.8	130.4	163.0	195.6	228.2	260.8	293.4
324	32.4	64.8	97.2	129.6	162.0	194.4	226.8	259.2	291.6
322	32.2	64.4	96.6	128.8	161.0	193.2	225.4	257.6	289.8
320	32.0	64.0	96.0	128.0	160.0	192.0	224.0	256.0	288.0
318	31.8	63.6	95.4	127.2	159.0	190.8	222.6	254.4	286.2
316	31.6	63.2	94.8	126.4	158.0	189.6	221.2	252.8	284.4
314	31.4	62.8	94.2	125.6	157.0	188.4	219.8	251.2	282.6
312	31.2	62.4	93.6	124.8	156.0	187.2	218.4	249.6	280.8
310	31.0	62.0	93.0	124.0	155.0	186.0	217.0	248.0	279.0
308	30.8	61.6	92.4	123.2	154.0	184.8	215.6	246.4	277.2
306	30.6	61.2	91.8	122.4	153.0	183.6	214.2	244.8	275.4
304	30.4	60.8	91.2	121.6	152.0	182.4	212.8	243.2	273.6
302	30.2	60.4	90.6	120.8	151.0	181.2	211.4	241.6	271.8
300	30.0	60.0	90.0	120.0	150.0	180.0	210.0	240.0	270.0
298	29.8	59.6	89.4	119.2	149.0	178.8	208.6	238.4	268.2
296	29.6	59.2	88.8	118.4	148.0	177.6	207.2	236.8	266.4
294	29.4	58.8	88.2	117.6	147.0	176.4	205.8	235.2	264.6
292	29.2	58.4	87.6	116.8	146.0	175.2	204.4	233.6	262.8
290	29.0	58.0	87.0	116.0	145.0	174.0	203.0	232.0	261.0
288	28.8	57.6	86.4	115.2	144.0	172.8	201.6	230.4	259.2
286	28.6	57.2	85.8	114.4	143.0	171.6	200.2	228.8	257.4
284	28.4	56.8	85.2	113.6	142.0	170.4	198.8	227.2	255.6
282	28.2	56.4	84.6	112.8	141.0	169.2	197.4	225.6	253.8
280	28.0	56.0	84.0	112.0	140.0	168.0	196.0	224.0	252.0

N	0	1	2	3	4	5	6	7	8	9	Diff.
160	**204120**	**204391**	**204663**	**204934**	**205204**	**205475**	**205746**	**206016**	**206286**	**206556**	271
1	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247	269
2	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921	267
3	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579	266
4	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221	264
5	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846	262
6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
7	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051	259
8	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630	258
9	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193	256
170	**230449**	**230704**	**230960**	**231215**	**231470**	**231724**	**231979**	**232234**	**232488**	**232742**	255
1	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276	253
2	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795	252
3	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300	250
4	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790	249
5	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266	248
6	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728	246
7	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176	245
8	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610	243
9	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031	242
180	**255273**	**255514**	**255755**	**255996**	**256237**	**256477**	**256718**	**256958**	**257198**	**257439**	241
1	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833	239
2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
3	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582	237
4	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937	235
5	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279	234
6	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609	233
7	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927	232
8	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232	230
9	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525	229
190	**278754**	**278982**	**279211**	**279439**	**279667**	**279895**	**280123**	**280351**	**280578**	**280806**	228
1	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
2	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332	226
3	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578	225
4	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812	223
5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
6	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246	221
7	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446	220
8	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635	219
9	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813	218

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
278	27.8	55.6	83.4	111.2	139.0	166.8	194.6	222.4	250.2
276	27.6	55.2	82.8	110.4	138.0	165.6	193.2	220.8	248.4
274	27.4	54.8	82.2	109.6	137.0	164.4	191.8	219.2	246.6
272	27.2	54.4	81.6	108.8	136.0	163.2	190.4	217.6	244.8
270	27.0	54.0	81.0	108.0	135.0	162.0	189.0	216.0	243.0
268	26.8	53.6	80.4	107.2	134.0	160.8	187.6	214.4	241.2
266	26.6	53.2	79.8	106.4	133.0	159.6	186.2	212.8	239.4
264	26.4	52.8	79.2	105.6	132.0	158.4	184.8	211.2	237.6
262	26.2	52.4	78.6	104.8	131.0	157.2	183.4	209.6	235.8
260	26.0	52.0	78.0	104.0	130.0	156.0	182.0	208.0	234.0
258	25.8	51.6	77.4	103.2	129.0	154.8	180.6	206.4	232.2
256	25.6	51.2	76.8	102.4	128.0	153.6	179.2	204.8	230.4
254	25.4	50.8	76.2	101.6	127.0	152.4	177.8	203.2	228.6
252	25.2	50.4	75.6	100.8	126.0	151.2	176.4	201.6	226.8
250	25.0	50.0	75.0	100.0	125.0	150.0	175.0	200.0	225.0
248	24.8	49.6	74.4	99.2	124.0	148.8	173.6	198.4	223.2
246	24.6	49.2	73.8	98.4	123.0	147.6	172.2	196.8	221.4
244	24.4	48.8	73.2	97.6	122.0	146.4	170.8	195.2	219.6
242	24.2	48.4	72.6	96.8	121.0	145.2	169.4	193.6	217.8
240	24.0	48.0	72.0	96.0	120.0	144.0	168.0	192.0	216.0
238	23.8	47.6	71.4	95.2	119.0	142.8	166.6	190.4	214.2
236	23.6	47.2	70.8	94.4	118.0	141.6	165.2	188.8	212.4
234	23.4	46.8	70.2	93.6	117.0	140.4	163.8	187.2	210.6
232	23.2	46.4	69.6	92.8	116.0	139.2	162.4	185.6	208.8
230	23.0	46.0	69.0	92.0	115.0	138.0	161.0	184.0	207.0

N	0	1	2	3	4	5	6	7	8	9	Diff.
200	**301030**	**301247**	**301464**	**301681**	**301898**	**302114**	**302331**	**302547**	**302764**	**302980**	217
1	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136	216
2	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282	215
3	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417	213
4	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542	212
5	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656	211
6	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760	210
7	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854	209
8	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938	208
9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	**322219**	**322426**	**322633**	**322839**	**323046**	**323252**	**323458**	**323665**	**323871**	**324077**	206
1	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131	205
2	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176	204
3	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211	203
4	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236	202
5	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253	
6	334454	334655	334856	335057	335257	335458	335658	335859	336059	336260	201
7	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257	200
8	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246	199
9	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225	198
220	**342423**	**342620**	**342817**	**343014**	**343212**	**343409**	**343606**	**343802**	**343999**	**344196**	197
1	344392	344589	344785	344981	345178	345374	345570	345766	345962	346157	196
2	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110	195
3	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054	194
4	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989	193
5	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916	
6	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834	192
7	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744	191
8	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646	190
9	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	**361728**	**361917**	**362105**	**362294**	**362482**	**362671**	**362859**	**363048**	**363236**	**363424**	188
1	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301	
2	365488	365675	365862	366049	366236	366423	366610	366796	366983	367169	187
3	367356	367542	367729	367915	368101	368287	368473	368659	368845	369030	186
4	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883	185
5	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728	184
6	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565	
7	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394	183
8	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216	182
9	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030	181

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
228	22.8	45.6	68.4	91.2	114.0	136.8	159.6	182.4	205.2
226	22.6	45.2	67.8	90.4	113.0	135.6	158.2	180.8	203.4
224	22.4	44.8	67.2	89.6	112.0	134.4	156.8	179.2	201.6
222	22.2	44.4	66.6	88.8	111.0	133.2	155.4	177.6	199.8
220	22.0	44.0	66.0	88.0	110.0	132.0	154.0	176.0	198.0
218	21.8	43.6	65.4	87.2	109.0	130.8	152.6	174.4	196.2
216	21.6	43.2	64.8	86.4	108.0	129.6	151.2	172.8	194.4
214	21.4	42.8	64.2	85.6	107.0	128.4	149.8	171.2	192.6
212	21.2	42.4	63.6	84.8	106.0	127.2	148.4	169.6	190.8
210	21.0	42.0	63.0	84.0	105.0	126.0	147.0	168.0	189.0
208	20.8	41.6	62.4	83.2	104.0	124.8	145.6	166.4	187.2
206	20.6	41.2	61.8	82.4	103.0	123.6	144.2	164.8	185.4
204	20.4	40.8	61.2	81.6	102.0	122.4	142.8	163.2	183.6
202	20.2	40.4	60.6	80.8	101.0	121.2	141.4	161.6	181.8
200	20.0	40.0	60.0	80.0	100.0	120.0	140.0	160.0	180.0
198	19.8	39.6	59.4	79.2	99.0	118.8	138.6	158.4	178.2
196	19.6	39.2	58.8	78.4	98.0	117.6	137.2	156.8	176.4
194	19.4	38.8	58.2	77.6	97.0	116.4	135.8	155.2	174.6
192	19.2	38.4	57.6	76.8	96.0	115.2	134.4	153.6	172.8
190	19.0	38.0	57.0	76.0	95.0	114.0	133.0	152.0	171.0
188	18.8	37.6	56.4	75.2	94.0	112.8	131.6	150.4	169.2
186	18.6	37.2	55.8	74.4	93.0	111.6	130.2	148.8	167.4
184	18.4	36.8	55.2	73.6	92.0	110.4	128.8	147.2	165.6
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0

N	0	1	2	3	4	5	6	7	8	9	Diff.
240	**380211**	**380392**	**380573**	**380754**	**380934**	**381115**	**381296**	**381476**	**381656**	**381837**	181
1	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636	180
2	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428	179
3	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212	178
4	387390	387568	387746	387924	388101	388279	388456	388634	388811	388989	
5	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759	177
6	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521	176
7	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277	
8	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025	175
9	396199	396374	396548	396722	396896	397071	397245	397419	397592	397766	174
250	**397940**	**398114**	**398287**	**398461**	**398634**	**398808**	**398981**	**399154**	**399328**	**399501**	173
1	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228	
2	401401	401573	401745	401917	402089	402261	402433	402605	402777	402949	172
3	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663	171
4	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370	
5	406540	406710	406881	407051	407221	407391	407561	407731	407901	408070	170
6	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764	169
7	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451	
8	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132	168
9	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806	167
260	**414973**	**415140**	**415307**	**415474**	**415641**	**415808**	**415974**	**416141**	**416308**	**416474**	
1	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135	166
2	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791	165
3	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439	
4	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082	164
5	423246	423410	423574	423737	423901	424065	424228	424392	424555	424718	
6	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349	163
7	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973	162
8	428135	428297	428459	428621	428783	428944	429106	429268	429429	429591	
9	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	**431364**	**431525**	**431685**	**431846**	**432007**	**432167**	**432328**	**432488**	**432649**	**432809**	
1	432969	433130	433290	433450	433610	433770	433930	434090	434249	434409	160
2	434569	434729	434888	435048	435207	435367	435526	435685	435844	436004	159
3	436163	436322	436481	436640	436799	436957	437116	437275	437433	437592	
4	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175	158
5	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752	
6	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323	157
7	442480	442637	442793	442950	443106	443263	443419	443576	443732	443889	
8	444045	444201	444357	444513	444669	444825	444981	445137	445293	445449	156
9	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003	155
280	**447158**	**447313**	**447468**	**447623**	**447778**	**447933**	**448088**	**448242**	**448397**	**448552**	
1	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095	154
2	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633	
3	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165	153
4	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692	
5	454845	454997	455150	455302	455454	455606	455758	455910	456062	456214	152
6	456366	456518	456670	456821	456973	457125	457276	457428	457579	457731	
7	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242	151
8	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748	
9	460898	461048	461198	461348	461499	461649	461799	461948	462098	462248	150

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
182	18.2	36.4	54.6	72.8	91.0	109.2	127.4	145.6	163.8
180	18.0	36.0	54.0	72.0	90.0	108.0	126.0	144.0	162.0
178	17.8	35.6	53.4	71.2	89.0	106.8	124.6	142.4	160.2
176	17.6	35.2	52.8	70.4	88.0	105.6	123.2	140.8	158.4
174	17.4	34.8	52.2	69.6	87.0	104.4	121.8	139.2	156.6
172	17.2	34.4	51.6	68.8	86.0	103.2	120.4	137.6	154.8
170	17.0	34.0	51.0	68.0	85.0	102.0	119.0	136.0	153.0
168	16.8	33.6	50.4	67.2	84.0	100.8	117.6	134.4	151.2
166	16.6	33.2	49.8	66.4	83.0	99.6	116.2	132.8	149.4
164	16.4	32.8	49.2	65.6	82.0	98.4	114.8	131.2	147.6
162	16.2	32.4	48.6	64.8	81.0	97.2	113.4	129.6	145.8
160	16.0	32.0	48.0	64.0	80.0	96.0	112.0	128.0	144.0
158	15.8	31.6	47.4	63.2	79.0	94.8	110.6	126.4	142.2
156	15.6	31.2	46.8	62.4	78.0	93.6	109.2	124.8	140.4

N	0	1	2	3	4	5	6	7	8	9	Diff.
290	**462398**	**462548**	**462697**	**462847**	**462997**	**463146**	**463296**	**463445**	**463594**	**463744**	
1	463893	464042	464191	464340	464490	464639	464788	464936	465085	465234	149
2	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719	
3	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200	148
4	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675	
5	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145	147
6	471292	471438	471585	471732	471878	472025	472171	472318	472464	472610	146
7	472756	472903	473049	473195	473341	473487	473633	473779	473925	474071	
8	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526	
9	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976	145
300	**477121**	**477266**	**477411**	**477555**	**477700**	**477844**	**477989**	**478133**	**478278**	**478422**	
1	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863	144
2	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	
3	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731	143
4	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157	
5	484300	484442	484585	484727	484869	485011	485153	485295	485437	485579	142
6	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997	
7	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410	141
8	488551	488692	488833	488974	489114	489255	489396	489537	489677	489818	
9	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	**491362**	**491502**	**491642**	**491782**	**491922**	**492062**	**492201**	**492341**	**492481**	**492621**	
1	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015	139
2	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406	
3	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791	
4	496930	497068	497206	497344	497483	497621	497759	497897	498035	498173	138
5	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550	
6	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922	137
7	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291	
8	502427	502564	502700	502837	502973	503109	503246	503382	503518	503655	136
9	503791	503927	504063	504199	504335	504471	504607	504743	504878	505014	
320	**505150**	**505286**	**505421**	**505557**	**505693**	**505828**	**505964**	**506099**	**506234**	**506370**	
1	506505	506640	506776	506911	507046	507181	507316	507451	507586	507721	135
2	507856	507991	508126	508260	508395	508530	508664	508799	508934	509068	
3	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411	134
4	510545	510679	510813	510947	511081	511215	511349	511482	511616	511750	
5	511883	512017	512151	512284	512418	512551	512684	512818	512951	513084	133
6	513218	513351	513484	513617	513750	513883	514016	514149	514282	514415	
7	514548	514681	514813	514946	515079	515211	515344	515476	515609	515741	
8	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064	132
9	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382	
330	**518514**	**518646**	**518777**	**518909**	**519040**	**519171**	**519303**	**519434**	**519566**	**519697**	131
1	519828	519959	520090	520221	520353	520484	520615	520745	520876	521007	
2	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314	
3	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616	130
4	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915	
5	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210	129
6	526339	526469	526598	526727	526856	526985	527114	527243	527372	527501	
7	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788	
8	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072	128
9	530200	530328	530456	530584	530712	530840	530968	531096	531223	531351	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
154	15.4	30.8	46.2	61.6	77.0	92.4	107.8	123.2	138.6
152	15.2	30.4	45.6	60.8	76.0	91.2	106.4	121.6	136.8
150	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0
148	14.8	29.6	44.4	59.2	74.0	88.8	103.6	118.4	133.2
146	14.6	29.2	43.8	58.4	73.0	87.6	102.2	116.8	131.4
144	14.4	28.8	43.2	57.6	72.0	86.4	100.8	115.2	129.6
142	14.2	28.4	42.6	56.8	71.0	85.2	99.4	113.6	127.8
140	14.0	28.0	42.0	56.0	70.0	84.0	98.0	112.0	126.0
138	13.8	27.6	41.4	55.2	69.0	82.8	96.6	110.4	124.2
136	13.6	27.2	40.8	54.4	68.0	81.6	95.2	108.8	122.4
134	13.4	26.8	40.2	53.6	67.0	80.4	93.8	107.2	120.6
132	13.2	26.4	39.6	52.8	66.0	79.2	92.4	105.6	118.8
130	13.0	26.0	39.0	52.0	65.0	78.0	91.0	104.0	117.0
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
340	**531479**	**531607**	**531734**	**531862**	**531990**	**532117**	**532245**	**532372**	**532500**	**532627**	
1	532754	532882	533009	533136	533264	533391	533518	533645	533772	533899	127
2	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167	
3	535294	535421	535547	535674	535800	535927	536053	536180	536306	536432	126
4	536558	536685	536811	536937	537063	537189	537315	537441	537567	537693	
5	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951	
6	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204	125
7	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454	
8	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701	
9	542825	542950	543074	543199	543323	543447	543571	543696	543820	543944	124
350	**544068**	**544192**	**544316**	**544440**	**544564**	**544688**	**544812**	**544936**	**545060**	**545183**	
1	545307	545431	545555	545678	545802	545925	546049	546172	546296	546419	
2	546543	546666	546789	546913	547036	547159	547282	547405	547529	547652	123
3	547775	547898	548021	548144	548267	548389	548512	548635	548758	548881	
4	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106	
5	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328	122
6	551450	551572	551694	551816	551938	552060	552181	552303	552425	552547	
7	552668	552790	552911	553033	553155	553276	553398	553519	553640	553762	121
8	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973	
9	555094	555215	555336	555457	555578	555699	555820	555940	556061	556182	
360	**556303**	**556423**	**556544**	**556664**	**556785**	**556905**	**557026**	**557146**	**557267**	**557387**	120
1	557507	557627	557748	557868	557988	558108	558228	558349	558469	558589	
2	558709	558829	558948	559068	559188	559308	559428	559548	559667	559787	
3	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
4	561101	561221	561340	561459	561578	561698	561817	561936	562055	562174	
5	562293	562412	562531	562650	562769	562887	563006	563125	563244	563362	
6	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548	
7	564666	564784	564903	565021	565139	565257	565376	565494	565612	565730	118
8	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909	
9	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084	
370	**568202**	**568319**	**568436**	**568554**	**568671**	**568788**	**568905**	**569023**	**569140**	**569257**	117
1	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426	
2	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592	
3	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755	116
4	572872	572988	573104	573220	573336	573452	573568	573684	573800	573915	
5	574031	574147	574263	574379	574494	574610	574726	574841	574957	575072	
6	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226	115
7	576341	576457	576572	576687	576802	576917	577032	577147	577262	577377	
8	577492	577607	577722	577836	577951	578066	578181	578295	578410	578525	
9	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669	114
380	**579784**	**579898**	**580012**	**580126**	**580241**	**580355**	**580469**	**580583**	**580697**	**580811**	
1	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950	
2	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085	
3	583199	583312	583426	583539	583652	583765	583879	583992	584105	584218	
4	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348	113
5	585461	585574	585686	585799	585912	586024	586137	586250	586362	586475	
6	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599	
7	587711	587823	587935	588047	588160	588272	588384	588496	588608	588720	112
8	588832	588944	589056	589167	589279	589391	589503	589615	589726	589838	
9	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
128	12.8	25.6	38.4	51.2	64.0	76.8	89.6	102.4	115.2
126	12.6	25.2	37.8	50.4	63.0	75.6	88.2	100.8	113.4
124	12.4	24.8	37.2	49.6	62.0	74.4	86.8	99.2	111.6
122	12.2	24.4	36.6	48.8	61.0	73.2	85.4	97.6	109.8
120	12.0	24.0	36.0	48.0	60.0	72.0	84.0	96.0	108.0
118	11.8	23.6	35.4	47.2	59.0	70.8	82.6	94.4	106.2
116	11.6	23.2	34.8	46.4	58.0	69.6	81.2	92.8	104.4
114	11.4	22.8	34.2	45.6	57.0	68.4	79.8	91.2	102.6

N	0	1	2	3	4	5	6	7	8	9	Diff.
390	**591065**	**591176**	**591287**	**591399**	**591510**	**591621**	**591732**	**591843**	**591955**	**592066**	
1	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175	111
2	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282	
3	594393	594503	594614	594724	594834	594945	595055	595165	595276	595386	
4	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487	
5	596597	596707	596817	596927	597037	597146	597256	597366	597476	597586	110
6	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681	
7	598791	598900	599009	599119	599228	599337	599446	599556	599665	599774	
8	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864	109
9	600973	601082	601191	601299	601408	601517	601625	601734	601843	601951	
400	**602060**	**602169**	**602277**	**602386**	**602494**	**602603**	**602711**	**602819**	**602928**	**603036**	
1	603144	603253	603361	603469	603577	603686	603794	603902	604010	604118	108
2	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197	
3	605305	605413	605521	605628	605736	605844	605951	606059	606166	606274	
4	606381	606489	606596	606704	606811	606919	607026	607133	607241	607348	
5	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419	107
6	608526	608633	608740	608847	608954	609061	609167	609274	609381	609488	
7	609594	609701	609808	609914	610021	610128	610234	610341	610447	610554	
8	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617	
9	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678	106
410	**612784**	**612890**	**612996**	**613102**	**613207**	**613313**	**613419**	**613525**	**613630**	**613736**	
1	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792	
2	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845	
3	615950	616055	616160	616265	616370	616476	616581	616686	616790	616895	105
4	617000	617105	617210	617315	617420	617525	617629	617734	617839	617943	
5	618048	618153	618257	618362	618466	618571	618676	618780	618884	618989	
6	619093	619198	619302	619406	619511	619615	619719	619824	619928	620032	
7	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072	104
8	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110	
9	622214	622318	622421	622525	622628	622732	622835	622939	623042	623146	
420	**623249**	**623353**	**623456**	**623559**	**623663**	**623766**	**623869**	**623973**	**624076**	**624179**	
1	624282	624385	624488	624591	624695	624798	624901	625004	625107	625210	103
2	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238	
3	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263	
4	627366	627468	627571	627673	627775	627878	627980	628082	628185	628287	
5	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308	102
6	629410	629512	629613	629715	629817	629919	630021	630123	630224	630326	
7	630428	630530	630631	630733	630835	630936	631038	631139	631241	631342	
8	631444	631545	631647	631748	631849	631951	632052	632153	632255	632356	101
9	632457	632559	632660	632761	632862	632963	633064	633165	633266	633367	
430	**633468**	**633569**	**633670**	**633771**	**633872**	**633973**	**634074**	**634175**	**634276**	**634376**	
1	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383	
2	635484	635584	635685	635785	635886	635986	636087	636187	636287	636388	
3	636488	636588	636688	636789	636889	636989	637089	637189	637290	637390	
4	637490	637590	637690	637790	637890	637990	638090	638190	638290	638389	100
5	638489	638589	638689	638789	638888	638988	639088	639188	639287	639387	
6	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382	
7	640481	640581	640680	640779	640879	640978	641077	641177	641276	641375	
8	641474	641573	641672	641771	641871	641970	642069	642168	642267	642366	
9	642465	642563	642662	642761	642860	642959	643058	643156	643255	643354	99

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
112	11.2	22.4	33.6	44.8	56.0	67.2	78.4	89.6	100.8
110	11.0	22.0	33.0	44.0	55.0	66.0	77.0	88.0	99.0
108	10.8	21.6	32.4	43.2	54.0	64.8	75.6	86.4	97.2
106	10.6	21.2	31.8	42.4	53.0	63.6	74.2	84.8	95.4
104	10.4	20.8	31.2	41.6	52.0	62.4	72.8	83.2	93.6
102	10.2	20.4	30.6	40.8	51.0	61.2	71.4	81.6	91.8
100	13.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
440	**643453**	**643551**	**643650**	**643749**	**643847**	**643946**	**644044**	**644143**	**644242**	**644340**	
1	644439	644537	644636	644734	644832	644931	645029	645127	645226	645324	
2	645422	645521	645619	645717	645815	645913	646011	646110	646208	646306	
3	646404	646502	646600	646698	646796	646894	646992	647089	647187	647285	98
4	647383	647481	647579	647676	647774	647872	647969	648067	648165	648262	
5	648360	648458	648555	648653	648750	648848	648945	649043	649140	649237	
6	649335	649432	649530	649627	649724	649821	649919	650016	650113	650210	
7	650308	650405	650502	650599	650696	650793	650890	650987	651084	651181	
8	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150	97
9	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116	
450	**653213**	**653309**	**653405**	**653502**	**653598**	**653695**	**653791**	**653888**	**653984**	**654080**	
1	654177	654273	654369	654465	654562	654658	654754	654850	654946	655042	
2	655138	655235	655331	655427	655523	655619	655715	655810	655906	656002	96
3	656098	656194	656290	656386	656482	656577	656673	656769	656864	656960	
4	657056	657152	657247	657343	657438	657534	657629	657725	657820	657916	
5	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870	
6	658965	659060	659155	659250	659346	659441	659536	659631	659726	659821	
7	659916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
8	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718	
9	661813	661907	662002	662096	662191	662286	662380	662475	662569	662663	
460	**662758**	**662852**	**662947**	**663041**	**663135**	**663230**	**663324**	**663418**	**663512**	**663607**	
1	663701	663795	663889	663983	664078	664172	664266	664360	664454	664548	
2	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487	94
3	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424	
4	666518	666612	666705	666799	666892	666986	667079	667173	667266	667360	
5	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293	
6	668386	668479	668572	668665	668759	668852	668945	669038	669131	669224	
7	669317	669410	669503	669596	669689	669782	669875	669967	670060	670153	93
8	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080	
9	671173	671265	671358	671451	671543	671636	671728	671821	671913	672005	
470	**672098**	**672190**	**672283**	**672375**	**672467**	**672560**	**672652**	**672744**	**672836**	**672929**	
1	673021	673113	673205	673297	673390	673482	673574	673666	673758	673850	
2	673942	674034	674126	674218	674310	674402	674494	674586	674677	674769	92
3	674861	674953	675045	675137	675228	675320	675412	675503	675595	675687	
4	675778	675870	675962	676053	676145	676236	676328	676419	676511	676602	
5	676694	676785	676876	676968	677059	677151	677242	677333	677424	677516	
6	677607	677698	677789	677881	677972	678063	678154	678245	678336	678427	
7	678518	678609	678700	678791	678882	678973	679064	679155	679246	679337	91
8	679428	679519	679610	679700	679791	679882	679973	680063	680154	680245	
9	680336	680426	680517	680607	680698	680789	680879	680970	681060	681151	
480	**681241**	**681332**	**681422**	**681513**	**681603**	**681693**	**681784**	**681874**	**681964**	**682055**	
1	682145	682235	682326	682416	682506	682596	682686	682777	682867	682957	
2	683047	683137	683227	683317	683407	683497	683587	683677	683767	683857	90
3	683947	684037	684127	684217	684307	684396	684486	684576	684666	684756	
4	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652	
5	685742	685831	685921	686010	686100	686189	686279	686368	686458	686547	
6	686636	686726	686815	686904	686994	687083	687172	687261	687351	687440	
7	687529	687618	687707	687796	687886	687975	688064	688153	688242	688331	
8	688420	688509	688598	688687	688776	688865	688953	689042	689131	689220	89
9	689309	689398	689486	689575	689664	689753	689841	689930	690019	690107	
490	**630196**	**690285**	**690373**	**690462**	**690550**	**690639**	**690728**	**690816**	**690905**	**690993**	
1	691081	691170	691258	691347	691435	691524	691612	691700	691789	691877	
2	691965	692053	692142	692230	692318	692406	692494	692583	692671	692759	
3	692847	692935	693023	693111	693199	693287	693375	693463	693551	693639	88
4	693727	693815	693903	693991	694078	694166	694254	694342	694430	694517	
5	694605	694693	694781	694868	694956	695044	695131	695219	695307	695394	
6	695482	695569	695657	695744	695832	695919	696007	696094	696182	696269	
7	696356	696444	696531	696618	696706	696793	696880	696968	697055	697142	
8	697229	697317	697404	697491	697578	697665	697752	697839	697926	698014	87
9	698100	698188	698275	698362	698449	698535	698622	698709	698796	698883	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
98	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4	88.2
96	9.6	19.2	28.8	38.4	48.0	57.6	67.2	76.8	86.4
94	9.4	18.8	28.2	37.6	47.0	56.4	65.8	75.2	84.6
92	9.2	18.4	27.6	36.8	46.0	55.2	64.4	73.6	82.8
90	9.0	18.0	27.0	36.0	45.0	54.0	63.0	72.0	81.0
88	8.8	17.6	26.4	35.2	44.0	52.8	61.6	70.4	79.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
500	**698970**	**699057**	**699144**	**699231**	**699317**	**699404**	**699491**	**699578**	**699664**	**699751**	
1	699838	699924	700011	700098	700184	700271	700358	700444	700531	700617	
2	700704	700790	700877	700963	701050	701136	701222	701309	701395	701482	
3	701568	701654	701741	701827	701913	701999	702086	702172	702258	702344	
4	702431	702517	702603	702689	702775	702861	702947	703033	703119	703205	
5	703291	703377	703463	703549	703635	703721	703807	703893	703979	704065	86
6	704151	704236	704322	704408	704494	704579	704665	704751	704837	704922	
7	705008	705094	705179	705265	705350	705436	705522	705607	705693	705778	
8	705864	705949	706035	706120	706206	706291	706376	706462	706547	706632	
9	706718	706803	706888	706974	707059	707144	707229	707315	707400	707485	
510	**707570**	**707655**	**707740**	**707826**	**707911**	**707996**	**708081**	**708166**	**708251**	**708336**	
1	708421	708506	708591	708676	708761	708846	708931	709015	709100	709185	85
2	709270	709355	709440	709524	709609	709694	709779	709863	709948	710033	
3	710117	710202	710287	710371	710456	710540	710625	710710	710794	710879	
4	710963	711048	711132	711217	711301	711385	711470	711554	711639	711723	
5	711807	711892	711976	712060	712144	712229	712313	712397	712481	712566	
6	712650	712734	712818	712902	712986	713070	713154	713238	713323	713407	
7	713491	713575	713659	713742	713826	713910	713994	714078	714162	714246	84
8	714330	714414	714497	714581	714665	714749	714833	714916	715000	715084	
9	715167	715251	715335	715418	715502	715586	715669	715753	715836	715920	
520	**716003**	**716087**	**716170**	**716254**	**716337**	**716421**	**716504**	**716588**	**716671**	**716754**	
1	716838	716921	717004	717088	717171	717254	717338	717421	717504	717587	
2	717671	717754	717837	717920	718003	718086	718169	718253	718336	718419	83
3	718502	718585	718668	718751	718834	718917	719000	719083	719165	719248	
4	719331	719414	719497	719580	719663	719745	719828	719911	719994	720077	
5	720159	720242	720325	720407	720490	720573	720655	720738	720821	720903	
6	720986	721068	721151	721233	721316	721398	721481	721563	721646	721728	
7	721811	721893	721975	722058	722140	722222	722305	722387	722469	722552	
8	722634	722716	722798	722881	722963	723045	723127	723209	723291	723374	
9	723456	723538	723620	723702	723784	723866	723948	724030	724112	724194	82
530	**724276**	**724358**	**724440**	**724522**	**724604**	**724685**	**724767**	**724849**	**724931**	**725013**	
1	725095	725176	725258	725340	725422	725503	725585	725667	725748	725830	
2	725912	725993	726075	726156	726238	726320	726401	726483	726564	726646	
3	726727	726809	726890	726972	727053	727134	727216	727297	727379	727460	
4	727541	727623	727704	727785	727866	727948	728029	728110	728191	728273	
5	728354	728435	728516	728597	728678	728759	728841	728922	729003	729084	81
6	729165	729246	729327	729408	729489	729570	729651	729732	729813	729893	
7	729974	730055	730136	730217	730298	730378	730459	730540	730621	730702	
8	730782	730863	730944	731024	731105	731186	731266	731347	731428	731508	
9	731589	731669	731750	731830	731911	731991	732072	732152	732233	732313	
540	**732394**	**732474**	**732555**	**732635**	**732715**	**732796**	**732876**	**732956**	**733037**	**733117**	
1	733197	733278	733358	733438	733518	733598	733679	733759	733839	733919	
2	733999	734079	734160	734240	734320	734400	734480	734560	734640	734720	80
3	734800	734880	734960	735040	735120	735200	735279	735359	735439	735519	
4	735599	735679	735759	735838	735918	735998	736078	736157	736237	736317	
5	736397	736476	736556	736635	736715	736795	736874	736954	737034	737113	
6	737193	737272	737352	737431	737511	737590	737670	737749	737829	737908	
7	737987	738067	738146	738225	738305	738384	738463	738543	738622	738701	
8	738781	738860	738939	739018	739097	739177	739256	739335	739414	739493	
9	739572	739651	739731	739810	739889	739968	740047	740126	740205	740284	79
550	**740363**	**740442**	**740521**	**740600**	**740678**	**740737**	**740836**	**740915**	**740994**	**741073**	
1	741152	741230	741309	741388	741467	741546	741624	741703	741782	741860	
2	741939	742018	742096	742175	742254	742332	742411	742489	742568	742647	
3	742725	742804	742882	742961	743039	743118	743196	743275	743353	743431	
4	743510	743588	743667	743745	743823	743902	743980	744058	744136	744215	
5	744293	744371	744449	744528	744606	744684	744762	744840	744919	744997	78
6	745075	745153	745231	745309	745387	745465	745543	745621	745699	745777	
7	745855	745933	746011	746089	746167	746245	746323	746401	746479	746556	
8	746634	746712	746790	746868	746945	747023	747101	747179	747256	747334	
9	747412	747489	747567	747645	747722	747800	747878	747955	748033	748110	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
86	8.6	17.2	25.8	34.4	43.0	51.6	60.2	68.8	77.4
84	8.4	16.8	25.2	33.6	42.0	50.4	58.8	67.2	75.6
82	8.2	16.4	24.6	32.8	41.0	49.2	57.4	65.6	73.8
80	8.0	16.0	24.0	32.0	40.0	48.0	56.0	64.0	72.0
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
560	**748188**	**748266**	**748343**	**748421**	**748498**	**748576**	**748653**	**748731**	**748808**	**748885**	
1	748963	749040	749118	749195	749272	749350	749427	749504	749582	749659	
2	749736	749814	749891	749968	750045	750123	750200	750277	750354	750431	
3	750508	750586	750663	750740	750817	750894	750971	751048	751125	751202	
4	751279	751356	751433	751510	751587	751664	751741	751818	751895	751972	77
5	752048	752125	752202	752279	752356	752433	752509	752586	752663	752740	
6	752816	752893	752970	753047	753123	753200	753277	753353	753430	753506	
7	753583	753660	753736	753813	753889	753966	754042	754119	754195	754272	
8	754348	754425	754501	754578	754654	754730	754807	754883	754960	755036	
9	755112	755189	755265	755341	755417	755494	755570	755646	755722	755799	
570	**755875**	**755951**	**756027**	**756103**	**756180**	**756256**	**756332**	**756408**	**756484**	**756560**	
1	756636	756712	756788	756864	756940	757016	757092	757168	757244	757320	76
2	757396	757472	757548	757624	757700	757775	757851	757927	758003	758079	
3	758155	758230	758306	758382	758458	758533	758609	758685	758761	758836	
4	758912	758988	759063	759139	759214	759290	759366	759441	759517	759592	
5	759668	759743	759819	759894	759970	760045	760121	760196	760272	760347	
6	760422	760498	760573	760649	760724	760799	760875	760950	761025	761101	
7	761176	761251	761326	761402	761477	761552	761627	761702	761778	761853	
8	761928	762003	762078	762153	762228	762303	762378	762453	762529	762604	75
9	762679	762754	762829	762904	762978	763053	763128	763203	763278	763353	
580	**763428**	**763503**	**763578**	**763653**	**763727**	**763802**	**763877**	**763952**	**764027**	**764101**	
1	764176	764251	764326	764400	764475	764550	764624	764699	764774	764848	
2	764923	764998	765072	765147	765221	765296	765370	765445	765520	765594	
3	765669	765743	765818	765892	765966	766041	766115	766190	766264	766338	
4	766413	766487	766562	766636	766710	766785	766859	766933	767007	767082	
5	767156	767230	767304	767379	767453	767527	767601	767675	767749	767823	
6	767898	767972	768046	768120	768194	768268	768342	768416	768490	768564	74
7	768638	768712	768786	768860	768934	769008	769082	769156	769230	769303	
8	769377	769451	769525	769599	769673	769746	769820	769894	769968	770042	
9	770115	770189	770263	770336	770410	770484	770557	770631	770705	770778	
590	**770852**	**770926**	**770999**	**771073**	**771146**	**771220**	**771293**	**771367**	**771440**	**771514**	
1	771587	771661	771734	771808	771881	771955	772028	772102	772175	772248	
2	772322	772395	772468	772542	772615	772688	772762	772835	772908	772981	
3	773055	773128	773201	773274	773348	773421	773494	773567	773640	773713	
4	773786	773860	773933	774006	774079	774152	774225	774298	774371	774444	73
5	774517	774590	774663	774736	774809	774882	774955	775028	775100	775173	
6	775246	775319	775392	775465	775538	775610	775683	775756	775829	775902	
7	775974	776047	776120	776193	776265	776338	776411	776483	776556	776629	
8	776701	776774	776846	776919	776992	777064	777137	777209	777282	777354	
9	777427	777499	777572	777644	777717	777789	777862	777934	778006	778079	
600	**778151**	**778224**	**778296**	**778368**	**778441**	**778513**	**778585**	**778658**	**778730**	**778802**	
1	778874	778947	779019	779091	779163	779236	779308	779380	779452	779524	
2	779596	779669	779741	779813	779885	779957	780029	780101	780173	780245	
3	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965	72
4	781037	781109	781181	781253	781324	781396	781468	781540	781612	781684	
5	781755	781827	781899	781971	782042	782114	782186	782258	782329	782401	
6	782473	782544	782616	782688	782759	782831	782902	782974	783046	783117	
7	783189	783260	783332	783403	783475	783546	783618	783689	783761	783832	
8	783904	783975	784046	784118	784189	784261	784332	784403	784475	784546	
9	784617	784689	784760	784831	784902	784974	785045	785116	785187	785259	
610	**785330**	**785401**	**785472**	**785543**	**785615**	**785686**	**785757**	**785828**	**785899**	**785970**	
1	786041	786112	786183	786254	786325	786396	786467	786538	786609	786680	71
2	786751	786822	786893	786964	787035	787106	787177	787248	787319	787390	
3	787460	787531	787602	787673	787744	787815	787885	787956	788027	788098	
4	788168	788239	788310	788381	788451	788522	788593	788663	788734	788804	
5	788875	788946	789016	789087	789157	789228	789299	789369	789440	789510	
6	789581	789651	789722	789792	789863	789933	790004	790074	790144	790215	
7	790285	790356	790426	790496	790567	790637	790707	790778	790848	790918	
8	790988	791059	791129	791199	791269	791340	791410	791480	791550	791620	
9	791691	791761	791831	791901	791971	792041	792111	792181	792252	792322	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
78	7.8	15.6	23.4	31.2	39.0	46.8	54.6	62.4	70.2
76	7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4
74	7.4	14.8	22.2	29.6	37.0	44.4	51.8	59.2	66.6
72	7.2	14.4	21.6	28.8	36.0	43.2	50.4	57.6	64.8
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0

N	0	1	2	3	4	5	6	7	8	9	Diff.
620	**792392**	**792462**	**792532**	**792602**	**792672**	**792742**	**792812**	**792882**	**792952**	**793022**	70
1	793092	793162	793231	793301	793371	793441	793511	793581	793651	793721	
2	793790	793860	793930	794000	794070	794139	794209	794279	794349	794418	
3	794488	794558	794627	794697	794767	794836	794906	794976	795045	795115	
4	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811	
5	795880	795949	796019	796088	796158	796227	796297	796366	796436	796505	
6	796574	796644	796713	796782	796852	796921	796990	797060	797129	797198	
7	797268	797337	797406	797475	797545	797614	797683	797752	797821	797890	
8	797960	798029	798098	798167	798236	798305	798374	798443	798513	798582	
9	798651	798720	798789	798858	798927	798996	799065	799134	799203	799272	69
630	**799341**	**799409**	**799478**	**799547**	**799616**	**799685**	**799754**	**799823**	**799892**	**799961**	
1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	
2	800717	800786	800854	800923	800992	801061	801129	801198	801266	801335	
3	801404	801472	801541	801609	801678	801747	801815	801884	801952	802021	
4	802089	802158	802226	802295	802363	802432	802500	802568	802637	802705	
5	802774	802842	802910	802979	803047	803116	803184	803252	803321	803389	
6	803457	803525	803594	803662	803730	803798	803867	803935	804003	804071	
7	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753	
8	804821	804889	804957	805025	805093	805161	805229	805297	805365	805433	68
9	805501	805569	805637	805705	805773	805841	805908	805976	806044	806112	
640	**806180**	**806248**	**806316**	**806384**	**806451**	**806519**	**806587**	**806655**	**806723**	**806790**	
1	806858	806926	806994	807061	807129	807197	807264	807332	807400	807467	
2	807535	807603	807670	807738	807806	807873	807941	808008	808076	808143	
3	808211	808279	808346	808414	808481	808549	808616	808684	808751	808818	
4	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492	
5	809560	809627	809694	809762	809829	809896	809964	810031	810098	810165	
6	810233	810300	810367	810434	810501	810569	810636	810703	810770	810837	
7	810904	810971	811039	811106	811173	811240	811307	811374	811441	811508	67
8	811575	811642	811709	811776	811843	811910	811977	812044	812111	812178	
9	812245	812312	812379	812445	812512	812579	812646	812713	812780	812847	
650	**812913**	**812980**	**813047**	**813114**	**813181**	**813247**	**813314**	**813381**	**813448**	**813514**	
1	813581	813648	813714	813781	813848	813914	813981	814048	814114	814181	
2	814248	814314	814381	814447	814514	814581	814647	814714	814780	814847	
3	814913	814980	815046	815113	815179	815246	815312	815378	815445	815511	
4	815578	815644	815711	815777	815843	815910	815976	816042	816109	816175	
5	816241	816308	816374	816440	816506	816573	816639	816705	816771	816838	
6	816904	816970	817036	817102	817169	817235	817301	817367	817433	817499	
7	817565	817631	817698	817764	817830	817896	817962	818028	818094	818160	
8	818226	818292	818358	818424	818490	818556	818622	818688	818754	818820	66
9	818885	818951	819017	819083	819149	819215	819281	819346	819412	819478	
660	**819544**	**819610**	**819676**	**819741**	**819807**	**819873**	**819939**	**820004**	**820070**	**820136**	
1	820201	820267	820333	820399	820464	820530	820595	820661	820727	820792	
2	820858	820924	820989	821055	821120	821186	821251	821317	821382	821448	
3	821514	821579	821645	821710	821775	821841	821906	821972	822037	822103	
4	822168	822233	822299	822364	822430	822495	822560	822626	822691	822756	
5	822822	822887	822952	823018	823083	823148	823213	823279	823344	823409	
6	823474	823539	823605	823670	823735	823800	823865	823930	823996	824061	
7	824126	824191	824256	824321	824386	824451	824516	824581	824646	824711	
8	824776	824841	824906	824971	825036	825101	825166	825231	825296	825361	65
9	825426	825491	825556	825621	825686	825751	825815	825880	825945	826010	
670	**826075**	**826140**	**826204**	**826269**	**826334**	**826399**	**826464**	**826528**	**826593**	**826658**	
1	826723	826787	826852	826917	826981	827046	827111	827175	827240	827305	
2	827369	827434	827499	827563	827628	827692	827757	827821	827886	827951	
3	828015	828080	828144	828209	828273	828338	828402	828467	828531	828595	
4	828660	828724	828789	828853	828918	828982	829046	829111	829175	829239	
5	829304	829368	829432	829497	829561	829625	829690	829754	829818	829882	
6	829947	830011	830075	830139	830204	830268	830332	830396	830460	830525	
7	830589	830653	830717	830781	830845	830909	830973	831037	831102	831166	
8	831230	831294	831358	831422	831486	831550	831614	831678	831742	831806	64
9	831870	831934	831998	832062	832126	832189	832253	832317	832381	832445	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
70	7.0	14.0	21.0	28.0	35.0	42.0	49.0	56.0	63.0
68	6.8	13.6	20.4	27.2	34.0	40.8	47.6	54.4	61.2
66	6.6	13.2	19.8	26.4	33.0	39.6	46.2	52.8	59.4
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8

N	0	1	2	3	4	5	6	7	8	9	Diff.
680	**832509**	**832573**	**832637**	**832700**	**832764**	**832828**	**832892**	**832956**	**833020**	**833083**	
1	833147	833211	833275	833338	833402	833466	833530	833593	833657	833721	
2	833784	833848	833912	833975	834039	834103	834166	834230	834294	834357	
3	834421	834484	834548	834611	834675	834739	834802	834866	834929	834993	
4	835056	835120	835183	835247	835310	835373	835437	835500	835564	835627	
5	835691	835754	835817	835881	835944	836007	836071	836134	836197	836261	
6	836324	836387	836451	836514	836577	836641	836704	836767	836830	836894	
7	836957	837020	837083	837146	837210	837273	837336	837399	837462	837525	
8	837588	837652	837715	837778	837841	837904	837967	838030	838093	838156	63
9	838219	838282	838345	838408	838471	838534	838597	838660	838723	838786	
690	**838849**	**838912**	**838975**	**839038**	**839101**	**839164**	**839227**	**839289**	**839352**	**839415**	
1	839478	839541	839604	839667	839729	839792	839855	839918	839981	840043	
2	840106	840169	840232	840294	840357	840420	840482	840545	840608	840671	
3	840733	840796	840859	840921	840984	841046	841109	841172	841234	841297	
4	841359	841422	841485	841547	841610	841672	841735	841797	841860	841922	
5	841985	842047	842110	842172	842235	842297	842360	842422	842484	842547	
6	842609	842672	842734	842796	842859	842921	842983	843046	843108	843170	
7	843233	843295	843357	843420	843482	843544	843606	843669	843731	843793	
8	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415	
9	844477	844539	844601	844664	844726	844788	844850	844912	844974	845036	
700	**845098**	**845160**	**845222**	**845284**	**845346**	**845408**	**845470**	**845532**	**845594**	**845656**	62
1	845718	845780	845842	845904	845966	846028	846090	846151	846213	846275	
2	846337	846399	846461	846523	846585	846646	846708	846770	846832	846894	
3	846955	847017	847079	847141	847202	847264	847326	847388	847449	847511	
4	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128	
5	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743	
6	848805	848866	848928	848989	849051	849112	849174	849235	849297	849358	
7	849419	849481	849542	849604	849665	849726	849788	849849	849911	849972	
8	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	
9	850646	850707	850769	850830	850891	850952	851014	851075	851136	851197	
710	**851258**	**851320**	**851381**	**851442**	**851503**	**851564**	**851625**	**851686**	**851747**	**851809**	
1	851870	851931	851992	852053	852114	852175	852236	852297	852358	852419	61
2	852480	852541	852602	852663	852724	852785	852846	852907	852968	853029	
3	853090	853150	853211	853272	853333	853394	853455	853516	853577	853637	
4	853698	853759	853820	853881	853941	854002	854063	854124	854185	854245	
5	854306	854367	854428	854488	854549	854610	854670	854731	854792	854852	
6	854913	854974	855034	855095	855156	855216	855277	855337	855398	855459	
7	855519	855580	855640	855701	855761	855822	855882	855943	856003	856064	
8	856124	856185	856245	856306	856366	856427	856487	856548	856608	856668	
9	856729	856789	856850	856910	856970	857031	857091	857152	857212	857272	
720	**857332**	**857393**	**857453**	**857513**	**857574**	**857634**	**857694**	**857755**	**857815**	**857875**	
1	857935	857995	858056	858116	858176	858236	858297	858357	858417	858477	
2	858537	858597	858657	858718	858778	858838	858898	858958	859018	859078	
3	859138	859198	859258	859318	859379	859439	859499	859559	859619	859679	60
4	859739	859799	859859	859918	859978	860038	860098	860158	860218	860278	
5	860338	860398	860458	860518	860578	860637	860697	860757	860817	860877	
6	860937	860996	861056	861116	861176	861236	861295	861355	861415	861475	
7	861534	861594	861654	861714	861773	861833	861893	861952	862012	862072	
8	862131	862191	862251	862310	862370	862430	862489	862549	862608	862668	
9	862728	862787	862847	862906	862966	863025	863085	863144	863204	863263	
730	**863323**	**863382**	**863442**	**863501**	**863561**	**863620**	**863680**	**863739**	**863799**	**863858**	
1	863917	863977	864036	864096	864155	864214	864274	864333	864392	864452	
2	864511	864570	864630	864689	864748	864808	864867	864926	864985	865045	
3	865104	865163	865222	865282	865341	865400	865459	865519	865578	865637	
4	865696	865755	865814	865874	865933	865992	866051	866110	866169	866228	
5	866287	866346	866405	866465	866524	866583	866642	866701	866760	866819	
6	866878	866937	866996	867055	867114	867173	867232	867291	867350	867409	59
7	867467	867526	867585	867644	867703	867762	867821	867880	867939	867998	
8	868056	868115	868174	868233	868292	868350	868409	868468	868527	868586	
9	868644	868703	868762	868821	868879	868938	868997	869056	869114	869173	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
64	6.4	12.8	19.2	25.6	32.0	38.4	44.8	51.2	57.6
62	6.2	12.4	18.6	24.8	31.0	37.2	43.4	49.6	55.8
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
740	**869232**	**869290**	**869349**	**869408**	**869466**	**869525**	**869584**	**869642**	**869701**	**869760**	
1	869818	869877	869935	869994	870053	870111	870170	870228	870287	870345	
2	870404	870462	870521	870579	870638	870696	870755	870813	870872	870930	
3	870989	871047	871106	871164	871223	871281	871339	871398	871456	871515	
4	871573	871631	871690	871748	871806	871865	871923	871981	872040	872098	
5	872156	872215	872273	872331	872389	872448	872506	872564	872622	872681	
6	872739	872797	872855	872913	872972	873030	873088	873146	873204	873262	
7	873321	873379	873437	873495	873553	873611	873669	873727	873785	873844	
8	873902	873960	874018	874076	874134	874192	874250	874308	874366	874424	58
9	874482	874540	874598	874656	874714	874772	874830	874888	874945	875003	
750	**875061**	**875119**	**875177**	**875235**	**875293**	**875351**	**875409**	**875466**	**875524**	**875582**	
1	875640	875698	875756	875813	875871	875929	875987	876045	876102	876160	
2	876218	876276	876333	876391	876449	876507	876564	876622	876680	876737	
3	876795	876853	876910	876968	877026	877083	877141	877199	877256	877314	
4	877371	877429	877487	877544	877602	877659	877717	877774	877832	877889	
5	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464	
6	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039	
7	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612	
8	879669	879726	879784	879841	879898	879956	880013	880070	880127	880185	
9	880242	880299	880356	880413	880471	880528	880585	880642	880699	880756	
760	**880814**	**880871**	**880928**	**880985**	**881042**	**881099**	**881156**	**881213**	**881271**	**881328**	
1	881385	881442	881499	881556	881613	881670	881727	881784	881841	881898	
2	881955	882012	882069	882126	882183	882240	882297	882354	882411	882468	57
3	882525	882581	882638	882695	882752	882809	882866	882923	882980	883037	
4	883093	883150	883207	883264	883321	883377	883434	883491	883548	883605	
5	883661	883718	883775	883832	883888	883945	884002	884059	884115	884172	
6	884229	884285	884342	884399	884455	884512	884569	884625	884682	884739	
7	884795	884852	884909	884965	885022	885078	885135	885192	885248	885305	
8	885361	885418	885474	885531	885587	885644	885700	885757	885813	885870	
9	885926	885983	886039	886096	886152	886209	886265	886321	886378	886434	
770	**886491**	**886547**	**886604**	**886660**	**886716**	**886773**	**886829**	**886885**	**886942**	**886998**	
1	887054	887111	887167	887223	887280	887336	887392	887449	887505	887561	
2	887617	887674	887730	887786	887842	887898	887955	888011	888067	888123	
3	888179	888236	888292	888348	888404	888460	888516	888573	888629	888685	
4	888741	888797	888853	888909	888965	889021	889077	889134	889190	889246	
5	889302	889358	889414	889470	889526	889582	889638	889694	889750	889806	56
6	889862	889918	889974	890030	890086	890141	890197	890253	890309	890365	
7	890421	890477	890533	890589	890645	890700	890756	890812	890868	890924	
8	890980	891035	891091	891147	891203	891259	891314	891370	891426	891482	
9	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039	
780	**892095**	**892150**	**892206**	**892262**	**892317**	**892373**	**892429**	**892484**	**892540**	**892595**	
1	892651	892707	892762	892818	892873	892929	892985	893040	893096	893151	
2	893207	893262	893318	893373	893429	893484	893540	893595	893651	893706	
3	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261	
4	894316	894371	894427	894482	894538	894593	894648	894704	894759	894814	
5	894870	894925	894980	895036	895091	895146	895201	895257	895312	895367	
6	895423	895478	895533	895588	895644	895699	895754	895809	895864	895920	
7	895975	896030	896085	896140	896195	896251	896306	896361	896416	896471	
8	896526	896581	896636	896692	896747	896802	896857	896912	896967	897022	
9	897077	897132	897187	897242	897297	897352	897407	897462	897517	897572	
790	**897627**	**897682**	**897737**	**897792**	**897847**	**897902**	**897957**	**898012**	**898067**	**898122**	55
1	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670	
2	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218	
3	899273	899328	899383	899437	899492	899547	899602	899656	899711	899766	
4	899821	899875	899930	899985	900039	900094	900149	900203	900258	900312	
5	900367	900422	900476	900531	900586	900640	900695	900749	900804	900859	
6	900913	900968	901022	901077	901131	901186	901240	901295	901349	901404	
7	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948	
8	902003	902057	902112	902166	902221	902275	902329	902384	902438	902492	
9	902547	902601	902655	902710	902764	902818	902873	902927	902981	903036	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
60	6.0	12.0	18.0	24.0	30.0	36.0	42.0	48.0	54.0
58	5.8	11.6	17.4	23.2	29.0	34.8	40.6	46.4	52.2
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6

N	0	1	2	3	4	5	6	7	8	9	Diff.
800	**903090**	**903144**	**903199**	**903253**	**903307**	**903361**	**903416**	**903470**	**903524**	**903578**	
1	903633	903687	903741	903795	903849	903904	903958	904012	904066	904120	
2	904174	904229	904283	904337	904391	904445	904499	904553	904607	904661	
3	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202	
4	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742	54
5	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281	
6	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820	
7	906874	906927	906981	907035	907089	907143	907196	907250	907304	907358	
8	907411	907465	907519	907573	907626	907680	907734	907787	907841	907895	
9	907949	908002	908056	908110	908163	908217	908270	908324	908378	908431	
810	**908485**	**908539**	**908592**	**908646**	**908699**	**908753**	**908807**	**908860**	**908914**	**908967**	
1	909021	909074	909128	909181	909235	909289	909342	909396	909449	909503	
2	909556	909610	909663	909716	909770	909823	909877	909930	909984	910037	
3	910091	910144	910197	910251	910304	910358	910411	910464	910518	910571	
4	910624	910678	910731	910784	910838	910891	910944	910998	911051	911104	
5	911158	911211	911264	911317	911371	911424	911477	911530	911584	911637	
6	911690	911743	911797	911850	911903	911956	912009	912063	912116	912169	
7	912222	912275	912328	912381	912435	912488	912541	912594	912647	912700	
8	912753	912806	912859	912913	912966	913019	913072	913125	913178	913231	
9	913284	913337	913390	913443	913496	913549	913602	913655	913708	913761	53
820	**913814**	**913867**	**913920**	**913973**	**914026**	**914079**	**914132**	**914184**	**914237**	**914290**	
1	914343	914396	914449	914502	914555	914608	914660	914713	914766	914819	
2	914872	914925	914977	915030	915083	915136	915189	915241	915294	915347	
3	915400	915453	915505	915558	915611	915664	915716	915769	915822	915875	
4	915927	915980	916033	916085	916138	916191	916243	916296	916349	916401	
5	916454	916507	916559	916612	916664	916717	916770	916822	916875	916927	
6	916980	917033	917085	917138	917190	917243	917295	917348	917400	917453	
7	917506	917558	917611	917663	917716	917768	917820	917873	917925	917978	
8	918030	918083	918135	918188	918240	918293	918345	918397	918450	918502	
9	918555	918607	918659	918712	918764	918816	918869	918921	918973	919026	
830	**919078**	**919130**	**919183**	**919235**	**919287**	**919340**	**919392**	**919444**	**919496**	**919549**	
1	919601	919653	919706	919758	919810	919862	919914	919967	920019	920071	
2	920123	920176	920228	920280	920332	920384	920436	920489	920541	920593	
3	920645	920697	920749	920801	920853	920906	920958	921010	921062	921114	52
4	921166	921218	921270	921322	921374	921426	921478	921530	921582	921634	
5	921686	921738	921790	921842	921894	921946	921998	922050	922102	922154	
6	922206	922258	922310	922362	922414	922466	922518	922570	922622	922674	
7	922725	922777	922829	922881	922933	922985	923037	923089	923140	923192	
8	923244	923296	923348	923399	923451	923503	923555	923607	923658	923710	
9	923762	923814	923865	923917	923969	924021	924072	924124	924176	924228	
840	**924279**	**924331**	**924383**	**924434**	**924486**	**924538**	**924589**	**924641**	**924693**	**924744**	
1	924796	924848	924899	924951	925003	925054	925106	925157	925209	925261	
2	925312	925364	925415	925467	925518	925570	925621	925673	925725	925776	
3	925828	925879	925931	925982	926034	926085	926137	926188	926240	926291	
4	926342	926394	926445	926497	926548	926600	926651	926702	926754	926805	
5	926857	926908	926959	927011	927062	927114	927165	927216	927268	927319	
6	927370	927422	927473	927524	927576	927627	927678	927730	927781	927832	
7	927883	927935	927986	928037	928088	928140	928191	928242	928293	928345	
8	928396	928447	928498	928549	928601	928652	928703	928754	928805	928857	
9	928908	928959	929010	929061	929112	929163	929215	929266	929317	929368	
850	**929419**	**929470**	**929521**	**929572**	**929623**	**929674**	**929725**	**929776**	**929827**	**929879**	
1	929930	929981	930032	930083	930134	930185	930236	930287	930338	930389	51
2	930440	930491	930542	930592	930643	930694	930745	930796	930847	930898	
3	930949	931000	931051	931102	931153	931204	931254	931305	931356	931407	
4	931458	931509	931560	931610	931661	931712	931763	931814	931865	931915	
5	931966	932017	932068	932118	932169	932220	932271	932322	932372	932423	
6	932474	932524	932575	932626	932677	932727	932778	932829	932879	932930	
7	932981	933031	933082	933133	933183	933234	933285	933335	933386	933437	
8	933487	933538	933589	933639	933690	933740	933791	933841	933892	933943	
9	933993	934044	934094	934145	934195	934246	934296	934347	934397	934448	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
56	5.6	11.2	16.8	22.4	28.0	33.6	39.2	44.8	50.4
54	5.4	10.8	16.2	21.6	27.0	32.4	37.8	43.2	48.6
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8

N	0	1	2	3	4	5	6	7	8	9	Diff.
860	**934498**	**934549**	**934599**	**934650**	**934700**	**934751**	**934801**	**934852**	**934902**	**934953**	
1	935003	935054	935104	935154	935205	935255	935306	935356	935406	935457	
2	935507	935558	935608	935658	935709	935759	935809	935860	935910	935960	
3	936011	936061	936111	936162	936212	936262	936313	936363	936413	936463	
4	936514	936564	936614	936665	936715	936765	936815	936865	936916	936966	
5	937016	937066	937116	937167	937217	937267	937317	937367	937418	937468	
6	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969	
7	938019	938069	938119	938169	938219	938269	938320	938370	938420	938470	50
8	938520	938570	938620	938670	938720	938770	938820	938870	938920	938970	
9	939020	939070	939120	939170	939220	939270	939320	939369	939419	939469	
870	**939519**	**939569**	**939619**	**939669**	**939719**	**939769**	**939819**	**939869**	**939918**	**939968**	
1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	
2	940516	940566	940616	940666	940716	940765	940815	940865	940915	940964	
3	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462	
4	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958	
5	942008	942058	942107	942157	942207	942256	942306	942355	942405	942455	
6	942504	942554	942603	942653	942702	942752	942801	942851	942901	942950	
7	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445	
8	943495	943544	943593	943643	943692	943742	943791	943841	943890	943939	
9	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433	
880	**944483**	**944532**	**944581**	**944631**	**944680**	**944729**	**944779**	**944828**	**944877**	**944927**	
1	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419	
2	945469	945518	945567	945616	945665	945715	945764	945813	945862	945912	
3	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403	
4	946452	946501	946551	946600	946649	946698	946747	946796	946845	946894	
5	946943	946992	947041	947090	947140	947189	947238	947287	947336	947385	
6	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875	49
7	947924	947973	948022	948070	948119	948168	948217	948266	948315	948364	
8	948413	948462	948511	948560	948608	948657	948706	948755	948804	948853	
9	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341	
890	**949390**	**949439**	**949488**	**949536**	**949585**	**949634**	**949683**	**949731**	**949780**	**949829**	
1	949878	949926	949975	950024	950073	950121	950170	950219	950267	950316	
2	950365	950414	950462	950511	950560	950608	950657	950706	950754	950803	
3	950851	950900	950949	950997	951046	951095	951143	951192	951240	951289	
4	951338	951386	951435	951483	951532	951580	951629	951677	951726	951775	
5	951823	951872	951920	951969	952017	952066	952114	952163	952211	952260	
6	952308	952356	952405	952453	952502	952550	952599	952647	952696	952744	
7	952792	952841	952889	952938	952986	953034	953083	953131	953180	953228	
8	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711	
9	953760	953808	953856	953905	953953	954001	954049	954098	954146	954194	
900	**954243**	**954291**	**954339**	**954387**	**954435**	**954484**	**954532**	**954580**	**954628**	**954677**	
1	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158	
2	955207	955255	955303	955351	955399	955447	955495	955543	955592	955640	
3	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120	
4	956168	956216	956265	956313	956361	956409	956457	956505	956553	956601	
5	956649	956697	956745	956793	956840	956888	956936	956984	957032	957080	48
6	957128	957176	957224	957272	957320	957368	957416	957464	957512	957559	
7	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038	
8	958086	958134	958181	958229	958277	958325	958373	958421	958468	958516	
9	958564	958612	958659	958707	958755	958803	958850	958898	958946	958994	
910	**959041**	**959089**	**959137**	**959185**	**959232**	**959280**	**959328**	**959375**	**959423**	**959471**	
1	959518	959566	959614	959661	959709	959757	959804	959852	959900	959947	
2	959995	960042	960090	960138	960185	960233	960280	960328	960376	960423	
3	960471	960518	960566	960613	960661	960709	960756	960804	960851	960899	
4	960946	960994	961041	961089	961136	961184	961231	961279	961326	961374	
5	961421	961469	961516	961563	961611	961658	961706	961753	961801	961848	
6	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322	
7	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795	
8	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268	
9	963316	963363	963410	963457	963504	963552	963599	963646	963693	963741	

Proportional Parts

Diff.	1	2	3	4	5	6	7	8	9
52	5.2	10.4	15.6	20.8	26.0	31.2	36.4	41.6	46.8
50	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2

N	0	1	2	3	4	5	6	7	8	9	Diff.
920	**963788**	**963835**	**963882**	**963929**	**963977**	**964024**	**964071**	**964118**	**964165**	**964212**	
1	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684	
2	964731	964778	964825	964872	964919	964966	965013	965061	965108	965155	
3	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625	
4	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095	47
5	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564	
6	966611	966658	966705	966752	966799	966845	966892	966939	966986	967033	
7	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501	
8	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969	
9	968016	968062	968109	968156	968203	968249	968296	968343	968390	968436	
930	**968483**	**968530**	**968576**	**968623**	**968670**	**968716**	**968763**	**968810**	**968856**	**968903**	
1	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369	
2	969416	969463	969509	969556	969602	969649	969695	969742	969789	969835	
3	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300	
4	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765	
5	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229	
6	971276	971322	971369	971415	971461	971508	971554	971601	971647	971693	
7	971740	971786	971832	971879	971925	971971	972018	972064	972110	972157	
8	972203	972249	972295	972342	972388	972434	972481	972527	972573	972619	
9	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082	
940	**973128**	**973174**	**973220**	**973266**	**973313**	**973359**	**973405**	**973451**	**973497**	**973543**	
1	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005	
2	974051	974097	974143	974189	974235	974281	974327	974374	974420	974466	
3	974512	974558	974604	974650	974696	974742	974788	974834	974880	974926	
4	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386	46
5	975432	975478	975524	975570	975616	975662	975707	975753	975799	975845	
6	975891	975937	975983	976029	976075	976121	976167	976212	976258	976304	
7	976350	976396	976442	976488	976533	976579	976625	976671	976717	976763	
8	976808	976854	976900	976946	976992	977037	977083	977129	977175	977220	
9	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678	
950	**977724**	**977769**	**977815**	**977861**	**977906**	**977952**	**977998**	**978043**	**978089**	**978135**	
1	978181	978226	978272	978317	978363	978409	978454	978500	978546	978591	
2	978637	978683	978728	978774	978819	978865	978911	978956	979002	979047	
3	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503	
4	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958	
5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	
6	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867	
7	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320	
8	981366	981411	981456	981501	981547	981592	981637	981683	981728	981773	
9	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226	
960	**982271**	**982316**	**982362**	**982407**	**982452**	**982497**	**982543**	**982588**	**982633**	**982678**	
1	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130	
2	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581	
3	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032	
4	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482	
5	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932	45
6	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382	
7	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830	
8	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279	
9	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727	
970	**986772**	**986817**	**986861**	**986906**	**986951**	**986996**	**987040**	**987085**	**987130**	**987175**	
1	987219	987264	987309	987353	987398	987443	987488	987532	987577	987622	
2	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068	
3	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514	
4	988559	988604	988648	988693	988737	988782	988826	988871	988916	988960	
5	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405	
6	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850	
7	989895	989939	989983	990028	990072	990117	990161	990206	990250	990294	
8	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738	
9	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182	

PROPORTIONAL PARTS

Diff.	1	2	3	4	5	6	7	8	9
48	4.8	9.6	14.4	19.2	24.0	28.8	33.6	38.4	43.2
46	4.6	9.2	13.8	18.4	23.0	27.6	32.2	36.8	41.4
44	4.4	8.8	13.2	17.6	22.0	26.4	30.8	35.2	39.6
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6	37.8

N	0	1	2	3	4	5	6	7	8	9	Diff.
980	**991226**	**991270**	**991315**	**991359**	**991403**	**991448**	**991492**	**991536**	**991580**	**991625**	
1	991669	991713	991758	991802	991846	991890	991935	991979	992023	992067	
2	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509	
3	992554	992598	992642	992686	992730	992774	992819	992863	992907	992951	
4	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392	
5	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833	
6	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273	
7	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713	44
8	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152	
9	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591	
990	**995635**	**995679**	**995723**	**995767**	**995811**	**995854**	**995898**	**995942**	**995986**	**996030**	
1	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468	
2	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906	
3	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343	
4	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779	
5	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216	
6	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652	
7	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087	
8	999131	999174	999218	999261	999305	999348	999392	999435	999479	999522	
9	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957	
1000	**000000**	**000043**	**000087**	**000130**	**000174**	**000217**	**000260**	**000304**	**000347**	**000391**	43

3. Areas of Circles, Diameters in Units and Eighths

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.0123	0.0491	0.1104	0.1963	0.3068	0.4418	0.6013
1	0.7854	0.9940	1.2272	1.4849	1.7671	2.0739	2.4053	2.7612
2	3.1416	3.5466	3.9761	4.4301	4.9087	5.4119	5.9396	6.4918
3	7.0686	7.6699	8.2958	8.9462	9.6211	10.321	11.045	11.793
4	12.566	13.364	14.186	15.033	15.904	16.800	17.721	18.665
5	19.635	20.629	21.648	22.691	23.758	24.850	25.967	27.109
6	28.274	29.465	30.680	31.919	33.183	34.472	35.785	37.122
7	38.485	39.871	41.282	42.718	44.179	45.664	47.173	48.707
8	50.265	51.849	53.456	55.088	56.745	58.426	60.132	61.862
9	63.617	65.397	67.201	69.029	70.882	72.760	74.662	76.589
10	78.540	80.516	82.516	84.541	86.590	88.664	90.763	92.886
11	95.033	97.205	99.402	101.62	103.87	106.14	108.43	110.75
12	113.10	115.47	117.86	120.28	122.72	125.19	127.68	130.19
13	132.73	135.30	137.89	140.50	143.14	145.80	148.49	151.20
14	153.94	156.70	159.48	162.30	165.13	167.99	170.87	173.78
15	176.71	179.67	182.65	185.66	188.69	191.75	194.83	197.93
16	201.06	204.22	207.39	210.60	213.82	217.08	220.35	223.65
17	226.98	230.33	233.71	237.10	240.53	243.98	247.45	250.95
18	254.47	258.02	261.59	265.18	268.80	272.45	276.12	279.81
19	283.53	287.27	291.04	294.83	298.65	302.49	306.35	310.24
20	314.16	318.10	322.06	326.05	330.06	334.10	338.16	342.25
21	346.36	350.50	354.66	358.84	363.05	367.28	371.54	375.83
22	380.13	384.46	388.82	393.20	397.61	402.04	406.49	410.97
23	415.48	420.00	424.56	429.13	433.74	438.36	443.01	447.69
24	452.39	457.11	461.86	466.64	471.44	476.26	481.11	485.98
25	490.87	495.79	500.74	505.71	510.71	515.72	520.77	525.84
26	530.93	536.05	541.19	546.35	551.55	556.76	562.00	567.27
27	572.56	577.87	583.21	588.57	593.96	599.37	604.81	610.27
28	615.75	621.26	626.80	632.36	637.94	643.55	649.18	654.84
29	660.52	666.23	671.96	677.71	683.49	689.30	695.13	700.98
30	706.86	712.76	718.69	724.64	730.62	736.62	742.64	748.69
31	754.77	760.87	766.99	773.14	779.31	785.51	791.73	797.98
32	804.25	810.54	816.86	823.21	829.58	835.97	842.39	848.83
33	855.30	861.79	868.31	874.85	881.41	888.00	894.62	901.26
34	907.92	914.61	921.32	928.06	934.82	941.61	948.42	955.25
35	962.11	969.00	975.91	982.84	989.80	996.78	1003.8	1010.8
36	1017.9	1025.0	1032.1	1039.2	1046.3	1053.5	1060.7	1068.0
37	1075.2	1082.5	1089.8	1097.1	1104.5	1111.8	1119.2	1126.7
38	1134.1	1141.6	1149.1	1156.6	1164.2	1171.7	1179.3	1186.9
39	1194.6	1202.3	1210.0	1217.7	1225.4	1233.2	1241.0	1248.8
40	1256.6	1264.5	1272.4	1280.3	1288.2	1296.2	1304.2	1312.2
41	1320.3	1328.3	1336.4	1344.5	1352.7	1360.8	1369.0	1377.2
42	1385.4	1393.7	1402.0	1410.3	1418.6	1427.0	1435.4	1443.8
43	1452.2	1460.7	1469.1	1477.6	1486.2	1494.7	1503.3	1511.9
44	1520.5	1529.2	1537.9	1546.6	1555.3	1564.0	1572.8	1581.6
d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8

4. Areas of Circles for Diameters in Units and Hundredths

d	0	1	2	3	4	5	6	7	8	9
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933
1.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.012	4.047	4.083	4.119
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.01	10.07	10.12
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35
5.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16
5.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98
5.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82
5.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67
d	0	1	2	3	4	5	6	7	8	9

4. Areas of Circles for Diameters in Units and Hundredths—*Continued*

d	0	1	2	3	4	5	6	7	8	9
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54
5.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43
5.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33
5.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25
5.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13
6.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09
6.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07
6.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07
6.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11
6.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15
6.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21
6.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28
6.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48
7.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60
7.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74
7.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89
7.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25
7.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45
7.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66
7.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89
7.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40
8.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68
8.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98
8.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29
8.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95
8.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31
8.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68
8.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07
8.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90
9.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33
9.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67 49	67.64	67.78
9.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25
9.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23
9.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75
9.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28
9.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82
9.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38
d	0	1	2	3	4	5	6	7	8	9

5. Natural Trigonometric Functions

Deg.	Min.	Sine	Covers	Cosec	Tan	Cotan	Secant	Versin	Cosine		
0	**0**	**0.00000**	**1.00000**	Infinite	**0.00000**	Infinite	**1.0000**	**0.00000**	**1.00000**	**90**	**0**
	15	.00436	.99564	229.18	.00436	229.18	1.0000	.00001	.99999		45
	30	.00873	.99127	114.59	.00873	114.59	1.0000	.00004	.99996		30
	45	.01309	.98691	76.397	.01309	76.390	1.0001	.00009	.99991		15
1	**0**	**.01745**	**.98255**	**57.299**	**.01746**	**57.290**	**1.0001**	**.00015**	**.99985**	**89**	**0**
	15	.02181	.97819	45.840	.02182	45.829	1.0002	.00024	.99976		45
	30	.02618	.97382	38.202	.02618	38.188	1.0003	.00034	.99966		30
	45	.03054	.96946	32.746	.03055	32.730	1.0005	.00047	.99953		15
2	**0**	**.03490**	**.96510**	**28.654**	**.03492**	**28.636**	**1.0006**	**.00061**	**.99939**	**88**	**0**
	15	.03926	.96074	25.471	.03929	25.452	1.0008	.00077	.99923		45
	30	.04362	.95638	22.926	.04366	22.904	1.0009	.00095	.99905		30
	45	.04798	.95202	20.843	.04803	20.819	1.0011	.00115	.99885		15
3	**0**	**.05234**	**.94766**	**19.107**	**.05241**	**19.081**	**1.0014**	**.00137**	**.99863**	**87**	**0**
	15	.05669	.94331	17.639	.05678	17.611	1.0016	.00161	.99839		45
	30	.06105	.93895	16.380	.06116	16.350	1.0019	.00187	.99813		30
	45	.06540	.93460	15.290	.06554	15.257	1.0021	.00214	.99786		15
4	**0**	**.06976**	**.93024**	**14.336**	**.06993**	**14.301**	**1.0024**	**.00244**	**.99756**	**86**	**0**
	15	.07411	.92589	13.494	.07431	13.457	1.0028	.00275	.99725		45
	30	.07846	.92154	12.745	.07870	12.706	1.0031	.00308	.99692		30
	45	.08281	.91719	12.076	.08309	12.035	1.0034	.00343	.99656		15
5	**0**	**.08716**	**.91284**	**11.474**	**.08749**	**11.430**	**1.0038**	**.00381**	**.99619**	**85**	**0**
	15	.09150	.90850	10.929	.09189	10.883	1.0042	.00420	.99580		45
	30	.09585	.90415	10.433	.09629	10.385	1.0046	.00460	.99540		30
	45	.10019	.89981	9.9812	.10069	9.9310	1.0051	.00503	.99497		15
6	**0**	**.10453**	**.89547**	**9.5668**	**.10510**	**9.5144**	**1.0055**	**.00548**	**.99452**	**84**	**0**
	15	.10887	.89113	9.1855	.10952	9.1309	1.0060	.00594	.99406		45
	30	.11320	.88680	8.8337	.11393	8.7769	1.0065	.00643	.99357		30
	45	.11754	.88246	8.5079	.11836	8.4490	1.0070	.00693	.99307		15
7	**0**	**.12187**	**.87813**	**8.2055**	**.12278**	**8.1443**	**1.0075**	**.00745**	**.99255**	**83**	**0**
	15	.12620	.87380	7.9240	.12722	7.8606	1.0081	.00800	.99200		45
	30	.13053	.86947	7.6613	.13165	7.5958	1.0086	.00856	.99144		30
	45	.13485	.86515	7.4156	.13609	7.3479	1.0092	.00913	.99086		15
8	**0**	**.13917**	**.86083**	**7.1853**	**.14054**	**7.1154**	**1.0098**	**.00973**	**.99027**	**82**	**0**
	15	.14349	.85651	6.9690	.14499	6.8969	1.0105	.01035	.98965		45
	30	.14781	.85219	6.7655	.14945	6.6912	1.0111	.01098	.98902		30
	45	.15212	.84788	6.5736	.15391	6.4971	1.0118	.01164	.98836		15
9	**0**	**.15643**	**.84357**	**6.3924**	**.15838**	**6.3138**	**1.0125**	**.01231**	**.98769**	**81**	**0**
	15	.16074	.83926	6.2211	.16286	6.1402	1.0132	.01300	.98700		45
	30	.16505	.83495	6.0589	.16734	5.9758	1.0139	.01371	.98629		30
	45	.16935	.83065	5.9049	.17183	5.8197	1.0147	.01444	.98556		15
10	**0**	**.17365**	**.82635**	**5.7588**	**.17633**	**5.6713**	**1.0154**	**.01519**	**.98481**	**80**	**0**
	15	.17794	.82206	5.6198	.18083	5.5301	1.0162	.01596	.98404		45
	30	.18224	.81776	5.4874	.18534	5.3955	1.0170	.01675	.98325		30
	45	.18652	.81348	5.3612	.18986	5.2672	1.0179	.01755	.98245		15
11	**0**	**.19081**	**.80919**	**5.2408**	**.19438**	**5.1446**	**1.0187**	**.01837**	**.98163**	**79**	**0**
	15	.19509	.80491	5.1258	.19891	5.0273	1.0196	.01921	.98079		45
	30	.19937	.80063	5.0158	.20345	4.9152	1.0205	.02008	.97992		30
	45	.20364	.79636	4.9106	.20800	4.8077	1.0214	.02095	.97905		15
12	**0**	**.20791**	**.79209**	**4.8097**	**.21256**	**4.7046**	**1.0223**	**.02185**	**.97815**	**78**	**0**
	15	.21218	.78782	4.7130	.21712	4.6057	1.0233	.02277	.97723		45
	30	.21644	.78356	4.6202	.22169	4.5107	1.0243	.02370	.97630		30
	45	.22070	.77930	4.5311	.22628	4.4194	1.0253	.02466	.97534		15
13	**0**	**.22495**	**.77505**	**4.4454**	**.23087**	**4.3315**	**1.0263**	**.02563**	**.97437**	**77**	**0**
	15	.22920	.77080	4.3630	.23547	4.2468	1.0273	.02662	.97338		45
	30	.23345	.76655	4.2837	.24008	4.1653	1.0284	.02763	.97237		30
	45	.23769	.76231	4.2072	.24470	4.0867	1.0295	.02866	.97134		15
14	**0**	**.24192**	**.75808**	**4.1336**	**.24933**	**4.0108**	**1.0306**	**.02970**	**.97030**	**76**	**0**
	15	.24615	.75385	4.0625	.25397	3.9375	1.0317	.03077	.96923		45
	30	.25038	.74962	3.9939	.25862	3.8667	1.0329	.03185	.96815		30
	45	.25460	.74540	3.9277	.26328	3.7983	1.0341	.03295	.96705		15
15	**0**	**.25882**	**.74118**	**3.8637**	**.26795**	**3.7320**	**1.0353**	**.03407**	**.96593**	**75**	**0**
		Cosine	Versin	Secant	Cotan	Tan	Cosec	Covers	Sine	Deg.	Min.

From 75° to 90° read from bottom of table upwards.

5. Natural Trigonometric Functions *(Continued)*

Deg.	Min.	Sine	Covers	Cosec	Tan	Cotan	Secant	Versin	Cosine		
15	**0**	**0.25882**	**0.74118**	**3.8637**	**0.26795**	**3.7320**	**1.0353**	**0.03407**	**0.96593**	**75**	**0**
	15	.26303	.73697	3.8018	.27263	3.6680	1.0365	.03521	.96479		45
	30	.26724	.73276	3.7420	.27732	3.6059	1.0377	.03637	.96363		30
	45	.27144	.72856	3.6840	.28203	3.5457	1.0390	.03754	.96246		15
16	**0**	**.27564**	**.72436**	**3.6280**	**.28674**	**3.4874**	**1.0403**	**.03874**	**.96126**	**74**	**0**
	15	.27983	.72017	3.5736	.29147	3.4308	1.0416	.03995	.96005		45
	30	.28402	.71598	3.5209	.29621	3.3759	1.0429	.04118	.95882		30
	45	.28820	.71180	3.4699	.30096	3.3226	1.0443	.04243	.95757		15
17	**0**	**.29237**	**.70763**	**3.4203**	**.30573**	**3.2709**	**1.0457**	**.04370**	**.95630**	**73**	**0**
	15	.29654	.70346	3.3722	.31051	3.2205	1.0471	.04498	.95502		45
	30	.30070	.69929	3.3255	.31530	3.1716	1.0485	.04628	.95372		30
	45	.30486	.69514	3.2801	.32010	3.1240	1.0500	.04760	.95240		15
18	**0**	**.30902**	**.69098**	**3.2361**	**.32492**	**3.0777**	**1.0515**	**.04894**	**.95106**	**72**	**0**
	15	.31316	.68684	3.1932	.32975	3.0326	1.0530	.05030	.94970		45
	30	.31730	.68270	3.1515	.33459	2.9887	1.0545	.05168	.94832		30
	45	.32144	.67856	3.1110	.33945	2.9459	1.0560	.05307	.94693		15
19	**0**	**.32557**	**.67443**	**3.0715**	**.34433**	**2.9042**	**1.0576**	**.05448**	**.94552**	**71**	**0**
	15	.32969	.67031	3.0331	.34921	2.8636	1.0592	.05591	.94409		45
	30	.33381	.66619	2.9957	.35412	2.8239	1.0608	.05736	.94264		30
	45	.33792	.66208	2.9593	.35904	2.7852	1.0625	.05882	.94118		15
20	**0**	**.34202**	**.65798**	**2.9238**	**.36397**	**2.7475**	**1.0642**	**.06031**	**.93969**	**70**	**0**
	15	.34612	.65388	2.8892	.36892	2.7106	1.0659	.06181	.93819		45
	30	.35021	.64979	2.8554	.37388	2.6746	1.0676	.06333	.93667		30
	45	.35429	.64571	2.8225	.37887	2.6395	1.0694	.06486	.93514		15
21	**0**	**.35837**	**.64163**	**2.7904**	**.38386**	**2.6051**	**1.0711**	**.06642**	**.93358**	**69**	**0**
	15	.36244	.63756	2.7591	.38888	2.5715	1.0729	.06799	.93201		45
	30	.36650	.63350	2.7285	.39391	2.5386	1.0748	.06958	.93042		30
	45	.37056	.62944	2.6986	.39896	2.5065	1.0766	.07119	.92881		15
22	**0**	**.37461**	**.62539**	**2.6695**	**.40403**	**2.4751**	**1.0785**	**.07282**	**.92718**	**68**	**0**
	15	.37865	.62135	2.6410	.40911	2.4443	1.0804	.07446	.92554		45
	30	.38268	.61732	2.6131	.41421	2.4142	1.0824	.07612	.92388		30
	45	.38671	.61329	2.5859	.41933	2.3847	1.0844	.07780	.92220		15
23	**0**	**.39073**	**.60927**	**2.5593**	**.42447**	**2.3559**	**1.0864**	**.07950**	**.92050**	**67**	**0**
	15	.39474	.60526	2.5333	.42963	2.3276	1.0884	.08121	.91879		45
	30	.39875	.60125	2.5078	.43481	2.2998	1.0904	.08294	.91706		30
	45	.40275	.59725	2.4829	.44001	2.2727	1.0925	.08469	.91531		15
24	**0**	**.40674**	**.59326**	**2.4586**	**.44523**	**2.2460**	**1.0946**	**.08645**	**.91355**	**66**	**0**
	15	.41072	.58928	2.4348	.45047	2.2199	1.0968	.08824	.91176		45
	30	.41469	.58531	2.4114	.45573	2.1943	1.0989	.09004	.90996		30
	45	.41866	.58134	2.3886	.46101	2.1692	1.1011	.09186	.90814		15
25	**0**	**.42262**	**.57738**	**2.3662**	**.46631**	**2.1445**	**1.1034**	**.09369**	**.90631**	**65**	**0**
	15	.42657	.57343	2.3443	.47163	2.1203	1.1056	.09554	.90446		45
	30	.43051	.56949	2.3228	.47697	2.0965	1.1079	.09741	.90259		30
	45	.43445	.56555	2.3018	.48234	2.0732	1.1102	.09930	.90070		15
26	**0**	**.43837**	**.56163**	**2.2812**	**.48773**	**2.0503**	**1.1126**	**.10121**	**.89879**	**64**	**0**
	15	.44229	.55771	2.2610	.49314	2.0278	1.1150	.10313	.89687		45
	30	.44620	.55380	2.2412	.49858	2.0057	1.1174	.10507	.89493		30
	45	.45010	.54990	2.2217	.50404	1.9840	1.1198	.10702	.89298		15
27	**0**	**.45399**	**.54601**	**2.2027**	**.50952**	**1.9626**	**1.1223**	**.10899**	**.89101**	**63**	**0**
	15	.45787	.54213	2.1840	.51503	1.9416	1.1248	.11098	.88902		45
	30	.46175	.53825	2.1657	.52057	1.9210	1.1274	.11299	.88701		30
	45	.46561	.53439	2.1477	.52612	1.9007	1.1300	.11501	.88499		15
28	**0**	**.46947**	**.53053**	**2.1300**	**.53171**	**1.8807**	**1.1326**	**.11705**	**.88295**	**62**	**0**
	15	.47332	.52668	2.1127	.53732	1.8611	1.1352	.11911	.88089		45
	30	.47716	.52284	2.0957	.54295	1.8418	1.1379	.12118	.87882		30
	45	.48099	.51901	2.0790	.54862	1.8228	1.1406	.12327	.87673		15
29	**0**	**.48481**	**.51519**	**2.0627**	**.55431**	**1.8040**	**1.1433**	**.12538**	**.87462**	**61**	**0**
	15	.48862	.51138	2.0466	.56003	1.7856	1.1461	.12750	.87250		45
	30	.49242	.50758	2.0308	.56577	1.7675	1.1490	.12964	.87036		30
	45	.49622	.50378	2.0152	.57155	1.7496	1.1518	.13180	.86820		15
30	**0**	**.50000**	**.50000**	**2.0000**	**.57735**	**1.7320**	**1.1547**	**.13397**	**.86603**	**60**	**0**
		Cosine	Versin	Secant	Cotan	Tan	Cosec	Covers	Sine	Deg.	Min.

From 60° to 75° read from bottom of table upwards.

5. Natural Trigonometric Functions (*Concluded*)

Deg.	Min	Sine	Covers	Cosec	Tan	Cotan	Secant	Versin	Cosine		
30	**0**	**0.50000**	**0.50000**	**2.0000**	**0.57735**	**1.7320**	**1.1547**	**0.13397**	**0.86603**	**60**	**0**
	15	.50377	.49623	1.9850	.58318	1.7147	1.1576	.13616	.86384		45
	30	.50754	.49246	1.9703	.58904	1.6977	1.1606	.13837	.86163		30
	45	.51129	.48871	1.9558	.59494	1.6808	1.1636	.14059	.85941		15
31	**0**	**.51504**	**.48496**	**1.9416**	**.60086**	**1.6643**	**1.1666**	**.14283**	**.85717**	**59**	**0**
	15	.51877	.48123	1.9276	.60681	1.6479	1.1697	.14509	.85491		45
	30	.52250	.47750	1 9139	.61280	1.6319	1.1728	.14736	.85264		30
	45	.52621	.47379	1.9004	.61882	1.6160	1.1760	.14965	.85035		15
32	**0**	**.52992**	**.47008**	**1 8871**	**.62487**	**1.6003**	**1.1792**	**.15195**	**.84805**	**58**	**0**
	15	.53361	.46639	1 8740	.63095	1 5849	1.1824	.15427	.84573		45
	30	.53730	.46270	1.8612	.63707	1.5697	1.1857	.15661	.84339		30
	45	.54097	.45903	1.8485	.64322	1.5547	1.1890	.15896	.84104		15
33	**0**	**.54464**	**45536**	**1.8361**	**.64941**	**1 5399**	**1.1924**	**.16133**	**.83867**	**57**	**0**
	15	.54829	.45171	1 8238	.65563	1.5453	1.1958	.16371	.83629		45
	30	.55194	.44806	1.8118	.66188	1.5108	1.1992	.16611	.83389		30
	45	.55557	.44443	1.7999	.66818	1.4966	1.2027	.16853	.83147		15
34	**0**	**.55919**	**.44081**	**1.7883**	**.67451**	**1.4826**	**1.2062**	**.17096**	**.82904**	**56**	**0**
	15	.56280	.43720	1 7768	.68087	1.4687	1 2098	.17341	.82659		45
	30	.56641	.43359	1.7655	.68728	1.4550	1.2134	.17587	.82413		30
	45	.57000	.43000	1.7544	.69372	1.4415	1.2171	.17835	.82165		15
35	**0**	**.57358**	**.42642**	**1 7434**	**.70021**	**1.4281**	**1.2208**	**.18085**	**.81915**	**55**	**0**
	15	.57715	.42285	1 7327	.70673	1.4150	1.2245	.18336	.81664		45
	30	.58070	.41930	1 7220	.71329	1.4019	1.2283	.18588	.81412		30
	45	.58425	.41575	1.7116	.71990	1.3891	1.2322	.18843	.81157		15
36	**0**	**.58779**	**.41221**	**1.7013**	**.72654**	**1.3764**	**1.2361**	**.19098**	**.80902**	**54**	**0**
	15	.59131	.40869	1.6912	.73323	1 3638	1.2400	.19356	.80644		45
	30	.59482	.40518	1.6812	.73996	1.3514	1 2440	.19614	.80386		30
	45	.59832	.40168	1.6713	.74673	1.3392	1.2480	.19875	.80125		15
37	**0**	**.60181**	**.39819**	**1.6616**	**.75355**	**1.3270**	**1.2521**	**.20136**	**.79864**	**53**	**0**
	15	.60529	.39471	1.6521	.76042	1.3151	1.2563	.20400	.79600		45
	30	.60876	.39124	1.6427	.76733	1.3032	1.2605	.20665	.79335		30
	45	.61222	.38778	1.6334	.77428	1.2915	1.2647	.20931	.79069		15
38	**0**	**.61566**	**.38434**	**1 6243**	**.78129**	**1.2799**	**1.2690**	**.21199**	**.78801**	**52**	**0**
	15	.61909	.38091	1.6153	.78834	1.2685	1.2734	.21468	.78532		45
	30	.62251	.37749	1.6064	.79543	1.2572	1.2778	.21739	.78261		30
	45	.62592	37408	1.5976	.80258	1.2460	1.2822	.22012	.77988		15
39	**0**	**.62932**	**37068**	**1 5890**	**.80978**	**1.2349**	**1.2868**	**.22285**	**.77715**	**51**	**0**
	15	.63271	.36729	1.5805	.81703	1.2239	1.2913	.22561	.77439		45
	30	.63608	.36392	1.5721	.82434	1.2131	1 2960	.22838	.77162		30
	45	.63944	.36056	1.5639	.83169	1.2024	1.3007	.23116	.76884		15
40	**0**	**.64279**	**.35721**	**1.5557**	**.83910**	**1.1918**	**1.3054**	**.23396**	**.76604**	**50**	**0**
	15	.64612	.35388	1.5477	.84656	1.1812	1.3102	.23677	.76323		45
	30	.64945	.35055	1.5398	.85408	1.1708	1.3151	.23959	.76041		30
	45	.65276	.34724	1.5320	.86165	1.1606	1.3200	.24244	.75756		15
41	**0**	**.65606**	**.34394**	**1.5242**	**.86929**	**1.1504**	**1.3250**	**.24529**	**.75471**	**49**	**0**
	15	.65935	.34065	1.5166	.87698	1.1403	1.3301	.24816	.75184		45
	30	.66262	.33738	1.5092	.88472	1.1303	1.3352	.25104	.74896		30
	46	.66588	.33412	1.5018	.89253	1.1204	1.3404	.25394	.74606		15
42	**0**	**.66913**	**.33087**	**1.4945**	**.90040**	**1.1106**	**1.3456**	**.25686**	**.74314**	**48**	**0**
	15	.67237	.32763	1.4873	.90834	1.1009	1.3509	.25978	.74022		45
	30	.67559	.32441	1.4802	.91633	1.0913	1.3563	.26272	.73728		30
	45	.67880	.32120	1.4732	.92439	1.0818	1.3618	.26566	.73432		15
43	**0**	**.68200**	**.31800**	**1.4663**	**.93251**	**1.0724**	**1.3673**	**.26865**	**.73135**	**47**	**0**
	15	.68518	.31482	1.4595	.94071	1.0630	1.3729	.27163	.72837		45
	30	.68835	.31165	1.4527	.94896	1.0538	1.3786	.27463	.72537		30
	45	.69151	.30849	1.4461	.95729	1.0446	1.3843	.27764	.72236		15
44	**0**	**.69466**	**.30534**	**1.4396**	**.96569**	**1.0355**	**1.3902**	**.28066**	**.71934**	**46**	**0**
	15	.69779	.30221	1.4331	.97416	1.0265	1.3961	.28370	.71630		45
	30	.70091	.29909	1.4267	.98270	1.0176	1.4020	.28675	.71325		30
	45	.70401	.29599	1.4204	.99131	1.0088	1.4081	.28981	.71019		15
45	**0**	**.70711**	**.29289**	**1.4142**	**.10000**	**1.0000**	**1.4142**	**.29289**	**.70711**	**45**	**0**
		Cosine	Versin	Secant	Cotan	Tan	Cosec	Covers	Sine	Deg.	Min.

From 45° to 60° read from bottom of table upwards.

6. Logarithmic Trigonometric Functions

Deg.	Sine	Cosec	Versin	Tangent	Cotan	Covers	Secant	Cosine	Deg
0	− ∞	+ ∞	− ∞	− ∞	+ ∞	10.00000	10.00000	10.00000	90
1	8.24186	11.75814	6.18271	8.24192	11.75808	9.99235	10.00007	9.99993	89
2	8.54282	11.45718	6.78474	8.54308	11.45692	9.98457	10.00026	9.99974	88
3	8.71880	11.28120	7.13687	8.71940	11.28060	9.97665	10.00060	9.99940	87
4	8.84358	11.15642	7.38667	8.84464	11.15536	9.96860	10.00106	9.99894	86
5	8.94030	11.05970	7.58039	8.94195	11.05805	9.96040	10.00166	9.99834	85
6	9.01923	10.98077	7.73863	9.02162	10.97838	9.95205	10.00239	9.99761	84
7	9.08589	10.91411	7.87238	9.08914	10.91086	9.94356	10.00325	9.99675	83
8	9.14356	10.85644	7.98820	9.14780	10.85220	9.93492	10.00425	9.99575	82
9	9.19433	10.80567	8.09032	9.19971	10.80029	9.92612	10.00538	9.99462	81
10	9.23967	10.76033	8.18162	9.24632	10.75368	9.91717	10.00665	9.99335	80
11	9.28060	10.71940	8.26418	9.28865	10.71135	9.90805	10.00805	9.99195	79
12	9.31788	10.68212	8.33950	9.32747	10.67253	9.89877	10.00960	9.99040	78
13	9.35209	10.64791	8.40875	9.36336	10.63664	9.88933	10.01128	9.98872	77
14	9.38368	10.61632	8.47282	9.39677	10.60323	9.87971	10.01310	9.98690	76
15	9.41300	10.58700	8.53243	9.42805	10.57195	9.86992	10.01506	9.98494	75
16	9.44034	10.55966	8.58814	9.45750	10.54250	9.85996	10.01716	9.98284	74
17	9.46594	10.53406	8.64043	9.48534	10.51466	9.84981	10.01940	9.98060	73
18	9.48998	10.51002	8.68969	9.51178	10.48822	9.83947	10.02179	9.97821	72
19	9.51264	10.48736	8.73625	9.53697	10.46303	9.82894	10.02433	9.97567	71
20	9.53405	10.46595	8.78037	9.56107	10.43893	9.81821	10.02701	9.97299	70
21	9.55433	10.44567	8.82230	9.58418	10.41582	9.80729	10.02985	9.97015	69
22	9.57358	10.42642	8.86223	9.60641	10.39359	9.79615	10.03283	9.96717	68
23	9.59188	10.40812	8.90034	9.62785	10.37215	9.78481	10.03597	9.96403	67
24	9.60931	10.39069	8.93679	9.64858	10.35142	9.77325	10.03927	9.96073	66
25	9.62595	10.37405	8.97170	9.66867	10.33133	9.76146	10.04272	9.95728	65
26	9.64184	10.35816	9.00521	9.68818	10.31182	9.74945	10.04634	9.95366	64
27	9.65705	10.34295	9.03740	9.70717	10.29283	9.73720	10.05012	9.94988	63
28	9.67161	10.32839	9.06838	9.72567	10.27433	9.72471	10.05407	9.94593	62
29	9.68557	10.31443	9.09823	9.74375	10.25625	9.71197	10.05818	9.94182	61
30	9.69897	10.30103	9.12702	9.76144	10.23856	9.69897	10.06247	9.93753	60
31	9.71184	10.28816	9.15483	9.77877	10.22123	9.68571	10.06693	9.93307	59
32	9.72421	10.27579	9.18171	9.79579	10.20421	9.67217	10.07158	9.92842	58
33	9.73611	10.26389	9.20771	9.81252	10.18748	9.65836	10.07641	9.92359	57
34	9.74756	10.25244	9.23290	9.82899	10.17101	9.64425	10.08143	9.91857	56
35	9.75859	10.24141	9.25731	9.84523	10.15477	9.62984	10.08664	9.91336	55
36	9.76922	10.23078	9.28099	9.86126	10.13874	9.61512	10.09204	9.90796	54
37	9.77946	10.22054	9.30398	9.87711	10.12289	9.60008	10.09765	9.90235	53
38	9.78934	10.21066	9.32631	9.89281	10.10719	9.58471	10.10347	9.89653	52
39	9.79887	10.20113	9.34802	9.90837	10.09163	9.56900	10.10950	9.89050	51
40	9.80807	10.19193	9.36913	9.92381	10.07619	9.55293	10.11575	9.88425	50
41	9.81694	10.18306	9.38968	9.93916	10.06084	9.53648	10.12222	9.87778	49
42	9.82551	10.17449	9.40969	9.95444	10.04556	9.51966	10.12893	9.87107	48
43	9.83378	10.16622	9.42918	9.96966	10.03034	9.50243	10.13587	9.86413	47
44	9.84177	10.15823	9.44818	9.98484	10.01516	9.48479	10.14307	9.85693	46
45	9.84949	10.15052	9.46671	10.00000	10.00000	9.46671	10.15052	9.84949	45
	Cosine	Secant	Covers	Cotan	Tangent	Versin	Cosec	Sine	

From 45° to 90° read from bottom of table upwards.

7. Properties of Numbers

Decimal Equivalents, Squares, Cubes, Three-halves Powers, Square Roots, Cube Roots, Fifth Roots Reciprocals, Circumference and Area of Circles

Number, N		N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
Fraction	Decimal								Circum.	Area
1/64	.015625	0.000244	$.381\times10^{-5}$	.1250	.2500	.00195	.4353	64.0	0.04909	.00019
1/32	.03125	.000977	$.305\times10^{-4}$	.1768	.3150	.00552	.5000	32.0	.09818	.00077
3/64	.046875	.002197	103×10^{-3}	.2165	.3606	.01015	.5422	18.8235	.14726	.00173
1/16	**.0625**	**.003906**	$.244\times10^{-3}$	**.2500**	**.3969**	**.01563**	**.5744**	**16.0**	**.19635**	**.00307**
5/64	.078125	.006104	477×10^{-3}	.2795	.4275	.02184	.6006	12.80	.24544	.00479
3/32	.09375	.008789	$.824\times10^{-3}$	.3062	.4543	.02871	.6229	10.6667	.29452	.00690
	.10	.010	.00100	.3162	.4642	.03162	.6310	10.0	.31416	.00785
7/64	.109375	.01196	.001308	.3307	.4782	.03617	.6424	9.1429	.34361	.00939
1/8	**.125**	**.01563**	**001953**	**.3536**	**.5000**	**.04419**	**.6598**	**8.0**	**.39270**	**.01227**
9/64	.140625	.01978	002782	.3750	.5200	.05273	.6755	7.1111	.44179	.01554
5/32	.15625	.02441	.003814	.3953	.5386	06176	.6899	6.40	.49087	.01917
11/64	.171875	.02954	005077	.4146	.5560	.07126	.7031	5.8182	.53996	.02320
3/16	**.1875**	**.03516**	**.006592**	**.4330**	**.5724**	**.08119**	**.7155**	**5.3333**	**.58905**	**.02761**
	.20	.040	.0080	.4472	.5848	.08944	.7248	5.0	.62832	.03142
13/64	.203125	.04126	.008381	.4507	.5878	.09155	.7270	4.9231	.63814	.03241
7/32	.21875	.04785	01047	.4677	.6025	.10231	.7379	4.5714	.68722	.03758
15/64	.234375	.05493	.01287	.4841	.6166	.11347	.7481	4.2667	.73631	.04314
1/4	**.250**	**.0625**	**.01563**	**.5000**	**.6300**	**.12500**	**.7579**	**4.0**	**.78540**	**.04909**
17/64	.265625	.07056	01874	.5154	.6428	.13690	.7671	3.7647	.83448	.05542
9/32	.28125	.07910	02225	.5303	.6552	.14916	.7759	3.5556	.88357	.06213
19/64	.296875	.08813	.02616	.5449	.6671	.16176	.7844	3.3684	.93266	.06922
	.30	.090	.0270	.5477	.6694	.16432	.7860	3.3333	.94248	.07069
5/16	**.3125**	**.09766**	**03052**	**.5590**	**.6786**	**.17469**	**.7925**	**3.2000**	**.98175**	**.07670**
21/64	.328125	.10767	03533	.5728	.6897	.18796	.8002	3.0476	1.0308	.08456
11/32	.34375	.11816	.04062	.5863	.7005	.20154	.8077	2.9091	1.0799	.09281
23/64	.359375	.12915	04641	.5995	.7110	.21544	.8149	2 7826	1.1290	.10143
3/8	**.375**	**.14063**	**05273**	**.6124**	**.7211**	**.22964**	**.8219**	**2.6667**	**1.1781**	**.11045**
25/64	.390625	.15259	.05961	.6250	.7310	.24414	.8286	2.5600	1.2272	.11984
	.40	.16	.0640	.6325	.7368	.25298	.8326	2.50	1.2566	.12566
13/32	.40625	.16504	.06705	.6374	.7406	.25894	.8351	2.4615	1.2763	.12962
27/64	.421875	.17798	07508	.6495	.7500	.27402	.8415	2.3704	1.3254	.13979
7/16	**.4375**	**.19141**	**08374**	**.6614**	**.7592**	**.28938**	**.8476**	**2.2857**	**1.3744**	**.15033**
29/64	.453125	.20532	.09304	.6732	.7681	.30502	.8536	2.2069	1.4235	.16126
15/32	.46875	.21973	.10300	.6847	.7768	.32093	.8594	2.1333	1.4726	.17257
31/64	.484375	.23462	.11364	.6960	.7854	.33711	.8650	2.0645	1.5217	.18427
1/2	**.50**	**.2500**	**.12500**	**.7071**	**.7937**	**.35355**	**.8706**	**2.0**	**1.5708**	**.19635**
33/64	.515625	.26587	.13709	.7181	.8019	.37025	.8759	1.9394	1.6199	.20881
17/32	.53125	.28223	.14993	.7289	.8099	.38721	.8812	1.8824	1.6690	.22166
35/64	.546875	.29907	.16355	.7395	.8178	.40442	.8863	1.8286	1.7181	.23489
9/16	**.5625**	**.31641**	**.17798**	**.7500**	**.8255**	**.42188**	**.8913**	**1.7778**	**1.7671**	**.24850**
37/64	.578125	.33423	.19323	.7604	.8331	.43957	.8962	1.7297	1.8162	.26250
19/32	.59375	.35254	.20932	.7706	.8405	.45751	.9010	1.6842	1.8653	.27688
	.60	.3600	.21600	.7746	.8434	.46476	.9029	1.6667	1.8850	.28274
39/64	.609375	.37134	22628	.7806	.8478	.47569	.9057	1.6410	1.9144	.29165
5/8	**625**	**.39063**	**.24414**	**7906**	**.8550**	**.49410**	**9103**	**1.6000**	**1.9635**	**.30680**
41/64	.640625	.41040	.26291	.8004	.8621	.51275	.9148	1.5610	2.0126	.32233
21/32	.65625	.43066	.28262	.8101	.8690	.53162	9192	1.5238	2.0617	.33824
43/64	671875	.45142	30330	.8197	.8759	.55072	9235	1.4884	2.1108	.35454
11/16	**.6875**	**.47266**	**.32495**	**.8292**	**.8826**	**.57005**	**9278**	**1.4545**	**2.1598**	**.37122**
	.70	.4900	.34300	.8367	.8879	.58566	.9312	1.4286	2.1991	.38485
45/64	.703125	.49438	.34761	.8385	.8892	.58959	.9320	1.4222	2.2089	.38829
23/32	71875	.51660	.37131	.8478	.8958	.60935	.9361	1.3913	2.2580	.40574
47/64	.734375	.53931	.39605	.8570	.9022	.62933	.9401	1.3617	2.3071	.42357
3/4	**.750**	**.56250**	**.42188**	**.8660**	**9086**	**.64952**	**.9441**	**1.3333**	**2.3562**	**.44179**
49/64	.765625	.58618	44879	.8750	.9148	.66992	9480	1.3061	2.4053	.46038
25/32	.78125	.61035	.47684	.8839	.9210	.69053	.9518	1.2800	2.4544	.47937
51/64	.796875	.63501	.50602	.8927	.9271	.71135	.9556	1.2549	2.5035	.49874
	.80	.6400	.51200	.8944	.9283	.71554	.9564	1.2500	2.5133	.50265
13/16	**.8125**	**.66016**	**.53638**	**.9014**	**.9331**	**73238**	**.9593**	**1.2308**	**2.5525**	**.51849**
53/64	.828125	.68579	.56792	.9100	.9391	.75361	.9630	1.2075	2.6016	.53862
27/32	.84375	.71191	.60067	.9186	.9449	.77503	.9666	1.1852	2.6507	.55914
55/64	.859375	.73853	.63467	.9270	.9507	.79666	.9702	1.1636	2.6998	.58004
7/8	**.875**	**.76563**	**.66992**	**.9354**	**.9565**	**.81849**	**9737**	**1.1429**	**2.7489**	**.60132**
57/64	.890625	.79321	.70645	.9437	.9621	.84051	9771	1.1228	2.7980	.62299
	.90	.81000	.72900	.9487	.9655	.85435	.9792	1.1111	2.8274	.63617
29/32	.90625	.82129	.74429	.9520	.9677	.86272	9805	1.1034	2.8471	.64504
59/64	.921875	.84985	.78346	.9601	.9733	.88513	.9839	1.0847	2.8962	.66747
15/16	**.9375**	**.87891**	**.82398**	**.9682**	**.9787**	**.90773**	**.9872**	**1.0667**	**2.9452**	**.69029**
61/64	.953125	.90845	.86587	.9763	.9841	.93053	.9905	1.0492	2.9943	.71349
31/32	.96875	.93848	.90915	.9843	.9895	.95349	.9937	1.0323	3.0434	.73708
63/64	.984375	.96899	.95385	.9922	.9948	.97666	.9969	1.0159	3.0925	.76104

7. **Properties of Numbers** (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
1.	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000000	3.1416	0.7854
1.125	1.2656	1.4238	1.0606	1.0400	1.1932	1.0238	.8888888	3.5343	.9940
1.25	1.5625	1.9531	1.1180	1.0772	1.3975	1.0456	.80000000	3.9270	1.2272
1.375	1.8906	2.5996	1.1726	1.1120	1.6123	1.0658	72727272	4.3197	1.4849
1.5	2.25	3.3750	1.2247	1.1447	1.8371	1.0845	.66666666	4.7124	1.7671
1.625	2.6406	4.2910	1.2748	1.1757	2.0715	1.1020	.61538462	5.1051	2.0739
1.75	3.0625	5.3594	1.3229	1.2051	2.3150	1.1186	.57142857	5.4978	2.4053
1.875	3.5156	6.5918	1.3693	1.2331	2.5675	1.1340	.53333333	5.8905	2.7612
2.	4.0000	8.0000	1.4142	1.2599	2.8284	1.1487	.50000000	6.2832	3.1416
2.125	4.5156	9.5957	1.4577	1.2856	3.0977	1.1627	.47058823	6.6759	3.5466
2.25	5.0625	11.3906	1.5000	1.3104	3.3750	1.1761	.44444444	7.0686	3.9761
2.375	5.6406	13.3965	1.5411	1.3342	3.6601	1.1889	.42105263	7.4613	4.4301
2.5	6.2500	15.6250	1.5811	1.3572	3.9529	1.2011	.40000000	7.8540	4.9087
2.625	6.8906	18.0879	1.6202	1.3795	4.2530	1.2129	.38095231	8.2467	5.4119
2.75	7.5625	20.7969	1.6583	1.4011	4.5604	1.2242	.36363636	8.6394	5.9396
2.875	8.2656	23.7637	1.6956	1.4219	4.8748	1.2352	.34782609	9.0321	6.4918
3.	9.0000	27.0000	1.7321	1.4422	5.1962	1.2457	.33333333	9.4248	7.0686
3.125	9.7656	30.5176	1.7678	1.4620	5.5243	1.2559	.32000000	9.8175	7.6699
3.25	10.5625	34.3281	1.8028	1.4813	5.8590	1.2658	.30769231	10.2102	8.2958
3.375	11.3906	38.4434	1.8371	1.5000	6.2003	1.2754	.29629629	10.6029	8.9462
3.5	12.2500	42.8750	1.8708	1.5183	6.5479	1.2847	.28571429	10.9956	9.6211
3.625	13.1406	47.6348	1.9039	1.5362	6.9018	1.2938	.27586207	11.3883	10.3206
3.75	14.0625	52.7344	1.9365	1.5536	7.2619	1.3026	.26666666	11.7810	11.0447
3.875	15.0156	58.1856	1.9685	1.5707	7.6279	1.3112	.25806452	12.1737	11.7932
4.	16.0000	64.0000	2.0000	1.5874	8.0000	1.3195	.25000000	12.5664	12.5664
4.125	17.0156	70.1895	2.0310	1.6038	8.3779	1.3277	.24242424	12.9591	13.3640
4.25	18.0625	76.7656	2.0616	1.6198	8.7616	1.3356	.23529412	13.3518	14.1863
4.375	19.1406	83.7402	2.0916	1.6355	9.1510	1.3434	.22857143	13.7445	15.0330
4.5	20.2500	91.1250	2.1213	1.6510	9.5460	1.3510	.22222222	14.1372	15.9043
4.625	21.3906	98.9317	2.1506	1.6661	9.9465	1.3584	.21621622	14.5299	16.8001
4.75	22.5625	107.1719	2.1795	1.6810	10.3524	1.3656	.21052632	14.9226	17.7205
4.875	23.7656	115.8574	2.2079	1.6956	10.7637	1.3728	.20512821	15.3153	18.6655
5.	25.0000	125.0000	2.2361	1.7100	11.1803	1.3797	.20000000	15.7080	19.6350
5.125	26.2656	134.6113	2.2638	1.7241	11.6022	1.3866	.19512195	16.1006	20.6289
5.25	27.5625	144.7031	2.2913	1.7380	12.0293	1.3933	.19047619	16.4933	21.6475
5.375	28.8906	155.2871	2.3184	1.7517	12.4614	1.3998	.18604651	16.8860	22.6906
5.5	30.2500	166.3750	2.3452	1.7652	12.8987	1.4063	.18181818	17.2787	23.7583
5.625	31.6406	177.9785	2.3727	1.7784	13.3409	1.4126	.17777777	17.6714	24.8505
5.75	33.0625	190.1094	2.3979	1.7915	13.7880	1.4188	.17391304	18.0641	25.9672
5.875	34.5156	202.7793	2.4238	1.8044	14.2400	1.4250	.17021277	18.4568	27.1085
6.	36.0000	216.0000	2.4495	1.8171	14.6969	1.4310	.16666666	18.8496	28.2743
6.125	37.5156	229.7832	2.4749	1.8297	15.1586	1.4369	.16326531	19.2422	29.4647
6.25	39.0625	244.1406	2.5000	1.8420	15.6250	1.4427	.16000000	19.6349	30.6796
6.375	40.6406	259.0840	2.5249	1.8542	16.0961	1.4484	.15686275	20.0276	31.9190
6.5	42.2500	274.6250	2.5495	1.8663	16.5718	1.4542	.15384615	20.4203	33.1831
6.625	43.8906	290.7754	2.5739	1.8781	17.0522	1.4596	.15094339	20.8130	34.4716
6.75	45.5625	307.5469	2.5981	1.8899	17.5370	1.4651	.14814815	21.2057	35.7847
6.875	47.2656	324.9512	2.6220	1.9015	18.0264	1.4705	.14545454	21.5984	37.1223
7.	49.0000	343.0000	2.6458	1.9129	18.5203	1.4758	.14285714	21.9911	38.4845
7.125	50.7656	361.7051	2.6693	1.9243	19.0186	1.4810	.14035088	22.3838	39.8712
7.25	52.5625	381.0781	2.6926	1.9354	19.5212	1.4862	.13793103	22.7765	41.2825
7.375	54.3906	401.1309	2.7157	1.9465	20.0283	1.4913	.13559322	23.1692	42.7183
7.5	56.2500	421.8750	2.7386	1.9574	20.5396	1.4963	.13333333	23.5619	44.1786
7.625	58.1406	443.3223	2.7613	1.9683	21.0552	1.5012	.13114754	23.9546	45.6635
7.75	60.0625	465.4844	2.7839	1.9789	21.5751	1.5061	.12903226	24.3473	47.1730
7.875	62.0156	488.3731	2.8063	1.9895	22.0992	1.5110	.12698413	24.7400	48.7069
8.	64.0000	512.0000	2.8284	2.0000	22.6274	1.5157	.12500000	25.1327	50.2655
8.125	66.0156	536.3770	2.8504	2.0104	23.1598	1.5204	.12307692	25.5254	51.8485
8.25	68.0625	561.5156	2.8723	2.0206	23.6963	1.5251	.12121212	25.9181	53.4562
8.375	70.1406	587.4278	2.8940	2.0308	24.2369	1.5297	.11940298	26.3108	55.0883
8.5	72.2500	614.1250	2.9155	2.0408	24.7816	1.5342	.11764706	26.7035	56.7450
8.625	74.3906	641.6192	2.9368	2.0508	25.3301	1.5387	.11594203	27.0962	58.4262
8.75	76.5625	669.9219	2.9580	2.0606	25.8828	1.5431	.11428571	27.4889	60.1320
8.875	78.7656	699.0450	2.9791	2.0704	26.4394	1.5475	.11267605	27.8816	61.8623
9.	81.0000	729.0000	3.0000	2.0801	27.0000	1.5518	.11111111	28.2743	63.6173
9.125	83.2656	759.7989	3.0207	2.0897	27.5645	1.5561	.10958904	28.6670	65.3966
9.25	85.5625	791.4531	3.0414	2.0992	28.1328	1.5604	.10810811	29.0597	67.2006
9.375	87.8906	823.9746	3.0619	2.1086	28.7050	1.5646	.10666666	29.4524	69.0291
9.5	90.2500	857.3750	3.0822	2.1179	29.2810	1.5687	.10526316	29.8451	70.8822
9.625	92.6406	891.6660	3.1024	2.1272	29.8608	1.5728	.10389610	30.2378	72.7597
9.75	95.0625	926.8594	3.1225	2.1363	30.4444	1.5769	.10256410	30.6305	74.6619
9.875	97.5156	962.9668	3.1425	2.1454	31.0317	1.5809	.10126582	31.0232	76.5886

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
10	**100**	**1000**	**3.1623**	**2.1544**	**31.623**	**1.5849**	**.10000000**	**31.4159**	**78.5398**
11	121	1331	3.3166	2.2240	36.483	1.6154	.09090909	34.5575	95.0332
12	144	1728	3.4641	2.2894	41.569	1.6438	.08333333	37.6991	113.0973
13	169	2197	3.6056	2.3513	46.873	1.6703	.07692308	40.8407	132.7323
14	196	2744	3.7417	2.4101	52.384	1.6953	.07142857	43.9823	153.9380
15	225	3375	3.8730	2.4662	58.095	1.7188	.06666667	47.1239	176.7146
16	256	4096	4.0000	2.5198	64.000	1.7411	.06250000	50.2654	201.0619
17	289	4913	4.1231	2.5713	70.093	1.7623	.05882353	53.4070	226.9801
18	324	5832	4.2426	2.6207	76.367	1.7826	.05555556	56.5486	254.4690
19	361	6859	4.3589	2.6684	82.819	1.8020	.05263158	59.6902	283.5287
20	**400**	**8000**	**4.4721**	**2.7144**	**89.443**	**1.8206**	**.05000000**	**62.8319**	**314.1593**
21	441	9261	4.5826	2.7589	96.235	1.8384	.04761905	65.9734	346.3606
22	484	10648	4.6904	2.8020	103.19	1.8556	.04545455	69.1150	380.1327
23	529	12167	4.7958	2.8439	110.30	1.8722	.04347826	72.2566	415.4756
24	576	13824	4.8990	2.8845	117.58	1.8882	.04166667	75.3982	452.3893
25	625	15625	5.0000	2.9240	125.00	1.9037	.04000000	78.5398	490.8739
26	676	17576	5.0990	2.9625	132.57	1.9186	.03846154	81.6813	530.9292
27	729	19683	5.1962	3.0000	140.30	1.9332	.03703704	84.8229	572.5553
28	784	21952	5.2915	3.0366	148.16	1.9473	.03571429	87.9645	615.7522
29	841	24389	5.3852	3.0723	156.17	1.9610	.03448276	91.1061	660.5198
30	**900**	**27000**	**5.4772**	**3.1072**	**164.32**	**1.9744**	**.03333333**	**94.2477**	**706.8583**
31	961	29791	5.5678	3.1414	172.60	1.9873	.03225806	97.3893	754.7676
32	1024	32768	5.6569	3.1748	181.02	2.0000	.03125000	100.5309	804.2477
33	1089	35937	5.7446	3.2075	189.57	2.0123	.03030303	103.6725	855.2986
34	1156	39304	5.8310	3.2396	198.25	2.0244	.02941176	106.8141	907.9203
35	1225	42875	5.9161	3.2711	207.06	2.0362	.02857143	109.9557	962.1127
36	1296	46656	6.0000	3.3019	216.00	2.0477	.02777778	113.0972	1017.8760
37	1369	50653	6.0828	3.3322	225.06	2.0589	.02702703	116.2388	1075.2101
38	1444	54872	6.1644	3.3620	234.25	2.0699	.02631579	119.3804	1134.1149
39	1521	59319	6.2450	3.3912	243.56	2.0807	.02564103	122.5220	1194.5906
40	**1600**	**64000**	**6.3246**	**3.4200**	**252.98**	**2.0913**	**.02500000**	**125.6636**	**1256.6371**
41	1681	68921	6.4031	3.4482	262.53	2.1016	.02439024	128.8052	1320.2543
42	1764	74088	6.4807	3.4760	272.19	2.1118	.02380952	131.9468	1385.4424
43	1849	79507	6.5574	3.5034	281.97	2.1218	.02325581	135.0884	1452.2012
44	1936	85184	6.6332	3.5303	291.86	2.1315	.02272727	138.2300	1520.5308
45	2025	91125	6.7082	3.5569	301.87	2.1411	.02222222	141.3716	1590.4313
46	2116	97336	6.7823	3.5830	311.99	2.1506	.02173913	144.5131	1661.9025
47	2209	103823	6.8557	3.6088	322.22	2.1598	.02127660	147.6547	1734.9445
48	2304	110592	6.9282	3.6342	332.55	2.1689	.02083333	150.7963	1809.5574
49	2401	117649	7.0000	3.6593	343.00	2.1779	.02040816	153.9379	1885.7410
50	**2500**	**125000**	**7.0711**	**3.6840**	**353.55**	**2.1867**	**.02000000**	**157.0795**	**1963.500**
51	2601	132651	7.1414	3.7084	364.21	2.1954	.01960784	160.2211	2042.820
52	2704	140608	7.2111	3.7325	374.98	2.2039	.01923077	163.3627	2123.716
53	2809	148877	7.2801	3.7563	385.85	2.2124	.01886792	166.5043	2206.183
54	2916	157464	7.3485	3.7798	396.82	2.2206	.01851852	169.6459	2290.221
55	3025	166375	7.4162	3.8030	407.89	2.2288	.01818182	172.7875	2375.829
56	3136	175616	7.4833	3.8259	419.07	2.2369	.01785714	175.9290	2463.008
57	3249	185193	7.5498	3.8485	430.35	2.2448	.01754386	179.0706	2551.758
58	3364	195112	7.6158	3.8709	441.72	2.2526	.01724138	182.2122	2642.079
59	3481	205379	7.6811	3.8930	453.19	2.2603	.01694915	185.3538	2733.970
60	**3600**	**216000**	**7.7460**	**3.9149**	**464.76**	**2.2679**	**.01666667**	**188.4954**	**2827.433**
61	3721	226981	7.8102	3.9365	476.43	2.2755	.01639344	191.6370	2922.466
62	3844	238328	7.8740	3.9579	488.19	2.2829	.01612903	194.7786	3019.070
63	3969	250047	7.9373	3.9791	500.05	2.2902	.01587302	197.9202	3117.245
64	4096	262144	8.0000	4.0000	512.00	2.2974	.01562500	201.0618	3216.990
65	4225	274625	8.0623	4.0207	524.05	2.3045	.01538462	204.2034	3318.307
66	4356	287496	8.1240	4.0412	536.19	2.3116	.01515152	207.3449	3421.194
67	4489	300763	8.1854	4.0615	548.42	2.3186	.01492537	210.4865	3525.652
68	4624	314432	8.2462	4.0817	560.74	2.3254	.01470588	213.6281	3631.680
69	4761	328509	8.3066	4.1016	573.16	2.3322	.01449275	216.7697	3739.280
70	**4900**	**343000**	**8.3666**	**4.1213**	**585.66**	**2.3389**	**.01428571**	**219.9113**	**3848.450**
71	5041	357911	8.4261	4.1408	598.26	2.3456	.01408451	223.0529	3959.191
72	5184	373248	8.4853	4.1602	610.94	2.3522	.01388889	226.1945	4071.503
73	5329	389017	8.5440	4.1793	623.71	2.3587	.01369863	229.3361	4185.386
74	5476	405224	8.6023	4.1983	636.57	2.3651	.01351351	232.4777	4300.839
75	5625	421875	8.6603	4.2172	649.52	2.3714	.01333333	235.6193	4417.864
76	5776	438976	8.7178	4.2358	662.55	2.3777	.01315789	238.7608	4536.459
77	5929	456533	8.7750	4.2543	675.68	2.3840	.01298701	241.9024	4656.625
78	6084	474552	8.8318	4.2727	688.88	2.3901	.01282051	245.0440	4778.361
79	6241	493039	8.8882	4.2908	702.17	2.3962	.01265823	248.1856	4901.669

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum	Circle (N = Diam.) Area
80	**6400**	**512000**	**8.9443**	**4.3089**	**715.54**	**2.4022**	**.01250000**	**251.327**	**5026.547**
81	6561	531441	9.0000	4.3267	729.00	2.4082	.01234568	254.469	5152.998
82	6724	551368	9.0554	4.3445	742.54	2.4141	.01219512	257.610	5281.016
83	6889	571787	9.1104	4.3621	756.17	2.4200	.01204819	260.752	5410.607
84	7056	592704	9.1652	4.3795	769.88	2.4258	.01190476	263.894	5541.770
85	7225	614125	9.2195	4.3968	783.66	2.4315	.01176471	267.035	5674.501
86	7396	636056	9.2736	4.4140	797.53	2.4372	.01162791	270.177	5808.805
87	7569	658503	9.3274	4.4310	811.49	2.4429	.01149425	273.318	5944.679
88	7744	681472	9.3808	4.4480	825.52	2.4485	.01136364	276.460	6082.124
89	7921	704969	9.4340	4.4647	839.63	2.4540	.01123596	279.602	6221.138
90	**8100**	**729000**	**9.4868**	**4.4814**	**853.82**	**2.4595**	**.01111111**	**282.743**	**6361.725**
91	8281	753571	9.5394	4.4979	868.09	2.4650	.01098901	285.885	6503.882
92	8464	778688	9.5917	4.5144	882.44	2.4705	.01086957	289.026	6647.610
93	8649	804357	9.6437	4.5307	896.86	2.4758	.01075269	292.168	6792.909
94	8836	830584	9.6954	4.5468	911.36	2.4810	.01063830	295.309	6939.778
95	9025	857375	9.7468	4.5629	925.95	2.4863	.01052632	298.451	7088.219
96	9216	884736	9.7980	4.5789	940.61	2.4915	.01041667	301.593	7238.230
97	9409	912673	9.8489	4.5947	955.34	2.4966	.01030928	304.734	7389.812
98	9604	941192	9.8995	4.6104	970.15	2.5018	.01020408	307.876	7542.962
99	9801	970299	9.9499	4.6261	985.04	2.5069	.01010101	311.017	7697.688
100	**10000**	**1000000**	**10.0000**	**4.6416**	**1000.0**	**2.5119**	**.01000000**	**314.159**	**7853.982**
101	10201	1030301	10.0499	4.6570	1015.0	2.5169	.00990099	317.301	8011.85
102	10404	1061208	10.0995	4.6723	1030.1	2.5219	.00980392	320.442	8171.28
103	10609	1092727	10.1489	4.6875	1045.3	2.5268	.00970874	323.584	8332.29
104	10816	1124864	10.1980	4.7027	1060.6	2.5317	.00961538	326.725	8494.87
105	11025	1157625	10.2470	4.7177	1075.9	2.5365	.00952381	329.867	8659.01
106	11236	1191016	10.2956	4.7326	1091.3	2.5413	.00943396	333.009	8824.73
107	11449	1225043	10.3441	4.7475	1106.8	2.5461	.00934579	336.150	8992.02
108	11664	1259712	10.3923	4.7622	1122.4	2.5509	.00925926	339.292	9160.88
109	11881	1295029	10.4403	4.7769	1138.0	2.5556	.00917431	342.433	9331.32
110	**12100**	**1331000**	**10.4881**	**4.7914**	**1153.7**	**2.5603**	**.00909091**	**345.575**	**9503.32**
111	12321	1367631	10.5357	4.8059	1169.5	2.5649	.00900901	348.716	9676.89
112	12544	1404928	10.5830	4.8203	1185.3	2.5695	.00892857	351.853	9852.03
113	12769	1442897	10.6301	4.8346	1201.2	2.5740	.00884956	355.000	10028.75
114	12996	1481544	10.6771	4.8488	1217.2	2.5786	.00877193	358.141	10207.03
115	13225	1520875	10.7238	4.8629	1233.2	2.5831	.00869565	361.283	10386.89
116	13456	1560896	10.7703	4.8770	1249.4	2.5876	.00862069	364.424	10568.32
117	13689	1601613	10.8167	4.8910	1265.5	2.5920	.00854701	367.566	10751.31
118	13924	1643032	10.8628	4.9049	1281.8	2.5964	.00847458	370.708	10935.88
119	14161	1685159	10.9087	4.9187	1298.1	2.6008	.00840336	373.849	11122.02
120	**14400**	**1728000**	**10.9545**	**4.9324**	**1314.5**	**2.6052**	**.00833333**	**376.991**	**11309.73**
121	14641	1771561	11.0000	4.9461	1331.0	2.6095	.00826446	380.132	11499.01
122	14884	1815848	11.0454	4.9597	1347.5	2.6138	.00819672	383.274	11689.86
123	15129	1860867	11.0905	4.9732	1364.1	2.6181	.00813008	386.416	11882.29
124	15376	1906624	11.1355	4.9866	1380.8	2.6223	.00806452	389.557	12076.28
125	15625	1953125	11.1803	5.0000	1397.5	2.6265	.00800000	392.699	12271.84
126	15876	2000376	11.2250	5.0133	1414.4	2.6307	.00793651	395.840	12468.98
127	16129	2048383	11.2694	5.0265	1431.2	2.6349	.00787402	398.982	12667.68
128	16384	2097152	11.3137	5.0397	1448.2	2.6390	.00781250	402.124	12867.96
129	16641	2146689	11.3578	5.0528	1465.2	2.6431	.00775194	405.265	13069.81
130	**16900**	**2197000**	**11.4018**	**5.0658**	**1482.2**	**2.6472**	**.00769231**	**408.407**	**13273.23**
131	17161	2248091	11.4455	5.0788	1499.4	2.6513	.00763359	411.548	13478.22
132	17424	2299968	11.4891	5.0916	1516.6	2.6553	.00757576	414.690	13684.77
133	17689	2352637	11.5326	5.1045	1533.8	2.6593	.00751880	417.831	13892.91
134	17956	2406104	11.5758	5.1172	1551.2	2.6633	.00746269	420.973	14102.61
135	18225	2460375	11.6190	5.1299	1568.6	2.6673	.00740741	424.115	14313.88
136	18496	2515456	11.6619	5.1426	1586.0	2.6712	.00735294	427.256	14526.72
137	18769	2571353	11.7047	5.1551	1603.6	2.6751	.00729927	430.398	14741.14
138	19044	2628072	11.7473	5.1676	1621.1	2.6790	.00724638	433.539	14957.12
139	19321	2685619	11.7898	5.1801	1638.8	2.6829	.00719424	436.681	15174.67
140	**19600**	**2744000**	**11.8322**	**5.1925**	**1656.5**	**2.6867**	**.00714286**	**439.823**	**15393.80**
141	19881	2803221	11.8743	5.2048	1674.3	2.6906	.00709220	442.964	15614.50
142	20164	2863288	11.9164	5.2171	1692.1	2.6944	.00704225	446.106	15836.77
143	20449	2924207	11.9583	5.2293	1710.0	2.6981	.00699301	449.247	16060.60
144	20736	2985984	12.0000	5.2415	1728.0	2.7019	.00694444	452.389	16286.01
145	21025	3048625	12.0416	5.2536	1746.0	2.7057	.00689655	455.531	16512.99
146	21316	3112136	12.0830	5.2656	1764.1	2.7094	.00684932	458.672	16741.54
147	21609	3176523	12.1244	5.2776	1782.2	2.7131	.00680272	461.814	16971.67
148	21904	3241792	12.1655	5.2896	1800.5	2.7168	.00675676	464.955	17203.36
149	22201	3307949	12.2066	5.3015	1818.8	2.7204	.00671141	468.097	17436.62

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Circle (N = Diam.) Area
150	**22500**	**3375000**	**12.2474**	**5.3133**	**1837.1**	**2.7241**	**.00666667**	**471.239**	**17671.46**
151	22801	3442951	12.2882	5.3251	1855.5	2.7277	.00662252	474.380	17907.86
152	23104	3511808	12.3288	5.3368	1874.0	2.7314	.00657895	477.522	18145.84
153	23409	3581577	12.3693	5.3485	1892.5	2.7349	.00653595	480.663	18385.38
154	23716	3652264	12.4097	5.3601	1911.1	2.7385	.00649351	483.805	18626.50
155	24025	3723875	12.4499	5.3717	1929.7	2.7420	.00645161	486.946	18869.19
156	24336	3796416	12.4900	5.3832	1948.4	2.7455	.00641026	490.088	19113.45
157	24649	3869893	12.5300	5.3947	1967.2	2.7490	.00636943	493.230	19359.28
158	24964	3944312	12.5698	5.4061	1986.0	2.7525	.00632911	496.371	19606.68
159	25281	4019679	12.6095	5.4175	2004.9	2.7560	.00628931	499.513	19855.65
160	**25600**	**4096000**	**12.6491**	**5.4288**	**2023.9**	**2.7595**	**.00625000**	**502.654**	**20106.19**
161	25921	4173281	12.6886	5.4401	2042.9	2.7629	00621118	505.796	20358.30
162	26244	4251528	12.7279	5.4514	2061 9	2.7663	00617284	508.938	20611.99
163	26569	4330747	12.7671	5.4626	2081.0	2.7697	.00613497	512.079	20867.24
164	26896	4410944	12.8062	5.4737	2100.2	2.7731	.00609756	515.221	21124.06
165	27225	4492125	12.8452	5.4848	2119 5	2.7765	.00606061	518.362	21382.46
166	27556	4574296	12.8841	5.4959	2138.8	2.7799	.00602410	521.504	21642.43
167	27889	4657463	12.9228	5 5069	2158.1	2 7832	.00598802	524.646	21903.96
168	28224	4741632	12.9615	5.5178	2177.5	2.7865	.00595238	527.787	22167.07
169	28561	4826809	13.0000	5.5288	2197.0	2.7898	.00591716	530.929	22431.75
170	**28900**	**4913000**	**13.0384**	**5.5397**	**2216.5**	**2.7931**	**.00588235**	**534.070**	**22698.00**
171	29241	5000211	13.0767	5.5505	2236.1	2.7964	00584795	537.212	22965.82
172	29584	5088448	13.1149	5 5613	2255.8	2 7997	.00581395	540.353	23235.21
173	29929	5177717	13.1529	5.5721	2275.5	2.8029	00578035	543.495	23506.18
174	30276	5268024	13.1909	5.5828	2295.2	2.8061	.00574713	546.637	23778.71
175	30625	5359375	13.2288	5.5934	2315.0	2.8094	.00571429	549.778	24052.81
176	30976	5451776	13.2665	5.6041	2334.9	2.8126	.00568182	552.920	24328.49
177	31329	5545233	13.3041	5.6147	2354.8	2.8158	.00564972	556.061	24605.73
178	31684	5639752	13.3417	5.6252	2374.8	2 8189	.00561798	559.203	24884.55
179	32041	5735339	13.3791	5.6357	2394.9	2.8221	.00558659	562.345	25164.94
180	**32400**	**5832000**	**13.4164**	**5.6462**	**2415.0**	**2.8252**	**.00555556**	**565.486**	**25446.90**
181	32761	5929741	13.4536	5.6567	2435.1	2.8284	.00552486	568.628	25730.42
182	33124	6028568	13.4907	5.6671	2455.3	2.8315	.00549451	571.769	26015.52
183	33489	6128487	13.5277	5.6774	2475.6	2.8346	.00546448	574.911	26302.19
184	33856	6229504	13.5647	5.6877	2495.9	2.8377	.00543478	578.053	26590.43
185	34225	6331625	13.6015	5.6980	2516.3	2.8408	00540541	581.194	26880.25
186	34596	6434856	13.6382	5.7083	2536.7	2.8438	.00537634	584.336	27171.63
187	34969	6539203	13.6748	5.7185	2557.2	2 8469	.00534759	587.477	27464.58
188	35344	6644672	13.7113	5.7287	2577.7	2.8499	.00531915	590.619	27759.11
189	35721	6751269	13.7477	5.7388	2598.3	2.8529	.00529101	593.761	28055.20
190	**36100**	**6859000**	**13.7840**	**5.7489**	**2619.0**	**2.8560**	**.00526316**	**596.902**	**28352.87**
191	36481	6967871	13.8203	5.7590	2639.7	2.8590	.00523560	600.044	28652.10
192	36864	7077888	13.8564	5.7690	2660.4	2.8619	.00520833	603.185	28952.91
193	37249	7189057	13.8924	5.7790	2681.2	2.8649	.00518135	606.327	29255.29
194	37636	7301384	13.9284	5.7890	2702.1	2.8679	.00515464	609.468	29559.24
195	38025	7414875	13.9642	5.7989	2723.0	2.8708	.00512821	612.610	29864.76
196	38416	7529536	14.0000	5.8088	2744.0	2.8738	.00510204	615.752	30171.85
197	38809	7645373	14.0357	5.8186	2765.0	2.8767	.00507614	618.893	30480.51
198	39204	7762392	14.0712	5.8285	2786.1	2.8796	.00505051	622.035	30790.74
199	39601	7880599	14.1067	5.8383	2807.2	2.8825	.00502513	625.176	31102.55
200	**40000**	**8000000**	**14.1421**	**5.8480**	**2828.4**	**2.8854**	**.00500000**	**628.318**	**31415.93**
201	40401	8120601	14.1774	5.8578	2849.7	2.8883	.00497512	631.460	31730.87
202	40804	8242408	14.2127	5 8675	2871.0	2.8911	.00495050	634.601	32047.39
203	41209	8365427	14.2478	5.8771	2892.3	2.8940	.00492611	637.743	32365.47
204	41616	8489664	14.2829	5.8868	2913.7	2.8968	.00490196	640.884	32685.13
205	42025	8615125	14.3178	5 8964	2935.2	2.8997	.00487805	644.026	33006.36
206	42436	8741816	14.3527	5.9059	2956.7	2.9025	.00485437	647.168	33329.16
207	42849	8869743	14.3875	5.9155	2978.2	2.9053	.00483092	650.309	33653.53
208	43264	8998912	14.4222	5.9250	2999.8	2.9081	.00480769	653.451	33979.47
209	43681	9129329	14.4568	5.9345	3021.5	2.9109	.00478469	656.592	34306.98
210	**44100**	**9261000**	**14.4914**	**5.9439**	**3043.2**	**2.9137**	**.00476190**	**659.734**	**34636.06**
211	44521	9393931	14.5258	5.9533	3065.0	2.9165	.00473934	662.875	34966.71
212	44944	9528128	14.5602	5.9627	3086.8	2.9192	.00471698	666.017	35298.94
213	45369	9663597	14.5945	5.9721	3108.7	2.9220	.00469484	669.159	35632.73
214	45796	9800344	14.6287	5.9814	3130.6	2.9247	00467290	672.300	35968.09
215	46225	9938375	14.6629	5.9907	3152.5	2.9274	.00465116	675.442	36305.03
216	46656	10077696	14.6969	6.0000	3174.5	2.9302	.00462963	678.583	36643.54
217	47089	10218313	14.7309	6.0092	3196.6	2.9329	.00460829	681.725	36983.61
218	47524	10360232	14.7648	6.0185	3218.7	2.9356	.00458716	684.867	37325.26
219	47961	10503459	14.7986	6.0277	3240.9	2.9383	.00456621	688.008	37668.48

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
220	**48400**	**10648000**	**14.8324**	**6.0368**	**3263.1**	**2.9409**	**.00454545**	**691.150**	**38013.27**
221	48841	10793861	14.8661	6.0459	3285.4	2.9436	.00452489	694.291	38359.63
222	49284	10941048	14.8997	6.0550	3307.7	2.9463	.00450450	697.433	38707.56
223	49729	11089567	14.9332	6.0641	3330.1	2.9489	.00448430	700.575	39057.07
224	50176	11239424	14.9666	6.0732	3352.5	2.9516	.00446429	703.716	39408.14
225	50625	11390625	15.0000	6.0822	3375.0	2.9542	.00444444	706.858	39760.78
226	51076	11543176	15.0333	6.0912	3397.5	2.9568	.00442478	709.999	40115.00
227	51529	11697083	15.0665	6.1002	3420.1	2.9594	.00440529	713.141	40470.78
228	51984	11852352	15.0997	6.1091	3442.7	2.9620	.00438596	716.283	40828.14
229	52441	12008989	15.1327	6.1180	3465.4	2.9646	.00436681	719.424	41187.07
230	**52900**	**12167000**	**15.1658**	**6.1269**	**3488.1**	**2.9672**	**.00434783**	**722.566**	**41547.56**
231	53361	12326391	15.1987	6.1358	3510.9	2.9698	.00432900	725.707	41909.63
232	53824	12487168	15.2315	6.1446	3533.7	2.9723	.00431034	728.849	42273.27
233	54289	12649337	15.2643	6.1534	3556.6	2.9749	.00429185	731.990	42638.48
234	54756	12812904	15.2971	6.1622	3579.5	2.9774	.00427350	735.132	43005.26
235	55225	12977875	15.3297	6.1710	3602.5	2.9800	.00425532	738.274	43373.61
236	55696	13144256	15.3623	6.1797	3625.5	2.9825	.00423729	741.415	43743.54
237	56169	13312053	15.3948	6.1885	3648.6	2.9850	.00421941	744.557	44115.03
238	56644	13481272	15.4272	6.1972	3671.7	2.9875	.00420168	747.698	44488.09
239	57121	13651919	15.4596	6.2058	3694.8	2.9900	.00418410	750.840	44862.73
240	**57600**	**13824000**	**15.4919**	**6.2145**	**3718.0**	**2.9926**	**.00416667**	**753.982**	**45238.93**
241	58081	13997521	15.5242	6.2231	3741.3	2.9950	.00414938	757.123	45616.71
242	58564	14172488	15.5563	6.2317	3764.6	2.9975	.00413223	760.265	45996.06
243	59049	14348907	15.5885	6.2403	3788.0	3.0000	.00411523	763.406	46376.98
244	59536	14526784	15.6205	6.2488	3811.4	3.0025	.00409836	766.548	46759.47
245	60025	14706125	15.6525	6.2573	3834.9	3.0049	.00408163	769.690	47143.52
246	60516	14886936	15.6844	6.2658	3858.4	3.0074	.00406504	772.831	47529.16
247	61009	15069223	15.7162	6.2743	3881.9	3.0098	.00404858	775.973	47916.36
248	61504	15252992	15.7480	6.2828	3905.5	3.0122	.00403226	779.114	48305.13
249	62001	15438249	15.7797	6.2912	3929.2	3.0147	.00401606	782.256	48695.47
250	**62500**	**15625000**	**15.8114**	**6.2996**	**3952.8**	**3.0171**	**.00400000**	**785.398**	**49087.39**
251	63001	15813251	15.8430	6.3080	3976.6	3.0195	.00398406	788.539	49480.87
252	63504	16003008	15.8745	6.3164	4000.4	3.0219	.00396825	791.681	49875.92
253	64009	16194277	15.9060	6.3247	4024.2	3.0243	.00395257	794.822	50272.55
254	64516	16387064	15.9374	6.3330	4048.1	3.0267	.00393701	797.964	50670.75
255	65025	16581375	15.9687	6.3413	4072.0	3.0291	.00392157	801.105	51070.52
256	65536	16777216	16.0000	6.3496	4096.0	3.0314	.00390625	804.247	51471.85
257	66049	16974593	16.0312	6.3579	4120.0	3.0338	.00389105	807.389	51874.76
258	66564	17173512	16.0624	6.3661	4144.1	3.0362	.00387597	810.530	52279.24
259	67081	17373979	16.0935	6.3743	4168.2	3.0385	.00386100	813.672	52685.29
260	**67600**	**17576000**	**16.1245**	**6.3825**	**4192.4**	**3.0418**	**.00384615**	**816.813**	**53092.92**
261	68121	17779581	16.1555	6.3907	4216.6	3.0432	.00383142	819.955	53502.11
262	68644	17984728	16.1864	6.3988	4240.8	3.0455	.00381679	823.097	53912.87
263	69169	18191447	16.2173	6.4070	4265.1	3.0478	.00380228	826.238	54325.21
264	69696	18399744	16.2481	6.4151	4289.5	3.0501	.00378788	829.380	54739.11
265	70225	18609625	16.2788	6.4232	4313.9	3.0524	.00377358	832.521	55154.59
266	70756	18821096	16.3095	6.4312	4338.3	3.0547	.00375940	835.663	55571.63
267	71289	19034163	16.3401	6.4393	4362.8	3.0570	.00374532	838.805	55990.25
268	71824	19248832	16.3707	6.4473	4387.3	3.0593	.00373134	841.946	56410.44
269	72361	19465109	16.4012	6.4553	4411.9	3.0616	.00371747	845.088	56832.20
270	**72900**	**19683000**	**16.4317**	**6.4633**	**4436.5**	**3.0639**	**.00370370**	**848.229**	**57255.53**
271	73441	19902511	16.4621	6.4713	4461.2	3.0662	.00369004	851.371	57680.43
272	73984	20123648	16.4924	6.4792	4485.9	3.0684	.00367647	854.512	58106.90
273	74529	20346417	16.5227	6.4872	4510.7	3.0707	.00366300	857.654	58534.94
274	75076	20570824	16.5529	6.4951	4535.5	3.0729	.00364964	860.796	58964.55
275	75625	20796875	16.5831	6.5030	4560.4	3.0752	.00363636	863.937	59395.74
276	76176	21024576	16.6132	6.5108	4585.3	3.0774	.00362319	867.079	59828.49
277	76729	21253933	16.6433	6.5187	4610.2	3.0796	.00361011	870.220	60262.82
278	77284	21484952	16.6733	6.5265	4635.2	3.0818	.00359712	873.362	60698.71
279	77841	21717639	16.7033	6.5343	4660.2	3.0840	.00358423	876.504	61136.18
280	**78400**	**21952000**	**16.7332**	**6.5421**	**4685.3**	**3.0863**	**.00357143**	**879.645**	**61575.22**
281	78961	22188041	16.7631	6.5499	4710.4	3.0885	.00355872	882.787	62015.82
282	79524	22425768	16.7929	6.5577	4735.6	3.0907	.00354610	885.928	62458.00
283	80089	22665187	16.8226	6.5654	4760.8	3.0928	.00353357	889.070	62901.75
284	80656	22906304	16.8523	6.5731	4786.0	3.0950	.00352113	892.212	63347.07
285	81225	23149125	16.8819	6.5808	4811.3	3.0972	.00350877	895.353	63793.97
286	81796	23393656	16.9115	6.5885	4836.7	3.0994	.00349650	898.495	64242.43
287	82369	23639903	16.9411	6.5962	4862.1	3.1015	.00348432	901.636	64692.46
288	82944	23887872	16.9706	6.6039	4887.5	3.1037	.00347222	904.778	65144.07
289	83521	24137569	17.0000	6.6115	4913.0	3.1058	.00346021	907.920	65597.24

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
290	**84100**	**24389000**	**17.0294**	**6.6191**	**4938.5**	**3.1080**	**.00344828**	**911.061**	**66051.99**
291	84681	24642171	17.0587	6.6267	4964.1	3.1101	.00343643	914.203	66508.30
292	85264	24897088	17.0880	6.6343	4989.7	3.1123	.00342466	917.344	66966.19
293	85849	25153757	17.1172	6.6419	5015.4	3.1144	.00341297	920.486	67425.65
294	86436	25412184	17.1464	6.6494	5041.1	3.1165	.00340136	923.627	67886.68
295	87025	25672375	17.1756	6.6569	5066.8	3.1186	.00338983	926.769	68349.28
296	87616	25934336	17.2047	6.6644	5092.6	3.1207	.00337838	929.911	68813.45
297	88209	26198073	17.2337	6.6719	5118.4	3.1228	.00336700	933.052	69279.19
298	88804	26463592	17.2627	6.6794	5144.3	3.1249	.00335570	936.194	69746.50
299	89401	26730899	17.2916	6.6869	5170.2	3.1270	.00334448	939.335	70215.38
300	**90000**	**27000000**	**17.3205**	**6.6943**	**5196.2**	**3.1291**	**.00333333**	**942.477**	**70685.83**
301	90601	27270901	17.3494	6.7018	5222.2	3.1312	.00332226	945.619	71157.86
302	91204	27543608	17.3781	6.7092	5248.2	3.1333	.00331126	948.760	71631.45
303	91809	27818127	17.4069	6.7166	5274.3	3.1354	.00330033	951.902	72106.62
304	92416	28094464	17.4356	6.7240	5300.4	3.1374	.00328947	955.043	72583.36
305	93025	28372625	17.4642	6.7313	5326.6	3.1395	.00327869	958.185	73061.66
306	93636	28652616	17.4929	6.7387	5352.8	3.1416	.00326797	961.327	73541.54
307	94249	28934443	17.5214	6.7460	5379.1	3.1436	.00325733	964.468	74022.99
308	94864	29218112	17.5499	6.7533	5405.4	3.1456	.00324675	967.610	74506.01
309	95481	29503629	17.5784	6.7606	5431.7	3.1477	.00323625	970.751	74990.60
310	**96100**	**29791000**	**17.6068**	**6.7679**	**5458.1**	**3.1497**	**.00322581**	**973.893**	**75476.76**
311	96721	30080231	17.6352	6.7752	5484.5	3.1518	.00321543	977.034	75964.50
312	97344	30371328	17.6635	6.7824	5511.0	3.1538	.00320513	980.176	76453.80
313	97969	30664297	17.6918	6.7897	5537.5	3.1558	.00319489	983.318	76944.67
314	98596	30959144	17.7200	6.7969	5564.1	3.1578	.00318471	986.459	77437.12
315	99225	31255875	17.7482	6.8041	5590.7	3.1598	.00317460	989.601	77931.13
316	99856	31554496	17.7764	6.8113	5617.3	3.1618	.00316456	992.742	78426.72
317	100489	31855013	17.8045	6.8185	5644.0	3.1638	.00315457	995.884	78923.88
318	101124	32157432	17.8326	6.8256	5670.7	3.1658	.00314465	999.026	79422.60
319	101761	32461759	17.8606	6.8328	5697.5	3.1678	.00313480	1002.167	79922.90
320	**102400**	**32768000**	**17.8885**	**6.8399**	**5724.3**	**3.1698**	**.00312500**	**1005.309**	**80424.77**
321	103041	33076161	17.9165	6.8470	5751.2	3.1718	.00311526	1008.450	80928.21
322	103684	33386248	17.9444	6.8541	5778.1	3.1737	.00310559	1011.592	81433.22
323	104329	33698267	17.9722	6.8612	5805.0	3.1757	.00309598	1014.734	81939.80
324	104976	34012224	18.0000	6.8683	5832.0	3.1777	.00308642	1017.875	82447.96
325	105625	34328125	18.0278	6.8753	5859.0	3.1796	.00307692	1021.017	82957.68
326	106276	34645976	18.0555	6.8824	5886.1	3.1816	.00306748	1024.158	83468.97
327	106929	34965783	18.0831	6.8894	5913.2	3.1835	.00305810	1027.300	83981.84
328	107584	35287552	18.1108	6.8964	5940.3	3.1855	.00304878	1030.442	84496.28
329	108241	35611289	18.1384	6.9034	5967.5	3.1874	.00303951	1033.583	85012.28
330	**108900**	**35937000**	**18.1659**	**6.9104**	**5994.7**	**3.1894**	**.00303030**	**1036.725**	**85529.86**
331	109561	36264691	18.1934	6.9174	6022.0	3.1913	.00302115	1039.866	86049.01
332	110224	36594368	18.2209	6.9244	6049.3	3.1932	.00301205	1043.008	86569.73
333	110889	36926037	18.2483	6.9313	6076.7	3.1951	.00300300	1046.149	87092.02
334	111556	37259704	18.2757	6.9382	6104.1	3.1970	.00299401	1049.291	87615.88
335	112225	37595375	18.3030	6.9451	6131.5	3.1989	.00298507	1052.433	88141.31
336	112896	37933056	18.3303	6.9521	6159.0	3.2009	.00297619	1055.574	88668.31
337	113569	38272753	18.3576	6.9589	6186.5	3.2028	.00296736	1058.716	89196.88
338	114244	38614472	18.3848	6.9658	6214.1	3.2047	.00295858	1061.857	89727.03
339	114921	38958219	18.4120	6.9727	6241.7	3.2066	.00294985	1064.999	90258.74
340	**115600**	**39304000**	**18.4391**	**6.9795**	**6269.3**	**3.2085**	**.00294118**	**1068.141**	**90792.03**
341	116281	39651821	18.4662	6.9864	6297.0	3.2103	.00293255	1071.282	91326.88
342	116964	40001688	18.4932	6.9932	6324.7	3.2122	.00292398	1074.424	91863.31
343	117649	40353607	18.5203	7.0000	6352.4	3.2141	.00291545	1077.565	92401.31
344	118336	40707584	18.5472	7.0068	6380.2	3.2160	.00290698	1080.707	92940.88
345	119025	41063625	18.5742	7.0136	6408.1	3.2178	.00289855	1083.849	93482.02
346	119716	41421736	18.6011	7.0203	6436.0	3.2197	.00289017	1086.990	94024.73
347	120409	41781923	18.6279	7.0271	6463.9	3.2216	.00288184	1090.132	94569.01
348	121104	42144192	18.6548	7.0338	6491.9	3.2234	.00287356	1093.273	95114.86
349	121801	42508549	18.6815	7.0406	6519.9	3.2253	.00286533	1096.415	95662.28
350	**122500**	**42875000**	**18.7083**	**7.0473**	**6547.9**	**3.2271**	**.00285714**	**1099.557**	**96211.28**
351	123201	43243551	18.7350	7.0540	6576.0	3.2289	.00284900	1102.698	96761.84
352	123904	43614208	18.7617	7.0607	6604.1	3.2308	.00284091	1105.840	97313.97
353	124609	43986977	18.7883	7.0674	6632.3	3.2326	.00283286	1108.981	97867.68
354	125316	44361864	18.8149	7.0740	6660.5	3.2345	.00282486	1112.123	98422.96
355	126025	44738875	18.8414	7.0807	6688.7	3.2363	.00281690	1115.264	98979.80
356	126736	45118016	18.8680	7.0873	6717.0	3.2381	.00280899	1118.406	99538.22
357	127449	45499293	18.8944	7.0940	6745.3	3.2399	.00280112	1121.548	100098.21
358	128164	45882712	18.9209	7.1006	6773.7	3.2417	.00279330	1124.689	100659.77
359	128881	46268279	18.9473	7.1072	6802.1	3.2435	.00278552	1127.831	101222.90

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	(Circle N = Diam.) Circum.	Area
360	**129600**	**46656000**	**18.9737**	**7.1138**	**6830.5**	**3.2453**	**.00277778**	**1130.973**	**101787.60**
361	130321	47045881	19.0000	7.1204	6859.0	3.2471	.00277008	1134.114	102353.87
362	131044	47437928	19.0263	7.1269	6887.5	3.2489	.00276243	1137.256	102921.72
363	131769	47832147	19.0526	7.1335	6916.1	3.2507	.00275482	1140.397	103491.13
364	132496	48228544	19.0788	7.1400	6944.7	3.2525	.00274725	1143.539	104062.12
365	133225	48627125	19.1050	7.1466	6973.3	3.2543	.00273973	1146.680	104634.67
366	133956	49027896	19.1311	7.1531	7002.0	3.2561	.00273224	1149.822	105208.80
367	134689	49430863	19.1572	7.1596	7030.7	3.2579	.00272480	1152.964	105784.49
368	135424	49836032	19.1833	7.1661	7059.5	3.2597	.00271739	1156.105	106361.76
369	136161	50243409	19.2094	7.1726	7088.3	3.2614	.00271003	1159.247	106940.60
370	**136900**	**50653000**	**19.2354**	**7.1791**	**7117.1**	**3.2632**	**.00270270**	**1162.389**	**107521.01**
371	137641	51064811	19.2614	7.1855	7146.0	3.2650	.00269542	1165.530	108102.99
372	138384	51478848	19.2873	7.1920	7174.9	3.2668	.00268817	1168.671	108686.54
373	139129	51895117	19.3132	7.1984	7203.9	3.2685	.00268097	1171.813	109271.66
374	139876	52313624	19.3391	7.2048	7232.8	3.2702	.00267380	1174.955	109858.35
375	140625	52734375	19.3649	7.2112	7261.8	3.2719	.00266667	1178.096	110446.62
376	141376	53157376	19.3907	7.2177	7290.9	3.2737	.00265957	1181.238	111036.45
377	142129	53582633	19.4165	7.2240	7320.0	3.2754	.00265252	1184.379	111627.86
378	142884	54010152	19.4422	7.2304	7349.2	3.2772	.00264550	1187.521	112220.83
379	143641	54439939	19.4679	7.2368	7378.4	3.2789	.00263852	1190.663	112815.38
380	**144400**	**54872000**	**19.4936**	**7.2432**	**7407.6**	**3.2807**	**.00263158**	**1193.804**	**113411.49**
381	145161	55306341	19.5192	7.2495	7436.8	3.2824	.00262467	1196.946	114009.18
382	145924	55742968	19.5448	7.2558	7466.1	3.2841	.00261780	1200.087	114608.44
383	146689	56181887	19.5704	7.2622	7495.4	3.2858	.00261097	1203.229	115209.27
384	147456	56623104	19.5959	7.2685	7524.8	3.2875	.00260417	1206.371	115811.67
385	148225	57066625	19.6214	7.2748	7554.2	3.2892	.00259740	1209.512	116415.64
386	148996	57512456	19.6469	7.2811	7583.7	3.2909	.00259067	1212.654	117021.18
387	149769	57960603	19.6723	7.2874	7613.2	3.2926	.00258398	1215.795	117628.30
388	150544	58411072	19.6977	7.2936	7642.7	3.2943	.00257732	1218.937	118236.98
389	151321	58863869	19.7231	7.2999	7672.3	3.2960	.00257069	1222.079	118847.24
390	**152100**	**59319000**	**19.7484**	**7.3061**	**7701.9**	**3.2977**	**.00256410**	**1225.220**	**119459.06**
391	152881	59776471	19.7737	7.3124	7731.5	3.2994	.00255754	1228.362	120072.46
392	153664	60236288	19.7990	7.3186	7761.2	3.3011	.00255102	1231.503	120687.42
393	154449	60698457	19.8242	7.3248	7790.9	3.3028	.00254453	1234.645	121303.96
394	155236	61162984	19.8494	7.3310	7820.7	3.3045	.00253807	1237.786	121922.07
395	156025	61629875	19.8746	7.3372	7850.5	3.3061	.00253165	1240.928	122541.75
396	156816	62099136	19.8997	7.3434	7880.3	3.3078	.00252525	1244.070	123163.00
397	157609	62570773	19.9249	7.3496	7910.2	3.3095	.00251889	1247.211	123785.82
398	158404	63044792	19.9499	7.3558	7940.1	3.3111	.00251256	1250.353	124410.21
399	159201	63521199	19.9750	7.3619	7970.0	3.3128	.00250627	1253.494	125036.17
400	**160000**	**64000000**	**20.0000**	**7.3681**	**8000.0**	**3.3145**	**.00250000**	**1256.636**	**125663.71**
401	160801	64481201	20.0250	7.3742	8030.0	3.3161	.00249377	1259.778	126292.81
402	161604	64964808	20.0499	7.3803	8061.1	3.3178	.00248756	1262.919	126923.48
403	162409	65450827	20.0749	7.3864	8090.2	3.3194	.00248139	1266.061	127555.73
404	163216	65939264	20.0998	7.3925	8120.3	3.3211	.00247525	1269.202	128189.55
405	164025	66430125	20.1246	7.3986	8150.5	3.3227	.00246914	1272.344	128824.93
406	164836	66923416	20.1494	7.4047	8180.7	3.3243	.00246305	1275.486	129461.89
407	165649	67419143	20.1742	7.4108	8210.9	3.3260	.00245700	1278.627	130100.42
408	166464	67917312	20.1990	7.4169	8241.2	3.3276	.00245098	1281.769	130740.52
409	167281	68417929	20.2237	7.4229	8271.5	3.3292	.00244499	1284.910	131382.19
410	**168100**	**68921000**	**20.2485**	**7.4290**	**8301.9**	**3.3308**	**.00243902**	**1288.052**	**132025.43**
411	168921	69426531	20.2731	7.4350	8332.3	3.3325	.00243309	1291.193	132670.24
412	169744	69934528	20.2978	7.4410	8362.7	3.3341	.00242718	1294.335	133316.63
413	170569	70444997	20.3224	7.4470	8393.2	3.3357	.00242131	1297.477	133964.58
414	171396	70957944	20.3470	7.4530	8423.7	3.3373	.00241546	1300.618	134614.10
415	172225	71473375	20.3715	7.4590	8454.2	3.3390	.00240964	1303.760	135265.20
416	173056	71991296	20.3961	7.4650	8484.8	3.3406	.00240385	1306.901	135917.86
417	173889	72511713	20.4206	7.4710	8515.4	3.3422	.00239808	1310.043	136572.10
418	174724	73034632	20.4450	7.4770	8546.0	3.3438	.00239234	1313.185	137227.91
419	175561	73560059	20.4695	7.4829	8576.7	3.3454	.00238663	1316.326	137885.29
420	**176400**	**74088000**	**20.4939**	**7.4889**	**8607.4**	**3.3470**	**.00238095**	**1319.468**	**138544.24**
421	177241	74618461	20.5183	7.4948	8638.2	3.3485	.00237530	1322.609	139204.76
422	178084	75151448	20.5426	7.5007	8669.0	3.3501	.00236967	1325.751	139866.85
423	178929	75686967	20.5670	7.5067	8699.8	3.3517	.00236407	1328.893	140530.51
424	179776	76225024	20.5913	7.5126	8730.7	3.3533	.00235849	1332.034	141195.74
425	180625	76765625	20.6155	7.5185	8761.6	3.3549	.00235294	1335.176	141862.54
426	181476	77308776	20.6398	7.5244	8792.5	3.3564	.00234742	1338.317	142530.92
427	182329	77854483	20.6640	7.5302	8823.5	3.3580	.00234192	1341.459	143200.86
428	183184	78402752	20.6882	7.5361	8854.5	3.3596	.00233645	1344.601	143872.38
429	184041	78953589	20.7123	7.5420	8885.6	3.3612	.00233100	1347.742	144545.46

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
430	**184900**	**79507000**	**20.7364**	**7.5478**	**8916.7**	**3.3627**	**.00232558**	**1350.885**	**145220.12**
431	185761	80062991	20.7605	7.5537	8947.8	3.3643	.00232019	1354.025	145896.35
432	186624	80621568	20.7846	7.5595	8979.0	3.3659	.00231481	1357.167	146574.15
433	187489	81182737	20.8087	7.5654	9010.1	3.3674	.00230947	1360.308	147253.52
434	188356	81746504	20.8327	7.5712	9041.4	3.3690	.00230415	1363.450	147934.46
435	189225	82312875	20.8567	7.5770	9072.7	3.3705	.00229885	1366.592	148616.97
436	190096	82881856	20.8806	7.5828	9104.0	3.3720	.00229358	1369.733	149301.05
437	190969	83453453	20.9045	7.5886	9135.3	3.3736	.00228833	1372.875	149986.70
438	191844	84027672	20.9284	7.5944	9166.7	3.3752	.00228311	1376.016	150673.92
439	192721	84604519	20.9523	7.6001	9198.1	3.3767	.00227790	1379.158	151362.72
440	**193600**	**85184000**	**20.9762**	**7.6059**	**9229.5**	**3.3783**	**.00227273**	**1382.300**	**152053.08**
441	194481	85766121	21.0000	7.6117	9261.0	3.3798	.00226757	1385.441	152745.02
442	195364	86350888	21.0238	7.6174	9292.5	3.3813	.00226244	1388.583	153438.53
443	196249	86938307	21.0476	7.6232	9324.1	3.3828	.00225734	1391.724	154133.60
444	197136	87528384	21.0713	7.6289	9355.7	3.3844	.00225225	1394.866	154830.25
445	198025	88121125	21.0950	7.6346	9387.3	3.3859	.00224719	1398.008	155528.47
446	198916	88716536	21.1187	7.6403	9419.0	3.3874	.00224215	1401.149	156228.26
447	199809	89314623	21.1424	7.6460	9450.7	3.3889	.00223714	1404.291	156929.62
448	200704	89915392	21.1660	7.6517	9482.4	3.3904	.00223214	1407.432	157632.55
449	201601	90518849	21.1896	7.6574	9514.2	3.3919	.00222717	1410.574	158337.05
450	**202500**	**91125000**	**21.2132**	**7.6631**	**9546.0**	**3.3935**	**.00222222**	**1413.716**	**159043.13**
451	203401	91733851	21.2368	7.6688	9577.8	3.3950	.00221729	1416.857	159750.77
452	204304	92345408	21.2603	7.6744	9609.6	3.3965	.00221239	1419.999	160459.99
453	205209	92959677	21.2838	7.6801	9641.5	3.3980	.00220751	1423.140	161170.77
454	206116	93576664	21.3073	7.6857	9673.5	3.3995	.00220264	1426.282	161883.13
455	207025	94196375	21.3307	7.6914	9705.5	3.4010	.00219780	1429.423	162597.05
456	207936	94818816	21.3542	7.6970	9737.5	3.4025	.00219298	1432.565	163312.55
457	208849	95443993	21.3776	7.7026	9769.5	3.4039	.00218818	1435.707	164029.62
458	209764	96071912	21.4009	7.7082	9801.6	3.4054	.00218341	1438.848	164748.26
459	210681	96702579	21.4243	7.7138	9833.8	3.4069	.00217865	1441.990	165468.47
460	**211600**	**97336000**	**21.4476**	**7.7194**	**9865.9**	**3.4084**	**.00217391**	**1445.131**	**166190.25**
461	212521	97972181	21.4709	7.7250	9898.1	3.4199	.00216920	1448.273	166913.60
462	213444	98611128	21.4942	7.7306	9930.3	3.4113	.00216450	1451.415	167638.52
463	214369	99252847	21.5174	7.7362	9962.6	3.4128	.00215983	1454.556	168365.02
464	215296	99897344	21.5407	7.7418	9994.8	3.4143	.00215517	1457.698	169093.08
465	216225	100544625	21.5639	7.7473	10027.	3.4158	.00215054	1460.839	169822.72
466	217156	101194696	21.5870	7.7529	10060.	3.4173	.00214592	1463.981	170553.92
467	218089	101847563	21.6102	7.7584	10092.	3.4187	.00214133	1467.123	171286.70
468	219024	102503232	21.6333	7.7639	10124.	3.4202	.00213675	1470.264	172021.05
469	219961	103161709	21.6564	7.7695	10157.	3.4217	.00213220	1473.406	172756.96
470	**220900**	**103823000**	**21.6795**	**7.7750**	**10189.**	**3.4231**	**.00212766**	**1476.547**	**173494.45**
471	221841	104487111	21.7025	7.7805	10222.	3.4246	.00212314	1479.689	174233.51
472	222784	105154048	21.7256	7.7860	10255.	3.4260	.00211864	1482.830	174974.14
473	223729	105823817	21.7486	7.7915	10287.	3.4275	.00211416	1485.972	175716.34
474	224676	106496424	21.7715	7.7970	10320.	3.4289	.00210970	1489.114	176460.12
475	225625	107171875	21.7945	7.8025	10352.	3.4304	.00210526	1492.255	177205.46
476	226576	107850176	21.8174	7.8079	10385.	3.4318	.00210084	1495.397	177952.37
477	227529	108531333	21.8403	7.8134	10418.	3.4332	.00209644	1498.538	178700.86
478	228484	109215352	21.8632	7.8188	10450.	3.4347	.00209205	1501.680	179450.91
479	229441	109902239	21.8861	7.8243	10483.	3.4361	.00208768	1504.822	180202.54
480	**230400**	**110592000**	**21.9089**	**7.8297**	**10516.**	**3.4375**	**.00208333**	**1507.963**	**180955.74**
481	231361	111284641	21.9317	7.8352	10549.	3.4390	.00207900	1511.105	181710.50
482	232324	111980168	21.9545	7.8406	10582.	3.4404	.00207469	1514.246	182466.84
483	233289	112678587	21.9773	7.8460	10615.	3.4418	.00207039	1517.388	183224.75
484	234256	113379904	22.0000	7.8514	10648.	3.4433	.00206612	1520.530	183984.23
485	235225	114084125	22.0227	7.8568	10681.	3.4447	.00206186	1523.671	184745.28
486	236196	114791256	22.0454	7.8622	10714.	3.4461	.00205761	1526.813	185507.90
487	237169	115501303	22.0681	7.8676	10747.	3.4475	.00205339	1529.954	186272.10
488	238144	116214272	22.0907	7.8730	10780.	3.4489	.00204918	1533.096	187037.86
489	239121	116930169	22.1133	7.8784	10813.	3.4504	.00204499	1536.238	187805.19
490	**240100**	**117649000**	**22.1359**	**7.8837**	**10847.**	**3.4518**	**.00204082**	**1539.380**	**188574.10**
491	241081	118370771	22.1585	7.8891	10880.	3.4532	.00203666	1542.521	189344.57
492	242064	119095488	22.1811	7.8944	10913.	3.4546	.00203252	1545.662	190116.62
493	243049	119823157	22.2036	7.8998	10946.	3.4560	.00202840	1548.804	190890.24
494	244036	120553784	22.2261	7.9051	10980.	3.4574	.00202429	1551.945	191665.43
495	245025	121287375	22.2486	7.9105	11013.	3.4588	.00202020	1555.087	192442.18
496	246016	122023936	22.2711	7.9158	11046.	3.4602	.00201613	1558.229	193220.51
497	247009	122763473	22.2935	7.9211	11080.	3.4616	.00201207	1561.370	194000.41
498	248004	123505992	22.3159	7.9264	11113.	3.4630	.00200803	1564.512	194781.89
499	249001	124251499	22.3383	7.9317	11147.	3.4643	.00200401	1567.653	195564.93

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
500	**250000**	**125000000**	**22.3607**	**7.9370**	**11180**	**3.4657**	**.00200000**	**1570.796**	**196349.54**
501	251001	125751501	22.3830	7.9423	11214	3.4671	.00199601	1573.937	197135.72
502	252004	126506008	22.4054	7.9476	11247	3.4685	.00199203	1577.078	197923.48
503	253009	127263527	22.4277	7.9528	11281	3.4699	.00198807	1580.220	198712.88
504	254016	128024064	22.4499	7.9581	11315	3.4713	.00198413	1583.361	199503.70
505	255025	128787625	22.4722	7.9634	11348	3.4726	.00198020	1586.503	200296.17
506	256036	129554216	22.4944	7.9686	11382	3.4740	.00197628	1589.645	201090.20
507	257049	130323843	22.5167	7.9739	11416	3.4754	.00197239	1592.786	201885.81
508	258064	131096512	22.5389	7.9791	11450	3.4768	.00196850	1595.928	202682.99
509	259081	131872229	22.5610	7.9843	11484	3.4781	.00196464	1599.069	203481.74
510	**260100**	**132651000**	**22.5832**	**7.9896**	**11517**	**3.4795**	**.00196078**	**1602.211**	**204282.06**
511	261121	133432831	22.6053	7.9948	11551	3.4808	.00195695	1605.352	205083.95
512	262144	134217728	22.6274	8.0000	11585	3.4822	.00195313	1608.494	205887.42
513	263169	135005697	22.6495	8.0052	11619	3.4836	.00194932	1611.636	206692.45
514	264196	135796744	22.6716	8.0104	11653	3.4849	.00194553	1614.777	207499.05
515	265225	136590875	22.6936	8.0156	11687	3.4863	.00194175	1617.919	208307.23
516	266256	137388096	22.7156	8.0208	11721	3.4876	.00193798	1621.060	209116.97
517	267289	138188413	22.7376	8.0260	11755	3.4890	.00193424	1624.202	209928.29
518	268324	138991832	22.7596	8.0311	11789	3.4904	.00193050	1627.344	210741.16
519	269361	139798359	22.7816	8.0363	11824	3.4917	.00192678	1630.485	211555.63
520	**270400**	**140608000**	**22.8035**	**8.0415**	**11858**	**3.4930**	**.00192308**	**1633.627**	**212371.66**
521	271441	141420761	22.8254	8.0466	11892	3.4944	.00191939	1636.768	213189.26
522	272484	142236648	22.8473	8.0517	11926	3.4957	.00191571	1639.910	214008.43
523	273529	143055667	22.8692	8.0569	11960	3.4970	.00191205	1643.052	214829.17
524	274576	143877824	22.8910	8.0620	11995	3.4984	.00190840	1646.193	215651.49
525	275625	144703125	22.9129	8.0671	12029	3.4997	.00190476	1649.335	216475.37
526	276676	145531576	22.9347	8.0723	12064	3.5010	.00190114	1652.476	217300.82
527	277729	146363183	22.9565	8.0774	12098	3.5024	.00189753	1655.618	218127.85
528	278784	147197952	22.9783	8.0825	12133	3.5037	.00189394	1668.760	218956.44
529	279841	148035889	23.0000	8.0876	12167	3.5050	.00189036	1661.901	219786.61
530	**280900**	**148877000**	**23.0217**	**8.0927**	**12202**	**3.5064**	**.00188679**	**1665.043**	**220618.34**
531	281961	149721291	23.0434	8.0978	12236	3.5077	.00188324	1668.184	221451.65
532	283024	150568768	23.0651	8.1028	12271	3.5090	.00187970	1671.326	222286.53
533	284089	151419437	23.0868	8.1079	12305	3.5103	.00187617	1674.467	223122.98
534	285156	152273304	23.1084	8.1130	12340	3.5116	.00187266	1677.609	223961.00
535	286225	153130375	23.1301	8.1180	12375	3.5130	.00186916	1680.751	224800.59
536	287296	153990656	23.1517	8.1231	12410	3.5143	.00186567	1683.892	225641.75
537	288369	154854153	23.1733	8.1281	12444	3.5156	.00186220	1687.034	226484.48
538	289444	155720872	23.1948	8.1332	12479	3.5169	.00185874	1690.175	227328.79
539	290521	156590819	23.2164	8.1382	12514	3.5182	.00185529	1693.317	228174.66
540	**291600**	**157464000**	**23.2379**	**8.1433**	**12549**	**3.5195**	**.00185185**	**1696.460**	**229022.10**
541	292681	158340421	23.2594	8.1483	12583	3.5208	.00184843	1699.600	229871.12
542	293764	159220088	23.2809	8.1533	12618	3.5221	.00184502	1702.742	230721.71
543	294849	160103007	23.3024	8.1583	12653	3.5234	.00184162	1705.883	231573.86
544	295936	160989184	23.3238	8.1633	12688	3.5247	.00183824	1709.025	232427.59
545	297025	161878625	23.3452	8.1683	12723	3.5260	.00183486	1712.167	233282.89
546	298116	162771336	23.3666	8.1733	12758	3.5273	.00183150	1715.308	234139.76
547	299209	163667323	23.3880	8.1783	12793	3.5286	.00182815	1718.450	234998.20
548	300304	164566592	23.4094	8.1833	12828	3.5299	.00182482	1721.591	235858.21
549	301401	165469149	23.4307	8.1882	12863	3.5311	.00182149	1724.733	236719.79
550	**302500**	**166375000**	**23.4521**	**8.1932**	**12899**	**3.5324**	**.00181818**	**1727.876**	**237582.94**
551	303601	167284151	23.4734	8.1982	12934	3.5337	.00181488	1731.016	238447.67
552	304704	168196608	23.4947	8.2031	12969	3.5350	.00181159	1734.158	239313.96
553	305809	169112377	23.5160	8.2081	13004	3.5363	.00180832	1737.299	240181.83
554	306916	170031464	23.5372	8.2130	13040	3.5376	.00180505	1740.441	241051.26
555	308025	170953875	23.5584	8.2180	13075	3.5388	.00180180	1743.582	241922.27
556	309136	171879616	23.5797	8.2229	13110	3.5401	.00179856	1746.724	242794.85
557	310249	172808693	23.6008	8.2278	13146	3.5414	.00179533	1749.866	243668.99
558	311364	173741112	23.6220	8.2327	13181	3.5426	.00179211	1753.007	244544.71
559	312481	174676879	23.6432	8.2377	13217	3.5439	.00178891	1756.149	245422.00
560	**313600**	**175616000**	**23.6643**	**8.2426**	**13252**	**3.5451**	**.00178571**	**1759.290**	**246300.86**
561	314721	176558481	23.6854	8.2475	13288	3.5464	.00178253	1762.432	247181.38
562	315844	177504328	23.7065	8.2524	13323	3.5477	.00177936	1765.574	248063.30
563	316969	178453547	23.7276	8.2573	13359	3.5490	.00177620	1768.715	248946.87
564	318096	179406144	23.7487	8.2621	13394	3.5502	.00177305	1771.857	249832.01
565	319225	180362125	23.7697	8.2670	13430	3.5515	.00176991	1774.998	250718.73
566	320356	181321496	23.7908	8.2719	13466	3.5527	.00176678	1778.140	251607.01
567	321489	182284263	23.8118	8.2768	13501	3.5540	.00176367	1781.282	252496.87
568	322624	183250432	23.8328	8.2816	13537	3.5553	.00176056	1784.423	253388.30
569	323761	184220009	23.8537	8.2865	13573	3.5565	.00175747	1787.565	254281.29

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
570	**324900**	**185193000**	**23.8747**	**8.2913**	**13609**	**3.5577**	**.00175439**	**1790.708**	**255175.86**
571	326041	186169411	23.8956	8.2962	13644	3.5590	.00175131	1793.848	256072.00
572	327184	187149248	23.9165	8.3010	13680	3.5602	.00174825	1796.989	256969.71
573	328329	188132517	23.9374	8.3059	13716	3.5615	.00174520	1800.131	257868.99
574	329476	189119224	23.9583	8.3107	13752	3.5627	.00174216	1803.273	258769.85
575	330625	190109375	23.9792	8.3155	13788	3.5640	.00173913	1806.414	259672.27
576	331776	191102976	24.0000	8.3203	13824	3.5652	.00173611	1809.556	260576.26
577	332929	192100033	24.0208	8.3251	13860	3.5664	.00173310	1812.697	261481.83
578	334084	193100552	24.0416	8.3300	13896	3.5677	.00173010	1815.839	262388.96
579	335241	194104539	24.0624	8.3348	13932	3.5689	.00172712	1818.981	263297.67
580	**336400**	**195112000**	**24.0832**	**8.3396**	**13968**	**3.5702**	**.00172414**	**1822.123**	**264207.94**
581	337561	196122941	24.1039	8.3443	14004	3.5714	.00172117	1825.264	265119.79
582	338724	197137368	24.1247	8.3491	14040	3.5726	.00171821	1828.405	266033.21
583	339889	198155287	24.1454	8.3539	14077	3.5738	.00171527	1831.547	266948.20
584	341056	199176704	24.1661	8.3587	14113	3.5751	.00171233	1834.689	267864.76
585	342225	200201625	24.1868	8.3634	14149	3.5763	.00170940	1837.830	268782.89
586	343396	201230056	24.2074	8.3682	14186	3.5775	.00170648	1840.972	269702.59
587	344569	202262003	24.2281	8.3730	14222	3.5787	.00170358	1844.113	270623.86
588	345744	203297472	24.2487	8.3777	14258	3.5799	.00170068	1847.255	271546.70
589	346921	204336469	24.2693	8.3825	14295	3.5812	.00169779	1850.397	272471.12
590	**348100**	**205379000**	**24.2899**	**8.3872**	**14331**	**3.5824**	**.00169492**	**1853.538**	**273397.10**
591	349281	206425071	24.3105	8.3919	14368	3.5836	.00169205	1856.680	274324.66
592	350464	207474688	24.3311	8.3967	14404	3.5848	.00168919	1859.821	275253.78
593	351649	208527857	24.3516	8.4014	14440	3.5860	.00168634	1862.963	276184.48
594	352836	209584584	24.3721	8.4061	14477	3.5872	.00168350	1866.104	277116.75
595	354025	210644875	24.3926	8.4108	14514	3.5884	.00168067	1869.246	278050.58
596	355216	211708736	24.4131	8.4155	14550	3.5896	.00167785	1872.388	278985.99
597	356409	212776173	24.4336	8.4202	14587	3.5908	.00167504	1875.529	279922.97
598	357604	213847192	24.4540	8.4249	14624	3.5920	.00167224	1878.671	280861.52
599	358801	214921799	24.4745	8.4296	14660	3.5932	.00166945	1881.812	281801.65
600	**360000**	**216000000**	**24.4949**	**8.4343**	**14697**	**3.5944**	**.00166667**	**1884.956**	**282743.34**
601	361201	217081801	24.5153	8.4390	14734	3.5956	.00166389	1888.096	283686.60
602	362404	218167208	24.5357	8.4437	14770	3.5958	.00166113	1891.237	284631.44
603	363609	219256227	24.5561	8.4484	14807	3.5980	.00165837	1894.379	285577.84
604	364816	220348864	24.5764	8.4530	14844	3.5992	.00165563	1897.520	286525.82
605	366025	221445125	24.5967	8.4577	14881	3.6004	.00165289	1900.662	287475.36
606	367236	222545016	24.6171	8.4623	14918	3.6016	.00165017	1903.804	288426.48
607	368449	223648543	24.6374	8.4670	14955	3.6028	.00164745	1906.945	289379.17
608	369664	224755712	24.6577	8.4716	14992	3.6040	.00164474	1910.087	290333.43
609	370881	225866529	24.6779	8.4763	15029	3.6052	.00164204	1913.228	291289.26
610	**372100**	**226981000**	**24.6982**	**8.4809**	**15066**	**3.6063**	**.00163934**	**1916.370**	**292246.66**
611	373321	228099131	24.7184	8.4856	15103	3.6075	.00163666	1919.511	293205.63
612	374544	229220928	24.7386	8.4902	15140	3.6087	.00163399	1922.653	294166.17
613	375769	230346397	24.7588	8.4948	15177	3.6099	.00163132	1925.795	295128.28
614	376996	231475544	24.7790	8.4994	15214	3.6111	.00162866	1928.936	296091.97
615	378225	232608375	24.7992	8.5040	15252	3.6122	.00162602	1932.078	297057.22
616	379456	233744896	24.8193	8.5086	15289	3.6134	.00162338	1935.219	298024.05
617	380689	234885113	24.8395	8.5132	15326	3.6146	.00162075	1938.361	298992.44
618	381924	236029032	24.8596	8.5178	15363	3.6158	.00161812	1941.503	299962.41
619	383161	237176659	24.8797	8.5224	15400	3.6169	.00161551	1944.644	300933.95
620	**384400**	**238328000**	**24.8998**	**8.5270**	**15437**	**3.6181**	**.00161290**	**1947.786**	**301907.05**
621	385641	239483061	24.9199	8.5316	15475	3.6192	.00161031	1950.927	302881.73
622	386884	240641848	24.9399	8.5362	15513	3.6204	.00160772	1954.069	303857.98
623	388129	241804367	24.9600	8.5408	15550	3.6216	.00160514	1957.211	304835.80
624	389376	242970624	24.9800	8.5453	15588	3.6227	.00160256	1960.352	305815.20
625	390625	244140625	25.0000	8.5499	15625	3.6239	.00160000	1963.494	306796.16
626	391876	245314376	25.0200	8.5544	15663	3.6250	.00159744	1966.635	307778.69
627	393129	246491883	25.0400	8.5590	15700	3.6262	.00159490	1969.777	308762.79
628	394384	247673152	25.0599	8.5635	15738	3.6274	.00159236	1972.919	309748.47
629	395641	248858189	25.0799	8.5681	15775	3.6285	.00158983	1976.060	310735.71
630	**396900**	**250047000**	**25.0998**	**8.5726**	**15812**	**3.6297**	**.00158730**	**1979.203**	**311724.53**
631	398161	251239591	25.1197	8.5772	15850	3.6309	.00158479	1982.343	312714.92
632	399424	252435968	25.1396	8.5817	15888	3.6320	.00158228	1985.485	313706.88
633	400689	253636137	25.1595	8.5862	15926	3.6331	.00157978	1988.626	314700.40
634	401956	254840104	25.1794	8.5907	15964	3.6343	.00157729	1991.768	315695.50
635	403225	256047875	25.1992	8.5952	16002	3.6354	.00157480	1994.910	316692.17
636	404496	257259456	25.2190	8.5997	16040	3.6366	.00157233	1998.051	317690.42
637	405769	258474853	25.2389	8.6043	16077	3.6377	.00156986	2001.193	318690.23
638	407044	259694072	25.2587	8.6088	16115	3.6389	.00156740	2004.334	319691.61
639	408321	260917119	25.2784	8.6132	16153	3.6400	.00156495	2007.476	320694.56

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
640	**409600**	**262144000**	**25.2982**	**8.6177**	**16191**	**3.6411**	**.00156250**	**2010.619**	**321699.09**
641	410881	263374721	25.3180	8.6222	16229	3.6423	.00156006	2013.759	322705.18
642	412164	264609288	25.3377	8.6267	16267	3.6435	.00155763	2016.901	323712.85
643	413449	265847707	25.3574	8.6312	16305	3.6446	.00155521	2020.042	324722.09
644	414736	267089984	25.3772	8.6357	16343	3.6457	.00155280	2023.184	325732.89
645	416025	268336125	25.3969	8.6401	16381	3.6468	.00155039	2026.326	326745.27
646	417316	269586136	25.4165	8.6446	16419	3.6479	.00154799	2029.467	327759.22
647	418609	270840023	25.4362	8.6490	16457	3.6499	.00154560	2032.609	328774.74
648	419904	272097792	25.4558	8.6535	16495	3 6502	.00154321	2035.750	329791.83
649	421201	273359449	25.4755	8.6579	16534	3.6513	.00154083	2038.892	330810.49
650	**422500**	**274625000**	**25.4951**	**8.6624**	**16572**	**3.6524**	**.00153846**	**2042.034**	**331830.72**
651	423801	275894451	25.5147	8.6668	16610	3.6536	.00153610	2045.175	332852.53
652	425104	277167808	25.5343	8.6713	16648	3.6547	.00153374	2048.317	333875.90
653	426409	278445077	25.5539	8.6757	16687	3.6558	.00153139	2051.458	334900.85
654	427716	279726264	25.5734	8.6801	16725	3.6569	.00152905	2054.600	335927.36
655	429025	281011375	25.5930	8.6845	16764	3.6580	.00152672	2057.741	336955.45
656	430336	282300416	25.6125	8.6890	16802	3.6592	.00152439	2060.883	337985.10
657	431649	283593393	25.6320	8.6934	16840	3.6603	.00152207	2064.025	339016.33
658	432964	284890312	25.6515	8.6978	16879	3.6614	.00151976	2067.166	340049.13
659	434281	286191179	25.6710	8.7022	16917	3.6625	.00151745	2070.308	341083.50
660	**435600**	**287496000**	**25.6905**	**8.7066**	**16956**	**3.6636**	**.00151515**	**2073.449**	**342119.44**
661	436921	288804781	25.7099	8.7110	16994	3.6647	.00151286	2076.591	343156.95
662	438244	290117528	25.7294	8.7154	17033	3.6658	.00151057	2079.733	344196.03
663	439569	291434247	25.7488	8.7198	17071	3.6669	.00150830	2082.874	345236.69
664	440896	292754944	25.7682	8.7241	17110	3.6680	.00150602	2086.016	346278.91
665	442225	294079625	25.7876	8.7285	17149	3.6691	.00150376	2089.157	347322.70
666	443556	295408296	25.8070	8.7329	17187	3.6702	.00150150	2092.299	348368.07
667	444889	296740963	25.8263	8.7373	17226	3.6713	.00149925	2095.441	349415.00
668	446224	298077632	25.8457	8.7416	17265	3.6724	.00149701	2098.582	350463.51
669	447561	299418309	25.8650	8.7460	17304	3.6735	.00149477	2101.724	351513.59
670	**448900**	**300763000**	**25.8844**	**8.7502**	**17343**	**3.6746**	**.00149254**	**2104.865**	**352565.24**
671	450241	302111711	25.9037	8.7547	17381	3.6757	.00149031	2108.007	353618.45
672	451584	303464448	25.9230	8.7590	17420	3.6768	.00148810	2111.148	354673.24
673	452929	304821217	25.9422	8.7634	17459	3.6779	.00148588	2114.290	355729.60
674	454276	306182024	25.9615	8.7677	17498	3.6790	.00148368	2117.432	356787.54
675	455625	307546875	25.9808	8.7721	17537	3.6801	.00148148	2120.573	357847.04
676	456976	308915776	26.0000	8.7764	17576	3.6812	.00147929	2123.715	358908.11
677	458329	310288733	26.0192	8.7807	17615	3.6823	.00147710	2126.856	359970.75
678	459684	311665752	26.0384	8.7850	17654	3.6834	.00147493	2129.998	361034.97
679	461041	313046839	26.0576	8.7893	17693	3.6845	.00147275	2133.140	362100.75
680	**462400**	**314432000**	**26.0768**	**8.7937**	**17732**	**3.6856**	**.00147059**	**2136.281**	**363168.11**
681	463761	315821241	26.0960	8.7980	17771	3.6866	.00146843	2139.423	364237.04
682	465124	317214568	26.1151	8.8023	17810	3.6877	.00146628	2142.564	365307.54
683	466489	318611987	26.1343	8.8066	17850	3.6888	.00146413	2145.706	366379.60
684	467856	320013504	26.1534	8.8109	17889	3.6899	.00146199	2148.848	367453.24
685	469225	321419125	26.1725	8.8152	17928	3.6909	.00145985	2151.989	368528.45
686	470596	322828856	26.1916	8.8194	17967	3.6920	.00145773	2155.131	369605.23
687	471969	324242703	26.2107	8.8237	18007	3.6931	.00145560	2158.272	370683.59
688	473344	325660672	26.2298	8.8280	18046	3.6942	.00145349	2161.414	371763.51
689	474721	327082769	26.2488	8.8323	18085	3.6953	.00145138	2164.556	372845.00
690	**476100**	**328509000**	**26.2679**	**8.8366**	**18125**	**3.6963**	**.00144928**	**2167.697**	**373928.07**
691	477481	329939371	26.2869	8.8408	18164	3.6974	.00144718	2170.839	375012.70
692	478864	331373888	26.3059	8.8451	18204	3.6985	.00144509	2173.980	376098.91
693	480249	332812557	26.3249	8.8493	18243	3.6995	.00144300	2177.122	377186.68
694	481636	334255384	26.3439	8.8536	18283	3.7006	.00144092	2180.263	378276.03
695	483025	335702375	26.3629	8.8578	18322	3.7016	.00143885	2183.405	379366.95
696	484416	337153536	26.3818	8.8621	18362	3.7027	.00143678	2186.547	380459.44
697	485809	338608873	26.4008	8.8663	18401	3.7038	.00143472	2189.688	381553.50
698	487204	340068392	26.4197	8.8706	18441	3.7049	.00143266	2192.830	382649.13
699	488601	341532099	26.4386	8.8748	18480	3.7059	.00143062	2195.971	383746.33
700	**490000**	**343000000**	**26.4575**	**8.8790**	**18520**	**3.7070**	**.00142857**	**2199.115**	**384845.10**
701	491401	344472101	26.4764	8.8833	18560	3.7080	.00142653	2202.255	385945.44
702	492804	345948408	26.4953	8.8875	18600	3.7091	.00142450	2205.396	387047.36
703	494209	347428927	26.5141	8.8917	18640	3.7101	.00142248	2208.538	388150.84
704	495616	348913664	26.5330	8.8959	18679	3.7112	.00142045	2211.679	389255.90
705	497025	350402625	26.5518	8.9001	18719	3.7123	.00141844	2214.821	390362.52
706	498436	351895816	26.5707	8.9043	18759	3.7133	.00141643	2217.963	391470.72
707	499849	353393243	26.5895	8.9085	18799	3.7144	.00141443	2221.104	392580.49
708	501264	354894912	26.6083	8.9127	18839	3.7154	.00141243	2224.246	393691.82
709	502681	356400829	26.6271	8.9169	18879	3.7165	.00141044	2227.387	394804.73

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Circle (N = Diam.) Area
710	**504100**	**357911000**	**26.6458**	**8.9211**	**18919**	**3.7175**	**.00140845**	**2230.529**	**395919.21**
711	505521	359425431	26.6646	8.9253	18959	3.7185	.00140647	2233.670	397035.26
712	506944	360944128	26.6833	8.9295	18999	3.7196	.00140449	2236.812	398152.89
713	508369	362467097	26.7021	8.9337	19039	3.7206	.00140252	2239.954	399272.08
714	509796	363994344	26.7208	8.9378	19079	3.7217	.00140056	2243.095	400392.84
715	511225	365525875	26.7395	8.9420	19119	3.7227	.00139860	2246.237	401515.18
716	512656	367061696	26.7582	8.9462	19159	3.7238	.00139665	2249.378	402639.08
717	514089	368601813	26.7769	8.9503	19199	3.7248	.00139470	2252.520	403764.56
718	515524	370146232	26.7955	8.9545	19239	3.7258	.00139276	2255.662	404891.60
719	516961	371694959	26.8142	8.9587	19280	3.7269	.00139082	2258.803	406020.22
720	**518400**	**373248000**	**26.8328**	**8.9628**	**19320**	**3.7279**	**.00138889**	**2261.946**	**407150.41**
721	519841	374805361	26.8514	8.9670	19360	3.7290	.00138696	2265.086	408282.17
722	521284	376367048	26.8701	8.9711	19400	3.7300	.00138504	2268.228	409415.50
723	522729	377933067	26.8887	8.9752	19440	3.7310	.00138313	2271.370	410550.40
724	524176	379503424	26.9072	8.9794	19481	3.7321	.00138122	2274.511	411686.87
725	525625	381078125	26.9258	8.9835	19521	3.7331	.00137931	2277.653	412824.91
726	527076	382657176	26.9444	8.9876	19562	3.7341	.00137741	2280.794	413964.52
727	528529	384240583	26.9629	8.9918	19602	3.7351	.00137552	2283.936	415105.71
728	529984	385828352	26.9815	8.9959	19643	3.7362	.00137363	2287.078	416248.46
729	531441	387420489	27.0000	9.0000	19683	3.7372	.00137174	2290.219	417392.79
730	**532900**	**389017000**	**27.0185**	**9.0041**	**19724**	**3.7382**	**.00136986**	**2293.361**	**418538.68**
731	534361	390617891	27.0370	9.0082	19764	3.7392	.00136799	2296.502	419686.15
732	535824	392223168	27.0555	9.0123	19805	3.7403	.00136612	2299.644	420835.19
733	537289	393832837	27.0740	9.0164	19845	3.7413	.00136426	2302.785	421985.79
734	538756	395446904	27.0924	9.0205	19886	3.7423	.00136240	2305.927	423137.97
735	540225	397065375	27.1109	9.0246	19927	3.7433	.00136054	2309.069	424291.72
736	541696	398688256	27.1293	9.0287	19967	3.7443	.00135870	2312.210	425447.04
737	543169	400315553	27.1477	9.0328	20008	3.7454	.00135685	2315.352	426603.94
738	544644	401947272	27.1662	9.0369	20049	3.7464	00135501	2318.493	427762.40
739	546121	403583419	27.1846	9.0410	20090	3.7474	.00135318	2321.635	428922.43
740	**547600**	**405224000**	**27.2029**	**9.0450**	**20130**	**3.7484**	**.00135135**	**2324.777**	**430084.03**
741	549081	406869021	27.2213	9.0491	20171	3.7494	00134953	2327.918	431247.21
742	550564	408518488	27.2397	9.0532	20212	3.7504	.00134771	2331.060	432411.95
743	552049	410172407	27.2580	9.0572	20253	3.7514	.00134590	2334.201	433578.27
744	553536	411830784	27.2764	9.0613	20294	3.7524	.00134409	2337.343	434746.16
745	555025	413493625	27.2947	9.0654	20335	3.7534	.00134228	2340.485	435915.62
746	556516	415160936	27.3130	9.0694	20376	3.7545	.00134048	2343.626	437086.64
747	558009	416832723	27.3313	9.0735	20417	3.7555	.00133869	2346.768	438259.24
748	559504	418508992	27.3496	9.0775	20458	3.7565	.00133690	2349.909	439433.41
749	561001	420189749	27.3679	9.0816	20499	3.7575	.00133511	2353.051	440609.16
750	**562500**	**421875000**	**27.3861**	**9.0856**	**20540**	**3.7585**	**.00133333**	**2356.193**	**441786.47**
751	564001	423564751	27.4044	9.0896	20581	3.7595	.00133156	2359.334	442965.35
752	565504	425259008	27.4226	9.0937	20622	3.7605	.00132979	2362.476	444145.80
753	567009	426957777	27.4408	9.0977	20663	3.7615	.00132802	2365.617	445327.83
754	568516	428661064	27.4591	9.1017	20704	3.7625	.00132626	2368.759	446511.42
755	570025	430368875	27.4773	9.1057	20745	3.7635	.00132450	2371.900	447696.59
756	571536	432081216	27.4955	9.1098	20787	3.7645	.00132275	2375.042	448883.32
757	573049	433798093	27.5136	9.1138	20828	3.7655	.00132100	2378.184	450071.63
758	574564	435519512	27.5318	9.1178	20869	3.7665	.00131926	2381.325	451261.51
759	576081	437245479	27.5500	9.1218	20910	3.7675	.00131752	2384.467	452452.96
760	**577600**	**438976000**	**27.5681**	**9.1258**	**20952**	**3.7685**	**.00131579**	**2387.608**	**453645.98**
761	579121	440711081	27.5862	9.1298	20993	3.7694	.00131406	2390.750	454840.57
762	580644	442450728	27.6043	9.1338	21035	3.7704	.00131234	2393.892	456036.73
763	582169	444194947	27.6225	9.1378	21076	3.7714	.00131062	2397.033	457234.46
764	583696	445943744	27.6405	9.1418	21117	3.7724	.00130890	2400.175	458433.77
765	585225	447697125	27.6586	9.1458	21159	3.7734	.00130719	2403.316	459634.64
766	586756	449455096	27.6767	9.1498	21200	3.7744	.00130548	2406.458	460837.08
767	588289	451217663	27.6948	9.1537	21242	3.7754	.00130378	2409.600	462041.10
768	589824	452984832	27.7128	9.1577	21283	3.7764	.00130208	2412.741	463246.69
769	591361	454756609	27.7308	9.1617	21325	3.7774	.00130039	2415.883	464453.84
770	**592900**	**456533000**	**27.7489**	**9.1657**	**21367**	**3.7784**	**.00129870**	**2419.026**	**465662.57**
771	594441	458314011	27.7669	9.1696	21408	3.7793	.00129702	2422.166	466872.87
772	595984	460099648	27.7849	9.1736	21450	3.7803	.00129534	2425.307	468084.74
773	597529	461889917	27.8029	9.1775	21492	3.7813	.00129366	2428.449	469298.18
774	599076	463684824	27.8209	9.1815	21533	3.7822	.00129199	2431.591	470513.19
775	600625	465484375	27.8388	9.1855	21575	3.7832	.00129032	2434.732	471729.77
776	602176	467288576	27.8568	9.1894	21617	3.7842	.00128866	2437.874	472947.92
777	603729	469097433	27.8747	9.1933	21658	3.7852	.00128700	2441.015	474167.65
778	605284	470910952	27.8927	9.1973	21700	3.7861	.00128535	2444.157	475388.94
779	606841	472729139	27.9106	9.2012	21742	3.7871	.00128370	2447.299	476611.81

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
780	**608400**	**474552000**	**27.9285**	**9.2052**	**21784**	**3.7881**	**.00128205**	**2450.440**	**477836.84**
781	609961	476379541	27.9464	9.2091	21826	3.7890	.00128041	2453.582	479062.25
782	611524	478211768	27.9643	9.2130	21868	3.7900	.00127877	2456.723	480289.83
783	613089	480048687	27.9821	9.2170	21910	3.7910	.00127714	2459.865	481518.97
784	614656	481890304	28.0000	9.2209	21952	3.7920	.00127551	2463.007	482749.69
785	616225	483736625	28.0179	9.2248	21994	3.7929	.00127389	2466.148	483981.98
786	617796	485587656	28.0357	9.2287	22036	3.7939	.00127226	2469.290	485215.84
787	619369	487443403	28.0535	9.2326	22078	3.7949	.00127065	2472.431	486451.28
788	620944	489303872	28.0713	9.2365	22120	3.7959	.00126904	2475.573	487688.28
789	622521	491169069	28.0891	9.2404	22162	3.7969	.00126743	2478.715	488926.85
790	**624100**	**493039000**	**28.1069**	**9.2443**	**22205**	**3.7978**	**.00126582**	**2481.856**	**490166.99**
791	625681	494913671	28.1247	9.2482	22247	3.7987	.00126422	2484.998	491408.71
792	627264	496793088	28.1425	9.2521	22289	3.7997	.00126263	2488.139	492651.99
793	628849	498677257	28.1603	9.2560	22331	3.8006	.00126103	2491.281	493896.85
794	630436	500566184	28.1780	9.2599	22373	3.8016	.00125945	2494.422	495143.28
795	632025	502459875	28.1957	9.2638	22416	3.8025	.00125786	2497.564	496391.27
796	633616	504358336	28.2135	9.2677	22458	3.8035	.00125628	2500.706	497640.84
797	635209	506261573	28.2312	9.2716	22500	3.8044	.00125471	2503.847	498891.98
798	636804	508169592	28.2489	9.2754	22543	3.8054	.00125313	2506.989	500144.69
799	638401	510082399	28.2666	9.2793	22585	3.8064	.00125156	2510.130	501398.97
800	**640000**	**512000000**	**28.2843**	**9.2832**	**22627**	**3.8073**	**.00125000**	**2513.272**	**502654.82**
801	641601	513922401	28.3019	9.2870	22670	3.8083	.00124844	2516.414	503912.25
802	643204	515849608	28.3196	9.2909	22712	3.8092	.00124688	2519.555	505171.24
803	644809	517781627	28.3373	9.2948	22755	3.8102	.00124533	2522.697	506431.80
804	646416	519718464	28.3549	9.2986	22797	3.8111	.00124378	2525.838	507693.94
805	648025	521660125	28.3725	9.3025	22840	3.8121	.00124224	2528.980	508957.64
806	649636	523606616	28.3901	9.3063	22883	3.8130	.00124069	2532.122	510222.92
807	651249	525557943	28.4077	9.3102	22925	3.8139	.00123916	2535.263	511489.77
808	652864	527514112	28.4253	9.3140	22968	3.8149	.00123762	2538.405	512758.19
809	654481	529475129	28.4429	9.3179	23010	3.8158	.00123609	2541.546	514028.18
810	**656100**	**531441000**	**28.4605**	**9.3217**	**23053**	**3.8168**	**.00123457**	**2544.688**	**515299.74**
811	657721	533411731	28.4781	9.3255	23096	3.8177	.00123305	2547.829	516572.87
812	659344	535387328	28.4956	9.3294	23138	3.8186	.00123153	2550.971	517847.57
813	660969	537367797	28.5132	9.3332	23181	3.8196	.00123001	2554.113	519123.84
814	662596	539353144	28.5307	9.3370	23224	3.8205	.00122850	2557.254	520401.68
815	664225	541343375	28.5482	9.3408	23267	3.8215	.00122699	2560.396	521681.10
816	665856	543338496	28.5657	9.3447	23310	3.8224	.00122549	2563.537	522962.08
817	667489	545338513	28.5832	9.3485	23352	3.8234	.00122399	2566.679	524244.63
818	669124	547343432	28.6007	9.3523	23395	3.8243	.00122249	2569.821	525528.76
819	670761	549353259	28.6182	9.3561	23438	3.8252	.00122100	2572.962	526814.46
820	**672400**	**551368000**	**28.6356**	**9.3599**	**23481**	**3.8262**	**.00121951**	**2576.104**	**528101.78**
821	674041	553387661	28.6531	9.3637	23524	3.8271	.00121803	2579.245	529390.56
822	675684	555412248	28.6705	9.3675	23567	3.8280	.00121655	2582.387	530680.97
823	677329	557441767	28.6880	9.3713	23610	3.8290	.00121507	2585.529	531972.95
824	678976	559476224	28.7054	9.3751	23653	3.8299	.00121359	2588.670	533266.50
825	680625	561515625	28.7228	9.3789	23696	3.8308	.00121212	2591.812	534561.62
826	682276	563559976	28.7402	9.3827	23740	3.8317	.00121065	2594.953	535858.32
827	683929	565609283	28.7576	9.3865	23783	3.8327	.00120919	2598.095	537156.58
828	685584	567663552	28.7750	9.3902	23826	3.8336	.00120773	2601.237	538456.41
829	687241	569722789	28.7924	9.3940	23869	3.8345	.00120627	2604.378	539757.82
830	**688900**	**571787000**	**28.8097**	**9.3978**	**23912**	**3.8355**	**.00120482**	**2607.520**	**541060.79**
831	690561	573856191	28.8271	9.4016	23955	3.8364	.00120337	2610.661	542365.34
832	692224	575930368	28.8444	9.4053	23999	3.8373	.00120192	2613.803	543671.46
833	693889	578009537	28.8617	9.4091	24042	3.8382	.00120048	2616.944	544979.15
834	695556	580093704	28.8791	9.4129	24085	3.8391	.00119904	2620.086	546288.40
835	697225	582182875	28.8964	9.4166	24128	3.8401	.00119760	2623.228	547599.23
836	698896	584277056	28.9137	9.4204	24172	3.8410	.00119617	2626.369	548911.63
837	700569	586376253	28.9310	9.4241	24215	3.8419	.00119474	2629.511	550225.61
838	702244	588480472	28.9482	9.4279	24259	3.8428	.00119332	2632.652	551541.15
839	703921	590589719	28.9655	9.4316	24302	3.8437	.00119190	2635.794	552858.26
840	**705600**	**592704000**	**28.9828**	**9.4354**	**24346**	**3.8446**	**.00119048**	**2638.936**	**554176.94**
841	707281	594823321	29.0000	9.4391	24389	3.8456	.00118906	2642.077	555497.20
842	708964	596947688	29.0172	9.4429	24432	3.8465	.00118765	2645.219	556819.02
843	710649	599077107	29.0345	9.4466	24476	3.8474	.00118624	2648.360	558142.42
844	712336	601211584	29.0517	9.4503	24520	3.8483	.00118483	2651.502	559467.39
845	714025	603351125	29.0689	9.4541	24563	3.8492	.00118343	2654.644	560793.92
846	715716	605495736	29.0861	9.4578	24607	3.8501	.00118203	2657.785	562122.03
847	717409	607645423	29.1033	9.4615	24650	3.8510	.00118064	2660.927	563451.71
848	719104	609800192	29.1204	9.4652	24694	3.8519	.00117925	2664.068	564782.96
849	720801	611960049	29.1376	9.4690	24738	3.8528	.00117786	2667.210	566115.78

7. Properties of Numbers (*Continued*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.) Circum.	Area
850	**722500**	**614125000**	**29.1548**	**9.4727**	**24782**	**3.8538**	**.00117647**	**2670.352**	**567450.17**
851	724201	616295051	29.1719	9.4764	24825	3.8547	.00117509	2673.493	568786.14
852	725904	618470208	29.1890	9.4801	24869	3.8556	.00117371	2676.635	570123.67
853	727609	620650477	29.2062	9.4838	24913	3.8565	.00117233	2679.776	571462.77
854	729316	622835864	29.2233	9.4875	24957	3.8574	.00117096	2682.918	572803.45
855	731025	625026375	29.2404	9.4912	25000	3.8582	.00116959	2686.059	574145.69
856	732736	627222016	29.2575	9.4949	25044	3.8592	.00116822	2689.201	575489.51
857	734449	629422793	29.2746	9.4986	25088	3.8601	.00116686	2692.343	576834.90
858	736164	631628712	29.2916	9.5023	25132	3.8610	.00116550	2695.484	578181.85
859	737881	633839779	29.3087	9.5060	25176	3.8619	.00116414	2698.626	579530.38
860	**739600**	**636056000**	**29.3258**	**9.5097**	**25220**	**3.8628**	**.00116279**	**2701.767**	**580880.48**
861	741321	638277381	29.3428	9.5134	25264	3.8637	.00116144	2704.909	582232.15
862	743044	640503928	29.3598	9.5171	25308	3.8646	.00116009	2708.051	583585.39
863	744769	642735647	29.3769	9.5207	25352	3.8655	.00115875	2711.192	584940.20
864	746496	644972544	29.3939	9.5244	25396	3.8664	.00115741	2714.334	586296.59
865	748225	647214625	29.4109	9.5281	25440	3.8673	.00115607	2717.475	587654.54
866	749956	649461896	29.4279	9.5317	25485	3.8682	.00115473	2720.617	589014.07
867	751689	651714363	29.4449	9.5354	25529	3.8691	.00115340	2723.759	590375.16
868	753424	653972032	29.4618	9.5391	25573	3.8700	.00115207	2726.900	591737.83
869	755161	656234909	29.4788	9.5427	25617	3.8708	.00115075	2730.042	593102.06
870	**756900**	**658503000**	**29.4958**	**9.5464**	**25661**	**3.8717**	**.00114943**	**2733.183**	**594467.87**
871	758641	660776311	29.5127	9.5501	25706	3.8726	.00114811	2736.325	595835.25
872	760384	663054848	29.5296	9.5537	25750	3.8735	.00114679	2739.466	597204.20
873	762129	665338617	29.5466	9.5574	25794	3.8744	.00114548	2742.608	598574.72
874	763876	667627624	29.5635	9.5610	25839	3.8753	.00114416	2745.750	599946.81
875	765625	669921875	29.5804	9.5647	25883	3.8762	.00114286	2748.891	601320.47
876	767376	672221376	29.5973	9.5683	25927	3.8771	.00114155	2752.033	602695.70
877	769129	674526133	29.6142	9.5719	25972	3.8780	.00114025	2755.174	604072.50
878	770884	676836152	29.6311	9.5756	26016	3.8789	.00113895	2758.316	605450.88
879	772641	679151439	29.6479	9.5792	26061	3.8797	.00113766	2761.458	606830.82
880	**774400**	**681472000**	**29.6648**	**9.5828**	**26105**	**3.8806**	**.00113636**	**2764.599**	**608212.34**
881	776161	683797841	29.6816	9.5865	26150	3.8815	.00113507	2767.741	609595.42
882	777924	686128968	29.6985	9.5901	26194	3.8823	.00113379	2770.882	610980.08
883	779689	688465387	29.7153	9.5937	26239	3.8832	.00113250	2774.024	612366.31
884	781456	690807104	29.7321	9.5973	26283	3.8841	.00113122	2777.166	613754.11
885	783225	693154125	29.7489	9.6010	26328	3.8850	.00112994	2780.307	615143.48
886	784996	695506456	29.7658	9.6046	26373	3.8859	.00112867	2783.449	616534.42
887	786769	697864103	29.7825	9.6082	26417	3.8868	.00112740	2786.590	617926.93
888	788544	700227072	29.7993	9.6118	26462	3.8877	.00112613	2789.732	619321.01
889	790321	702595369	29.8161	9.6154	26507	3.8885	.00112486	2792.874	620716.66
890	**792100**	**704969000**	**29.8329**	**9.6190**	**26551**	**3.8894**	**.00112360**	**2796.015**	**622113.89**
891	793881	707347971	29.8496	9.6226	26596	3.8902	.00112233	2799.157	623512.68
892	795664	709732288	29.8664	9.6262	26641	3.8911	.00112108	2802.298	624913.04
893	797449	712121957	29.8831	9.6298	26686	3.8920	.00111982	2805.440	626314.98
894	799236	714516984	29.8998	9.6334	26730	3.8929	.00111857	2808.581	627718.49
895	801025	716917375	29.9166	9.6370	26775	3.8937	.00111732	2811.723	629123.56
896	802816	719323136	29.9333	9.6406	26820	3.8946	.00111607	2814.865	630530.21
897	804609	721734273	29.9500	9.6442	26865	3.8955	.00111483	2818.006	631938.43
898	806404	724150792	29.9666	9.6477	26910	3.8963	.00111359	2821.148	633348.22
899	808201	726572699	29.9833	9.6513	26955	3.8972	.00111235	2824.289	634759.58
900	**810000**	**729000000**	**30.0000**	**9.6549**	**27000**	**3.8981**	**.00111111**	**2827.431**	**636172.51**
901	811801	731432701	30.0167	9.6585	27045	3.8989	.00110988	2830.573	637587.01
902	813604	733870808	30.0333	9.6620	27090	3.8998	.00110865	2833.714	639003.09
903	815409	736314327	30.0500	9.6656	27135	3.9007	.00110742	2836.856	640420.73
904	817216	738763264	30.0666	9.6692	27180	3.9015	.00110619	2839.997	641839.95
905	819025	741217625	30.0832	9.6727	27225	3.9024	.00110497	2843.139	643260.73
906	820836	743677416	30.0998	9.6763	27270	3.9032	.00110375	2846.281	644683.09
907	822649	746142643	30.1164	9.6799	27316	3.9041	.00110254	2849.422	646107.01
908	824464	748613312	30.1330	9.6834	27361	3.9050	.00110132	2852.564	647532.51
909	826281	751089429	30.1496	9.6870	27406	3.9059	.00110011	2855.705	648959.58
910	**828100**	**753571000**	**30.1662**	**9.6905**	**27451**	**3.9067**	**.00109890**	**2858.847**	**650388.22**
911	829921	756058031	30.1828	9.6941	27497	3.9076	.00109769	2861.988	651818.43
912	831744	758550528	30.1993	9.6976	27542	3.9084	.00109649	2865.130	653250.21
913	833569	761048497	30.2159	9.7012	27587	3.9093	.00109529	2868.272	654683.56
914	835396	763551944	30.2324	9.7047	27632	3.9101	.00109409	2871.413	656118.48
915	837225	766060875	30.2490	9.7082	27678	3.9110	.00109290	2874.555	657554.98
916	839056	768575296	30.2655	9.7118	27723	3.9118	.00109170	2877.696	658993.04
917	840889	771095213	30.2820	9.7153	27769	3.9127	.00109051	2880.838	660432.68
918	842724	773620632	30.2985	9.7188	27814	3.9135	.00108932	2883.980	661873.88
919	844561	776151559	30.3150	9.7224	27859	3.9144	.00108814	2887.121	663316.66

7. Properties of Numbers *(Continued)*

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle (N = Diam.)	
								Circum.	Area
920	**846400**	**778688000**	**30.3315**	**9.7259**	**27905**	**3.9152**	**.00108696**	**2890.265**	**664761.01**
921	848241	781229961	30.3480	9.7294	27950	3.9161	.00108578	2893.404	666206.92
922	850084	783777448	30.3645	9.7329	27996	3.9169	.00108460	2896.546	667654.41
923	851929	786330467	30.3809	9.7364	28042	3.9178	.00108342	2899.688	669103.47
924	853776	788889024	30.3974	9.7400	28087	3.9186	.00108225	2902.829	670554.10
925	855625	791453125	30.4138	9.7435	28133	3.9194	.00108108	2905.971	672006.30
926	857476	794022776	30.4302	9.7470	28179	3.9203	.00107991	2909.112	673460.08
927	859329	796597983	30.4467	9.7505	28224	3.9212	.00107875	2912.254	674915.42
928	861184	799178752	30.4631	9.7540	28270	3.9220	.00107759	2915.396	676372.33
929	863041	801765089	30.4795	9.7575	28315	3.9229	.00107643	2918.537	677830.82
930	**864900**	**804357000**	**30.4959**	**9.7610**	**28361**	**3.9237**	**.00107527**	**2921.679**	**679290.87**
931	866761	806954491	30.5123	9.7645	28407	3.9246	.00107411	2924.820	680752.50
932	868624	809557568	30.5287	9.7680	28453	3.9254	.00107296	2927.962	682215.69
933	870489	812166237	30.5450	9.7715	28499	3.9262	.00107181	2931.103	683680.46
934	872356	814780504	30.5614	9.7750	28544	3.9271	.00107066	2934.245	685146.80
935	874225	817400375	30.5778	9.7785	28590	3.9279	.00106952	2937.387	686614.71
936	876096	820025856	30.5941	9.7819	28636	3.9288	.00106838	2940.528	688084.19
937	877969	822656953	30.6105	9.7854	28682	3.9296	.00106724	2943.670	689555.24
938	879844	825293672	30.6268	9.7889	28728	3.9304	.00106610	2946.811	691027.86
939	881721	827936019	30.6431	9.7924	28774	3.9313	.00106496	2949.953	692502.05
940	**883600**	**830584000**	**30.6594**	**9.7959**	**28820**	**3.9321**	**.00106383**	**2953.095**	**693977.82**
941	885481	833237621	30.6757	9.7993	28866	3.9329	.00106270	2956.236	695455.15
942	887364	835896888	30.6920	9.8028	28912	3.9338	.00106157	2959.378	696934.06
943	889249	838561807	30.7083	9.8063	28958	3.9346	.00106045	2962.519	698414.53
944	891136	841232384	30.7246	9.8097	29004	3.9354	.00105932	2965.661	699896.58
945	893025	843908625	30.7409	9.8132	29050	3.9363	.00105820	2968.803	701380.19
946	894916	846590536	30.7571	9.8167	29096	3.9371	.00105708	2971.944	702865.38
947	896809	849278123	30.7734	9.8201	29142	3.9379	.00105597	2975.086	704352.14
948	898704	851971392	30.7896	9.8236	29189	3.9388	.00105485	2978.227	705840.47
949	900601	854670349	30.8058	9.8270	29235	3.9396	.00105374	2981.369	707330.37
950	**902500**	**857375000**	**30.8221**	**9.8305**	**29281**	**3.9404**	**.00105263**	**2984.511**	**708821.84**
951	904401	860085351	30.8383	9.8339	29327	3.9413	.00105152	2987.652	710314.88
952	906304	862801408	30.8545	9.8374	29374	3.9421	.00105042	2990.794	711809.50
953	908209	865523177	30.8707	9.8408	29420	3.9429	.00104932	2993.935	713305.68
954	910116	868250664	30.8869	9.8443	29466	3.9438	.00104822	2997.077	714803.43
955	912025	870983875	30.9031	9.8477	29513	3.9446	.00104712	3000.218	716302.76
956	913936	873722816	30.9192	9.8511	29559	3.9454	.00104603	3003.360	717803.66
957	915849	876467493	30.9354	9.8546	29605	3.9462	.00104493	3006.502	719306.12
958	917764	879217912	30.9516	9.8580	29652	3.9471	.00104384	3009.643	720810.16
959	919681	881974079	30.9677	9.8614	29698	3.9479	.00104275	3012.785	722315.77
960	**921600**	**884736000**	**30.9839**	**9.8648**	**29745**	**3.9487**	**.00104167**	**3015.926**	**723822.95**
961	923521	887503681	31.0000	9.8683	29791	3.9495	.00104058	3019.068	725331.70
962	925444	890277128	31.0161	9.8717	29838	3.9503	.00103950	3022.210	726842.02
963	927369	893056347	31.0322	9.8751	29884	3.9512	.00103842	3025.351	728353.91
964	929296	895841344	31.0483	9.8785	29931	3.9520	.00103734	3028.493	729867.37
965	931225	898632125	31.0644	9.8819	29977	3.9528	.00103627	3031.634	731382.40
966	933156	901428696	31.0805	9.8854	30024	3.9536	.00103520	3034.776	732899.01
967	935089	904231063	31.0966	9.8888	30070	3.9544	.00103413	3037.918	734417.18
968	937024	907039232	31.1127	9.8922	30117	3.9553	.00103306	3041.059	735936.93
969	938961	909853209	31.1288	9.8956	30164	3.9561	.00103199	3044.201	737458.24
970	**940900**	**912673000**	**31.1448**	**9.8990**	**30210**	**3.9569**	**.00103093**	**3047.342**	**738981.18**
971	942841	915498611	31.1609	9.9024	30257	3.9577	.00102987	3050.484	740505.59
972	944784	918330048	31.1769	9.9058	30304	3.9585	.00102881	3053.625	742031.62
973	946729	921167317	31.1929	9.9092	30351	3.9593	.00102775	3056.767	743559.22
974	948676	924010424	31.2090	9.9126	30398	3.9602	.00102669	3059.909	745088.39
975	950625	926859375	31.2250	9.9160	30444	3.9610	.00102564	3063.050	746619.13
976	952576	929714176	31.2410	9.9194	30491	3.9618	.00102459	3066.192	748151.44
977	954529	932574833	31.2570	9.9227	30538	3.9626	.00102354	3069.333	749685.32
978	956484	935441352	31.2730	9.9261	30585	3.9634	.00102249	3072.475	751220.78
979	958441	938313739	31.2890	9.9295	30632	3.9642	.00102145	3075.617	752757.80
980	**960400**	**941192000**	**31.3050**	**9.9329**	**30679**	**3.9650**	**.00102041**	**3078.760**	**754296.40**
981	962361	944076141	31.3209	9.9363	30726	3.9658	.00101937	3081.900	755836.59
982	964324	946966168	31.3369	9.9396	30773	3.9666	.00101833	3085.041	757378.30
983	966289	949862087	31.3528	9.9430	30820	3.9674	.00101729	3088.183	758921.61
984	968256	952763904	31.3688	9.9464	30867	3.9682	.00101626	3091.325	760466.48
985	970225	955671625	31.3847	9.9497	30914	3.9691	.00101523	3094.466	762012.93
986	972196	958585256	31.4006	9.9531	30961	3.9699	.00101420	3097.608	763560.95
987	974169	961504803	31.4166	9.9565	31008	3.9707	.00101317	3100.749	765110.54
988	976144	964430272	31.4325	9.9598	31055	3.9715	.00101215	3103.891	766661.70
989	978121	967361669	31.4484	9.9632	31102	3.9723	.00101112	3107.033	768214.44

7. Properties of Numbers (*Concluded*)

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	$N^{3/2}$	$\sqrt[5]{N}$	$\frac{1}{N}$	Circle ($N = D$)	
								Circum.	Area
990	**980100**	**970299000**	**31.4643**	**9.9666**	**31150**	**3.9731**	**.00101010**	**3110.174**	**769768.74**
991	982081	973242271	31.4802	9.9699	31197	3.9739	.00100908	3113.316	771324.61
992	984064	976191488	31.4960	9.9733	31244	3.9747	.00100806	3116.457	772882.06
993	986049	979146657	31.5119	9.9766	31291	3.9755	.00100705	3119.599	774441.07
994	988036	982107784	31.5278	9.9800	31339	3.9763	.00100604	3122.740	776001.66
995	990025	985074875	31.5436	9.9833	31386	3.9771	.00100503	3125.882	777563.82
996	992016	988047936	31.5595	9.9866	31433	3.9779	.00100402	3129.024	779127.54
997	994009	991026973	31.5753	9.9900	31480	3.9787	.00100301	3132.165	780692.84
998	996004	994011992	31.5911	9.9933	31528	3.9795	.00100200	3135.307	782259.71
999	998001	997002999	31.6070	9.9967	31575	3.9803	.00100100	3138.448	783828.15
1000	**1000000**	**1000000000**	**31.6228**	**10.0000**	**31623**	**3.9811**	**.00100000**	**3141.593**	**785398.16**

8. Napierian Logarithms of Numbers from 1 to 119

n	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	−∞	0.0000	0 6931	1.0986	1.3863	1.6094	1.7918	1.9459	2.0794	2.1972
1	2.3026	2 3979	2.4849	2.5649	2.6391	2.7081	2.7726	2.8332	2.8904	2 9444
2	2.9957	3.0445	3.0910	3.1355	3.1781	3.2189	3.2581	3.2958	3.3322	3.3673
3	3.4012	3.4340	3.4657	3.4965	3.5264	3.5553	3.5835	3.6109	3.6376	3.6636
4	3.6889	3.7136	3.7377	3.7612	3.7842	3.8067	3.8286	3.8501	3.8712	3.8918
5	3.9120	3.9318	3.9512	3.9703	3.9890	4.0073	4.0254	4.0430	4.0604	4.0775
6	4.0943	4.1109	4.1271	4.1431	4.1589	4.1744	4.1897	4.2047	4.2195	4.2341
7	4.2485	4.2627	4.2767	4.2905	4.3041	4.3175	4.3307	4.3438	4.3567	4.3694
8	4.3820	4.3944	4.4067	4.4188	4.4308	4 4427	4.4543	4.4659	4.4773	4.4886
9	4.4998	4.5109	4.5218	4.5326	4.5433	4.5539	4.5643	4.5747	4.5850	4.5951
10	4.6052	4.6151	4.6250	4.6347	4.6444	4.6540	4.6634	4.6728	4.6821	4.6913
11	4.7005	4.7095	4.7185	4.7274	4.7362	4.7449	4.7536	4.7622	4.7707	4.7791

9. Multipliers for Transferring Logarithms

Common to Napierian			Napierian to Common		
1	2.302585093	Example: Find Nap log of 105. Com log 105 = 2.02119 2 4.605170 .02 46052 1 2303 1 230 9 207 Nap log 105 = 4.65396	1	0.434294482	Example: Find number corresponding to Nap log $\bar{1}$.6078 .6 0.26058 07 304 8 35 +0.26397 $\bar{1}$ −0.43429 Com log = $\bar{1}$.8297 Number = 0.6756
2	4.605170186		2	0.868588964	
3	6.907755279		3	1.302883446	
4	9.210340372		4	1.737177928	
5	11.512925465		5	2.171472410	
6	13.815510558		6	2.605766891	
7	16.118095651		7	3.040061373	
8	18.420680744		8	3.474355855	
9	20.723265837		9	3.908650337	

10. Multipliers for Finding Lengths of Circular Arcs

	Degrees	Minutes	Seconds	
1	0.017453293	0.000290888	0.000004848	Example: Find length of arc for a central angle of 48° 45′ in circle of 12 ft radius. 40° 0.698132 8° .139626 40′ .011636 5′ .001454 0.85085 12 Length = 10.210 ft
2	0.034906585	0.000581776	0.000009696	
3	0.052359878	0.000872665	0.000014544	
4	0.069813170	0.001163553	0.000019393	
5	0.087266463	0.001454441	0.000024241	
6	0.104719755	0.001745329	0.000029089	
7	0.122173048	0.002036217	0.000033937	
8	0.139626340	0.002327106	0.000038785	
9	0.157079633	0.002617994	0.000043633	

11. Circumferences of Circles (Diameters in Units and Tenths)

d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.314	0.628	0.942	1.257	1.571	1.885	2.199	2.513	2.827
1	3.142	3.456	3.770	4.084	4.398	4.712	5.027	5.341	5.655	5.969
2	6.283	6.597	6.912	7.226	7.540	7.854	8.168	8.482	8.796	9.111
3	9.425	9.739	10.05	10.37	10.68	11.00	11.31	11.62	11.94	12.25
4	12.57	12.88	13.19	13.51	13.82	14.14	14.45	14.77	15.08	15.39
5	15.71	16.02	16.34	16.65	16.96	17.28	17.59	17.91	18.22	18.54
6	18.85	19.16	19.48	19.79	20.11	20.42	20.73	21.05	21.36	21.68
7	21.99	22.31	22.62	22.93	23.25	23.56	23.88	24.19	24.50	24.82
8	25.13	25.45	25.76	26.08	26.39	26.70	27.02	27.33	27.65	27.96
9	28.27	28.59	28.90	29.22	29.53	29.85	30.16	30.47	30.79	31.10
10	31.42	31.73	32.04	32.36	32.67	32.99	33.30	33.62	33.93	34.24
11	34.56	34.87	35.19	35.50	35.81	36.13	36.44	36.76	37.07	37.38
12	37.70	38.01	38.33	38.64	38.96	39.27	39.58	39.90	40.21	40.53
13	40.84	41.15	41.47	41.78	42.10	42.41	42.73	43.04	43.35	43.67
14	43.98	44.30	44.61	44.92	45.24	45.55	45.87	46.18	46.50	46.81
15	47.12	47.44	47.75	48.07	48.38	48.69	49.01	49.32	49.64	49.95
16	50.27	50.58	50.89	51.21	51.52	51.84	52.15	52.46	52.78	53.09
17	53.41	53.72	54.04	54.35	54.66	54.98	55.29	55.61	55.92	56.23
18	56.55	56.86	57.18	57.49	57.81	58.12	58.43	58.75	59.06	59.38
19	59.69	60.00	60.32	60.63	60.95	61.26	61.58	61.89	62.20	62.52

12. Circumferences of Circles (Diameter in Units and Eighths)

d	0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0.0000	0.3927	0.7854	1.1781	1.5708	1.9635	2.3562	2.7489
1	3.1416	3.5343	3.9270	4.3197	4.7124	5.1051	5.4978	5.8905
2	6.2832	6.6759	7.0686	7.4613	7.8540	8.2467	8.6394	9.0321
3	9.4248	9.8175	10.210	10.603	10.996	11.388	11.781	12.174
4	12.566	12.959	13.352	13.744	14.137	14.530	14.923	15.315
5	15.708	16.101	16.493	16.886	17.279	17.671	18.064	18.457
6	18.850	19.242	19.635	20.028	20.420	20.813	21.206	21.598
7	21.991	22.384	22.777	23.169	23.562	23.955	24.347	24.740
8	25.133	25.525	25.918	26.311	26.704	27.096	27.489	27.882
9	28.274	28.667	29.060	29.452	29.845	30.238	30.631	31.023
10	31.416	31.809	32.201	32.594	32.987	33.379	33.772	34.165
11	34.558	34.950	35.343	35.736	36.128	36.521	36.914	37.306
12	37.699	38.092	38.485	38.877	39.270	39.663	40.055	40.448
13	40.841	41.233	41.626	42.019	42.412	42.804	43.197	43.590
14	43.982	44.375	44.768	45.160	45.553	45.946	46.338	46.731
15	47.124	47.517	47.909	48.302	48.695	49.087	49.480	49.873
16	50.265	50.658	51.051	51.444	51.836	52.229	52.622	53.014
17	53.407	53.800	54.192	54.585	54.978	55.371	55.763	56.156
18	56.549	56.941	57.334	57.727	58.119	58.512	58.905	59.298
19	59.690	60.083	60.475	60.868	61.261	61.654	62.046	62.439

13. Decimal Equivalents of Common Fractions

The given decimals are the parts of inches corresponding to fraction of inches in first column; also, the parts of feet for the fraction of inches in third column.

	0.0052	1/16		0.2552	3 1/16		0.5052	6 1/16		0.7552	9 1/16
	0.0104	1/8		0.2604	3 1/8		0.5104	6 1/8		0.7604	9 1/8
1/64	0.015625	3/16	17/64	0.265625	3 3/16	33/64	0.515625	6 3/16	49/64	0.765625	9 3/16
	0.0208	1/4		0.2708	3 1/4		0.5208	6 1/4		0.7708	9 1/4
	0.0260	5/16		0.2760	3 5/16		0.5260	6 5/16		0.7760	9 5/16
1/32	0.03125	3/8	9/32	0.28125	3 3/8	17/32	0.53125	6 3/8	25/32	0.78125	9 3/8
	0.0364	7/16		0.2865	3 7/16		0.5364	6 7/16		0.7865	9 7/16
	0.0417	1/2		0.2917	3 1/2		0.5417	6 1/2		0.7917	9 1/2
3/64	0.046875	9/16	19/64	0.296875	3 9/16	35/64	0.546875	6 9/16	51/64	0.796875	9 9/16
	0.0521	5/8		0.3021	3 5/8		0.5521	6 5/8		0.8021	9 5/8
	0.0573	11/16		0.3073	3 11/16		0.5573	6 11/16		0.8073	9 11/16
1/16	0.0625	3/4	5/16	0.3125	3 3/4	9/16	0.5625	6 3/4	13/16	0.8125	9 3/4
	0.0677	13/16		0.3177	3 13/16		0.5677	6 13/16		0.8177	9 13/16
	0.0729	7/8		0.3229	3 7/8		0.5729	6 7/8		0.8229	9 7/8
5/64	0.078125	15/16	21/64	0.328125	3 15/16	37/64	0.578125	6 15/16	53/64	0.828125	9 15/16
	0.0833	1		0.3333	4		0.5833	7		0.8333	10
	0.0885	1 1/16		0.3385	4 1/16		0.5885	7 1/16		0.8385	10 1/16
3/32	0.09375	1 1/8	11/32	0.34375	4 1/8	19/32	0.59375	7 1/8	27/32	0.84375	10 1/8
	0.0990	1 3/16		0.3490	4 3/16		0.5990	7 3/16		0.8490	10 3/16
	0.1042	1 1/4		0.3542	4 1/4		0.6042	7 1/4		0.8542	10 1/4
7/64	0.109375	1 5/16	23/64	0.359375	4 5/16	39/64	0.609375	7 5/16	55/64	0.859375	10 5/16
	0.1146	1 3/8		0.3646	4 3/8		0.6146	7 3/8		0.8646	10 3/8
	0.1198	1 7/16		0.3698	4 7/16		0.6198	7 7/16		0.8698	10 7/16
1/8	0.1250	1 1/2	3/8	0.3750	4 1/2	5/8	0.6250	7 1/2	7/8	0.8750	10 1/2
	0.1302	1 9/16		0.3802	4 9/16		0.6302	7 9/16		0.8802	10 9/16
	0.1354	1 5/8		0.3854	4 5/8		0.6354	7 5/8		0.8854	10 5/8
9/64	0.140625	1 11/16	25/64	0.390625	4 11/16	41/64	0.640625	7 11/16	57/64	0.890625	10 11/16
	0.1458	1 3/4		0.3958	4 3/4		0.6458	7 3/4		0.8958	10 3/4
	0.1510	1 13/16		0.4010	4 13/16		0.6510	7 13/16		0.9010	10 13/16
5/32	0.15625	1 7/8	13/32	0.40625	4 7/8	21/32	0.65625	7 7/8	29/32	0.90625	10 7/8
	0.1615	1 15/16		0.4114	4 15/16		0.6615	7 15/16		0.9115	10 15/16
	0.1667	2		0.4167	5		0.6667	8		0.9167	11
11/64	0.171875	2 1/16	27/64	0.421875	5 1/16	43/64	0.671875	8 1/16	59/64	0.921875	11 1/16
	0.1771	2 1/8		0.4271	5 1/8		0.6771	8 1/8		0.9271	11 1/8
	0.1823	2 3/16		0.4323	5 3/16		0.6823	8 3/16		0.9323	11 3/16
3/16	0.1875	2 1/4	7/16	0.4375	5 1/4	11/16	0.6875	8 1/4	15/16	0.9375	11 1/4
	0.1927	2 5/16		0.4427	5 5/16		0.6927	8 5/16		0.9427	11 5/16
	0.1979	2 3/8		0.4479	5 3/8		0.6979	8 3/8		0.9479	11 3/8
13/64	0.203125	2 7/16	29/64	0.453125	5 7/16	45/64	0.703125	8 7/16	61/64	0.953125	11 7/16
	0.2083	2 1/2		0.4583	5 1/2		0.7083	8 1/2		0.9583	11 1/2
	0.2135	2 9/16		0.4635	5 9/16		0.7135	8 9/16		0.9635	11 9/16
7/32	0.21875	2 5/8	15/32	0.46875	5 5/8	23/32	0.71875	8 5/8	31/32	0.96875	11 5/8
	0.2240	2 11/16		0.4740	5 11/16		0.7240	8 11/16		0.9740	11 11/16
	0.2292	2 3/4		0.4792	5 3/4		0.7292	8 3/4		0.9792	11 3/4
15/64	0.234375	2 13/16	31/64	0.484375	5 13/16	47/64	0.734375	8 13/16	63/64	0.984375	11 13/16
	0.2395	2 7/8		0.4896	5 7/8		0.7396	8 7/8		0.9896	11 7/8
	0.2448	2 15/16		0.4948	5 15/16		0.7448	8 15/16		0.9948	11 15/16
1/4	0.2500	3	1/2	0.5000	6	3/4	0.7500	9	1	1.0000	12

14. Product of Fractions Expressed in Decimals

	1	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16
1/16	.0625	.0039														
1/8	.1250	.0078	.0156													
3/16	.1875	.0117	.0234	.0351												
1/4	.2500	.0156	.0313	.0469	.0625											
5/16	.3125	.0195	.0391	.0587	.0781	.0977										
3/8	.3750	.0234	.0469	.0705	.0937	.1172	.1406									
7/16	.4375	.0273	.0547	.0823	.1093	.1367	.1641	.1914								
1/2	.5000	.0313	.0625	.0938	.1250	.1562	.1875	.2188	.2500							
9/16	.5625	.0352	.0703	.1056	.1406	.1757	.2110	.2462	.2813	.3164						
5/8	.6250	.0391	.0781	.1172	.1562	.1952	.2343	.2734	.3125	.3516	.3906					
11/16	.6875	.0430	.0859	.1289	.1718	.2148	.2578	.3007	.3438	.3867	.4297	.4727				
3/4	.7500	.0469	.0938	.1406	.1875	.2344	.2813	.3281	.3750	.4219	.4686	.5156	.5625			
13/16	.8125	.0508	.1016	.1523	.2031	.2539	.3047	.3555	.4063	.4570	.5078	.5586	.6094	.6601		
7/8	.8750	.0547	.1094	.1640	.2187	.2734	.3281	.3828	.4375	.4922	.5469	.6016	.6563	.7109	.7656	
15/16	.9375	.0586	.1172	.1757	.2343	.2929	.3515	.4102	.4688	.5274	.5859	.6445	.7031	.7617	.8203	.8789
1	1.000	.0625	.1250	.1875	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	.8125	.8750	.9375

15. Weights and Measures

(See also Circular 47, U S Bur of Standards, on Units of Weights and Measures, July, 1914)

Avoirdupois Weight

Grains	Drams	Ounces	Pounds	Hund'd-wt	Net tons	Gross tons
1	0.03657	0.002286	0.000143	0.00000127	0.00000007134	0.0000000637
27.34375	1	0 0625	0.003906	0.00003488	0.000001953	0.000001744
437.5	16	1	0.0625	0.00055804	0 00003125	0.00002790
7 000	256	16	1	0.0089286	0.0005	0.0004464
784 000	28 672	1 792	112	1	0.056	0.05
14 000 000	512 000	32 000	2 000	17.857	1	0.89286
15 680 000	573 440	35 840	2 240	20	1.12	1

1 lb avoir = 1.215278 lb troy. 1 net ton = 2 000 lb = 0.892857 gross ton.

Troy Weight

Grains	Pennyweight	Ounces	Pounds
1	0.041667	0.0020833	0.0001736
24	1	0.05	0.0041667
480	20	1	0.0833333
5 760	240	12	1

1 lb troy = 0.822857 lb avoir. 175 oz troy = 192 oz avoir.

Apothecaries' Weight

Grains	Scruples	Drams	Ounces	Pounds
1	0.05	0.016667	0.0020833	0.000173611
20	1	0.333333	0.0416667	0.0034722
60	3	1	0.125	0.0104167
480	24	8	1	0.0833333
5 760	288	96	12	1

The lb, oz, and gr are same as in troy wt. Avoir gr = troy gr = apothecaries' gr.

15. Weights and Measures (*Continued*)

Density

Lb per cu in	Lb per cu ft	Net tons per cu yd	Lb per U S gallon	Gm per cu cm
1	1 728	23.328	230.999	27.6826
0.0005787	1	0.0135	0.13368	0.01602
0.042867	74.074	1	9.9023	1.18655
0.004329	7.4805	0.10099	1	0.11983
0.036127	62.4281	0.84278	8.3454	1

Linear Measure

Inches	Feet	Yards	Rods	Furlongs	Statute miles	Meters
1	0.08333	0.02778	0.0050505	0.00012626	0.00001578	0.0254
12	1	0.33333	0.0606061	0.00151515	0.00018939	0.3048
36	3	1	0.1818182	0.00454545	0.00056818	0.9144
198	16.5	5.5	1	0.025	0.003125	5.029
7 920	660	220	40	1	0.125	201.16
63 360	5 280	1 760	320	8	1	1609.4
39.37	3.2808	1.0936	0.1988	0.0000777	0.0006214	1

Rope and Cable Measure

Inches	Spans	Feet	Fathoms	Cable lengths	Statute miles	Meters
1	0.1111	0.08333	0.013889	0.0001157	0.00001578	0.0254
9	1	0.75	0.16667	0.001042	0.000142	0.2286
12	1.3333	1	0.125	0.001389	0.0001894	0.3048
72	8	6	1	0.008333	0.0011364	1.8288
8 640	960	720	120	1	0.136368	219.456
63 360	7 040	5 280	880	7.3333	1	1 609.4
39.37	4.377	3.2808	0.547	0.0455	0.0006214	1

Nautical Measure

1 nautical mile, as adopted by the U S Coast and Geodetic Survey, equals the length of one minute of arc of a great circle of a sphere the surface of which equals that of the earth = 6 080.204 ft = 1.1516 statute miles. 1 league = 3 nautical miles = 18 240.613 ft.

Gunter's Chain

Inches	Links	Feet	Rods	Chains	Statute miles	Meters
1	0.1263	0.08333	0.0050505	0.001263	0.00001578	0.0254
7.92	1	0.66	0.04	0.01	0.000125	0.2012
12	1.7156	1	0.060606	0.01515	0.0001894	0.3048
198	25	16.5	1	0.25	0.003125	5.029
792	100	66	4	1	0.0125	20.12
63 360	8 000	5 280	320	80	1	1609.4
39.37	4.971	3.2808	0.1988	0.04971	0.0006214	1

Square or Land Measure

Sq inches	Sq feet	Sq yards	Sq rods	Acres	Sq miles	Sq meters
1	0.006944	0.0007716	....	....	....	0.0006452
144	1	0.111111	....	....	....	0.0929
1 296	9.0	1	0.03306	0.0002066	....	0.8361
39 204	272.25	30.25	1	0.00625	0.00000977	25.29
6 272 640	43 560	4 840	160	1	0.0015625	4 047
....	27 878 400	3 097 600	102 400	640	1	2 590 000
1 550	10 764	1.196	0.03954	0.0002471	....	1

1 sq rood = 40 sq rods. 1 acre = 4 sq roods. 1 sq acre = 208.71 ft sq.

15. Weights and Measures (*Continued*)

Cubic or Solid Measure

Cu in	Cu ft	Cu yd	Cu meters
1	0.0005787	0.000021433	0.000016387
1 728	1	0.037037	0.02832
46 656	27	1	0.7646
61 020	35.31	1.3079	1

1 cord wood = 128 cu ft = 4 by 4 by 8 ft. 1 perch of masonry = 24.75 cu ft = 16.5 by 1.5 by 1 ft; usually taken as 25 cu ft.

Dry Measure

Pints	Quarts	Gallons	Pecks	Bushels	Cu in	Cu ft	Cu meters
1	0.50	0.125	0.0625	0.015625	33.6003	0.01944	0.0005506
2	1	0.25	0.125	0.03125	67.2006	0.03889	0.001101
8	4	1	0.50	0.125	268.8025	0.1556	0.004405
16	8	2	1	0.25	537.605	0.3111	0.00881
64	32	8	4	1	2 150.42	1.2444	0 03524
0.029782	0.014881	0.0037202	0.0018601	0.00046503	1	0.0005787	0.000016387
51.4281	25.7141	6.42851	3.21426	0.80356	1 728	1	0.02832
1 816.173	908.086	227.022	113.511	28.3777	61 020	35.31	1

1 heaped bushel = 1.25 struck bushel, and the cone must be not less than 6 in high.

Liquid Measure

Gills	Pints	Quarts	Gallons *	Barrels	Cu in	Cu ft	Cu meters
1	0.25	0.125	0.03125	0.000992	7.21875	0.004177	0.0001183
4	1	0.5	0.125	0.003968	28.875	0.01671	0.0004732
8	2	1	0.25	0.007937	57.75	0.03342	0 0009464
32	8	4	1	0.031746	231	0.1337	0.003785
1 008	252	126	31.5	1	7 276.5	4.2109	0.1192
0.13853	0.034632	0.017316	0.004329	0.00013743	1	0.0005787	0.000016387
239.377	59.8443	29.9221	7.48053	0.23748	1 728	1	0.02832
8 453.542	2 113.385	1 056.693	264.1732	8.38645	61 020	35.31	1

* U S gal. British imperial gal = 277.410 cu in, or 10 lb avoir of pure water at 62° F and barom at 30 in. British imperial gal = 1.20091 U S gal. 1 fluid dram = 60 minims = 0.125 fluid-oz = 0.0078125 pint. 1 fluid oz = 480 minims = 8 drams = 0.0625 pint.

Metric System

Measures of Length, Capacity, and Weight

Length	Kilometer	Hecto-meter	Decameter	Meter	Decimeter	Centimeter	Millimeter
Capacity	Kiloliter or Stere	Hecto-liter or decistere	Decaliter or centi-stere	Liter or millistere	Deciliter	Centiliter	Milliliter
Weight	Kilo-gramme	Hecto-gramme	Deca-gramme	Gramme	Deci-gramme	Centi-gramme	Milli-gramme
	1	10	100	1 000	10 000	100 000	1 000 000
		1	10	100	1 000	10 000	100 000
			1	10	100	1 000	10 000
				1	10	100	1 000
				0.1	1	10	100
				0.01	0.1	1	10
				0 001	0.01	0.1	1

1 myriameter = 10 kilometers = 10 000 meters. 1 tonne = 1 000 kilogrammes = 100 quintals = 10 myriagrammes. 1 gramme = 1 cu centimeter of distilled water at its max density at sea level in latitude of Paris and barom at 760 millimeters. 1 liter = 1 cubic decimeter.

15. Weights and Measures (*Concluded*)

Square or Surface Measure

Square kilometer	Square hectometer or hectare	Square decameter or are	Square meter or centiare	Square decimeter	Square centimeter	Square millimeter
	100	10 000	1 000 000			
	1	100	10 000	1 000 000		
	0.01	1	100	10 000	1 000 000	
	0.0001	0.01	1	100	10 000	1 000 000
	0.000001	0.0001	0.01	1	100	10 000
		0.000001	0.0001	0.01	1	100
			0.000001	0.0001	0.01	1

1 sq myriameter = 100 sq kilometers = 100 000 000 sq meters.

Cubic Measure

Cubic decameter	Cubic meter or stere	Cubic decimeter or liter	Cubic centimeter	Cubic millimeter
1	1 000	1 000 000	1 000 000 000	
0.001	1	1 000	1 000 000	1 000 000 000
0.000001	0.001	1	1 000	1 000 000
0.000000001	0.000001	0.001	1	1 000
	0.000000001	0.000001	0.001	1

1 cu meter = 1 kiloliter = 1 stere.

16. Fractions of an Inch to Millimeters

16ths	32nds	64ths	Millimeter	16ths	32nds	64ths	Millimeter	16ths	32nds	64ths	Millimeter
		1	0.397			23	9.128			45	17.859
	1	2	0.794	6	12	24	9.525		23	46	18.256
		3	1.191			25	9.922			47	18.653
1	2	4	1.588		13	26	10.319	12	24	48	19.050
		5	1.984			27	10.716			49	19.447
	3	6	2.381	7	14	28	11.113		25	50	19.844
		7	2.778			29	11.509			51	20.241
2	4	8	3.175		15	30	11.906	13	26	52	20.638
		9	3.572			31	12.303			53	21.034
	5	10	3.969	8	16	32	12.700		27	54	21.431
		11	4.366			33	13.097			55	21.828
3	6	12	4.763		17	34	13.494	14	28	56	22.225
		13	5.159			35	13.891			57	22.622
	7	14	5.556	9	18	36	14.288		29	58	23.019
		15	5.953			37	14.684			59	23.416
4	8	16	6.350		19	38	15.081	15	30	60	23.813
		17	6.747			39	15.478			61	24.209
	9	18	7.144	10	20	40	15.875		31	62	24.606
		19	7.541			41	16.272			63	25.003
5	10	20	7.938		21	42	16.669	16	32	64	25.400
		21	8.334			43	17.066				
	11	22	8.731	11	22	44	17.463				

17. Conversion Table of Measures (C. H. Burnside)

		1	2	3	4	5	6	7	8	9
Linear Measure	In to centimeters (cm)	2.5400	5.0800	7.6200	10.160	12.700	15.240	17.780	20.320	22.860
	Ft to meters	0.3048	0.6096	0.9144	1.2192	1.5240	1.8288	2.1336	2.4384	2.7432
	Yards to meters	0.9144	1.8288	2.7432	3.6576	4.5720	5.4864	6.4008	7.3152	8.2296
	Fathoms to meters	1.8288	3.6576	5.4864	7.3152	9.1440	10.973	12.802	14.630	16.459
	Statute miles to kilom	1.6093	3.2187	4.8280	6.4374	8.0467	9.6561	11.265	12.875	14.484
	Nautical miles to kilom	1.8533	3.7066	5.5600	7.4133	9.2666	11.120	12.973	14.826	16.679
	Cm to in	0.3937	0.7874	1.1811	1.5748	1.9685	2.3622	2.7559	3.1496	3.5433
	Meters (m) to ft	3.2808	6.5617	9.8425	13.123	16.404	19.685	22.966	26.247	29.527
	Meters to yards	1.0936	2.1872	3.2808	4.3744	5.4681	6.5617	7.6553	8.7489	9.8425
	Meters to fathoms	0.5468	1.0936	1.6404	2.1872	2.7340	3.2808	3.8276	4.3744	4.9212
	Km to statute miles	0.6214	1.2427	1.8641	2.4855	3.1069	3.7282	4.3496	4.9710	5.5923
	Km to nautical miles	0.5396	1.0791	1.6187	2.1583	2.6979	3.2374	3.7770	4.3166	4.8561
Square Measure	Sq in to sq cm	6.4516	12.903	19.355	25.807	32.258	38.710	45.161	51.613	58.065
	Sq ft to sq m	0.0929	0.1858	0.2787	0.3716	0.4645	0.5574	0.6503	0.7432	0.8361
	Sq yards to sq m	0.8361	1.6723	2.5084	3.3445	4.1807	5.0168	5.8529	6.6890	7.5252
	Acres to hectares	0.4047	0.8094	1.2141	1.6187	2.0234	2.4281	2.8328	3.2375	3.6422
	Sq stat miles to sq km	2.5900	5.1800	7.7700	10.360	12.950	15.540	18.130	20.720	23.310
	Sq cm to sq in	0.1550	0.3100	0.4650	0.6200	0.7750	0.9300	1.0850	1.2400	1.3950
	Sq m to sq ft	10.764	21.528	32.292	43.055	53.819	64.583	75.347	86.111	96.875
	Sq m to sq yd	1.1960	2.3920	3.5880	4.7839	5.9799	7.1759	8.3719	9.5679	10.764
	Hectares to acres	2.4710	4.9421	7.4131	9.8842	12.355	14.826	17.297	19.768	22.239
	Sq km to sq stat miles	0.3861	0.7722	1.1583	1.5444	1.9305	2.3166	2.7027	3.0888	3.4749
Cubic Measure	Cu in to cu cm	16.387	32.774	49.162	65.549	81.936	98.323	114.71	131.10	147.48
	Cu ft to cu m	0.0283	0.0566	0.0850	0.1133	0.1416	0.1699	0.1982	0.2265	0.2549
	Cu yd to cu m	0.7646	1.5291	2.2937	3.0582	3.8228	4.5874	5.3519	6.1165	6.8810
	Cu cm to cu in	0.0610	0.1220	0.1831	0.2441	0.3051	0.3661	0.4272	0.4882	0.5492
	Cu m to cu ft	35.314	70.629	105.94	141.26	176.57	211.89	247.20	282.51	317.83
	Cu m to cu yd	1.3079	2.6159	3.9238	5.2318	6.5397	7.8477	9.1556	10.464	11.772
Capacity and Volume	Fluid drams to milliliters	3.6966	7.3932	11.090	14.786	18.483	22.180	25.876	29.573	33.269
	Fluid oz to liters	0.0296	0.0591	0.0887	0.1183	0.1479	0.1774	0.2070	0.2366	0.2662
	Liq quarts to liters	0.9463	1.8927	2.8390	3.7853	4.7317	5.6780	6.6243	7.5707	8.5170
	Dry quarts to liters	1.1012	2.2024	3.3036	4.4048	5.5060	6.6072	7.7084	8.8096	9.9108
	U S bush to hectoliters	0.3524	0.7048	1.0572	1.4095	1.7619	2.1143	2.4667	2.8191	3.1715
	U S liq barrels to kiloliters	0.1192	0.2385	0.3577	0.4770	0.5962	0.7154	0.8347	0.9539	1.0732
	Cu ft to hectoliters	0.2832	0.5663	0.8495	1.1327	1.4158	1.6990	1.9821	2.2653	2.5484
	Milliliters to fluid drams	0.2705	0.5410	0.8116	1.0821	1.3526	1.6231	1.8936	2.1641	2.4347
	Liters to fluid oz	33.815	67.629	101.44	135.26	169.07	202.89	236.70	270.52	304.33
	Liters to liquid quarts	1.0567	2.1134	3.1701	4.2268	5.2836	6.3403	7.3970	8.4537	9.5104
	Liters to dry quarts	0.9081	1.8162	2.7243	3.6324	4.5405	5.4486	6.3567	7.2648	8.1729
	Hectoliters to U S bush	2.8378	5.6756	8.5135	11.351	14.189	17.027	19.865	22.703	25.540
	Kiloliters to U S liq barrels	8.3864	16.773	25.159	33.546	41.932	50.319	58.705	67.092	75.478
	Hectoliters to cu ft	3.5315	7.0631	10.595	14.126	17.658	21.189	24.721	28.252	31.784
Weight, Force, and Mass	Grains to grams	0.0648	0.1296	0.1944	0.2592	0.3240	0.3888	0.4536	0.5184	0.5832
	Oz [avoir] to gm	28.350	56.699	85.049	113.40	141.75	170.10	198.45	226.80	255.15
	Oz [troy] to gm	31.103	62.207	93.310	124.41	155.52	186.62	217.72	248.83	279.93
	Lb [avoir] to kg	0.4536	0.9072	1.3608	1.8144	2.2680	2.7215	3.1751	3.6287	4.0823
	Net tons to tonnes [metric]	0.9072	1.8144	2.7215	3.6287	4.5359	5.4431	6.3503	7.2575	8.1647
	Gross tons to tonnes [net]	1.0160	2.0321	3.0481	4.0642	5.0802	6.0963	7.1123	8.1284	9.1444
	Gm to grains	15.432	30.865	46.297	61.729	77.162	92.594	108.03	123.46	138.89
	Gm to oz [avoir]	0.0353	0.0705	0.1058	0.1411	0.1764	0.2116	0.2469	0.2822	0.3175
	Gm to oz [troy]	0.0322	0.0643	0.0965	0.1286	0.1608	0.1929	0.2251	0.2572	0.2894
	Kg to lb [avoir]	2.2046	4.4092	6.6139	8.8185	11.023	13.228	15.432	17.637	19.842
	Tonnes [metric] to net ton	1.1023	2.2046	3.3069	4.4092	5.5116	6.6139	7.7162	8.8185	9.9208
	Tonnes [metric] to gross ton	0.9842	1.9684	2.9526	3.9368	4.9210	5.9052	6.8894	7.8737	8.8579
Speed	Ft per sec to cm per sec	30.480	60.960	91.440	121.92	152.40	182.88	213.36	243.84	274.32
	Ft per sec to m per min	18.288	36.576	54.864	73.152	91.440	109.73	128.02	146.30	164.59
	Ft per min to cm per sec	0.5080	1.0160	1.5240	2.0320	2.5400	3.0480	3.5560	4.0640	4.5720
	Knots to km per hr	1.8533	3.7066	5.5600	7.4133	9.2666	11.120	12.973	14.826	16.679
	Cm per sec to ft per sec	0.0328	0.0656	0.0984	0.1312	0.1640	0.1968	0.2297	0.2625	0.2953
	M per min to ft per sec	0.0547	0.1094	0.1640	0.2187	0.2734	0.3281	0.3828	0.4374	0.4921
	Cm per sec to ft per min	1.9685	3.9370	5.9055	7.8740	9.8425	11.811	13.780	15.748	17.716
	Kilom per hr to knots	0.5396	1.0792	1.6188	2.1584	2.6980	3.2376	3.7772	4.3167	4.8563

17. Conversion Table of Measures (*Concluded*)

		1	2	3	4	5	6	7	8	9
Force per Length, Area, and Volume	Lb per lin ft to kg per m	1.4881	2.9763	4.4644	5.9526	7.4407	8.9289	10.417	11.905	13.393
	Lb/sq in to kg/sq cm	0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
	Lb per sq ft to kg per sq m	4.8825	9.7650	14.647	19.530	24.412	29.294	34.177	39.059	43.942
	Net tons per sq in to tonnes per sq cm	0.1406	0.2812	0.4218	0.5625	0.7031	0.8437	0.9843	1.1249	1.2655
	Net tons per sq ft to tonnes per sq m	9.7651	19.530	29.295	39.060	48.826	58.591	68.356	78.121	87.886
	Lb/cu in to gm/cu cm	27.680	55.359	83.039	110.72	138.40	166.08	193.76	221.44	249.12
	Lb per cu ft to kg per cu m	16.018	32.037	48.055	64.073	80.092	96.110	112.13	128.15	144.16
	Kg per m to lb per lin ft	0.6720	1.3439	2.0159	2.6879	3.3599	4.0318	4.7038	5.3758	6.0477
	Kg per sq cm to lb per sq in	14.223	28.446	42.669	56.892	71.115	85.338	99.561	113.78	128.01
	Kg per sq m to lb per sq ft	0.2048	0.4096	0.6145	0.8193	1.0241	1.2289	1.4337	1.6385	1.8434
	Tonnes per sq m to net tons per sq ft	0.1024	0.2048	0.3072	0.4096	0.5120	0.6144	0.7168	0.8192	0.9216
	Gm/cu cm to lb/cu in	0.0361	0.0723	0.1084	0.1445	0.1806	0.2168	0.2529	0.2890	0.3252
	Kg per cu m to lb per cu ft	0.0624	0.1249	0.1873	0.2497	0.3121	0.3746	0.4370	0.4994	0.5619
	Tonnes per sq cm to net tons per sq in	7.1117	14.223	21.335	28.447	35.558	42.670	49.782	56.894	64.005
Work	Ft-lb to meter-kg	0.1382	0.2765	0.4148	0.5530	0.6913	0.8295	0.9678	1.1060	1.2443
	Ft-lb to joules	1.3563	2.7126	4.0689	5.4251	6.7814	8.1377	9.4940	10.850	12.207
	Meter-kg to ft-lb	7.2330	14.466	21.699	28.932	36.165	43.398	50.631	57.864	65.097
	Joules to ft-lb	0.7373	1.4746	2.2119	2.9492	3.6866	4.4239	5.1612	5.8985	6.6358
Power	Hp to kilowatts (kw)	0.7457	1.4914	2.2371	2.9828	3.7285	4.4742	5.2199	5.9656	6.7113
	Hp to cheval-vapeur	1.0139	2.0277	3.0416	4.0555	5.0694	6.0832	7.0971	8.1110	9.1248
	Ft-lb per sec to watts	1.3563	2.7126	4.0689	5.4251	6.7814	8.1377	9.4940	10.850	12.207
	Lb per hp to kg per cheval-vapeur	0.4474	0.8947	1.3421	1.7895	2.2369	2.6842	3.1316	3.5790	4.0263
	Kw to hp	1.3410	2.6820	4.0231	5.3641	6.7051	8.0461	9.3871	10.728	12.069
	Cheval-vapeur to hp	0.9863	1.9726	2.9590	3.9453	4.9316	5.9179	6.9042	7.8905	8.8769
	Watts to ft-lb per sec	0.7373	1.4746	2.2119	2.9492	3.6866	4.4239	5.1612	5.8985	6.6358
	Kg/cheval-vapeur to lb/hp	2.2353	4.4706	6.7058	8.9411	11.176	13.412	15.647	17.882	20.118
Heat	B t u to calories	0.252	0.504	0.756	1.008	1.260	1.512	1.764	2.016	2.268
	B t u to ft-lb [mean]	777.52	1555.0	2332.6	3110.1	3887.6	4665.1	5442.6	6220.2	6997.7
	B t u/lb to calories/kg	0.5556	1.1111	1.6667	2.2223	2.7778	3.3334	3.8890	4.4445	5.0001
	B t u per sq ft to calories per sq m	2.7126	5.4252	8.1378	10.850	13.563	16.276	18.988	21.701	24.413
	B t u per cu ft to calories per cu m	8.8993	17.799	26.698	35.597	44.496	53.396	62.295	71.194	80.093
	Calories to B t u	3.9683	7.9366	11.905	15.873	19.842	23.810	27.778	31.746	35.715
	Ft-lb to B t u [mean]	1.286*	2.572*	3.858*	5.145*	6.431*	7.717*	9.003*	1.029†	1.158†
	Calories/kg to B t u/lb	1.8000	3.5999	5.3999	7.1998	8.9998	10.800	12.600	14.400	16.200
	Calories per sq m to B t u per sq ft	0.3687	0.7373	1.1060	1.4746	1.8433	2.2119	2.5806	2.9492	3.3179
	Calories per cu m to B t u per cu ft	0.1124	0.2247	0.3371	0.4495	0.5618	0.6742	0.7866	0.8990	1.0113

* (10^{-3}) † (10^{-2}).

1 joule = 10 million ergs = 10 million dyne-centimeters.
1 knot = 1 nautical mile = 6 080 ft per hour.
1 horse power = 550 ft-lb per second.
1 kilowatt = 1 000 watts = 1 000 joules per sec.
1 cheval-vapeur = 75 kilogram-meters per sec.
1 B t u = heat necessary to raise 1 lb of water 1° F.
1 calorie = heat necessary to raise 1 kg of water 1° C.

18. Cubic Feet and Gallon Equivalents

Cu ft	Gallons equivalent to cu ft	Cu ft equivalent to gallons	Cu ft	Gallons equivalent to cu ft	Cu ft equivalent to gallons
0.1	0.75	0.0134	600	4 488.3	80.208
.2	1.50	.0267	700	5 236.4	93.576
.3	2.24	.0401	800	5 984.4	106.944
.4	2.99	.0535	900	6 732.5	120.312
.5	3.74	.0668	1 000	7 480.5	133.681
.6	4.49	.0802	2 000	14 961.0	267.361
.7	5.24	.0936	3 000	22 441.6	401.042
.8	5.98	.1069	4 000	29 922.1	534.722
.9	6.73	.1203	5 000	37 402.6	668.403
1.0	7.48	.134	6 000	44 883.1	802.083
2	14.96	.267	7 000	52 363.6	935.764
3	22.44	.401	8 000	59 844.2	1 069.444
4	29.92	.535	9 000	67 324.7	1 203.125
5	37.40	.668	10 000	74 805.2	1 336.806
6	44.88	.802	20 000	149 610.4	2 673.61
7	52.36	.936	30 000	224 415.6	4 010.42
8	59.84	1.069	40 000	299 220.8	5 347.22
9	67.32	1.203	50 000	374 025.9	6 684.03
10	74.80	1.337	60 000	448 831.1	8 020.83
20	149.6	2.674	70 000	523 636.3	9 357.64
30	224.4	4.010	80 000	598 441.5	10 694.44
40	299.2	5.347	90 000	673 246.7	12 031.25
50	374.0	6.684	100 000	748 051.9	13 368.1
60	448.8	8.021	200 000	1 496 103.8	26 736.1
70	523.6	9.358	300 000	2 244 155.7	40 104.2
80	598.4	10.694	400 000	2 992 207.6	53 472.2
90	673.2	12.031	500 000	3 740 259.5	66 840.3
100	748.0	13.368	600 000	4 488 311.4	80 208.3
200	1 496.1	26.736	700 000	5 236 363.3	93 576.4
300	2 244.2	40.104	800 000	5 984 415.2	106 944.4
400	2 992.2	53.472	900 000	6 732 467.1	120 312.5
500	3 740.3	66 840	1 000 000	7 480 519.0	133 680.6

19. Chinese Measures

Following is the system of weights and measures adopted by China in pursuance of the imperial rescript of August 28, 1908.

MEASURES OF LENGTH

Chinese		Metric	
1 Hao	equals	0.0032	Centimeter
1 Li	"	0.032	"
1 Fen	"	0.32	"
1 Ts'un	"	3.2	Centimeters
1 Ch'ih	"	0.32	Meter
1 Pu	"	1.6	Meters
1 Chang	"	3.2	"

MEASURES OF SURFACE

Chinese		Metric	
1 Hao	equals	0.006144	Are
1 Li	"	0.06144	"
1 Fen	"	0.6144	"
1 Fang pu	"	0 0256	"
1 Mu	"	6.144	Ares
1 Ch'ing	"	614.4	"

MEASURES OF CAPACITY

Chinese		Metric	
1 Shao	equals	0.0104	Liter
1 Ko	"	0.1035	"
1 Sheng	"	1.0355	Liters
1 Tou	"	10.355	"
1 Hu	"	51.7734	"
1 Tan or Picul	"	103.5469	"

MEASURES OF WEIGHT

Chinese		Metric	
1 Hao	equals	0.0037301	Gram
1 Li	"	0.037301	"
1 Fen	"	0.37301	"
1 Ch'ien	"	3.7301	Grams
1 Liang	"	37.301	"
1 Chin (catty)	"	596.816	"

20. Japanese Measures

Kwan, = 3.75 kg or 8.2673336 lb avoir, is the unit of mass. It is divided into 1 000 mommes, the momme into 10 funs, the fun into 10 rins, the rin into 10 mos, and the mo into 10 shis.

Shaku is the unit of length, equal to 10 ÷ 33 (0.30303) meter, or 0.9941919 U S ft. It is decimally divided into sun, bu, rin, mo, and shi. Multiples of the shaku are: the ken = 6 shaku; the cho = 60 ken, and the ri = 36 cho, or 12 960 shaku.

The shaku (land measure) = 0.00033 are (metric), or 0.988417534 sq ft, 10 shaku = 1 go, 10 go = 1 bu or tsubo, 30 bu = 1 sé, 10 sé = 1 tan, and 10 tan = 1 cho.

The shaku (capacity measure) = 0.01804 liter, or 0.0180616 U S liquid quart, 10 shaku = 1 go, 10 go = 1 sho, 10 sho = 1 to, and 10 to = 1 koku (4.963 bushels). Metric weights and measures are recognized as legal in accordance with the above equivalents.

21. Russian Measures (E. L. Stenger)

Quantity	Equivalent	Quantity	Equivalent	Quantity	Equivalent
Vershok	1 750 in 44.45 mm	Sq mile	2 2758 sq verst 2 590 sq kilom	100 poods	1 8056 short ton 1 6122 long " 1 6380 met "
Russian ft	1 0 English ft 0 3048 meter	Sq arshine	256 0 sq vershok 5 4444 sq ft		
Arshine	16 0 vershok 28 0 in 2 3333 ft	Desiatine	24 000 sq sagene 2 7001 acre	Doli	44.435 mg 0 686 grain
Sagene	3 0 arshine 7 0 ft	Vedro	10 0 krushkar 12 299 liter 750 51 cu in 3 249 U S gal 2 7067 Imp gal	Zolotnik	0 3333 lot 96 0 doli 4 2659 gm 65 833 grain 0 13715 troy oz 0 15047 av oz
Verst	500 sagene 3 500 ft 0 66288 mile 1 06680 kilom	Funt	0 9028 lb 409 503 gm		
Meter	1 4061 arshine 0 4687 sagene 0 000937 verst	Pood	40 0 funt 36 113 lb 16 3805 kg	Cubic sagene	343 0 cu ft 2 6797 cord

22. Spanish-American Measures

Arroba = 25.4 lb av = $^1/_4$ quintál = 11.51 kg (also, in Spain, 1 arroba = 1 cántara = 4.2 U S gal approx).

Tonelada = 20 quintales = 2 032.2 lb av = 921.8 kg.

Vara = 2.75 ft (Perú), 2.841 ft (Argentina), 2.782 ft (Spain).

Estaca (Perú) = 167 meters sq = 27 889 sq meters.

Fanega (Perú) = 1.65 acre (varying locally); also = 141.43 lb av = 64.14 kg; also = 1.599 bushel (in Mex = 2.577 bushels).

Pertenencia, or mining claim, in Mexico and in general throughout Spanish America, is a solid of indefinite depth, underlying a square surface area of 100 meters on a side (Mexican Mining Law of June 4, 1892; unchanged since).

23. Miscellaneous Measures

Board measure. Commercial boards (unplaned or green lumber) are assumed to be 1 in thick, but are often less; rough, dry boards, about $^7/_8$ in. On basis of 1 in thickness, number of feet of board measure in a stick of squared timber = length (ft) × breadth (ft) × thickness (in).

Cubic measures. 1 cord fire-wood = 4 × 4 × 8 ft = 128 cu ft. 1 perch of stone = 1.5 ft wide × 1 ft high × 16.5 ft long = 24.75 cu ft. As the perch varies in different regions, it should be checked by local practice.

Structural steel. Sec 43, Art 26, contains brief tables of standard sizes and weights. For complete tables, see steel manufacturers' catalogues.

Miner's inch of water. See Sec 10, Art 123 and Sec 38, Art 20.

Ship measurements are of 5 kinds. In U S and British countries, the ton = 2 240 lb; where metric ton is used, 2 204.6 lb.

(1) *Displacement* is total wt in tons, of a vessel and contents. Displacement "light" = wt of vessel without stores, bunker fuel or cargo; displacement "loaded" = total wt, including last 3 items.

(2) *Cargo tonnage* is expressed on basis of either "weight" or "cubic measurement." "Measurement ton" is usually 40 cu ft (as in U S); in some countries, 42 cu ft.

(3) *Gross tonnage* applies to vessel itself, not to cargo, and is the contents in cu ft of its closed-in spaces, divided by 100 (assuming a vessel-ton to be 100 cu ft). "Register" states both gross and net tonnage, and is about the same under U S and British rules.

(4) *Net tonnage* is gross tonnage, less deductions for space occupied by crew, engine room and fuel, and for navigation; that is, the space available for passengers and cargo. As a ton of cargo usually occupies much less than 100 cu ft, see (2), cargo tonnage generally exceeds both net and gross tonnage.

(5) Deadweight tonnage is the number of tons a vessel can carry, of cargo, stores and fuel. It is the difference between tons of water displaced when vessel is "light" and when submerged to "load line" (which is often shown on vessel's side by the "Plimsoll mark"). Capacity for weight cargo is less than dead-weight tonnage.

Examples of relative measurements. (*a*) An ordinary freight steamer: net tonnage, 4 000; gross tonnage, 6 000; deadweight capacity, 10 000 tons; displacement, loaded, 13 350 tons. (*b*) Oil tanker, Standard Oil Co of Calif, launched 1938: net tonnage, 4 934; gross tonnage, 8 298; deadweight capac, 12 800 tons; displacement, loaded, 17 320 tons.

24. Values of $1, at — % Compound Interest, for — Years
(Adapted from tables in "Principles of Finance," by F. C. Kent, 1927)

n	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000
2	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100
3	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310
4	1.1255	1.1698	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641
5	1.1593	1.2166	1.2763	1.3382	1.4025	1.4693	1.5386	1.6105
6	1.1940	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716
7	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487
8	1.2668	1.3686	1.4774	1.5938	1.7182	1.8509	1.9926	2.1436
9	1.3048	1.4233	1.5513	1.6895	1.8384	1.9990	2.1719	2.3579
10	1.3439	1.4802	1.6289	1.7908	1.9671	2.1589	2.3674	2.5937
11	1.3842	1.5394	1.7103	1.8983	2.1048	2.3316	2.5804	2.8531
12	1.4258	1.6010	1.7958	2.0122	2.2522	2.5182	2.8127	3.1384
13	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523
14	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975
15	1.5580	1.8009	2.0789	2.3965	2.7590	3.1722	3.6425	4.1772
16	1.6047	1.8730	2.1829	2.5403	2.9522	3.4259	3.9703	4.5950
17	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545
18	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599
19	1.7535	2.1068	2.5269	3.0256	3.6165	4.3157	5.1417	6.1159
20	1.8061	2.1911	2.6533	3.2071	3.8697	4.6609	5.6044	6.7275
21	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002
22	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403
23	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543
24	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497
25	2.0938	2.6658	3.3863	4.2919	5.4274	6.8485	8.6231	10.8347
26	2.1566	2.7725	3.5557	4.5494	5.8073	7.3963	9.3991	11.9182
27	2.2213	2.8834	3.7334	4.8223	6.2139	7.9881	10.2451	13.1100
28	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210
29	2.3566	3.1186	4.1161	5.4184	7.1142	9.3173	12.1722	15.8631
30	2.4273	3.2434	4.3219	5.7435	7.6122	10.0626	13.2677	17.4494
31	2.5001	3.3731	4.5380	6.0881	8.1451	10.8677	14.4618	19.1943
32	2.5751	3.5080	4.7649	6.4534	8.7153	11.7371	15.7633	21.1138
33	2.6523	3.6484	5.0032	6.8406	9.3253	12.6760	17.1820	23.2251
34	2.7319	3.7943	5.2533	7.2510	9.9781	13.6901	18.7284	25.5477
35	2.8139	3.9461	5.5160	7.6861	10.6766	14.7853	20.4140	28.1024
36	2.8983	4.1039	5.7918	8.1472	11.4239	15.9682	22.2512	30.9127
37	2.9852	4.2681	6.0814	8.6361	12.2236	17.2456	24.2538	34.0039
38	3.0748	4.4388	6.3855	9.1542	13.0793	18.6253	26.4367	37.4043
39	3.1670	4.6164	6.7047	9.7035	13.9948	20.1153	28.8160	41.1448
40	3.2620	4.8010	7.0399	10.2857	14.9744	21.7245	31.4094	45.2592
41	3.3599	4.9931	7.3920	10.9029	16.0227	23.4625	34.2363	49.7852
42	3.4607	5.1928	7.7616	11.5570	17.1442	25.3395	37.3175	54.7637
43	3.5645	5.4005	8.1497	12.2504	18.3443	26.3666	40.6761	60.2401
44	3.6714	5.6165	8.5571	12.9854	19.6284	29.5560	44.3369	66.2641
45	3.7816	5.8412	8.9850	13.7646	21.0024	31.9204	48.3273	72.8905
46	3.8950	6.0748	9.4342	14.5905	22.4726	34.4741	52.6767	80.1795
47	4.0019	6.3178	9.9060	15.4659	24.0457	37.2320	57.4176	88.1975
48	4.1322	6.5705	10.4013	16.3939	25.7289	40.2106	62.5852	97.0172
49	4.2562	6.8333	10.9213	17.3775	27.5299	43.4274	68.2179	106.7189
50	4.3839	7.1067	11.4674	18.4201	29.4570	46.9016	74.3575	117.3908

The formula for computing the above table is: $V = A(1 + R)^n$, where V = final value, A = amount invested, R = interest rate, and n = number of years.

Example. If $1 be invested, at 3% compound interest, for a period of 15 years, $V = 1(1.03)^{15}$, whence, $\log V = 15 \log (1.03) = 0.1926$, and V = $1.56.

Conversely, to find the *present value* of an investment, the payment of which is due at the end of a term of years, at a given rate of compound interest, the above formula becomes,

$A = \frac{V}{(1 + R)^n}$ (see Table 25).

25. Present Value of $1, Due at End of — Years, at Compound Interest
(Condensed from Kent's Tables, 2nd Ed, 1927)

Years	3%	4%	5%	6%	7%	8%	9%	10%
1	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091
2	.9426	.9245	.9070	.8900	.8734	.8573	.8417	.8264
3	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513
4	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830
5	.8626	.8219	.7835	.7472	.7130	.6806	.6499	.6209
6	.8375	.7903	.7462	.7050	.6663	.6302	.5963	.5645
7	.8131	.7599	.7107	.6650	.6227	.5835	.5470	.5131
8	.7894	.7307	.6768	.6274	.5820	.5403	.5019	.4665
9	.7664	.7026	.6446	5919	.5439	.5002	.4604	.4241
10	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855
11	7224	.6496	5847	.5268	.4751	.4289	.3875	.3505
12	.7014	.6246	.5568	.4970	.4440	3971	.3555	.3186
13	.6809	.6006	.5303	.4688	.4150	.3677	.3262	.2897
14	.6611	.5775	5051	.4423	.3878	.3405	.2992	.2633
15	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394
16	.6232	.5339	.4581	.3936	.3387	.2919	.2519	.2176
17	.6050	.5134	.4363	.3714	.3166	.2703	.2311	.1978
18	.5874	.4936	.4155	3503	.2959	.2502	.2120	.1798
19	.5703	4746	.3957	.3305	.2765	.2317	.1945	.1635
20	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486
21	.5375	.4388	.3589	.2941	.2415	1986	.1637	.1351
22	.5219	.4219	.3418	.2775	.2257	.1839	.1502	.1228
23	.5067	.4057	.3256	.2618	.2109	.1703	1378	.1117
24	.4919	.3901	.3101	.2470	.1971	.1577	.1264	.1015
25	.4776	.3751	.2953	.2330	.1842	1460	.1160	.0923
26	.4637	.3607	.2812	.2198	.1722	1352	.1064	.0839
27	.4502	.3468	.2678	.2074	.1609	.1252	.0976	.0763
28	.4371	.3335	.2551	.1956	.1504	.1159	.0895	.0693
29	.4243	.3206	.2429	.1845	.1406	.1073	.0821	.0630
30	4120	3083	.2314	1741	1314	0994	.0754	.0573

Formula for computing this table is: $A = \frac{1}{(1+R)^n}$, where A = present value of the investment (represented by the amounts in tables at different rates of interest), V = final value, or $1, R = interest rate, and n = number of years. Note that this formula is the *converse* of that used to compute Table 24.

Example. What sum must be invested at 3% compound interest to produce $1 at end of 20 years? A = unknown investment, or present value, V = $1, R = 3%, and n = 20; hence, $A = \frac{1}{(1.03)^{20}}$. Since $\log A = \log 1 - 20 \log (1.03) = 9.7432 - 10$, A = $0.5537.

To find the present value of any other sum, multiply that sum by present value of $1 for required number of years, as in this table.

26. Present Value of an Annual Dividend of $1, for a Term of — Years, at · · %
(Based on Inwood's Tables, 30th Ed, 1920)

Years	3%	4%	5%	6%	7%	8%	9%	10%
1	0.971	0.961	0.952	0.943	0.935	0.926	0.917	0.909
2	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.735
3	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.417	5.242	5.076	4.917	4.766	4.623	4.486	4.355
7	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495
12	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606
16	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824
17	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022
18	13.753	12.659	11.690	10.828	10.059	9.372	8.756	8.201
19	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365
20	14.877	13.590	12.462	11.470	10.594	9.818	9.129	8.514
21	15.415	14.029	12.821	11.764	10.835	10.017	9.292	8.649
22	15.937	14.451	13.163	12.042	11.061	10.201	9.442	8.771
23	16.444	14.857	13.489	12.303	11.272	10.371	9.580	8.883
24	16.935	15.247	13.799	12.550	11.469	10.529	9.707	8.985
25	17.413	15.622	14.094	12.783	11.654	10.675	9.823	9.077
26	17.877	15.983	14.375	13.003	11.826	10.810	9.929	9.161
27	18.327	16.330	14.643	13.210	11.987	10.935	10.027	9.237
28	18.764	16.663	14.898	13.406	12.137	11.051	10.116	9.307
29	19.188	16.984	15.141	13.591	12.278	11.158	10.198	9.370
30	19.600	17.292	15.372	13.765	12.409	11.258	10.274	9.427

Formula for computing above table is: $a_n = \frac{1 - v^n}{i}$, where a_n = present value of $1 per annum for n years, v^n = present value of an investment that will produce $1, at i given dividend rate, in n years, and i = dividend rate.

Example. If n = 10 years, i = 6%, and v^n = 0.5584 (see Table 25):

$$a_n = \frac{1 - 0.5584}{0.06} = \frac{0.4416}{0.06} = \$7.36.$$

27. Present Value of an Annual Dividend of \$1 for n Years, That Will Yield $x\%$ Simple Interest, and also Provide Annual Sums Which, if Invested at 4% Compound Interest, Will Replace Original Investment (From Inwood and Kent; quoted by Hoover, "Principles of Mining" and Finlay, "Cost of Mining")

Years	5%	6%	7%	8%	9%	10%
1	.95	.94	.93	.92	.92	.91
2	1.85	1.82	1.78	1.75	1.72	1.69
3	2.70	2.63	2.56	2.50	2.44	2.38
4	3.50	3.38	3.27	3.17	3.07	2.98
5	4.26	4.09	3.93	3.78	3.64	3.51
6	4.98	4.74	4.53	4.33	4.15	3.99
7	5.66	5.36	5.09	4.84	4.62	4.41
8	6.31	5.93	5.60	5.30	5.04	4.79
9	6.92	6.47	6.08	5.73	5.42	5.14
10	7.50	6.98	6.52	6.12	5.77	5.45
11	8.05	7.45	6.94	6.49	6.09	5.74
12	8.58	7.90	7.32	6.82	6.39	6.00
13	9.08	8.32	7.68	7.13	6.66	6.24
14	9.55	8.72	8.02	7.42	6.91	6.46
15	10.00	9.09	8.34	7.79	7.14	6.67
16	10.43	9.45	8.63	7.95	7.36	6.86
17	10.85	9.78	8.91	8.18	7.56	7.03
18	11.24	10.10	9.17	8.40	7.75	7.19
19	11.61	10.40	9.42	8.61	7.93	7.34
20	11.96	10.68	9.65	8.80	8.09	7.49
21	12.30	10.95	9.87	8.99	8.24	7.62
22	12.62	11.21	10.08	9.16	8.39	7.74
23	12.93	11.45	10.28	9.32	8.52	7.85
24	13.23	11.68	10.46	9.47	8.65	7.96
25	13.51	11.90	10.64	9.61	8.77	8.06
26	13.78	12.11	10.80	9.75	8.88	8.16
27	14.04	12.31	10.96	9.88	8.99	8.25
28	14.28	12.50	11.11	10.00	9.09	8.33
29	14.52	12.68	11.25	10.11	9.18	8.41
30	14.74	12.85	11.38	10.22	9.27	8.49

Table 27 is computed by the formula:

$$P = \frac{R^n - 1}{r + r'(R^n - 1)}, \text{ where}$$

P = present value of annual dividend; n = number of years that the dividend is received; r = one year's interest on \$1 at 4% = 0.04; $R = 1 + r = 1.04$; R^n = value of \$1 at 4% compound interest for n years, which may be taken directly from Table 24 (thus avoiding use of logarithms); r' = one year's interest on \$1 at $x\%$.

Example. Find present value of an annual dividend of \$100 000 over 20 years at 8% simple interest, and replacing capital by reinvestment of an annual sum at 4% compound interest. Then $r = 0.04$, $R = 1.04$, $r' = 0.08$ and $n = 20$; whence

$$P = \frac{1.04^{20} - 1}{.04 + .08\,(1.04^{20} - 1)} = 8.804$$

which, multiplied by \$100 000 gives the present value required = \$880 400.

Obviously, any desired interest rate other than 4% may be used for the sinking fund, by using the appropriate values for r and R in the formula.

28. Miscellaneous Problems in Present Value Computations

(By James F. McClelland, Vice-Pres of Phelps Dodge Corporation)

1. Present value P of a deferred annuity or annual dividend of \$1, to begin at the end of p years and continue for an additional n years, allowing compound interest at $x\%$. Let r = one year's interest on \$1 at $x\%$; $R = 1 + r$. Then

$$P = \frac{1}{r}\left(\frac{1}{R^p} - \frac{1}{R^{n+p}}\right)$$

In this formula, $\frac{1}{R^p}$ and $\frac{1}{R^{n+p}}$ are the present values, at $x\%$ compound interest, of \$1 due in p years and $n + p$ years respectively and may be taken directly from Table 25.

2. Present value P of a deferred annuity or annual dividend of \$1, to begin at the end of p years and continue for an additional n years, to yield interest on capital (present value of annuity) at $x\%$ and to return capital through investment of balance of annuity at $y\%$ compound interest. Let r = one year's interest on \$1 at $y\%$; $R = 1 + r$; r' = one year's interest on \$1 at $x\%$. Then

$$P = \frac{R^n - 1}{(1 + r')^p[r + r'(R^n - 1)]}$$

In this formula, R^n is the value of \$1 at $y\%$ compound interest for n years; $(1 + r')^p$ is the value of \$1 at $x\%$ compound interest for p years. These values may be taken directly from Table 24.

3. Present value of a series of irregular annual dividends is the sum of present values of each dividend and may be computed from Table 25. Problems also arise in which it is desired to determine present value of a series of irregular annual dividends, allowing

for interest on capital at one interest rate x, and return of capital at a different rate of interest y. Then

$$P = \frac{S}{1 + \frac{r'}{r}(R^n - 1)}$$

where P = present value; r = one year's interest on \$1 at y%; $R = 1 + r$; r' = one year's interest on \$1 at x%; n = number of years over which the dividends are received; S = value of the series of irregular annual dividends at compound interest in n years at the rate y. In using this formula the value of each dividend must first be computed, using Table 24 for this purpose. S is the sum of such values.

Example. Assume irregular dividends as listed in the accompanying table, when $n = 5$ yr; interest rate desired on investment, $x = 6\%$; $r' = 0.06$; and interest rate y on sinking fund = 4%; whence, $R = 1 + r = 1.04$. S is found as follows:

$$P = \frac{\$6\,934\,070}{1 + \frac{.06}{.04}(1.04^5 - 1)} = \$5\,233\,663,$$

as proved in the table below.

Year	Dividend received	Value of \$1 at 4% at end of n years* (from Table 24)	Value of dividend at 4% at end of n years (col 2 × col 3)
1st	\$1 330 800	1.1698	\$1 556 770
2nd	1 426 100	1.1249	1 604 220
3rd	1 208 700	1.0816	1 307 330
4th	1 208 700	1.0400	1 257 050
5th	1 208 700	1.0000	1 208 700
		S =	\$6 934 070

* If dividends are received at end of each year, interest will accumulate on the first dividend for $n - 1$ years; on the second dividend for $n - 2$ years, etc.

Year	Dividend	Annual interest desired = 6% on \$5 233 663	Balance for sinking fund	Amount of sinking fund at end of 5th year
1	\$1 330 800	\$314 020	\$1 016 780	\$1 189 430
2	1 426 100	314 020	1 112 080	1 250 979
3	1 208 700	314 020	894 680	967 686
4	1 208 700	314 020	894 680	930 467
5	1 208 700	314 020	894 680	894 680
				\$5 233 242

NOTE. If the series of irregular annual dividends is deferred, first find P at end of the deferment period, as above. The present value of the deferred series is the present value of P. That is, P should be discounted for the number of years of deferment.

Estimates of future income from mining properties ordinarily involve judgment as to future metal prices, market demand and numerous variables affecting the time at which income will be produced. It should be borne in mind that present values, calculated from such estimates of income, reflect these judgment factors. A series of present-value calculations, based on estimates of future income under different assumptions as to the variable factors, is useful as an aid to judgment in determining a cash price for a mining property. Present-value calculations are invaluable in determining the relative values of two or more mining properties, especially in the case of mergers, where payment for the properties is made in capital stock of the resulting corporation. See also remarks in connection with Table 5, Sec 25.

29. Values of Foreign Monetary Units

Following equivalents, in terms of U S "money of account," were established by the Secy of the Treasury on Jan 1, 1940, for estimating the value of currencies of different countries (at par for gold units; non-gold units have no fixed par with gold). The Secretary's circulars are issued quarterly. This list comprises countries in which mining is carried on.

Country	Monetary Unit	Value in U S money	Remarks
Argentine Republic	Peso	$1.6335	Valuation is of gold peso. Paper nominally convertible at 44% of face value. Conversion suspended Dec 16, 1929
Australia	Pound	8.2397	Control of gold exports authorized Dec 17, 1929
Belgium	Belga	.1695	Decree of Mch 31, 1936; one belga equals 5 Belgian francs
Bolivia	Boliviano	.6180	Conversion of notes into gold suspended Sep 23, 1931
Brazil	Milreis	.0606	Based upon official rate for milreis as announced by Bank of Brazil. Conversion of Stabilization-Office notes into gold suspended Nov 22, 1930
British Honduras	Dollar	1.6931	Conversion of notes suspended
Bulgaria	Lev	.0122	Exchange control established Oct 15, 1931
Canada	Dollar	1.6931	Embargo on export of gold, Oct 19, 1931; redemption of Dominion notes in gold suspended Apl 10, 1933
Chile	Peso	.2060	Valuation is of gold peso; received for conversion at 4 paper pesos per gold peso. Conversion suspended July 30, 1931
China	Yuan	....	Silver standard abandoned Nov 3, 1935; bank notes legal tender under Currency Board Control; exchange for British currency fixed at about 1sh 2 1/2d, or about 29 1/2¢ U S per yuan
Hong Kong	Dollar	.2325	Treasury notes and notes of the three banks of issue made legal tender by silver nationalization ordinance of Dec 5, 1935
Colombia	Peso	.5714	Obligation to sell gold suspended Sep 24, 1931. New gold content of 0 56424 gram gold, 9/10 fine, effective Nov 30, 1938
Costa Rica	Colon	.7879	Conversion of notes into gold suspended Sep 18, 1914; exchange control estab Jan 16, 1932
Cuba	Peso	1.0000	By law of May 25, 1934
Dominican Republic	Dollar	1.6931	U S money is principal circulating medium
Ecuador	Sucre	.3386	Conversion of notes into gold suspended Feb 9, 1932
Egypt	Pound (100 piasters)	8.3692	Conversion of notes into gold suspended Sep 21, 1931
Finland	Markka	.0426	Conversion of notes into gold suspended Oct 12, 1931
France	Franc	....	Monetary law of Oct 1, 1936, providing for gold content of franc, superseded by decree of June 30, 1937, which ultimately will fix gold content of franc
Germany	Reichsmark	.4033	Exchange control established July 13, 1931
Great Britain	Pound. Demand, June, 1940, $3.75–$3.81	8.2397	Obligation to sell gold at legal monetary par suspended Sep 21, 1931
Greece	Drachma	.0220	Conversion of notes into gold suspended Apl 26, 1932
Guatemala	Quetzal	1.6931	Conversion of notes into gold suspended Mch 8, 1933
Haiti	Gourde	.2000	National bank notes redeemable on demand in U S dollars
Honduras	Lempira	.8466	Gold exports prohibited Mch 27, 1931; lempira circulates as equivalent of half of U S dollar
Hungary	Pengo	.2961	Exchange control established July 17, 1931
India (British)	Rupee	.6180	Obligation to sell gold at legal monetary par suspended Sep 21, 1931
Indo-China	Piaster	....	Piaster pegged to French franc at rate of 1 piaster = 10 francs; conversion into gold suspended Oct 2, 1936

Values of Foreign Monetary Units—*Continued*

Country	Monetary Unit	Value in U S money	Remarks
Italy	Lira	.0526	New gold content of 46.77 milligrams fine gold per lira established Oct 5, 1936
Japan	Yen (par, 0.844)	.2343	Embargo on gold exports Dec 13, 1931
Mexico	Peso	....	Decree of Aug 28, 1936 left the monetary unit to be later defined by law
Netherlands and colonies	Guilder (florin)	.6806	Suspension of convertibility of notes into gold and restrictions placed on free gold exports Sep 26, 1936; gold export prohibition repealed by decree June 28, 1938
Newfoundland	Dollar	1.6931	Newfoundland and Canadian notes legal tender
New Zealand	Pound	8.2397	Conversion of notes into gold suspended and export of gold restricted Aug 5, 1914; exchange regulations, Dec 1931
Nicaragua	Cordoba	1.6933	Embargo on gold exports Nov 13, 1931
Panamá	Balboa	1.0000	U S money is principal circulating medium
Paraguay	Peso (Argentine)	1.6335	Paraguayan paper currency used; exchange control established June 28, 1932
Persia (Iran)	Rial	.0824	Obligation to pay out gold deferred Mch 13, 1932; exchange control estab Mch 1, 1936
Perú	Sol	.4740	Conversion of notes into gold suspended May 18, 1932
Philippine Islands	Peso	.5000	By act approved Mch 16, 1935
Portugal	Escudo	*.0749	Gold exchange standard suspended Dec 31, 1931
Rumania	Leu	.010127	Exchange control established May 18, 1932
Salvador	Colon	.8466	Conversion of notes into gold suspended Oct 7, 1931
Spain	Peseta	....	....
Straits Settlements	Dollar	.9613	British pound and Straits dollar and half dollar legal tender
Sweden	Krona	.4537	Conversion of notes into gold suspended Sep 29, 1931
Thailand (Siam)	Baht (Tical)	.7491	Conversion of notes into gold suspended May 11, 1932
Turkey	Piaster	.0744	100 piasters equal to the Turkish pound; exchange control established Feb 26, 1930
Union of South Africa	Pound	8.2397	Conversion of notes into gold suspended Dec 28, 1932
Union of Soviet Republics	Chevronets	8.7123	....
Uruguay	Peso	.6583	Conversion of notes into gold suspended Aug 2, 1914; exchange control established Sep 7, 1931. New gold content of .585018 gram of gold per peso established Jan 12, 1938
Venezuela	Bolivar	.3267	Exchange control established Dec 12, 1936
Yugoslavia	Dinar	.0298	Exchange control established Oct 7, 1931

NOTE—Values in column 3 are based on present arbitrary value of gold in the U S, now (1940) approx $35 per oz.

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Light figures refer to Volume I. **Bold-face figures refer to Volume II.**